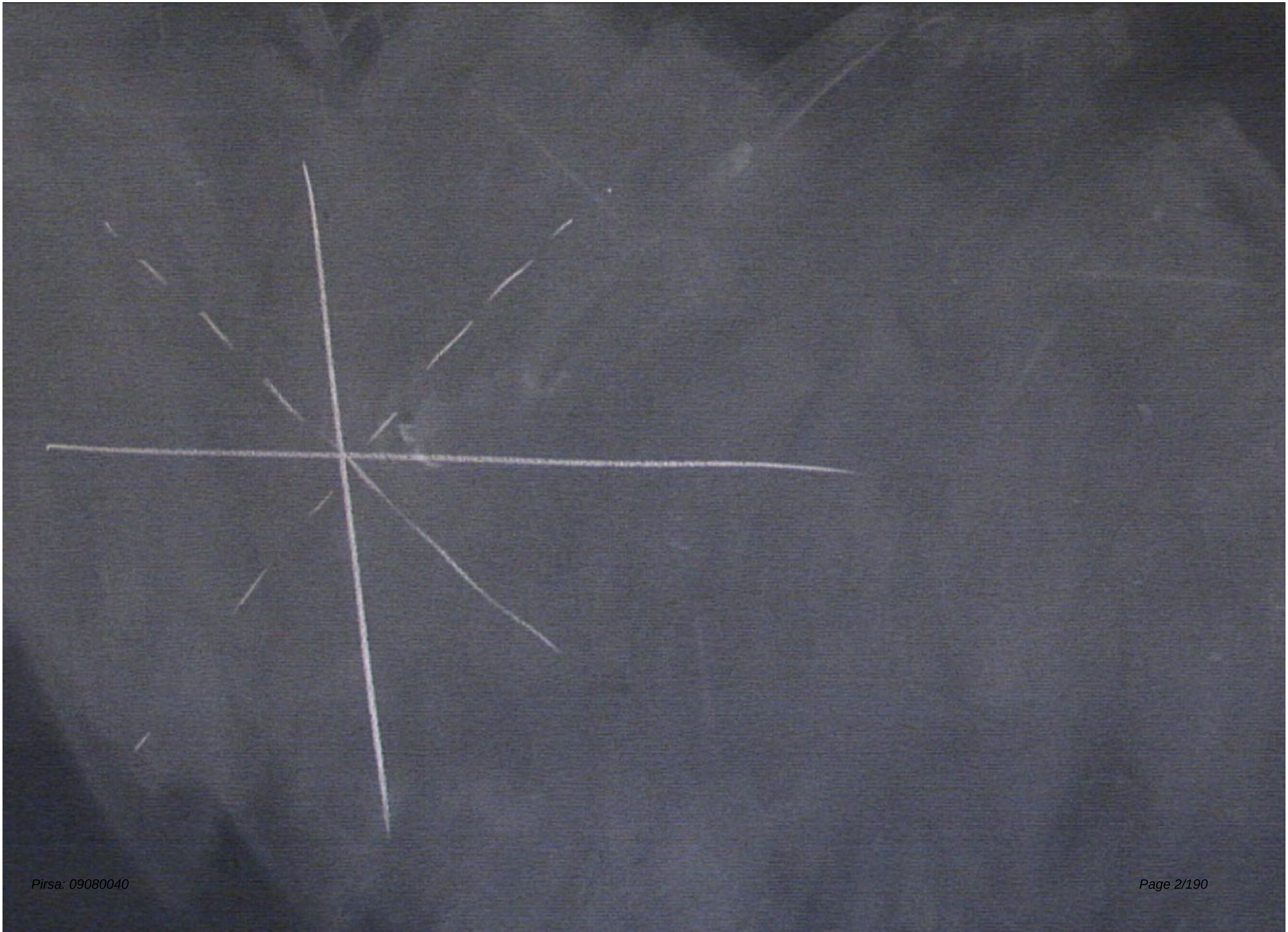
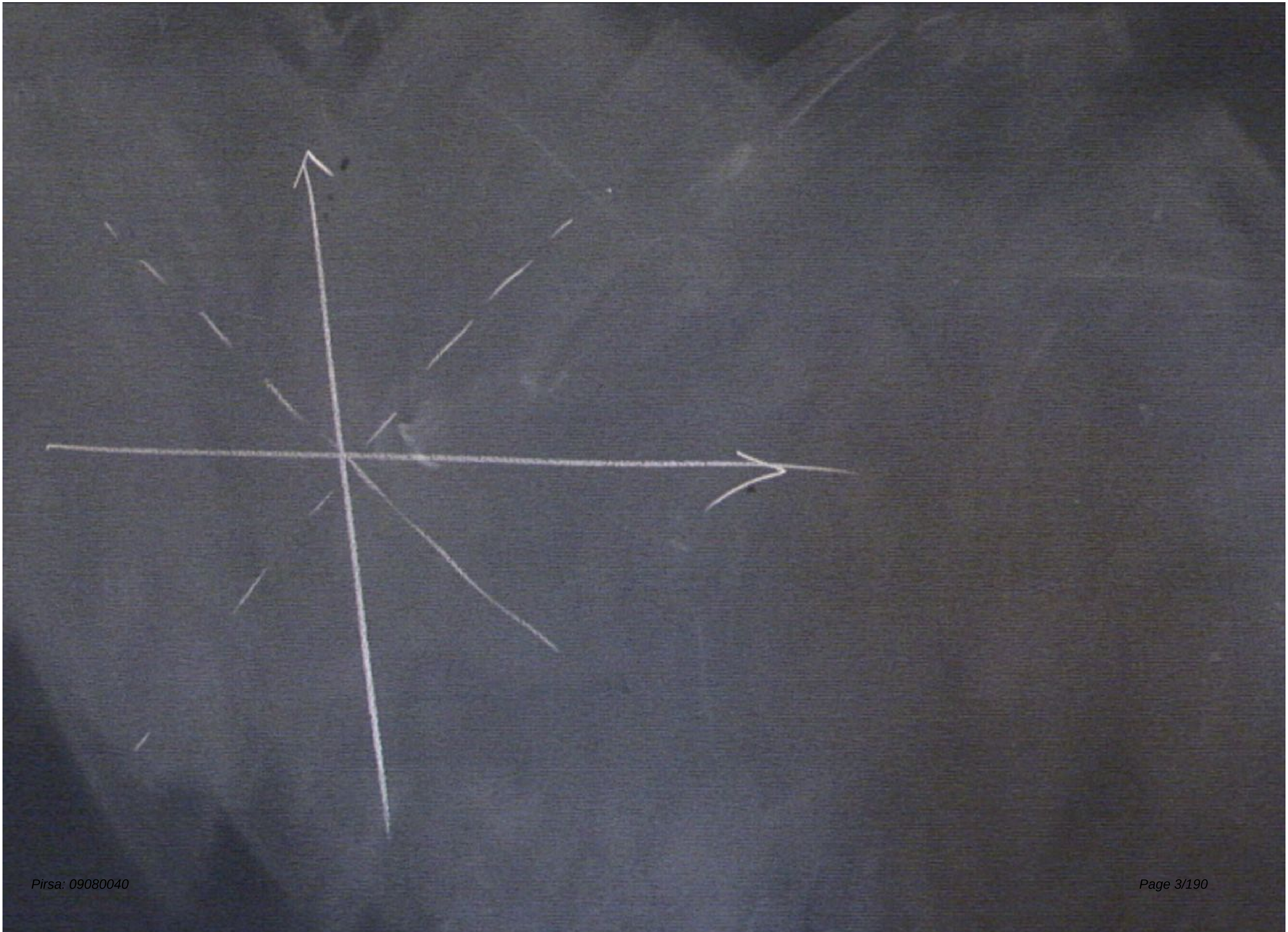
Title: PSI - Research Skills 3B

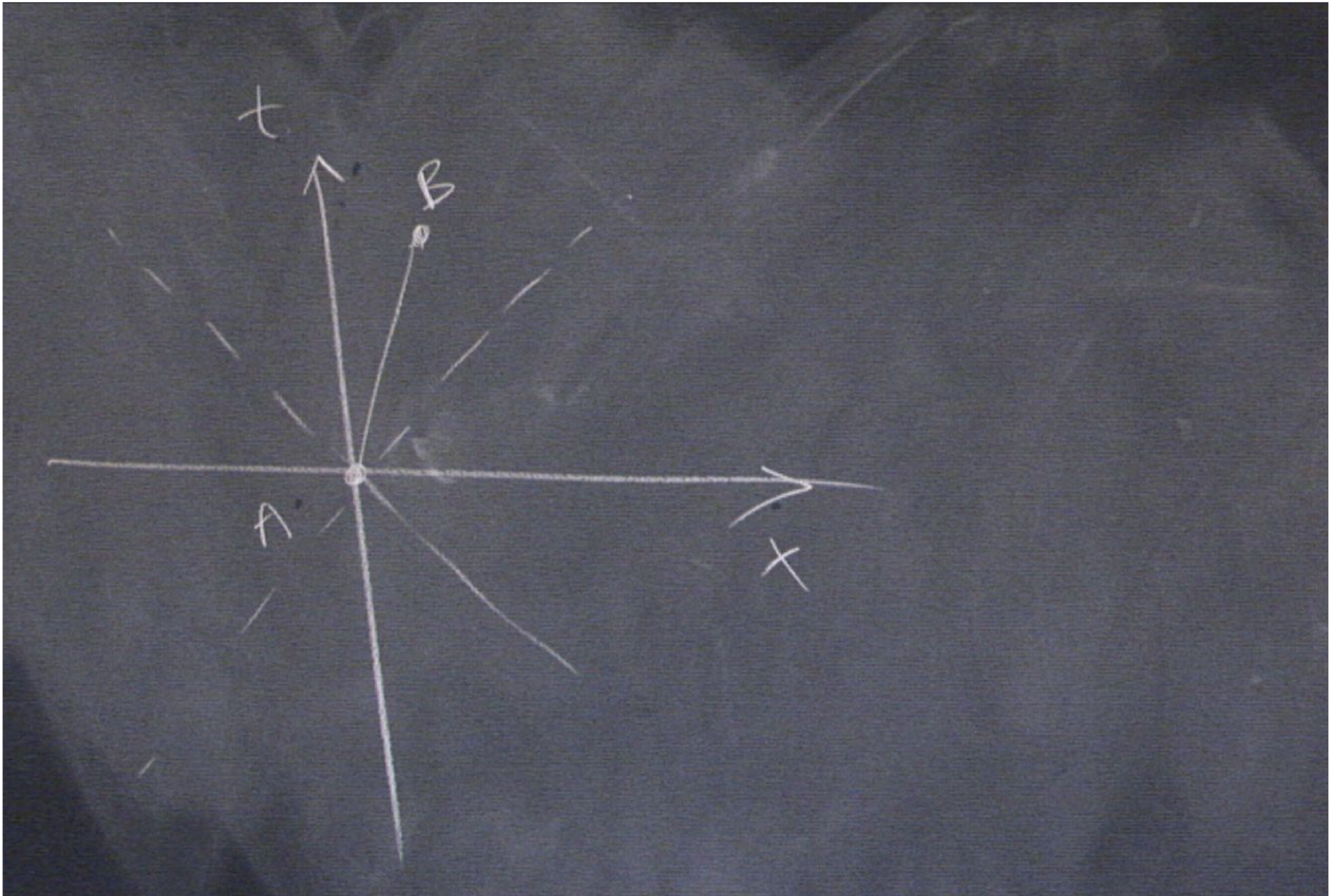
Date: Aug 26, 2009 11:00 AM

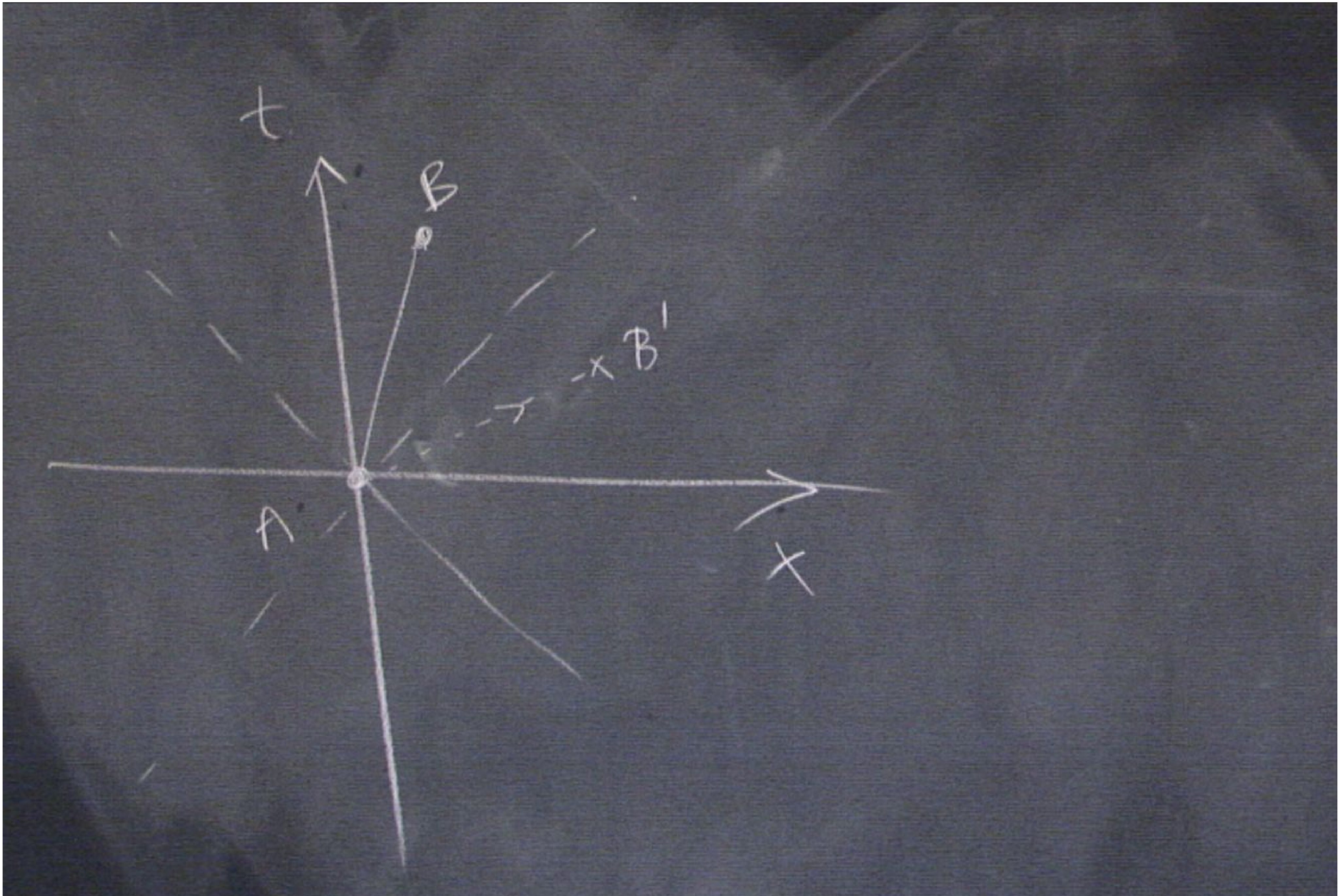
URL: http://pirsa.org/09080040

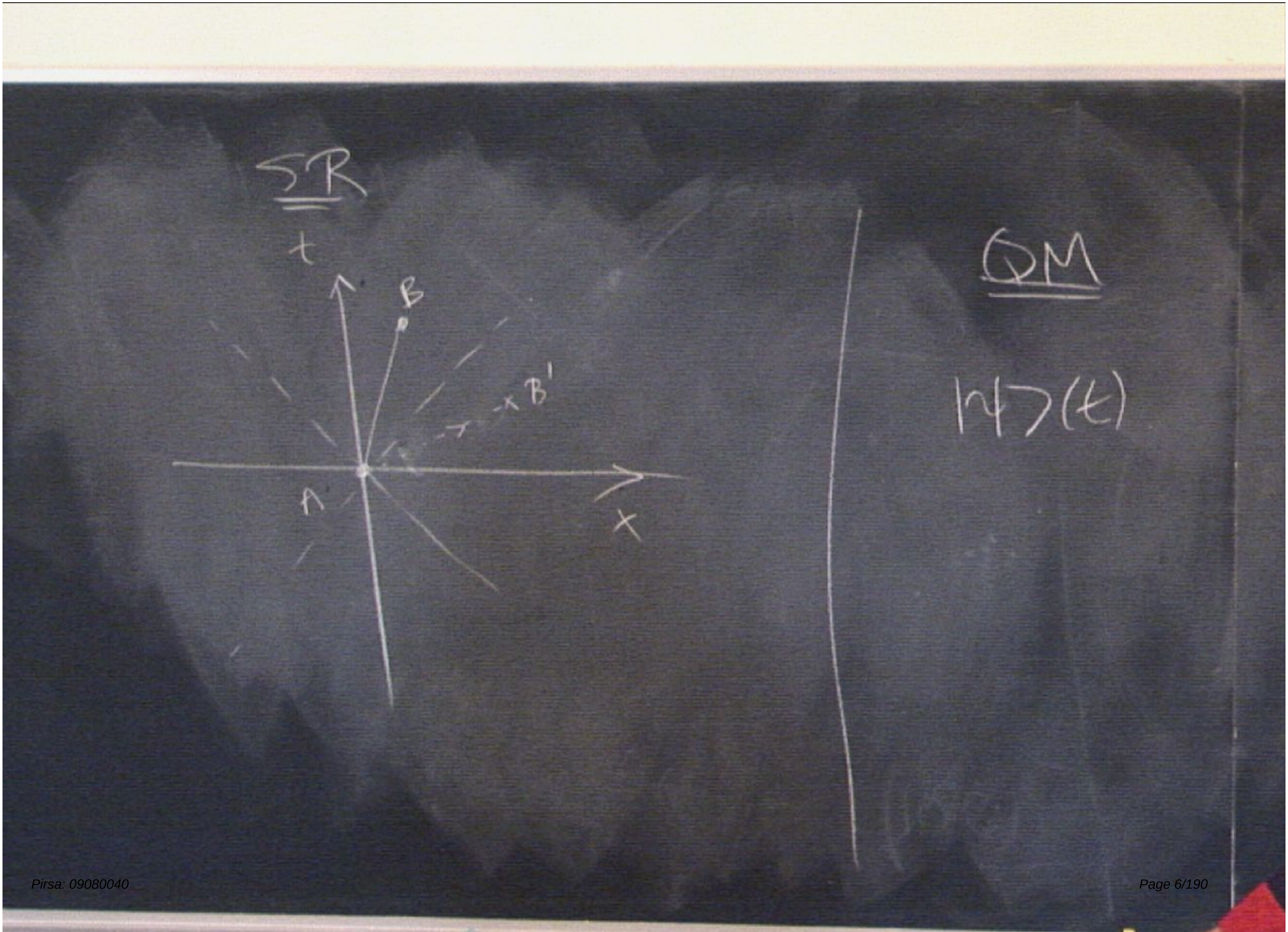
Abstract:

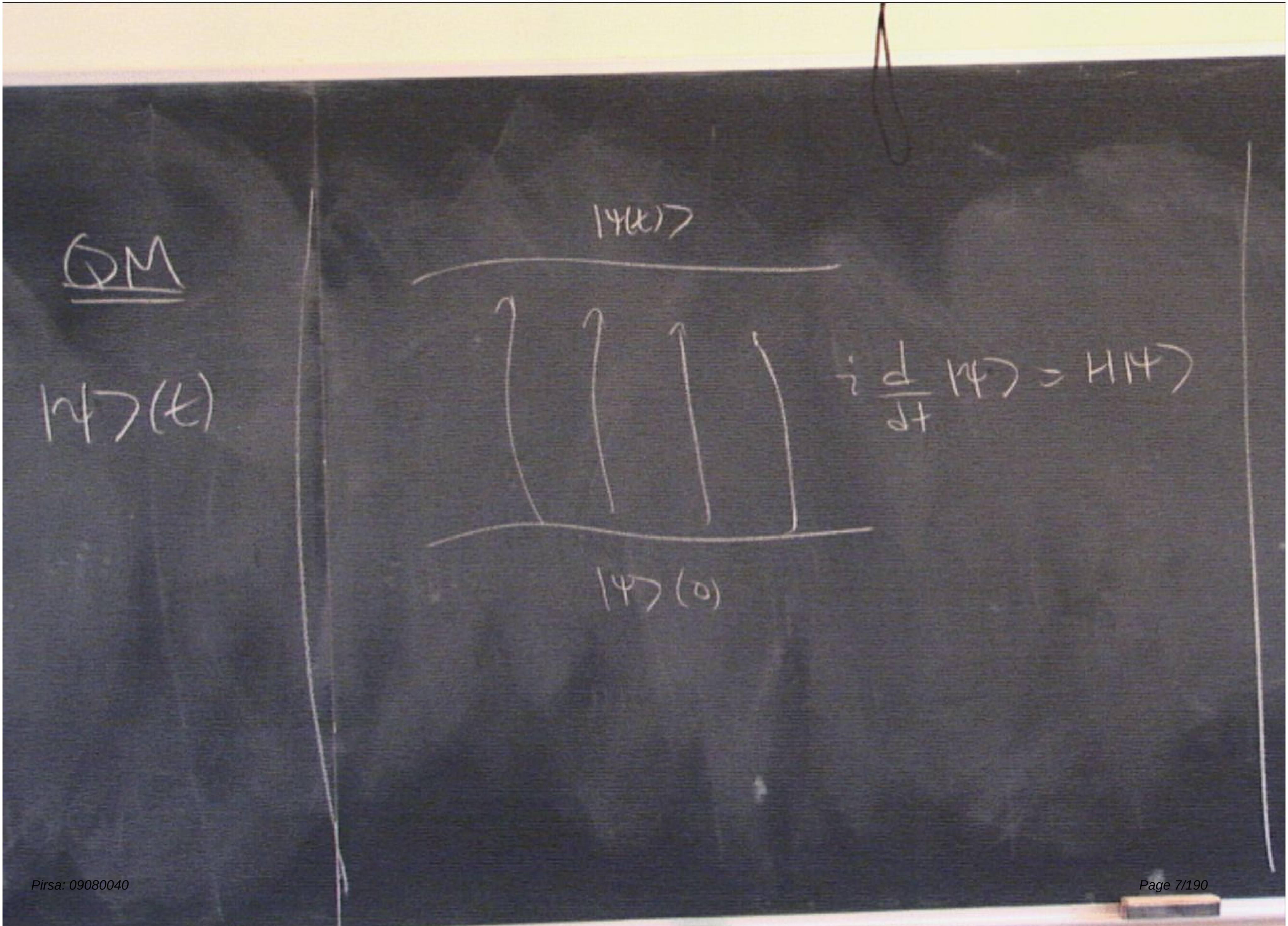


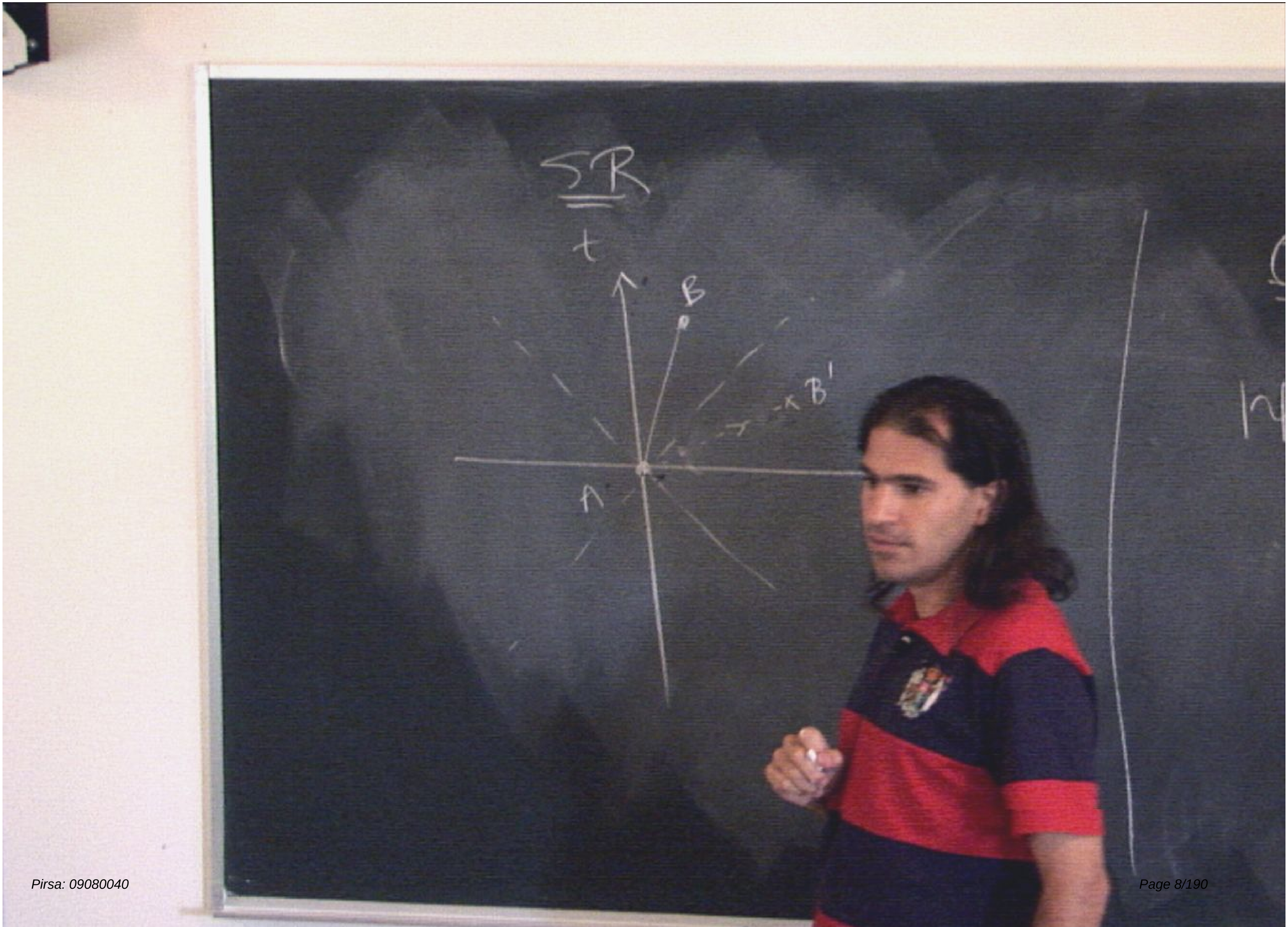


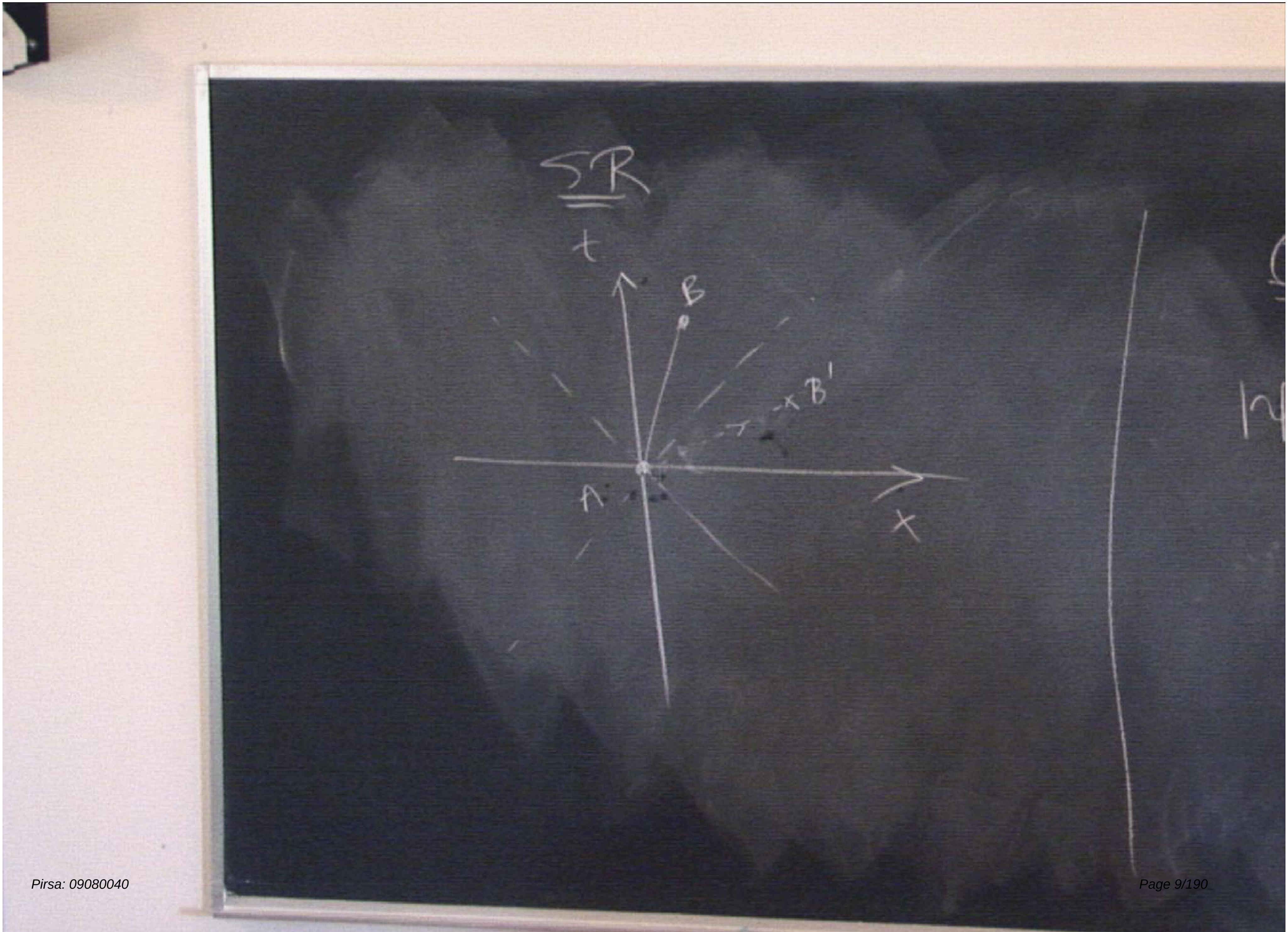

t
B
A
x

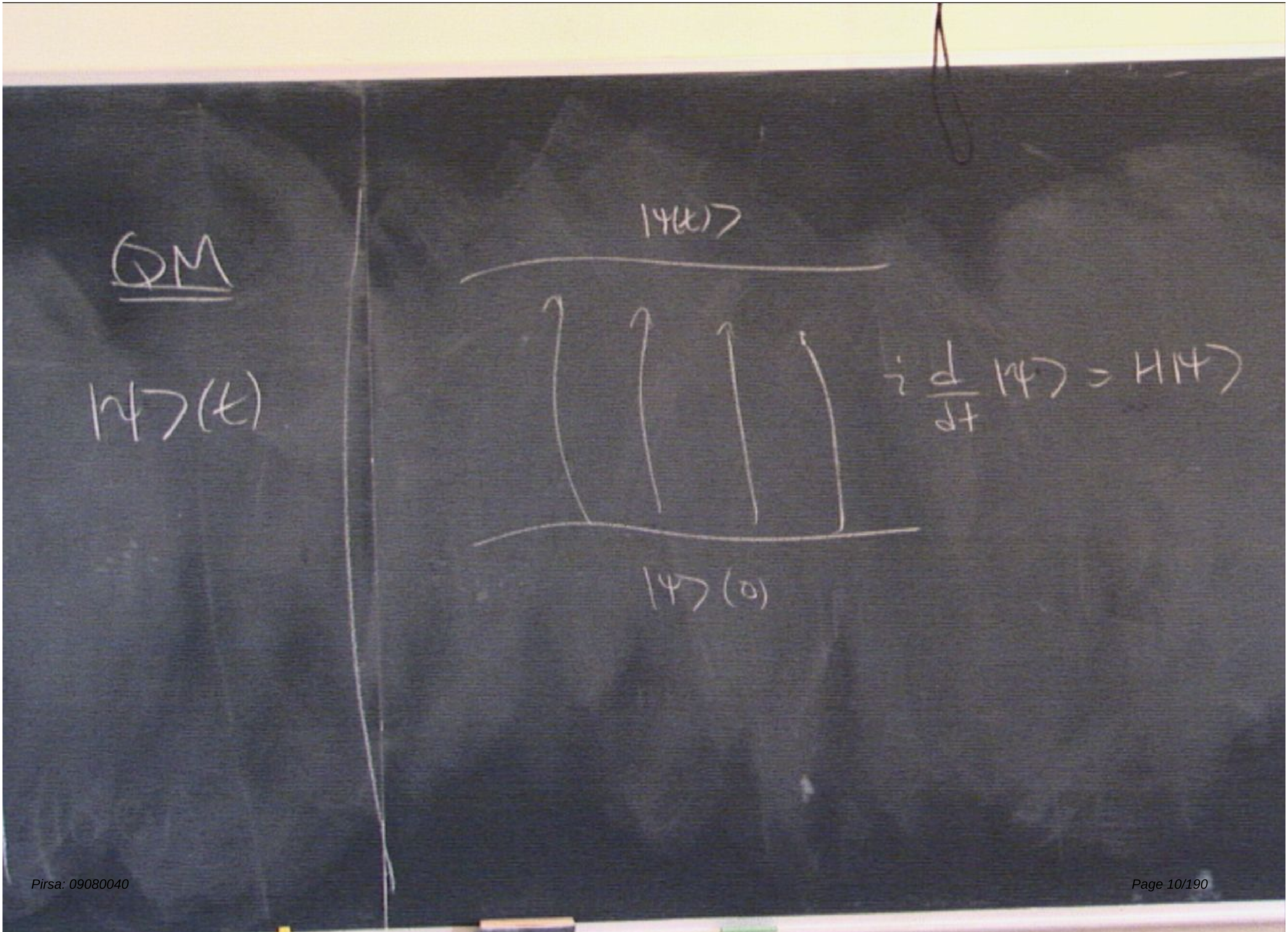

t
B
B'
A
x


SR
t
B
B'
A
x
QM
|ψ⟩(t)


QM
|ψ⟩(t)
|ψ(t)⟩
i d/dt |ψ⟩ = H|ψ⟩
|ψ⟩(0)


SR
t
B
B'
A


SR
t
B
B'
A
x


QM
$|\psi\rangle(t)$
$|\psi(t)\rangle$
$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle$
$|\psi\rangle(0)$

$$|\vec{p}, \sigma\rangle$$

$$|0\rangle$$

$$|\vec{p}, \sigma\rangle$$

$$|\vec{p}_1, \sigma_1, \vec{p}_2, \sigma$$

$$\vdots$$

$|\vec{p}, \sigma\rangle$

$|0\rangle$

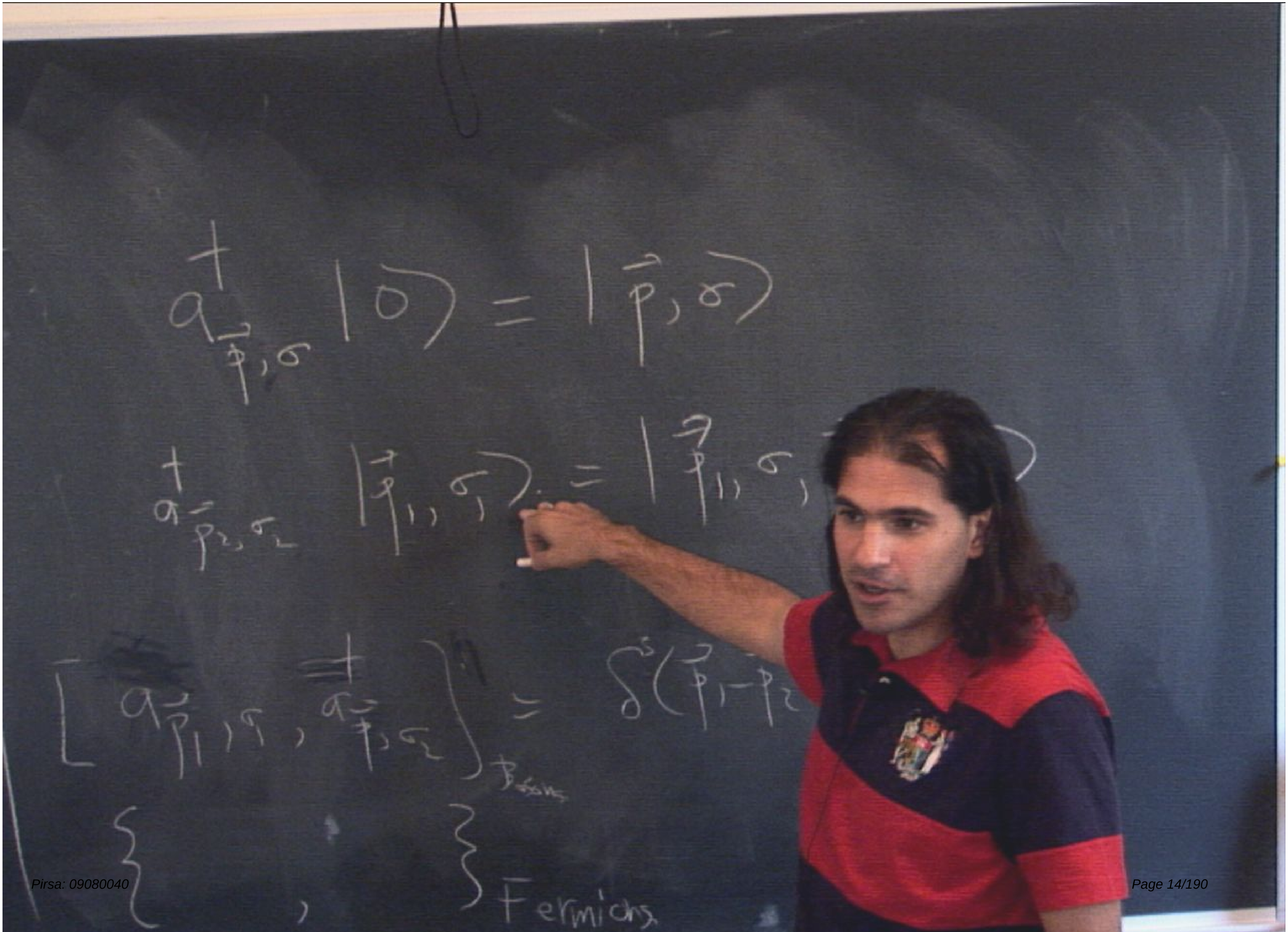
$|\vec{p}, \sigma\rangle$

$|\vec{p}_1, \sigma_1, \vec{p}_2, \sigma_2\rangle$

$\vdots$

$$a^\dagger_{\vec{p},\sigma}|0\rangle = |\vec{p},\sigma\rangle$$

$$a^\dagger_{\vec{p}_2,\sigma_2}|\vec{p}_1,\sigma_1\rangle = |\vec{p}_1,\sigma_1,\vec{p}_2,\sigma_2\rangle$$


$a^\dagger_{\vec p,\sigma}|0\rangle = |\vec p,\sigma\rangle$
$a^\dagger_{\vec p_2,\sigma_2}|\vec p_1,\sigma_1\rangle = |\vec p_1,\sigma_1,$
$[a_{\vec p_1,\sigma_1}, a^\dagger_{\vec p_2,\sigma_2}]_{Bosons} = \delta^3(\vec p_1-\vec p_2$
$\{\ ,\ \}_{Fermions}$

$$a^\dagger_{\vec{p},\sigma}|0\rangle = |\vec{p},\sigma\rangle$$

$$a^\dagger_{\vec{p}_2,\sigma_2}|\vec{p}_1,\sigma_1\rangle = |\vec{p}_1,\sigma_1,\vec{p}_2,\sigma_2\rangle$$

$$\left[a_{\vec{p}_1,\sigma_1}, a^\dagger_{\vec{p}_2,\sigma_2}\right]_{\text{Bosons}} = \delta^3(\vec{p}_1-\vec{p}_2)\,\delta_{\sigma_1,\sigma_2}$$

$$\{\ ,\ \}_{\text{Fermions}}$$

$$a^{\dagger}_{\vec{p},\sigma} |0\rangle = |\vec{p},\sigma\rangle$$

$$a^{\dagger}_{\vec{p}_2,\sigma_2} |\vec{p}_1,\sigma_1\rangle = |\vec{p}_1,\sigma_1,\vec{p}_2,\sigma_2\rangle$$

$$\left[a_{\vec{p}_1,\sigma_1} , a^{\dagger}_{\vec{p},\sigma_2} \right]_{\text{Bosons}} = \delta^3(\vec{p}_1-\vec{p}_2)\, \delta_{\sigma_1,\sigma_2}$$

$$\{ \quad , \quad \}_{\text{Fermions}}$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma}\right)$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma}\right)$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma}\right)$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma} \right)$$

$$\int d^3x\ \mathcal{H}(x)$$

$$\int d^3x\, d^3y\ \mathcal{H}(x,y)$$

$$H\left(a_{\vec{p},\sigma} \; a^{\dagger}_{\vec{p},\sigma} \right)$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma}\right)$$

$$\varphi(\vec{x}) = \int d^3p\ a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}}$$

$$H\left(a_{\vec{p},\sigma}\;\; a^{\dagger}_{\vec{p},\sigma}\right)$$

$$\varphi(\vec{x}) = \int d^3p \;\; a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}}.$$

$$\downarrow$$

$$a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{a}}.$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma} \right)$$

$$\varphi(\vec{x}) = \int d^3p\ a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}}$$

$$\downarrow$$

$$a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{a}}$$

$$\varphi(\vec{x}) \to \varphi(\vec{x}+\vec{a}).$$

$$H\left(a_{\vec{p},\sigma},\ a^{\dagger}_{\vec{p},\sigma} \right)$$

$$\varphi(\vec{x}) = \int d^3p\ a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}}$$

$$\downarrow$$

$$a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{a}}$$

$$\varphi(\vec{x}) \longrightarrow \varphi(\vec{x}+\vec{a})$$

$$H = \int d^3x\ \mathcal{H}\left(\left[\varphi(\vec{x}),\ \varphi^{\dagger}(\vec{x}) \right] \right)$$

$$H\left(a_{\vec{p},\sigma} \quad a^{\dagger}_{\vec{p},\sigma} \right)$$

$$\varphi(\vec{x}) = \int d^3p \; a_{\vec{p}} \, e^{i\vec{p}\cdot\vec{x}}$$

$$\downarrow$$

$$a_{\vec{p}} \, e^{i\vec{p}\cdot\vec{a}}$$

$$H = \int d^3x \; \mathcal{H}\left(\left[\varphi(\vec{x}), \varphi^{\dagger}(\vec{x}) \right] \right)$$

$$H\left(a_{\vec{p},\sigma} \quad a^{\dagger}_{\vec{p},\sigma} \right)$$

$$\varphi(\vec{x}) = \int d^3p \; a_{\vec{p}} \, e^{i\vec{p}\cdot\vec{x}}$$

$$\downarrow$$

$$a_{\vec{p}} \, e^{i\vec{p}\cdot\vec{a}}$$

$$\varphi(\vec{x}) \longrightarrow \varphi(\vec{x}+\vec{a})$$

$$H = \int d^3x \cdot \mathcal{H}\left(\left[\varphi(\vec{x}), \varphi^{\dagger}(\vec{x}) \right] \right)$$

