Title: Entanglement detection with bounded reference frames

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Abstract: Violation of local realism can be probed by theory–independent tests, such as Bell’s inequality experiments. There, a common assumption is the existence of perfect, classical, reference frames, which allow for the specification of measurement settings with arbitrary precision. However, if the reference frames are ``bounded'', only limited precision can be attained. We expect then that the finiteness of the reference frames limits the observability of genuine quantum features. Using spin coherent states as reference frames, we determined their minimal size necessary to violate Bell’s inequalities in entangled systems ranging from qubits to macroscopic dimensions. In the latter, the reference frame’s size must be quadratically larger than that of the system. Lacking such large reference frames, precludes quantum phenomena from appearing in everyday experience.

Entanglement detection with bounded reference frames

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Reference:

F. Costa, N. Harrigan, T. Rudolph, Č. Brukner,
arXiv:0902.0935

Perimeter Institute,
August '09

Reference frames in quantum mechanics

Reference frames in quantum mechanics

- Physical states are defined relative to a reference frame

  e.g. position with respect to a Cartesian frame, time with respect to a clock, etc...

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- What if the reference frame itself is a quantum system?

  Relative properties between quantum mechanical systems

Outline

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- Reference frames in quantum mechanics: a brief review.

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- GHZ states and Mermin inequalities.
- Higher spins and classicality.
- Conclusions.

Comunicating a direction without a shared reference frame

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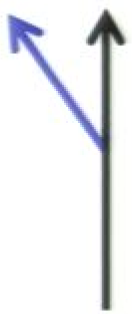
Relative states can be encoded without an external reference frame

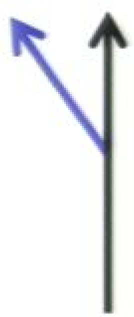
Formalizing reference frames



$$|\psi\rangle$$

Formalizing reference frames

Change of reference frame induces a passive transformation



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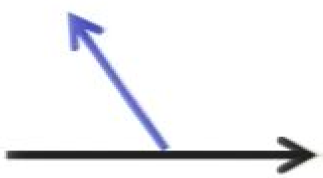

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Transformations of a RF form a group

e.g. rotations-SU(2)

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e.g. rotations-SU(2)

Passive transformations are a representation of the group
e.g. spin

Quantum states in absence of a reference frame

$$\rho$$

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Example: 3 spin-1/2 particles with no external RF

$\uparrow \quad \uparrow \quad \uparrow$

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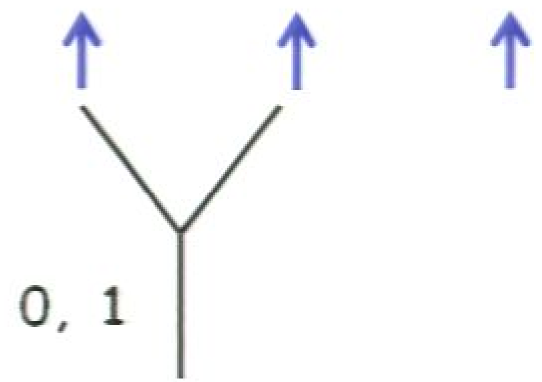

The total spin of any subset can be measured with no external RF

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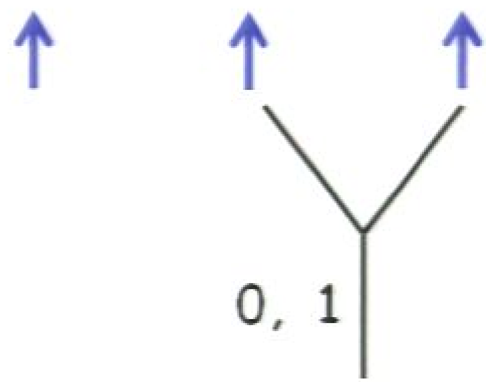

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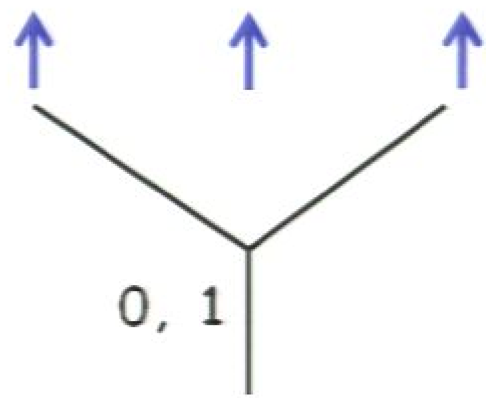

