

Title: PSI - Research Skills 4A

Date: Aug 27, 2009 09:00 AM

URL: <http://pirsa.org/09080041>

Abstract:



$$\frac{\quad}{P^2 - M^2}$$

$$\frac{P^2 - M^2}{1}$$

$$\frac{P^2 - (m_e + m_p E_B)^2}{1}$$

$$\frac{1}{P^2 - M^2 + iM\Gamma}$$

M

1

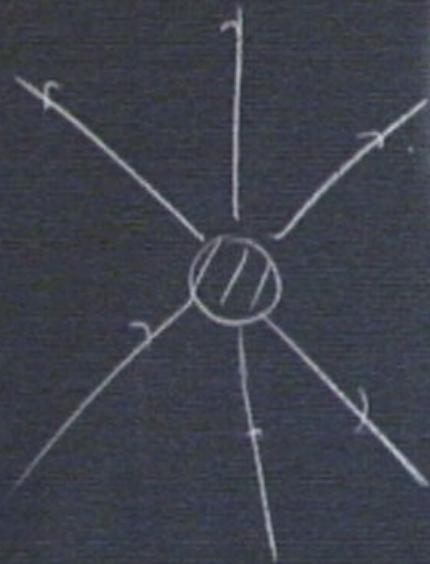
$$p^2 - (m_e + m_p E_B)^2$$

Massless Particle Scatt. Amplitude.

Massless Particle Scatt. Amplitude.



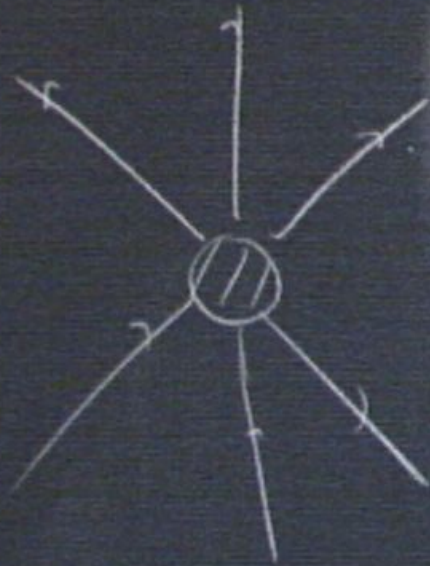
Massless Particle Scatt. Amplitude.



$$P_{in}^{\mu} = (E_{in}, \vec{p}_{in})$$

$$P_{out}^{\mu} = (E_{out}, \vec{p}_{out})$$

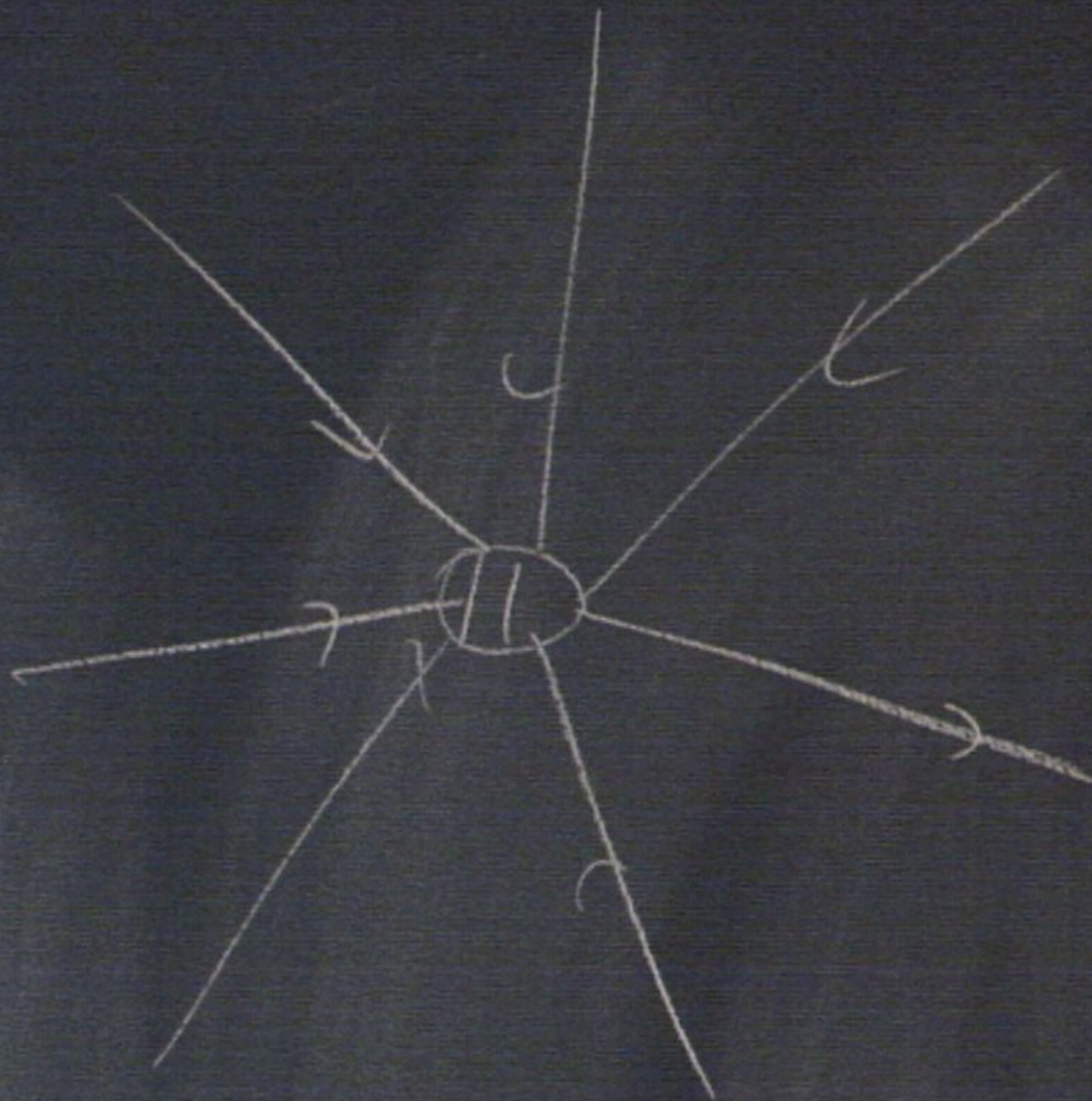
Massless Particle Scatt. Amplitude.

