

Title: Undergrad Research Project Talk

Date: Aug 06, 2009 02:00 PM

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Abstract:

TRAINING NON-GAUSSIANITY FOR AN ARBITRARY INITIAL S

CONSTRAINING NON-GAUSSIANITY FOR AN ARBITRARY

$$\langle T^{00} \rangle \ll M_{\text{Pl}}^2 H^2.$$

CONSTRAINING NON-GAUSSIANITY FOR AN ARBITRARY

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OUTLINE

1. INFLATION
2. π LAGRANGIAN
3. NON GAUSSIANITIES AS

$$\langle T^{00} \rangle \ll M_{\text{Pl}}^2 H^2$$

OUTLINE

I. INFLATION

TI LAGRANGIAN

NON GAUSSIANITIES AS A PROBE OF INITIAL STATE

DENSITY MATRICES

$$\langle T^{00} \rangle \ll M_{\text{pl}}^2 H^2$$

OUTLINE

1. INFLATION

2. π LAGRANGIAN

3. GAUSSIANITIES AS A PROBE OF INITIAL STATE

COVARIANCE MATRICES

CONSTRAINTS FROM BACKREACTION

I. INFLATION

1. INFLATION

$$\text{FRW } ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + d\Omega^2 \right]$$

$$\text{FRIEDMANN } \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2}$$

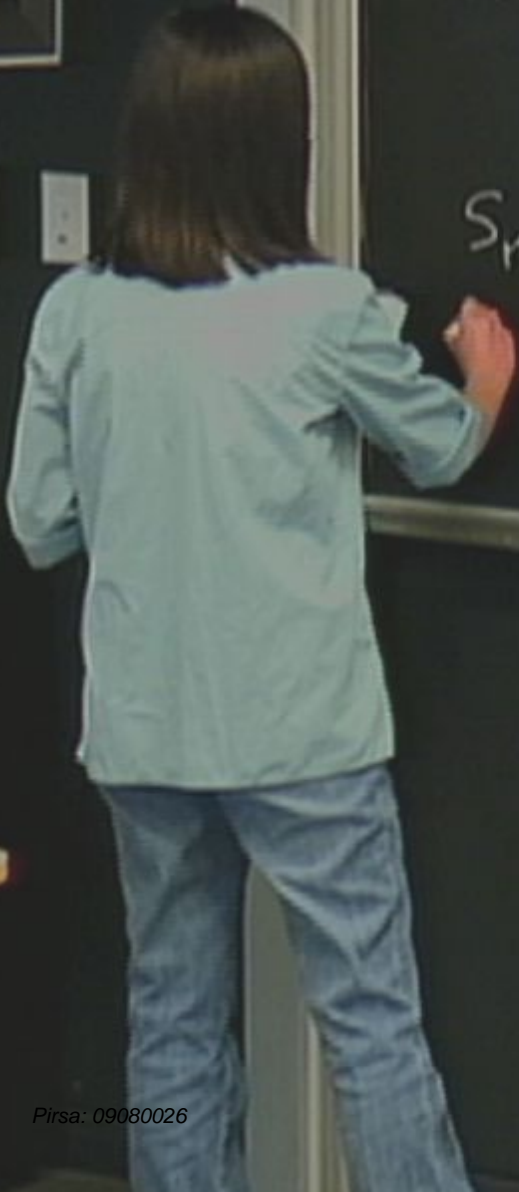
1. INTRODUCTION

FRW $ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + d\Omega^2 \right]$

FRIEDMANN $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2}$ $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$a = e^{Ht}$ $H = \text{constant}$ $\rho = \text{constant}$ $\nabla_{\mu} T^{\mu\nu} = 0$ $\frac{p}{\rho} < -\frac{1}{3}$

$S_H = \int d^4x \sqrt{|g|} \left(\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - V(\varphi) \right)$



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$\nabla_\mu T_{\nu}^{\mu} = 0$

$\frac{p}{\rho} < -\frac{1}{3}$

$\rho = \text{constant}$

$p = -\rho$

$S_M = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right)$

$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} \left(\frac{1}{2} \partial^\sigma \varphi \partial_\sigma \varphi + V(\varphi) \right)$

