Title: How to Know the Universe from a Hole in the Ground

Date: Aug 04, 2009 09:00 AM

URL: http://pirsa.org/09080023

Abstract: We understand the history of our universe very well but remain ignorant on one key question: what is most of the universe actually made of? Beautiful measurements, by satellites, balloon-basted observatories, the Hubble telescope and ground-based telescopes have allowed us to accurately trace this history of the history of the ordinary matter we are made of. Yet these measurements also show us that most of the universe is dark - that is to say it cannot be seen visibly no matter how bright a light is shone on it. I will discuss why we think that 95% of the universe is dark and will show how we are trying to directly observe dark matter. I will explain what it is like to do science underground and why we need to be so deep to make these measurements.

Outline

NOLAB

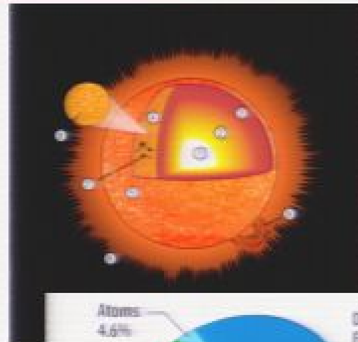
ow does the sun shine?
ow do we measure what we can't see?
ooking for Dark Matter

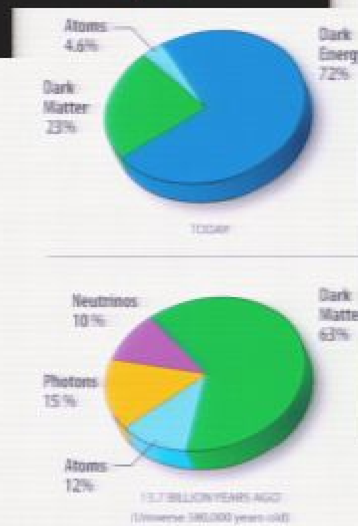
e Big Picture

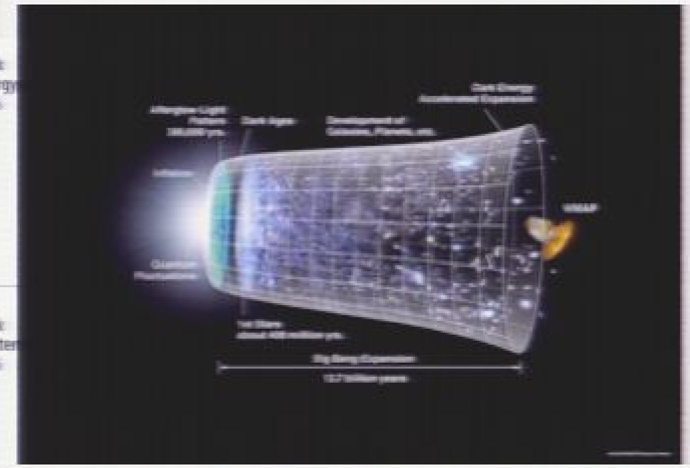
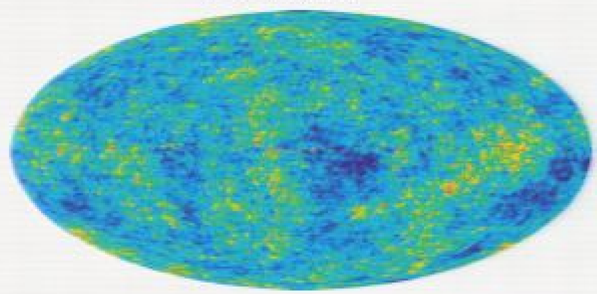
hat is the Universe made of?
ow has it changed over time?
ow can we know?

Day In the Life

ow Hugh Lippincott, Luc Gatien
d I spent July 4, 2008.



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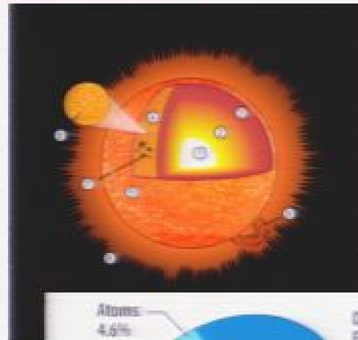

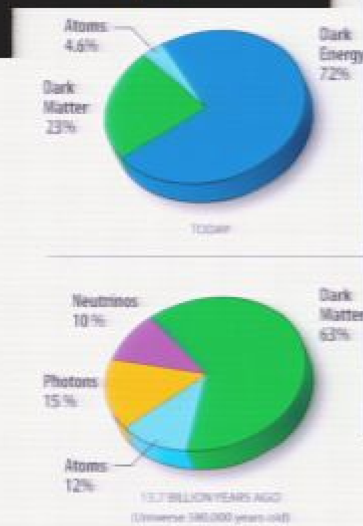

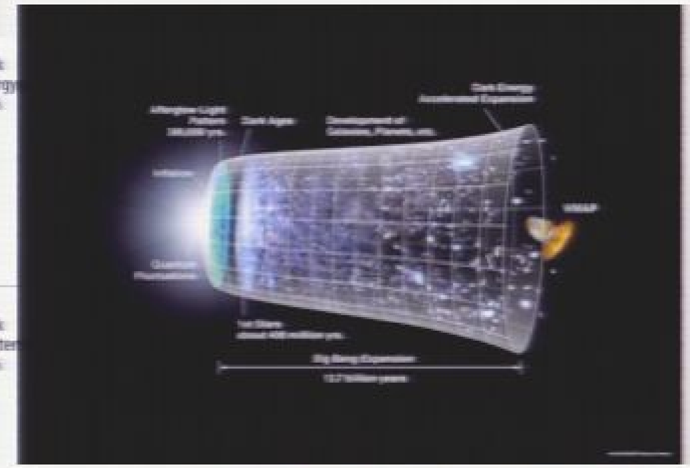

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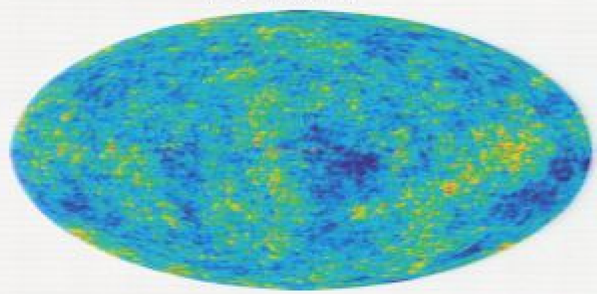

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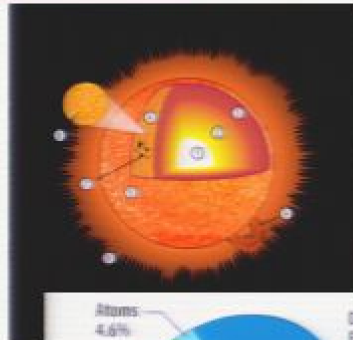
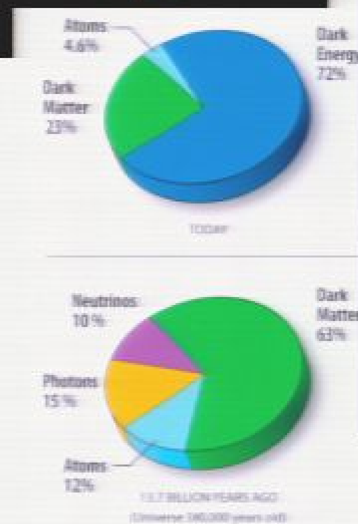
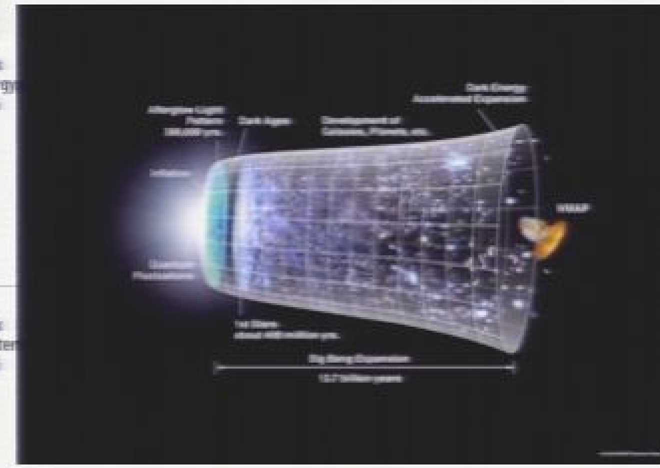


SNOLAB

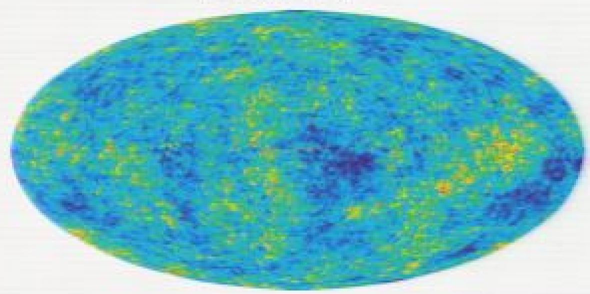
How does the sun shine?
How do we measure what we can't see?
Looking for Dark Matter

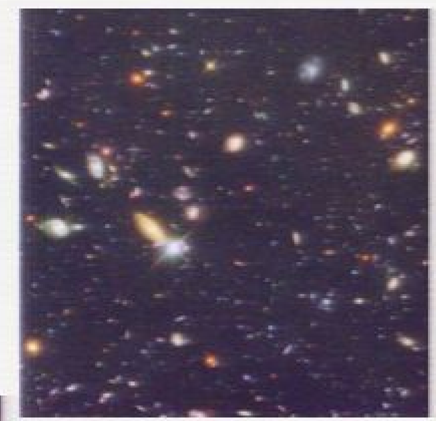




The Big Picture

What is the Universe made of?
How has it changed over time?
How can we know?





A Day In the Life

How Hugh Lippincott, Luc Gatien
and I spent July 4, 2008.

How The Sun Shines

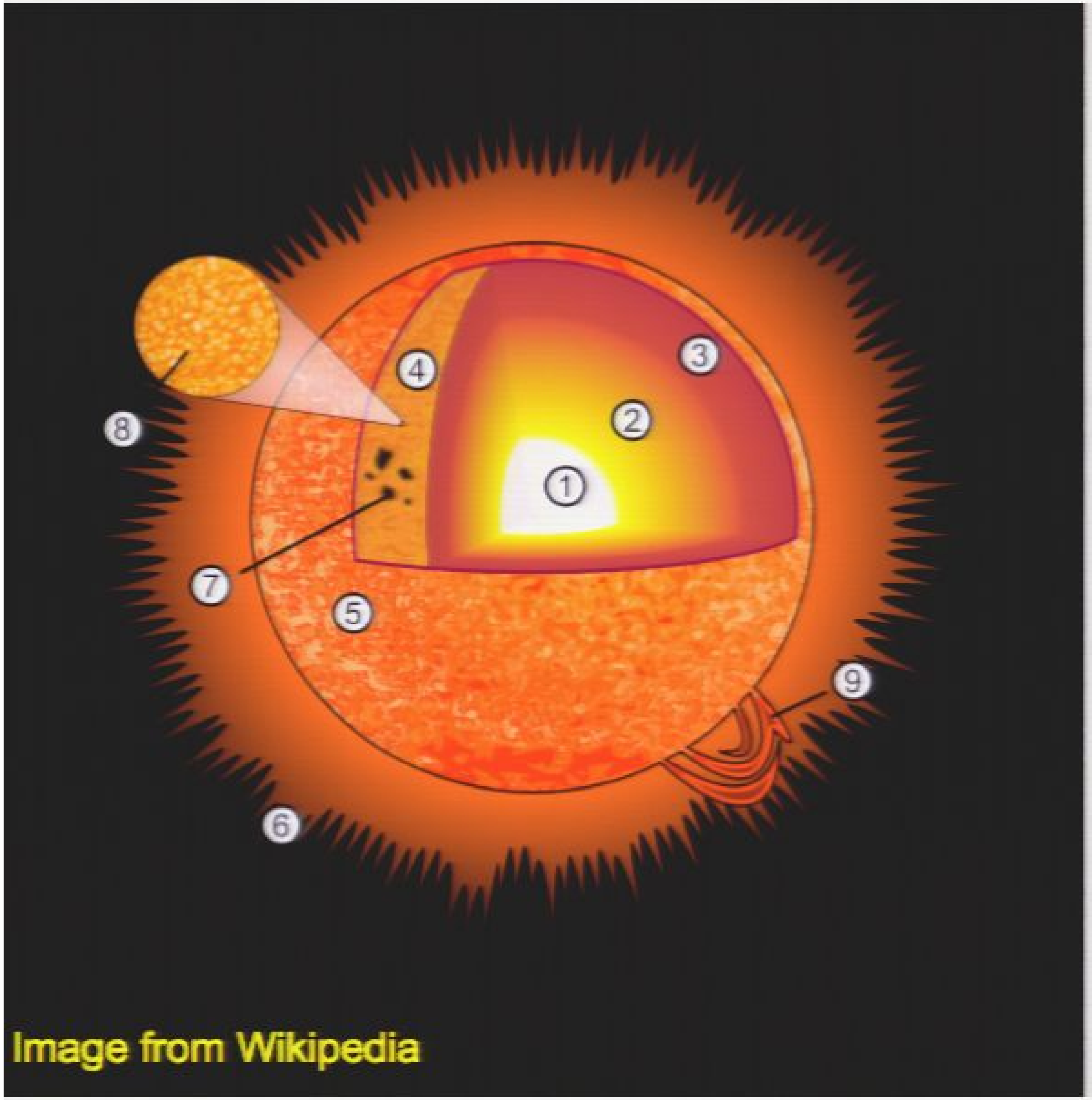


Image from Wikipedia

Nuclear reactions in core produce heat/neutrinos

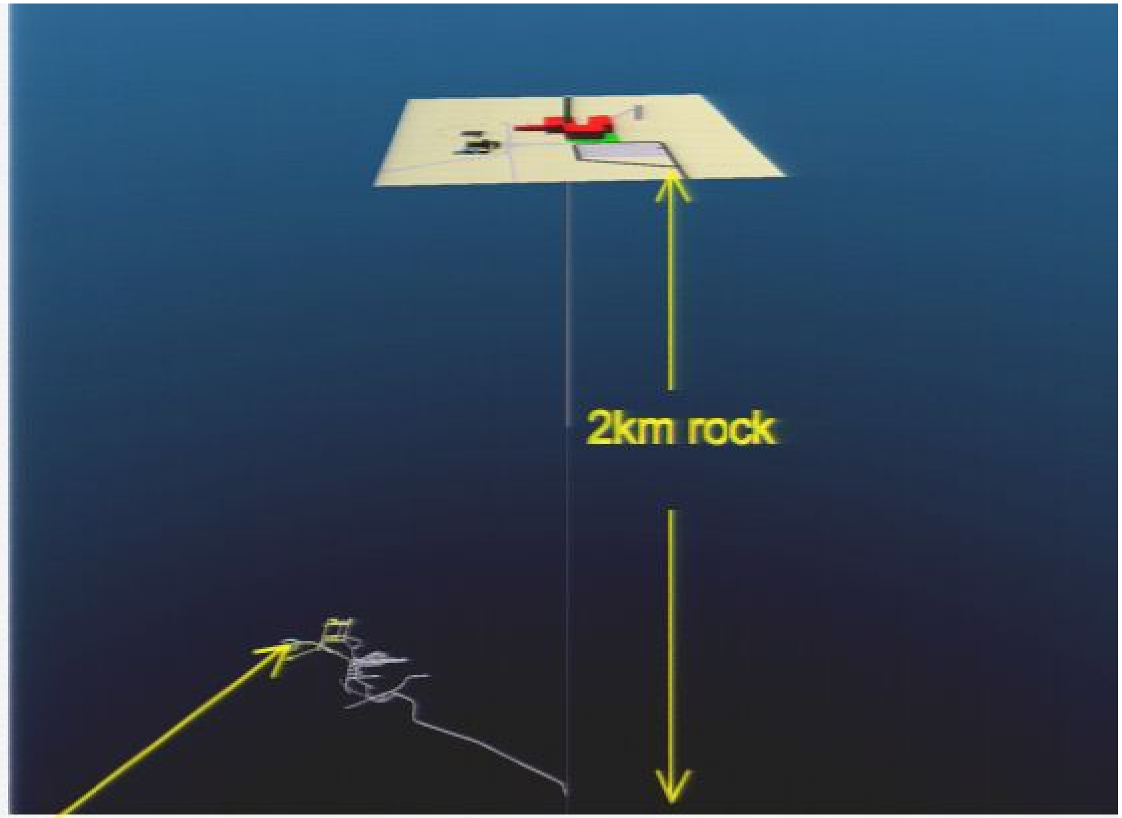
Heat works it way to surface by radiation and convection (the details of this is the hard part)

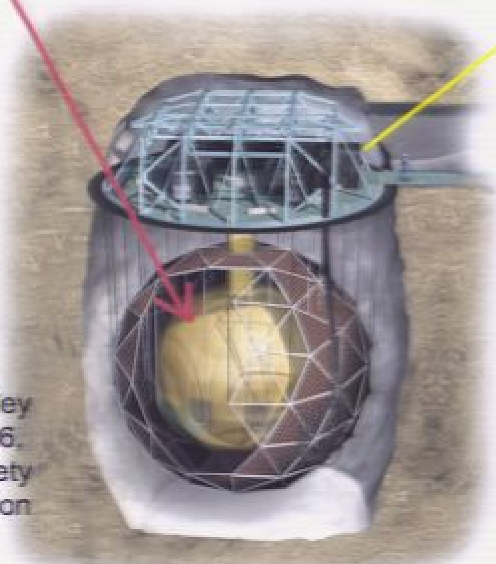
Heat comes of surface as radiation (aka sunlight)

Before neutrinos all data came from looking at the sun's surface

- size
- surface temperature
- composition
- …

This first experiments to measure solar neutrinos (Homestake and Kamioka) measured 1/3 to 1/2 of the expected number.

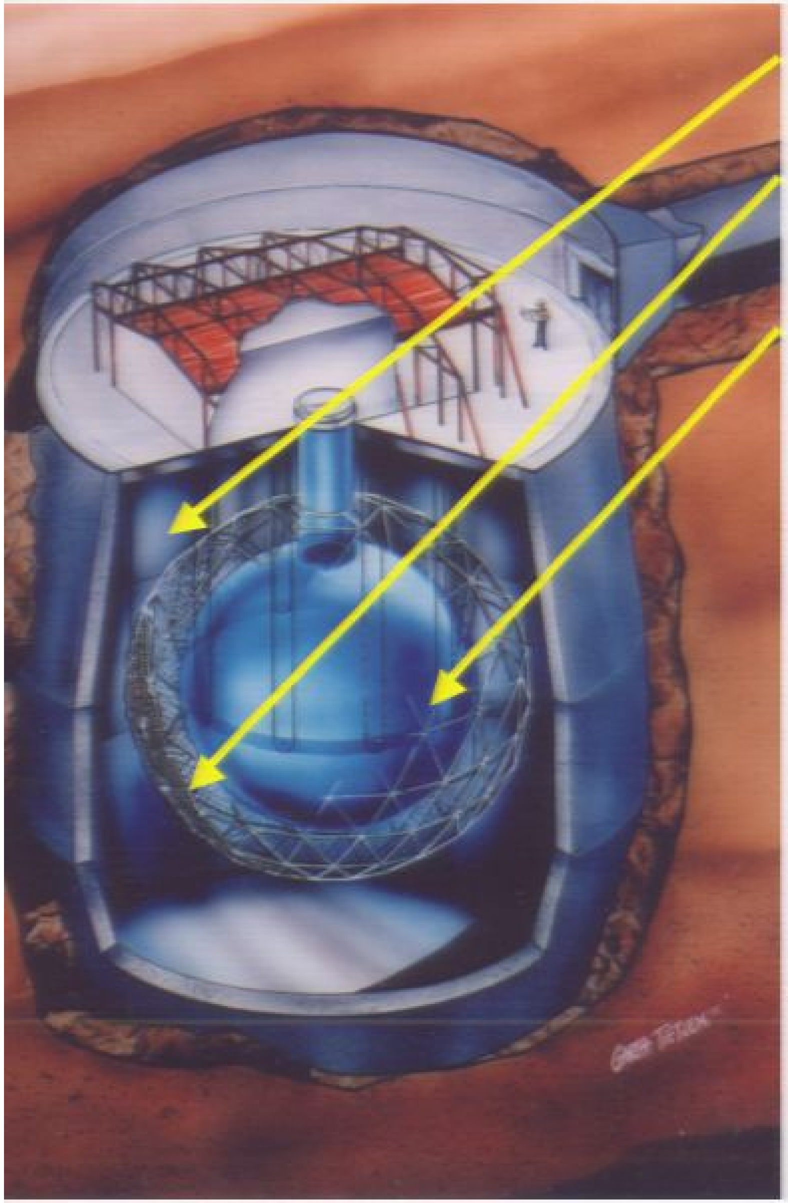




Professor Herb Chen (UC Irvine) realized that heavy water was a great target for solar neutrinos:

1. Count all types of neutrinos and hence directly measure nuclear reactions in the core of the sun.
2. At the same time measure only the type of neutrinos created in the sun.
3. And answer why the number of neutrinos measured until SNO didn't jive with solar models.

