Title: Candidates for Principles of Quantumness

Date: Aug 15, 2009 09:00 AM

URL: http://pirsa.org/09080014

Abstract: Quantum Mechanics (QM) is a beautiful simple mathematical structure--- Hilbert spaces and operator algebras---with an unprecedented predicting power in the whole physical domain. However, after more than a century from its birth, we still don't have a "principle" from which to derive the mathematical framework. The situation is similar to that of Lorentz transformations before the advent of the relativity principle. The invariance of the physical law with the reference system and the existence of a limiting velocity, are not just physical principles: they are mandatory operational principles without which one cannot do experimental Physics. And it is a very seductive idea to think that QM could be derived from some other principle of such epistemological kind, which is either indispensable or crucial in dramatically reducing the experimental complexity. Indeed, the large part of the formal structure of QM is a set of formal tools for describing the process of gathering information in any experiment, independently on the particular physics involved. It is mainly a kind of "information theory", a theory about our knowledge of physical entities rather than about the entities themselves. If we strip off such informational part from the theory, what would be left should be a "principle of the quantumness" from which QM should be derived. In my talk I will analyze the consequences of two possible candidates for the principle of quantumness: 1) PFAITH: the existence of a pure bipartite state by which we can calibrate all local tests and prepare all bipartite states by local tests; 2) PURIFY: the existence of a purification for all states. We will consider the two postulates within the general context of probabilistic theories---also called test-theories. Within test-theories we will introduce the notion of "time-cascade" of tests, which entails the identifications "events=transformations" and "evolution=conditioning", and derive the general matrix-algebra representation of such theories, with particular focus on theories that satisfy the "local discriminability principle". Some of the concepts will be illustrated in some specific test-theories, including the usual cases of classical and quantum mechanics, the extended versions of the PR boxes, the so-called "spin-factors", and quantum mechanics on a real (instead of complex) Hilbert spaces. After the brief tutorial on test-theories, I will analyze all the consequences of the two candidate postulates. We will see how postulate PFAITH implies the "local observability principle" and the tensor-product structure for the linear spaces of states and effects, along with a remarkable list of additional features that are typically quantum, including purification for some states, the impossibility of bit commitment, and many others. We will see how the postulate is not satisfied by classical mechanics, and a stronger version of the postulate also exclude theories where we cannot have teleportation, e.g. PR-boxes. Finally we will analyze the consequences of postulate PURIFY, and show how it is equivalent to the possibility of dilating any probabilistic transformation on a system to a deterministic invertible transformation on the system interacting with an ancilla. Therefore PURIFY is equivalent to the general principle that "every transformation can be in-principle inverted, having sufficient control on the environment". Using a simple diagrammatic representation we will see how PURIFY implies general theorems as: 1) deterministic full teleportation; 2) inverting a transformation upon an input state (i.e. error-correction) is equivalent to the fact that environment and reference remain uncorrelated; 3) inverting some transformations by reading the environment; etc. We will see that some non-quantum theories (e.g. QM on real Hilbert spaces) still satisfy PURIFY. Finally I will address the problem on how to prove that a test-theory is quantum. One would need to show that also the "effects" of the theory---not just the transformations---make a matrix algebra. A way of deriving the "multiplication" of effects is to identify them with atomic events. This can be done assuming the atomicity of evolution in conjunction with the Choi-Jamiolkowski isomorphism. Suggested readings: 1. arXiv:0807.4383, to appear in "Philosophy of Quantum Information and Entanglement", Eds A. Bokulich and G. Jaeger (Cambridge University Press, Cambridge UK, in press) 2. G. Chiribella, G. M. D'Ariano, and

P. Perinotti (in preparation) 3. G. M. D'Ariano, A. Tosini (in preparation http://www.qubit.it

QUit
quantum information
theory group

CANDIDATES FOR PRINCIPLES OF QUANTUMNESS

Giacomo Mauro D'Ariano

Pavia University

Reconstructing Quantum Theory
Perimeter Institute, Waterloo, CA, August 9-16 2009

arXiv:0807.4383: in *Philosophy of Quantum Information and Entanglement*, Eds A. Bokulich and G. Jaeger (CUP, Cambridge UK, 2009)

arXiv:0908.1583 with G. Chiribella and P.Perinotti

PRINCIPLES OF QUANTUMNESS

QM: probabilistic theory satisfying:

1. Causality
2. Local observability
3. Conservation of information

OUTLINE

OUTLINE

1. Short lecture on:

OUTLINE

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   Causal Theories with Local Discriminability

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2. Postulates PFAITH, FAITHE, SUPERFAITH

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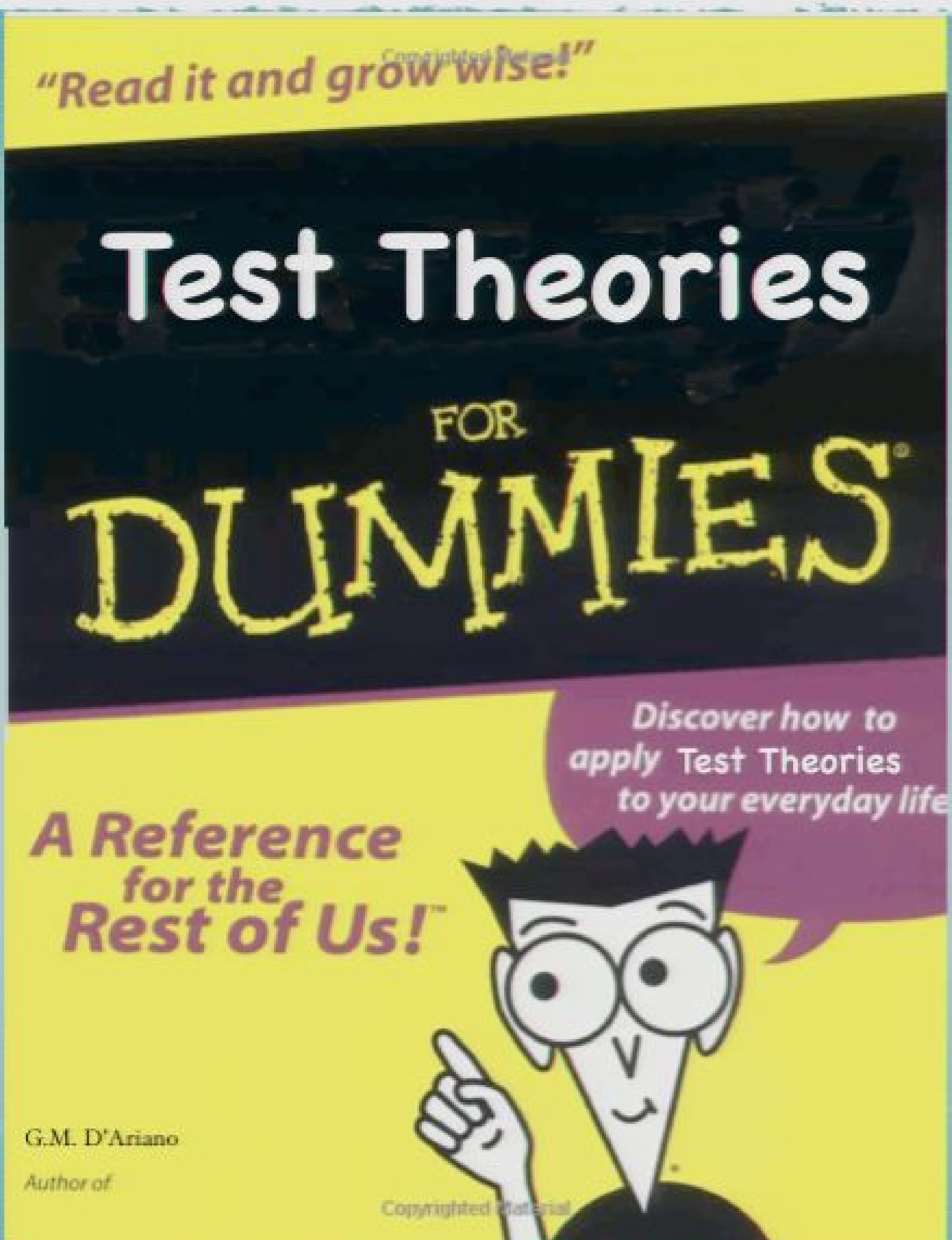
4. How to prove that the theory is QM



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Test Theories

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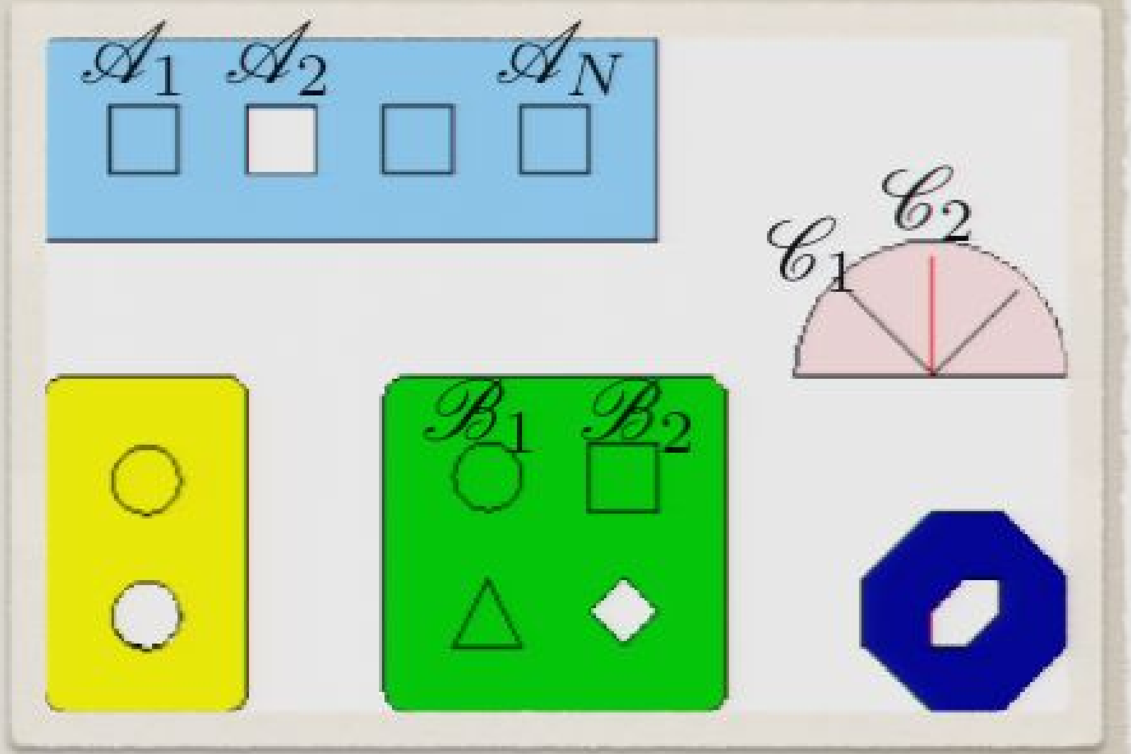
G.M. D'Ariano

Author of

TESTS

Test: $\mathbb{A} \equiv \{\mathscr{A}_j\}$ set of outcomes $\mathscr{A}_j$



- The same event occur in different tests
- Deterministic test = single-utcome

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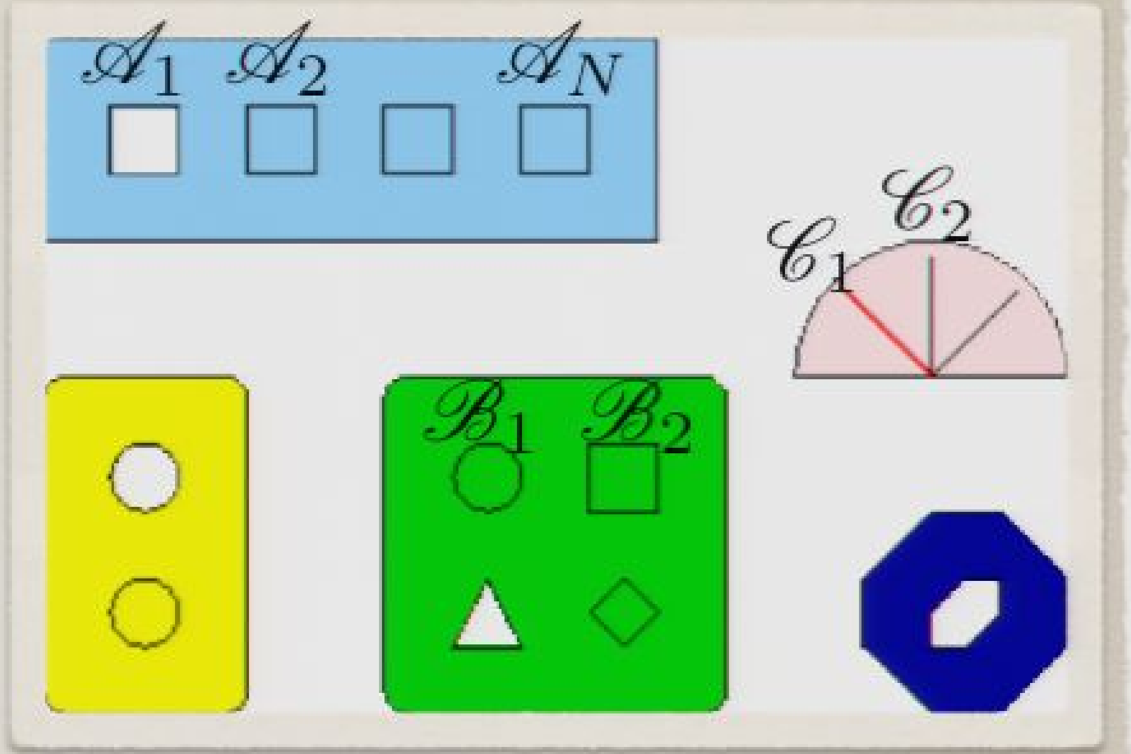