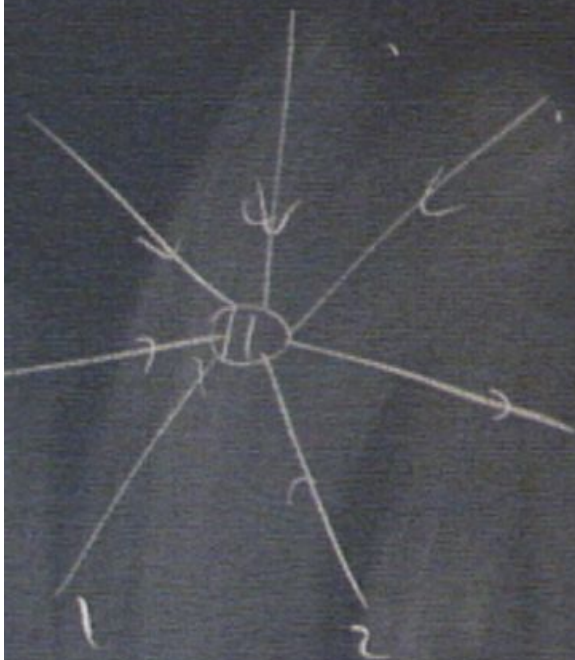


$$p_{in}^{\mu} = (E_{in}, \vec{p}_{in})$$

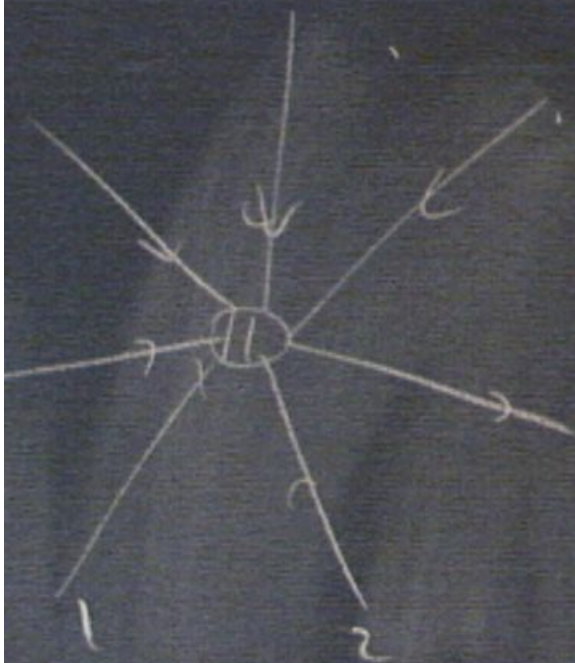
$$p_{out}^{\mu} = (E_{out}, \vec{p}_{out})$$

$$E \geq 0$$





$$M(p_1^{\mu}, \dots, p_n^{\mu})$$



$$M(p_1^{\mu}, \dots, p_n^{\mu})$$

$$M((\wedge p)_1, \dots, (\wedge p)_n) = M(p_1, \dots, p_n)$$



$$M(p_1^{\mu}, \dots, p_n^{\mu})$$

$$M(\bigwedge p_1, \dots, \bigwedge p_n) = M(p_1, \dots, p_n)$$

Massless Particle Scatt. Amplitude

$$M(p_1, \dots, p_n) = \delta^4\left(\sum_i p_i\right)$$

Massless Particle Scatt. Amplitude

$$M(p_1, \dots, p_n) = \delta^4\left(\sum_i p_i\right) \times \mathcal{M}(p_1, \dots, p_n).$$

