Title: A reconstruction of quantum mechanics from quantum logics with unique conditional probabilities

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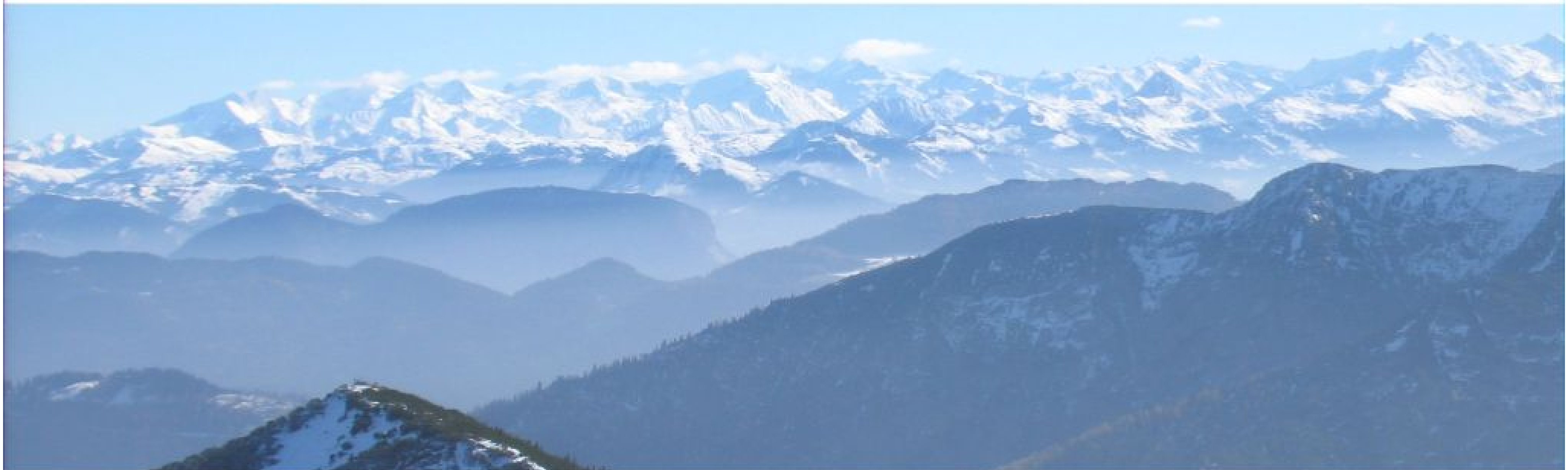
Abstract: The starting point of the reconstruction process is a very simple quantum logical structure on which probability measures (states) and conditional probabilities are defined. This is a generalization of Kolmogorov's measure-theoretic approach to probability theory. In the general framework, the conditional probabilities need neither exist nor be uniquely determined if they exist. Postulating their existence and uniqueness becomes the major step in the reconstruction process. A certain new mathematical structure can then be derived, and examples immediately reveal that probability conditionalization is identical with the Lüders - von Neumann measurement process. Some further postulates bring us to Jordan algebras, and the consideration of composite systems finally shows why these algebras must be the self-adjoint parts of von Neumann algbras such that they can be represented as linear operators on Hilbert spaces over the complex numbers. This is why the approach gets ahead of other ones that are not able to justify the need for the complex Hilbert space or the Jordan operator algebras. The mathematical structure of quantum mechanics can thus be reconstructed from a few probabilistic basic principles and becomes a non-Boolean extension of classical probability theory. Its link to physics is that probability conditionalization in this structure is identical with the Lüders - von Neumann measurement process.

Gerd Niestegge



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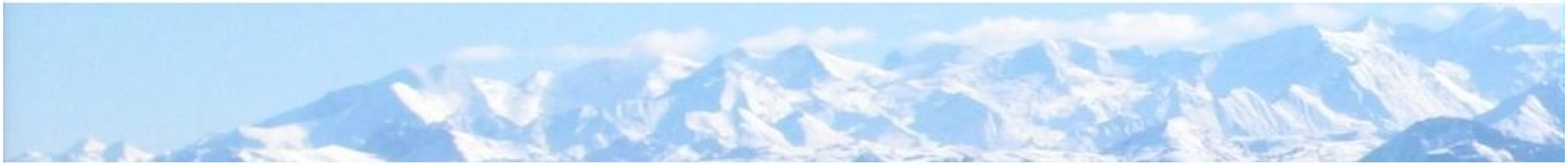
Reconstructing Quantum Theory
Workshop August 9 - 16, 2009, at the Perimeter Institute
Waterloo, Canada

gerd.niestegge@web.de

Classical probability theory

Quantum mechanics
$A = L_h(H)$

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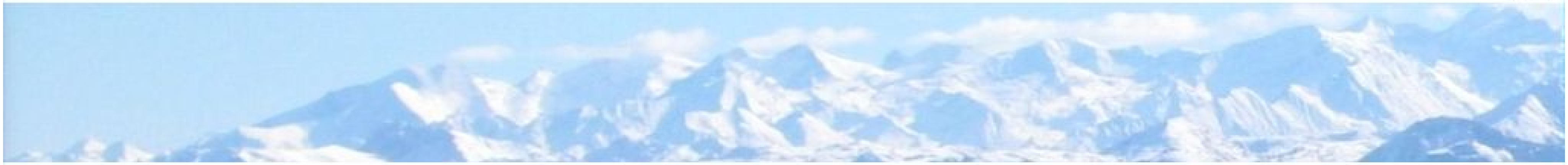

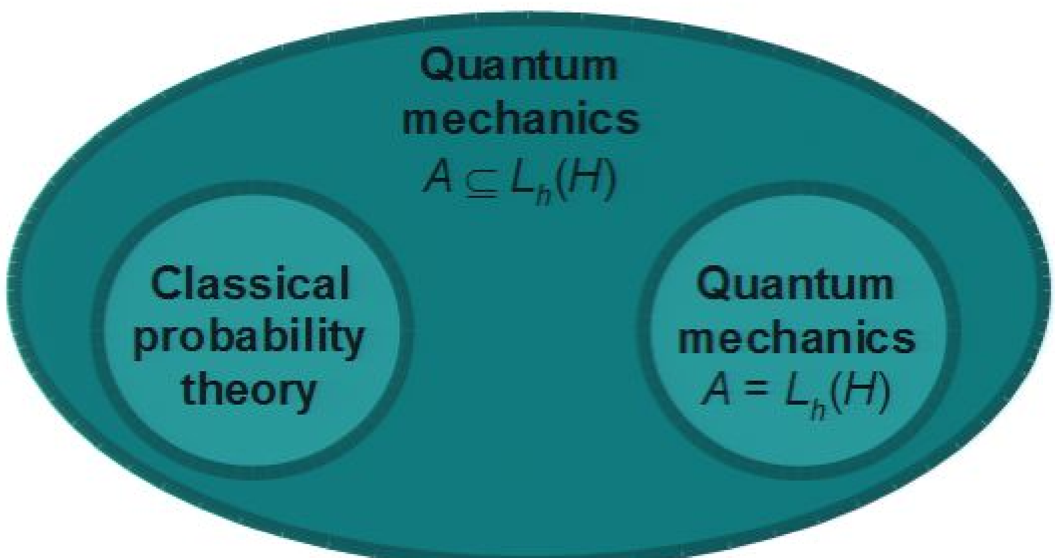
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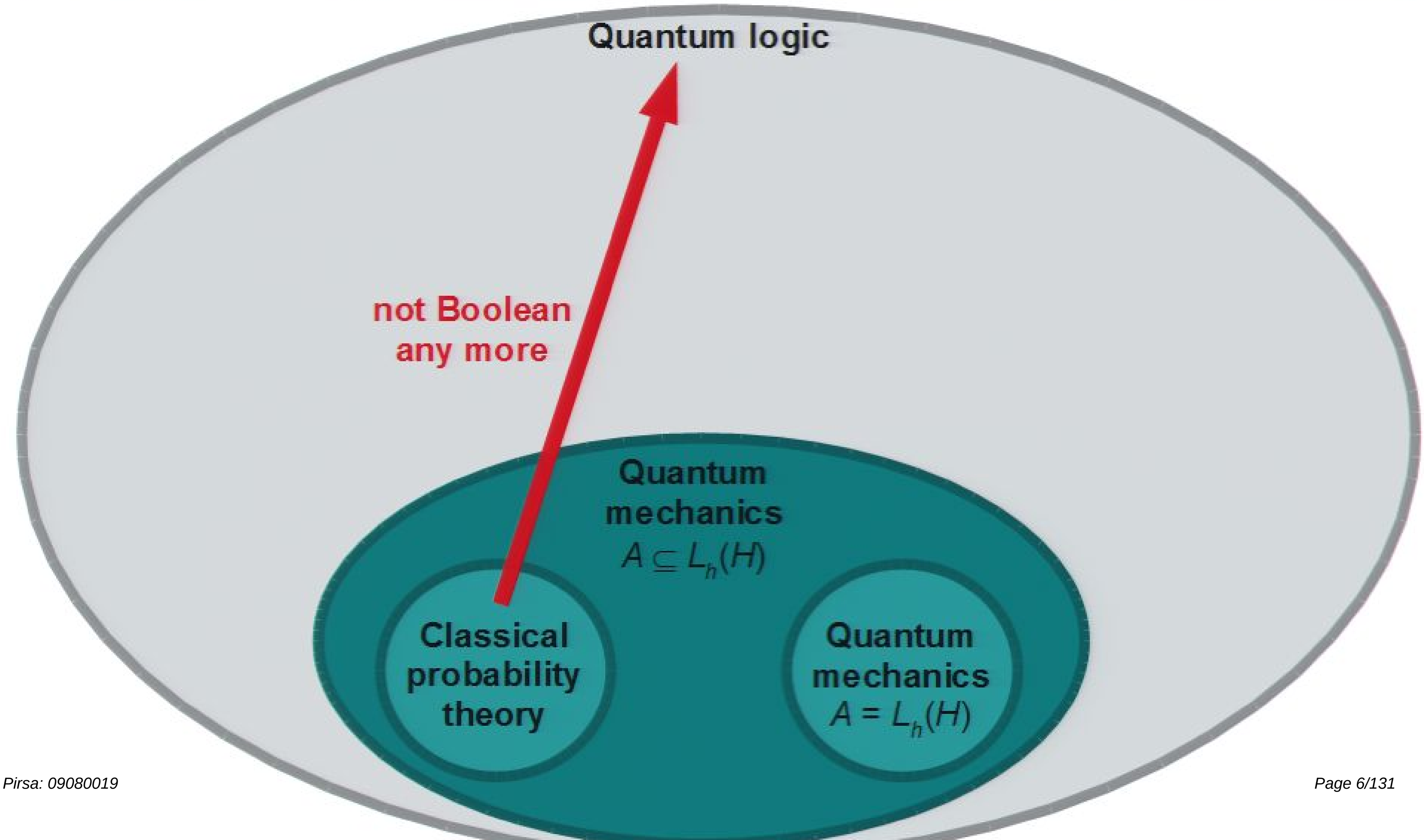
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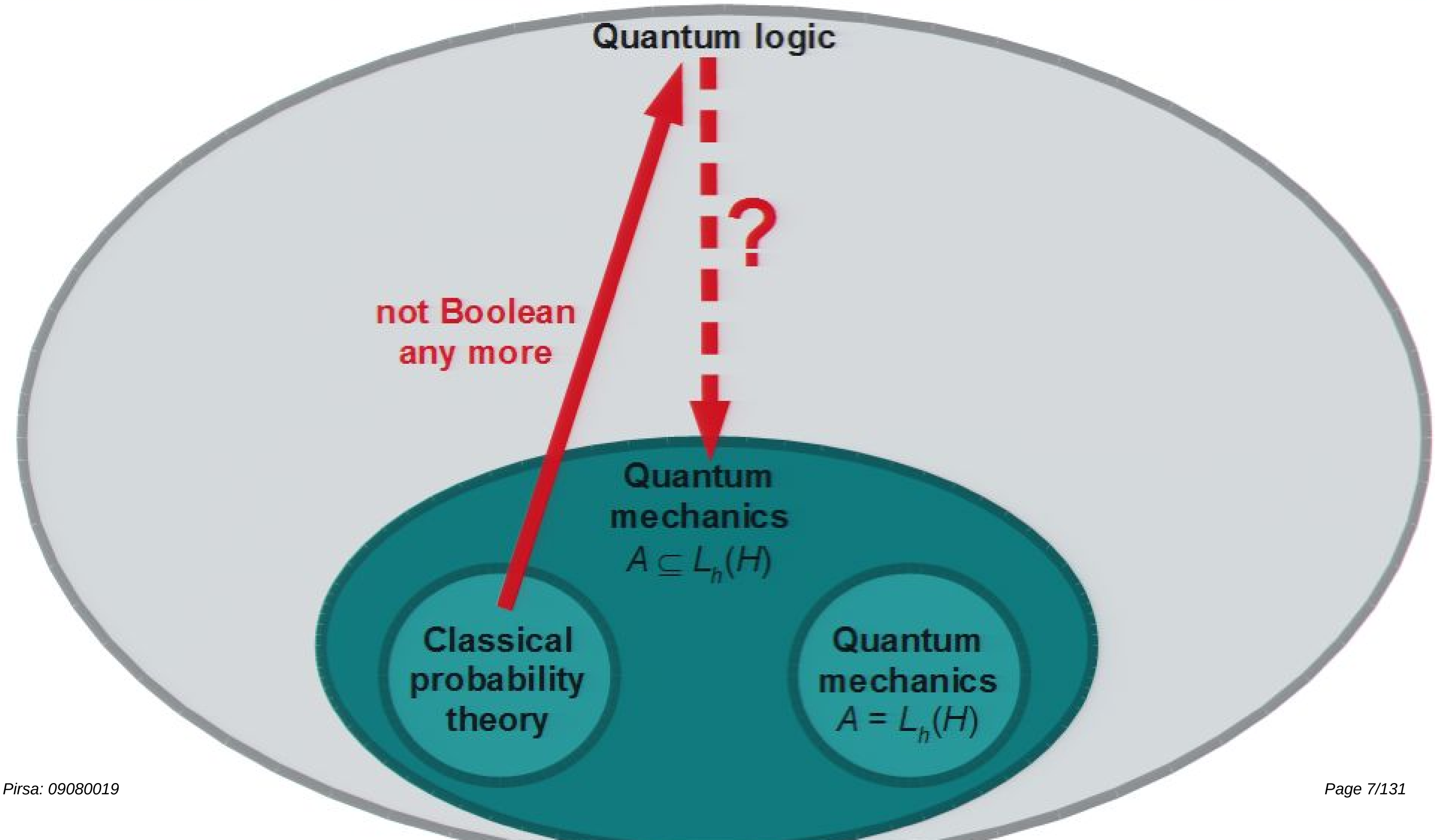
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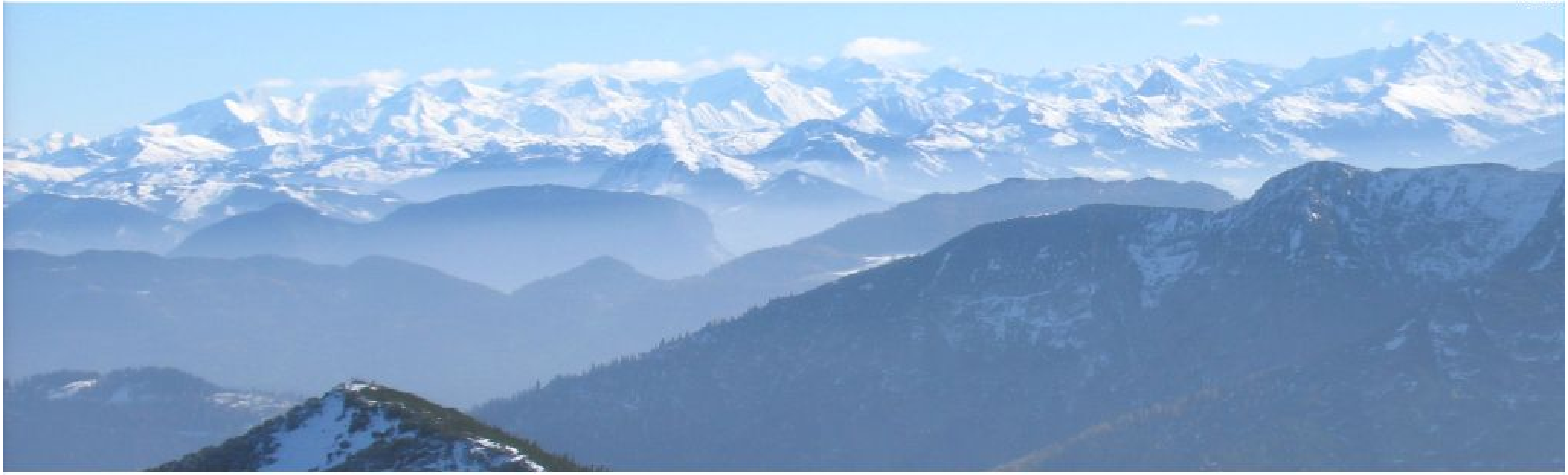

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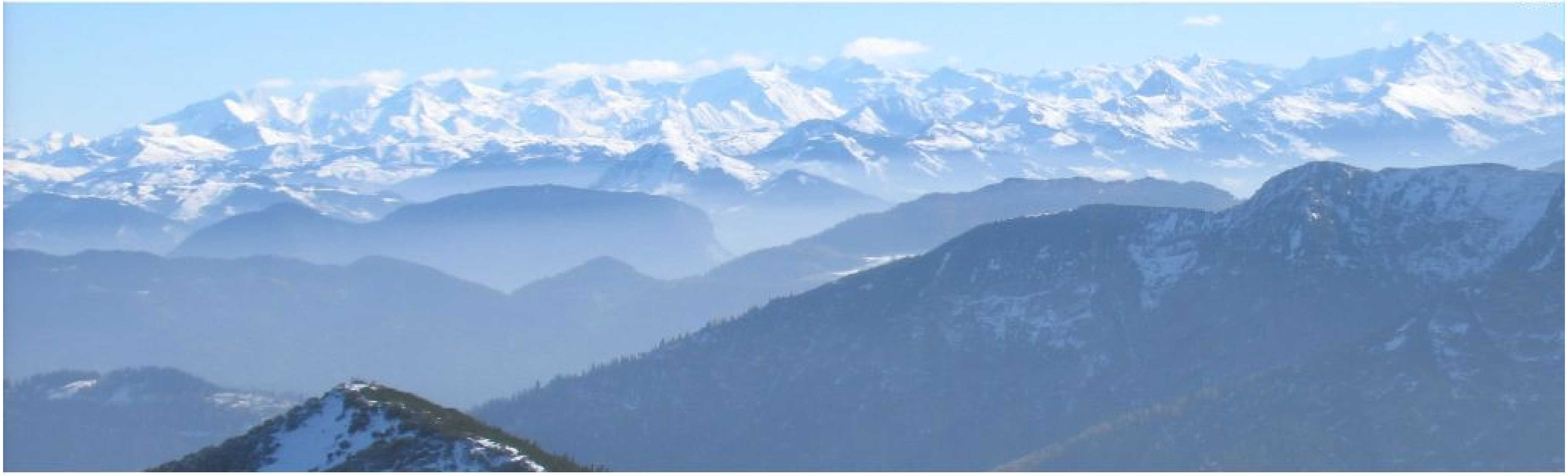