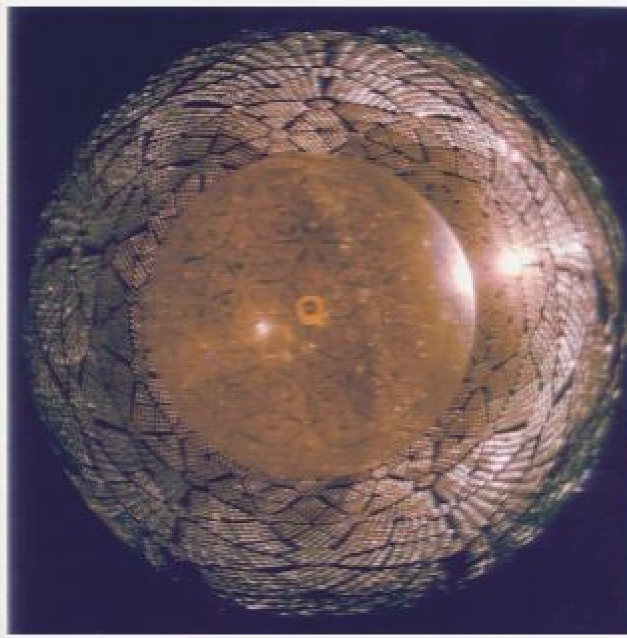
SNO



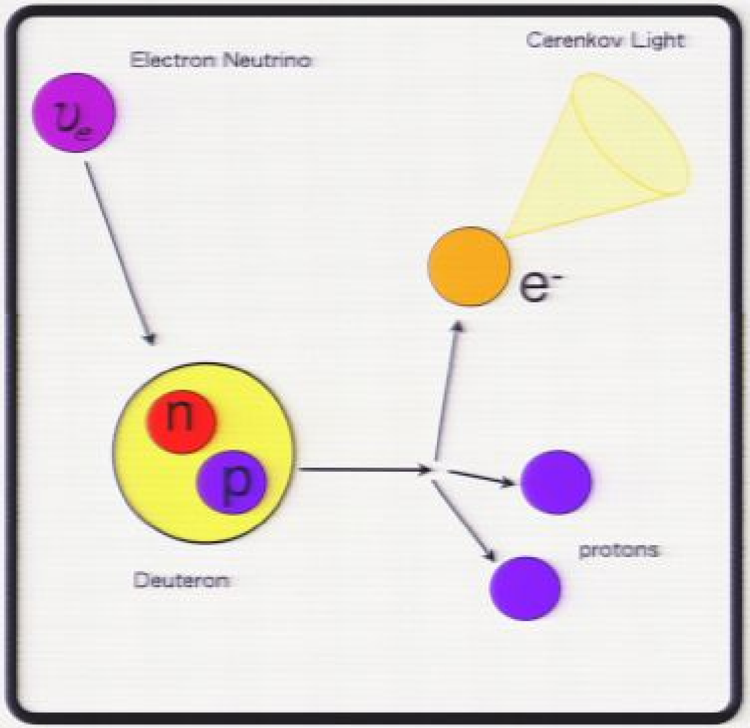
Light water shielding

9600 photomultiplier tube eyes

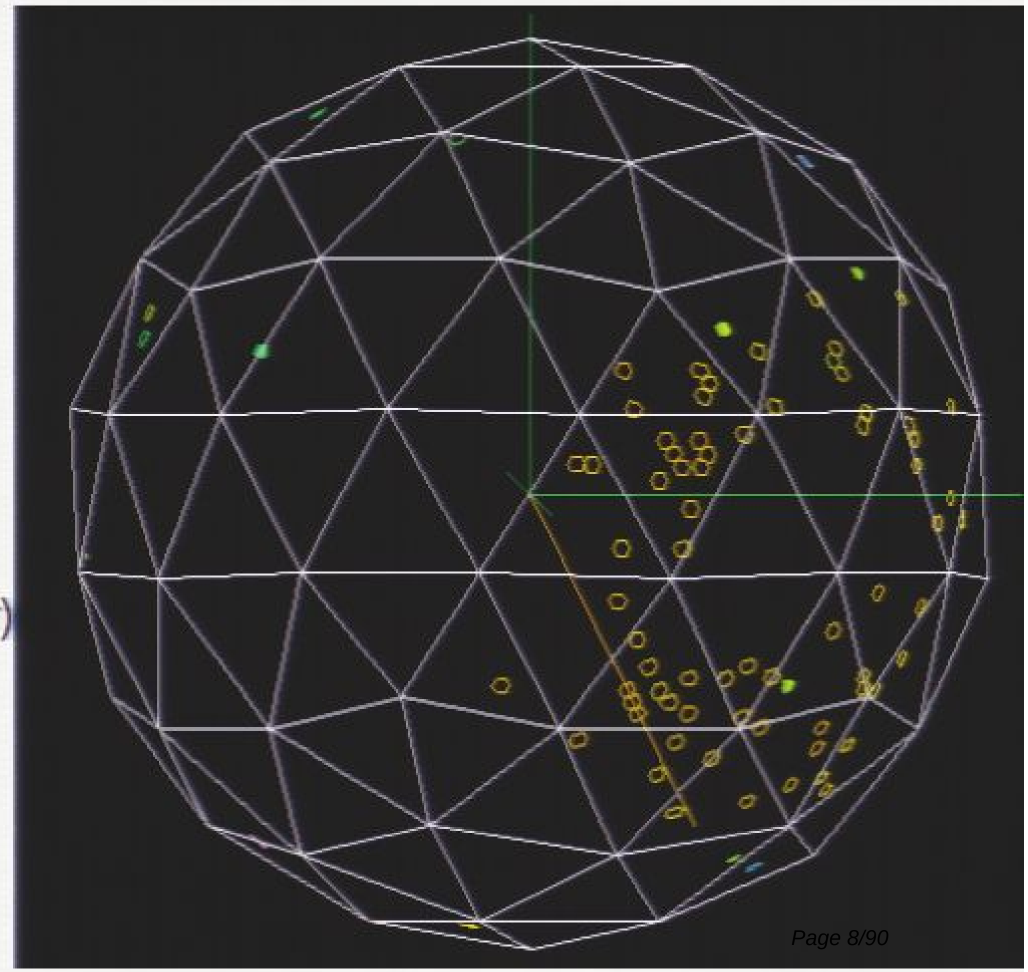
1000 tonnes heavy water



Detecting Neutrinos



1- Electron neutrino hits deuteron
2- Reaction creates 2 protons and an electron
3- Electron is moving faster than light in water
(not faster than light, faster than light in water)
4- Fast electron creates Cherenkov light
- an optical version of a sonic boom.
5- Light travels to photomultiplier *eyes*



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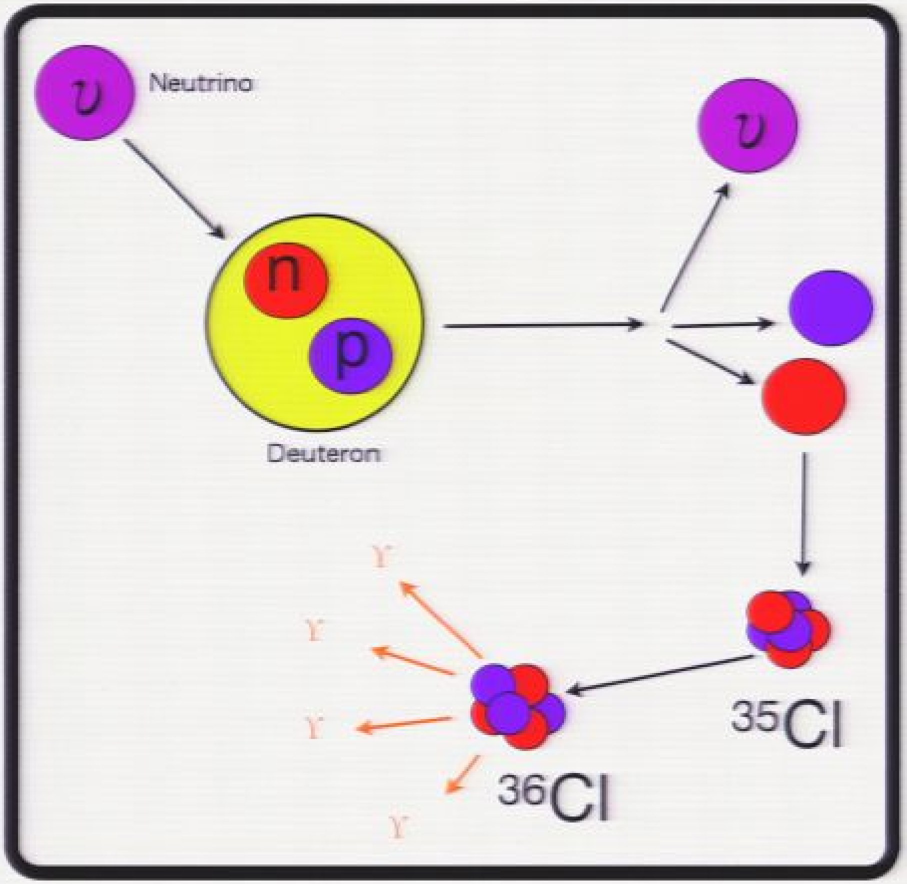


Neutral current reaction measures all neutrino types equally.

Creates free neutron which captures on Cl (in salt) or in independent data set on ^{3}He neutron counters.

SNO Results

- The total number of solar neutrinos jives with solar models:
  - We understand how the sun shines.
- *Electron* neutrinos are 30.1 +/- 3.3 % of the total in SNO:
  - Neutrinos change flavour
  - Neutrinos have mass (Exactly how much mass is unknown but the differences between the masses is known.)
  - Neutrinos are **not** the **dark matter**

For details see: http://sno.phy.queensu.ca/

Backing Up to Look at the Big Picture

A Strange Situation:

What is the history of the universe?

This we know **- and have measured -** with accuracy of about 1% for the age of the universe.

What is the geometry of the universe?

Again, with 1% accuracy on the scale of the observable universe we **have measured** that the angles in a triangle add to 180 degrees.

What is the universe made of?

5% known - 95% unknown

What is Dark Matter?

- It is matter that does not interact electrically or magnetically.
- This is true of neutrinos - but neutrinos are only a tiny fraction of the dark universe

- Why do we think there is dark matter?

When I was a Graduate Student

- (It wasn't *that* long ago)
- In 1991 took a class called "Physics of the Early Universe"
  - The detailed calculation we did was how much Deuterium, Helium and Lithium got made just after the Big Bang
  - Why? It was the only thing we could calculate **and compare to data.**
- Then observational cosmology erupted.
  - **COSMOLOGY IS NOW DATA-DRIVEN**

The Data

1. Cosmic Microwave Background
2. Large-Scale Structure
3. Big Bang NucleoSynthesis (Amount of Helium and Lithium in Universe compared to Hydrogen)
4. Gravitational Lensing
5. Type 1A Supernovae (which I'll not talk about today)

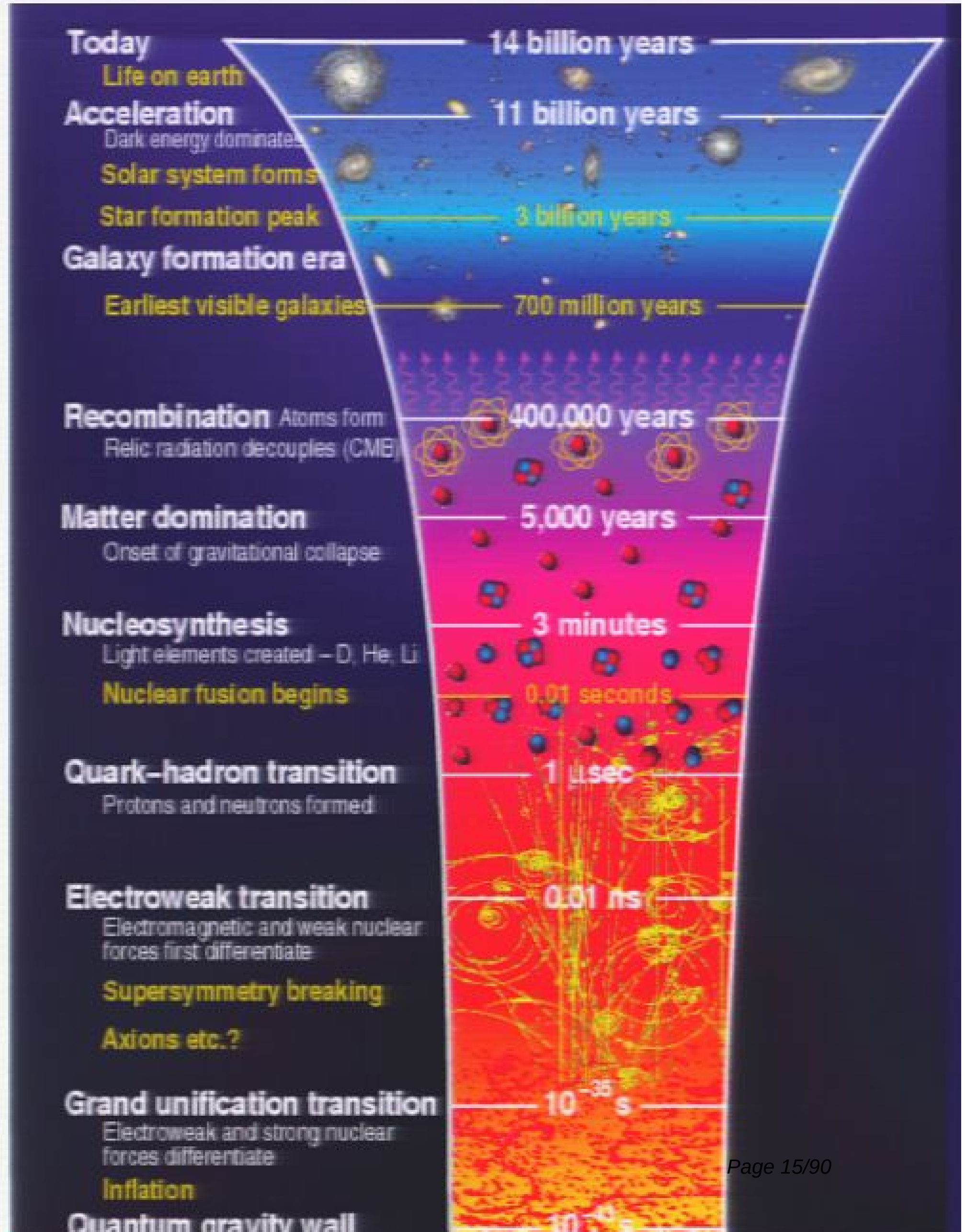
History

Large-scale structure

Cosmic Background Radiation

Big Bang Nucleosynthesis

Figure taken from ESA "bluebook"
for the Plank mission.



Before the Universe Became Transparent

- Driving in a thick fog is dangerous
  - even though there is lots of light
- When the universe was young (a 130 000 years or so) and hot
  - Hydrogen and helium gas that made up the universe was ionized.
  - Lots of light, but it kept scattering off those pesky charged things in its way.
- The interactions between the photons and the ionized gas kept the two in thermal equilibrium.
- These processes are completely understood.

- The universe expanded and cooled…

The Universe Became Transparent

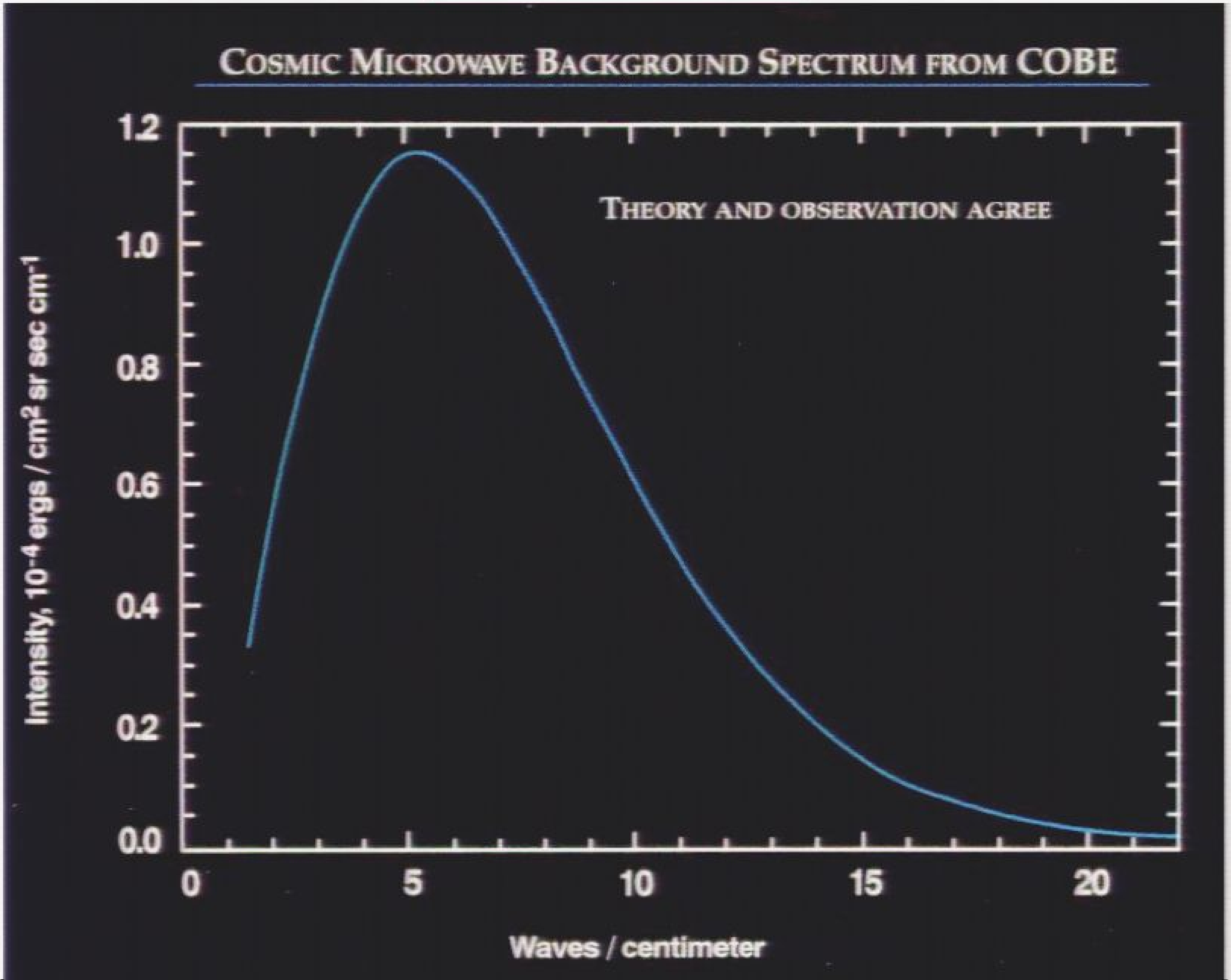
There is gas in the room but we can still see each other.