$$M((\lambda p)_1, \dots, (\lambda p)_n) = M(p_1, \dots, p_n) \text{ scalar.}$$

$$M^{h_1 \dots h_n} [p_1, \dots, p_n]$$

$h_1 \dots h_n [p_1, \dots, p_n]$



$$M^{h_1 \dots h_n} [p_1, \dots, p_n]$$



$$M^{h_1 \dots h_n} [\wedge p_1, \dots, \wedge p_n].$$

$$\neq M^{h_1 \dots h_n} [p_1, \dots, p_n]$$

MI

$[p_1, \dots, p_n]$

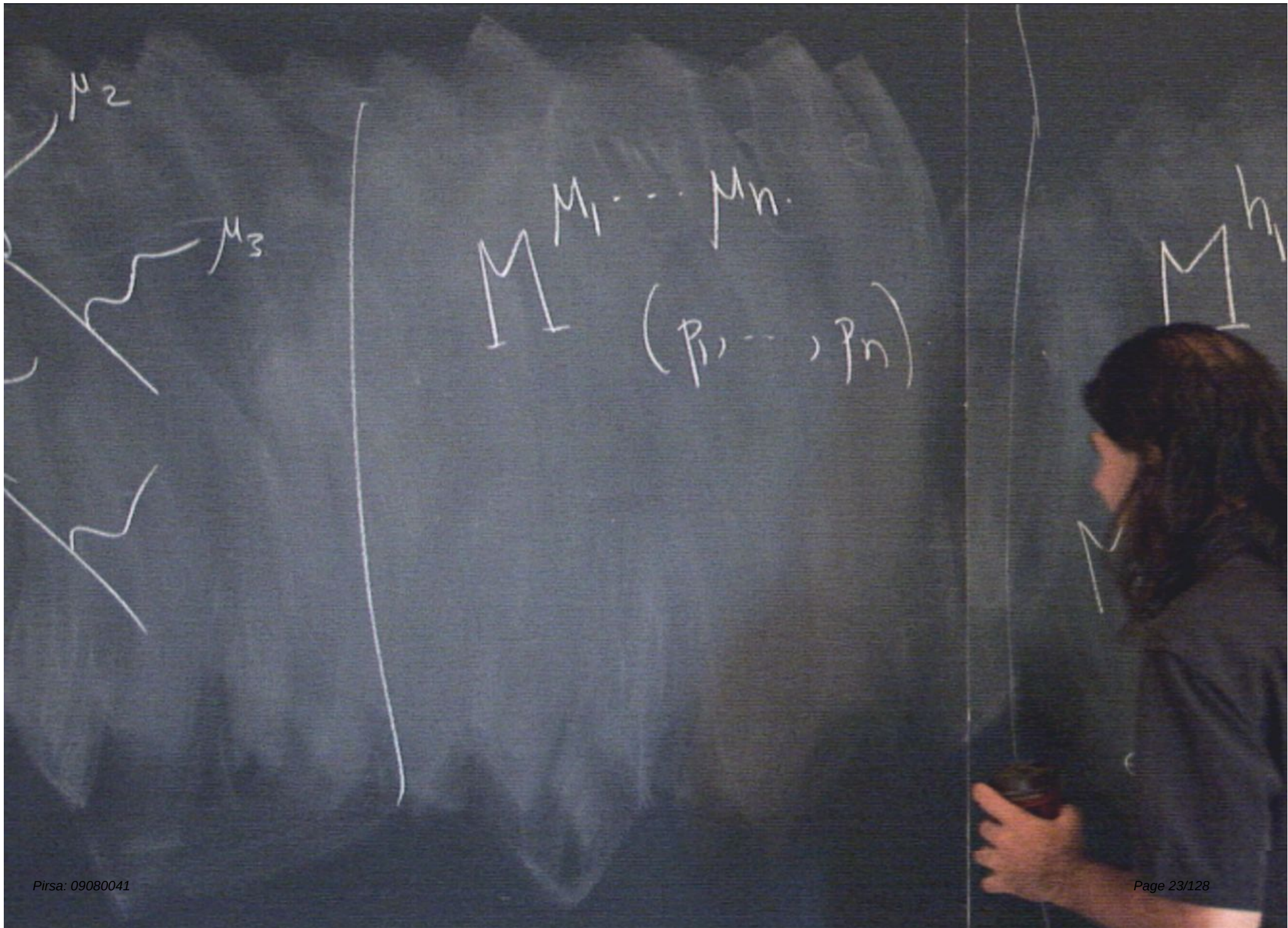
$n, n!$

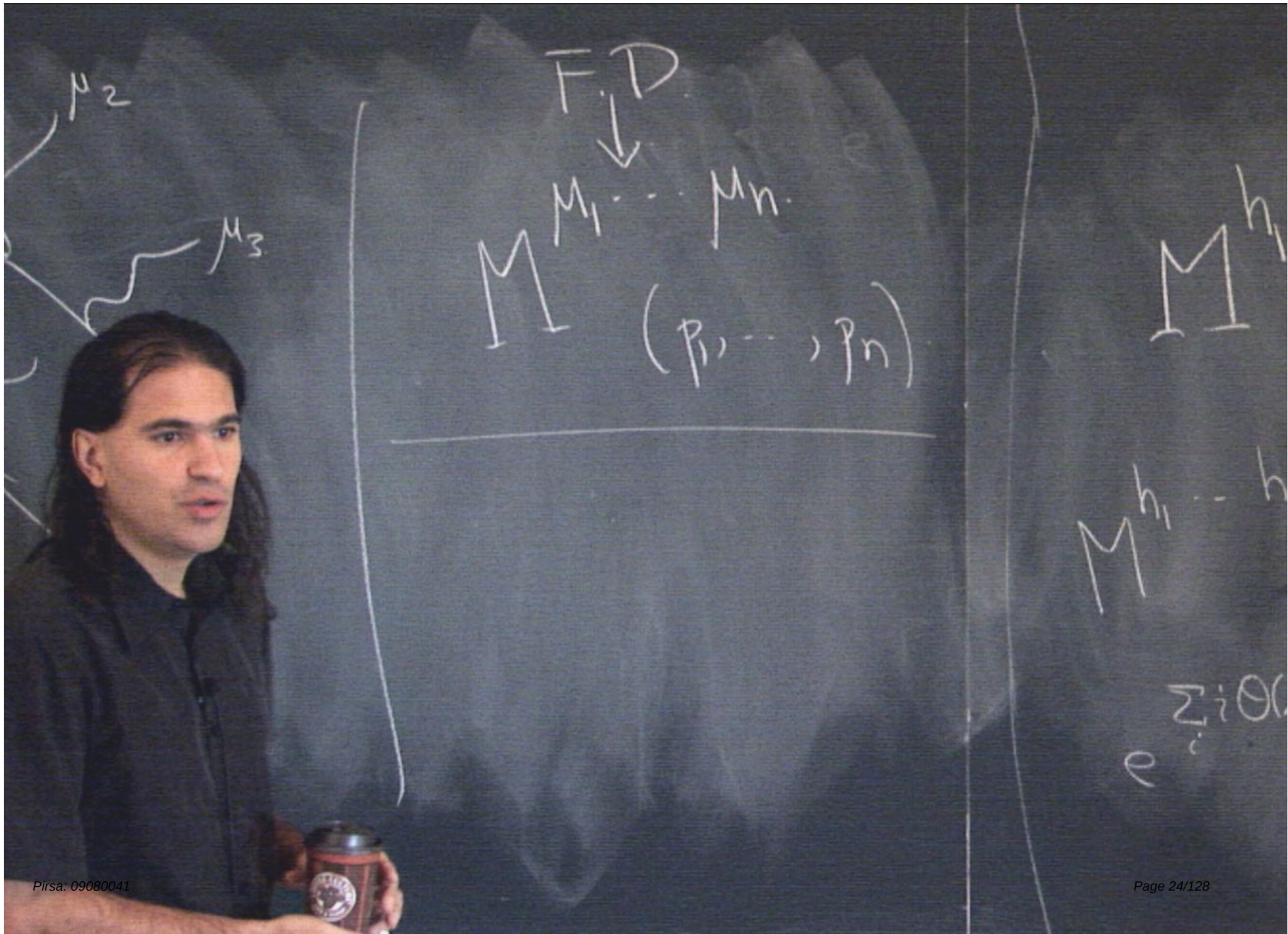
$$M^{h_1 \dots h_n} [\wedge p_1, \dots, \wedge p_n].$$

$$e^{\sum_i i \Theta(\wedge, p_i) \cdot h_i} M^{h_1 \dots h_n} [p_1, \dots, p_n]$$









