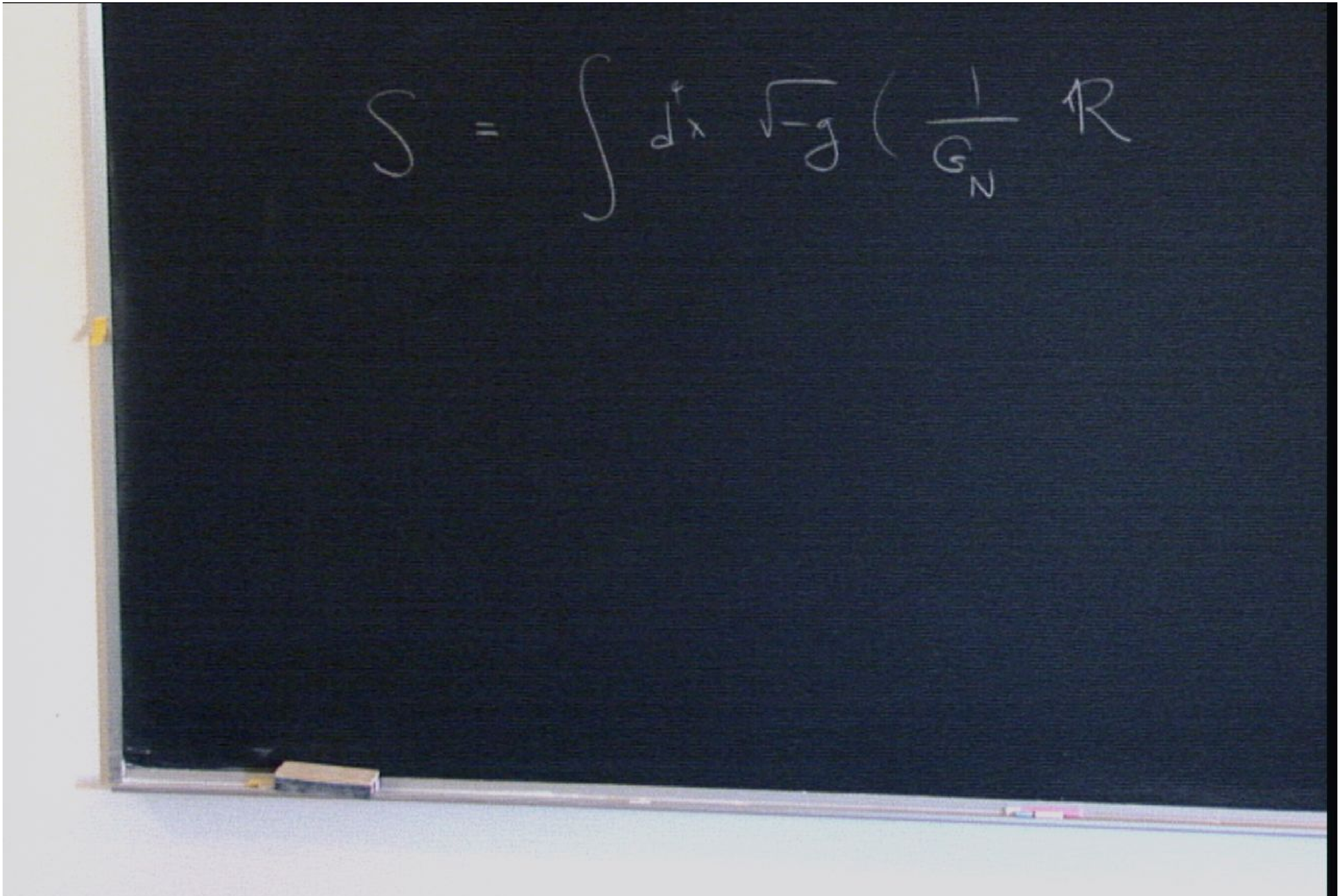
Title: PSI - Research Skills 2A

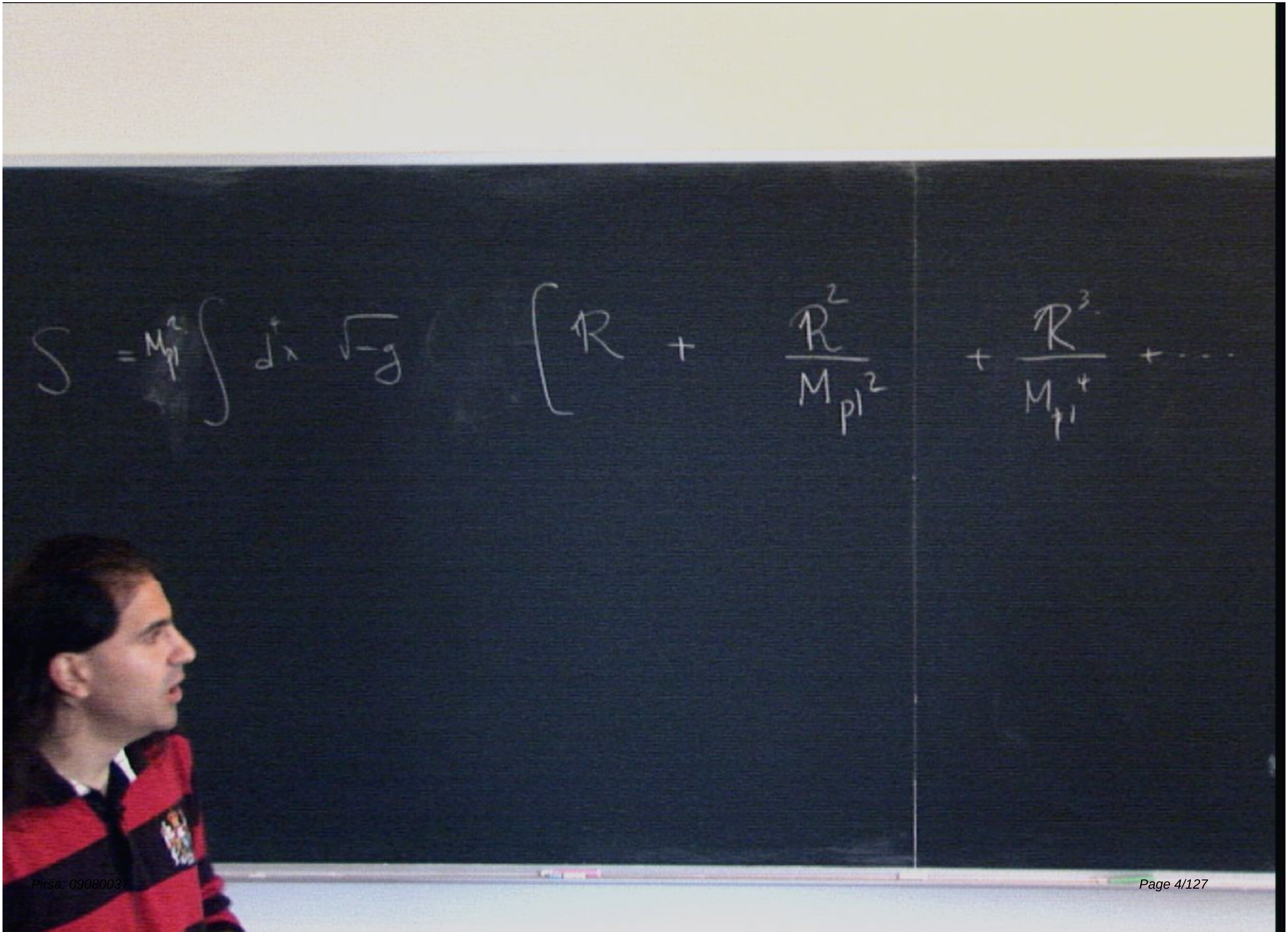
Date: Aug 25, 2009 09:00 AM

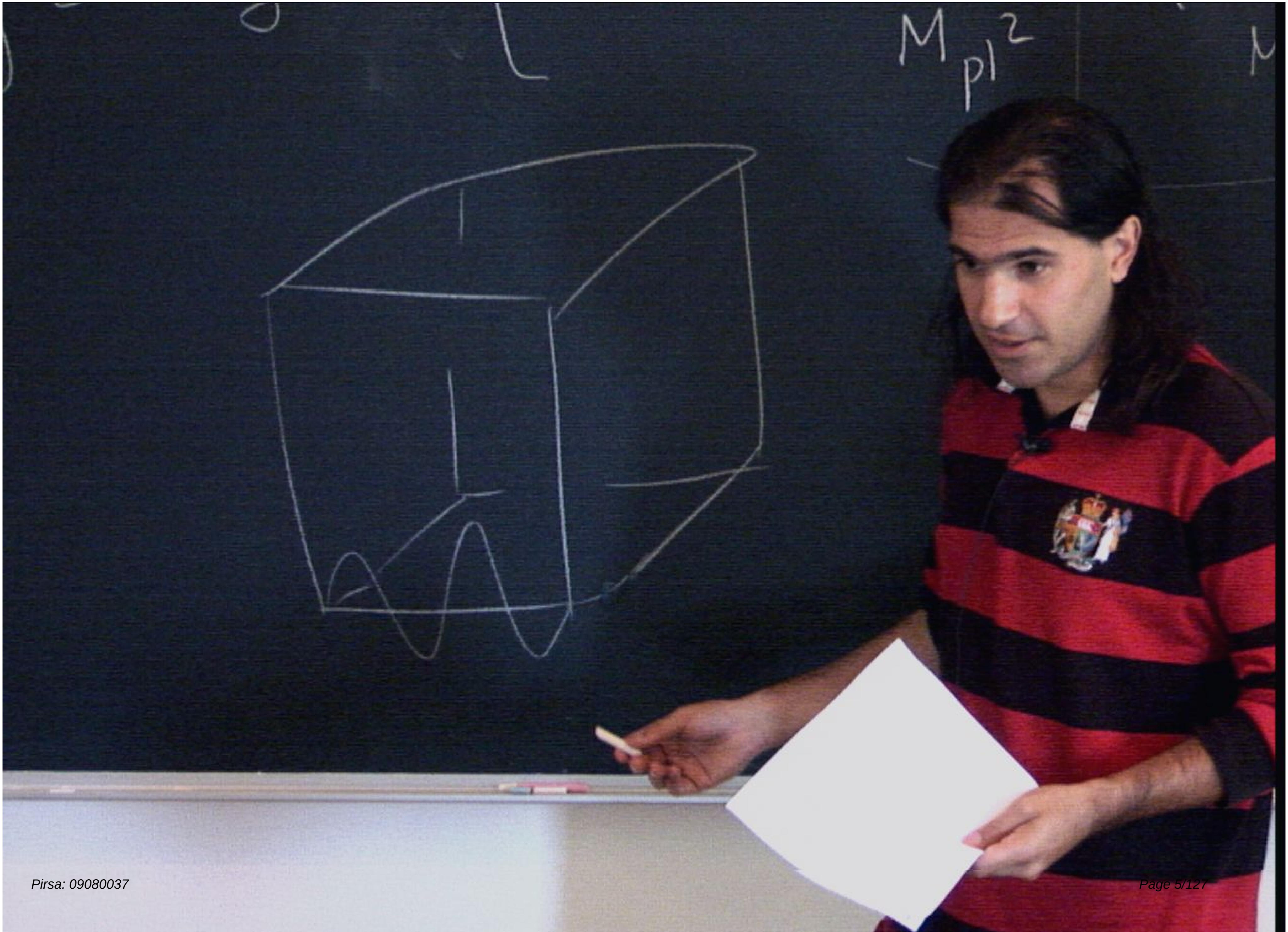
URL: http://pirsa.org/09080037

Abstract:


$S = \int d^4x \sqrt{-g} \left(\frac{1}{G_N} R \right.$

$$S = M_{pl}^2 \int d^4x \sqrt{-g} \left[R + \frac{R^2}{M_{pl}^2} + \frac{R^3}{M_{pl}^4} \right.$$


$S = M_{pl}^2 \int d^4x \sqrt{-g} \left[R + \frac{R^2}{M_{pl}^2} + \frac{R^3}{M_{pl}^4} + \cdots \right.$


M_{pl}^2

M_{pl}^2 M_{pl}^4

$$\omega_{(n_1,n_2,n_3)} = \sqrt{\frac{n_1^2}{L^2}+\frac{n_2^2}{L^2}+\frac{n_3^2}{L^2}}$$

$$|0\rangle_{n_1,n_2,n_3}$$

$$a, a^\dagger$$

$$a_{n_1,n_2,n_3}, \quad a^\dagger_{n_1,n_2,n_3}$$

$$|0\rangle_{n_1,n_2,n_3}.$$

$$a^\dagger_{n_1,n_2,n_3}|0\rangle = |1\rangle_{n_1,n_2,n_3}.$$

$$a,\ a^\dagger$$

$$a_{n_1,n_2,n_3},\ a^\dagger_{n_1,n_2,n_3}$$

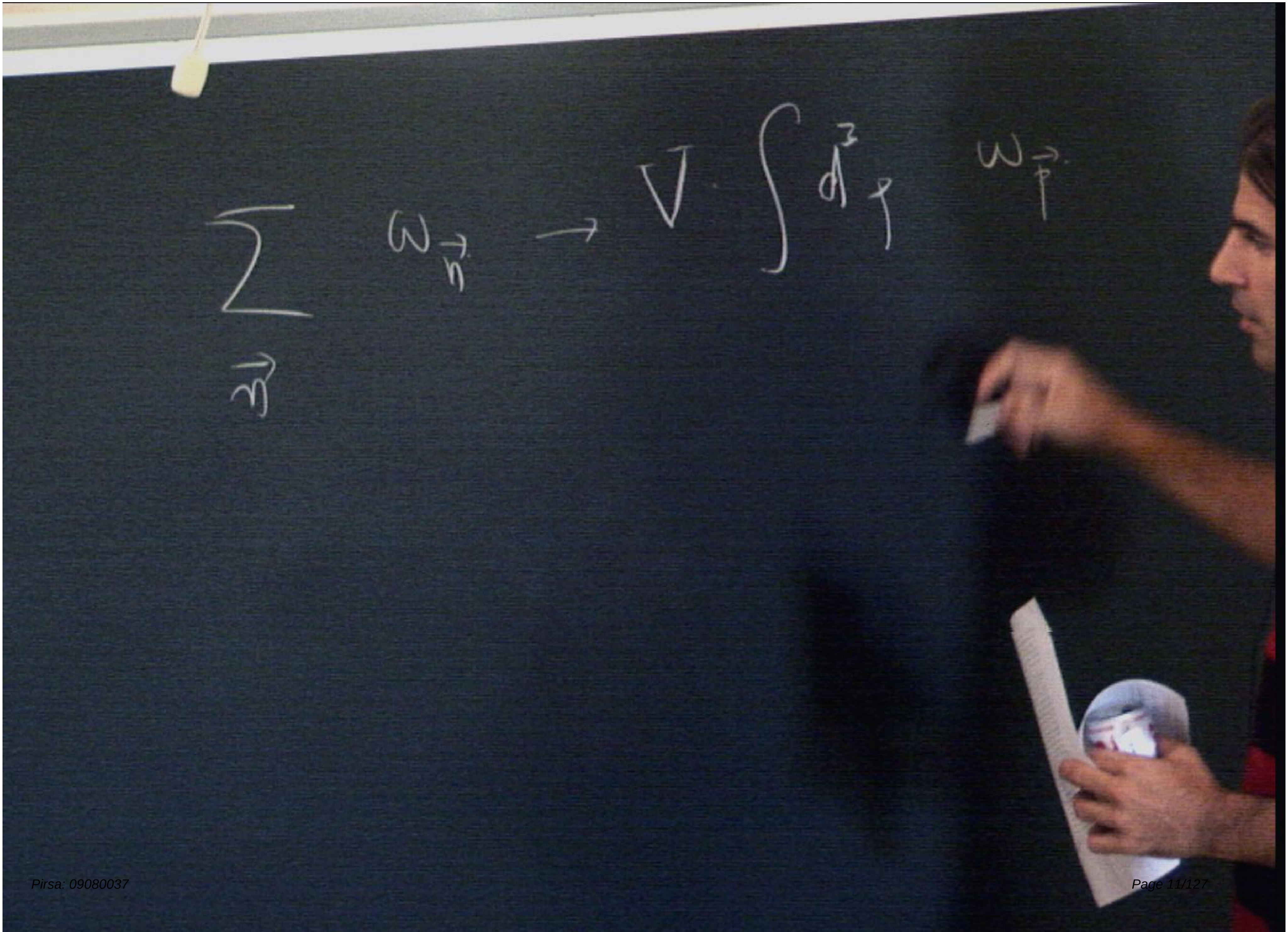
$$S = M_{pl} \int d x \sqrt{-g}$$

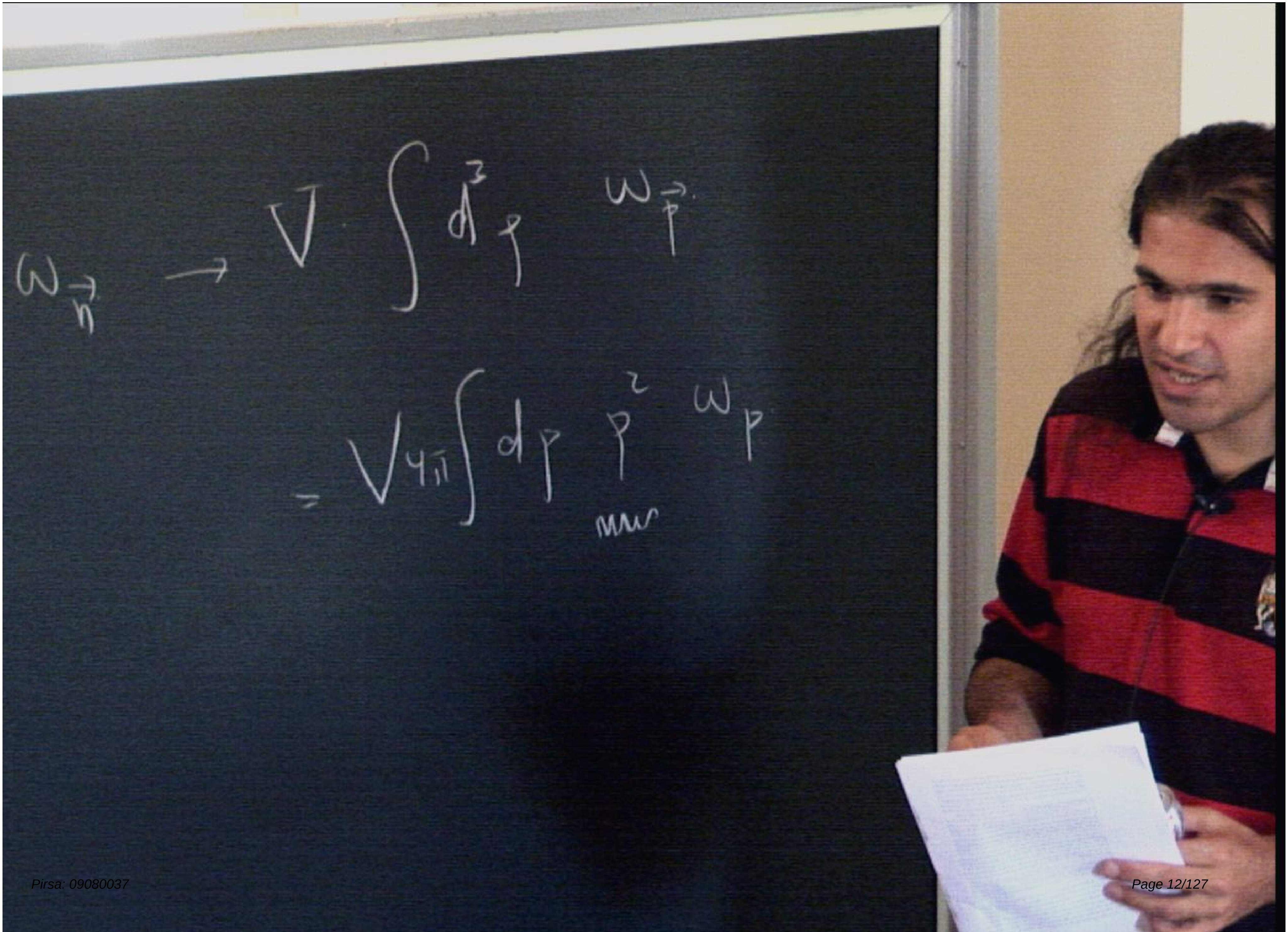
$$H = \sum_{n_1, n_2, n_3} H^{HO(n_1, n_2, n_3)}.$$

$$= \sum_{n_1, n_2, n_3} \left(\frac{1}{2} \hbar \omega_{n_1, n_2, n_3} + a^\dagger_{n_1 n_2 n_3} a_{n_1 n_2 n_3} \right).$$

$$= \sum_{n_1, n_2, n_3} H$$

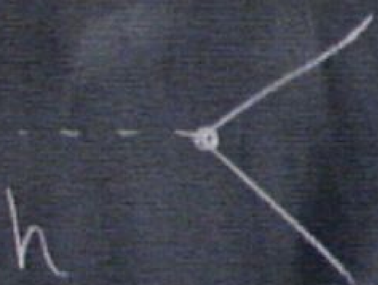
$$\sum_{n_1, n_2, n_3} \left(\frac{1}{2} \hbar \omega_{n_1, n_2, n_3} + a^{\dagger}_{n_1, n_2, n_3} a_{n_1, n_2, n_3} \right).$$


$\sum_{\vec{n}} \omega_{\vec{n}} \rightarrow V \cdot \int d^3 p \; \omega_{\vec{p}}$


$\omega_{\vec{n}} \to V \int d^3p \; \omega_{\vec{p}}$
$= V 4\pi \int dp \; p^2 \; \omega_p$

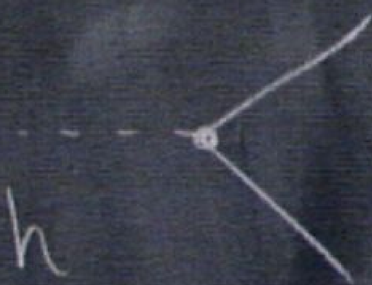
$$\int dp \; p^2 \left(\sqrt{p^2 + m_B^2} - \sqrt{p^2 + m_F^2} \right)$$

$$\int dp \; p^2 \left(\sqrt{p^2 + m_B^2} - \sqrt{p^2 + m_F^2} \right)$$



$$m \sim \lambda h$$

$$\int dp \; p^2 \left(\sqrt{p^2 + m_B^2} - \sqrt{p^2 + m_F^2} \right)$$

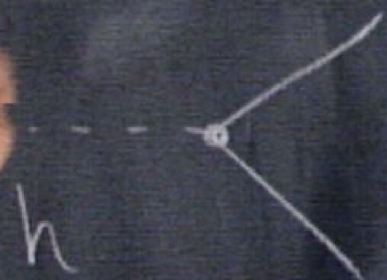


$$m \sim \lambda h$$

$$m_B^2 \sim \lambda^2 h^2 + \Delta m^2$$

$$m_F^2 \sim \lambda^2 h^2$$

$$\int dp \; p^2 \left(\sqrt{p^2 + m_B^2} - \sqrt{p^2 + m_F^2} \right)$$



$$m \sim \lambda h$$

$$m_B^2 \sim \lambda^2 h^2 + \Delta m^2$$

$$m_F^2 \sim \lambda^2 h^2$$

$$V \sim \Delta m^2 \rho_{max}^2 + \dots$$

$$+ \Delta m^2 h^2$$

$$+ \dots$$

$$\frac{n_1^2}{L^2} + \frac{n_2^2}{L^2} + \frac{n_3^2}{L^2}$$

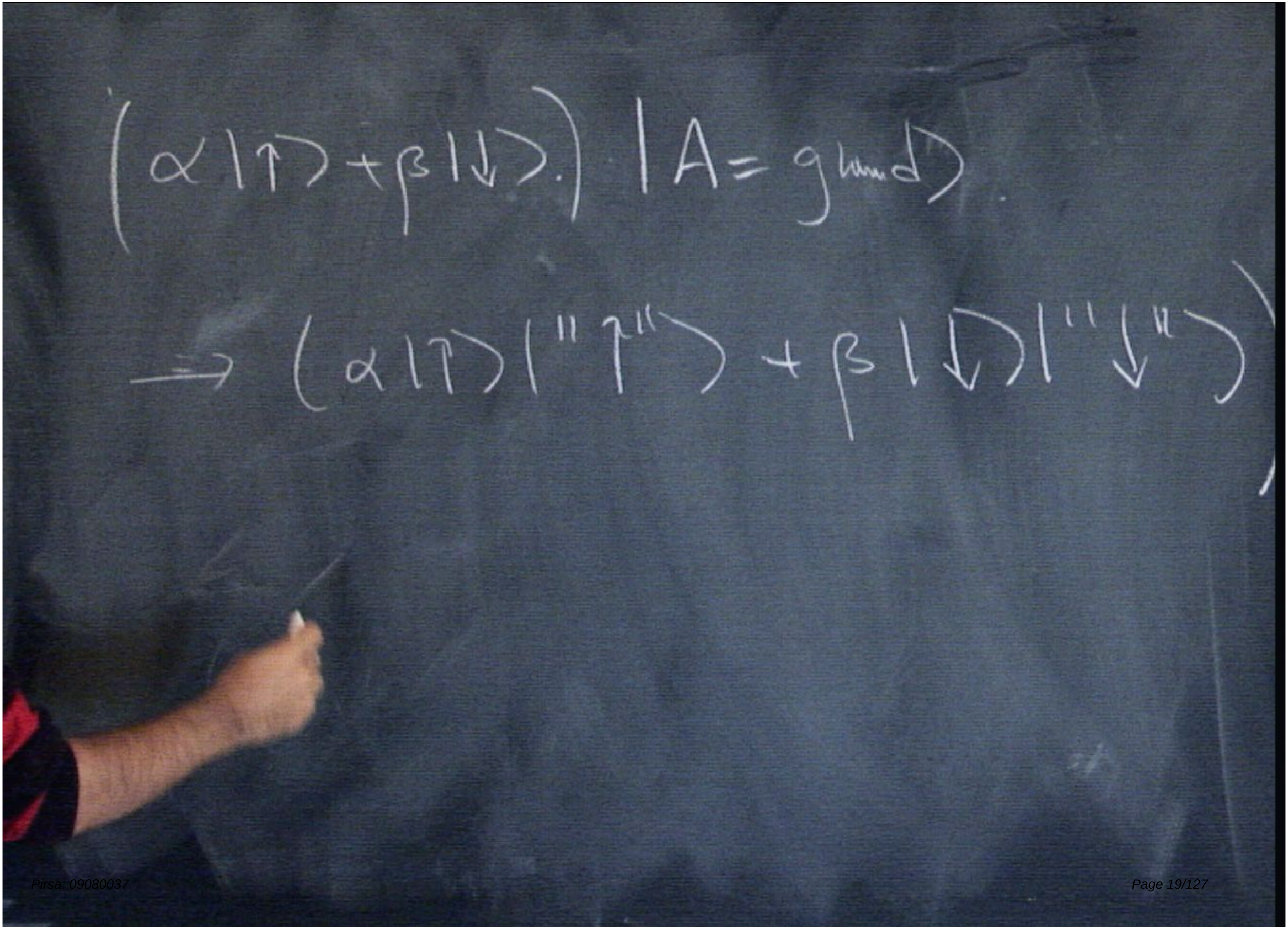
$$V \sim \Delta m^2 \, \rho_{max}^2 + \dots$$

$$+ \, \Delta m^2 \, h^2$$

$$+ \dots$$

$$\frac{n_1^2}{L^2} + \frac{n_2^2}{L^2} + \frac{n_3^2}{L^2}$$

a


$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\,|A = \text{ground}\rangle$
$\rightarrow (\alpha|\uparrow\rangle|``\uparrow"\rangle + \beta|\downarrow\rangle|``\downarrow"\rangle)$

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\,|A = \text{ground}\rangle$$

$$\Rightarrow (\alpha|\uparrow\rangle|``\uparrow"\rangle + \beta|\downarrow\rangle|``\downarrow"\rangle)$$

$\langle ``\uparrow" | ``\downarrow" \rangle$ is miniscule.