The universe cooled and hydrogen and helium became neutral atoms.

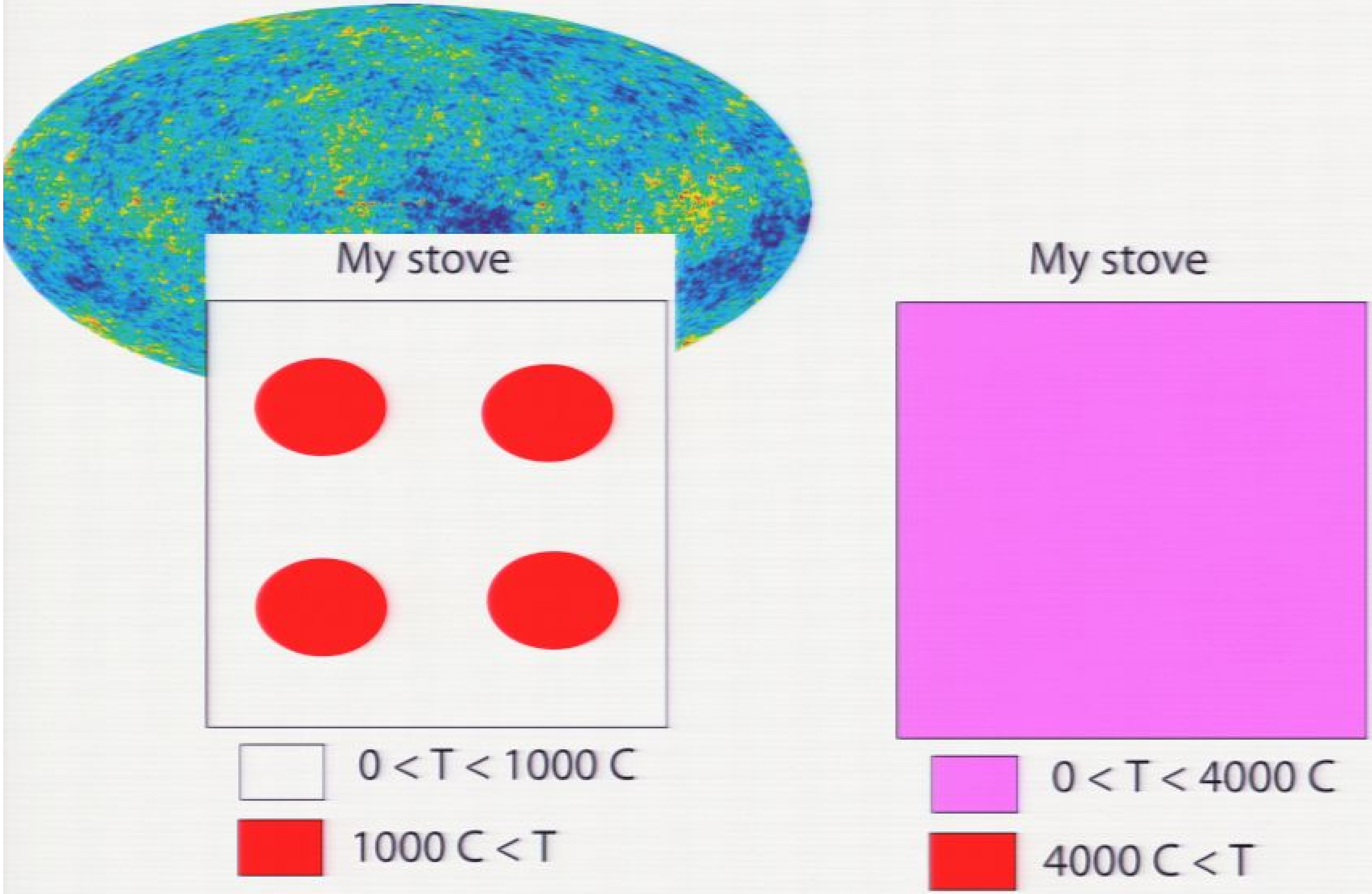
The light was able to free stream.

What colour was it?

COBE Data


COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE
THEORY AND OBSERVATION AGREE
Intensity, 10^{-4} ergs / cm^2 sr sec cm^{-1}
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Waves / centimeter

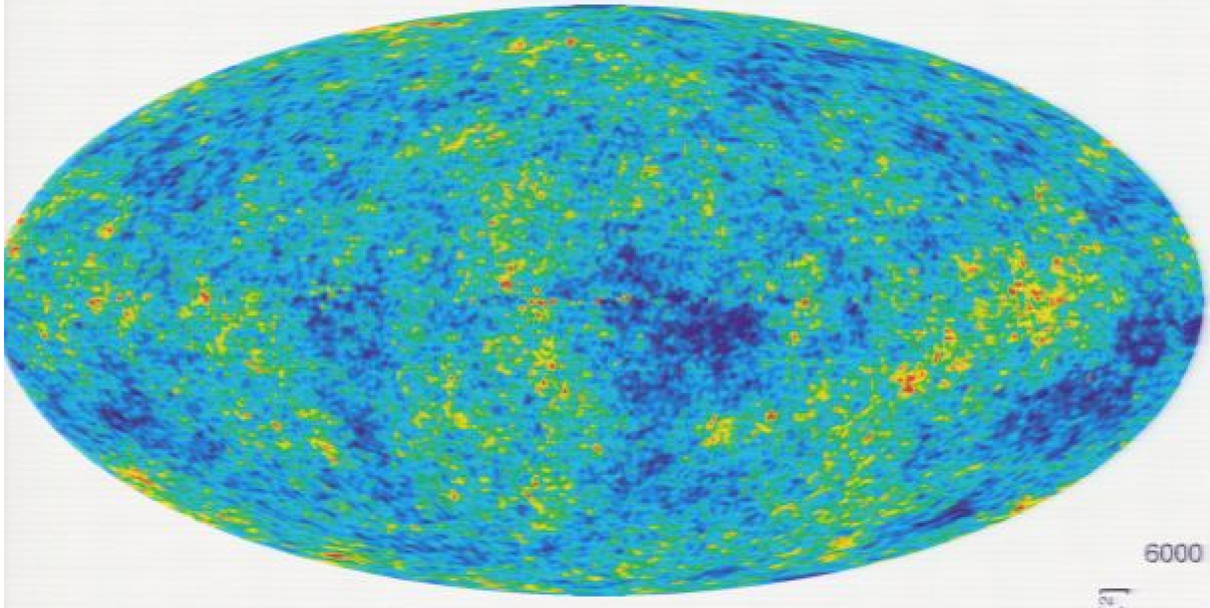
Cosmic Microwave Background



Figures of stove by Hugh Lippincott
from physicsformom.blogspot.com

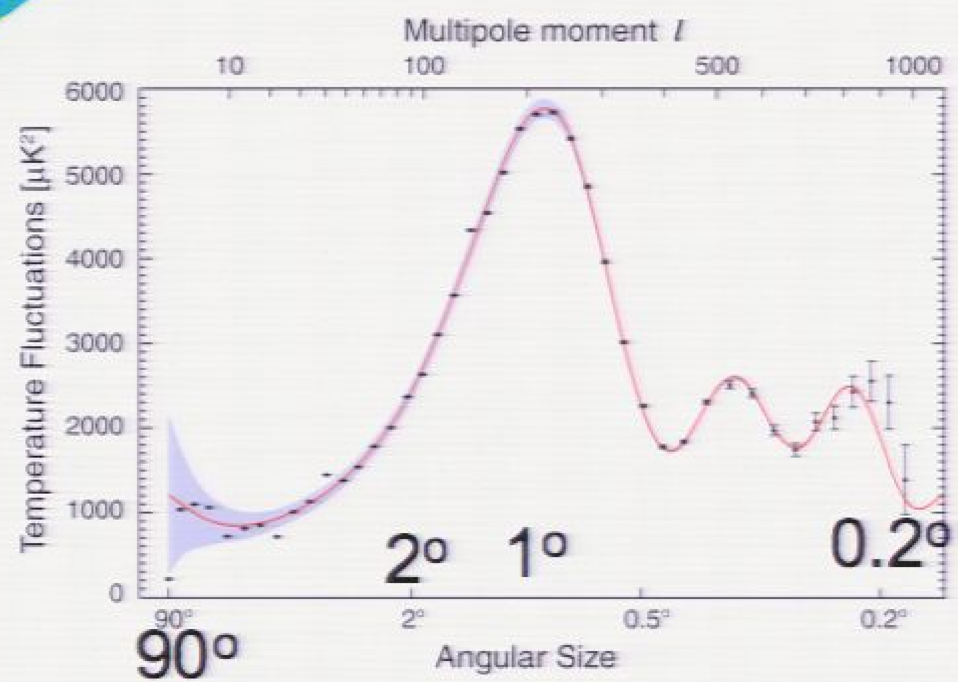
Data from NASA
WMAP satelite

Cosmic Microwave Background



Temperature map of whole sky:
2.725 K Average
Blue = Cooler by 0.0002 degrees
Red = Warmer by 0.0002 degrees

Smooth to ~1 part in 10,000.



- The tiny variations are caused by pressure waves in the Hydrogen and Helium plasma.
- Allows measurement of density, pressure, … of gas just as atoms were formed.

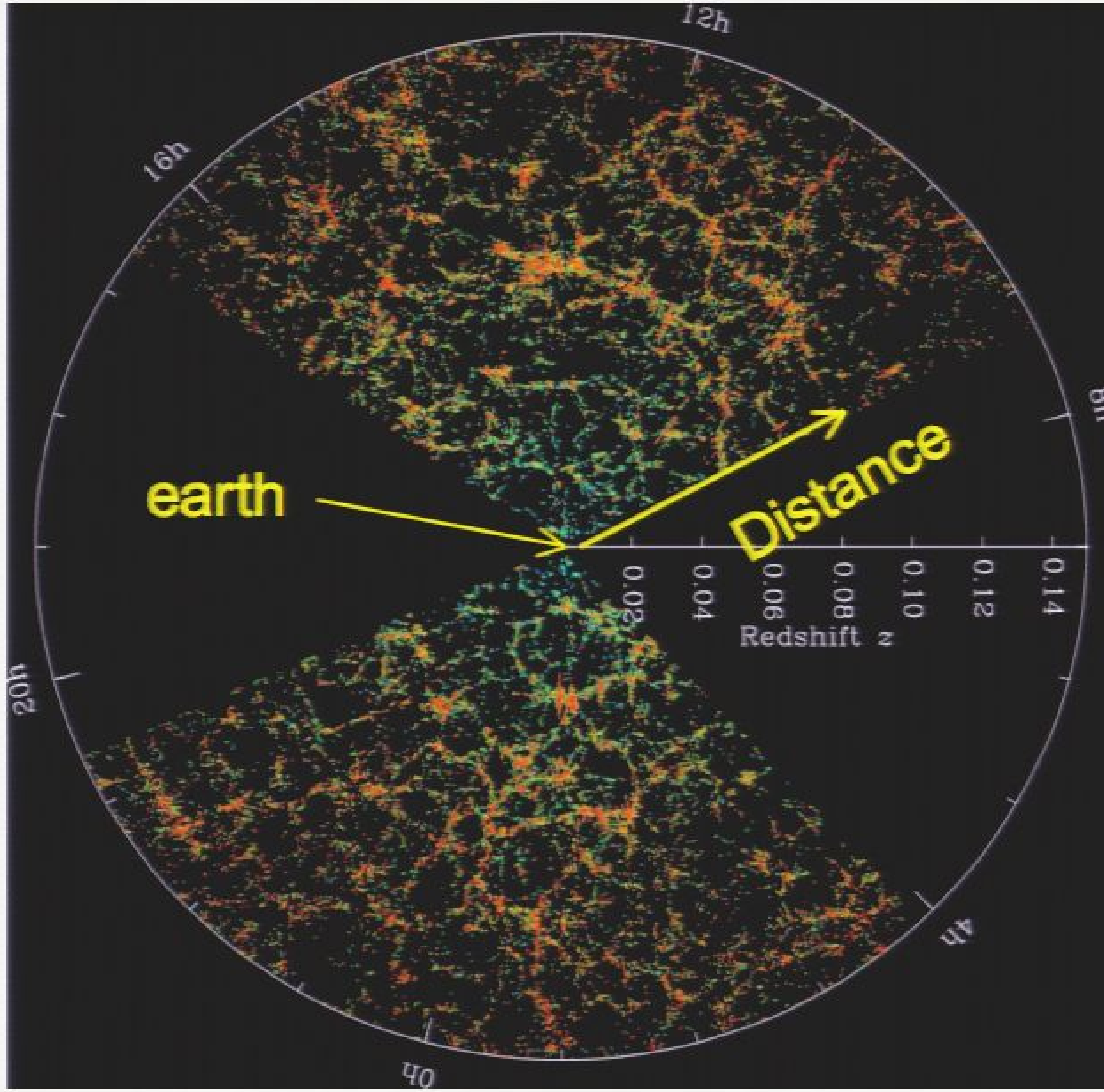
Data from NASA WMAP satellite

Structure "Now"

One slice through the sky by Sloan Digital Sky Survey.

Notice the universe is not smooth. It has a sponge-like texture.

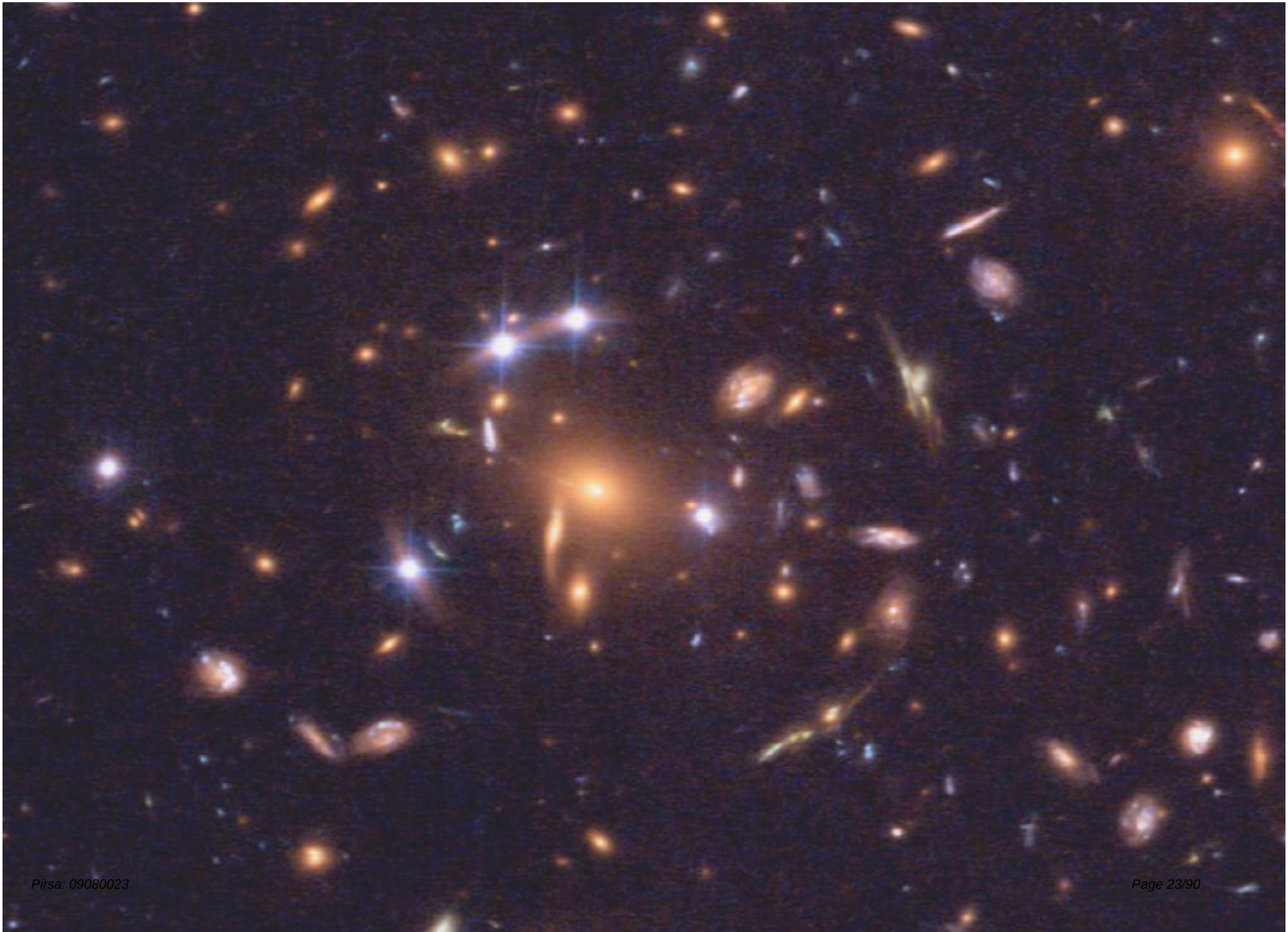
Dark matter seeds that clumping.