μ_2

μ_3

F.D



$M, \mu_1, \dots, \mu_n$

$(p_1, \dots, p_n)$

$\in_{\mu_1(p_1)}^{h_1} \in_{\mu_2(p_2)}^{h_2} \dots \in_{\mu_n(p_n)}^{h_n}$

$M, \mu_1, \dots, \mu_n (p_1, \dots, p_n)$

M^{h_1}

$M^{h_1} \dots h_n$

$e^{\sum i \theta_i}$

$$\in_{\mu} (\wedge p)$$

$$\neq (\wedge e)(p) \cdot e^{\partial(\wedge, p)}$$

$$\in_{\mu_1(p)}^h \in_{\mu_1(p)}^h$$

$$M^M, \dots$$

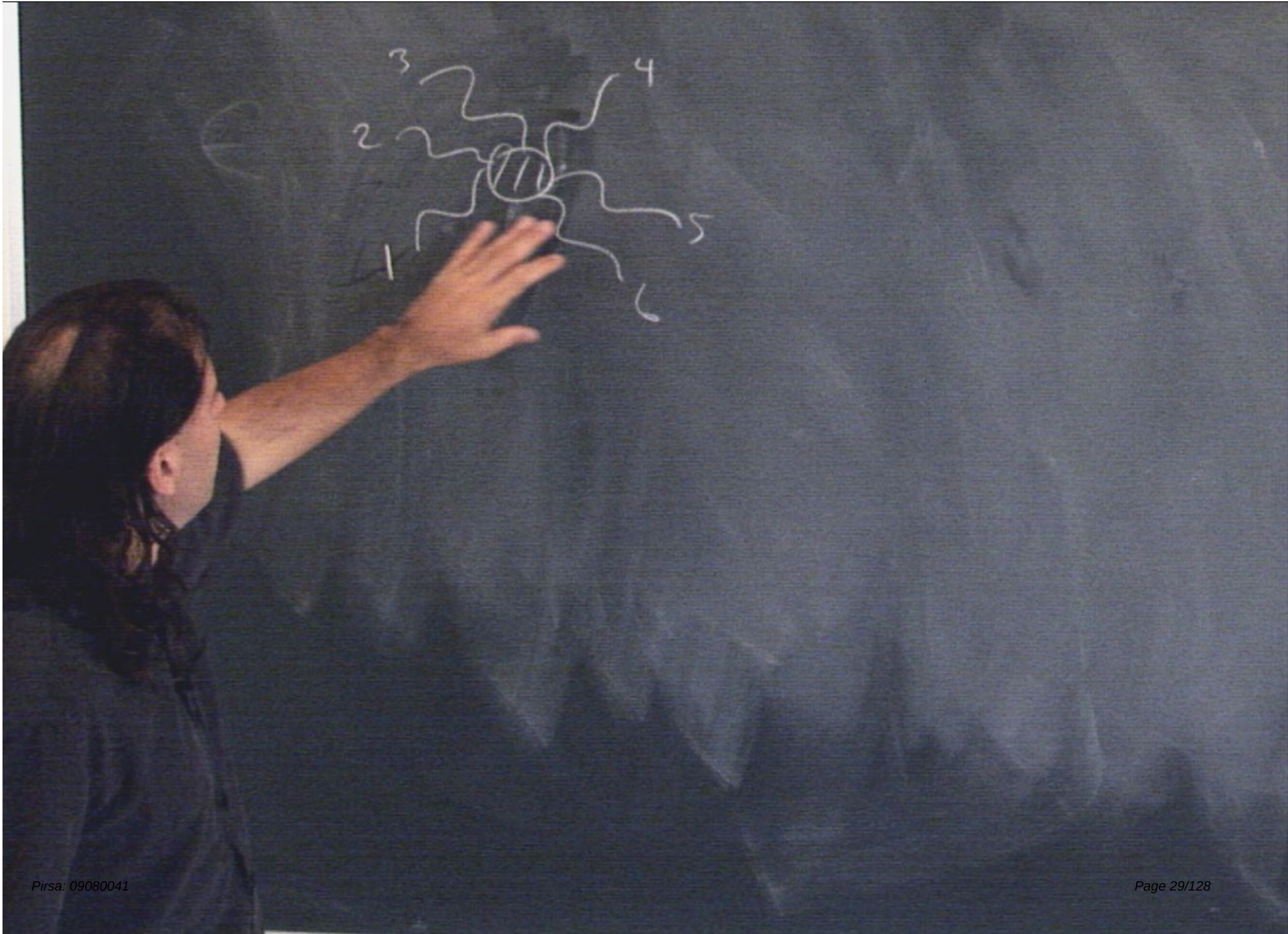
$$\in_{\mu} (\wedge p)$$

$$\neq (\wedge e)(p) \quad e \quad \text{! } \theta(\wedge, p)$$

$$\in_{\mu, (p)}^h \in$$

M











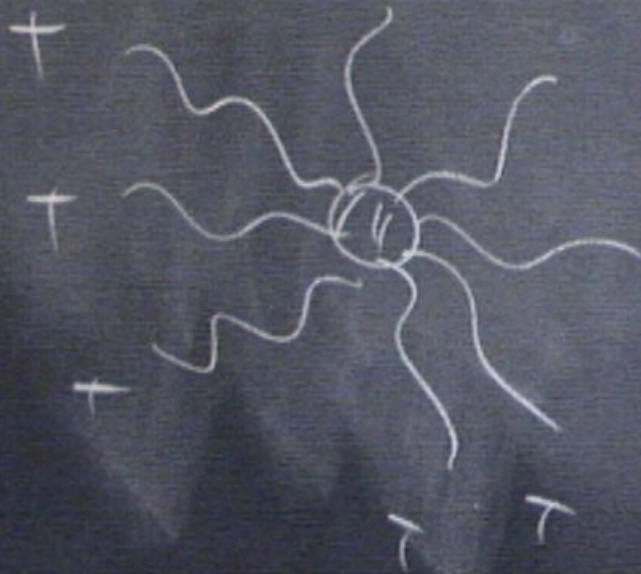
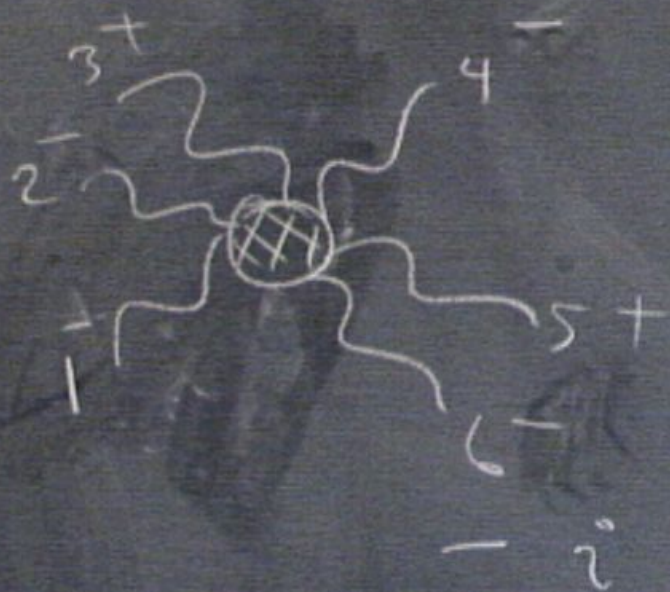
$$\frac{\langle 46 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

M^h



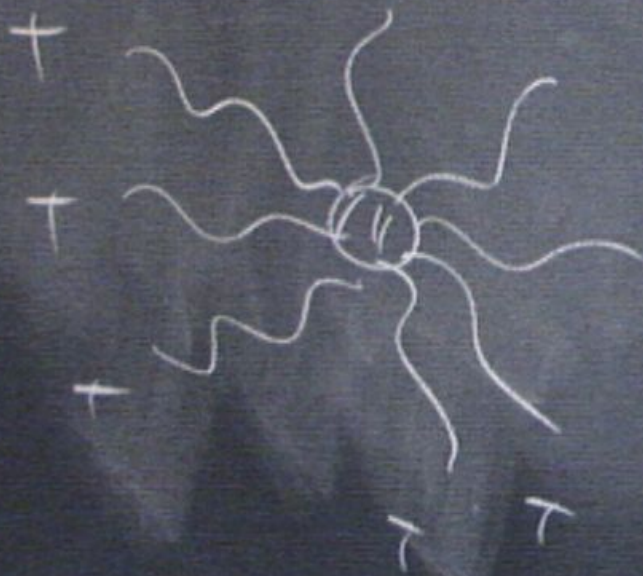
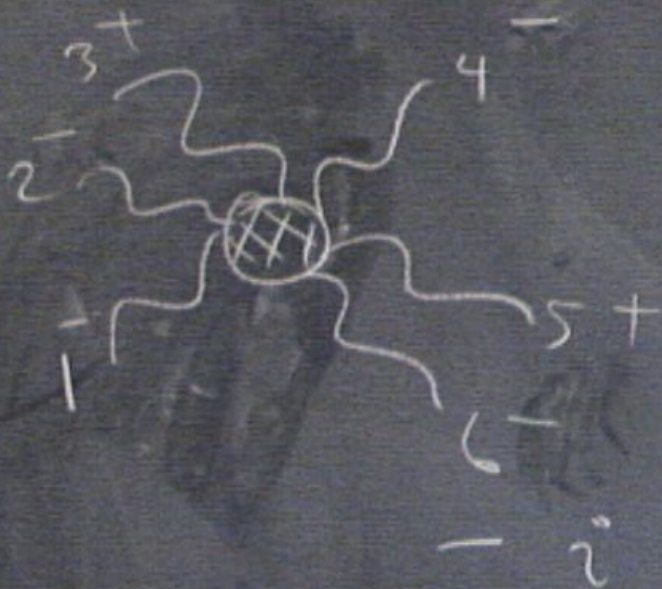
$$\frac{\langle 46 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle}$$

$$\frac{[35]^4}{[12][23] \dots [61]}$$



12:

$$\frac{\langle 46 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle}$$



12:

$$\frac{\langle 46 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle}$$

MHV Amplitudes

$$= \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$



$$\frac{\langle 46 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

MHV Amplitudes

$$= \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

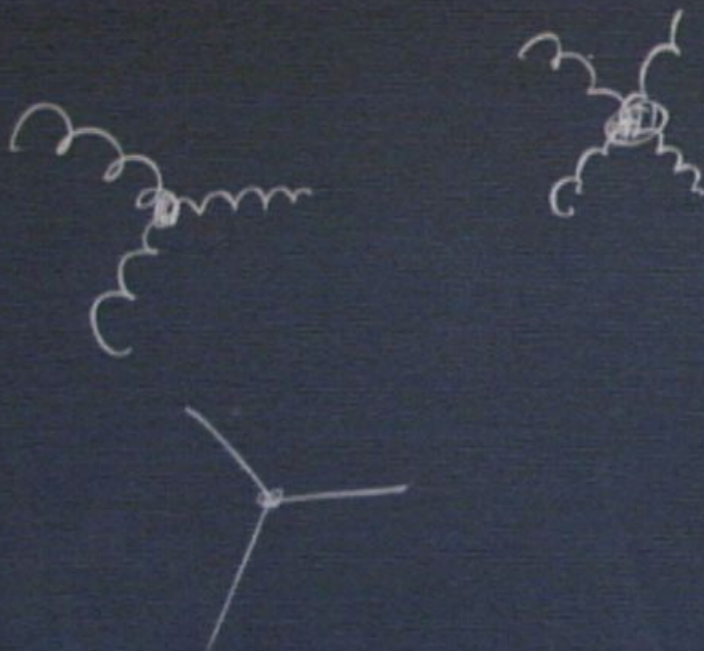
1/2:







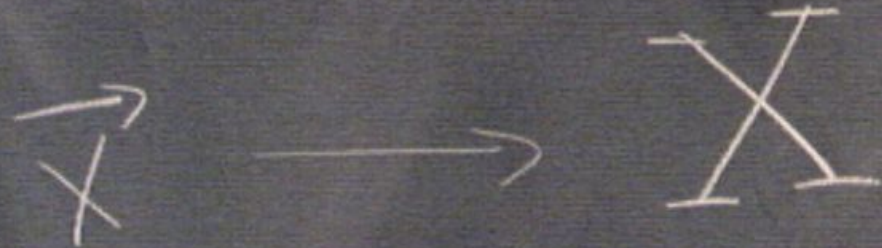




Lorentz Group in 4D

Rotations in 3D

Rotations in 3D.