$(\vec{x})$,

$\varphi^\dagger(\vec{x})$

$$H = \sum \frac{\vec{p}^{\,2}}{2m} a^\dagger_{\vec{p}} a_{\vec{p}}$$

$$H = \sum \frac{\vec{p}^{\,2}}{2m} a^{\dagger}_{\vec{p}} a_{\vec{p}}$$

$$= \int d^3x \frac{(\nabla\phi)^{\dagger}(\nabla\phi)}{2m}$$

$(\vec{x})$,

$\varphi^{\dagger}(\vec{x})$

$(\vec{x}),$

$\varphi^\dagger(\vec{x})$

$$H = \sum \frac{\vec{p}^{\,2}}{2m} a^\dagger_{\vec{p}} a_{\vec{p}}$$

$$= \int d^3x \, \frac{(\nabla\phi)^\dagger(\nabla\phi)}{2m}$$

$(\vec{x}),$

$\varphi^\dagger(\vec{x})$

$$H = \int d^3p \; \frac{\vec{p}^{\,2}}{2m} \; a^\dagger_{\vec{p}} a_{\vec{p}}$$

$$= \int d^3x \; \frac{(\nabla\phi)^\dagger (\nabla\phi)}{2m}$$

$$H = \int d^3p \, \frac{\vec{p}^{\,2}}{2m} \, a^\dagger_{\vec{p}} a_{\vec{p}}$$

$$= \int d^3x \, \frac{(\nabla\phi)^\dagger(\nabla\phi)}{2m} - c\left(\phi^\dagger\phi\right)^2$$

$$\varphi = \int d^3p \, e^{i\vec{p}\cdot\vec{x}} \, a_{\vec{p}}$$

$$\varphi^\dagger = \int d^3p \, e^{-i\vec{p}\cdot\vec{x}} \, a^\dagger_{\vec{p}}$$

$$i \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

$$|\psi\rangle = e^{-iH_0 t}|\psi\rangle_I$$

$$H = H_0 + H_{int}$$

$$i \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

$$|\psi\rangle = e^{-iH_0 t}|\psi\rangle_I$$

$$i\hbar \frac{d|\psi\rangle_I}{dt} = H^I_{int}(t)\,|\psi\rangle_I(t)$$

$$\downarrow$$

$$e^{-iH_0 t} H_{int}\, e^{iH_0 t}$$

$$H = H_0 + H_{int}$$

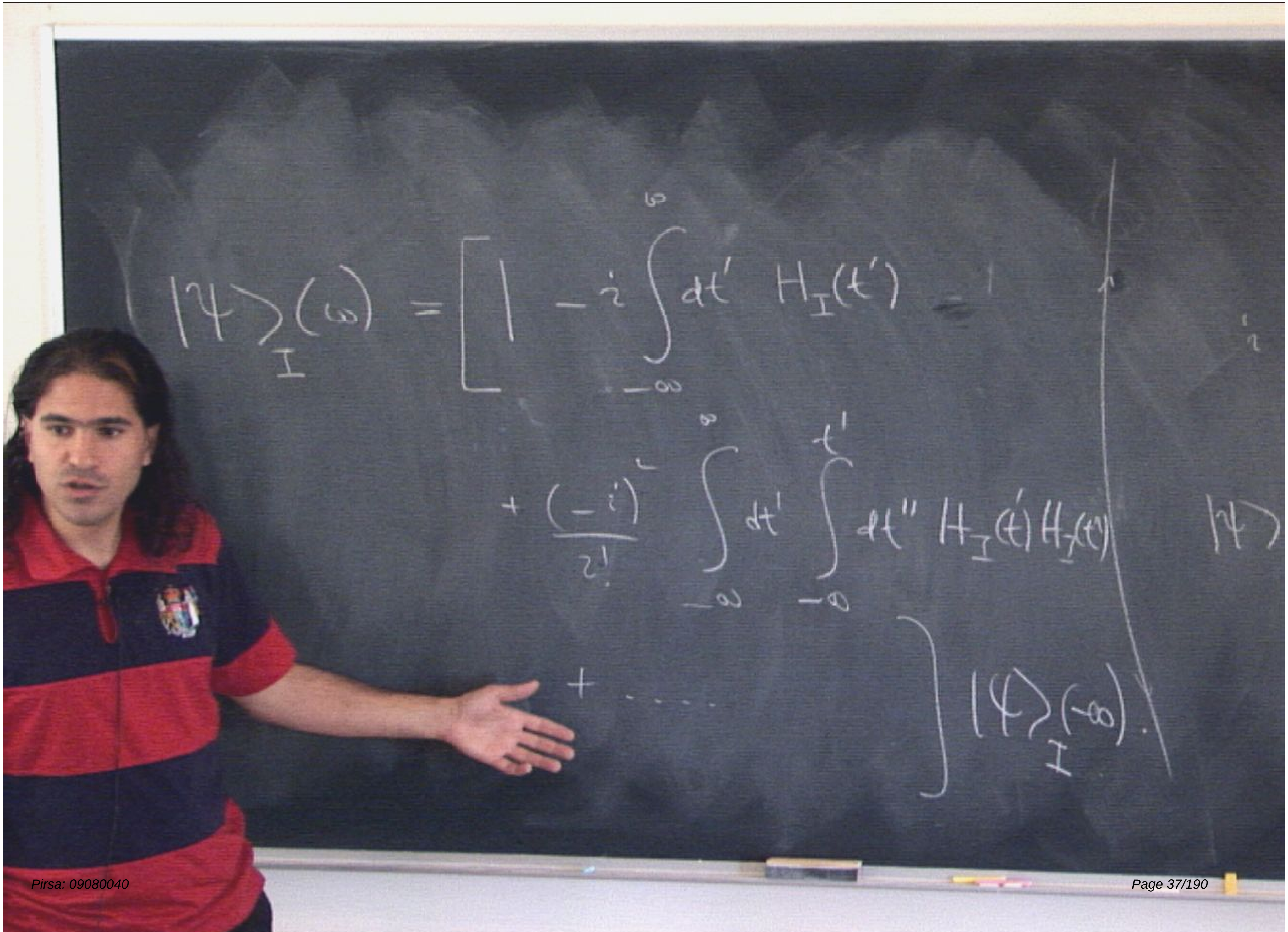
$$i \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

$$|\psi\rangle = e^{-iH_0 t}|\psi\rangle_I$$

$$i\hbar \frac{d|\psi\rangle_I}{dt} = H^I_{int}(t)\,|\psi\rangle_I(t)$$

$$\downarrow$$

$$e^{-iH_0 t} H_{int}\, e^{iH_0 t}$$


$|\psi\rangle_I(\infty) = \Big[1 - i\int_{-\infty}^{\infty} dt' \, H_I(t')$
$+ \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{t'} dt'' \, H_I(t') H_I(t'')$
$+ \cdots \Big] |\psi\rangle_I(-\infty).$
$|\psi\rangle$

$$|\psi\rangle_I(\infty) = \Big[1 - i\int_{-\infty}^{\infty} dt'\, H_I(t') + \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{t'} dt''\, H_I(t')\,H_I(t'') + \cdots \Big]\, |\psi\rangle_I(-\infty).$$

i

$|\psi\rangle$

$$H = H_0 + H_{int}$$

$$\frac{\psi\rangle}{t} = H|\psi\rangle$$

$$= e^{-iH_0 t}|\psi\rangle_I$$

$$i\hbar \frac{d|\psi\rangle_I}{dt} = H^I_{int}(t)\,|\psi\rangle_I(t)$$

$$|\psi\rangle_I(\infty) = \Big[1 - i\int_{-\infty}^{\infty} dt'\, H_I(t') + (-i)^2 \int_{-\infty}^{\infty} dt' \int_{-\infty}^{t'} dt''\, H_I(t')\, H_I(t'') + \cdots \Big] |\psi\rangle_I(-\infty).$$

$$|\psi_I(\infty)\rangle = T\, e^{-i\int_{-\infty}^{\infty} dt'\, H^I_{int}(t')}\, |\psi\rangle(-\infty)$$

$$T\, A(t_1)\, B(t_2) \equiv \begin{cases} A(t_1)\, B(t_2) & \text{if } t_1 > t_2 \\ B(t_2)\, A(t_1) & \text{if } t_2 > t_1 \end{cases}$$

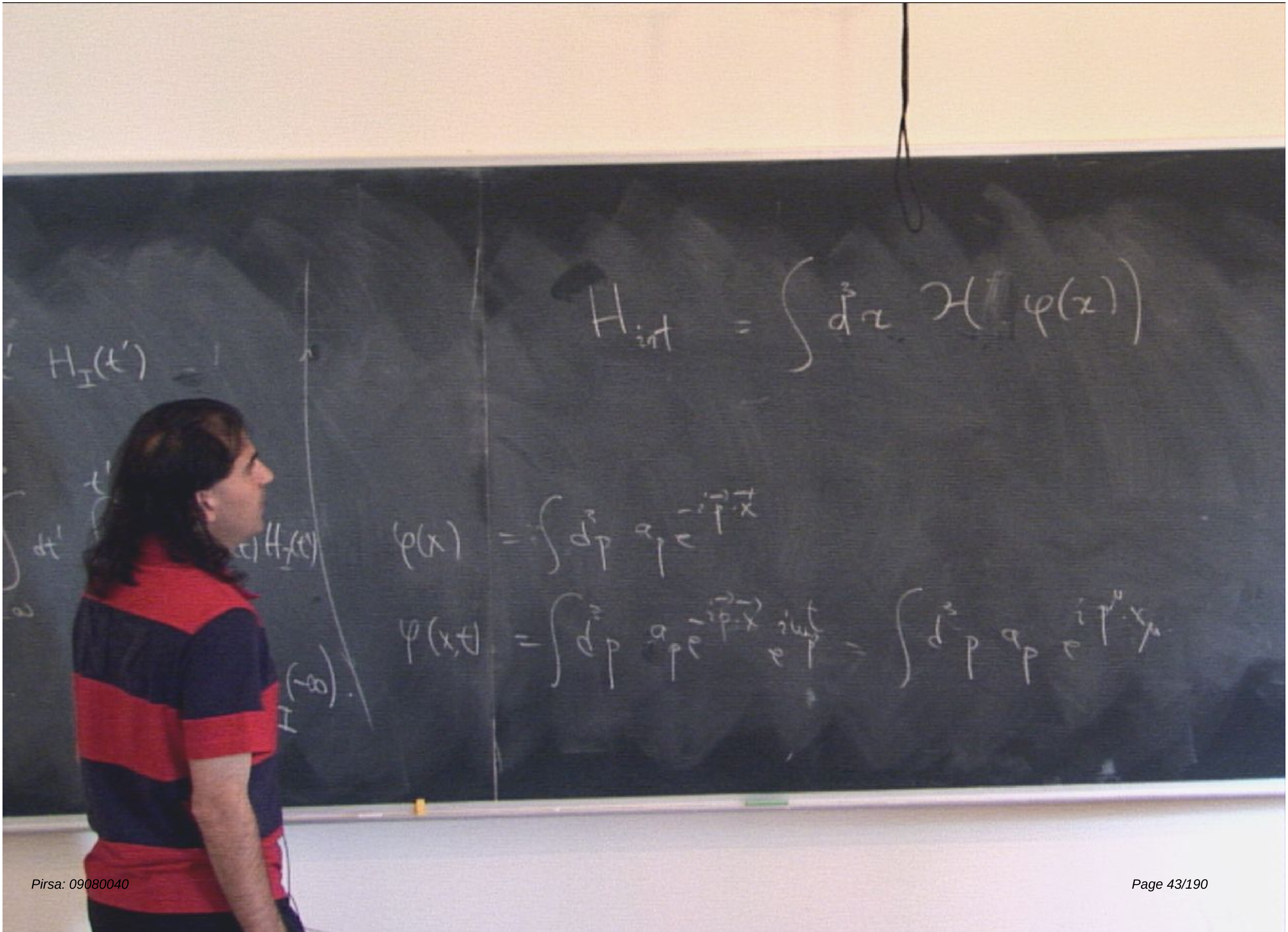
$$H_{int} = \int d^3x \, \mathcal{H}(t) \varphi(x$$

" $H_I(t') H_I(t'')$

$|\psi\rangle_I(-\infty)$

$$\varphi(x) = \int d^3p \, a_p e^{-i\vec{p}\cdot\vec{x}}$$

$$\varphi(x,t) = \int d^3p \, a_p e^{-i\vec{p}\cdot\vec{x}} e^{i\omega_p t} = \int d^3p \, a_p e^{i p^\mu}$$


$H_I(t')$
$H_{int} = \int d^3x\, \mathcal{H}(\varphi(x))$
$\varphi(x) = \int d^3p\, a_p e^{-i\vec{p}\cdot\vec{x}}$
$\varphi(x,t) = \int d^3p\, a_p e^{-i\vec{p}\cdot\vec{x}} e^{i\omega_p t} = \int d^3p\, a_p e^{i p^\mu x_\mu}$

$$H_{int} = \int d^3x \; \mathcal{H}(\varphi(x,t))$$

$\vec{x}$

a

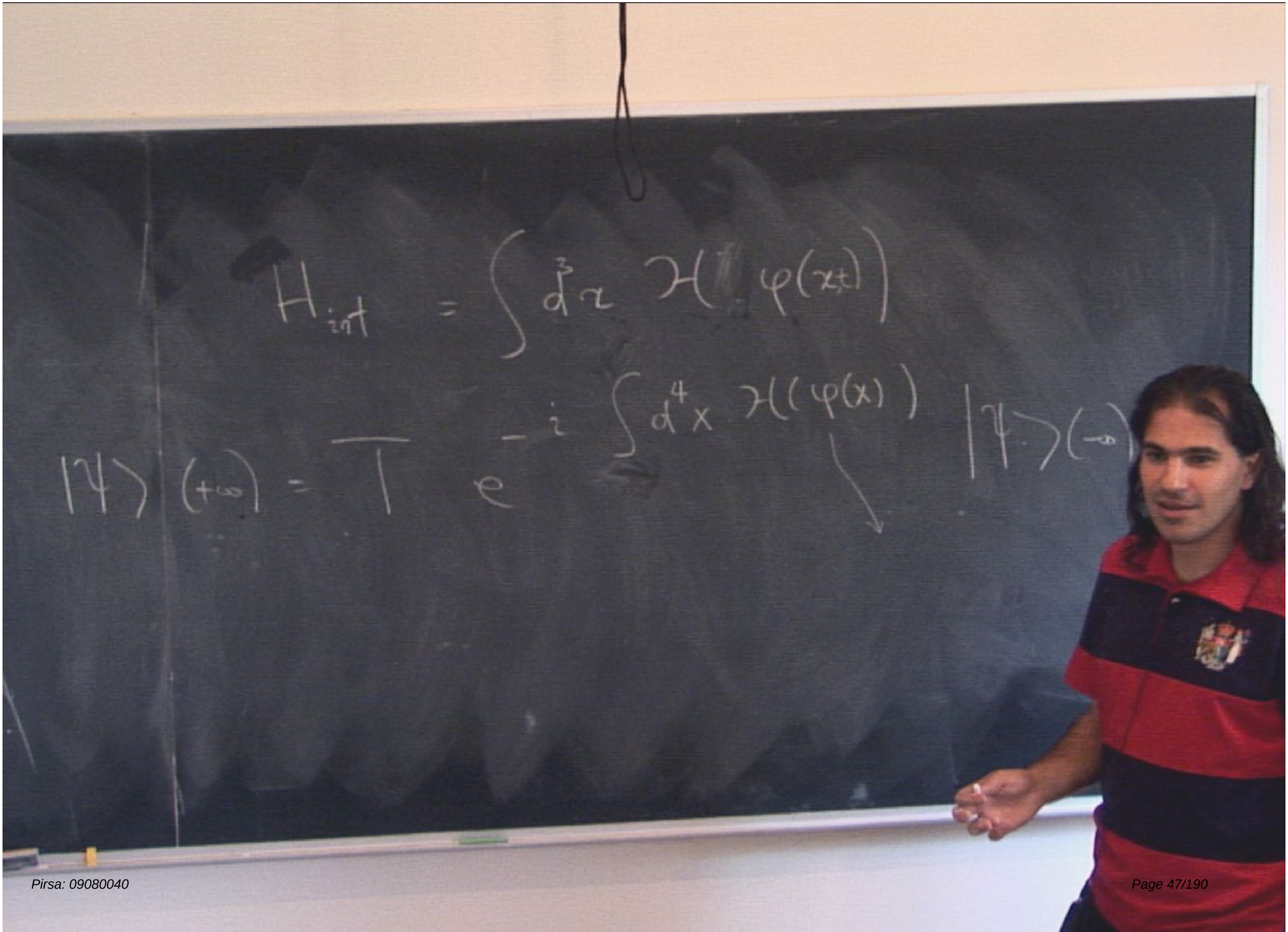
$$d^3p \; a_p \; e^{i p^\mu \cdot x_\mu}$$

$$H_{int} = \int d^3x \, \mathcal{H}(\varphi(x,t))$$

$$|\Psi\rangle(+\infty) = \overline{T} \; e^{-i\int_{-\infty}^{\infty} d^4x \, \mathcal{H}(\varphi(x))}$$

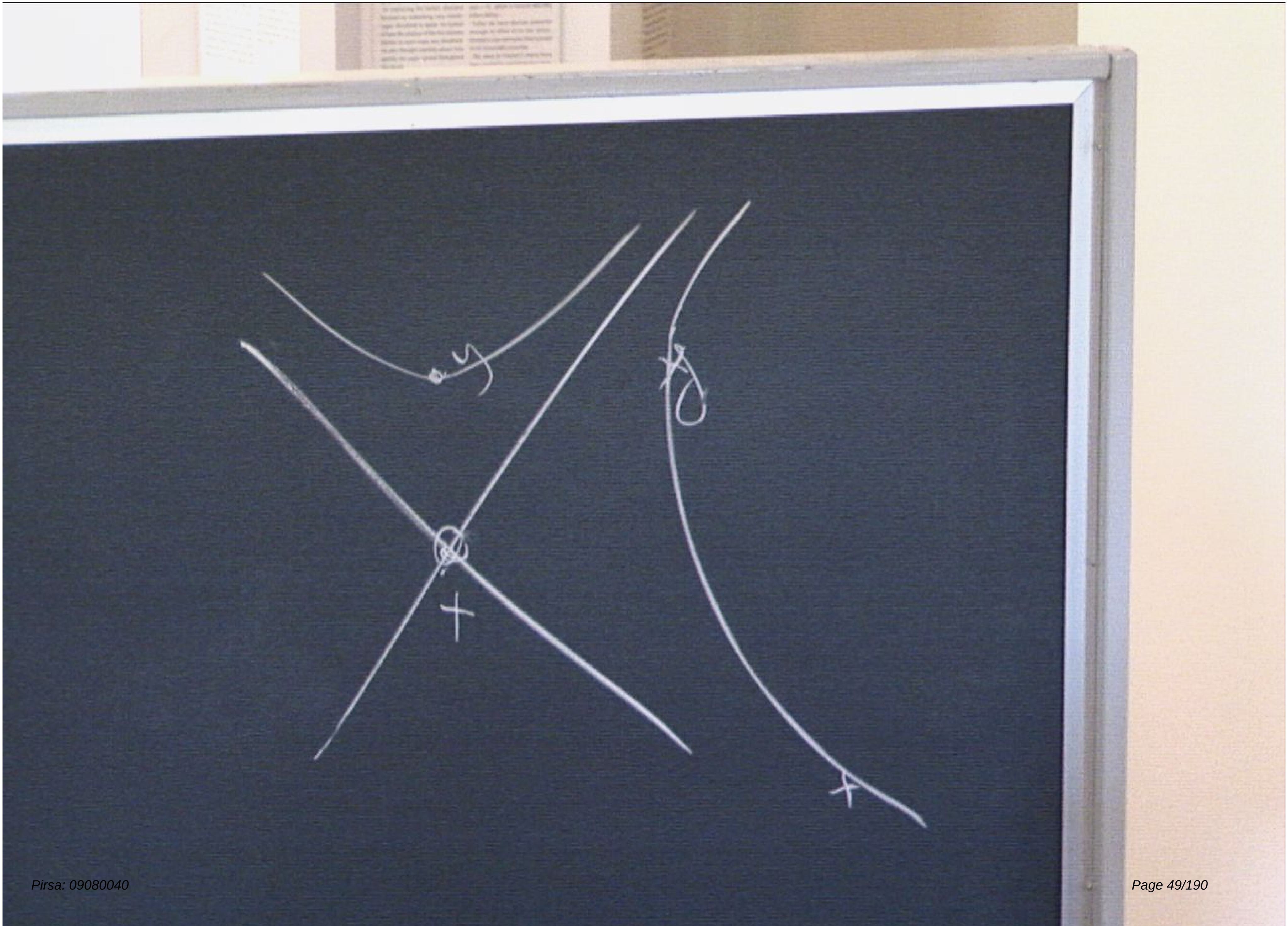
$$H_{int} = \int d^3x \, \mathcal{H}(\varphi(x,t))$$

$$|\psi\rangle(+\infty) = T \, e^{-i\int_{-\infty}^{\infty} d^4x \, \mathcal{H}(\varphi(x))} \, |\psi\rangle(-\infty)$$


$H_{int} = \int d^3x\, \mathcal{H}(\varphi(x,t))$
$|\psi\rangle(+\infty) = T\, e^{-i\int d^4x\, \mathcal{H}(\varphi(x))}\, |\psi\rangle(-\infty)$

$$H_{int} = \int d^3x \, \mathcal{H}(\varphi(x,t))$$

$$|\psi\rangle(+\infty) = T \, e^{-i\int d^4x \, \mathcal{H}(\varphi(x))} \, |\psi\rangle(-\infty)$$



$$H_{int} = \int d^3x\, \mathcal{H}(\varphi(x,t))$$

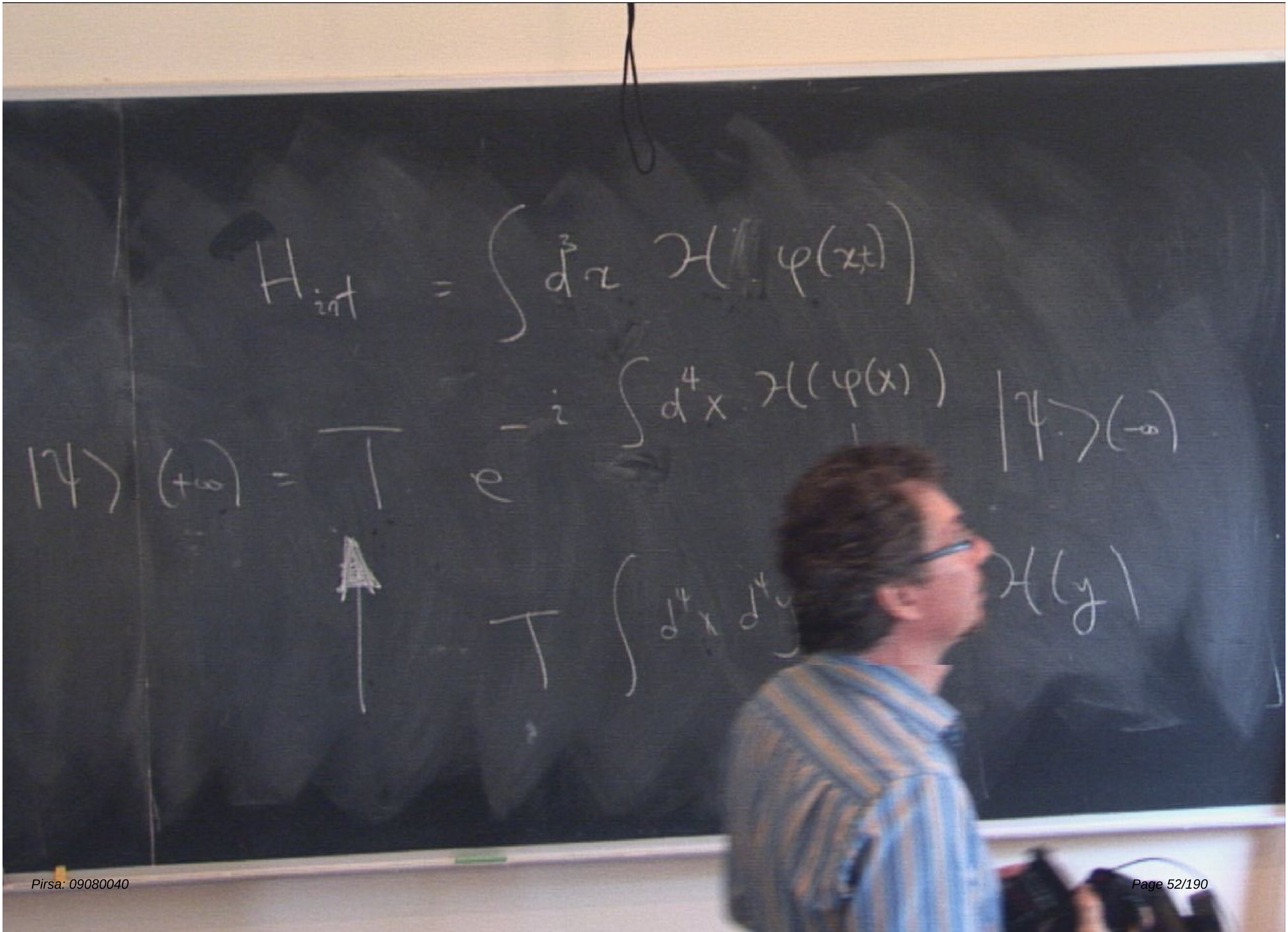
$$|\psi\rangle(+\infty) = T\, e^{-i\int d^4x\, \mathcal{H}(\varphi(x))}\, |\psi\rangle(-\infty)$$

$$T\int d^4x\, d^4y\, \mathcal{H}(x)\, \mathcal{H}(y)$$

$$H_{int} = \int d^3x \, \mathcal{H}(\varphi(x,t))$$

$$|\psi\rangle(+\infty) = T \, e^{-i\int d^4x \, \mathcal{H}(\varphi(x))} \, |\psi\rangle(-\infty)$$

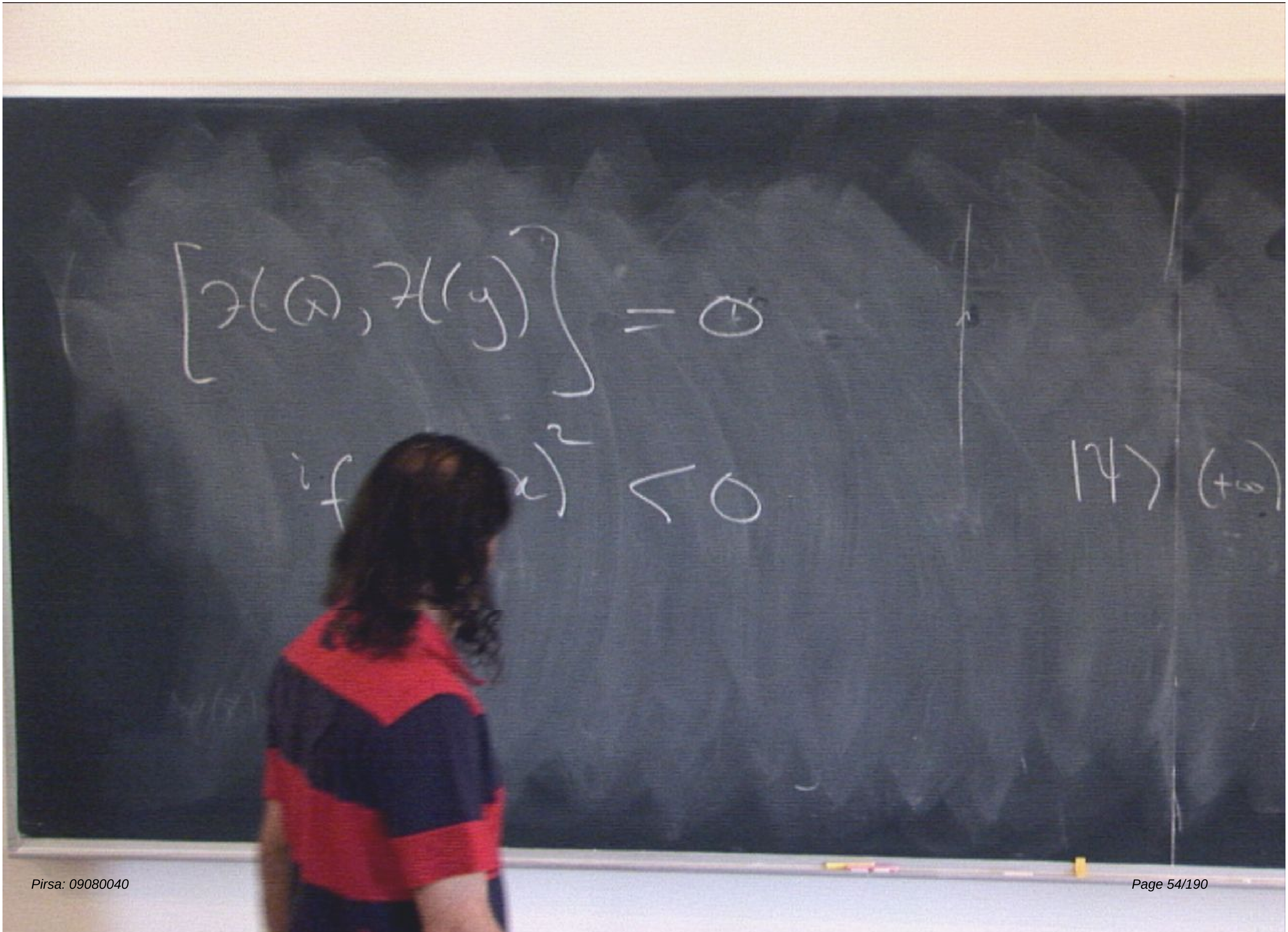
$$T \int d^4x \, d^4y \, \mathcal{H}(x) \, \mathcal{H}(y)$$

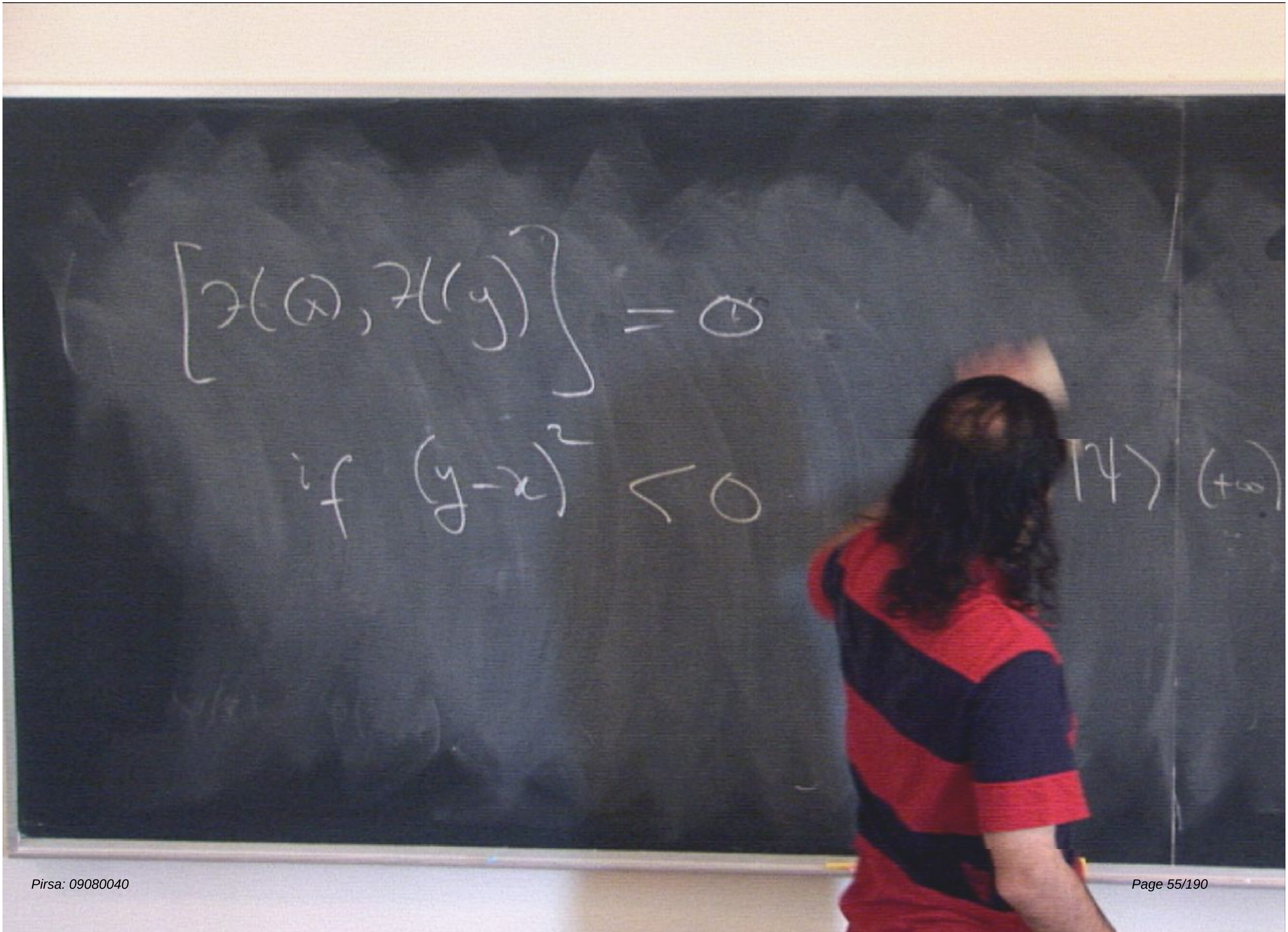

$H_{int} = \int d^3x \, \mathcal{H}(\varphi(x,t))$
$|\psi\rangle(+\infty) = T \, e^{-i\int d^4x \, \mathcal{H}(\varphi(x))} \, |\psi\rangle(-\infty)$
$T \int d^4x \, d^4y$
$\mathcal{H}(y)$

$$H_{int} = \int d^3x\, \mathcal{H}(\varphi(x,t))$$

$$|\psi\rangle(+\infty) = T\, e^{-i\int d^4x\, \mathcal{H}(\varphi(x))}\, |\psi\rangle(-\infty)$$

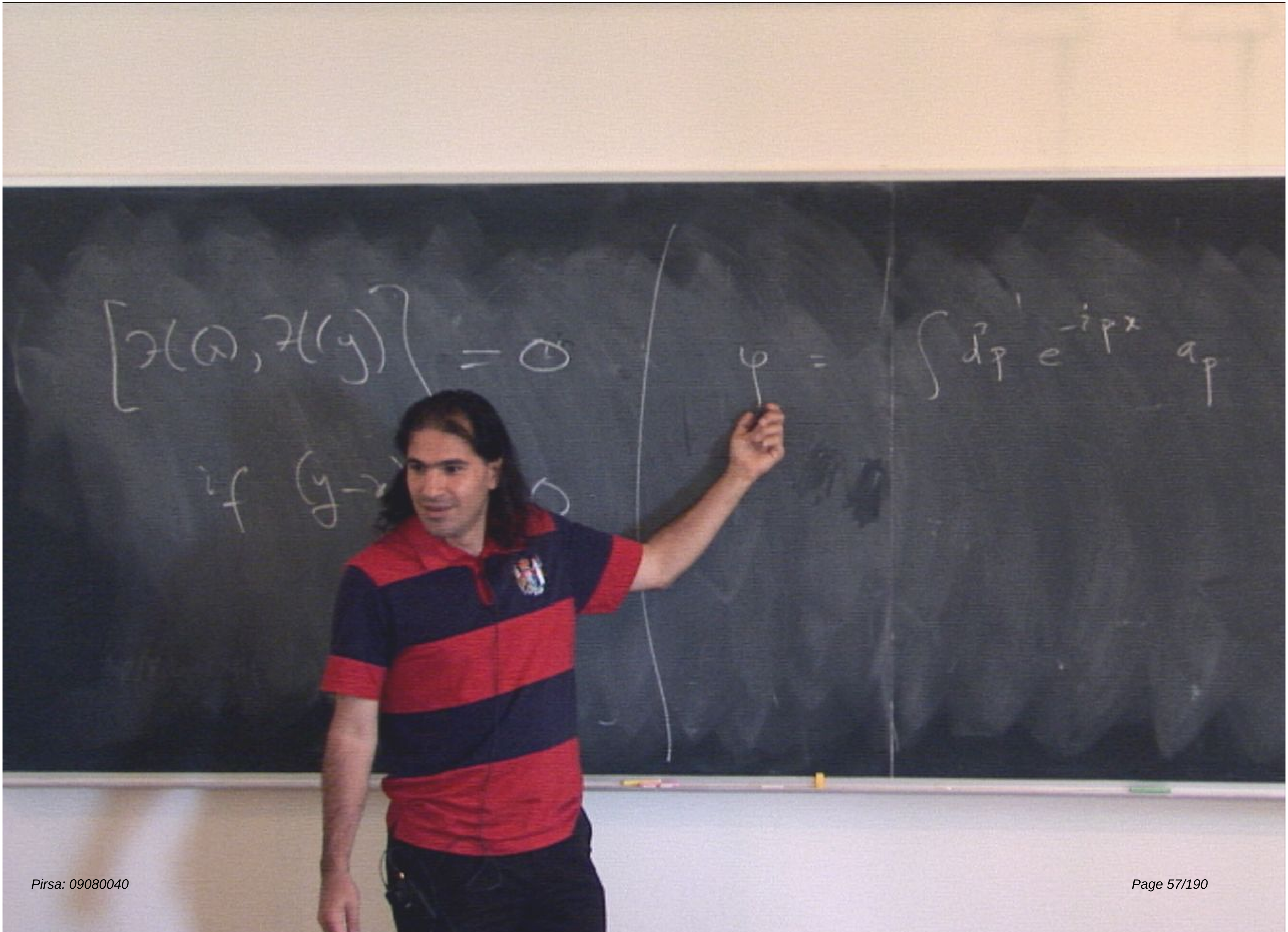
$$T \int d^4x\, d^4y\, \mathcal{H}(x)\, \mathcal{H}(y)$$

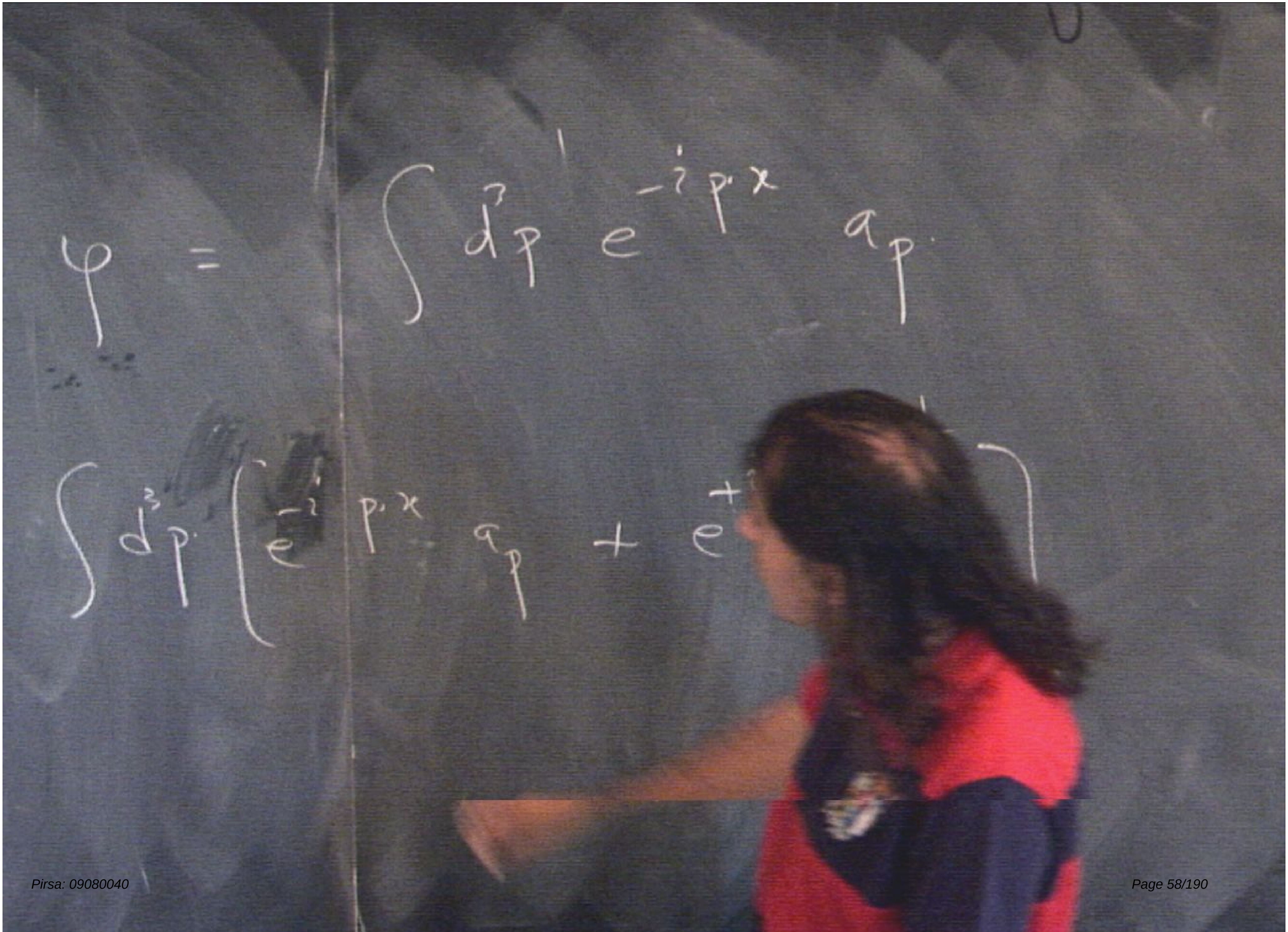

[H(x), H(y)] = 0
x)^2 < 0
|ψ⟩ (+∞)

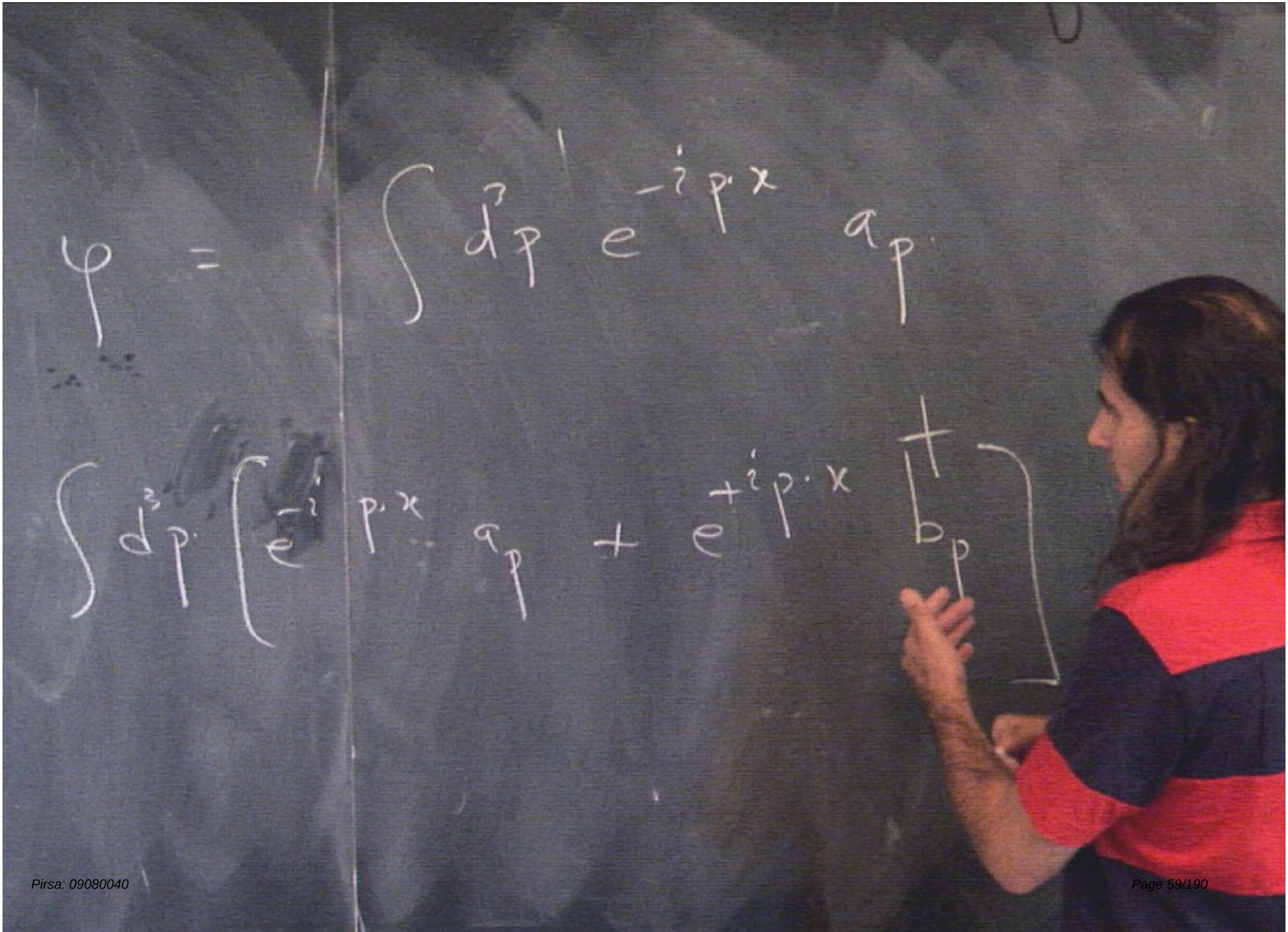

[H(x), H(y)] = 0
if (y−x)^2 < 0
|ψ⟩ (+∞)

$$[\mathcal{H}(x), \mathcal{H}(y)] = 0$$

$$\text{if } (y-x)^2 < 0$$


$[\mathcal{H}(x), \mathcal{H}(y)] = 0$
if $(y-x$
$\varphi = \int d^3p\, e^{-ipx}\, a_p$


$\varphi = \int d^3p \, e^{-ip\cdot x} \, a_p$
$\int d^3p \left(e^{-ip\cdot x} a_p + e^{+} \right)$


$\varphi = \int d^3p \, e^{-ip\cdot x} a_p$
$\int d^3p \left(e^{-ip\cdot x} a_p + e^{+ip\cdot x} b_p^\dagger \right)$

$$\varphi = \int d^3p \; e^{-ip\cdot x} \, a_p$$

$$\int d^3p \left[e^{-ip\cdot x} \, a_p + e^{+ip\cdot x} \, b_p^{\dagger} \right]$$

$$\varphi = \int d^3p \; e^{-ip\cdot x} \, a_p$$

$$\int d^3p \left[e^{-ip\cdot x} \, a_p + e^{+ip\cdot x} \, b_p^{\dagger} \right]$$

$$\varphi = \int d^3p \, e^{-ip\cdot x} \, a_p$$

$$\int d^3p \left[e^{-ip\cdot x} \, a_p + e^{+ip\cdot x} \, b_p^\dagger \right]$$

$$\varphi = \int d^3p \; e^{-ip\cdot x} \, a_p$$

$$\int d^3p \left[e^{-ip\cdot x} \, a_p + e^{+ip\cdot x} \, a_p^\dagger \right]$$

$$\varphi = \int d^3p \; e^{-ip\cdot x} \, a_p$$

$$\int d^3p \left[e^{-ip\cdot x} \, a_p + e^{+ip\cdot x} \, a_p^\dagger \right]$$