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\,|A = \text{ground}\rangle.$$

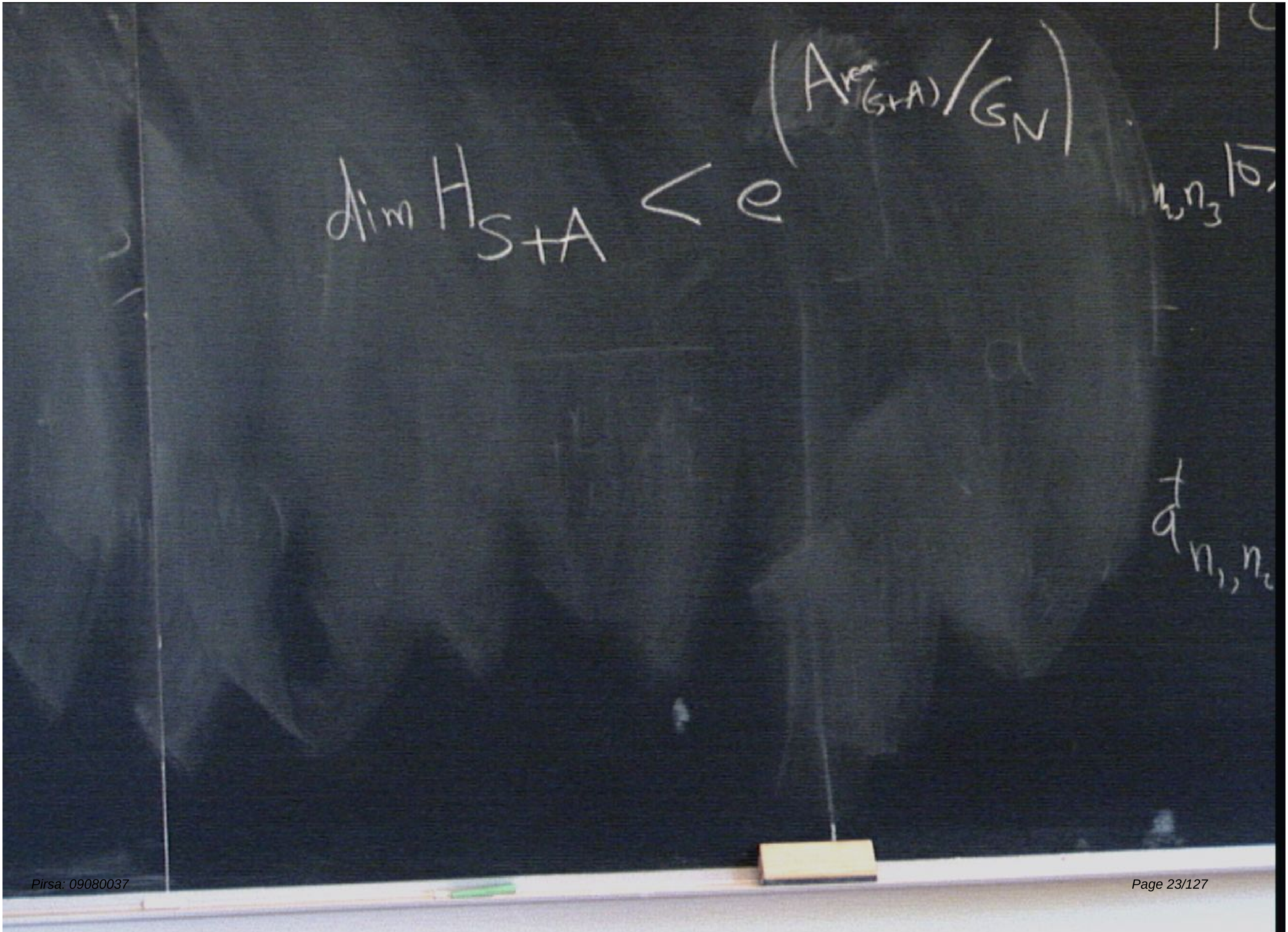
$$\rightarrow (\alpha|\uparrow\rangle|``\uparrow"\rangle + \beta|\downarrow\rangle|``\downarrow"\rangle)$$

$$\langle ``\uparrow" | ``\downarrow" \rangle \sim (1-\varepsilon)^N.$$

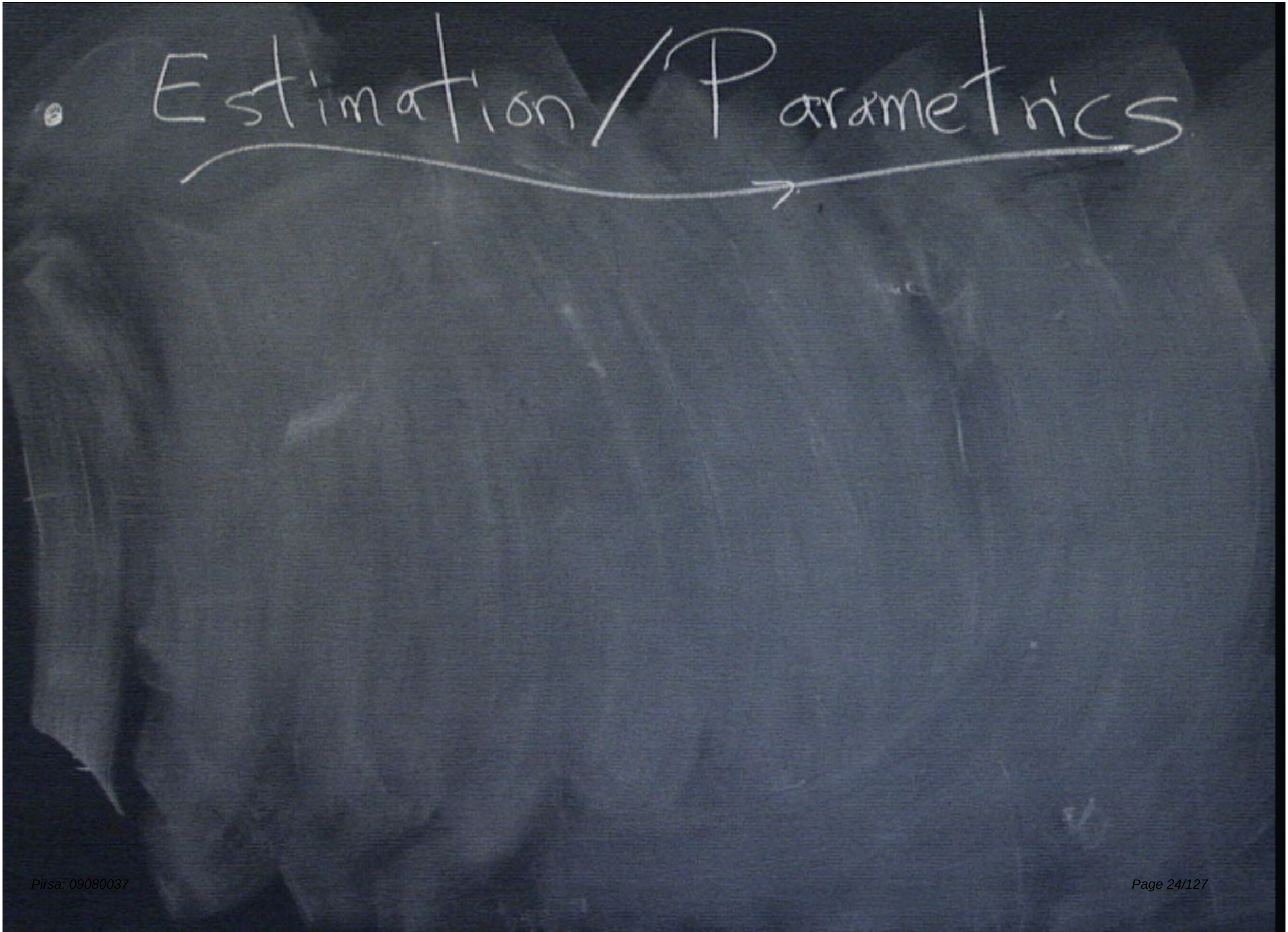
$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)\,|A = \text{ground}\rangle$$

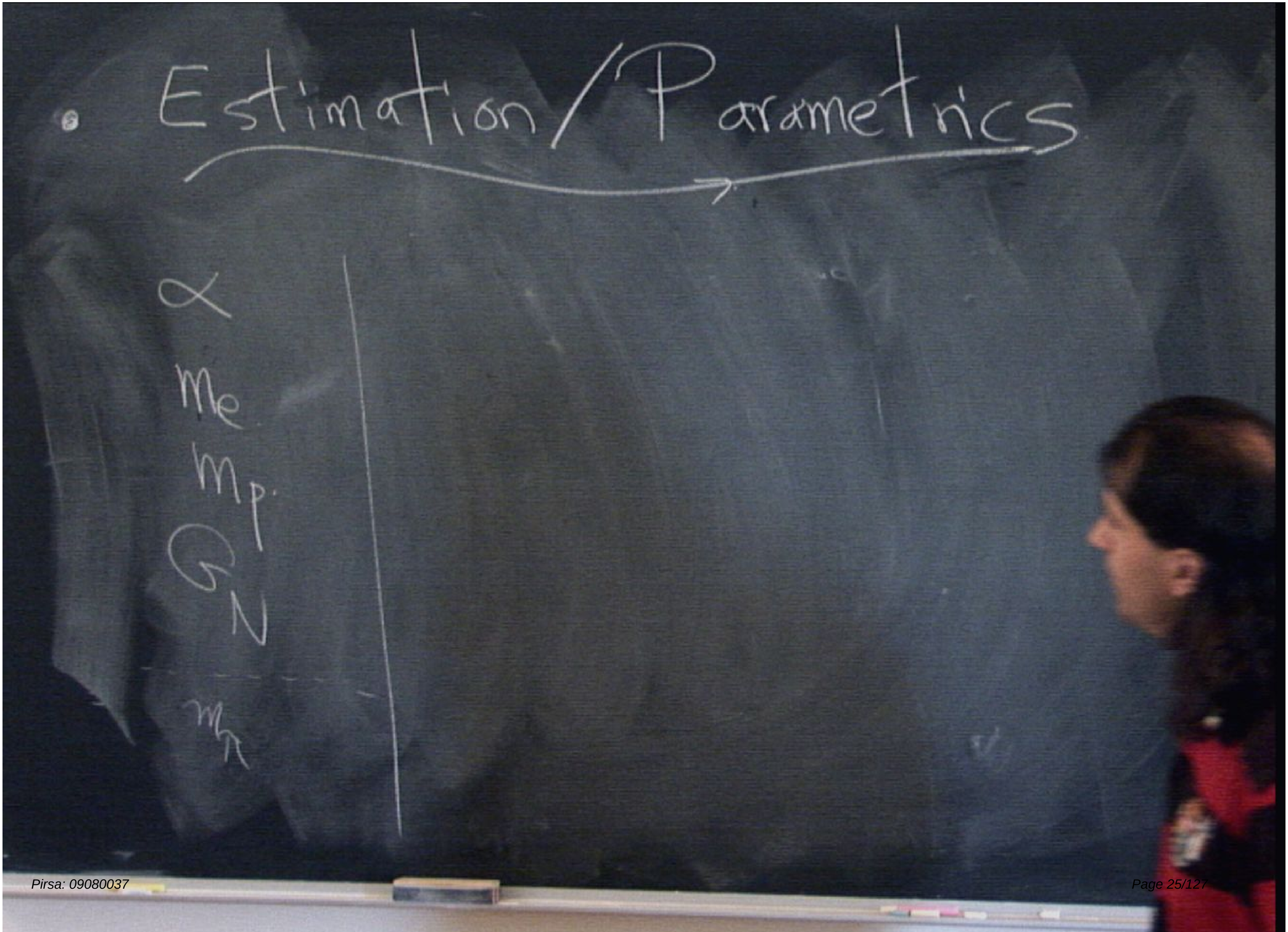
$$\Rightarrow (\alpha|\uparrow\rangle|``\uparrow"\rangle + \beta|\downarrow\rangle|``\downarrow"\rangle)$$

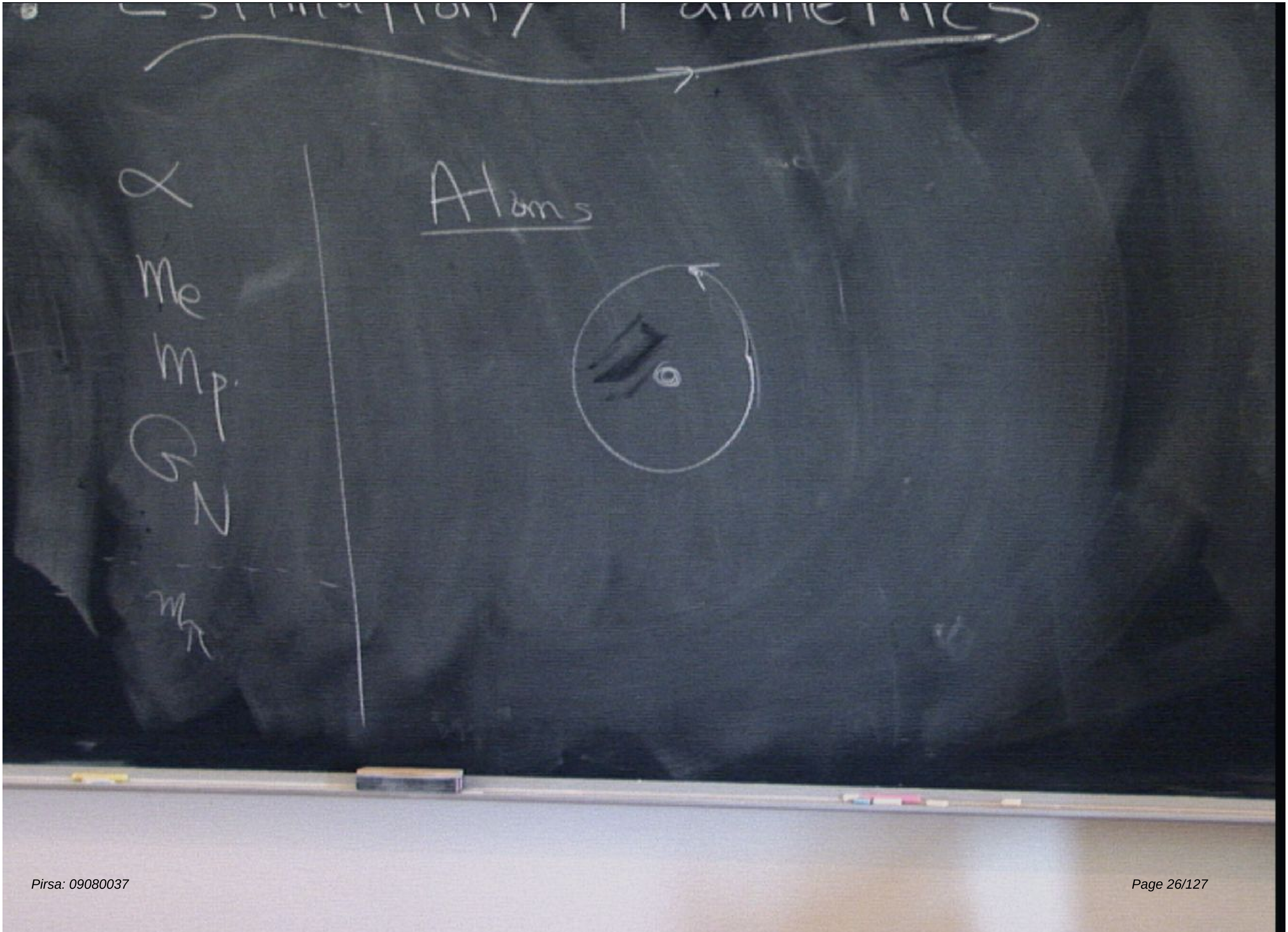
$$\langle ``\uparrow" | ``\downarrow" \rangle \sim (1-\varepsilon)^N < \left(\frac{1}{\dim \mathcal{H}_A}\right)$$

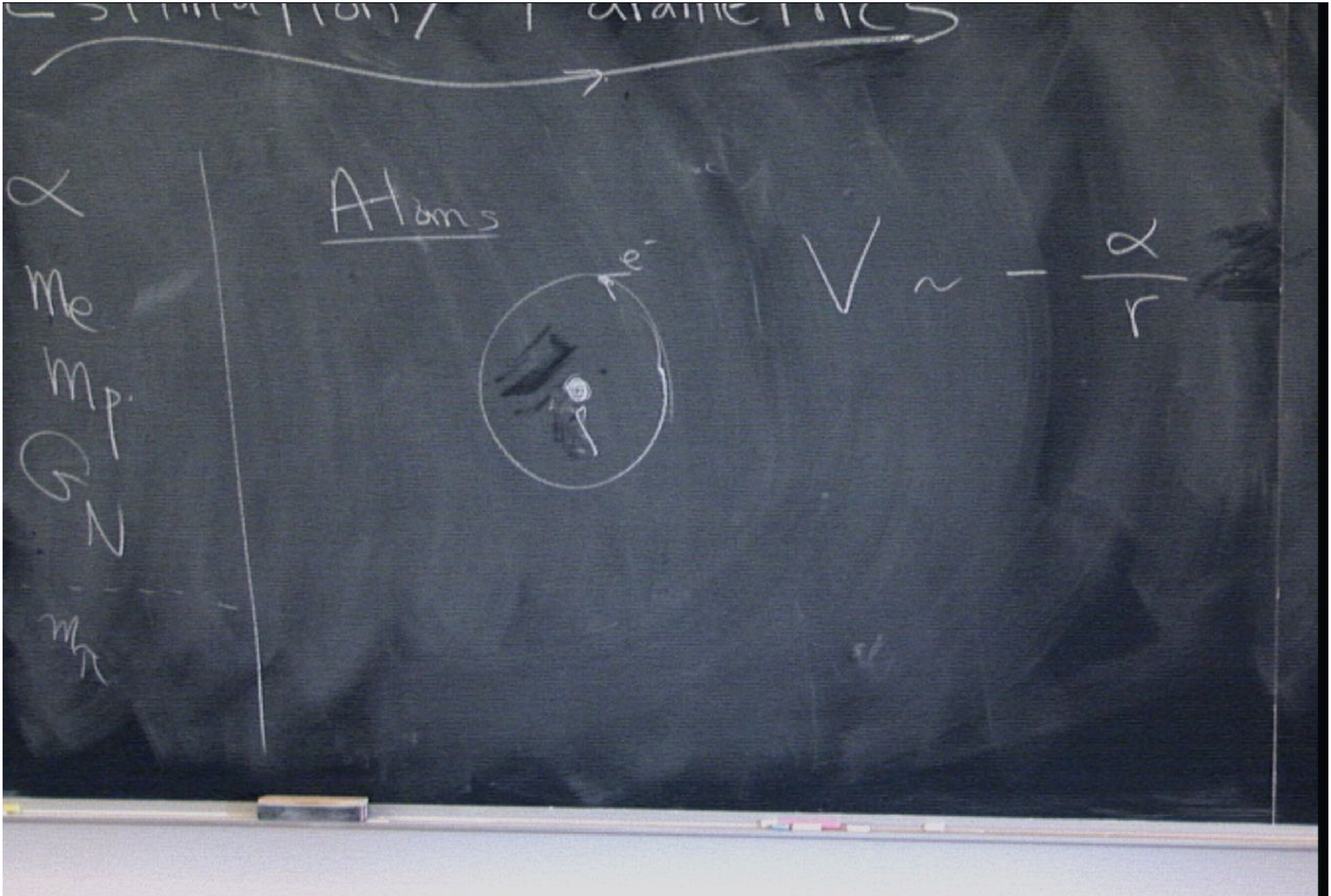


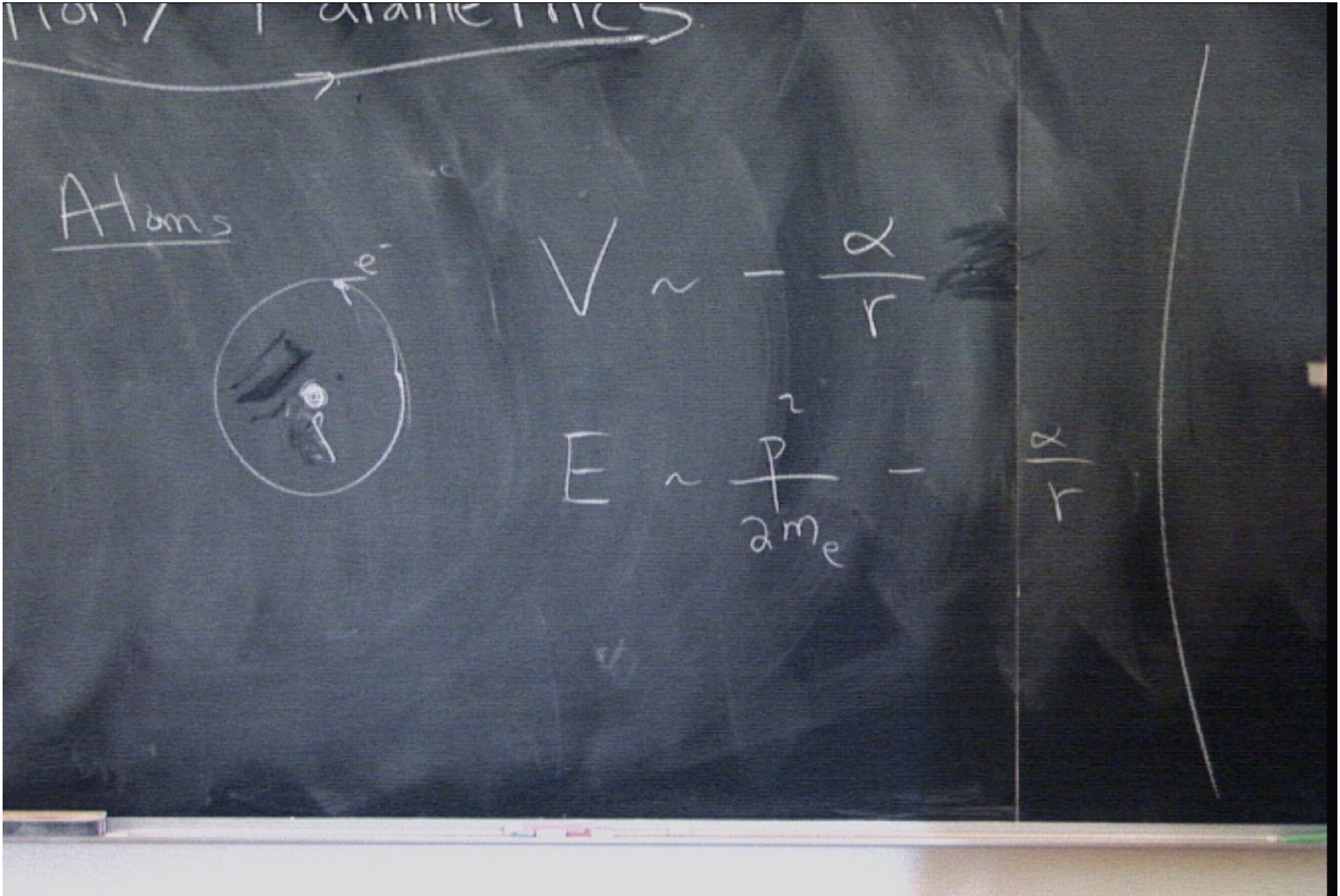
dim H_{S+A} < e^(Area_(S+A) / G_N)

