

Title: Summer Undergraduate Student Talk

Date: Aug 14, 2009 02:00 PM

URL: <http://pirsa.org/09080030>

Abstract:

ZoA in light of Density

Reconstruction from peculiar velocities

1) Density fluctuations

2) Linear matter model

γ reconstruction

ZoA in light of Density

Reconstruction from peculiar velocities

- 1) Density fluctuations
- 2) Linear matter model
- 3) Density reconstruction

ZoA in light of Density

Reconstruction from peculiar velocities

- 1) Density fluctuations
- 2) Linear matter mode
- 3) Density reconstru

1) What are the concerns?

ZoA in light of Density

Reconstruction from p
velocities

1) Density fluctuation

2) Linear momentum

3) Density

- 1) What are the concerns?
- 2)

ZoA in light of Density

Reconstruction from pe.
velocities

- 1) Density fluctuati
- 2) Linear matter
- 3) Density reco

1) What are the concenes?
2) dynamics?

ZoA in light of Density

Reconstruction
velo solidar

- 1) Density f... ions
- 2) Linear model
- 3) Den... ruction

- 1) What are the concerns?
- 2) —||— dynamics?

ZoA in light of Density

Reconstruction from peculiar velocities

- 1) Density fluctuations
- 2) Linear matter
- 3) Density recon

1) What are the concerns?

2) —"— dynamics?

⇒ dark energy model



velocities

- 1) Density fluctuations
- 2) Linear matter model
- 3) Density reconstruction

2) — " —

dynamics?

⇒ dark energy model



model

ZoA in light of Density

Reconstruction from peculiar velocities

- 1) Density fluctuations
- 2) Linear matter model
- 3) Density reconse

1) What are the constraints?

2) —"— dynamics?

⇒ dark energy model



⇒ modified gravity

ZoA in light of Density

Reconstruction from peculiar velocities

- 1) Density fluctuations
- 2) Linear matter
- 3) Density

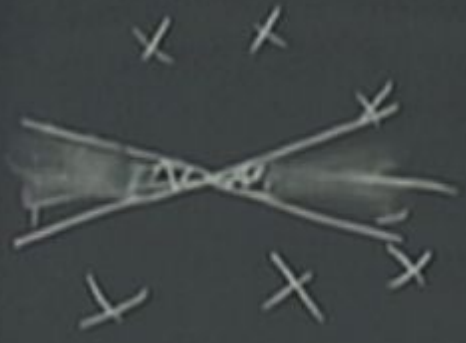
- 1) What are the concerns?
- 2) — " — dynamics?

⇒ dark energy model



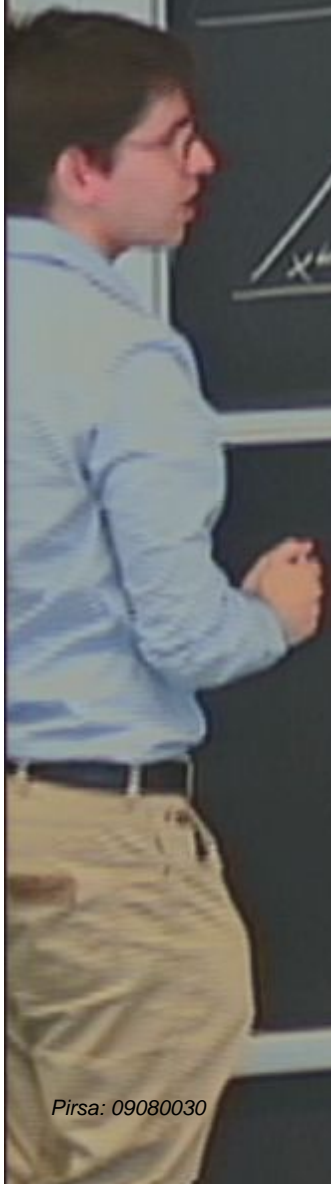
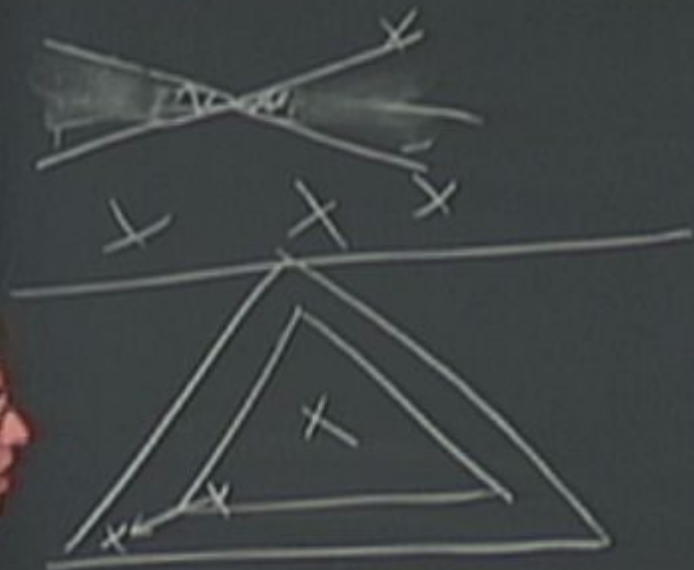
⇒ modified gravity