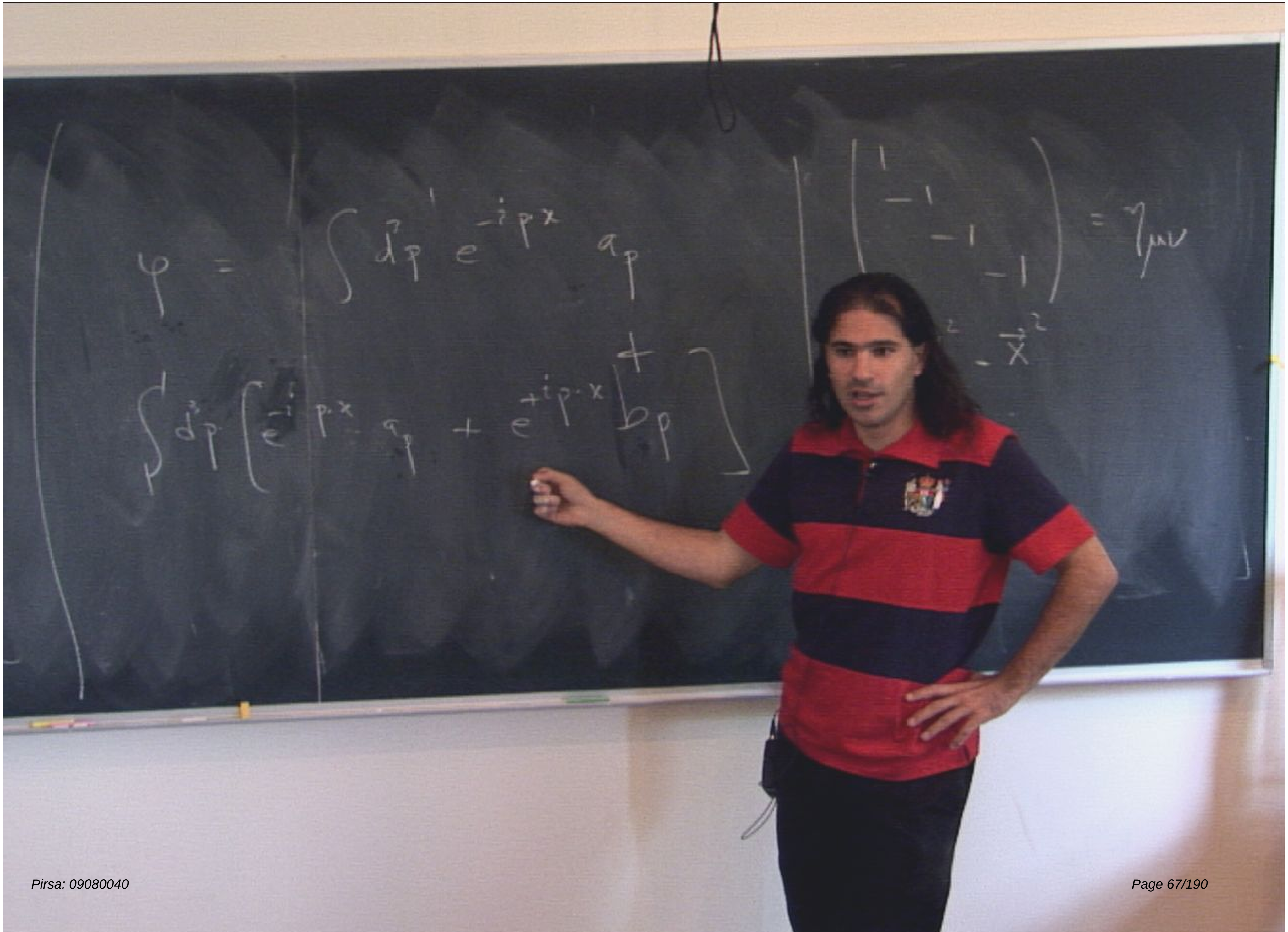
$$\varphi = \int d^3p \; e^{-ip\cdot x} \, a_p$$

$$+$$

$$\int d^3p \left(e^{-ip\cdot x} \, a_p + e^{+ip\cdot x} \, b_p \right)$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \eta_{\mu\nu}$$

$$x^2 = t^2 - \vec{x}^2$$

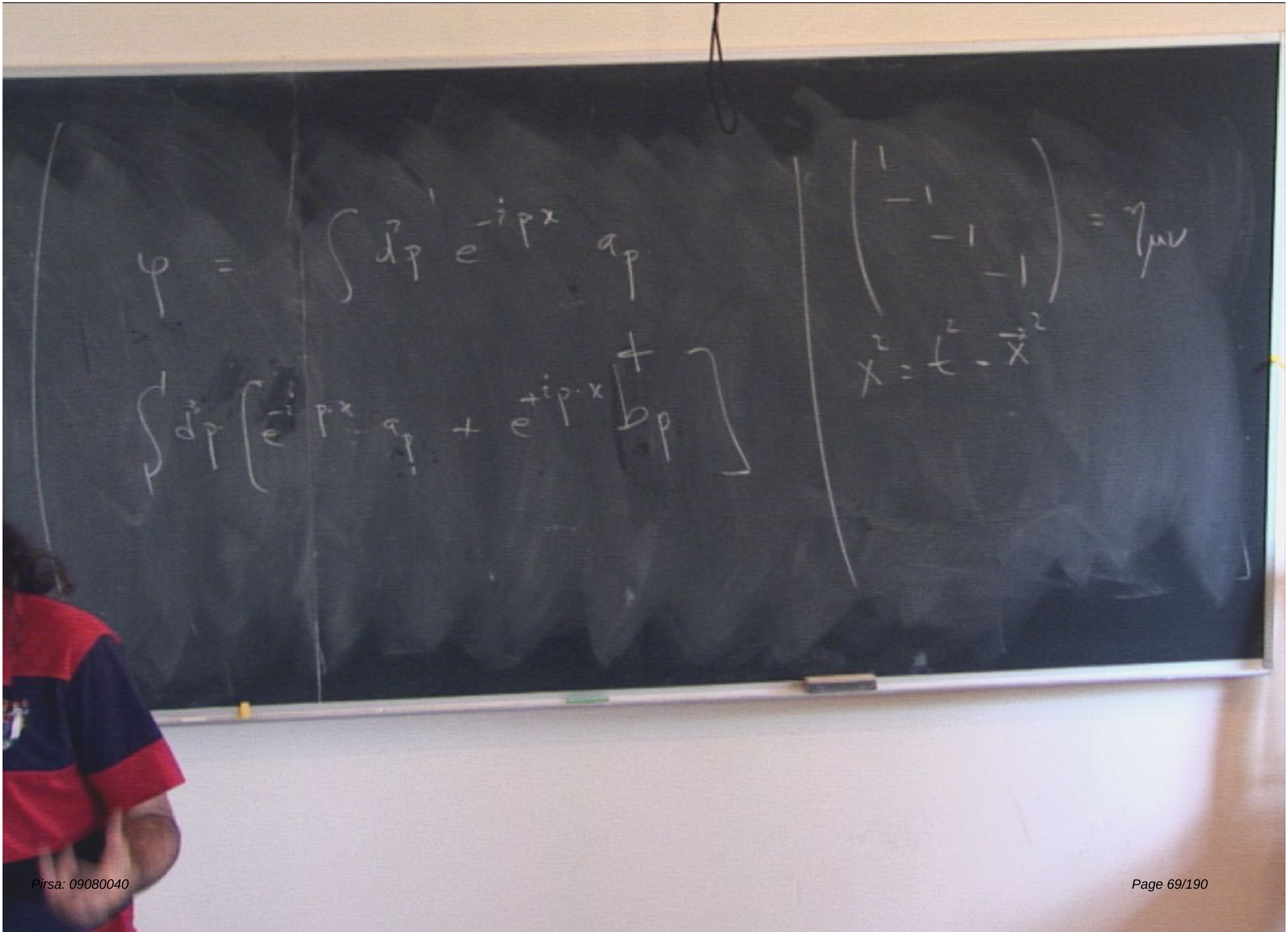

φ = ∫ d³p e^{-ipx} a_p +
∫ d³p [e^{-ip·x} a_p + e^{+ip·x} b_p^†]
(1, −1, −1, −1) = η_μν

$$\varphi = \int d^3p \, e^{-ip\cdot x} a_p$$

$$\int d^3p \left[e^{-ip\cdot x} a_p + e^{+ip\cdot x} b_p^\dagger \right]$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \eta_{\mu\nu}$$

$$x^2 = t^2 - \vec{x}^2$$


$\varphi = \int d^3p \, e^{-ipx} a_p$
$\int d^3p \left[e^{-ip\cdot x} a_p + e^{+ip\cdot x} b_p^\dagger \right]$
$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \eta_{\mu\nu}$
$x^2 = t^2 - \vec{x}^2$

$$\varphi = \int d^3p \, e^{-ipx} a_p$$

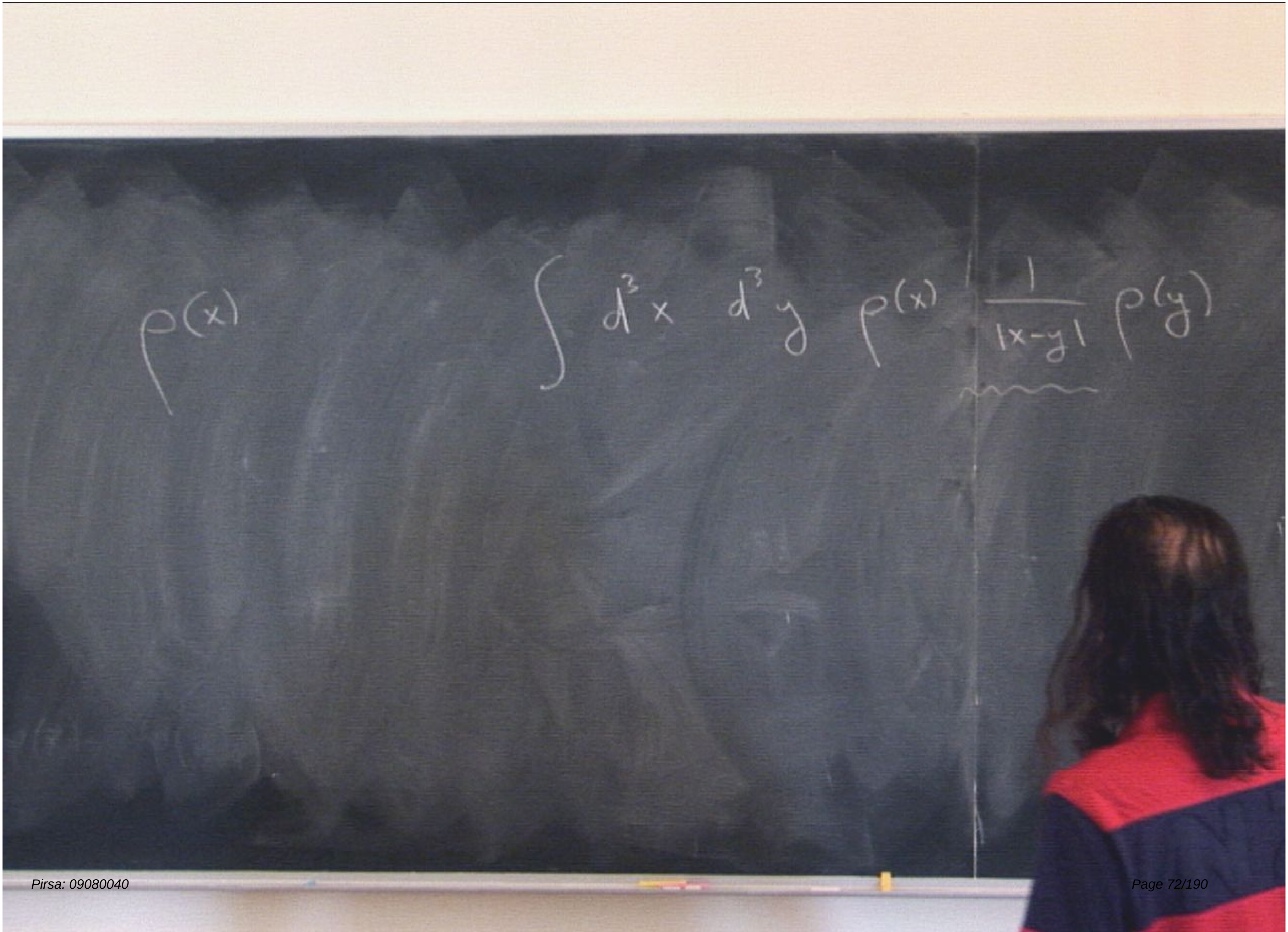
$$\int d^3p \left[e^{-ip\cdot x} a_p + e^{+ip\cdot x} b_p^\dagger \right]$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \eta_{\mu\nu}$$

$$x^2 = t^2 - \vec{x}^2$$

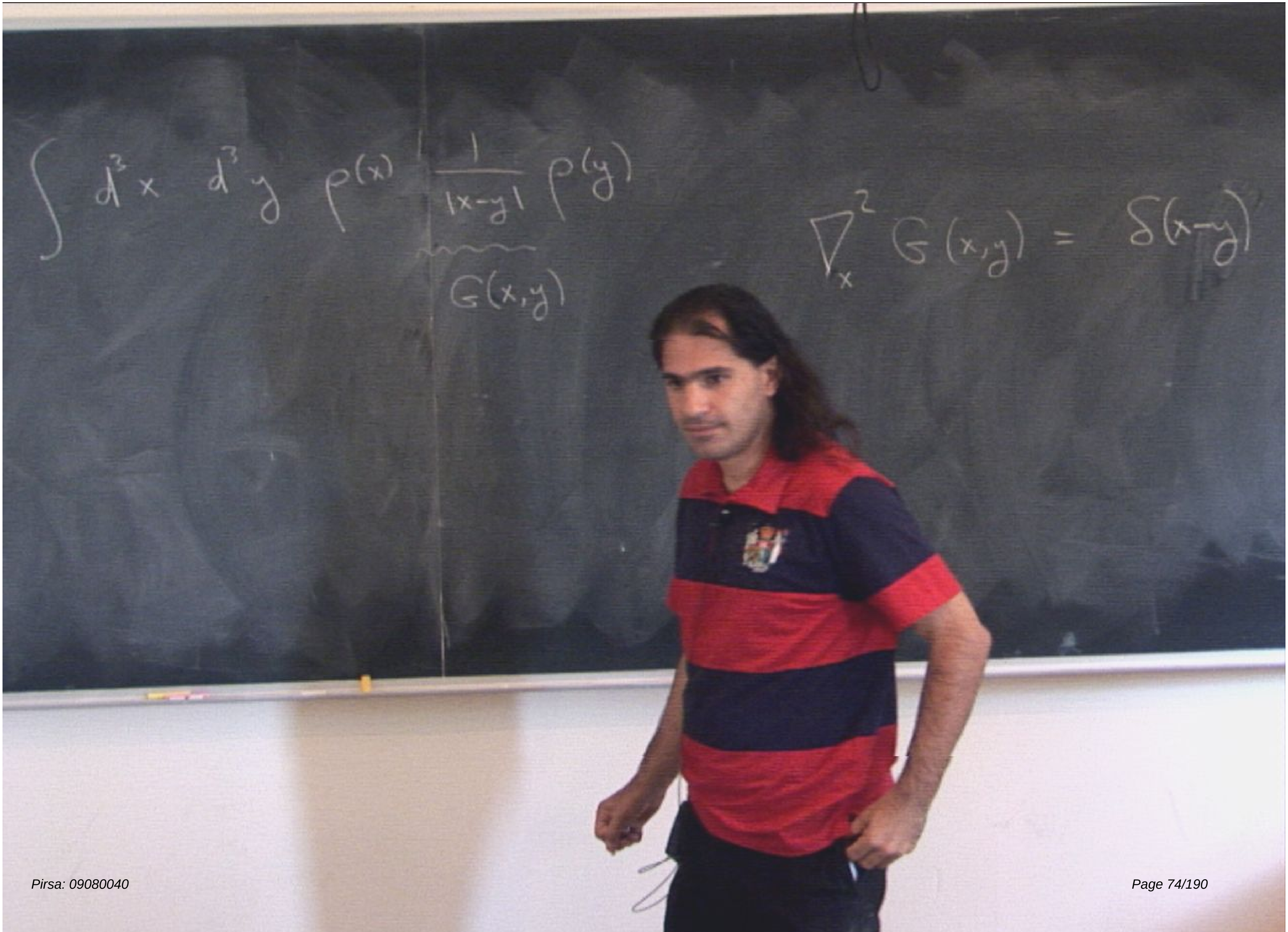
$\rho(x)$

$$\int d^3x \, d^3y \, \rho(x) \frac{1}{|x-y|} \rho(y)$$


$\rho(x)$
$\int d^3x \, d^3y \, \rho(x) \frac{1}{|x-y|} \rho(y)$

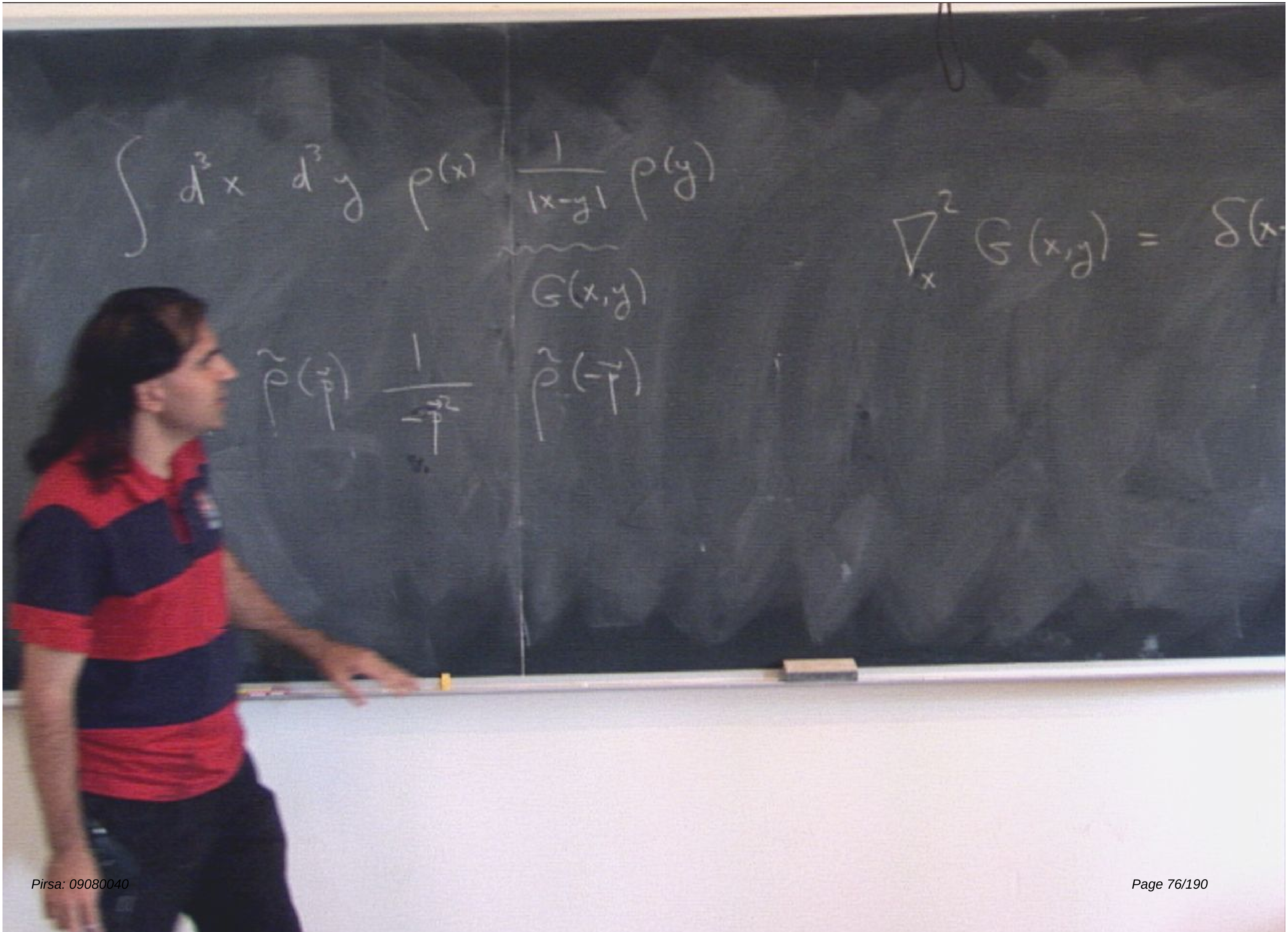
$$\int d^3x \; d^3y \; \rho(x) \underbrace{\frac{1}{|x-y|}}_{G(x,y)} \rho(y)$$

$$\nabla_x^2 G(x,y) = \delta(x-y)$$


$\int d^3x \; d^3y \; \rho(x) \underbrace{\frac{1}{|x-y|}}_{G(x,y)} \rho(y)$
$\nabla_x^2 G(x,y) = \delta(x-y)$

$$\int d^3x \; d^3y \; \rho(x) \underbrace{\frac{1}{|x-y|}}_{G(x,y)} \rho(y)$$

$$\nabla_x^2 \, G(x,y) = \delta(x-y)$$


$\int d^3x \, d^3y \, \rho(x) \frac{1}{|x-y|} \rho(y)$
$G(x,y)$
$\tilde{\rho}(\vec{p}) \frac{1}{-\vec{p}^2} \tilde{\rho}(-\vec{p})$
$\nabla_x^2 G(x,y) = \delta(x$

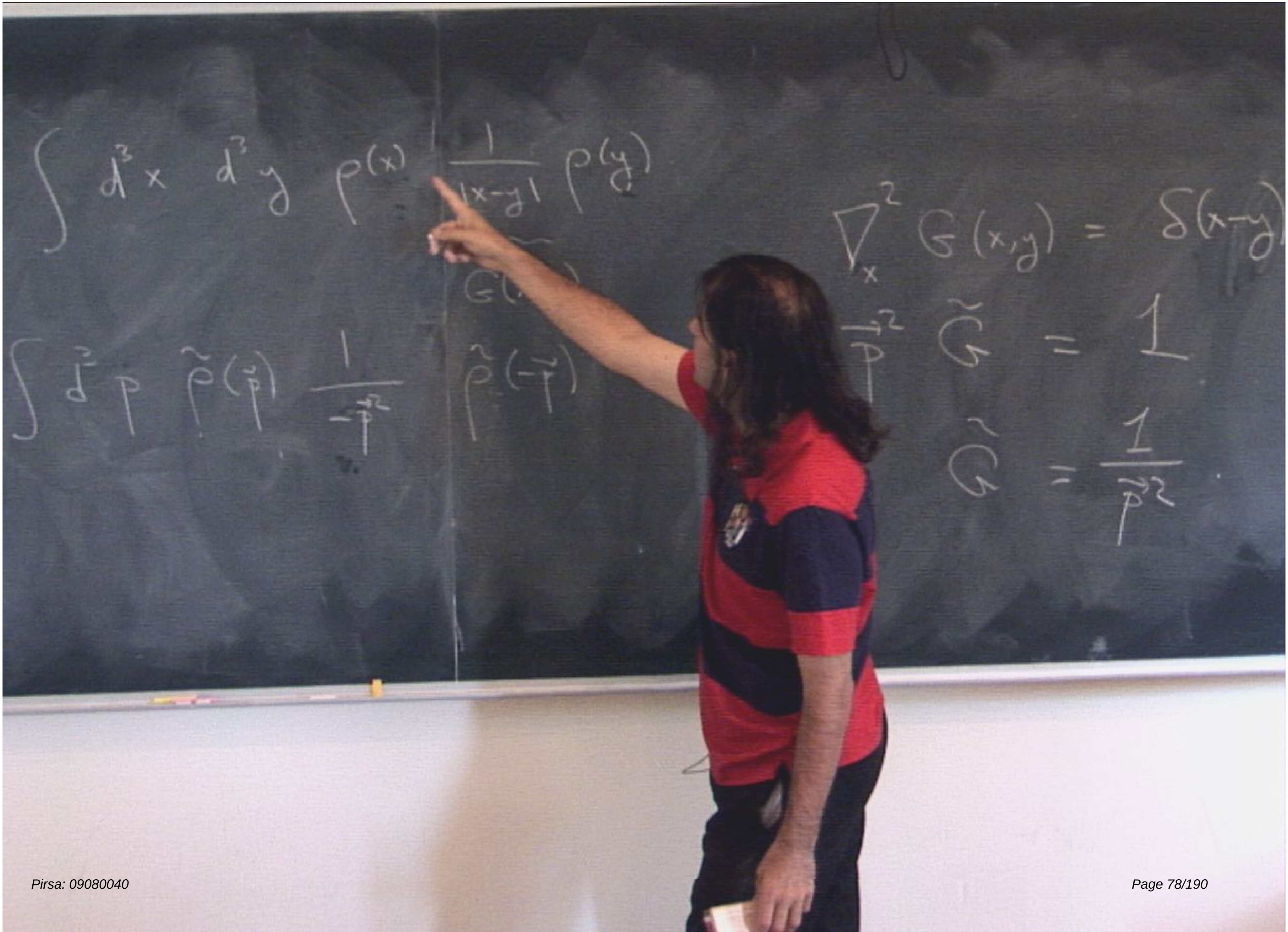
$$\int d^3x \; d^3y \; \rho(x) \underbrace{\frac{1}{|x-y|}}_{G(x,y)} \rho(y)$$

$$\int d^3p \; \tilde{\rho}(\vec{p}) \frac{1}{-\vec{p}^2} \tilde{\rho}(-\vec{p})$$

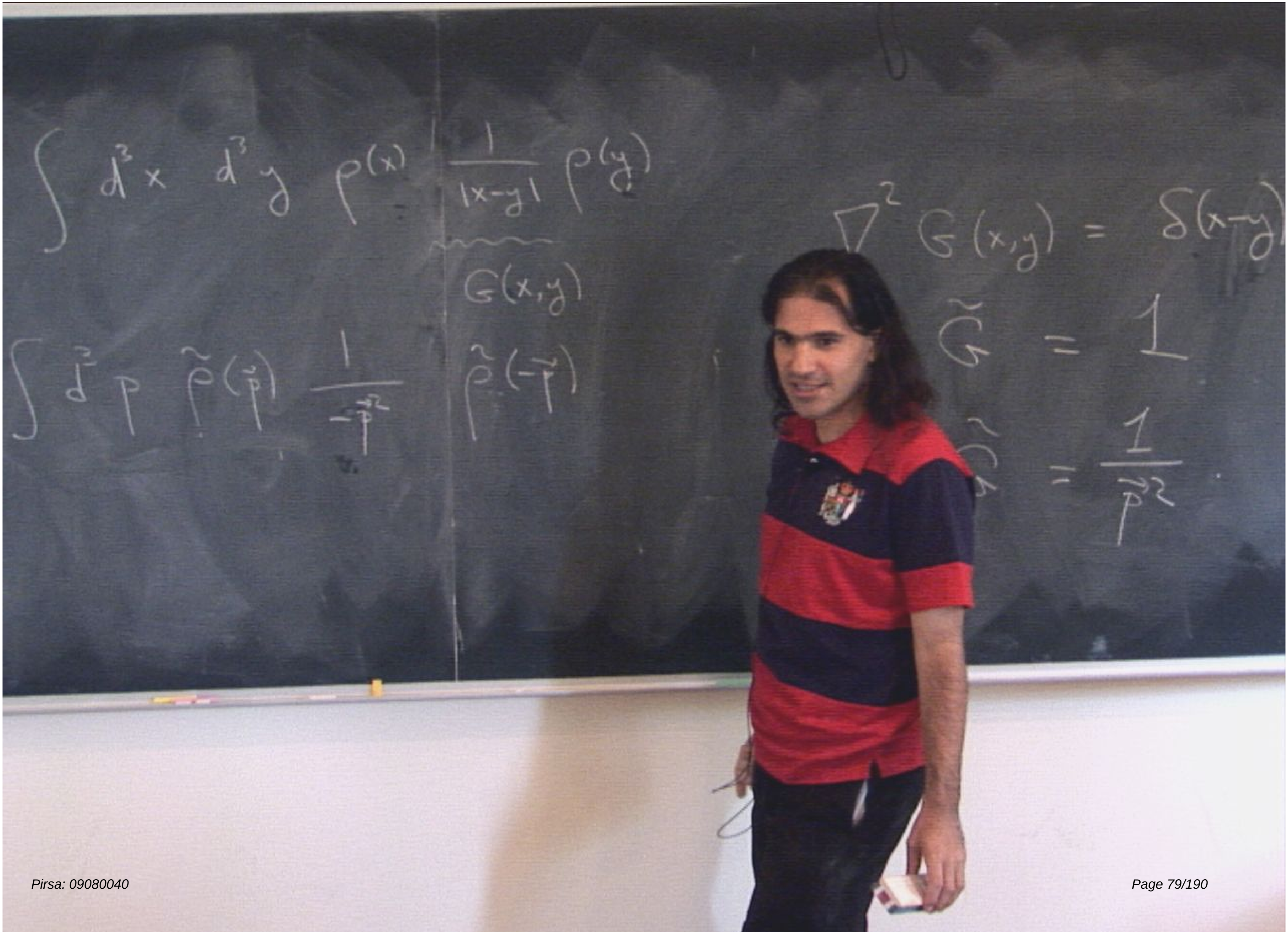
$$\nabla_x^2 \, G(x,y) = \delta(x-y)$$

$$\vec{p}^2 \, \tilde{G} = 1$$

$$\tilde{G} = \frac{1}{\vec{p}^2} \, .$$



$\int d^3x \, d^3y \, \rho(x) \frac{1}{|x-y|} \rho(y)$
$\nabla_x^2 G(x,y) = \delta(x-y)$
$\int d^3p \, \tilde{\rho}(\vec{p}) \frac{1}{-\vec{p}^2} \tilde{\rho}(-\vec{p})$
$\vec{p}^2 \tilde{G} = 1$
$\tilde{G} = \frac{1}{\vec{p}^2}$



$\int d^3x \; d^3y \; \rho(x) \underbrace{\frac{1}{|x-y|}}_{G(x,y)} \rho(y)$
$\int d^3p \; \tilde{\rho}(\vec{p}) \frac{1}{-\vec{p}^2} \tilde{\rho}(-\vec{p})$
$\nabla^2 G(x,y) = \delta(x-y)$
$\tilde{G} = 1$
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$$\int d^3x \; d^3y \; \rho(x) \underbrace{\frac{1}{|x-y|}}_{G(x,y)} \rho(y)$$

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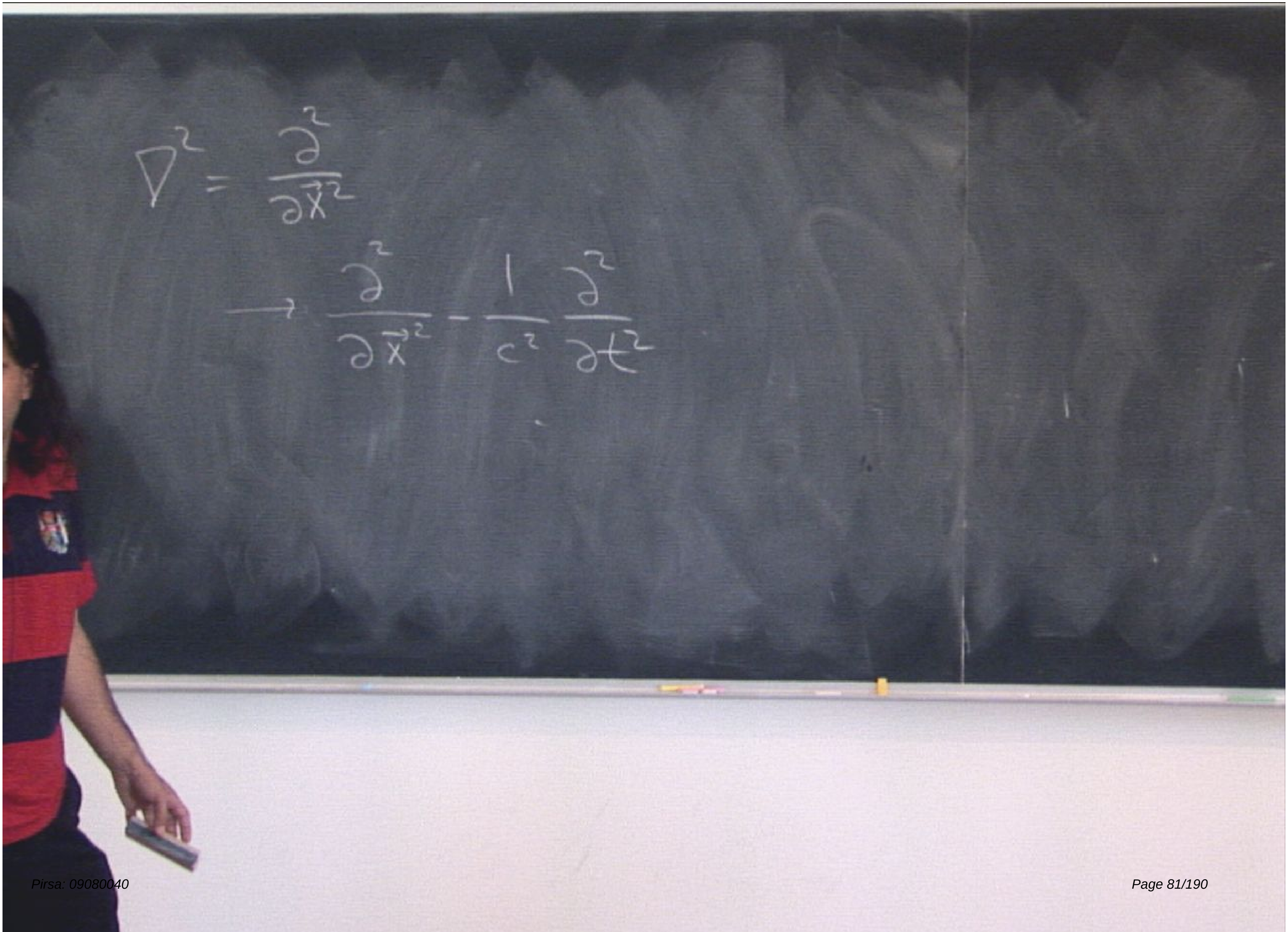
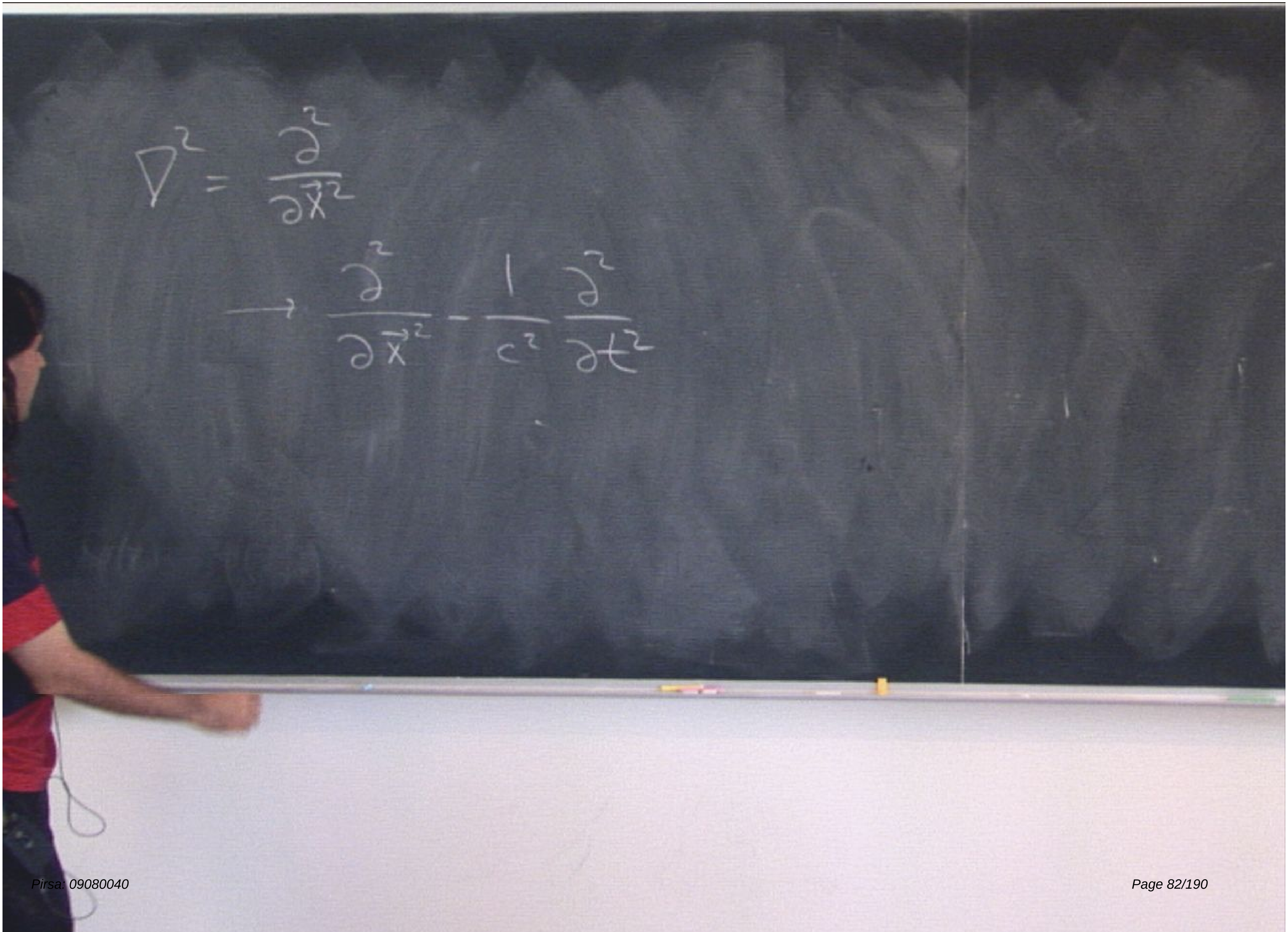
$$\nabla_x^2 G(x,y) = \delta(x-y)$$

$$\vec{p}^2 \tilde{G} = 1$$

$$\tilde{G} = \frac{1}{\vec{p}^2}$$


$\nabla^2 = \frac{\partial^2}{\partial \vec{x}^2}$
$\longrightarrow \frac{\partial^2}{\partial \vec{x}^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$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$$\vec{x}^2$$

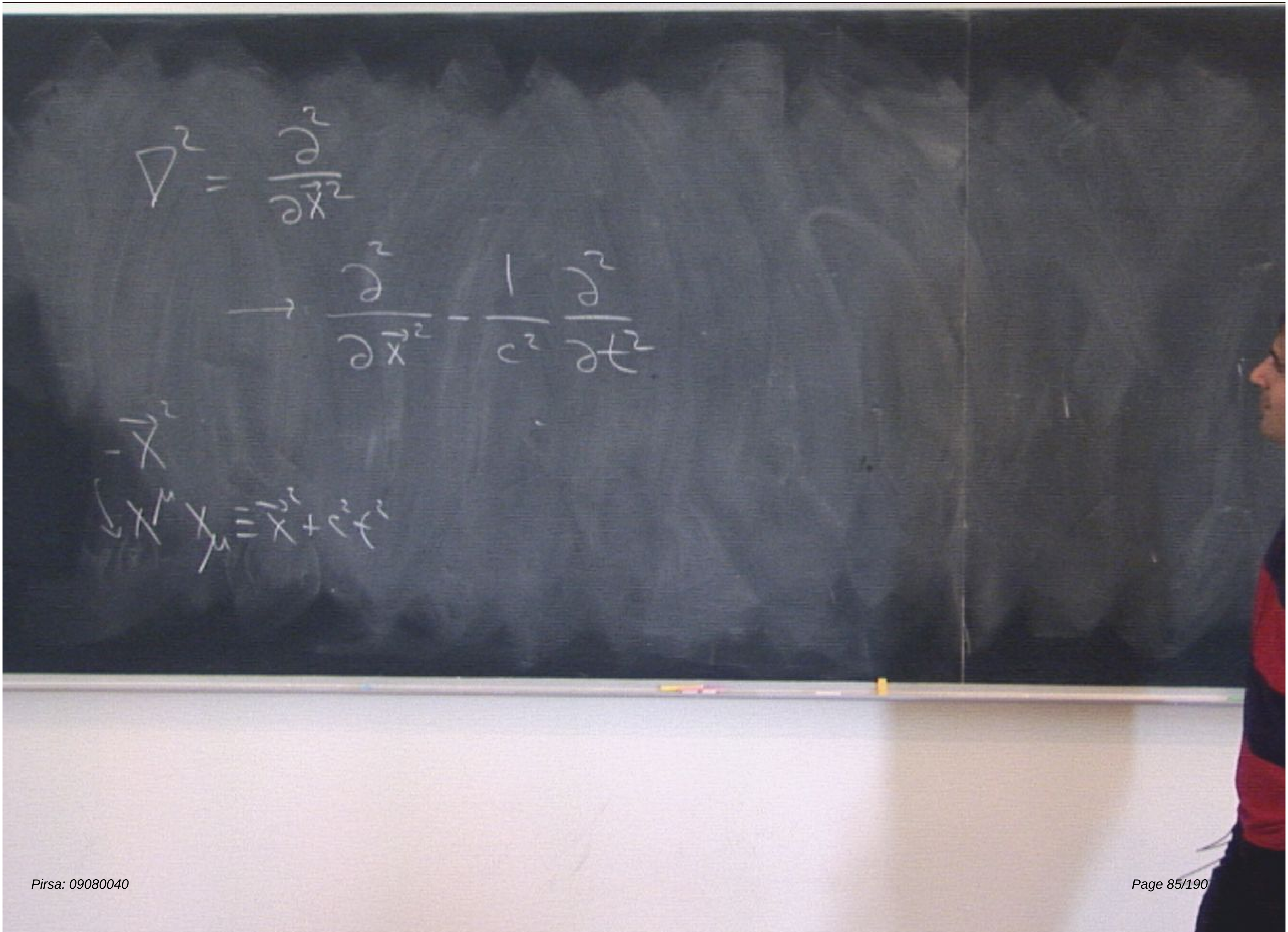
$$x^\mu x_\mu = \vec{x}^2 - c^2 t^2$$

$$\nabla^2 = \frac{\partial^2}{\partial \vec{x}^2}$$

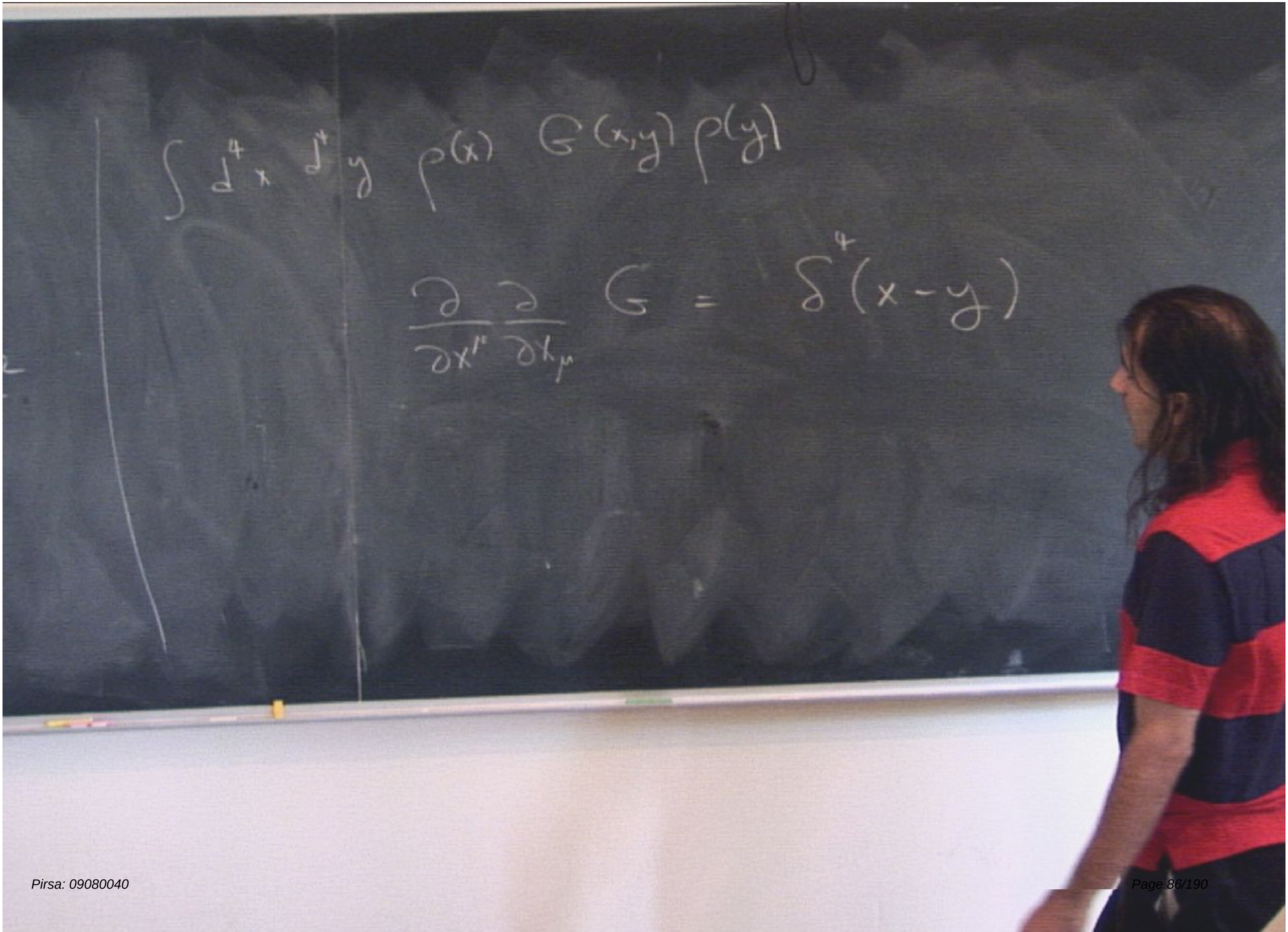
$$\longrightarrow \frac{\partial^2}{\partial \vec{x}^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

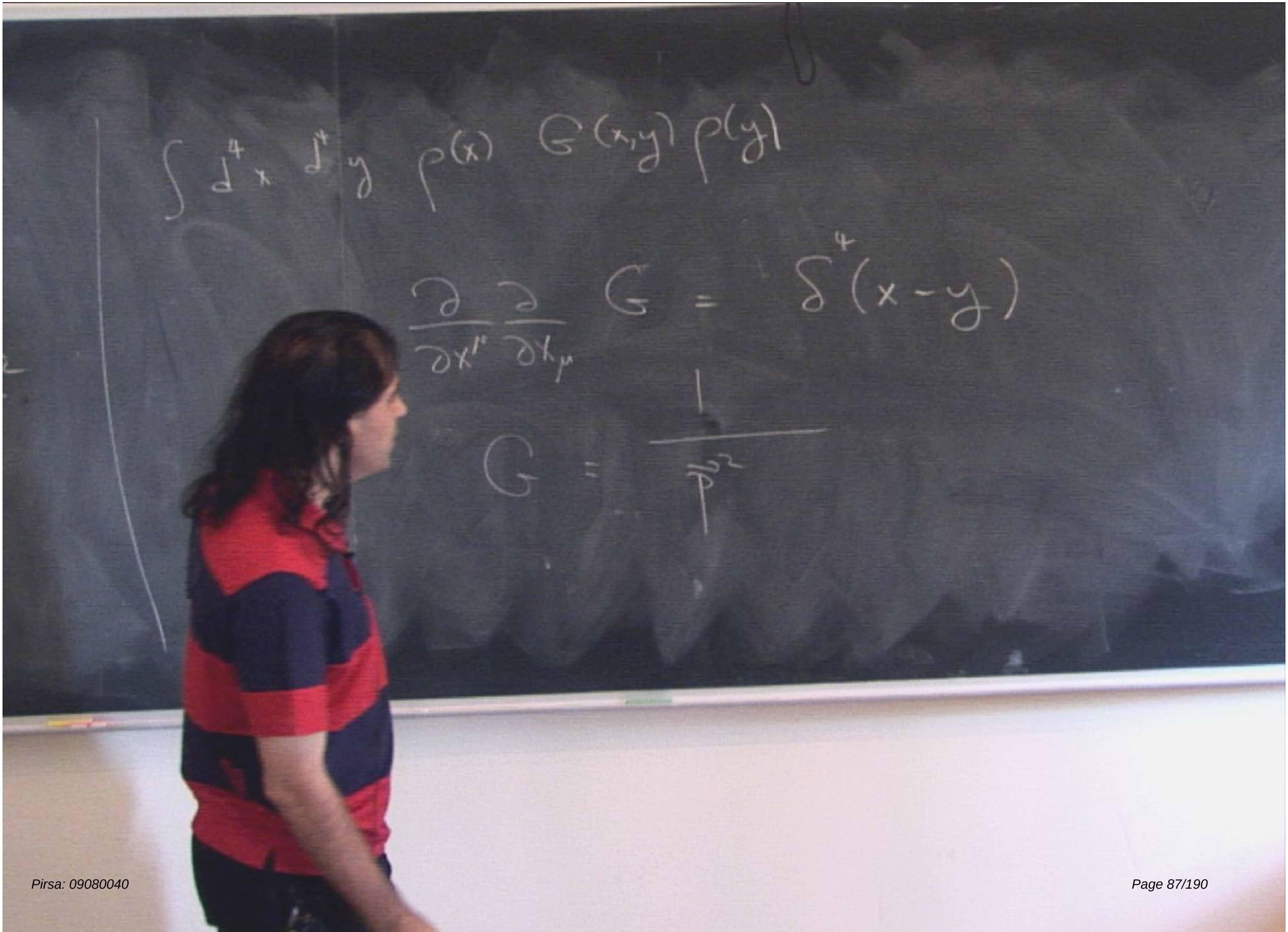
$$\vec{x}^2$$

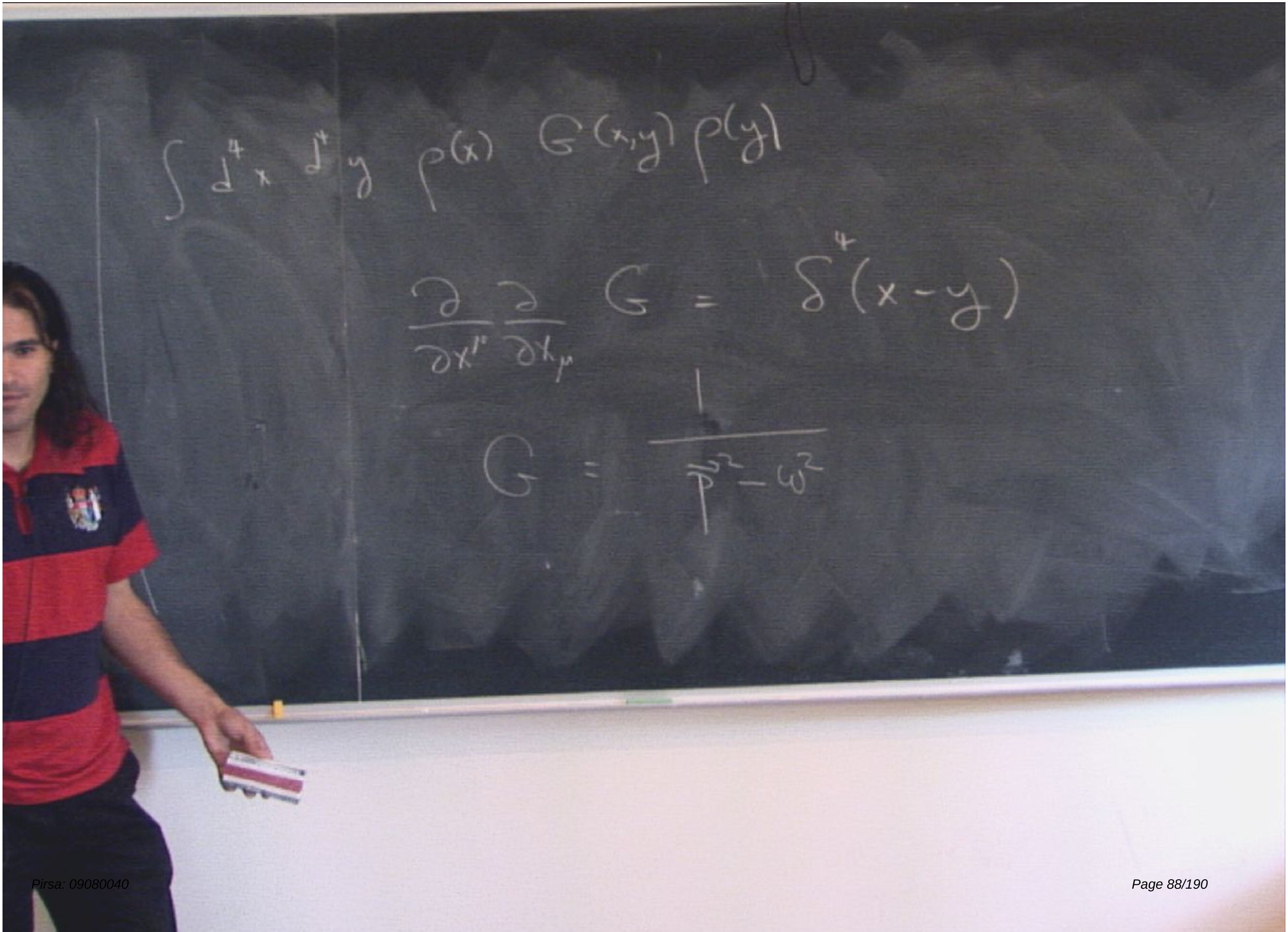
$$x^\mu x_\mu = \vec{x}^2 - c^2 t^2$$



$\nabla^2 = \frac{\partial^2}{\partial \vec{x}^2}$
$\longrightarrow \frac{\partial^2}{\partial \vec{x}^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$
$-\vec{x}^2$
$x^\mu x_\mu \equiv \vec{x}^2 + c^2 t^2$