Rotations in 3D.

$$\vec{x} \longrightarrow X = \vec{\sigma} \cdot \vec{x}$$

Rotations in 3D.

$$\vec{X} \longrightarrow X = \vec{\sigma} \cdot \vec{X}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow X = \begin{pmatrix} z & x+iy \\ x-iy & -z \end{pmatrix}$$

ions in 3D.

$$\vec{X} \longrightarrow X = \vec{\sigma} \cdot \vec{X}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow X = \begin{pmatrix} z & x+iy \\ x-iy & -z \end{pmatrix}$$

completely general
traceless hermitian matrix

ions in 3D.

$$\vec{X} \longrightarrow X = \vec{\sigma} \cdot \vec{X}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow X = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix}$$

completely general
+ traceless hermitian matrix

$$\sigma_x x + \sigma_y y + \sigma_z z$$

in 3D.

$$X = \vec{\sigma} \cdot \vec{X}$$

$$X = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix}$$

completely general

Traces hermitian matrix.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

→

$$X = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix}$$

completely gener
+ traceless hermiti

σ_i

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}$$

$$\vec{x}$$



$$X = \vec{\sigma} \cdot \vec{x}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$X = \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}$$

completely general
+ traceless hermitian

$$\det X =$$

$\vec{x}$ $\longrightarrow$ X

$$= \vec{\sigma} \cdot \vec{x}$$

 $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\longrightarrow$ X

$$= \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}$$

completely general
+ traceless hermitian

$$\det X = -z^2 - (x^2 + y^2)$$

$$= -\vec{x}^2$$

$$X \rightarrow U^\dagger X U$$

$$\vec{X} \rightarrow U^\dagger \vec{X} U$$

$$\vec{\sigma} \cdot \vec{X}$$

$$U^\dagger \vec{X} U = \vec{\sigma} \cdot \vec{X}'$$

$$\det U^\dagger X U$$

$$= \det X$$

$$\vec{x} \longrightarrow$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow$$

$$\det X = -z^2 -$$

$$= -\vec{x}^2$$

$$\det U^\dagger X U$$

$$= \det \vec{X}$$

$$-\vec{X}'^2 = -\vec{X}^2$$

$$\vec{X} \longrightarrow$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow$$

$$\det \vec{X} = -z^2$$

$$= -\vec{X}^2$$

$$XU$$

$$\det U^T X U$$

$$= \det X$$

$$-X'^2 = -X^2$$

$$\Rightarrow X' = R(U)X$$

$$\vec{x}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\det X$$

$$X^M = (x^0, \vec{x}).$$

$$= (x^0, \vec{x})$$

$$= (1, \vec{\sigma})$$

$$x^\mu g_\mu = \begin{pmatrix} x^0 + x^3 \\ x^0 - x^3 \end{pmatrix}$$

$$X^M = (x^0, \vec{x})$$

$$\sigma_\mu = (1, -\vec{\sigma})$$

$$X^\mu \sigma_\mu =$$

$$X^\mu \sigma_\mu = \begin{pmatrix} X^0 + X^3 & X^1 - iX^2 \\ X^1 + iX^2 & X^0 - X^3 \end{pmatrix}$$

$$X^\mu \sigma_\mu = \begin{pmatrix} X^0 + X^3 & X^1 - iX^2 \\ X^1 + iX^2 & X^0 - X^3 \end{pmatrix}$$

$$\equiv \not{X}$$

completely general 2×2 hermit. matrix

$$X^\mu \sigma_\mu = \begin{pmatrix} X_0 + X_3 & X_1 - iX_2 \\ X_1 + iX_2 & X_0 - X_3 \end{pmatrix} = X$$

$$\det X = X_0^2 - X_3^2 - (X_1^2 + X_2^2)$$

completely general

$$\begin{pmatrix} A & Z + iW \\ Z - iW & B \end{pmatrix}$$

$$X^\mu \sigma_\mu = \begin{pmatrix} X_0 + iX_3 & X_1 + iX_2 \\ X_1 - iX_2 & X_0 - iX_3 \end{pmatrix}$$

$$\det X = X_0^2 - X_3^2 - (X_1^2 + X_2^2)$$

$$= X^2$$

→ L⁺ X L

$X \rightarrow L^+ X L$

$$X \rightarrow L^{\dagger} X L$$

L : any 2×2 complex matrix.
 (with $\det L = 1$)

$$X \rightarrow L^T X L \quad \left| \begin{array}{l} \det X \\ = \det(L^T X L) \end{array} \right.$$

any 2×2 complex matrix.

(with $\det L = 1$)

$$GL(2, \mathbb{C})$$

$$x^\mu \sigma_\mu$$

$$\det X$$

$$GL(2, \mathbb{C})$$

$$SL(2, \mathbb{C})$$

$$x^\mu \sigma_\mu$$

$$\det X$$

$$GL(2, \mathbb{C})$$

$$SL(2, \mathbb{C})$$

Lorentz group.

$$x^\mu \sigma_\mu$$

$$\det X$$

L α β

L_{α}^{β}

$\alpha = 1,2$
 β

L_{α}^{β}

f_{α}

$\lambda = 1, 2$
 β

$$L_{\alpha}^{\beta}$$

$$\begin{matrix} \alpha & -1, 2 \\ \beta & \end{matrix}$$

$$f_{\alpha}$$

$$f'_{\alpha} = L_{\alpha}^{\beta} f_{\beta}$$

$$L_{\alpha}^{\beta}$$

$$\vec{\alpha} \rightarrow 1, 2$$

$$\psi_{\alpha}$$

$$\psi'_{\alpha} = L_{\alpha}^{\beta} \psi_{\beta}$$

Spinor rep.
of L.G.

$$\begin{pmatrix} L & \beta \\ \alpha & \end{pmatrix}^* = \begin{pmatrix} L^* & \\ & \alpha \end{pmatrix} \begin{pmatrix} \beta \\ \end{pmatrix}$$

$$\begin{pmatrix} L & \beta \\ \alpha & \end{pmatrix}^* = \begin{pmatrix} L^* & \\ & \alpha \end{pmatrix} \begin{pmatrix} \dot{\beta} \\ \dot{\alpha} \end{pmatrix}$$

$$\left(L_{\alpha}^{\beta} \right)^* = \left(L^* \right)_{\dot{\alpha}}^{\dot{\beta}} \quad ;$$

$$\overline{f'}_{\dot{\alpha}} = \left(L^* \right)_{\dot{\alpha}}^{\dot{\beta}} \overline{f}_{\dot{\beta}}$$

$$\left(L_{\alpha}^{\beta} \right)^* = \left(L^* \right)_{\dot{\alpha}}^{\dot{\beta}}$$

$$\overline{\psi}'_{\dot{\alpha}} = \left(L^* \right)_{\dot{\alpha}}^{\dot{\beta}} \overline{\psi}_{\dot{\beta}}$$

