Title: Undergrad Research Project Talk

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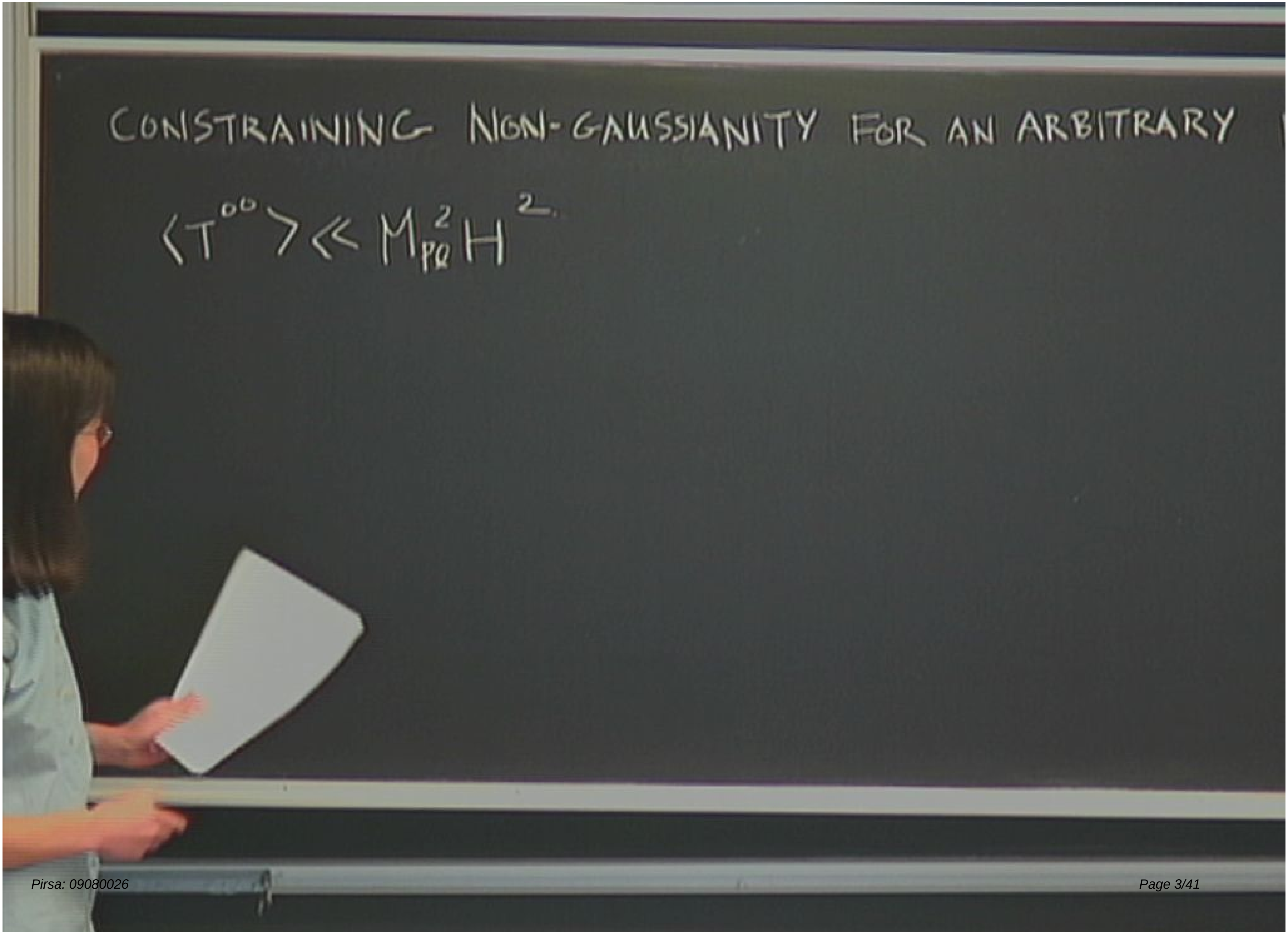
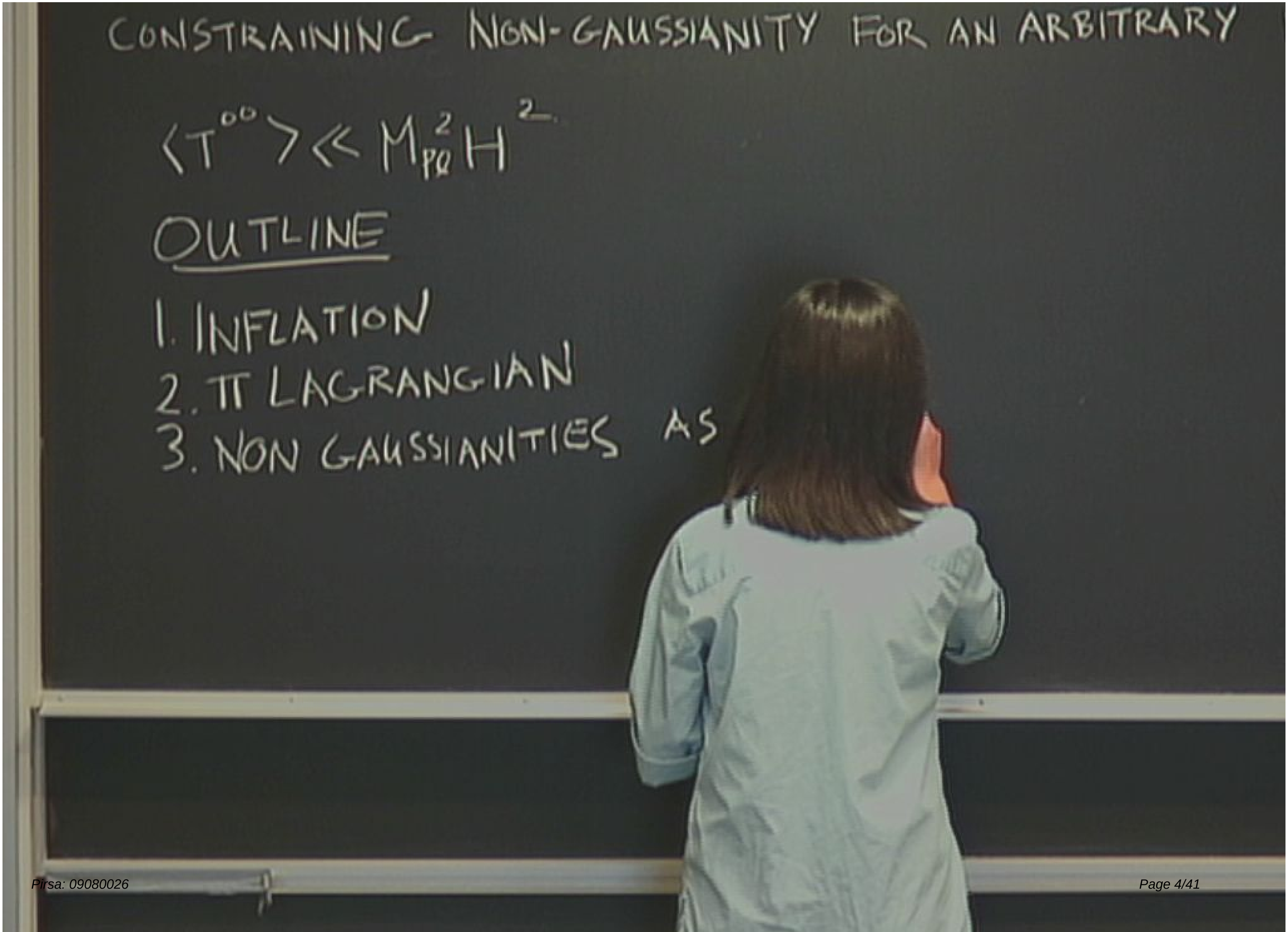
Abstract:

STRAINING NON-GAUSSIANITY FOR AN ARBITRARY INITIAL S


CONSTRAINING NON-GAUSSIANITY FOR AN ARBITRARY
$\langle T^{00} \rangle \ll M_{Pl}^2 H^2.$


CONSTRAINING NON-GAUSSIANITY FOR AN ARBITRARY
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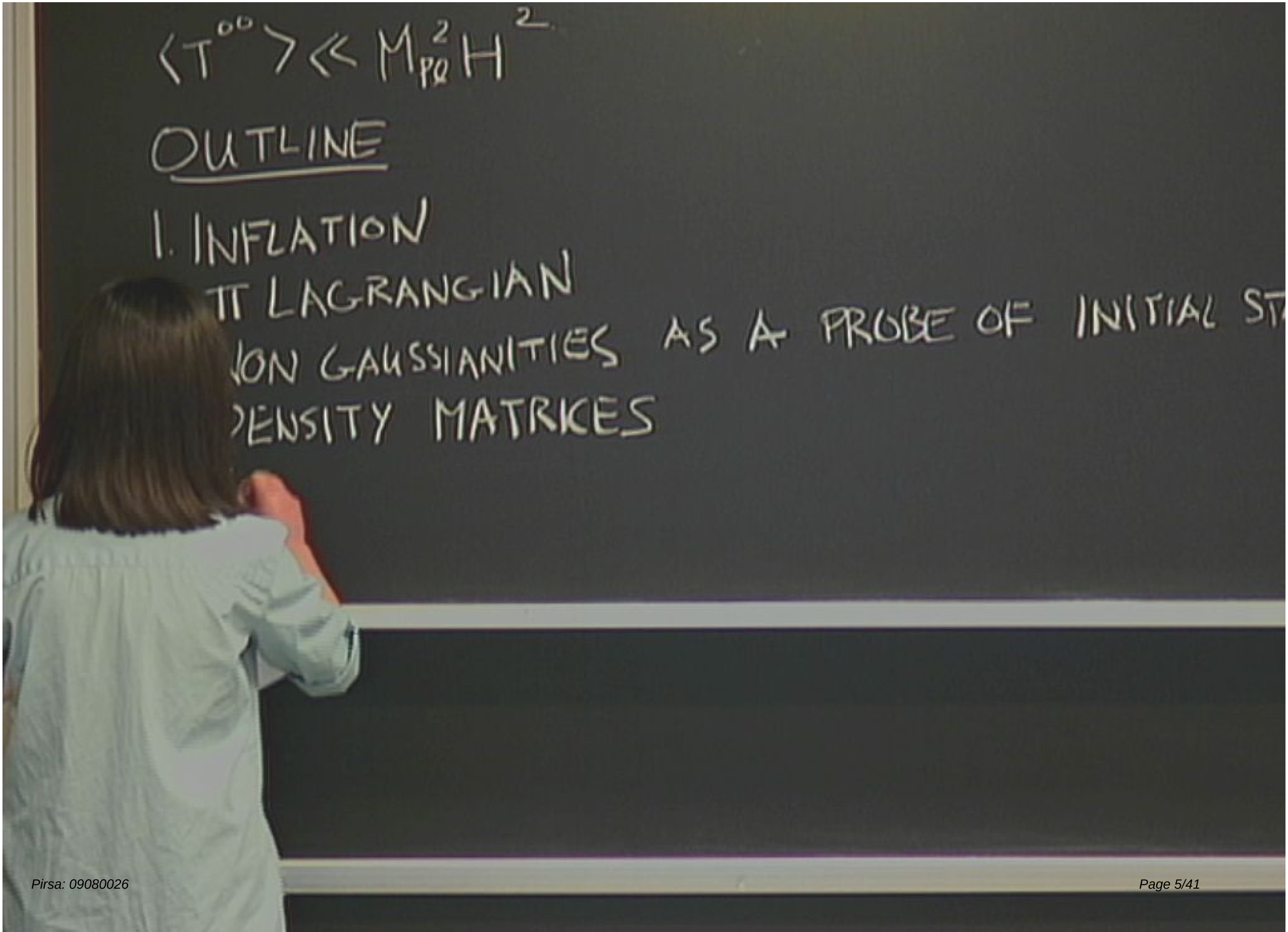
OUTLINE
1. INFLATION
2. π LAGRANGIAN
3. NON GAUSSIANITIES AS


