Title: Demarcating probability theories by their degree of agent-dependency

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Abstract: Recent advances in quantum computation and quantum information theory have led to revived interest in, and cross-fertilisation with, foundational issues of quantum theory. In particular, it has become apparent that quantum theory may be interpreted as but a variant of the classical theory of probability and information. While the two theories may at first sight appear widely different, they actually share a substantial core of common properties; and their divergence can be reduced to a single attribute only, their respective degree of agent-dependency. I propose a mathematical description for this ?degree of agent-dependency? and show how assuming different values allows one to derive the classical and the quantum case from their common core. Finally, I explore ? and eventually dismiss ? the possibility that beyond quantum theory there might be other variants of classical probability theory that are relevant to physics.

Demarcating probability theories by their degree of agent-dependency

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arXiv:0710.2119v2 [quant-ph]

Reconstructing Quantum Theory Conference
Perimeter Institute, August 9-16, 2009

Quantum theory and classical probability are often seen as two very different theories...

Major differences

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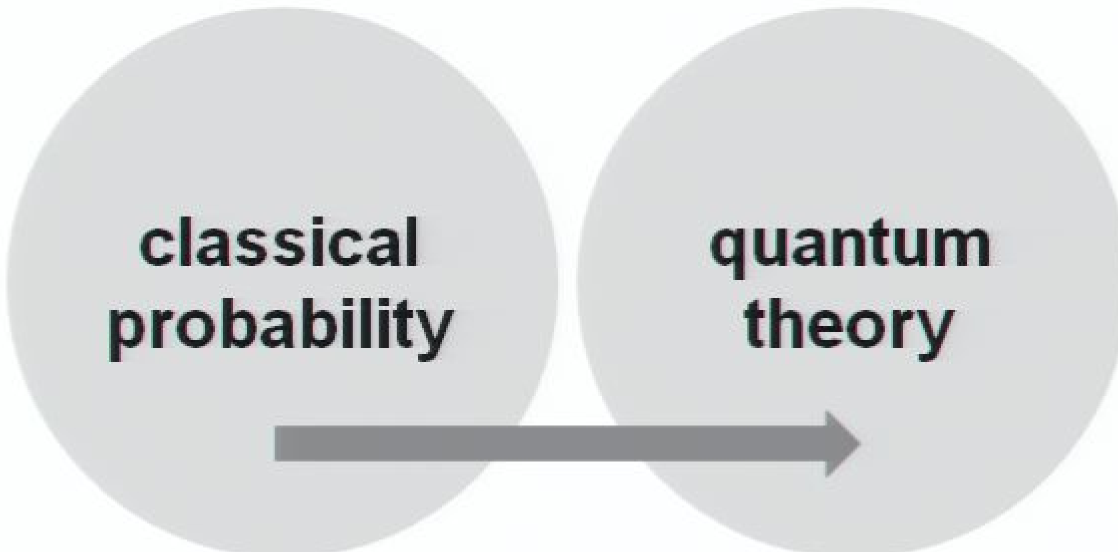
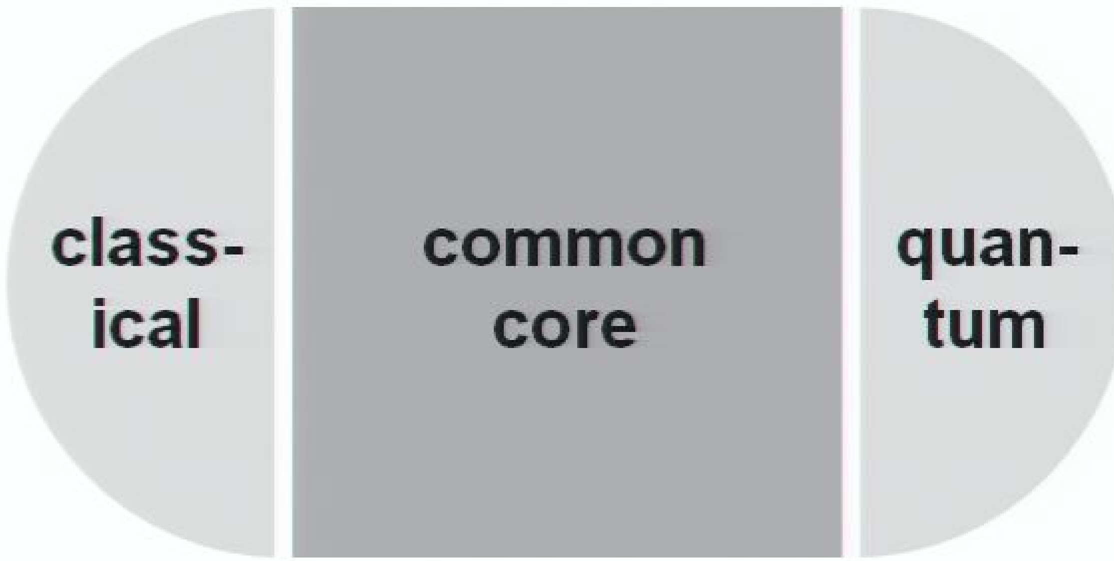
Major differences



- amplitudes, interference
- non-commutativity
- uncertainty relations
- entanglement
- violation of Bell inequalities
- Kochen-Specker theorem
- ...

...yet they share a substantial common core

Overlap



Both classical and quantum theory deal with propositions and logical relations

Logical structure

Classical concept	Quantum analog	Generic name
Sample space	Hilbert space	Proposition system
Subset	Subspace	Hypothesis, proposition
Element	1-dim. subspace (ray)	Most accurate hypothesis
Empty set	Zero	Absurd hypothesis
Disjointedness	Orthogonality	Contradiction, exclusion
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- orthomodular poset / orthoalgebra / test space
- **weaker than classical:** $\wedge$, $\vee$ defined iff jointly decidable

Probabilities satisfy sum and Bayes rules – central to ensure consistency

Reasoning in the face of uncertainty

Bayesian probability

- embodies agent‘s state of knowledge
- degree of belief rather than limit of relative frequency
- can be legitimately assigned not just to ensembles but also to individual systems

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Consistency

Different ways of using the same information must lead to the same conclusions, irrespective of the particular path chosen

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Cox 1946, Jaynes 2003

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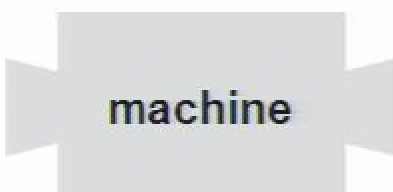
quantum:
only if probabilities pertain to propositions that are jointly decidable

Cox 1946, Jaynes 2003

Agents can intervene by selection or interrogation

Agent intervention

Generic setup



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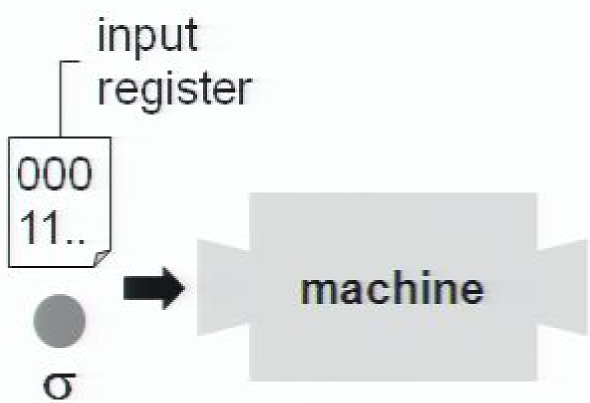

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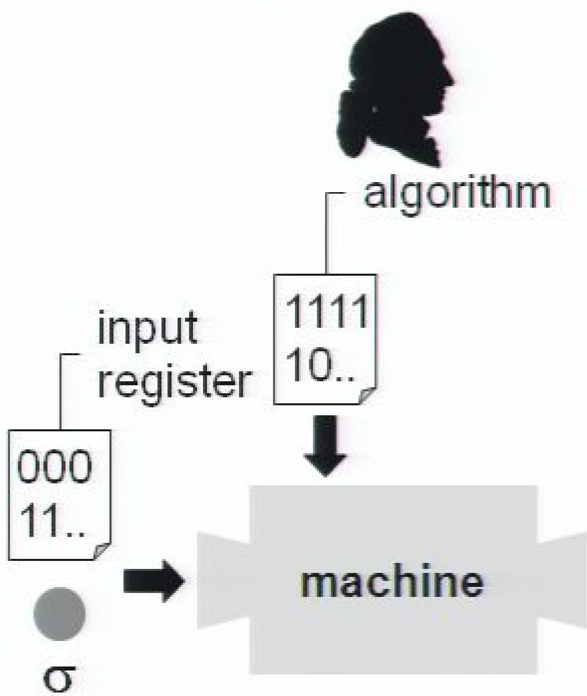

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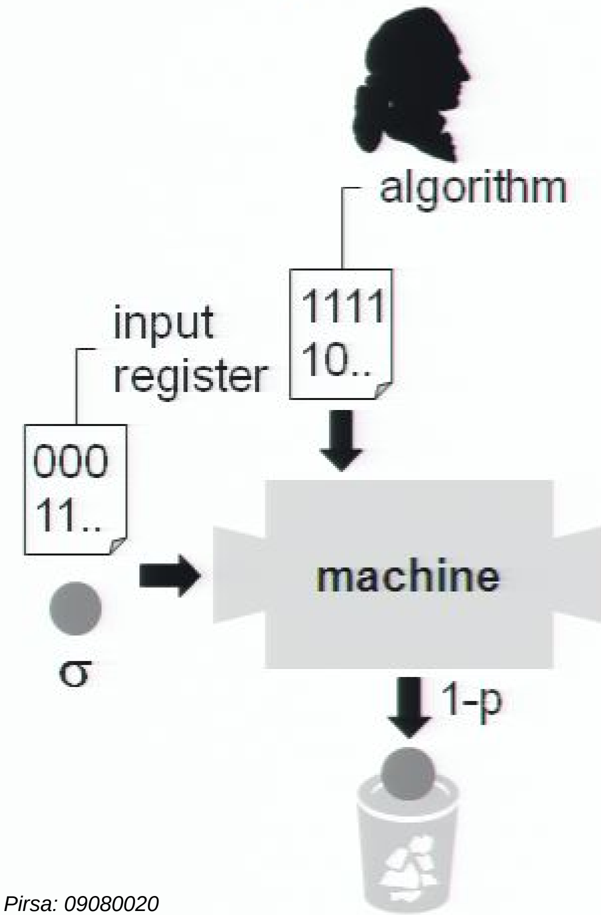