A reconstruction of quantum mechanics from quantum logics with unique conditional probabilities

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(2) A new mathematical structure

(3) Examples – The link to quantum measurement

(4) A further postulate for the conditional probabilities

(5) Observables and two further postulates

(6) Composite systems and a last postulate

(7) Conclusions

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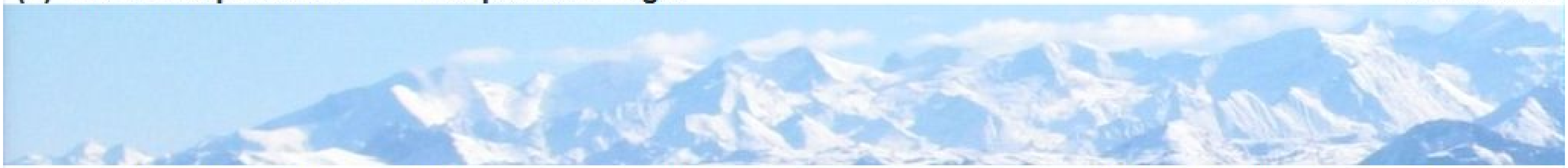

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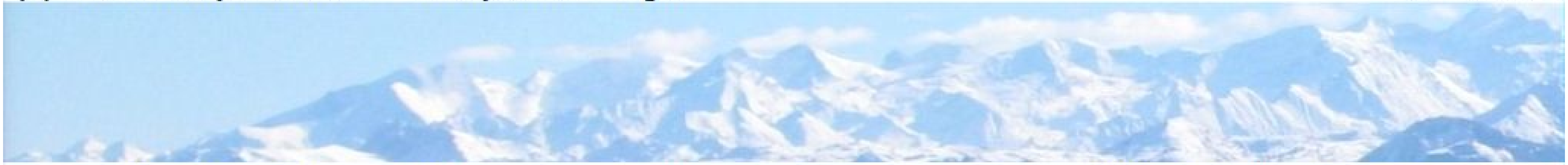

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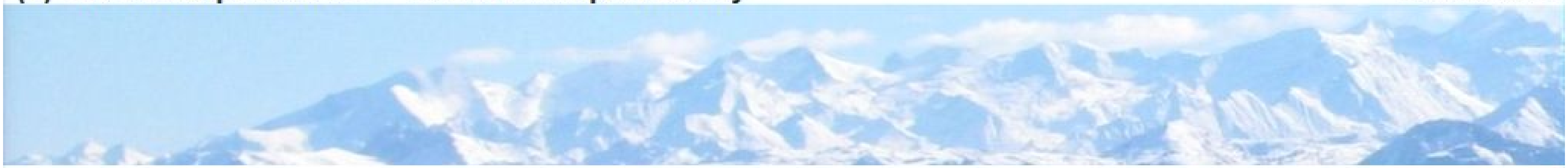

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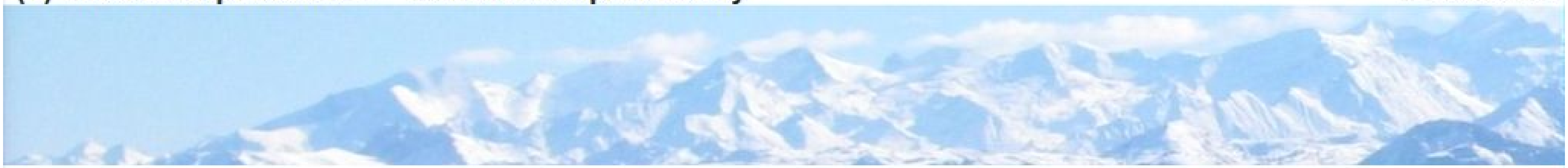

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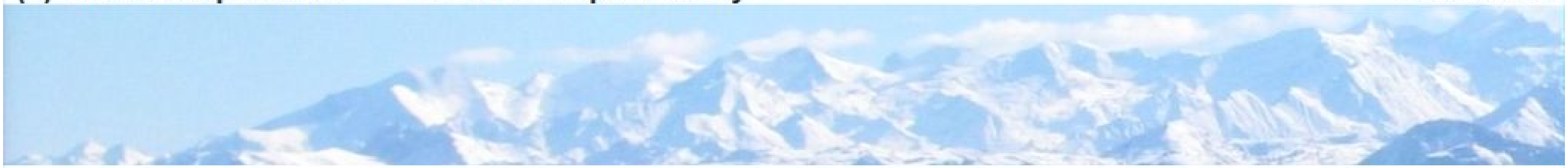

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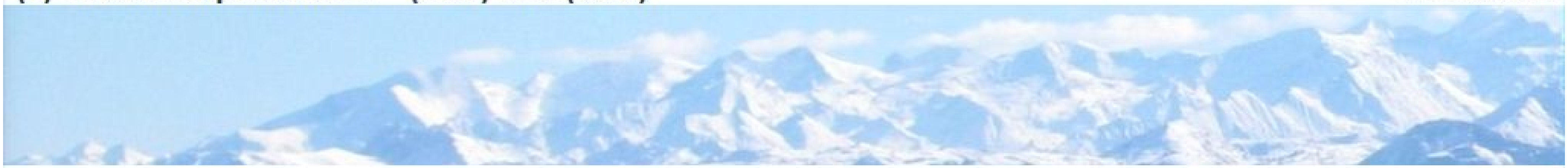

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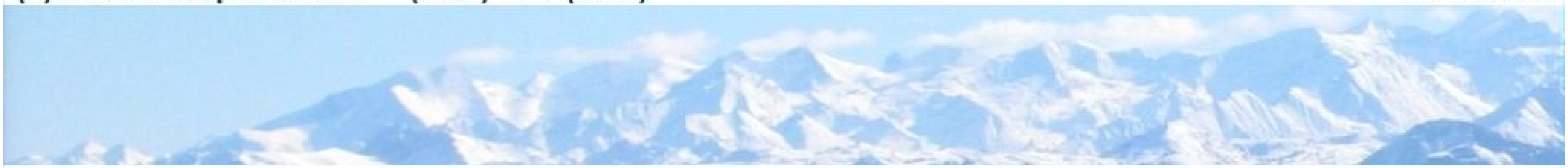

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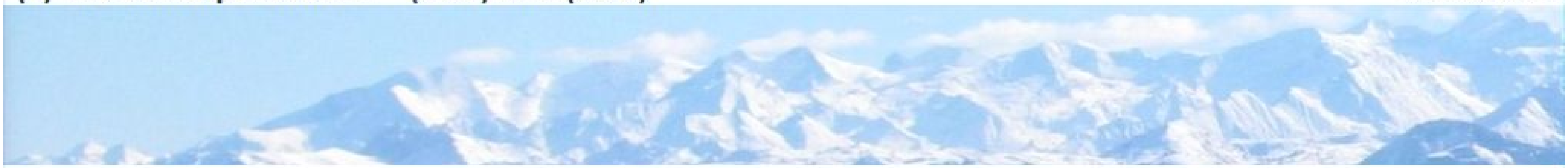

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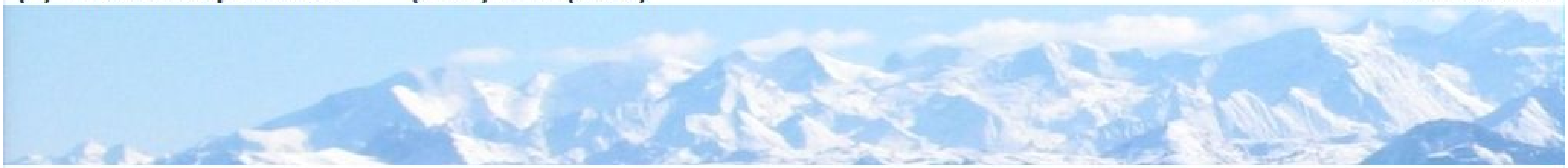

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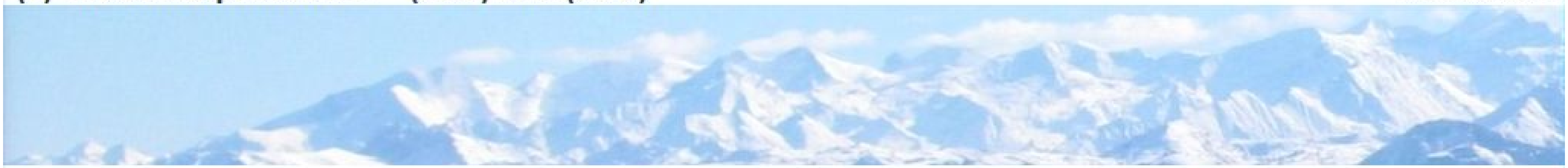

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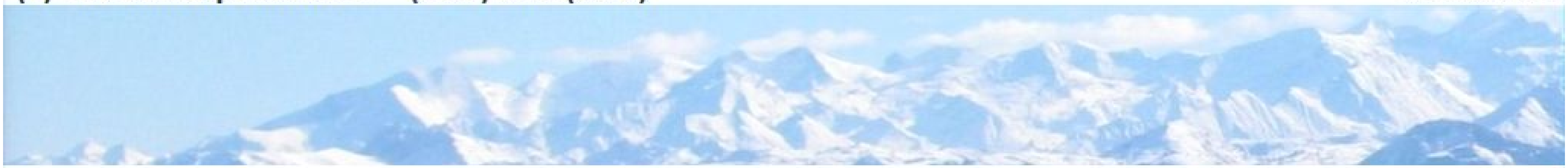

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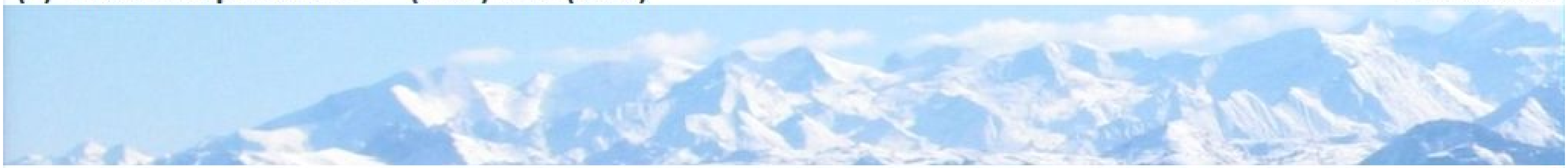

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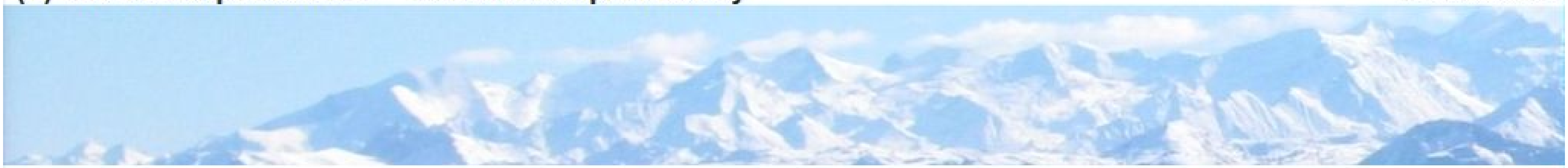

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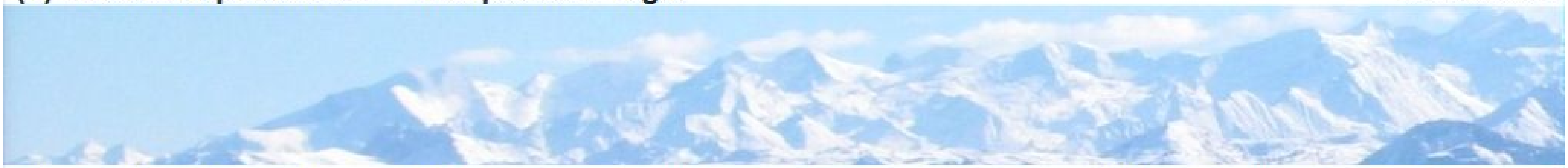

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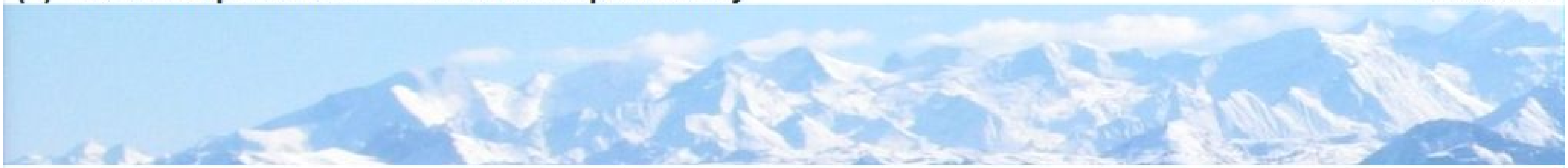

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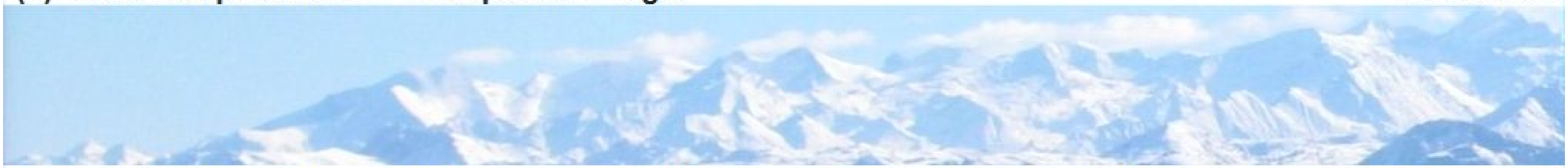

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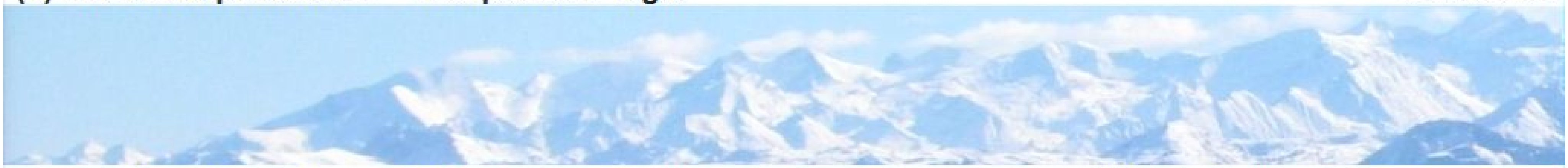

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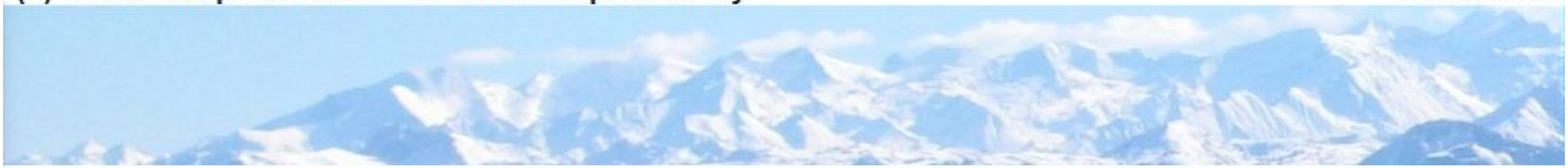

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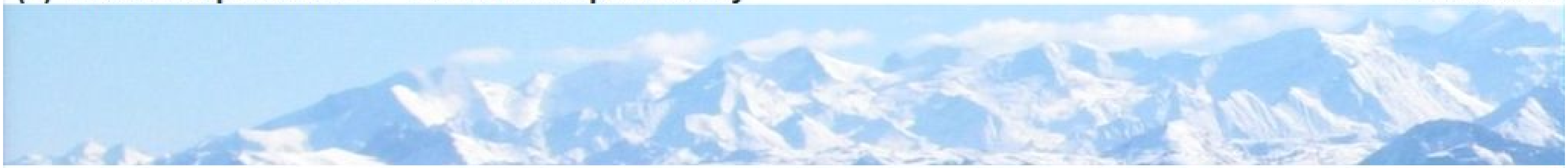

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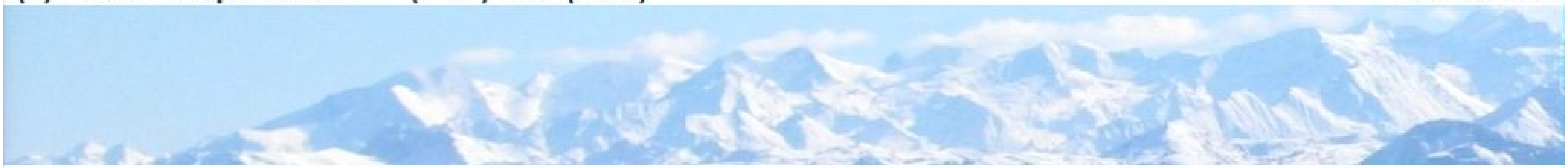

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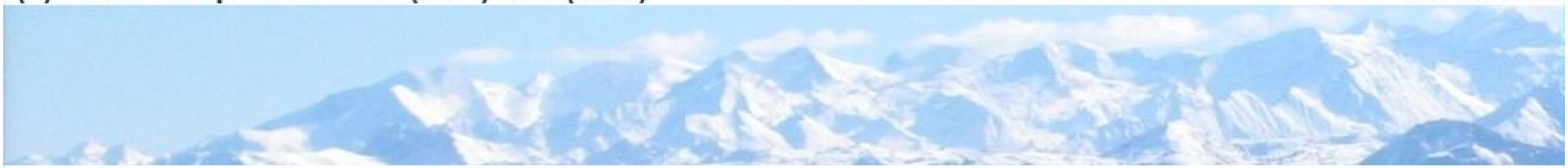

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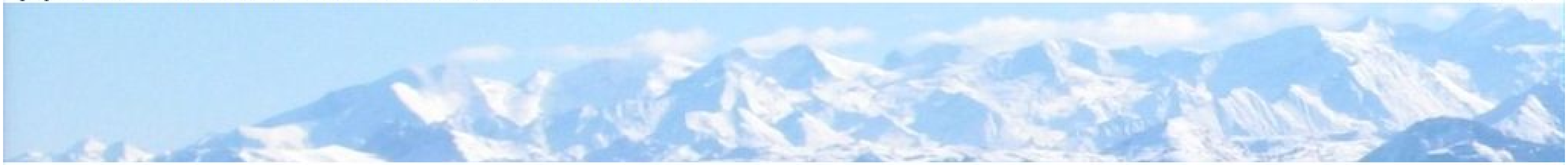
(2) A new mathematical structure – Theorem 1



Theorem 1: Assume that E satisfies (UC1) and (UC2). Then E can be embedded in the unit interval $[0,1]$ of an order unit space A with predual V such that the following conditions hold:

NOTE: *An order unit space is a Banach space with a partial ordering* $\leq$ *and an order unit* 1.