Another spinor rep.

$$\sigma^\mu \rightarrow L^\dagger \sigma^\mu L$$

$$L_\alpha^\beta$$

$$f_\alpha$$

$$f_\alpha = L_\alpha^\beta$$

One Spinor
 f_α

$$\sigma^M \rightarrow L^+ \sigma^M L$$

$$(\sigma^M x_M)$$

$$L_\alpha^\beta$$

$$f_\alpha$$

$$f_\alpha =$$

One

$$P^M, P^2 = 0$$

$$\rightarrow L^\dagger \sigma^\mu L$$

$$\propto \dot{\alpha}$$

$$P^\mu, P^2 = 0$$

$$(\sigma \cdot p)$$

$$\rightarrow L^\dagger \sigma^\mu L$$

$$P^\mu, P^2 = 0$$

$$(\sigma \cdot p)$$

$$P \propto \dot{x}$$

$$\sigma^\mu L \quad P^\mu, \quad P^2 = 0$$

$$(\sigma \cdot p) \quad P \propto \dot{x}$$

$$\det(\sigma \cdot p) = -p^2 = 0$$

$$P_{\alpha\alpha} = \lambda_{\alpha} \tilde{\lambda}_{\alpha}$$

$$P_{\alpha\alpha} = \lambda_{\alpha} \tilde{\lambda}_{\alpha}$$

$$M_{ij} = V_i w_j$$

$$P_{\alpha\alpha} = \lambda_{\alpha} \tilde{\lambda}_{\alpha}$$

$$M_{ij} = V_i^{(1)} \omega_j^{(1)} + \dots + V_i^{(n)} \omega_j^{(n)}$$

$$P_{\alpha\alpha} = \lambda_{\alpha} \tilde{\lambda}_{\alpha}$$

$$M_{ij} = V_i^{(1)} \omega_j^{(1)} + \dots + V_i^{(n)} \omega_j^{(n)}$$

$$M = \sum |n\rangle \langle n| \lambda_n$$

$$P_{\alpha} \dot{\alpha} = \lambda_{\alpha} \dot{\alpha}$$

For real momenta, $(\tilde{\lambda}_{\alpha}) = (\lambda_{\alpha})^*$

$$T \propto \dot{\alpha} = \lambda_{\alpha} \dot{\lambda}_{\alpha}$$

For real momenta, $(\tilde{\lambda}_{\dot{\alpha}}) = (\lambda_{\alpha})^*$

$$\begin{bmatrix} E + p_z & 0 \\ 0 & E - p_z \end{bmatrix} = \begin{bmatrix} 2E & 0 \\ 0 & 0 \end{bmatrix}$$

$$T \alpha \dot{\alpha} = \lambda_{\alpha} \lambda_{\dot{\alpha}}$$

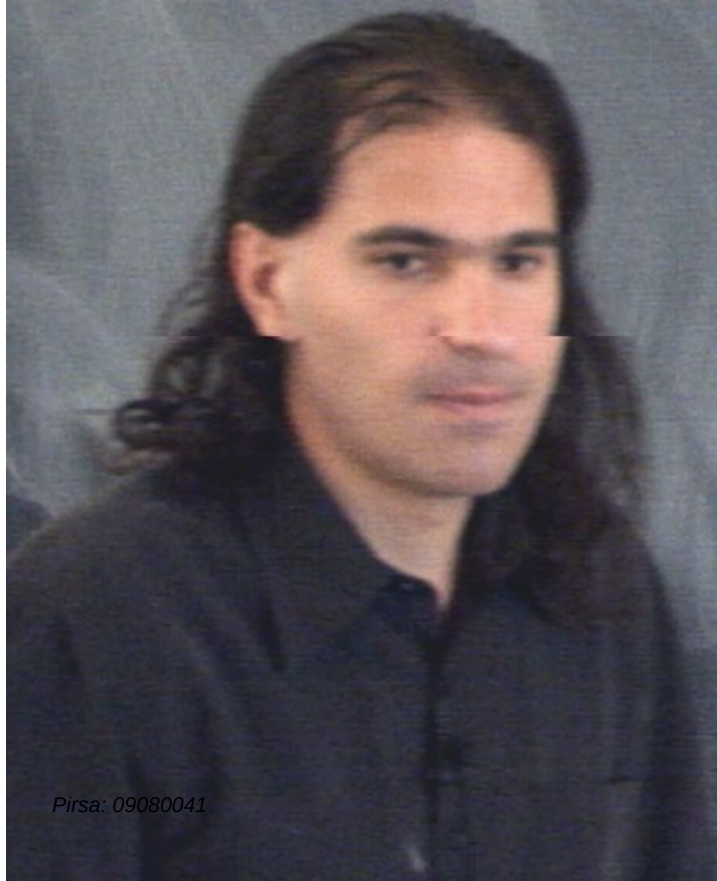
or real momenta, $(\tilde{\lambda}_{\dot{\alpha}}) = (\lambda_{\alpha})^*$

$$\begin{bmatrix} E+p_z & 0 \\ 0 & E-p_z \end{bmatrix} = \begin{bmatrix} 2E & 0 \\ 0 & 0 \end{bmatrix} \quad \lambda = \begin{pmatrix} \sqrt{2E} \\ 0 \end{pmatrix}$$

$$\tilde{\lambda} = \begin{pmatrix} 0 \\ \sqrt{2E} \end{pmatrix}$$

$$\psi_1$$

$$\psi_2$$



$$\psi^1_\alpha$$

$$\psi^2_\alpha$$

$$(\psi^1 \psi^2) = \epsilon^{\alpha\beta} \psi^1_\alpha \psi^2_\beta$$

$$\frac{1}{\alpha} \psi^2_\alpha$$

$$(\psi^1 \psi^2) = \epsilon^{\alpha\beta} \psi^1_\alpha \psi^2_\beta$$

$$\epsilon^{\alpha\beta} = 0 \text{ if } \alpha = \beta$$

$$= 1 \quad \alpha=1, \beta=2$$

$$= -1 \quad \alpha=2, \beta=1$$

$$\psi^2_\alpha$$

$$\epsilon^{\alpha\beta} = 0 \text{ if } \alpha = \beta$$

$$= 1 \quad \alpha=1, \beta=2$$

$$= -1 \quad \alpha=2, \beta=1$$

$$\psi^2 = \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta$$

$$\psi^1_\alpha$$

$$\psi^2_\alpha$$

$$\langle \psi^1 \psi^2 \rangle = \epsilon^{\alpha\beta} / \psi^1_\alpha \psi^2_\beta$$

$$\psi^1_\alpha$$

$$\psi^2_\alpha$$

①

$$\langle \psi^1 \psi^2 \rangle = \epsilon^{\alpha\beta} | \psi^1_\alpha \psi^2_\beta |$$

$$\bar{\psi}^1_\alpha, \bar{\psi}^2_\alpha$$

$$[\bar{\psi}^1 \bar{\psi}^2] = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}^1_{\dot{\alpha}} \bar{\psi}^2_{\dot{\beta}}$$

$$P_{\alpha\alpha} = \lambda_{\alpha} \lambda_{\alpha}$$

$$P_{\alpha\alpha} = \lambda_{\alpha} \lambda_{\alpha}$$

$$\lambda_{\alpha} \rightarrow e^{i\theta} \lambda_{\alpha}$$

$$\tilde{\lambda}_{\alpha} \rightarrow e^{-i\theta} \tilde{\lambda}_{\alpha}$$

$P_{\alpha\alpha}$ is

invariant.