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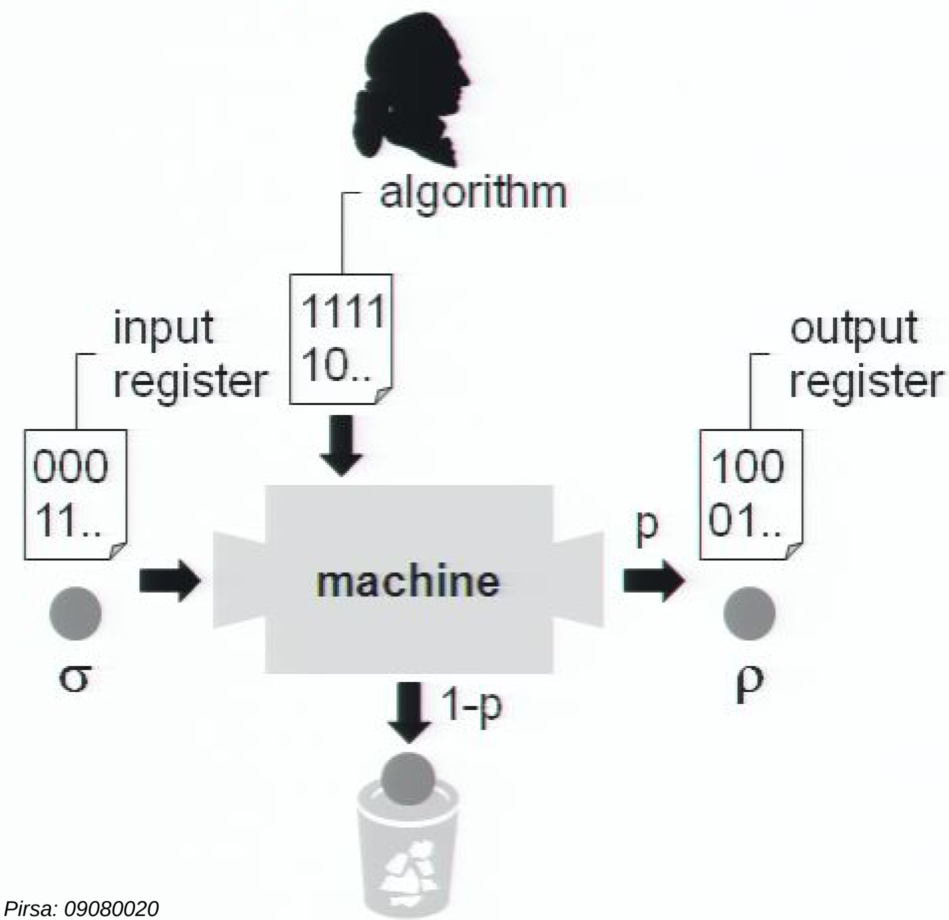

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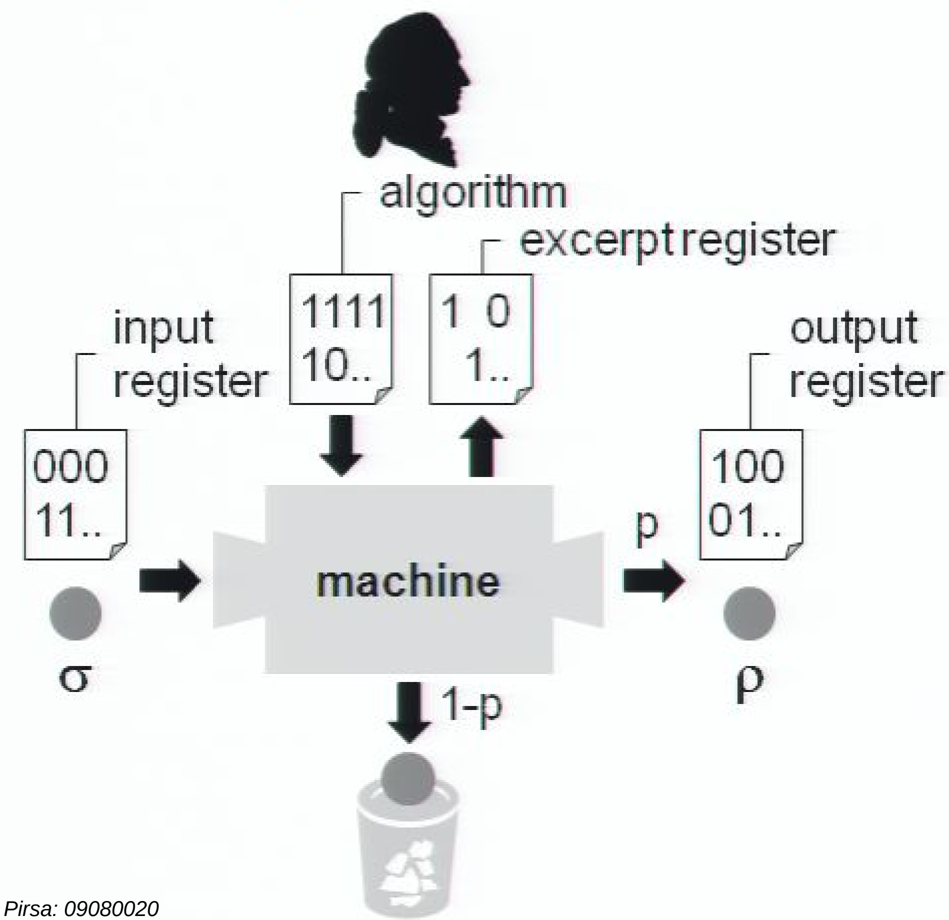

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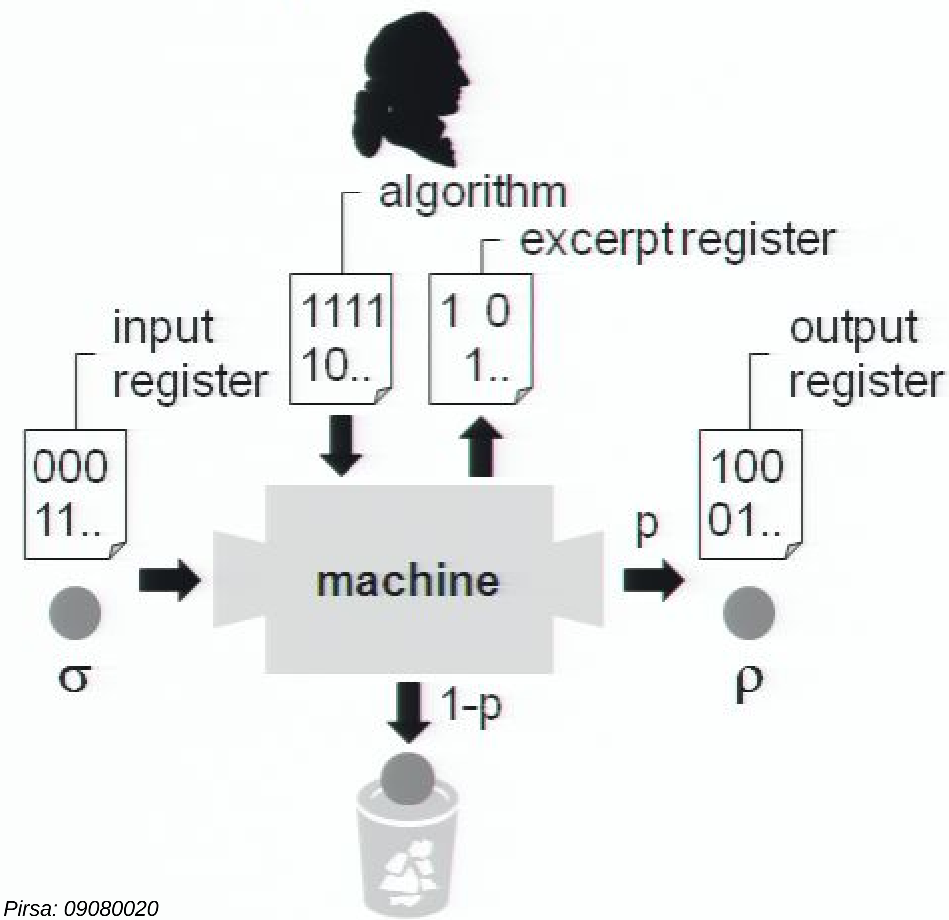

Two basic algorithms

$\mathcal{S}$ Selection

- sequence of measurements
- register = record of results
- selection rules, possibly stochastic
- keep or discard the system

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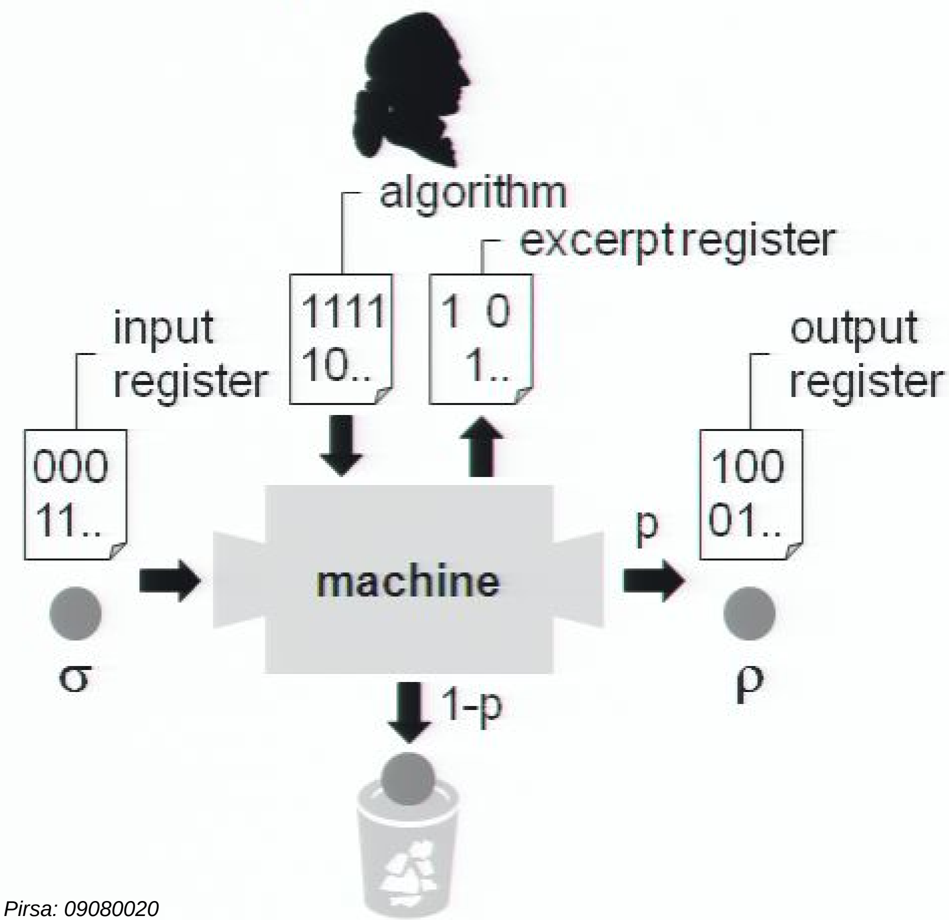

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$\mathcal{I}$ **Interrogation**

- finite or infinite sequence of measurements, possibly stochastic
- register = record of results
- measurement setups may be controlled by intermediate results (feed forward, as in 1-way q. comp.)
- no waste

Arbitrary selection rules are allowed – selection can prepare arbitrary states

Most general selection

normalisation $\mathrm{prob}(s|\mathcal{S},m_n\ldots m_1,\sigma)^{-1}$

record remains internal to machine

sequence of n measurement setups

record of results (string of integers)

selected

$$\mathrm{prob}(x|\rho) \propto \sum_{i_n\ldots i_1} \mathrm{prob}(x|i_n\ldots i_1, m_n\ldots m_1,\sigma)\cdot\mathrm{prob}(i_n\ldots i_1|m_n\ldots m_1,\sigma)\cdot\mathrm{prob}(s|i_n\ldots i_1,\mathcal{S})$$