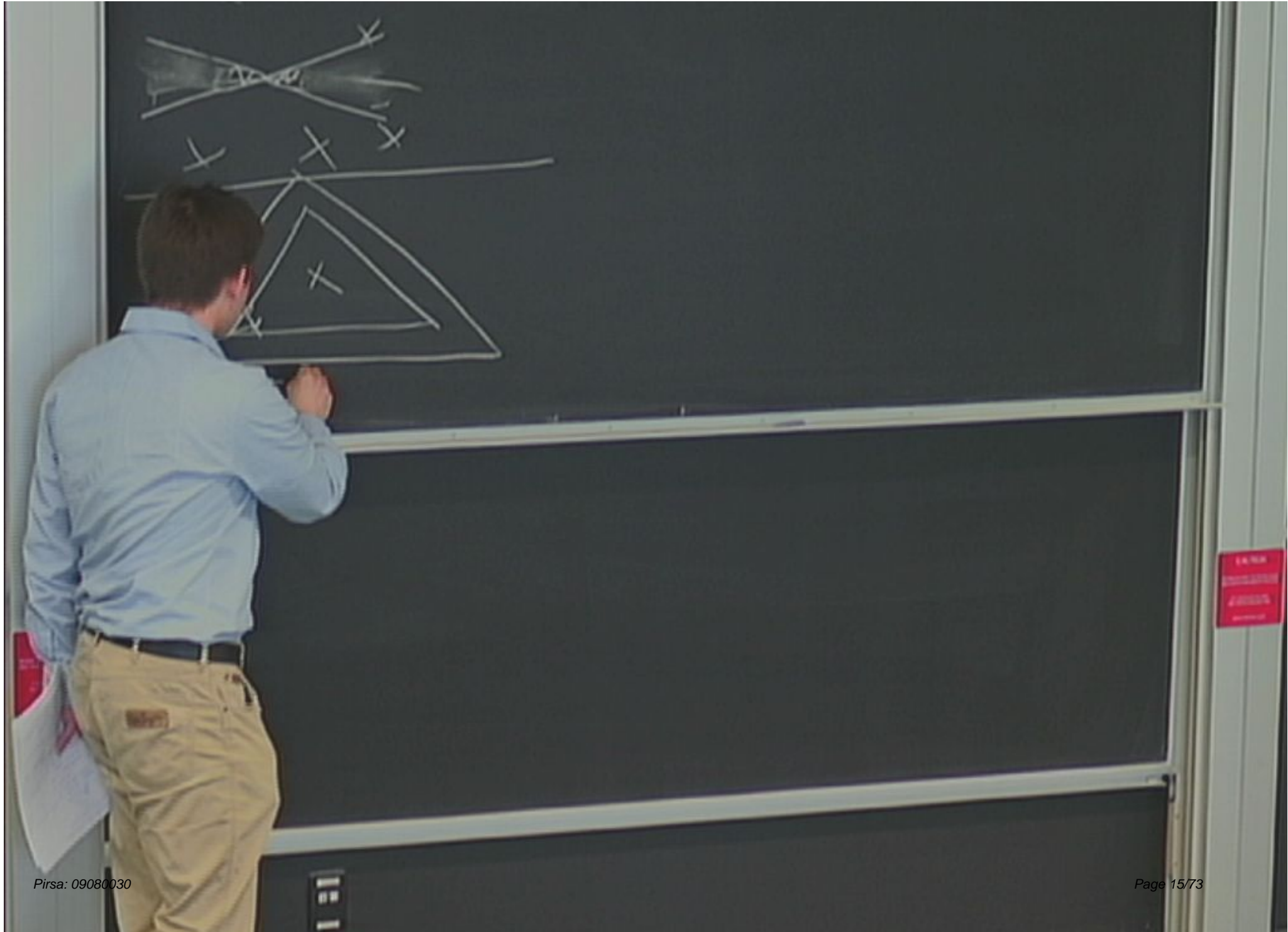


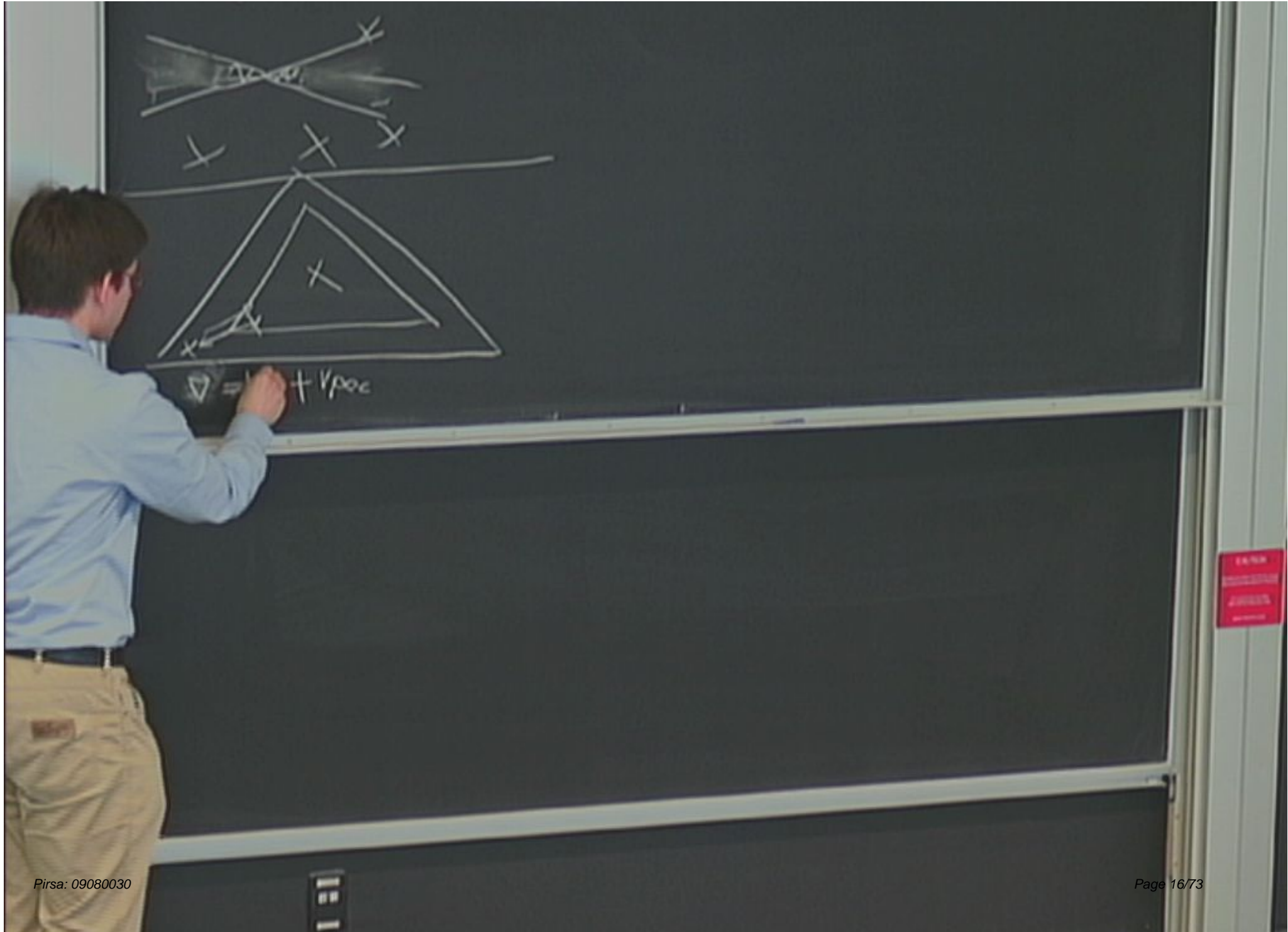


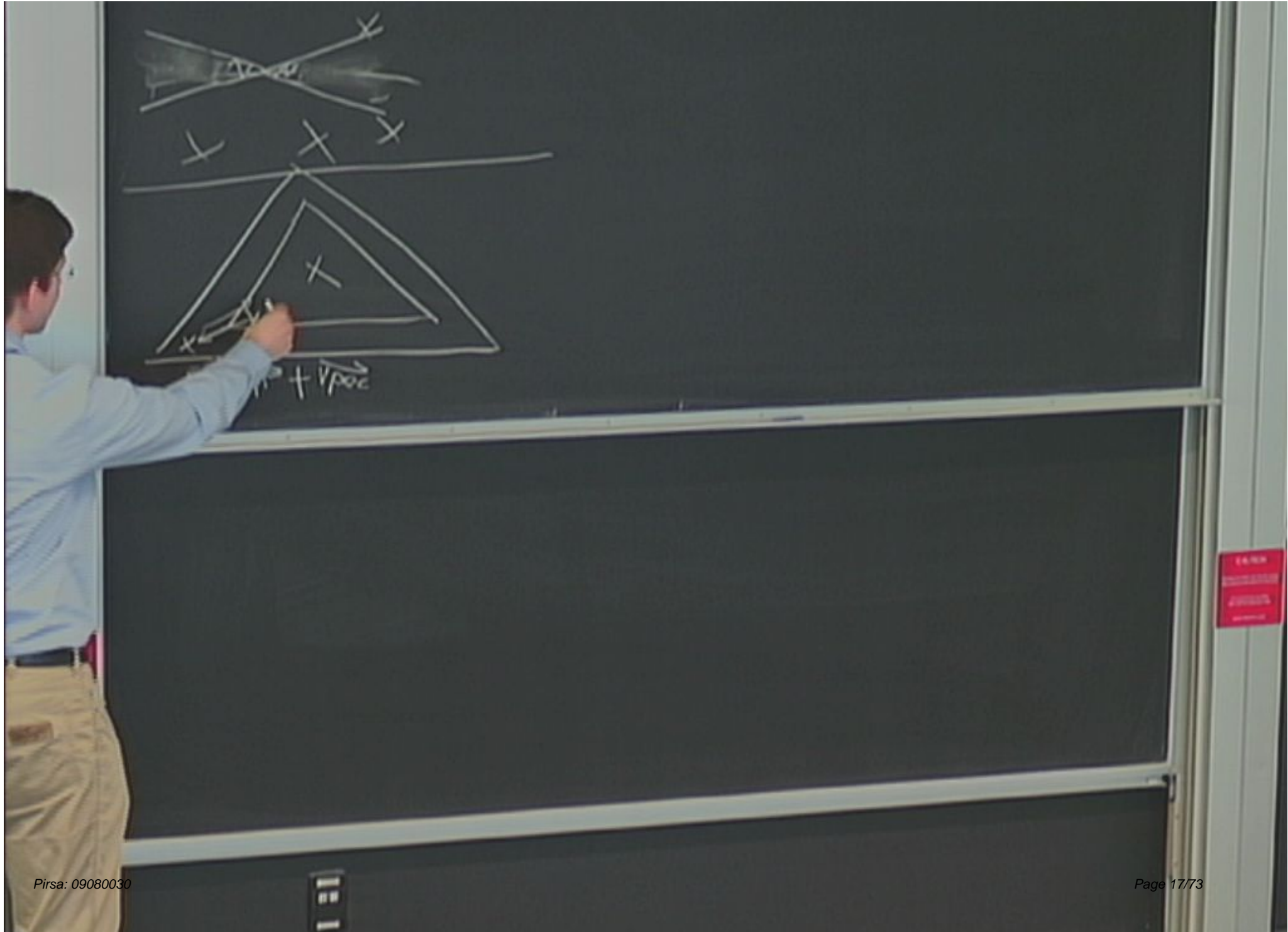
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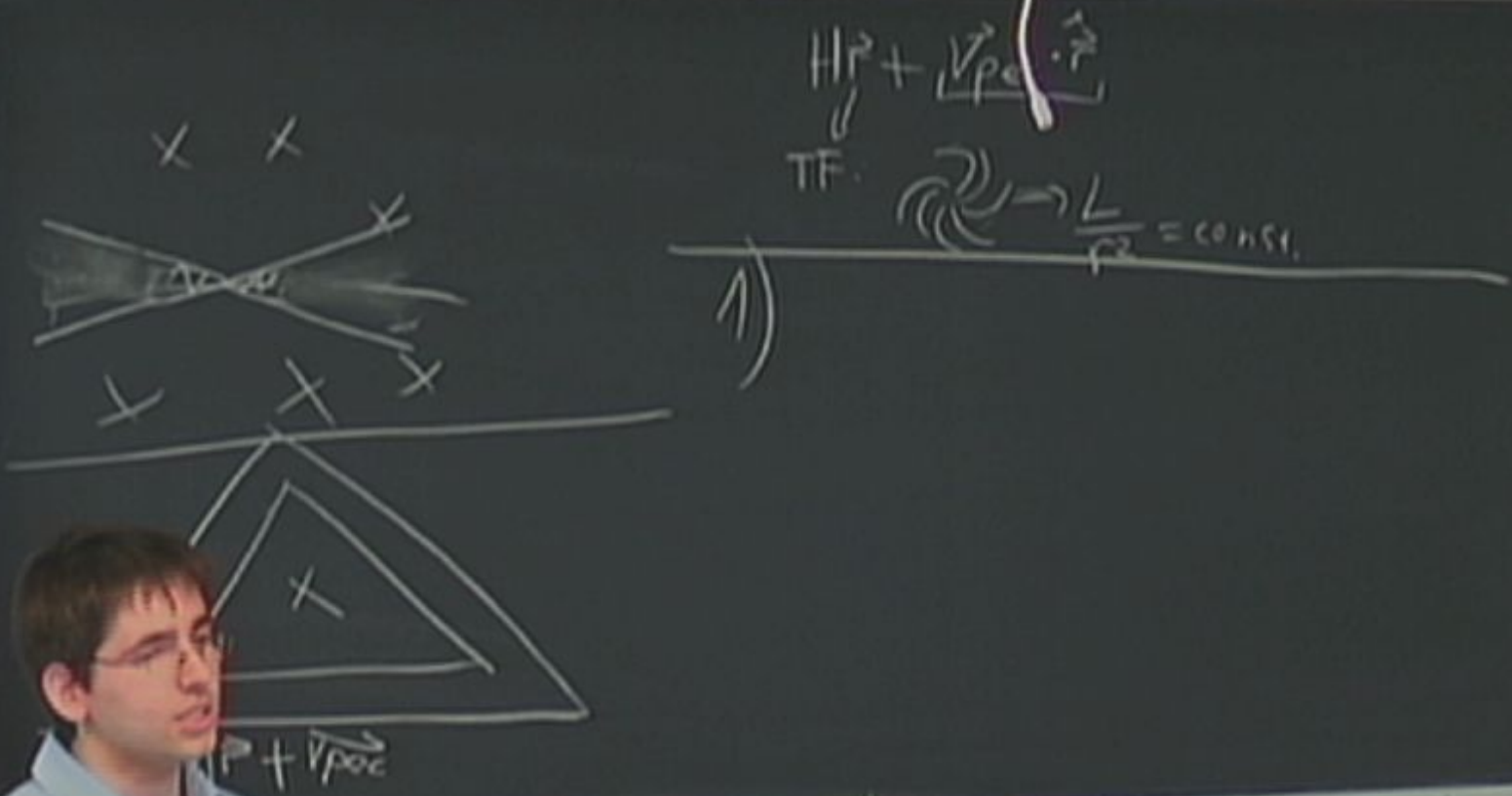


$$H\vec{r} + \vec{v}_{pec} \cdot \hat{r}$$