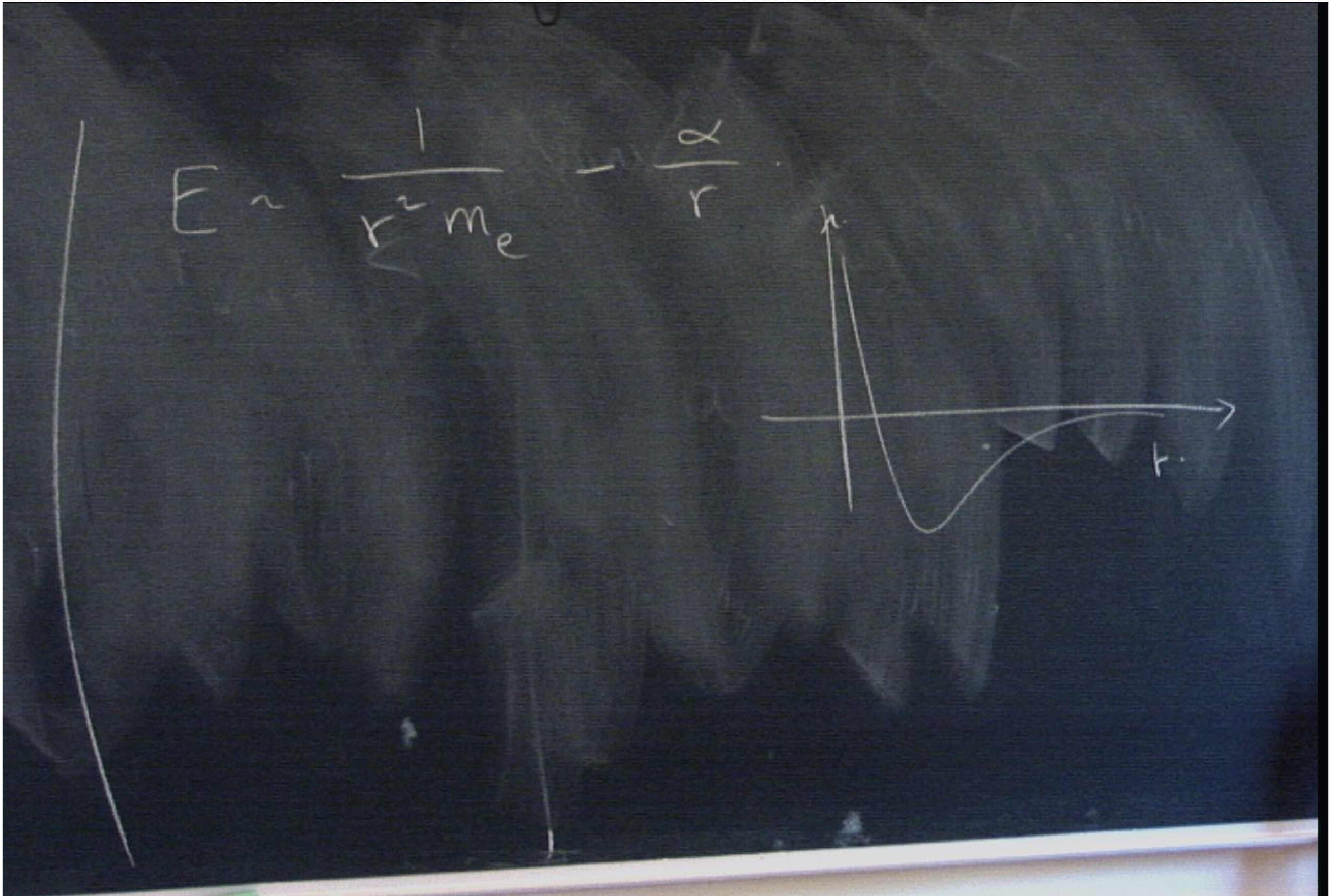

• Estimation / Parametrics

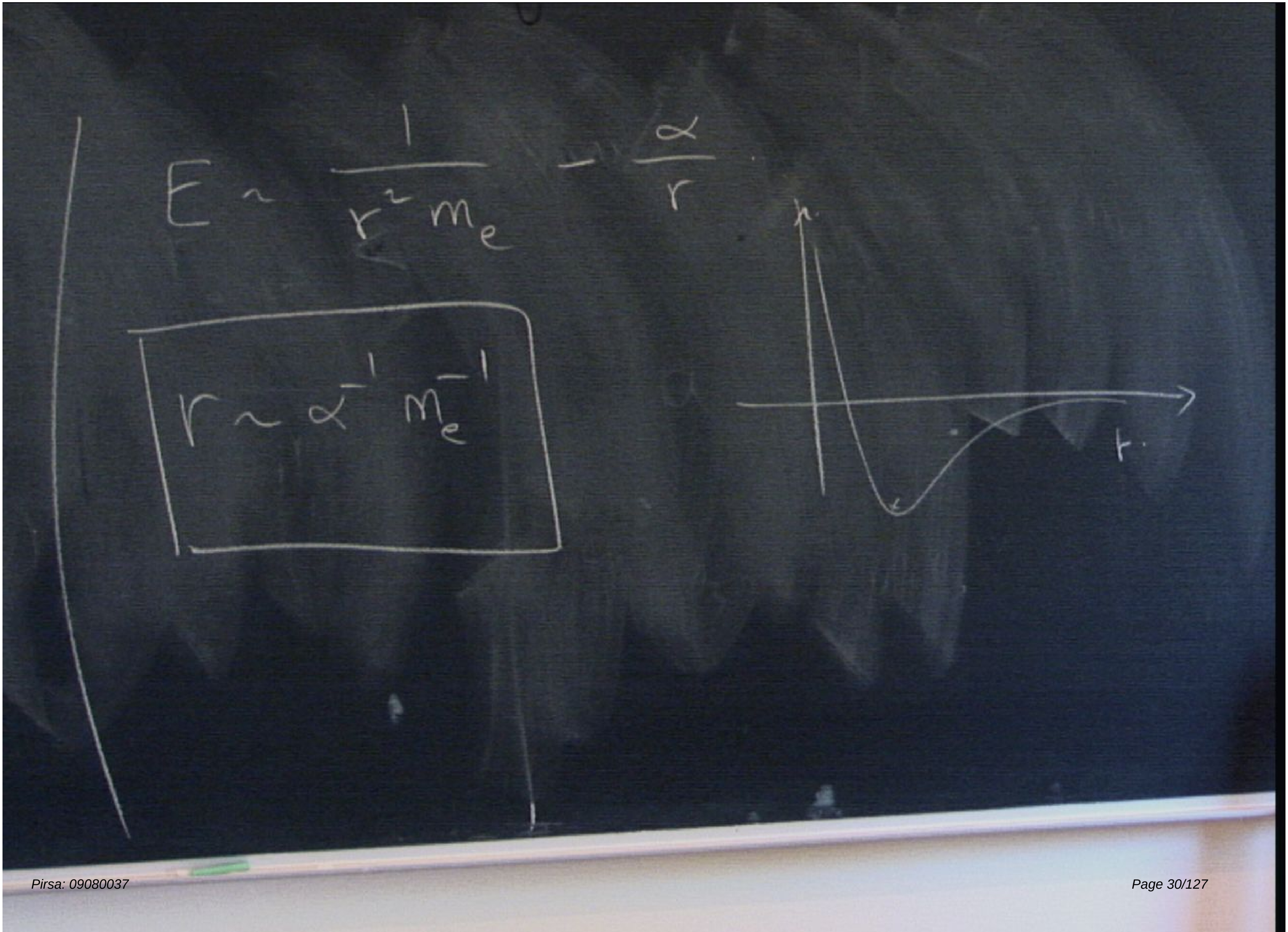

Estimation/Parametrics
α
me
mp
G
N


Atoms
α
me
mp
GN
mπ


Atoms
e⁻
V ~ − α / r
α
me
mp
G
N


Atoms
e⁻
V ~ − α/r
E ~ p²/2mₑ − α/r


$E \sim \frac{1}{r^2 m_e} - \frac{\alpha}{r}$
r


$E \sim \frac{1}{r^2 m_e} - \frac{\alpha}{r}$
$r \sim \alpha^{-1} m_e^{-1}$
r

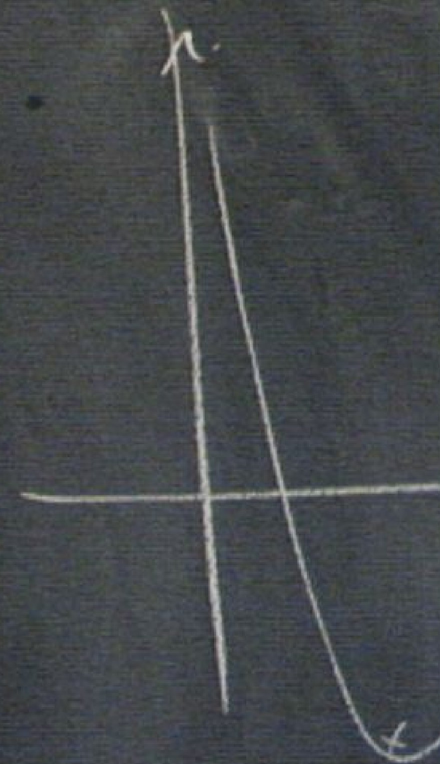
$2m$

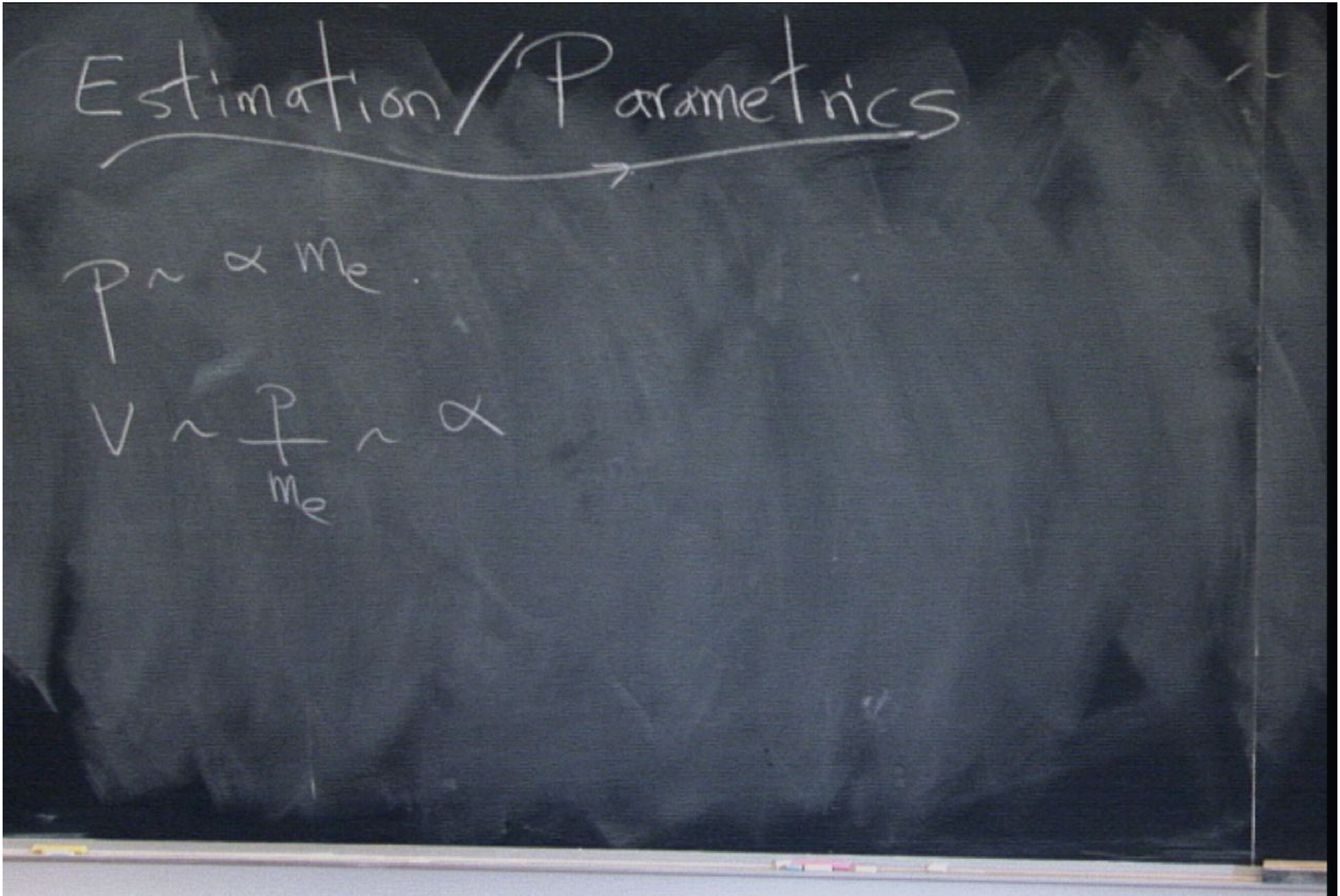
$$p \propto \frac{1}{r}$$

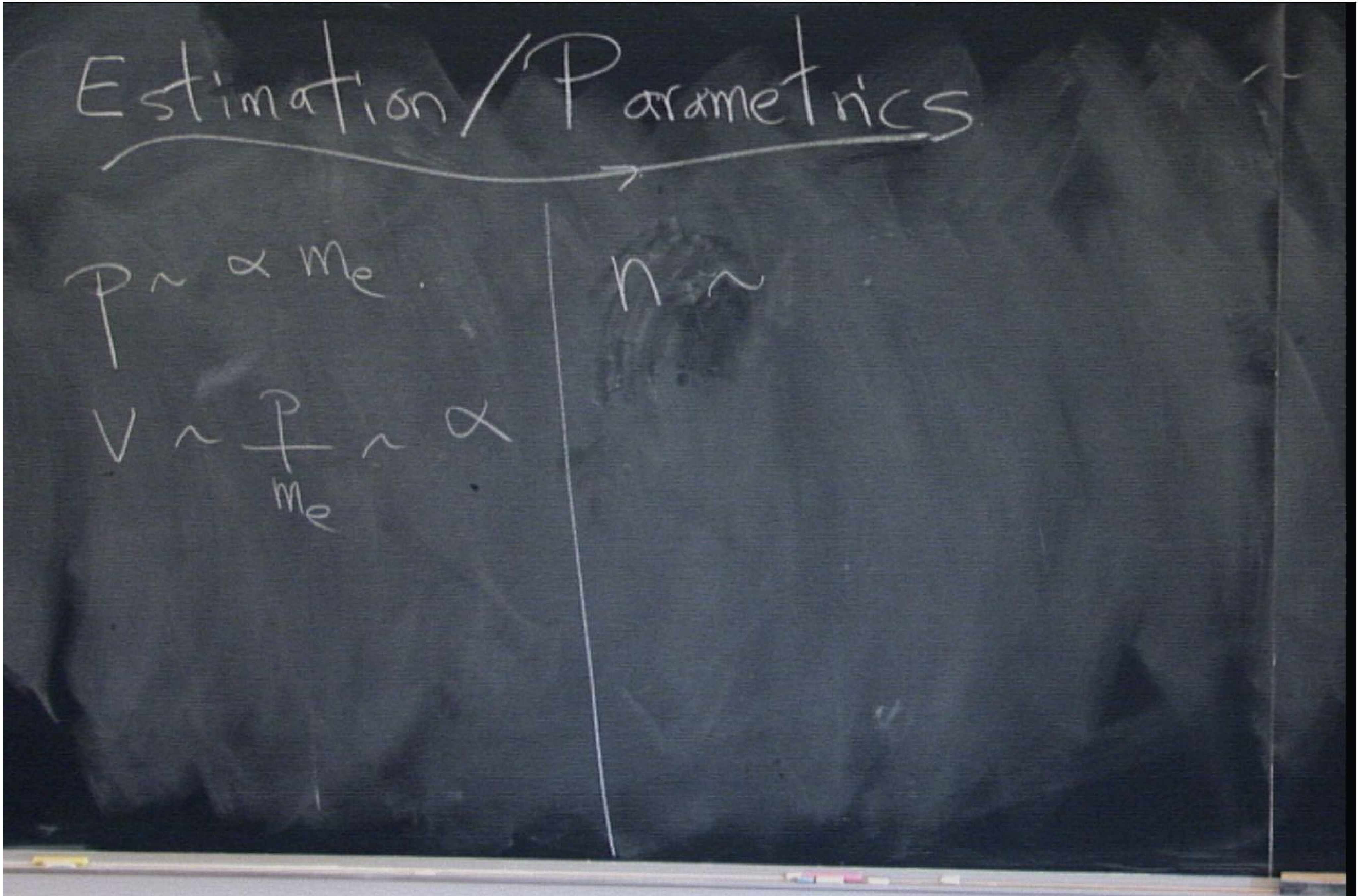
$E \sim \quad r^2 m_e \quad r$

$$r_{atom} \sim \alpha^{-1} m_e^{-1}$$

$$E_{atomic} \sim \alpha^2 m_e$$




Estimation / Parametrics
$p \sim \alpha m_e$
$v \sim \frac{p}{m_e} \sim \alpha$


Estimation/Parametrics
p ~ α m_e
V ~ p/m_e ~ α
n ~

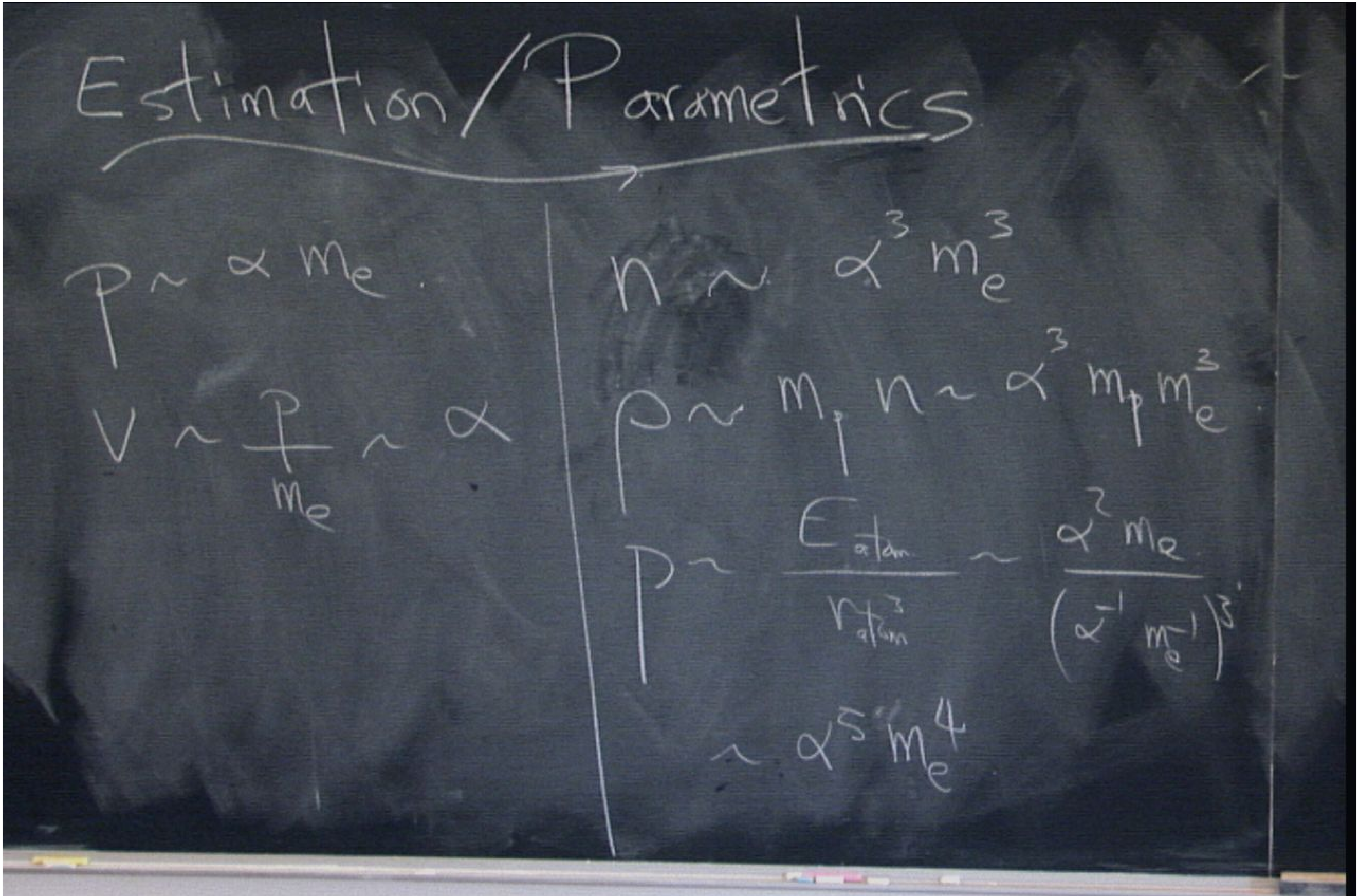
Estimation / Parametrics

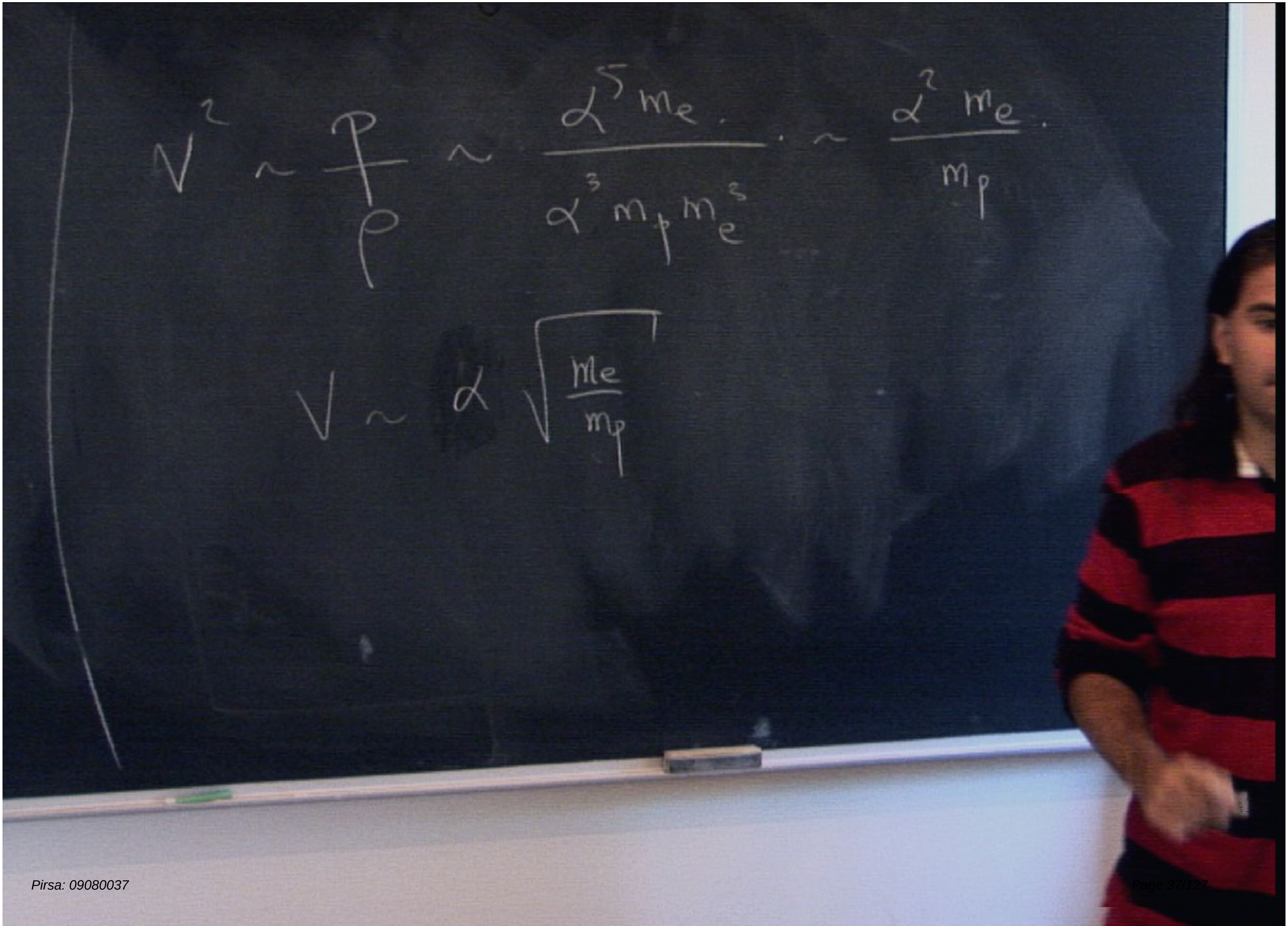
$p \sim \alpha m_e$.

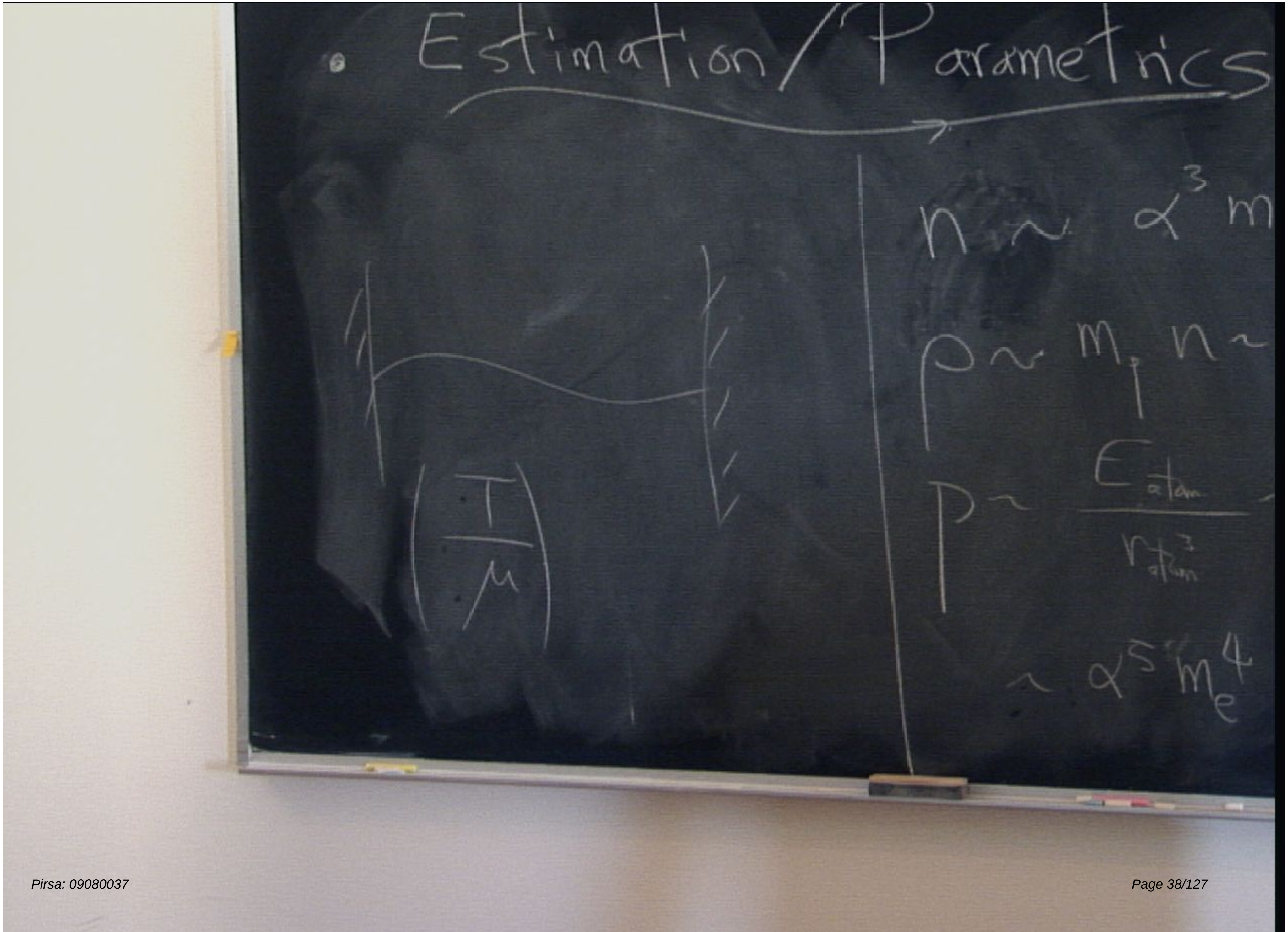
$v \sim \frac{p}{m_e} \sim \alpha$

$n \sim \alpha^3 m_e^3$

$\rho \sim m_p n \sim \alpha^3 m_p m_e^3$.


Estimation / Parametrics
$p \sim \alpha m_e$
$v \sim \frac{p}{m_e} \sim \alpha$
$n \sim \alpha^3 m_e^3$
$\rho \sim m_p n \sim \alpha^3 m_p m_e^3$
$P \sim \frac{E_{atom}}{r_{atom}^3} \sim \frac{\alpha^2 m_e}{(\alpha^{-1} m_e^{-1})^3}$
$\sim \alpha^5 m_e^4$


$V^2 \sim \frac{P}{\rho} \sim \frac{\alpha^5 m_e}{\alpha^3 m_p m_e^3} \sim \frac{\alpha^2 m_e}{m_p}$
$V \sim \alpha \sqrt{\frac{m_e}{m_p}}$


Estimation / Parametrics
n ~ α³ m
ρ ~ m n ~
p ~ E_atom / r_atom³
~ α⁵ m_e⁴
(T / μ)

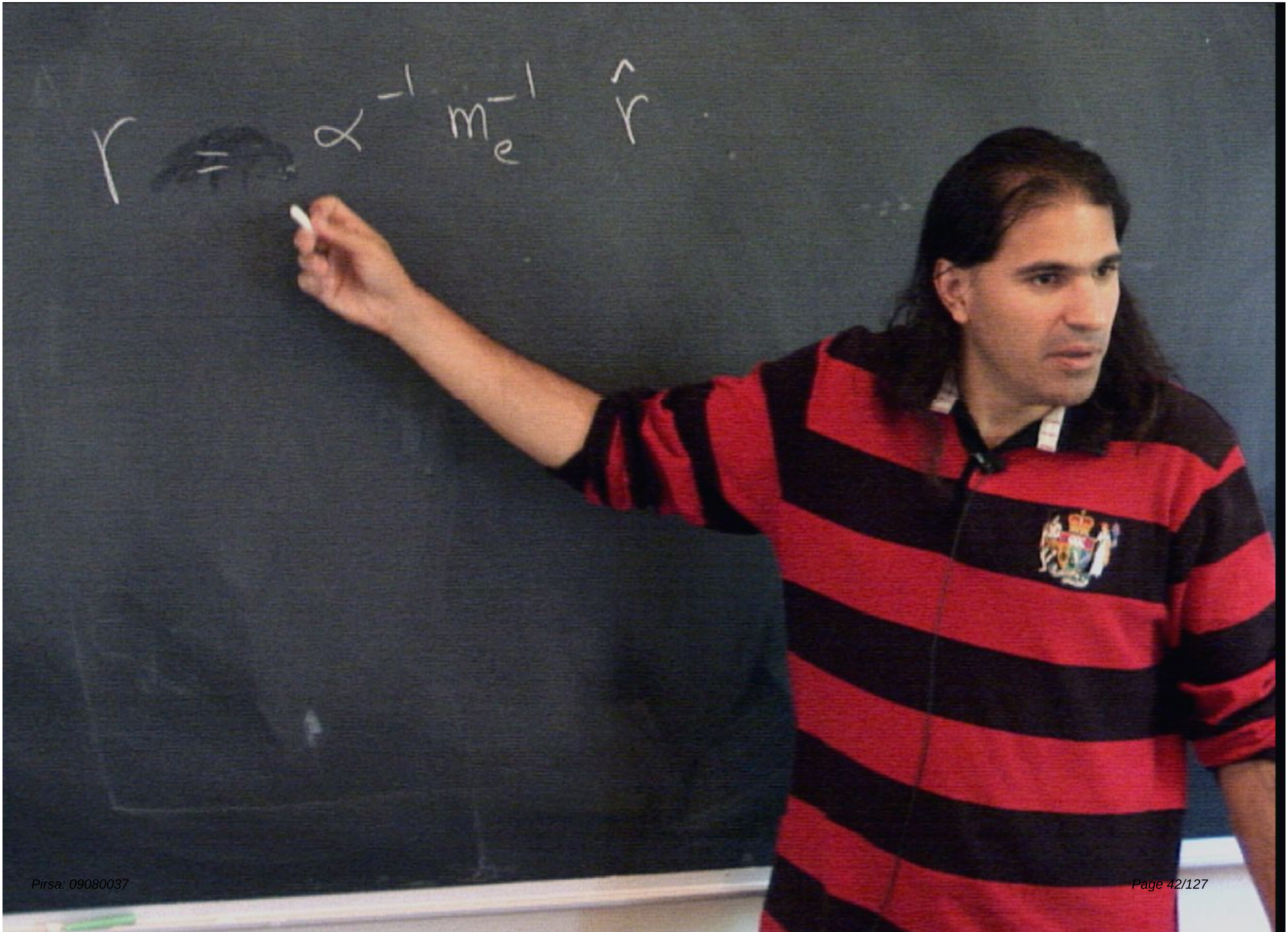
$$\frac{P}{\rho} \sim \frac{\alpha^5 m_e}{\alpha^3 m_p m_e^3} \sim \frac{\alpha^2 m_e}{m_p}$$

$$\sim \alpha \sqrt{\frac{m_e}{m_p}}$$

$$v \sim \sigma$$

$$= \left(\frac{p^2}{2m} - \frac{\alpha}{r} \right)$$

$$H = \left(\frac{p^2}{2m} - \frac{\alpha}{r} \right)$$


$r = \alpha^{-1} m_e^{-1} \hat{r}$

$$r = \alpha^{-1} m_e^{-1} \hat{r}$$

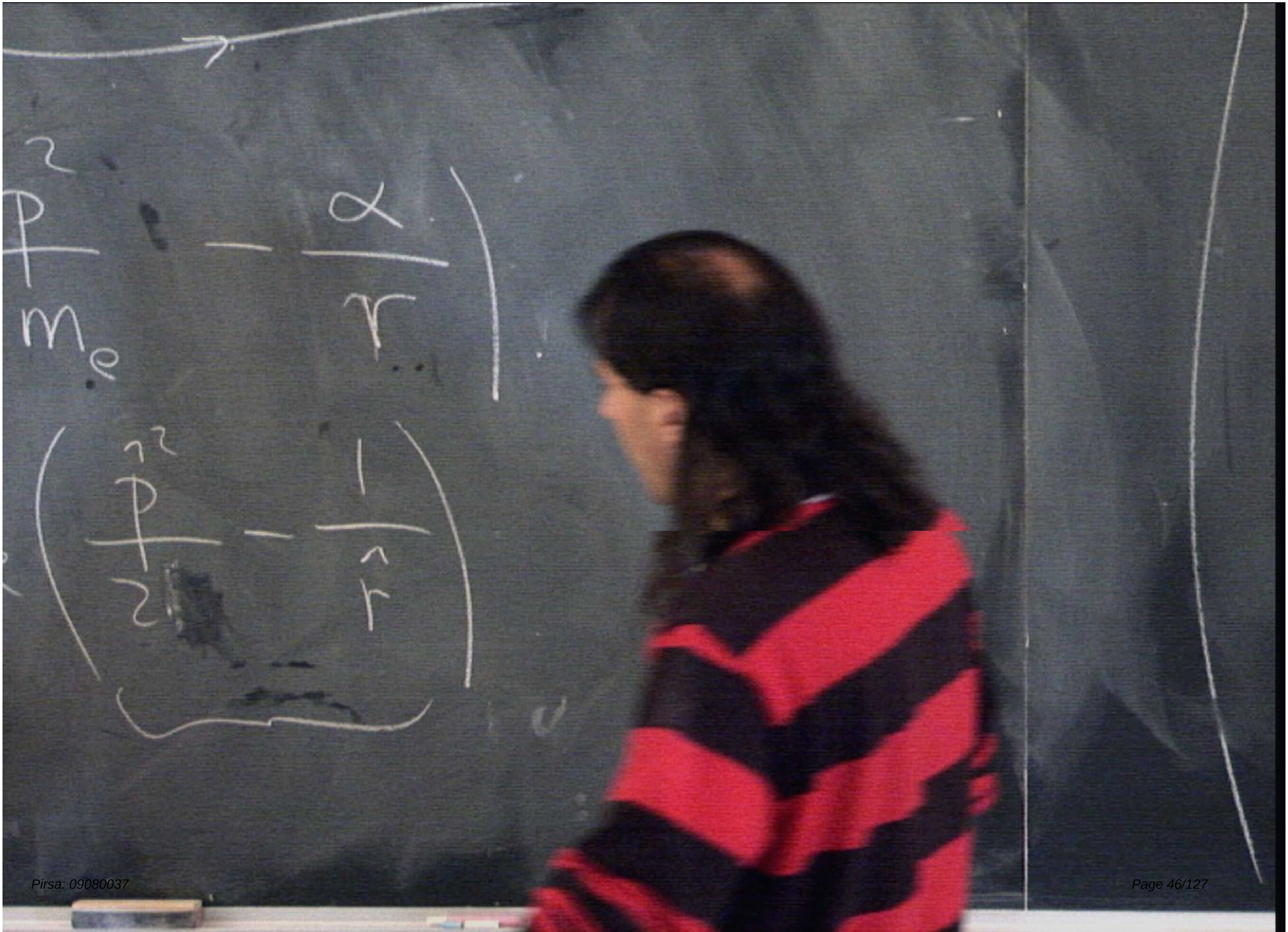
$$p = \alpha \, m_e \, \hat{p}$$

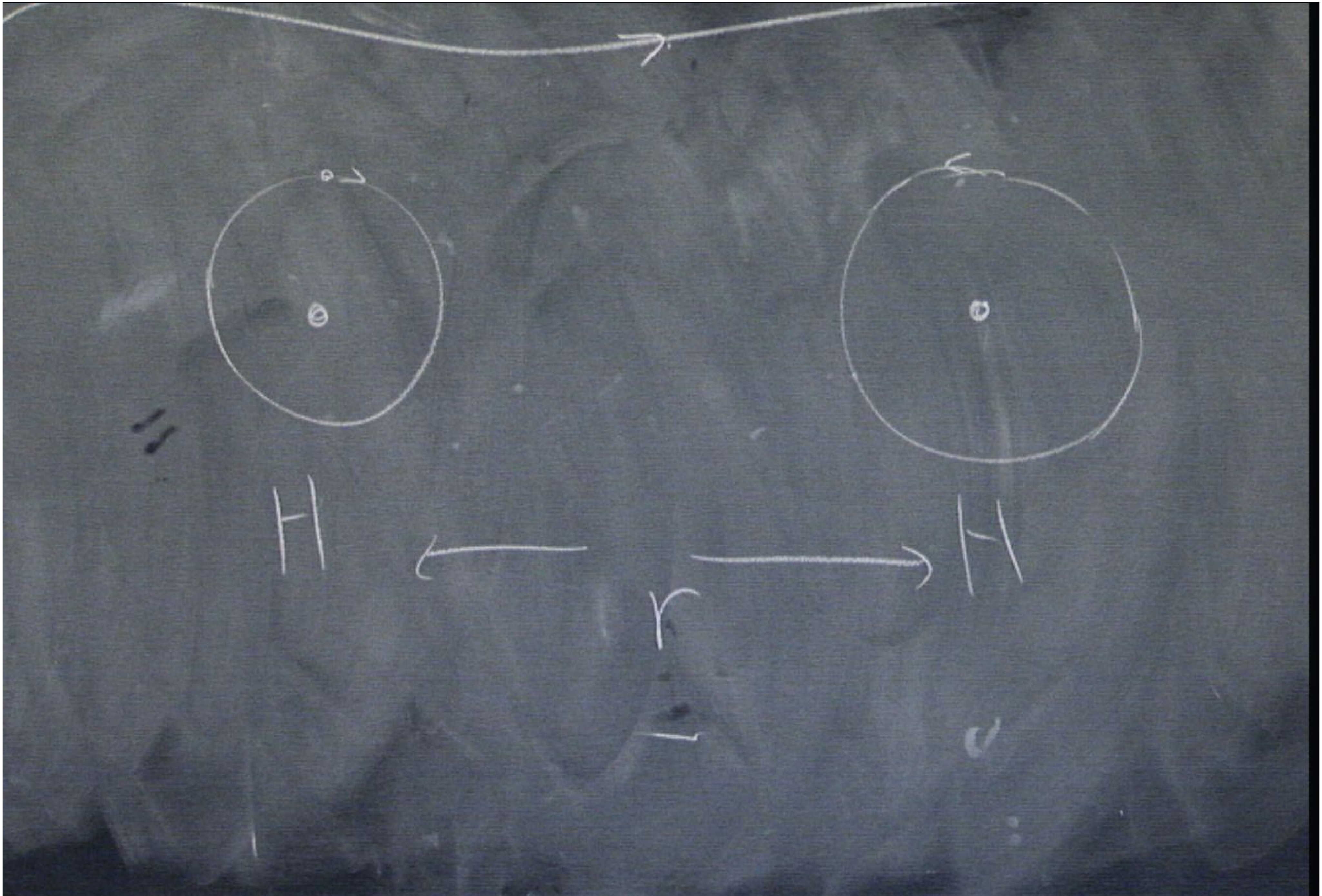
$$H = \left(\frac{p^2}{2m_e} - \frac{\alpha}{r} \right)$$

$$= \alpha^2 m_e \left(\frac{\hat{p}^2}{2} - \frac{1}{\hat{r}} \right)$$

$$H = \left(\frac{p^2}{2m_e} - \frac{\alpha}{r} \right)$$

$$= \alpha^2 m_e \left(\frac{\hat{p}^2}{2} - \frac{1}{\hat{r}} \right)$$




H
r
H

$$E \sim \frac{d_1}{r^3}$$

$$E \sim \frac{d_1}{r^3}.$$

$$H_{int} \sim \frac{\vec{d}_1 \cdot \vec{d}_2}{4\pi r^3}.$$

r^3

$$H_{int} \sim \frac{\vec{d}_1 \cdot \vec{d}_2}{4\pi r^3}$$

$$\langle \text{ground} | H_{int} | \text{ground} \rangle$$

$$\frac{\langle \text{ground} | H_{int} | E_n \rangle \langle E_n | H_{int} | \text{ground} \rangle}{E_n - E_{\text{ground}}}$$

$$\frac{\langle \text{ground} | H_{int} | E_n \rangle \langle E_n | H_{int} | \text{ground} \rangle}{E_n - E_{\text{ground}}}$$

$$\sim \frac{1}{r^6} (d^2)(d^2)$$

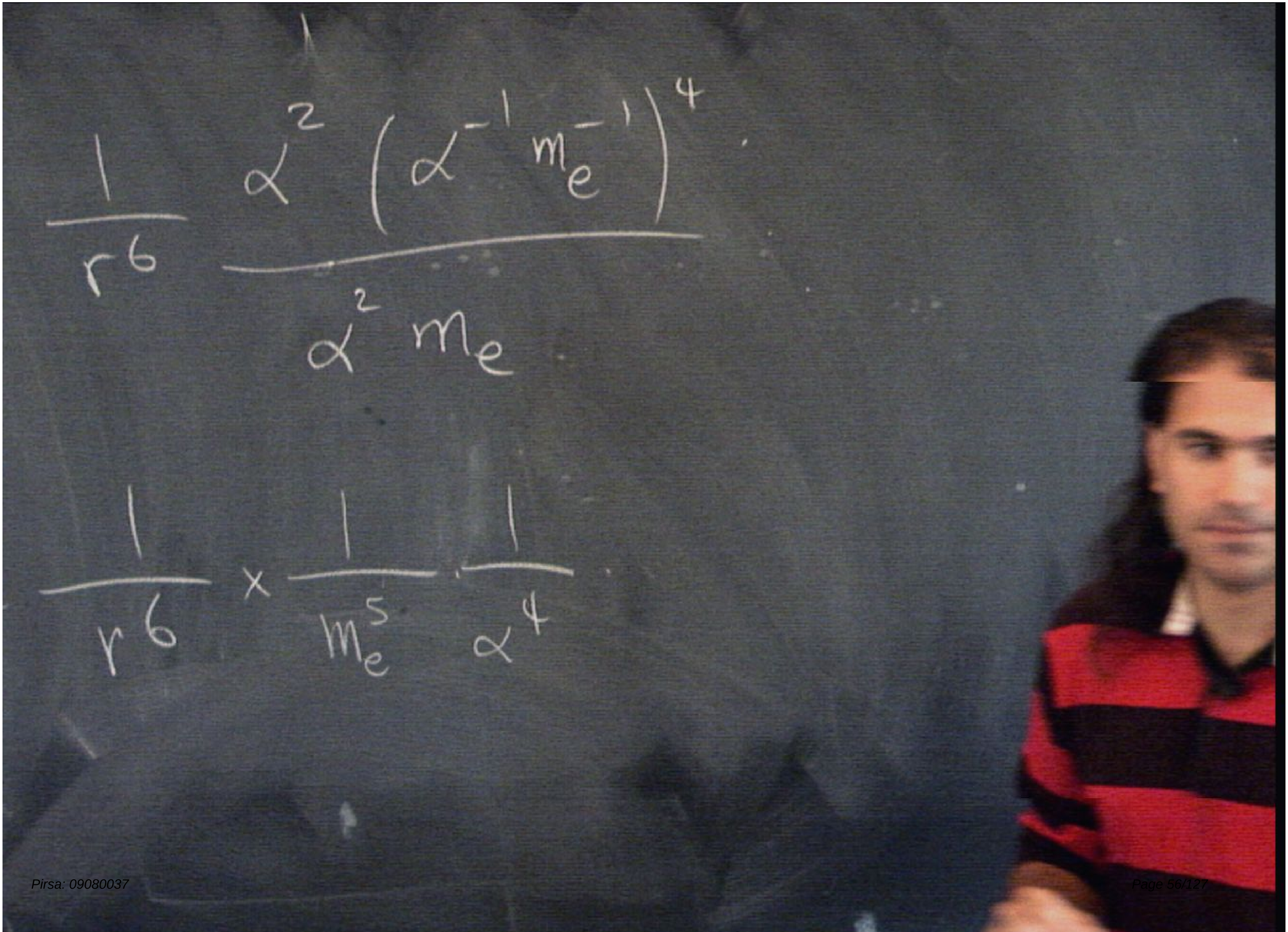
$$\frac{\langle \text{ground} | H_{int} | E_n \rangle \langle E_n | H_{int} | \text{ground} \rangle}{E_n - E_{\text{ground}}}$$