$$R_{\alpha\dot{\alpha}} = \lambda_{\alpha}^{\dot{\beta}} \lambda_{\dot{\alpha}}^{\beta}$$

$$k_{\alpha\dot{\alpha}} = \lambda_{\alpha}^k \lambda_{\dot{\alpha}}^k$$

$$P = \wedge k$$

$$K_{\alpha\dot{\alpha}} = \lambda_{\alpha}^{\dot{K}} \lambda_{\dot{\alpha}}^{\dot{K}}$$

$$= \Lambda(p) K$$

$$\lambda_{\alpha}^{\dot{P}} = L_{\alpha}^{\beta} (\Lambda(p)) \lambda_{\beta}^{\dot{K}}$$

λ_x Δp

$$\lambda_{\alpha} \wedge P \neq L_{\alpha}^{\beta}(\wedge) \lambda_{\beta} P$$

$$\gamma_{\alpha}^{\Delta P} = L_{\alpha}^{\beta}(\Lambda) \gamma_{\beta}^P \times e^{i\Theta(\Lambda, P)}$$

$$\begin{array}{c} \lambda_1 \lambda_2 \dots \lambda_p \\ \vdots \\ \lambda_2 \end{array}$$

=

1, 2, ...

$$\langle \lambda_1, \lambda_2 \rangle$$

$$\begin{matrix} & \Delta P \\ \nearrow & \\ \lambda & \\ \vdots & \end{matrix}$$

=

1, 2, ...

$$\langle \lambda_1, \lambda_2 \rangle = E^{\alpha_3} \lambda_{1,0}$$

$$\bigwedge_{\alpha} \Delta P = L'_{\alpha}(\wedge)$$

1, 2, ...

$$\langle \lambda_1, \lambda_2 \rangle = \bigoplus_{\beta} \lambda_{1\alpha} \lambda_{2\beta}$$

$$\gamma_{\alpha}^{\Lambda p} = L_{\alpha}^{\beta}(\Lambda) \gamma_{\beta}^p \times \frac{e^{i\Theta(\Lambda, p)}}{e}$$

$$\langle \lambda_1 \lambda_2 \rangle = \epsilon^{\alpha\beta} \gamma_{1\alpha} \gamma_{2\beta}$$

particle of helicity $-\frac{1}{2}$

$$\langle \lambda_1 \lambda_2 \rangle \rightarrow \langle \lambda_1 \lambda_2 \rangle e^{i\Theta_1} e^{i\Theta_2}$$

$$[\lambda_1, \lambda_2] \rightarrow e^{-i\theta_1} e^{-i\theta_2}$$

1, 2, ...

$$[\lambda_1, \lambda_2] \rightarrow e^{-i\theta_1} e^{-i\theta_2} [\lambda_1, \lambda_2]$$

helicity $+\frac{1}{2}$

1, 2, ...

$M^{h_1 \dots h_m}$

$$M^{h_1 \dots h_n}(\lambda_1, \tilde{\lambda}_1; \lambda_2, \tilde{\lambda}_2; \dots, \lambda_n, \tilde{\lambda}_n)$$

$$M^{h_1 \dots h_n}(\lambda_1, \tilde{\lambda}_1; \lambda_2, \tilde{\lambda}_2; \dots, \lambda_n, \tilde{\lambda}_n)$$

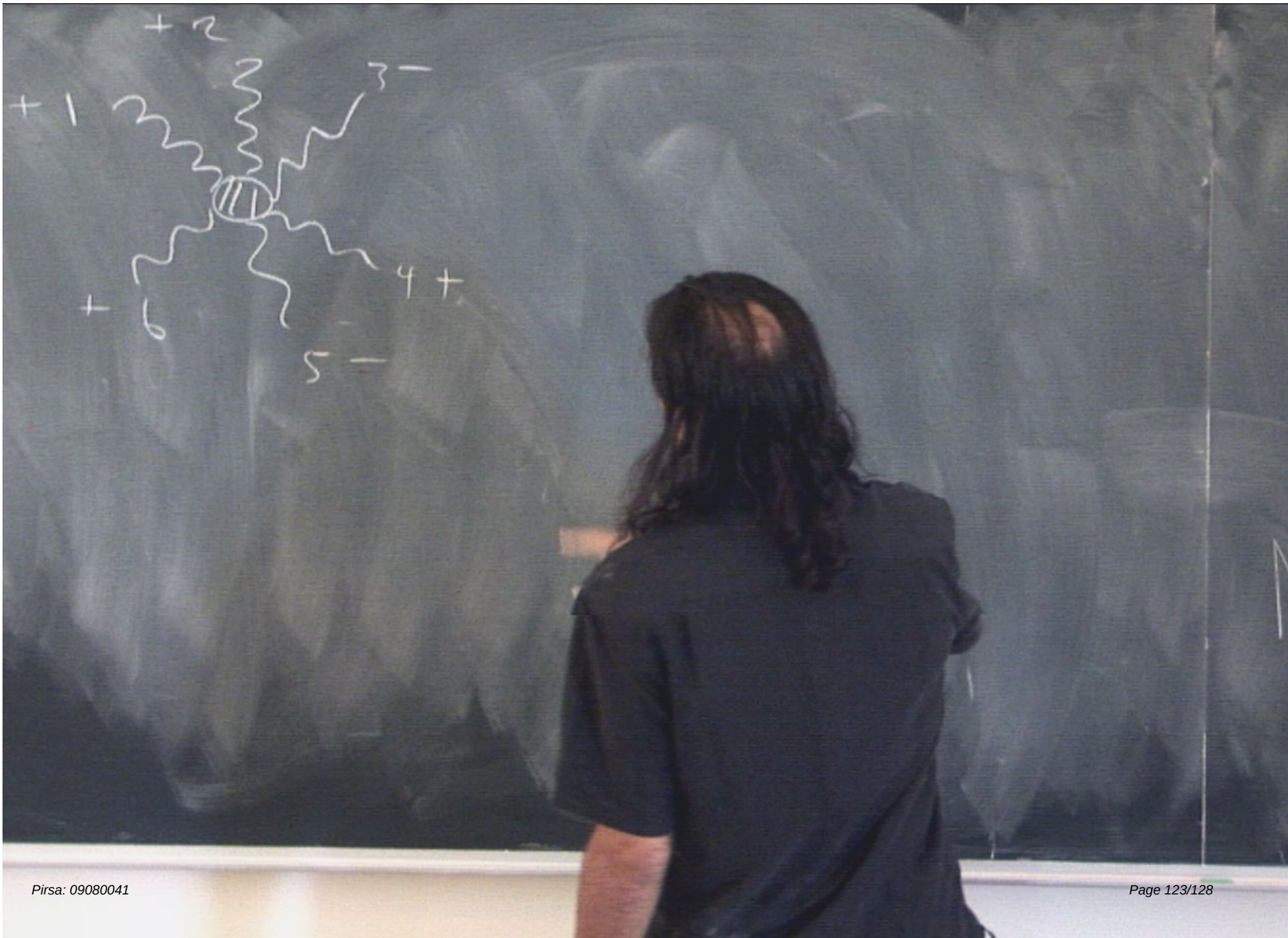
built out of $\langle ij \rangle, [ij], \dots$