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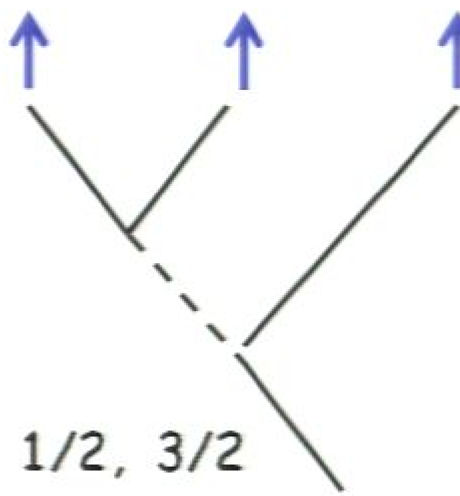

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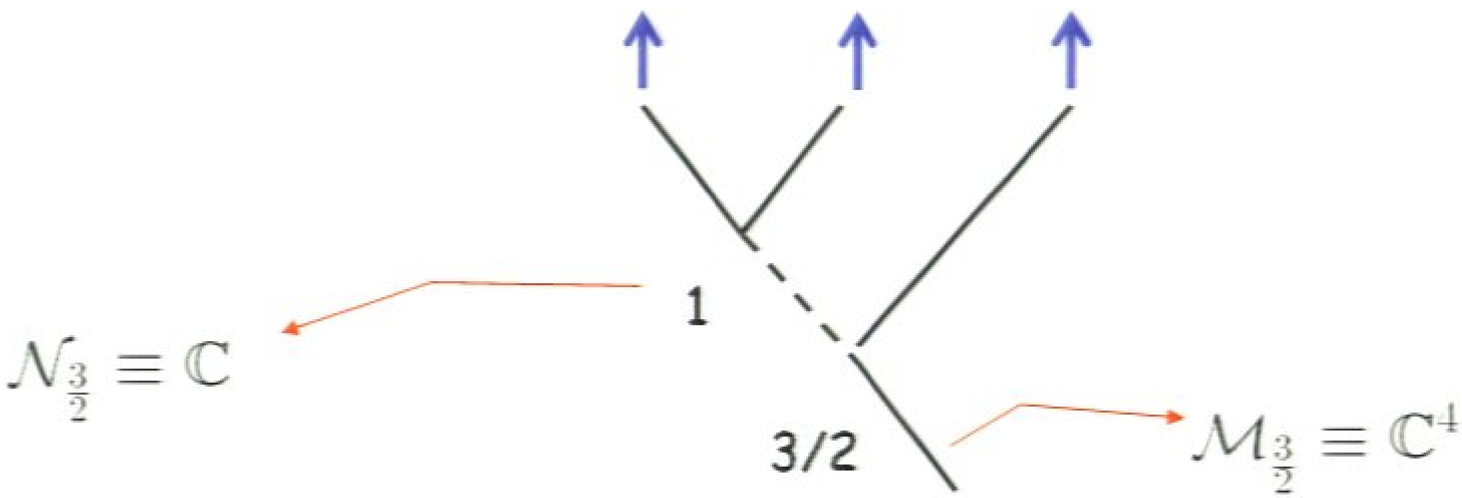

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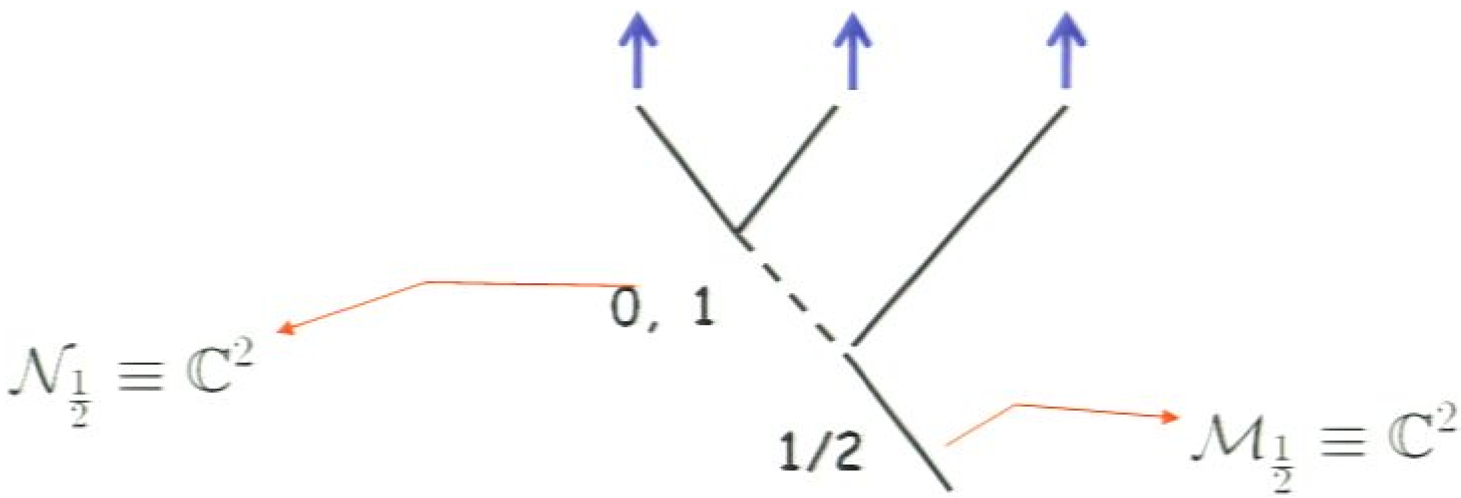