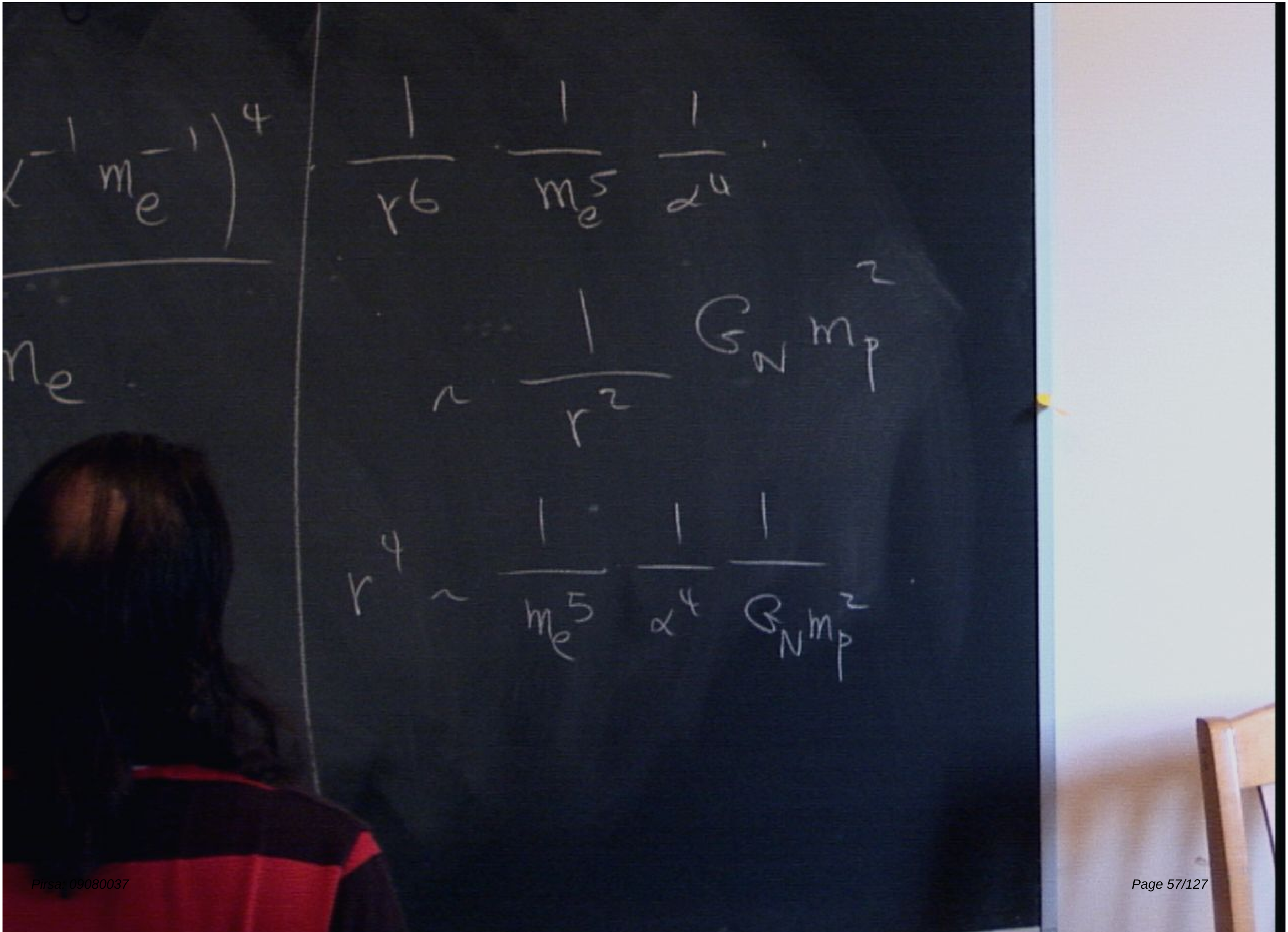
$$\sim \frac{1}{r^6} \frac{(d^2)(d^2)}{E_{\text{atom}}}$$

$$\frac{\langle \text{ground}|H_{int}|E_n\rangle\langle E_n|H_{int}|\text{ground}\rangle}{E_n - E_{ground}}$$

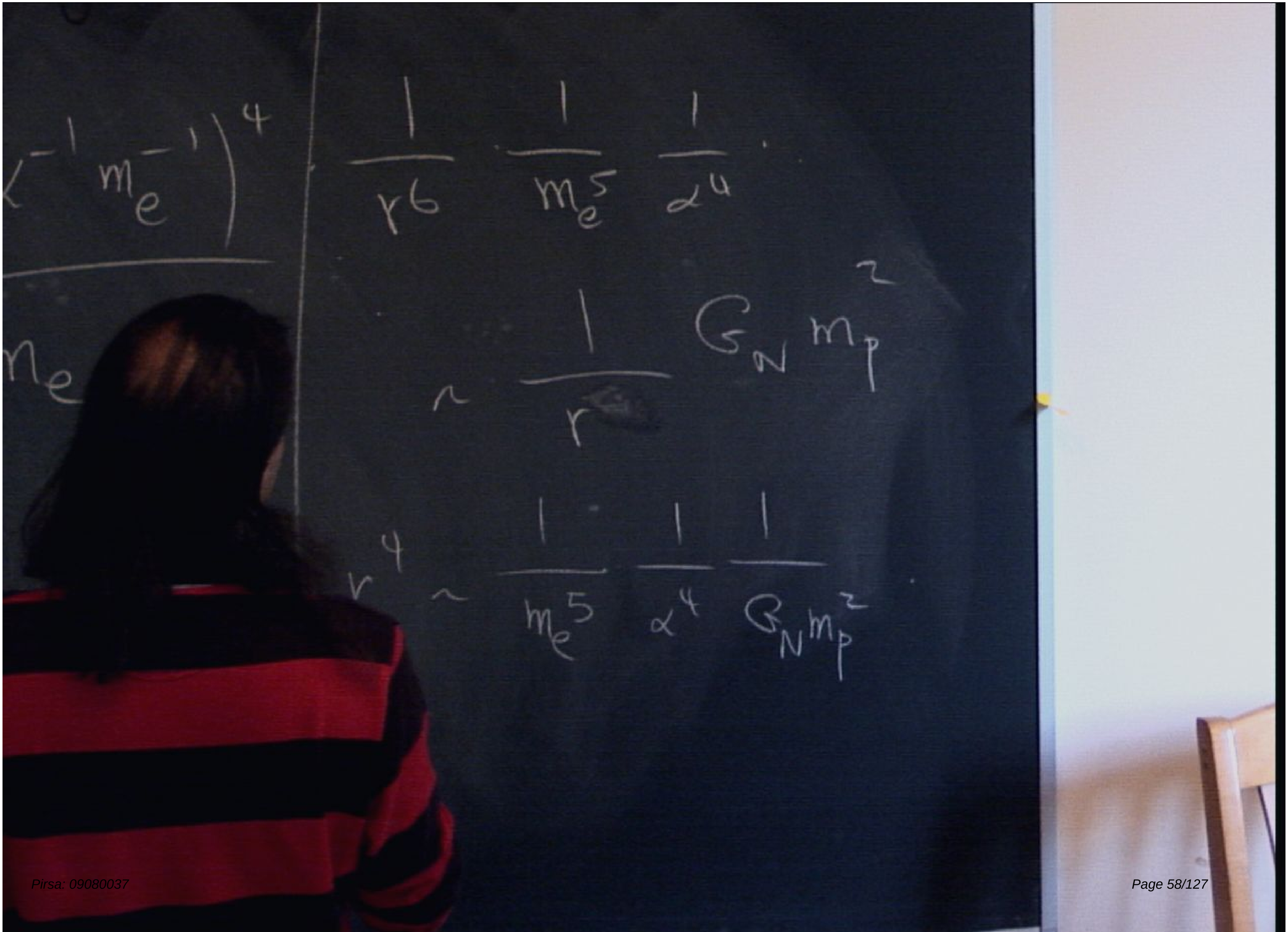
$$d \sim e\, r_{atom}$$

$$\sim \frac{1}{(4\pi)^2 r^6}\,\frac{(d^2)(d^2)}{E_{atom}} \sim \frac{1}{r^6}\left(\frac{e^2}{4\pi}\right)^2$$


$\frac{1}{r^6} \frac{\alpha^2 (\alpha^{-1} m_e^{-1})^4}{\alpha^2 m_e}$
$\frac{1}{r^6} \times \frac{1}{m_e^5} \cdot \frac{1}{\alpha^4}$



$\frac{1}{r^6} \frac{1}{m_e^5} \frac{1}{\alpha^4}$
$\sim \frac{1}{r^2} G_N m_p^2$
$r^4 \sim \frac{1}{m_e^5} \frac{1}{\alpha^4} \frac{1}{G_N m_p^2}$


$\frac{1}{r^6} \cdot \frac{1}{m_e^5} \frac{1}{\alpha^4}$
$\sim \frac{1}{r} \; G_N m_p^2$
$r^4 \sim \frac{1}{m_e^5} \cdot \frac{1}{\alpha^4} \frac{1}{G_N m_p^2}$

$$\left(\alpha^{-1} m_e^{-1}\right)^4$$

$$m_e$$

$$\frac{1}{r^6} \cdot \frac{1}{m_e^5} \, \frac{1}{\alpha^4} \cdot$$

$$\sim \frac{1}{r} \, G_N m_p^2$$

$$r^5 \sim \frac{1}{m_e^5} \, \frac{1}{\alpha^4} \, \frac{1}{G_N m_p^2}$$

$$r \sim \frac{1}{m_e} \times \left(\frac{1}{\alpha^4} \, \frac{1}{G_N m_p^2} \right)^{1/5}$$

$r \sim 10^{-11}$ cm.

$\times \left[10^{-8} \right.$

$\frac{1}{r^6} \quad \frac{1}{m_e^5} \quad \frac{1}{\alpha}$

$\sim \frac{1}{r} \quad ($

$r^5 \sim \frac{1}{m_e^5} \frac{1}{\alpha^4}$

$r \sim \frac{1}{m_e} \times \left(\frac{1}{\alpha^4} \right.$

$$r \sim 10^{-11}\ \text{cm}$$

$$\times \left[10^{-8}\, 10^{-38}\right]^{1/5}$$

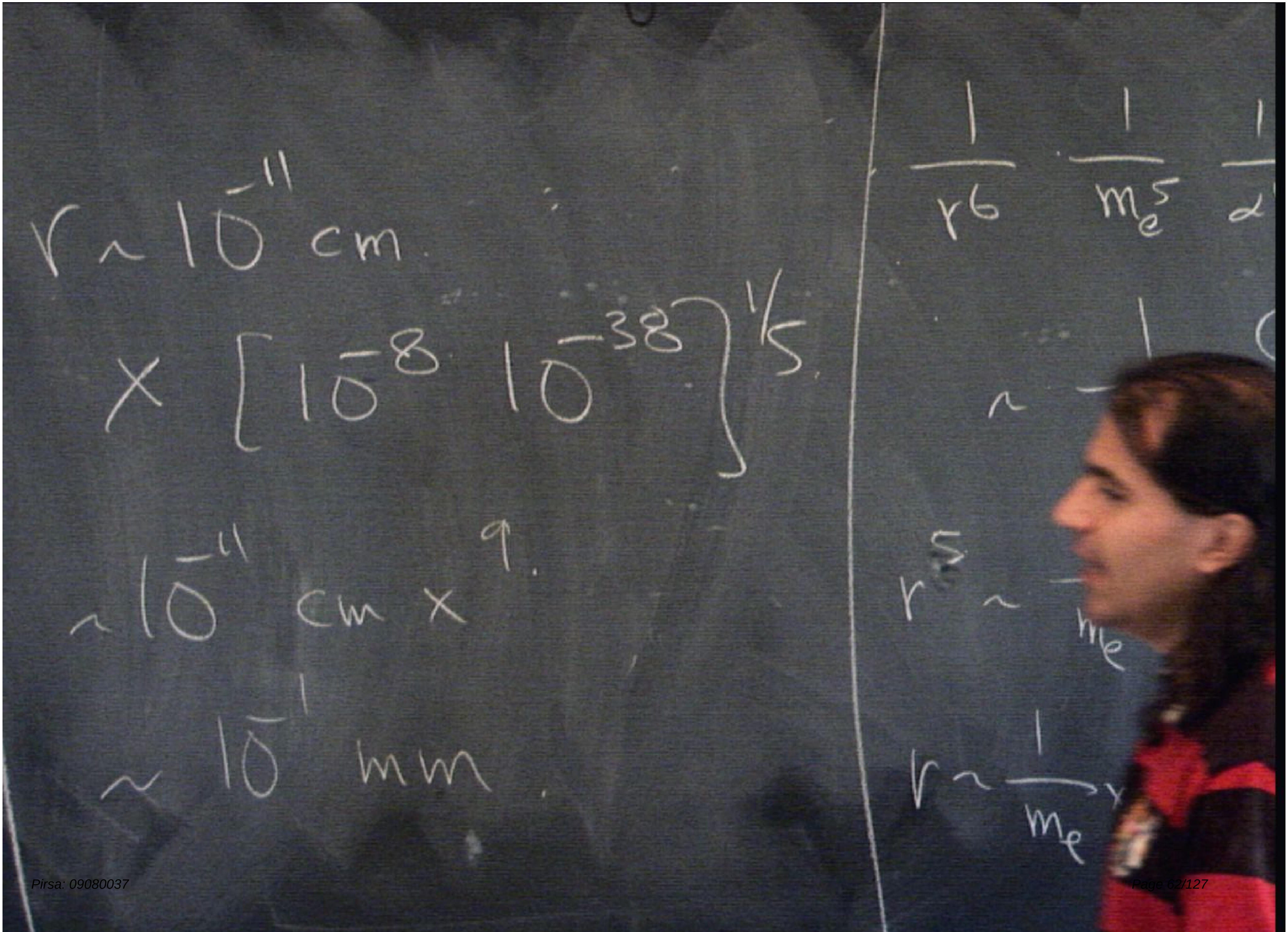
$$\sim 10^{-11}\ \text{cm} \times$$

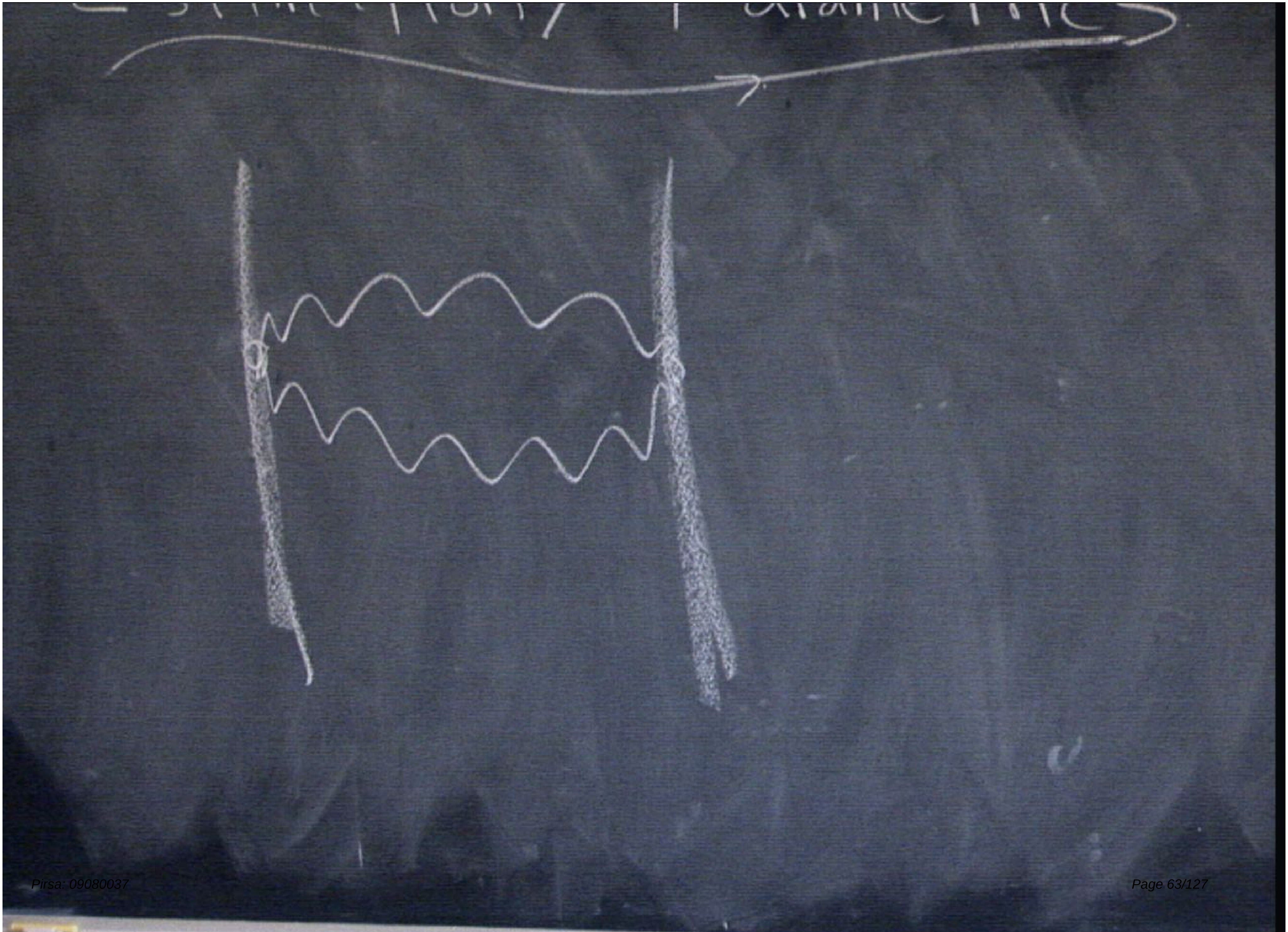
$$\frac{1}{r^6} \cdot \frac{1}{m_e^5}\ \frac{1}{\alpha}$$

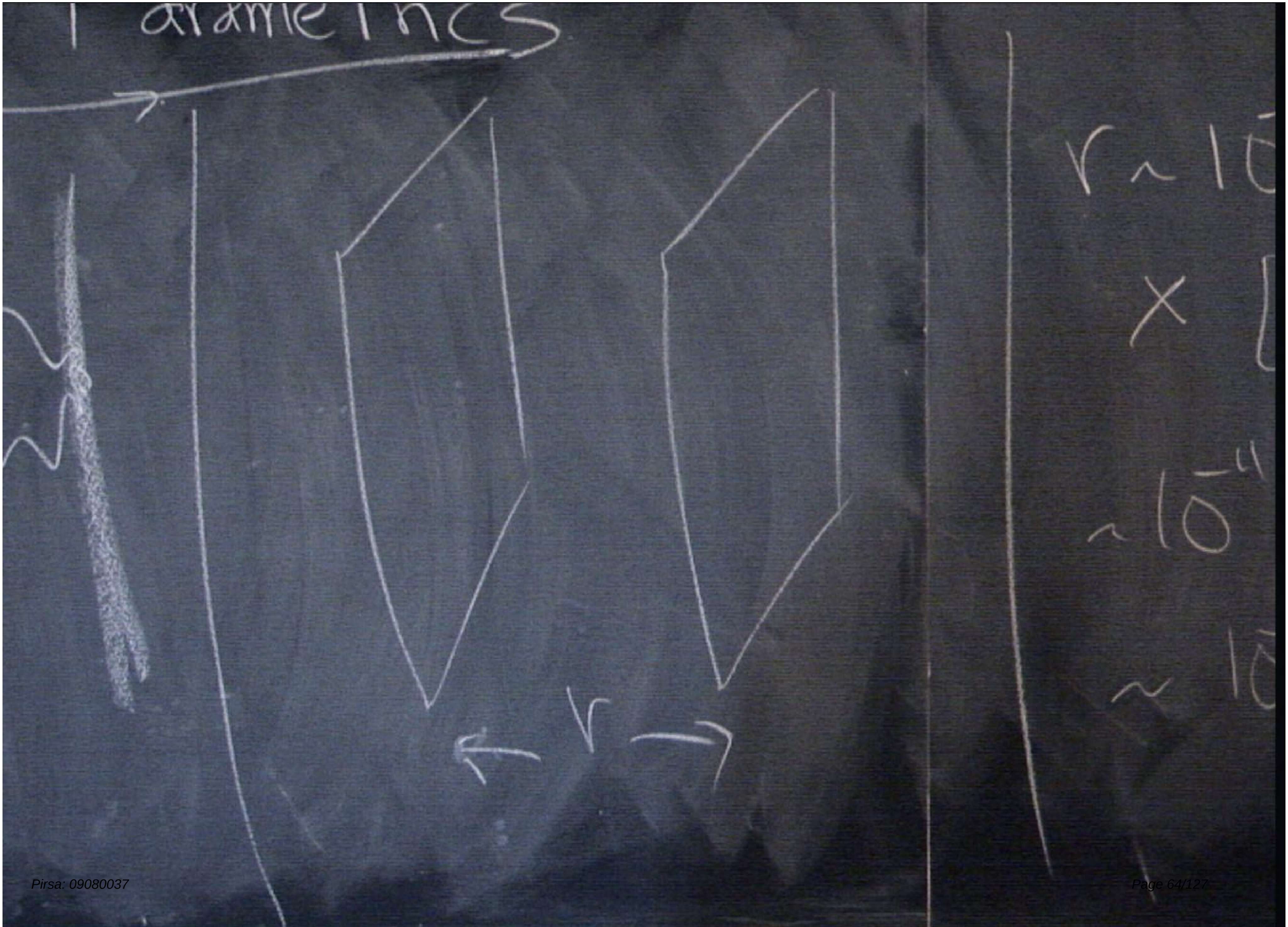
$$\sim \frac{1}{r}$$

$$r^5 \sim \frac{1}{m_e^5}\ \frac{1}{\alpha^4}$$

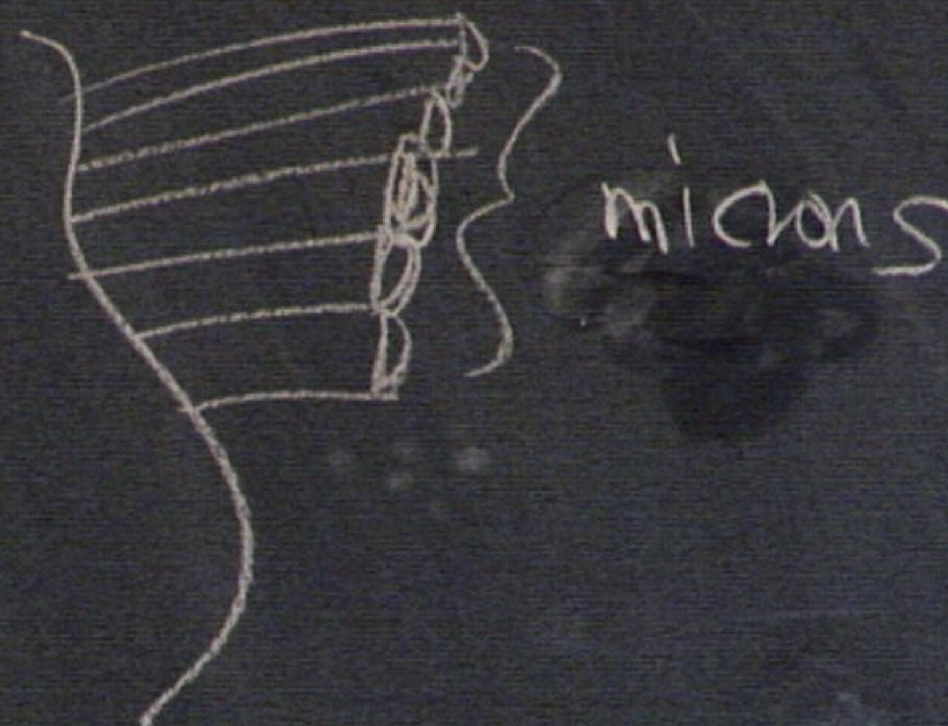
$$r \sim \frac{1}{m_e} \times \left(\frac{1}{\alpha^4}\right.$$

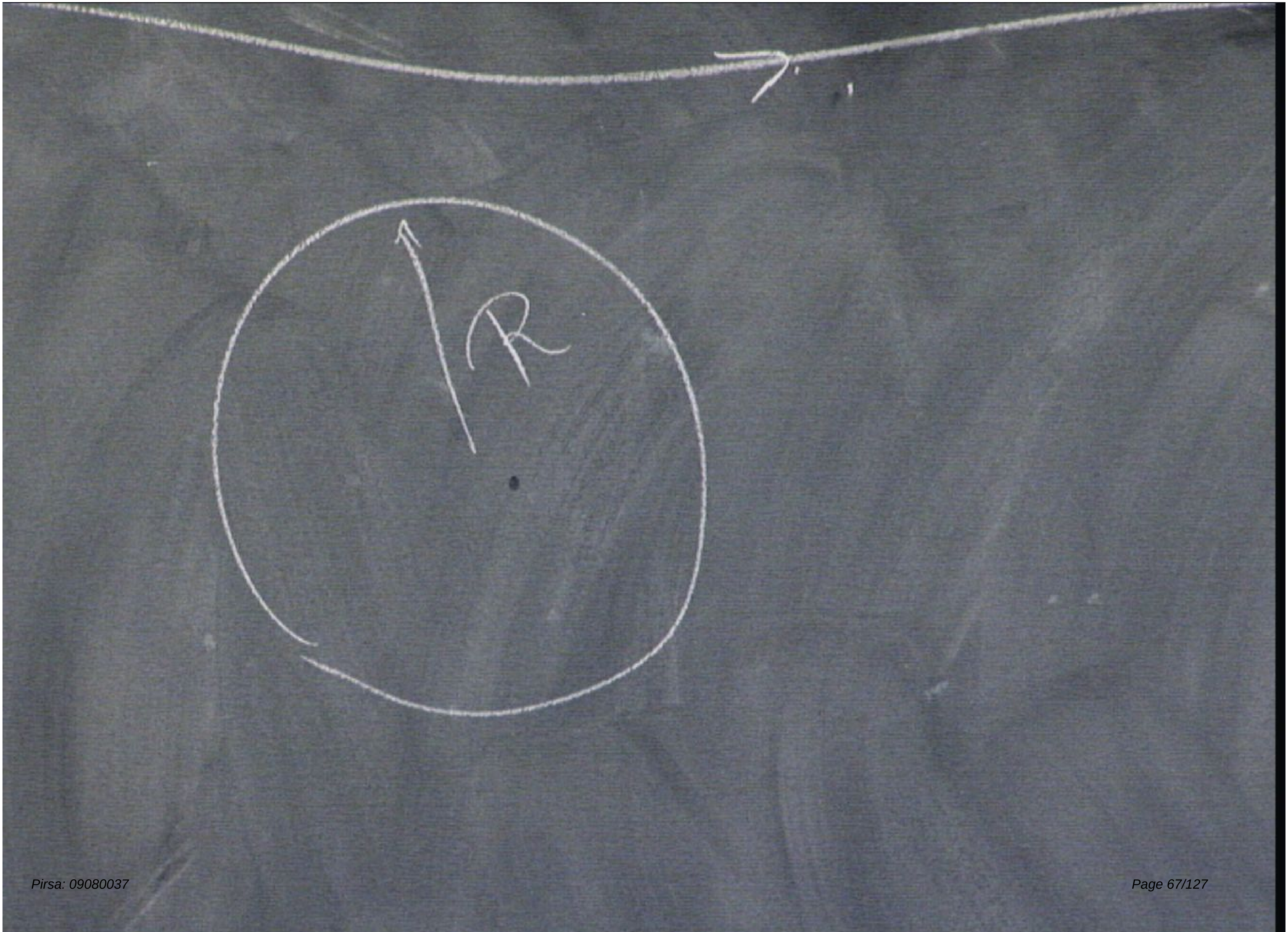

r ~ 10^-11 cm
× [10^-8 · 10^-38]^1/5
~ 10^-11 cm × 9
~ 10^-1 mm
1/r6 · 1/m_e^5
r ~ 1/m_e ×