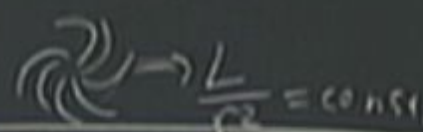
$$H \dot{r}^2 + \underbrace{V_{\text{pec}}}_{\text{TF}} \cdot \hat{r}^2 \rightarrow \frac{L}{r^2} = \text{const.}$$





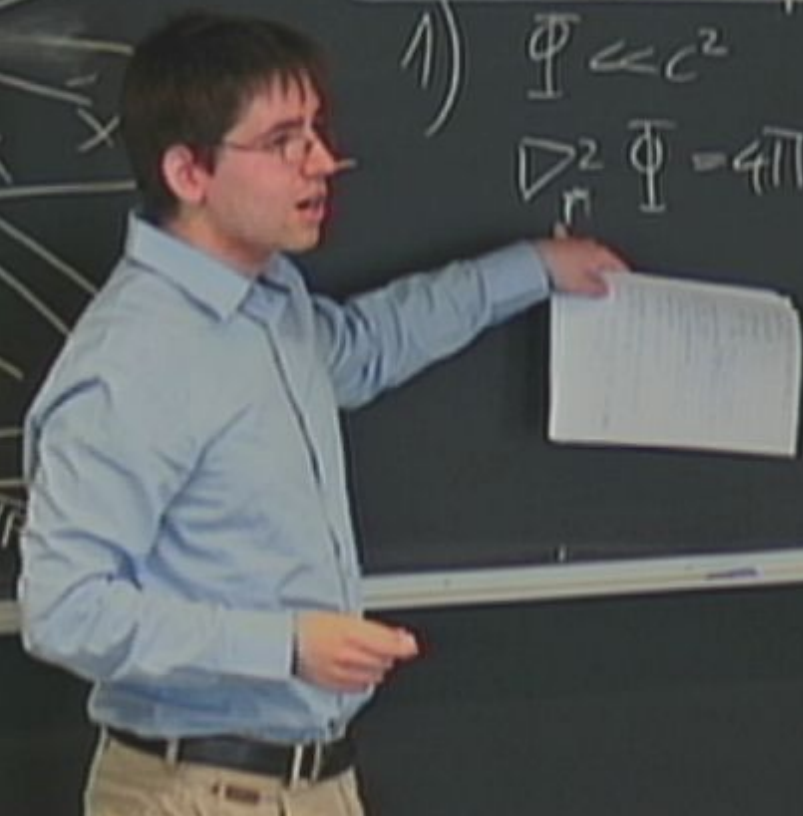


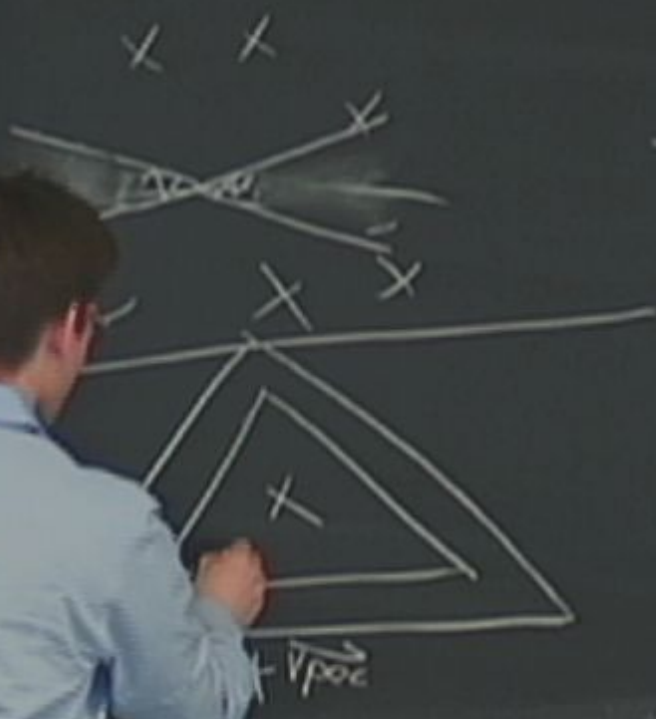
$$H\vec{p} + \sqrt{p_{\text{pec}} \cdot \vec{p}}$$