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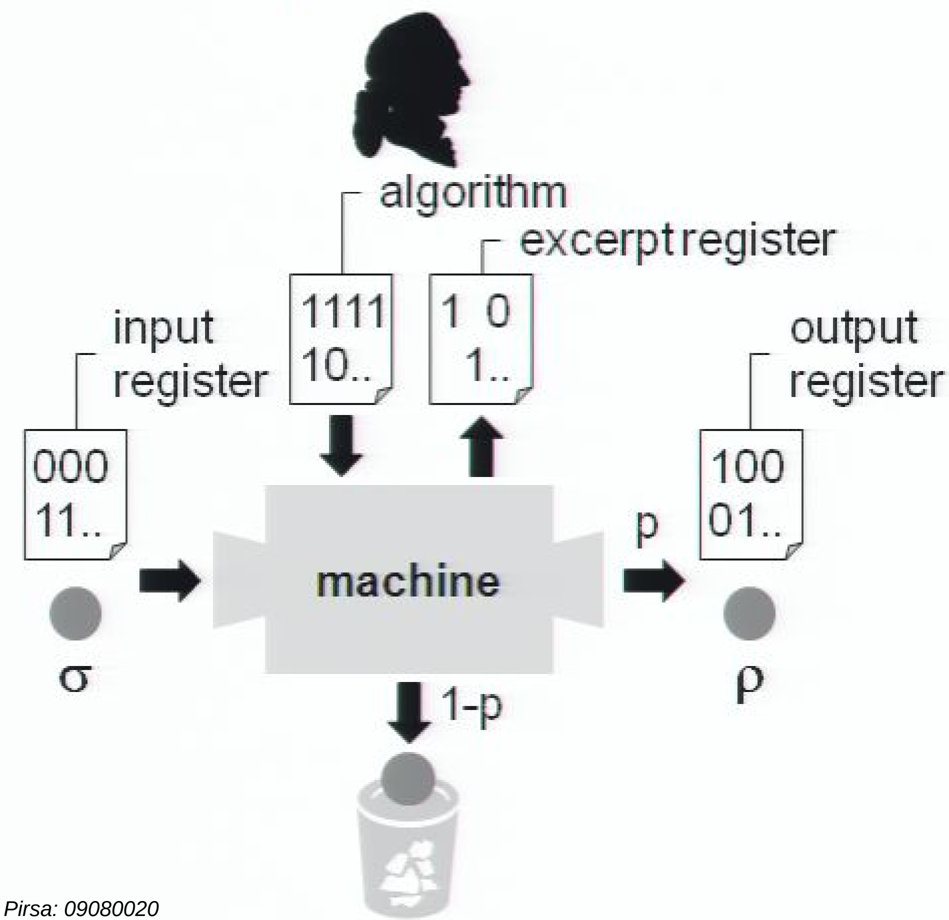

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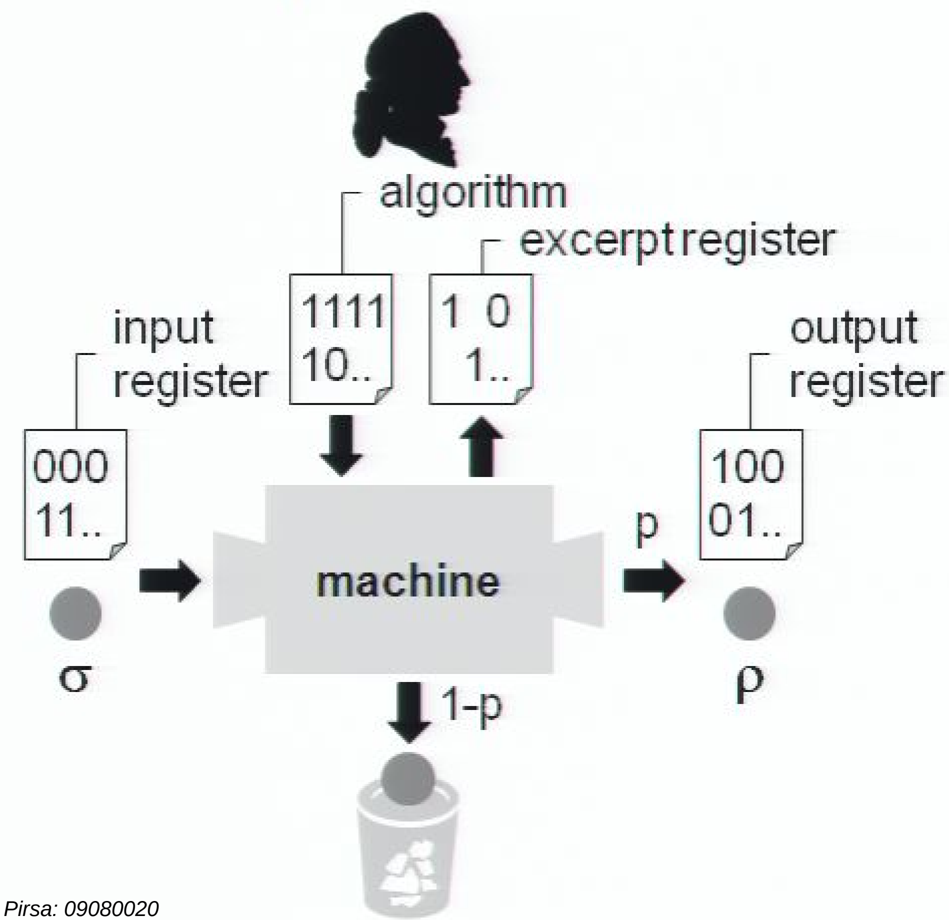

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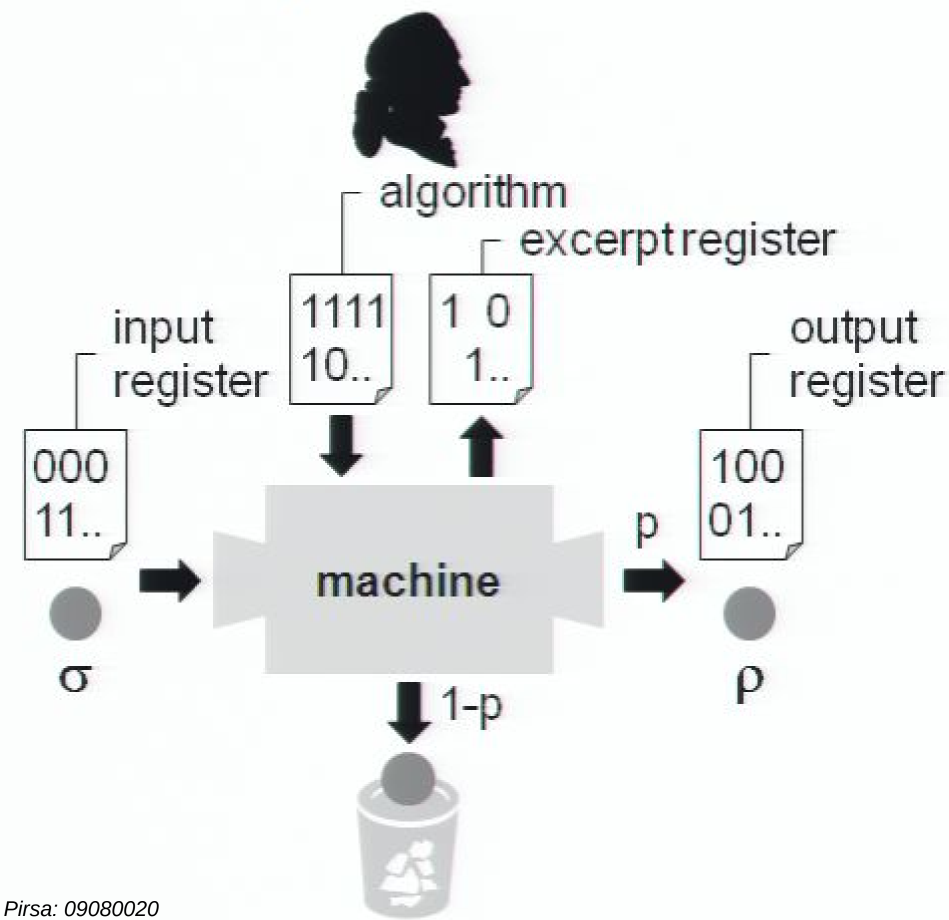

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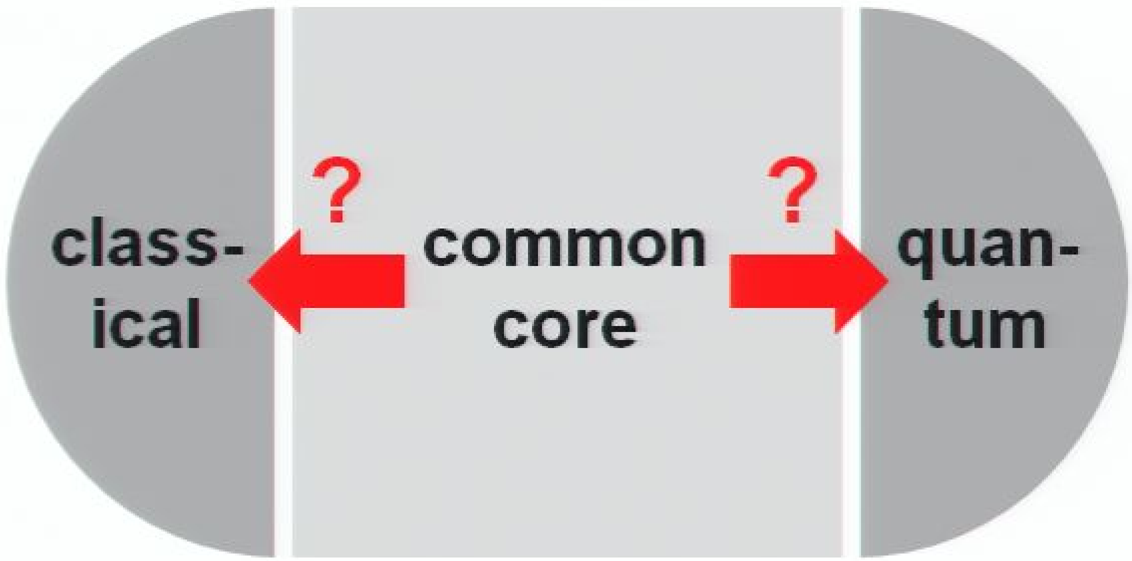
Convexity

- arbitrary mixtures constitute valid posteriors
- states form a convex set

Which additional assumptions are needed to arrive at the classical and quantum case, respectively?