Then $\{a \in A : \|a\| \leq 1\} = [-1,1] := \{a \in A : -1 \leq a \leq 1\}$

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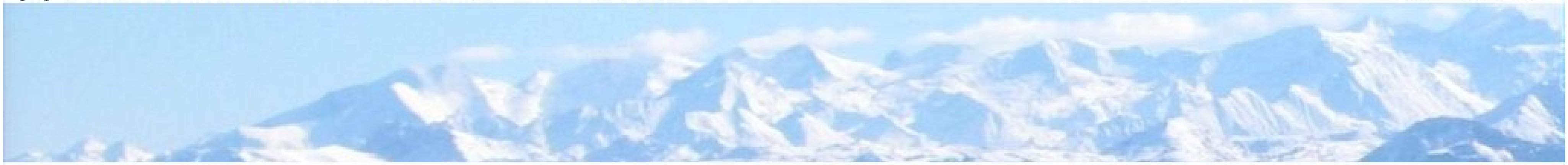

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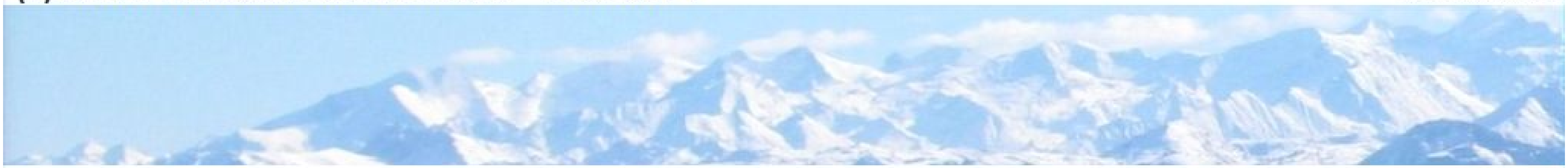

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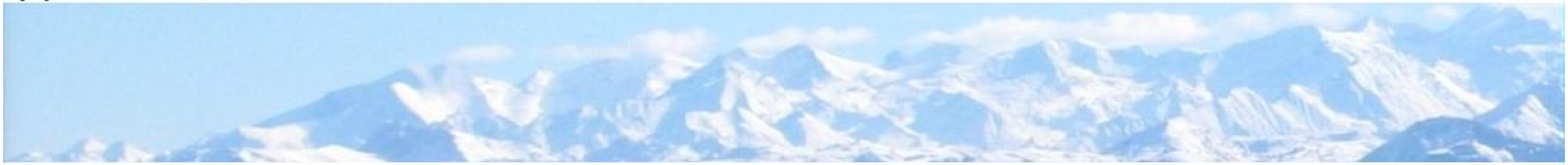

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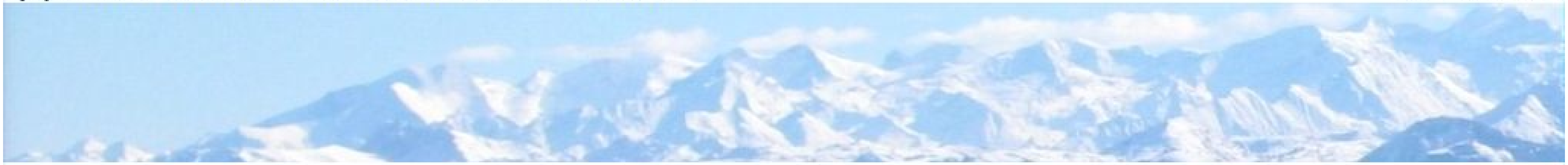

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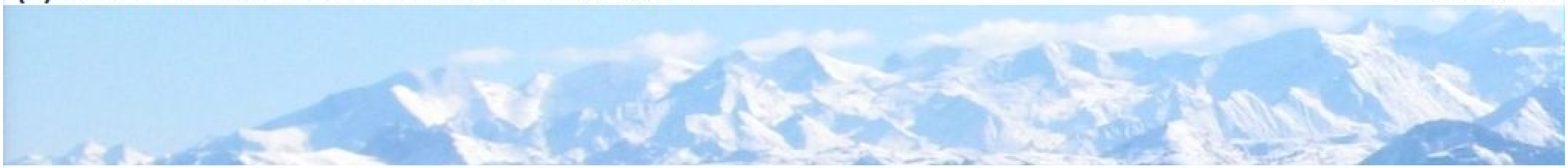

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Proof: Some simple functional analysis [2008b].

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(2) A new mathematical structure – Objective conditional probability

The conditional probability of an event *f* under *e* in a σ-additive state μ with μ(*e*)>0 has the shape:

$$\mu(f|e) = \frac{1}{\mu(e)} \hat{\mu}(U_e(f)).$$

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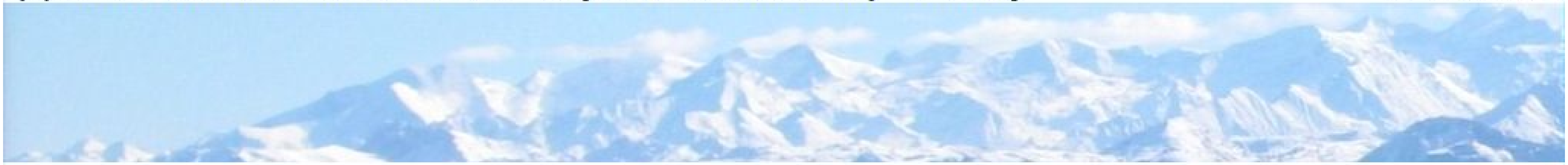

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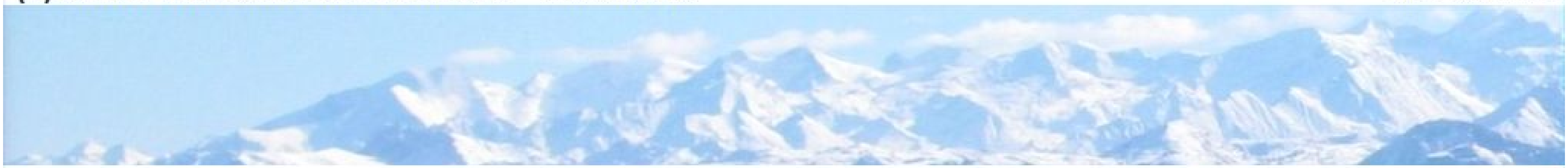
If $U_e(f)=se$ with a real number *s*, then $\mu(f|e)=s$ for each state with $\mu(e)>0$. This **objective conditional probability** is denoted by

$$\mathbb{P}(f|e).$$

(2) A new mathematical structure – Questions

- Do concrete examples exist for this new mathematical structure?



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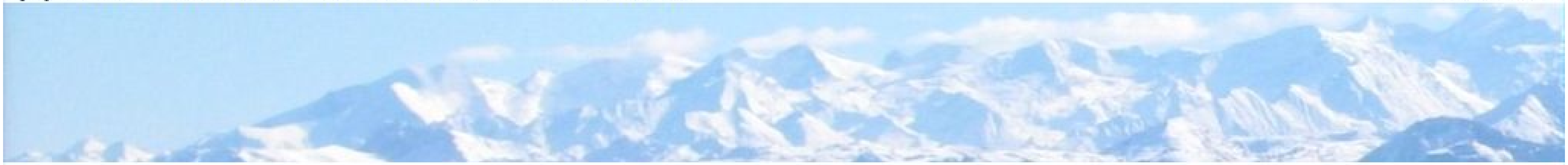

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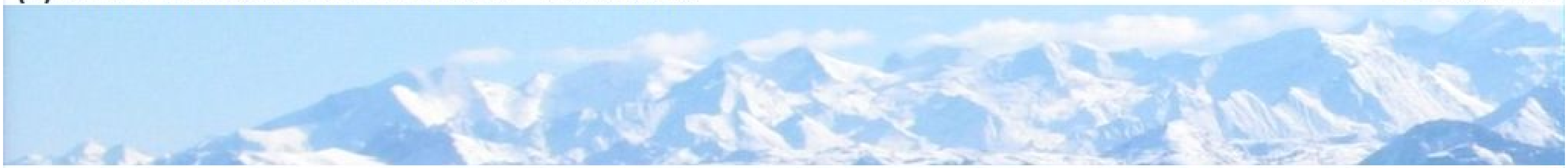

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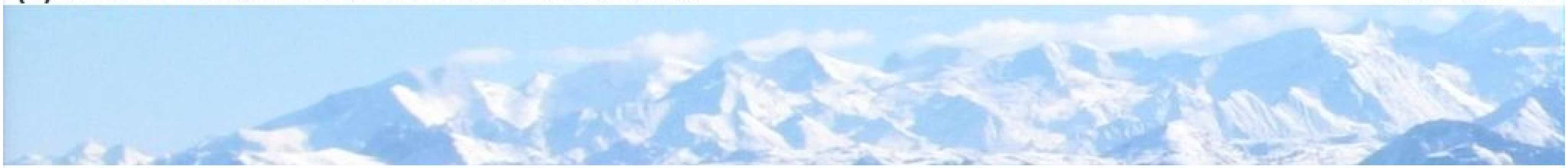

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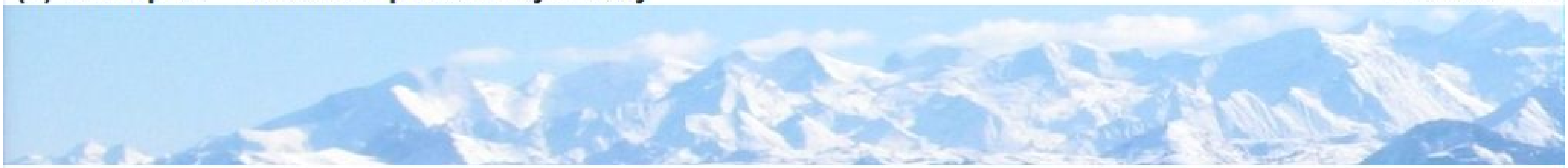
(3) Examples – Classical probability theory



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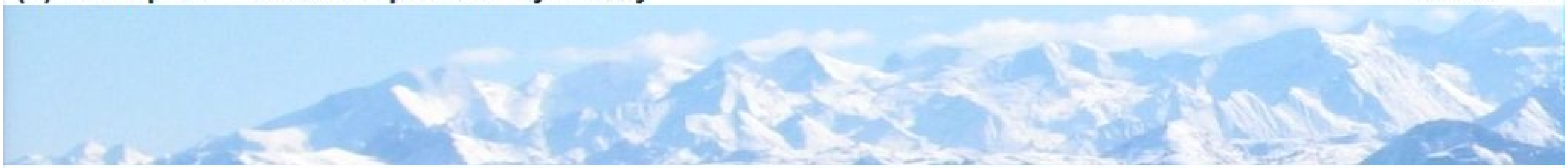

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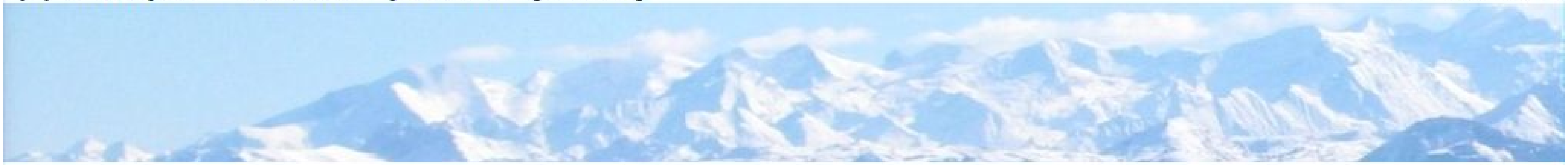

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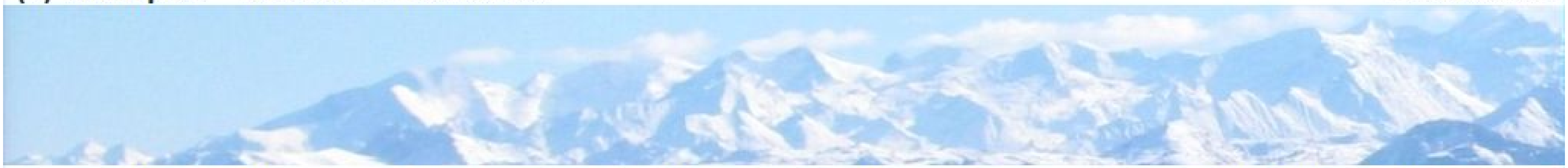
All events are compatible.

$\mathbb{P}(f|e)= 0$ for $e \cap f=\emptyset$ and $\mathbb{P}(f|e)= 1$ for $e \leq f$. Other cases are not possible and $\mathbb{P}(f|e)$ can never assume a value in the open unit interval.

(3) Examples – Quantum mechanics



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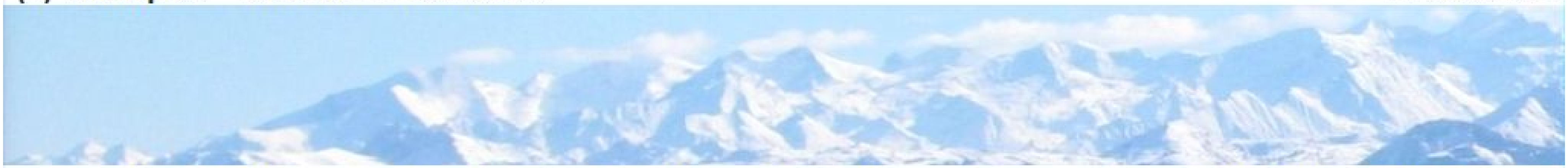

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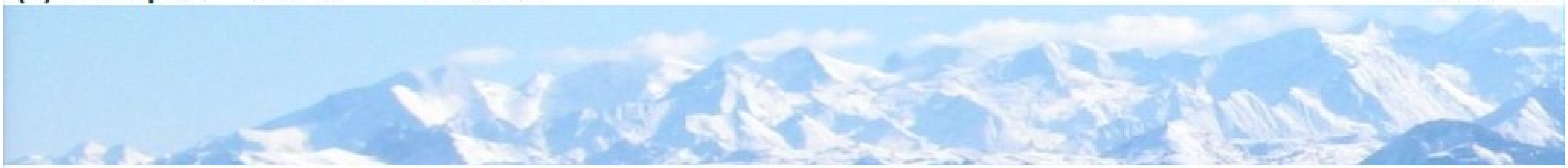

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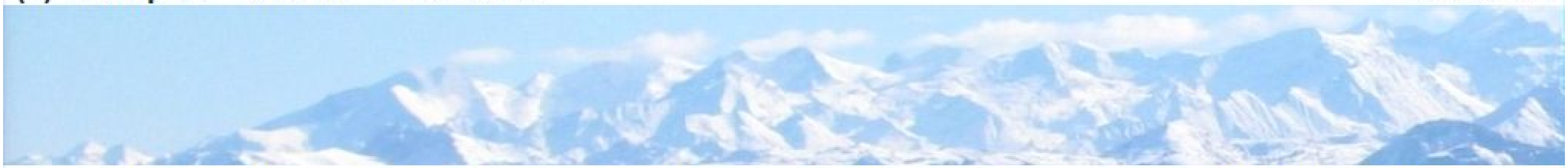

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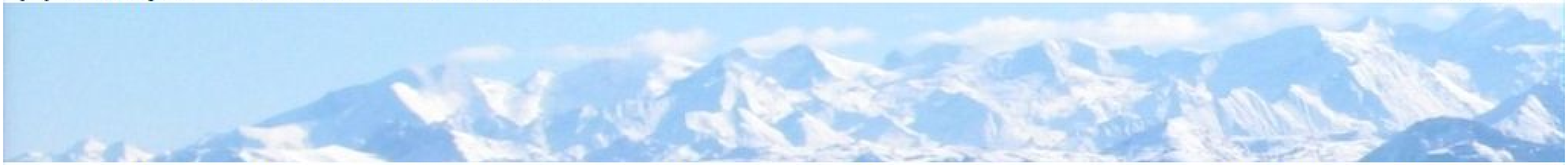

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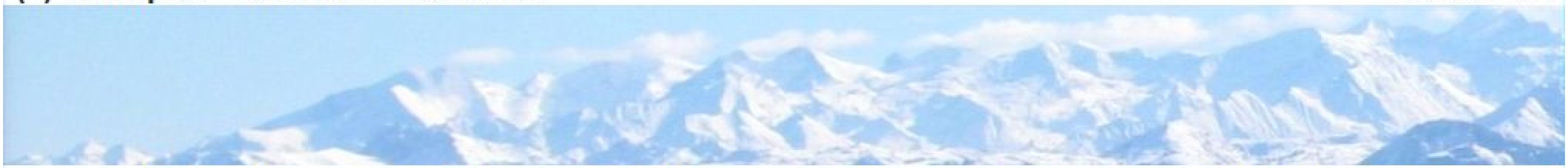

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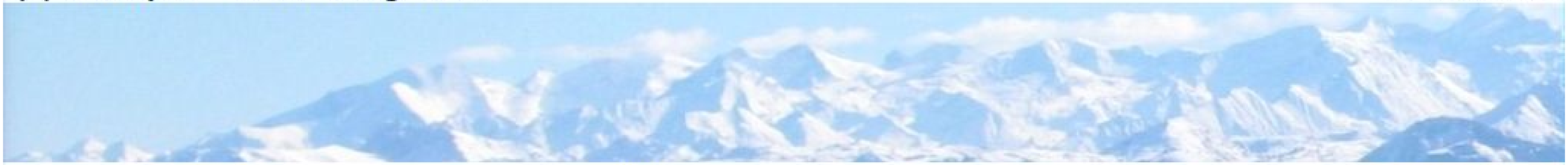
For $e = |\eta\rangle\langle\eta|$ and $f = |\xi\rangle\langle\xi|$ we get

$$\mathbb{P}(f|e) = |\langle\eta|\xi\rangle|^2,$$

(3) Examples – Jordan algebras



Assume that E is the projection lattice of a JBW algebra w/o type I_2 part. Then (UC1) as well as (UC2) hold and

$$U_e(f) = \{e,f,e\}$$

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The events e and f are compatible iff they operator-commute.

$$\mathbb{P}(f|e) = s \quad \text{iff} \quad \{e,f,e\} = se.$$

NOTES: (a) The so-called Jordan triple product is defined as $\{a,b,c\}:=a\circ(b\circ c)-b\circ(c\circ a)+c\circ(a\circ b)$ and, for idempotent elements, we get $\{e,f,e\}:=2e\circ(e\circ f)-e\circ f$.
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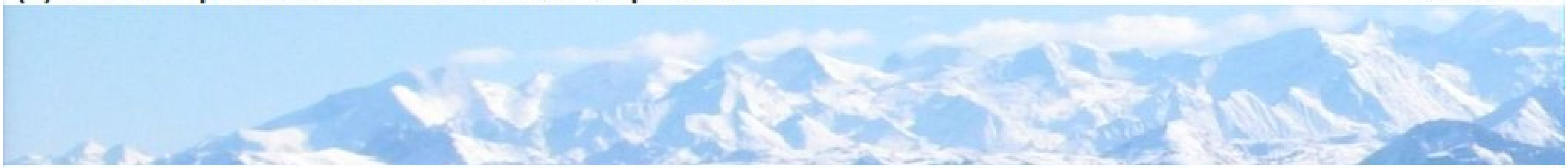
The exceptional JBW algebras now provide a type of examples that cannot be represented on a Hilbert

(4) A further postulate for the conditional probabilities - 1



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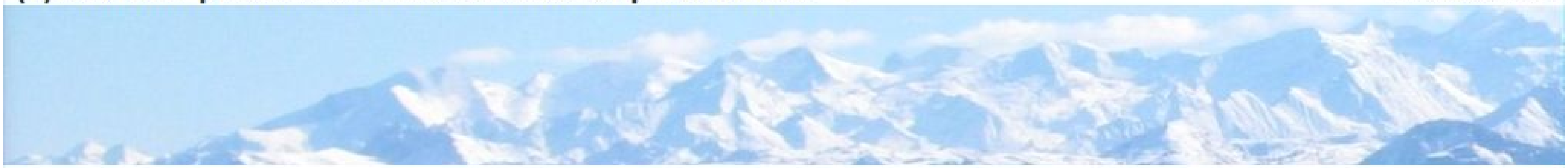

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E.M. Alfsen and F.W. Shultz's interpretation (1979): „*The probability of the exclusive disjunction of two system properties is independent of the order of the measurements of the two system properties*".