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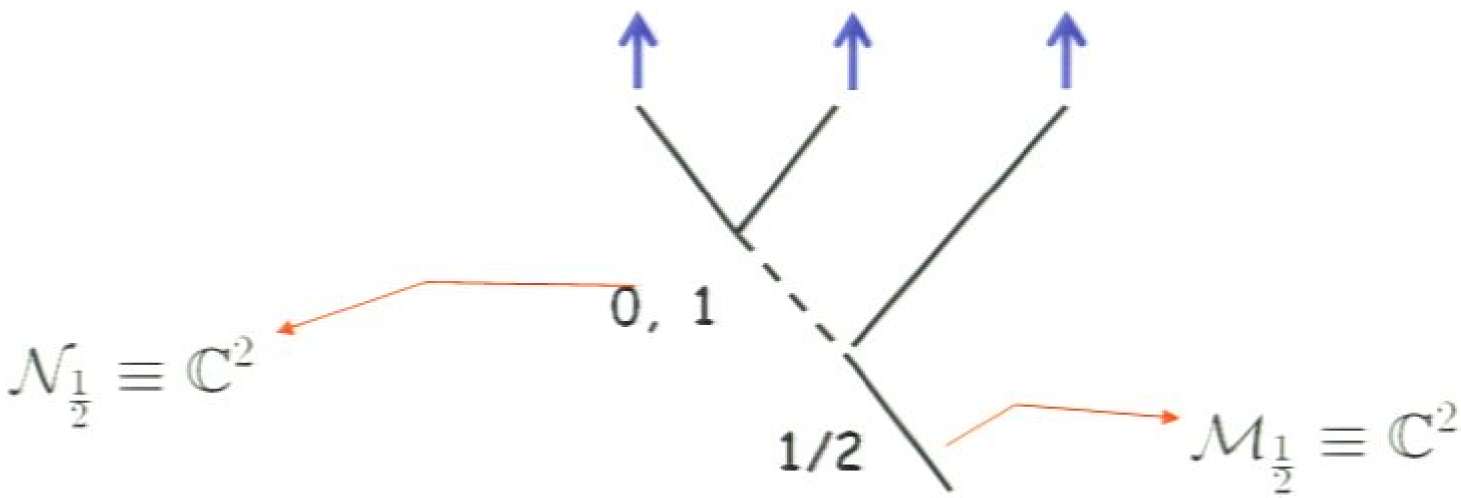