TF.  $\frac{L}{r^2} = \text{const.}$

1) $\Phi \ll c^2$

$$\nabla_r^2 \Phi = 4\pi g \left(\rho + \frac{3p}{a} \right) - \Delta$$



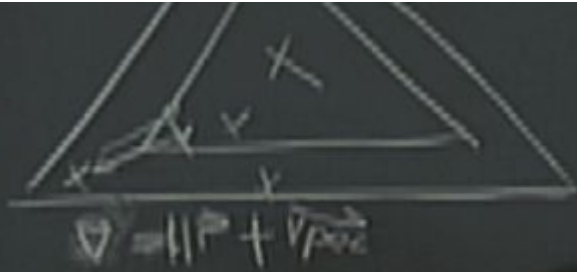


$$H\dot{r}^2 + \underbrace{V_{pec} \cdot \dot{r}}_{TF} = \text{const.}$$

$$1) \quad \Phi \ll c^2$$

$$\nabla_{\vec{r}}^2 \Phi = 4\pi G \left(\rho + \frac{3p}{a} \right) - \Delta$$

$$\vec{r} = a(t) \vec{x}$$



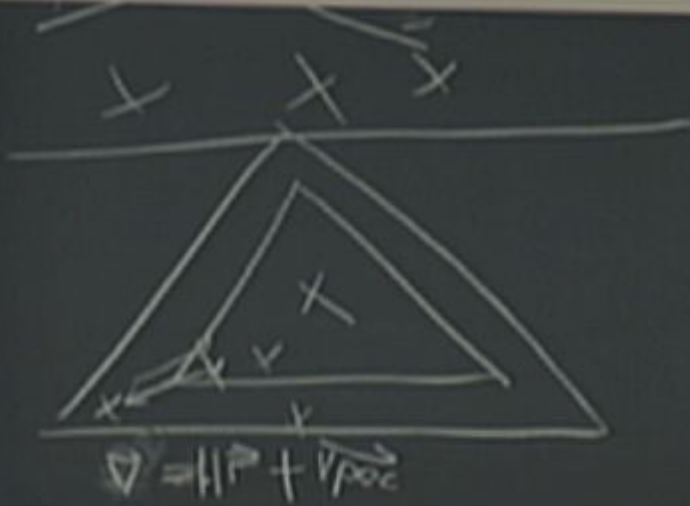
$$\vec{v} = v_H + v_V$$

$$= \frac{v}{H} H + \frac{v}{V} V$$



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$$\nabla^2 \Phi = 4\pi G \left(\rho + \frac{3}{2} \rho \right) - \Delta$$

$$\vec{r} = \text{alt } \vec{x}$$

$$\vec{u} = a_1 \vec{x}_1 + a_2 \vec{x}_2$$

$$= \frac{a_1}{H} \vec{r}_1 + \frac{a_2}{H} \vec{r}_2$$

$$\frac{d^2 q}{dt^2} + \left(\frac{2\pi b}{a} + \frac{3\pi b(t)}{a} - \frac{\Delta}{4\pi S} \right)$$

$$= -\frac{4\pi}{3} \frac{\alpha(\rho_b(t) + \frac{3\rho_b(t)}{a} - \frac{\Delta}{4\pi r})}{\rho_b(t)} - 1$$

$$\begin{aligned}
 \frac{q}{\tau^2} &= -\frac{4\pi\epsilon_0}{3} \text{ca} \left(\rho_b(t) + \frac{3\rho_b(t)}{a} - \frac{\Delta}{4\pi\epsilon_0} \right) \\
 \rho &= \frac{\rho(\vec{x}, t)}{\rho_b(t)} - 1
 \end{aligned}$$

$$\frac{d^2 q}{dt^2} = -\frac{4\pi\epsilon_0 a^2}{3} \left(\rho_b(t) + \frac{3\rho_b(t)}{a} - \frac{\Delta}{4\pi\epsilon_0} \right)$$

$$j = \frac{\rho(\vec{x}, t)}{\rho_b(t)} - 1$$

$$\nabla_{\vec{x}}^2 \phi = 4\pi\epsilon_0 a^2 (\rho(\vec{x}, t) - \rho_b(t)), \quad \phi = \Phi + \frac{1}{2} a \ddot{x}^2$$

$$\frac{d^2 q}{dt^2} = -\frac{4\pi\epsilon_0 a^2}{3} \left(\rho_b(t) + \frac{3\rho_b(t)}{a} - \frac{\Delta}{4\pi\epsilon_0} \right)$$

$$J = \frac{\rho(\vec{x}, t)}{\rho_b(t)} - 1$$

$$\nabla_{\vec{x}}^2 \phi = 4\pi\epsilon_0 a^2 (\rho(\vec{x}, t) - \rho_b(t)), \quad \phi = \Phi + \frac{1}{2} a \ddot{x} z^2$$

$$\nabla_{\vec{x}}^2 \phi = 4\pi\epsilon_0 a^2 \rho_b J$$

$$\frac{d^2 q}{dt^2} = -\frac{4\pi\epsilon_0 \sigma a (p_b(t) + \frac{3p_b(t)}{a} - \frac{\Delta}{4\pi\epsilon_0})}{3}$$

$$J = \frac{p(x,t)}{p_b(t)} - 1$$

$$\nabla_x^2 \phi = 4\pi\epsilon_0 a^2 (p(x,t) - p_b(t)), \quad \phi = \Phi + \frac{1}{2} a \ddot{x}^2$$

$$\nabla_x^2 \phi = 4\pi\epsilon_0 a^2 p_b J$$

$$2) \quad dN = f(\vec{x}, \vec{p}, t) d^3p d^3x$$

$$\rho(\vec{x}, t) = m a^{-3} \int d^3p f$$

ρ

$$2) \quad dN = f(\vec{x}, \vec{p}, t) d^3p d^3x$$

$$g(\vec{x}, t) = m^{-3} \int d^3p f$$

$$\rho(\vec{x}, t) = \rho_b(t) (1 + \delta\rho(\vec{x}, t))$$

$$2) \quad dN = f(\vec{x}, \vec{p}, t) d^3p d^3x$$

$$g(\vec{x}, t) = m a^{-3} \int d^3p f$$

$$g(\vec{x}, t) = \rho_b(t) (1 + \delta g(\vec{x}, t))$$

$$\boxed{\rho_b(t) \propto a^{-3}}$$

$$\rho(\vec{x}, t) = \rho_b(t) (1 + \delta \vec{x}, t)$$
$$\rho_b(t) \propto a^{-3}$$

Ideal fluid

$$2) \quad dN = f(\vec{x}, \vec{p}, t) d^3p d^3x$$

$$g(\vec{x}, t) = m a^{-3} \int d^3p f$$

$$g(\vec{x}, t) = \rho_b(t) (1 + \delta g(\vec{x}, t))$$

$$\boxed{\rho_b(t) \propto a^{-3}}$$

Ideal fluid/Vlasov eq.

$$\frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = \frac{1}{a^2} \nabla \cdot (\dots \rightarrow \phi)$$

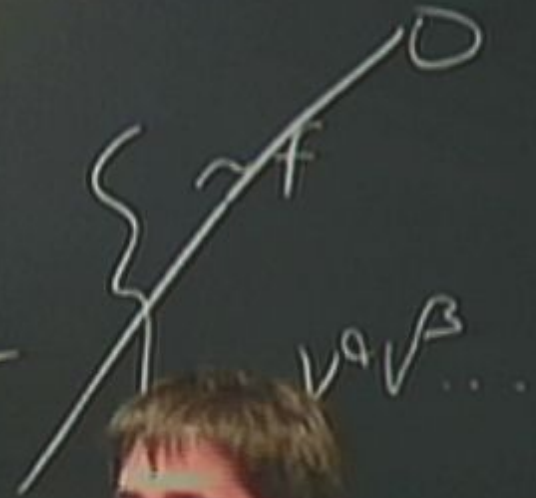
$$2) \quad dN = f(\vec{x}, \vec{p}, t) d^3p d^3x$$

$$g(\vec{x}, t) = m a^{-3} \int d^3p f$$

$$\frac{\rho(\vec{x}, t) = \rho_b(t) (1 + \delta \rho(\vec{x}, t))}{\rho_b(t) \propto a^{-3}}$$