$\int d^4x \, d^4y \, \rho(x) \, G(x,y) \, \rho(y)$
$\frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} G = \delta^4(x-y)$

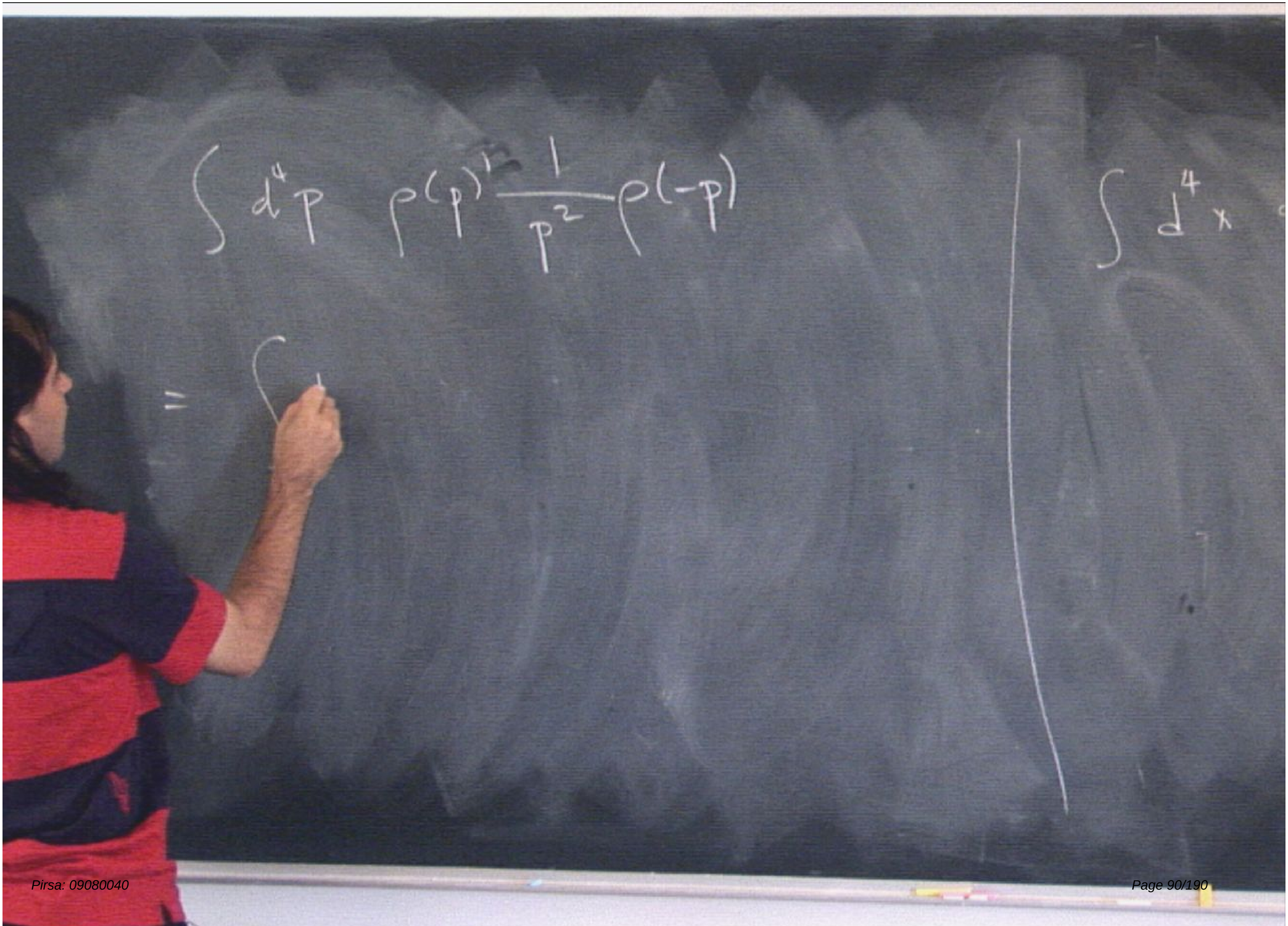

$\int d^4x \, d^4y \, \rho(x) \, G(x,y) \, \rho(y)$
$\frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} G = \delta^4(x-y)$
$G = \frac{1}{\partial^2}$

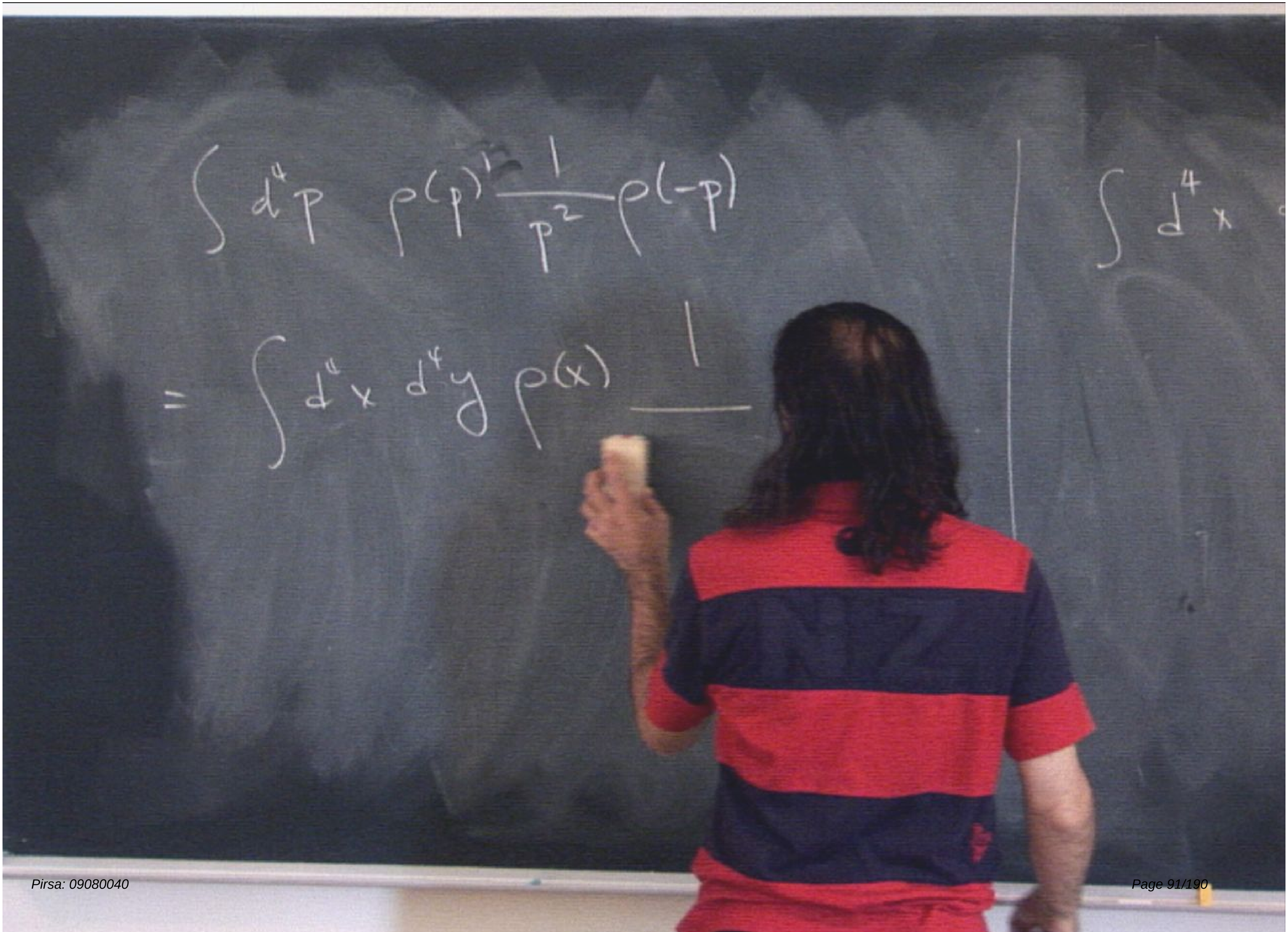

$\int d^4x\, d^4y\, \rho(x)\, G(x,y)\, \rho(y)$
$\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x_\mu} G = \delta^4(x-y)$
$G = \frac{1}{\vec{p}^2 - \omega^2}$

$$\int d^4x \, d^4y \, \rho(x) \, G(x,y) \, \rho(y)$$

$$\frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} G = \delta^4(x-y)$$

$$G = \frac{1}{p^2 - \omega^2} = \frac{1}{p^\mu p_\mu}$$


$\int d^4p \;\; \rho(p) \frac{1}{p^2} \rho(-p)$
$= \int$
$\int d^4x$

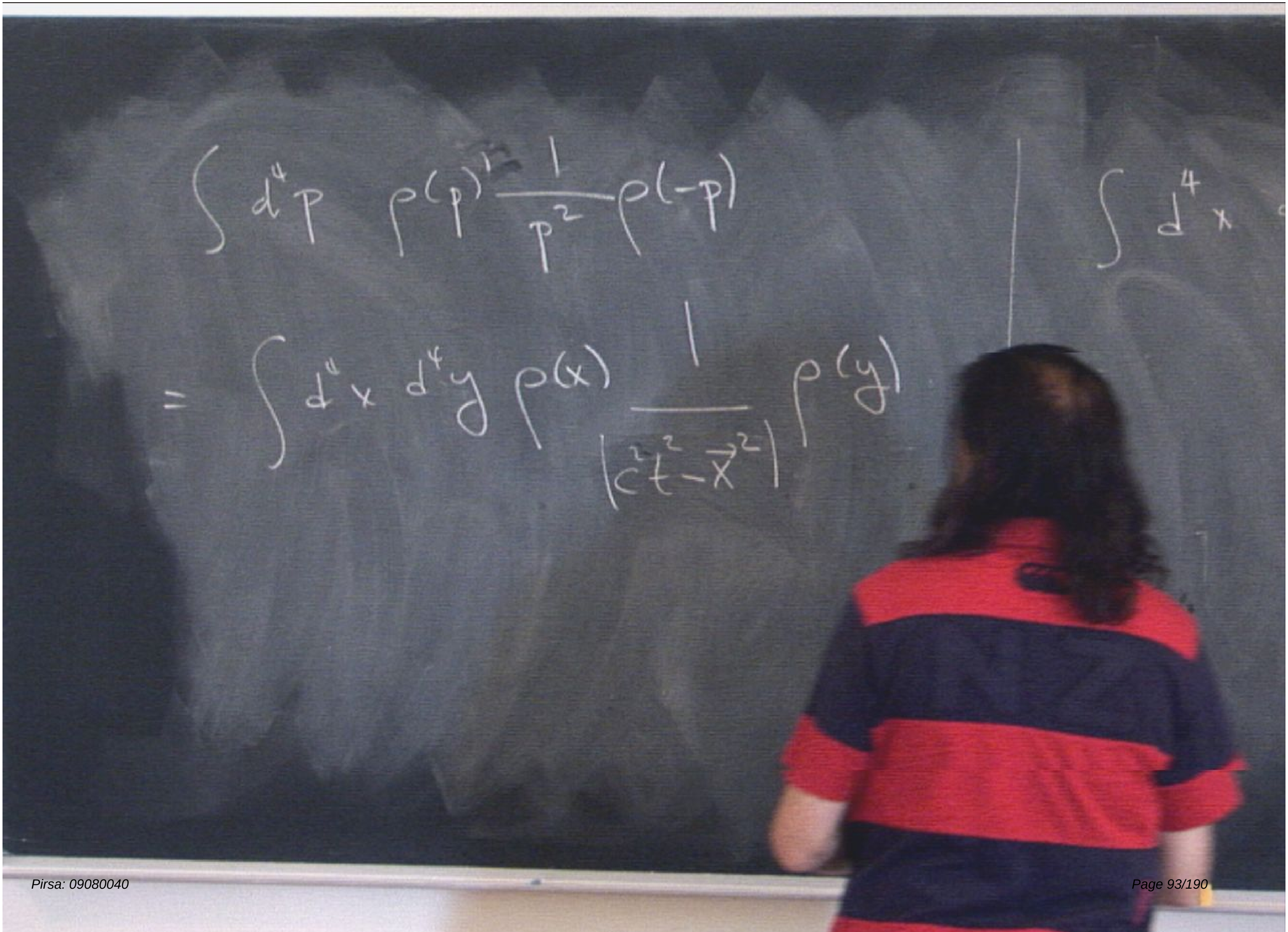


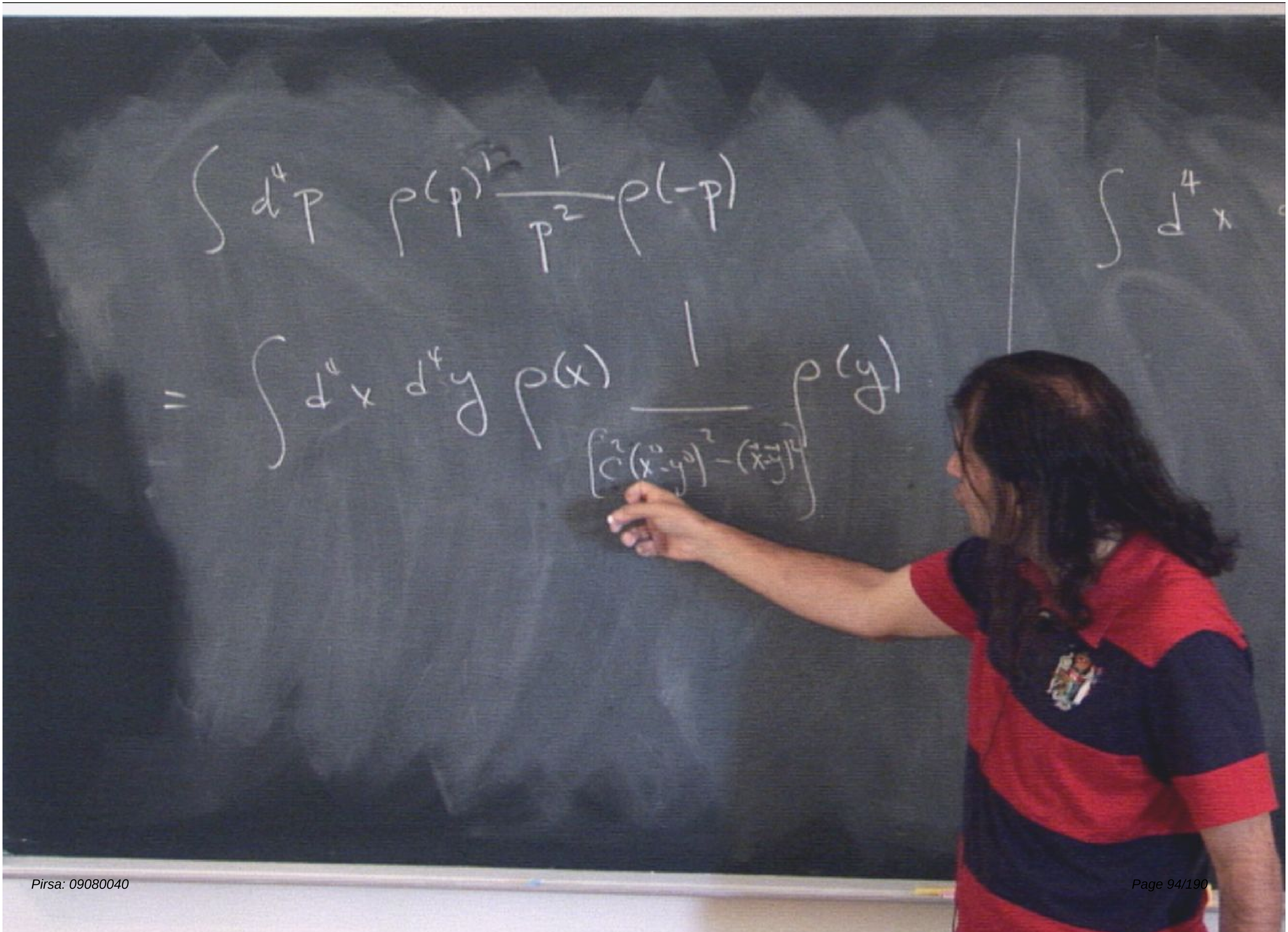
$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$
$= \int d^4x \, d^4y \, \rho(x) \, \frac{1}{\quad}$
$\int d^4x$

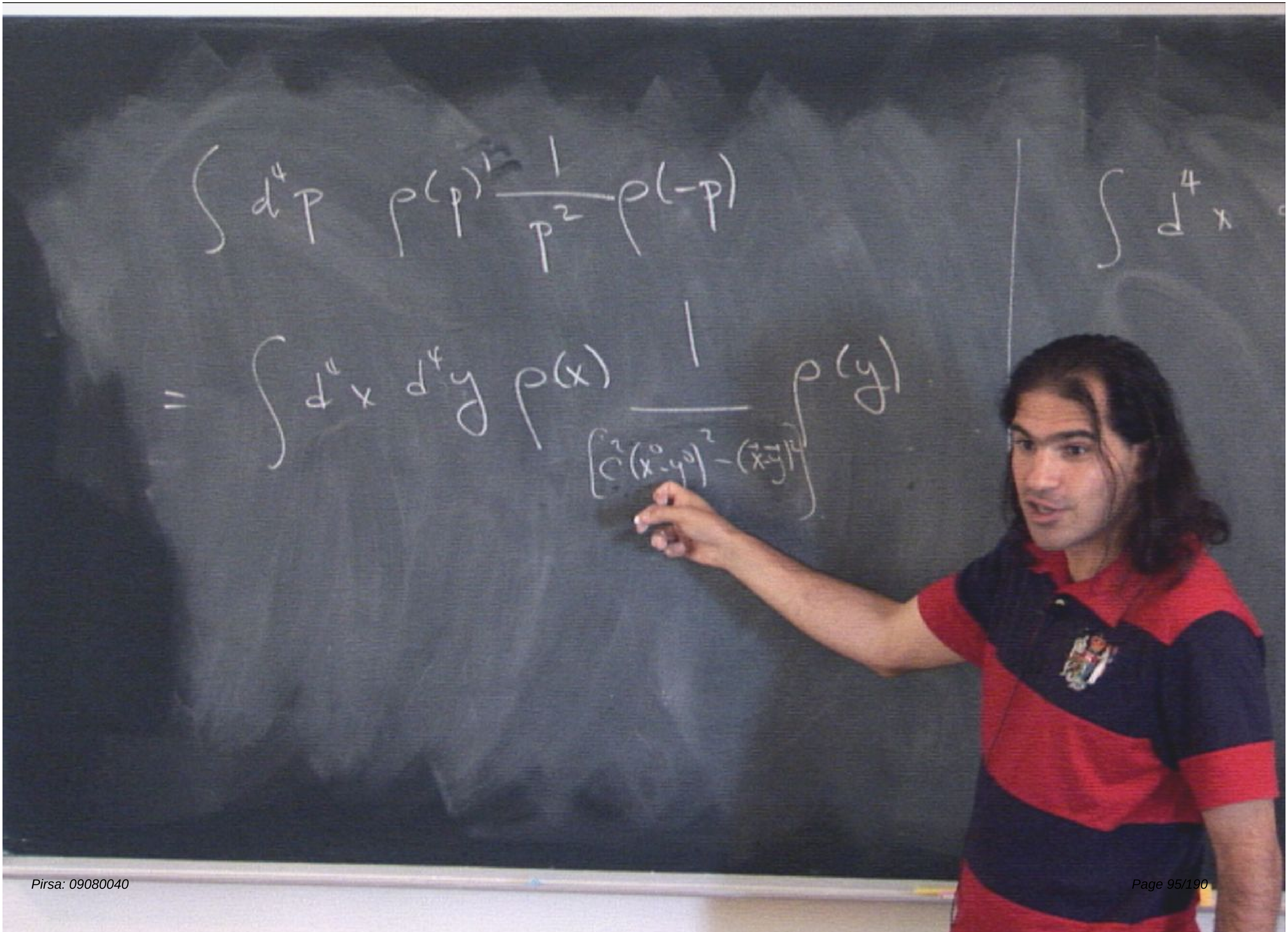
$$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$$

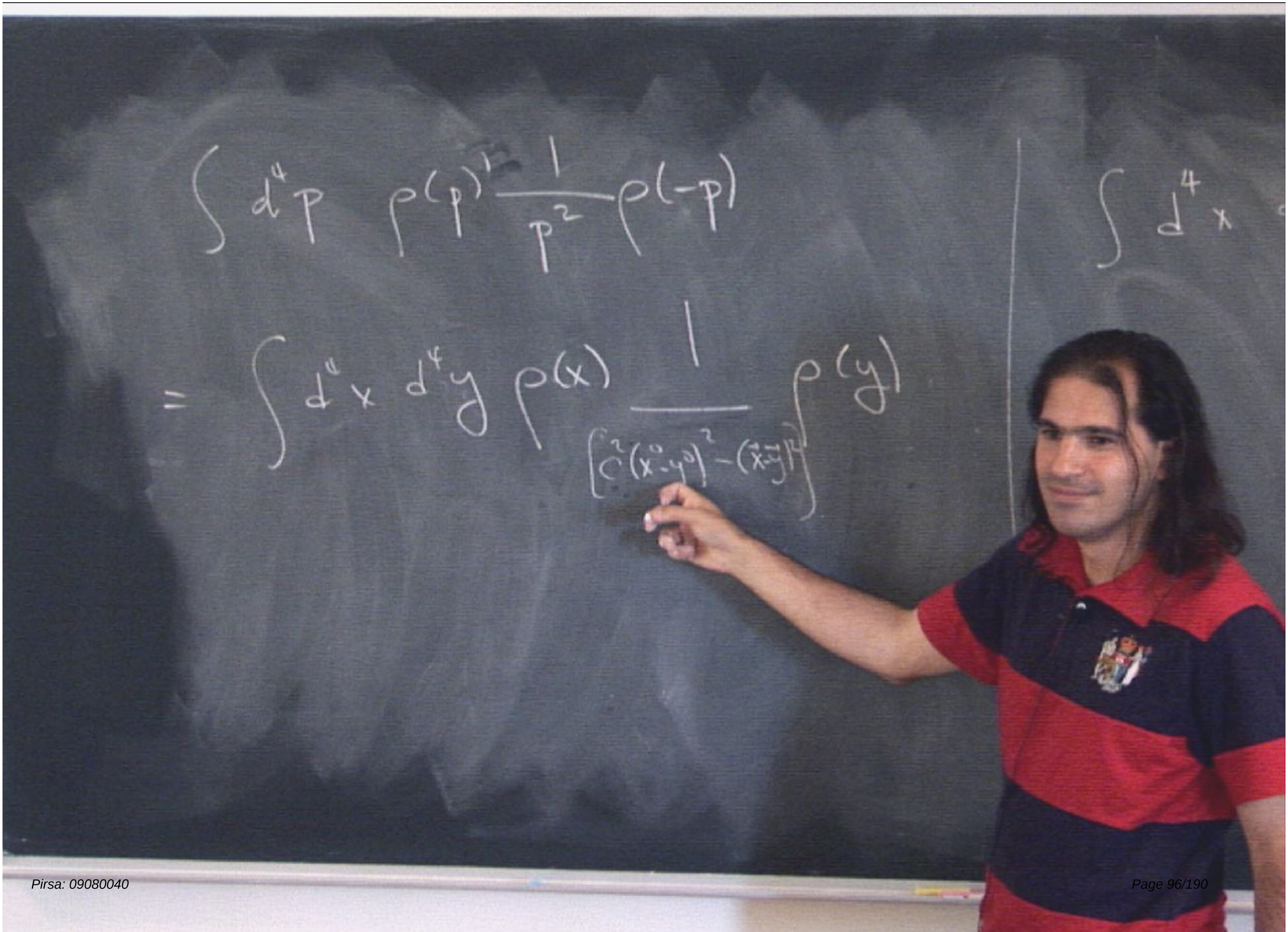
$$= \int d^4x \, d^4y \, \rho(x) \frac{1}{|c^2t^2 - \vec{x}^2|} \rho(y)$$

$$\int d^4x$$


$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$
$= \int d^4x \, d^4y \, \rho(x) \frac{1}{|c^2t^2 - \vec{x}^2|} \rho(y)$
$\int d^4x$


$\int d^4p \;\rho(p) \frac{1}{p^2} \rho(-p)$
$= \int d^4x \, d^4y \, \rho(x) \frac{1}{\left[c^2(x^0-y^0)^2 - (\vec{x}-\vec{y})^2\right]} \rho(y)$
$\int d^4x$


$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$
$= \int d^4x \, d^4y \; \rho(x) \frac{1}{\left[c^2(x^0-y^0)^2 - (\vec{x}-\vec{y})^4\right]} \rho(y)$


$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$
$= \int d^4x \, d^4y \; \rho(x) \frac{1}{\left[c^2(x^0-y^0)^2 - (\vec{x}-\vec{y})^2\right]} \rho(y)$
$\int d^4x$

$$\int d^4p \;\rho(p)\frac{1}{p^2}\rho(-p)$$

$$= \int d^4x\, d^4y\, \rho(x)\frac{1}{\left[c^2(x^0-y^0)^2-(\vec{x}-\vec{y})^2\right]}\rho(y)$$

$$\frac{1}{|x-y|} \longrightarrow \frac{1}{X^2}$$

$$\int d^4x$$

$$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$$

$$= \int d^4x \, d^4y \, \rho(x) \frac{1}{\left[c^2 (x^0 - y^0)^2 - (\vec{x} - \vec{y})^2\right]} \rho(y)$$

$$\frac{1}{|x-y|} \longrightarrow \frac{1}{x^2} \; , \quad \frac{1}{\vec{p}^2} \longrightarrow \frac{1}{(p^\mu p_\mu)}$$

$$\int d^4x$$

$$\int d^4p \;\; \rho(p) \frac{1}{p^2} \rho(-p)$$

$$\int d^4x$$

$$= \int d^4x \, d^4y \, \rho(x) \frac{1}{\left[c^2(x^0-y^0)^2 - (\vec{x}-\vec{y})^2\right]} \rho(y)$$

$$\frac{1}{|x-y|} \longrightarrow \frac{1}{x^2} \, , \quad \frac{1}{\vec{p}^2} \longrightarrow \frac{1}{(p^\mu p_\mu)} .$$

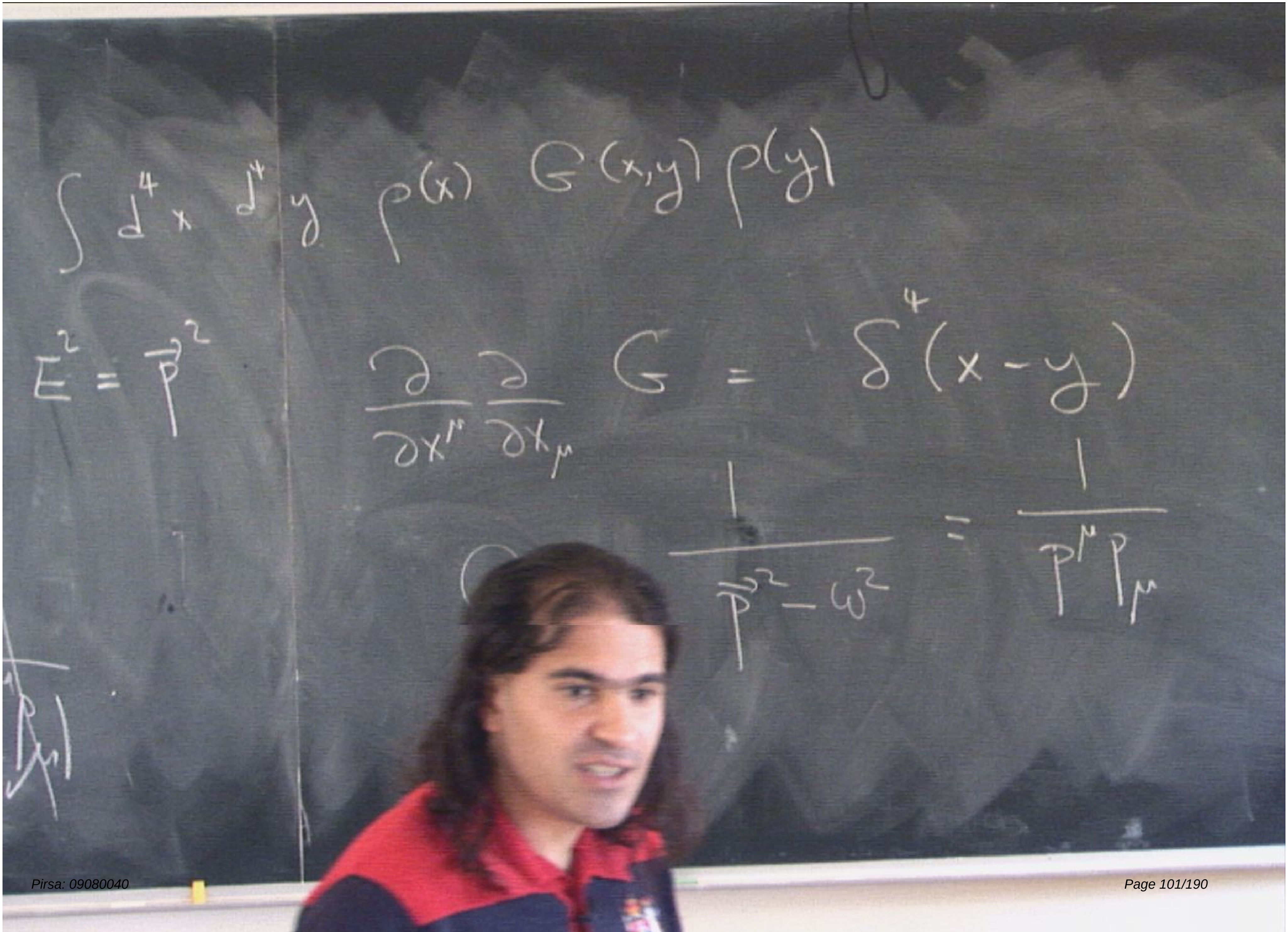
$$\int d^4p \; \rho(p) \frac{1}{p^2} \rho(-p)$$

$$= \int d^4x \, d^4y \, \rho(x) \frac{1}{\left[c^2(x^0-y^0)^2 - (\vec{x}-\vec{y})^2\right]} \rho(y)$$

$$\frac{1}{|x-y|} \rightarrow \frac{1}{x^2} \, , \quad \frac{1}{\vec{p}^{\,2}} \rightarrow \frac{1}{(p^\mu p_\mu)}$$

$$\int d^4x$$

$$E^2 = \vec{p}^{\,2}$$

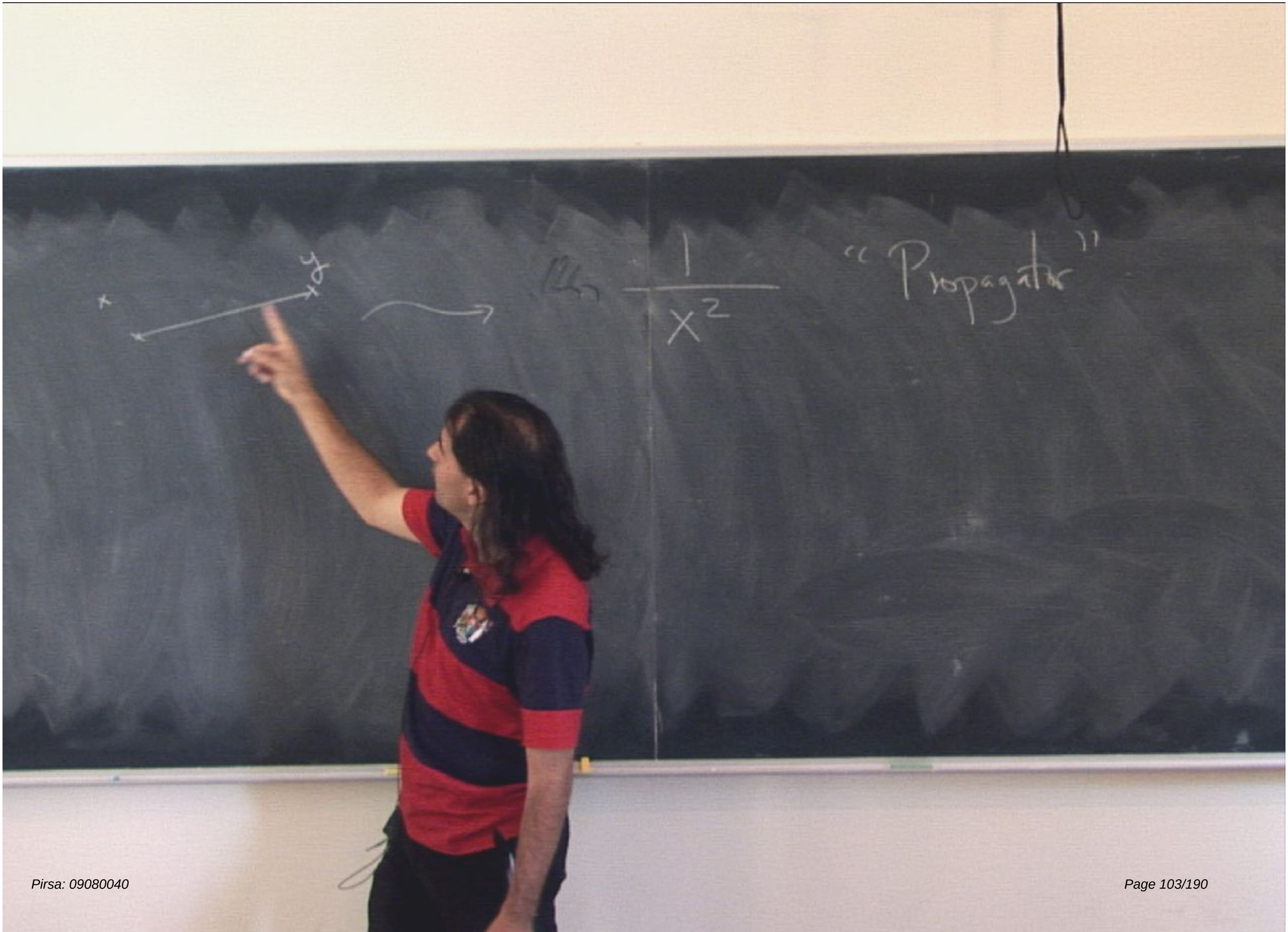

$\int d^4x \, d^4y \, \rho(x) \, G(x,y) \, \rho(y)$
$E^2 = \vec{p}^{\,2}$
$\frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} G = \delta^4(x-y)$
$G = \frac{1}{\vec{p}^{\,2} - \omega^2} = \frac{1}{p^\mu p_\mu}$

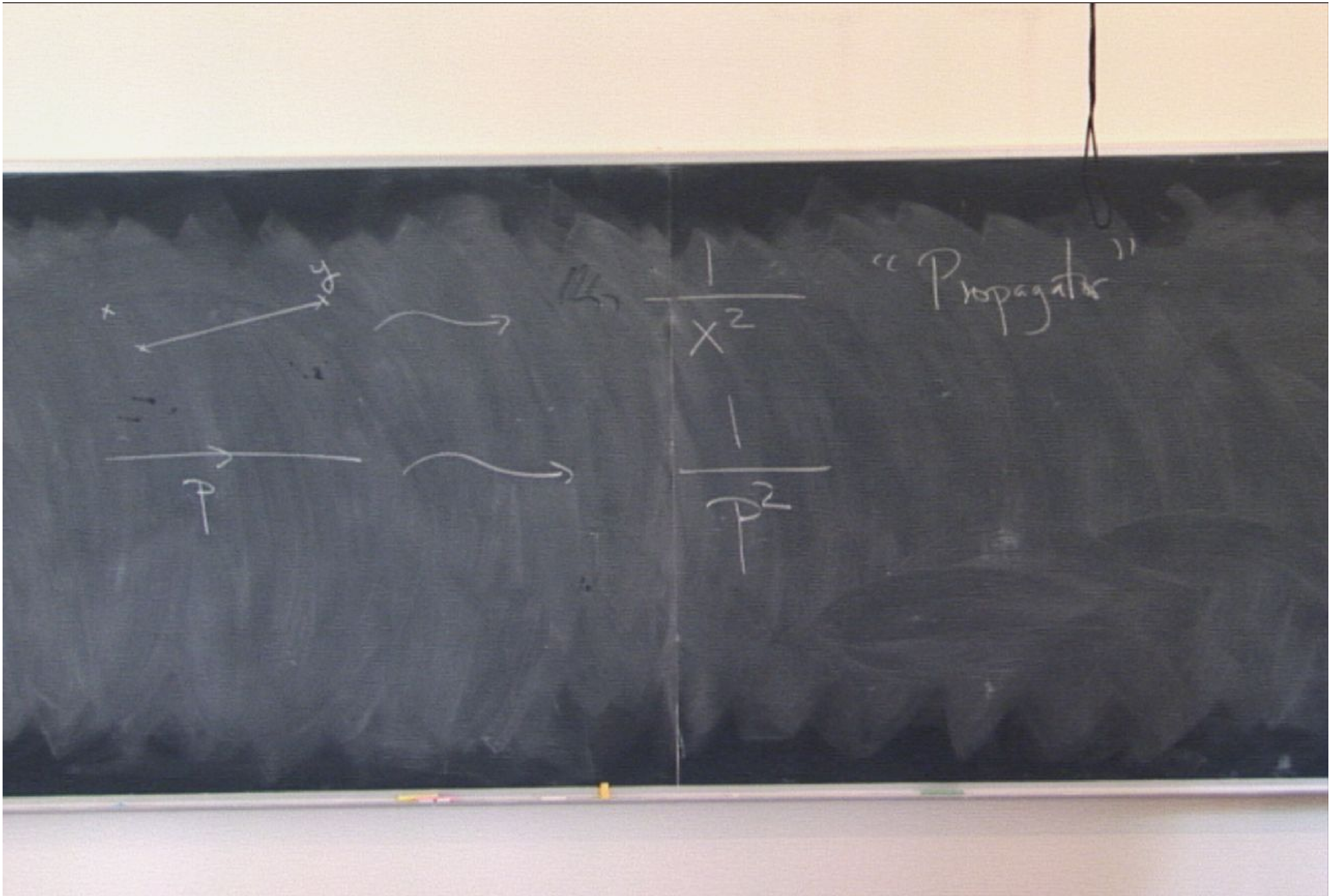
$$\int d^4x \, d^4y \, \rho(x) \, G(x,y) \, \rho(y)$$

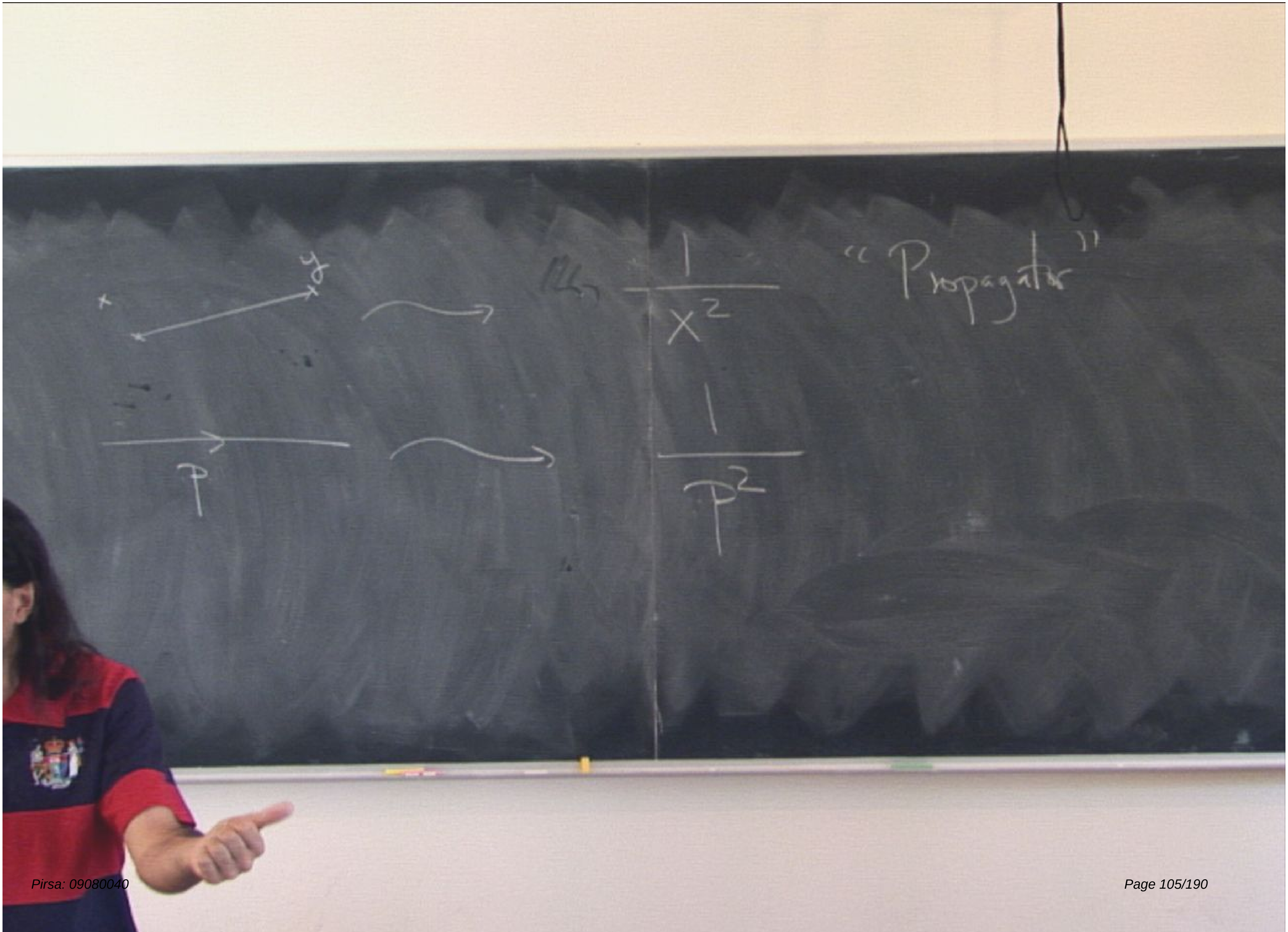
$$E^2 = \vec{p}^{\,2}$$

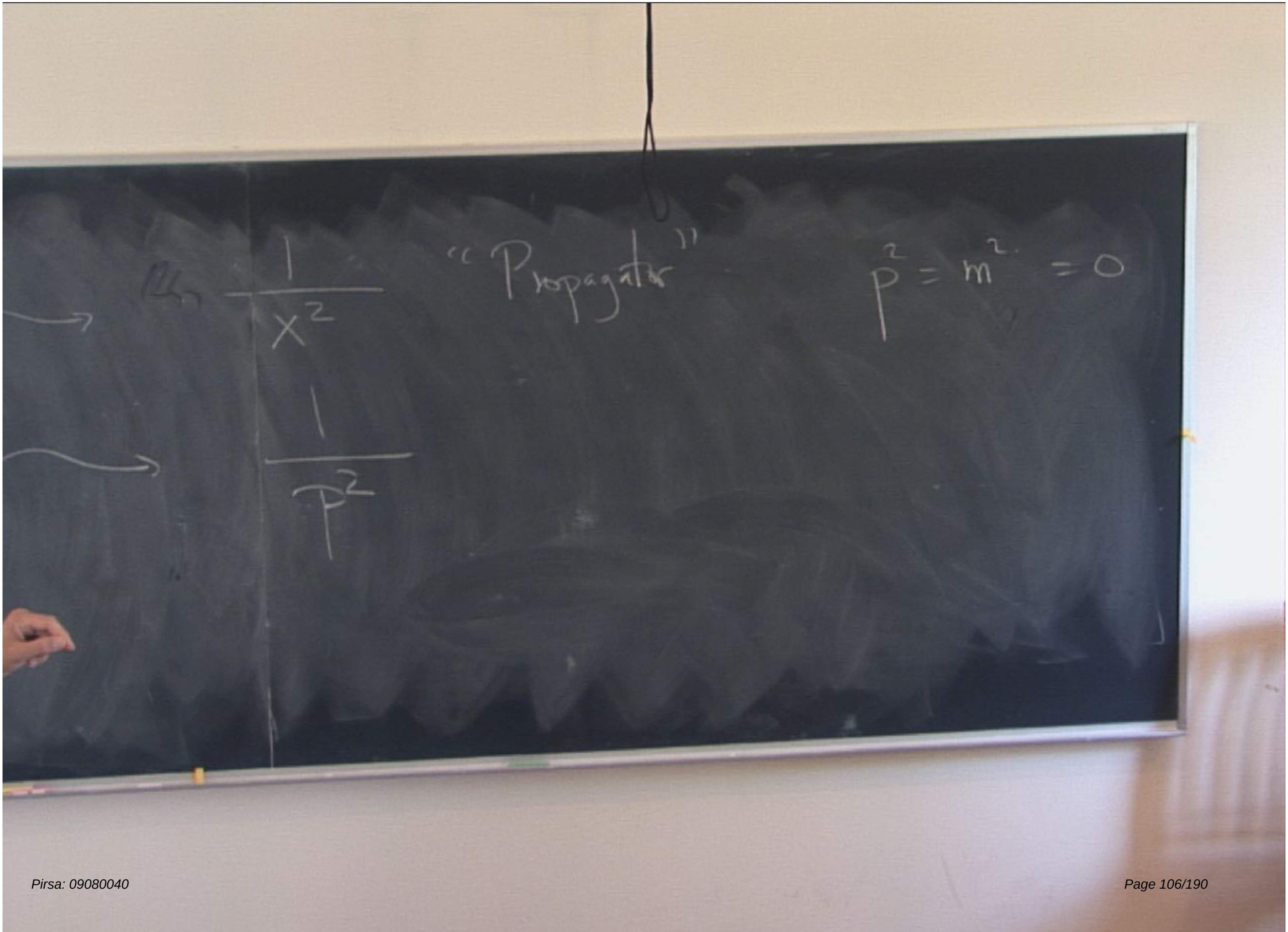
$$\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x_\mu} G = \delta^4(x-y)$$