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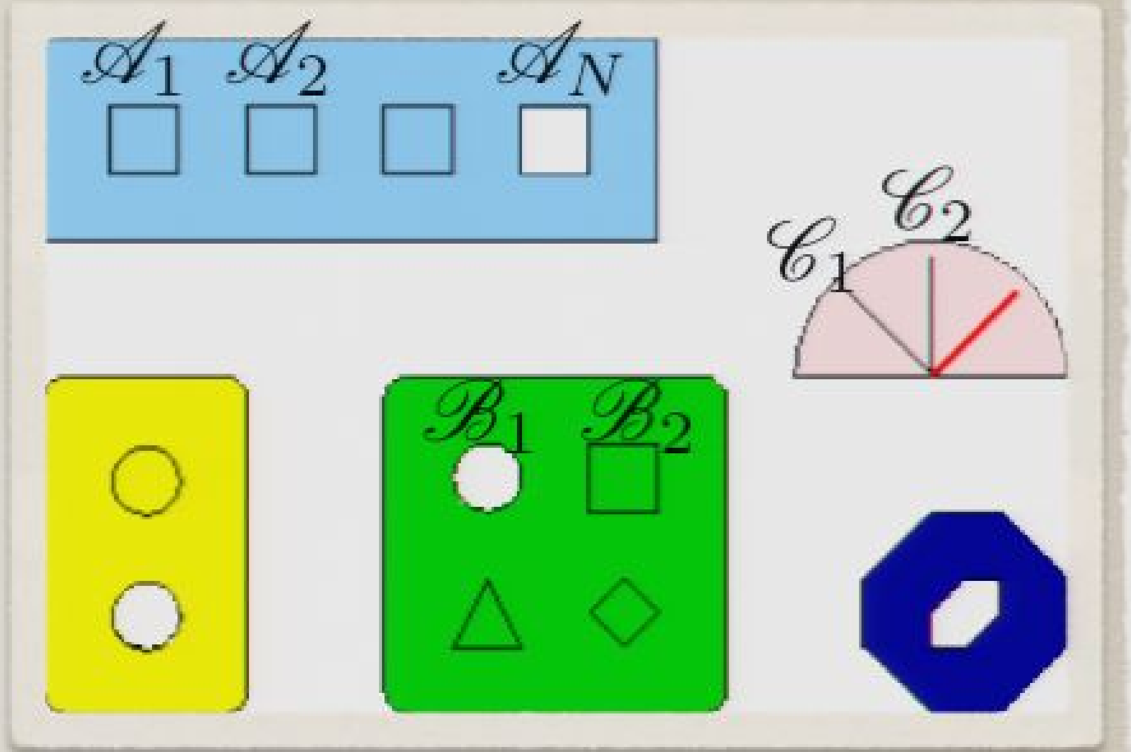

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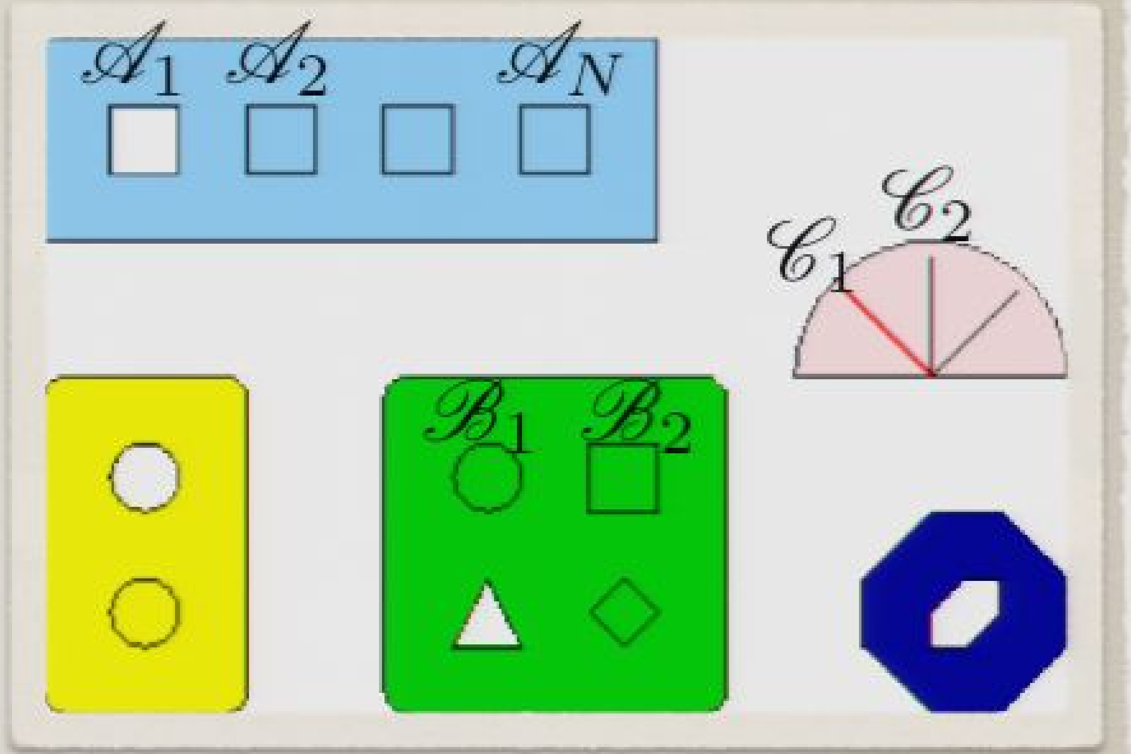

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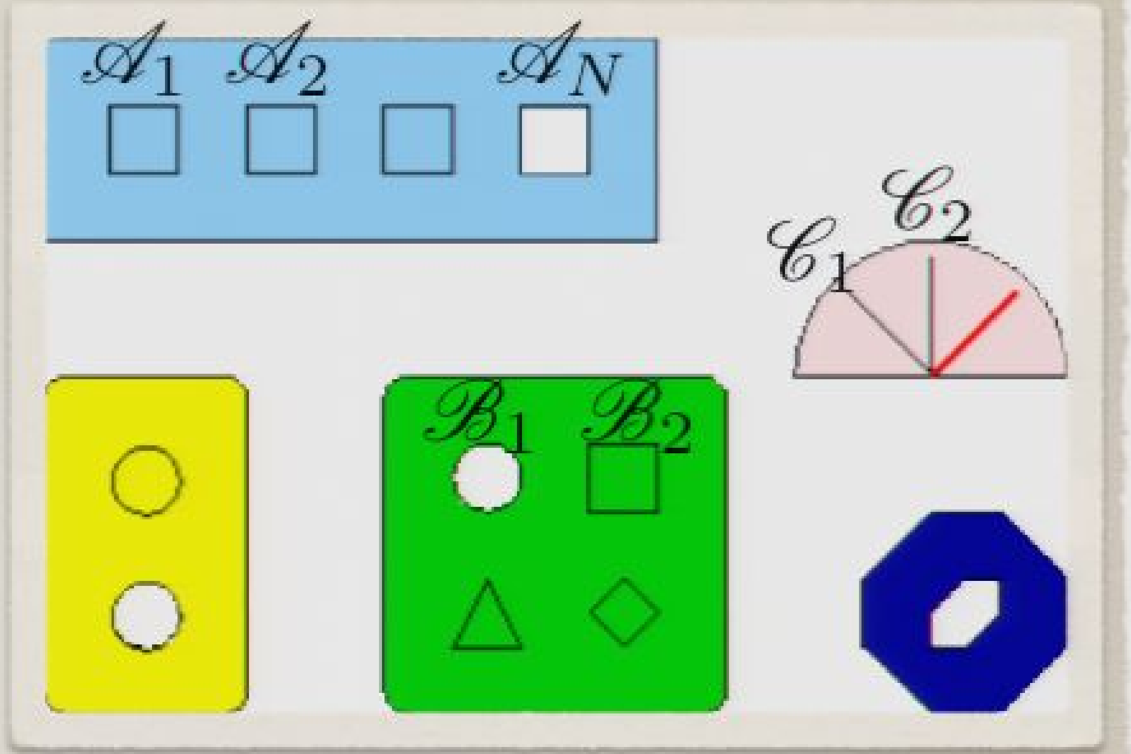

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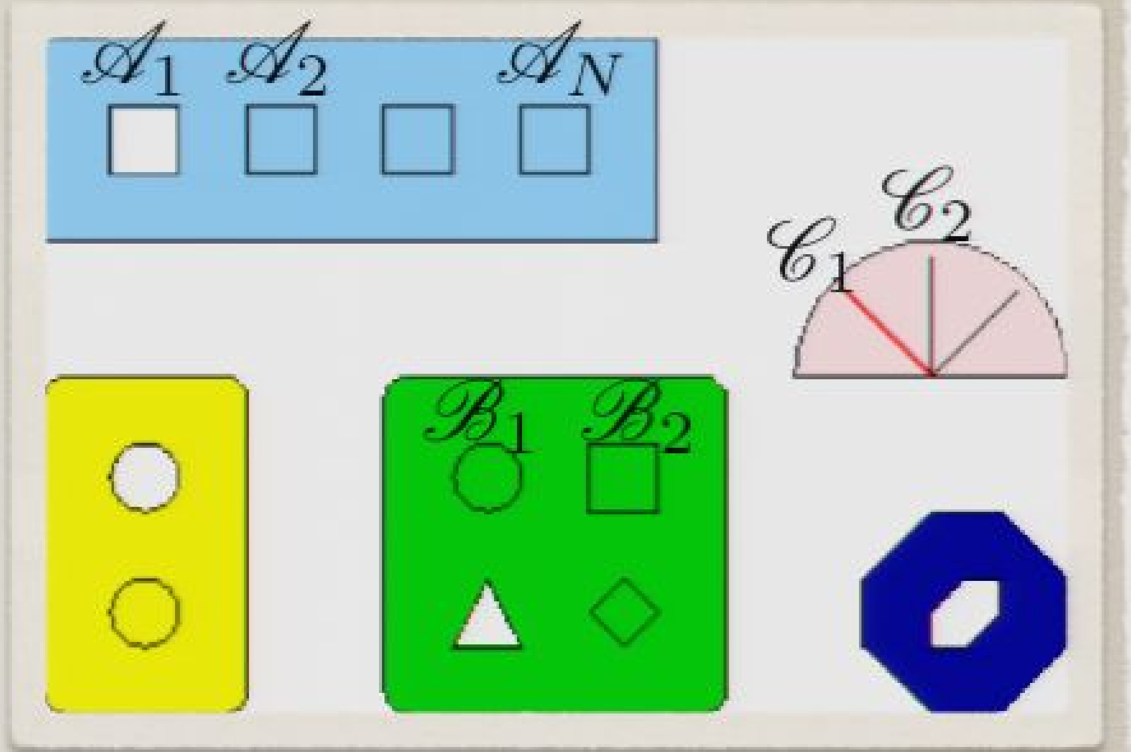

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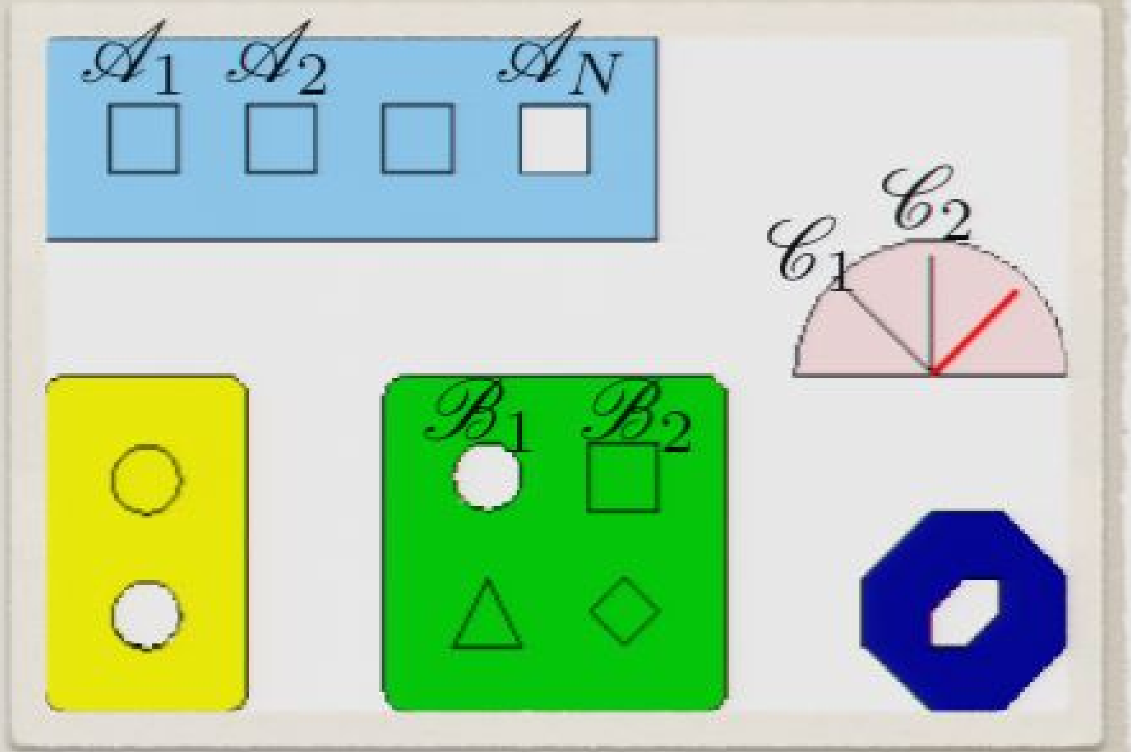

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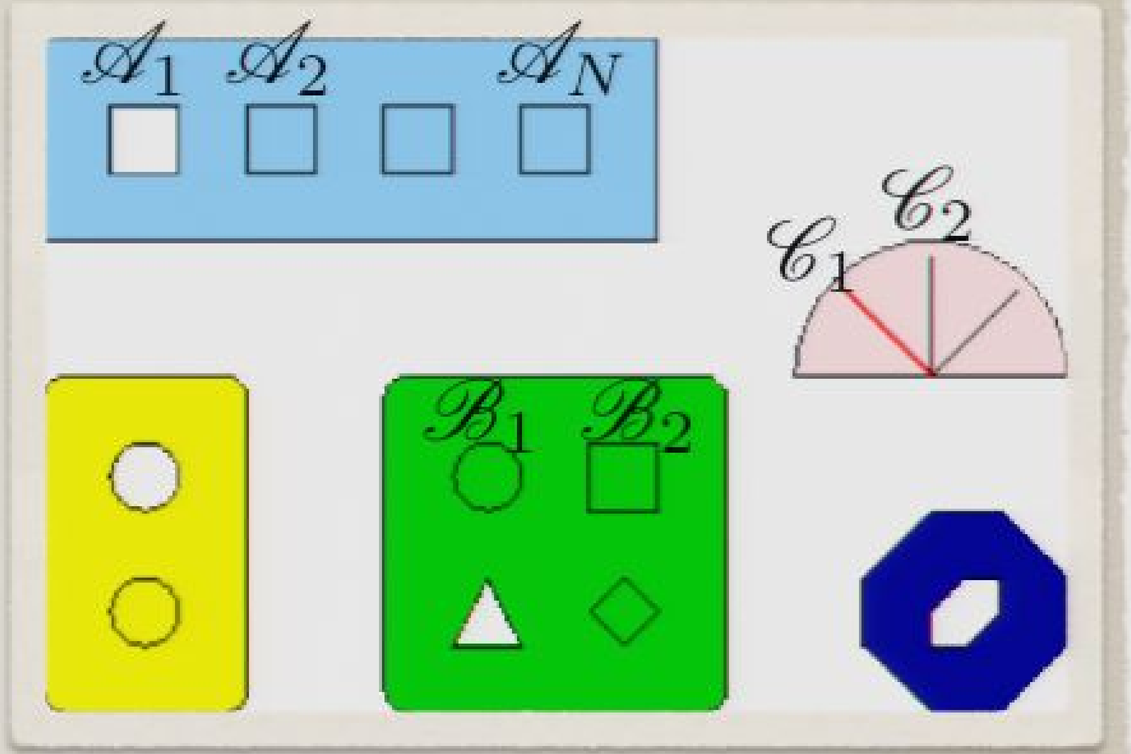