$\langle T^{00} \rangle \ll M_{pl}^2 H^2$.
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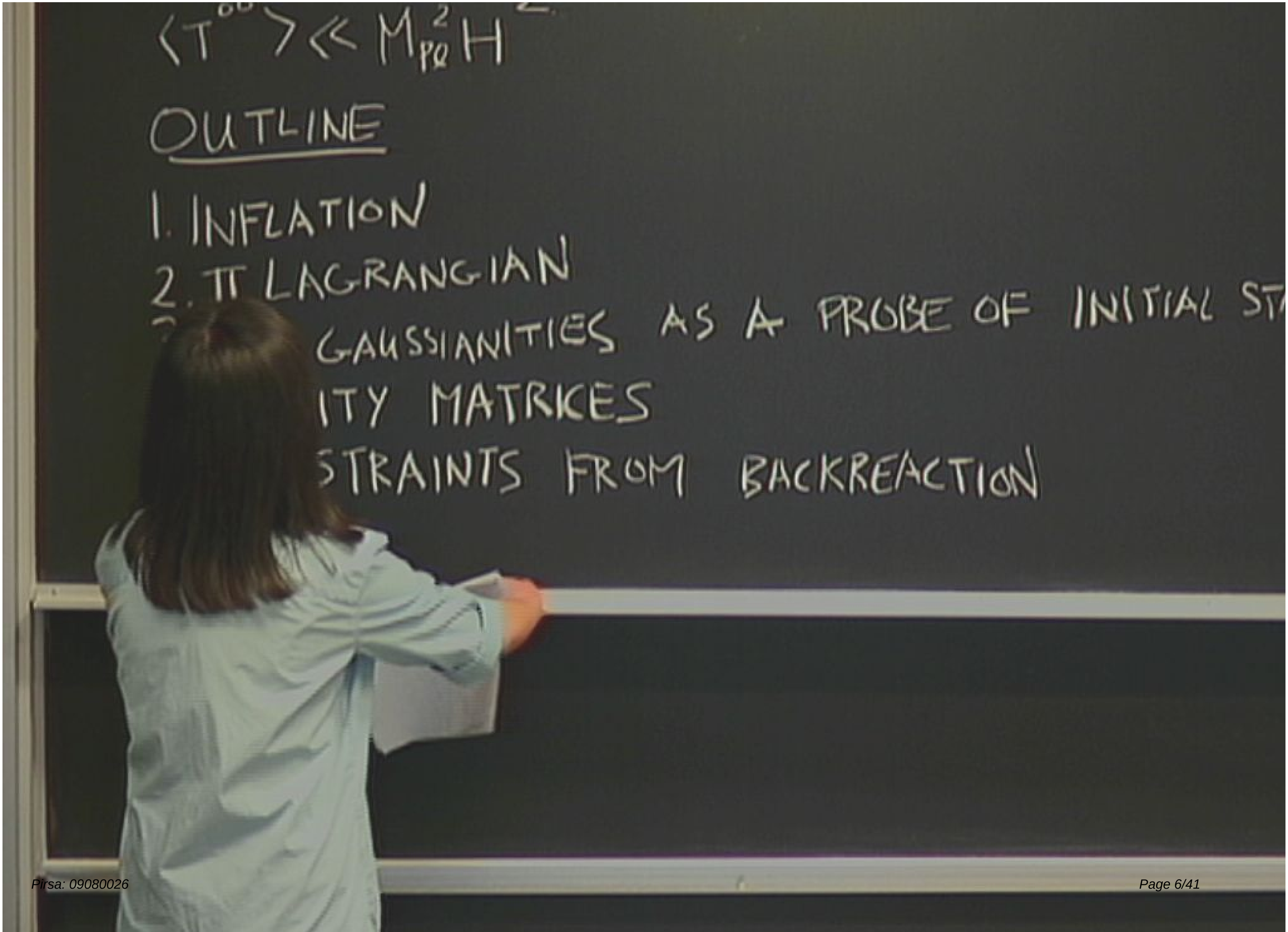
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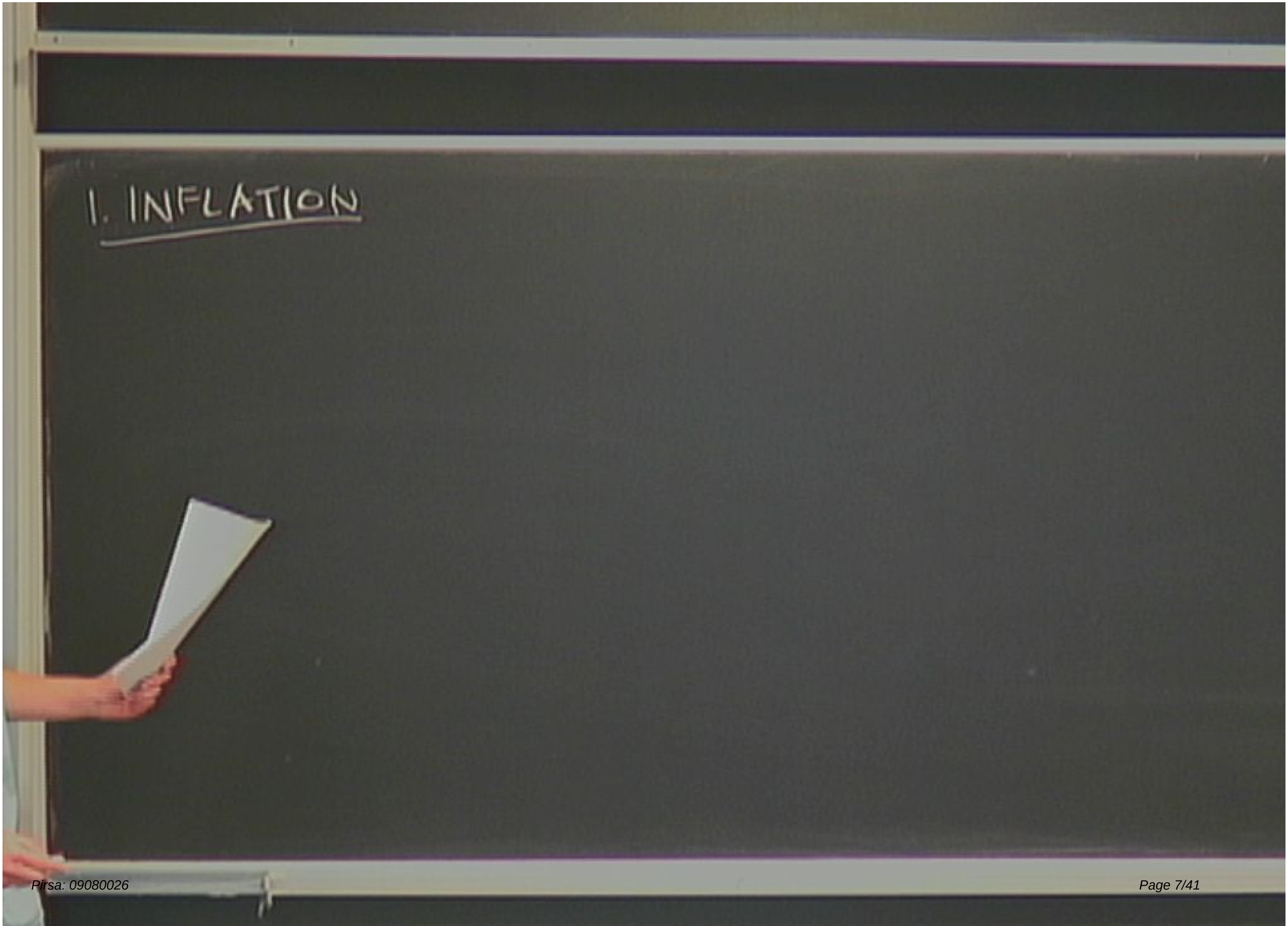
1. INFLATION

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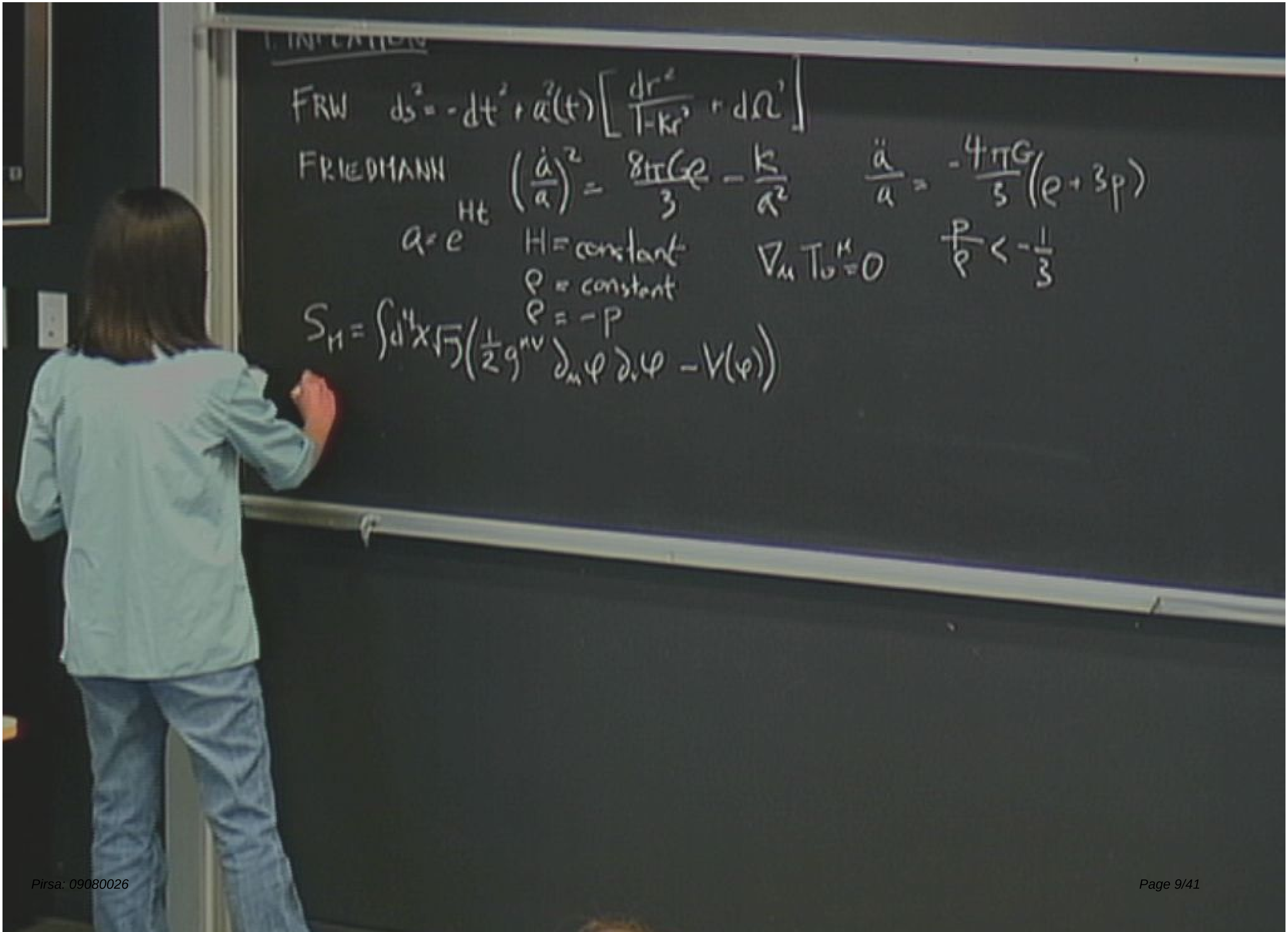
FRW $$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + d\Omega^2\right]$$

FRIEDMANN $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$


FRW $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + d\Omega^2\right]$
FRIEDMANN $\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$
$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
$a = e^{Ht}$
H = constant
ρ = constant
$\rho = -p$
$\nabla_\mu T^\mu_\nu = 0$
$\frac{p}{\rho} < -\frac{1}{3}$
$S_M = \int d^4x \sqrt{-g}\left(\frac{1}{2} g^{\mu\nu} \partial_\mu\varphi \partial_\nu\varphi - V(\varphi)\right)$

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$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\varphi\,\partial_\sigma\varphi + V(\varphi)\right)$

$\rho = \frac{\dot{\varphi}^2}{2} + V(\varphi)$

$p = \frac{\dot{\varphi}^2}{2} - V(\varphi)$

1. INFLATION

FRW $\quad ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + d\Omega^2\right]$