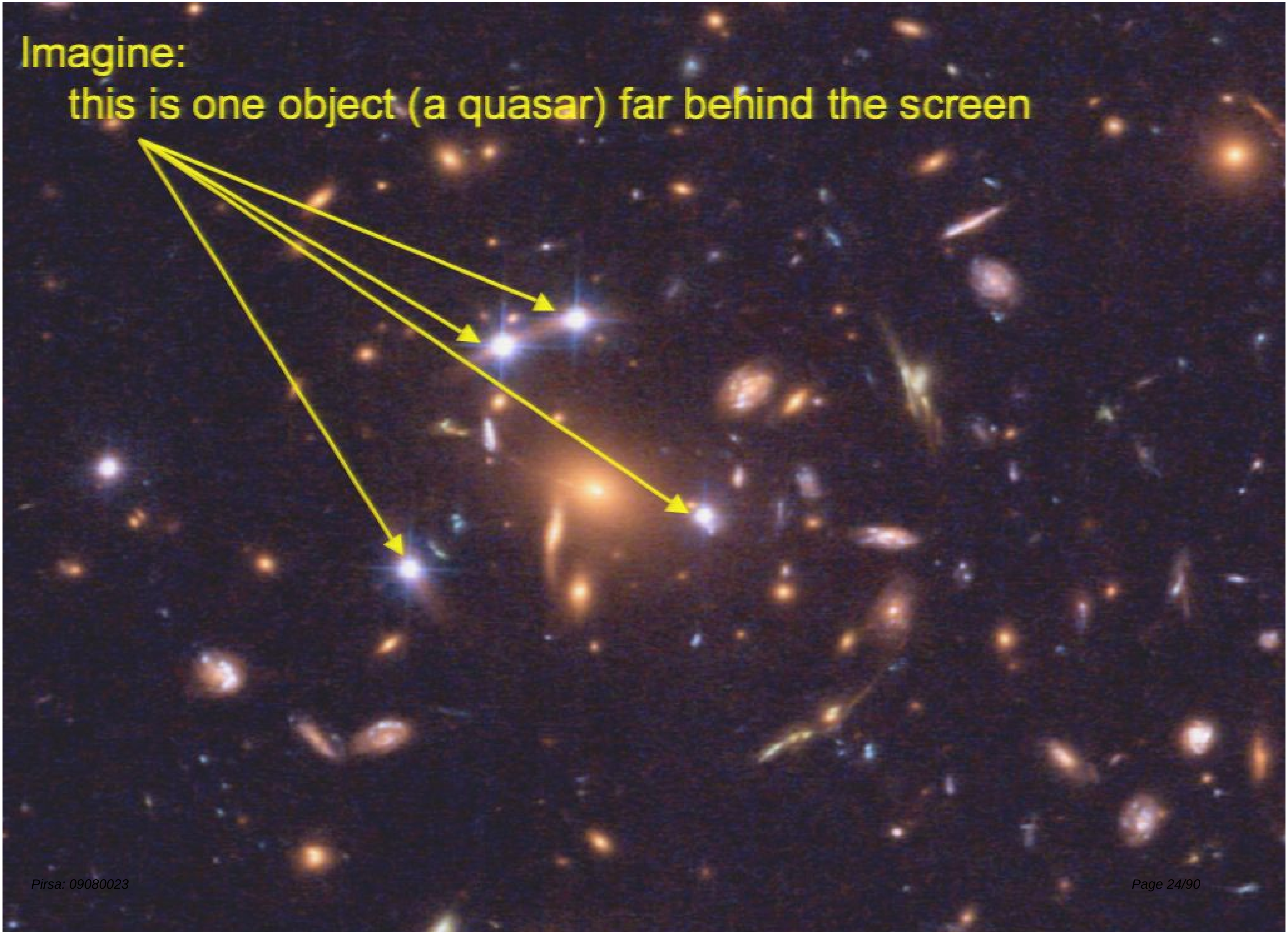


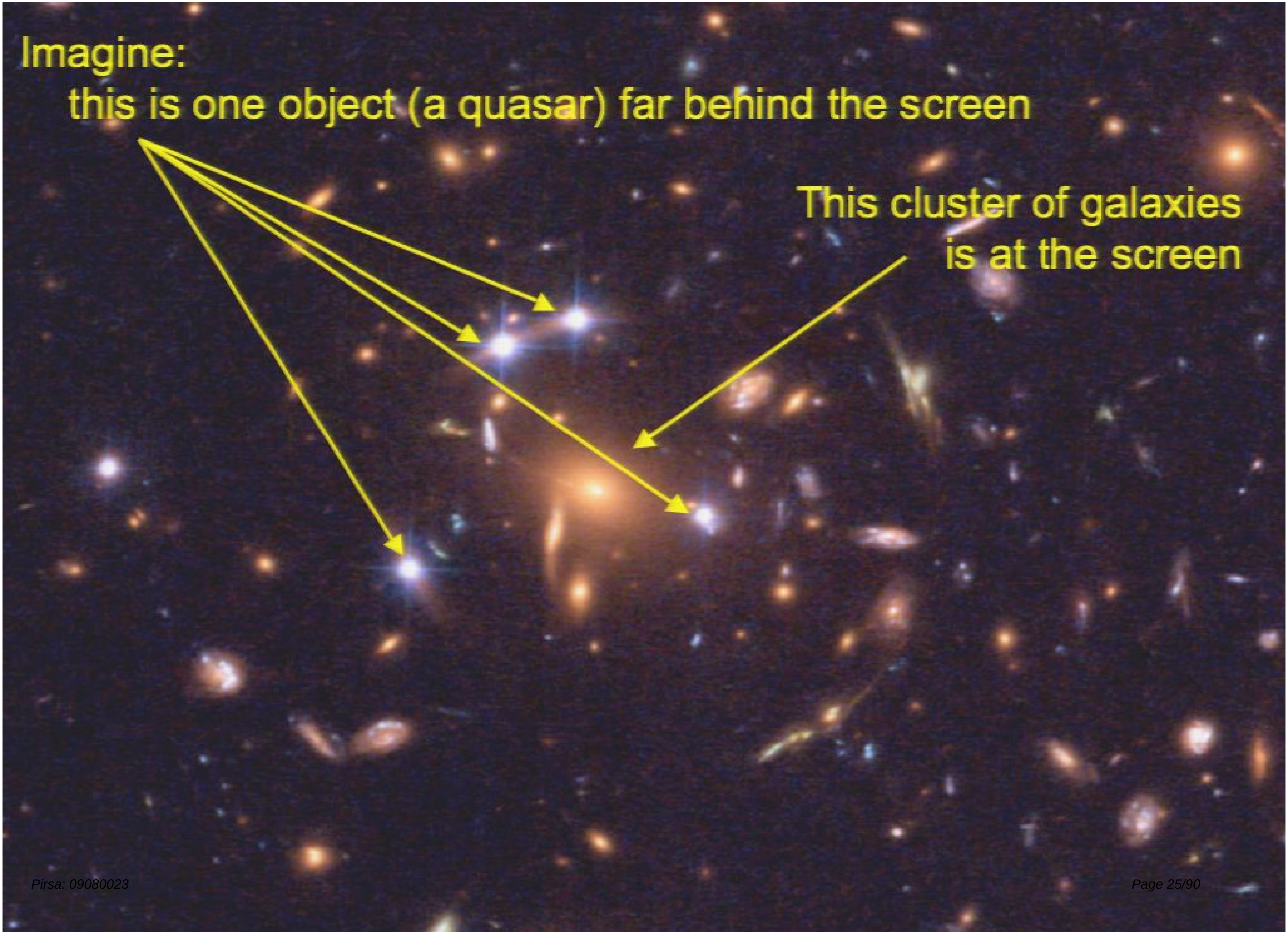
Rotation of Galaxies

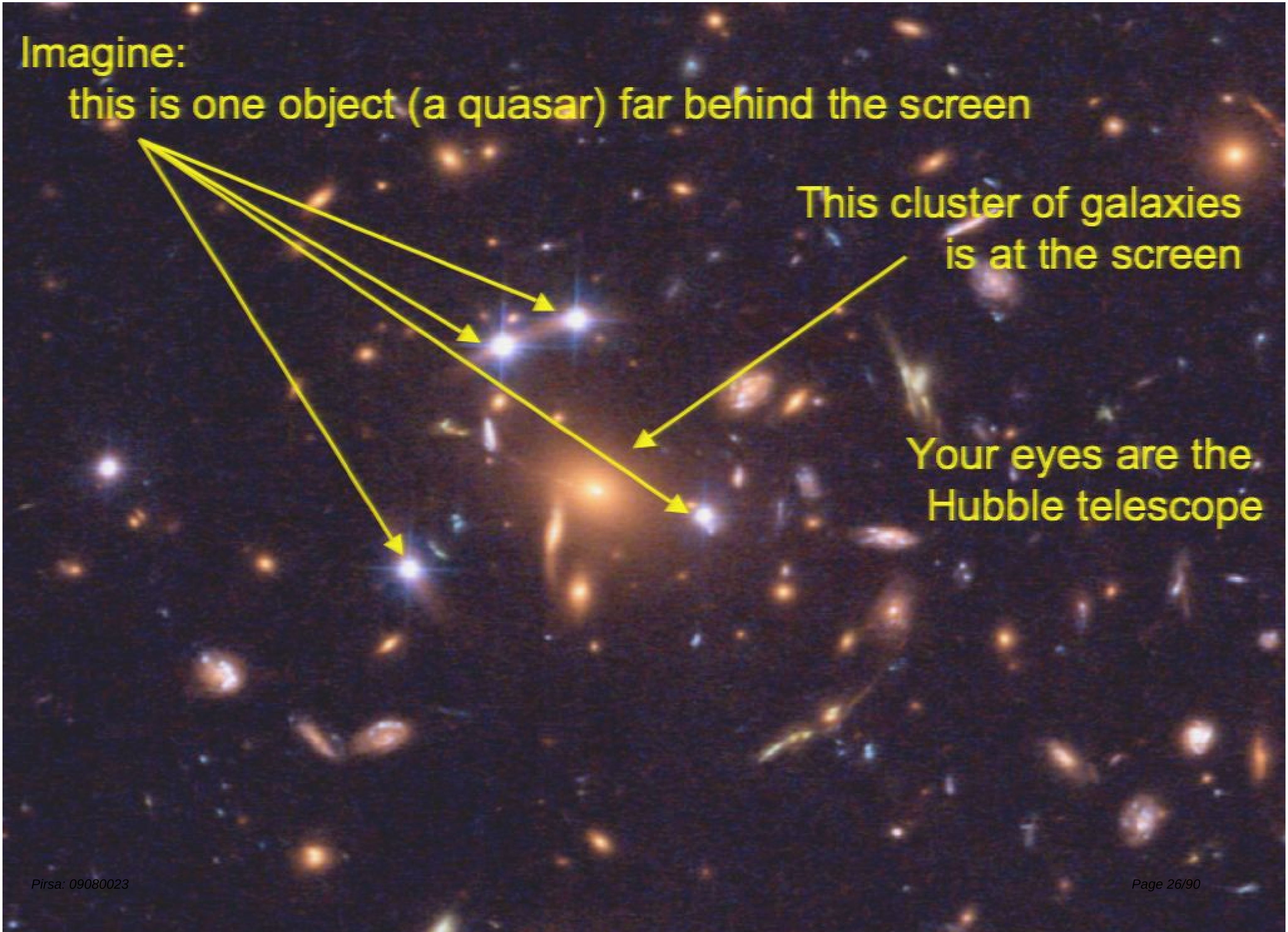
- Stars in spiral galaxies at outer edge of galaxies are orbiting far too fast.
  - What holds the galaxy together?

- Orbiting as if there is a dark component to the galaxy that extends beyond the visible disk.




Imagine:
this is one object (a quasar) far behind the screen


Imagine:
this is one object (a quasar) far behind the screen
This cluster of galaxies
is at the screen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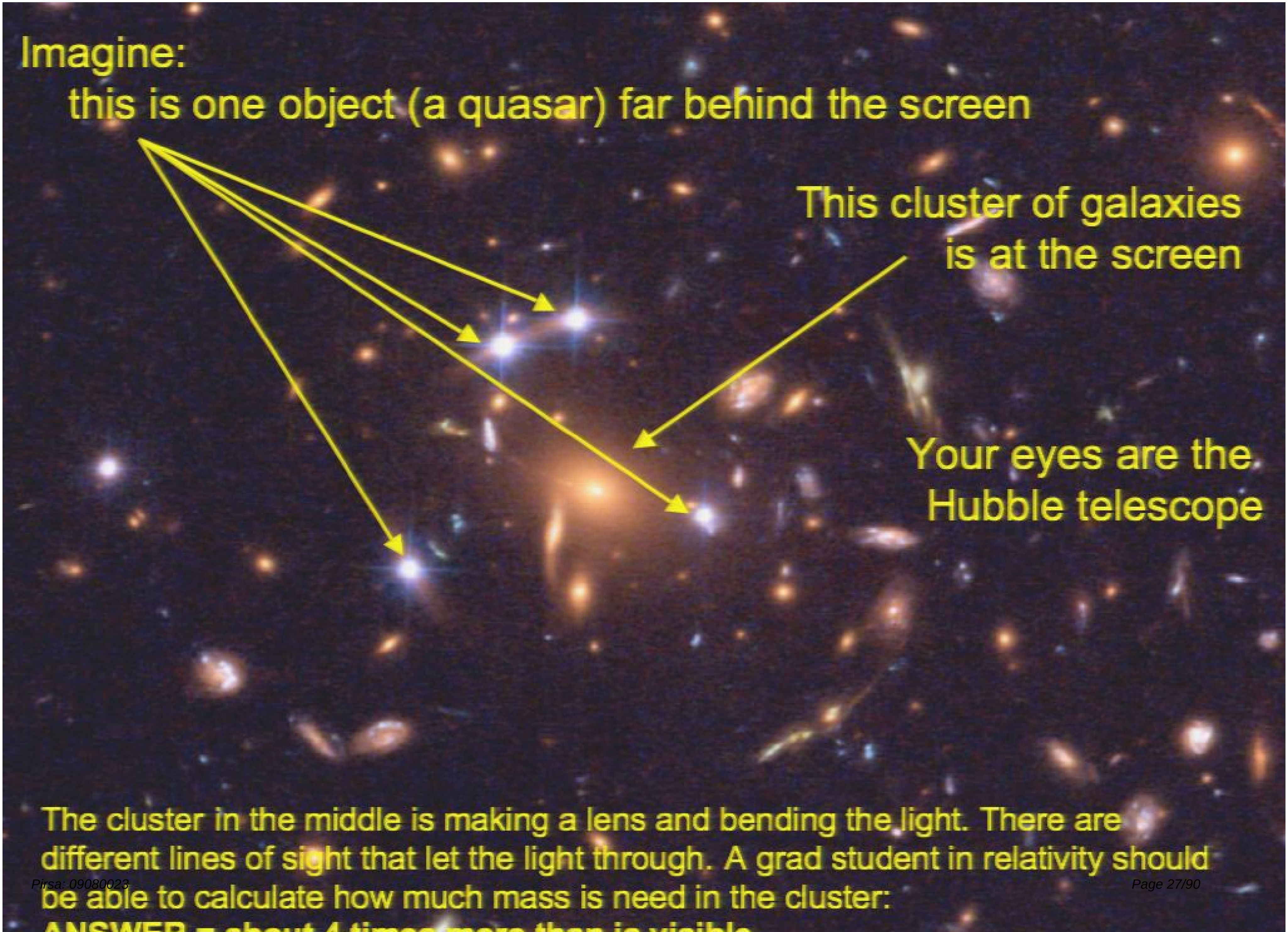
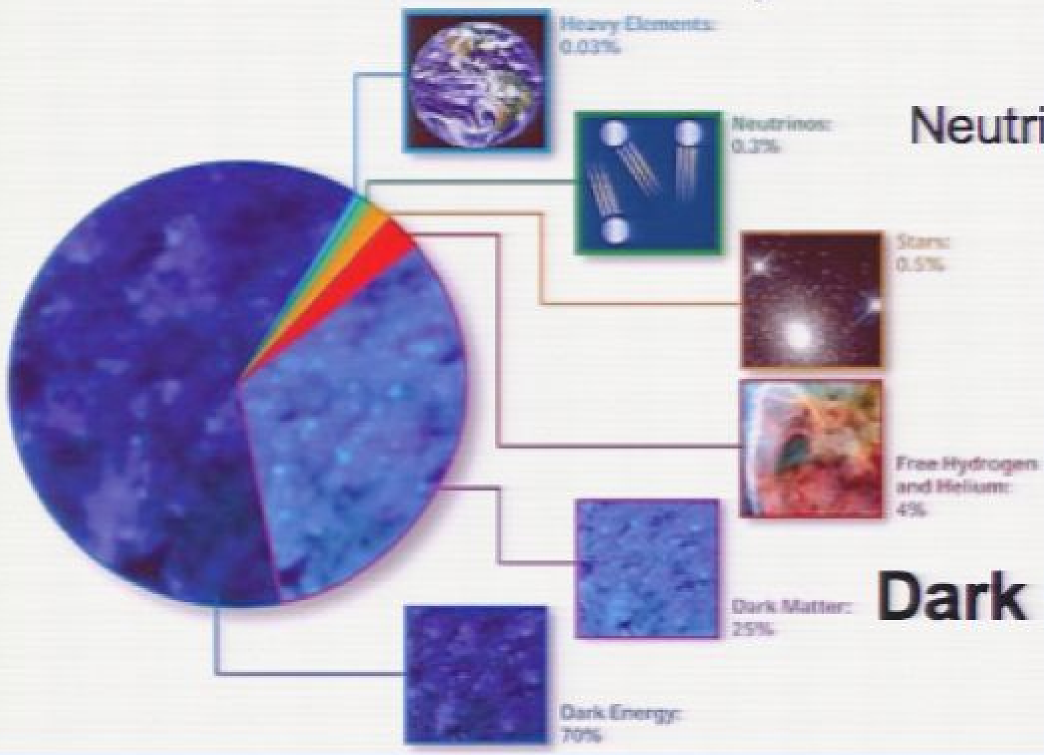
Your eyes are the
Hubble telescope


Imagine:
this is one object (a quasar) far behind the screen
This cluster of galaxies
is at the screen
Your eyes are the
Hubble telescope
The cluster in the middle is making a lens and bending the light. There are
different lines of sight that let the light through. A grad student in relativity should
be able to calculate how much mass is need in the cluster:

Cosmic Pie Chart

Heavy elements



Neutrinos

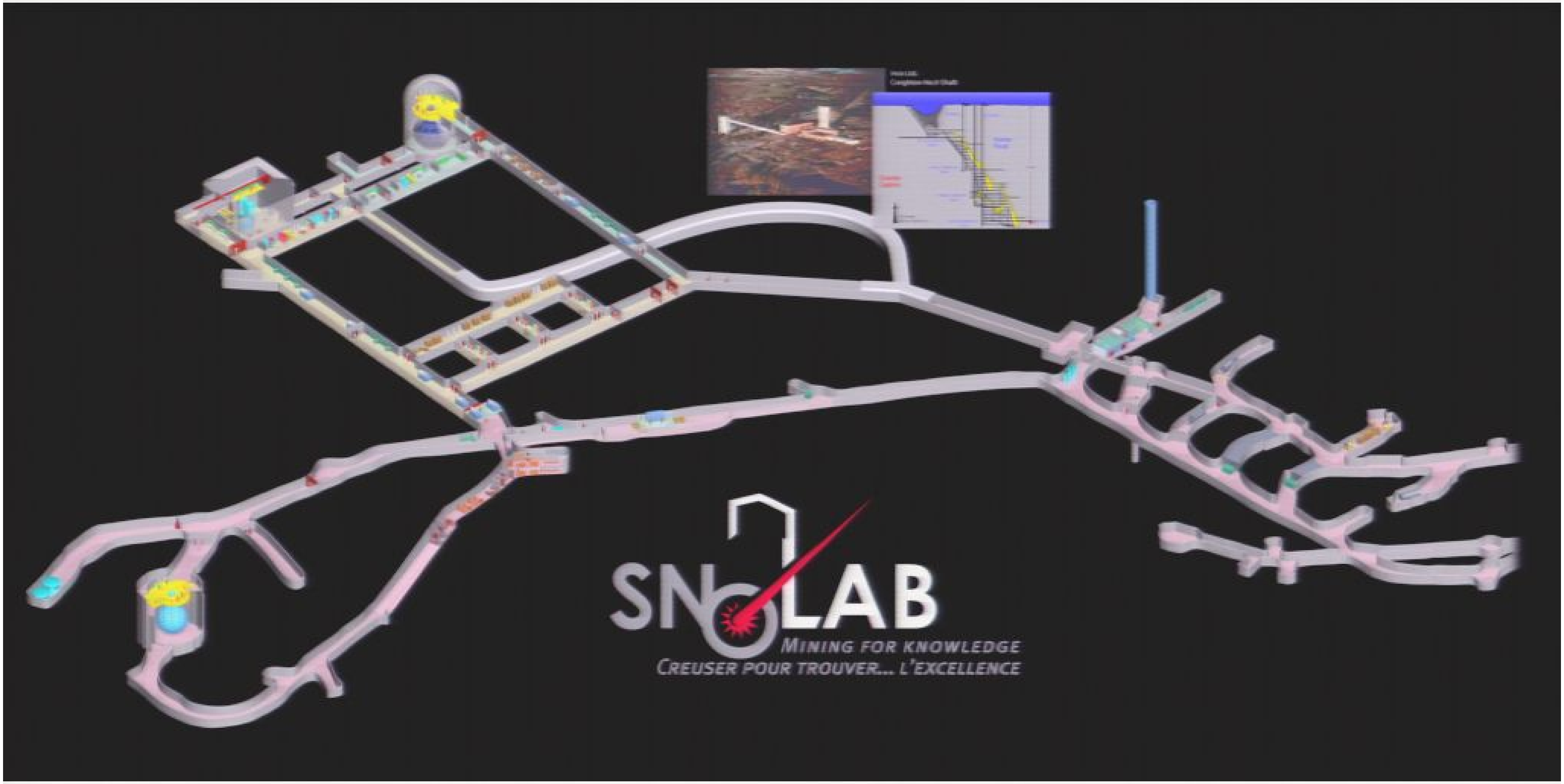
Stars

Free Hydrogen and Helium

Dark Matter

Dark energy (truly weird)

SNO to SNOLAB



Direct Search for Dark Matter

- One of the pillars of the SNOLAB program.