Ideal fluid/Vlasov eq.

$$\frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = \frac{1}{a^2} \nabla \cdot ((1 + \delta) \nabla \phi) +$$



$$\underline{k < 1}$$

$$\frac{\partial^2 \sigma}{\alpha^2} + 2 \frac{\dot{\alpha}}{\alpha} \frac{\partial \sigma}{\partial t} = \frac{1}{\sigma} \nabla^2 \psi$$

$$k < 1$$

$$\frac{\partial^2 \sigma}{\alpha^2} + 2 \frac{\dot{\alpha}}{\alpha} \frac{\partial \sigma}{\partial t} = 4 \pi G \rho \alpha^2 \sigma$$

Ω :

$k \ll 1$

$$\frac{\partial^2 \delta}{\partial x^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = 4\pi G \rho$$

$\Omega: H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G \rho}{3} + \frac{\Lambda}{3}$

$$k < 1$$

$$\frac{\partial^2 \sigma}{\alpha^2} + 2 \frac{\dot{\alpha}}{\alpha} \frac{\partial \sigma}{\partial t} = 4\pi S \rho_b \dot{\alpha}^2 \sigma$$

$$\Omega: H^2 = \frac{\dot{\alpha}^2}{\alpha^2} = \frac{8\pi S \rho_b}{3} - \frac{K^2}{\alpha^2} + \Lambda = \frac{8\pi S \rho_b}{3} \frac{\Delta}{R(t)}$$

$$k \ll 1$$

$$\frac{\partial^2 \delta}{\alpha^2} + 2 \frac{\dot{\alpha}}{\alpha} \frac{\partial \delta}{\partial t} = 4\pi G \rho_b \alpha^2 \delta$$

$$\Omega: H^2 = \frac{\dot{\alpha}^2}{\alpha^2} = \frac{8\pi G \rho_b}{3} - \frac{v^2}{\alpha^2} + \frac{\Lambda}{3} \equiv \frac{8\pi G \rho_b}{3} \frac{1}{\Omega(t)}$$

$$\Omega(t) = \frac{\rho_b}{H^2} \left(\frac{8\pi G}{3} \rho_b \right)$$

$$k \ll 1$$

$$\frac{\partial^2 \sigma}{\alpha^2} + 2 \frac{\dot{\alpha}}{\alpha} \frac{\partial \sigma}{\partial t} = 4\pi S \rho_b \dot{\alpha}^2 \sigma$$

$$\Omega: H^2 = \frac{\dot{\alpha}^2}{\alpha^2} = \frac{8\pi S \rho_b}{3} - \frac{\rho_{\text{rad}}}{\alpha^2} + \Lambda \rightarrow 0 \equiv \frac{8\pi S \rho_b}{3} \frac{1}{\Omega(t)}$$

$$\Omega(t) = \frac{\rho_b}{H^2} \left(\frac{8\pi S}{3} \right) \rho_b \equiv 0$$

$k < 1$

$$\frac{\partial^2 \sigma}{\alpha^2} + 2 \frac{\dot{\alpha}}{\alpha} \frac{\partial \sigma}{\partial t} = 4\pi S \rho_b \dot{\alpha}^2 \sigma$$

$$\Omega: H^2 = \dot{\alpha}^2 = \frac{8}{3}\pi S \rho_b - \frac{\rho_{\text{rad}}}{\alpha} + \Lambda \rightarrow \frac{8}{3}\pi S \rho_b \frac{1}{\Omega(t)}$$

$$\Omega(t) = \frac{\rho_b}{H^2 / \left(\frac{8}{3}\pi S \right) \rho_b, \text{FD}}$$

$$\ddot{a}^2 = \frac{8}{3} \pi G \rho_b a^2$$
$$\Rightarrow a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$



$$d^2 = \frac{2}{3} \pi G \rho_b d^2$$

$$\rightarrow a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4}{3} \frac{\partial \psi}{\partial x} = \frac{2}{3\pi^2} \psi$$

$$d^2 = \frac{2}{3} \pi G \rho_b d^2$$

$$a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$

$$\frac{\psi}{a^2} + \frac{4}{3} \frac{\partial \psi}{\partial a} = \frac{2}{3\pi c} \psi$$

$$\psi(\vec{x}, t) = \underbrace{A(\vec{x})}_{\psi_1} t^{2/3} + \underbrace{B(\vec{x})}_{\psi_2} t^{-1}$$

$$z=1: \ddot{a}^2 = \frac{2}{3} \pi G \rho_b a^2$$

$$\Rightarrow a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4}{3x} \frac{\partial \psi}{\partial x} = \frac{2}{3\pi^2} \psi$$

$$\psi(x,t) = \underbrace{A(x)}_{D_1} t^{2/3} + \underbrace{B(x)}_{D_2} t^{-1}$$

$$\Omega=1: \alpha^2 = \frac{2}{3} \pi S_{\text{RB}} \alpha^2$$

$$\Rightarrow \alpha \propto t^{2/3}, \quad 6\pi S_{\text{RB}} t^2 = 1$$

$$\frac{\partial^2 \delta}{\partial x^2} + \frac{4}{3x} \frac{\partial \delta}{\partial x} = \frac{2}{3t^2} \delta$$

$$\delta(\vec{x}, t) = A(\vec{x}) t^{2/3} + \text{c.c.}$$

$$x = \left| \frac{2}{3} \pi S_{\text{RB}} t^2 \right|^{-1/2}$$

$$t^2 = \frac{3}{2} \pi S_{\text{RB}} t^2 \left| \frac{2}{3} \pi S_{\text{RB}} t^2 \right|^{-1/2}$$

5) $\delta \vec{x}$

$$\Omega = 1: \alpha^2 = \frac{2}{3} \pi S_{\text{RB}} \alpha^2$$

$$\Rightarrow \alpha \propto t^{2/3}, \quad 6\pi S_{\text{RB}} t^2 = 1$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4}{3x} \frac{\partial \psi}{\partial x} = \frac{2}{3t^2} \psi$$

$$\psi(\vec{x}, t) = A(\vec{x}) t^{2/3} + B(\vec{x}) t^{-1/3}$$

$$x = \left| \frac{2}{3} \pi S_{\text{RB}} t^2 - 1 \right|$$

$$t^2 = \frac{3}{2\pi S_{\text{RB}}} \left(\frac{x}{t} + 1 \right)$$

$$5) \quad \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} = -\frac{\psi}{\alpha} = \psi$$

$$\Omega=1: \dot{a}^2 = \frac{8}{3} \pi \epsilon_0 \rho_0 a^2$$

$$\Rightarrow a \propto t^{2/3}, \quad \dot{a} = 1$$

$$\frac{\partial^2 \sigma}{\partial t^2} + \frac{4}{3} \frac{\partial \sigma}{\partial t} = \sigma$$

$$\sigma(\vec{x}, t) = A(\vec{x}) B(\vec{x}) e^{-t}$$

$$x = \sqrt{R^{-1}(t)}$$

$$H^2 = \frac{8}{3} \pi \epsilon_0 \rho_0$$

$$5) \quad \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{\nabla \phi}{a} = \vec{g}$$

$$\frac{\nabla \cdot \vec{v}}{a} + \frac{\partial \sigma}{\partial t} = 0$$

$$\nabla^2 \phi = 4\pi \epsilon_0 \rho_0 a^3 \sigma$$

$$\Omega=1: \dot{a}^2 = \frac{8}{3} \pi G \rho_b a^2$$

$$\Rightarrow a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$

$$\frac{\partial^2 \sigma}{\partial x^2} + \frac{4}{3x} \frac{\partial \sigma}{\partial x} = \frac{2}{3x^2} \sigma$$

$$\sigma(\vec{x}, t) = A(\vec{x}) e^{i/\lambda} + B(\vec{x}) e^{-i/\lambda}$$

$$x = \left| \frac{R^{-1}(t) - 1}{1 - H_0 R(t)} \right|$$

$$H^2 = \frac{8}{3} \pi G \rho_b (1+x)$$

$$5) \quad \frac{\partial \vec{v}}{\partial t} + \frac{d}{dt} \vec{v} = -\frac{\nabla \phi}{a} = \vec{g}$$