Key question



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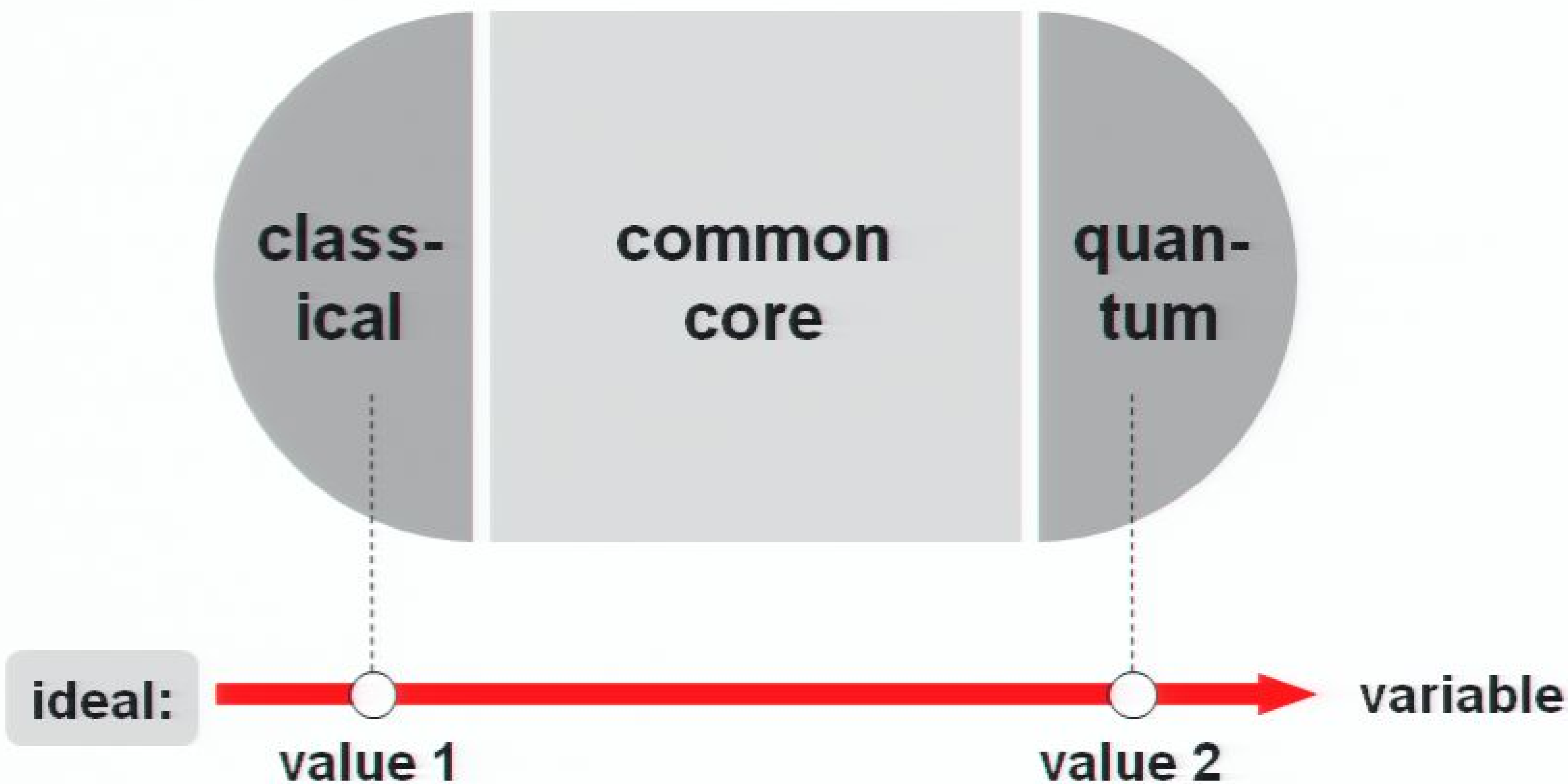

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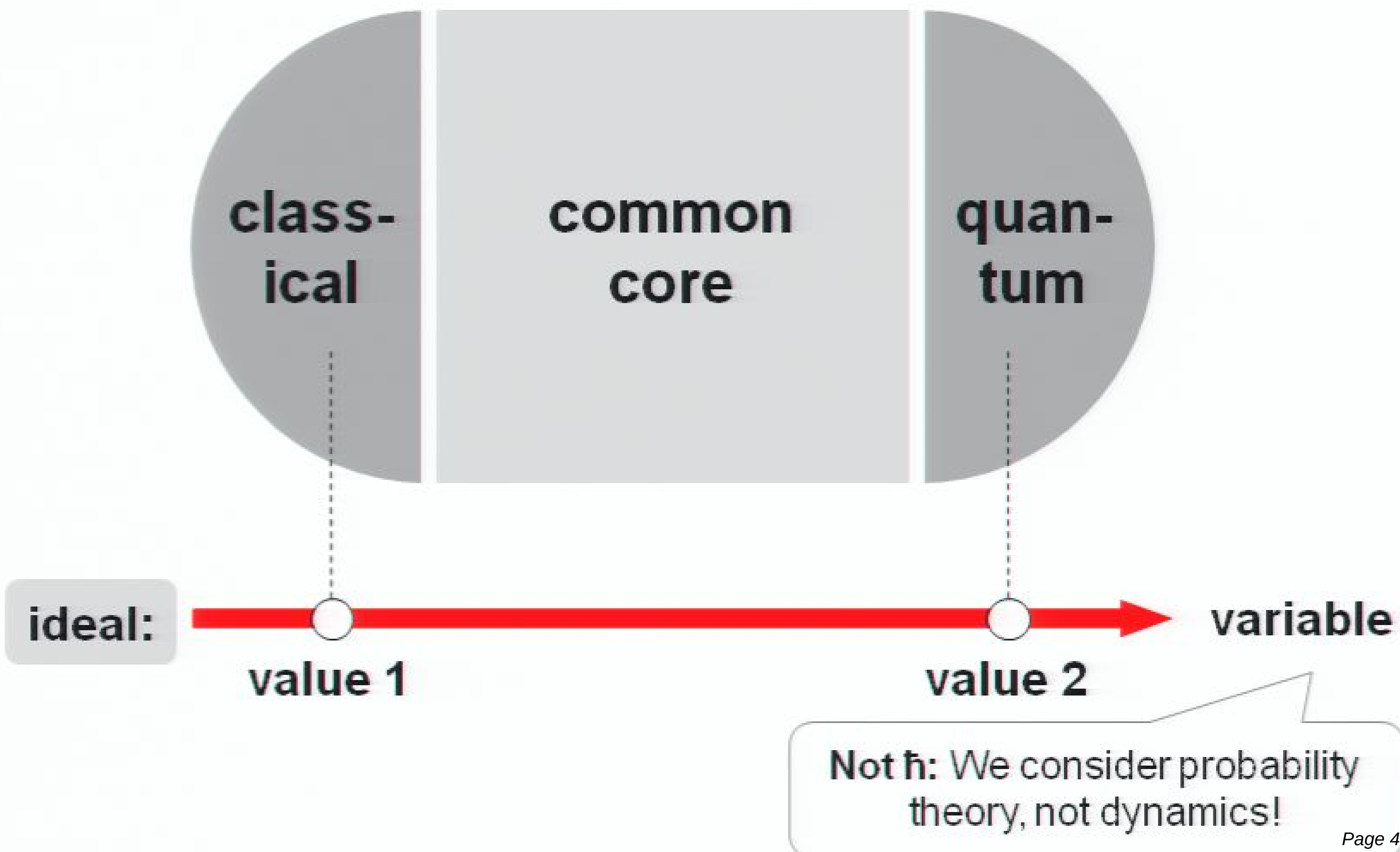

Only in the quantum case it is possible to prepare arbitrary states by mere interrogation

Response to interrogation

Hidden, deterministic, static interrogation

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Classical invariability

- Interrogation has no effect:

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based on
joint decidability

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Quantum malleability

- Interrogation can steer a system from any pure state to any other pure state:

  $\forall\ e,f\ \exists\ \mathcal{I}:\ \text{prob}(x|\mathcal{I},e) = \text{prob}(x|f) \quad \forall\ x$

v. Neumann 1932

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- Preceding interrogation can make any measurement emulate any other measurement of the same resolution:

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based on
quantum Zeno effect

Responsiveness to interrogation adds another layer to agent-dependency

Agent-dependency

Classical

Quantum

Responsiveness to interrogation adds another layer to agent-dependency

Agent-dependency

Classical

Orthodox: **data** → **conclusion**

Quantum

Responsiveness to interrogation adds another layer to agent-dependency

Agent-dependency

(shaded ellipse): agent-dependent

Classical

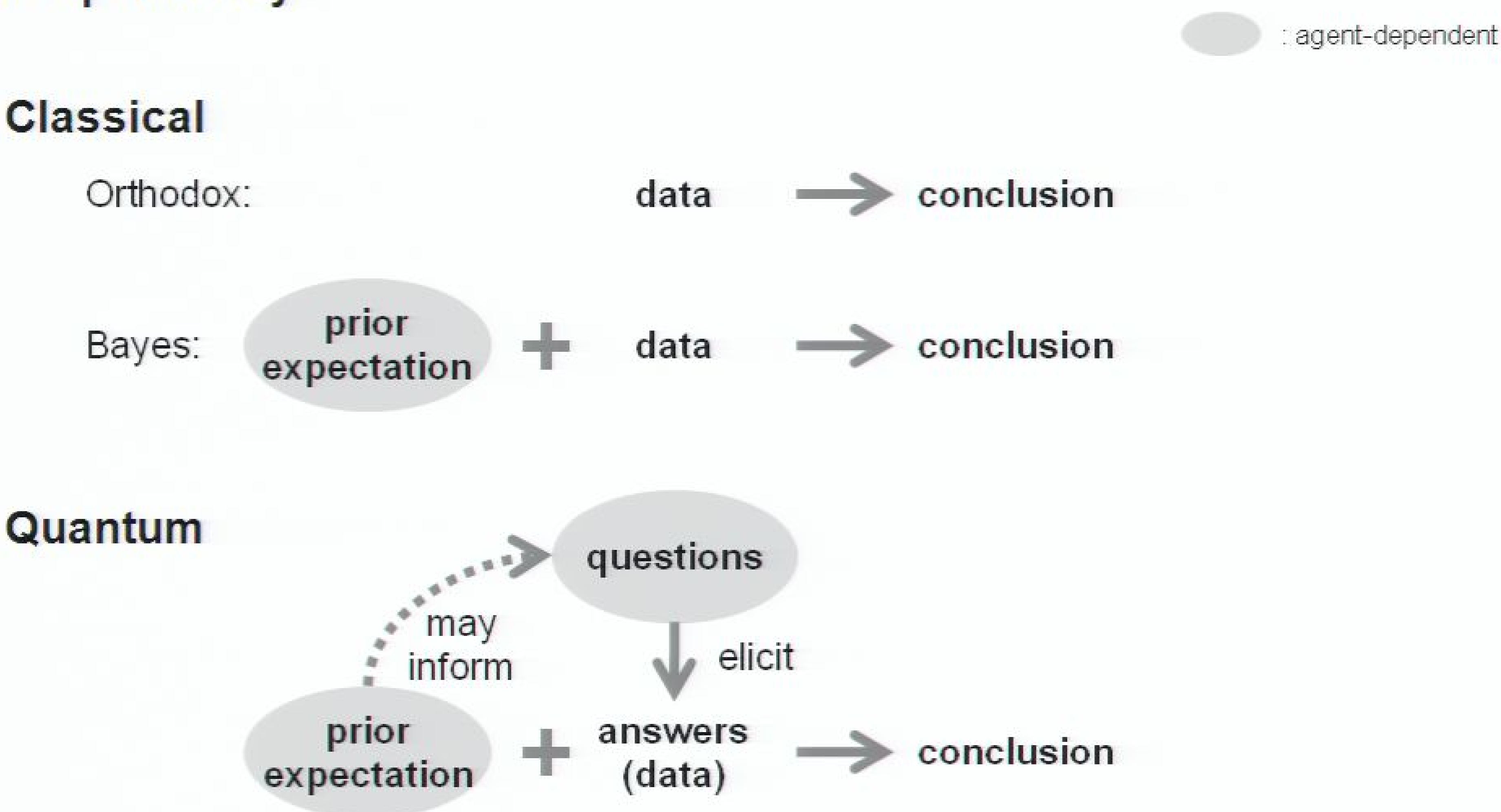
Orthodox: **data** → **conclusion**

Bayes: **prior expectation** + **data** → **conclusion**

Quantum

Responsiveness to interrogation adds another layer to agent-dependency

Agent-dependency



In quantum theory conclusions depend on questions asked

Wheeler's game of 20 questions (1/2)

About the game of twenty questions. You recall how it goes—one of the after-dinner party sent out of the living room, the others agreeing on a word, the one fated to be questioner returning and starting his questions. « Is it a living object? » « No. » « Is it here on earth? » « Yes. » So the questions go from respondent to respondent around the room until at length the word emerges: victory if in twenty tries or less; otherwise, defeat.

Then comes the moment when we are fourth to be sent from the room. We are locked out unbelievably long. On finally being readmitted, we find a smile on everyone's face, sign of a joke or a plot. We innocently start our questions. At first the answers come quickly. Then each question begins to take longer in the answering—strange, when the answer itself is only a simple « yes » or « no ». At length, feeling hot on the trail, we ask, « Is the word 'cloud'? » « Yes », comes the reply, and everyone bursts out laughing. When we were out of the room, they explain, they had agreed not to agree in advance on any word at all. Each one around the circle could respond « yes » or « no » as he pleased to whatever question we put to him. But however he replied he had to have a word in mind compatible with his own reply—and with all the replies that went before. No wonder some of those decisions between « yes » and « no » proved so hard!