FRIEDMANN $\quad \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} \qquad \frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$a = e^{Ht} \qquad H = \text{constant} \qquad \nabla_\mu T^\mu_\nu = 0 \qquad \frac{p}{\rho} < -\frac{1}{3} \qquad V(\varphi) \gg \frac{1}{2}\dot\varphi^2$

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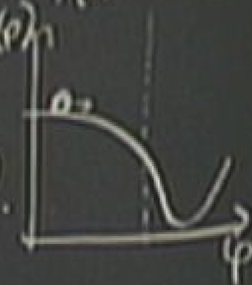

1. INFLATION

FRW $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + d\Omega^2\right]$

FRIEDMANN $\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}$ $\qquad \frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$a = e^{Ht}$ $\qquad H = \text{constant}$ $\qquad \nabla_\mu T^\mu_\nu = 0$ $\qquad \frac{p}{\rho} < -\frac{1}{3}$ $\qquad V(\varphi) \gg \frac{1}{2}\dot\varphi^2$

H^{-1} = Hubble radius $\qquad \rho = \text{constant}$

$\rho = -p$

$S_M = \int d^4x \sqrt{-g}\left(\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - V(\varphi)\right)$

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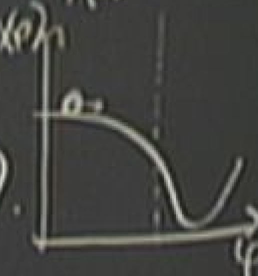
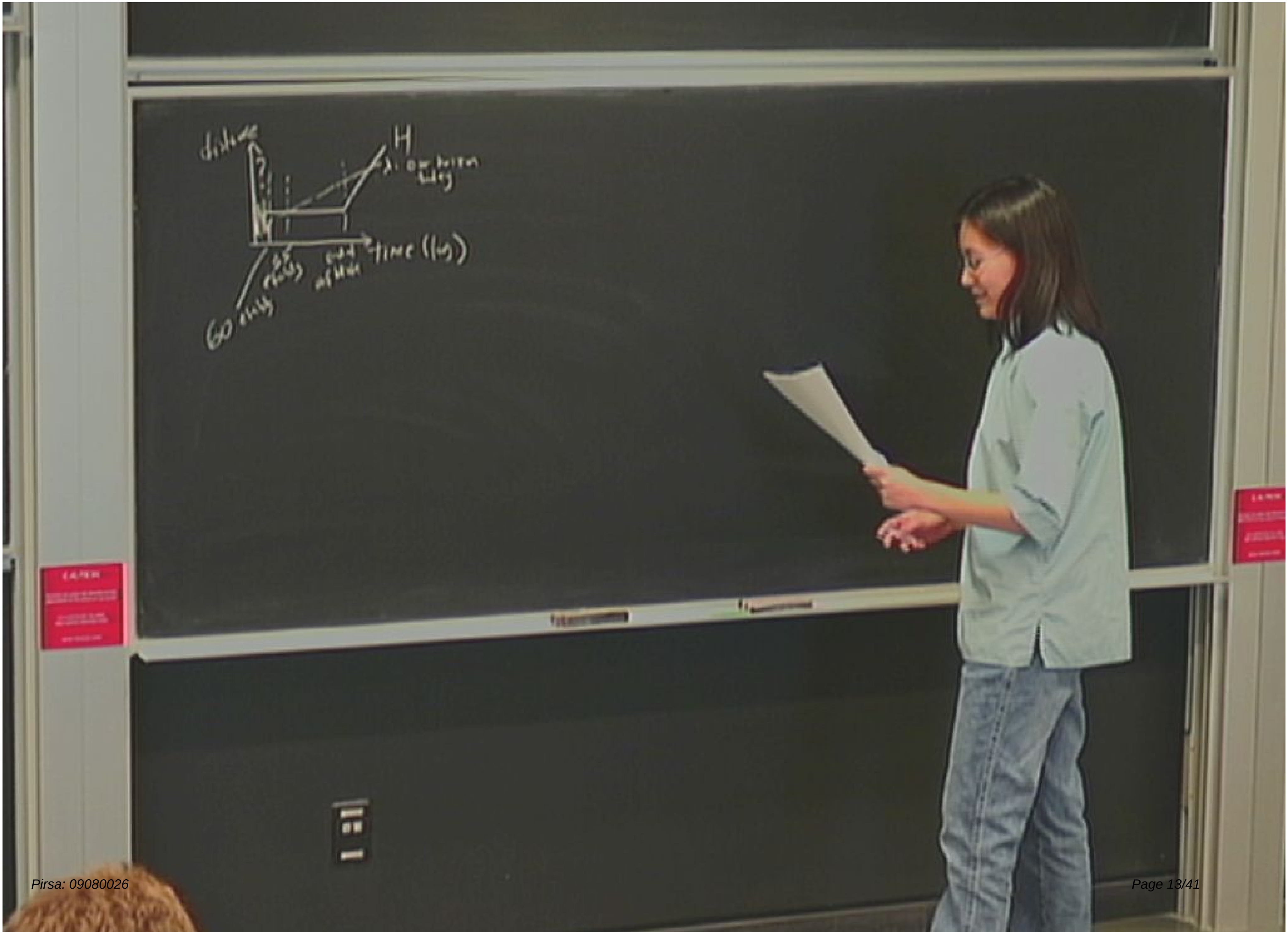
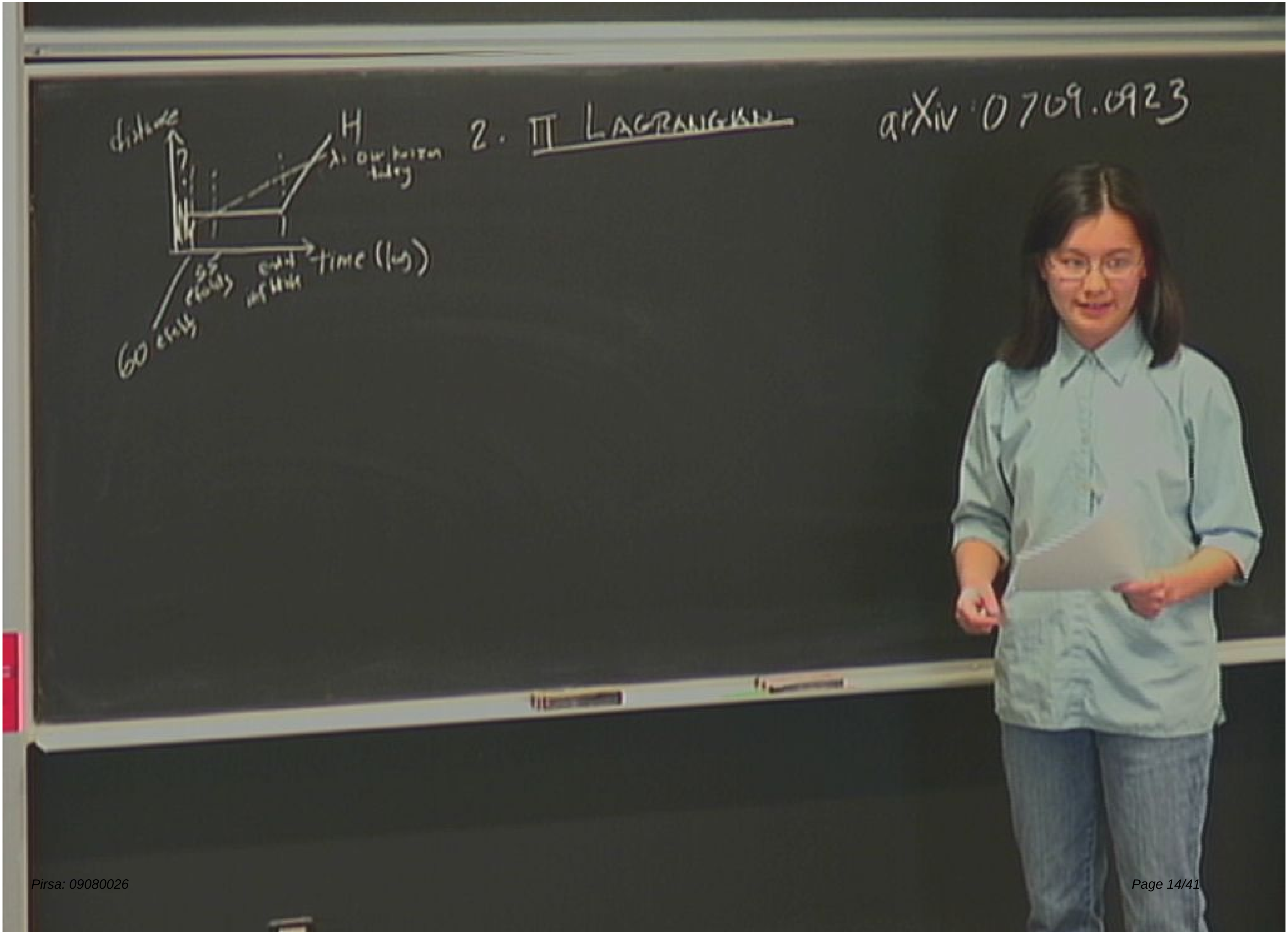
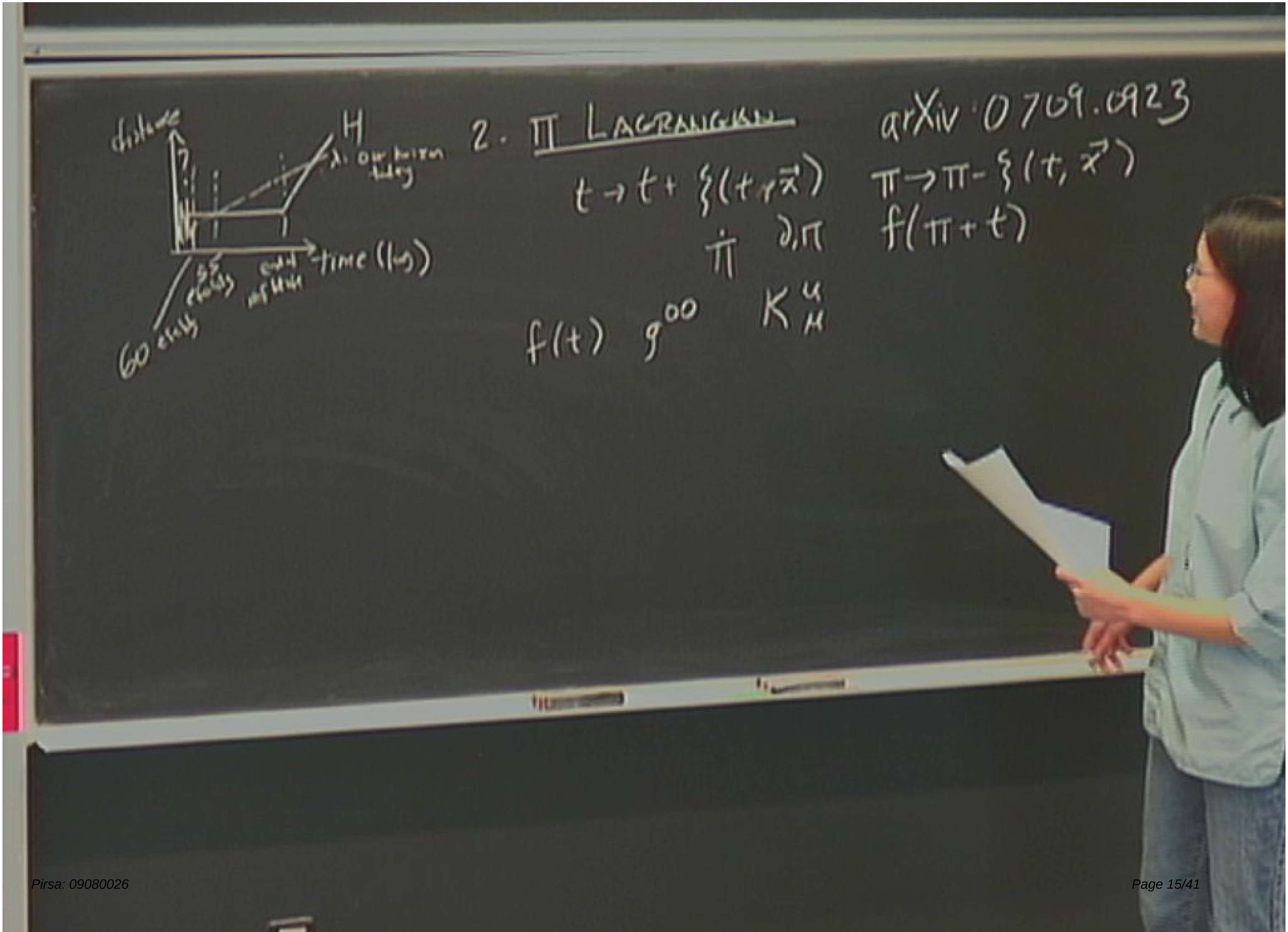