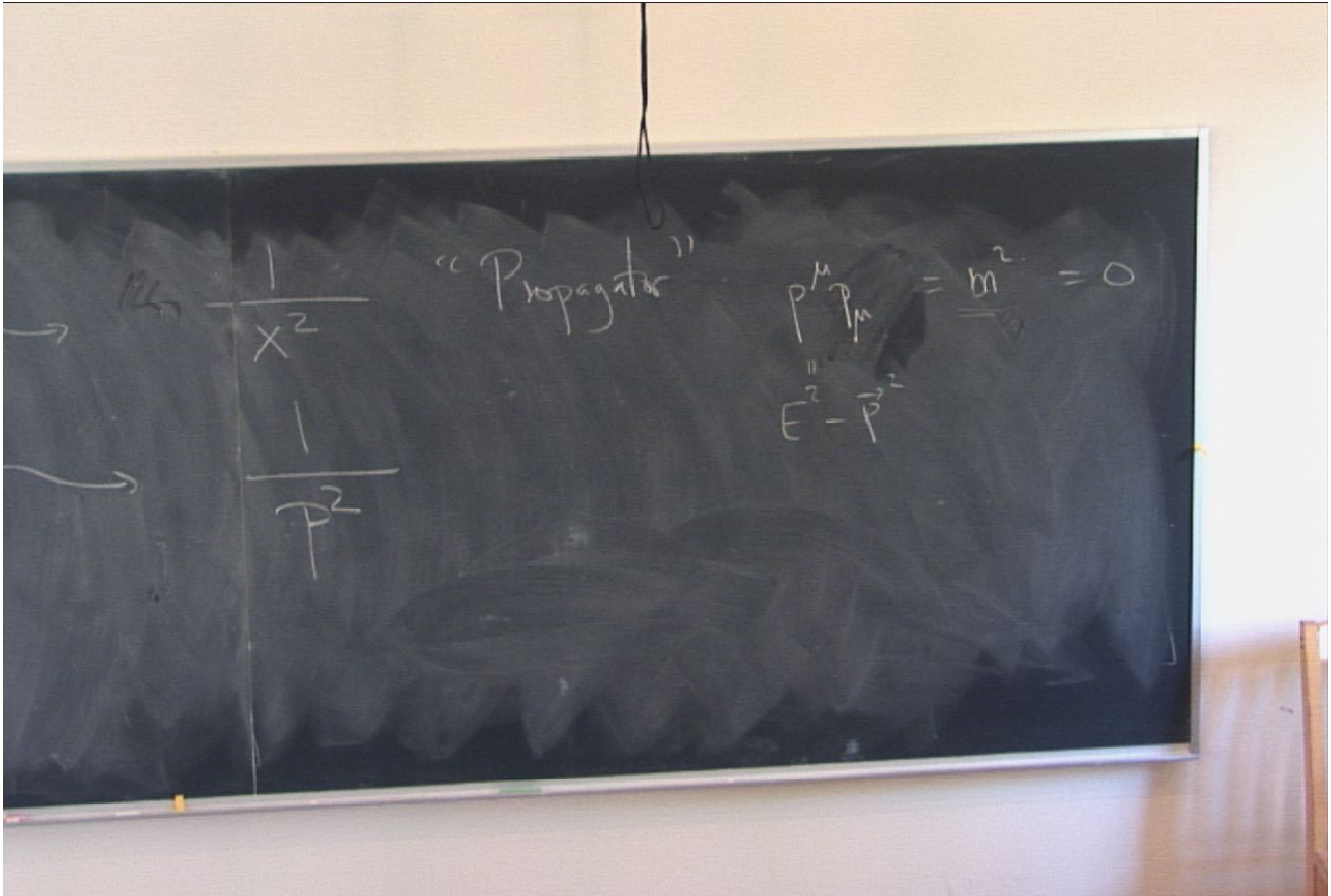
$$G = \frac{1}{\vec{p}^{\,2} - \omega^2} = \frac{1}{p^\mu p_\mu}$$

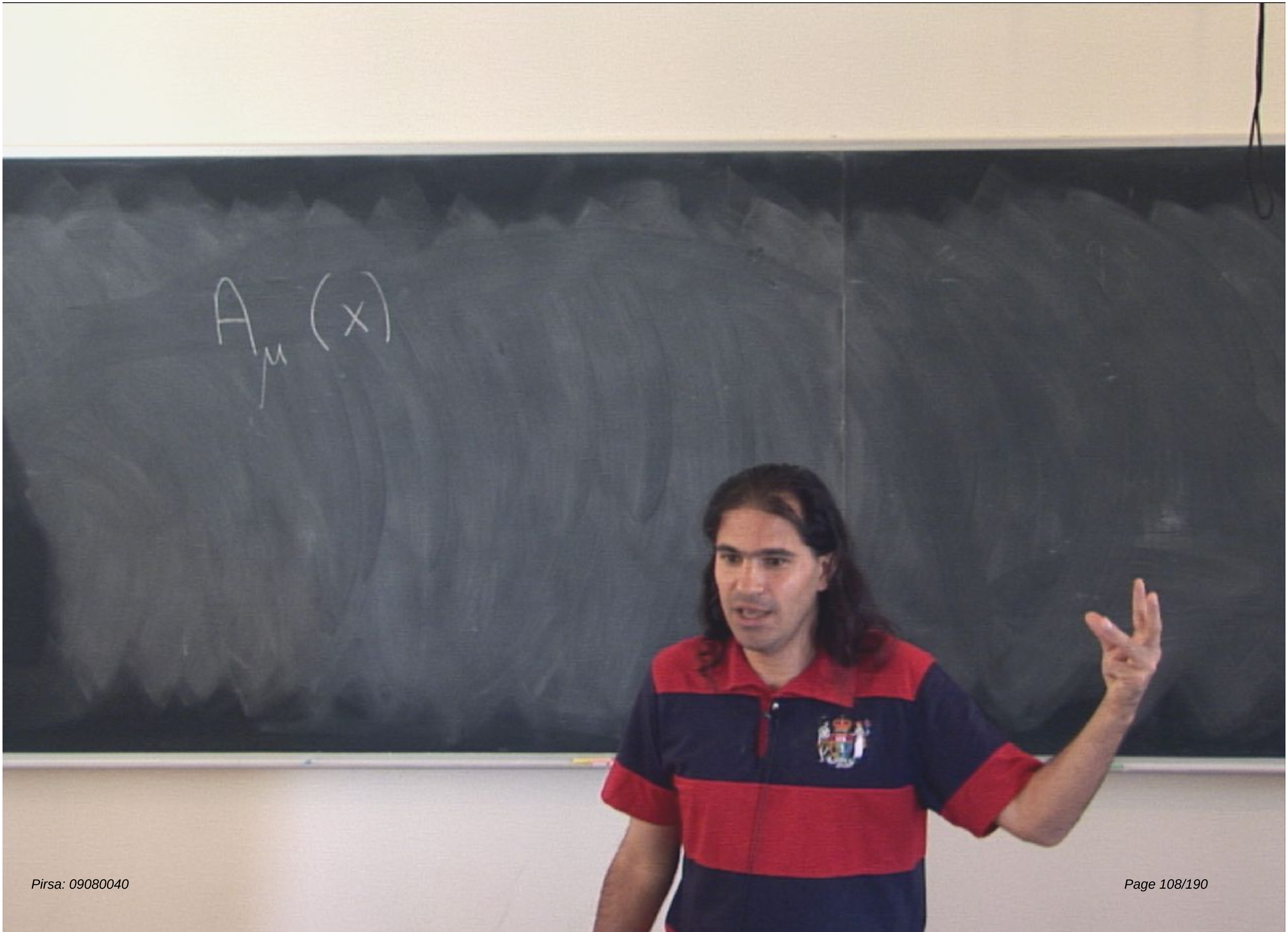

x
y
$\frac{1}{x^2}$
"Propagator"

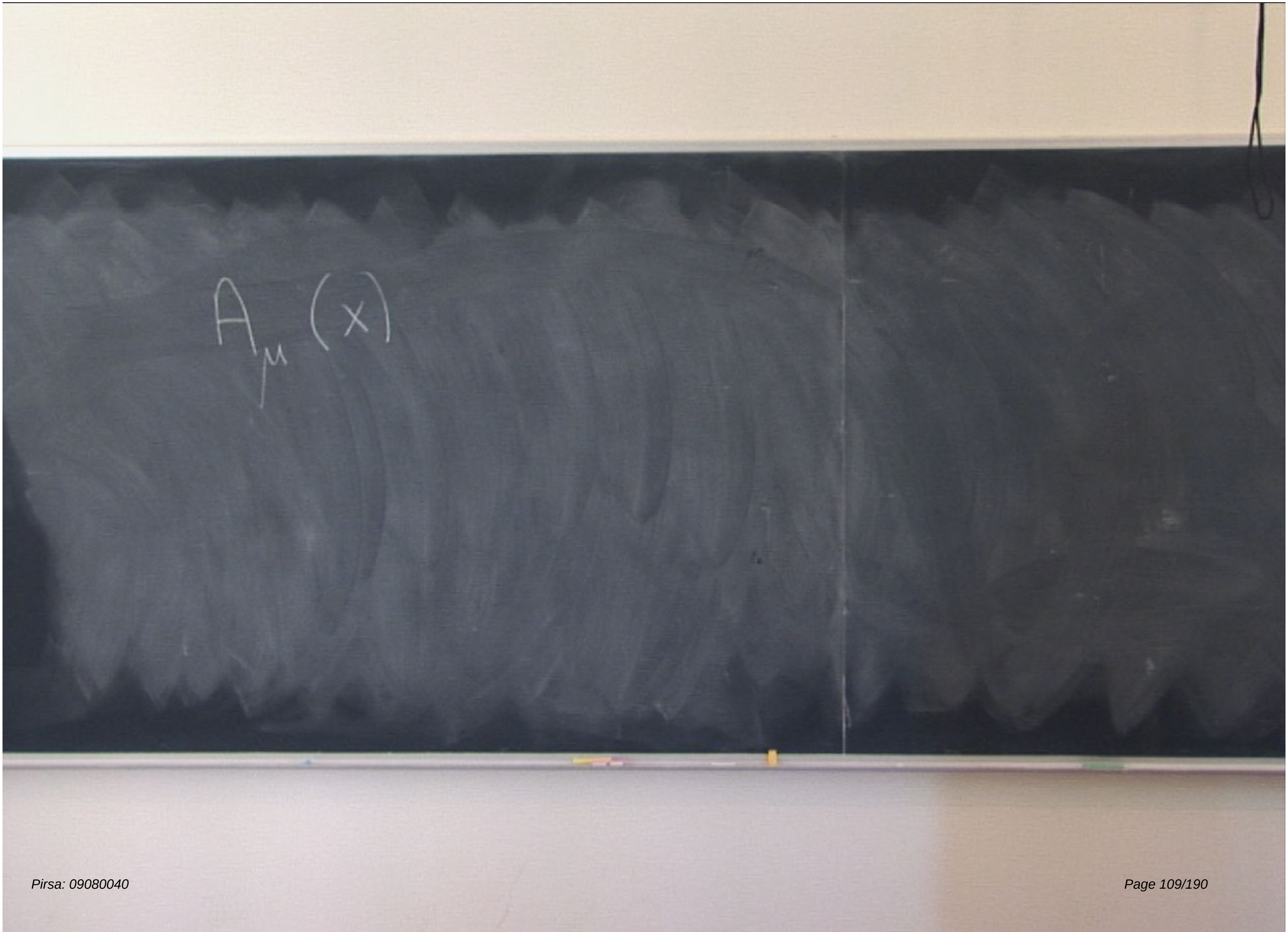

x
y
p
$\frac{1}{x^2}$
$\frac{1}{p^2}$
"Propagator"

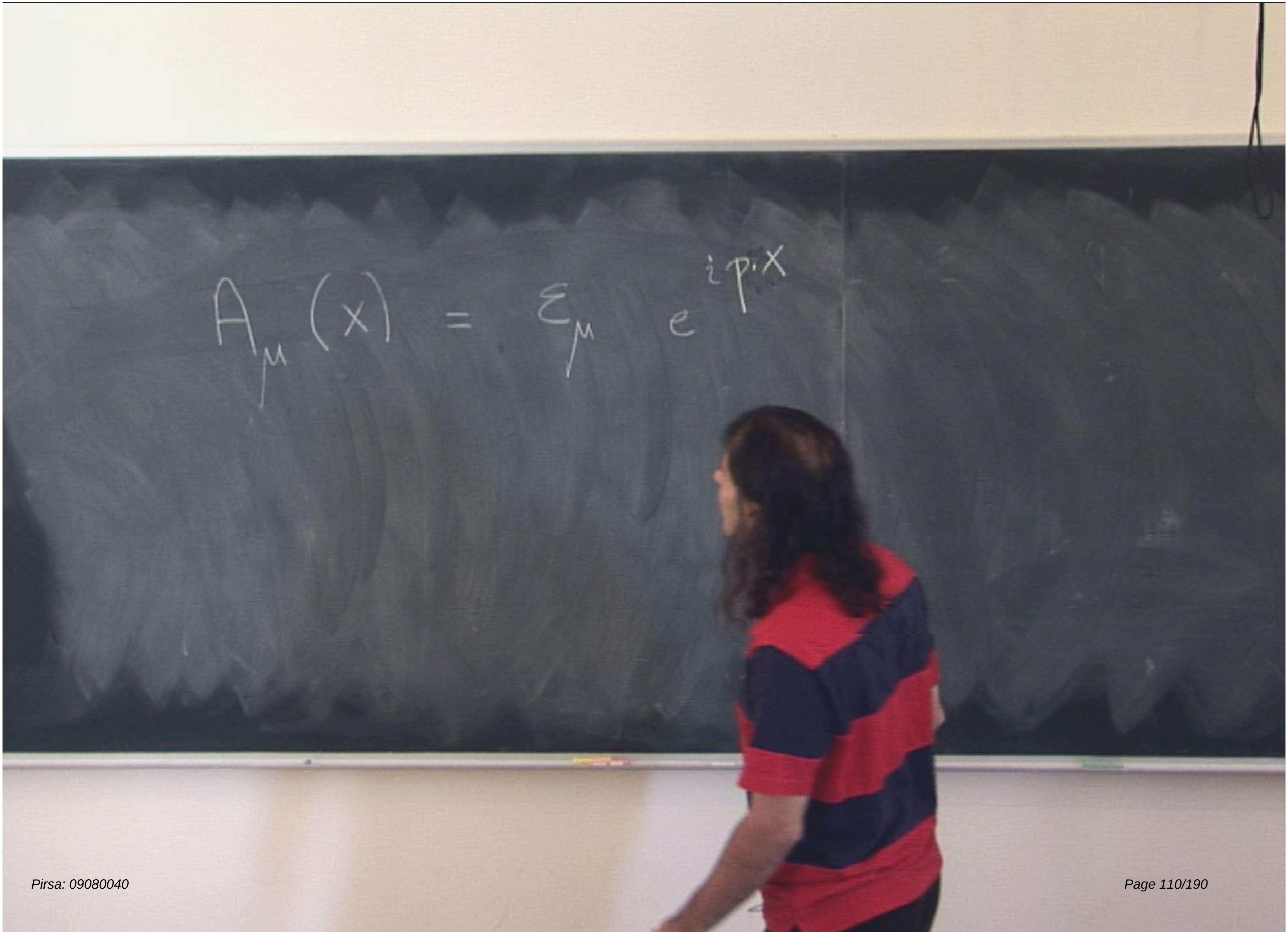

x
y
p
$\frac{1}{x^2}$
$\frac{1}{p^2}$
“Propagator”


$\frac{1}{x^2}$
"Propagator"
$p^2 = m^2 = 0$
$\frac{1}{p^2}$


1/x²
1/p²
"Propagator"
p^μ p_μ = m² = 0
E² − p⃗²


$A_\mu(x)$


$A_\mu(x)$


$A_\mu(x) = \varepsilon_\mu \, e^{ip\cdot x}$

$$A_\mu(x) = \varepsilon_\mu \, e^{i p \cdot x}$$

polarization vector

$$A_\mu(x) = \varepsilon_\mu \, e^{i p \cdot x}$$

polarization vector.

<u>Massive Spin 1</u>

$$p^\mu \epsilon_\mu = 0$$

$$A_\mu(x) = \varepsilon_\mu \, e^{ip\cdot x}$$

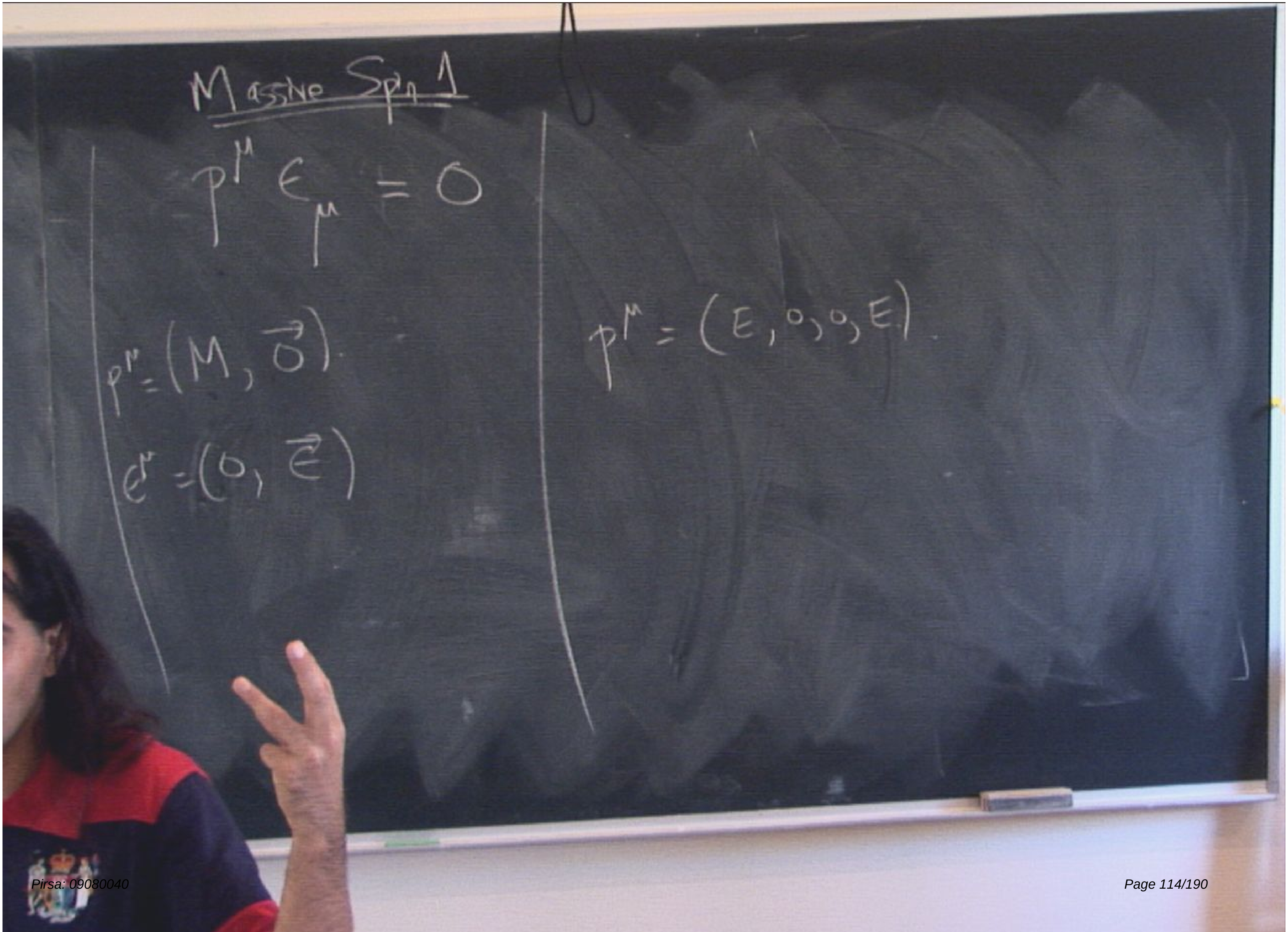
polarization vector

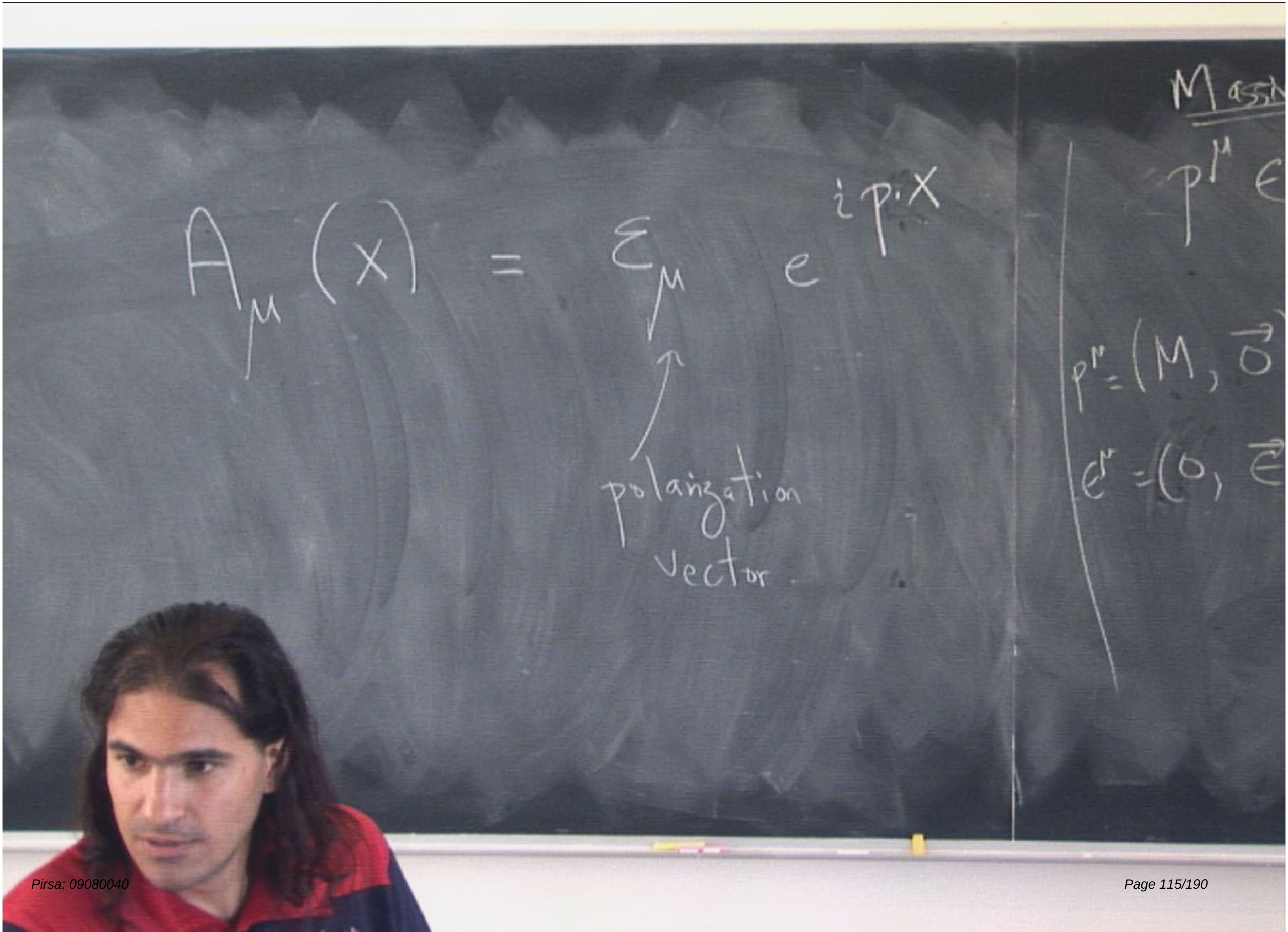
Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$p^\mu = (M, \vec{0})$$

$$\epsilon^\mu = (0, \vec{\epsilon})$$


Massive Spin 1
$p^\mu \epsilon_\mu = 0$
$p^\mu = (M, \vec{0})$
$\epsilon^\mu = (0, \vec{\epsilon})$
$p^\mu = (E, 0, 0, E)$


$A_\mu(x) = \varepsilon_\mu \, e^{ip\cdot x}$
polarization
vector
$p^\mu = (M, \vec{0}$
$\varepsilon^\mu = (0, \vec{\varepsilon}$

$$A_\mu(x) = \varepsilon_\mu \, e^{i p \cdot x}$$

polarization vector

Massive Sp

$$p^\mu \epsilon_\mu =$$

$$p^\mu = (M, \vec{0})$$

$$\epsilon^\mu = (0, \vec{\epsilon})$$

Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$p^\mu = (M, \vec{0})$$

$$\epsilon^\mu = (0, \vec{\epsilon})$$

Declaration

ϵ_μ and $(\epsilon_\mu + \alpha p_\mu)$

Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$p^\mu = (M, \vec{0})$$

$$\epsilon^\mu = (0, \vec{\epsilon})$$

Declaration

$$\epsilon_\mu \text{ and } (\epsilon_\mu + \alpha p_\mu)$$

Describe the same state.

Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$p^\mu = (M, \vec{0})$$

$$\epsilon^\mu = (0, \vec{\epsilon})$$

Declaration

ϵ_μ and $(\epsilon_\mu + \alpha p_\mu)$

Describe the same state.

$$p^\mu \epsilon_\mu = p^\mu \cdot (\epsilon_\mu + \alpha p_\mu).$$

Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$p^\mu = (M, \vec{0})$$

$$\epsilon^\mu = (0, \vec{\epsilon})$$

Declaration

ϵ_μ and $(\epsilon_\mu + \alpha p_\mu)$.

Describe the same state.

$$p^\mu \epsilon_\mu = p^\mu \times (\epsilon_\mu + \alpha p_\mu).$$

$$A_\mu(x) = \epsilon_\mu \, e^{ip\cdot x}$$

polarization vector.

$p^\mu =$

ϵ^μ

Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$(M, \vec{0})$$

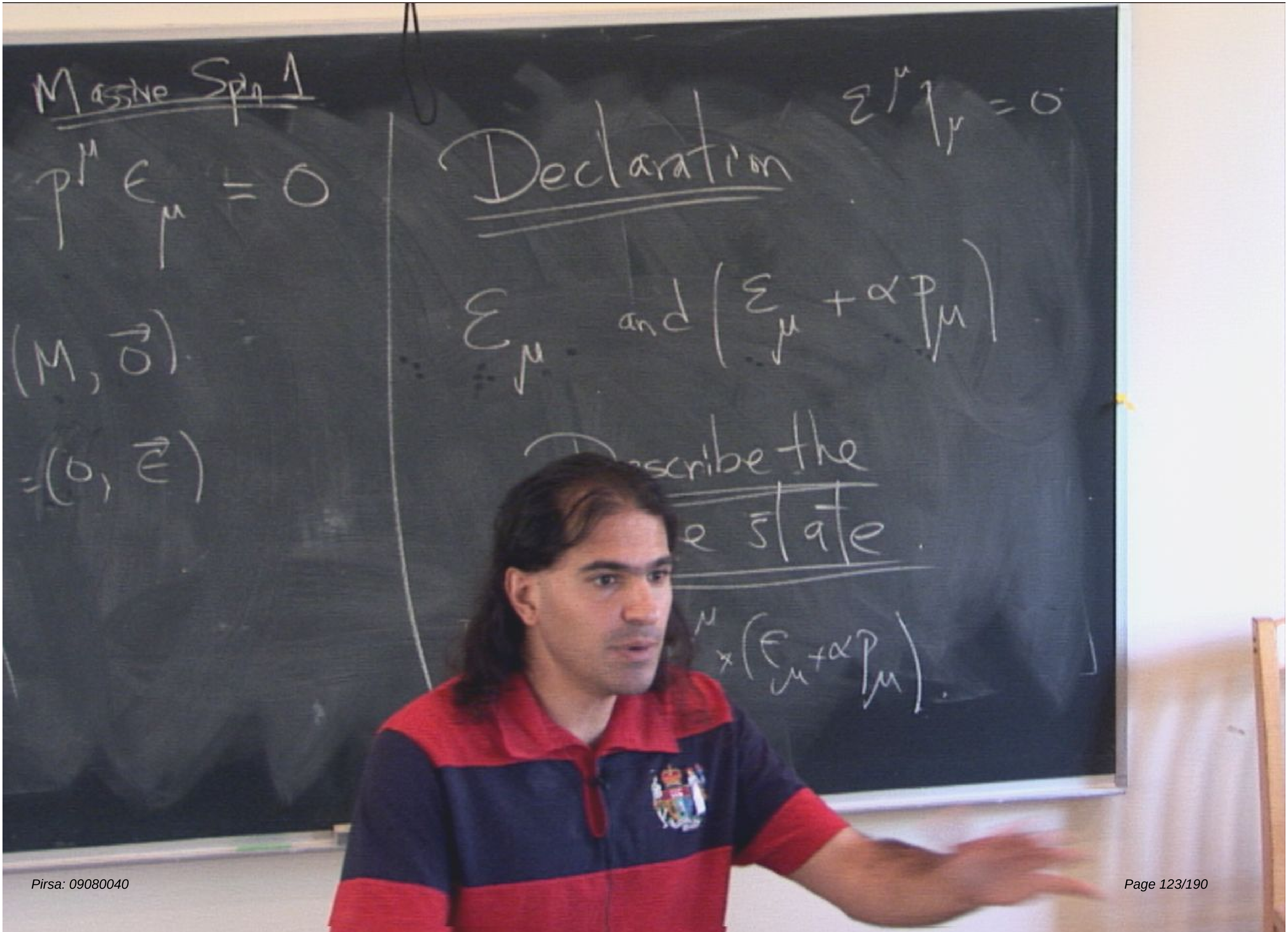
$$= (0, \vec{\epsilon})$$

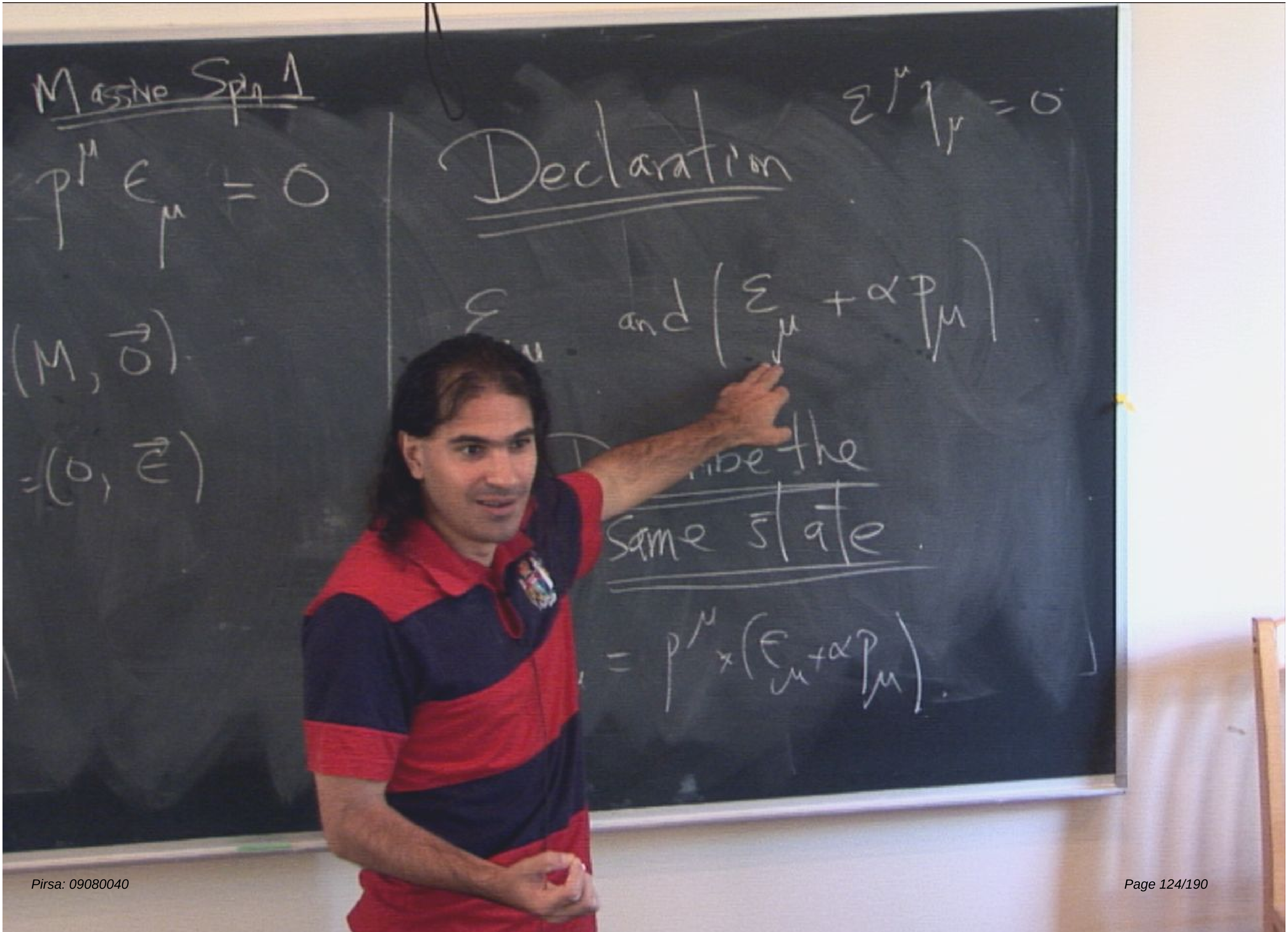
Declaration

ϵ_μ and $(\epsilon_\mu + \alpha p_\mu)$

Describe the same state.

$$p^\mu \epsilon_\mu = p^\mu \times (\epsilon_\mu + \alpha p_\mu).$$


Massive Spin 1
$p^\mu \epsilon_\mu = 0$
$(M, \vec{0})$
$= (0, \vec{\epsilon})$
Declaration
$\epsilon^\mu p_\mu = 0$
ϵ_μ and $(\epsilon_\mu + \alpha p_\mu)$
scribe the
e state
$(\epsilon_\mu + \alpha p_\mu)$.



Massive Spin 1
Declaration
$\varepsilon^\mu p_\mu = 0$
$p^\mu \epsilon_\mu = 0$
$(M, \vec{0})$
$=(0, \vec{\epsilon})$
and $(\varepsilon_\mu + \alpha p_\mu)$
the
same state
$= p^\mu \times (\epsilon_\mu + \alpha p_\mu)$

Massive Spin 1

$$p^\mu \epsilon_\mu = 0$$

$$(M, \vec{0})$$

$$= (0, \vec{\epsilon})$$

$$\epsilon^\mu p_\mu = 0$$

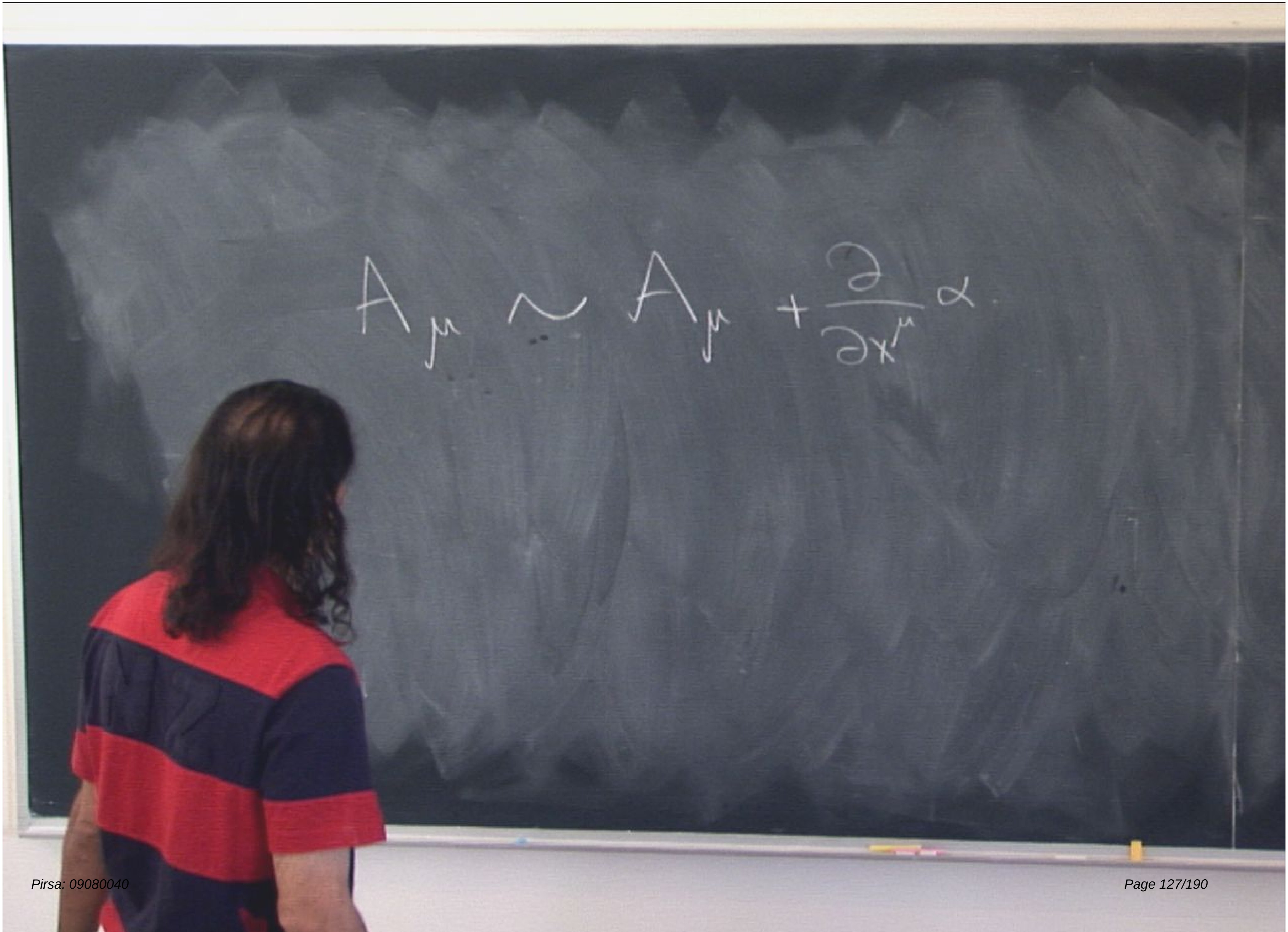
Declaration

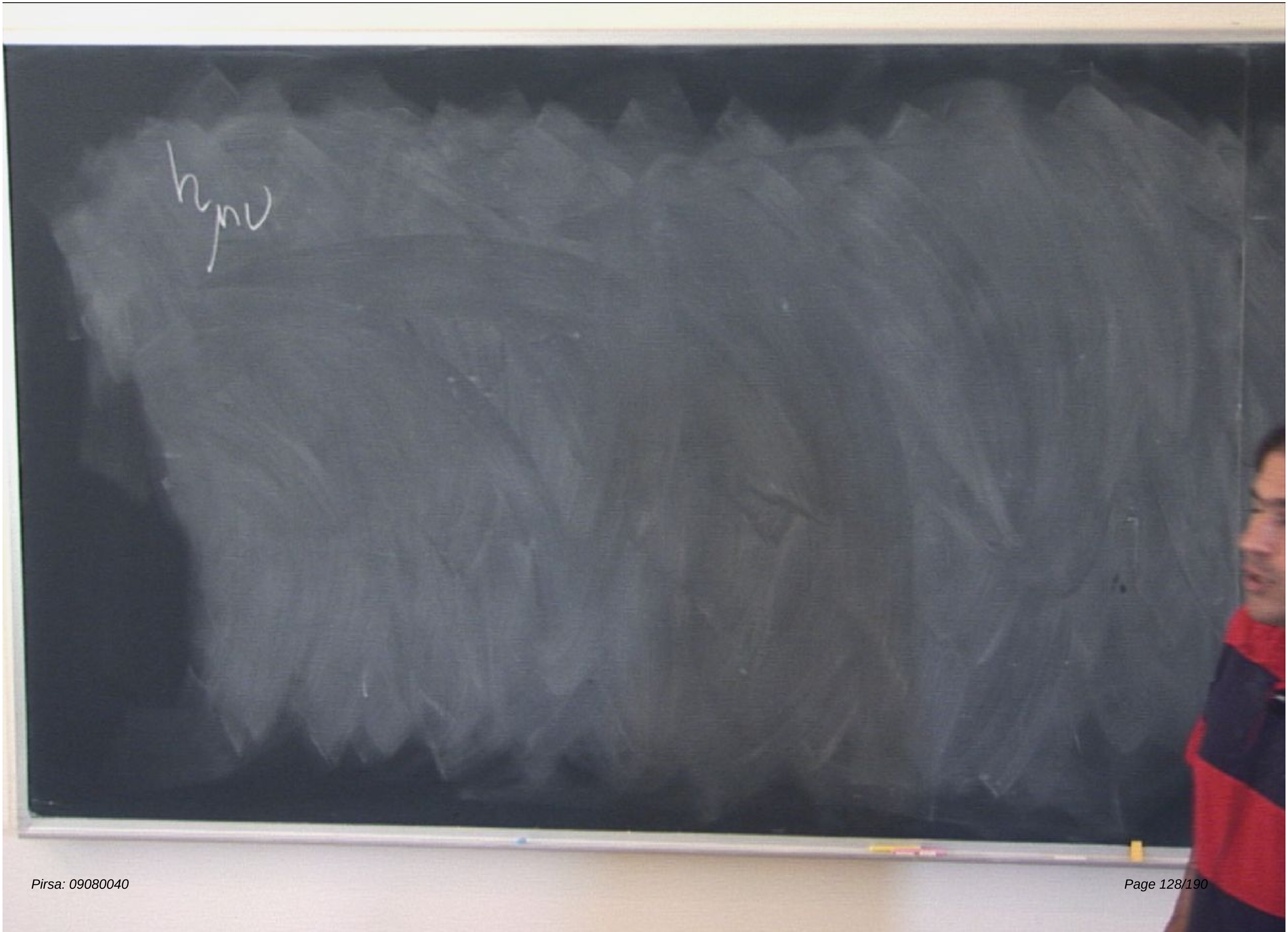
ϵ_μ and $(\epsilon_\mu + \alpha p_\mu)$

Describe the same state.