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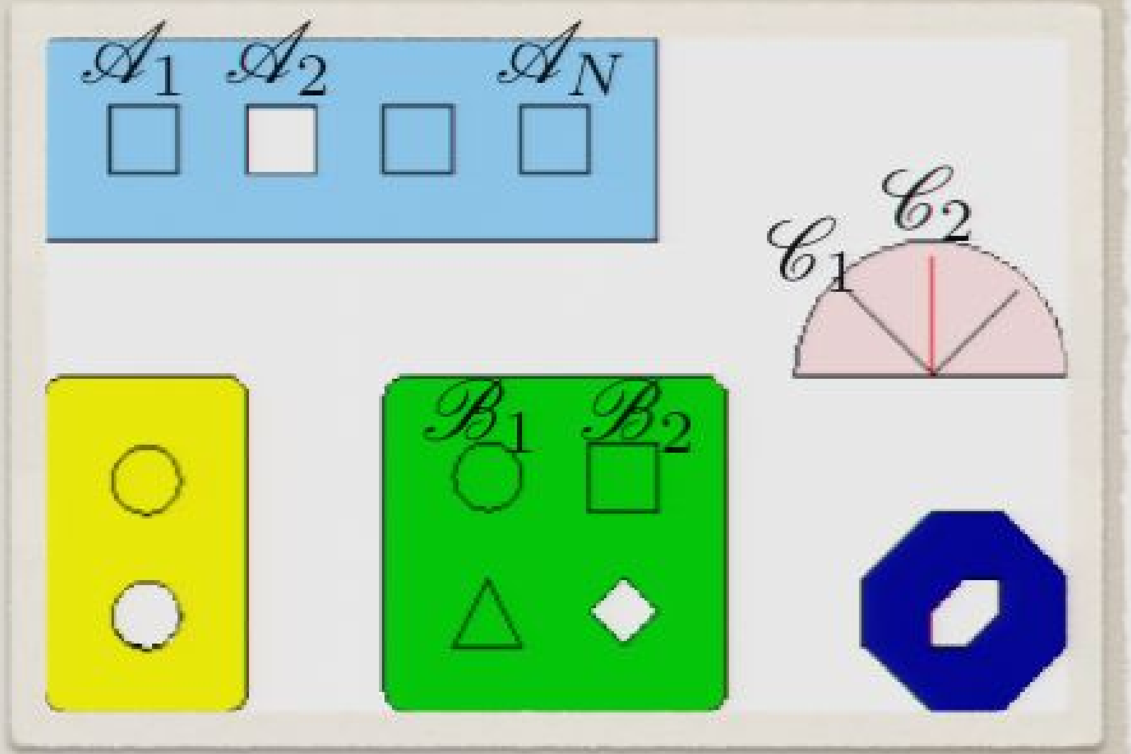

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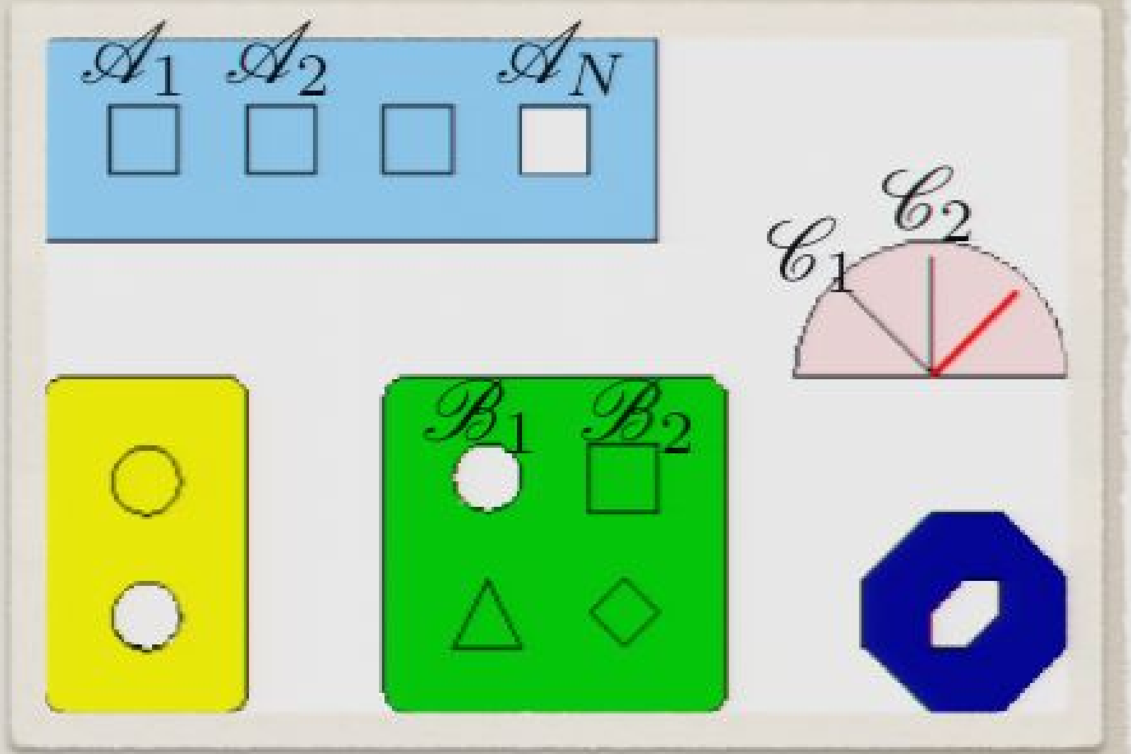

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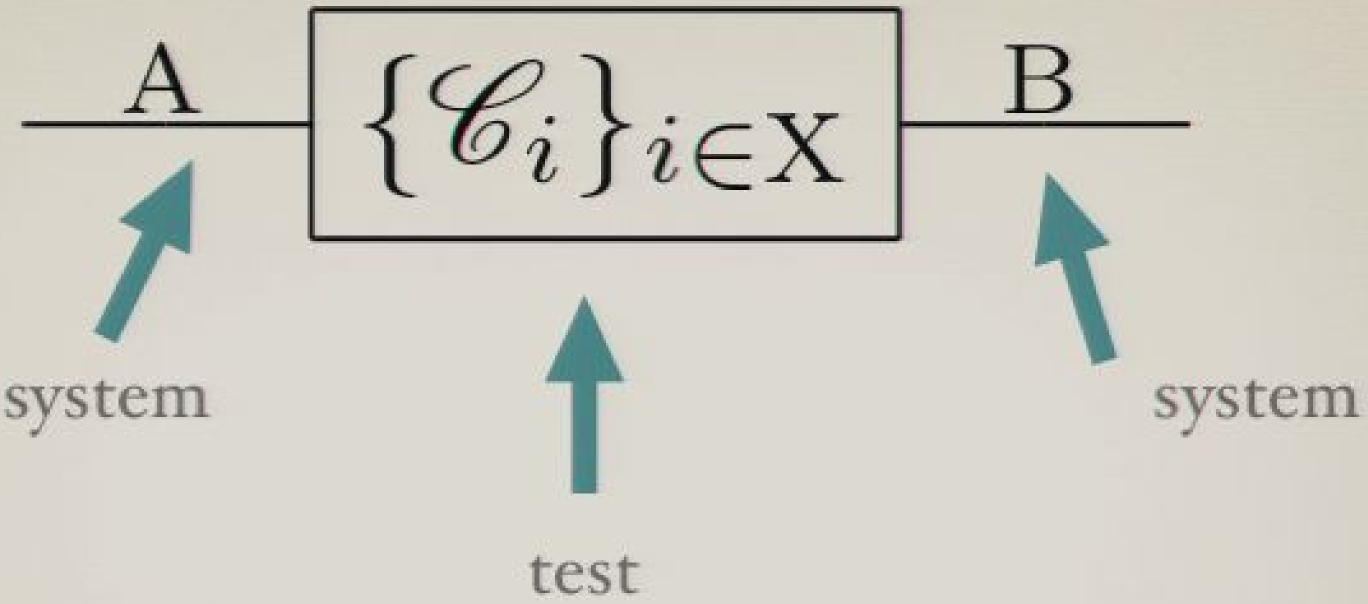
Coarse-graining of events: $\mathscr{A} \cup \mathscr{B}$

$$\mathbb{A} = \{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3\} \underset{\text{Refinement}}{\overset{\text{Coarse-graining}}{\rightleftarrows}} \mathbb{A}' = \{\mathscr{A}_1, \mathscr{A}_2 \cup \mathscr{A}_3\}$$

TESTS



A

$\{\mathscr{C}_i\}_{i \in \mathrm{X}}$

B

system

test

system

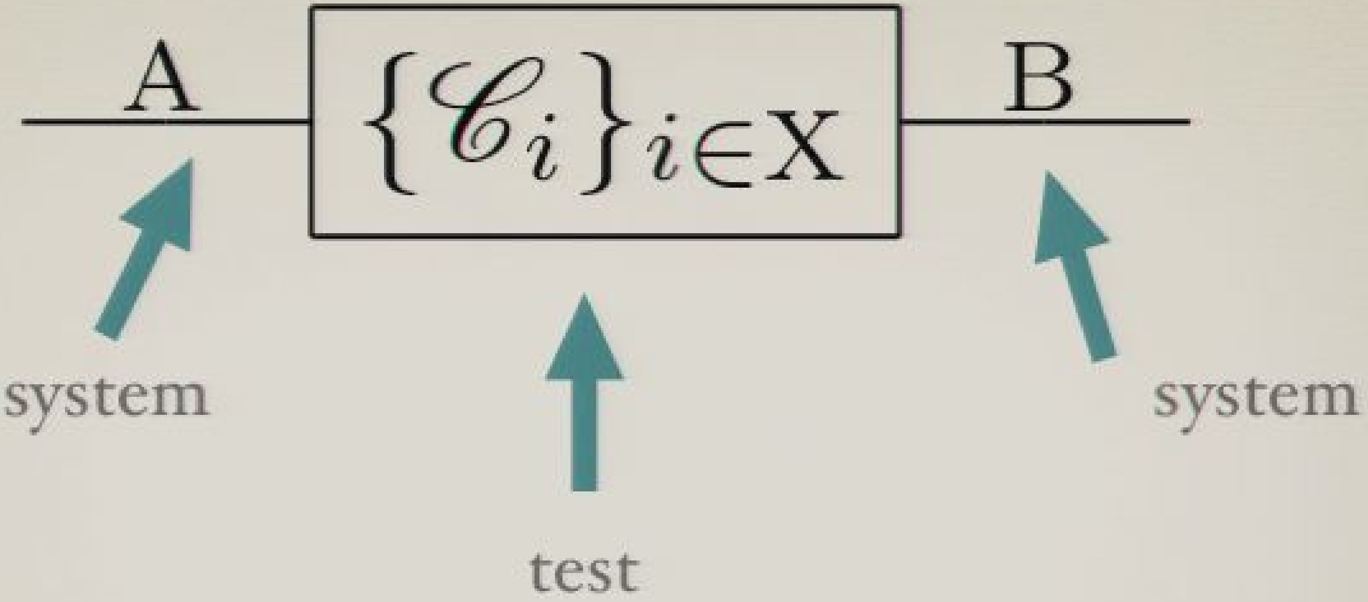
event:

A

$\mathscr{C}_i$

B

TESTS



A $\{\mathscr{C}_i\}_{i\in X}$ B

system

test

system

event:

A $\mathscr{C}_i$ B

STATES

State ω: probability rule $\omega(\mathscr{A})$ for any possible event $\mathscr{A}$ in any test

Normalization: $$\sum_{\mathscr{A}_j \in \mathbb{A}} \omega(\mathscr{A}_j) = 1$$

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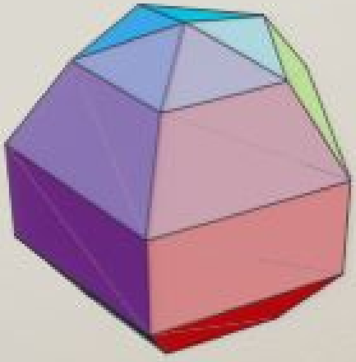
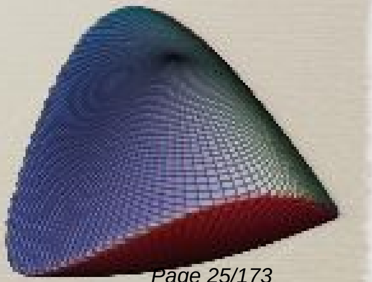
Normalization: $$\sum_{\mathscr{A}_j \in \mathbb{A}} \omega(\mathscr{A}_j) = 1$$

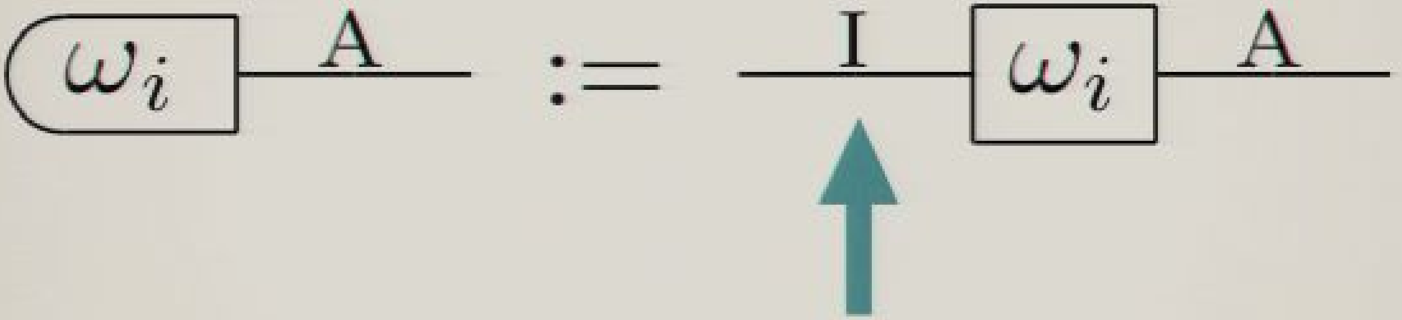
Convex set of states: $\mathfrak{S}$





STATES

Preparation tests: states as preparation events

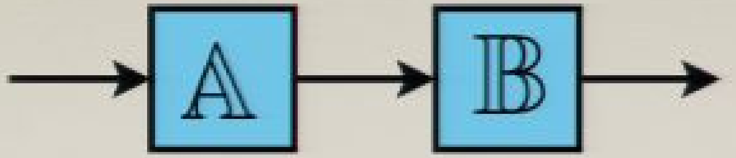


trivial system

(states normalized to probability of preparation)