Wheeler 1983 („Law without law")

In quantum theory conclusions depend on questions asked

Wheeler's game of 20 questions (2/2)

1

The word does not already exist „out there" – rather, information about the word is brought into being through the questions raised

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...like in quantum theory:

- There is no preexisting reality that is merely revealed, rather than influenced, by the act of measurement
- The image of reality that emerges through acts of measurement reflects as much the history of intervention as it reflects the external world

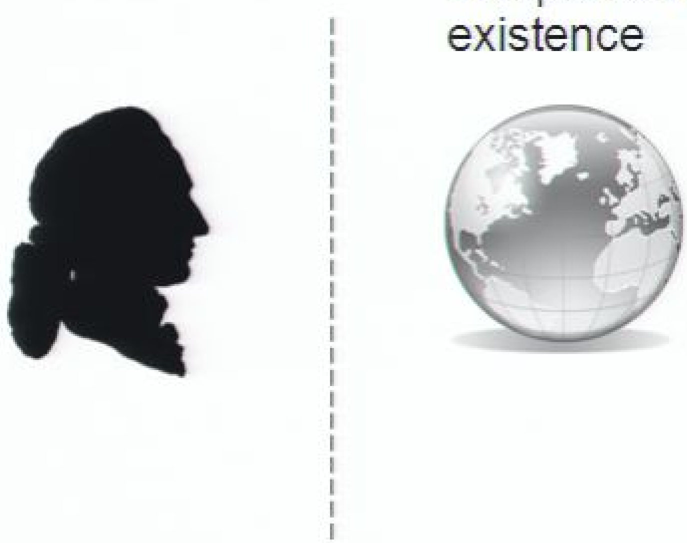
Wheeler 1983 („Law without law")

This new form of agent-dependency leads to a radically different world view

World view

Classical	Quantum

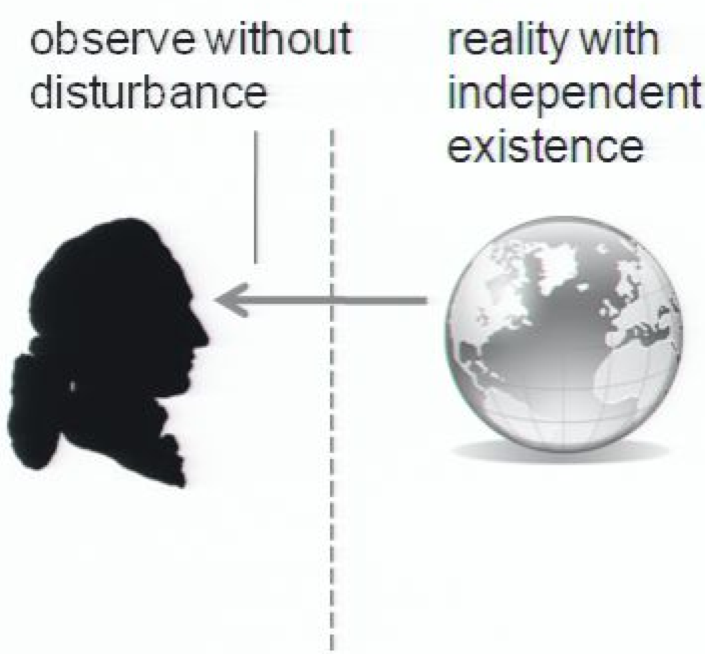
reality with
independent
existence



This new form of agent-dependency leads to a radically different world view

World view

Classical	Quantum



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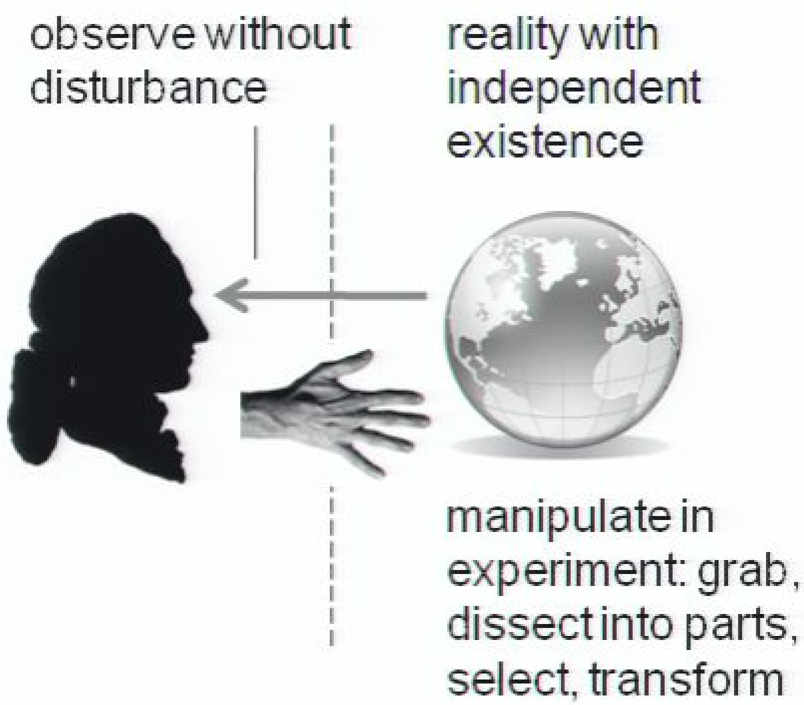

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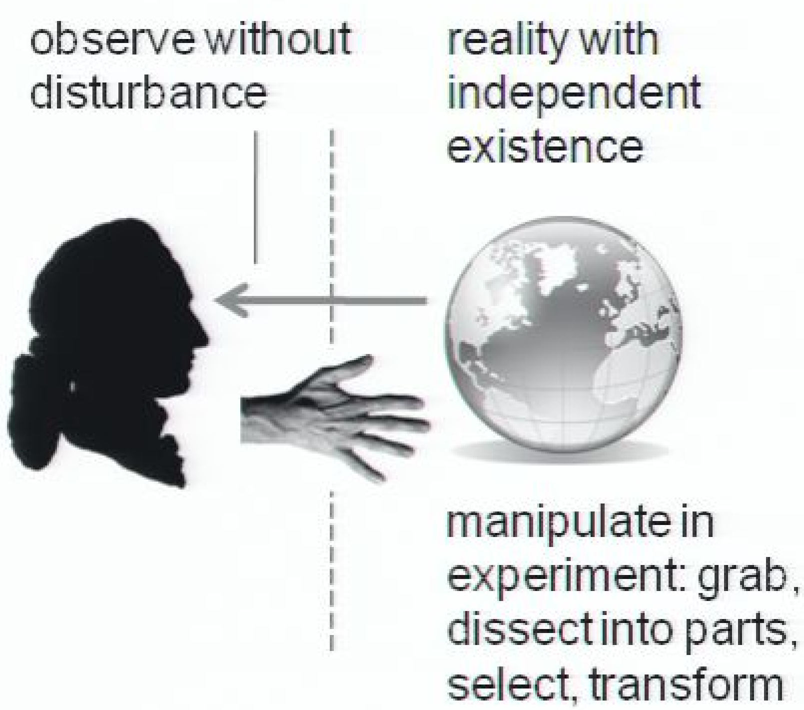

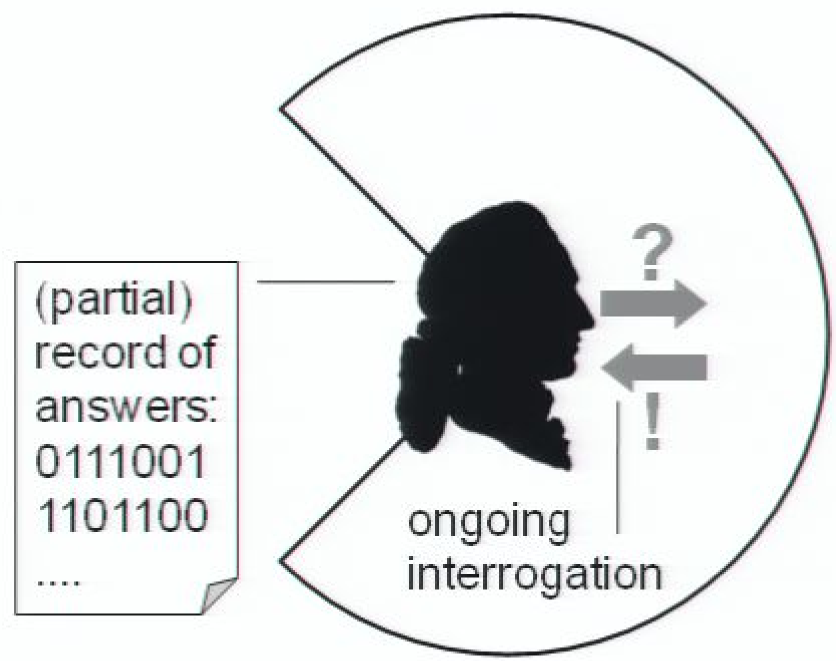

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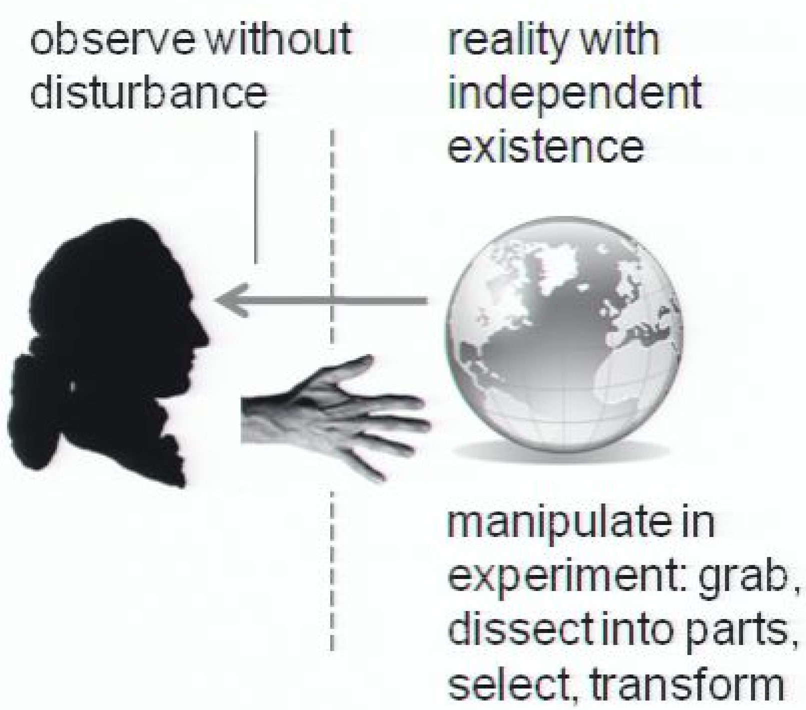

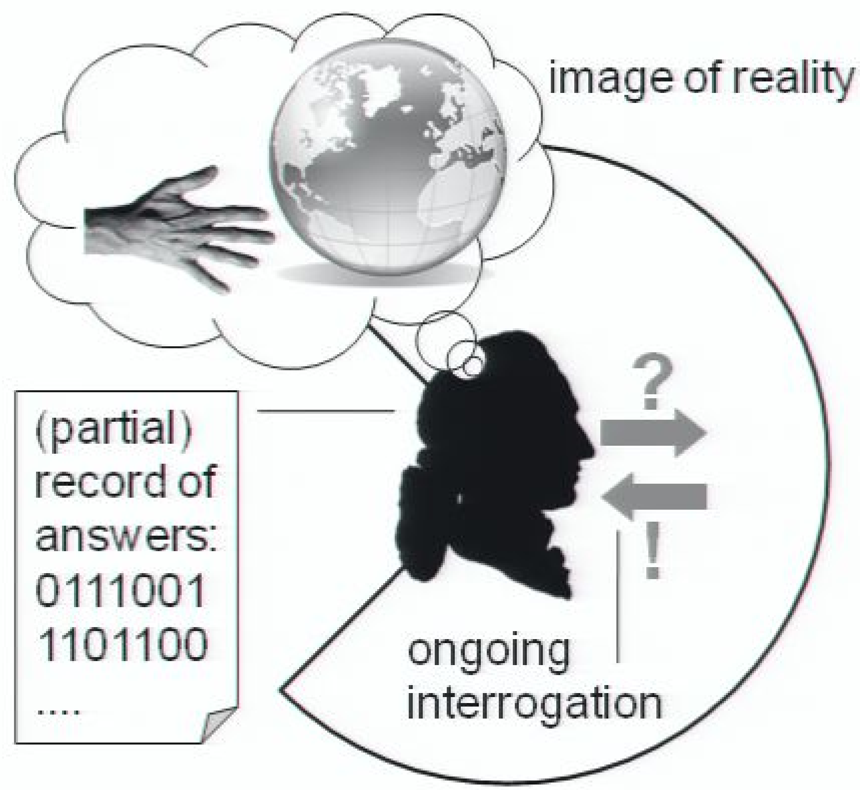

Can agent-dependency serve as a foundational principle for quantum theory?