$\rho = \frac{\dot{\varphi}^2}{2} + V(\varphi)$

$p = \frac{\dot{\varphi}^2}{2} - V(\varphi)$

1. INFLATION

FRW $ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + d\Omega^2 \right]$

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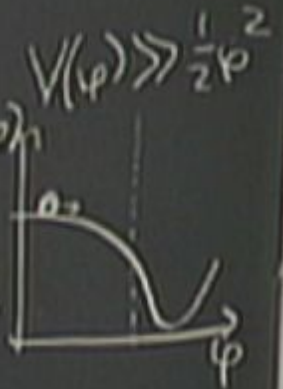
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FRIEDMANN $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$ $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$H^{-1} = H_{\text{Hubble radius}}$ $a = e^{Ht}$ $H = \text{constant}$ $\nabla_{\mu} T_{\nu}^{\mu} = 0$

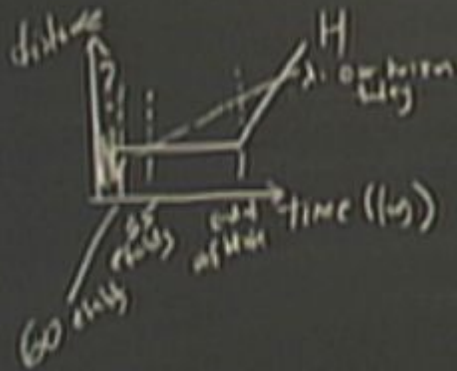
$\frac{p}{\rho} < -\frac{1}{3}$

$\rho = \text{constant}$ $\rho = -p$

$S_M = \int d^4x \sqrt{|g|} \left(\frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi - V(\varphi) \right)$

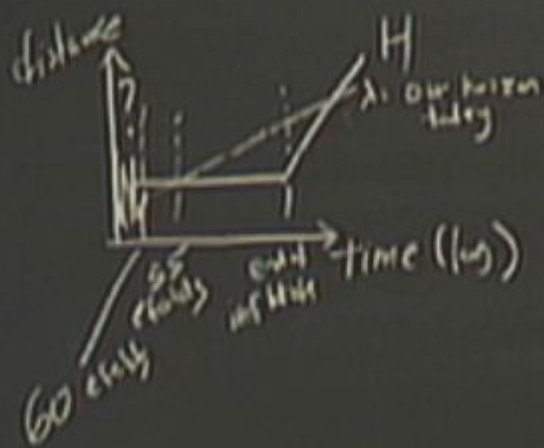
$T_{\mu\nu} = \partial_{\mu} \varphi \partial_{\nu} \varphi - g_{\mu\nu} \left(\frac{1}{2} \partial^{\sigma} \varphi \partial_{\sigma} \varphi + V(\varphi) \right)$

$\rho = \frac{1}{2} \dot{\varphi}^2 + V(\varphi)$ $p = \frac{1}{2} \dot{\varphi}^2 - V(\varphi)$