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Quantum reference frames

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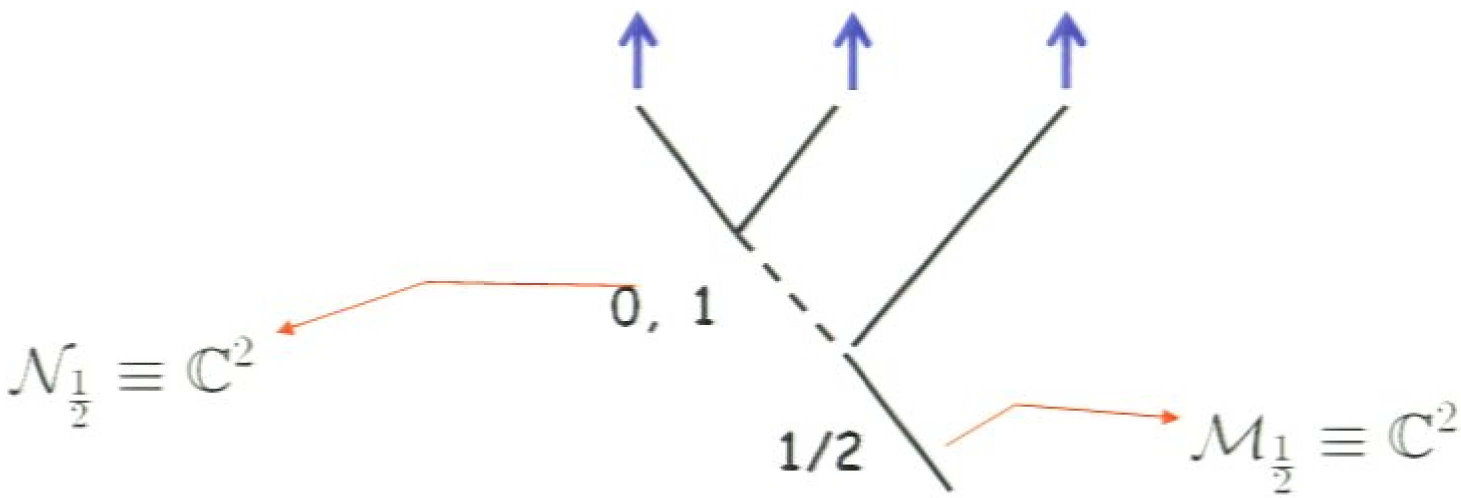

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e.g. q = total spin

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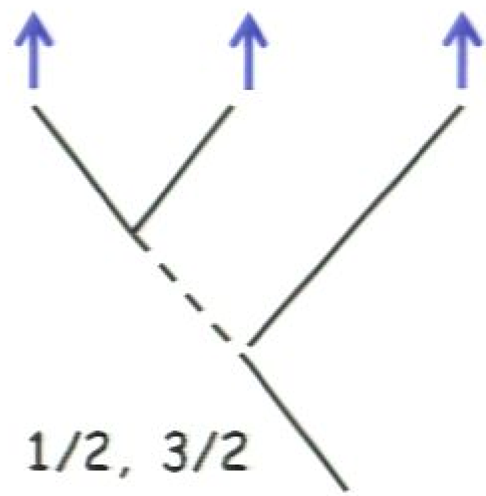

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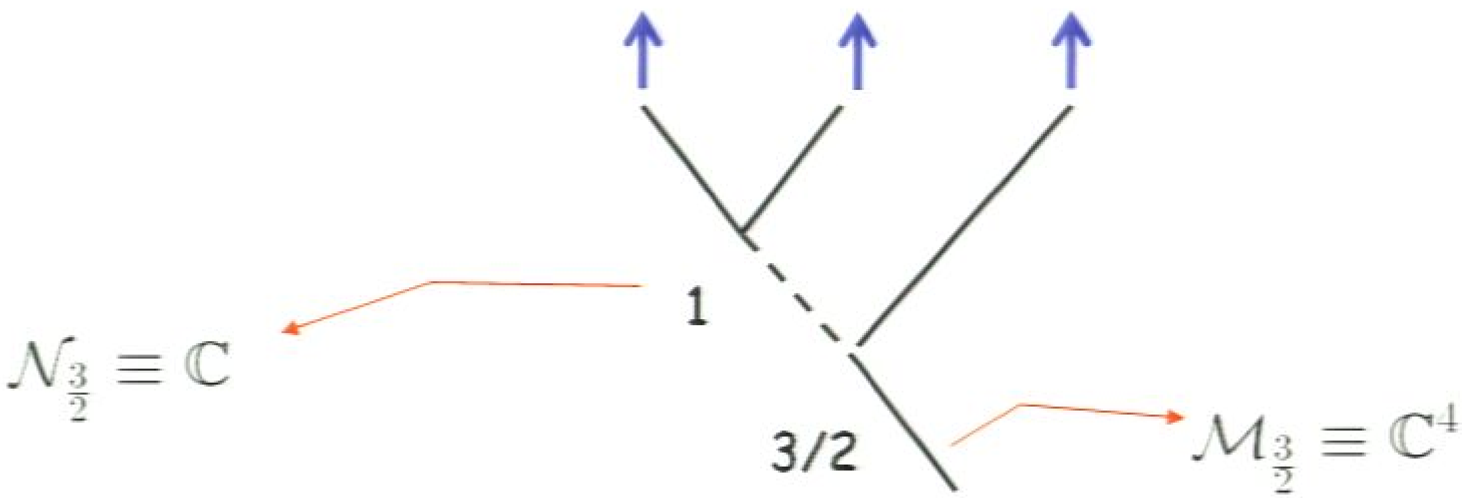