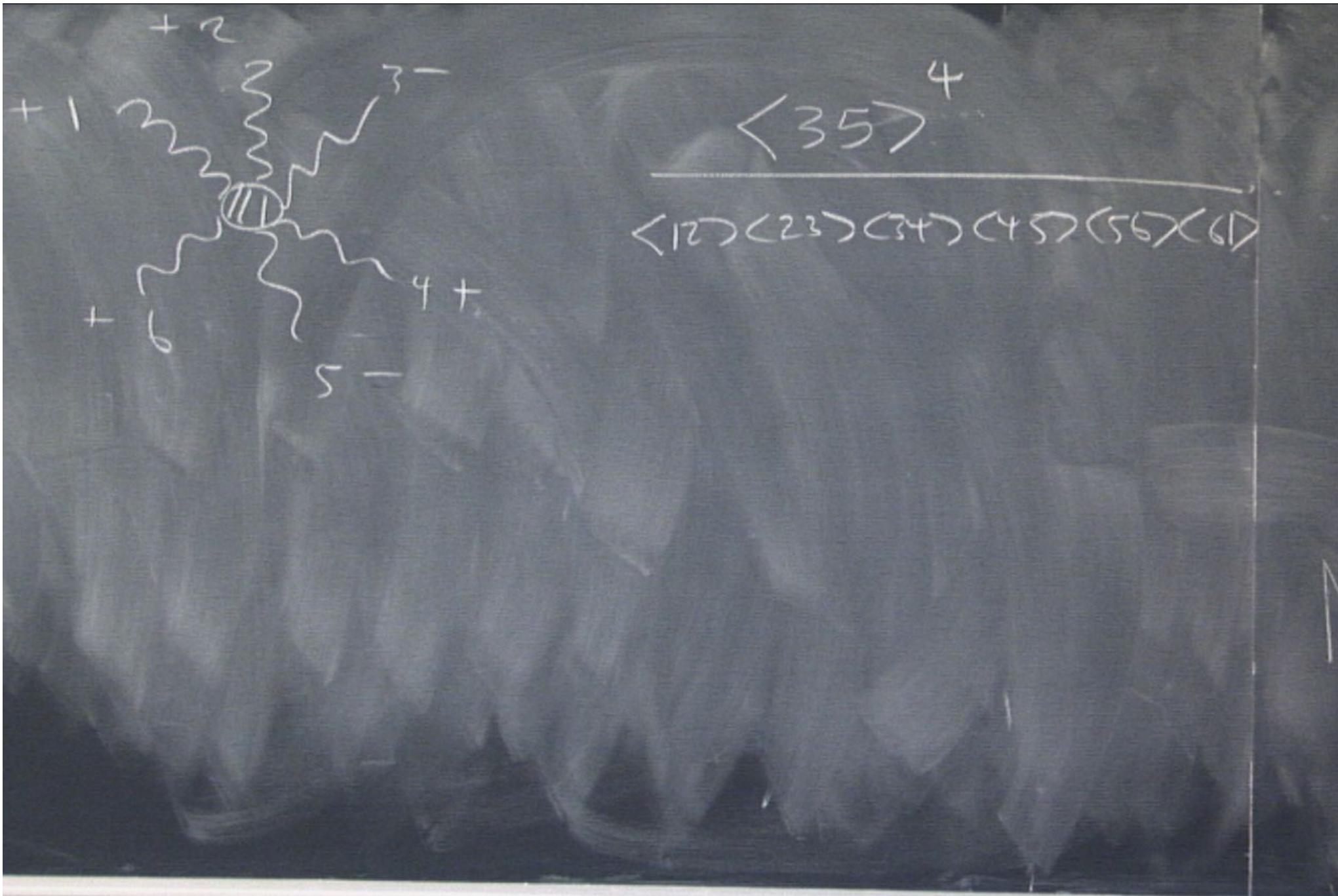
$$M^{h_1 \dots h_n}(\lambda_1, \tilde{\lambda}_1; \lambda_2, \tilde{\lambda}_2; \dots, \lambda_n, \tilde{\lambda}_n)$$

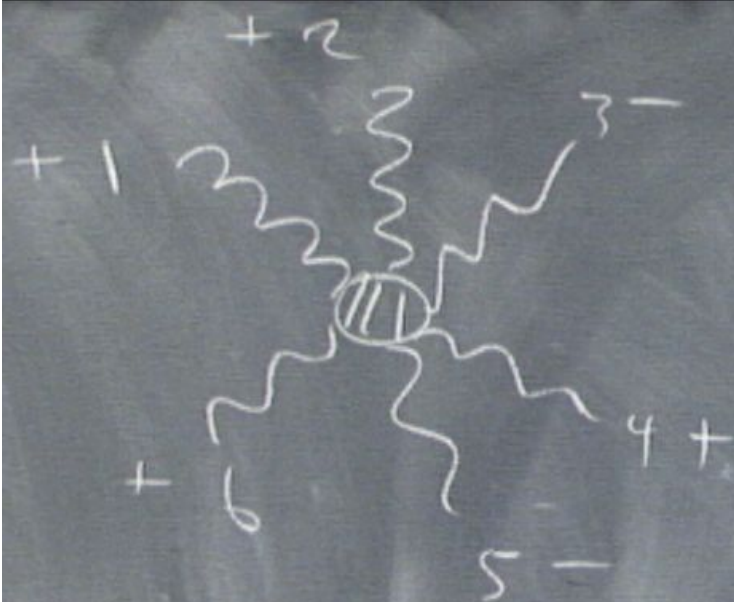
built out of $\langle ij \rangle, [ij], \dots$

$$M^{h_1 \dots h_n}(e^{i\theta_1} \lambda_1, e^{i\theta_1} \tilde{\lambda}_1; e^{i\theta_2} \lambda_2, e^{i\theta_2} \tilde{\lambda}_2; \dots)$$

$$= e^{i2h_1 \theta_1} e^{i2h_2 \theta_2} \dots e^{i2h_n \theta_n} M(\lambda_1, \tilde{\lambda}_1, \dots, \lambda_n, \tilde{\lambda}_n)$$



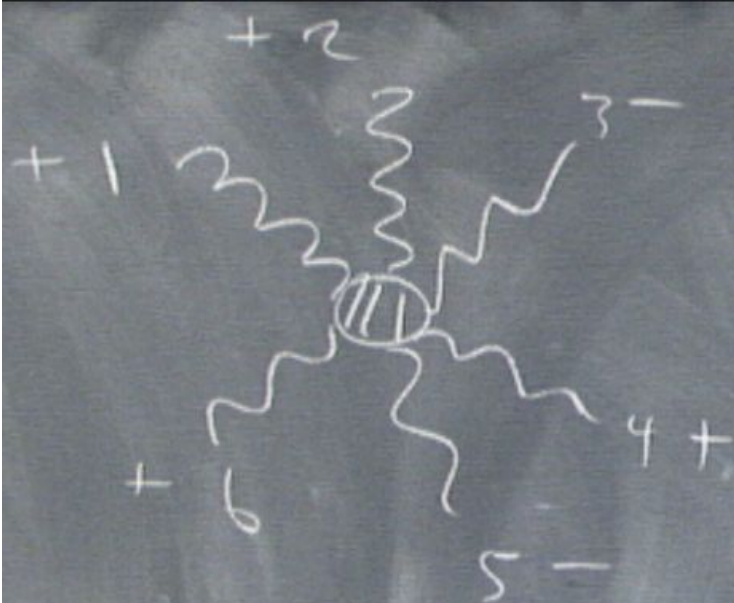




$$M = \frac{\langle 35 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

$$\lambda_3 \rightarrow e^{i0} \lambda_3$$

$$M \rightarrow e^{+2i0} M$$



$$M = \frac{\langle 35 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

$$\lambda_3 \rightarrow e^{i0} \lambda_3$$

$$M \rightarrow e^{+2i\theta} M$$

$$= e^{-2i\theta}$$