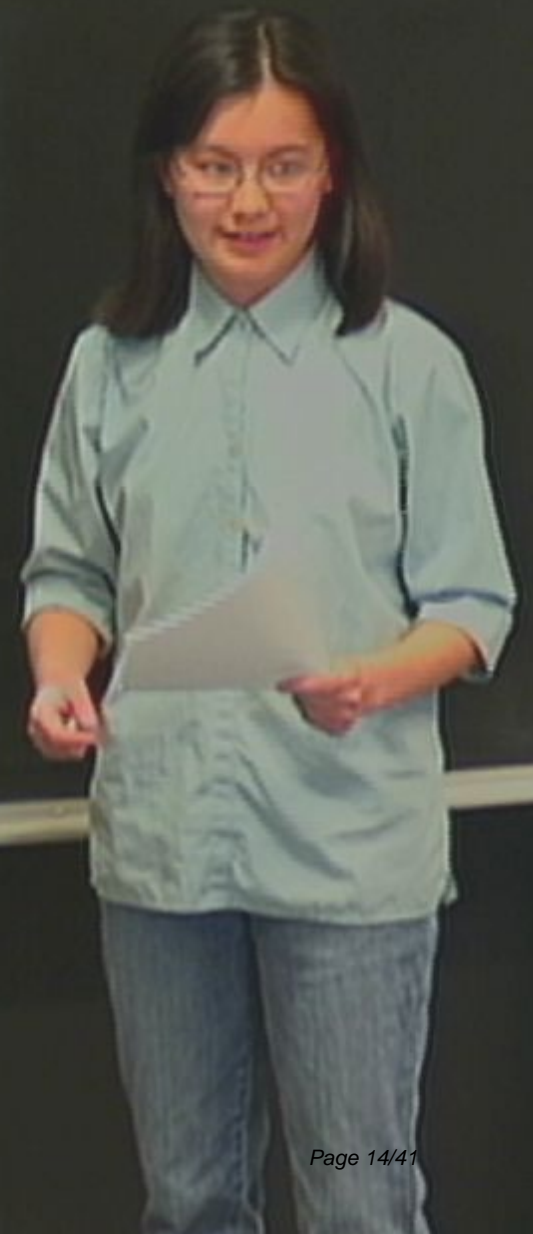
CAUTION
DO NOT TOUCH THE BOARD
OR THE SURROUNDING AREA

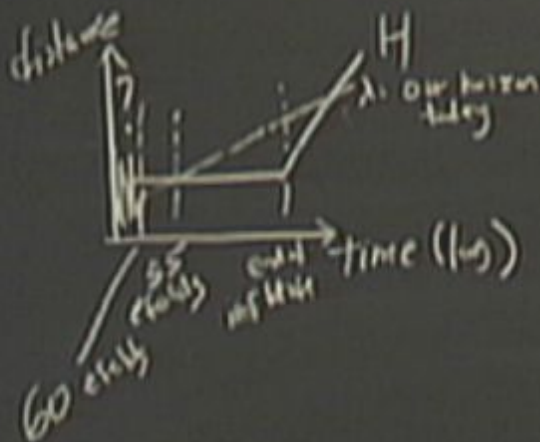
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2. LAGRANGE