Note that the exclusive disjunction is not an event or proposition.

E.M. Alfsen and F.W. Shultz consider "beams" and "filters" instead of conditional probability and use their so-called P-projections which are different from the U_e-projections. To make the U_e-projections P-Projections, a postulate is needed which is redundant in the reconstruction process presented here.

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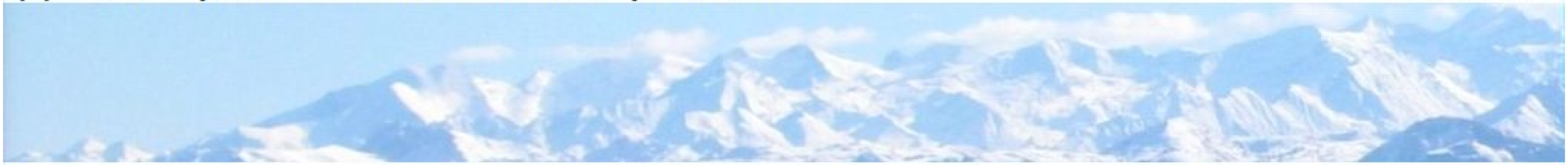

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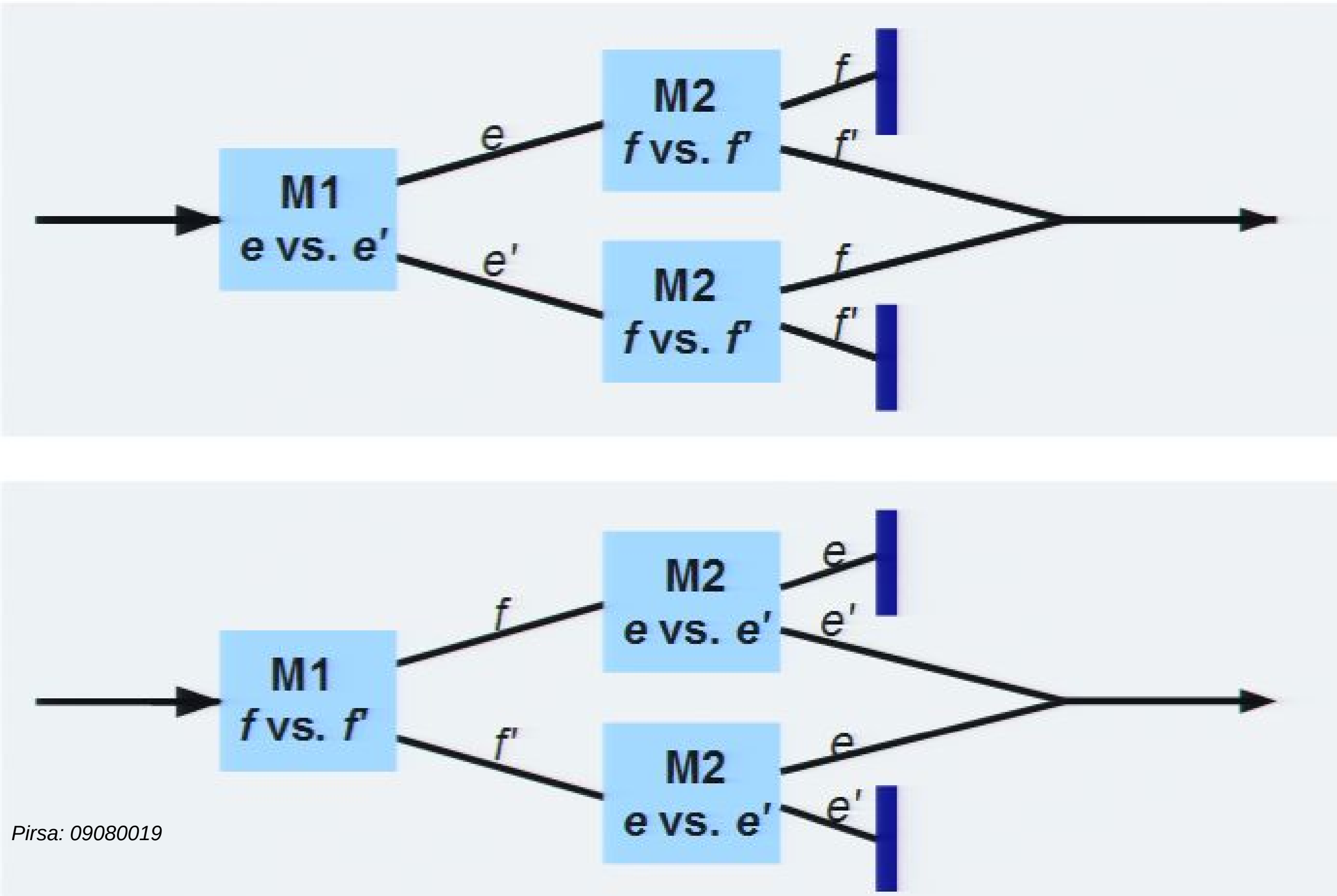

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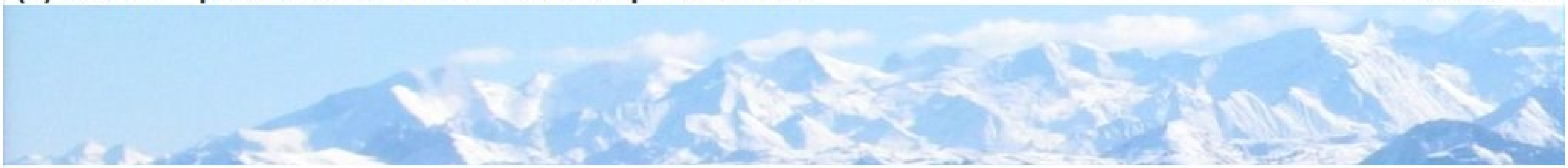

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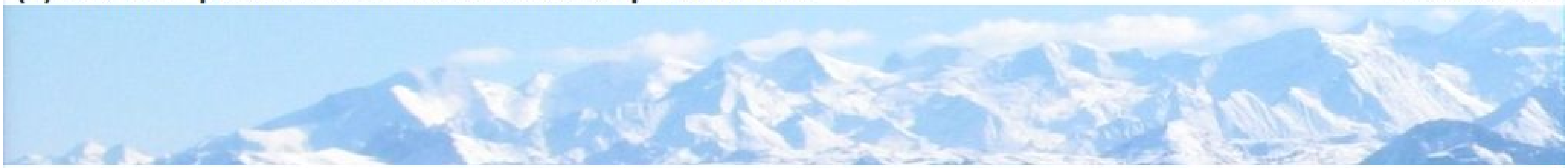

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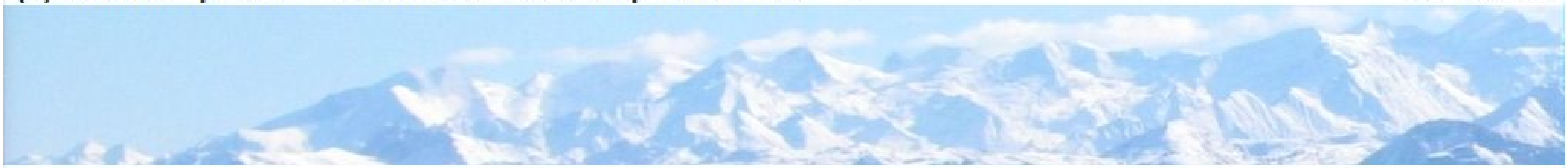

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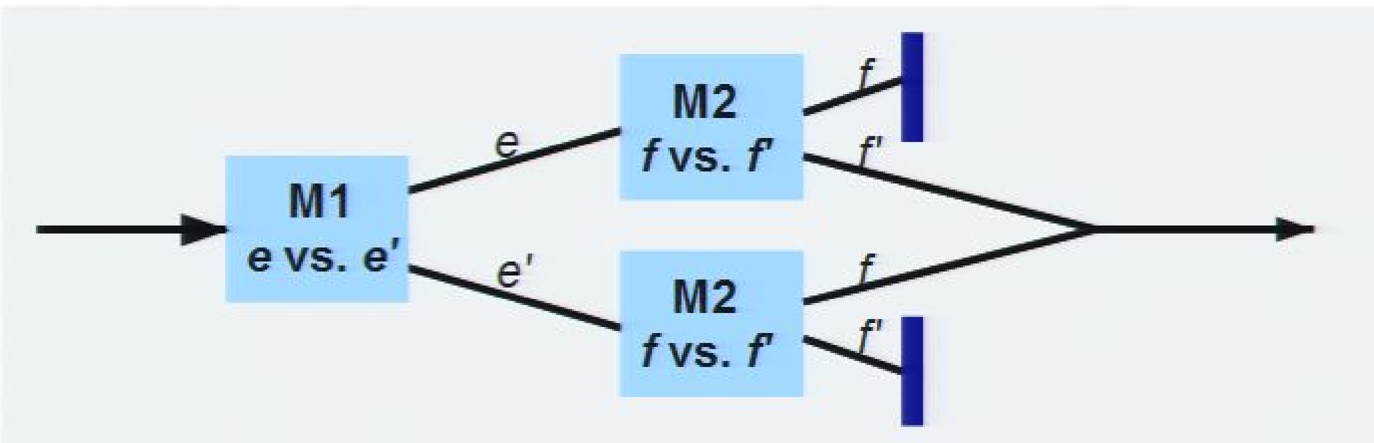

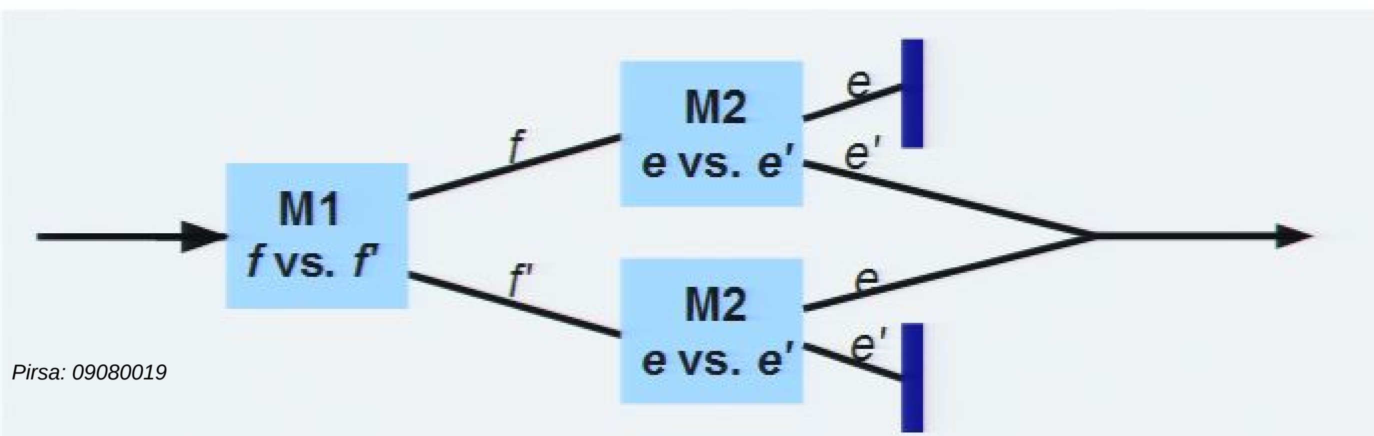

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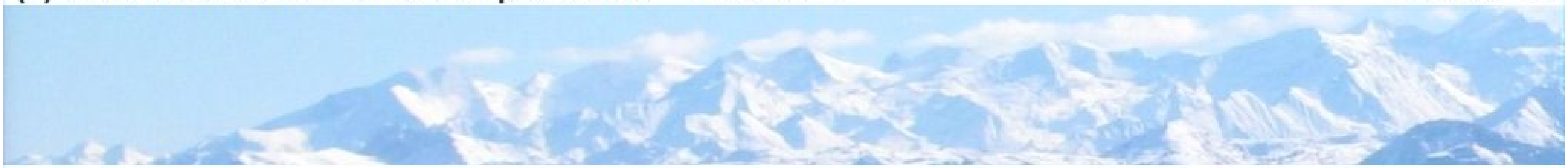
(5) Observables and two further postulates – Theorem 3



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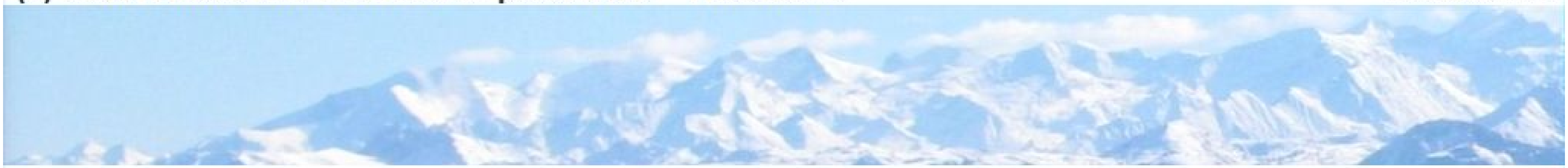
The <u>probability distribution</u> of X in the σ-additive state μ is denoted by μ^X: $\mu^X(B) := \mu(e_B)$. The <u>expectation value</u> of X is the measure integral $\int t \, d\mu^X$.

(5) Observables and two further postulates – Theorem 3



A real <u>observable</u> X is an abstract spectral measure allocating an event $e_B \in E$ to each Borel subset B of the real numbers. Then: {bounded real observables} $\leftrightarrow$ {elements of A which have a spectral decomposition in E}.

The <u>probability distribution</u> of X in the σ-additive state μ is denoted by μ^X: $\mu^X(B) := \mu(e_B)$. The <u>expectation value</u> of X is the measure integral $\int t \, d\mu^X$.

(A2) For each pair of events $e, f \in E$ there is a real observable X such that $\mu(f|e)\mu(e)$ is identical with the expectation value of X for all σ-additive states μ.

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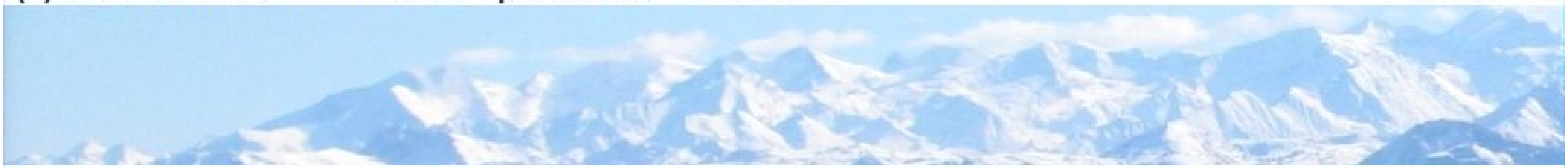

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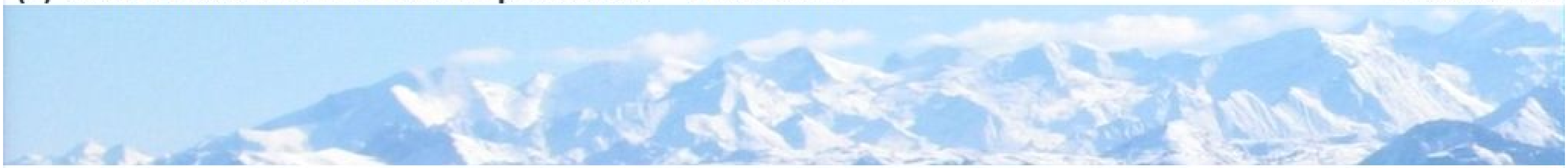

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Theorem 2: Then the order unit space A from Theorem 1 can be equipped with a (generally non-associative) commutative multiplication operation $\circ$ such that A becomes a JBW algebra and E is a weakly dense subset of the projection lattice in A.