$$p^\mu \epsilon_\mu = p^\mu \times (\epsilon_\mu + \alpha p_\mu).$$

$$A_\mu \sim A_\mu + \partial_\mu \alpha$$


$A_\mu \sim A_\mu + \frac{\partial}{\partial x^\mu}\alpha$

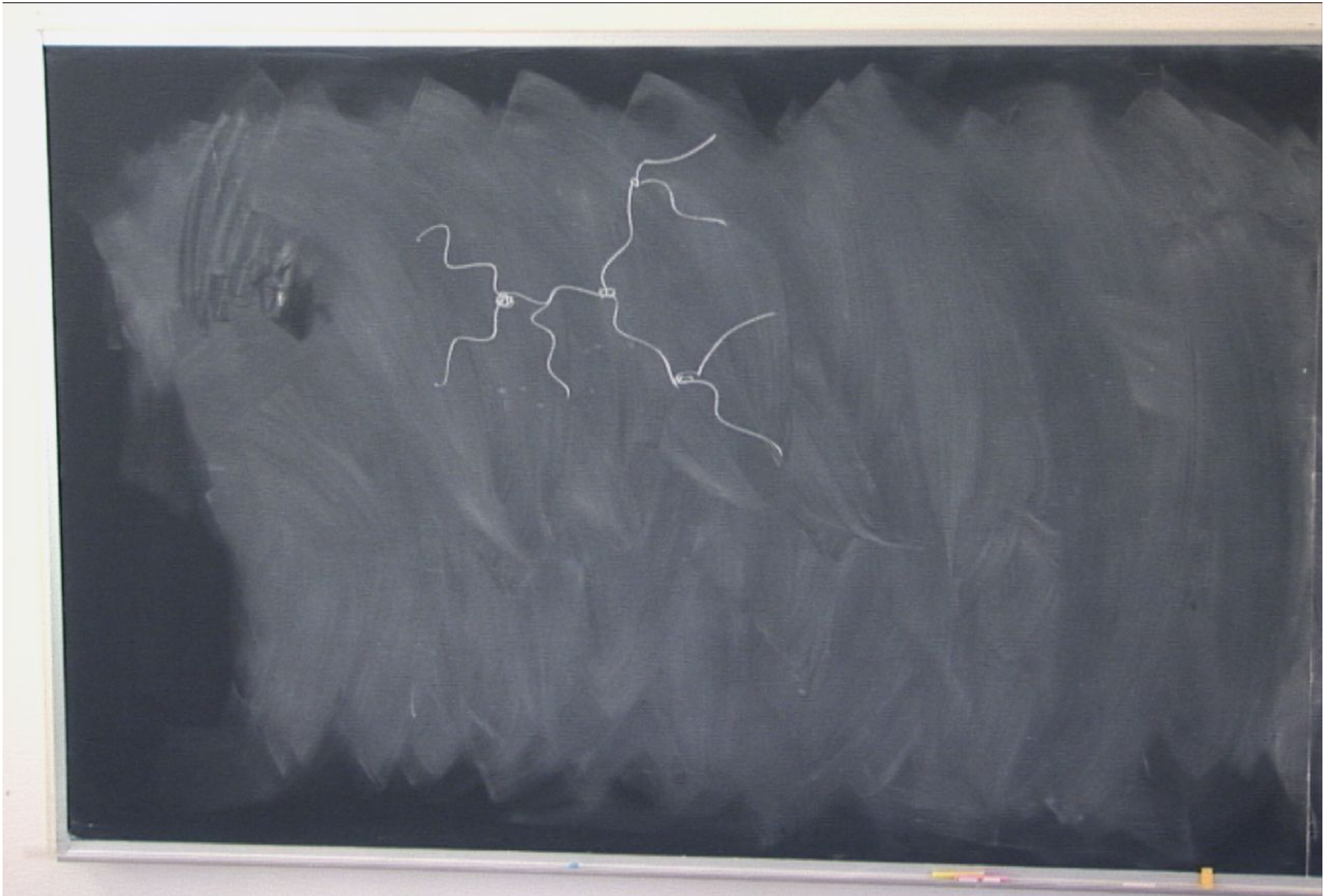

$h_{\mu\nu}$

$$\epsilon_{\mu\nu}\,, \qquad p^{\mu}\epsilon_{\mu\nu} = 0$$

$$\epsilon_{\mu\nu} \sim \epsilon_{\mu\nu} + p_{\mu}\alpha_{\nu} + p_{\nu}\alpha_{\mu}\,.$$

$$\epsilon_{\mu\nu}\,, \quad p^{\mu}\epsilon_{\mu\nu} = 0$$

$$\epsilon_{\mu\nu} \sim \epsilon_{\mu\nu} + p_{\mu}\alpha_{\nu} + p_{\nu}\alpha_{\mu}.$$



$$M^{\mu_1 \cdots \mu_n}$$

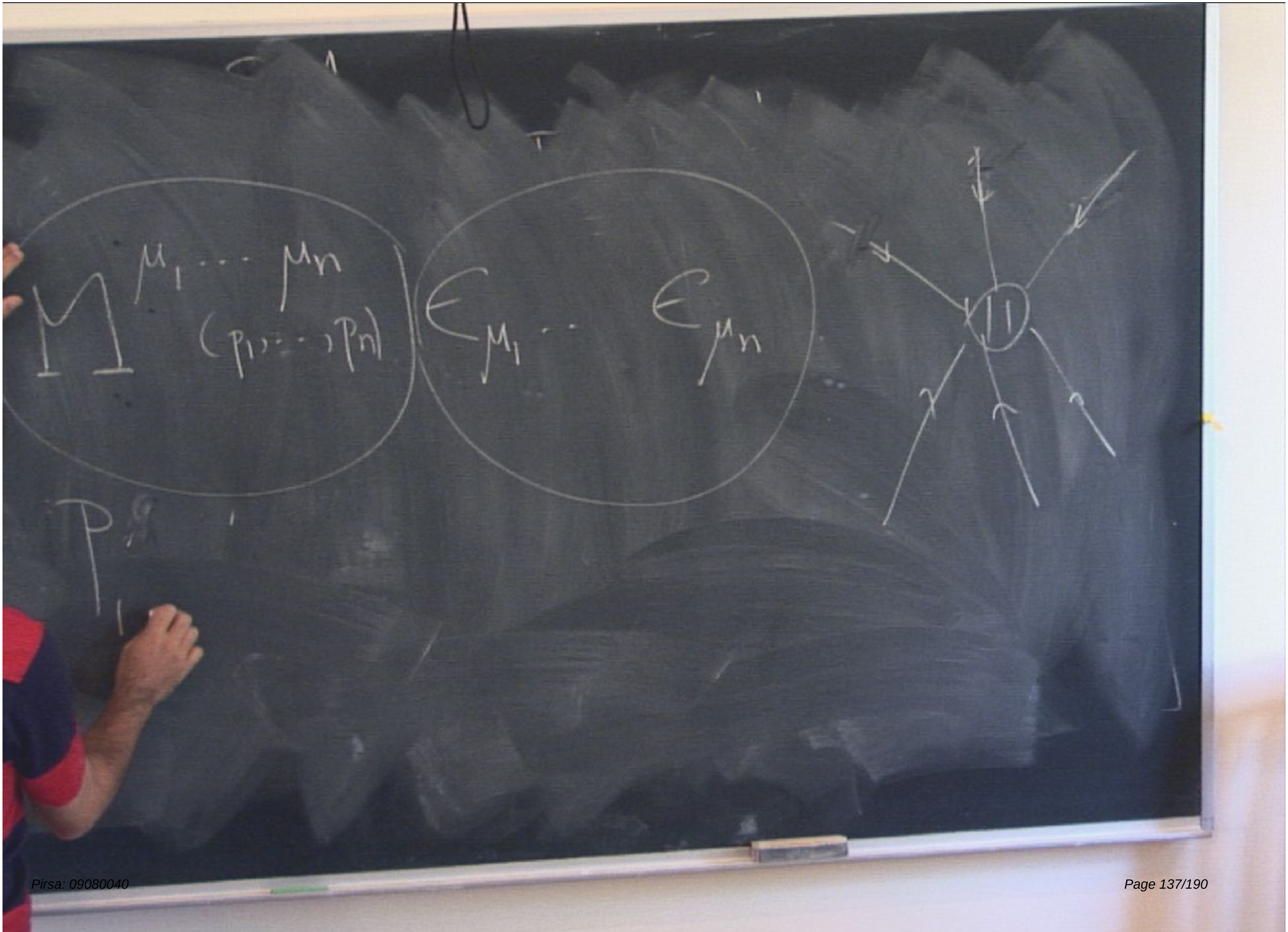
$M^{\mu_1 \dots \mu_n}$ $\epsilon_{\mu_1 \dots} \epsilon_{\mu_n}$

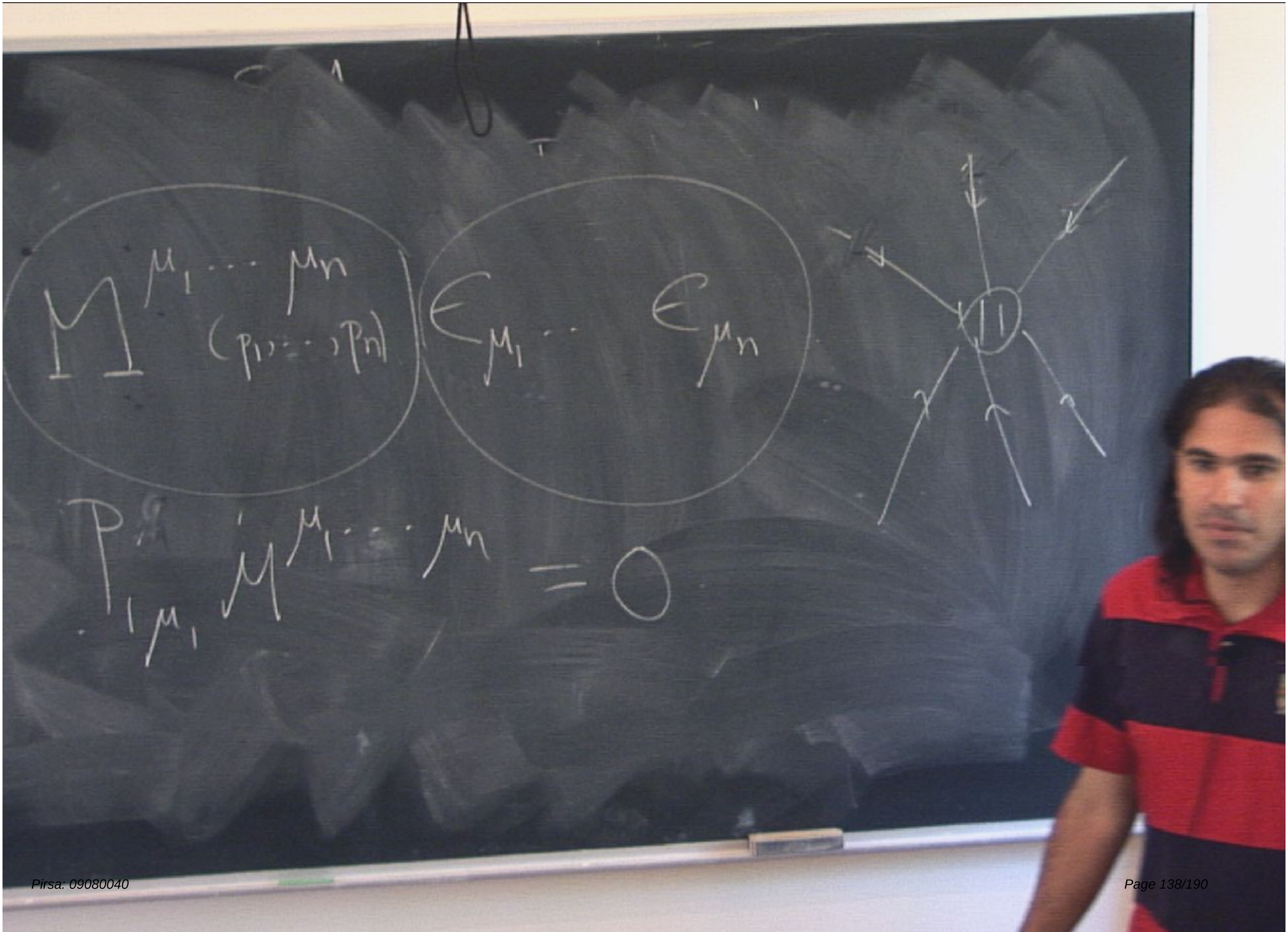
$$M^{\mu_1 \cdots \mu_n}_{(p_1, \ldots, p_n)} \epsilon_{\mu_1} \cdots \epsilon_{\mu_n} .$$

$$P$$

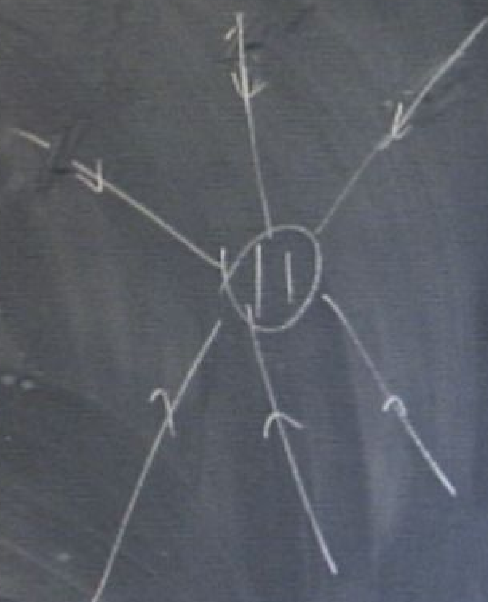
$$M^{\mu_1 \cdots \mu_n}(p_1, \ldots, p_n)\, \epsilon_{\mu_1} \cdots \epsilon_{\mu_n}$$

P


$M^{\mu_1 \cdots \mu_n}(p_1, \ldots, p_n)$
$\epsilon_{\mu_1} \cdots \epsilon_{\mu_n}$
P_1

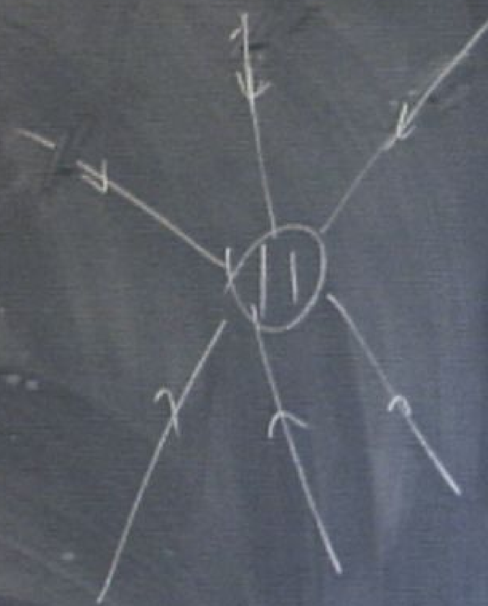

$M^{\mu_1 \cdots \mu_n}(p_1, \ldots, p_n) \; \epsilon_{\mu_1} \cdots \epsilon_{\mu_n}$
$p_{1\mu_1} M^{\mu_1 \cdots \mu_n} = 0$

$$M^{\mu_1 \cdots \mu_n}(p_1, \ldots, p_n)\, \epsilon_{\mu_1} \cdots \epsilon_{\mu_n}$$



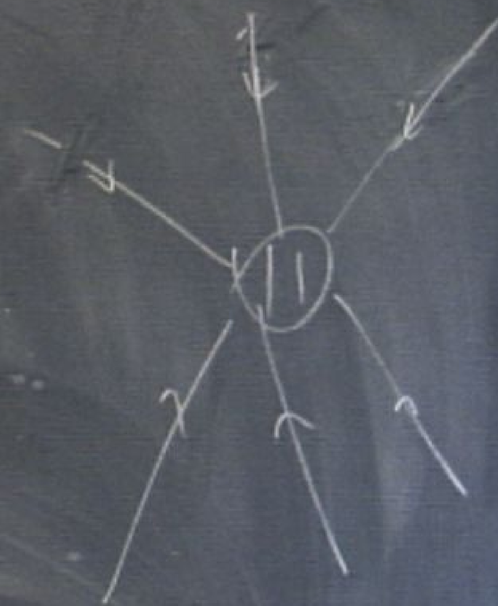
$$p_{1\mu_1} M^{\mu_1 \cdots \mu_n} = 0$$

$$M^{\mu_1 \cdots \mu_n}(p_1, \dots, p_n) \quad \epsilon_{\mu_1} \cdots \epsilon_{\mu_n}$$

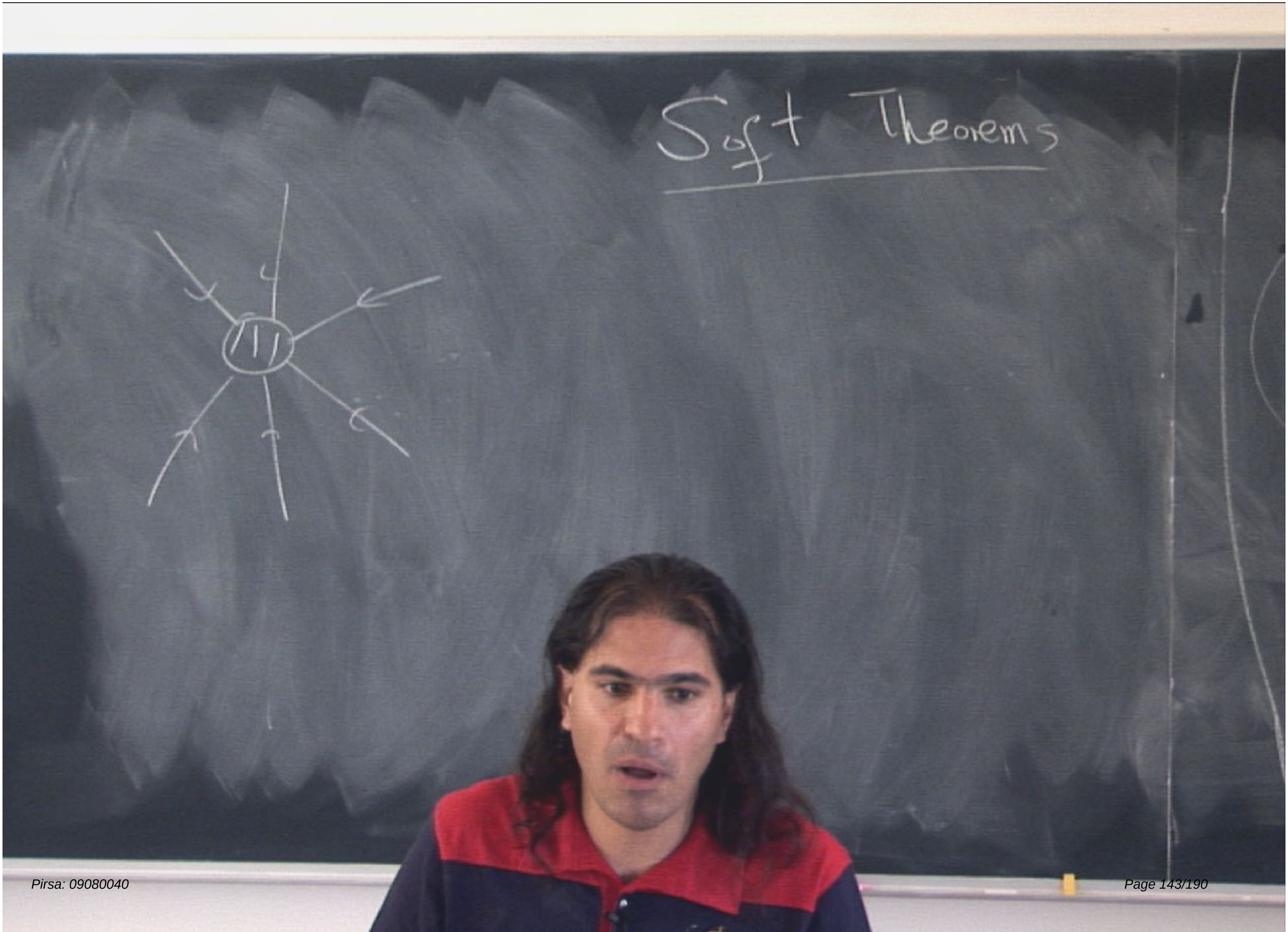


$$p_{1\mu_1} M^{\mu_1 \cdots \mu_n} = 0$$

$$M^{\mu_1 \cdots \mu_n}(p_1, \dots, p_n)\, \epsilon_{\mu_1} \cdots \epsilon_{\mu_n}$$

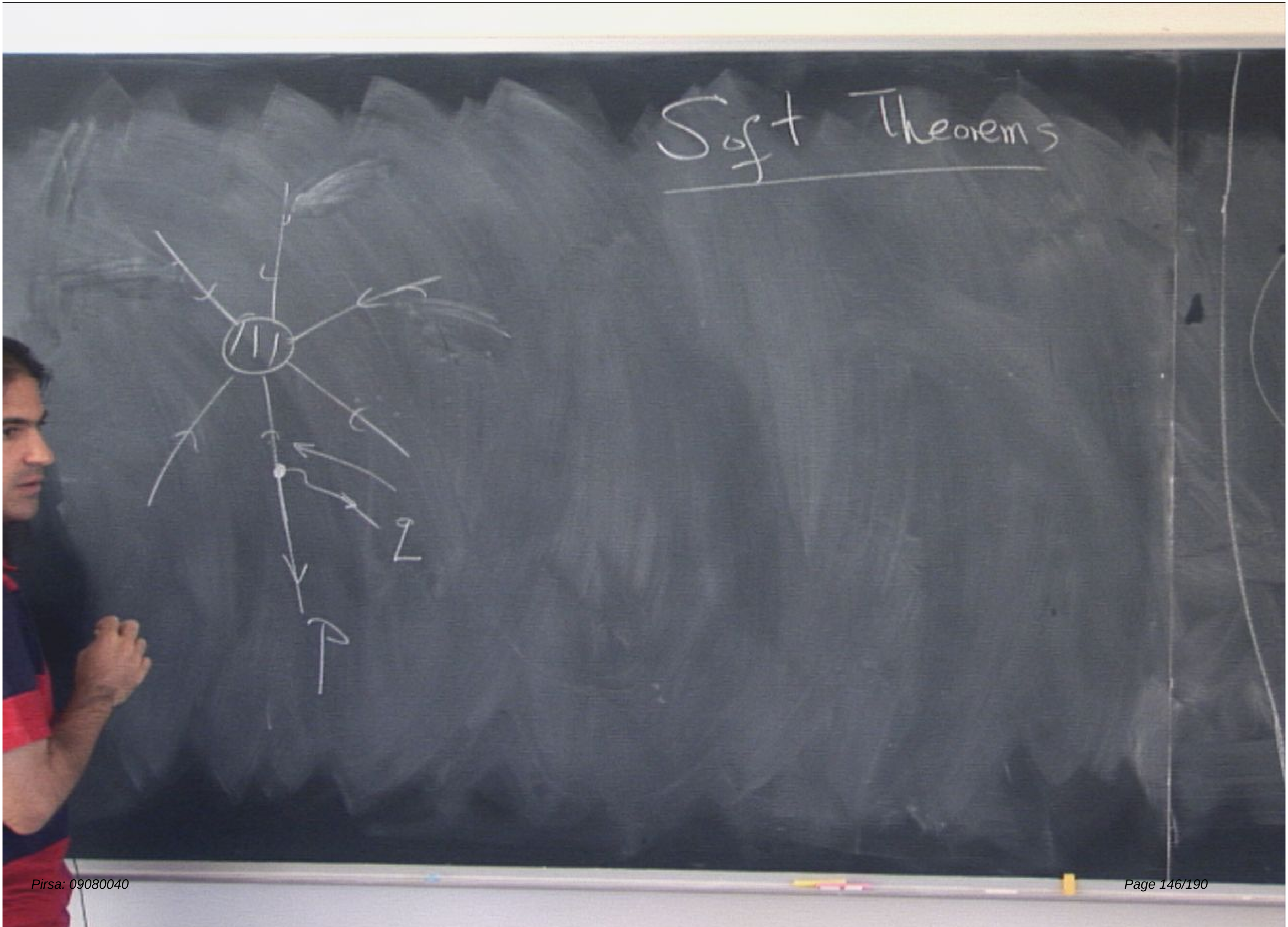


$$p_{1\mu_1} M^{\mu_1 \cdots \mu_n} = 0$$

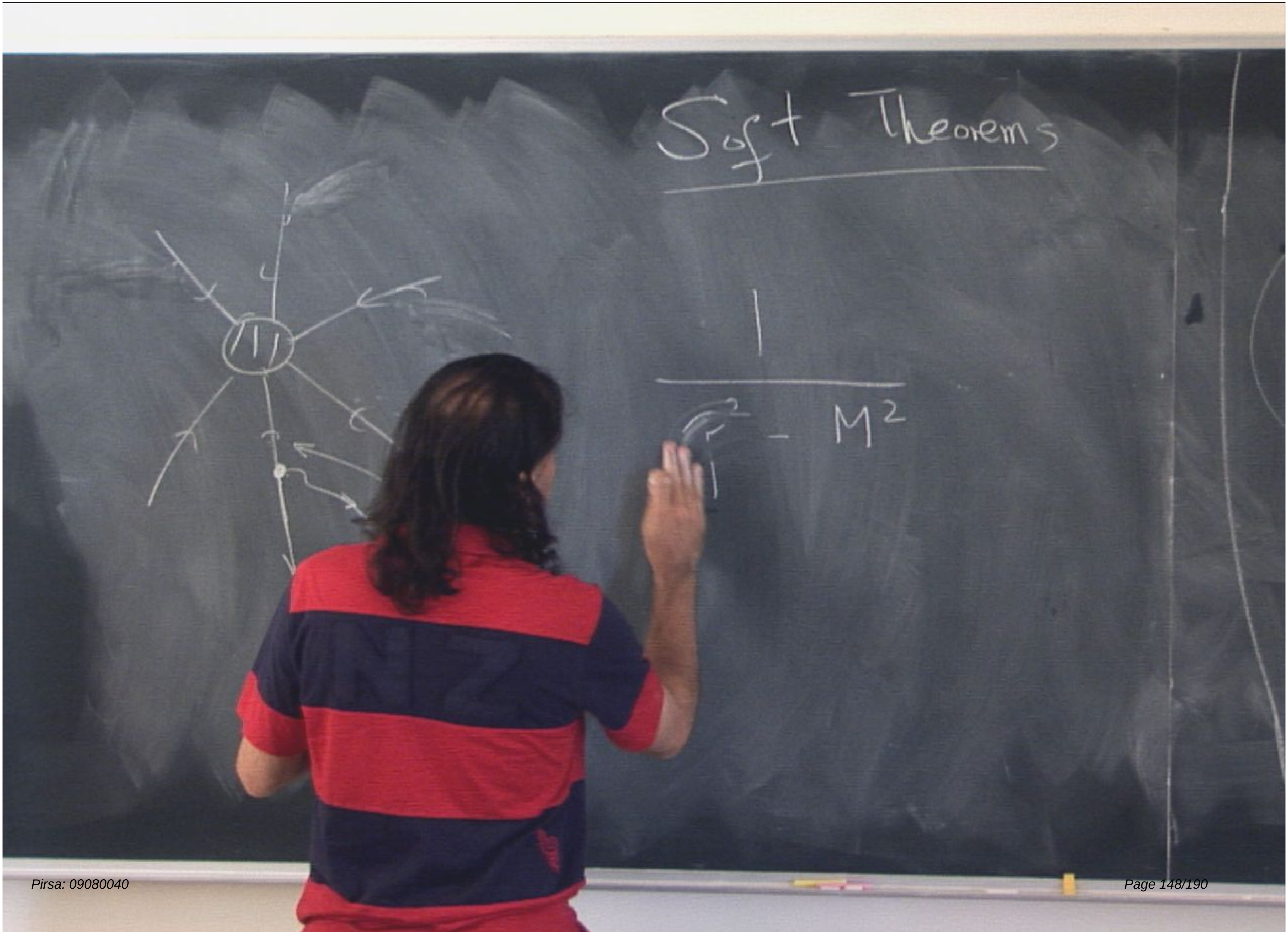

Soft Theorems

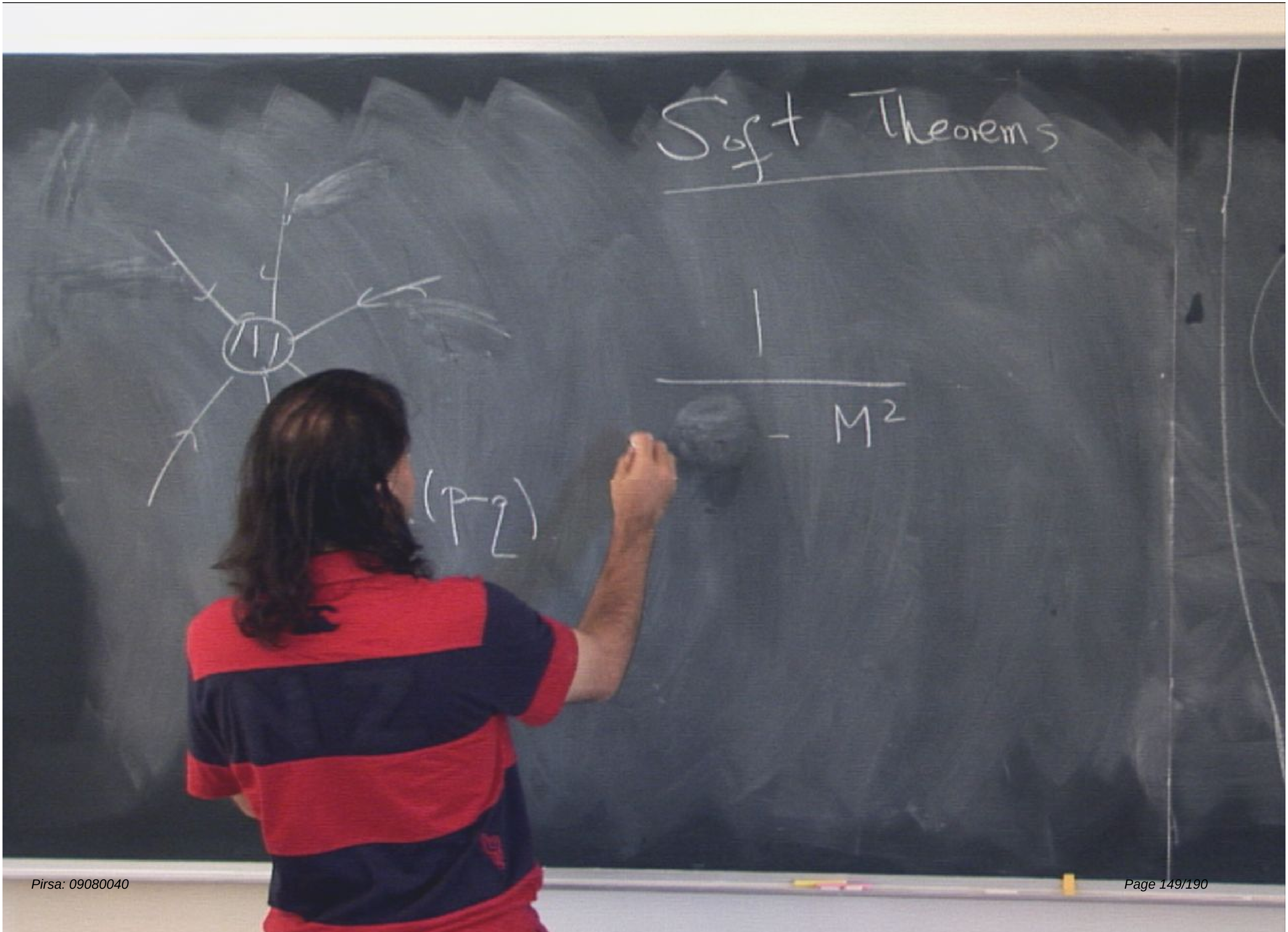
Soft Theorems


Soft Theorems
z

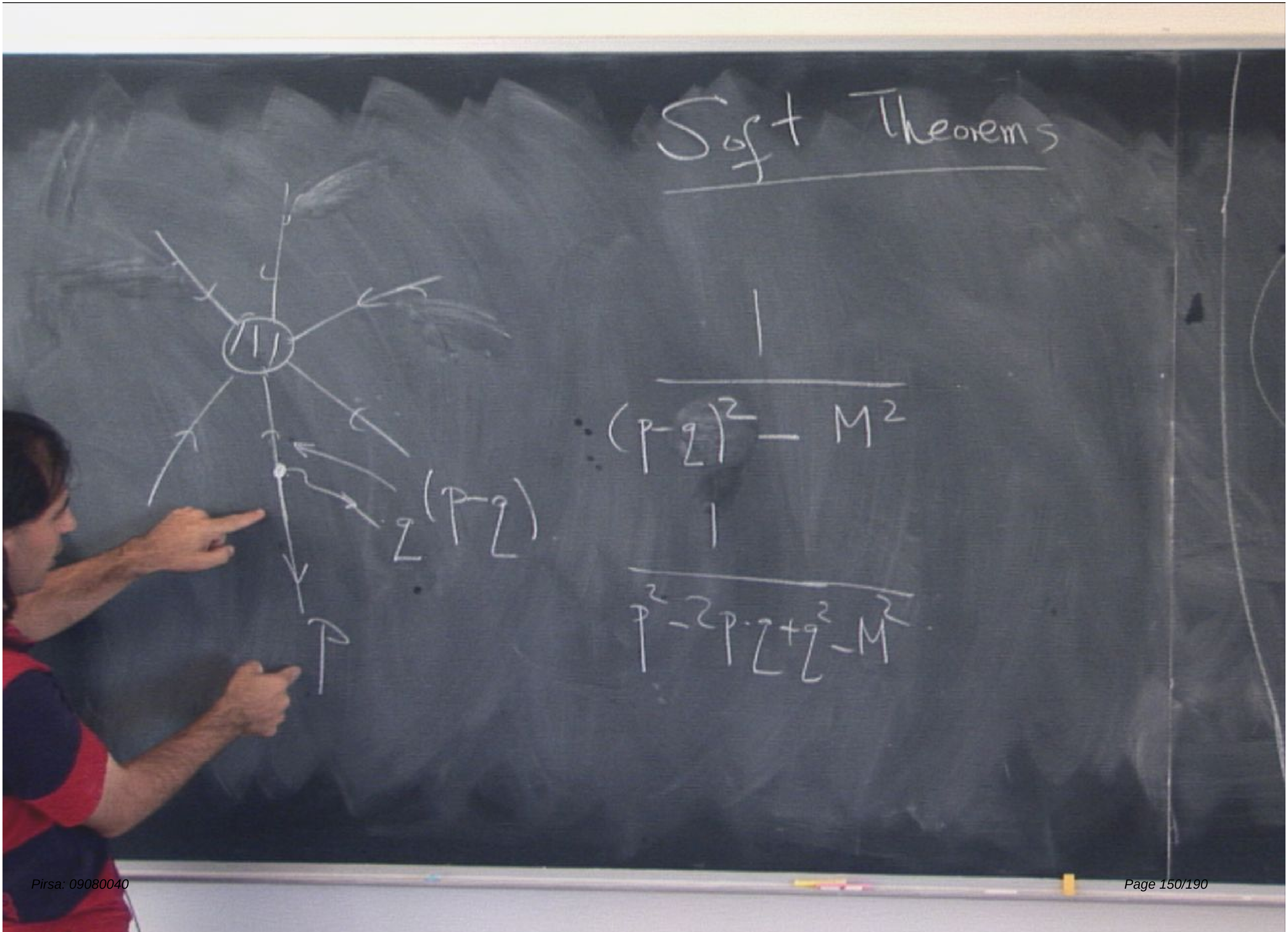

Soft Theorems
q
p


Soft Theorems

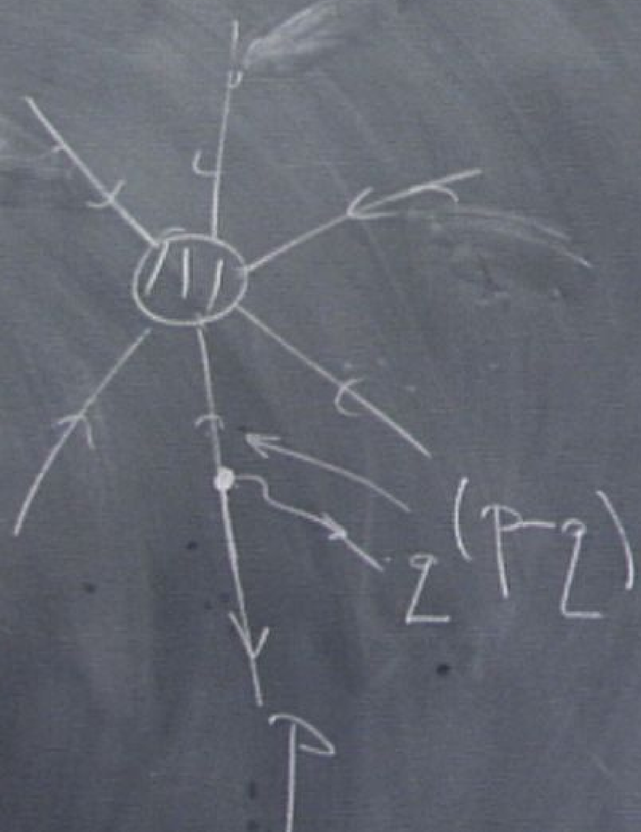

Soft Theorems



Soft Theorems
1
M²
(p-q)


Soft Theorems
q
$(p-q)$
p
$\frac{1}{(p-q)^2 - M^2}$
$\frac{1}{p^2-2p\cdot q+q^2-M^2}$

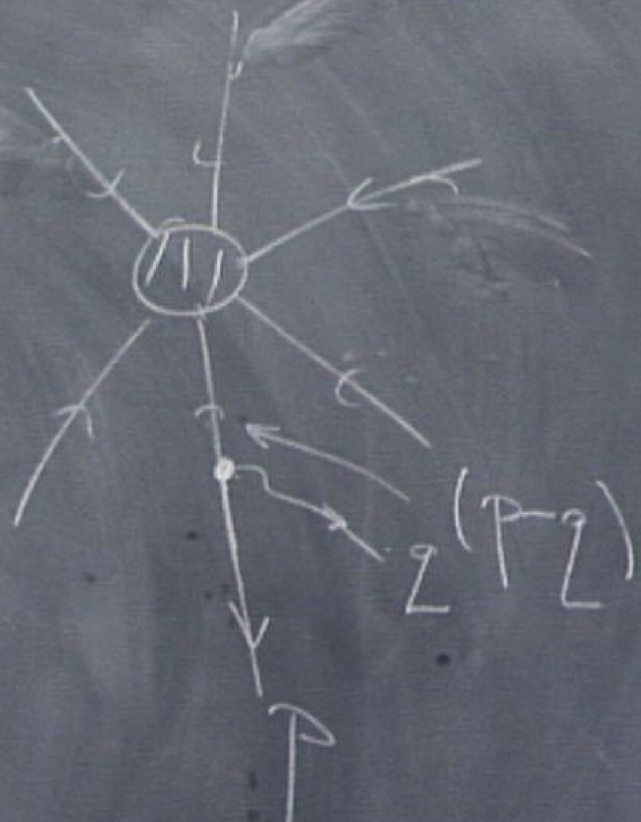
Soft Theorems



$$\frac{1}{(p-q)^2 - M^2}$$

$$\frac{1}{\cancel{p^2} - 2p\cdot q + q^2 - \cancel{M^2}}$$

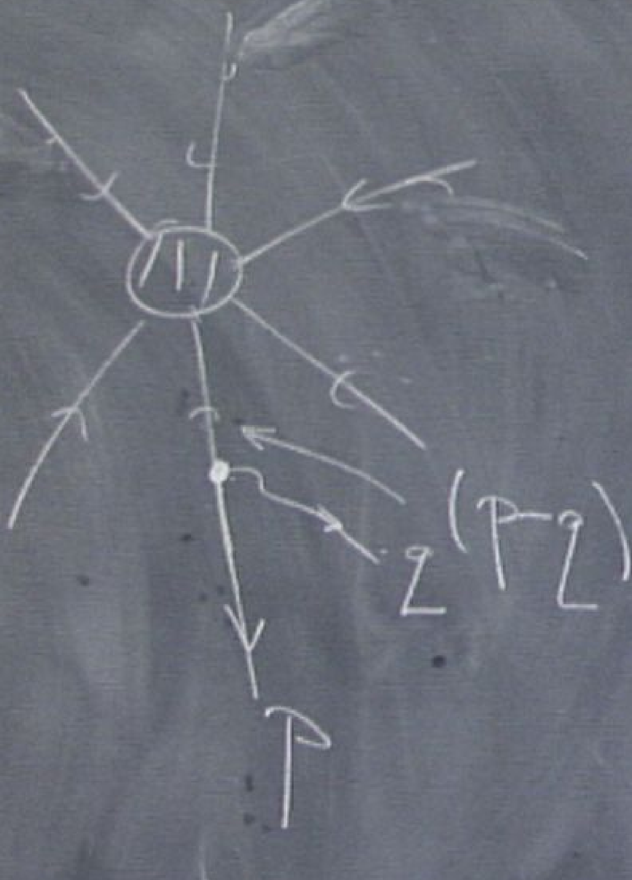
Soft Theorems



$$\frac{1}{(p-q)^2 - M^2}$$

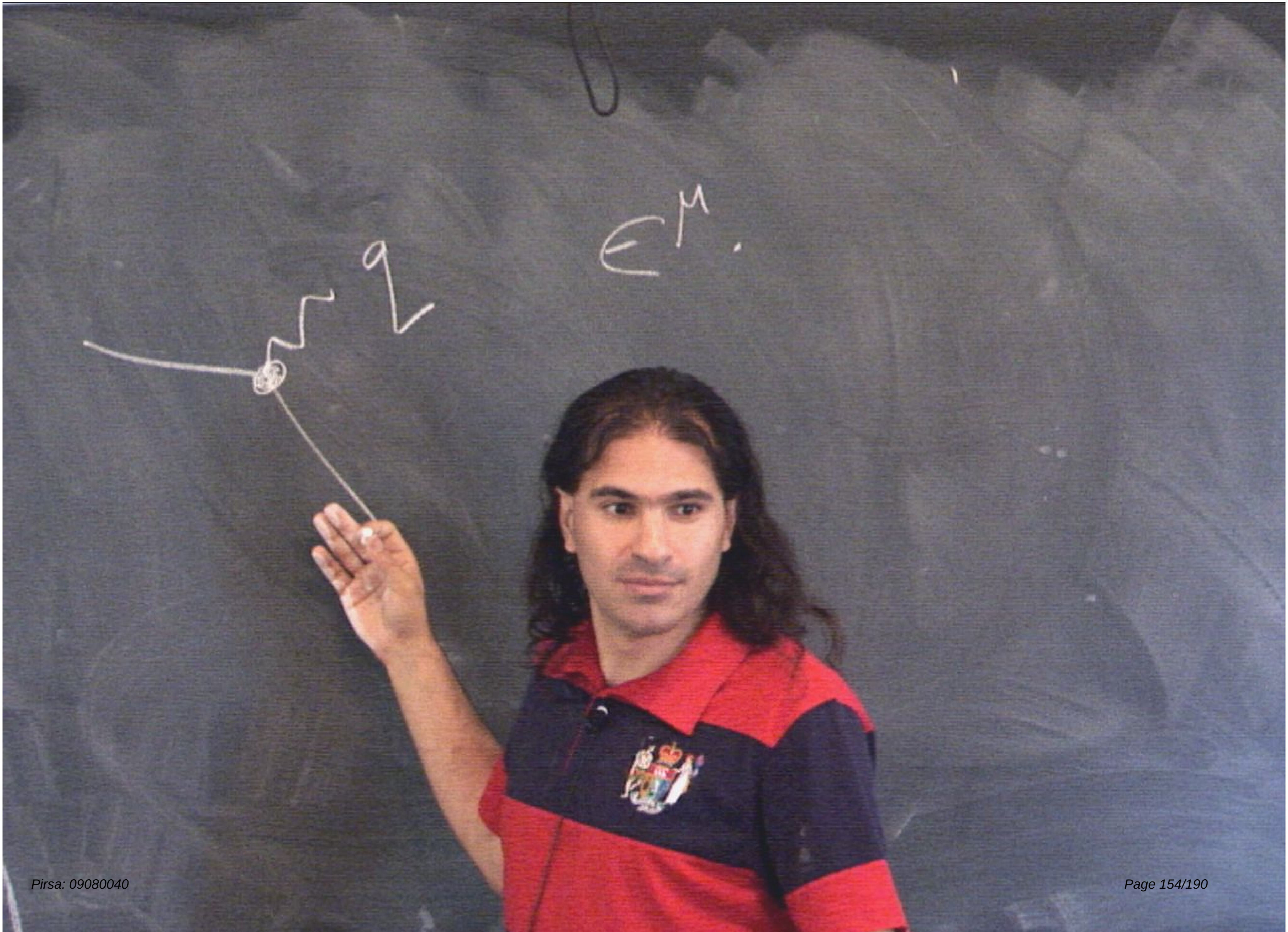
$$\frac{1}{\cancel{p^2} - 2p\cdot q + \cancel{q^2} - \cancel{M^2}}$$