$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\varphi\,\partial_\sigma\varphi + V(\varphi)\right)$





2. Π LAGRANGIAN
arXiv 0709.0923
H
time (log)
60 efolds


distance
H
time (log)
60 efolds
2. π Lagrangian
arXiv:0709.0923
t → t + ξ(t, x⃗)
π → π − ξ(t, x⃗)
π̇
∂ᵢπ
f(π + t)
f(t)
g⁰⁰
K^μ_μ

2. π Lagrangian

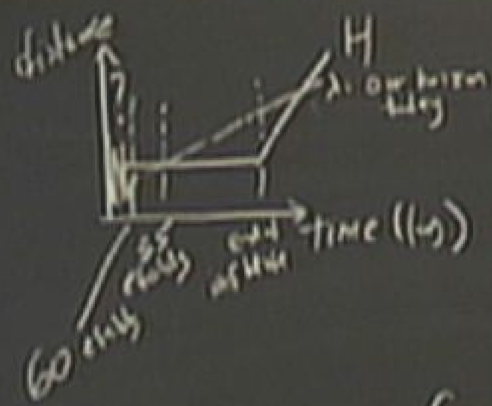
arXiv 0709.0923

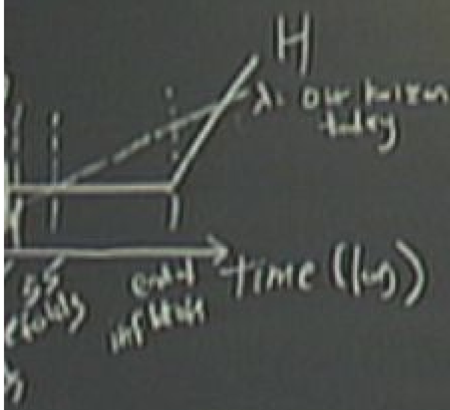
$$t \to t + \xi(t, \vec{x}) \qquad \pi \to \pi - \xi(t, \vec{x})$$

$$\dot{\pi} \qquad \partial_i \pi \qquad f(\pi + t)$$

$$f(t) \qquad g^{00} \qquad K^{\mu}_{\mu}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 \right.$$





2. π LAGRANGIAN

arXiv:0709.0923

$t \to t + \xi(t,\vec{x})$ $\quad$ $\pi \to \pi - \xi(t,\vec{x})$

$\dot{\pi}$ $\quad$ $\partial_i \pi$ $\quad$ $f(\pi + t)$

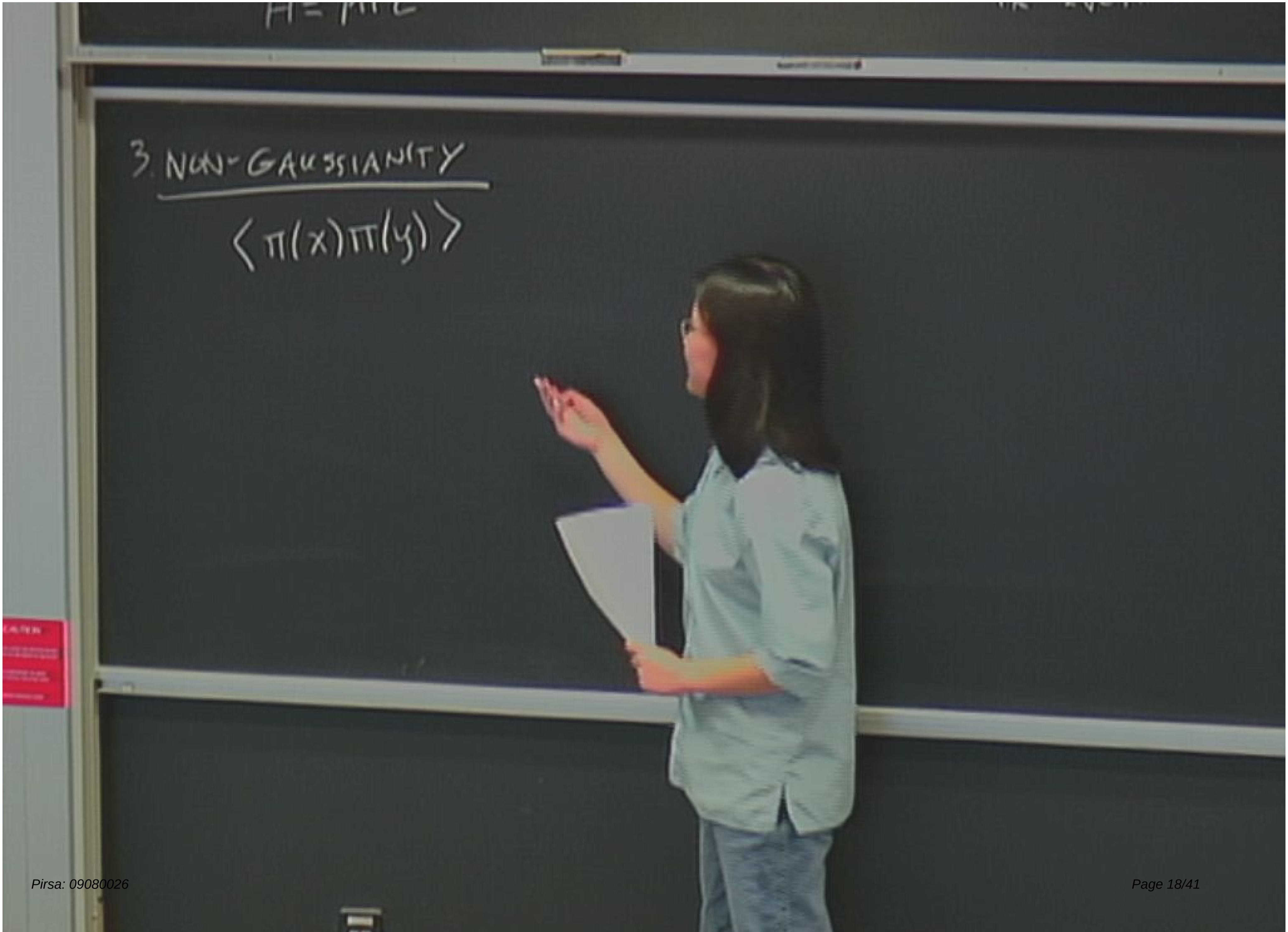
$f(t)$ $\quad$ g^{00} $\quad$ K^{μ}_{μ}

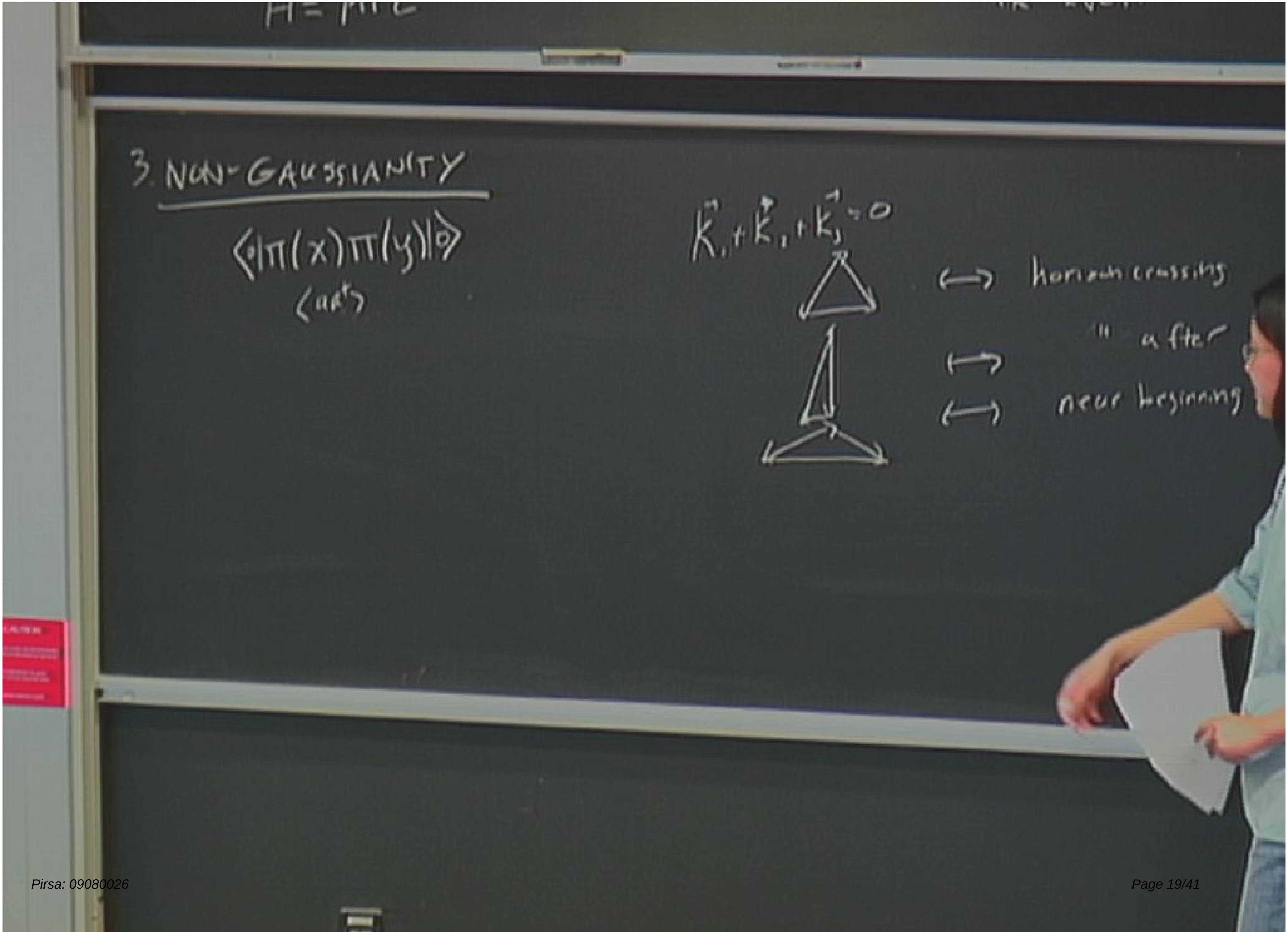
$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2} M_{Pl}^2 \dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i \pi)^2}{a^2}\right) - \frac{4}{3} M_3^4 \dot{\pi}^3\right.$$