Proof: details in [2008b],
brief sketch on next slide

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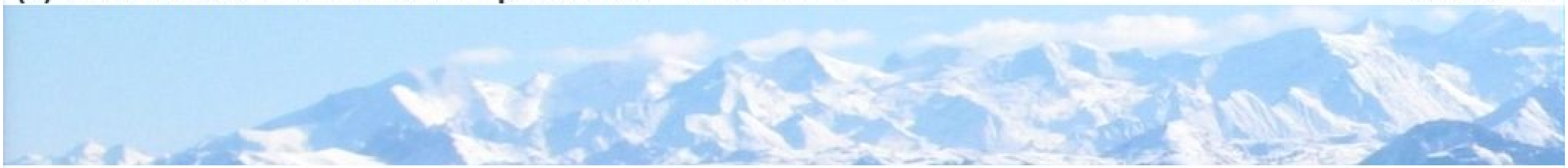

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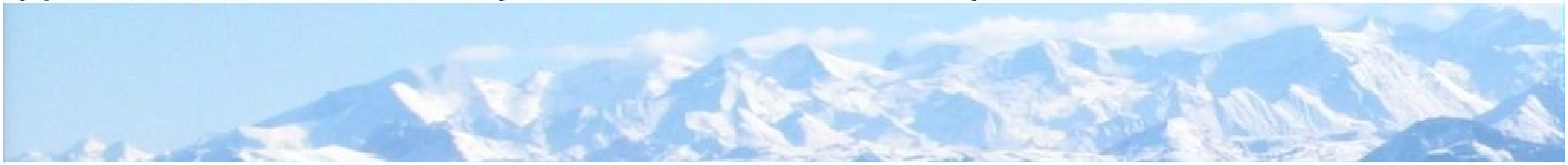
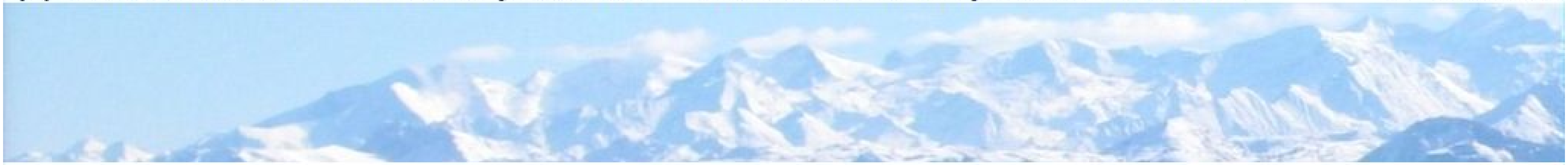
NOTE: *Reconsidering the 3 examples, the monotone sequentially complete subalgebra generated by E (or the || ||-closed linear hull of E) is identical with the measurable functions, the von Neumann algebra, the JBW algebra,*



- (A3) makes the bounded real observables form a linear subspace of *A*.

(5) Observables and two further postulates – Brief sketch of the proof of Theorem 3



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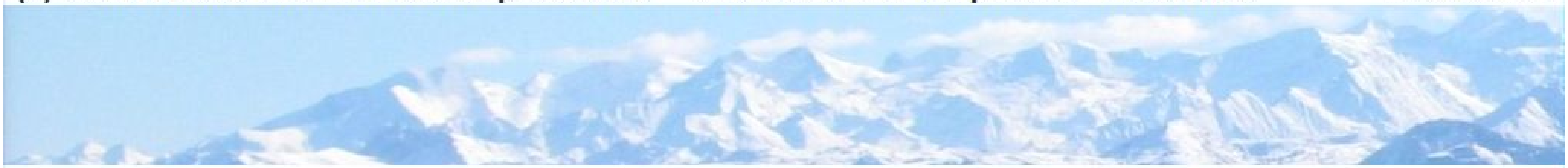

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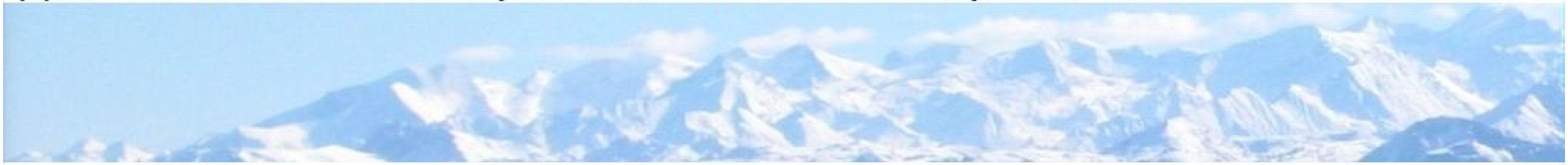

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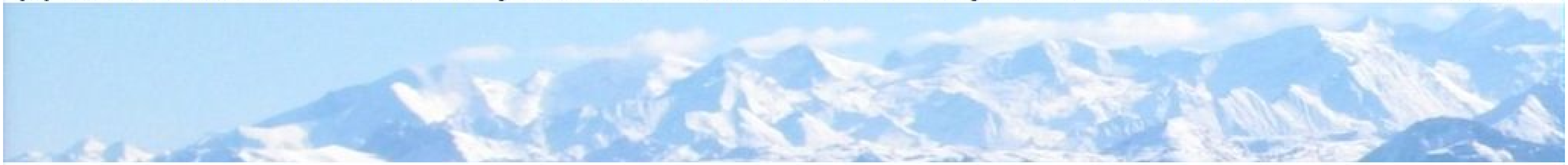

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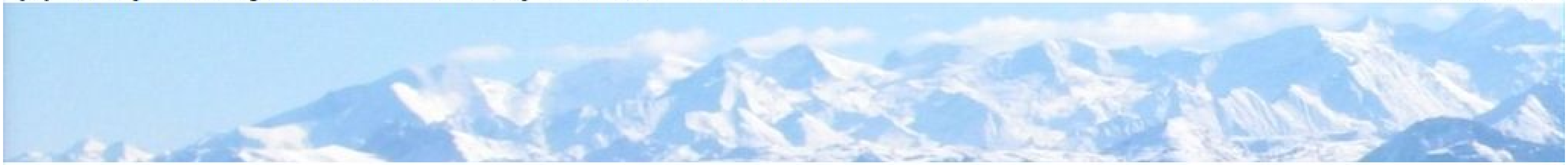
(6) Composite systems and a last postulate – The model



(C) A model $E \otimes F$ of a composite system consisting of two single systems each modelled by E and F, respectively, shall satisfy the following conditions:

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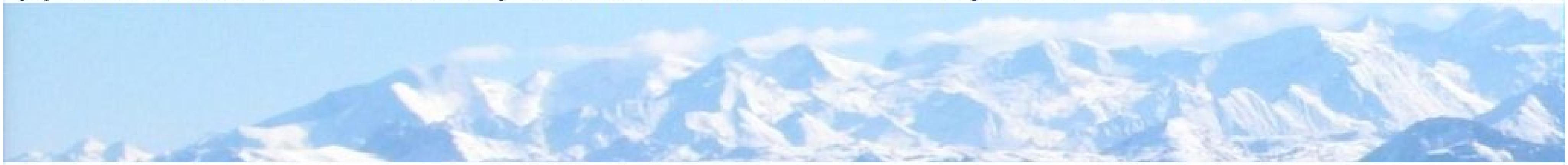

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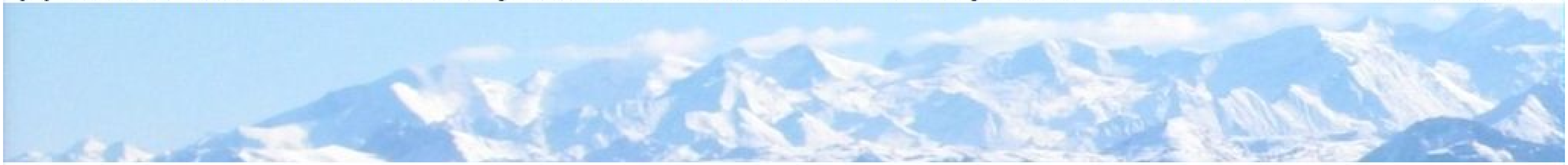

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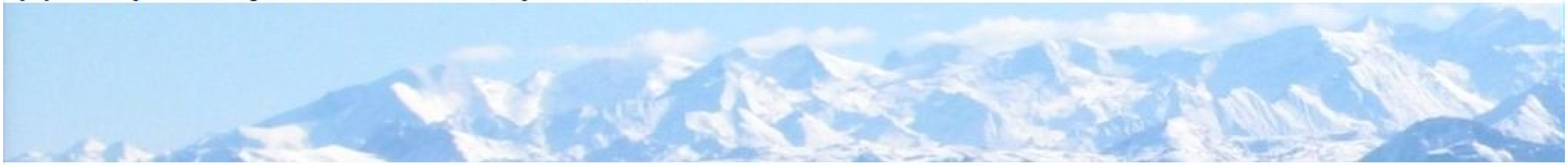

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(i) For each pair of events $e \in E$ and $f \in F$ there is an event $e \otimes f \in E \otimes F$ *such that* $e \otimes 0 = 0 \otimes f = 0$, $\mathbb{1} \otimes \mathbb{1} = \mathbb{1}$, and $e_1 \otimes f_1 \perp e_2 \otimes f_2$ whenever $e_1 \perp e_2$ or $f_1 \perp f_2$. Moreover the maps $e \rightarrow e \otimes f$ with f fixed and $f \rightarrow e \otimes f$ with e fixed are σ-additive.

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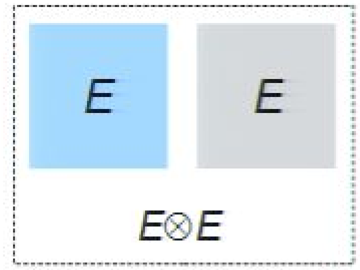
(ii) For each separately σ-additive state ρ on $E \times F$, there is one and only one σ-additive state ρ' on $E \otimes F$ with $\rho(e,f) = \rho'(e \otimes f)$.

The second condition is motivated by the consideration of the joint distribution of a generalized E-valued and an F-valued observable under a certain compatibility condition [2004b].

(6) Composite systems and a last postulate – Theorem 3

When does $E \otimes E$ exist and satisfy (UC1), (UC2), (A1), (A2) and (A3)?



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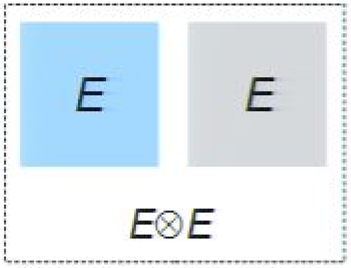

Theorem 3:

It does exist, if E is the projection lattice of a Jordan matrix algebra constructed over the complex numbers, but it is not unique. One of the two solutions is the projection lattice of the tensor product of the corresponding C*-algebras (not the tensor product of the Jordan algebras).

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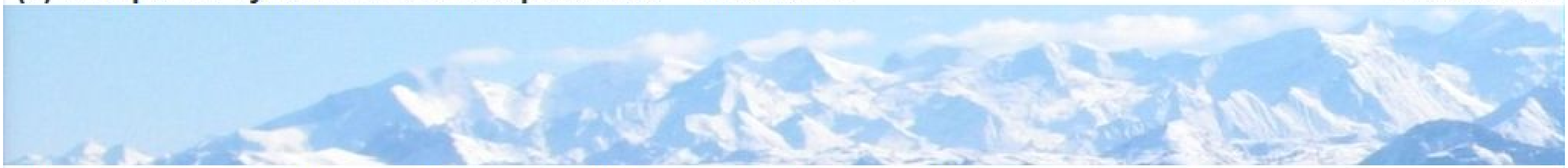

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It does not exist, if *E* is the projection lattice of a Jordan matrix algebra constructed over the real numbers, quaternions or octonions.

Proof: long, many lemmas needed, calculation and comparison of dimensions, details in [2004b]

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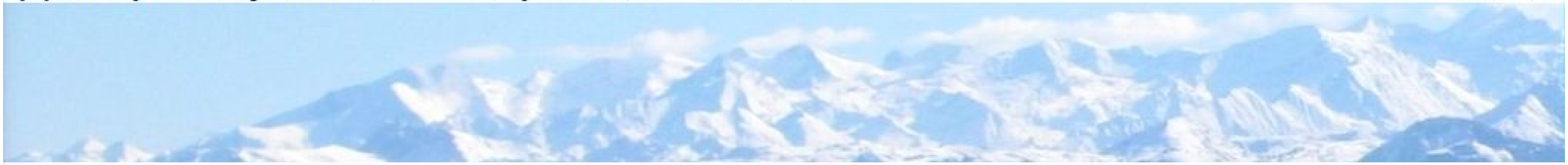

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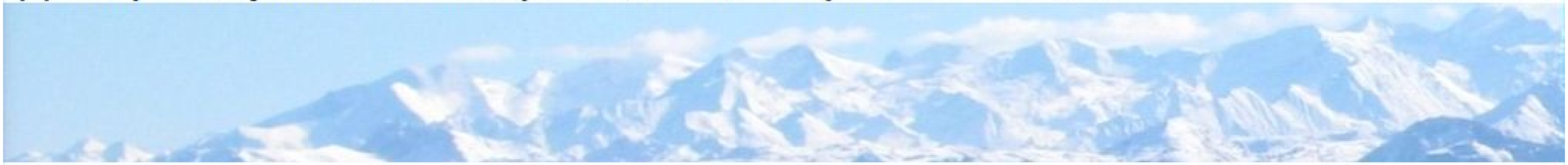
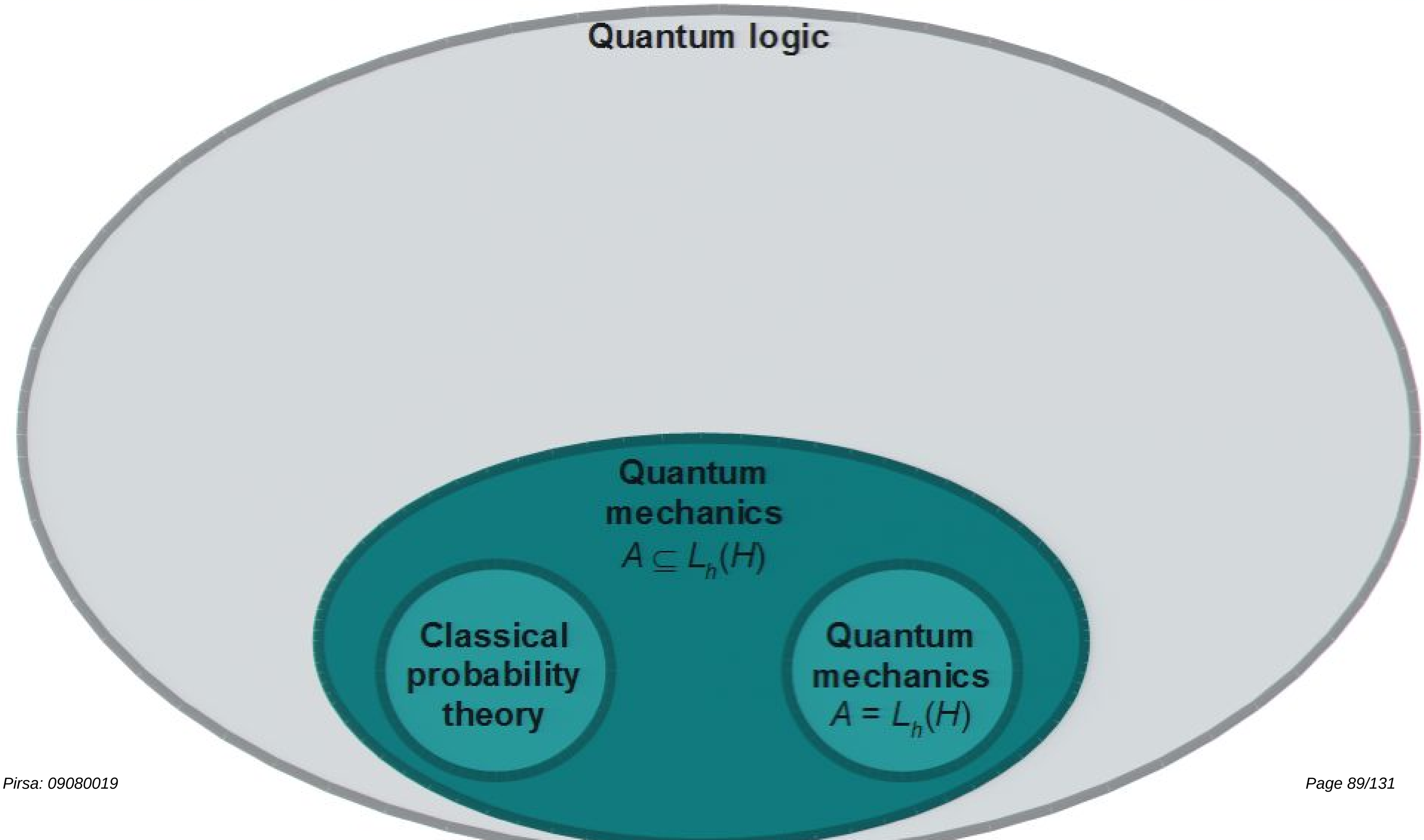
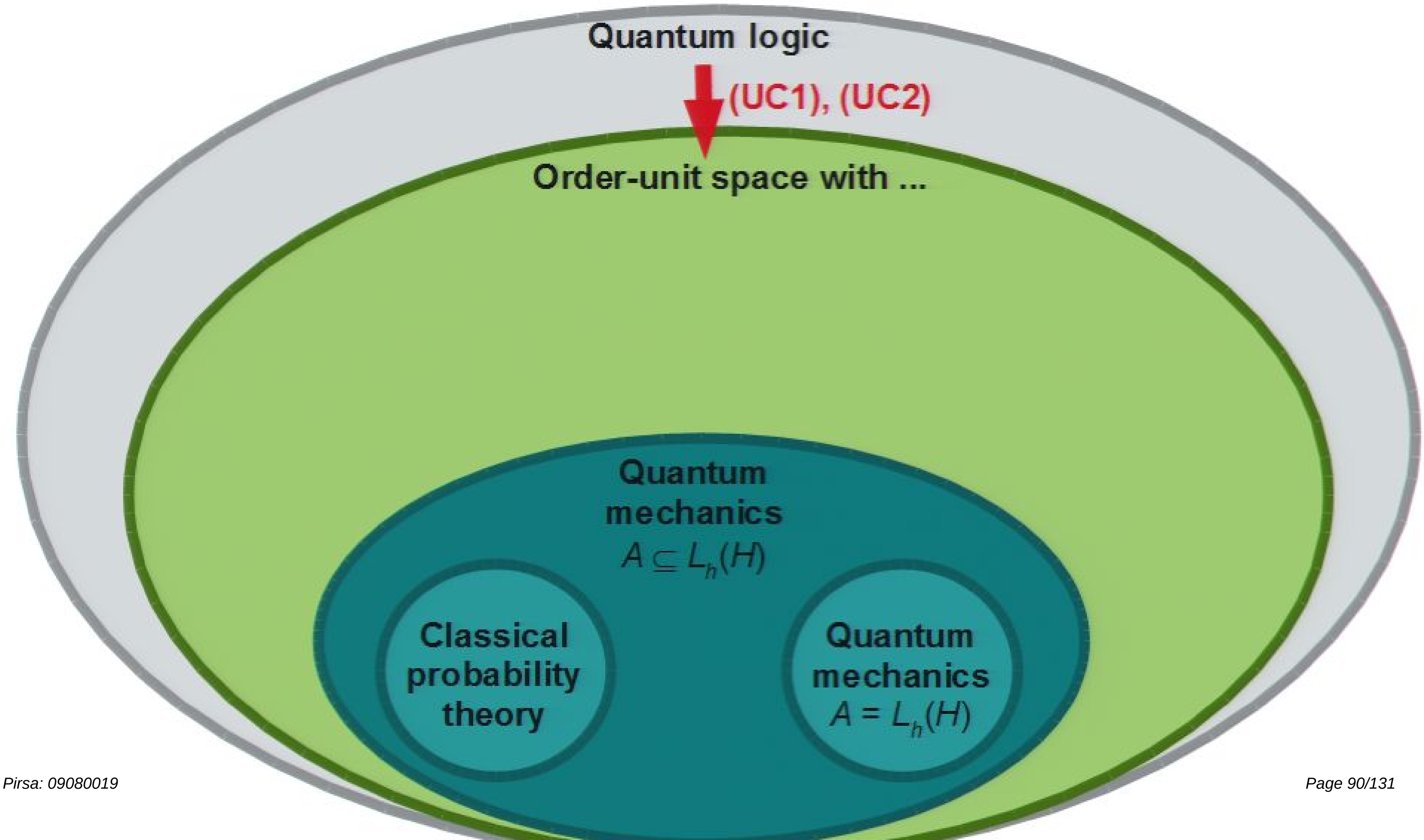
Thus postulate (C) rules out not only the exceptional Jordan algebra constructed over the octonions, but also the Jordan matrix algebras constructed over the reals or quaternions such that only the complex numbers remain – at least in the finite-dimensional case.