CASCADES OF TESTS

Time-cascade:



$\mathbb{B} \circ \mathbb{A} = \{\mathscr{B}_j \circ \mathscr{A}_i\}$ cascade of tests $\mathbb{A} = \{\mathscr{A}_i\},\ \mathbb{B} = \{\mathscr{B}_j\},$

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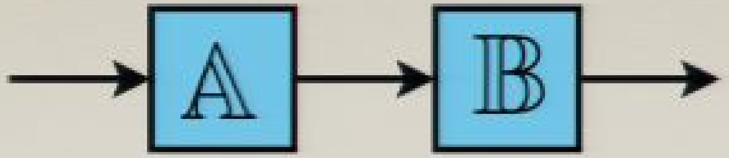

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collection of joined events with the following rule for marginals:

$$\sum_{\mathscr{B}_j\in\mathbb{B}}\omega(\mathscr{B}_j\circ\mathscr{A})=:f(\mathbb{B},\mathscr{A})\equiv\omega(\mathscr{A}),\quad\forall\,\mathbb{B},\,\mathscr{A},\,\omega$$

NSF (No signaling from the future)

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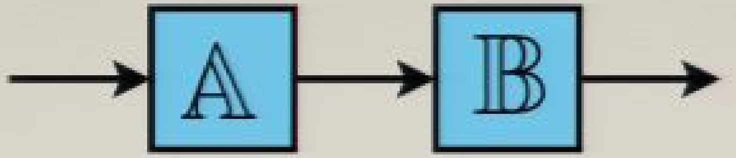

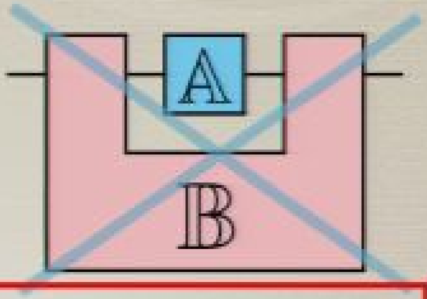

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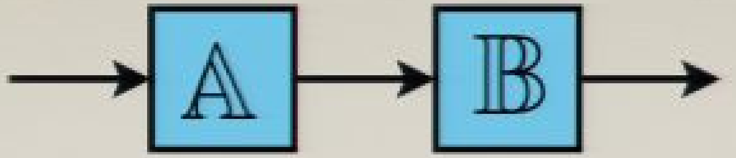

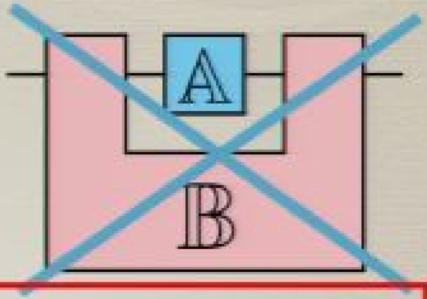

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composition of events $\mathscr{B} \circ \mathscr{A}$ + notion of conditional state

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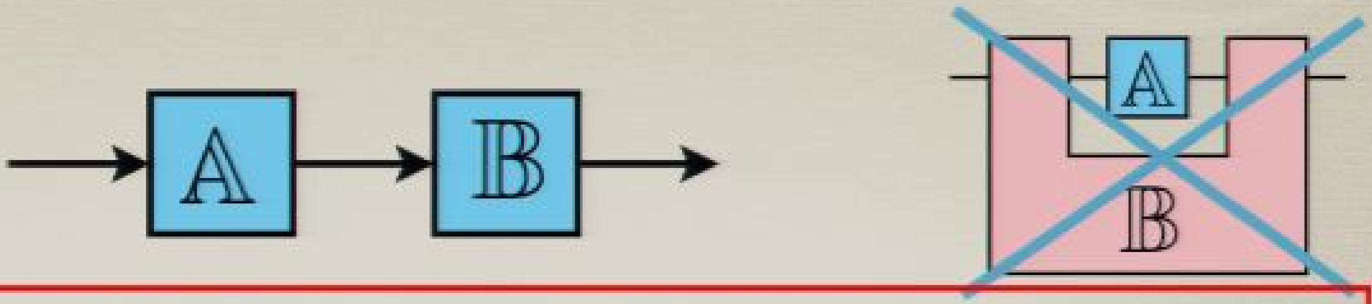

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$$\omega_{\mathscr{A}}(\mathscr{B}) = \frac{\omega(\mathscr{B} \circ \mathscr{A})}{\omega(\mathscr{A})}$$

CASCADES OF TESTS

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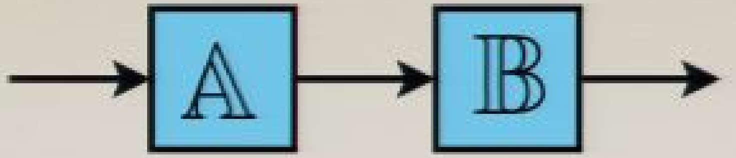

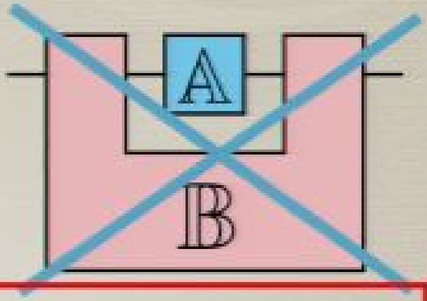

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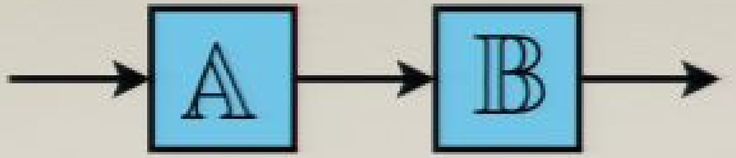

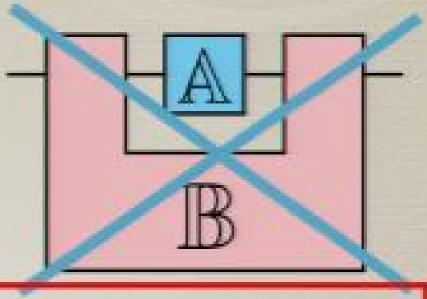

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events ≡ transformations + linearity of evolution

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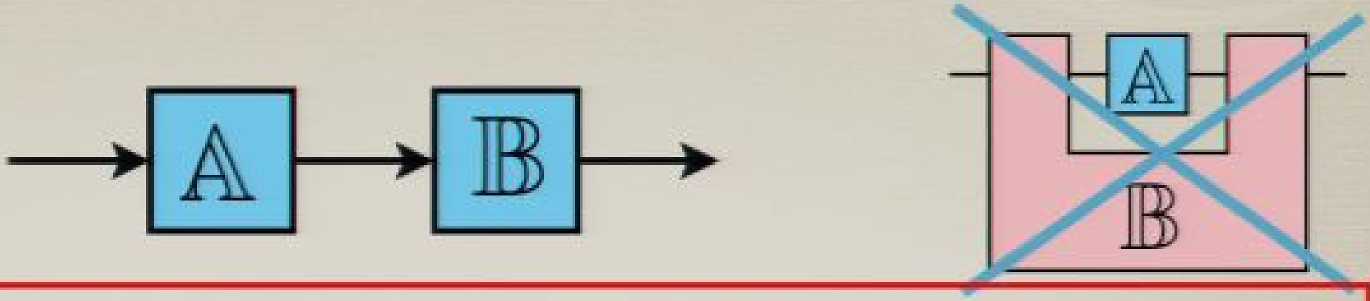

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➡ events $\equiv$ transformations + linearity of evolution

variable

$$\mathscr{A}\,\omega := \omega(\cdot \circ \mathscr{A})$$

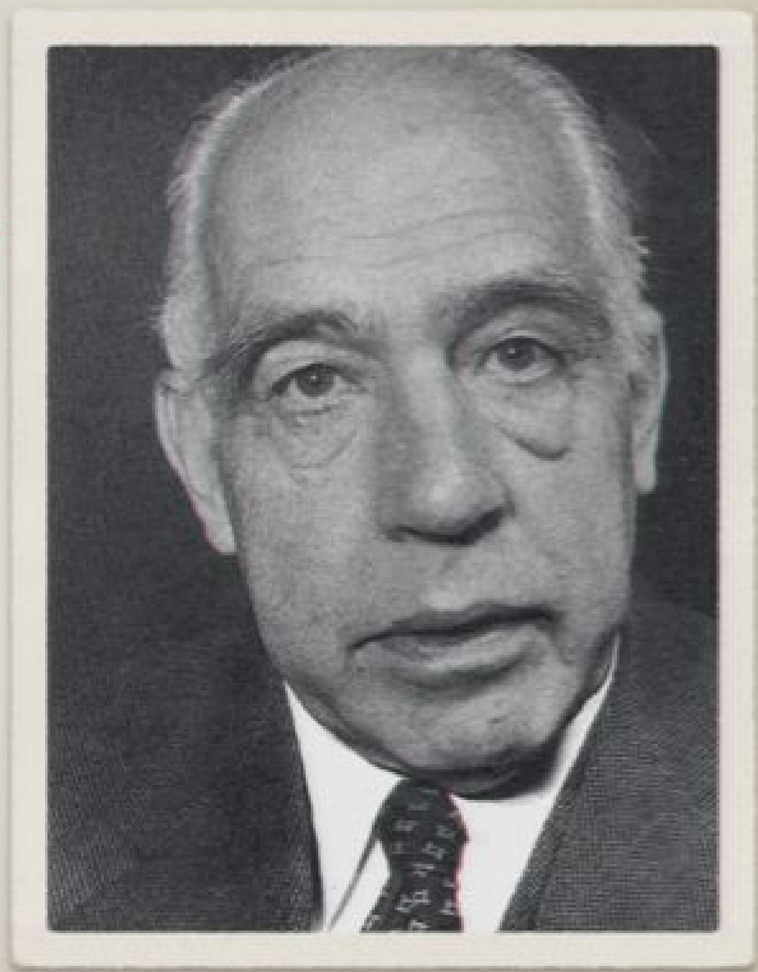
SYSTEM

$$\mathsf{S} = \{\omega_1, \omega_2, \ldots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \ldots\}$$

Copenhagen

collection of tests closed under

- coarse-graining
- conditioning
- cascading
- convex combination



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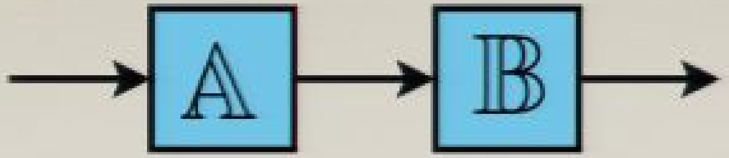

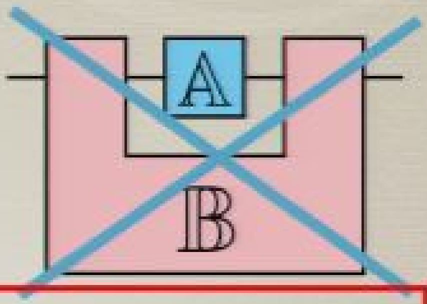

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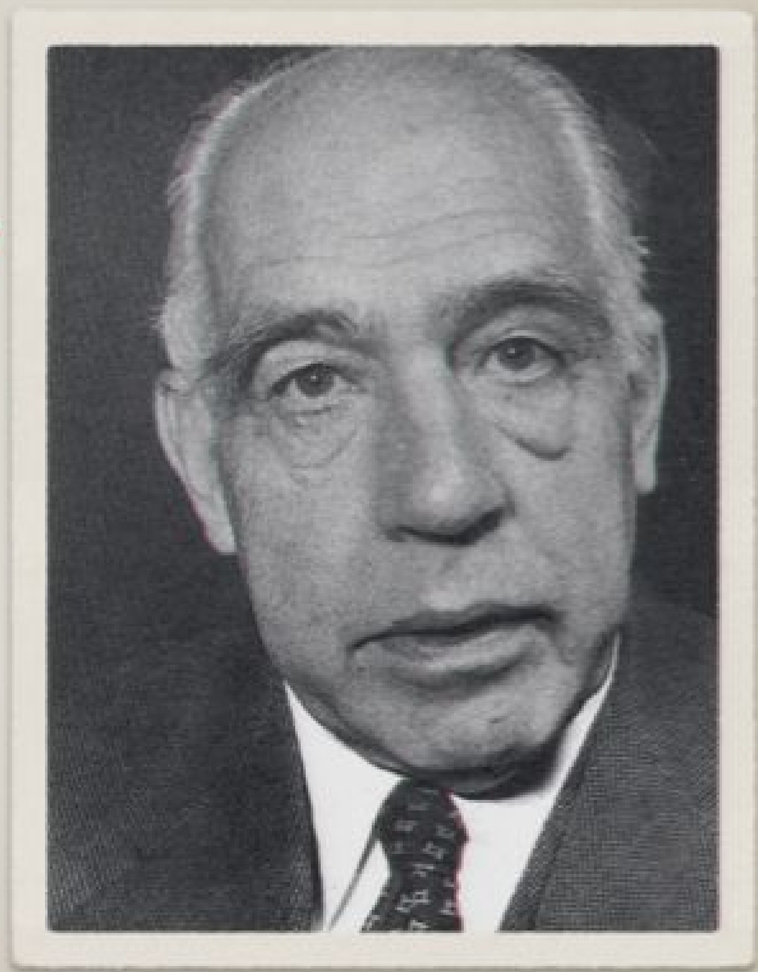
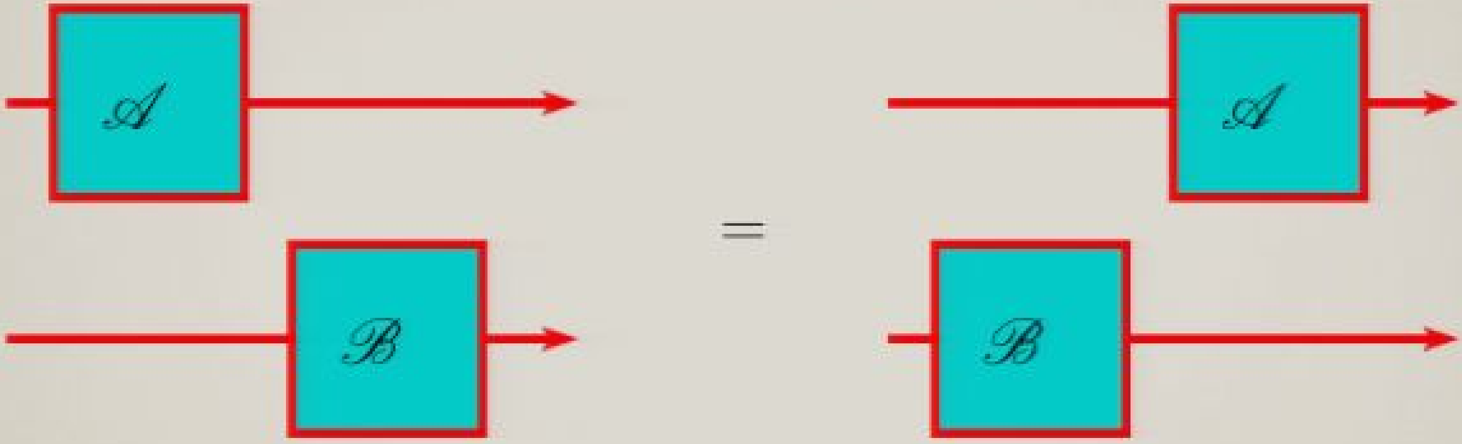