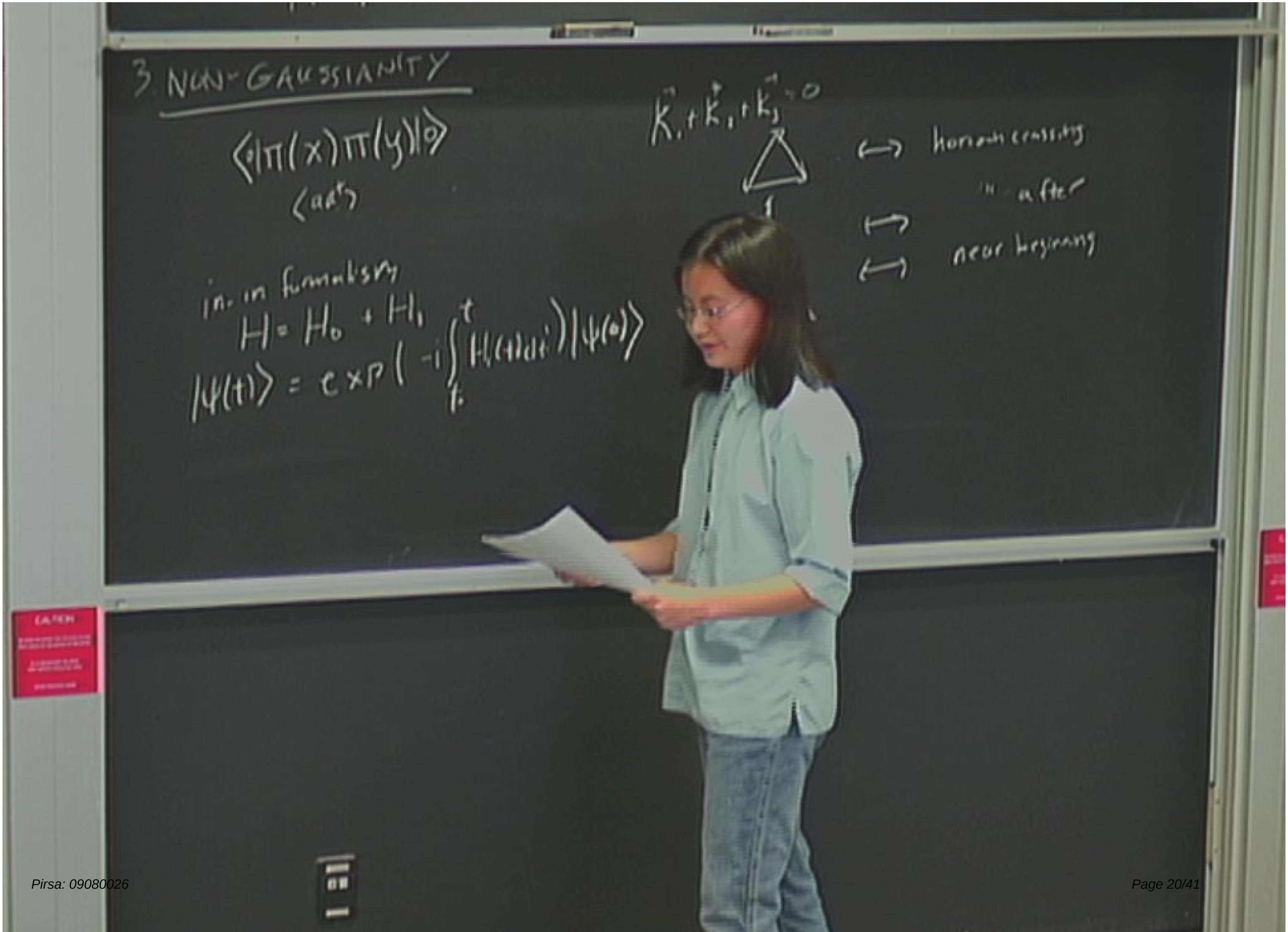
$$\pi = \int \frac{d^3k}{(2\pi)^3}\left(u_k^* \hat{a}_k^+ + u_k \hat{a}_k\right)$$

$$\zeta = -H\pi$$

$$u_k = \frac{i}{2\sqrt{\epsilon k^3 c_s}\, M_{Pl}}\left(1 + iky c_s\right.$$


3. NON-GAUSSIANITY
⟨π(x)π(y)⟩


3. NON-GAUSSIANITY
⟨0|π(x)π(y)|0⟩
k1 + k2 + k3 = 0
horizon crossing
" after
near beginning


3. NON-GAUSSIANITY
in-in formalism
H = H0 + H1
horizon crossing
after
near beginning

3. NON-GAUSSIANITY

$$\langle 0|\pi(x)\pi(y)|0\rangle$$

$$\langle a a^\dagger\rangle$$

$$\vec{k}_1+\vec{k}_2+\vec{k}_3=0$$

$\longleftrightarrow$ horizon crossing

" after

$\longleftrightarrow$

$\longleftrightarrow$ near beginning

in-in formalism

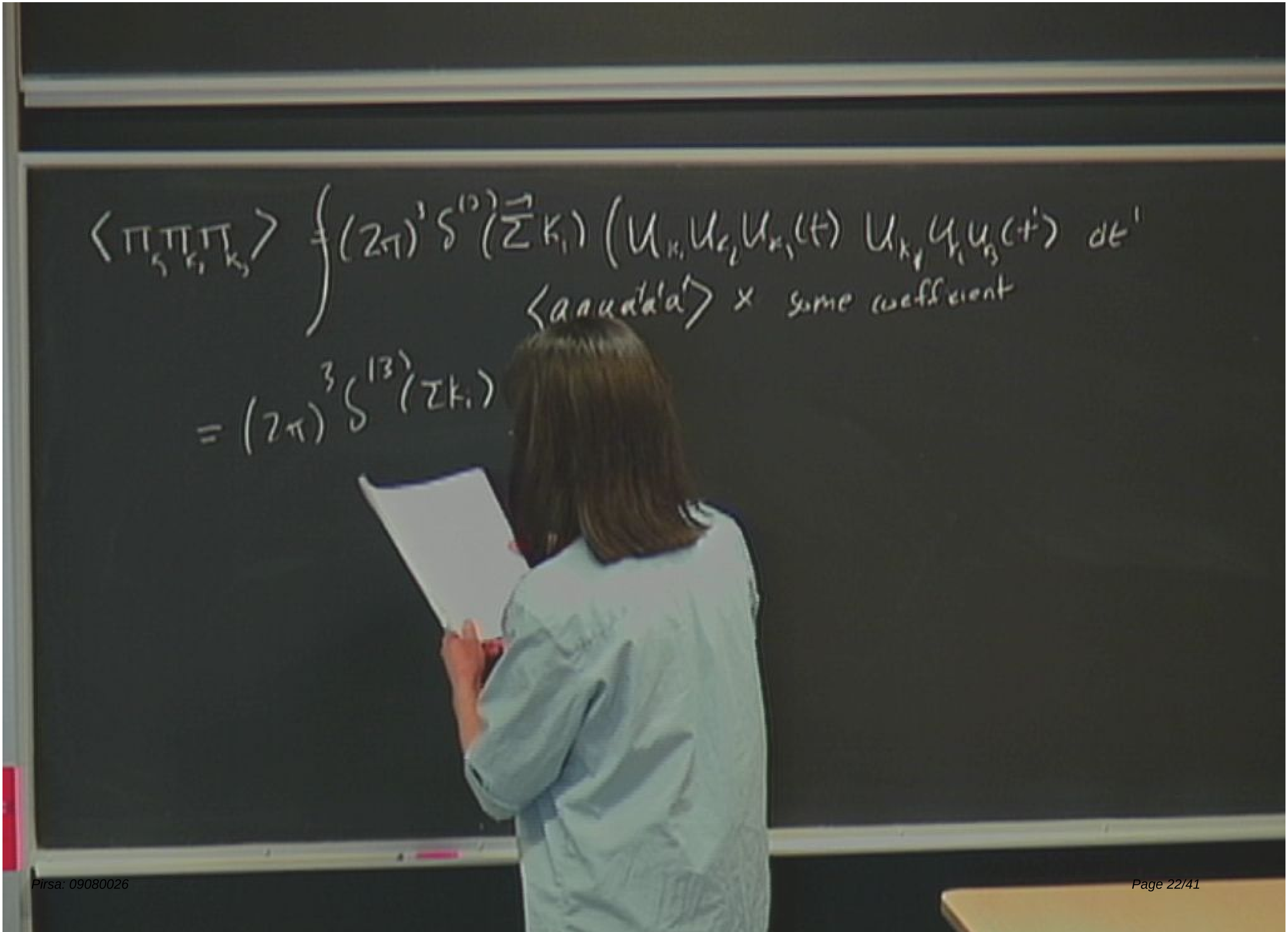
$$H = H_0 + H_1$$

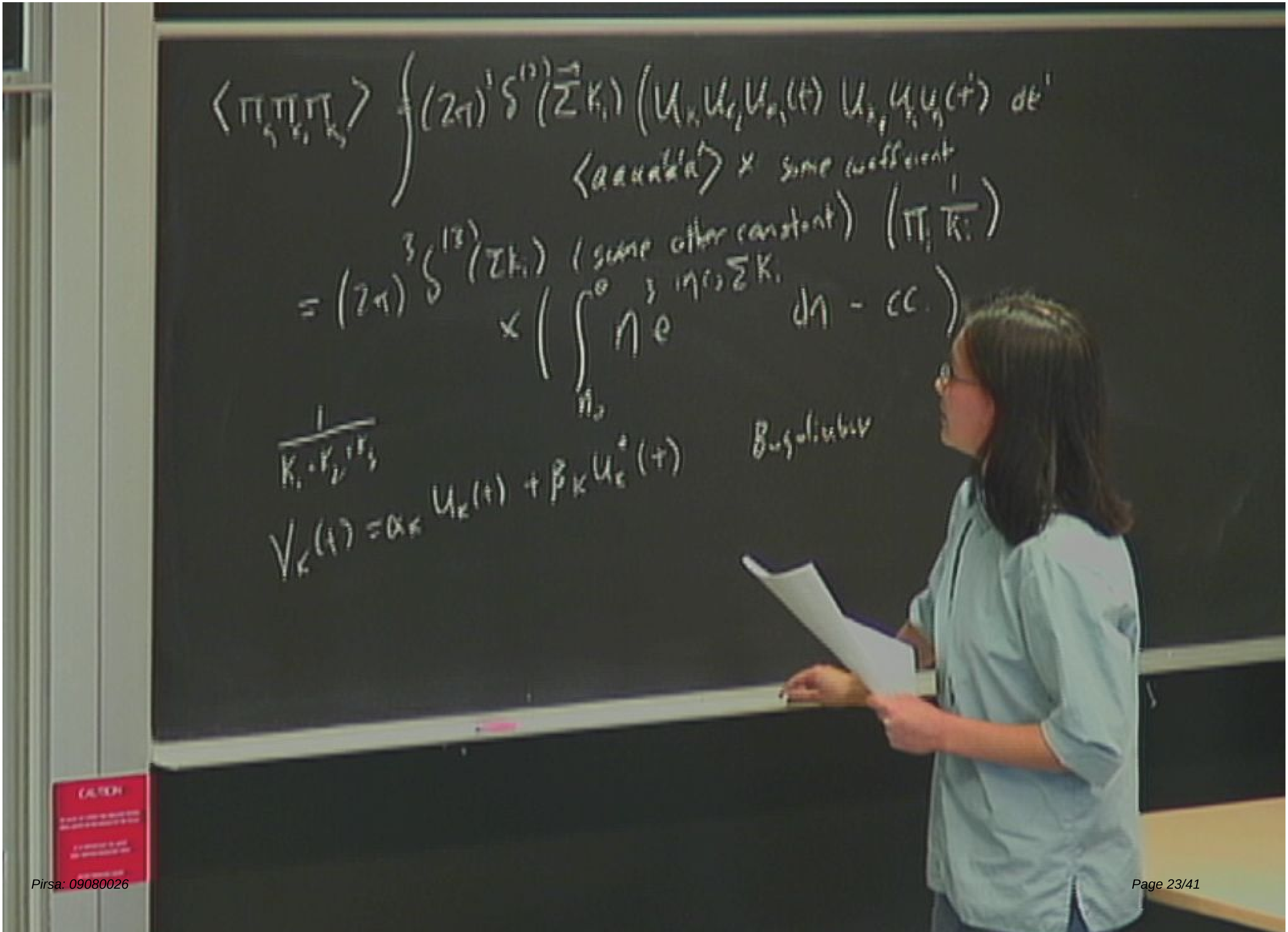
$$|\psi(t)\rangle = \exp\left(-i\int_{t_0}^{t} H_1(t')dt'\right)|\psi(0)\rangle$$