Quantum logic
Quantum mechanics
$A \subseteq L_h(H)$
Classical probability theory
Quantum mechanics
$A = L_h(H)$

(6) Composite systems and a last postulate – The complete reconstruction



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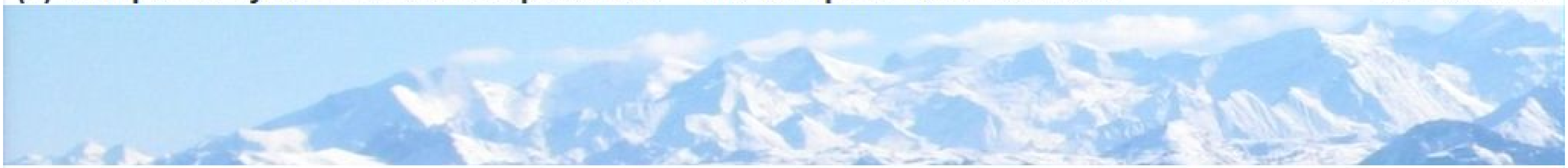
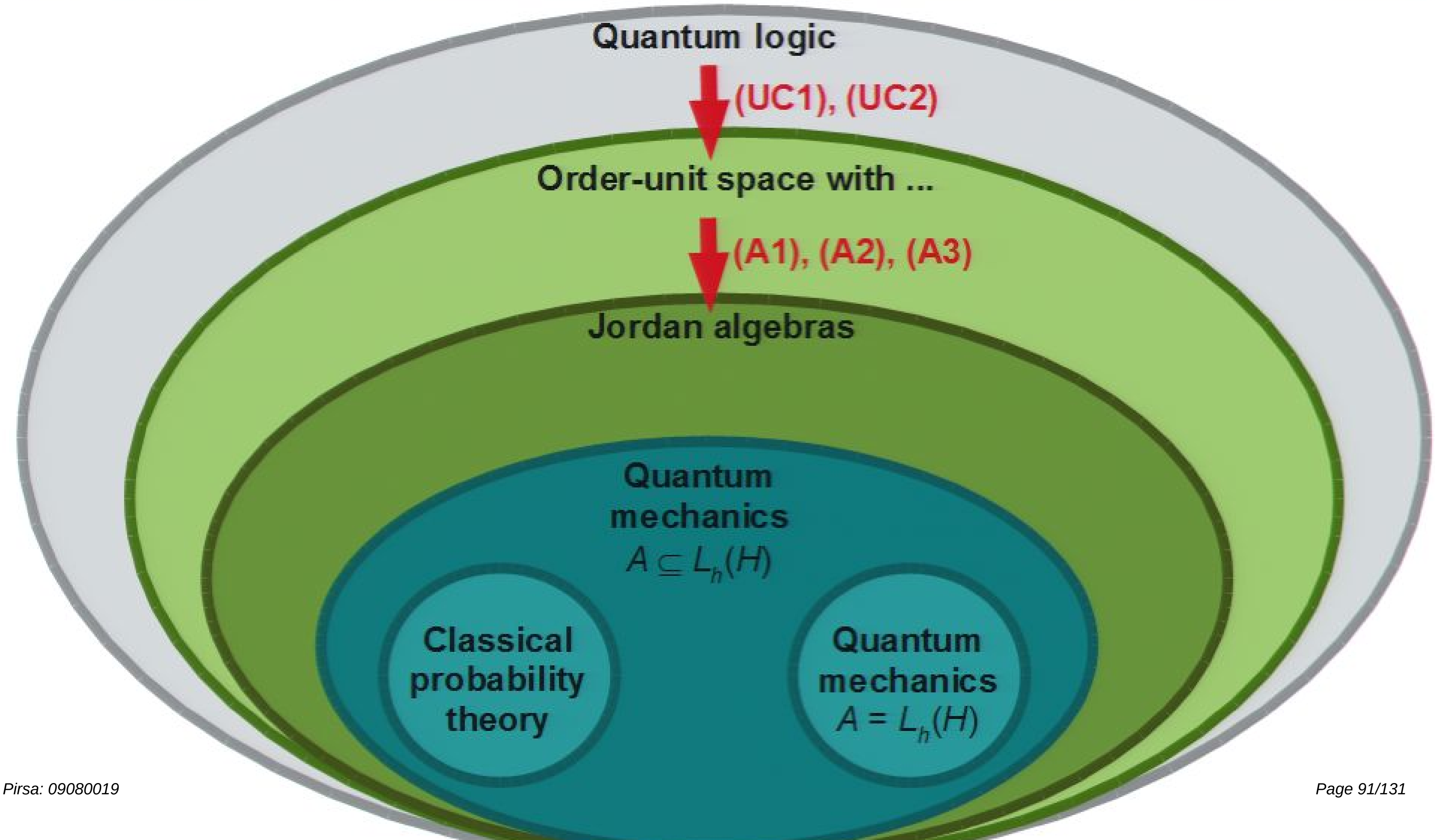
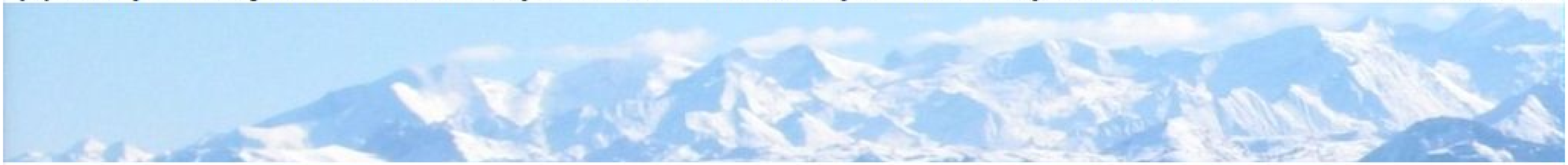




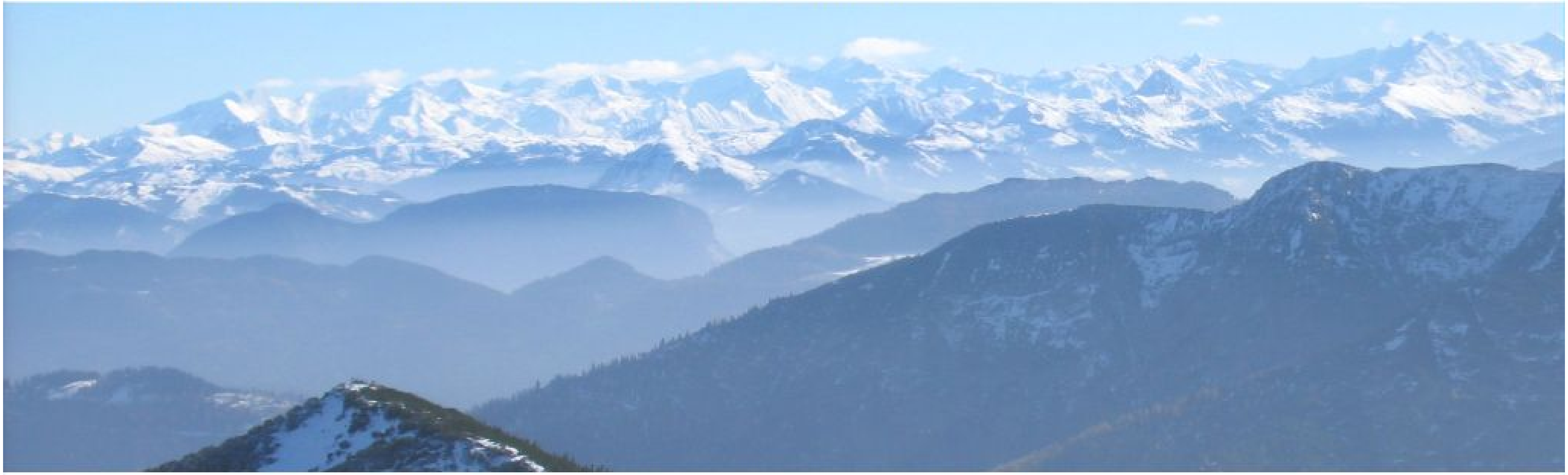
(6) Composite systems and a last postulate – The examples and the postulates



	Basic postulates (UC1), (UC2)	Additional postulates (A1), (A2, (A3)	Composite systems (C)	Complex Hilbert space represention
Projection lattice in a von Neumann algebra	✔	✔	✔	✔
Projection lattice in a special JBW algebra	✔	✔		✔
Projection lattice in an exceptional JBW algebra	✔	✔		
???*	✔			

*: *Are (A1), (A2), (A3) redundant?*

(7) Conclusions – The reconstruction of quantum mechanics



The reconstruction process has brought us stepwise

(1) to a new mathematical structure,

(2) to abstract Jordan algebras, and

(3) to operator algebras on Hilbert spaces over the complex numbers.

The approach gets ahead of other ones that are not able to justify the need for the complex Hilbert space or operator algebras.

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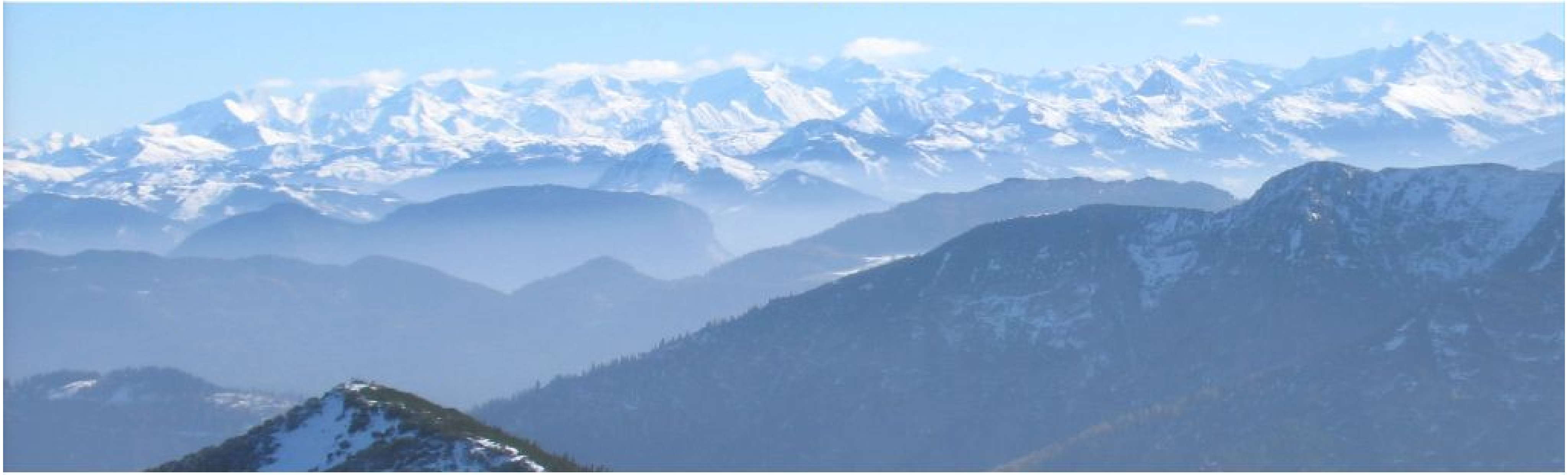

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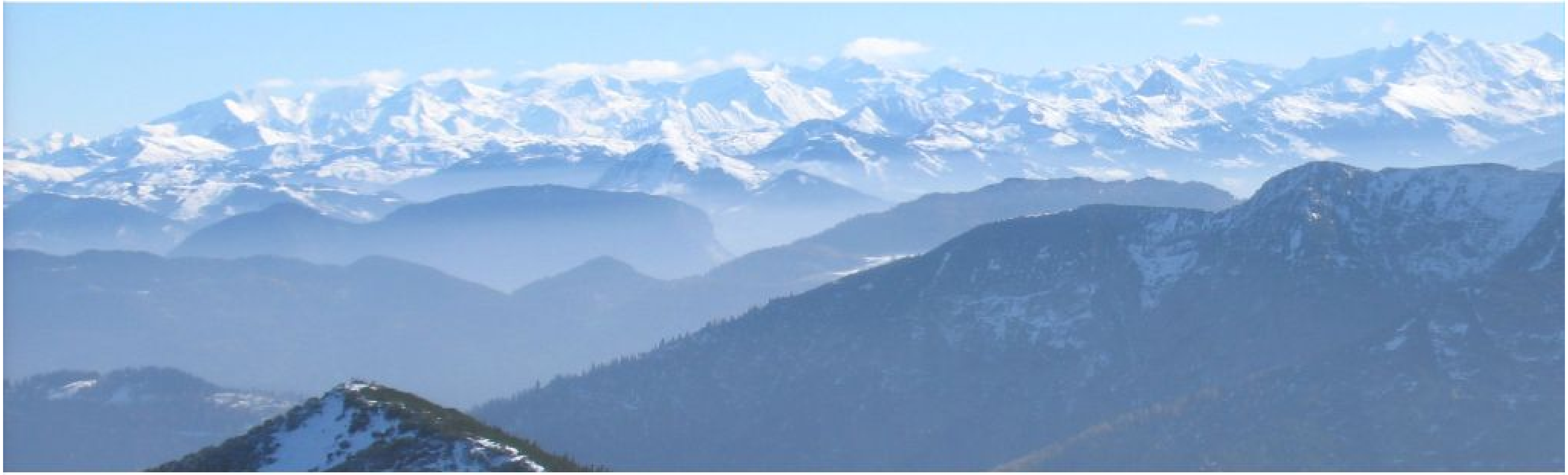
The approach gets ahead of other ones that are not able to justify the need for the complex Hilbert space or operator algebras.

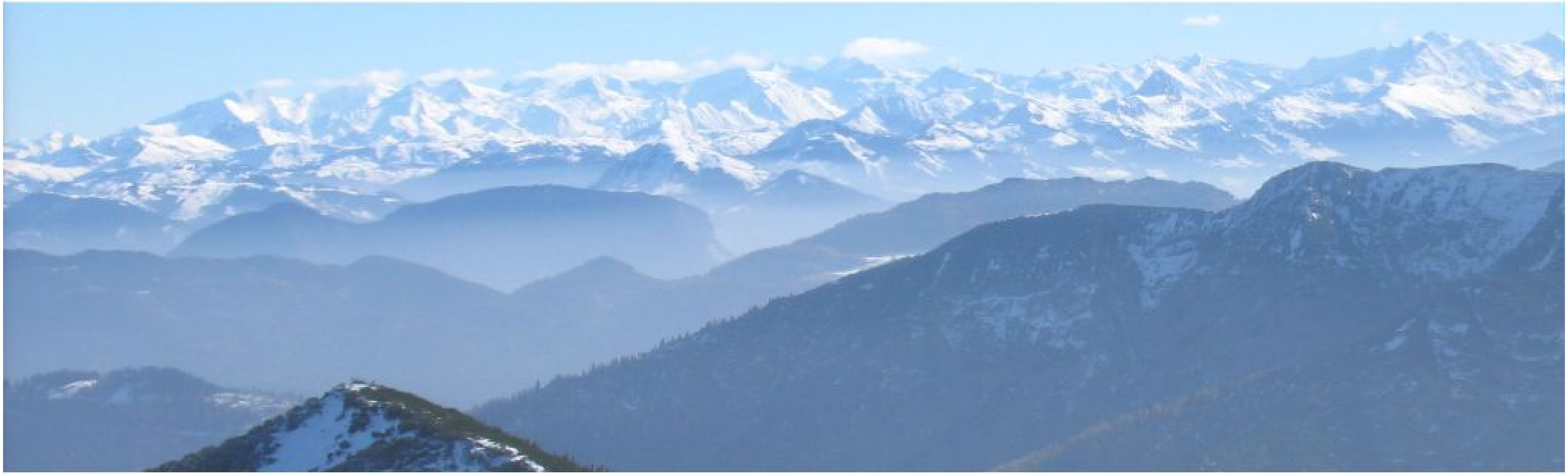
The mathematical structure of quantum mechanics has been reconstructed from a few probabilistic basic principles and becomes a non-Boolean extension of classical probability theory. Its link to physics is that probability conditionalization becomes identical with the Lüders - von Neumann measurement process as soon as the event system is not Boolean any more.