arXiv:0709.0923





2. PII LAGRANGIAN

$$t \rightarrow t + \delta(t, \vec{x})$$

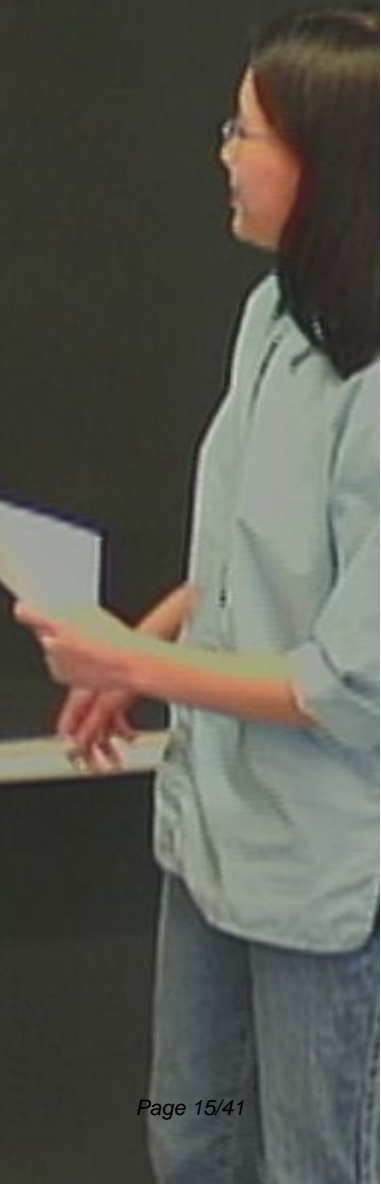
$$\dot{\pi} \quad \partial_t \pi$$

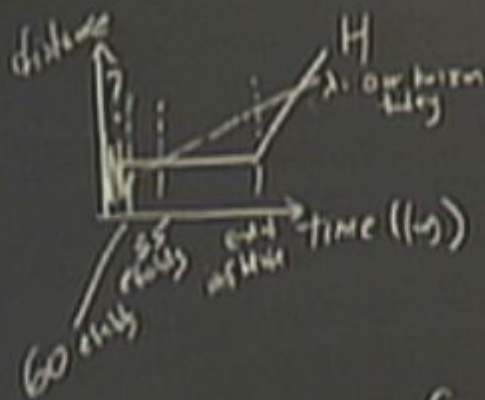
$$f(t) \quad g^{00} \quad K_M^u$$

arXiv:0709.0923

$$\pi \rightarrow \pi - \delta(t, \vec{x})$$

$$f(\pi + t)$$





2. II LAGRANGIAN

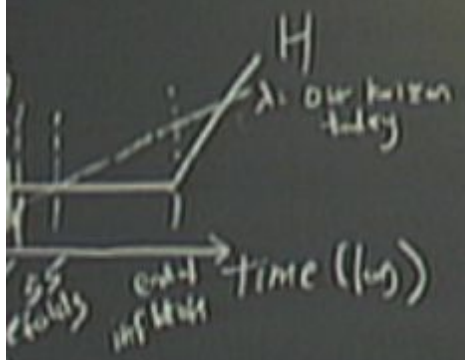
arXiv:0709.0923

$$t \rightarrow t + \xi(t, \vec{x})$$

$$\pi \rightarrow \pi - \xi(t, \vec{x})$$

$$f(\pi + t)$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{Pl}^2 \dot{H} \left(\frac{\dot{\pi}^2}{a^2} - \frac{(\partial_{\mu\mu})^2}{a^4} \right) + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^2 - \dot{\pi} \frac{(\partial_{\mu\mu})^2}{a^4} \right) - \frac{4}{3} M_3^4 \dot{\pi}^2 \right]$$



2. π LAGRANGIAN

arXiv: 0709.0923

$$t \rightarrow t + \zeta(t, \vec{x})$$

$$\pi \rightarrow \pi - \zeta(t, \vec{x})$$

$$\dot{\pi} \quad \partial_i \pi$$

$$f(\pi + t)$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2M_2^4 \left(\dot{\pi}^2 + \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \pi^3 \right]$$

$$\pi = \int \frac{d^3k}{(2\pi)^3} \left(U_k^* \hat{a}_k + U_k \hat{a}_k \right)$$

$$\zeta = -H\pi$$

$$U_k = \frac{1}{2\sqrt{\epsilon} k^3 c_s} \text{Hein}(1 + ik\eta c_s)$$

H = MLE

3. NON-GAUSSIANITY

$$\langle \pi(x)\pi(y) \rangle$$

3. NON-GAUSSIANITY

$$\langle \pi(x) \pi(y) \rangle$$

$\langle \text{arr} \rangle$

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$



\leftrightarrow horizon crossing
" after
 \leftrightarrow near beginning

3. NON-GAUSSIANITY

$$\langle \phi | \pi(x) \pi(y) | \phi \rangle$$

(a,b)

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$



↔ horizontal crossing

" " after

↔

near beginning

in. in formalism

$$H = H_0 + H_1$$

$$|\psi(t)\rangle = \exp\left(-i \int_0^t H_1(t') dt'\right) |\psi(0)\rangle$$

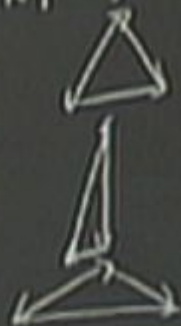


3. NON-GAUSSIANITY

$$\langle 0 | \pi(x) \pi(y) | 0 \rangle$$

(at)