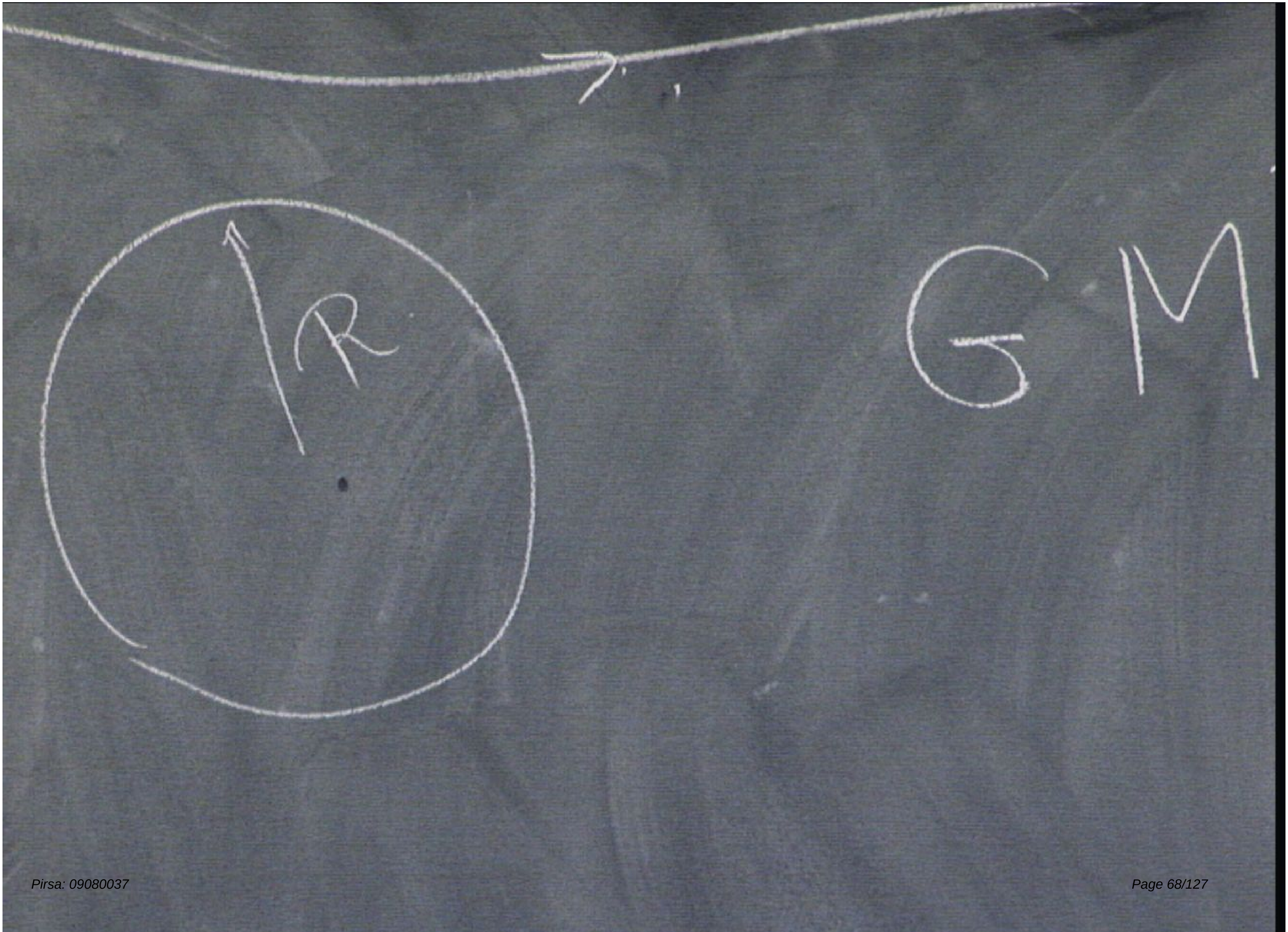



← r →

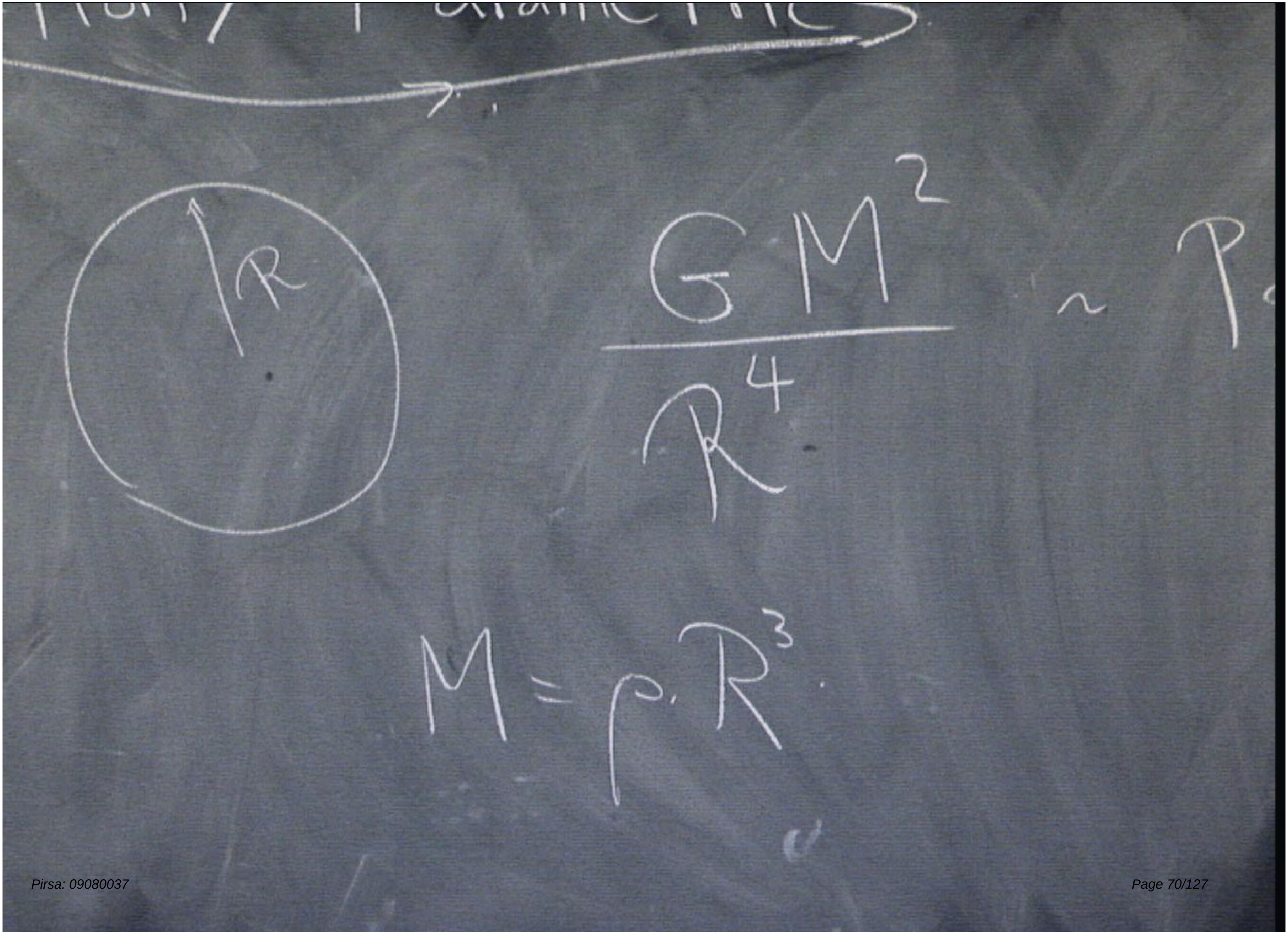
$$P = -\frac{1}{r^4}$$


microns


R


R
GM


R
GM^2
R^4


R
$\frac{GM^2}{R^4} \sim P$
$M = \rho \cdot R^3$

grav $\sim G\rho^2 R^2$ ...

$$\mathrm{grav} \sim G\rho_{matter}^2 R^2$$

$$G\rho_{atoms}^2 R^2 \sim P_{atom}$$

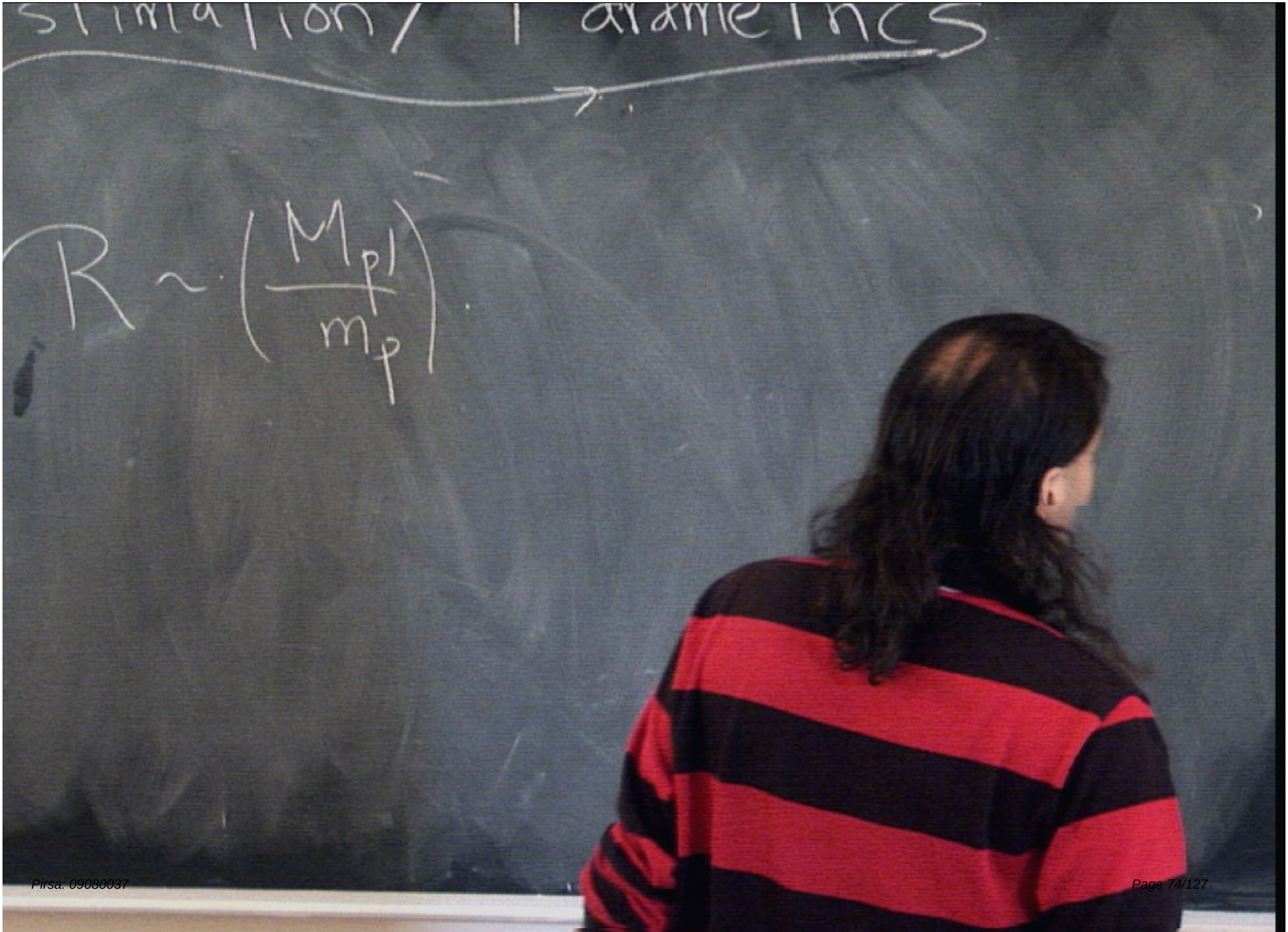
$$R^2 \sim \frac{\alpha^2 m_e}{(\alpha^{-1} m_e^{-1})^3} \cdot \frac{1}{G\,(m_p(\alpha^{-1} m_e^{-1})^3}$$

$$G = \frac{1}{M_{pl}^2}.$$

$$G \rho_{matter}^2 R^2$$

$$\rho_{atoms}^2 R^2 \sim P_{atom}$$

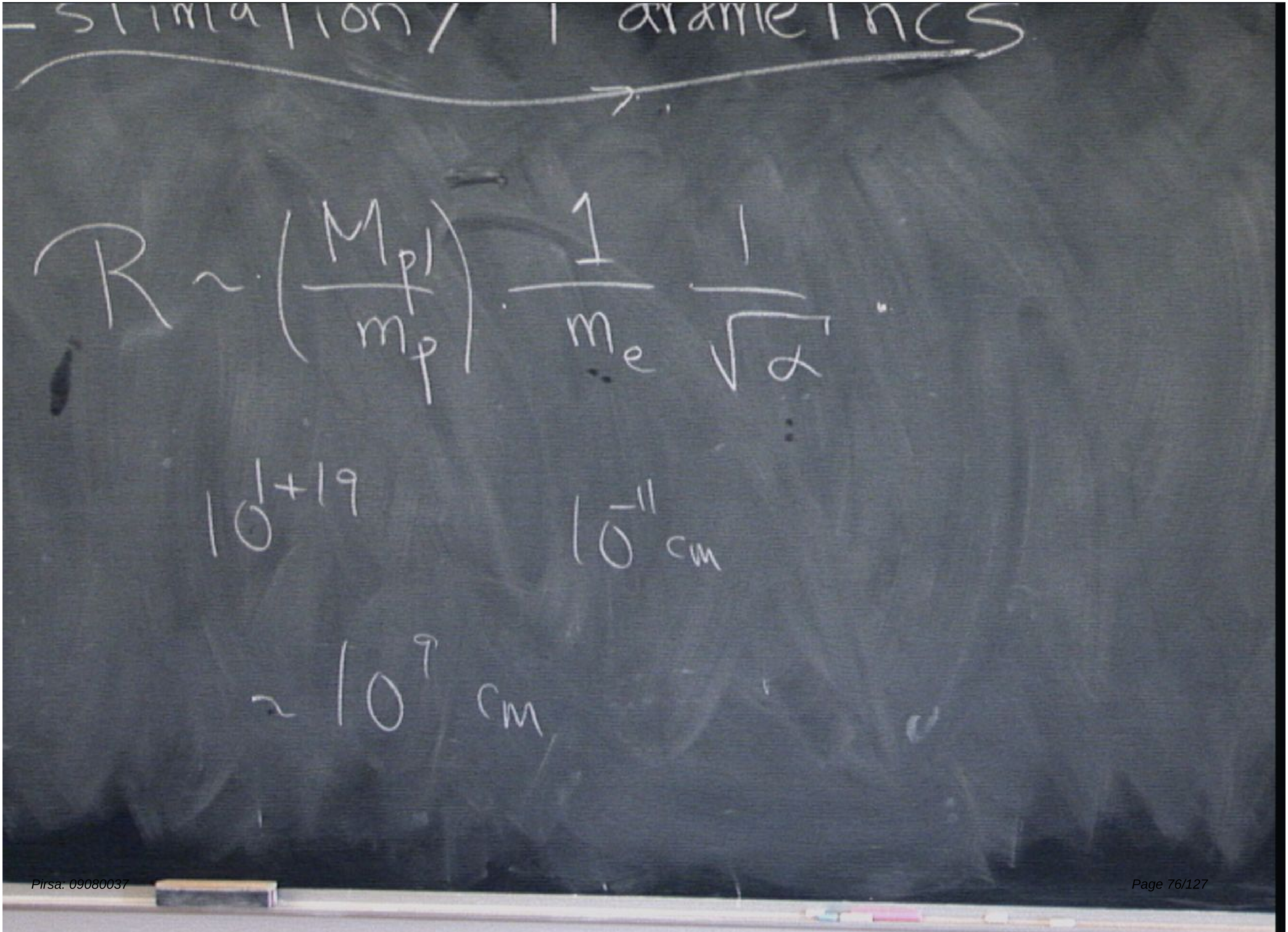
$$R^2 \sim \frac{\alpha^2 m_e}{(\alpha^{-1} m_e^{-1})^3} \cdot \frac{1}{G (m_p \alpha^3 m_e^3)^2} \sim \frac{M_{pl}^2}{m_p^2} \frac{1}{m_e^2} \frac{1}{\alpha}.$$


$R \sim \left(\frac{M_{pl}}{m_p}\right)$

Estimation / Parametrics

$$R \sim \left(\frac{M_{pl}}{m_p}\right) \frac{1}{m_e} \frac{1}{\sqrt{\alpha}}$$

10^{1+19} 10^{-11} cm


stimation / Parametrics
$R \sim \left(\frac{M_{pl}}{m_p}\right) \frac{1}{m_e} \frac{1}{\sqrt{\alpha}}$
10^{1+19}
10^{-11} cm
$\sim 10^{9}$ cm

Estimation / Parametrics

$$R \sim \left(\frac{M_{pl}}{m_p}\right) \frac{1}{m_e} \frac{1}{\sqrt{\alpha}}$$

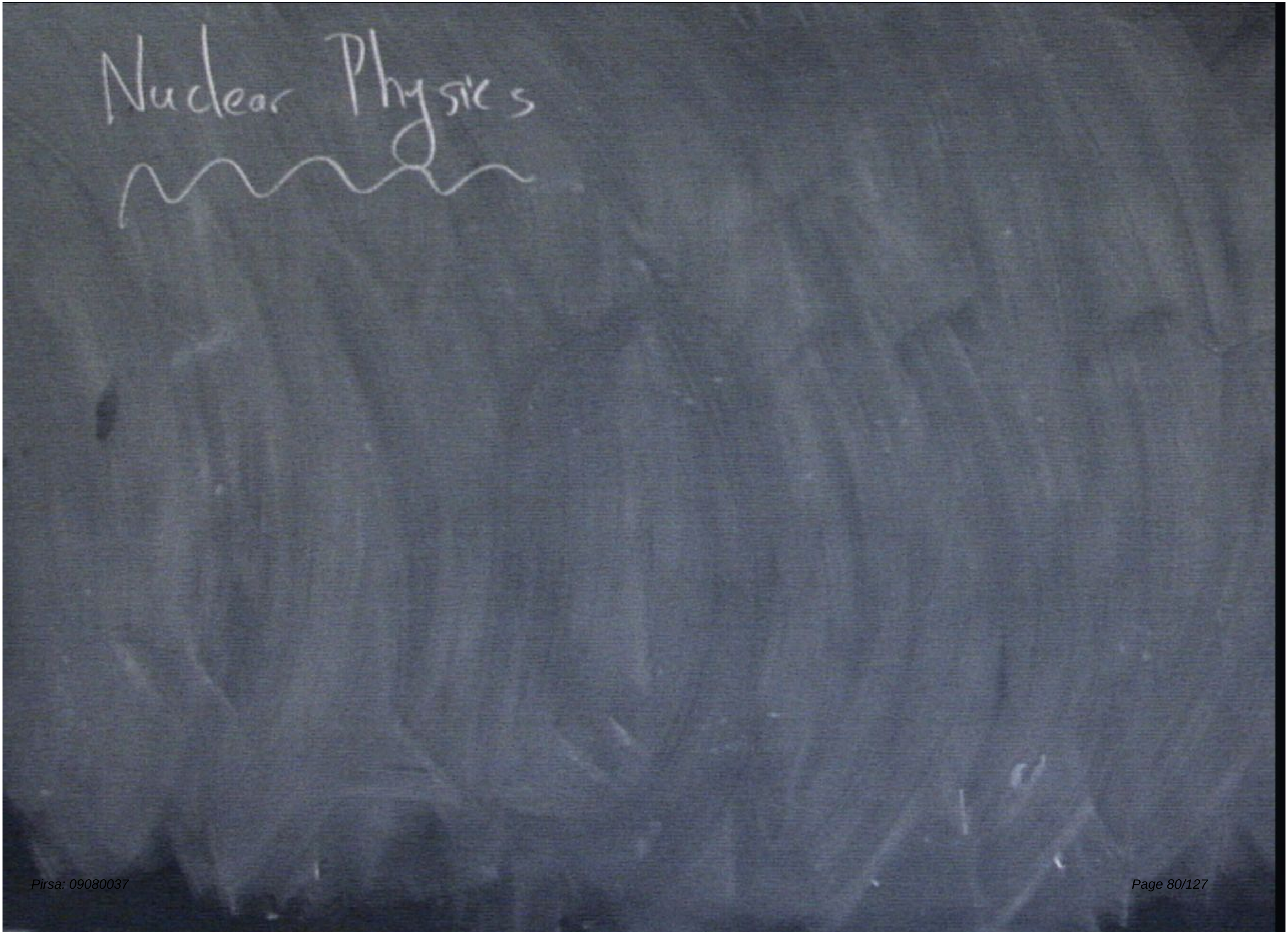
$$\sqrt{\alpha} \left(\frac{M_{pl}}{m_p}\right) \left(m_e^{-1} \alpha^{-1}\right) \sim \sqrt{\alpha} \left(\frac{M_{pl}}{m_p}\right) r_{atom}$$

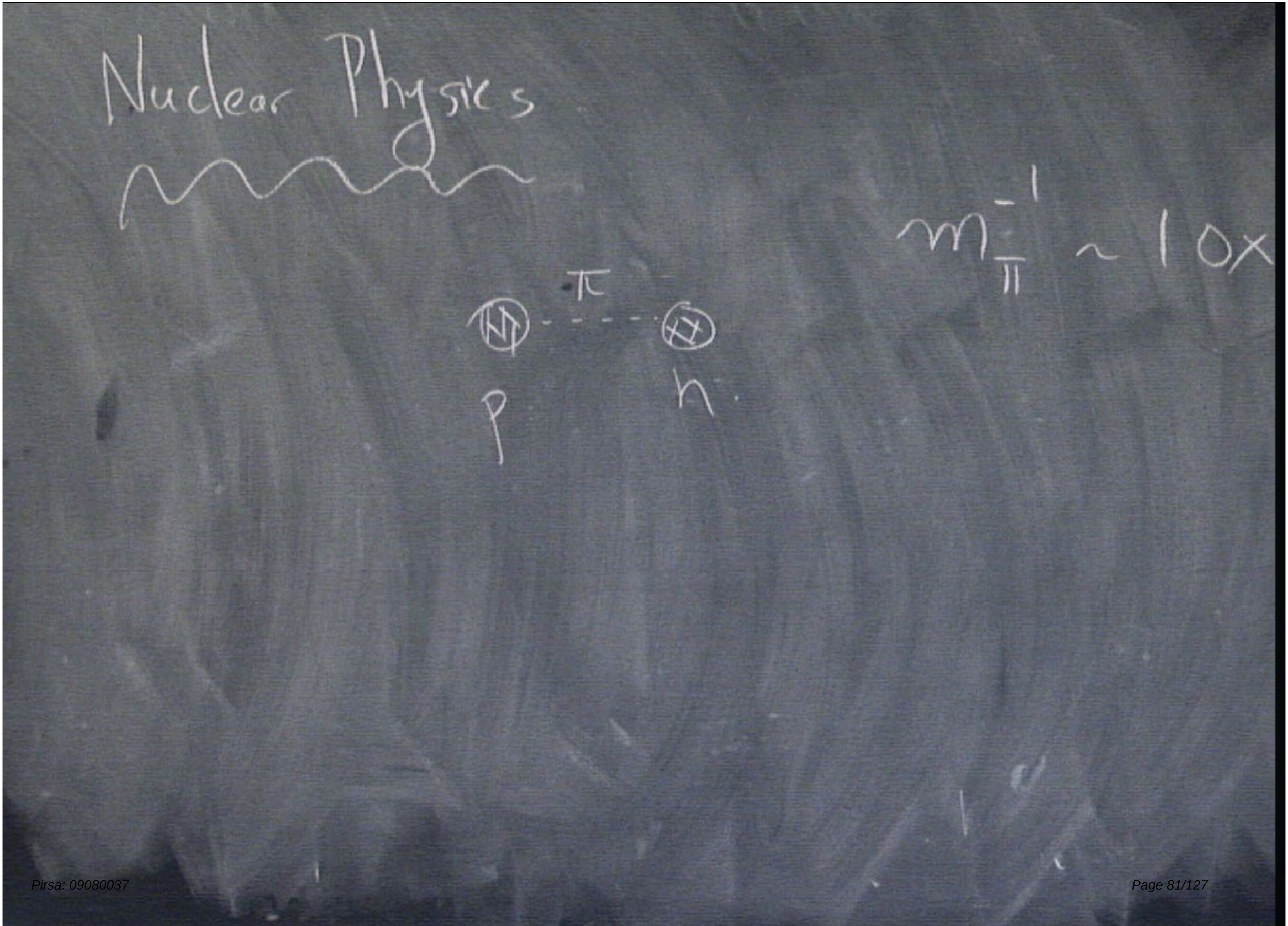
$$\alpha_G = G_N m_p^2 .$$

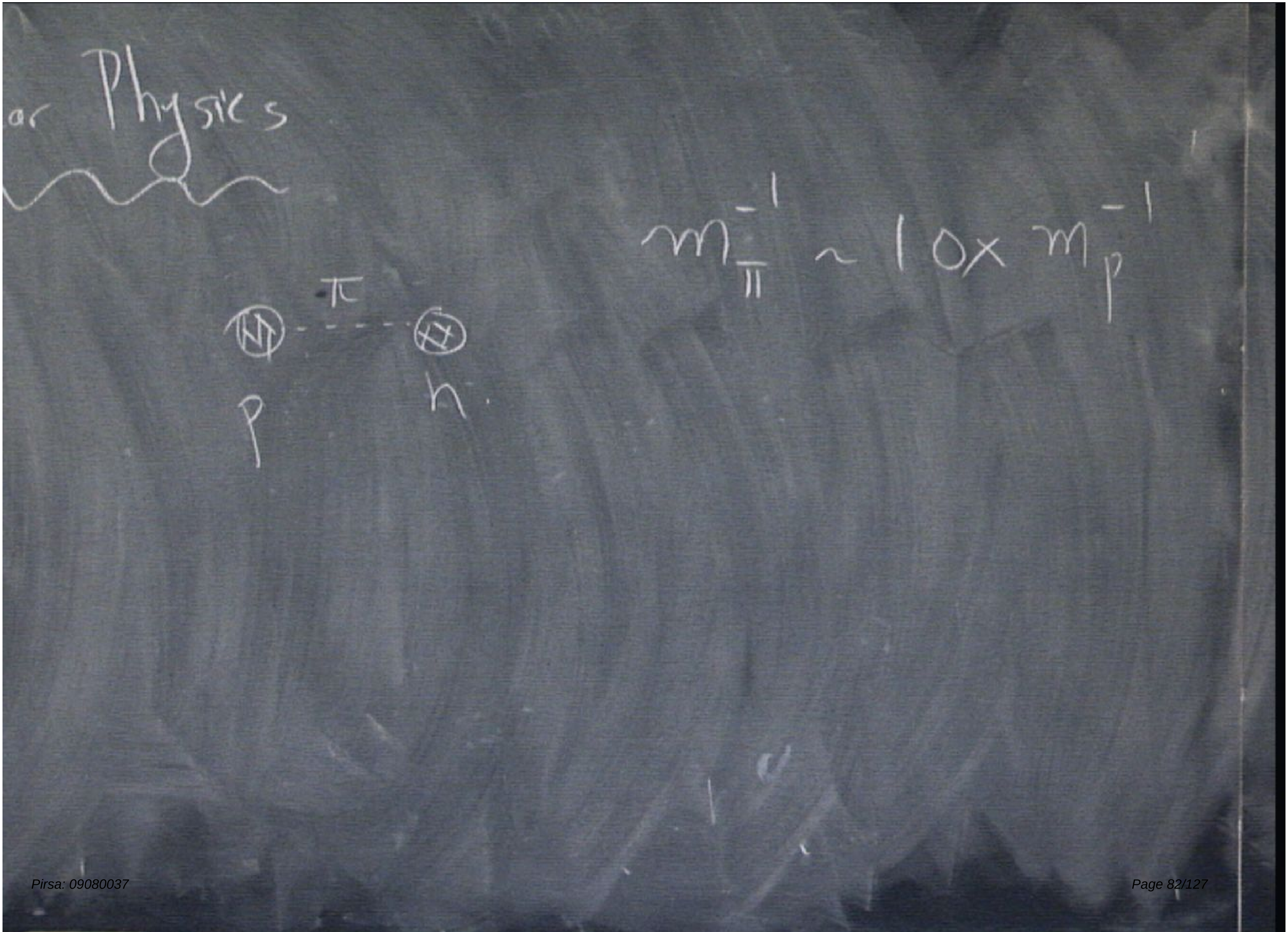
$$R \sim \left(\frac{M_{pl}}{m_p}\right) \frac{1}{m_e} \frac{1}{\sqrt{\alpha}}$$