$$\frac{\nabla \cdot \vec{v}}{a} + \dots = 0$$

$$\Rightarrow \rho_b a^2 \sigma$$

$$\Omega = 1: \dot{a}^2 = \frac{8}{3} \pi S \rho_b a^2$$

$$\Rightarrow a \propto t^{2/3}, \quad 6\pi \quad 1$$

$$\frac{\partial^2 \sigma}{\partial t^2} + \frac{4}{3} \frac{\partial \sigma}{\partial t} = -\sigma$$

$$\sigma(\vec{x}, t) = A B(\vec{x}) e^{-t}$$

$$x = \sqrt{R^{-1}(t)}$$

$$H^2 = \frac{8}{3} \pi S \rho_b$$

$$5) \quad \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{\nabla \phi}{a} = \vec{g}$$

$$\frac{\nabla \cdot \vec{v}}{a} + \frac{\partial \sigma}{\partial t} = 0$$

$$\nabla^2 \phi = 4\pi S \rho_b a^2 \sigma$$

$$\vec{v} = -a \frac{\partial}{\partial \vec{x}} \left(\frac{\nabla \cdot \vec{v}}{4\pi S \rho_b a^2} \right) + \frac{\vec{F}(\vec{x})}{a(t)}$$

$$\Omega=1: \dot{a}^2 = \frac{8}{3} \pi \rho_b a^2$$

$$\Rightarrow a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$

$$\frac{\partial^2 \delta}{\partial x^2} + \frac{4}{3x} \frac{\partial \delta}{\partial x} = \frac{2}{3x^2} \delta$$

$$\delta(\vec{x}, t) = A(\vec{x}) t^{-1}$$

$$x = \sqrt{2} R^{-1/2} t$$

$$H^2 = \frac{8}{3} \pi G \rho_b$$

$$5) \quad \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{\nabla \phi}{a} = \vec{g}$$

$$\frac{\nabla \cdot \vec{v}}{a} + \frac{\partial \delta}{\partial t} = 0$$

$$\nabla^2 \phi = 4\pi G \rho_b a^2 \delta$$

$$\nabla \cdot \vec{v} = -a \frac{\partial \delta}{\partial t} \left(\frac{\vec{x} \cdot \vec{x}}{4\pi G \rho_b a^2} \right) + \frac{\vec{v} \cdot \vec{x}}{a} \frac{\nabla \cdot \vec{v}}{a}$$

$$\Omega = 1: \dot{a}^2 = \frac{8}{3} \pi G \rho_b a^2$$

$$\Rightarrow a \propto t^{2/3}, \quad 6\pi G \rho_b t^2 = 1$$

$$\frac{\partial^2 \delta}{\partial x^2} + \frac{4}{3x} \frac{\partial \delta}{\partial x} = \frac{2}{3t^2} \delta$$

$$\delta(\vec{x}, t) = A(\vec{x}) t^{2/3} + B(\vec{x}) t^{-1}$$

$$x = \left| \frac{\vec{r}(t) - \vec{r}_1}{a(t)} \right|$$

$$H^2 = \frac{8}{3} \pi G \rho_b \frac{1}{a^2(t)}$$

$$5) \quad \frac{\partial \dot{\delta}}{\partial t} + \frac{\dot{a}}{a} \delta = -\frac{\nabla^2 \phi}{a} = \frac{2}{3} \delta$$

$$\frac{\nabla^2 \phi}{a} + \frac{\partial \delta}{\partial t} = 0$$

$$\nabla^2 \phi = 4\pi G \rho_b a^2 \delta$$

$$\frac{\partial \delta}{\partial t} + \frac{\dot{a}}{a} \delta = -a \frac{\nabla^2}{a^2} \left(\frac{\delta}{4\pi G \rho_b a} \right) + \frac{\nabla^2 \phi}{a}$$

$$\frac{\partial \delta}{\partial t} + \frac{\dot{a}}{a} \delta = 4\pi G \rho_b a$$

$$\vec{g} = -\frac{\nabla\phi}{a} = G\sigma_b \alpha \int d^3x' \delta(\vec{r}' - t) \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = G_s \rho_b a \int d^3x' \delta(\vec{r}' - t) \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$
$$\vec{g}_r \propto \rho_b a D_a(t)$$



$$\vec{g} = -\frac{\nabla\phi}{a} = \frac{G\delta_b}{a} \int d^3x' \delta(\vec{r}' - t) \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$
$$\vec{g} \propto \frac{G\delta_b}{a} D_q(t)$$
$$\vec{v} =$$

$$\vec{g} = -\frac{\nabla\phi}{a} = \frac{1}{4\pi\epsilon_0} \alpha \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g} \propto \frac{1}{r^2} \hat{r}$$

$$\vec{v} = \frac{\vec{g}}{4\pi\epsilon_0} \frac{1}{Q} \frac{\partial Q}{\partial t}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = \epsilon_0 \epsilon_b \alpha \int d^3x' \delta(\vec{r}' - t) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\epsilon_b \alpha D_q(t)$$

$$= \frac{2 + \vec{g}}{3HR} \quad f = \frac{a}{D} \frac{dD}{da}, \quad H = \frac{a}{a}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = G\sigma_b \alpha \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g}_r \propto \sigma_b \alpha D_q(t)$$

$$\vec{v} = \frac{\vec{g}_r}{4\pi\sigma_b} \frac{1}{Q} \frac{\partial D_q}{\partial t}$$

$$= \frac{2\vec{g}_r}{3HR} \quad f = \frac{a}{D} \frac{dD}{da} \quad H = \frac{\dot{a}}{a}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = G\sigma_b \alpha \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$

$$\sigma_b \alpha D_a(t)$$

$$\frac{\vec{g}_r}{4\pi\sigma_b} \frac{1}{Q} \frac{\partial D_r}{\partial t}$$

$$\frac{2+\vec{g}}{2Hr} \quad f = \frac{a}{D} \frac{dD}{da} \quad H = \frac{a}{a}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = \frac{1}{4\pi\epsilon_0} \alpha \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g} \propto \frac{1}{a} \frac{\partial D_a(t)}{\partial t}$$

$$\vec{v} = \frac{\vec{g}}{4\pi\epsilon_0} \frac{1}{Q} \frac{\partial D_a}{\partial t}$$

$$= \frac{2\vec{g}}{3H^2} \quad f = \frac{a}{D} \frac{dD}{da} \quad H = \frac{\dot{a}}{a}$$

$$\vec{v} = \frac{H\vec{g}}{4H} \int d^3x' \frac{\delta(\vec{r}' - \vec{r}) (\vec{r}' - \vec{r})}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = \frac{1}{4\pi\epsilon_0} \alpha \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g} \propto \frac{1}{r^2} \hat{r}$$

$$\vec{v} = \frac{\vec{g}}{4\pi\epsilon_0} \frac{1}{Q} \frac{\partial Q}{\partial t}$$

$$= \frac{2\vec{g}}{3Hr} \quad f = \frac{1}{D} \frac{dD}{da} \quad H = \frac{\dot{a}}{a}$$

$$\vec{v} = \frac{H\vec{g}}{4H} \int d^3x' \frac{\delta(\vec{r}' - \vec{r}) (\vec{r}' - \vec{r})}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g} = -\frac{\nabla\phi}{a} = \frac{1}{4\pi\epsilon_0} \alpha \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{g}_r \propto \frac{1}{r^2} \frac{\partial D_r}{\partial t}$$

$$\vec{v} = \frac{\vec{g}_r}{4\pi\epsilon_0} \frac{1}{Q} \frac{\partial D_r}{\partial t}$$

$$= \frac{2\vec{g}_r}{3Hr} \quad f = \frac{a}{D} \frac{dD}{da}, \quad H = \frac{\dot{a}}{a}$$