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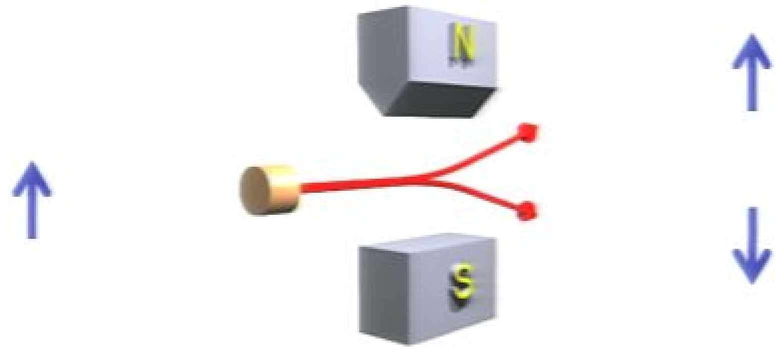
Quantum reference frames

Quantum reference frames

Measure the projection of a spin ½ particle along a given axis



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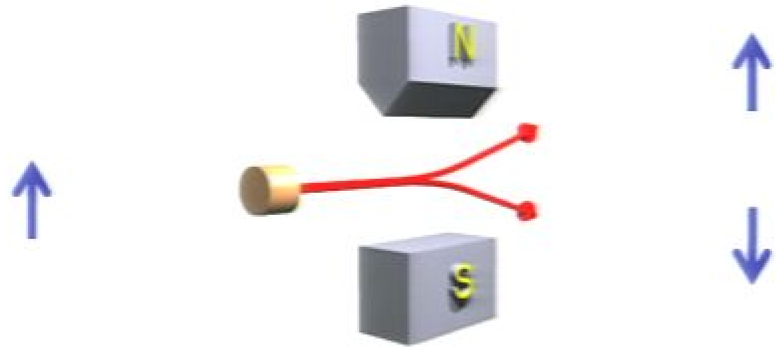

Use a spin j coherent state as reference frame

$|j, j\rangle$

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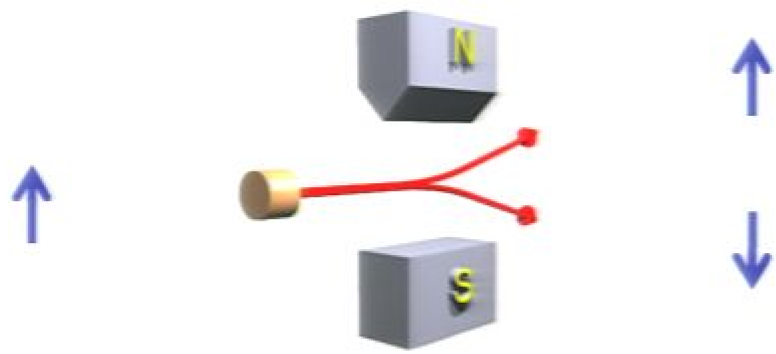

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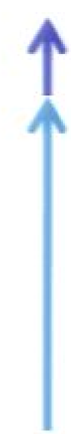

$|j+\frac{1}{2}, j+\frac{1}{2}\rangle$

We measure the **total** angular momentum of the **joint state**

The total angular momentum can be either $j+\frac{1}{2}$ or $j-\frac{1}{2}$

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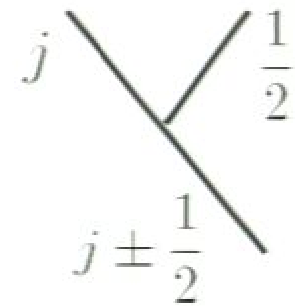
We can interpret:



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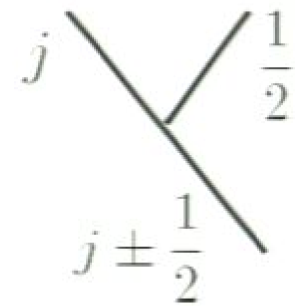

$j + \frac{1}{2}$ $\longrightarrow$ Spin was up

$j - \frac{1}{2}$

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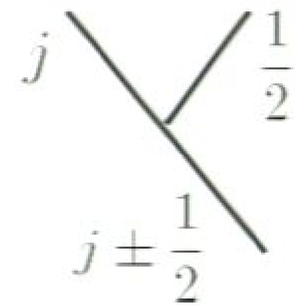

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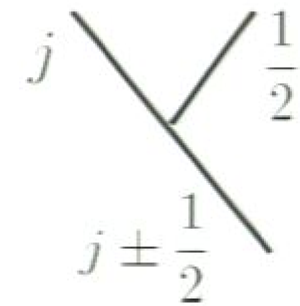