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$



↔ horizon crossing

" " after

↔

↔ near horizon

in-in formalism

$$H = H_0 + H_1$$

$$|\psi(t)\rangle = \exp\left(-i \int_t^t H_1(t') dt'\right) |\psi(0)\rangle$$

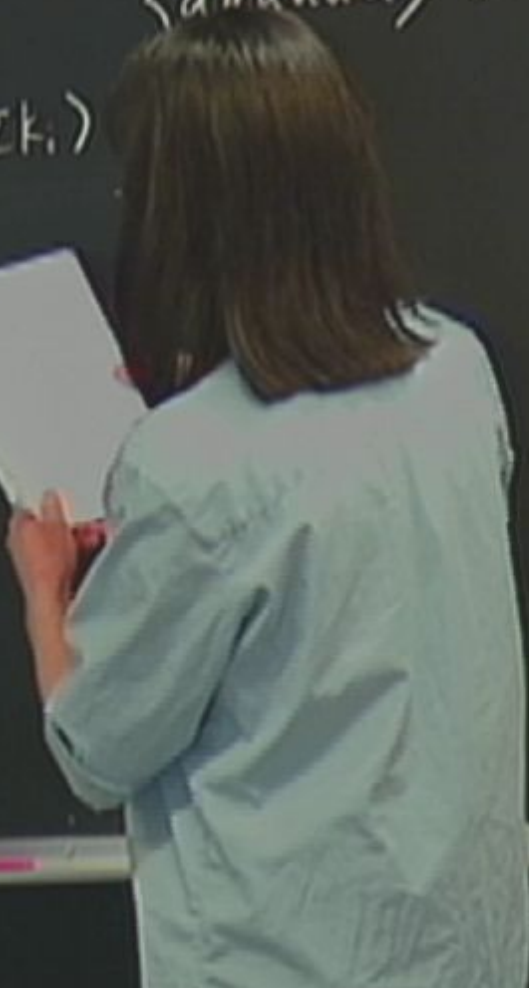
$$\begin{aligned} \langle \psi(t) | \hat{A}(t) | \psi(t) \rangle &= \langle 0 | \bar{T} \exp\left(i \int_0^t H_1(t') dt'\right) \hat{A}(t) T \exp\left(-i \int_0^t H_1(t') dt'\right) | 0 \rangle \\ &= \langle 0 | \hat{A}(0) | 0 \rangle - i \int_0^t dt' \langle 0 | [\hat{A}(t), H_1(t')] | 0 \rangle + \dots \end{aligned}$$

$\hat{A} = \pi \pi \pi$

$$\langle \pi_{k_1} \pi_{k_2} \pi_{k_3} \rangle \int (2\pi)^3 \delta^{(3)}(\sum k_i) (U_{k_1} U_{k_2} U_{k_3}(t) U_{k_4} U_{k_5} U_{k_6}(t')) dt'$$

$\langle a a u a' a' \rangle \times \text{some coefficient}$

$$= (2\pi)^3 \delta^{(3)}(\sum k_i)$$



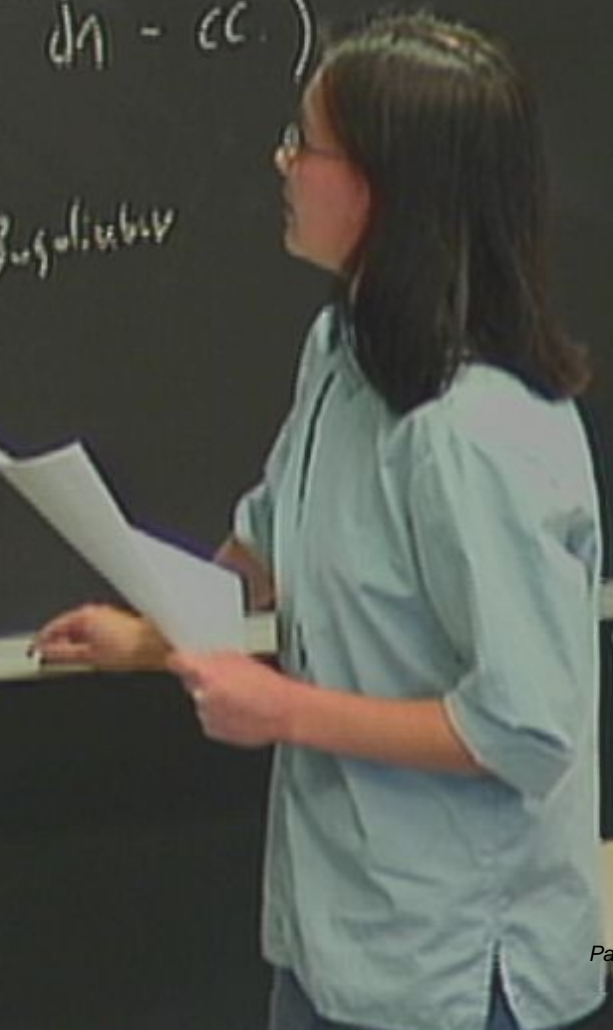
$$\langle \pi_{\xi} \pi_{\eta} \pi_{\zeta} \rangle \left\{ (2\pi)^3 \delta^{(3)}(\vec{\Sigma} K_i) (U_{\xi}, U_{\eta}, U_{\zeta}(t) U_{\xi}, U_{\eta}, U_{\zeta}(t')) dt' \right.$$