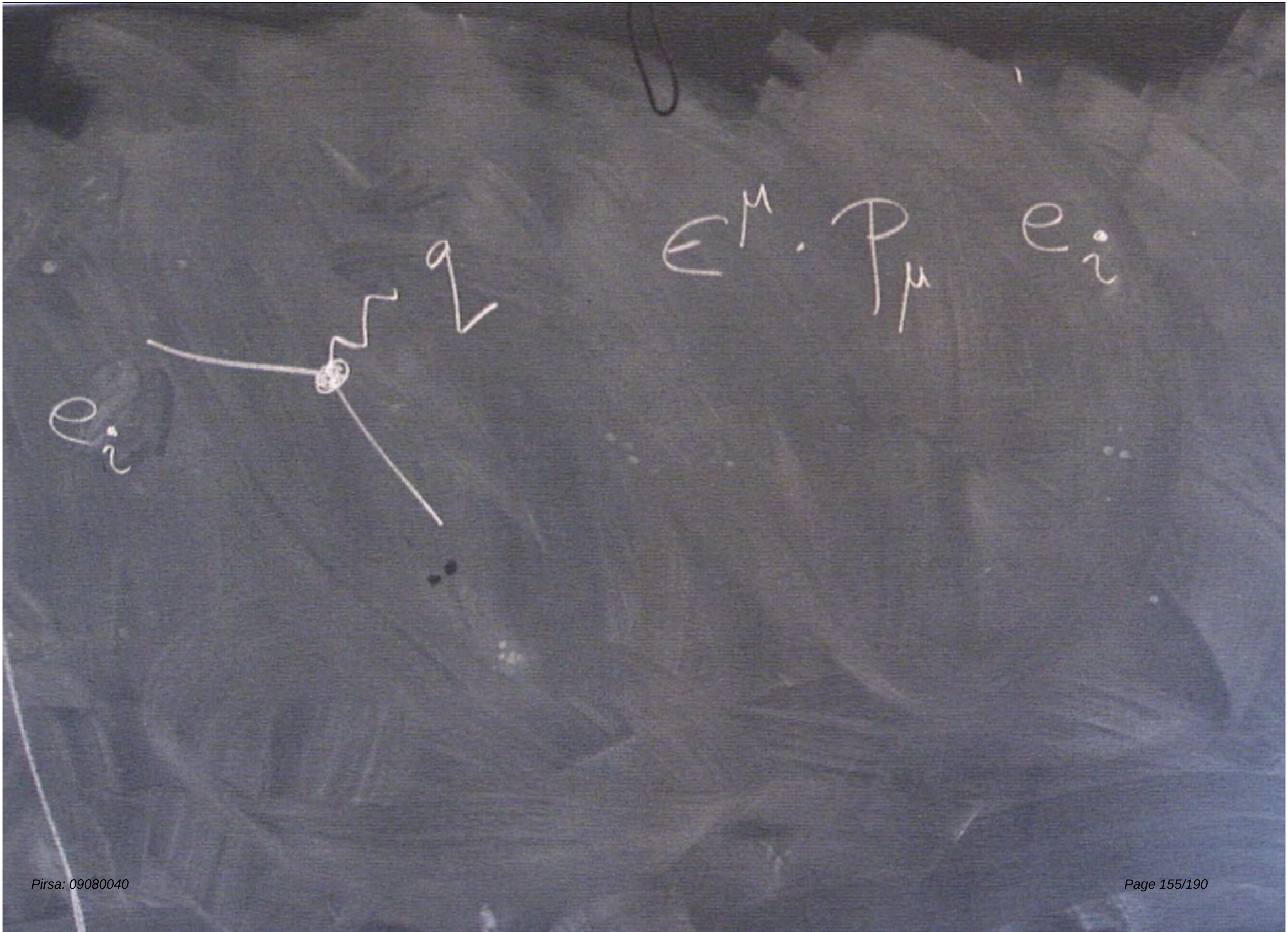
Soft Theorems

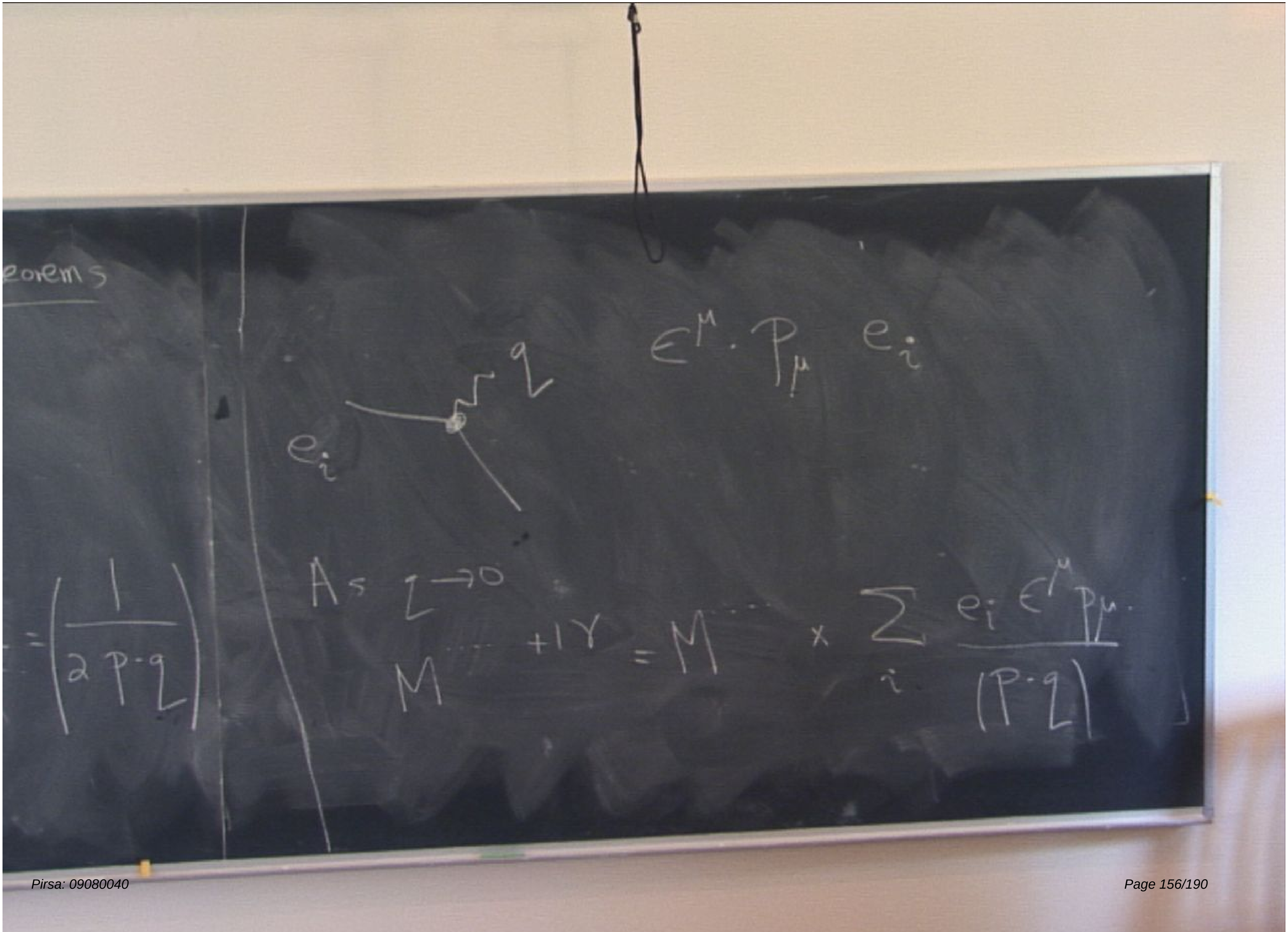


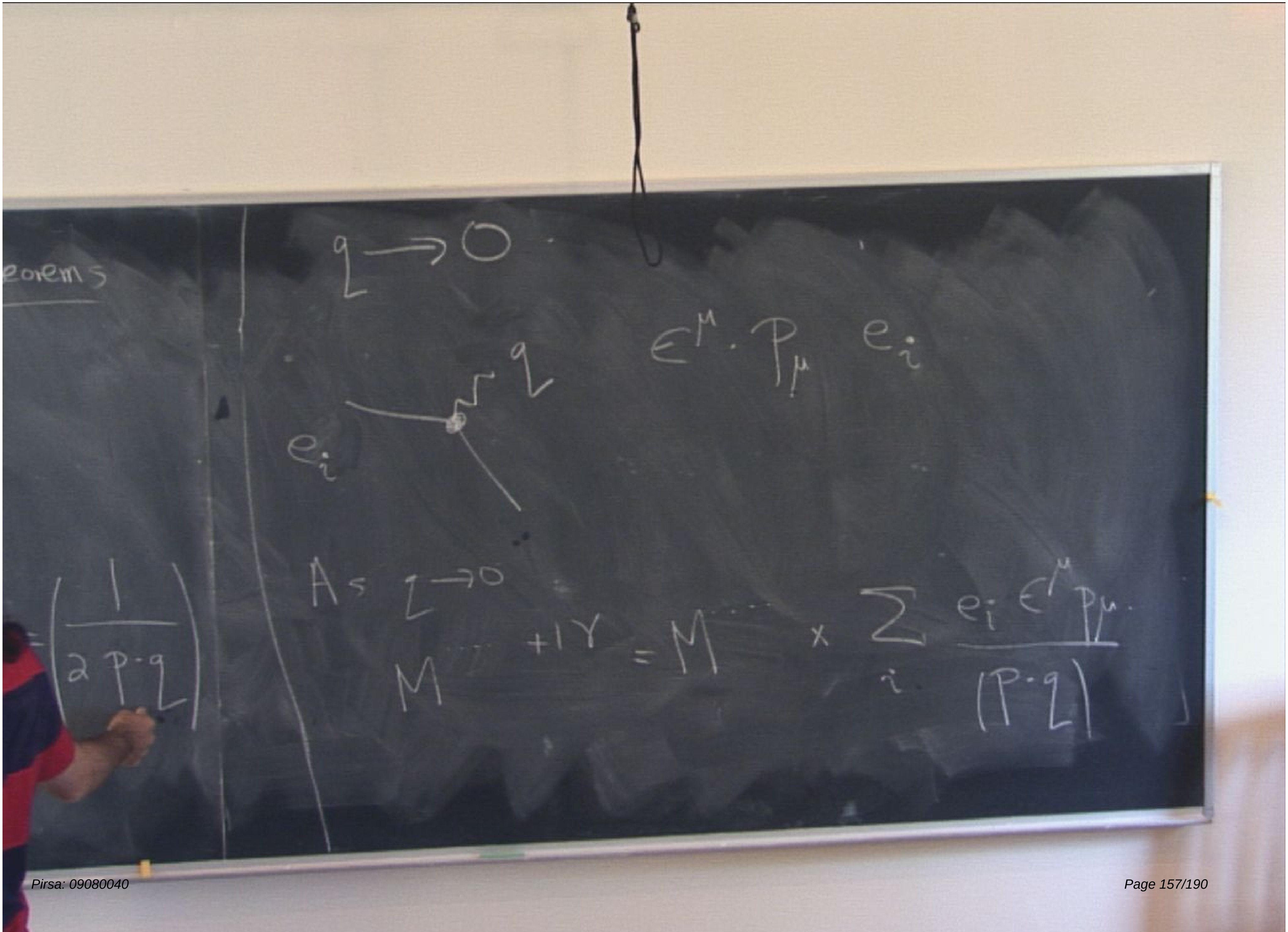
$$\frac{1}{(p-q)^2 - M^2}$$

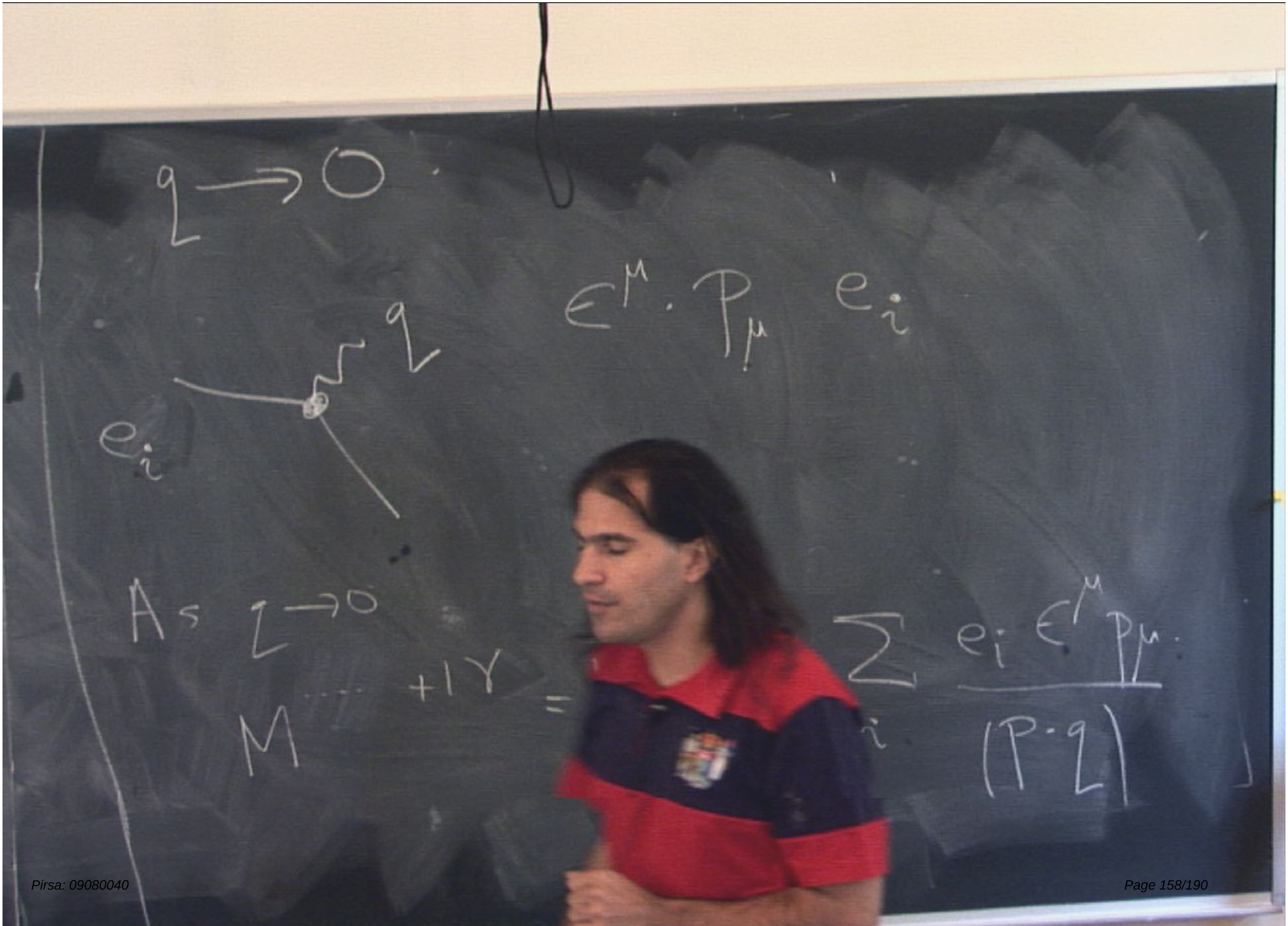
$$\frac{1}{\cancel{p^2} - 2p\cdot q + \cancel{q^2} - \cancel{M^2}} = \left(\frac{1}{2p\cdot q}\right)$$


q
ε^μ.

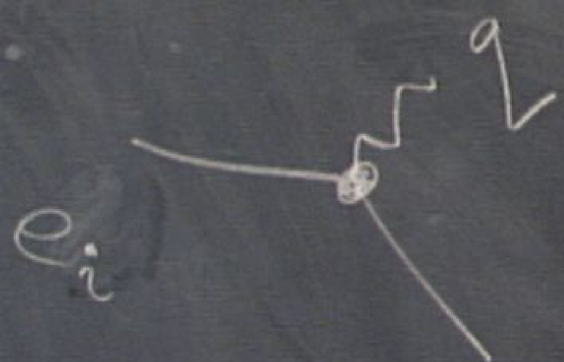

e_i
q
$\epsilon^\mu \cdot p_\mu \quad e_i$


eorems
q
e_i
ε^μ · P_μ   e_i
As q → 0
= (1 / 2 P·q)
M + 1γ = M × Σ_i e_i ε^μ P_μ / (P·q)


eorems
q → 0
ε^μ · P_μ   e_i
q
e_i
(1 / 2 p·q)
As q → 0
M + 1γ = M × Σ_i e_i ε^μ p_μ / (P·q)


q → 0
ε^μ · p_μ
e_i
q
e_i
As q → 0
M ... +1γ = Σ_i e_i ε^μ p_μ / (p · q)

$q \to 0$



$\epsilon^\mu \cdot p_\mu \quad e_i$

As $q \to 0$

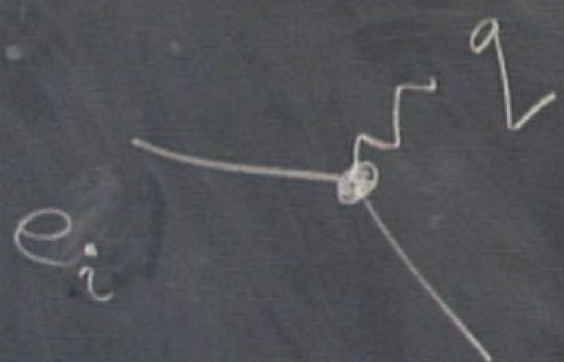
$$M^{\cdots + 1\gamma} = M^{\cdots} \times \sum_i \frac{e_i \, \epsilon^\mu \cdot p_\mu}{(p \cdot q)}$$

$\epsilon^\mu \to q^\mu$ should give

$q \to 0$

$\epsilon^\mu \cdot p_\mu \quad e_i$

$$\boxed{\sum_i e_i = 0}$$



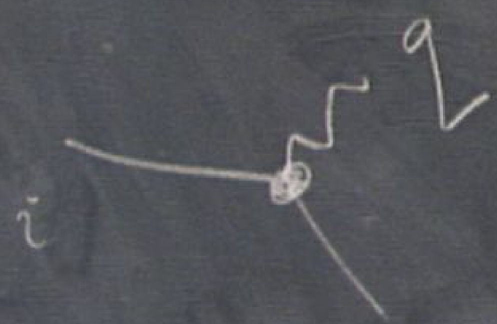
As $q \to 0$

$$M^{\cdots + 1\gamma} = M^{\cdots} \times \sum_i \frac{e_i \, \epsilon^\mu p_{i\mu}}{(p_i \cdot q)}$$

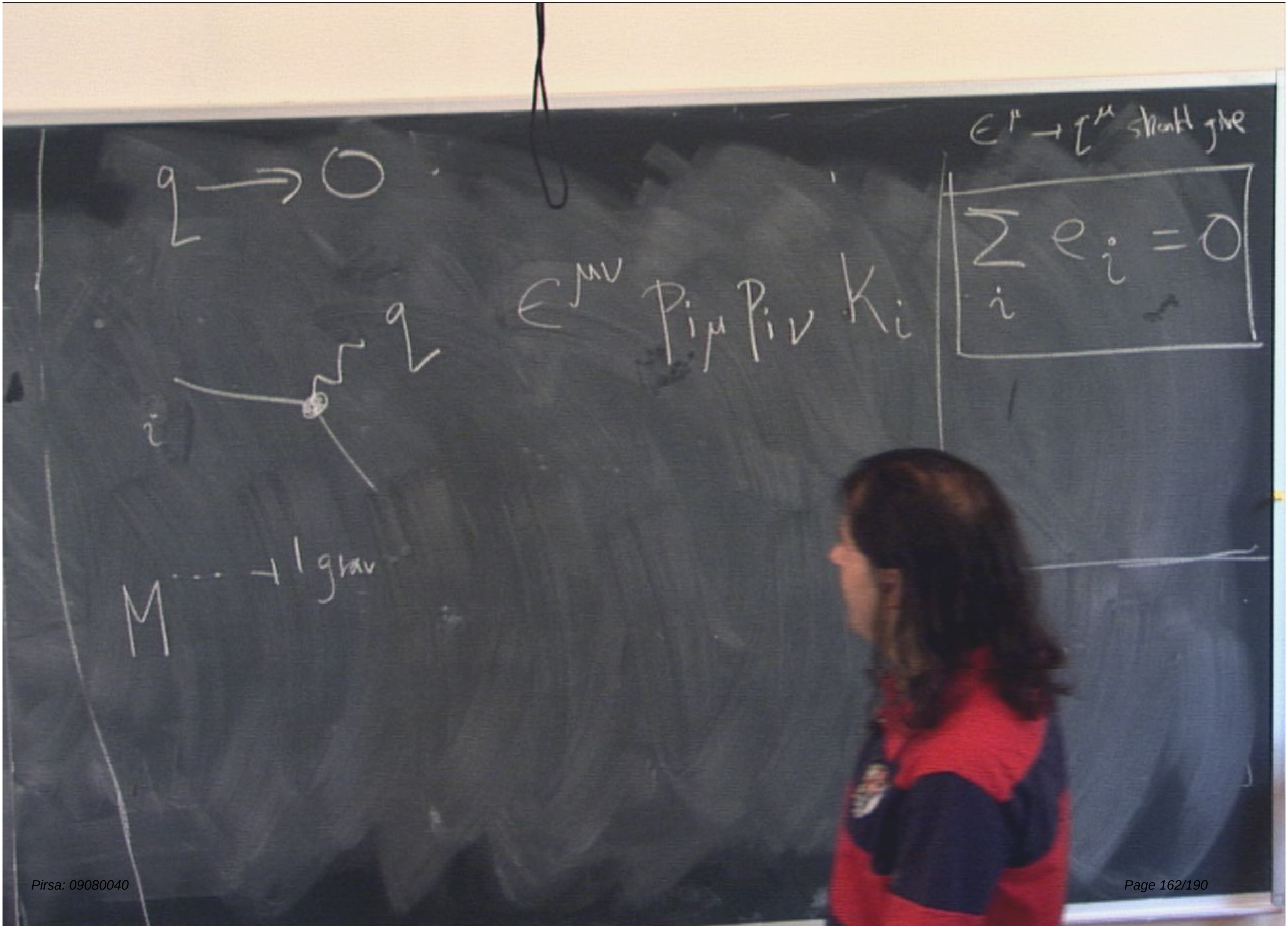
$\epsilon^\mu \to \xi^\mu$ shouldn't give

$q \to 0$.

$$\boxed{\sum_i e_i = 0}$$



$\epsilon^{\mu\nu} p_{i\mu} p_{i\nu}$.

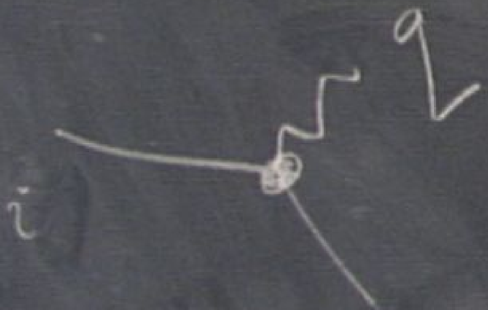

$q \to 0$
$\epsilon^{\mu} \to \epsilon^{\mu}$ should give
$\sum_i e_i = 0$
$\epsilon^{\mu\nu} p_{i\mu} p_{i\nu} K_i$
q
i
$M \cdots \to l_{grav}$

$\epsilon^{\mu} \to \epsilon^{\mu}$ should give

$q \to 0$

$$\boxed{\sum_i e_i = 0}$$

$$\epsilon^{\mu\nu} p_{i\mu} p_{i\nu} K_i$$



$$M^{\cdots \to 1\,\mathrm{grav}} = M^{\cdots} \times \sum_i \frac{\epsilon^{\mu\nu} p_{i\mu} p_{i\nu} K_i}{(p_i \cdot q)}$$

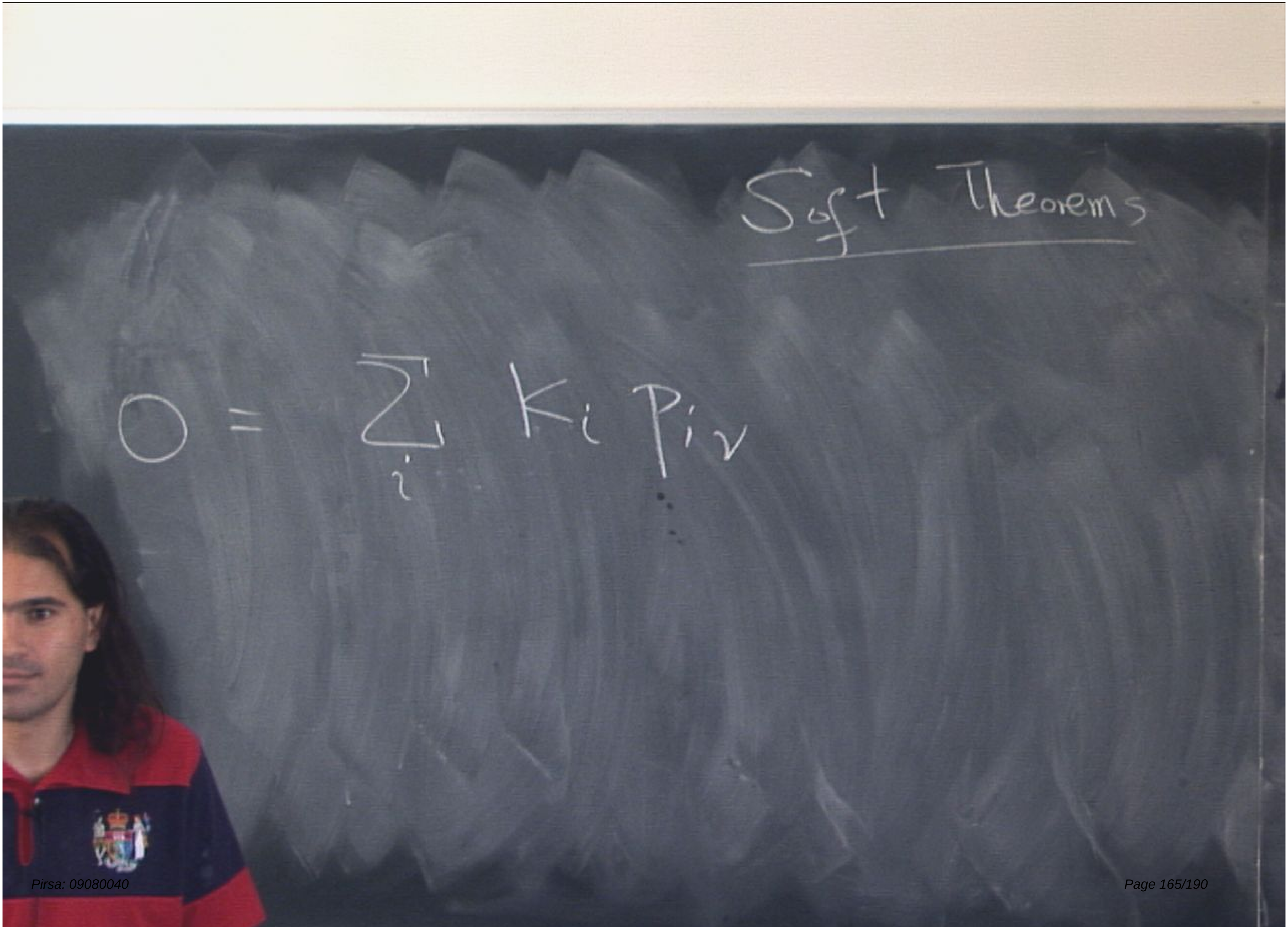
$\epsilon^{\mu} \to \xi^{\mu}$ should give

$$\boxed{\sum_i e_i = 0}$$

$q \to 0$.

$$\epsilon^{\mu\nu} p_{i\mu} p_{i\nu} K_i$$

$$M^{\cdots + 1\,\mathrm{grav}} = M^{\cdots} \times \sum_i \frac{\epsilon^{\mu\nu} p_{i\mu} p_{i\nu} K_i}{(p_i \cdot q)}$$


Soft Theorems
$0 = \sum_i k_i \, p_{i\nu}$

Soft Theorems

$$0 = \sum_i k_i p_{i\nu}$$

Soft Theorems

$$0 = \sum_i k_i \, p_{i\nu}$$

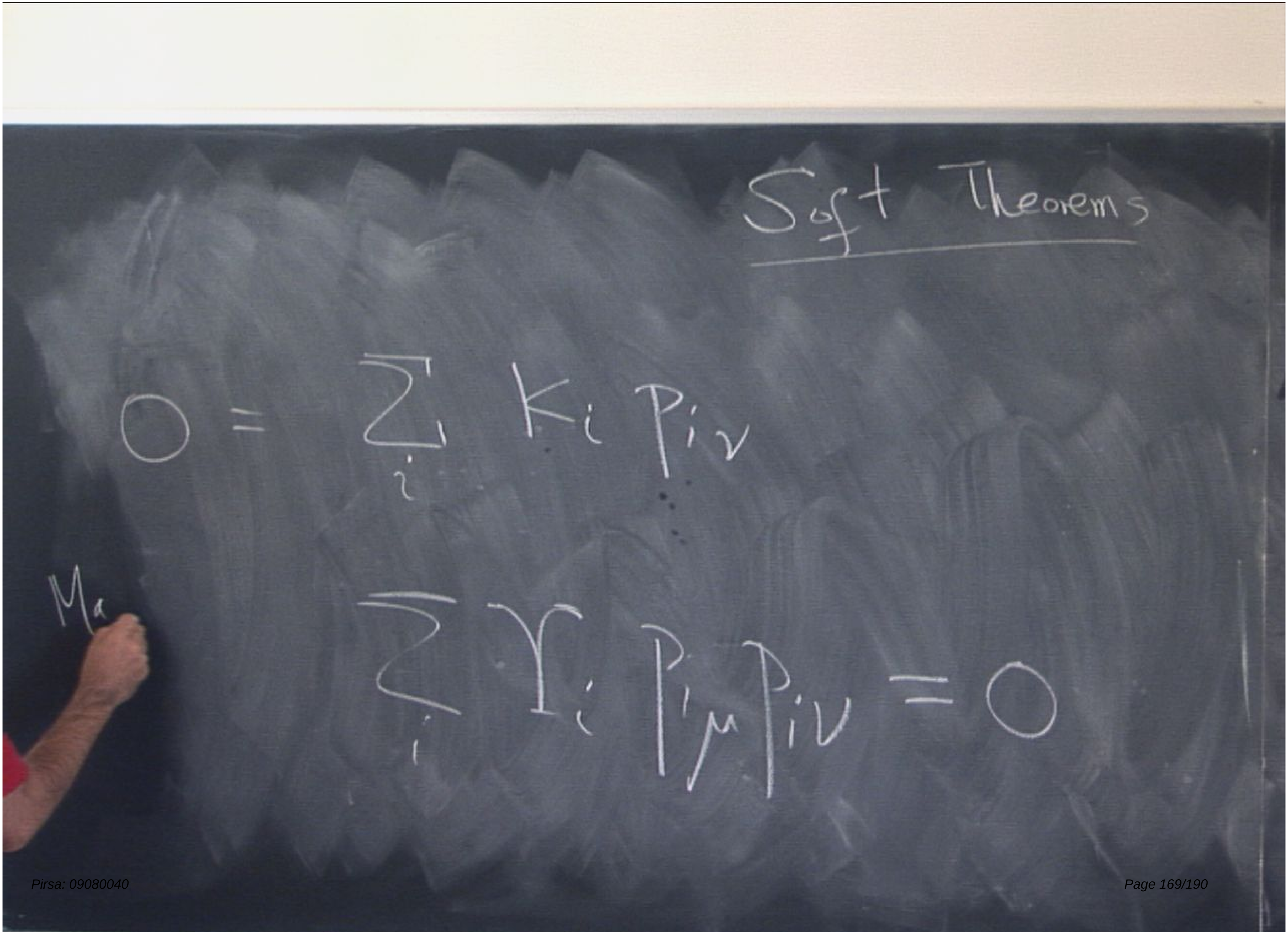
$$\sum_i p_{i\nu} = 0$$

Soft Theorems

$$0 = \sum_i K_i \, p_{i\nu}$$

$$\sum_i p_{i\nu} = 0$$

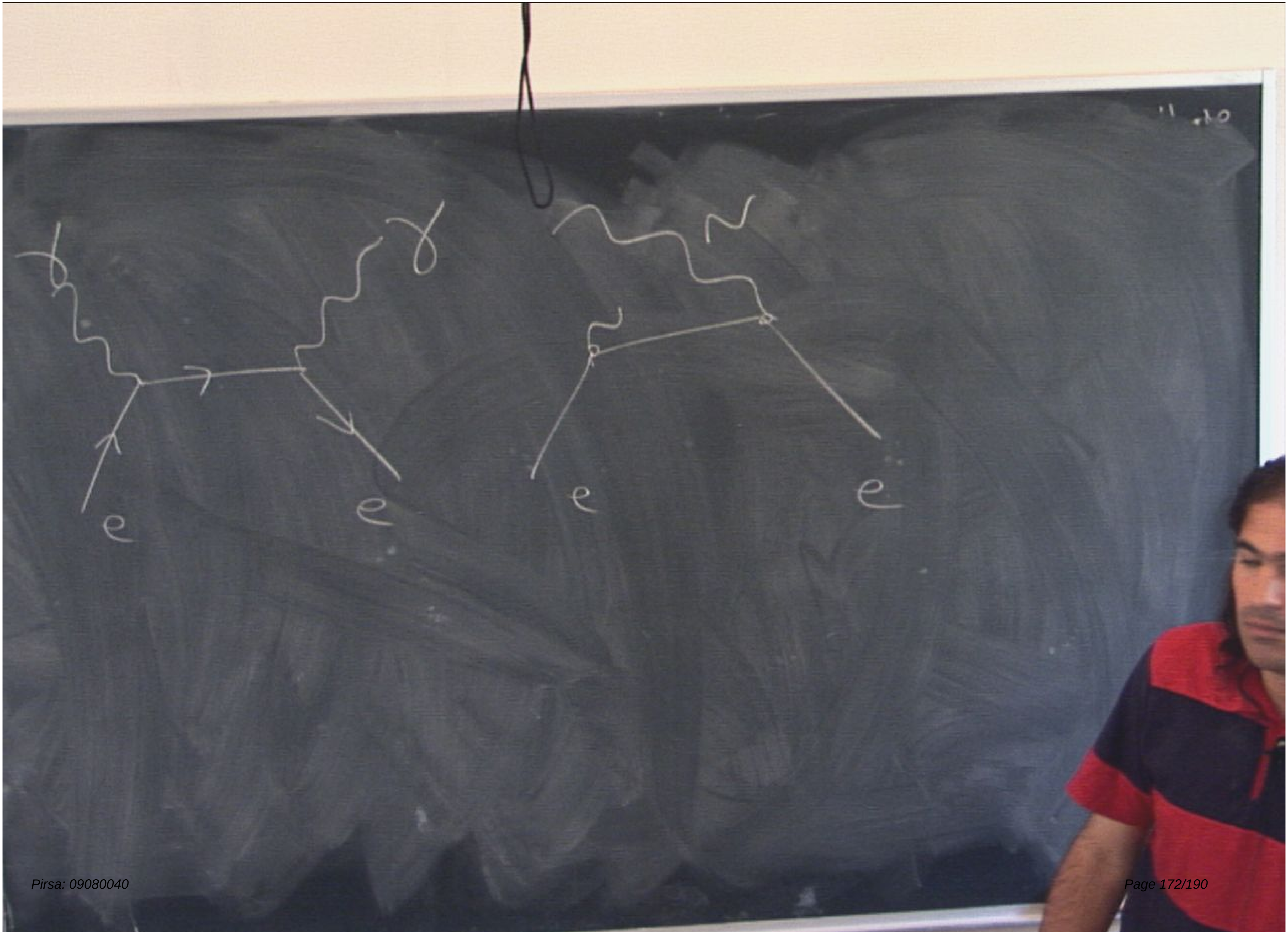
$\Longrightarrow K_i$ is universal

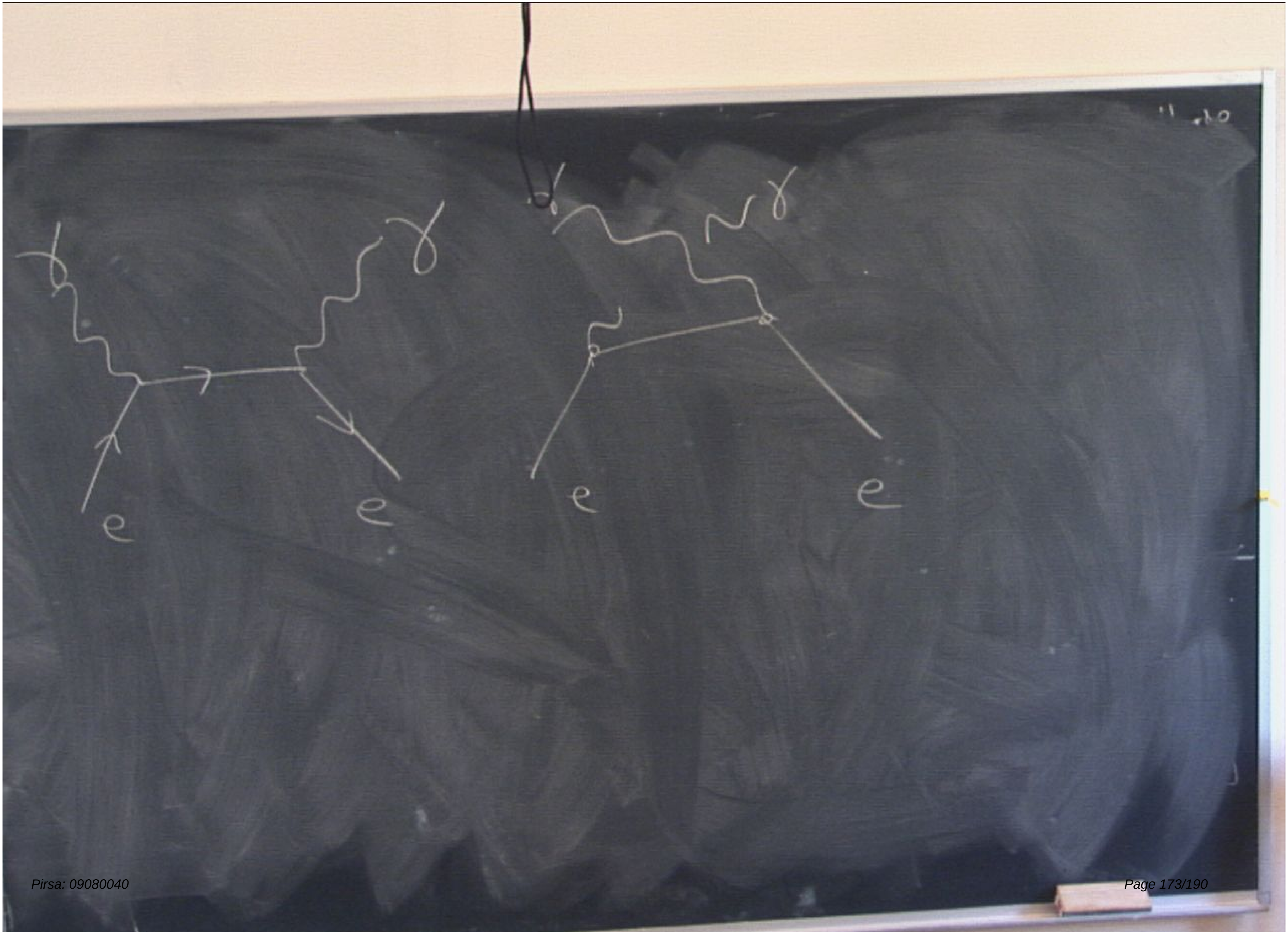

Soft Theorems
$0 = \sum_i \kappa_i p_{i\nu}$
M_a
$\sum_i \eta_i p_{i\mu} p_{i\nu} = 0$

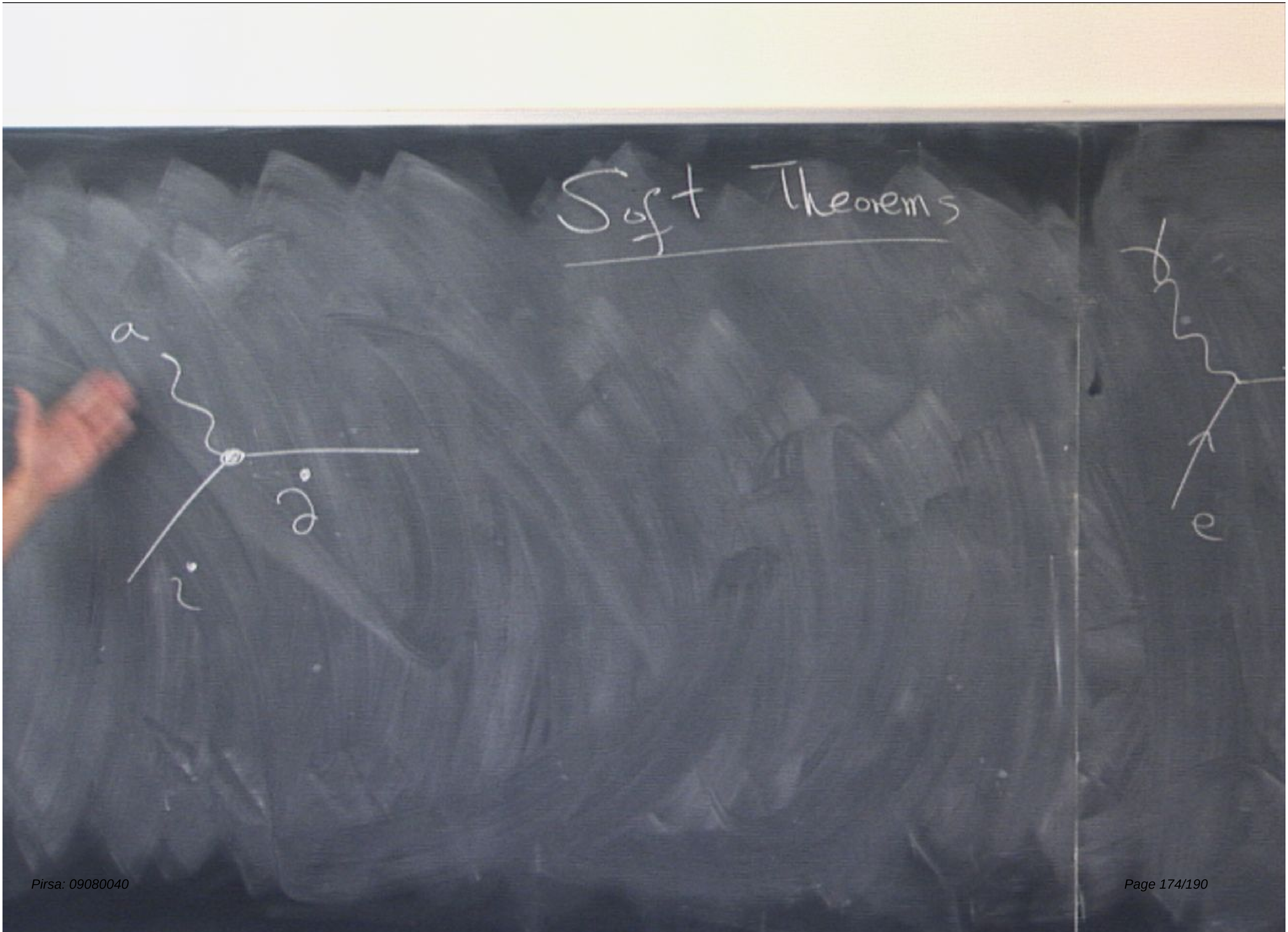
Soft Theorems

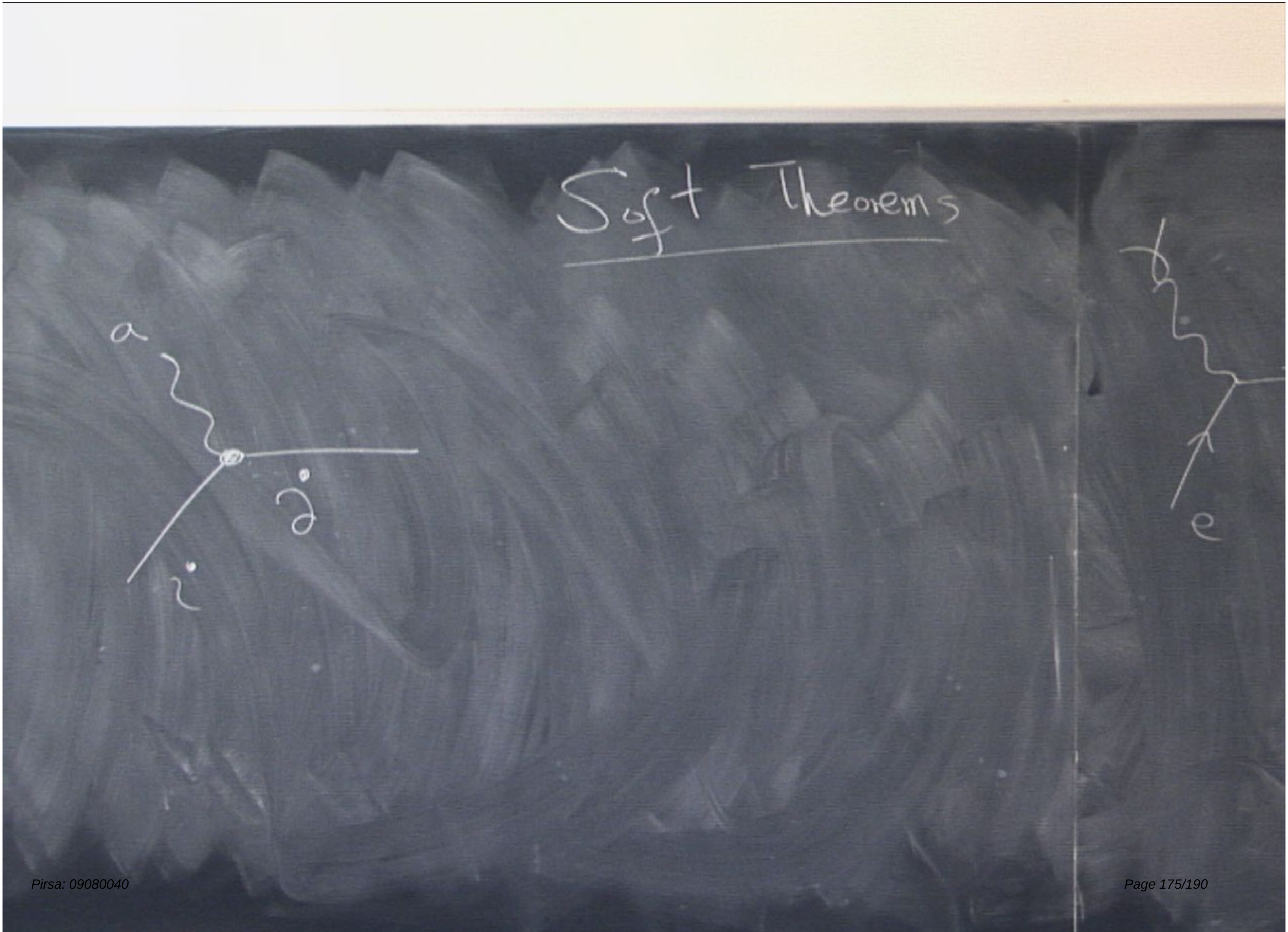
$$0 = \sum_i \kappa_i \, p_{i\nu}$$

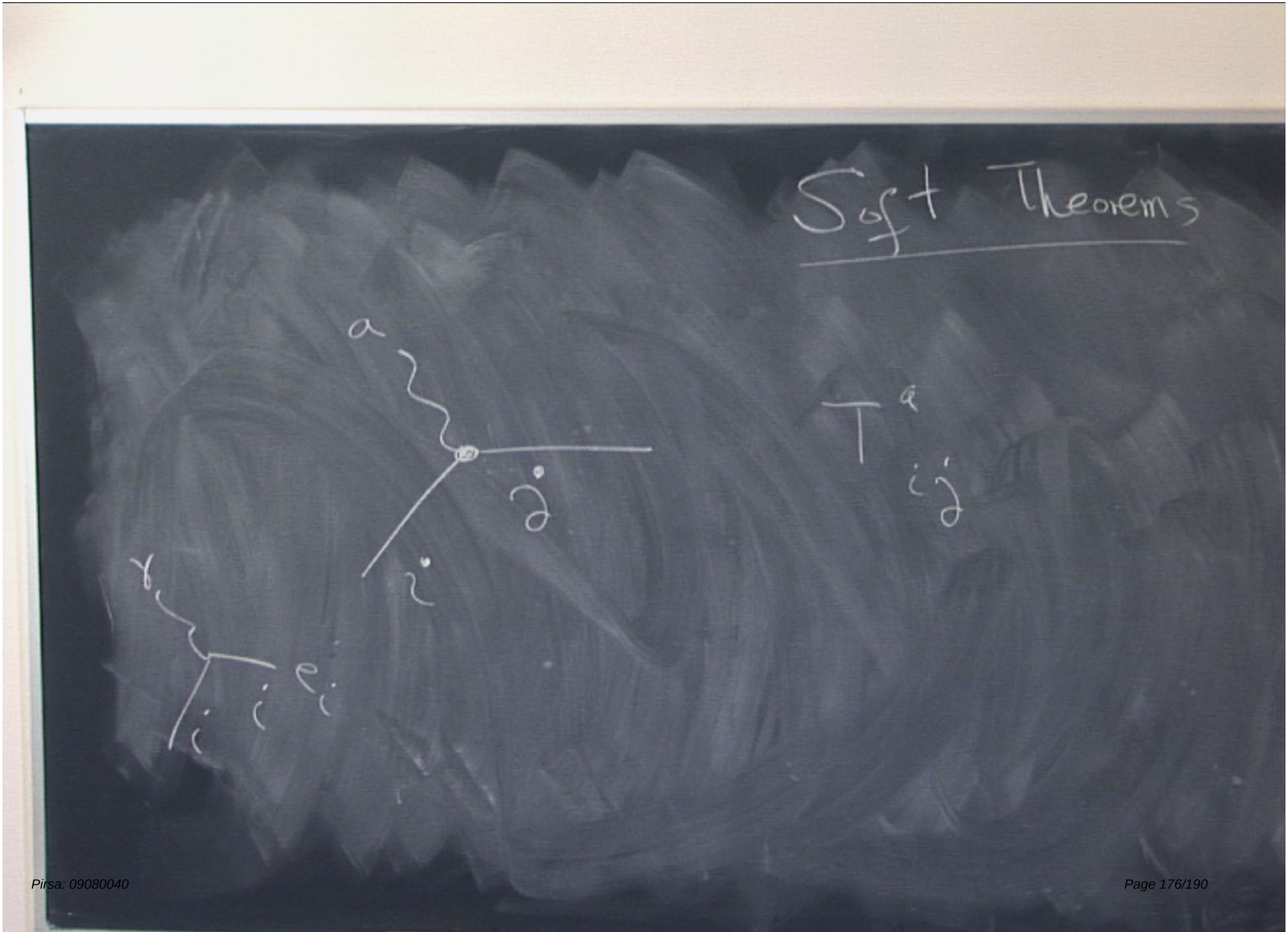
Massless spin 3 : $$\sum_i \gamma_i \, p_{i\mu} p_{i\nu} = 0$$