Conjecture

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Mathematical apparatus		
Physical principle		

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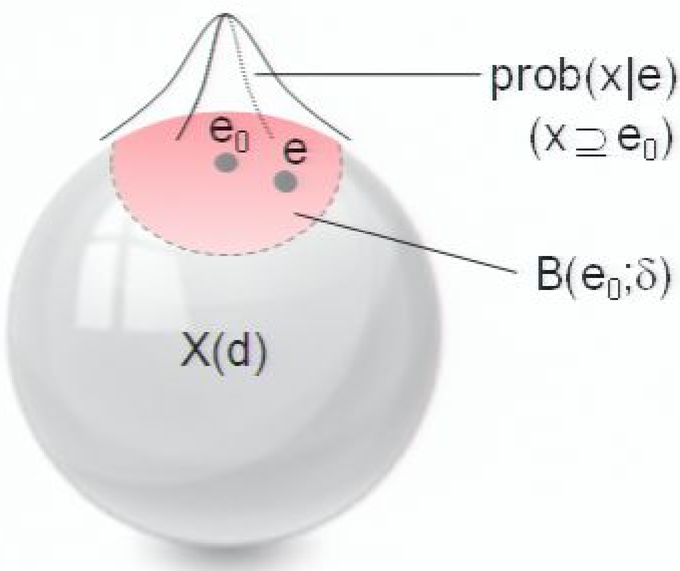


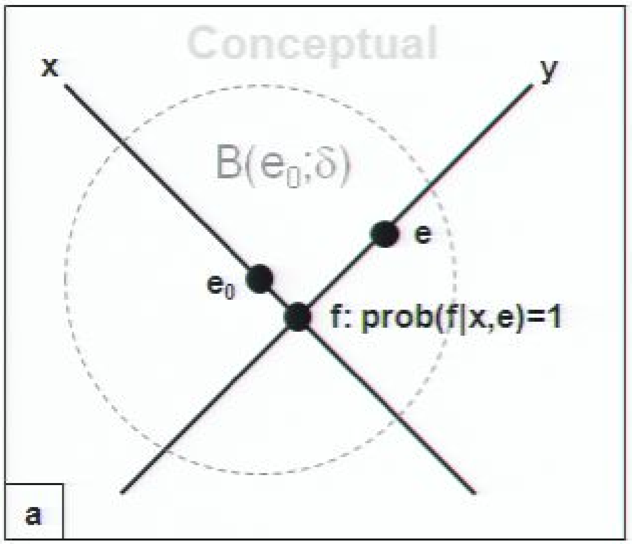
Continuity condition:

$\forall \varepsilon > 0 \ \exists \delta > 0: \ \text{prob}(x|e) > 1-\varepsilon \ \forall e \in B(e_0;\delta)$

⇔ Robustness under small preparation inaccuracies

The continuity condition imposes tight constraints on the manifold dimension

Idea of proof



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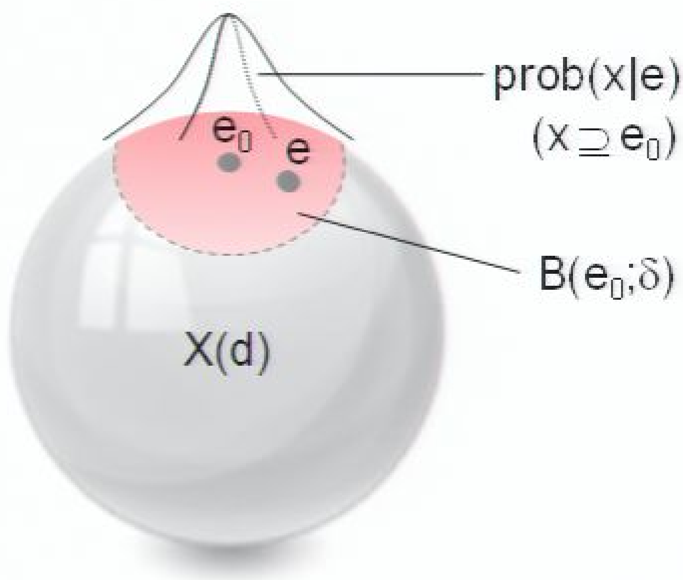

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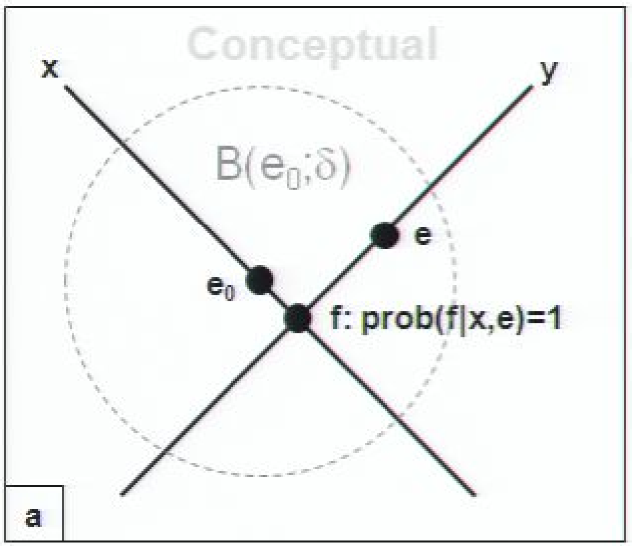

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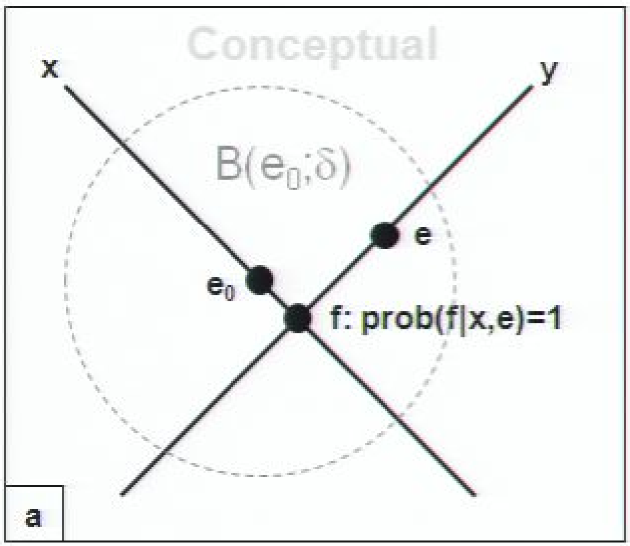

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- Fixed: a, x, e_0
- Two equivalent ways to specify variable e:
  - directly as a refinement of a
  - first f as a refinement of „base" x, then e as a refinement of „fiber" y

$\Rightarrow \dim X(d) = \dim X(k) + \dim X(d-k+1)$

- Initial conditions: $\dim X(0) = \dim X(1) = 0$

$$\dim X(d) = \dim X(2) \cdot (d-1)$$

Continuity in combination with common core leads to quantum theory in Hilbert space over R, C or H

Analysis of symmetry group

Definition

Automorphisms of a proposition system preserve

- logical relations $\perp, \subseteq, \setminus$
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Irreducible building block

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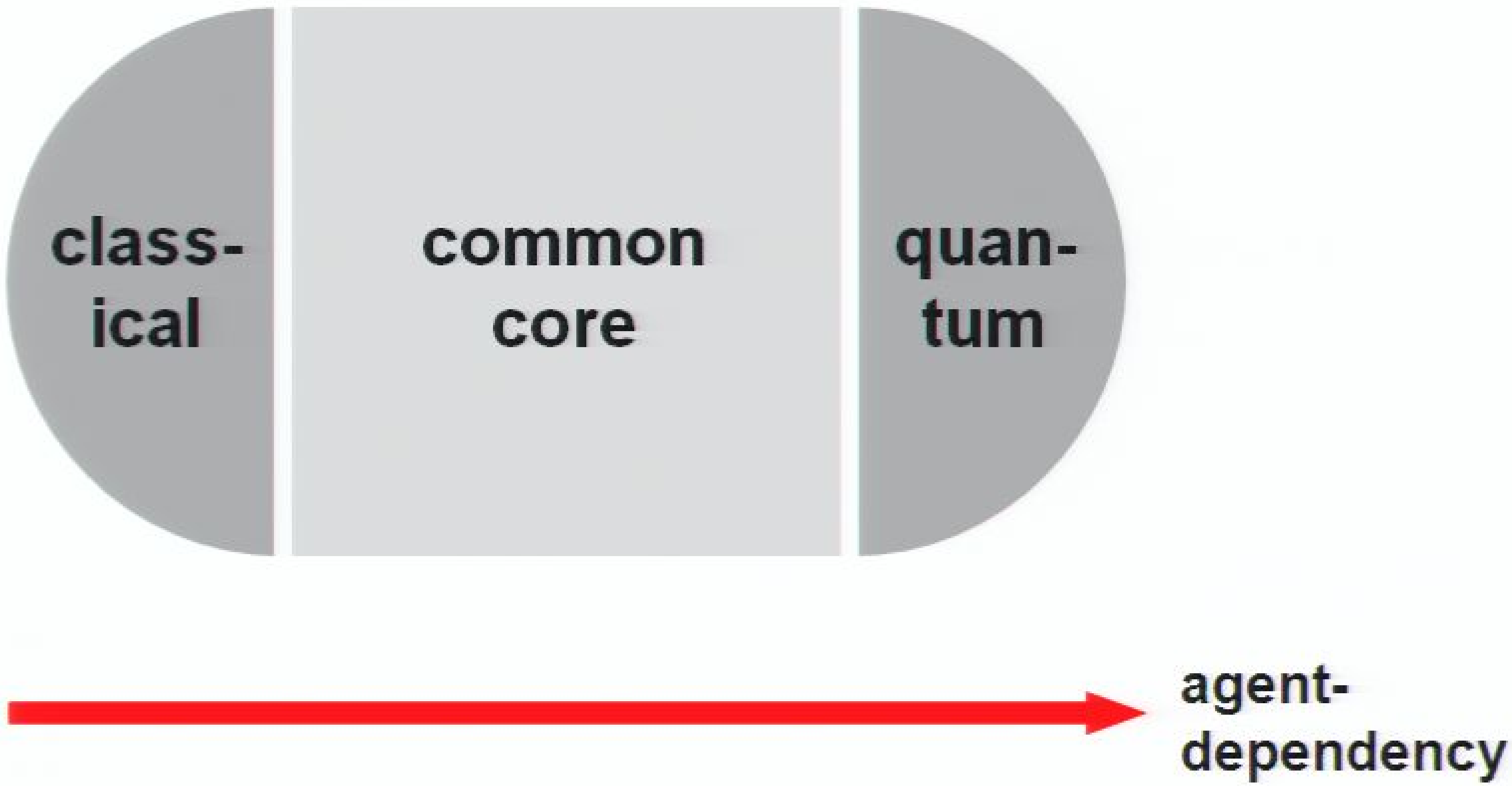
$$G(d) \in \{SO(n \cdot d), U(n \cdot d), Sp(n \cdot d) \mid n \in N\}$$