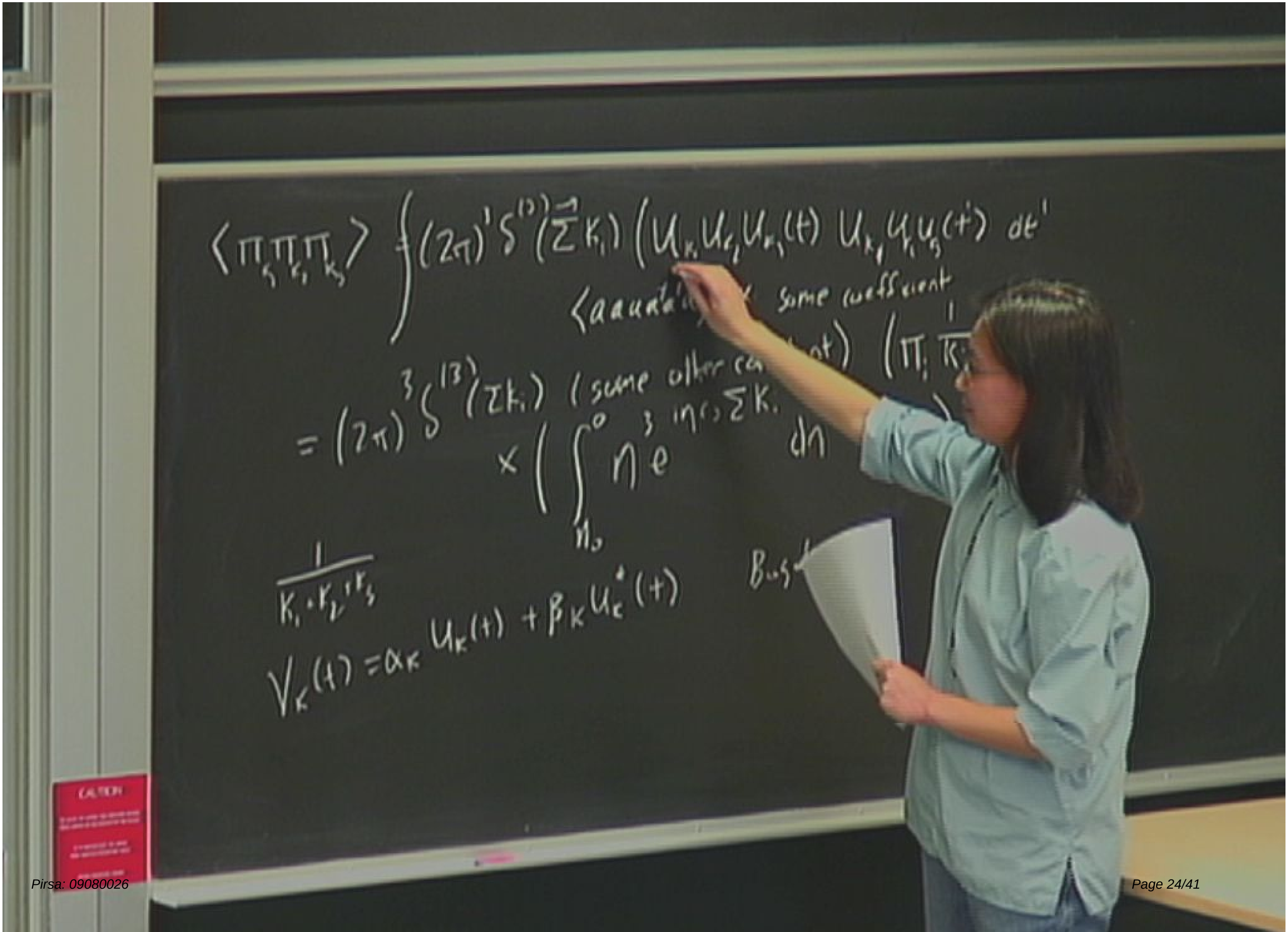
$$\langle\psi(t)|\hat{A}(t)|\psi(t)\rangle = \langle 0|\,\bar{T}\exp\left(i\int_{-\infty}^{t} H_1(t')dt'\right)\hat{A}(t)\,T\exp\left(-i\int_{-\infty}^{t} H_1(t')dt'\right)|0\rangle$$

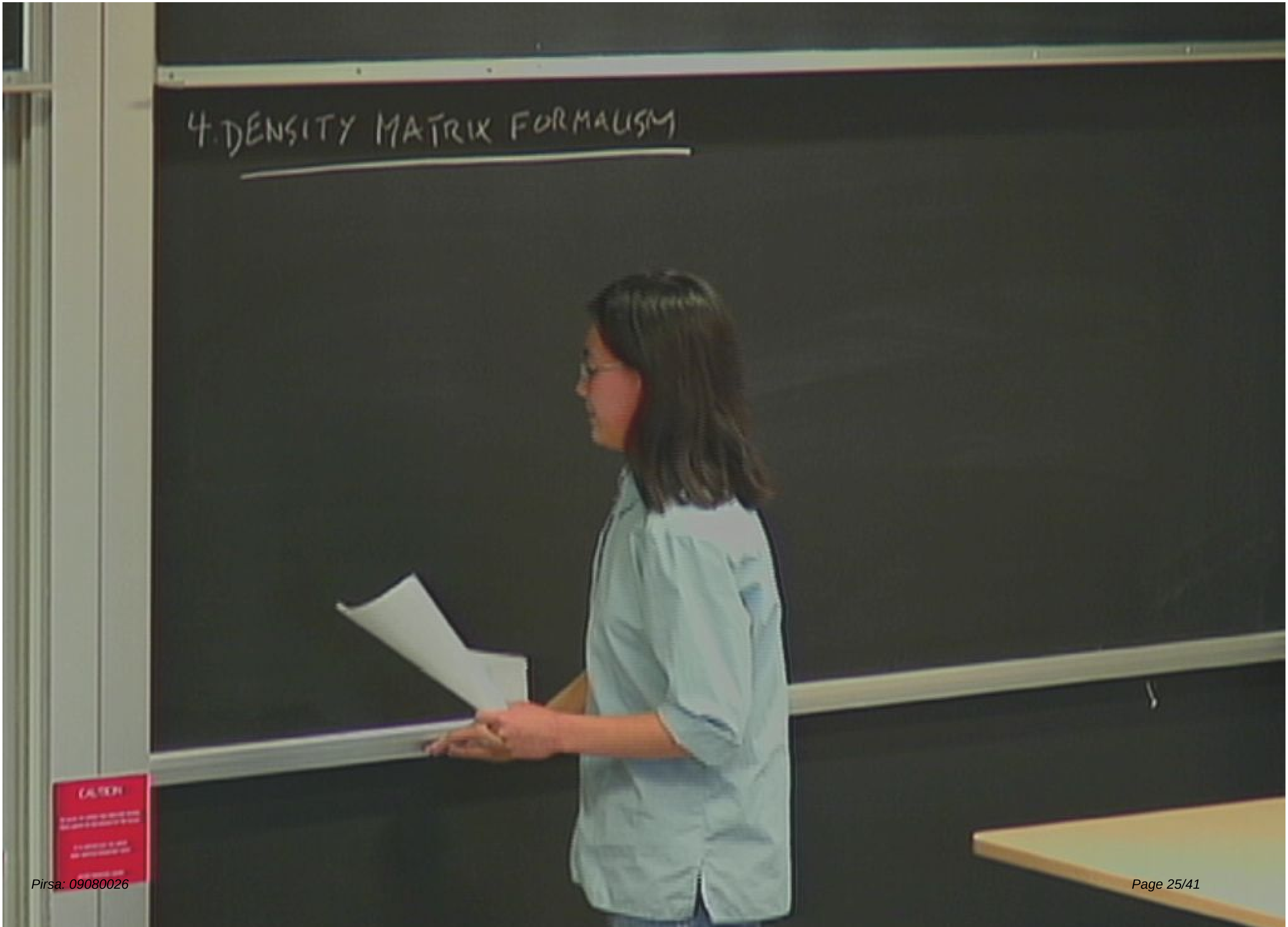
$$= \langle 0|\hat{A}|0\rangle - i\int_{-\infty}^{t} dt'\,\langle 0|[\hat{A}(t), H_{int}(t')]|0\rangle + \ldots$$

$$\hat{A} = \pi\pi\pi$$


$\langle \pi_{k_1}\pi_{k_2}\pi_{k_3} \rangle \int (2\pi)^3 \delta^{(3)}(\sum \vec{k}_i) \left(u_{k_1}u_{k_2}u_{k_3}(t)\, u_{k_1}u_{k_2}u_{k_3}(t') \right) dt'$
$\langle a a a a^\dagger a^\dagger a^\dagger \rangle \times$ some coefficient
$= (2\pi)^3 \delta^{(3)}(\sum k_i)$


⟨π_k1 π_k2 π_k3⟩ ∫(2π)^3 δ^(3)(Σ k_i) (u_k1 u_k2 u_k3(t) u*_k1 u*_k2 u*_k3(t') dt'
⟨a a a a† a† a†⟩ × some coefficient
= (2π)^3 δ^(3)(Σk_i) (some other constant) (Π_i 1/k_i)
× (∫_η0^0 η e^(iη Σk_i) dη - cc.)
1/(k1+k2+k3)
Bogoliubov
V_k(t) = α_k u_k(t) + β_k u*_k(t)
CAUTION


$\langle \pi_{k_1}\pi_{k_2}\pi_{k_3} \rangle \int (2\pi)^3 \delta^{(3)}(\sum \vec{k}_i) \left(U_{k_1} U_{k_2} U_{k_3}(t)\, U_{k_1} U_{k_2} U_{k_3}(t') \, dt' \right.$
some coefficient
$= (2\pi)^3 \delta^{(3)}(\sum k_i)$ (some other const)
$\times \left(\int_{\eta_0}^{0} \eta^3 e^{i\eta \sum k_i} d\eta \right.$
$\frac{1}{k_1+k_2+k_3}$
$V_k(t) = \alpha_k U_k(t) + \beta_k U_k^*(t)$


4. DENSITY MATRIX FORMALISM

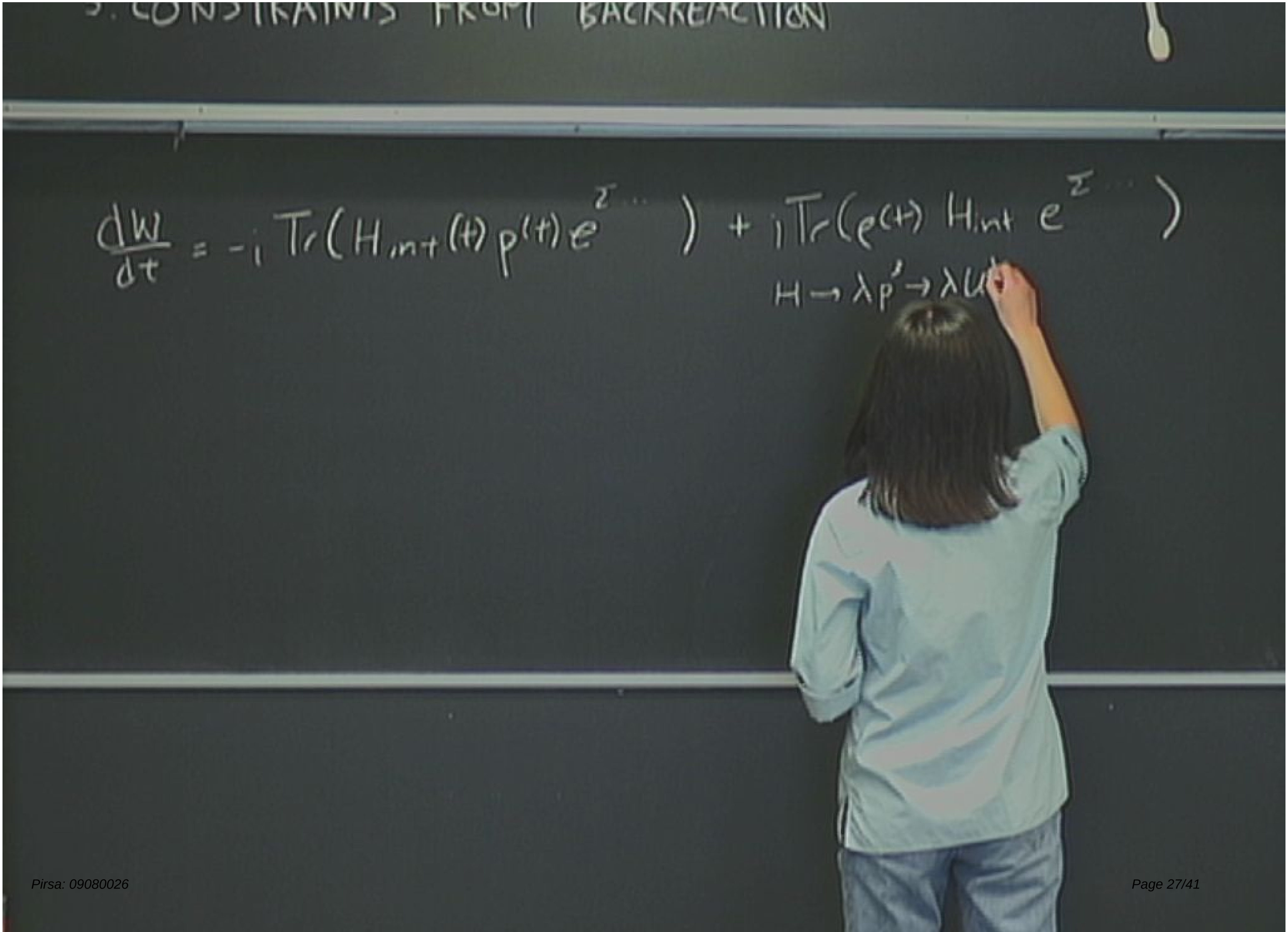
4. DENSITY MATRIX FORMALISM

$$\rho = \sum p_i |\psi_i\rangle\langle\psi_i| \qquad \langle \mathcal{O} \rangle = \mathrm{Tr}(\rho\, \mathcal{O})$$

$$\langle a_{k_1} a_{k_2} \rangle \qquad \langle a_1 a_2^\dagger a_3^\dagger \rangle$$

$$W[\{\alpha_k\},\{\alpha_k^*\}] = \mathrm{Tr}\left(\rho\, e^{\sum_k \alpha_k^* \hat{a}_k^\dagger} e^{\sum_k \alpha_k \hat{a}_k}\right)$$

$$\langle \hat{a}_{k_1} \rangle = \mathrm{Tr}(\rho\, \hat{a}_{k_1}) = \left.\frac{\delta W}{\delta \alpha_1}\right|_{\alpha=\alpha^*=0}$$