$j + \frac{1}{2}$ → Spin was up

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But we have three possible states for the joint system:

→ $|\frac{1}{2}\rangle \otimes |j, j\rangle$

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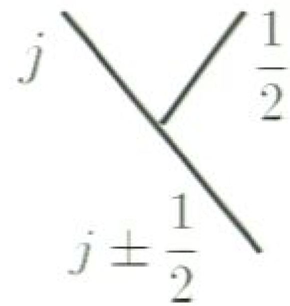

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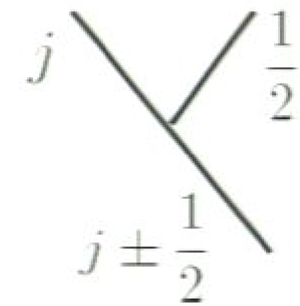

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In one case we guess wrong!

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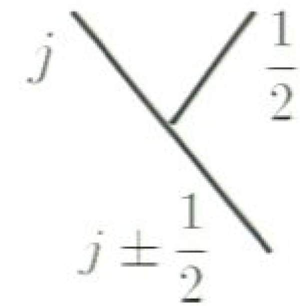

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In one case we guess wrong!

The probability for the wrong guess vanishes for $j \to \infty$

(Classical reference frame)

Spin measurement along „direction“

Possible Outcomes: $j = |j_{RF} - j_S|, \ldots, j_{RF} + j_S$

Outcome: $j = j_{RF} + m$

Projector: $\hat{\Pi}_{j_{RF}+m} = \sum_{m'=-j_{RF}-m}^{j_{RF}+m} |j_{RF} + m, m'\rangle\langle j_{RF} + m, m'|$

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Quantum Reference Frame in a coherent state
Effective POVM:

$$\hat{P}^{j_{RF}}_{j_S\, m} = \langle j_{RF}|\hat{\Pi}_{j_{RF}+m}|j_{RF}\rangle \xrightarrow{j_{RF} \to \infty} |j_S, m\rangle\langle j_S, m| \equiv \hat{\Pi}_m$$

↑ Von Neumann spin projection

Violation of Bell´s inequalities with quantum reference frames

$$S := \left| E(\vec{\alpha}_1, \vec{\beta}_1) + E(\vec{\alpha}_1, \vec{\beta}_2) + E(\vec{\alpha}_2, \vec{\beta}_1) - E(\vec{\alpha}_2, \vec{\beta}_2) \right| \leq 2,$$

Violation of Bell´s inequalities with quantum reference frames

$|j_1, j_1\rangle$ $\qquad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$ $\qquad |j_2, j_2\rangle$

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$$E^{j_1 j_2}(\vartheta) = \frac{1 - 4 j_1 j_2 \cos\vartheta}{(2j_1+1)(2j_2+1)} \xrightarrow{j_1, j_2 \to \infty} E(\vartheta) = -\cos\vartheta$$

What is the minimum size of j_1, j_2 for which Bell's inequalities are violated?

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$$S_{j_1, j_2} = 2 \left| \frac{1 + 4\sqrt{2} j_1 j_2}{(2j_1+1)(2j_2+1)} \right|$$

What is the minimum size of j_1, j_2 for which Bell's inequalities are violated?

Answer: $j_1 = j_2 = \frac{5}{2}$

Interlude

Effective decoherence

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YES……………………

Bartlett, Rudolph, Spekkens, Turner '09

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$\mathcal{I}[\rho] = \rho$ $\mathcal{G}$ = twirling operator

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Can we apply this to our case?

Yes, be we find a different result...

Bell inequalities under effective decoherence

Singlet state $$|\psi^-\rangle\langle\psi^-| = \frac{1}{4}\left(\mathbb{I} - \vec{\sigma}^A \cdot \vec{\sigma}^B\right)$$

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Apply an indepentent effective decoherence on each side

$$\mathcal{E}_{j_1} \otimes \mathcal{E}_{j_2}(|\psi^-\rangle\langle\psi^-|) = \frac{1}{4}\left[\mathbb{I} - \frac{j_1 j_2}{(j_1+1)(j_2+1)}\vec{\sigma}^A \cdot \vec{\sigma}^B\right]$$

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Correlation function
$$E_{j_1 j_2}(\vartheta) = -\frac{j_1 j_2}{(j_1+1)(j_2+1)}\cos\vartheta$$

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Effective decoherence map $$\mathcal{E}_j(\rho) = \left(\frac{j}{j+1}\mathcal{I} + \frac{1}{j+1}\mathcal{G}\right)[\rho]$$

$\mathcal{I}[\rho] = \rho$ $\quad$ $\mathcal{G}$ = twirling operator

Can we apply this to our case?