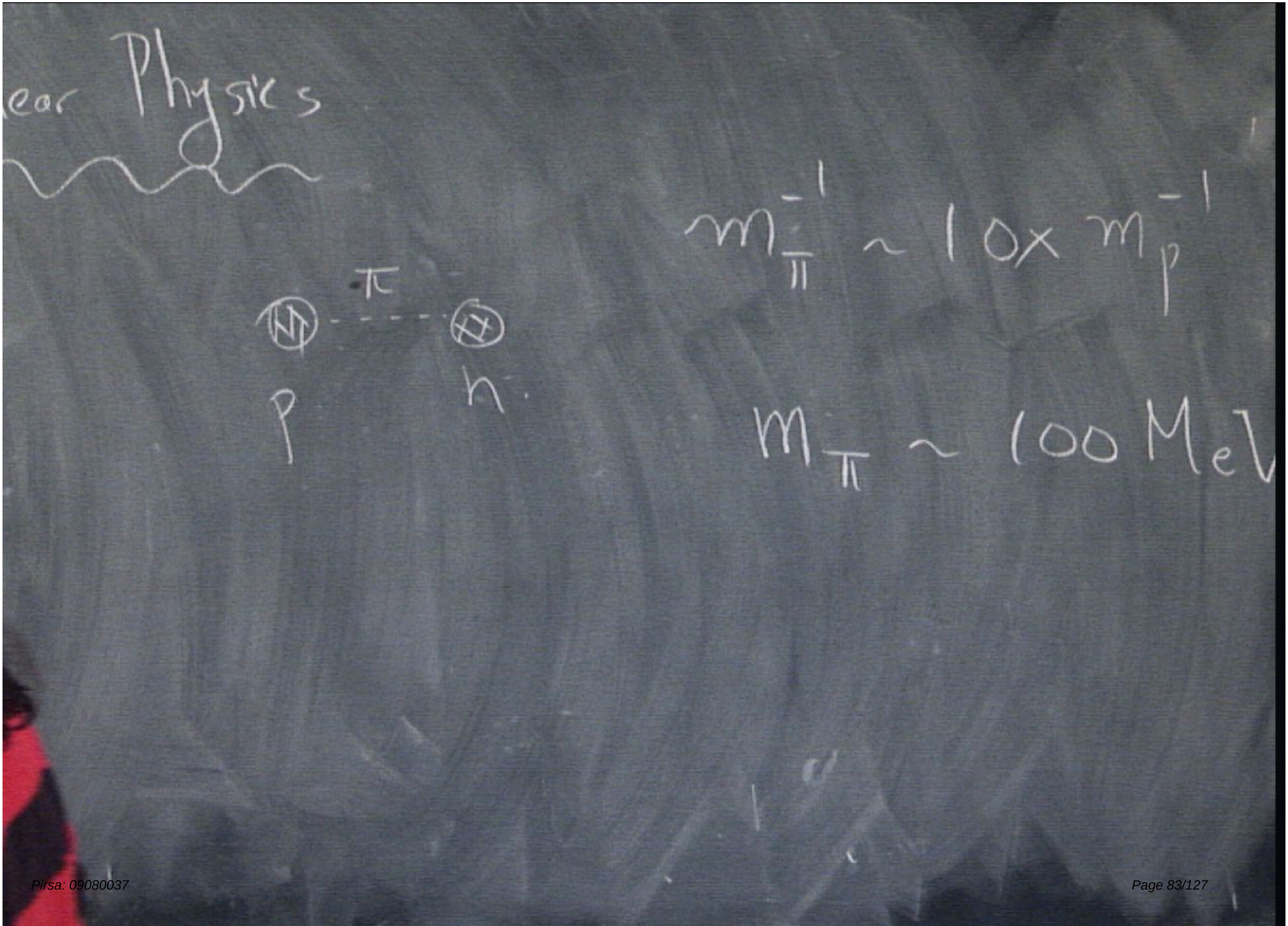
$$\sqrt{\alpha} \left(\frac{M_{pl}}{m_p}\right) \left(m_e^{-1} \alpha^{-1}\right) \sim \sqrt{\alpha} \left(\frac{M_{pl}}{m_p}\right) r_a$$

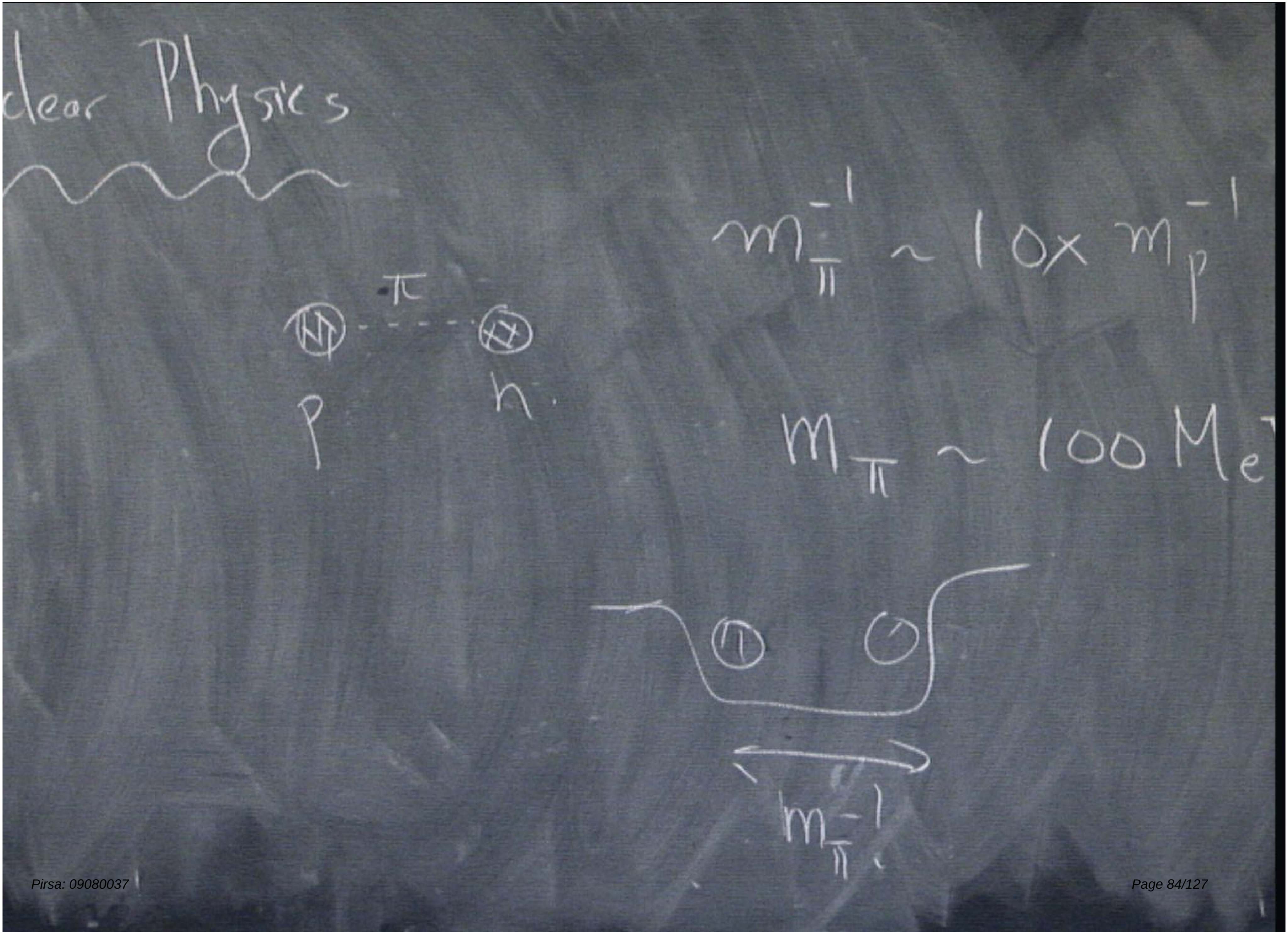
$$\sim \sqrt{\frac{\alpha}{\alpha_G}}\, r_{atom}$$

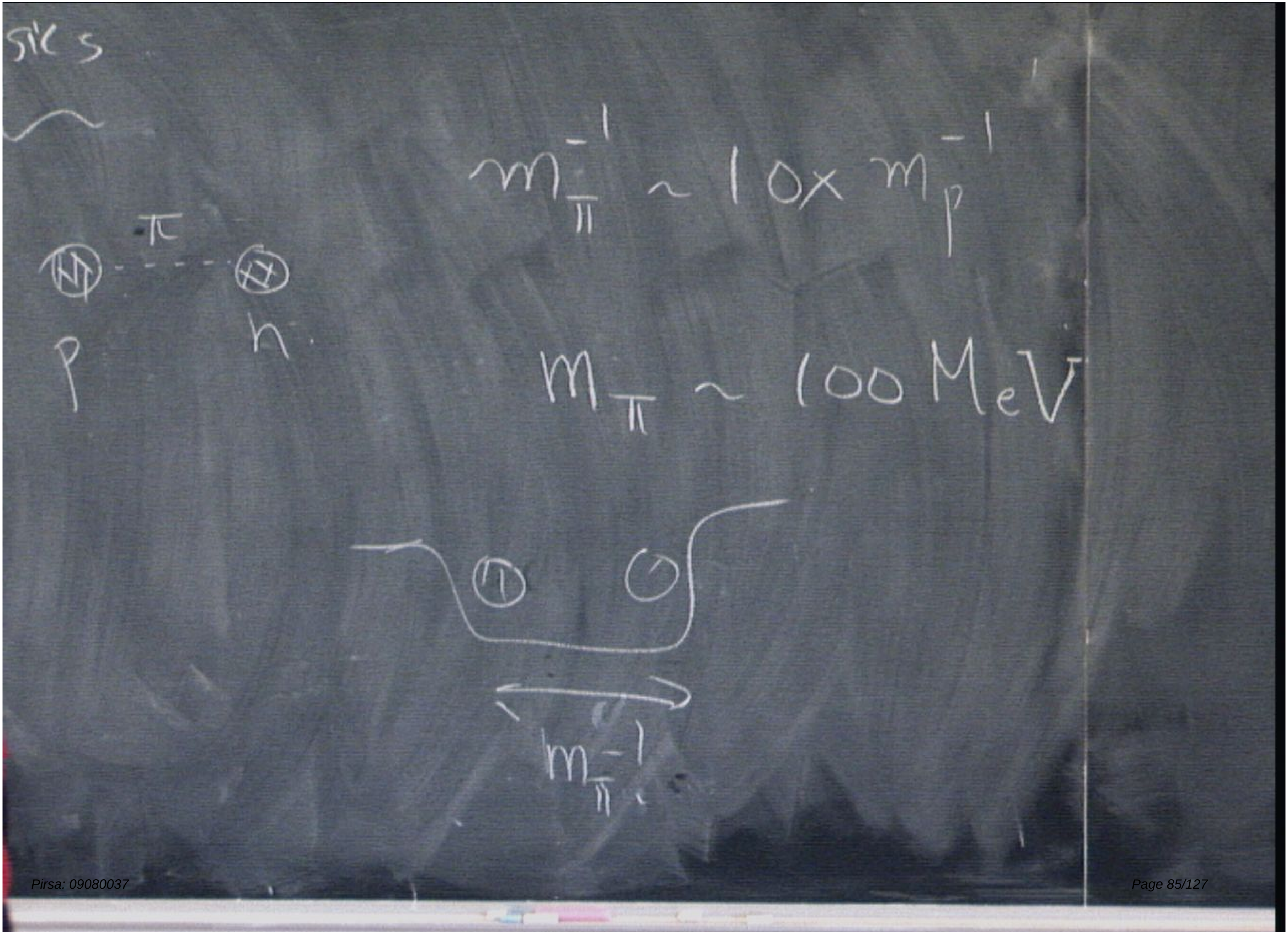

Nuclear Physics


Nuclear Physics
π
p
n
$m_\pi^{-1} \sim 10x$

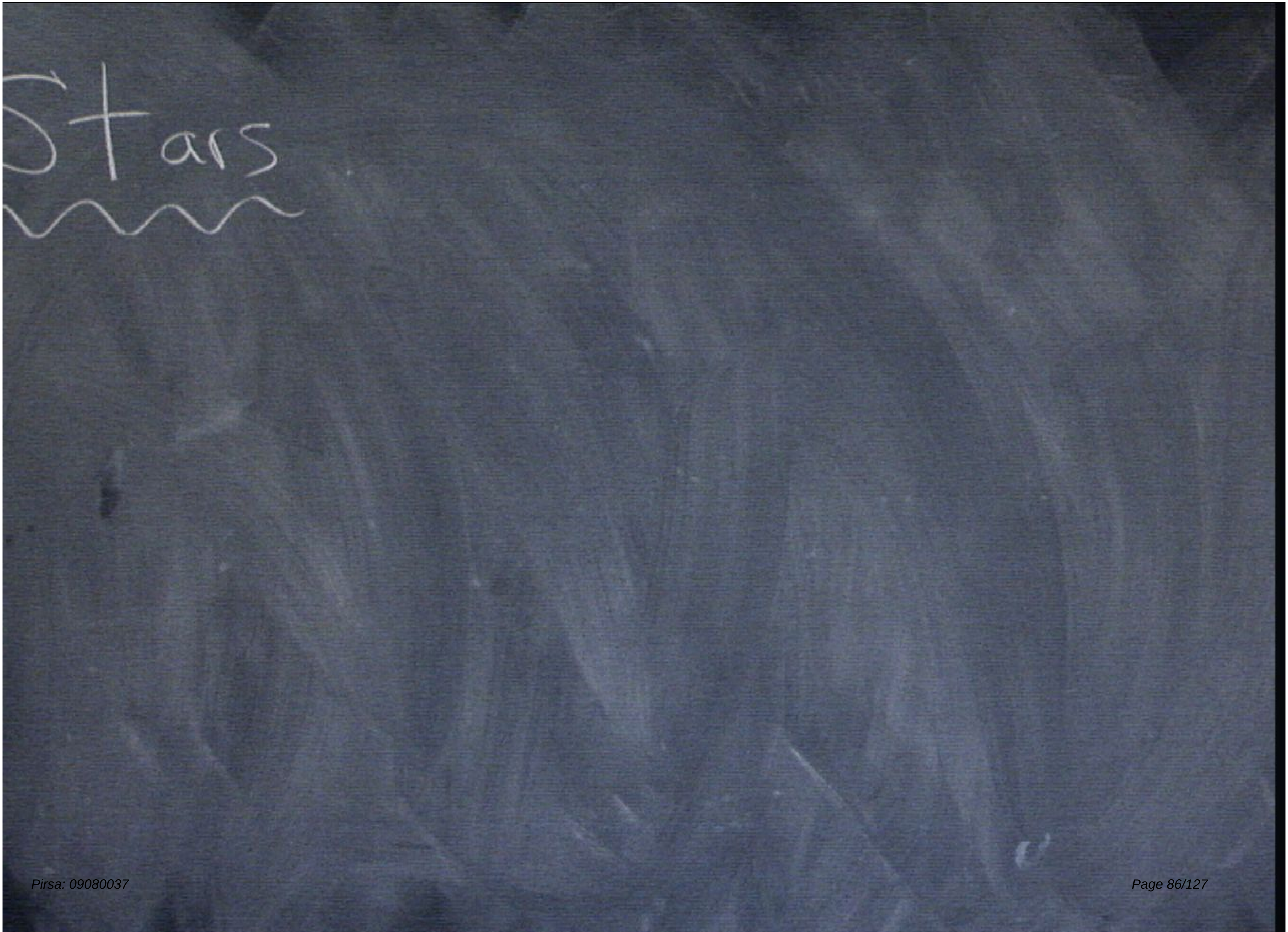

ar Physics
π
p
n
$m_\pi^{-1} \sim 10 \times m_p^{-1}$


...ear Physics
m_π^{-1} ~ 10x m_p^{-1}
π
p
n
m_π ~ 100 MeV


clear Physics
π
p
n
$m_\pi^{-1} \sim 10 \times m_p^{-1}$
$m_\pi \sim 100$ Me
m_π^{-1}



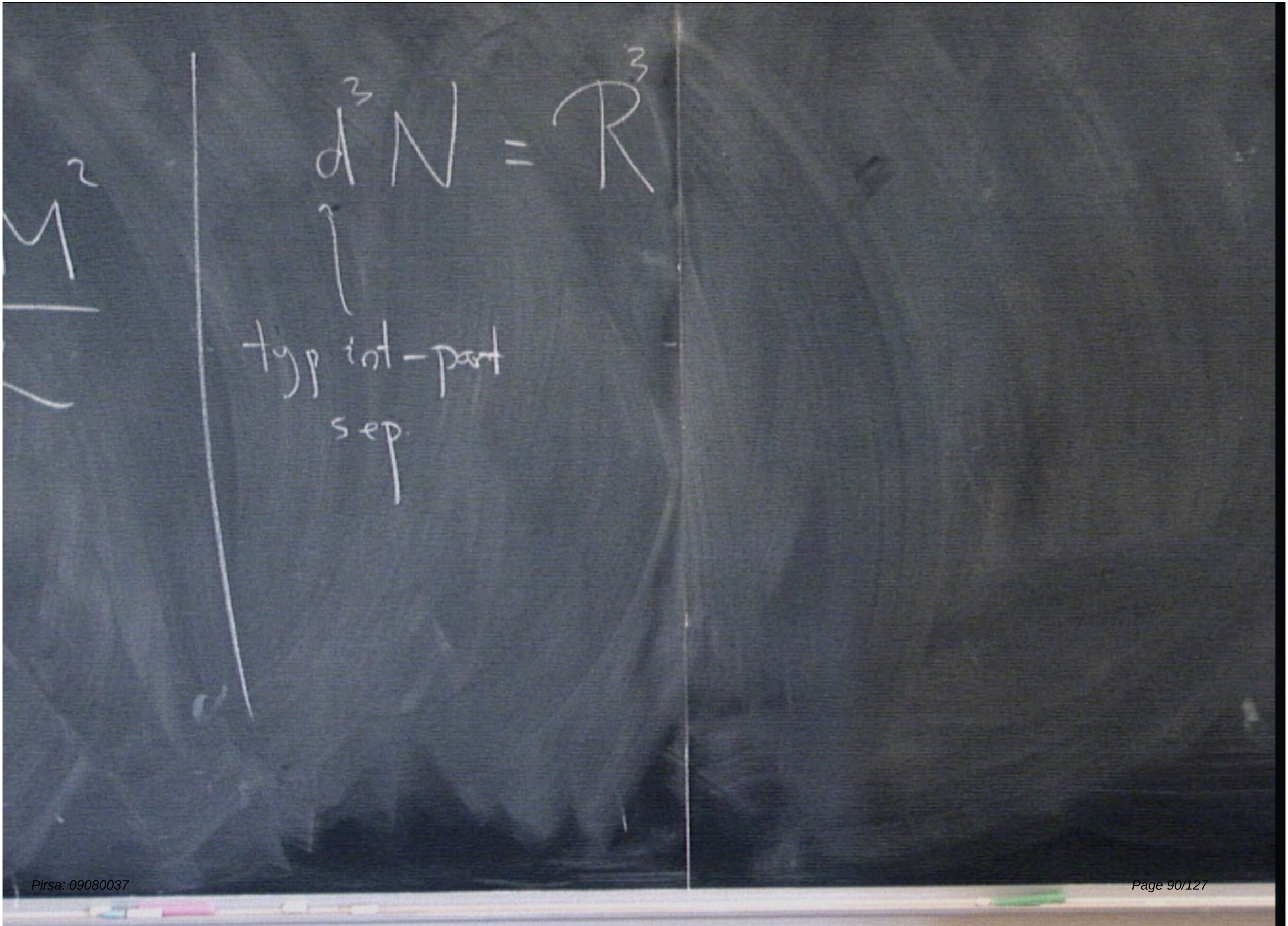
π
p
n
m_π^-1 ~ 10x m_p^-1
m_π ~ 100 MeV
m_π^-1


Stars


N T


$\bar{N} T \sim \frac{GM^2}{R}$

$$d^3N = R^3$$


$d^3 N = R^3$
typ int-part
sep.

$$N T \sim \frac{G M^2}{R}$$

$$T \sim \frac{G m_p^2 N^2}{R} \sim \frac{G m_p^2 N^{2/3}}{d}$$

$$T \sim \frac{GM^2}{R}$$

$$\frac{G m_p^2 N}{R} \sim \frac{G m_p^2 N^{2/3}}{d}$$

$$d^3 N = R^3$$

typ int-part sep.

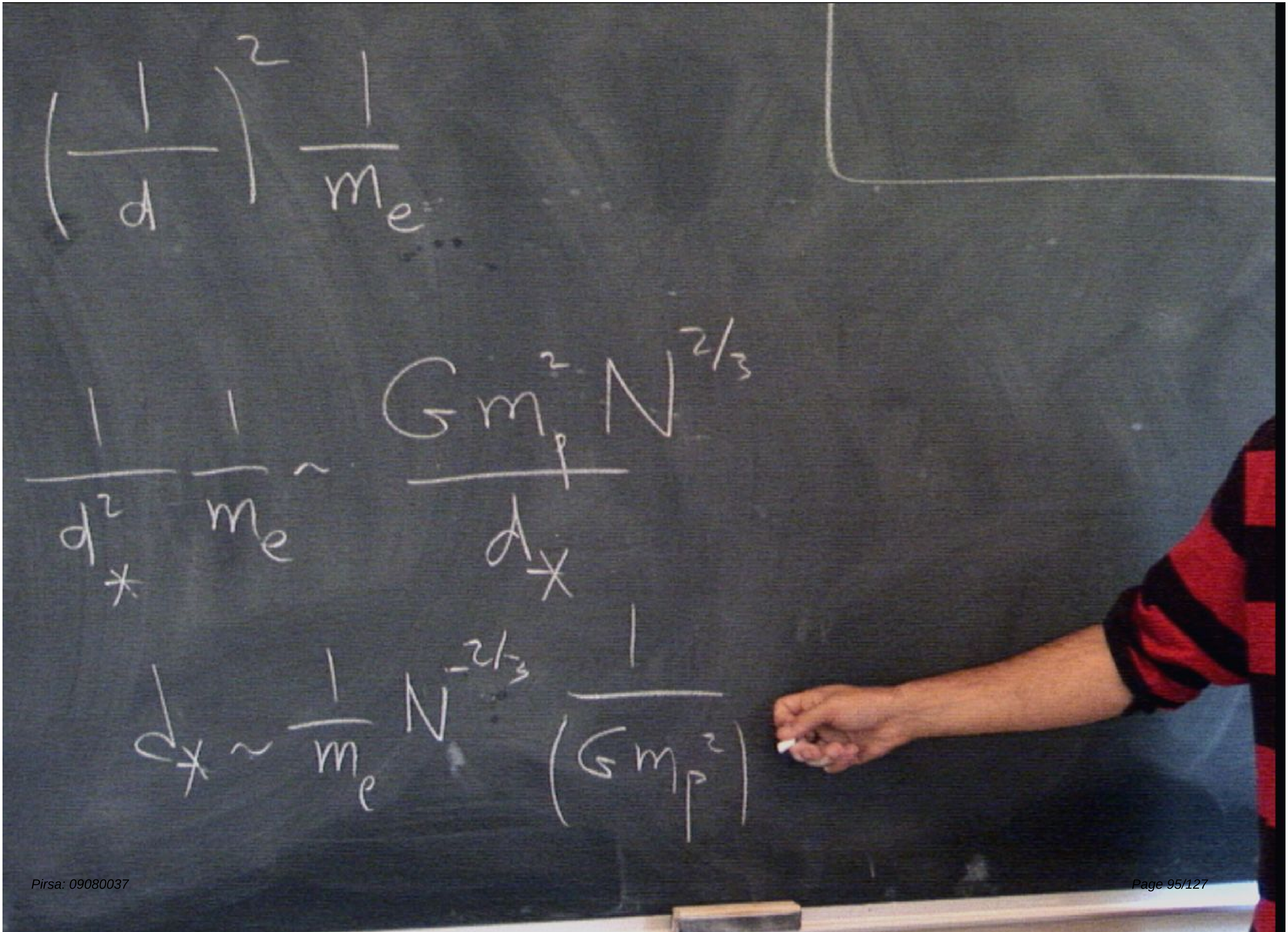
$$N T \sim \frac{G M^2}{R}$$

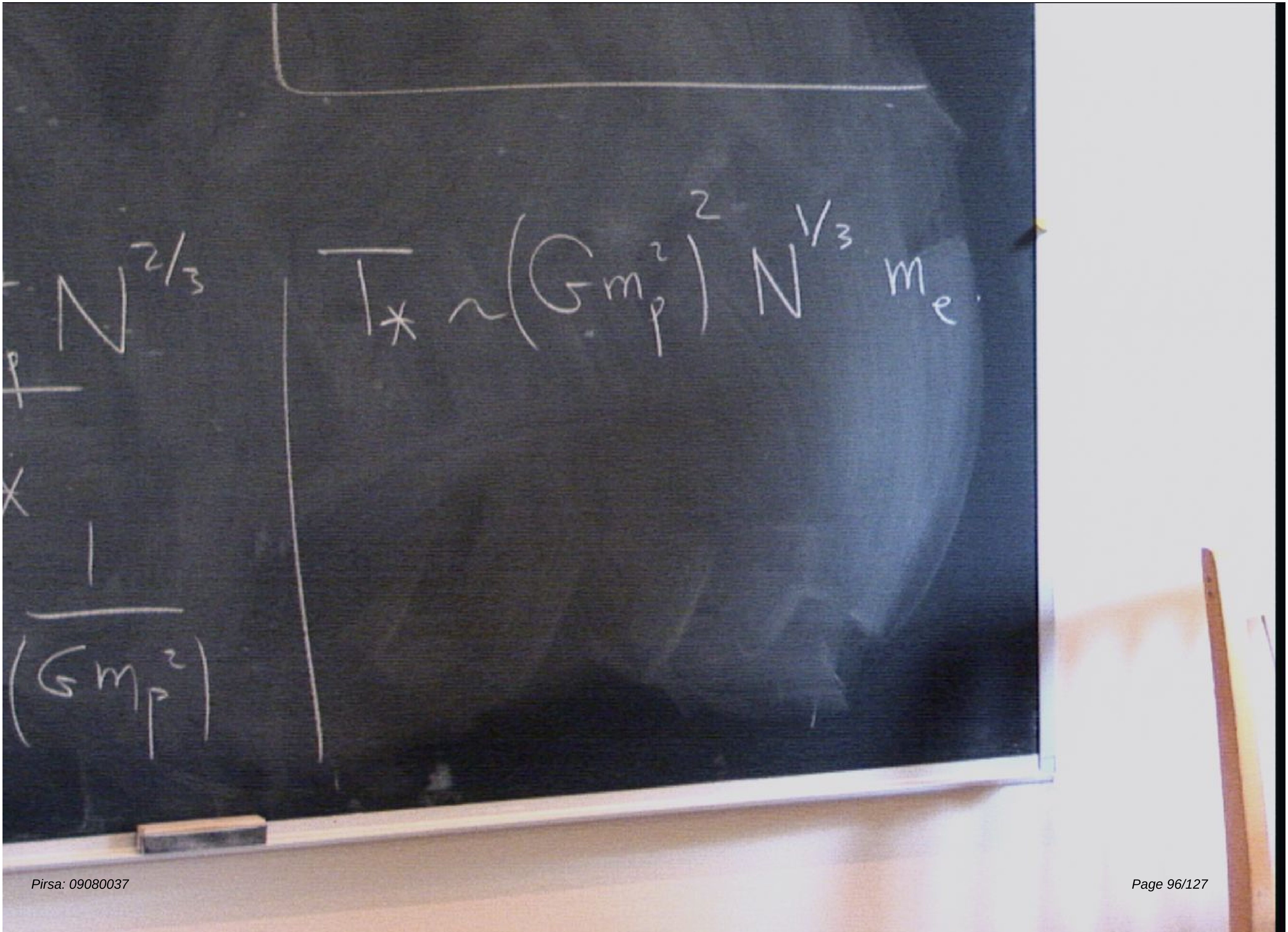
$$T \sim \frac{G m_p^2 N}{R} \sim \frac{G m_p^2 N^{2/3}}{d}$$

$d^3 N$

d: typ int-part sep.

$$\left(\frac{1}{d}\right)^2 \frac{1}{m_e}$$


$\left(\frac{1}{d}\right)^2 \frac{1}{m_e}$
$\frac{1}{d_*^2}\frac{1}{m_e} \sim \frac{G m_p^2 N^{2/3}}{d_*}$
$d_* \sim \frac{1}{m_e} N^{-2/3} \frac{1}{(G m_p^2)}$


T* ~ (Gmp²)² N^(1/3) me

$$E \sim \alpha\, m_p$$

$\leftarrow m_p^{-1} \rightarrow$

$$E \sim \alpha \, m_p$$

$$T_{nuc} \sim \alpha^2 m_p \left(\text{Gamow} \right)$$

$$\frac{1}{m_e}$$

$$\frac{G m_p^2 N^{2/3}}{\lambda_x}$$

$$\frac{1}{m_e} N^{-2/3} \frac{1}{(G m_p^2)}$$

$$T_* \sim (G m_p^2)^2 N^{4/3} m_e$$

$$(Gm_p^2)^2 N^{4/3} m_e > \alpha^2 m_p$$

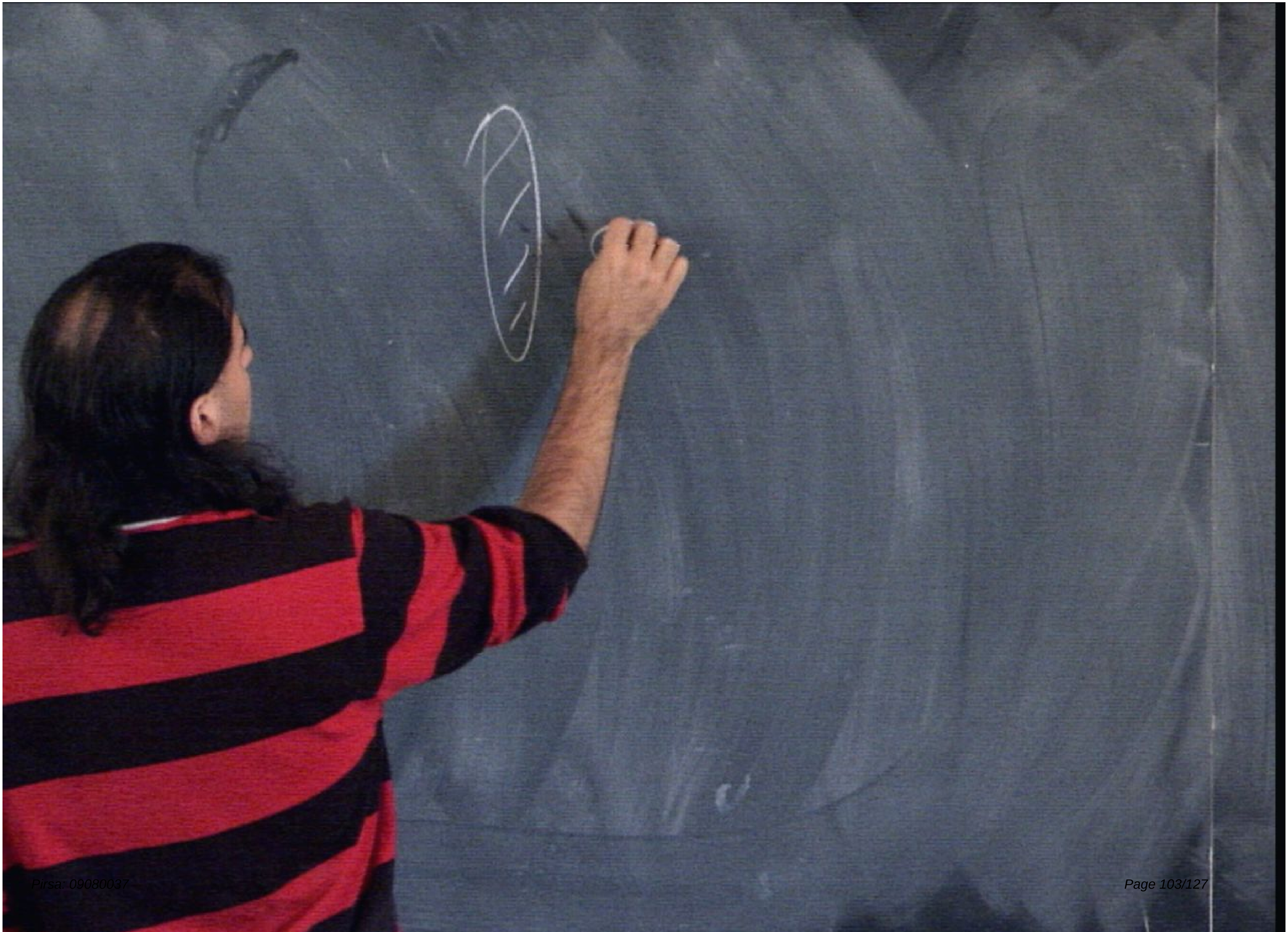
$$(Gm_p^2)^2 N^{4/3} m_e > \alpha^2 m_p$$