5. CONSTRAINTS FROM BACKREACTION
dW/dt = −i Tr(H_int(t) ρ(t) e^Σ…) + i Tr(ρ(t) H_int e^Σ…)
H → λρ' → λU

$$\frac{dW}{dt} = -i\,\mathrm{Tr}\left(H_{int}(t)\,\rho(t)\,e^{\Sigma \cdots}\right) + i\,\mathrm{Tr}\left(\rho(t)\, H_{int}\, e^{\Sigma}\right.$$

$$H \to \lambda p^3 \to \lambda \alpha^3 a^+$$

$$H_{int} = \iiint \frac{d^9 k}{(2\pi)^3} \lambda(k_1, k_2, k_3)\, p_{k_1} p_{k_2} p_{k_3}\, \delta(\Sigma \vec{k})$$

$$\boxed{i \frac{dW}{dt} \iiint \frac{d^9 k}{(2\pi)^3} \sum_{s_i \in \pm} \alpha^+_{k_1} \frac{\delta^2 W}{\delta \alpha_{k_2} \delta \alpha_{k_3}} + \text{cyc} + \alpha^+_{k_1} \alpha^+_{k_2} \frac{\delta W}{\delta \alpha_3} + \text{cyc} + \alpha_1^+ \alpha}$$

5. CONSTRAINTS FROM BACKREACTION

$$\frac{dW}{dt} = -i\,\mathrm{Tr}\left(H_{int}(t)\,\rho(t)\,e^{\Sigma \cdots}\right) + i\,\mathrm{Tr}\left(\rho(t)\,H_{int}\,e^{\Sigma \cdots}\right)$$

$$H \to \lambda p^3 \to \lambda a^\dagger a^\dagger \qquad 1 + \Sigma a_n a_n + \frac{\Sigma}{2}$$

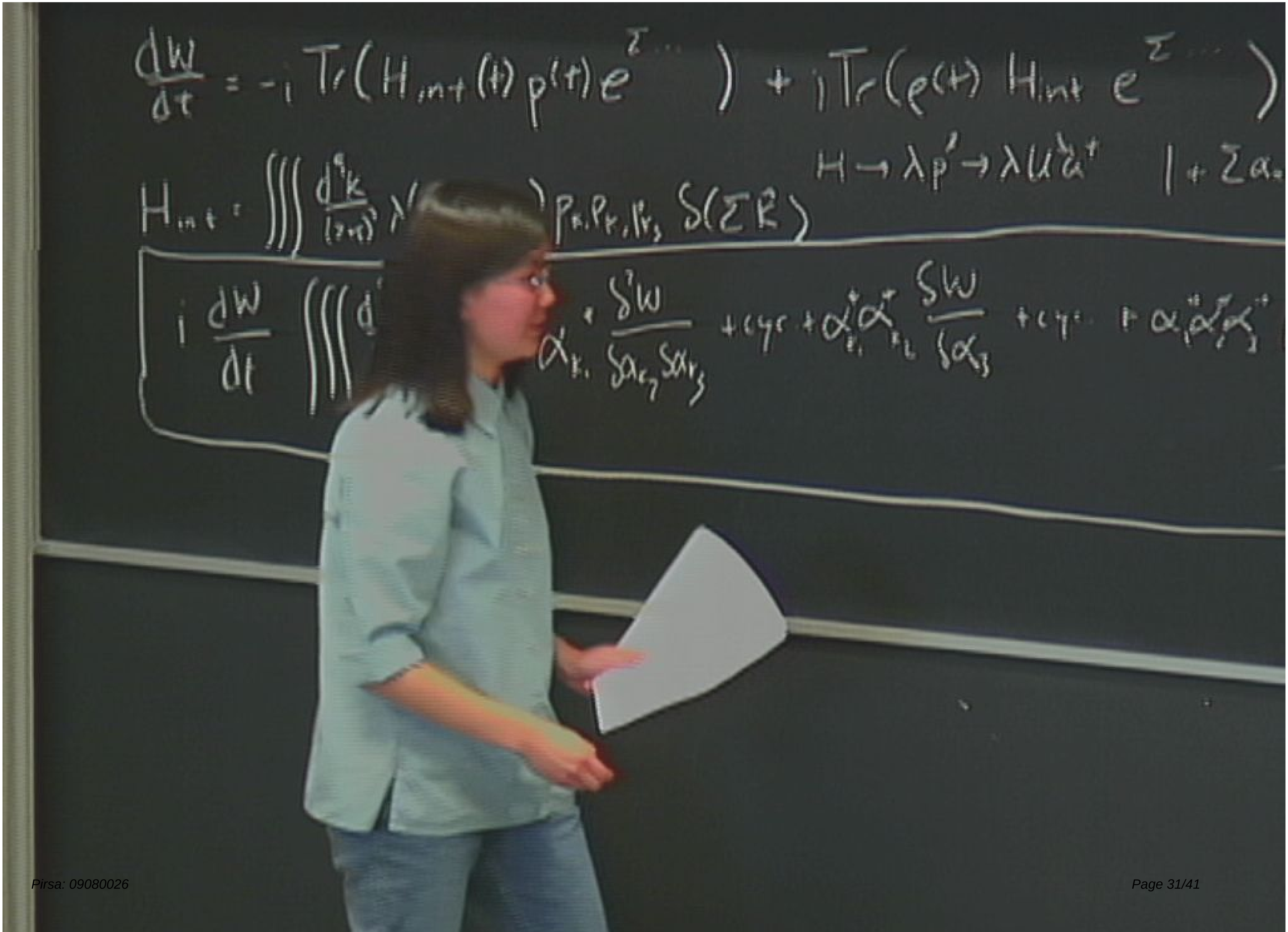
$$H_{int} = \iiint \frac{d^3k}{(2\pi)^3}\,\lambda(k_1,k_2,k_3)\,p_{k_1}p_{k_2}p_{k_3}\,\delta(\Sigma \vec{k})$$

$$i\,\frac{dW}{dt} \iiint \frac{d^3k}{(2\pi)^3} \sum_{s_i \in \pm} \alpha_{k_1}^{*}\,\frac{\delta^2 W}{\delta\alpha_{k_2}\delta\alpha_{k_3}} + \text{cyc} + \alpha_{k_1}^{*}\alpha_{k_2}^{*}\,\frac{\delta W}{\delta\alpha_3} + \text{cyc} + \alpha_1^{*}\alpha_2^{*}\alpha_3^{*}\,W\Big)\,\lambda$$

$H = p\pi r^2$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1, k_2, k_3)}_{Tr(\rho a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} \alpha^*_1 \alpha^*_2 \alpha^*_3 + \underbrace{C(k_1, k_2, k_3)}_{Tr(\rho a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} \alpha^*_{k_1} \alpha^*_{k_2} \alpha_{k_3} + \text{cyc.} + cc$$



dW/dt = -i Tr(H_int(t) ρ(t) e^Σ...) + i Tr(ρ(t) H_int e^Σ...)
H → λρ' → λUα†
H_int = ∭ d³k/(2π)³ λ( ) ρ_k1 ρ_k2 ρ_k3 δ(Σ k)
i dW/dt ∭
δ²W/δα_k2 δα_k3 + cyc + α_k1† α_k2† δW/δα_3 + cyc + α_1† α_2† α_3†

$H = p\pi - L$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1,k_2,k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3} + \underbrace{C(k_1,k_2,k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3} + \mathrm{cyc.} + \mathrm{cc}$$

3. NON GAUSSIANITIES AS A PROBE OF INITIAL STATE
4. DENSITY MATRICES
5. CONSTRAINTS FROM BACKREACTION

$$\frac{dW}{dt} = -i\,\mathrm{Tr}\left(H_{int}(t)\,\rho(t)\,e^{\Sigma \cdots}\right) + i\,\mathrm{Tr}\left(\rho(t)\,H_{int}\,e^{\Sigma \cdots}\right)$$

$$H_{int} = \iiint \frac{d^3k}{(2\pi)^3}\,\lambda(k_1,k_2,k_3)\,p_{k_1}p_{k_2}p_{k_3}\,\delta(\Sigma k_i)$$

$$H \to \lambda p^3 \to \lambda a^\dagger a^\dagger a^\dagger \qquad | + \Sigma \alpha_i a_i + \frac{\alpha^2}{2!}$$

$$\boxed{i\frac{dW}{dt} \iiint \frac{d^3k}{(2\pi)^3} \sum_{S_3 P} \alpha^*_{k_1} \frac{\delta W}{\delta\alpha_{k_2}\delta\alpha_{k_3}} + \text{cyc} + \alpha^*_{k_1}\alpha^*_{k_2}\frac{\delta W}{\delta\alpha_{k_3}} + \text{cyc} + \alpha^*\alpha^*\alpha^* W)\,\lambda\,\delta}$$

$$H = p\dot{\pi} - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1,k_2,k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} \alpha^\dagger_1 \alpha^\dagger_2 \alpha^\dagger_3 + \underbrace{C(k_1,k_2,k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} \alpha^\dagger_{k_1} \alpha^\dagger_{k_2} \alpha_{k_3} + \text{cyc.} + cc$$

λ

maximize $\langle \pi\pi\pi \rangle \big|_{late}$

$$H = p\pi - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1, k_2, k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} \alpha^\dagger_{1} \alpha^\dagger_{2} \alpha^\dagger_{3} + \underbrace{C(k_1, k_2, k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} \alpha^\dagger_{k_1} \alpha^\dagger_{k_2} \alpha_{k_3} + \text{cyc.} + cc$$

maximize $\langle \pi\pi\pi \rangle\big|_{\text{late}}$

$H = p\pi - L$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1, k_2, k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} \hat{a}_{1} \hat{a}_{2} \hat{a}_{3} + \underbrace{C(k_1, k_2, k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} \hat{a}_{k_1} \hat{a}_{k_2} \hat{a}_{k_3} + \text{cyc.} + cc$$

$\lambda \downarrow$

maximize $\langle \pi\pi\pi \rangle \big|_{\text{late}}$

$H = p\pi - L$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1,k_2,k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} a^\dagger_1 a^\dagger_2 a^\dagger_3 + \underbrace{C(k_1,k_2,k_3)}_{\mathrm{Tr}(\rho\, a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3} + \text{cyc.} + \text{cc}$$