Yes, be we find a different result...

Violation of Bell´s inequalities with quantum reference frames

$|j_1, j_1\rangle$ $\qquad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right) \qquad$ $|j_2, j_2\rangle$

$$S := \left|E(\vec{\alpha}_1, \vec{\beta}_1) + E(\vec{\alpha}_1, \vec{\beta}_2) + E(\vec{\alpha}_2, \vec{\beta}_1) - E(\vec{\alpha}_2, \vec{\beta}_2)\right| \leq 2.$$

$$E^{j_1 j_2}(\vartheta) = \frac{1 - 4 j_1 j_2 \cos\vartheta}{(2j_1+1)(2j_2+1)} \xrightarrow{j_1, j_2 \to \infty} E(\vartheta) = -\cos\vartheta$$

$$S_{j_1, j_2} = 2\left|\frac{1 + 4\sqrt{2} j_1 j_2}{(2j_1+1)(2j_2+1)}\right|$$

What is the minimum size of j_1, j_2 for which Bell's inequalities are violated?

Answer: $j_1 = j_2 = \frac{5}{2}$

Interlude

Effective decoherence

Is it possible to „trace out" the reference frame and just work with the system?

YES......................

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Bell inequalities under effective decoherence

Singlet state $$|\psi^-\rangle\langle\psi^-| = \frac{1}{4}\left(\mathbb{I} - \vec{\sigma}^A \cdot \vec{\sigma}^B\right)$$

Apply an indepentent effective decoherence on each side

$$\mathcal{E}_{j_1} \otimes \mathcal{E}_{j_2}(|\psi^-\rangle\langle\psi^-|) = \frac{1}{4}\left[\mathbb{I} - \frac{j_1 j_2}{(j_1+1)(j_2+1)}\vec{\sigma}^A \cdot \vec{\sigma}^B\right]$$

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Larger than before

Mermin‘s inequalities

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N observers measure spin ½ particles and have RFs of length j

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$$M := \left| \sum_{x_1,\ldots,x_N=0,1} \cos\left[\frac{\pi}{2}(x_1+\cdots+x_N)\right] E(\hat{\alpha}_{x_1},\ldots,\hat{\alpha}_{x_N}) \right| \leq 2^{\frac{N-1}{2}}$$

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In all cases: **(Quantum Bound)/(Classical Bound) = Exp(N)**

Peres Inequalities

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Local realistic bound for a pair of arbitrarily large spins

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Spin-0 State: $|\Psi^-_{j_S}\rangle := \frac{1}{\sqrt{2j_S+1}} \sum_{m=-j_S}^{j_S} (-1)^{j_S-m} |j_S, m\rangle |j_S, -m\rangle$

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$$\theta_1 - \theta_2 = \theta_2 - \theta_3 = \theta_3 - \theta_4 = x/(2j+1).$$

$$\frac{3 \sin x}{x} - \frac{\sin 3x}{3x} = 2.481 > 2 \quad \text{for: } x = 1.054$$

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Angle θ must be given with precision better than 1/j

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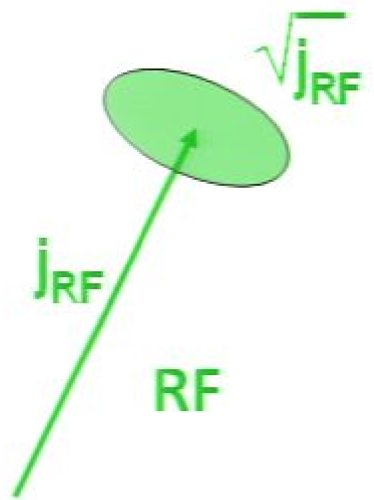

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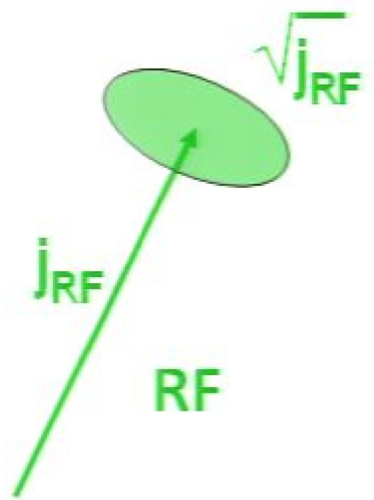