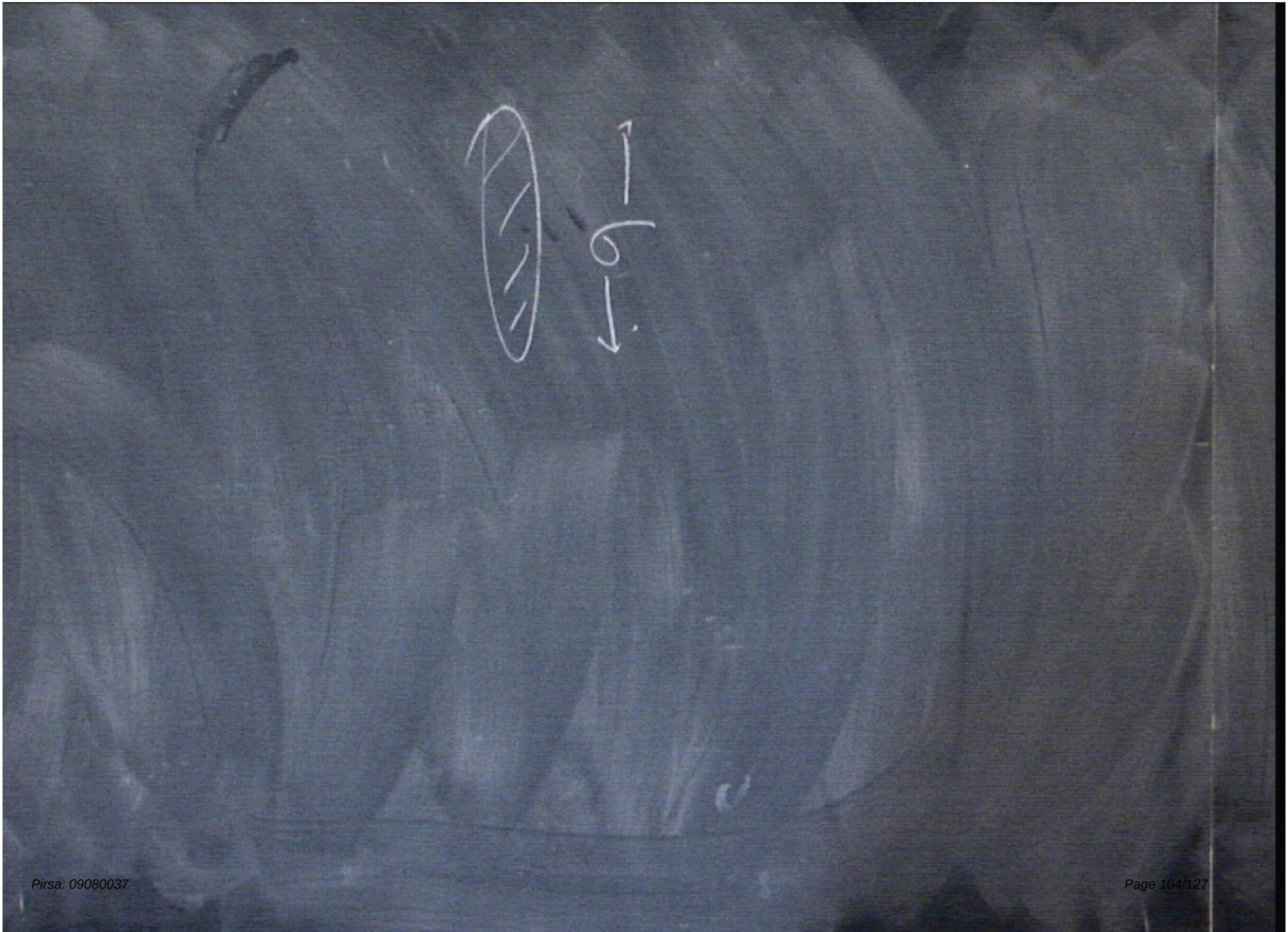
$$\boxed{N > \left(\frac{\alpha^2 m_p}{m_e}\right)^{3/4} \left(\frac{1}{Gm_p^2}\right)^{3/2}}$$

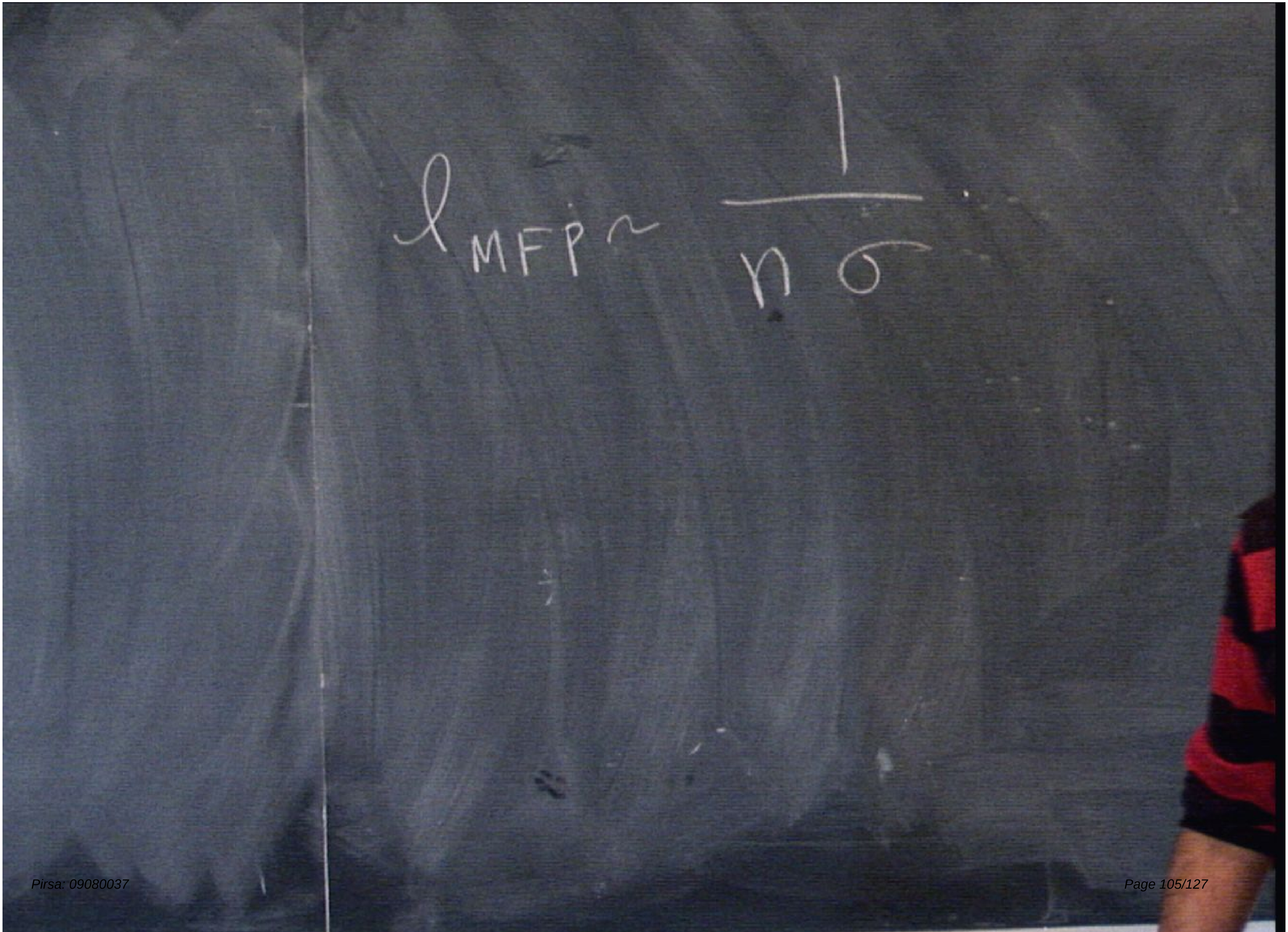
$$(Gm_p^2)^2 N^{4/3} m_e > \alpha^2 m_p$$

$$N > \left(\frac{\alpha^2 m_p}{m_e}\right)^{3/4} \left(\frac{1}{Gm_p^2}\right)^{3/2}$$

$$\sim 10^{57}$$



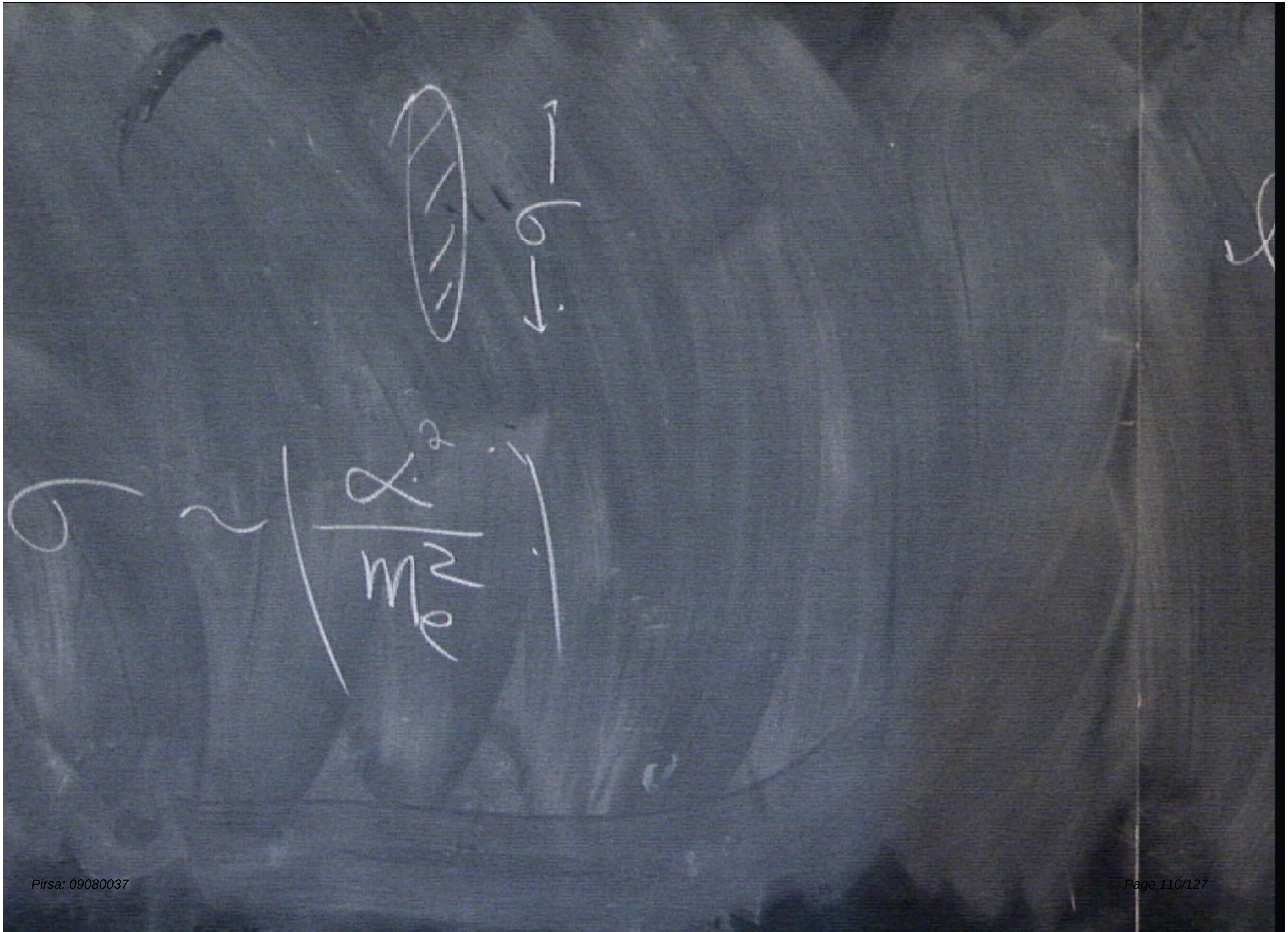

σ


$l_{MFP} \sim \frac{1}{n\sigma}$


σ ~

$$\sigma \sim \frac{\alpha}{m_e^2}$$

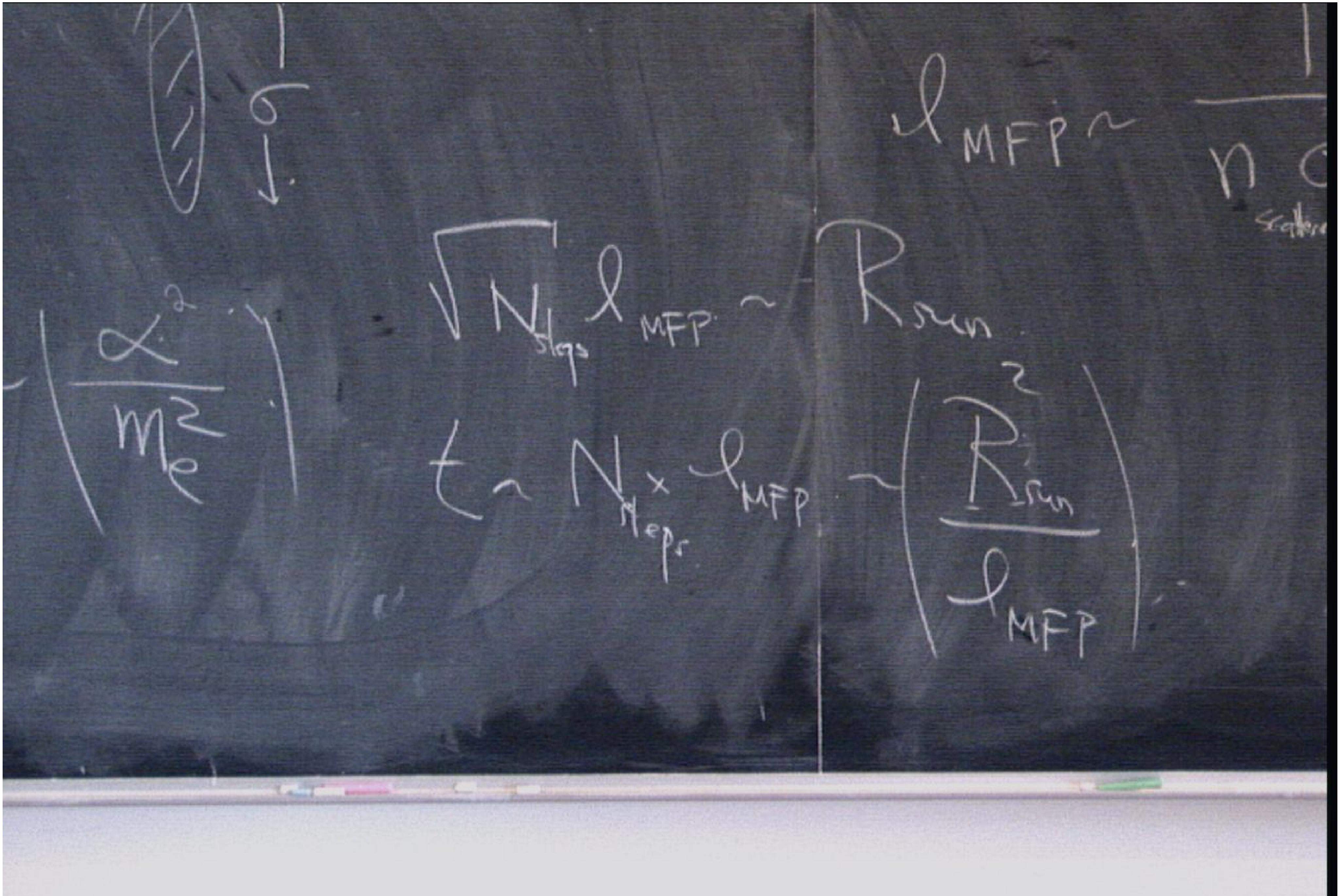
$$\sigma \sim \left(\frac{\alpha}{m_e^2} \right)$$

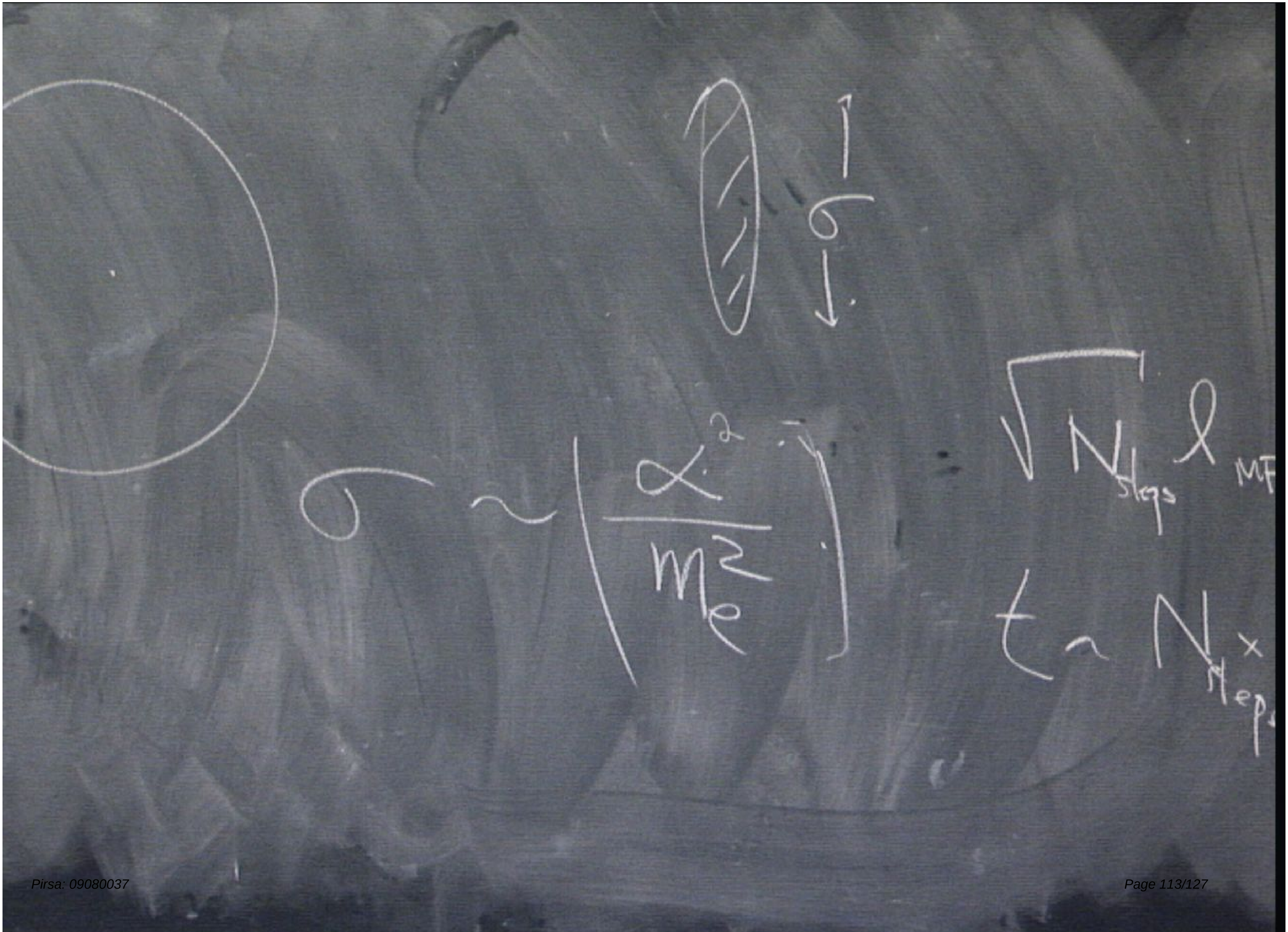

σ
$\sigma \sim \left(\frac{\alpha^2}{m_e^2}\right)$

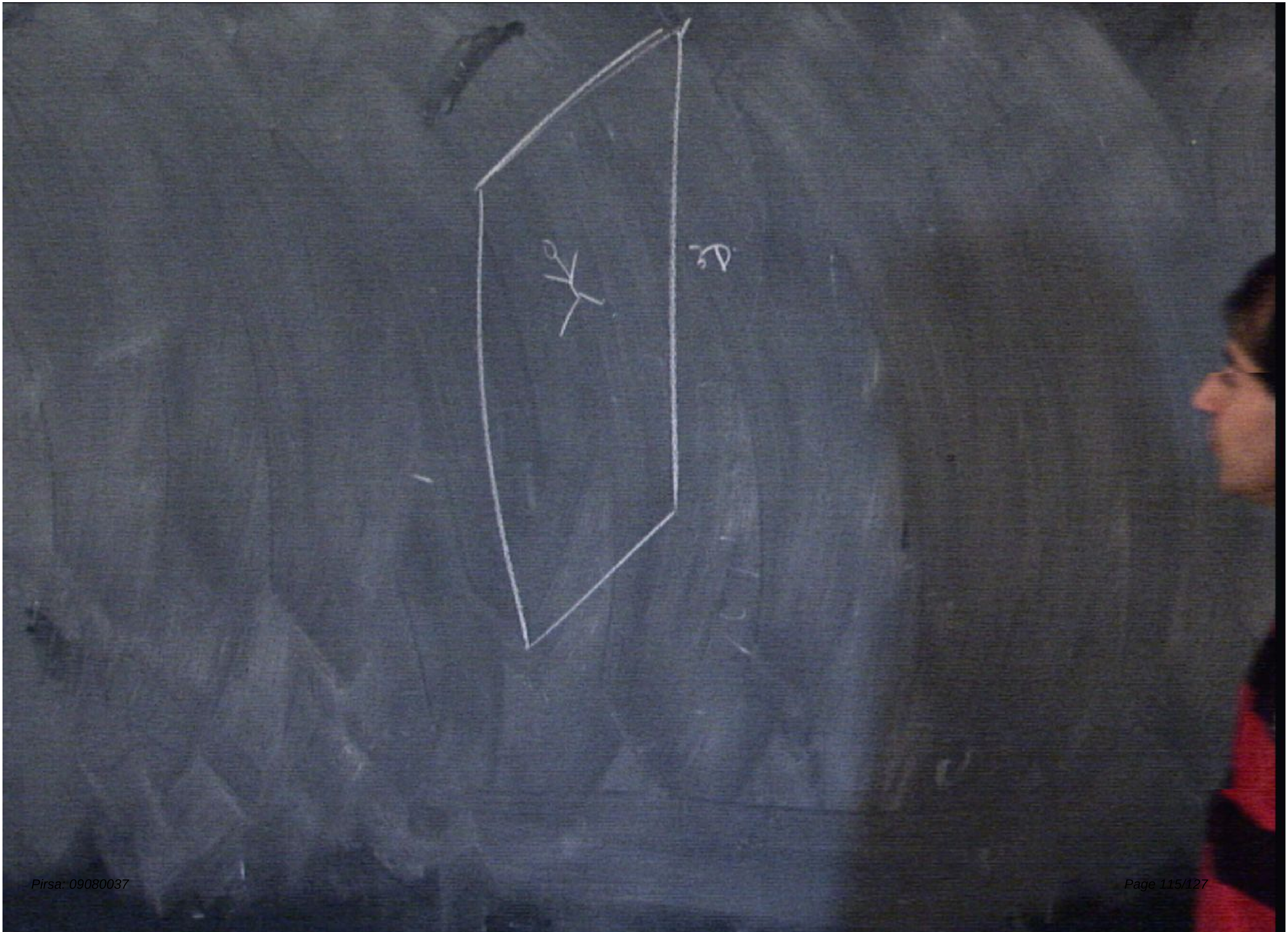
$$l_{MFP} \sim$$

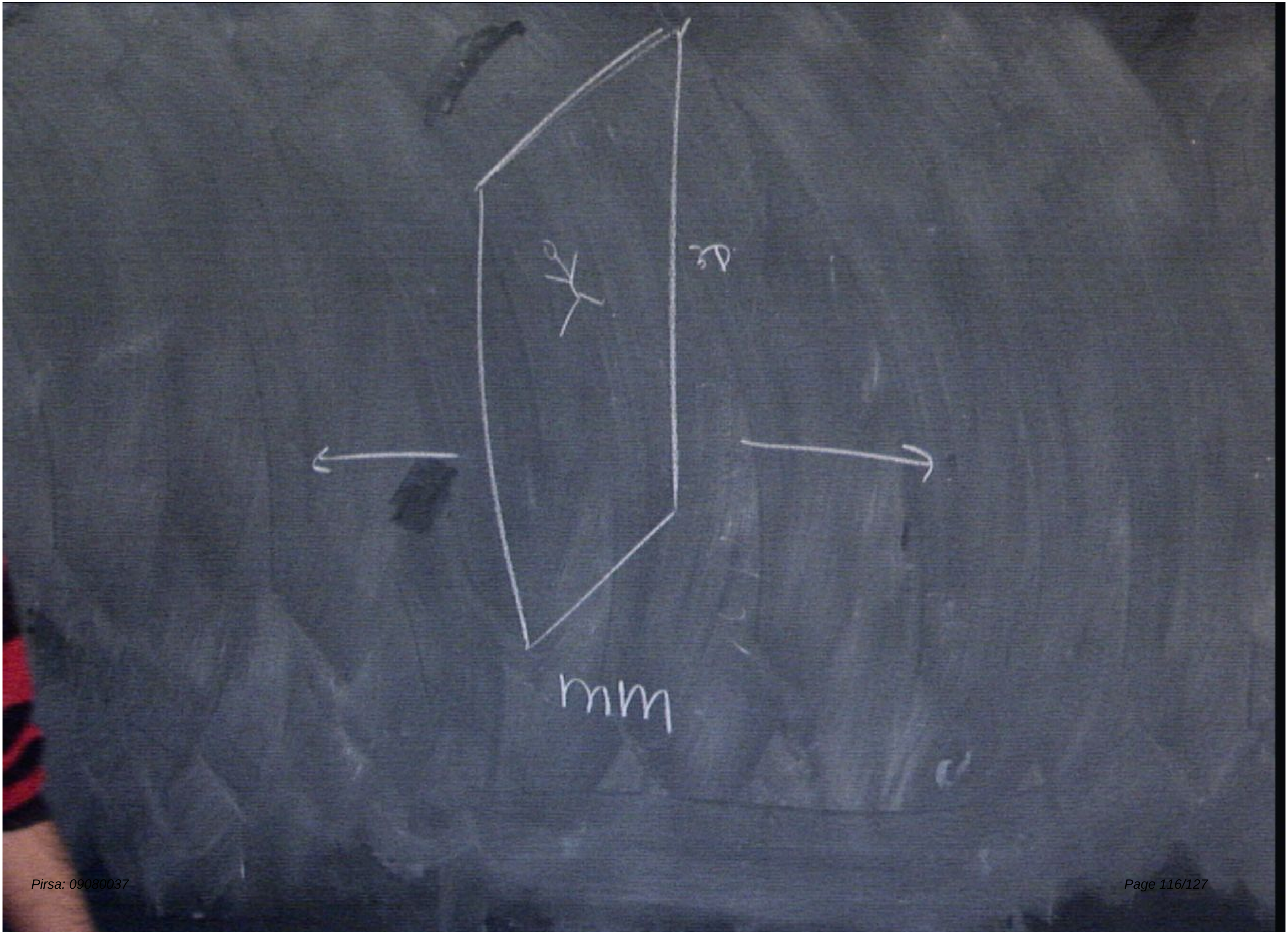
$$\sim \left(\frac{\alpha^2}{m_e^2} \right)$$

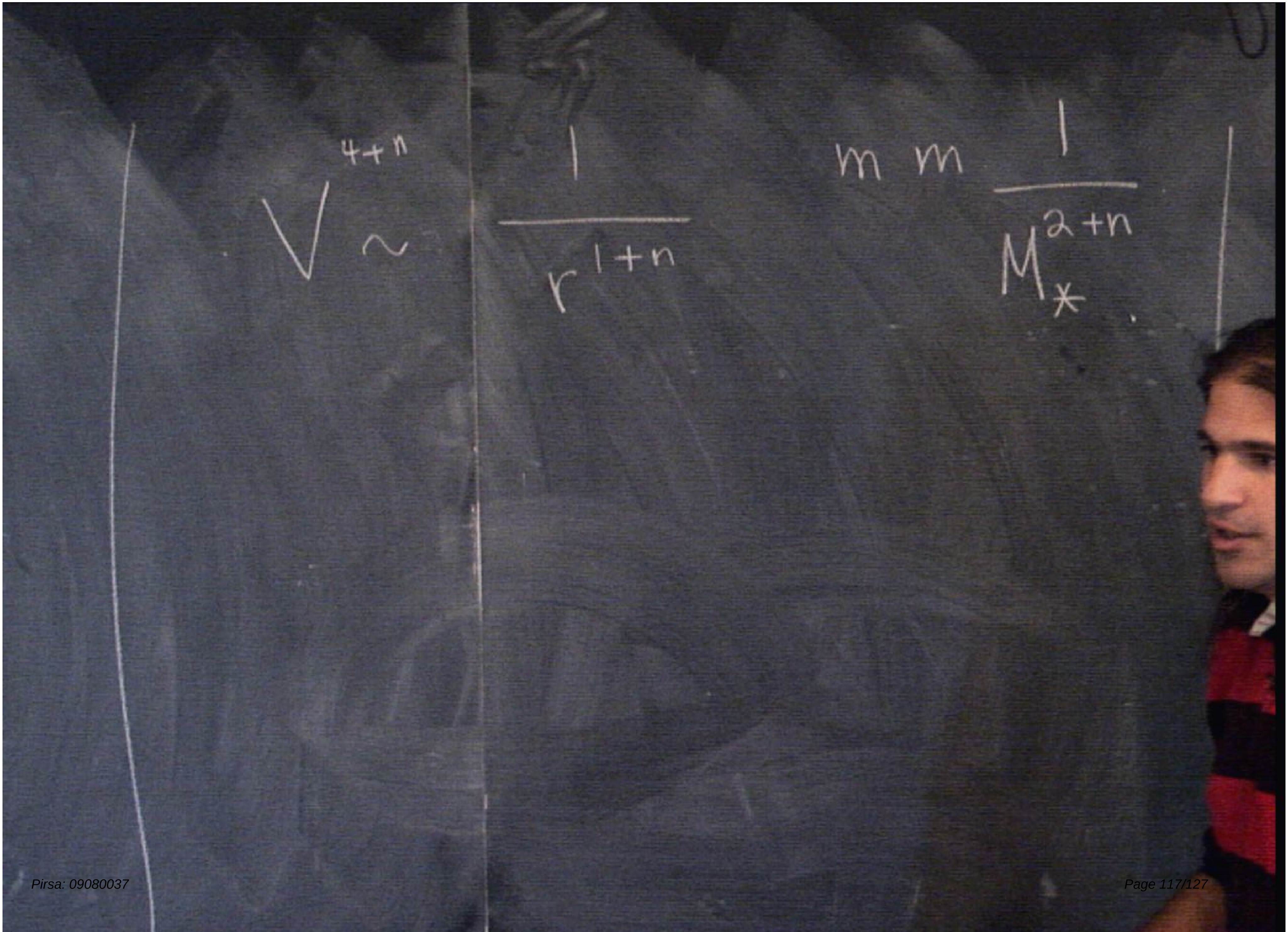
$$\sqrt{N}\, l_{MFP} \sim R_{sun}$$


$\ell_{MFP} \sim \frac{1}{n\,\sigma}$
scatter
$\sqrt{N_{steps}}\,\ell_{MFP} \sim R_{sun}$
$\left(\frac{\alpha^2}{m_e^2}\right)$
$t \sim N_{steps} \times \ell_{MFP} \sim \left(\frac{R_{sun}}{\ell_{MFP}}\right)^2$