$\lambda \downarrow$

maximize $\langle \pi\pi\pi \rangle |_{\text{late}}$

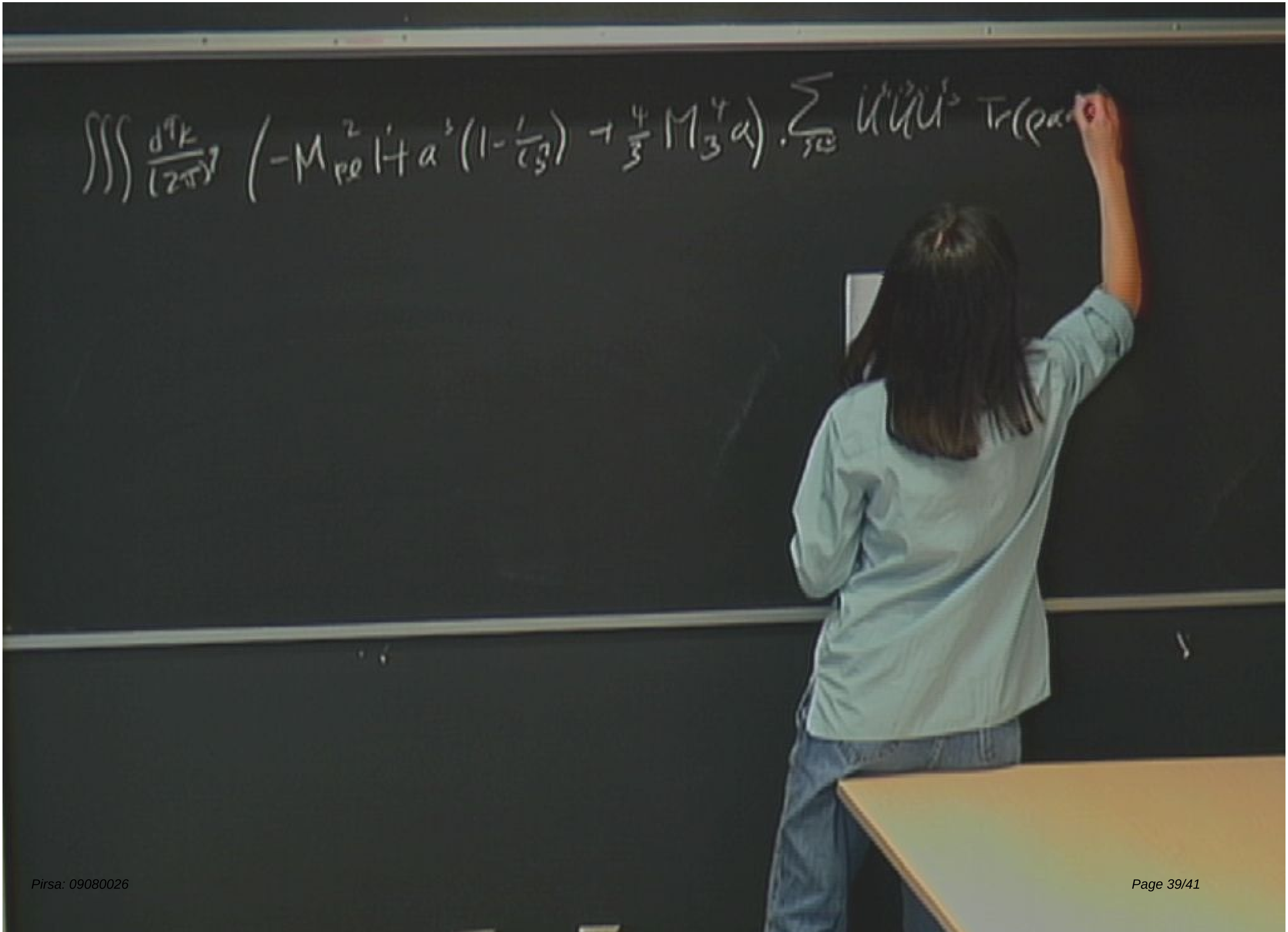
$H = p\pi - \mathcal{L}$

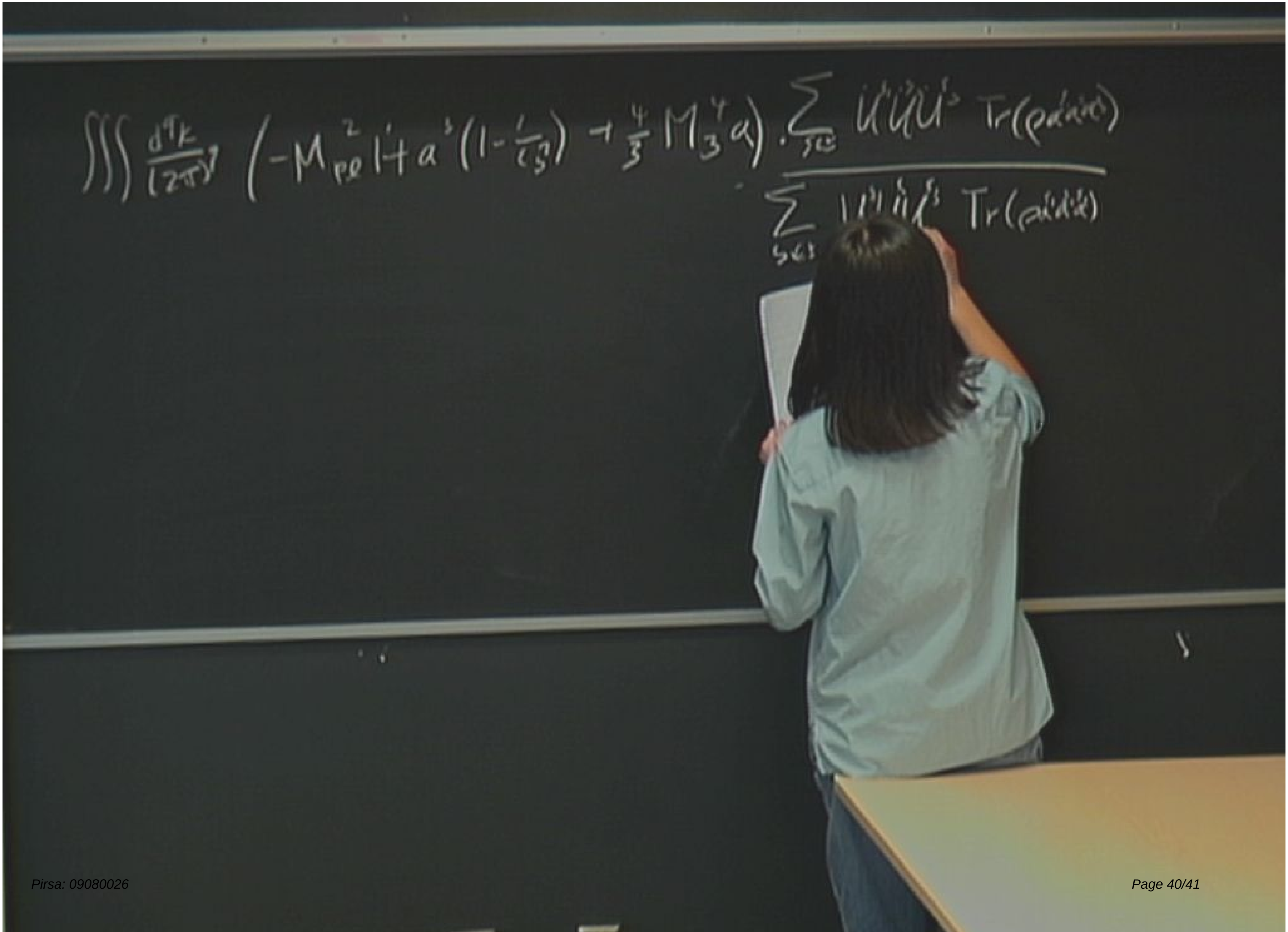
5. CONSTRAINTS FROM BACKREACTION

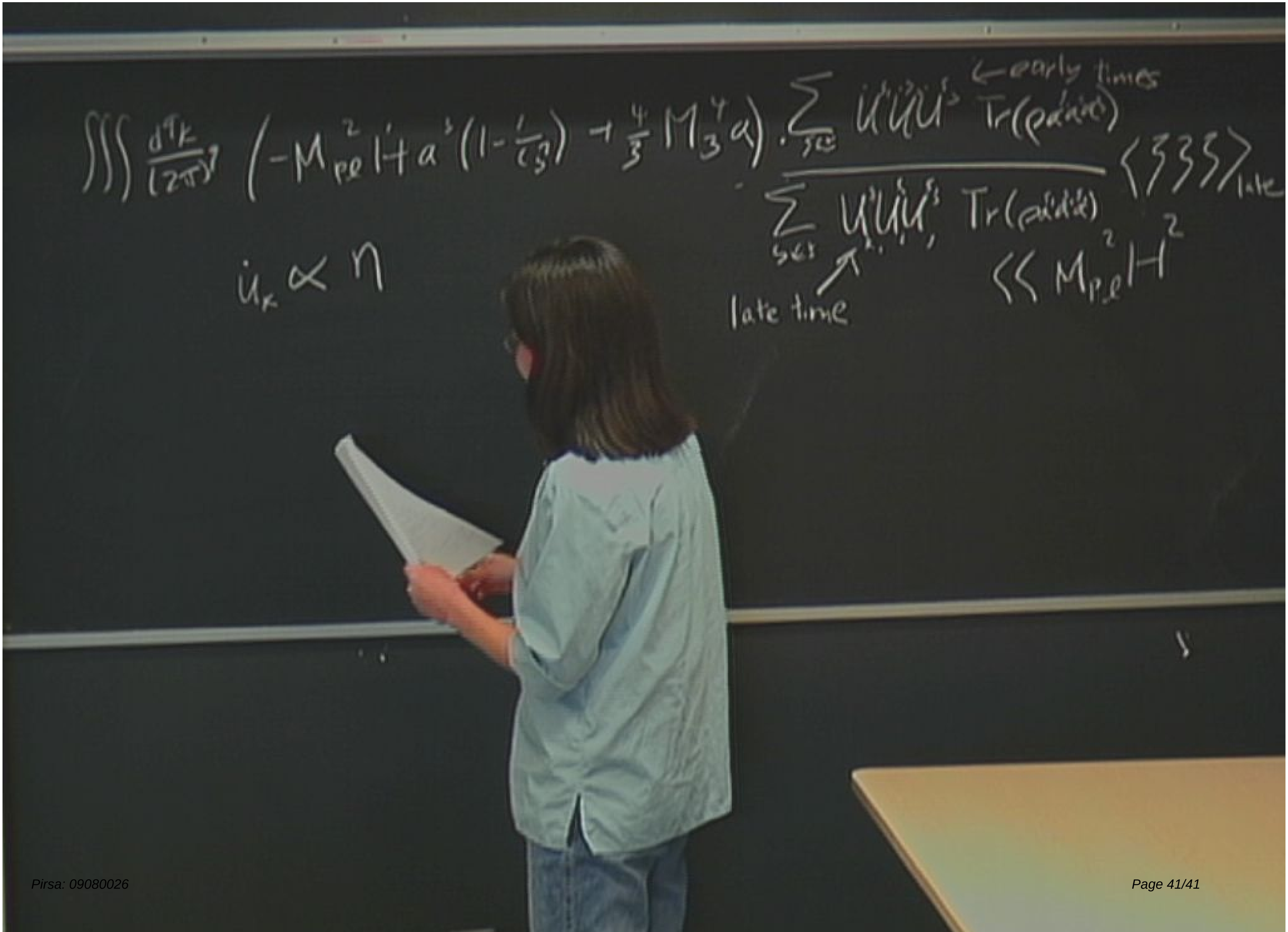
$$W = 1 + \sum_{k_1 k_2 k_3} \underbrace{B(k_1, k_2, k_3)}_{\mathrm{Tr}(\rho a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3})} \hat{\alpha}_1 \hat{\alpha}_2 \hat{\alpha}_3 + \underbrace{C(k_1 k_2 k_3)}_{\mathrm{Tr}(\rho a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3})} \alpha^\dagger_{k_1} \alpha^\dagger_{k_2} \alpha_{k_3} + \text{cyc.} + \text{cc}$$

$\lambda \downarrow$

maximize $\langle \pi\pi\pi \rangle|_{\text{late}}$


∭ d⁴k/(2π)³ (−M_pl² H' + a³(1 − 1/c_s²) + 4/3 M_3⁴ a) · Σ u u u Tr(ρ




early times
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