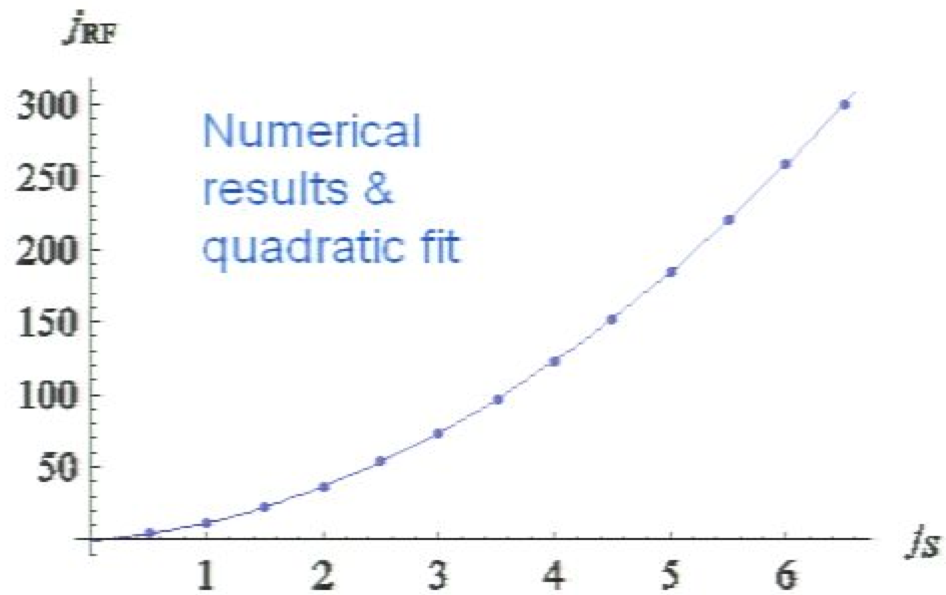

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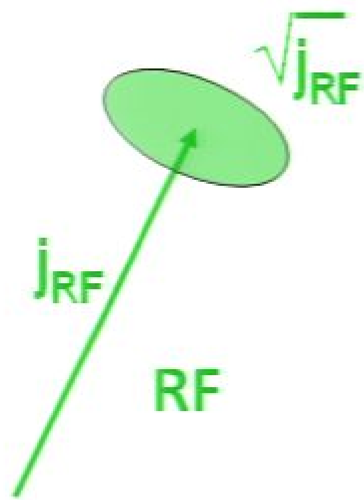

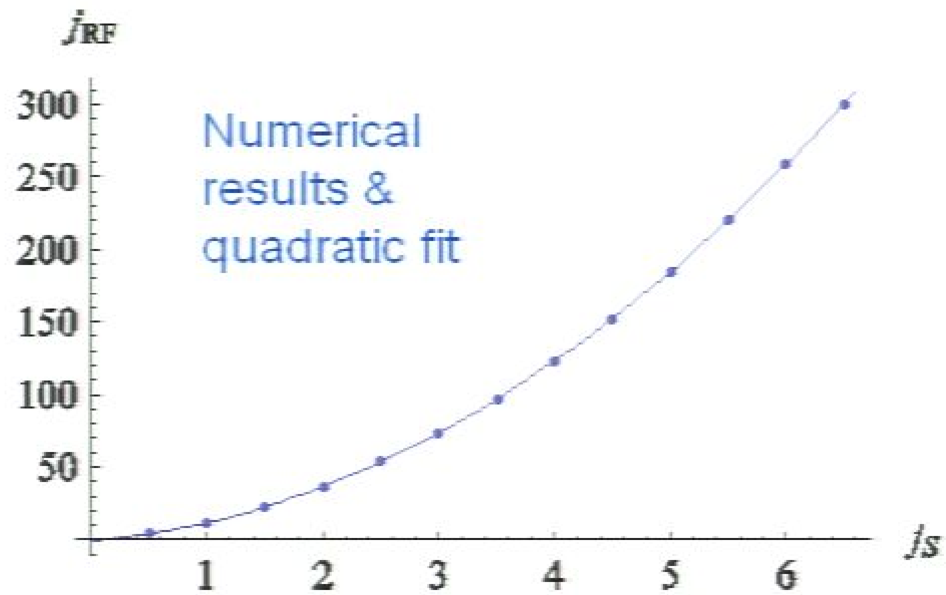

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Decoherence: To see entanglement of macroscopic environment quadratically larger RF needed → Never realized in practice

Conclusions:

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Possible explanation of **classicality.**

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New... ?

Thank you for your attention!