- Dark Matter
  - Exists around galaxies
  - Should be some around ours
  - Must have been created in early universe so it should interact in some ways with ordinary matter
  - Interaction rates needed consistent with a weak-force interaction (similar to neutrinos)
  - Requires a similar skill set in the workers as a neutrino experiment

SNOLAB's First 2 Dark Matter Experiments

Picasso

- DM strikes Fluorine atom in liquid drop of C_4F_{10}
- Droplet boils and expands sending pressure shockwave through gel holding droplet.

DEAP-1

- DM strikes argon atom in 7 kg of liquid
- Recoiling atom causes argon to scintillate in hard UV
- Wavelength shifting coatings convert UV to visible.

The Hard Question To Answer

If you see a signal in an experiment that looks like a dark-matter interaction *how do you know it is not a conventional interaction that mimics the dark-matter signal?*

(In math: What is the probability we measure a given number of background events in an experiment.)

We want to develop large-experiments with ~0.1 background events per tonne of detector per year.

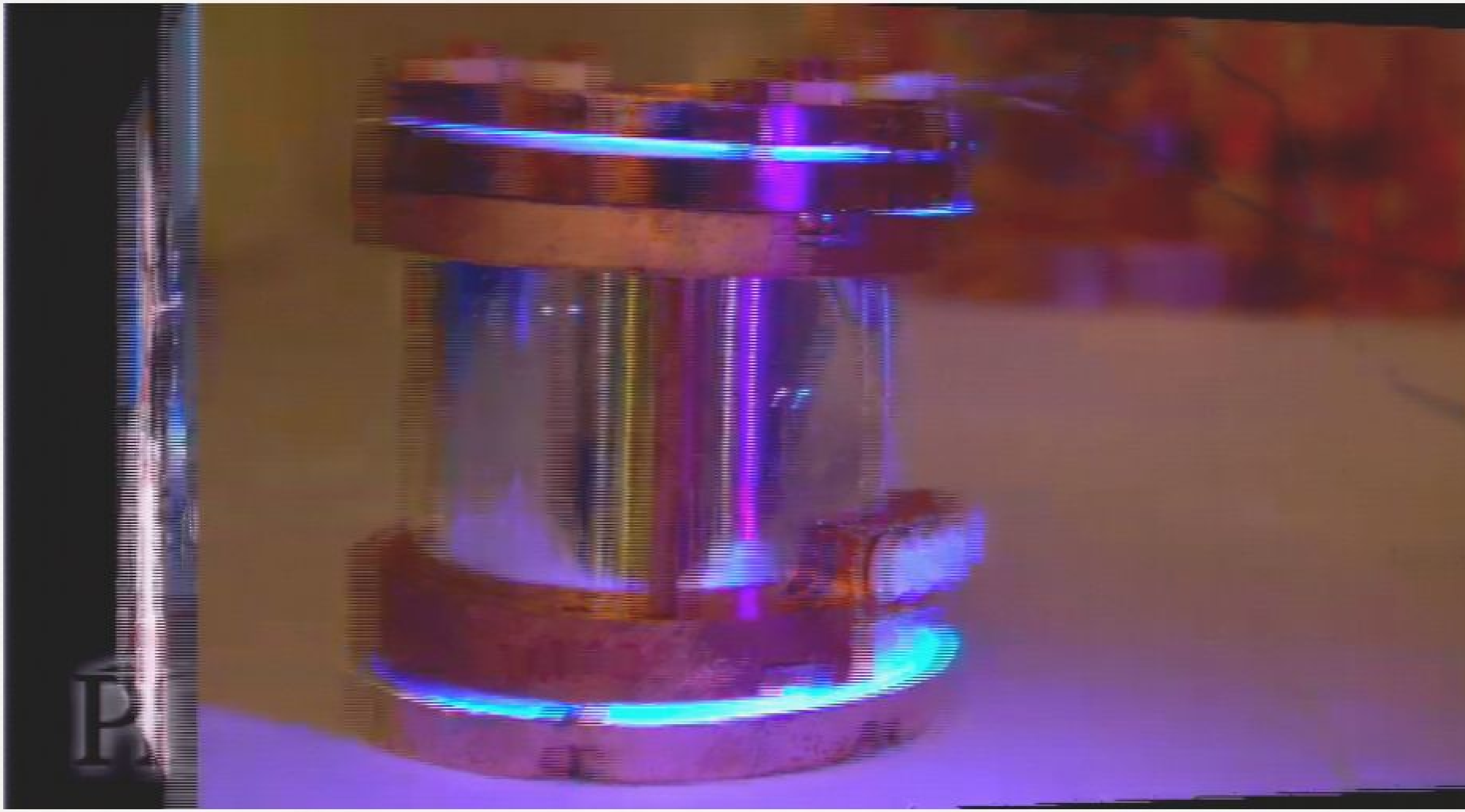
Backgrounds to a Dark-Matter Search

1. Cosmic-ray-induced events

2. Radon and radon daughters. (Radon has a radioactive daughter with a 22 year half life.)

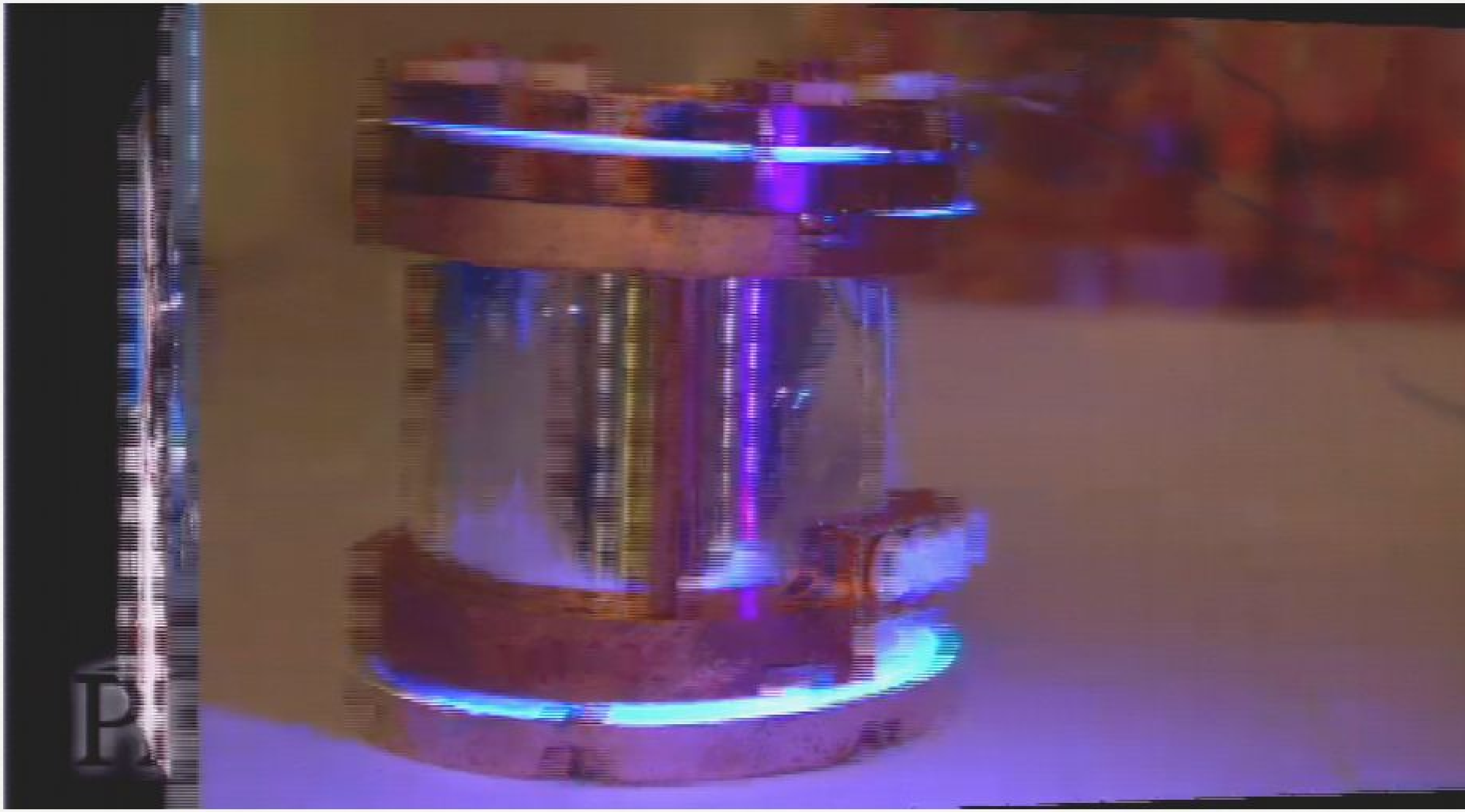
3. Neutrons

Underground Science



Video from the Perimeter Institute

Underground Science



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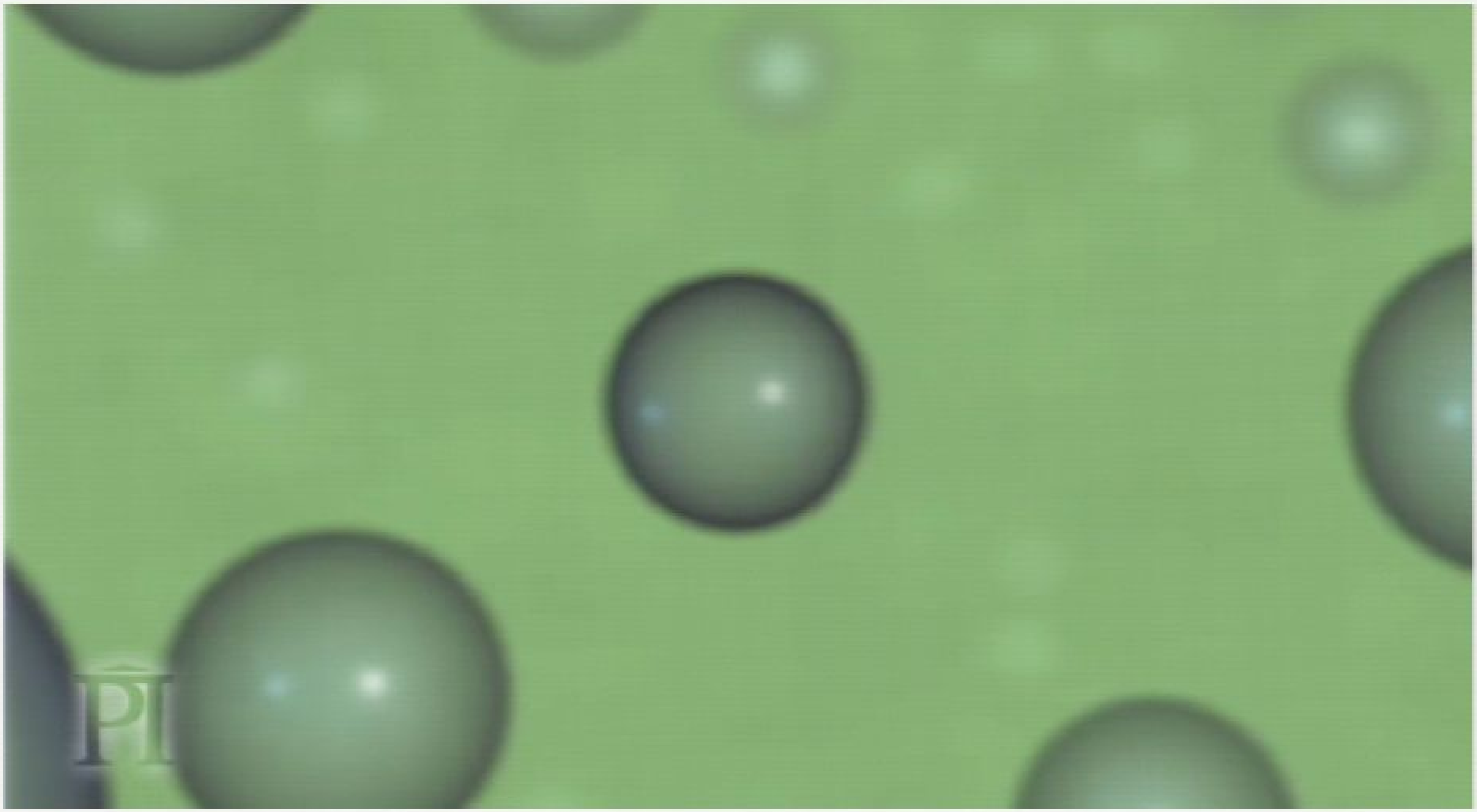

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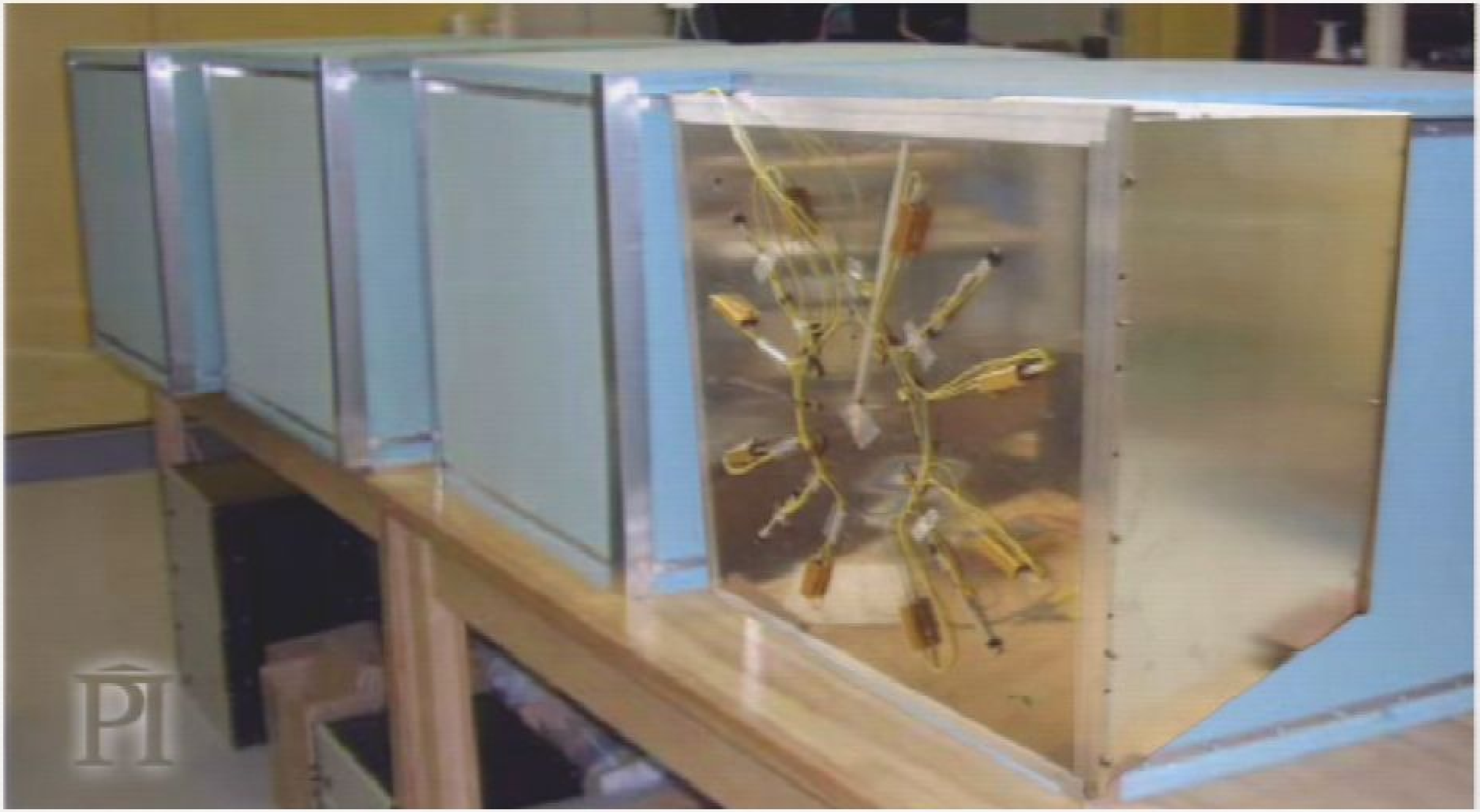

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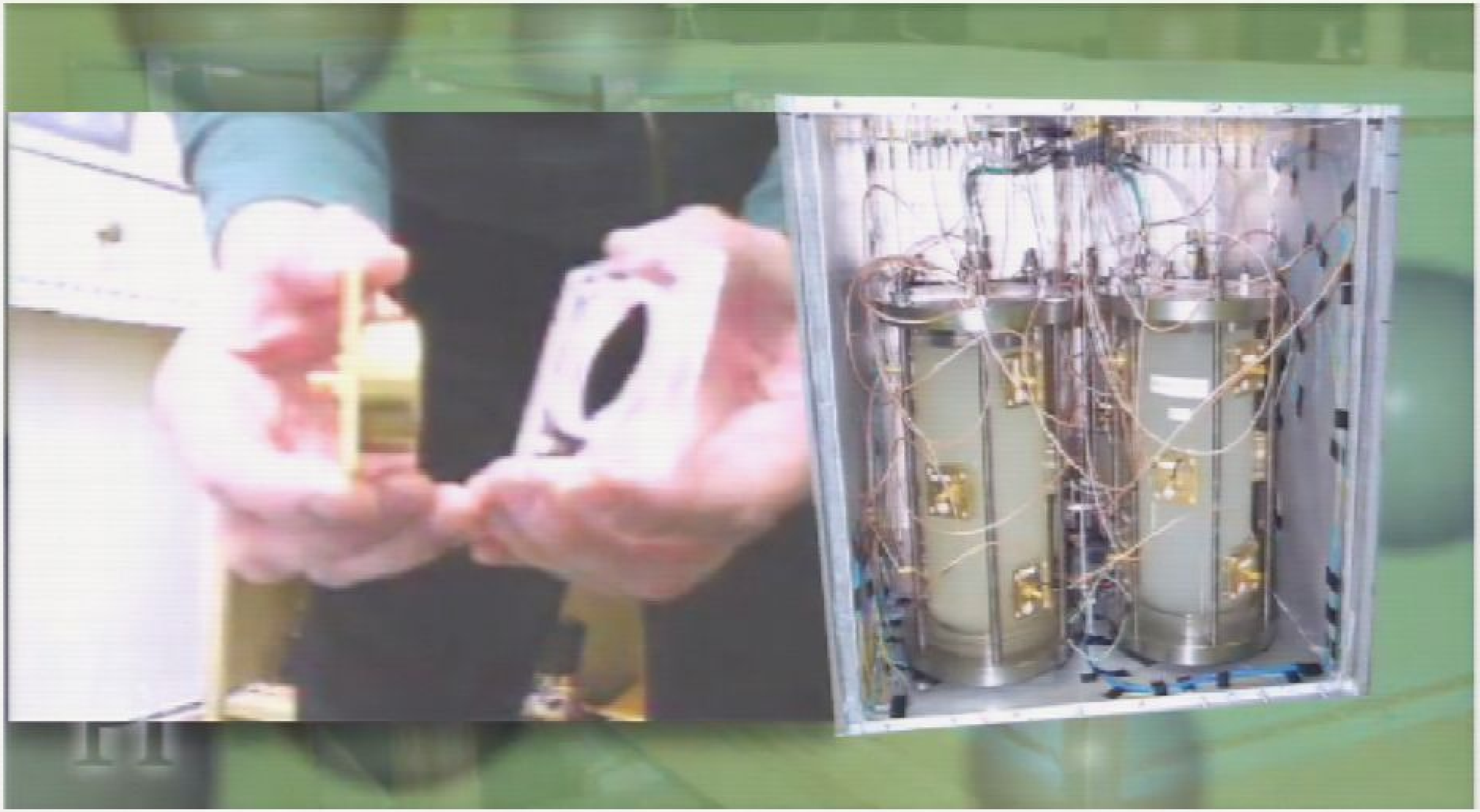