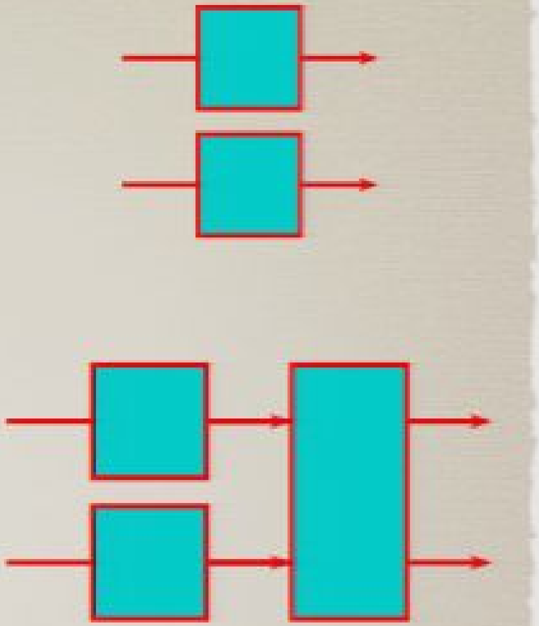
INDEPENDENT SYSTEMS

Two systems are independent if on each system it is possible to perform all their tests as local tests, i.e. such that on every joint state one has the commutativity of the transformations from different systems



$$\mathscr{A}^{(1)} \circ \mathscr{B}^{(2)} = \mathscr{B}^{(2)} \circ \mathscr{A}^{(1)}$$

MULTIPARTITE SYSTEMS



We compose the two systems A and B into the bipartite system AB considered as a new system containing all local tests $\mathsf{A} \times \mathsf{B}$ plus other tests, and closing w.r.t. coarse graining, convex combination and cascading:

$$\boxed{\mathsf{AB} \supseteq \mathsf{A} \times \mathsf{B}}$$

Nonlocal tests: $\mathsf{AB} \setminus \mathsf{A} \times \mathsf{B}$

Equivalence classes for transformations

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Two transformations $\mathscr{A}$ and $\mathscr{B}$ are conditioning equivalent if

$$\omega_{\mathscr{A}} = \omega_{\mathscr{B}} \qquad \forall \omega \in \mathfrak{S}$$

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Two transformations $\mathscr{A}$ and $\mathscr{B}$ are probabilistically equivalent if

$$\omega(\mathscr{A}) = \omega(\mathscr{B}) \quad \forall \omega \in \mathfrak{S}$$

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Conditioning-equivalence class

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Probabilistic-equivalence class

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Probabilistic-equivalence class

A transformation is completely specified by the two classes:

$$\boxed{\mathscr{A}\omega = \omega(\mathscr{A})\omega_{\mathscr{A}}}$$

EFFECTS

Effect: equivalence class of transformations occurring with the same probability for all states. a effect of $\mathscr{A}$:

$$\forall \omega \in \mathfrak{S} : \quad \omega(\mathscr{A}) \equiv \omega(a)$$

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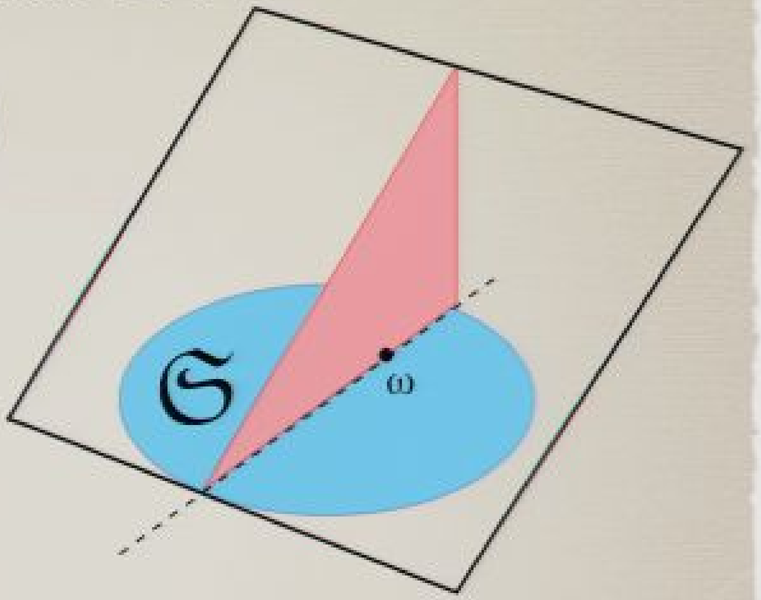

Duality: effects are positive linear functionals ≤1 over states.

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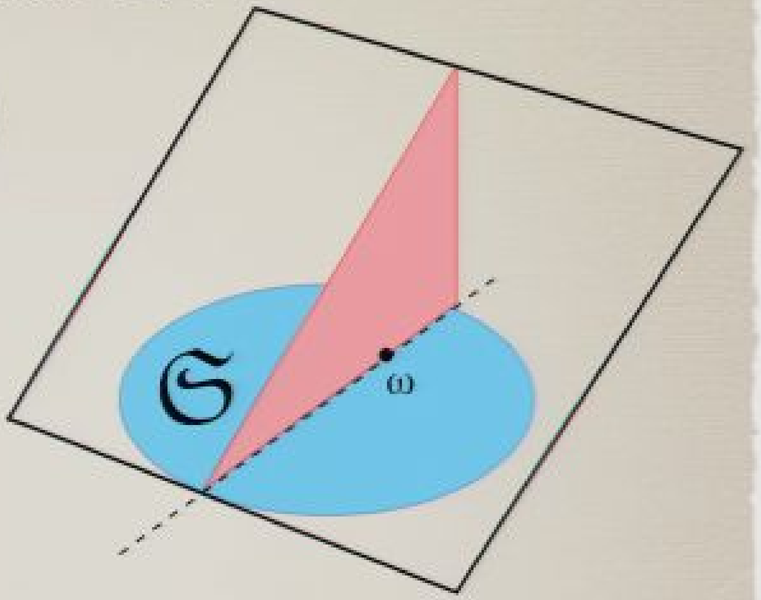

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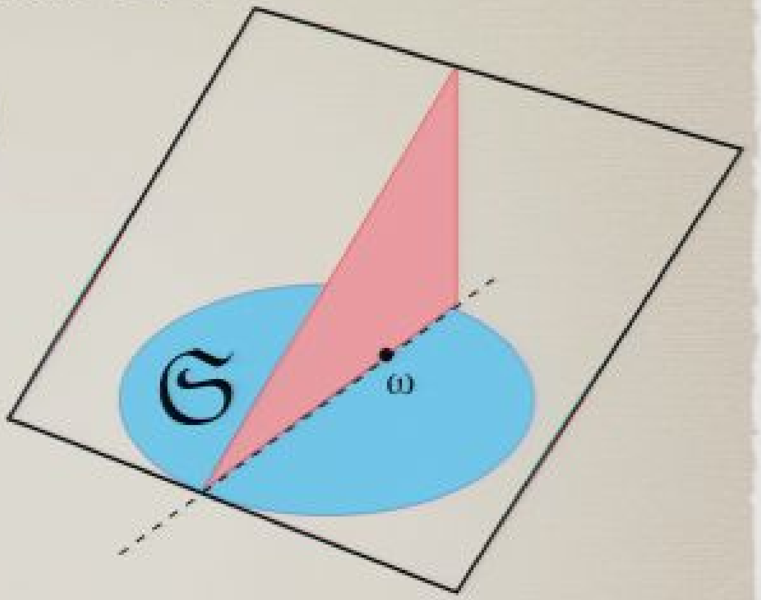

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Transformations act linearly on effects (Heisenberg Picture)

Convex set of effects: $\mathfrak{E}$

Observation tests: effects as events

NOTATION *a la Dirac*

$$p_{kji} := (a_k|_{\mathrm{B}}\, \mathscr{C}_j\, |\rho_i)_{\mathrm{A}}$$

$|\omega)_{\mathrm{A}}$ state

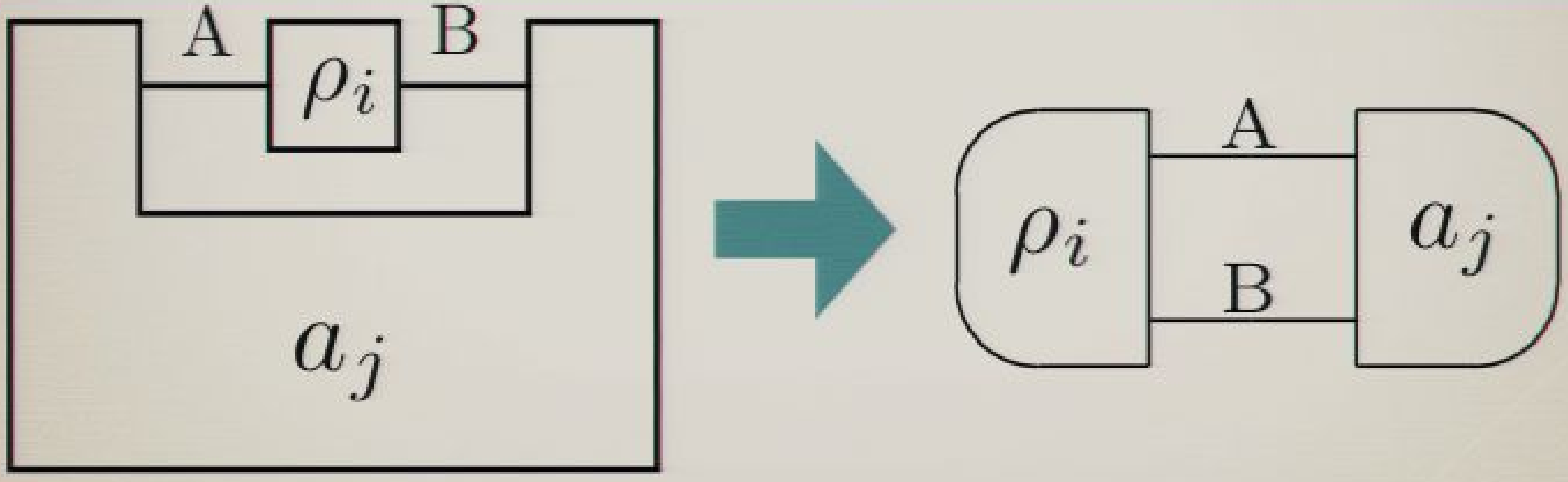
$(a|\omega) := \omega(a)$

probability

$(a|_{\mathrm{A}}$ effect

CAUSALITY

CAUSAL THEORIES



NON CAUSAL THEORIES

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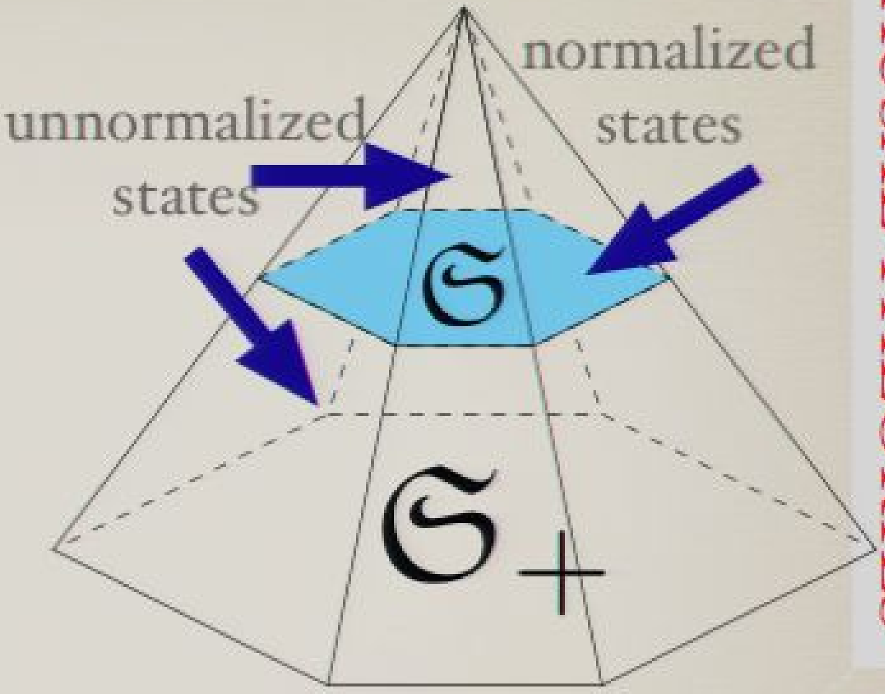

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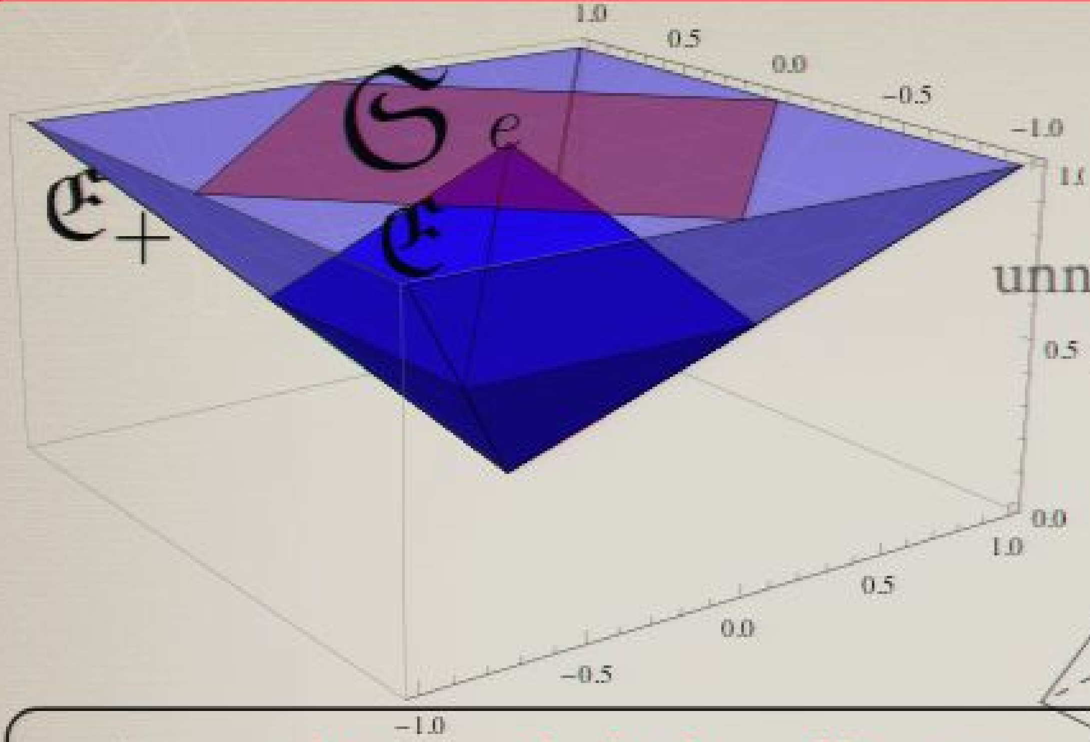