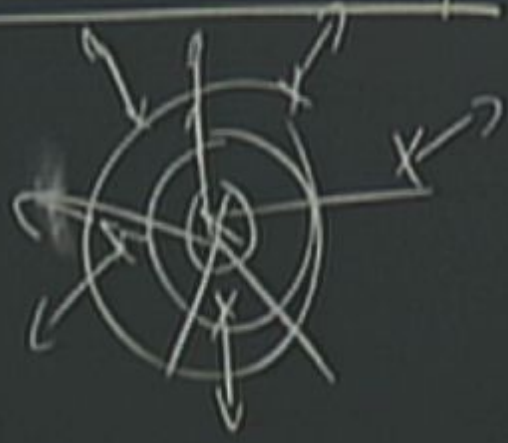
$$\vec{v} = \frac{Hf}{4\pi} \int d^3x' \frac{\delta(\vec{r}' - \vec{r}) (\vec{r}' - \vec{r})}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{V} = \frac{H_0 c^2}{4\pi} \nabla \int d^3x' \frac{\rho(\vec{x}')}{|\vec{x}' - \vec{x}|}$$

$$\vec{V} = \frac{H_{\text{eff}}}{4\pi} \nabla \left(\int d^3x' \frac{\rho(\vec{x}')}{|\vec{x}' - \vec{x}|} \right)$$
$$\nabla \cdot \vec{V} = -H_{\text{eff}} \rho(\vec{x})$$

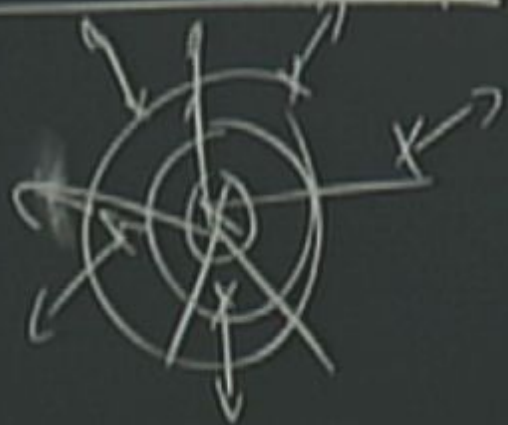
$$\vec{V} = \frac{H \text{ of } \nabla \int d^3x' \frac{\delta(\vec{x}')}{|\vec{x}' - \vec{x}|}}{4\pi} \quad | \quad \vec{V} = -\nabla \phi_v$$

$$\nabla \cdot \vec{V} = -H \text{ of } \delta(\vec{x})$$



$$\vec{V} = \frac{1}{4\pi\epsilon_0} \nabla \int d^3x' \frac{\rho(\vec{x}')}{|\vec{x}' - \vec{x}|}$$

$$\nabla \cdot \vec{V} = -\frac{1}{\epsilon_0} \rho(\vec{x})$$

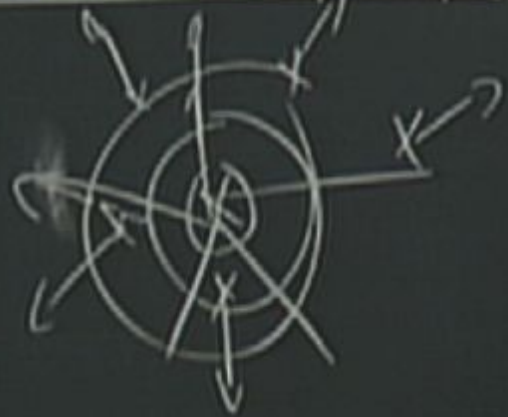


$$\vec{V} = -\nabla \phi_V$$

$$\phi_V = \int_0^{\infty} V(r, \theta, \phi) dr'$$

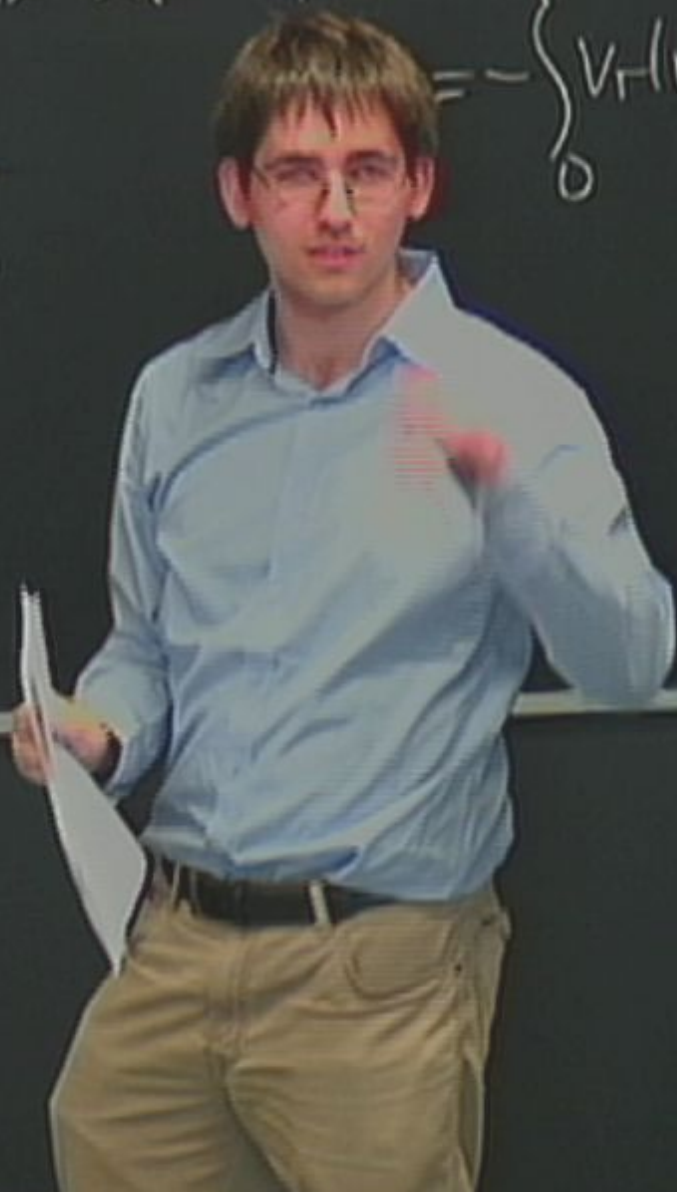
$$\vec{V} = \frac{H \text{at}}{4\pi} \nabla \left(\int d^3x' \frac{\delta(\vec{x}')}{|\vec{x}' - \vec{x}|} \right)$$

$$\nabla \cdot \vec{V} = -H \text{at } \delta(\vec{x})$$



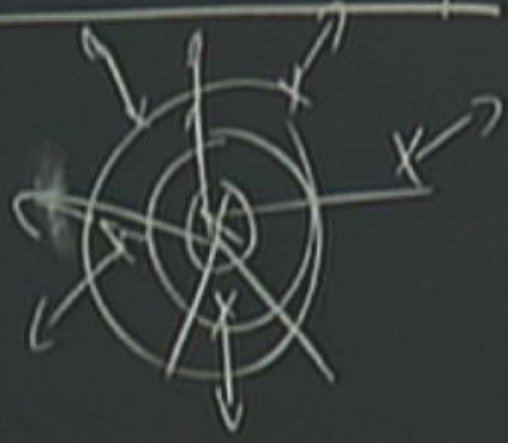
$$\vec{V} = -\nabla \phi_V$$

$$= - \int_0^{\infty} V(r, \theta, \phi) dr'$$



$$\vec{V} = \frac{H \text{ at}}{4\pi} \nabla \int d^3x' \frac{\delta(\vec{x}')}{|\vec{x}' - \vec{x}|}$$