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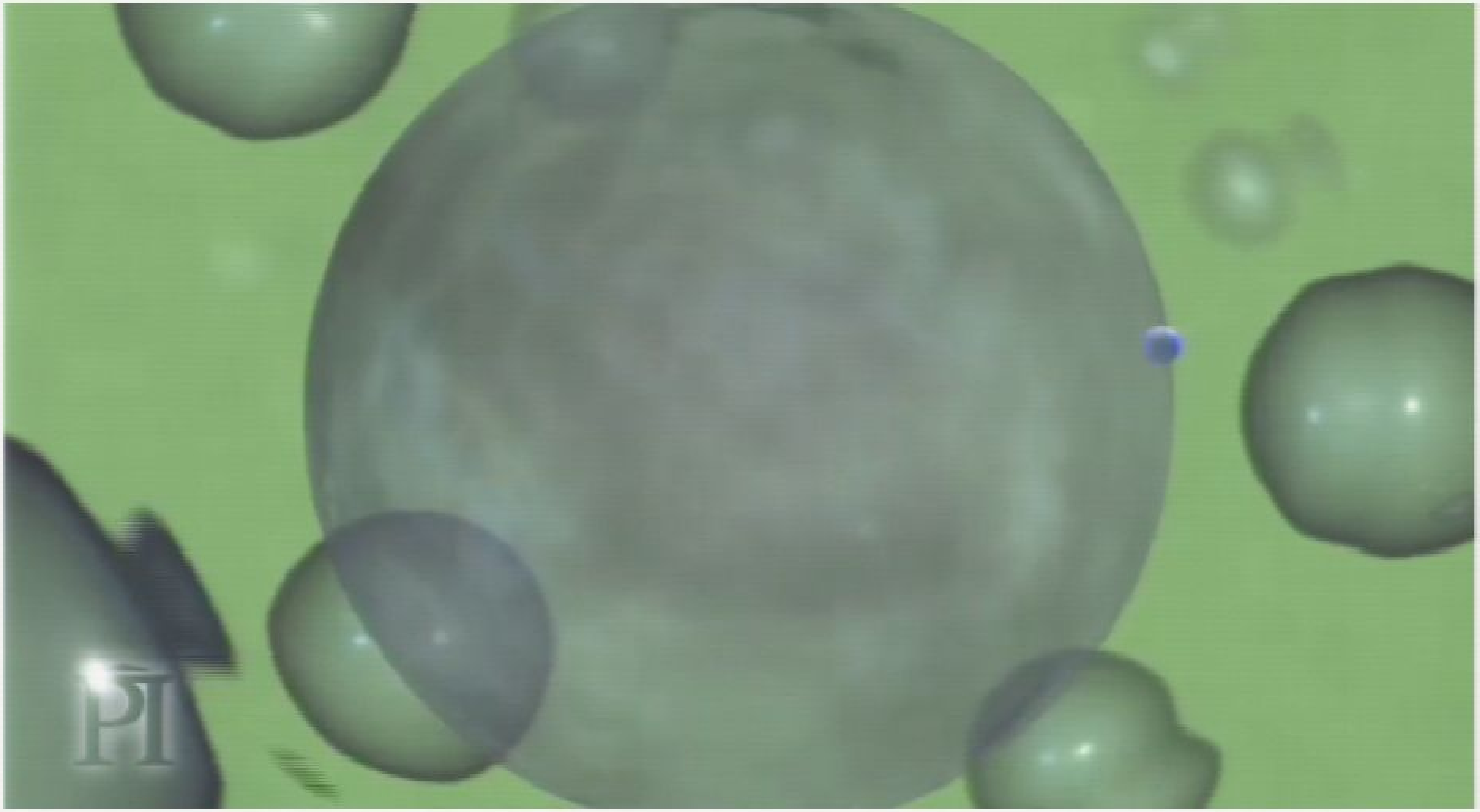

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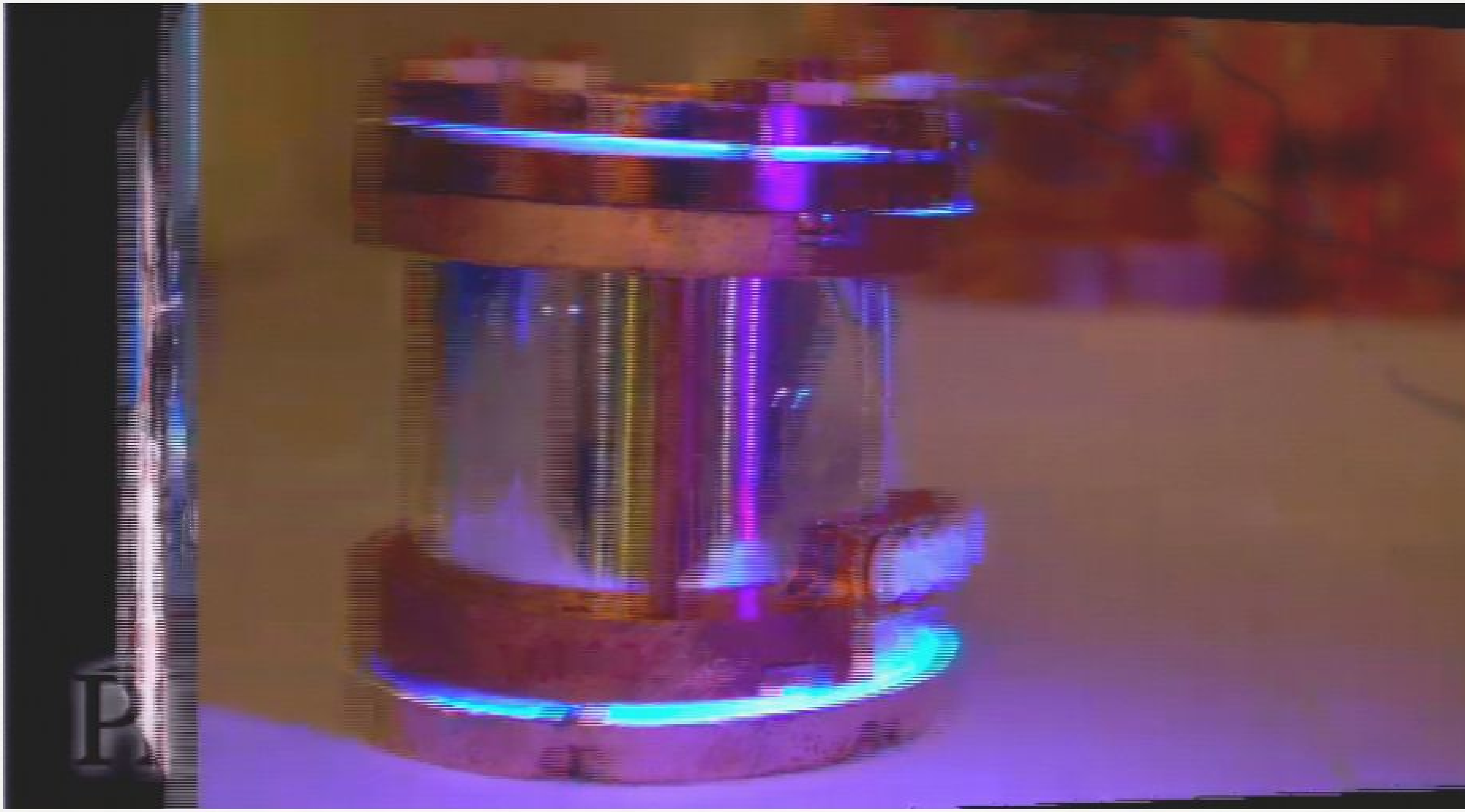

Video from the Perimeter Institute

DEAP/CLEAN Program

DEAP-1:

- 7 kg prototype experiment
- Run at Queen's for background suppression studies and calibration. (published)
- Now at SNOLAB for continued background studies, DM search

microCLEAN:

- 4 kg prototype run with LAr and LNe at Yale
- Backgrounds and calibrations measured (published).

DEAP-3600:

- 3600 kg liquid argon for dark matter
- primary emphasis of Canadian collaborators in short term

miniCLEAN-360:

- 360 kg liquid argon for dark matter and prototyping neon for particle astrophysics
- primary emphasis of US collaborators in short term

DEAP/CLEAN Program

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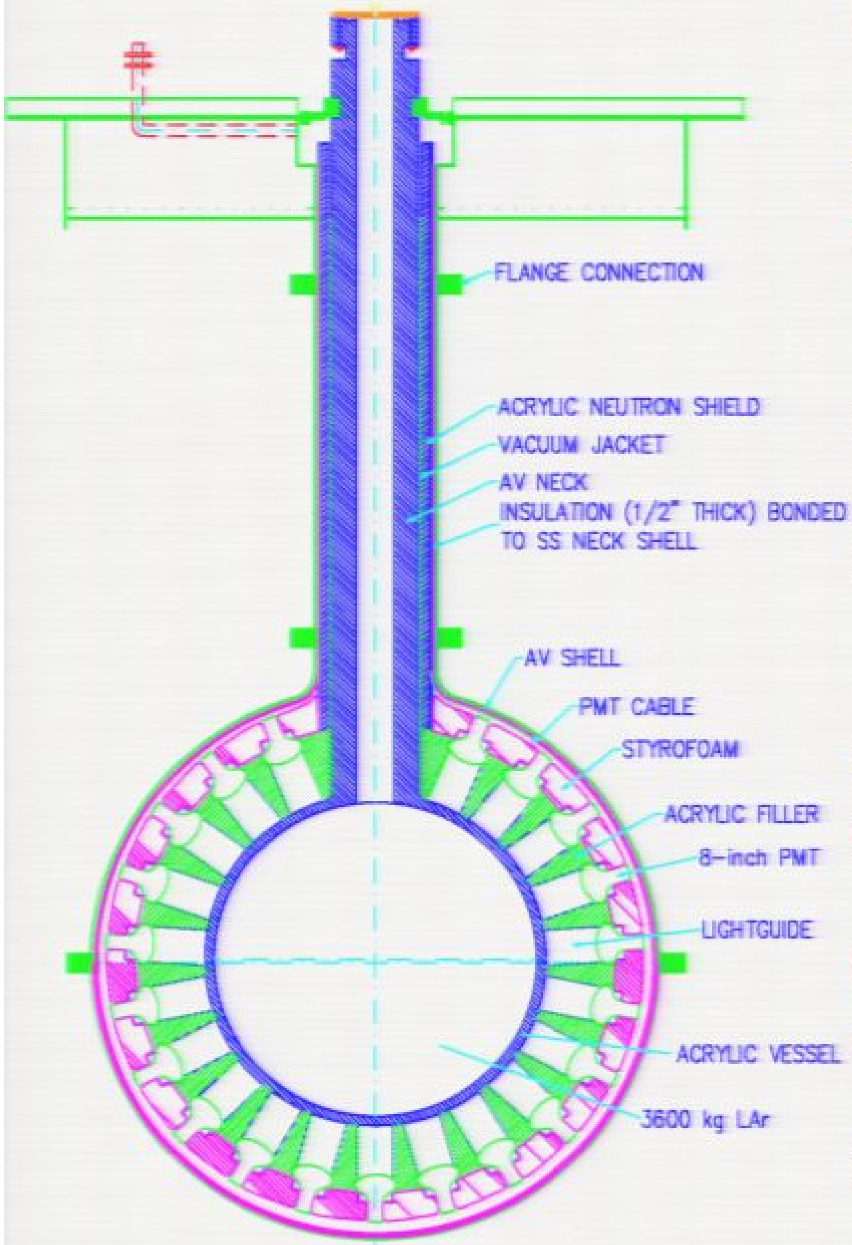
DEAP 3600 detector



Design is driven by background requirements

85 cm radius acrylic sphere contains
3600 kg LAr
(55 cm, 1000 kg fiducial)

266 8" PMTs (warm)

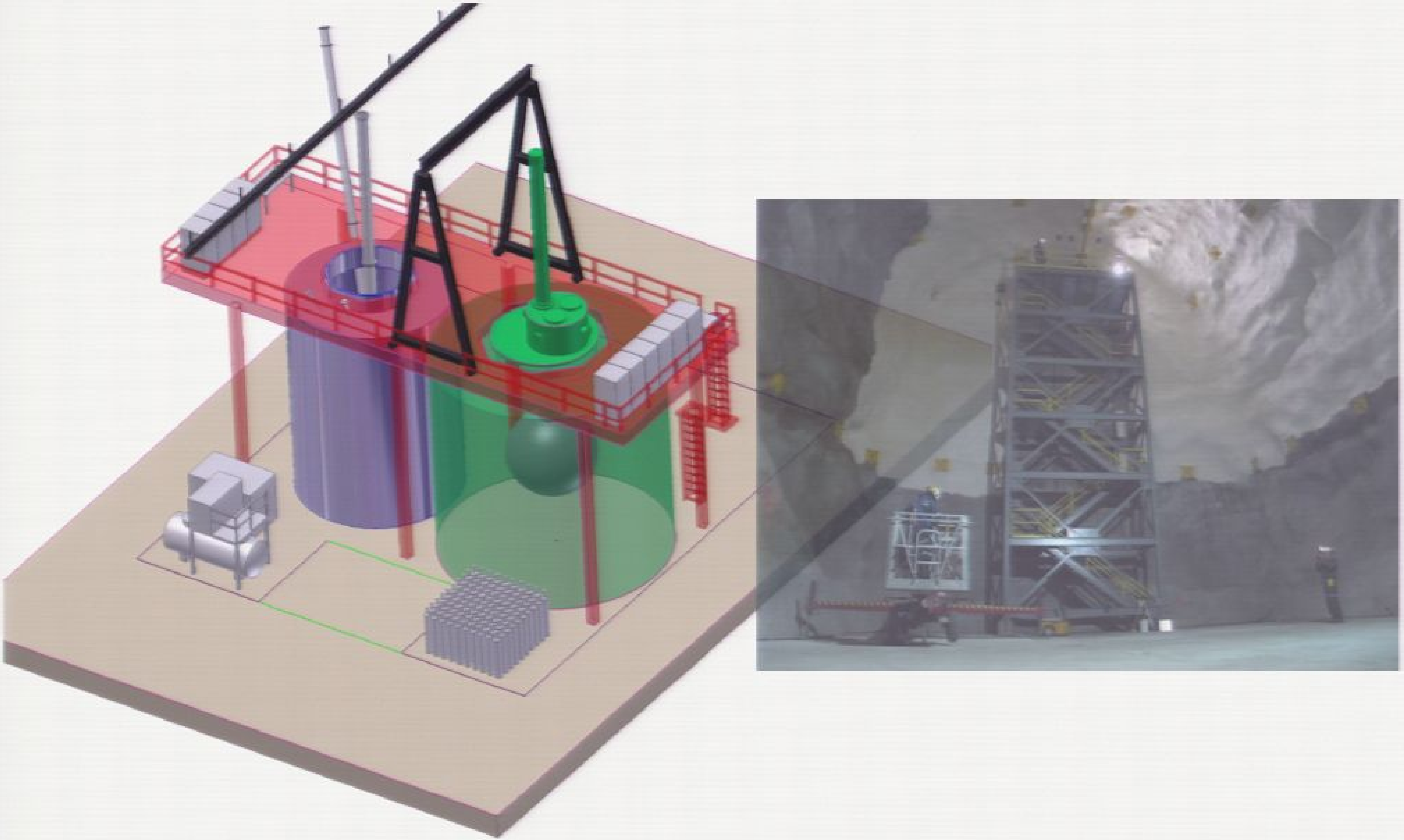
50 cm acrylic light guides and fillers for neutron shielding (from PMTs)

Steel shell for safety to prevent cryogen/water mixing (AV failure)

Only LAr, acrylic, and WLS (10 g) inside of neutron shield (minimizes Rn emanation)

8.5 m diameter water shielding tank

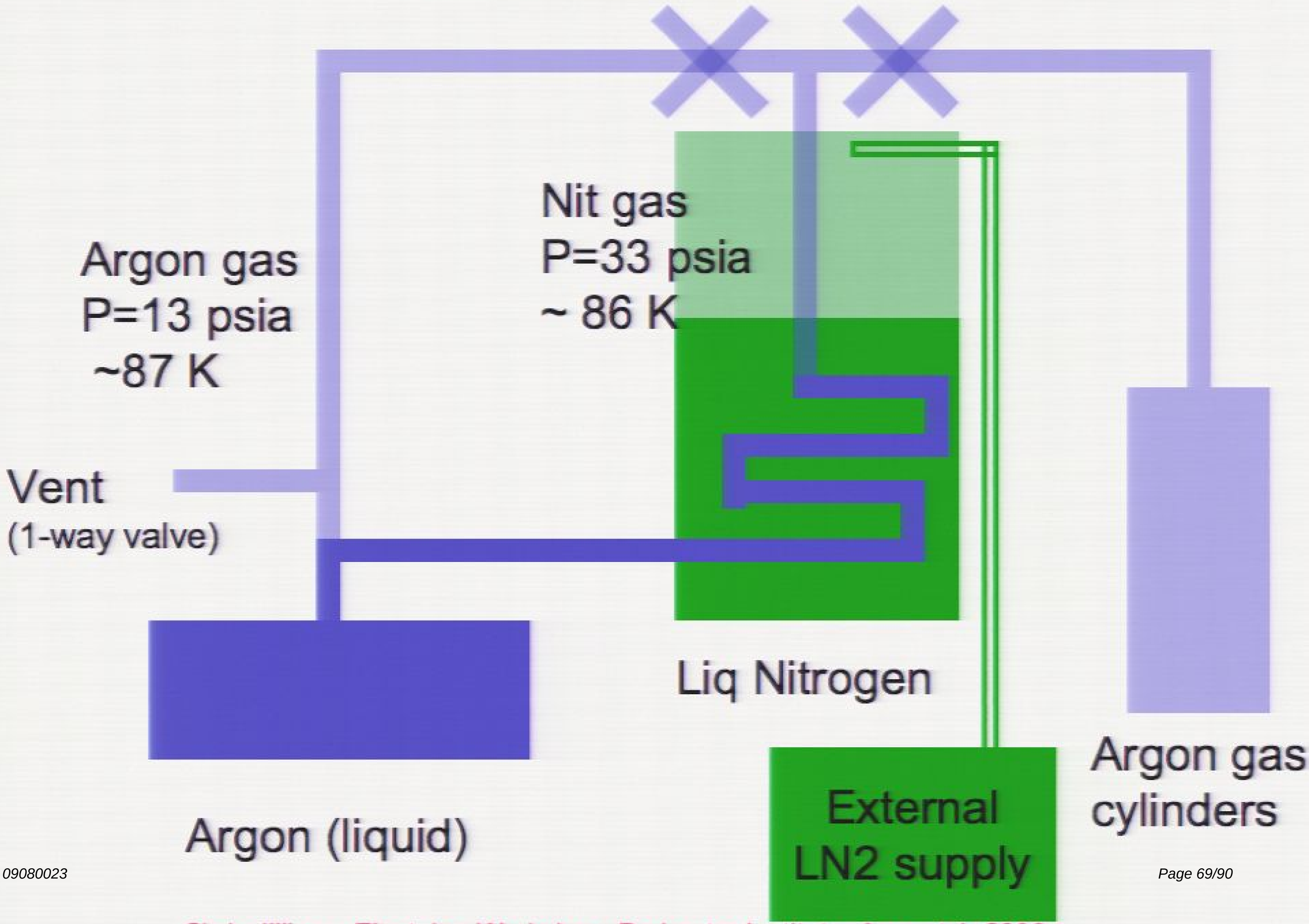
DEAP-3600 and miniCLEAN-360



An Optimists View

- The LHC will discover a new weakly-interacting particle - maybe supersymmetric, maybe not.
- The properties of that particle will jive with what is need for dark matter: heavy, but not too heavy; correct interaction strengths to get produced I the right amounts.
- Experiments at SNOLAB and other underground facilities will measure the number of these particles in our galaxy.
- We will then be able to understand ~25% of our universe.