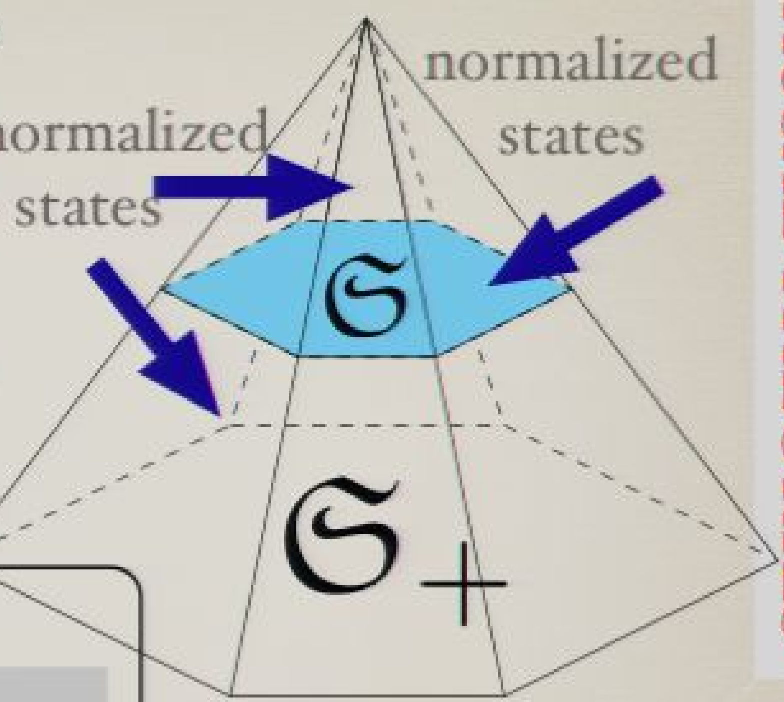

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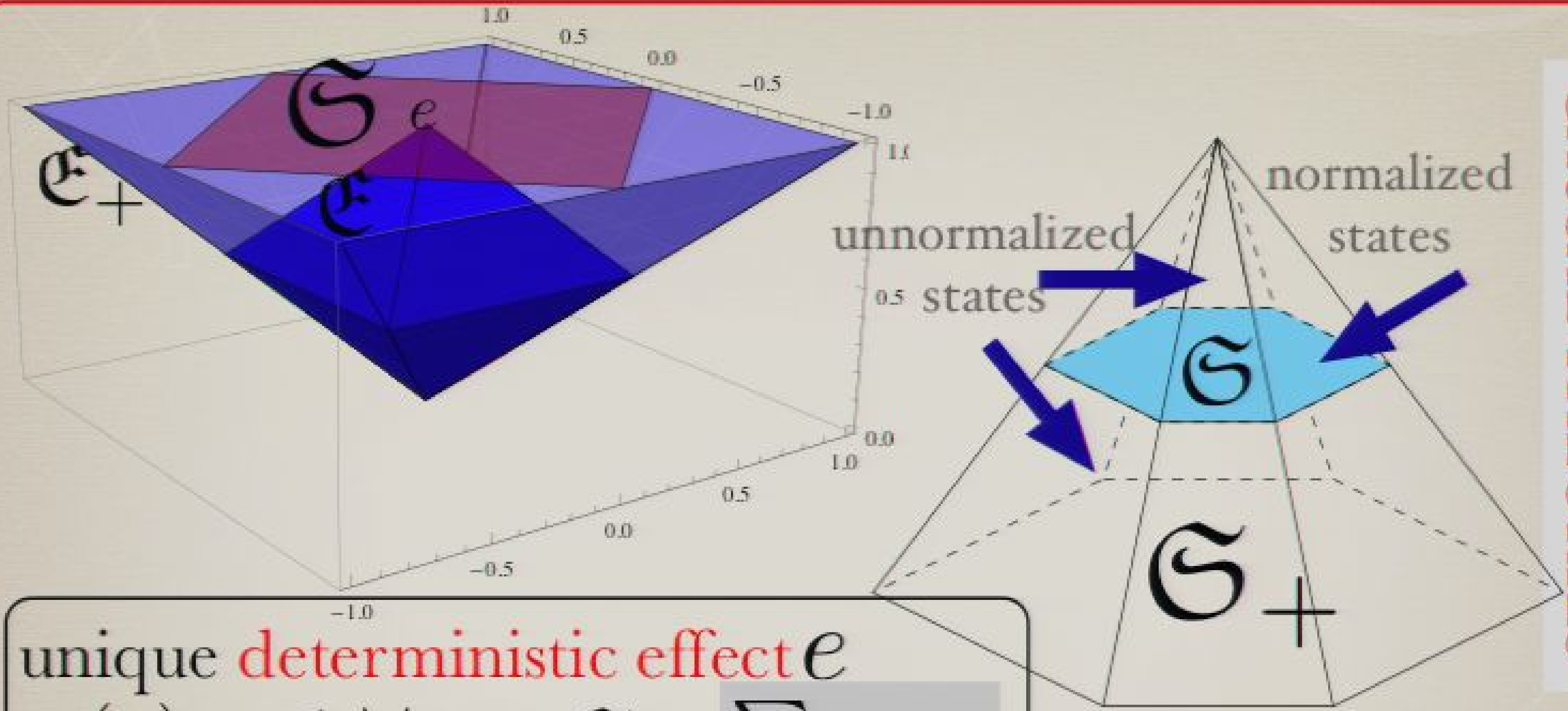

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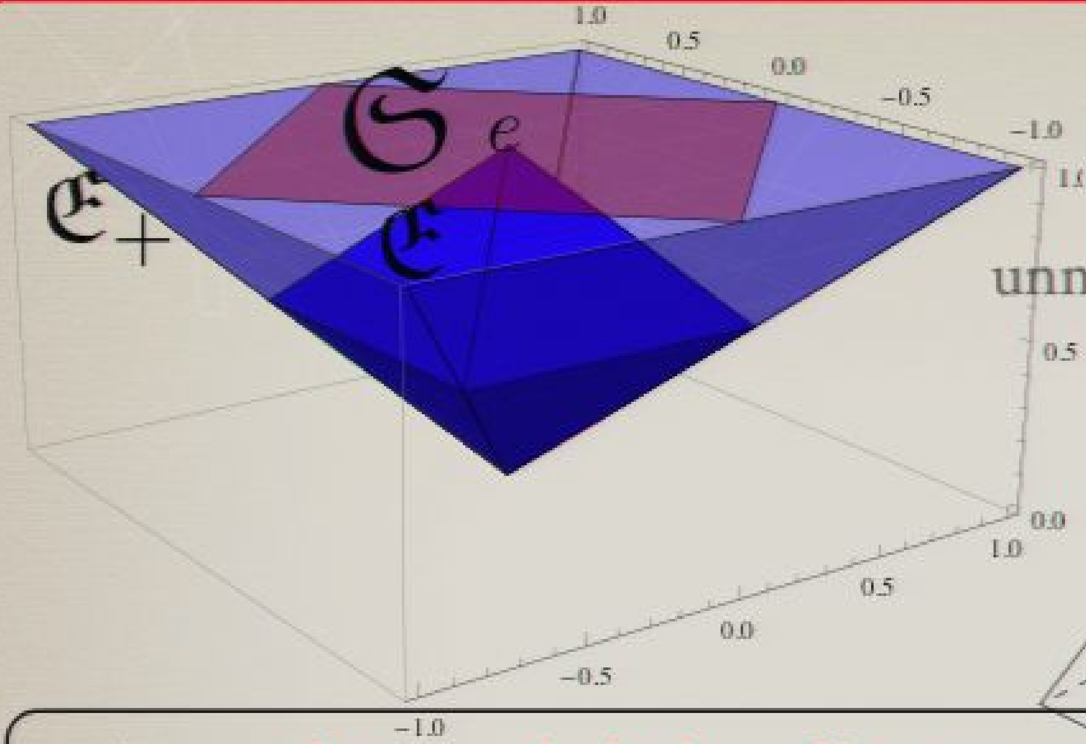

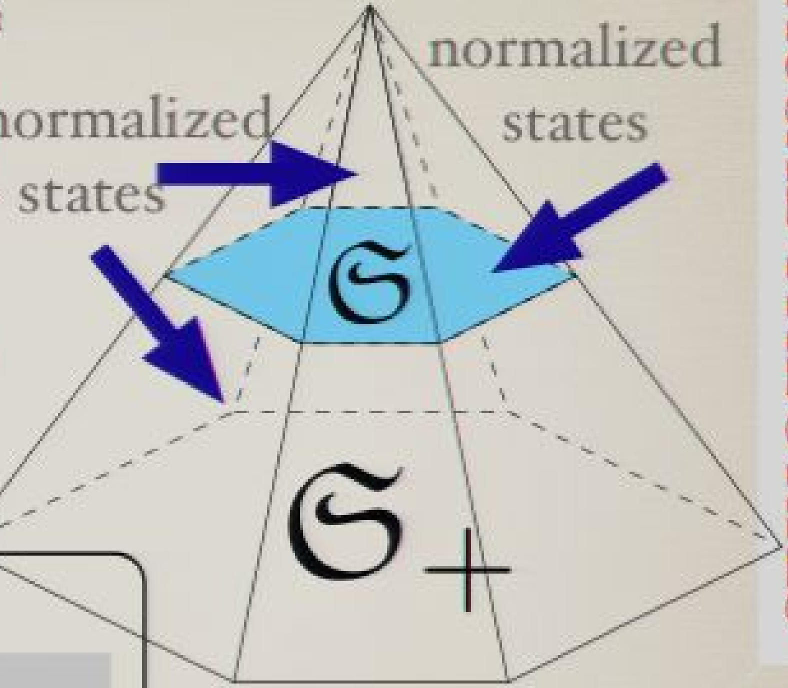

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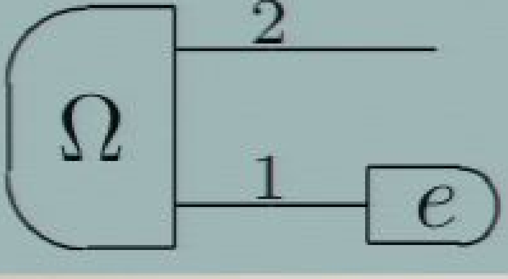

In Dirac notation:

$$(e|_1\,|\Omega)_{12} = |\omega)_2$$

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QM: probabilistic theory satisfying:

1. Causality
2. Local observability
3. Conservation of information

arXiv:0807.4383: in *Philosophy of Quantum Information and Entanglement*, Eds A. Bokulich and G. Jaeger (CUP, Cambridge UK, 2009)

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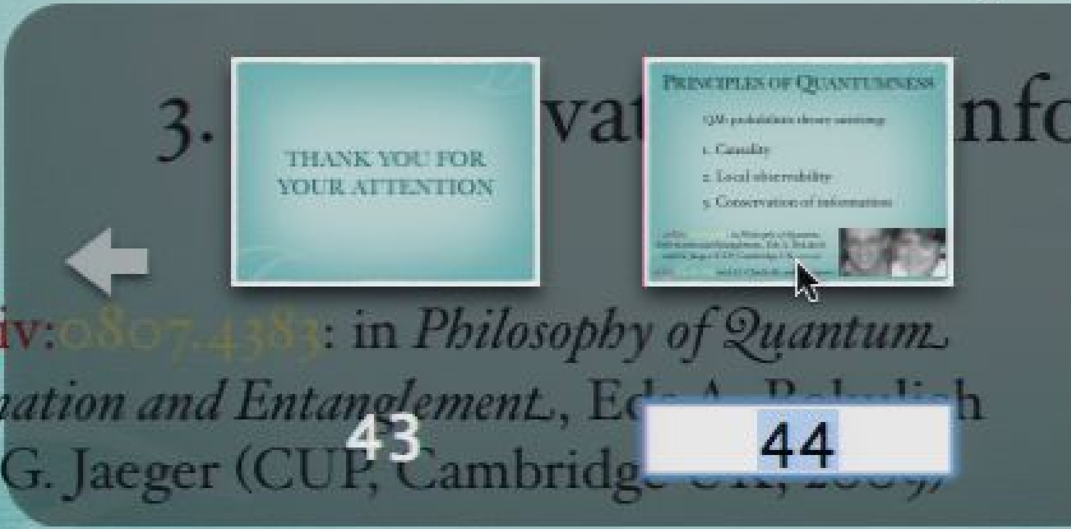

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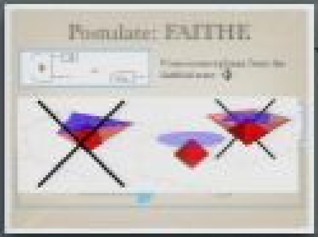

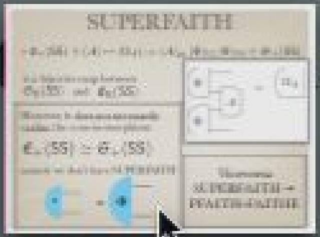

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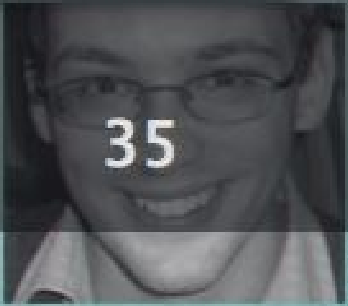

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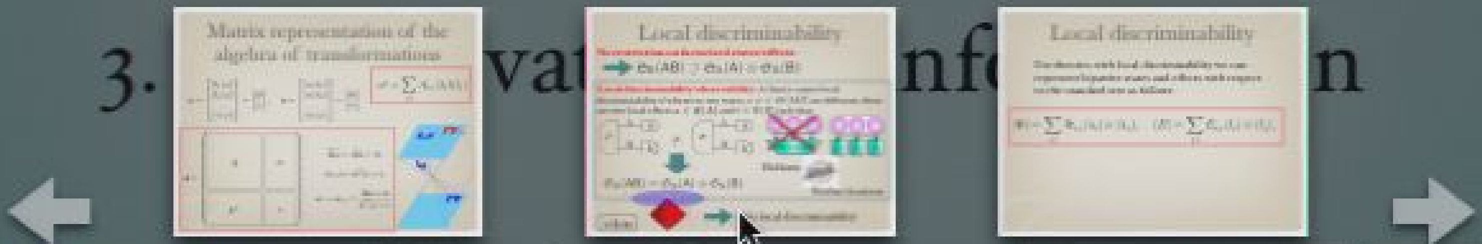