Classical and quantum case derive from common core via additional assumptions on agent-dependency

Conclusion



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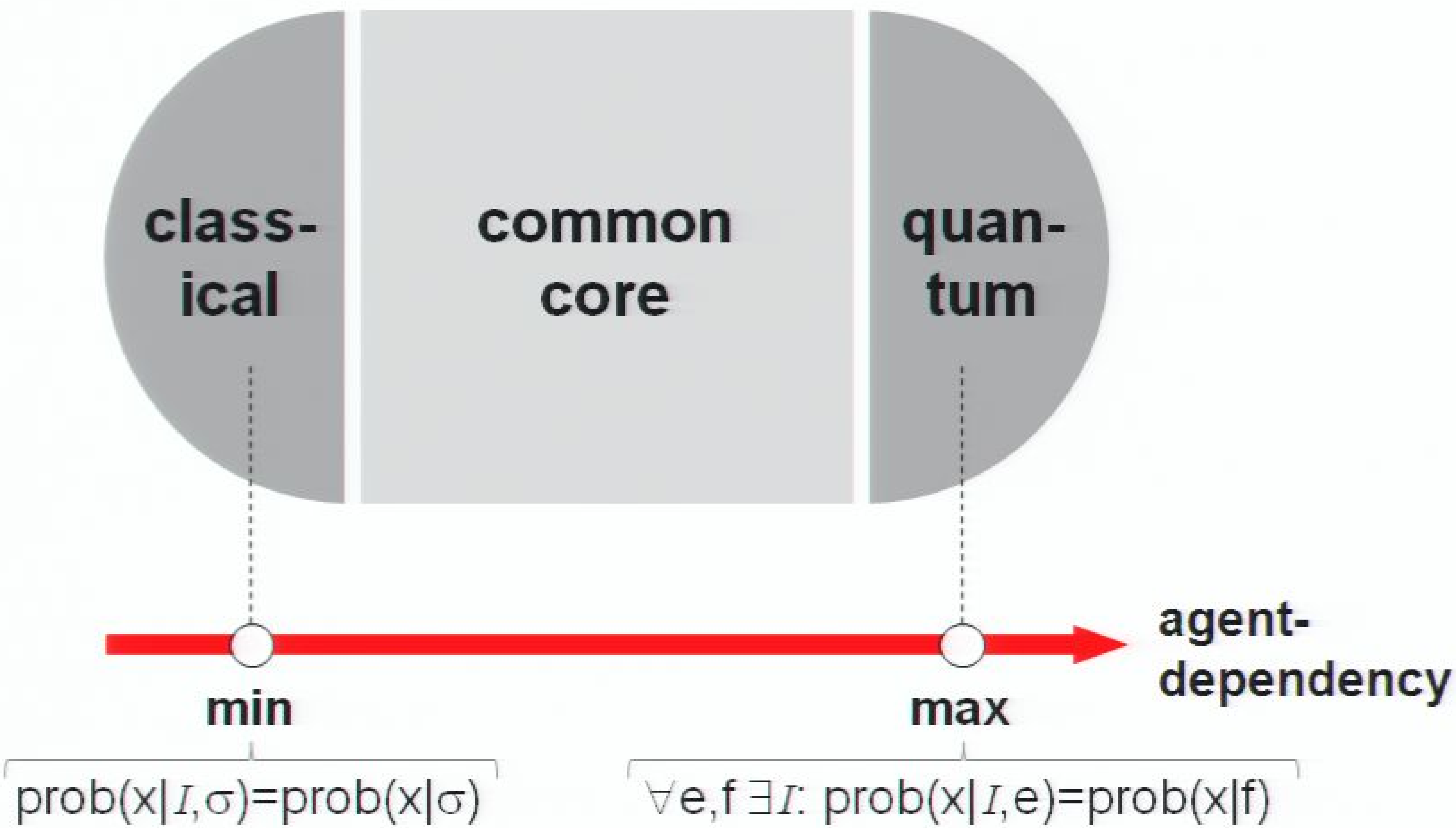

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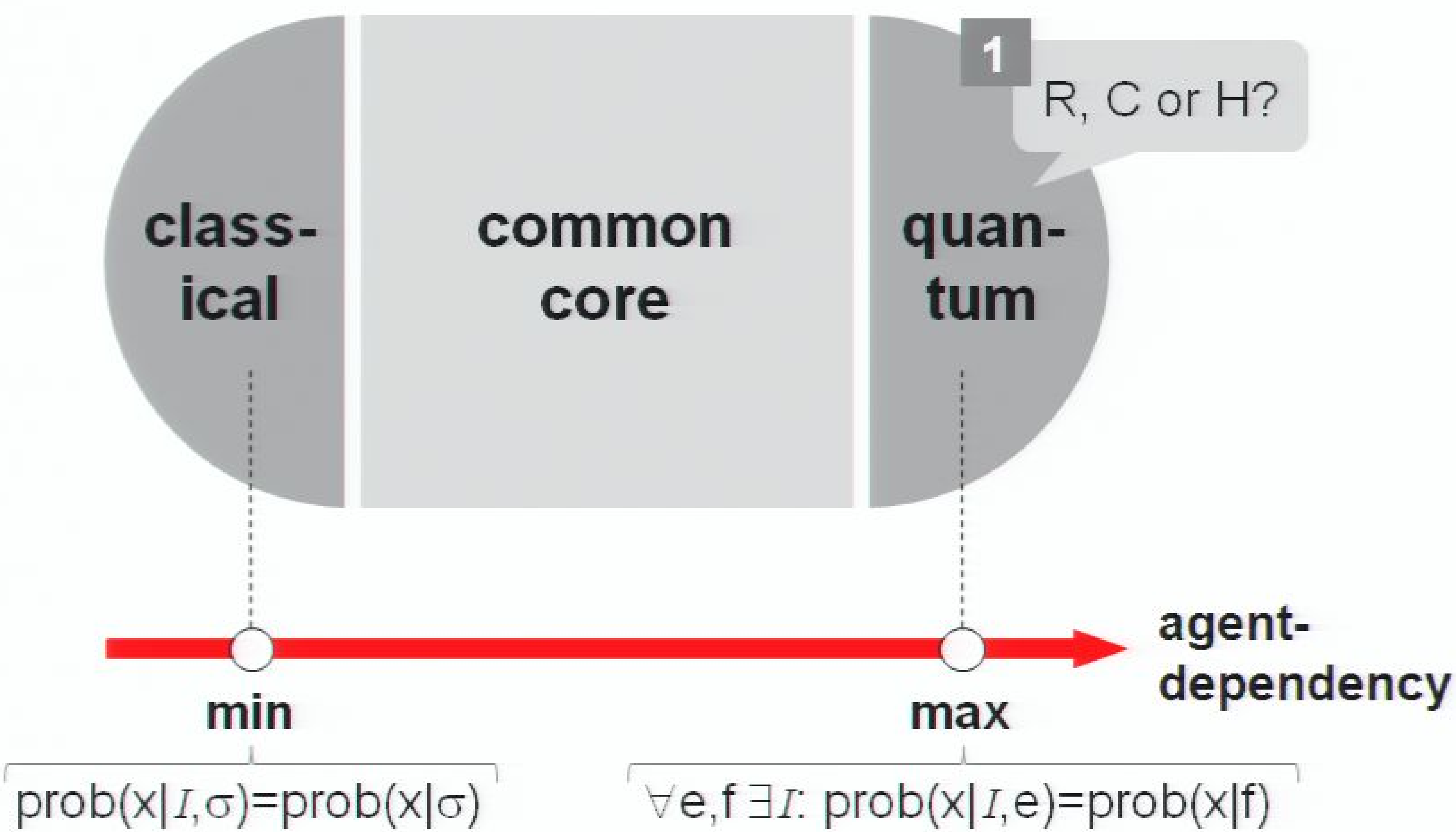

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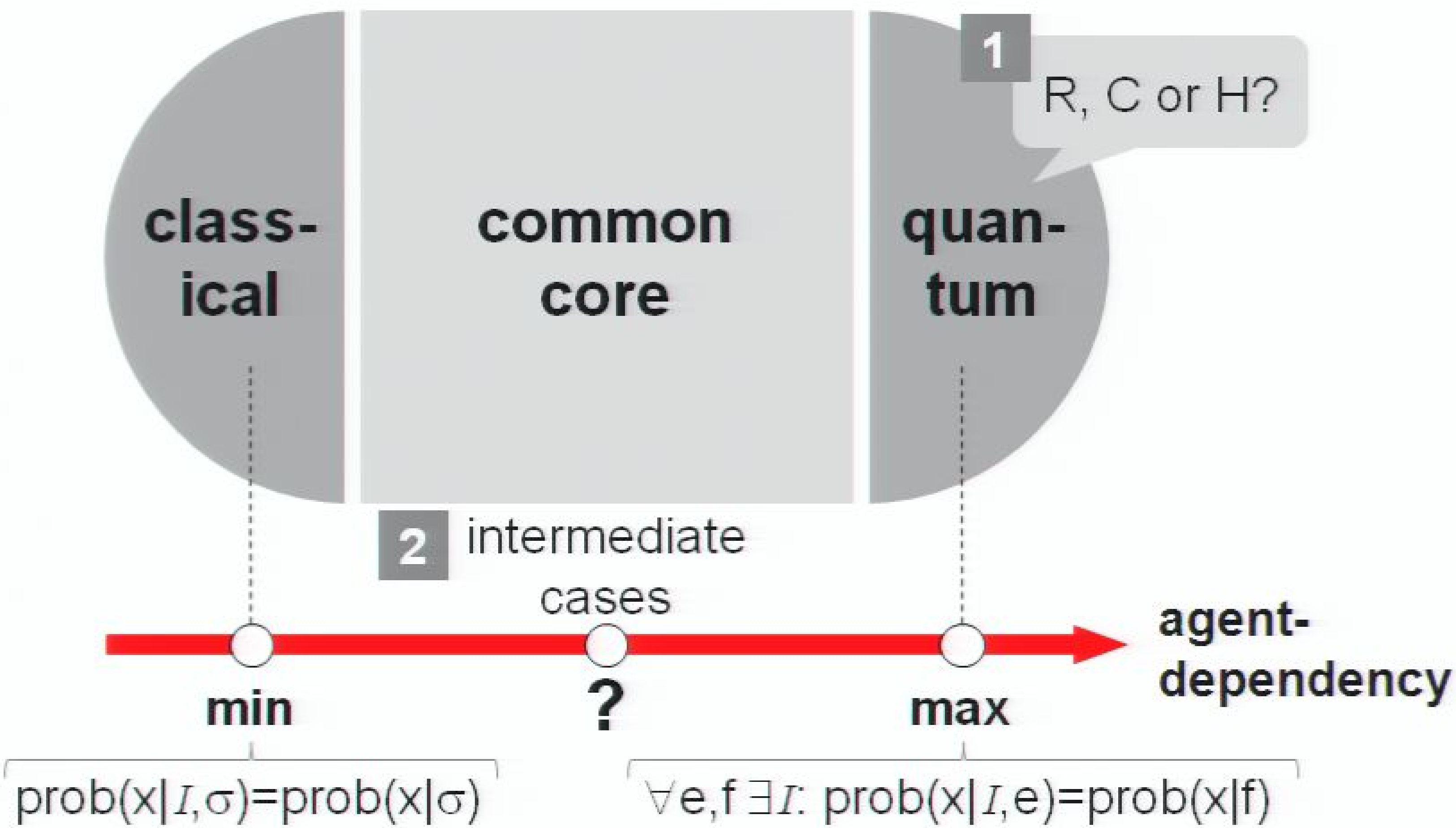