A Few Basics


Nit gas
P=33 psia
~ 86 K
Argon gas
P=13 psia
~87 K
Vent
(1-way valve)
Liq Nitrogen
Argon gas
cylinders
External
LN2 supply
Argon (liquid)

A Day in the Life

- I want to describe a day that ended up going very well but had lots of problems in it. I think it illustrates a lot of what goes on when we work well.

- This day was a day on the DEAP-1 prototype detector.

- DEAP-1 is a testbed for many technologies that will be used on the larger scale DEAP-3600 detector.

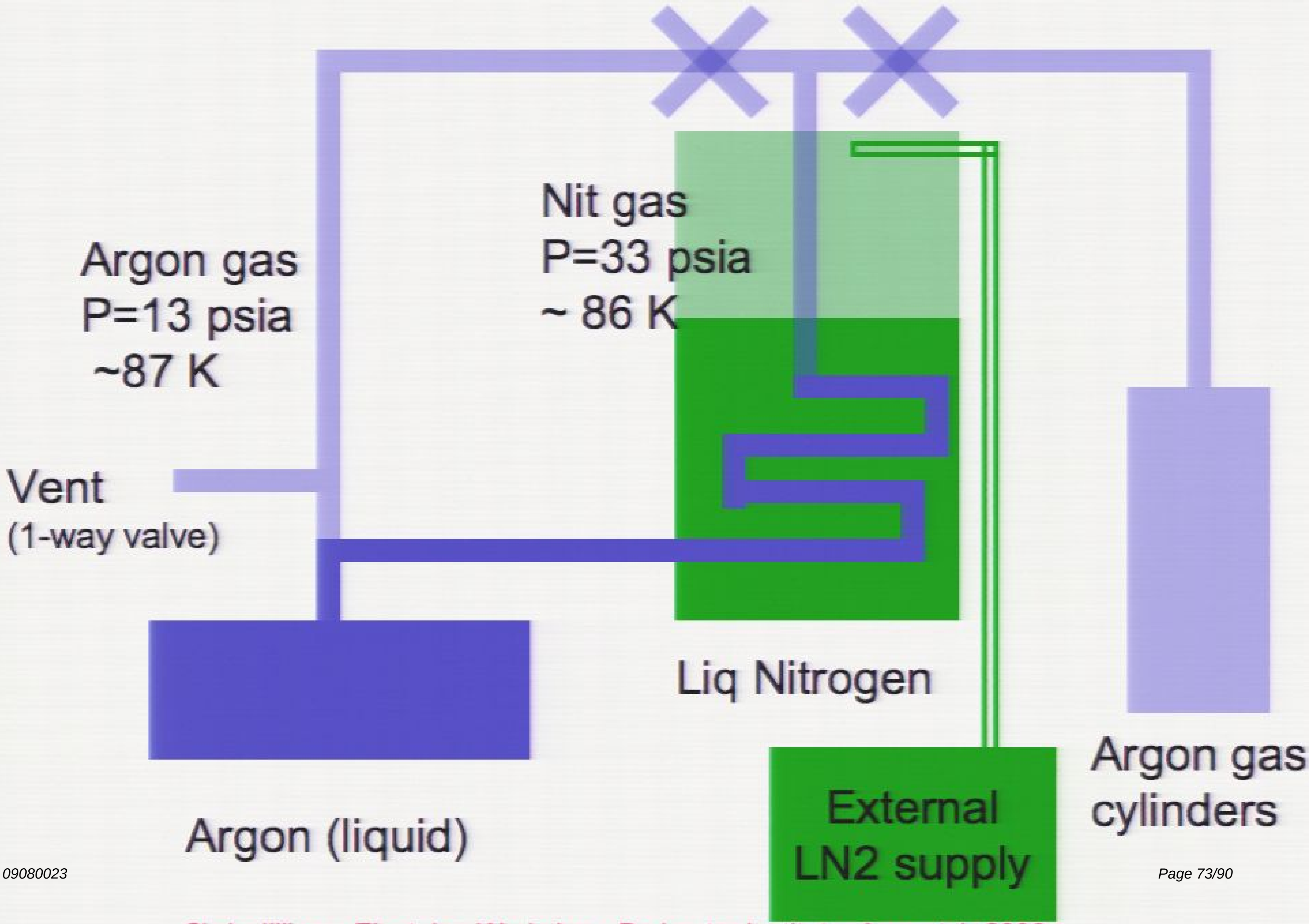
An Optimists View

- The LHC will discover a new weakly-interacting particle - maybe supersymmetric, maybe not.
- The properties of that particle will jive with what is need for dark matter: heavy, but not too heavy; correct interaction strengths to get produced I the right amounts.
- Experiments at SNOLAB and other underground facilities will measure the number of these particles in our galaxy.
- We will then be able to understand ~25% of our universe.

A Day in the Life

- I want to describe a day that ended up going very well but had lots of problems in it. I think it illustrates a lot of what goes on when we work well.
- This day was a day on the DEAP-1 prototype detector.
- DEAP-1 is a testbed for many technologies that will be used on the larger scale DEAP-3600 detector.

A Few Basics



Nit gas
P=33 psia
~ 86 K
Argon gas
P=13 psia
~87 K
Vent
(1-way valve)
Liq Nitrogen
Argon gas
cylinders
External
LN2 supply
Argon (liquid)


11.09.2007


12.04.2007

On July 3

- DEAP-1 was pumped down to high vacuum.
- DEAP-1 was room temperature
- We started filling DEAP-1
- Argon hit the warm surfaces and boiled and vented
- We left it filling
- Overnight DEAP-1 started to get cold and started to fill

July 4, 2008

- 3 of us for DEAP caught 6:30 cage underground.
- In lab at 7:15.
- System to refill LN2 system failed. Quickly swap 2 dewars of LN2.
  - Why did one of the two dewars on the system fail.
  - Guessed reason. Fixed. Too bad we guessed wrong.

LN2 Fail and Fix

Re-tested system - while filling detector.
Heard horrible clicking sound here

Valve in Backwards!

No manual valve between bad valve and volume of LN2 for cooling.

Meanwhile

- As detector filled with argon we were able to start measuring signals.
- Automated data analysis system was recording data and shipping it to a computer farm (HPCVL) at Queen's University.
- Automated analysis started.

- Summer student made a series of plots from data as it streamed in (as did postdoc at Queen's) - Many phone calls.
- We knew within 1-2 hours the data were still good and argon pure.

Fill Finished About 1PM

- Surprisingly close to schedule given that liquid nitrogen fail effectively stopped fill for 2 hours.
- 4 hours to prepare shielding. We use plastic wand water to surround detector to keep backgrounds away.

- Fit plastic blocks around bits of plumbing.
- Stack 22-litre boxes of water.

What About Prep and Planning?

We had lots: there was 10 pages of thought-out procedure carried out on July 3. That is why we succeeded despite mistakes.

DEAP Procedure Manual
Date: June 27, 2008

Version: 2.0
Section: 6.6
Page: 6.6-1

6.6 Checklist For Argon Vacuum State

Checklist carried out by: Chris Sillings	Date and time: July 3, 2008 9:35 AM

This checklist is designed to ensure the argon system is at vacuum and sealed and ready for cooldown.

Valve	Expected Setting	Actual Setting
MV-001	Closed	Open - Closed
MV-002	Closed	Open - Closed
Regulator Setting	Closed	Closed
Regulator Outlet Valve	Closed	Open - Closed
MV-005	Closed	Open - Closed
MV-006	Closed	Open - Closed
MV-007	Closed	Open - Closed
MV-009	Closed	Open - Closed
Getter	Bypass	Off / On / Bypass / Purifying
Gas flow setpoint (slow controls)	0	✓
Actual gas flow	~0	✓
MV-031	Open	Open - Closed
PCV-032	70-90 psig	80 psig
MV-028	Closed	Open - Closed
MV-029	Notch 5	Open - Notch 1 - 2 - 3 - 4 - 5 - 6
MV-030	Closed open	Open - Closed - Removed
VTP-003	Removed	Off - At Speed - Fault - Removed
VRP-004	Removed	Off - On - Removed
Flute Temperature	> 260 K	✓
Neck Temperature	> 260	✓
Window 1 Temperature	> 260	✓
Window 2 Temperature	> 260	✓
Neck heater check done?		✓
PMT A heater check done?		✓
PMT B heater check done?		✓

* To test all-metal valve during fill

DEAP Procedure Manual
Date: July 3, 2008

Version: 2.0
Section: 5.16
Page: 5.16-1

5.16 Cooldown Procedure: Argon System

Procedure carried out by: Chris, Hugh & Shawn	Date and time: July 3, 2008 13:05

5.16.1 Purpose

This is the detector fill procedure. Start this procedure about 2PM to finish the following morning.

Do not under any circumstances do anything to the argon high-pressure manifold without approval of the principal investigator or the detector manager.

5.16.2 Pre-Fill Checks

1. Verify that PMT heaters are installed and working. (*Individually* turn on the heaters and look for a rise in temperature.) — See checklist 6-6
2. Verify that neck heater is installed and working. (Set heater to 38C and verify the temperature goes there.) — Was @ 44C. Turned to 40 C.
3. Carry out the argon-vacuum checklist (6.6). — See 9:35 AM checklist
4. Verify that the LN2 supply dewar is nearly full and pressurized.
5. Ensure the liquid nitrogen temperature is 87K. Colder than this and we risk freezing argon. (82 K)
6. Make sure that neck heater is set to control temperature to 25-30 C. **Monitor the neck and PMT temperature.** The neck temperature is read off the meter on top of teh slow-controls rack.
7. The SAES getter heater should be at *Set* temperature and the *Life Status Good*.

5.16.3 Procedure

5.16.3.1 Start of Fill

1. Fully decrease PCV-003 to zero pressure. — already
2. Set flow-control using slow-controls screen to 5 slpm.
3. Open valve MV-001.
4. Open valve MV-002.
5. Ensure the SAES valves are in purification mode. — Done (was bypass)
6. Ensure MV-009 is open.

13:13 Opened MV-002, opened MV-009 13:15 Opened MV-005

Fill Finished About 1PM

- Surprisingly close to schedule given that liquid nitrogen fail effectively stopped fill for 2 hours.
- 4 hours to prepare shielding. We use plastic wand water to surround detector to keep backgrounds away.

- Fit plastic blocks around bits of plumbing.
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MV-005	Closed	Open - Closed
MV-006	Closed	Open - Closed
MV-007	Closed	Open - Closed
MV-009	Closed	Open - Closed
Getter	Bypass	Off / On / Bypass / Purifying
Gas flow setpoint (slow controls)	0	✓
Actual gas flow	zero	✓
MV-031	Open	Open - Closed
PCV-032	70-90 psig	80 psig
MV-028	Closed	Open - Closed
MV-029	Notch 5	Open - Notch 1 - 2 - 3 - 4 - 5 - 6
MV-030	~~Closed~~ open	Open - Closed - Removed
VTP-003	Removed	Off - At Speed - Fault - Removed
VRP-004	Removed	Off - On - Removed
Flute Temperature	> 260 K	✓
Neck Temperature	> 260	✓
Window 1 Temperature	> 260	✓
Window 2 Temperature	> 260	✓
Neck heater check done?		✓
PMT A heater check done?		✓
PMT B heater check done?		✓

* To test all-metal valve during fill

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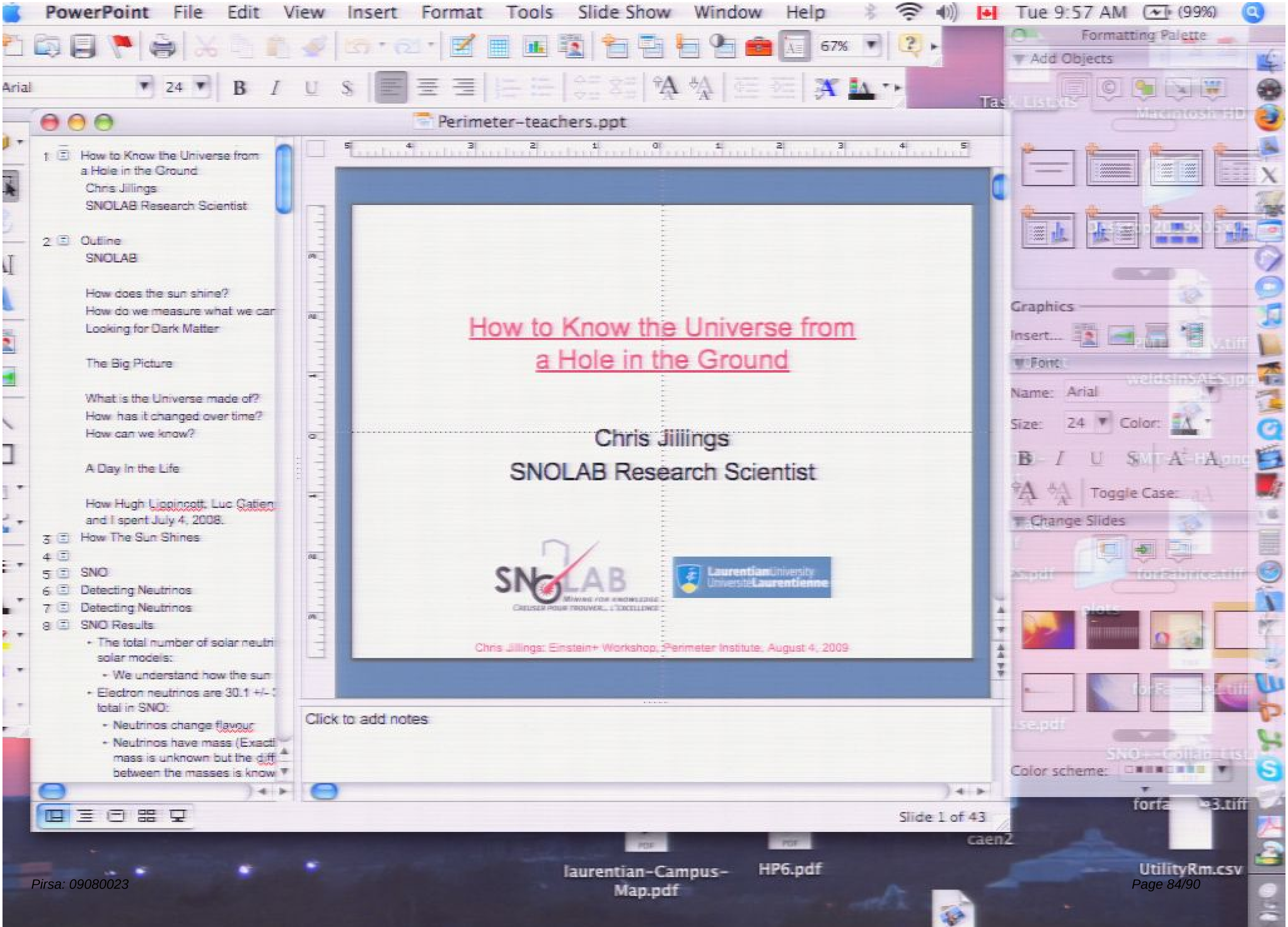
4. Open valve MV-002.
5. Ensure the SAES valves are in purification mode.
6. Ensure MV-009 is open.

Why Did I Pick This Day?

- I worked with
  - a grad student (Hugh from Yale)
  - an undergrad (Luc, L.U. student)
  - with people off-site (Bei at Queen's)
  - with engineers and techs.
- We
  - Plumbed (fixing the LN2 fail)
  - Followed procedures very carefully
  - Improvised
  - Took data
  - Analyzed data

It is many more ordinary days rolled into one.