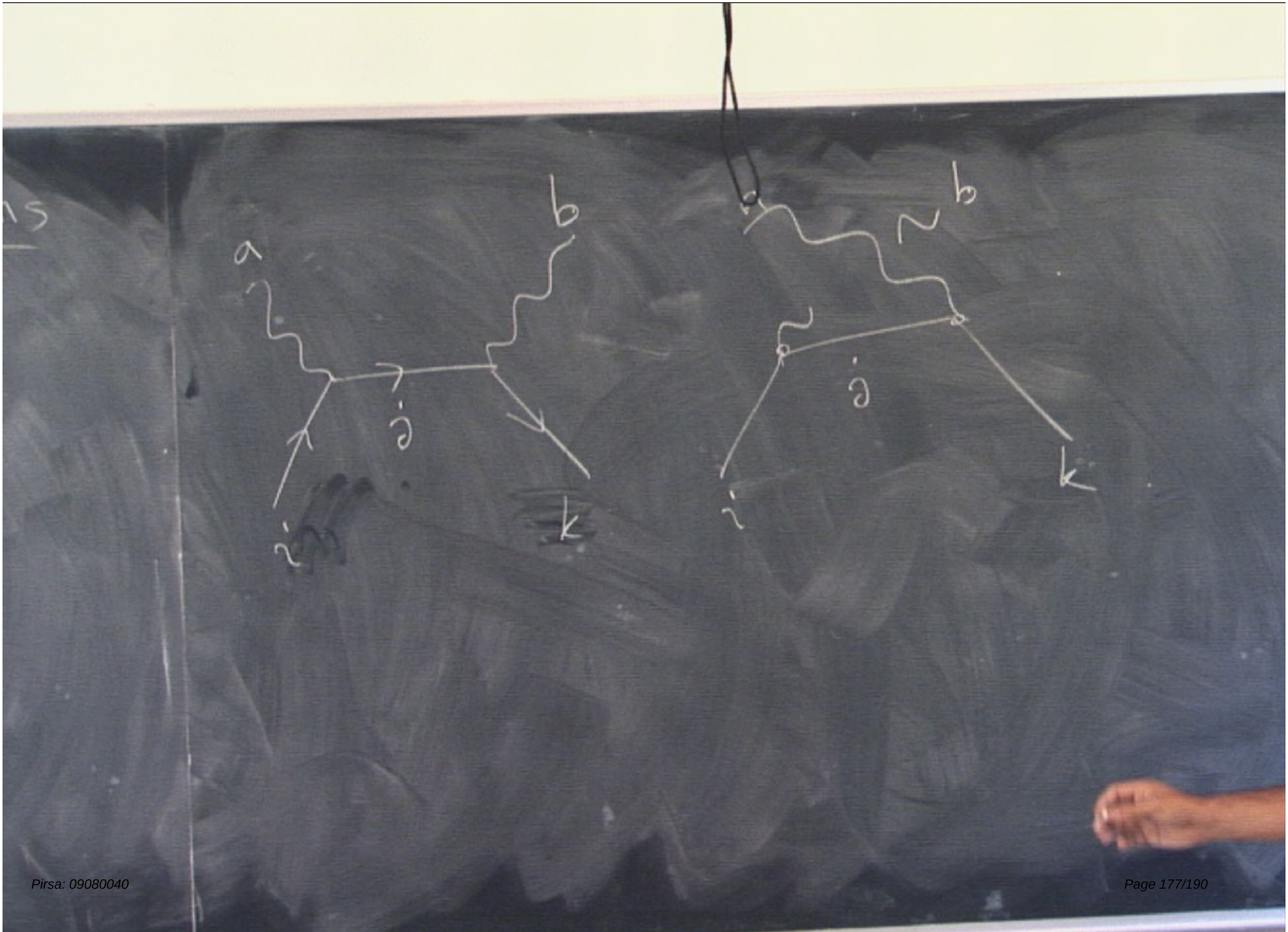

γ
γ
e
e
e
e

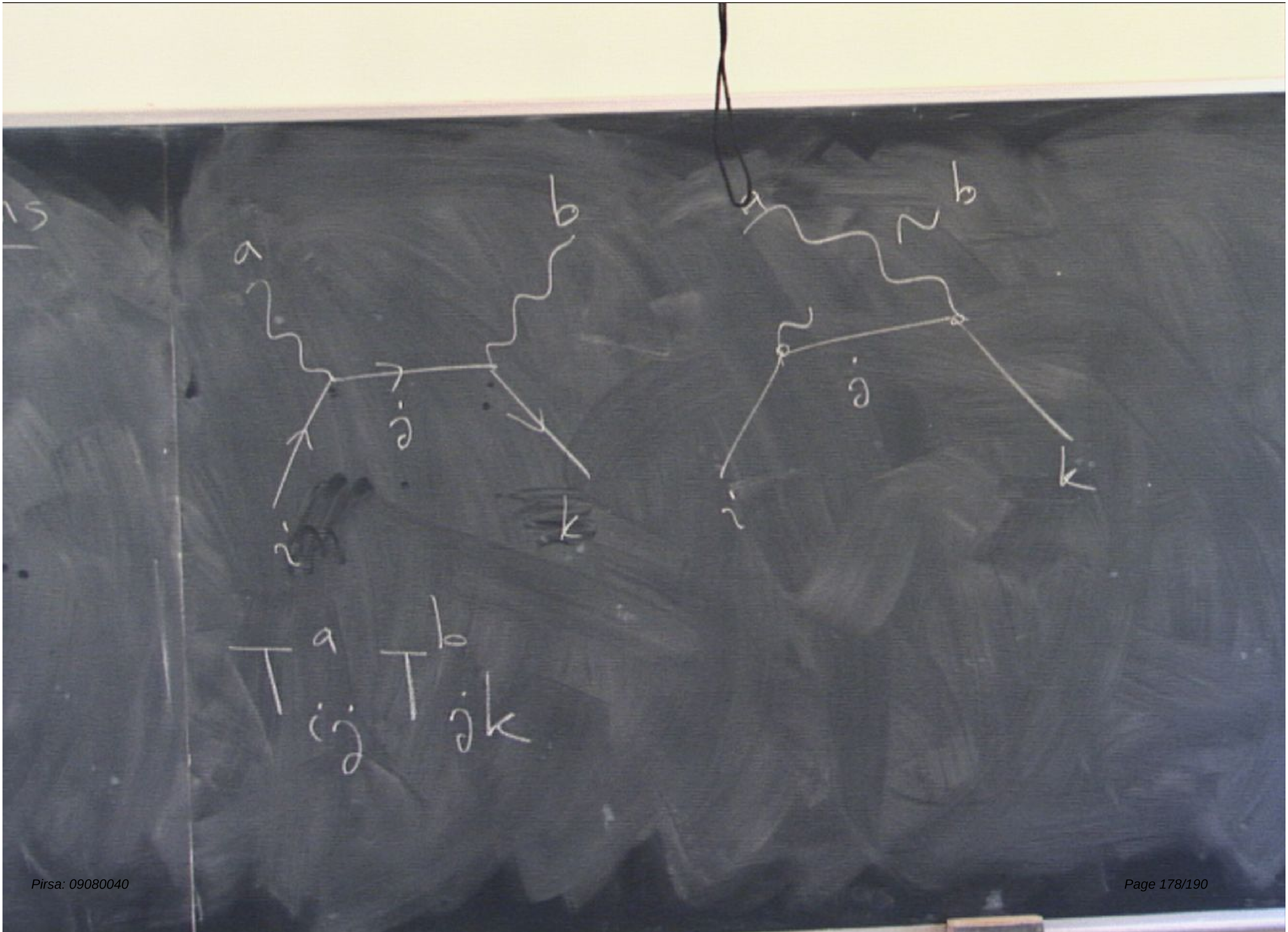

γ
γ
e
e
γ
γ
e
e

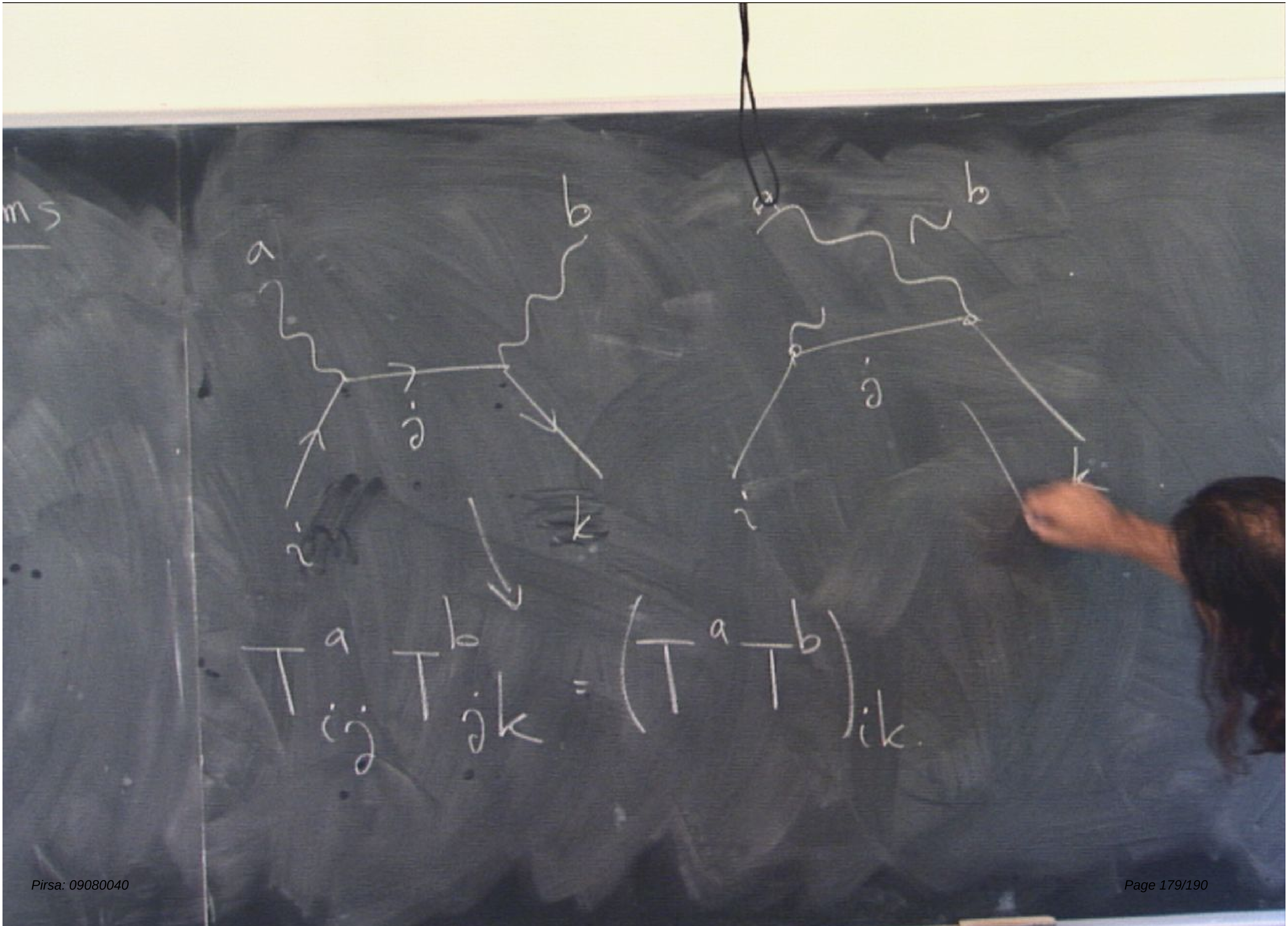

Soft Theorems
a
e

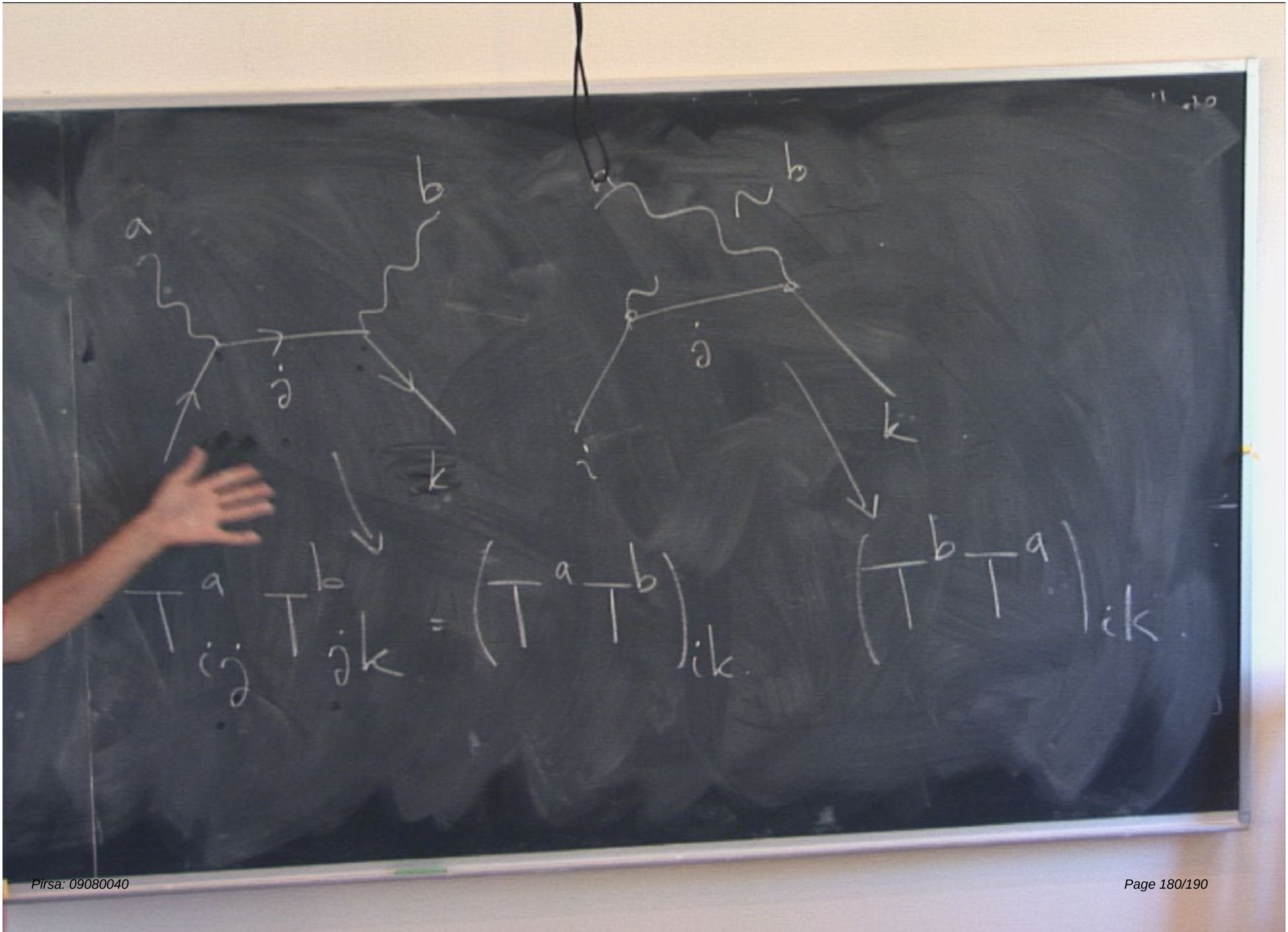

Soft Theorems
a
e

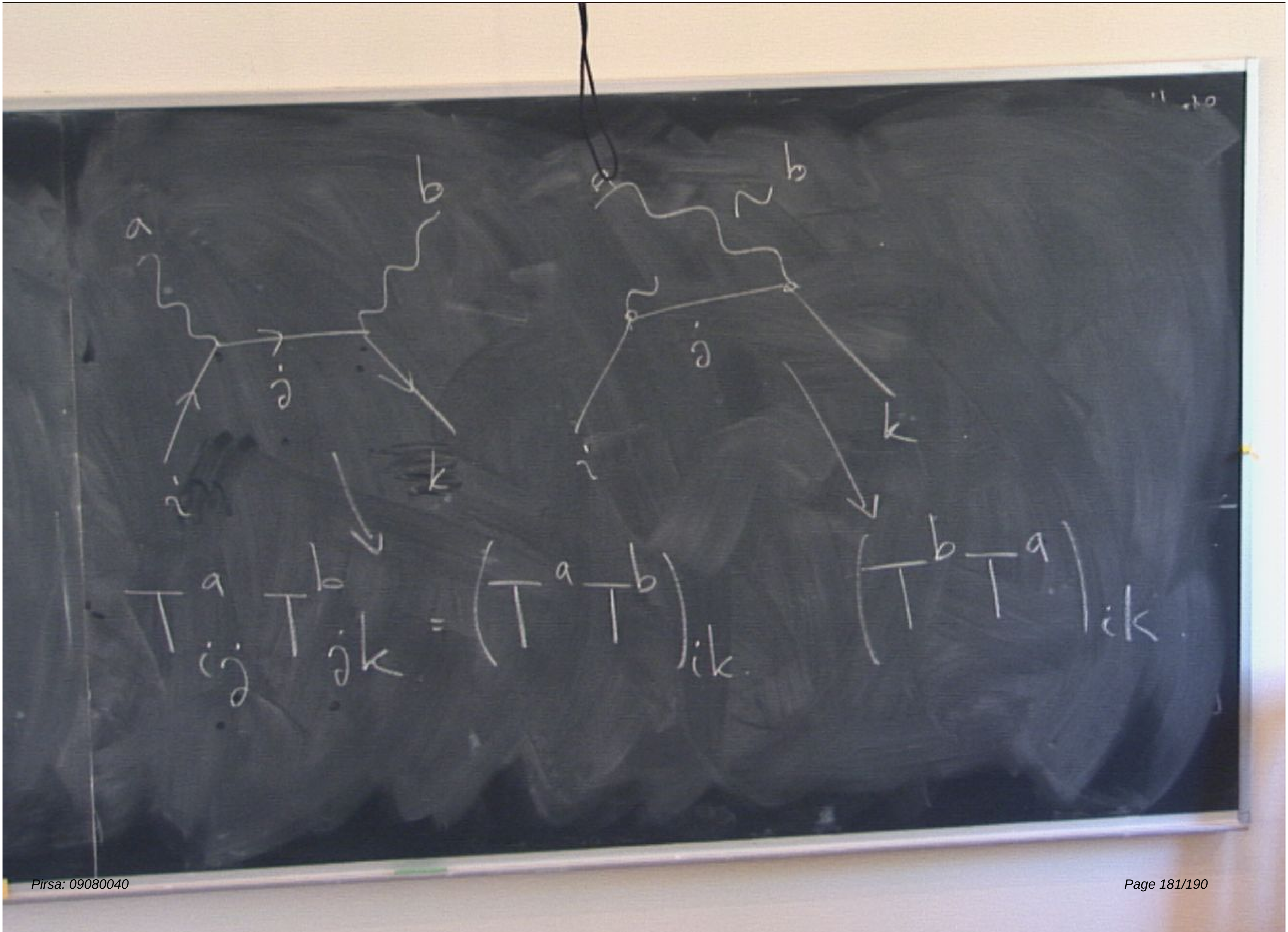

Soft Theorems


a
b
j
i
k
b
j
i
k


a
b
j
i
k
$T^a_{ij} T^b_{jk}$

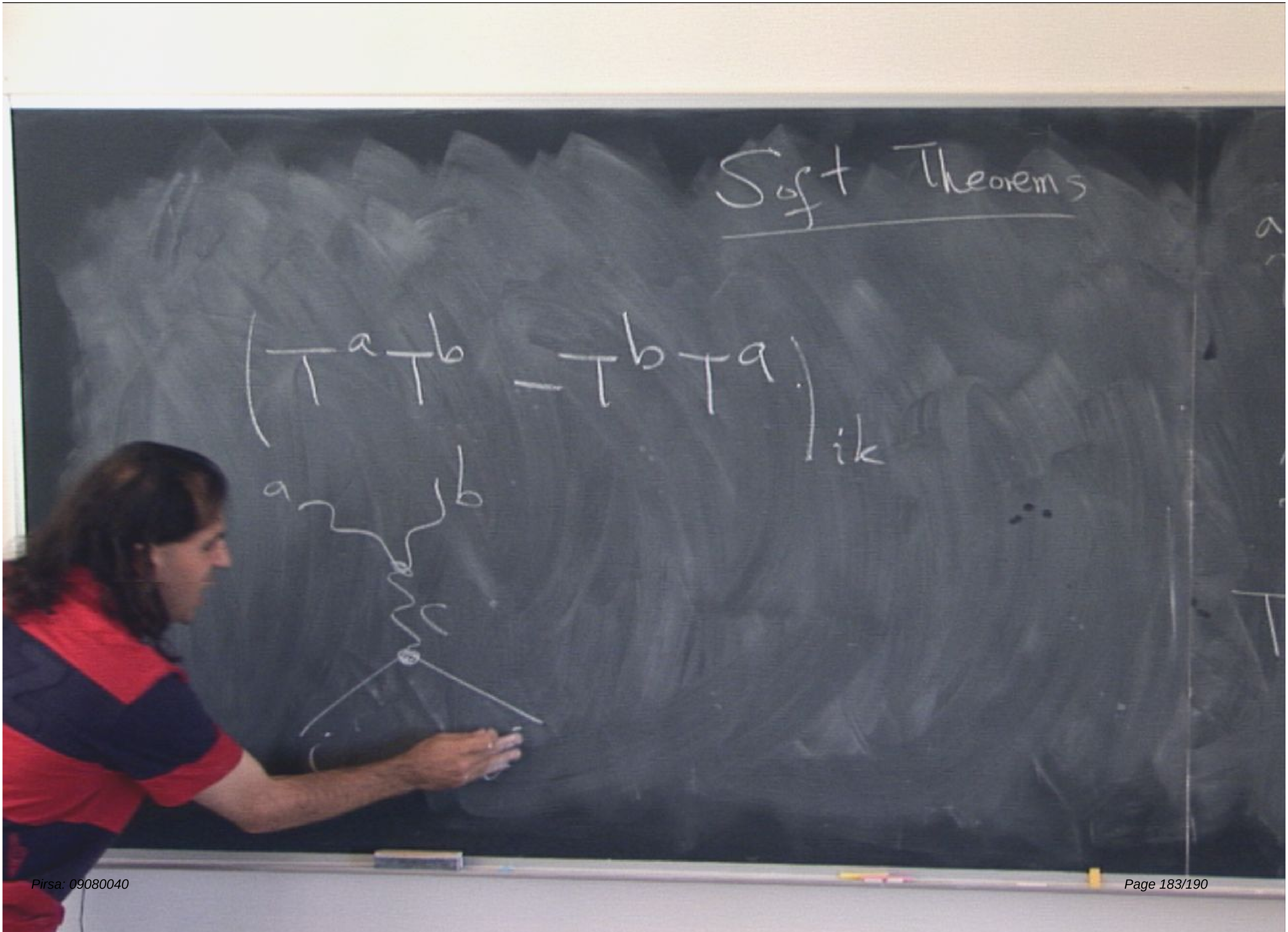

ms
a
b
j
i
k
$T^a_{ij} T^b_{jk} = (T^a T^b)_{ik}$

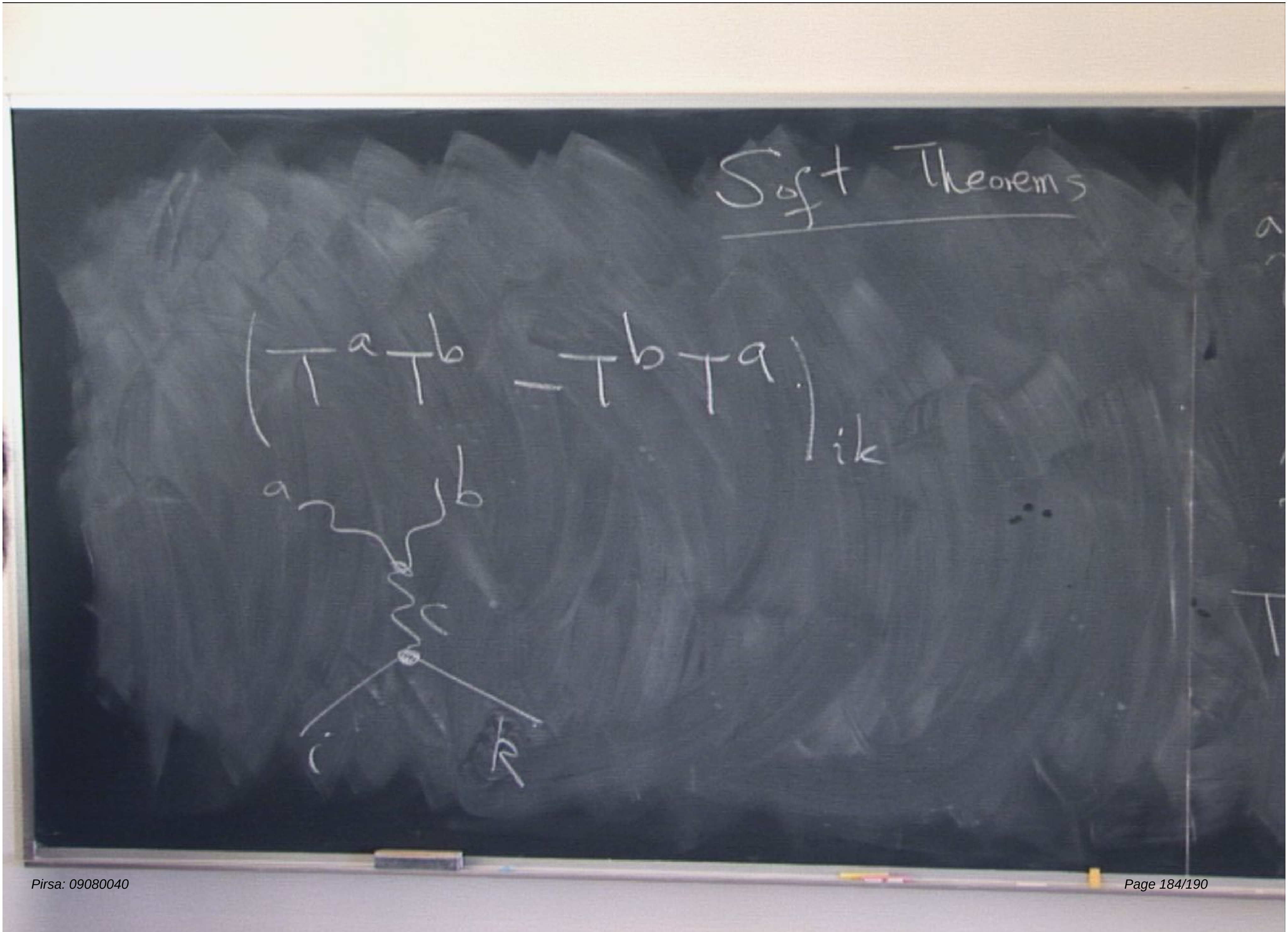

a
b
j
k
b
j
i
k
$T^a_{ij} T^b_{jk} = (T^a T^b)_{ik}$
$(T^b T^a)_{ik}$

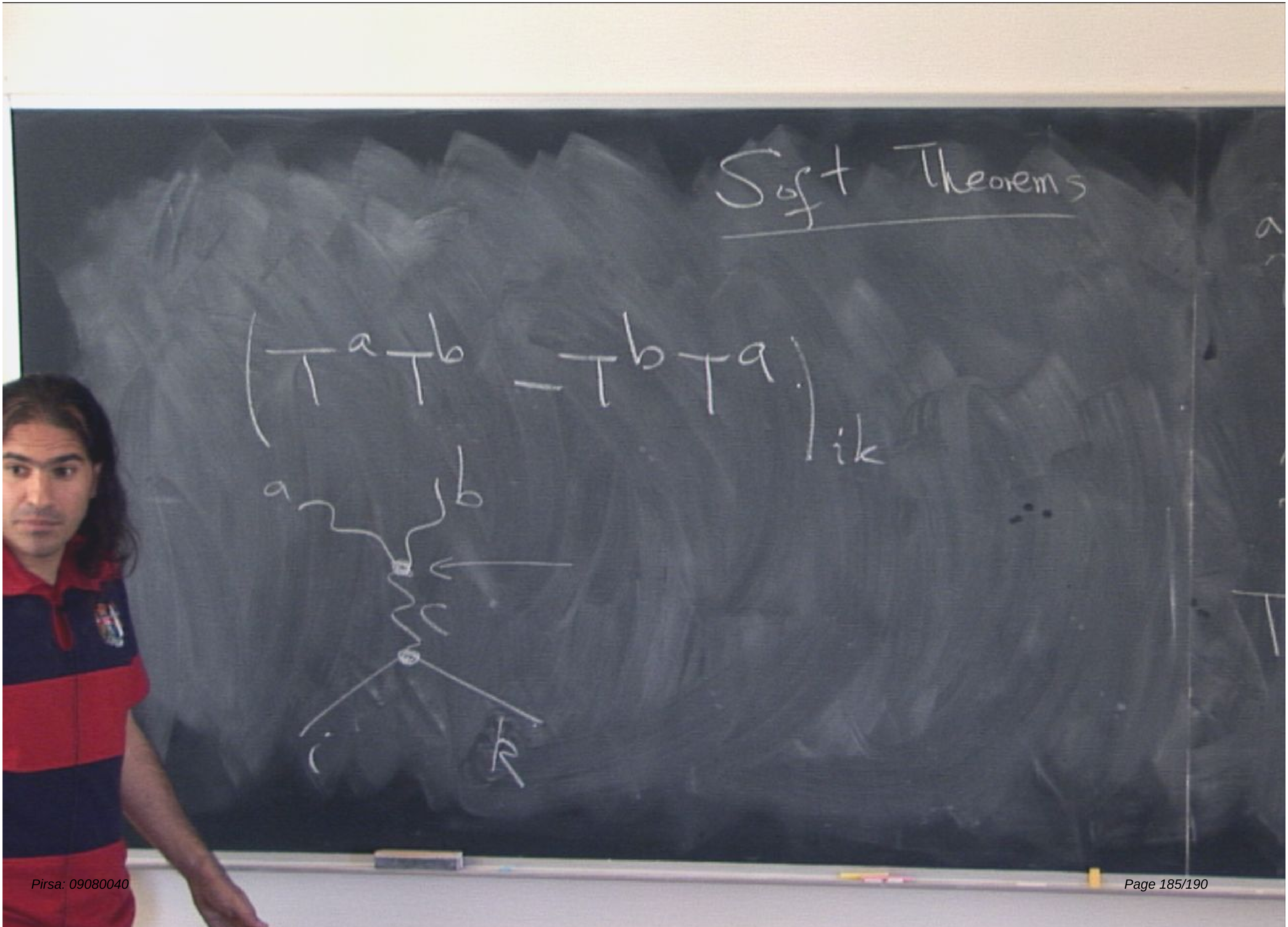

a
b
i
j
k
$T^a_{ij} T^b_{jk} = (T^a T^b)_{ik}$
$(T^b T^a)_{ik}$

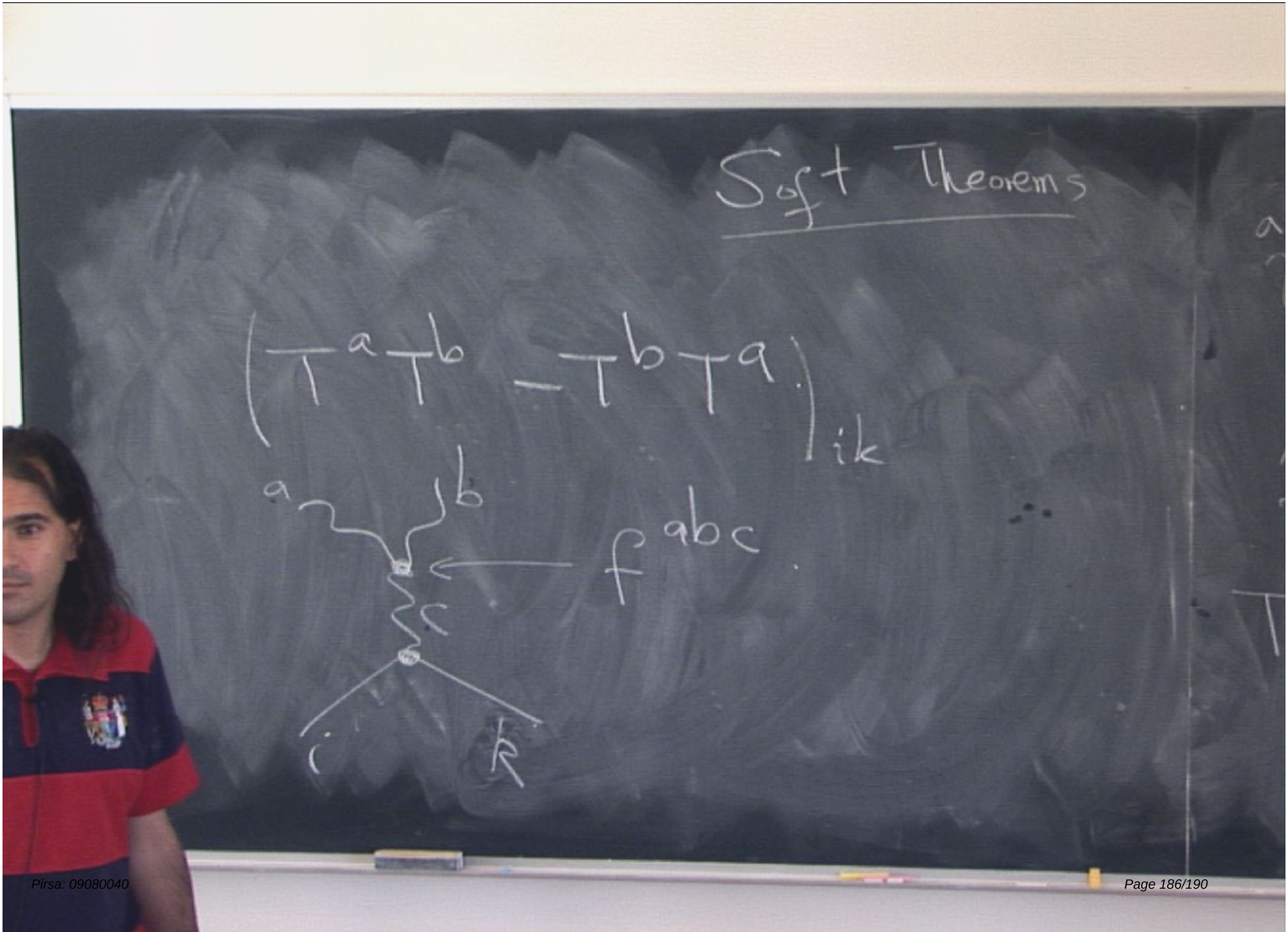
Soft Theorems

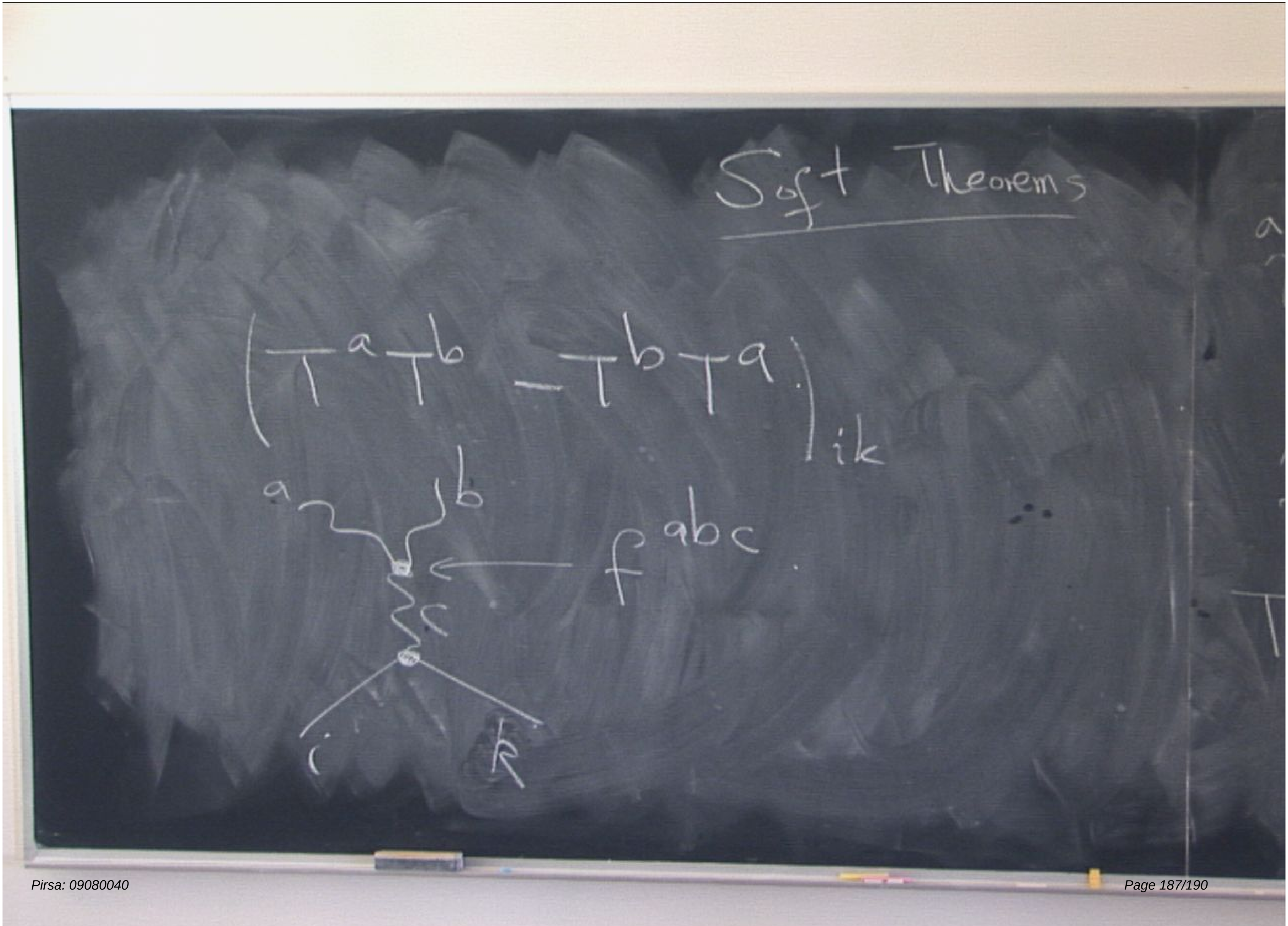
$$\left(T^a T^b - T^b T^a\right)_{ik}$$


Soft Theorems
$(T^a T^b - T^b T^a)_{ik}$
a
b
c


Soft Theorems
$(T^a T^b - T^b T^a)_{ik}$
a
b
c
i
k

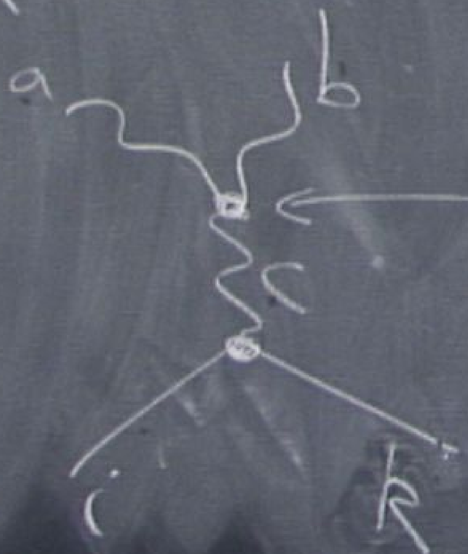

Soft Theorems
$(T^aT^b - T^bT^a)_{ik}$
a
b
c
i
k


Soft Theorems
$(T^a T^b - T^b T^a)_{ik}$
a
b
c
f^{abc}
i
k

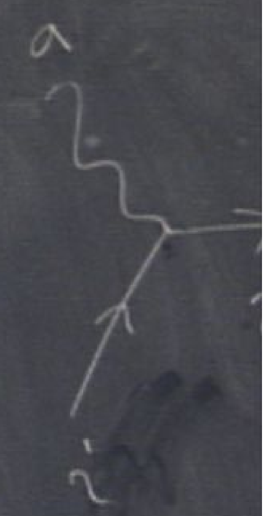

Soft Theorems
$(T^aT^b - T^bT^a)_{ik}$
a
b
f^{abc}
c
i
k

Soft Theorems

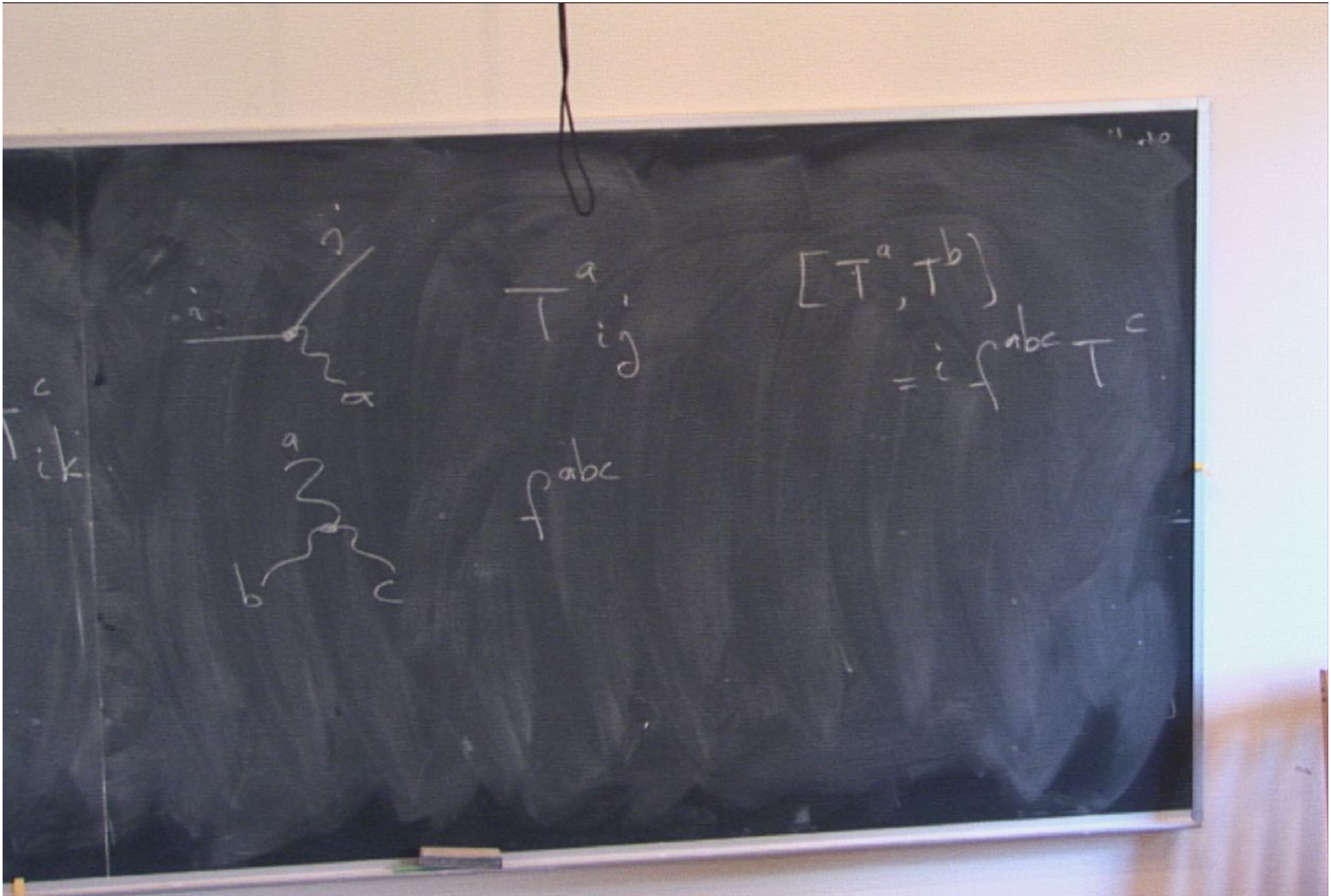
$$\left(T^a T^b - T^b T^a\right)_{ik} = f^{abc} T^c_{ik}$$

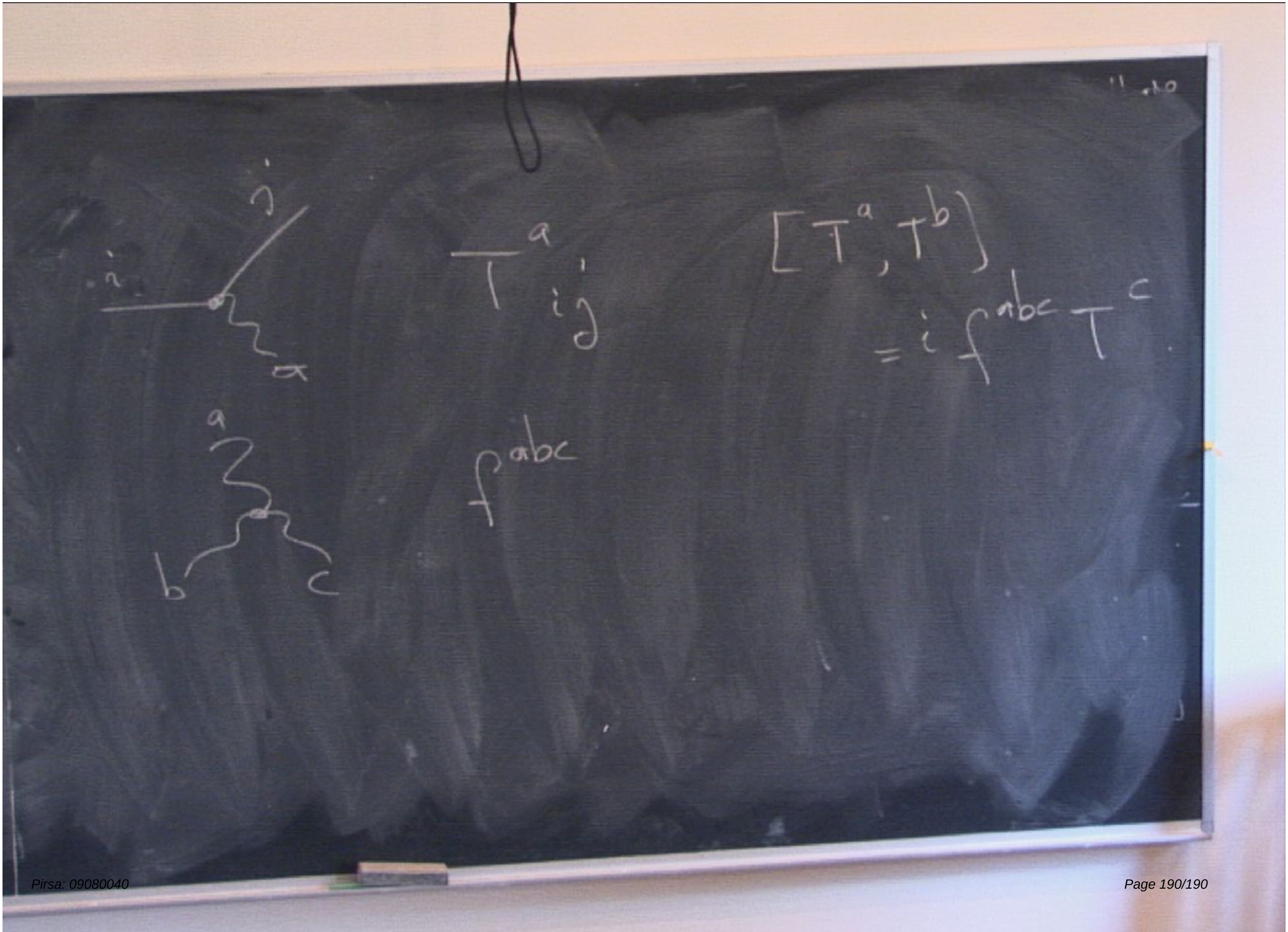


f^{abc}



T^a_{ij} T


T^a_{ij}
$[T^a, T^b] = i f^{abc} T^c$
f^{abc}
T^c_{ik}



j
i
a
T^a_{ij}
$[T^a, T^b]$
$= i f^{abc} T^c$
a
b
c
f^{abc}