- **Objective conditional probabilities**



- Objective conditional probabilities
- Iterated conditioning/measurement series

(7) Conclusions – Further features of the model



- Objective conditional probabilities
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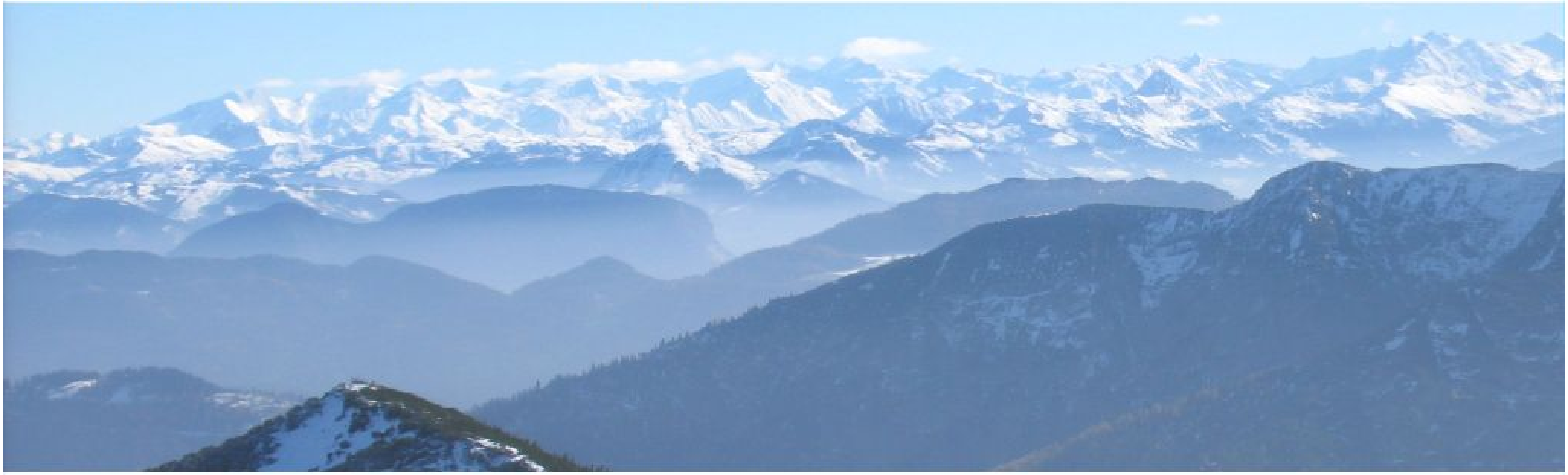
(7) Conclusions – Further features of the model

- Objective conditional probabilities
- Iterated conditioning/measurement series
- Wave-like interference
- Compatibility (different descriptions and different types)



- Objective conditional probabilities
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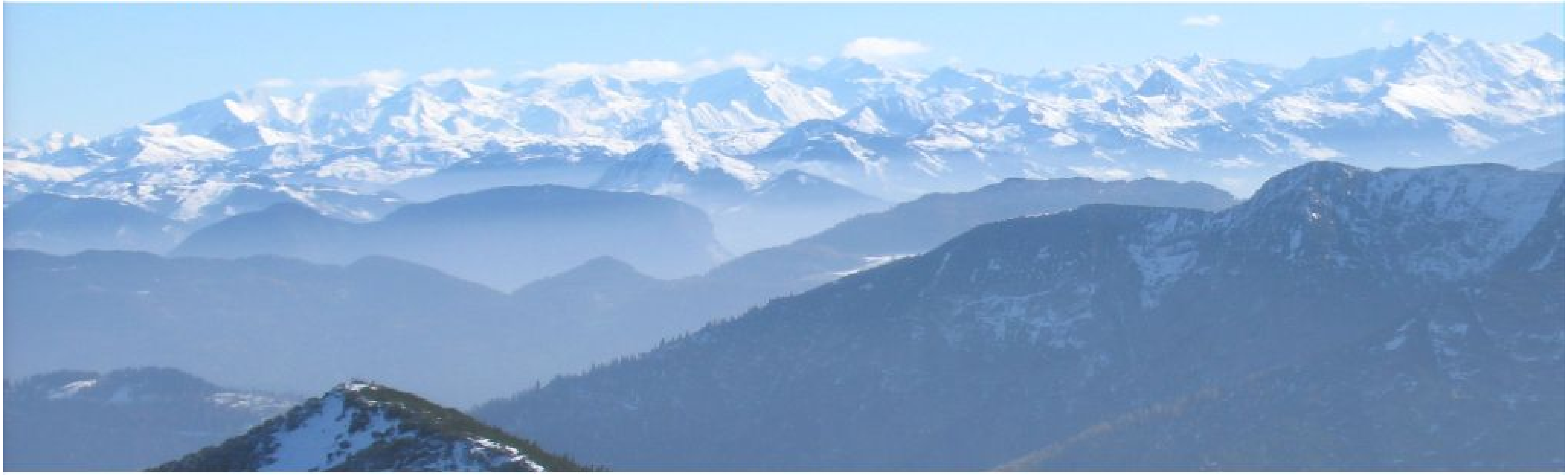
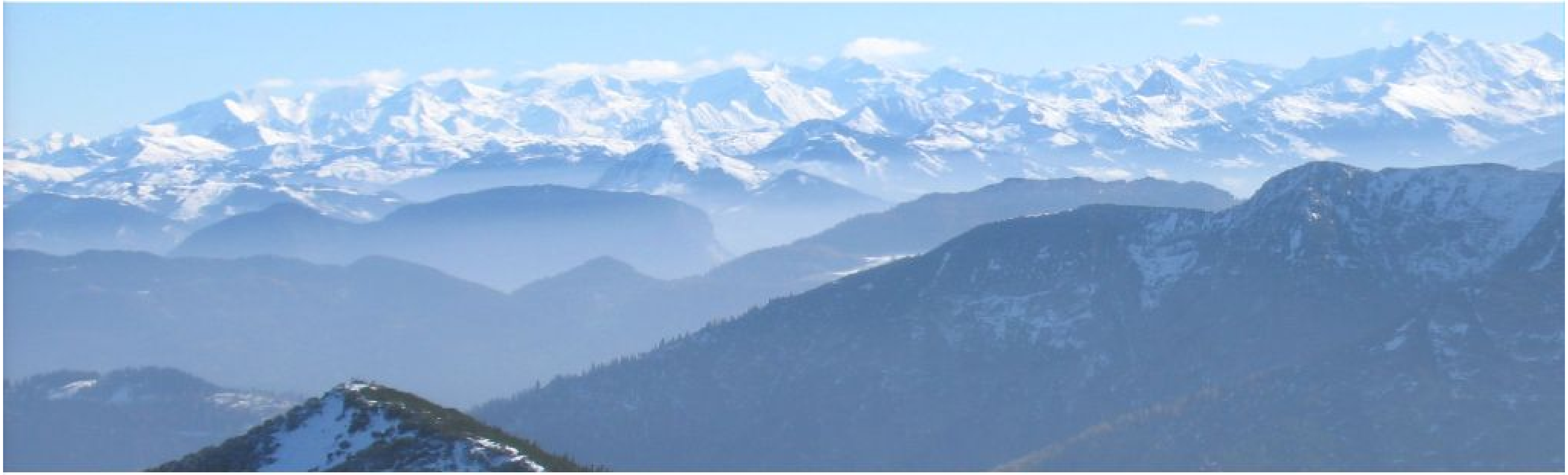
- Wave-like interference
- Compatibility (different descriptions and different types)
- Joint distributions of observables/simultaneous measurability



- Objective conditional probabilities
- Iterated conditioning/measurement series
- Wave-like interference
- Compatibility (different descriptions and different types)
- Joint distributions of observables/simultaneous measurability
- Conditional expectations

(7) Conclusions – Historical remarks



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(7) Conclusions – Historical remarks

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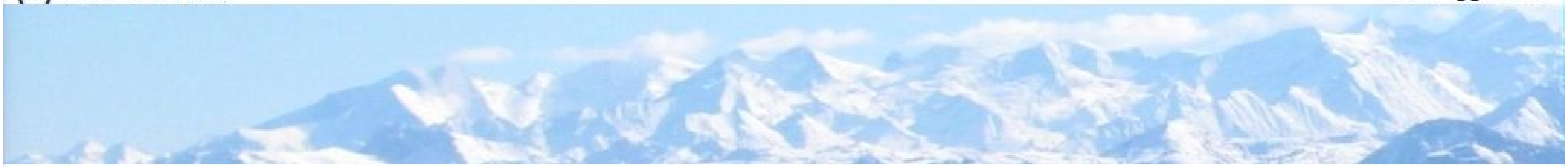
- W. Guz (1980) considered a similar approach including (A1), (A2) and (A3), but proved a result like Theorem 2 only for the finite events in an atomic quantum logic (i.e., only for the finite-dimensional cases). This is based on earlier results by J. Gunson (1967, "filtering process").

(8) References



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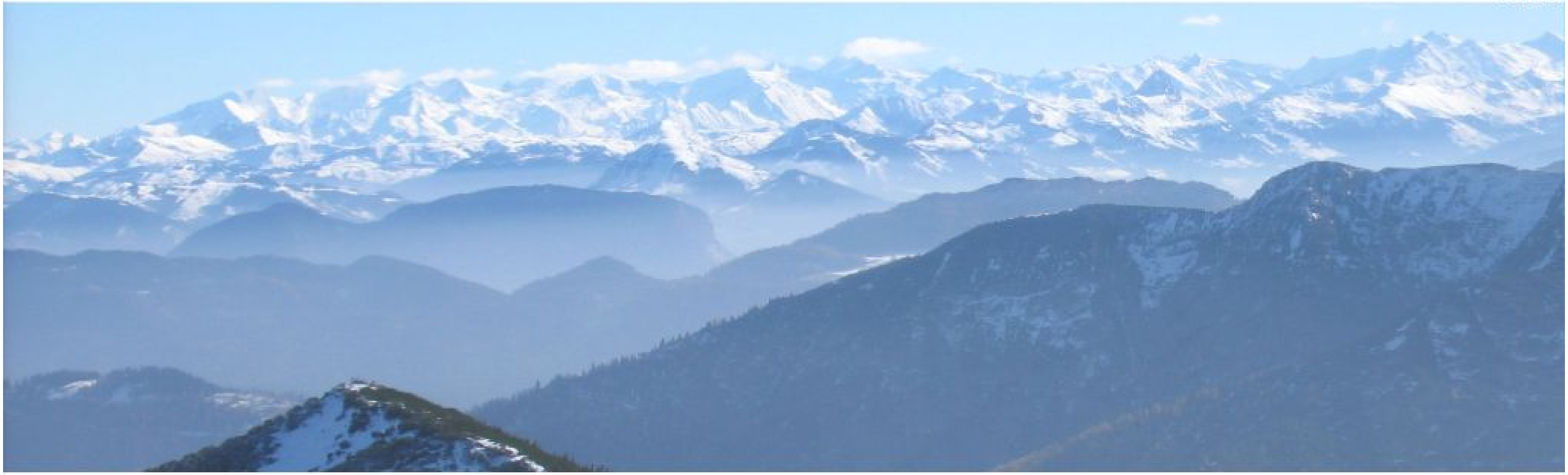

A reconstruction of quantum mechanics from quantum logics with unique conditional probabilities

Thank you for your kind attention!

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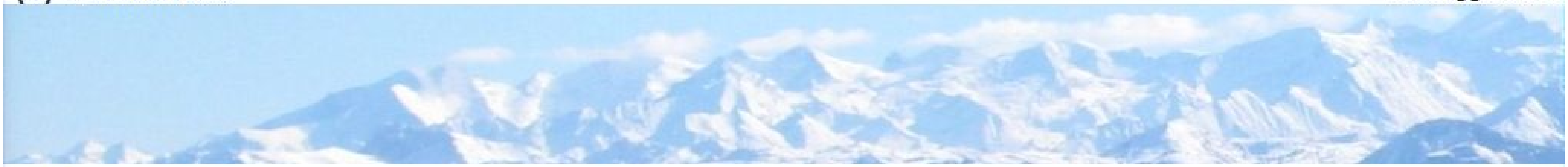
(8) References



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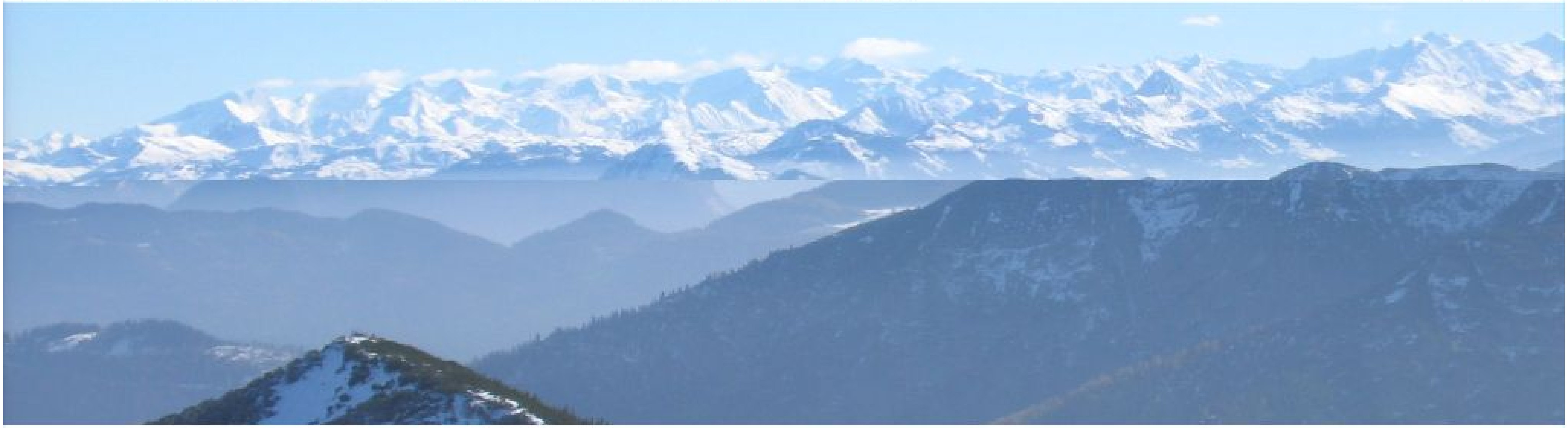
(7) Conclusions – Historical remarks

- Link between conditional probabilities and quantum measurement: J.C.T. Poole (1968), W. Guz (1974-81), J. Bub (1977), et al.

(7) Conclusions – The reconstruction of quantum mechanics



The reconstruction process has brought us stepwise

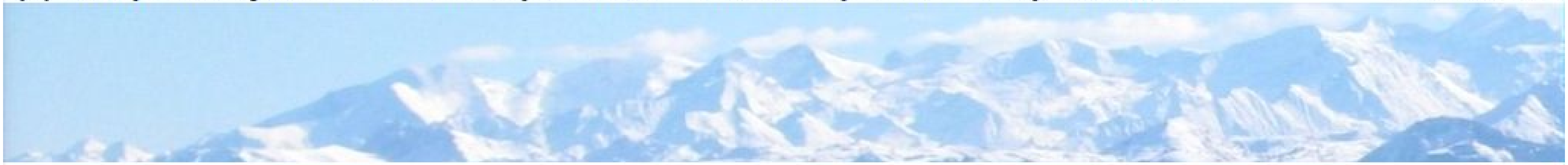
(1) to a new mathematical structure,

(2) to abstract Jordan algebras, and

(3) to operator algebras on Hilbert spaces over the complex numbers.

The approach gets ahead of other ones that are not able to justify the need for the complex Hilbert space or operator algebras.

(6) Composite systems and a last postulate – The examples and the postulates

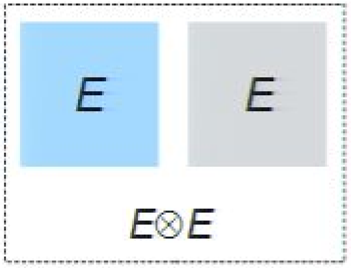


	Basic postulates (UC1), (UC2)	Additional postulates (A1), (A2, (A3)	Composite systems (C)	Complex Hilbert space represention
Projection lattice in a von Neumann algebra	✔	✔	✔	✔
Projection lattice in a special JBW algebra	✔	✔		✔
Projection lattice in an exceptional JBW algebra	✔	✔		
???*	✔			

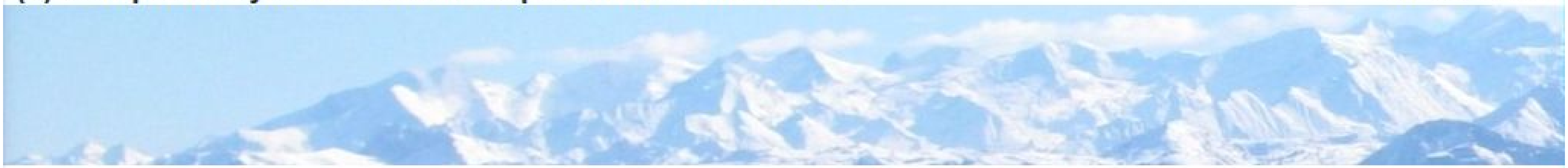
*: *Are (A1), (A2), (A3) redundant?*

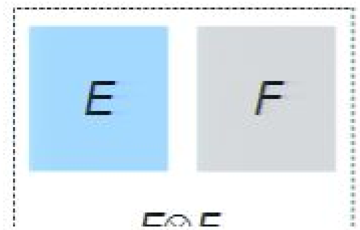
(6) Composite systems and a last postulate – Theorem 3

When does $E \otimes E$ exist and satisfy (UC1), (UC2), (A1), (A2) and (A3)?



(6) Composite systems and a last postulate – The model





(C) A model $E\otimes F$ of a composite system consisting of two single systems each modelled by E and F respectively, shall satisfy the following conditions:

(6) Composite systems and a last postulate – The model

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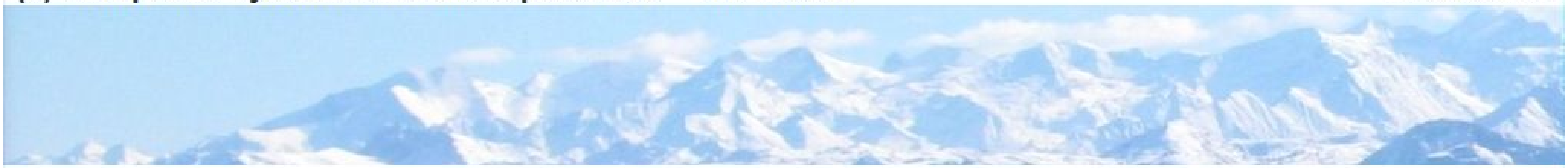


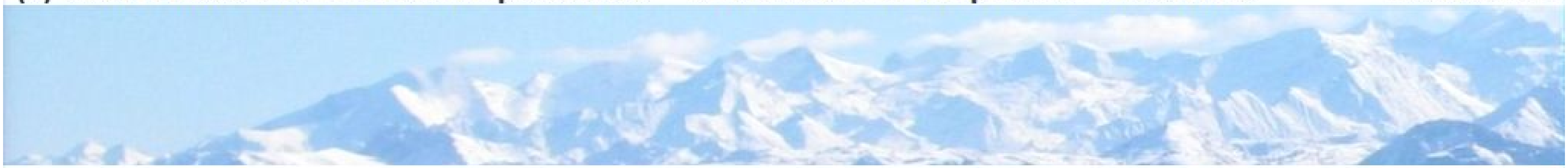
(C) A model $E\otimes F$ of a composite system consisting of two single systems each modelled by E and F, respectively, shall satisfy the following conditions:

(i) For each pair of events $e\in E$ and $f\in F$ there is an event $e\otimes f\in E\otimes F$ *such that* $e\otimes 0=0\otimes f=0$, $\mathbb{1}\otimes\mathbb{1}=\mathbb{1}$, and $e_1\otimes f_1\perp e_2\otimes f_2$ whenever $e_1\perp e_2$ or $f_1\perp f_2$. Moreover the maps $e\rightarrow e\otimes f$ with f fixed and $f\rightarrow e\otimes f$ with e fixed are σ-additive.