$\langle a a a a \rangle \times$ some coefficient

$$= (2\pi)^3 \delta^{(3)}(\vec{\Sigma} K_i) (\text{some other constant}) \left(\pi_i \frac{1}{K_i} \right) \times \left(\int_{\eta_2}^0 \eta e^{3 \eta \zeta \Sigma K_i} d\eta - cc. \right)$$

$$\frac{1}{K_i \cdot K_j \cdot K_k}$$

$$V_K(t) = \alpha_K U_K(t) + \beta_K U_K^*(t) \quad \text{Bogoliubov}$$



$$\langle \pi_{k_1} \pi_{k_2} \pi_{k_3} \rangle \left\{ (2\pi)^3 \delta^{(3)}(\sum k_i) (U_{k_1} U_{k_2} U_{k_3}(t) U_{k_1} U_{k_2} U_{k_3}(t)) dt' \right.$$

< a quadratic in some coefficient

$$= (2\pi)^3 \delta^{(3)}(\sum k_i) \left(\text{some other constant} \right) \left(\pi_{k_1} \pi_{k_2} \pi_{k_3} \right)$$

$$\times \left(\int_{n_0}^0 n e^{i n \tau} d\tau \right)$$

$$\frac{1}{k_1 \cdot k_2 \cdot k_3}$$

$$V_k(t) = \alpha_k U_k(t) + \beta_k U_k^*(t)$$

Bus



4. DENSITY MATRIX FORMALISM



4. DENSITY MATRIX FORMALISM

$$\rho = \sum p_i |\psi_i\rangle \langle \psi_i| \quad \langle O \rangle = \text{Tr}(\rho O)$$

$$\langle a_r, a_r \rangle \quad \langle a_r^\dagger a_r^\dagger \rangle$$

$$W[\{\alpha_k\}, \{\alpha_k^*\}] = \text{Tr} \left(\rho e^{\sum_k \alpha_k^\dagger \hat{a}_k^\dagger} e^{\sum_k \alpha_k \hat{a}_k} \right)$$

$$\langle \hat{a}_{k_1} \rangle = \text{Tr}(\rho \hat{a}_{k_1}) = \left. \frac{\delta W}{\delta \alpha_{k_1}} \right|_{\alpha = \alpha^* = 0}$$

5. CONSTRAINTS FROM BACKREACTION

$$\frac{dW}{dt} = -i \text{Tr}(H_{\text{int}}(t) \rho(t) e^{\Sigma}) + i \text{Tr}(e^{\Sigma} H_{\text{int}} e^{-\Sigma})$$

$H \rightarrow \lambda \rho' \rightarrow \lambda \mathcal{U}$

$$\frac{dW}{dt} = -i \text{Tr}(H_{int}(t) \rho(t) e^{\Sigma}) + i \text{Tr}(e^{\Sigma} H_{int} e^{\Sigma})$$

$$H_{int} = \iiint \frac{d^3k}{(2\pi)^3} \lambda(k_1, k_2, k_3) \rho_{k_1} \rho_{k_2} \rho_{k_3} \delta(\Sigma k) \quad H \rightarrow \lambda p^3 \rightarrow \lambda u^3 a^\dagger$$

$$i \frac{dW}{dt} \iiint \frac{d^3k}{(2\pi)^3} \sum_{S_i \in \pm} \alpha_{k_1}^+ \frac{\delta W}{\delta \alpha_{k_1}^+ \delta \alpha_{k_2}^+ \delta \alpha_{k_3}^+} + \text{cyc} + \alpha_{k_1}^+ \alpha_{k_2}^+ \frac{\delta W}{\delta \alpha_{k_3}^+} + \text{cyc} + \alpha_{k_1}^+ \alpha_{k_3}^+ \frac{\delta W}{\delta \alpha_{k_2}^+} + \text{cyc}$$

5. CONSTRAINTS FROM BACKREACTION

$$\frac{dW}{dt} = -i \text{Tr}(H_{int}(t) \rho(t) e^{\Sigma \dots}) + i \text{Tr}(e^{ct}) H_{int} e^{\Sigma \dots}$$