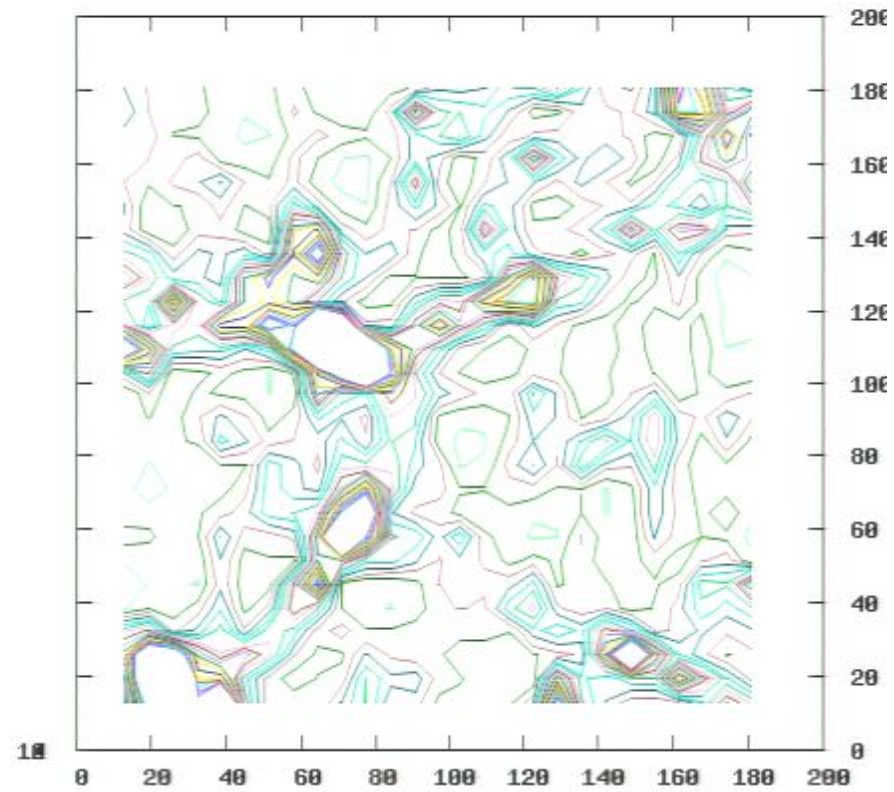
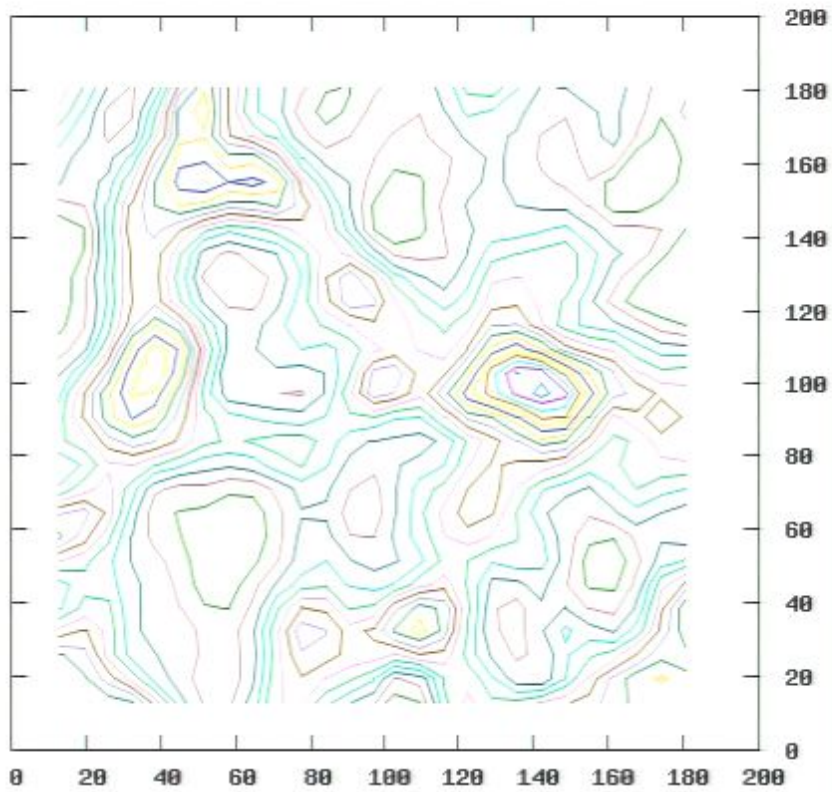
$$\nabla \cdot \vec{V} = -H \text{ at } \delta(\vec{x})$$



$$\vec{V} = -\nabla \phi_V$$

$$\phi_V(\vec{x}) = - \int_0^{\infty} V_H(r, \theta, \phi) dr'$$





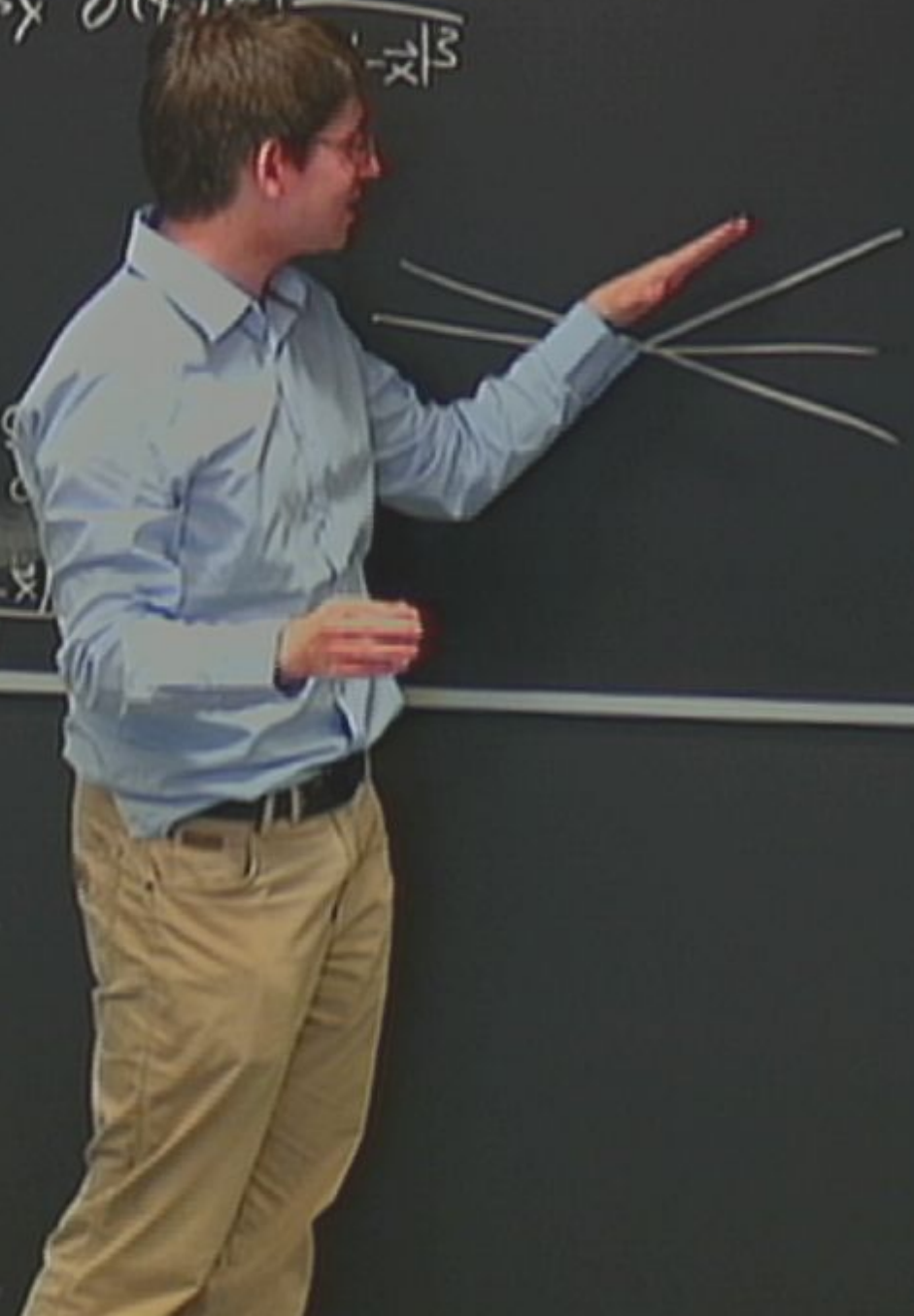
$$\vec{g} = -\frac{\nabla\Phi}{a} = G\gamma_b a \int d^3x' \rho(\vec{x}', t) \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$

$$\vec{g}_x \propto \gamma_b a D_a(t)$$

$$\vec{v} = \frac{\vec{g}_x}{4\pi\gamma_b} \frac{1}{R} \frac{\partial D_a}{\partial t}$$

$$= \frac{2+\gamma}{3H_0 R} \quad \tau = \frac{1}{H_0}$$

$$\vec{v} = \frac{H_0 \vec{x}}{4H} \int d^3x' \rho(\vec{x}') \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$



$$\vec{B} = -\frac{\nabla\phi}{a} - \frac{c}{4\pi a} \int d^3x' \delta(\vec{r}' - \vec{r}) \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int d^3x' \frac{\vec{j}(\vec{r}', t')}{|\vec{r}' - \vec{r}|}$$

$$\frac{d\phi}{da}, \quad H = \frac{a}{a'}$$