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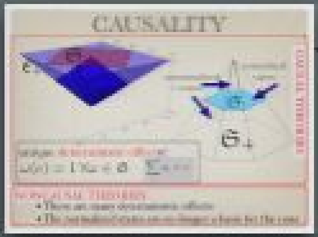

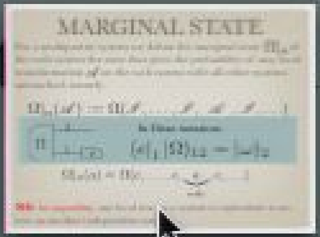

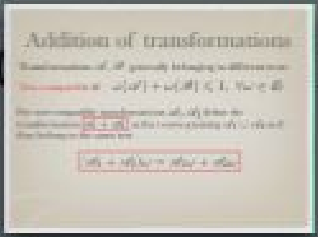

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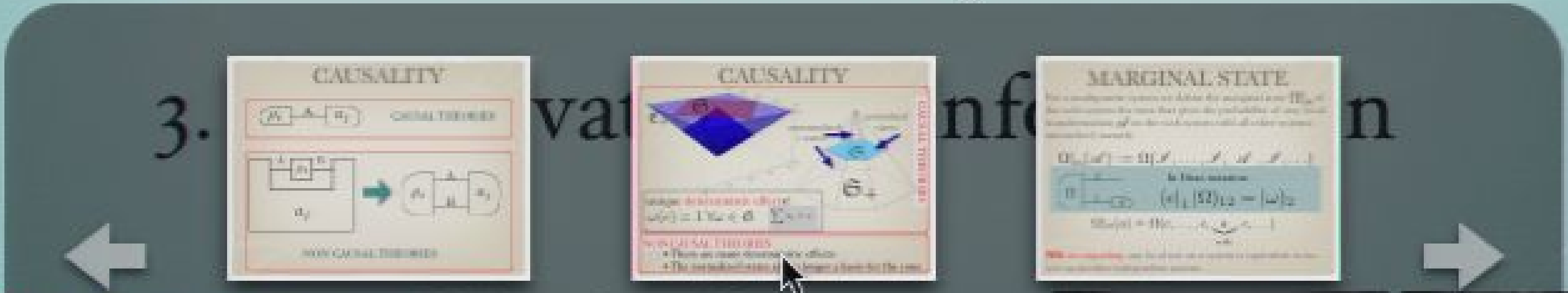

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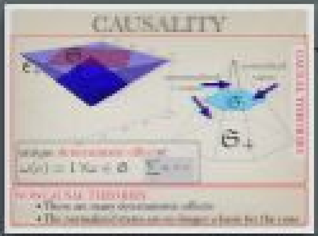

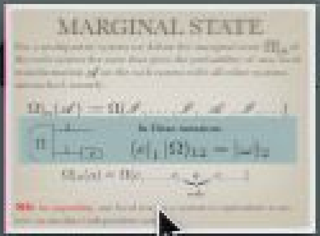

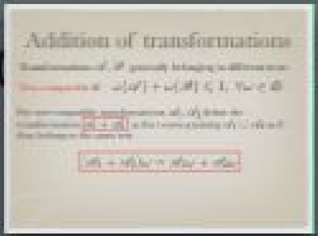

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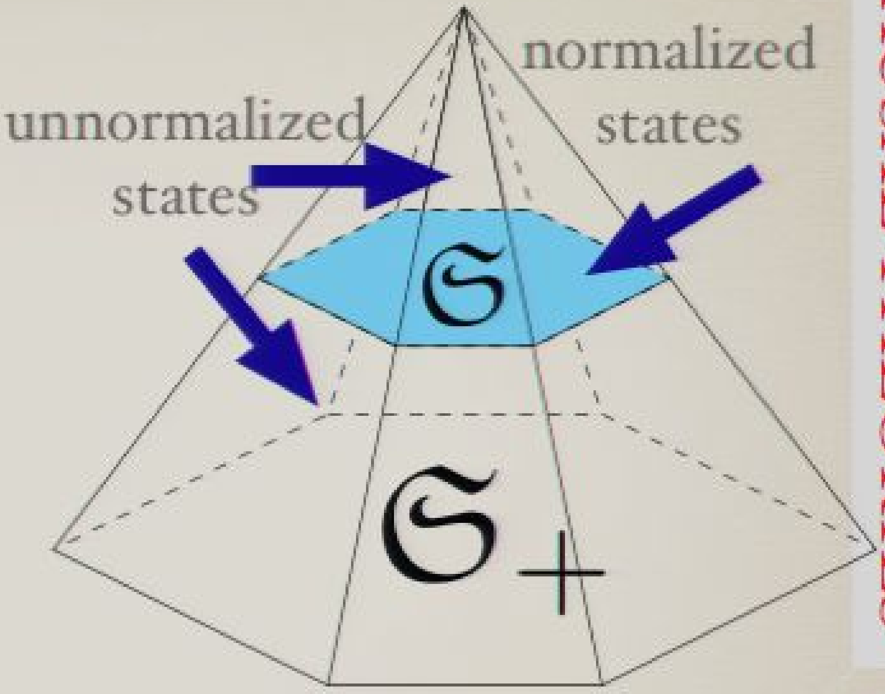

CAUSAL THEORIES

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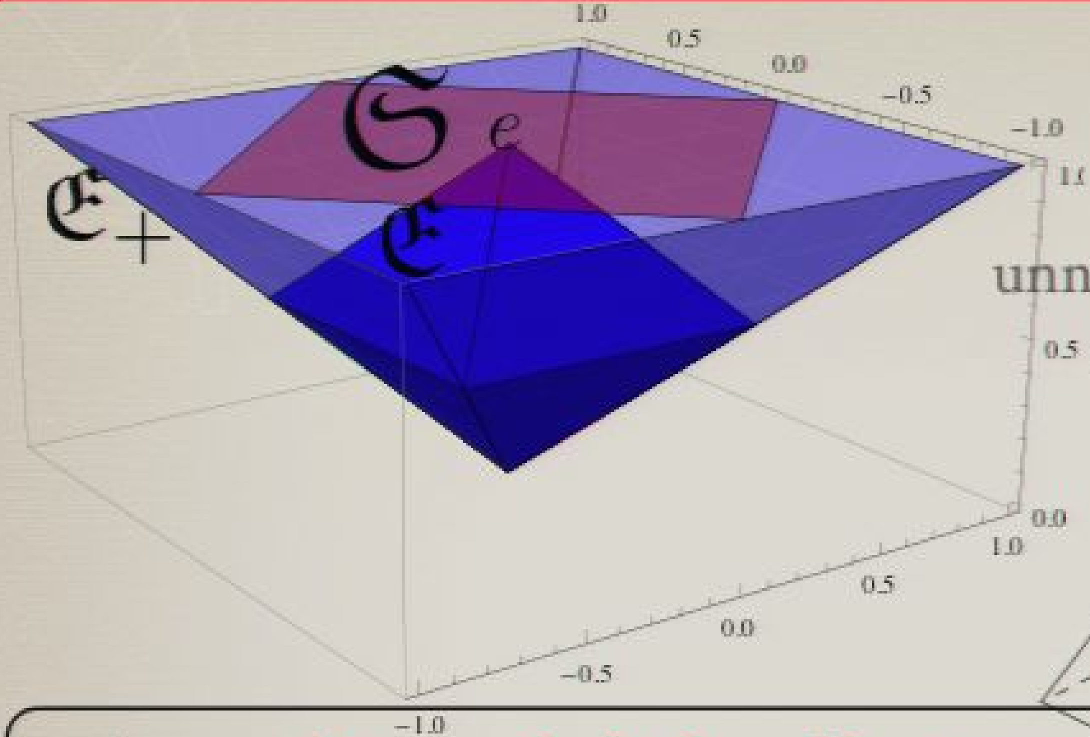

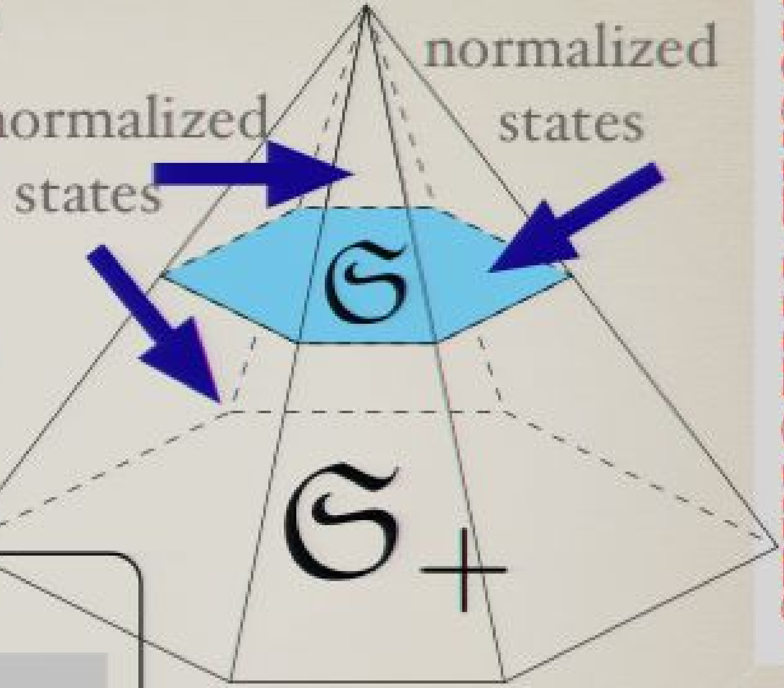

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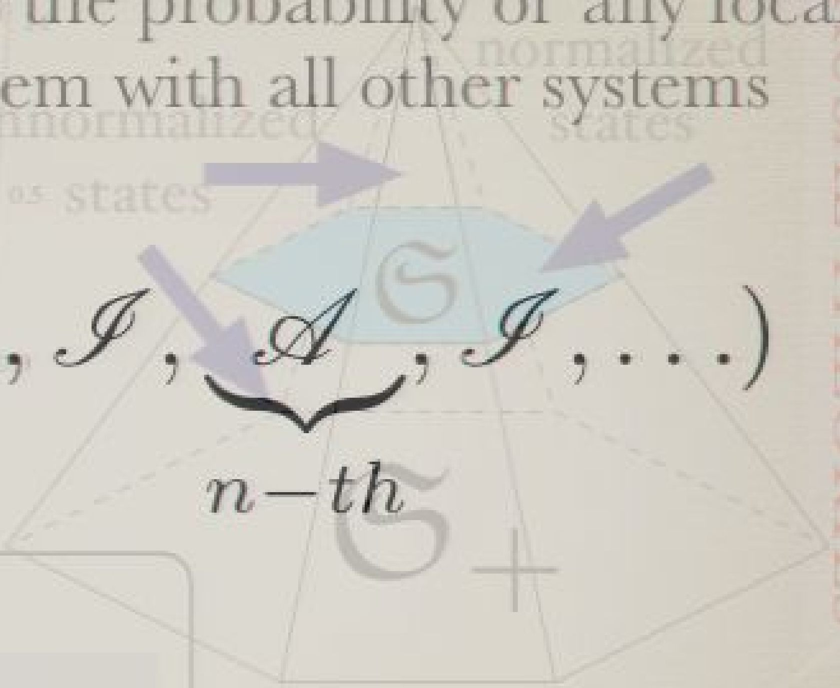

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unnormalized states

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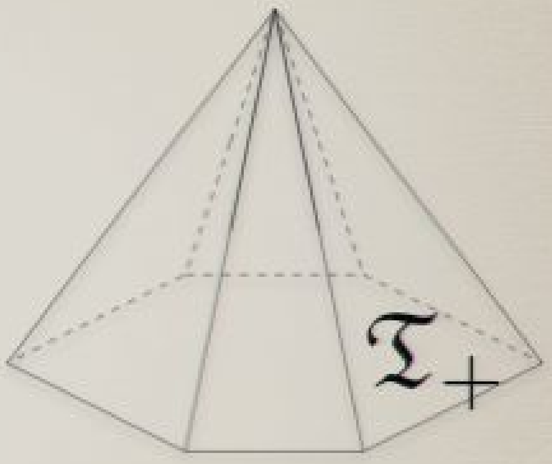

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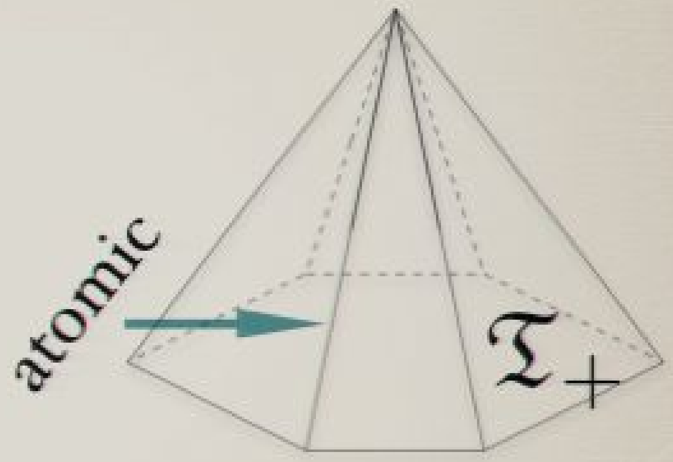

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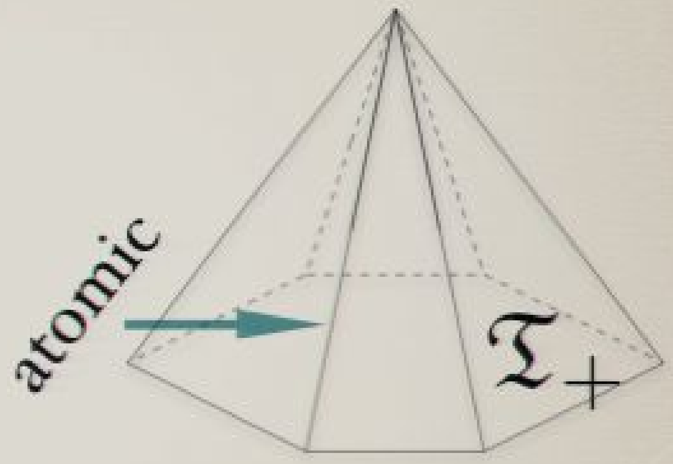

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- The identity transformation $\mathscr{I}$ is not necessarily atomic!