$H \rightarrow \lambda p^2 \rightarrow \lambda u \dot{u}^+ \quad | + \sum \alpha_n a_n + \frac{\sigma}{2}$

$$H_{int} = \iiint \frac{d^3k}{(2\pi)^3} \lambda(k_1, k_2, k_3) p_{k_1} p_{k_2} p_{k_3} \delta(\Sigma R)$$

$$i \frac{dW}{dt} \iiint \frac{d^3k}{(2\pi)^3} \sum_{\Sigma_i \in \dots} \alpha_{k_i} \left(\frac{\delta W}{\delta \alpha_{k_1} \delta \alpha_{k_2} \delta \alpha_{k_3}} + c_{\gamma} + \alpha_{k_1}^+ \alpha_{k_2}^+ \frac{\delta W}{\delta \alpha_{k_3}} + c_{\gamma} + \alpha_{k_1}^+ \alpha_{k_3}^+ W \right) \lambda$$

$$H = p\dot{r} - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K, K_1, K_2, K_3} \underbrace{B(K, K_1, K_2, K_3)}_{\text{Tr}(\rho \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger)} \alpha_1^\dagger \alpha_2^\dagger \alpha_3^\dagger + \underbrace{C(K, K_1, K_2, K_3)}_{\text{Tr}(\rho \hat{a}_1 \hat{a}_2 \hat{a}_3)} \alpha_1 \alpha_2 \alpha_3 + \text{cyc.} + \alpha$$



$$\frac{dW}{dt} = -i \text{Tr}(H_{int}(t) \rho(t) e^{\Sigma \dots}) + i \text{Tr}(e^{\Sigma \dots} H_{int} e^{\Sigma \dots})$$

$$H_{int} = \iiint \frac{d^3k}{(2\pi)^3} \lambda^{(k)} \rho_k, p_k, p_{k_3} S(\Sigma R) \quad H \rightarrow \lambda p' \rightarrow \lambda u \alpha^+ \quad | + \Sigma \alpha_+$$

$$i \frac{dW}{dt} \iiint d^3k \left[\frac{\delta W}{\delta \alpha_k} + c \gamma_c + \alpha_i^+ \alpha_{i_2}^+ \frac{\delta W}{\delta \alpha_3} + c \gamma_c + \alpha_i^+ \alpha_{i_2}^+ \right]$$



$$H = p\dot{r} - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K, K_1, K_2} \underbrace{B(K, K_1, K_2)}_{\text{Tr}(\rho \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2)} \alpha_1^\dagger \alpha_1 \alpha_2^\dagger \alpha_2 + \underbrace{C(K, K_1, K_2)}_{\text{Tr}(\rho \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2)} \alpha_1^\dagger \alpha_1 \alpha_2^\dagger \alpha_2 + c.c. + \dots$$



3. NON GAUSSIANTIES AS A PRODUCT OF INITIAL STATES
4. DENSITY MATRICES
5. CONSTRAINTS FROM BACKREACTION

$$\frac{dW}{dt} = -i \text{Tr}(H_{int}(t) \rho(t) e^{\mathcal{Z}}) + i \text{Tr}(\rho(t) H_{int} e^{\mathcal{Z}})$$

$$H_{int} = \iiint \frac{d^3k}{(2\pi)^3} \lambda(k_1, k_2, k_3) p_1 p_2 p_3 S(\Sigma R)$$

$H \rightarrow \lambda p' \rightarrow \lambda u \dot{u}^{\dagger} \quad | + \sum a_{\alpha} \alpha \cdot \frac{\sigma}{2}$

$$i \frac{dW}{dt} \left(\iiint \frac{d^3k}{(2\pi)^3} \sum_{\Sigma R} \alpha_1 \cdot \frac{\delta W}{\delta \alpha_1 \delta \alpha_2 \delta \alpha_3} + i \gamma + i \alpha_1^{\dagger} \alpha_2^{\dagger} \alpha_3^{\dagger} \frac{\delta W}{\delta \alpha_1} + i \gamma + i \alpha_1^{\dagger} \alpha_2^{\dagger} \alpha_3^{\dagger} W \right) \lambda S$$

EACH
STUDENT
SHOULD
HAVE
ONE

$$H = p\pi - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K, K_2, K_3} B(K, K_2, K_3) \alpha_1^+ \alpha_2^+ \alpha_3^- + \underbrace{C(K, K_2, K_3)}_{\text{Tr}(\rho \hat{a}_1^+ \hat{a}_2^+ \hat{a}_3^-)} \alpha_1^+ \alpha_2^+ \alpha_3^- + \text{cyc.} + \text{cc}$$