(ii) For each separately σ-additive state ρ on $E\times F$, there is one and only one σ-additive state ρ' on $E\otimes F$ with $\rho(e,f)=\rho'(e\otimes f)$.

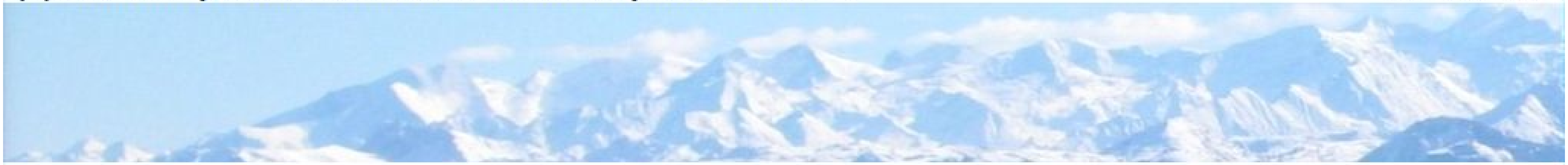
The second condition is motivated by the consideration of the joint distribution of a generalized E-valued and an F-valued observable under a certain compatibility condition [2004b].

(5) Observables and two further postulates – Brief sketch of the proof of Theorem 3

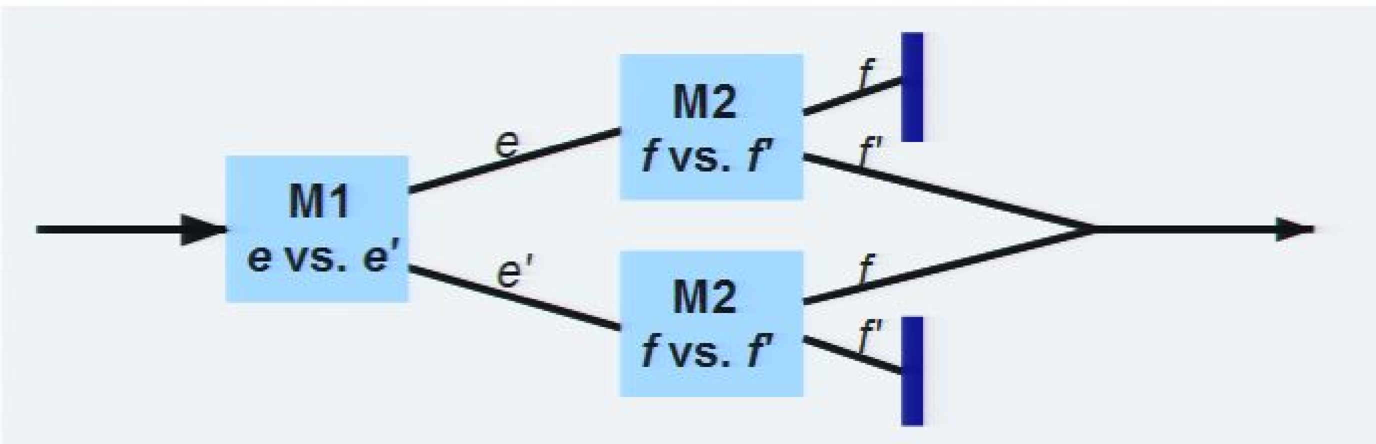


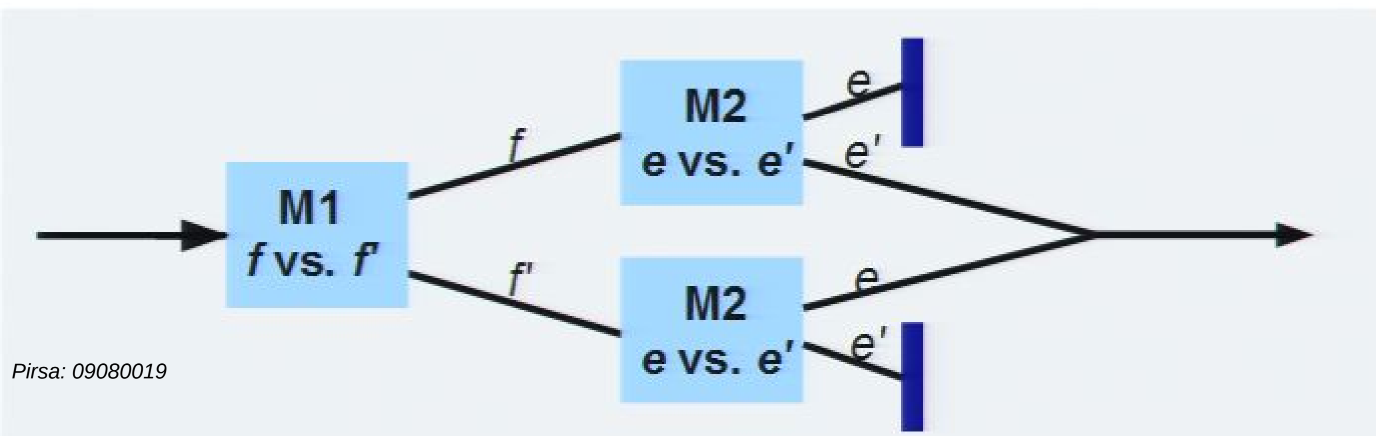
- (A3) makes the bounded real observables form a linear subspace of *A*.

(4) A further postulate for the conditional probabilities - 2



(A1) $\mu(f'|e)\mu(e) + \mu(f|e')\mu(e') = \mu(e'|f)\mu(f) + \mu(e|f')\mu(f')$





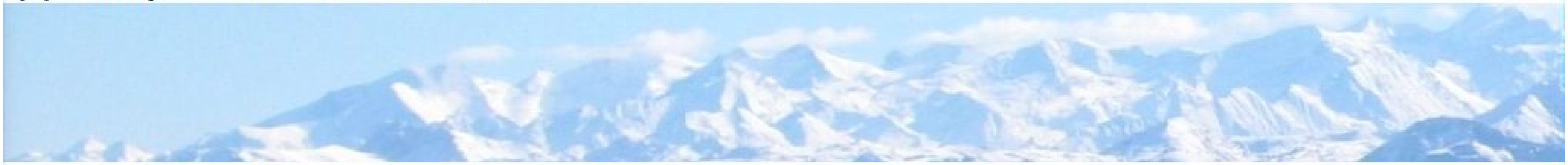
(A1) says that

the probability that a particle passes the upper measuring arrangement

is identical with

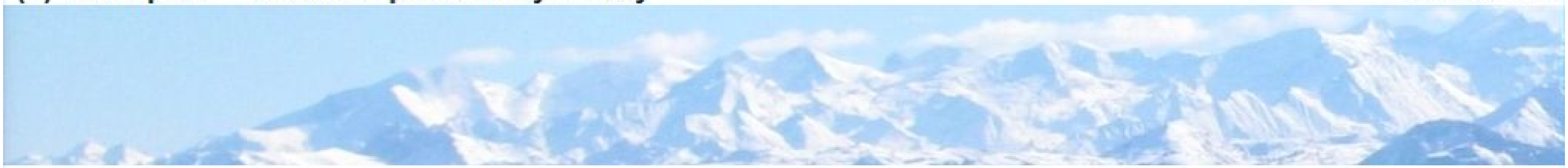
the probability that a particle passes the lower measuring arrangement.

(3) Examples – Quantum mechanics



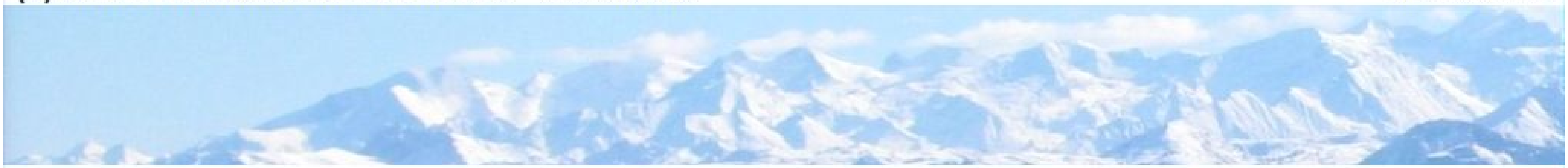
Assume that E is the projection lattice of a von Neumann algebra w/o type I_2 part. Then (UC1) as well as (UC2) hold and

(3) Examples – Classical probability theory



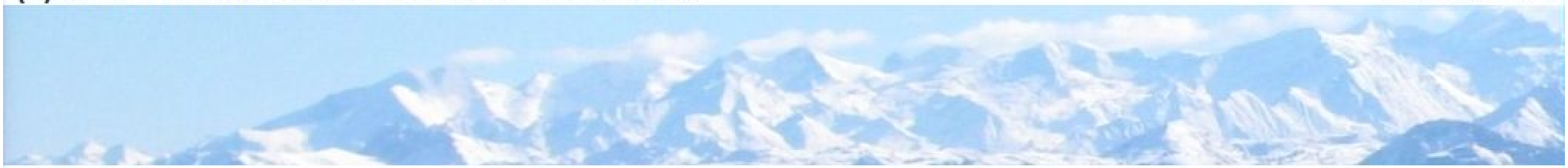
Assume that E is a σ-algebra (consisting of subsets of a certain set). Then (UC1) as well as (UC2) hold and

(2) A new mathematical structure – Questions



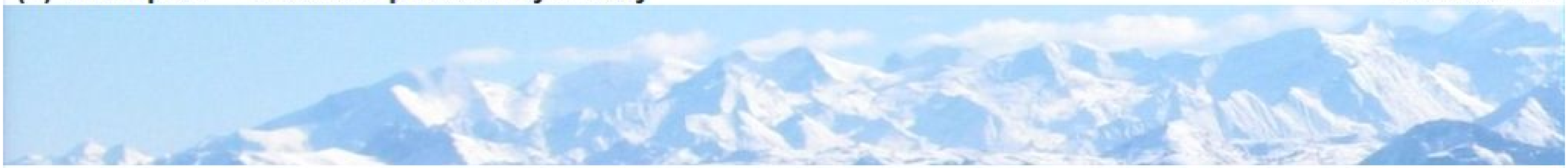
- Do concrete examples exist for this new mathematical structure?

(2) A new mathematical structure – Questions



- Do concrete examples exist for this new mathematical structure?
- What is the shape of the U_e-projections then?
- How do the conditional probabilities look?
- What is compatibility?
- What is objective conditional probability?

(3) Examples – Classical probability theory



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(3) Examples – Classical probability theory

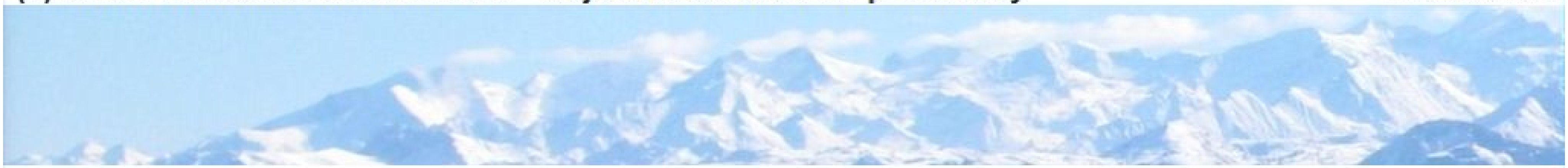
Assume that E is a σ-algebra (consisting of subsets of a certain set). Then (UC1) as well as (UC2) hold and

$$U_e(f) = e \cap f$$

$$\mu(f|e) = \mu(e \cap f)/\mu(e)$$

All events are compatible.

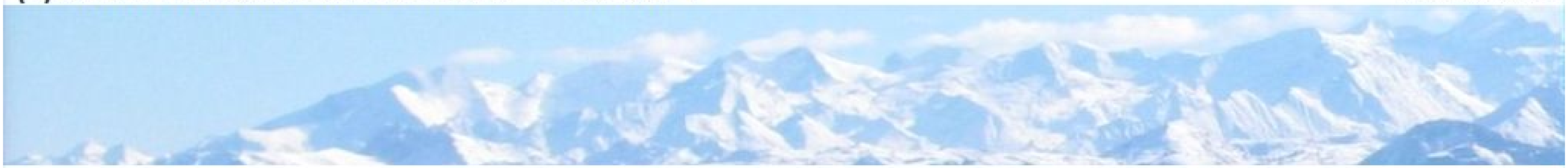
(2) A new mathematical structure – Objective conditional probability



The conditional probability of an event *f* under *e* in a σ-additive state μ with μ(*e*)>0 has the shape:

$$\mu(f|e) = \frac{1}{\mu(e)} \hat{\mu}(U_e(f)).$$

(2) A new mathematical structure – Theorem 1



Theorem 1: Assume that *E* satisfies (UC1) and (UC2). Then *E* can be embedded in the unit interval $[0,1]$ of an order unit space *A* with predual *V* such that the following conditions hold:

NOTE: *An order unit space is a Banach space with a partial ordering* $\leq$ *and an order unit* 1.

Then $\{a \in A : \|a\| \leq 1\} = [-1,1] := \{a \in A : -1 \leq a \leq 1\}$

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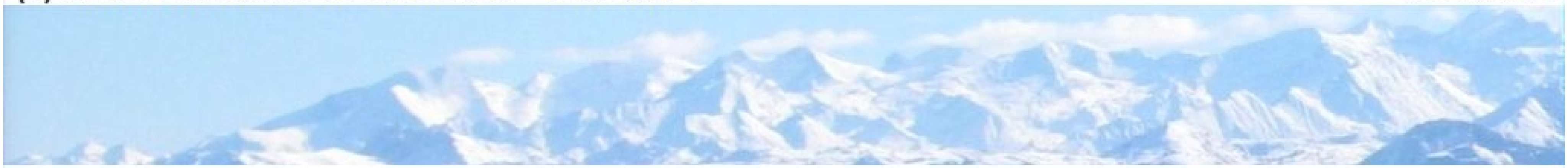

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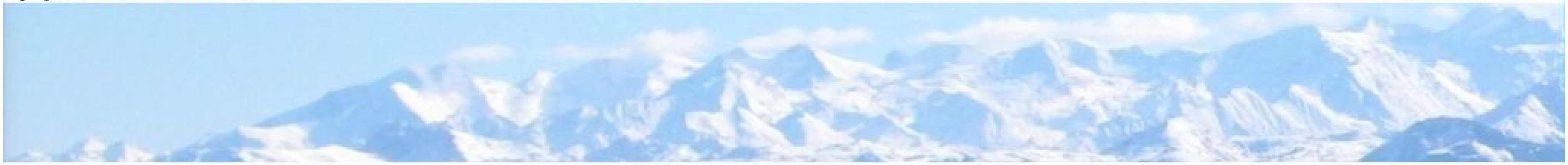

Theorem 1: Assume that E satisfies (UC1) and (UC2). Then E can be embedded in the unit interval $[0,1]$ of an order unit space A with predual V such that the following conditions hold:

(i) A is the weakly closed linear hull of E, and each σ-additive state μ on E has a weakly continuous positive linear extension $\hat{\mu}$ to A.

(ii) For each event $e \in E$ there is there is a weakly continuous positive linear projection $U_e: A \rightarrow A$ such that

- $U_e(1) = e$,

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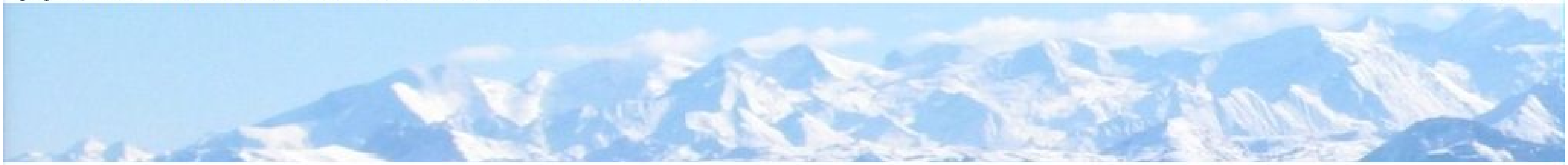

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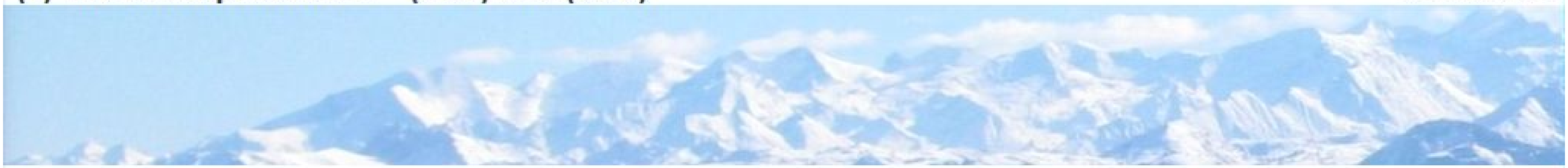
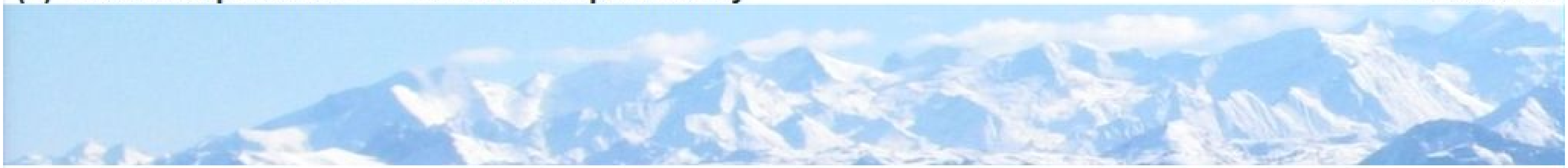
(1) The basic postulates – (UC1) and (UC2)

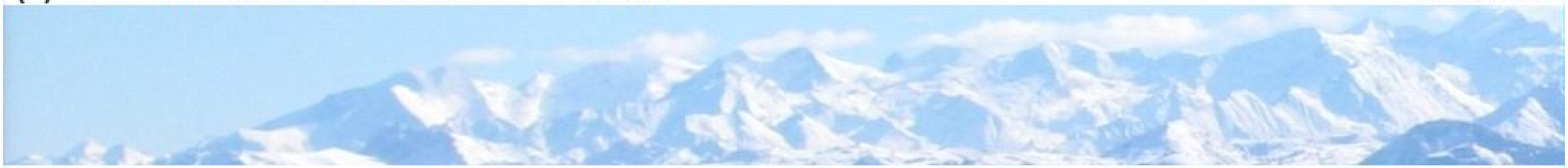


(UC1) For each pair of events $e \neq f$, there exists a σ-additive state μ with $\mu(e) \neq \mu(f)$.

(1) The basic postulates – Conditional probability



A (σ-additive) <u>state</u> μ is a map $\mu: E \rightarrow [0,1]$ which is orthogonally additive (σ-additive) and fulfils $\mu(\mathbb{1})=1$.



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Proof: Some simple functional analysis [2008b].

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