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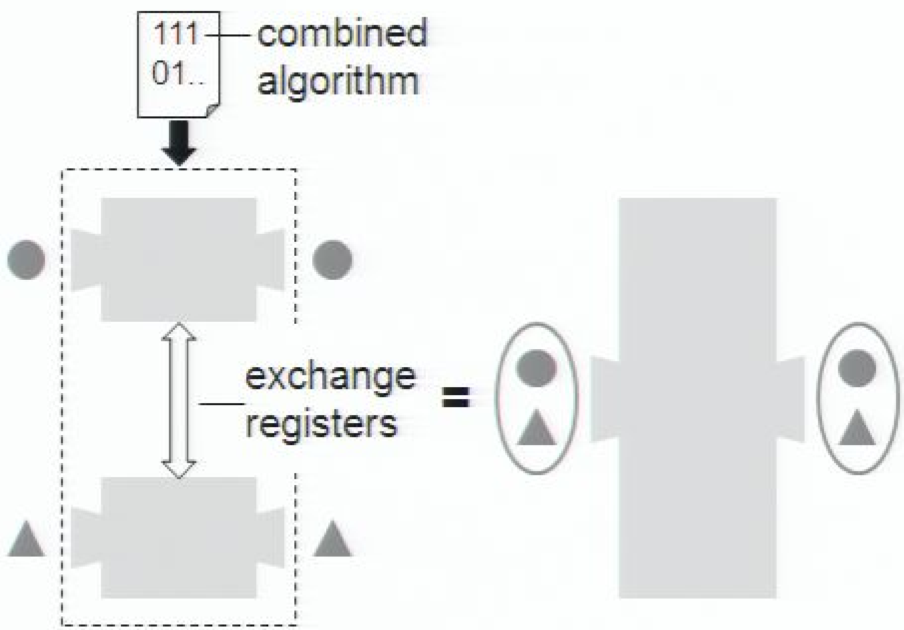
Composition of agent interventions

Composition of interventions yields an intervention on the composite



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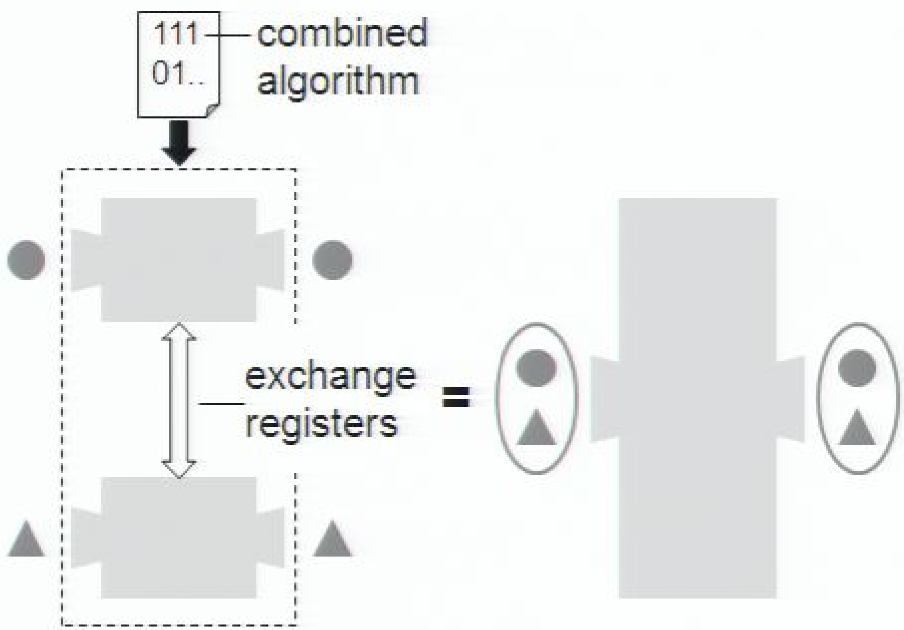

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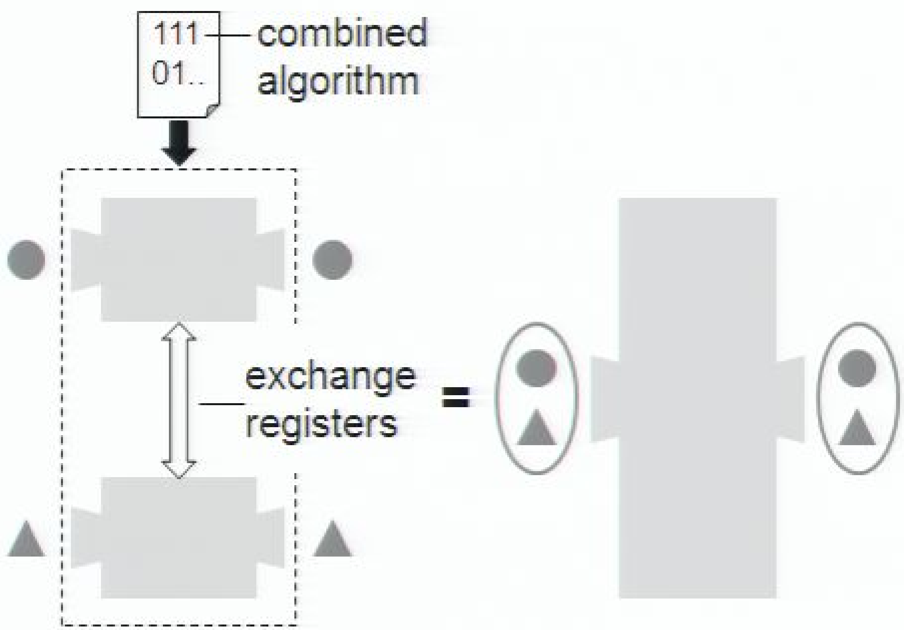

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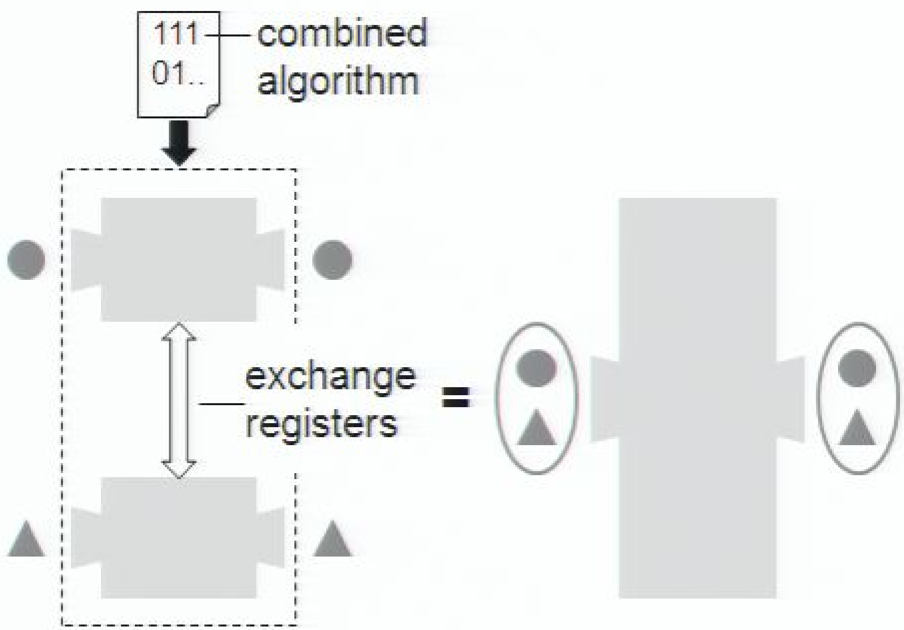

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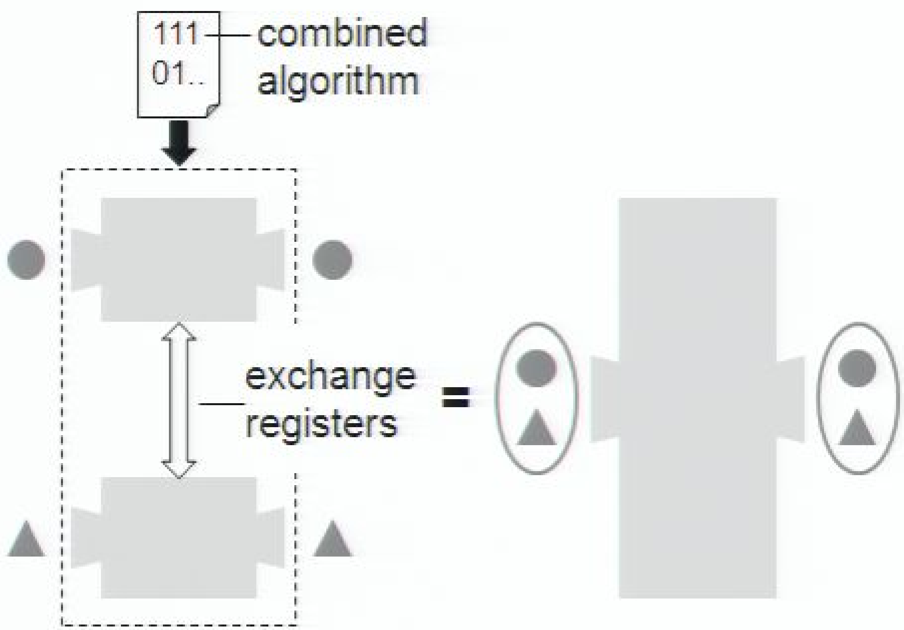

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higher-power theories, $\mu \geq 3$ (Hardy)	$\dim G(d) = d^{\mu} + (\dim G(1) - 1) \cdot d$	probabilities exhibit discontinuities; physical meaning of group?

Further details:

arXiv:0710.2119v2 [quant-ph]

Only in the quantum case it is possible to prepare arbitrary states by mere interrogation

Response to interrogation

Hidden, deterministic, static interrogation

Classical invariability	Quantum malleability
▪ Interrogation has no effect: $\text{prob}(x\vert\mathcal{I},\sigma) = \text{prob}(x\vert\sigma) \quad \forall\ \mathcal{I}, x, \sigma$ based on **joint decidability**	▪ Interrogation can steer a system from any pure state to any other pure state: $\forall\ e,f\ \exists\ \mathcal{I}:\ \text{prob}(x\vert\mathcal{I},e) = \text{prob}(x\vert f) \quad \forall\ x$ v. Neumann 1932 ▪ Preceding interrogation can make any measurement emulate any other measurement of the same resolution: $\forall\ \{x_i\},\{y_i\} \in M(\{k_i\})\ \exists\ \mathcal{I}:\ \text{prob}(y_i\vert\mathcal{I},\sigma) = \text{prob}(x_i\vert\sigma) \quad \forall\ i,\sigma$ based on **quantum Zeno effect**