\downarrow
 \downarrow

maximize $\langle \pi \pi \pi \rangle |_{\text{late}}$



$$H = p\pi + L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K, K_2, K_3} B(K, K_2, K_3) \alpha_1^+ \alpha_2^+ \alpha_3^- + \underbrace{C(K, K_2, K_3)}_{\text{Tr}(\rho \hat{a}_1^+ \hat{a}_2^+ \hat{a}_3^-)} \alpha_1^+ \alpha_2^+ \alpha_3^- + c_4 r + c_5$$

\downarrow
 \downarrow

maximize $\langle \pi \pi \pi \rangle |_{\text{late}}$

CAUTION

INFORMATION ON THE USE OF
 THIS BOARD IS AVAILABLE AT
 THE BOTTOM OF THE BOARD.
 FOR MORE INFORMATION
 CONTACT THE BOARD MANAGER.

$$H = p\pi - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K, K_2, K_3} B(K, K_2, K_3) \alpha_1^+ \alpha_2^+ \alpha_3^- + \underbrace{C(K, K_2, K_3)}_{\text{Tr}(\rho \alpha_1^+ \alpha_2^+ \alpha_3^-)} \alpha_1^+ \alpha_2^+ \alpha_3^- + c_4 r. + c_5$$

$\downarrow \downarrow$ $\text{Tr}(\rho \alpha_1^+ \alpha_2^+ \alpha_3^-)$

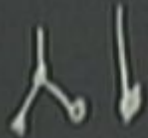
$$\text{maximize } \langle \pi \pi \pi \rangle |_{\text{late}}$$



$$H = p\pi - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K, K_1, K_2} B(K, K_1, K_2) \alpha_1^+ \alpha_1^- + \underbrace{C(K, K_1, K_2)}_{\text{Tr}(\rho \hat{a}_1^+ \hat{a}_1^-)} \alpha_{K_1}^+ \alpha_{K_2}^- + c_4 r + c$$



maximize $\langle \pi \pi \pi \rangle |_{\text{late}}$



$$H = p\pi - L$$

5. CONSTRAINTS FROM BACKREACTION

$$W = 1 + \sum_{K_1, K_2, K_3} B(K_1, K_2, K_3) \alpha_1^+ \alpha_2^+ \alpha_3^- + \underbrace{C(K_1, K_2, K_3)}_{\text{Tr}(\rho \alpha_1^+ \alpha_2^+ \alpha_3^-)} \alpha_1^+ \alpha_2^+ \alpha_3^- + c_4 r. + c$$

\downarrow
 \downarrow

$\text{Tr}(\rho \alpha_1^+ \alpha_2^+ \alpha_3^-)$

maximize $\langle \pi \pi \pi \rangle |_{\text{late}}$

CAUTION

$$\iiint \frac{d^3k}{(2\pi)^3} \left(-M_{\text{pl}}^2 \left[1 + a^3 \left(1 - \frac{1}{c_s^2} \right) + \frac{4}{3} M_{\text{pl}}^4 a \right] \cdot \sum_{\text{pol}} \hat{k}^i \hat{k}^j \text{Tr}(\rho_{\text{pol}}) \right)$$

$$\iiint \frac{d^3k}{(2\pi)^3} \left(-M_{\text{pl}}^2 \left[1 + a^3 \left(1 - \frac{1}{c_s^2} \right) + \frac{4}{3} M_{\text{pl}}^4 a \right] \cdot \frac{\sum_{\text{sets}} \mathbf{k}^i \mathbf{k}^j \mathbf{k}^k \text{Tr}(\rho_{\text{dust}})}{\sum_{\text{sets}} |\mathbf{k}|^3 \text{Tr}(\rho_{\text{dust}})} \right)$$

$$\int \frac{d^4 k}{(2\pi)^4} \left(-M_{pl}^2 + a^3 \left(1 - \frac{1}{c_3} \right) + \frac{4}{3} M_3^4 a \right) \frac{\sum_{\text{set}} u^i u^j \text{Tr}(\rho_{ij})}{\sum_{\text{set}} u^i u^j \text{Tr}(\rho_{ij})} \langle \dots \rangle_{\text{late}}$$

← early times

late time

$\ll M_{pl}^2 H^2$

$$u_k \propto \eta$$