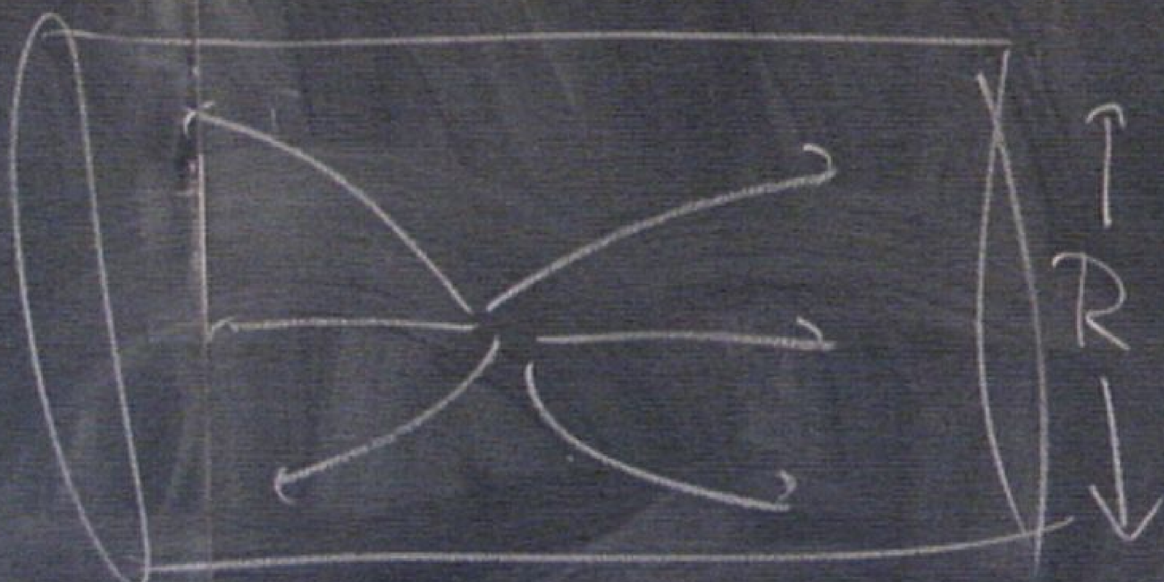

σ
σ ~ (α² / me²)
√N_steps l_MF
t ∝ N_steps ×

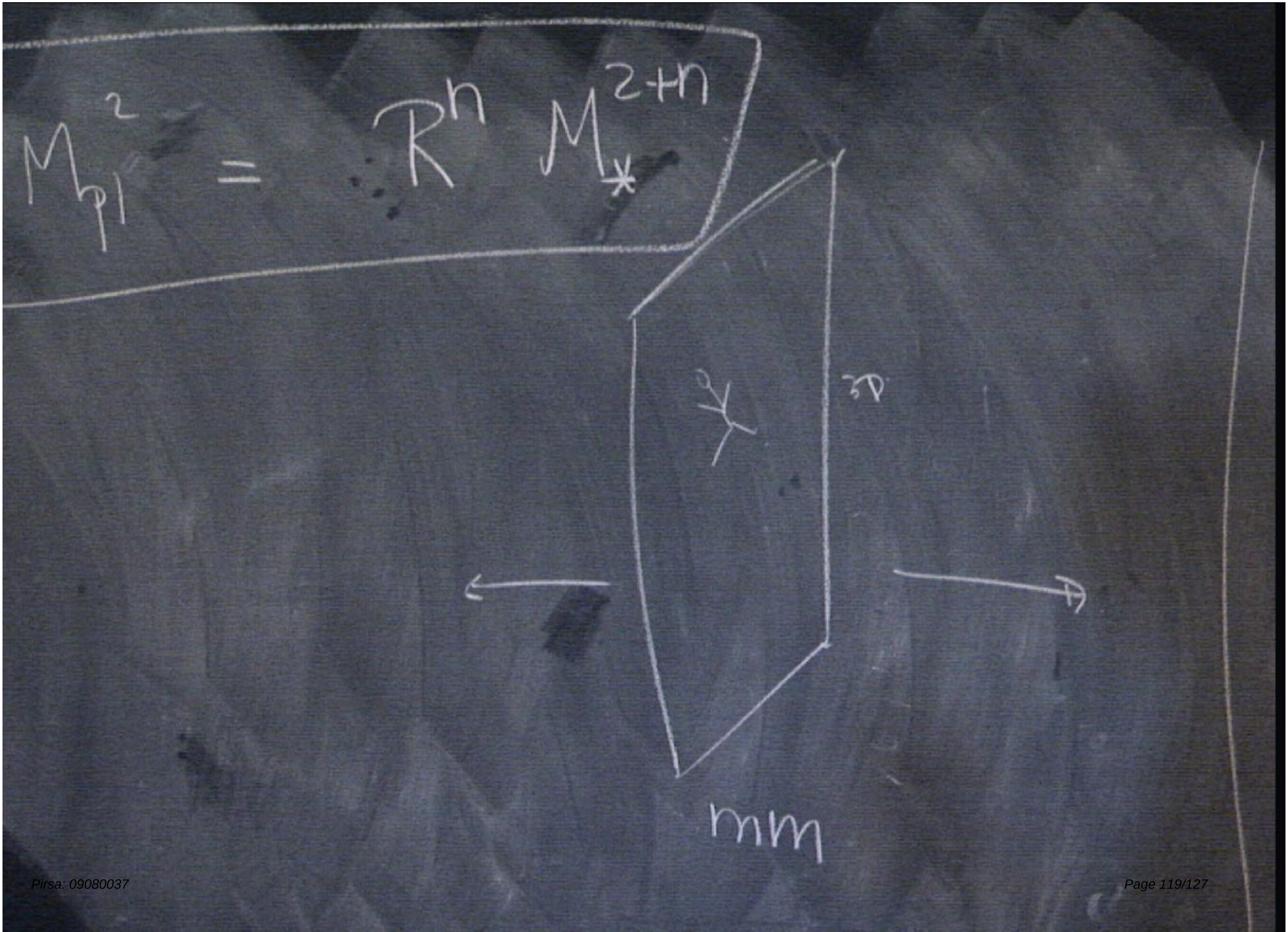


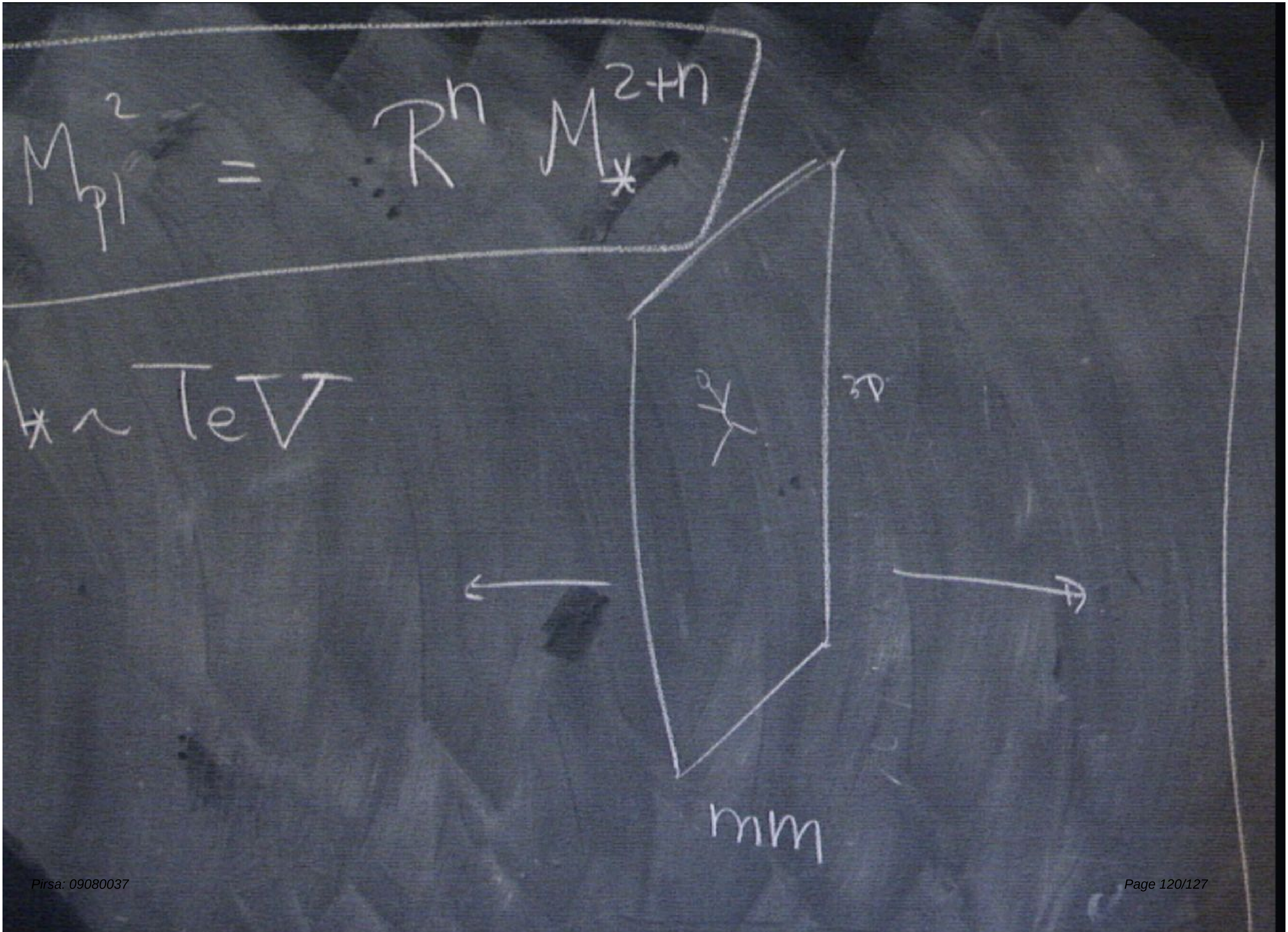

mm


$V^{4+n} \sim \frac{1}{r^{1+n}} \, m \, m \, \frac{1}{M_*^{2+n}}$

$$V^{4+n} \sim \frac{1}{r^{1+n}} \; m \; m \; \frac{1}{\left(M_*^{2+n}\right)}$$




$M_{pl}^2 = R^n M_*^{2+n}$
3D
mm


$M_{pl}^2 = R^n M_*^{2+n}$
$M_* \sim$ TeV
3D
mm

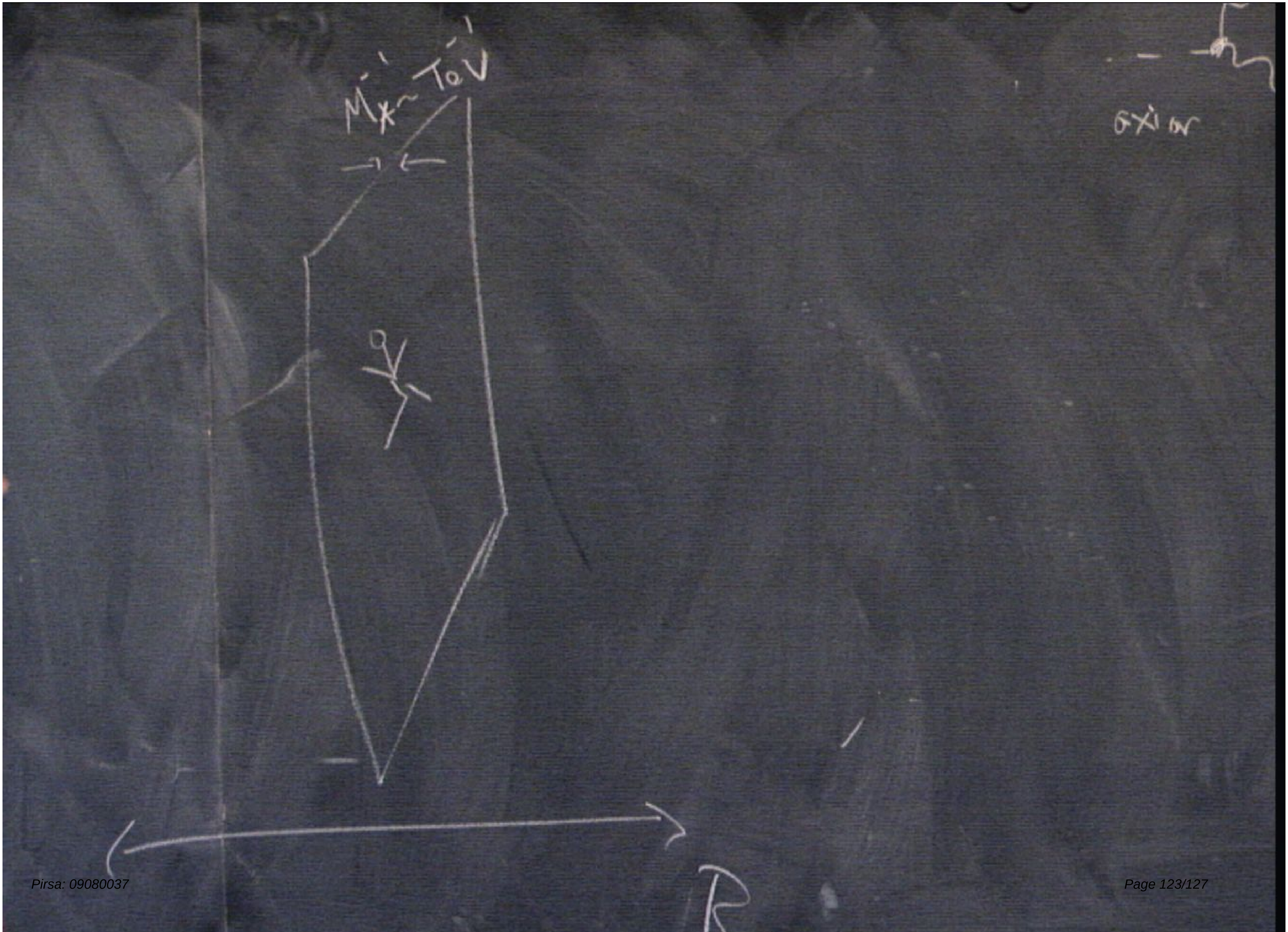
$\sim \text{TeV}$

$$R \sim 10^{\frac{38}{n}} \; 10^{-\frac{3(2+n)}{n}} \; \text{GeV}^{-1}$$

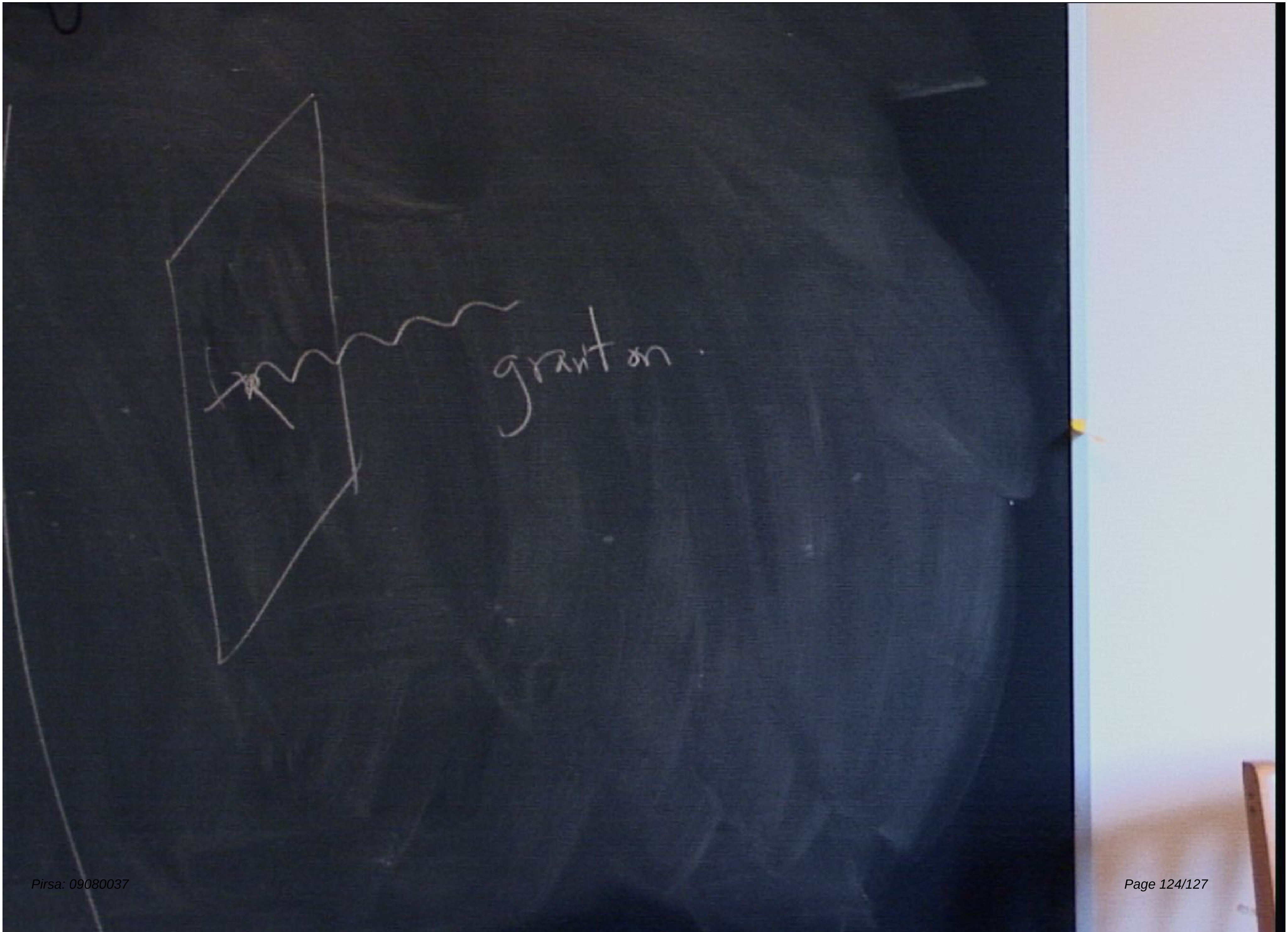
$$M_* \sim \text{TeV}$$

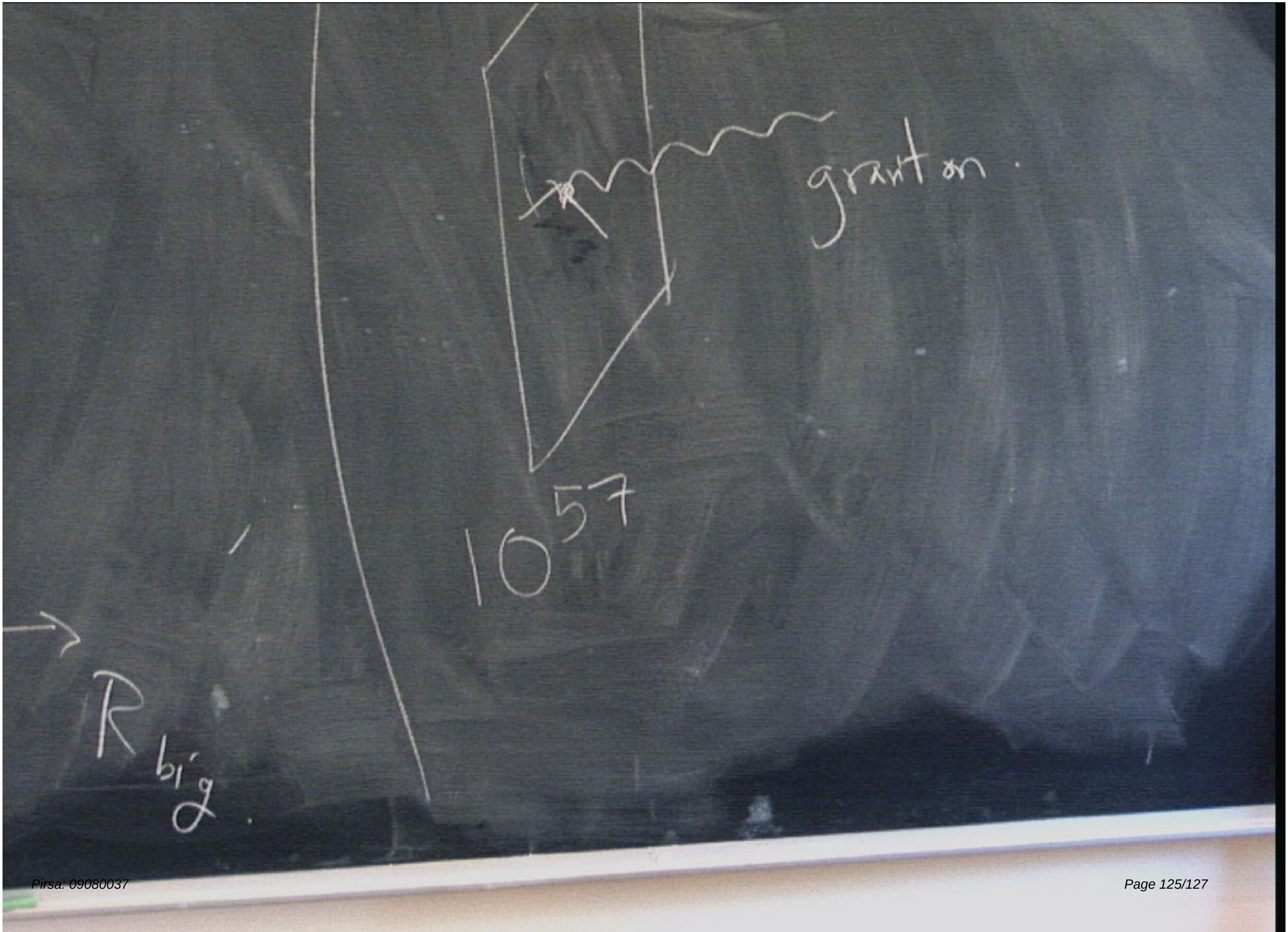
$$R \sim 10^{\frac{38}{n}} \, 10^{-\frac{3(2+n)}{n}} \, \text{GeV}^{-1}$$

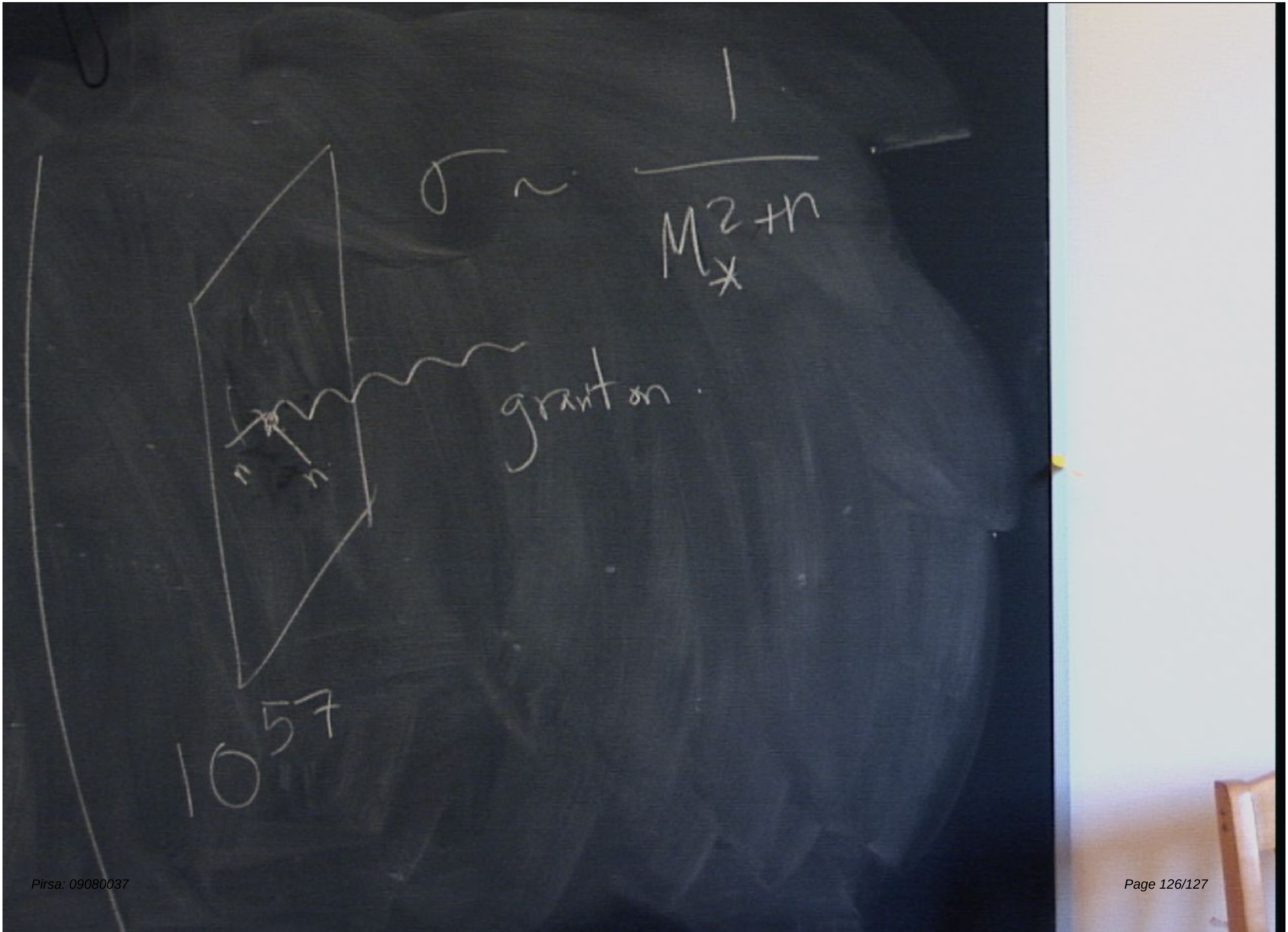
$$n=2 \quad R \sim 1\,\text{mm}.$$

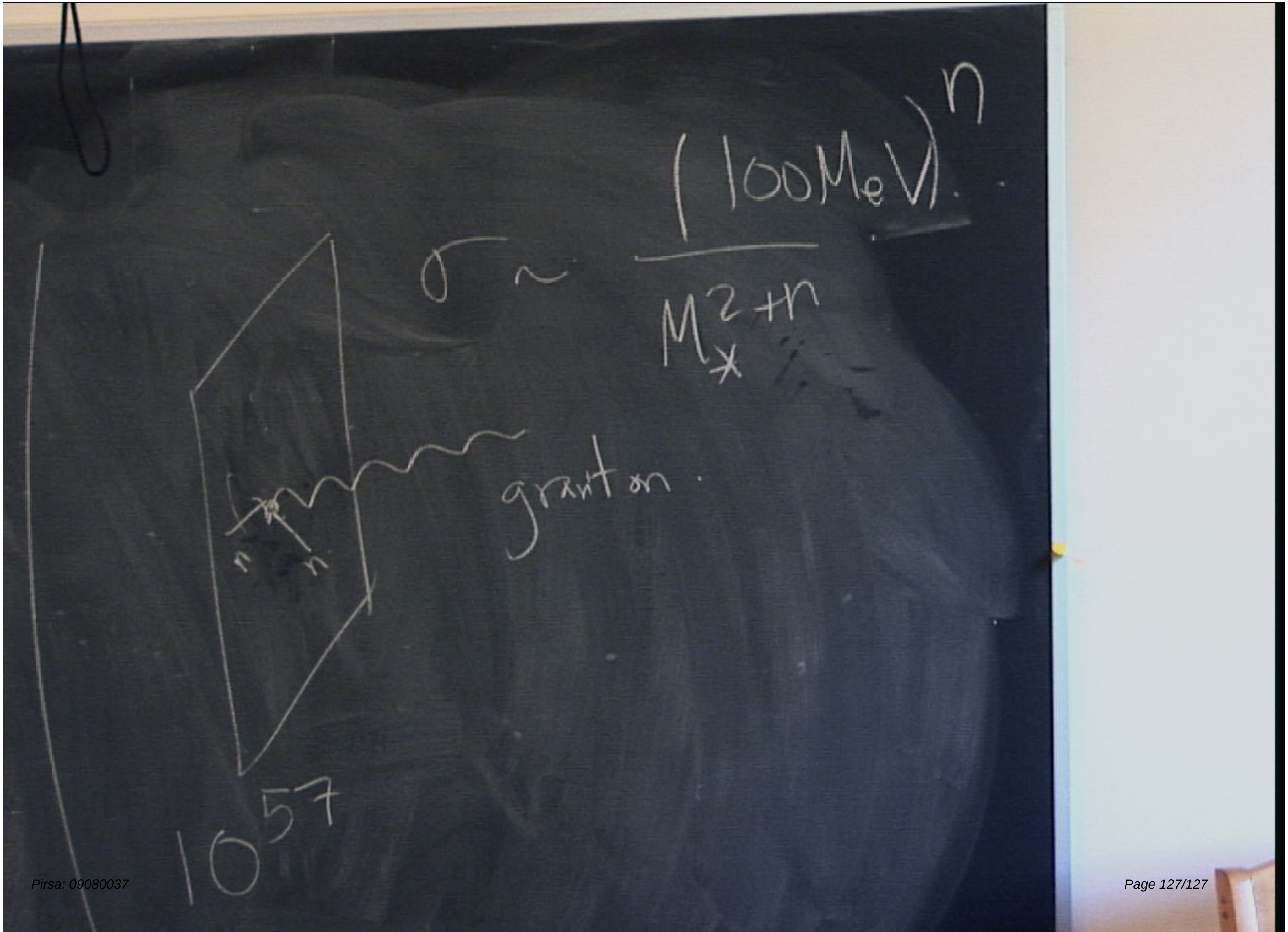


M ~ TeV
R


graviton


graviton.
10^{57}
R_{big}.


σ ~ 1 / M*^(2+n)
graviton
n
n
10^57


σ ~ (100MeV)^n / M_*^{2+n}
graviton
n
n
10^57