And one of the reasons I really like my job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How to Know the Universe from a Hole in the Ground
Chris Jillings
SNOLAB Research Scientist
Outline
SNOLAB
How does the sun shine?
Looking for Dark Matter
The Big Picture
What is the Universe made of?
How has it changed over time?
How can we know?
A Day In the Life
How The Sun Shines
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Detecting Neutrinos
Detecting Neutrinos
SNO Results
Neutrinos change flavour
How to Know the Universe from
a Hole in the Ground
Chris Jillings
SNOLAB Research Scientist
Chris Jillings: Einstein+ Workshop, Perimeter Institute, August 4, 2009
Click to add notes
Slide 1 of 43
Formatting Palette
Add Objects
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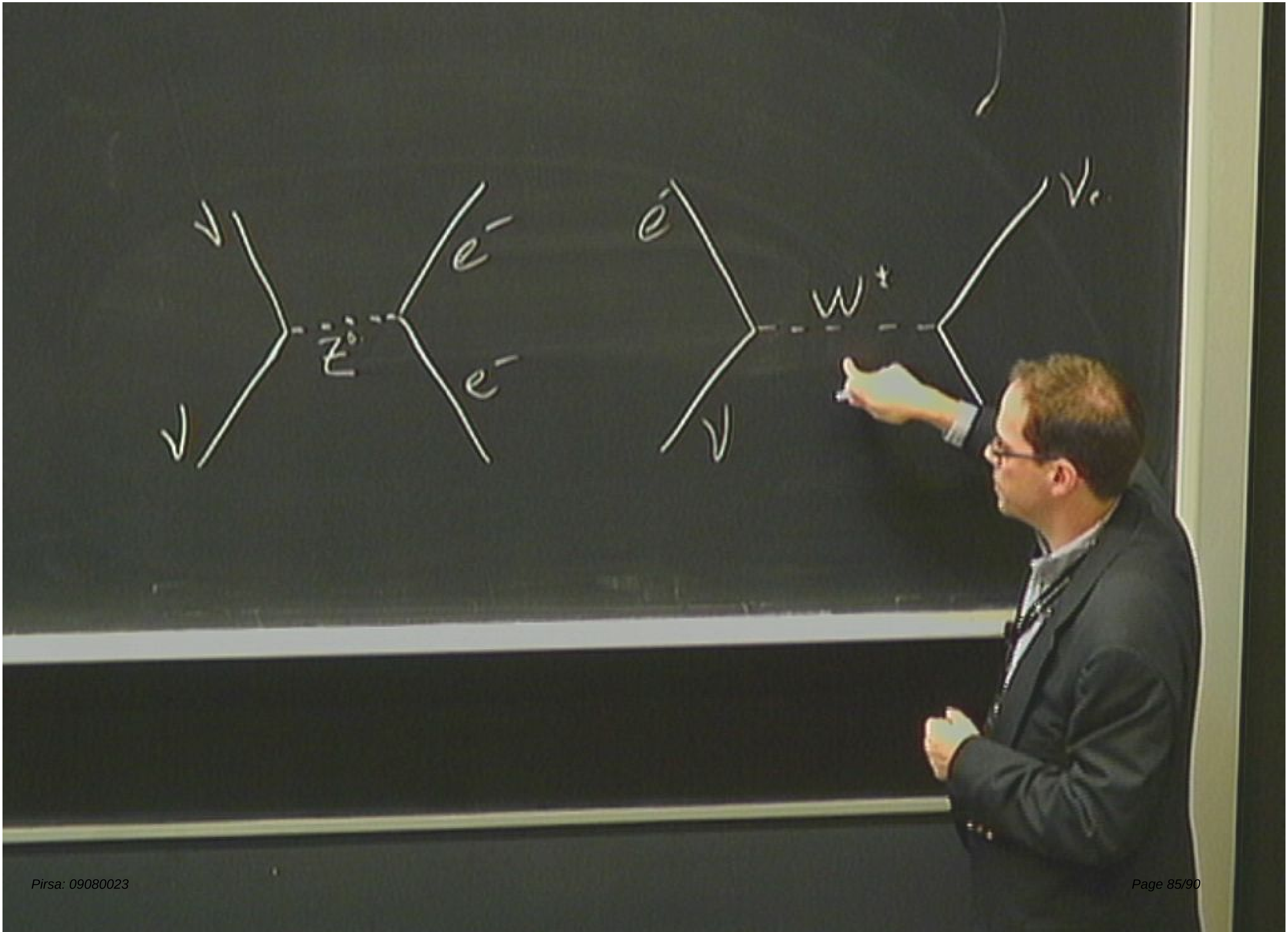
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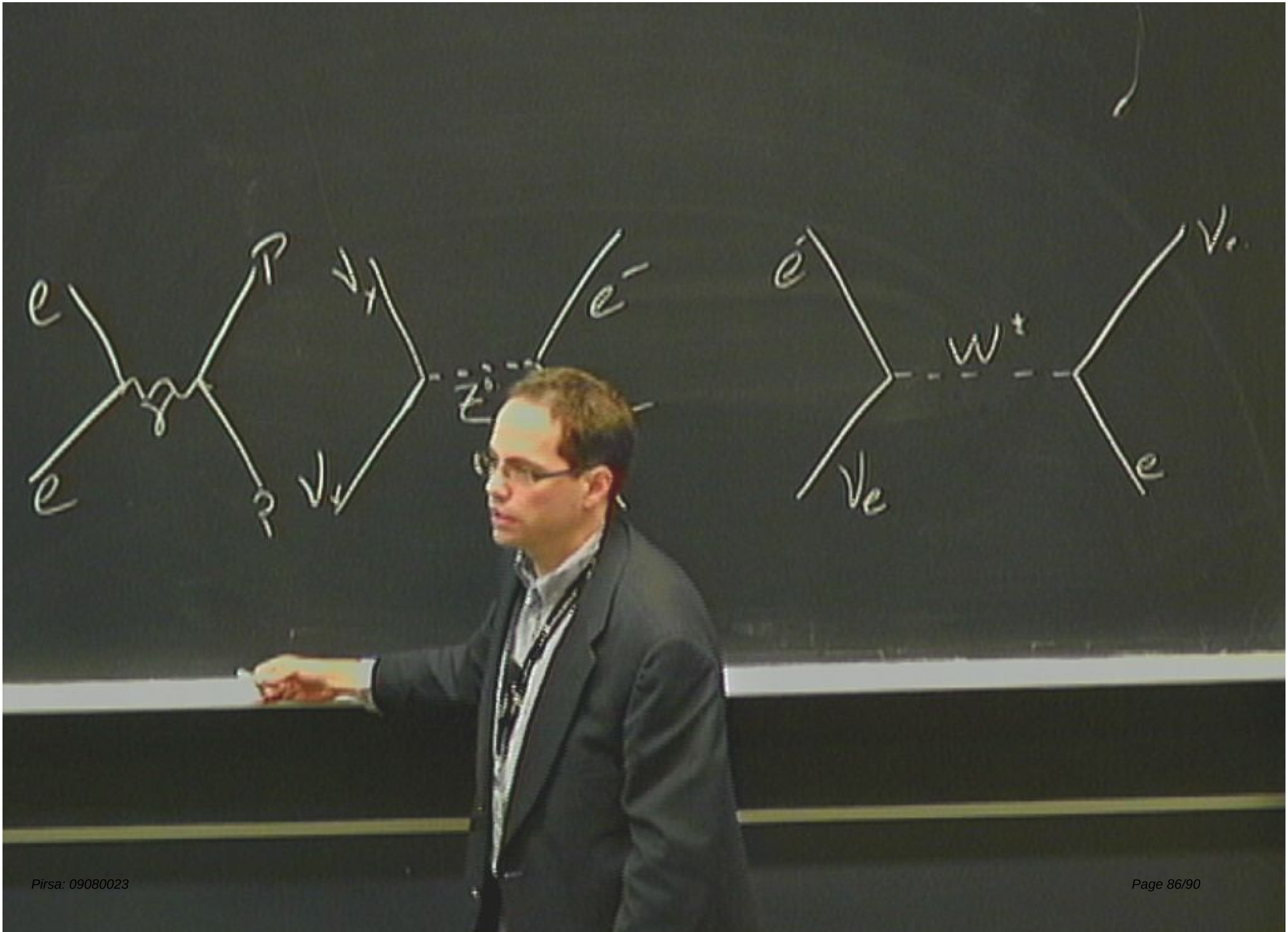

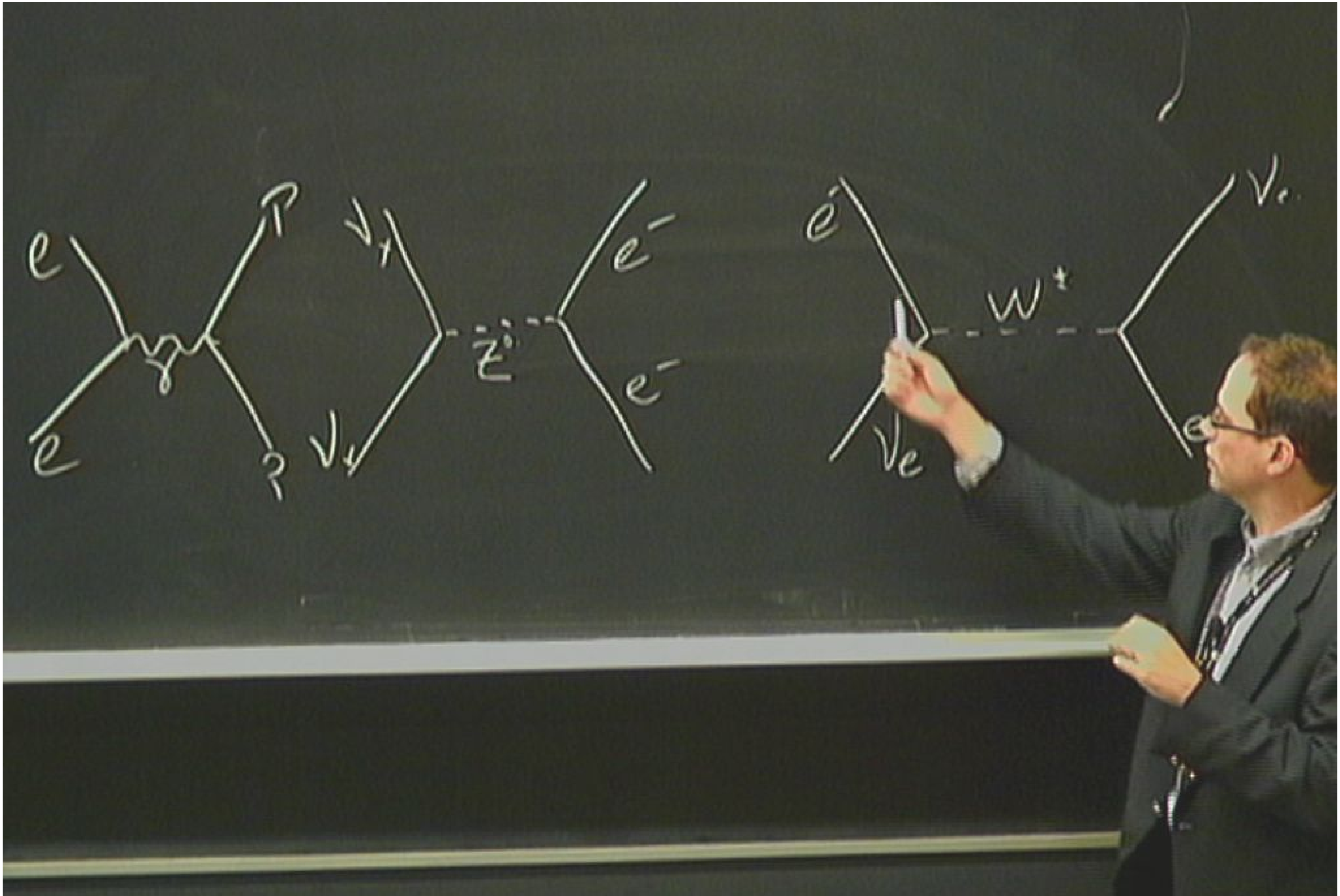

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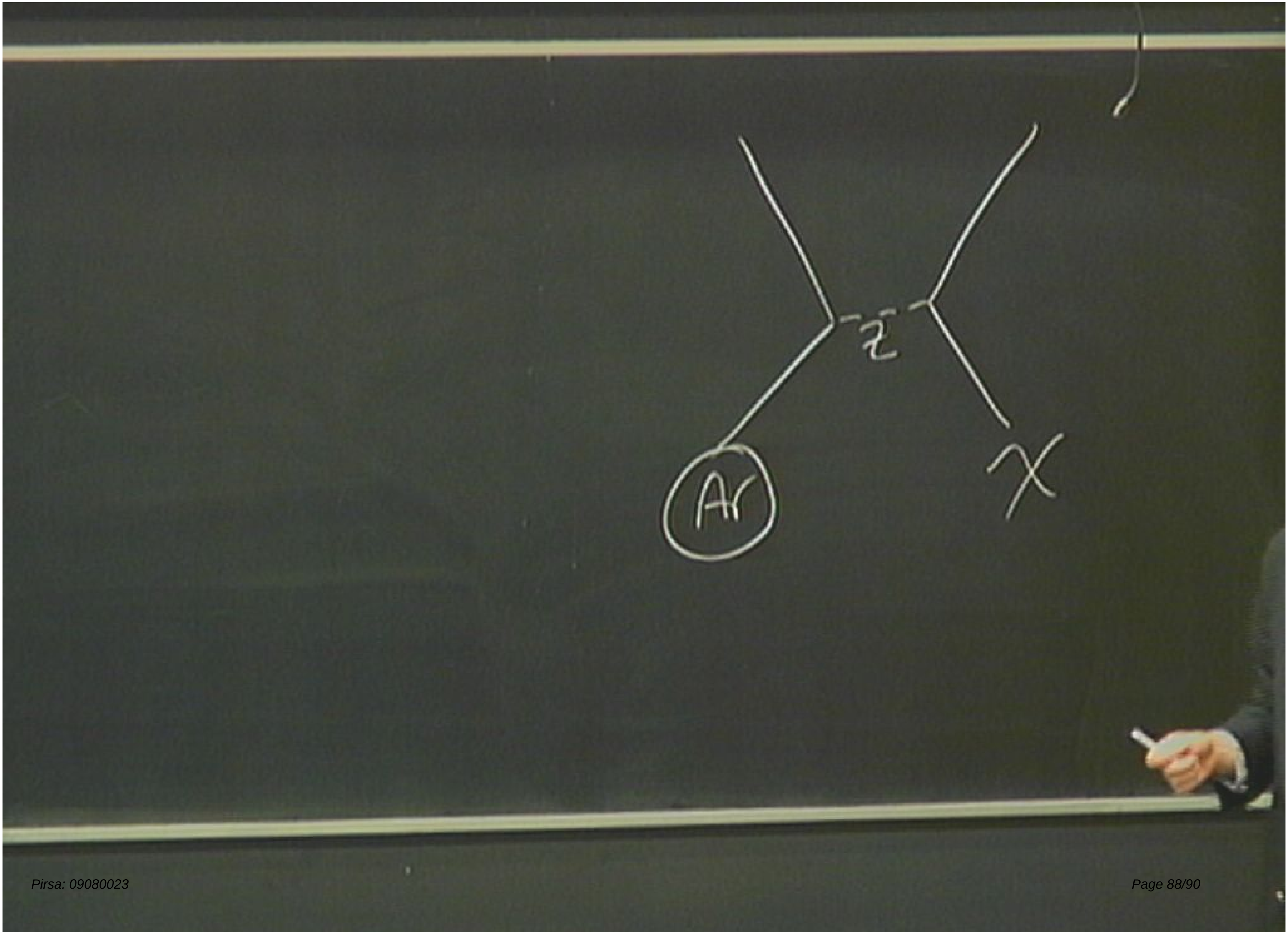
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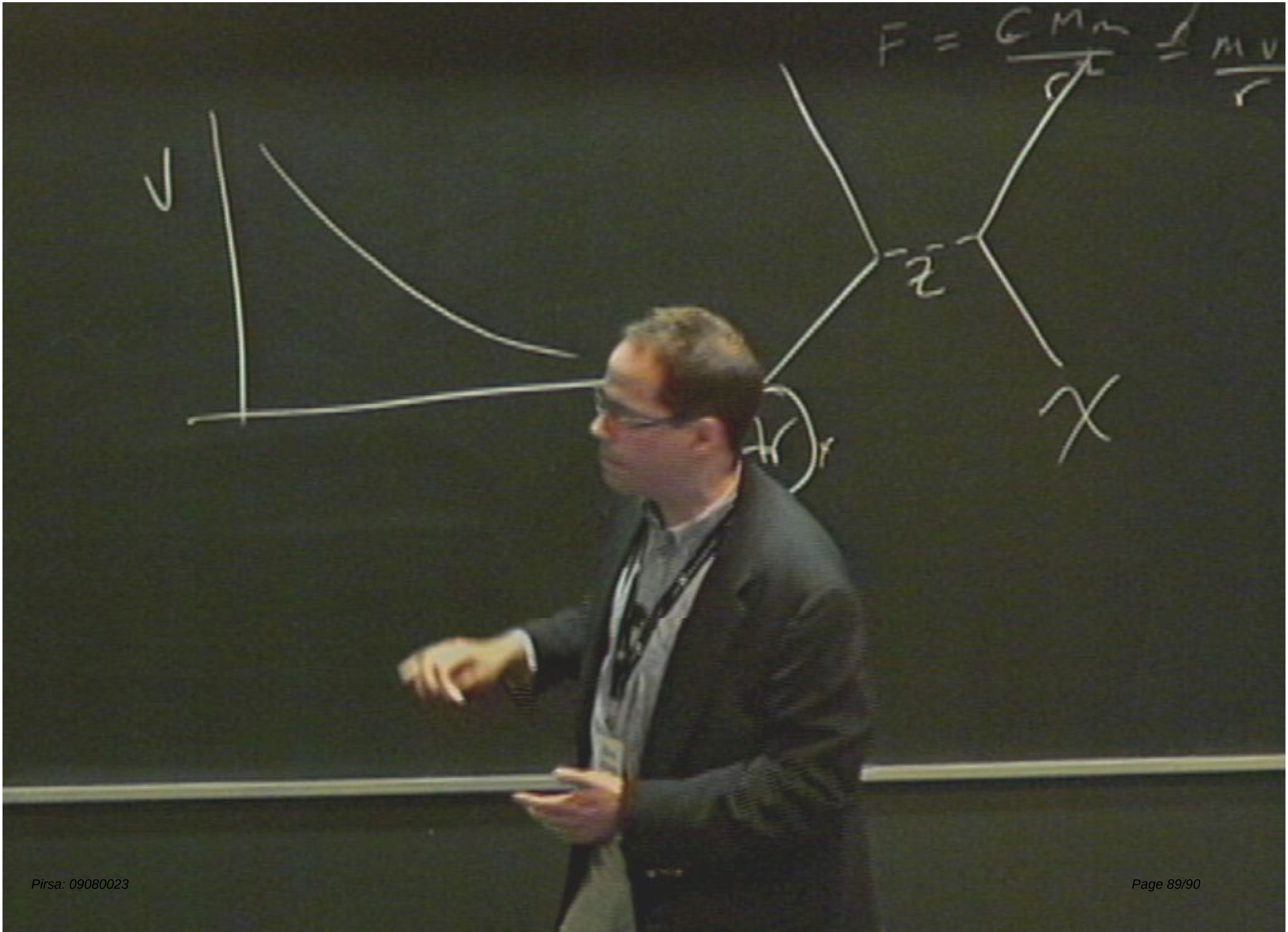

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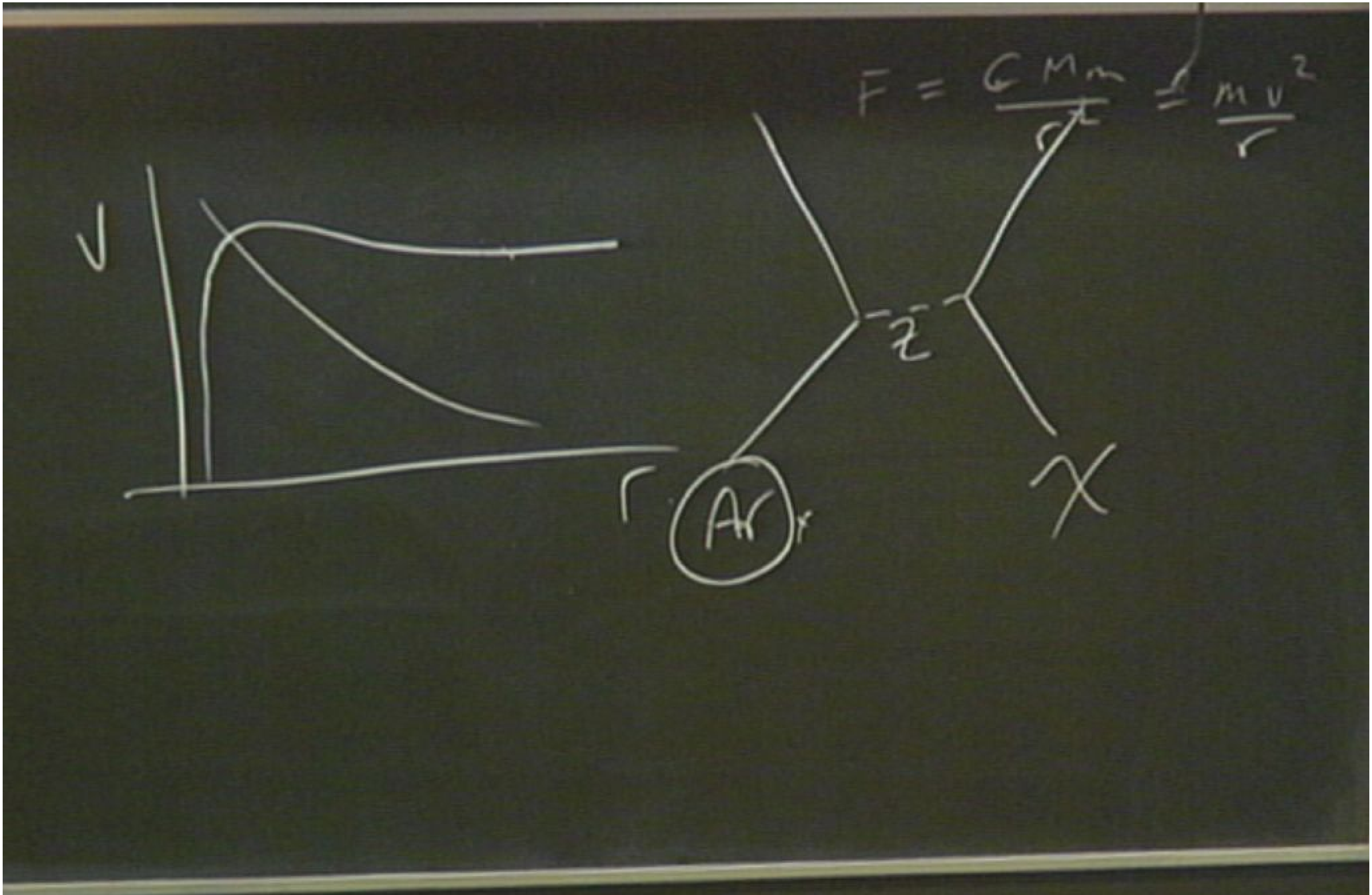
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