Matrix-algebra of transformations

Complete sum and rescaling to make the set of transformations a real **matrix algebra** $\mathfrak{T}_{\mathbb{R}}$ over the real linear space of effects/states $\mathfrak{S}_{\mathbb{R}}$ $\mathfrak{E}_{\mathbb{R}}$

$$\mathfrak{S}_{\mathbb{R}} := \mathsf{Span}_{\mathbb{R}}\mathfrak{S}, \ \ldots$$

STANDARD REFERENCE-TEST

$$\boxed{\mathbb{S} = \{\mathscr{S}_i\}, \quad \mathscr{S}_i = |\lambda_i)(l_i|}$$

$\{\lambda_i\}$ minimal effect-separating set of states

$\{l_i\}$ minimal state-separating set of effects (info-complete)

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$$(l_i|\,\lambda_j) = \delta_{ij}$$

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$$(l_i|\lambda_j) = \delta_{ij}$$

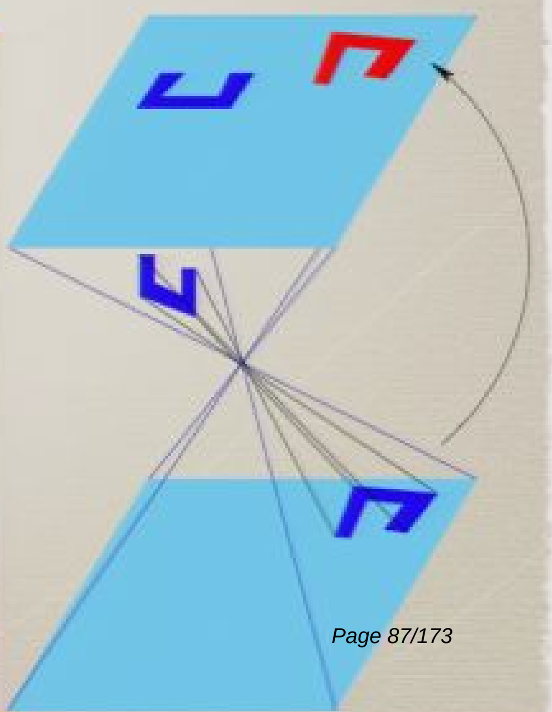
- take the last element $l_N \equiv e$ and correspondingly λ_N giving the direction of the cone axis of $\mathfrak{S}_+$

Matrix representation of the algebra of transformations

$$\boldsymbol{\omega} = \begin{bmatrix} (l_1|\omega) \\ (l_2|\omega) \\ \dots \\ (e|\omega) \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\omega}} \\ \hat{\omega} \end{bmatrix}, \quad \boldsymbol{a} = \begin{bmatrix} (a|\lambda_1) \\ (a|\lambda_2) \\ \dots \\ (a|\chi) \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{a}} \\ \hat{a} \end{bmatrix}$$

$$\boxed{\mathscr{A} = \sum_{ij} A_{ij} \, |\lambda_i)(l_j|}$$

$$\boldsymbol{A} = \left(\begin{array}{c|c} \hat{\boldsymbol{A}} & \hat{\boldsymbol{\alpha}} \\ \hline \hat{\boldsymbol{a}}^T & \hat{a} \end{array} \right), \qquad \begin{aligned} \widehat{\boldsymbol{A}\boldsymbol{\omega}} &= \hat{\boldsymbol{A}}\hat{\boldsymbol{\omega}} + \hat{\boldsymbol{\alpha}}, \\ (a|\omega) &= \hat{\boldsymbol{a}}^T\hat{\boldsymbol{\omega}} + \hat{a}, \\ \hat{\boldsymbol{\omega}} \to \hat{\boldsymbol{\omega}}_{\mathscr{A}} &= \frac{\hat{\boldsymbol{A}}\hat{\boldsymbol{\omega}} + \hat{\boldsymbol{\alpha}}}{\hat{\boldsymbol{a}}^T\hat{\boldsymbol{\omega}} + \hat{a}}. \end{aligned}$$

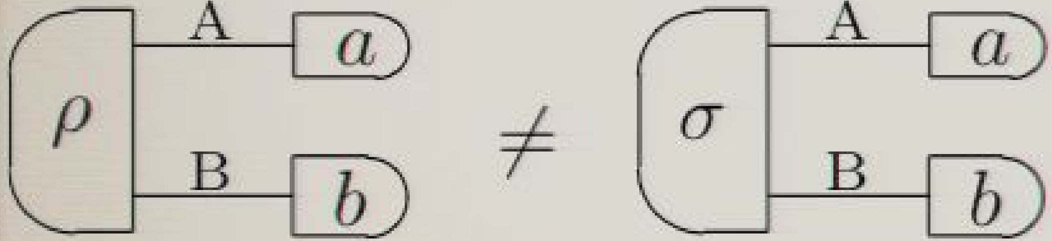


Local discriminability

No restriction on factorized states/effects

➡ $\mathfrak{S}_{\mathbb{R}}(\mathrm{AB}) \supseteq \mathfrak{S}_{\mathbb{R}}(\mathrm{A}) \otimes \mathfrak{S}_{\mathbb{R}}(\mathrm{B})$

Local discriminability/observability: A theory enjoys local discriminability if whenever two states $\rho, \sigma \in \mathfrak{S}(\mathrm{AB})$ are different, there are two local effects $a \in \mathfrak{E}(\mathrm{A})$ and $b \in \mathfrak{E}(\mathrm{B})$ such that



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➡ $\mathfrak{S}_{\mathbb{R}}(\mathsf{AB}) \supseteq \mathfrak{S}_{\mathbb{R}}(\mathsf{A}) \otimes \mathfrak{S}_{\mathbb{R}}(\mathsf{B})$

Local discriminability/observability: A theory enjoys local discriminability if whenever two states $\rho, \sigma \in \mathfrak{S}(\mathrm{AB})$ are different, there are two local effects $a \in \mathfrak{E}(\mathrm{A})$ and $b \in \mathfrak{E}(\mathrm{B})$ such that

⬇

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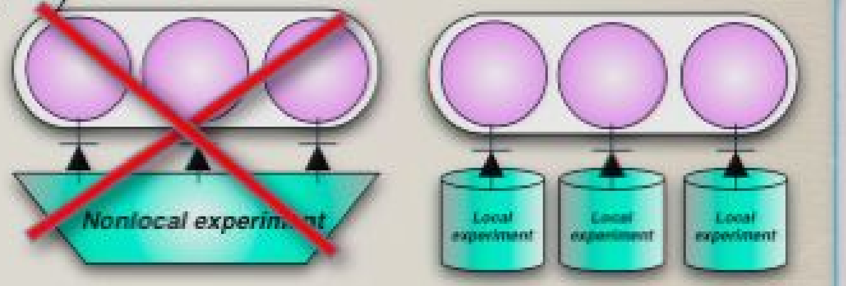

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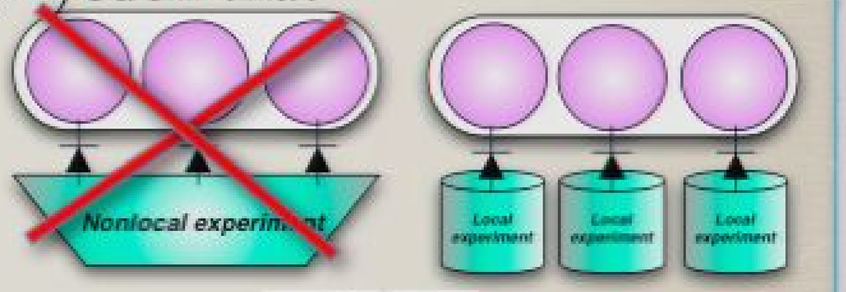

Holism

Reductionism

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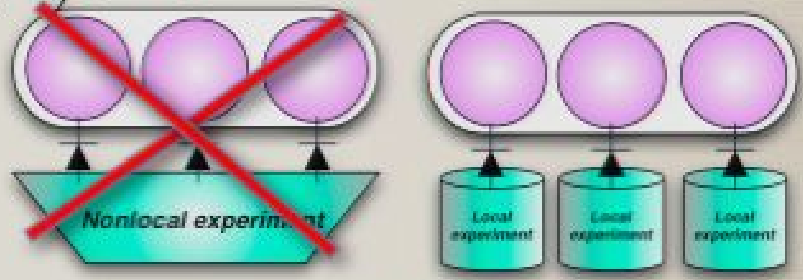

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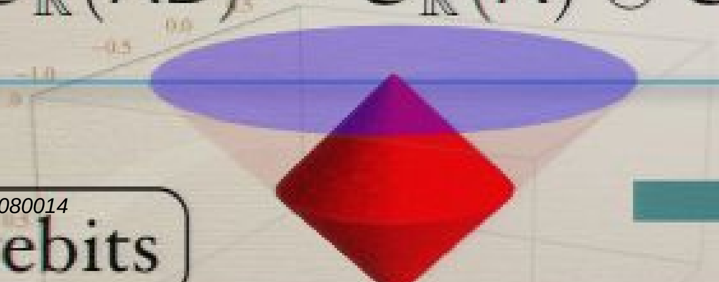


rebits

No local discriminability

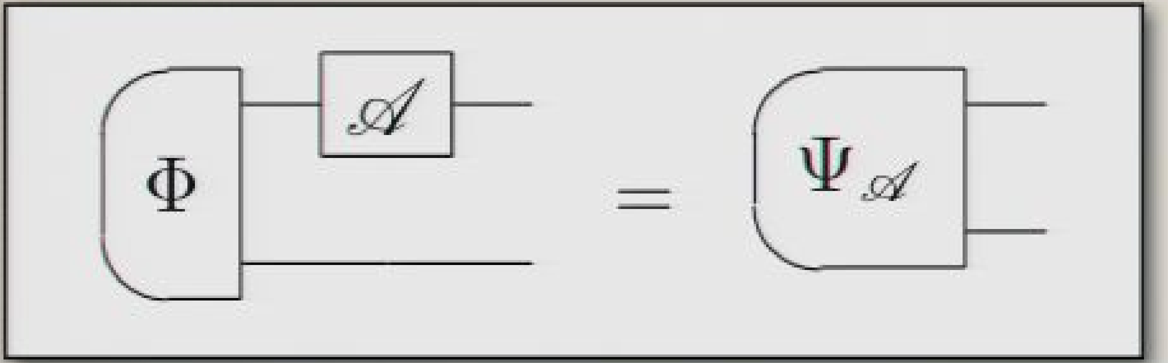
Local discriminability

For theories with local discriminability we can represent bipartite states and effects with respect to the standard test as follows

$$|\Psi) = \sum_{ij} \Psi_{ij} \, |\lambda_i) \otimes |\lambda_j), \quad (E| = \sum_{ij} E_{ij} \, (l_i| \otimes (l_j| \, ,$$

FAITHFUL STATES

A state Φ of a bipartite system is dynamically faithful when the output state $\mathscr{A}_1 |\Phi)_{12}$ from a local transformation $\mathscr{A}$ on one system is in 1-to-1 correspondence with the transformation $\mathscr{A}$



calibrability of tests

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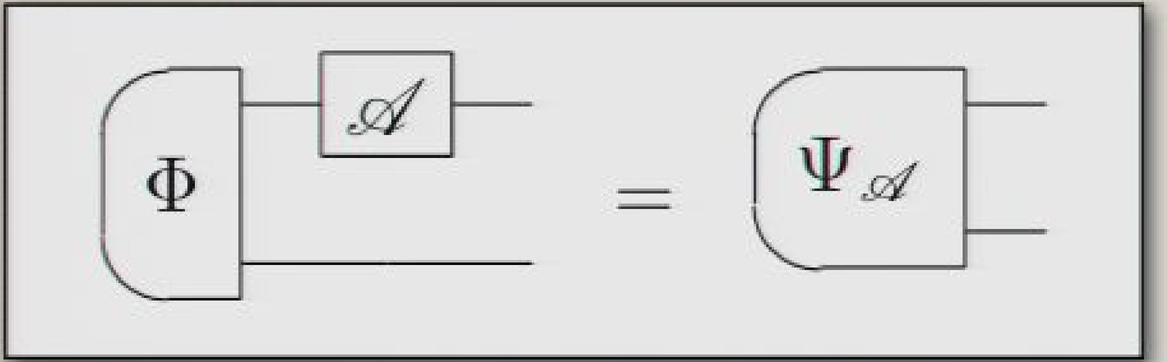
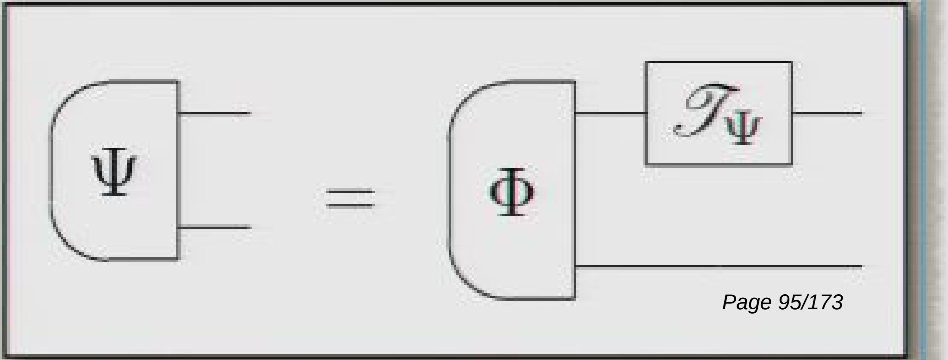


calibrability of tests

A state Φ of a bipartite system is preparationally faithful if every joint state Ψ can be achieved by a suitable local transformation $\mathscr{T}_\Psi$ on one system occurring with nonzero probability

local state-preparability



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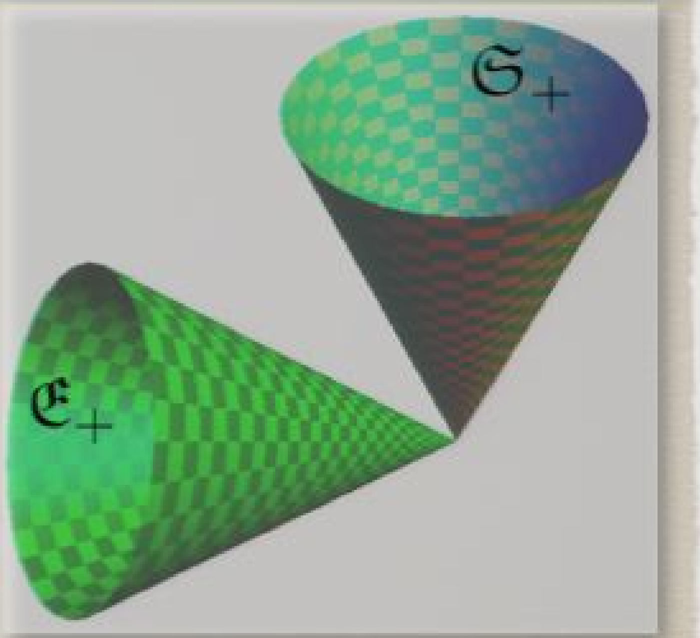

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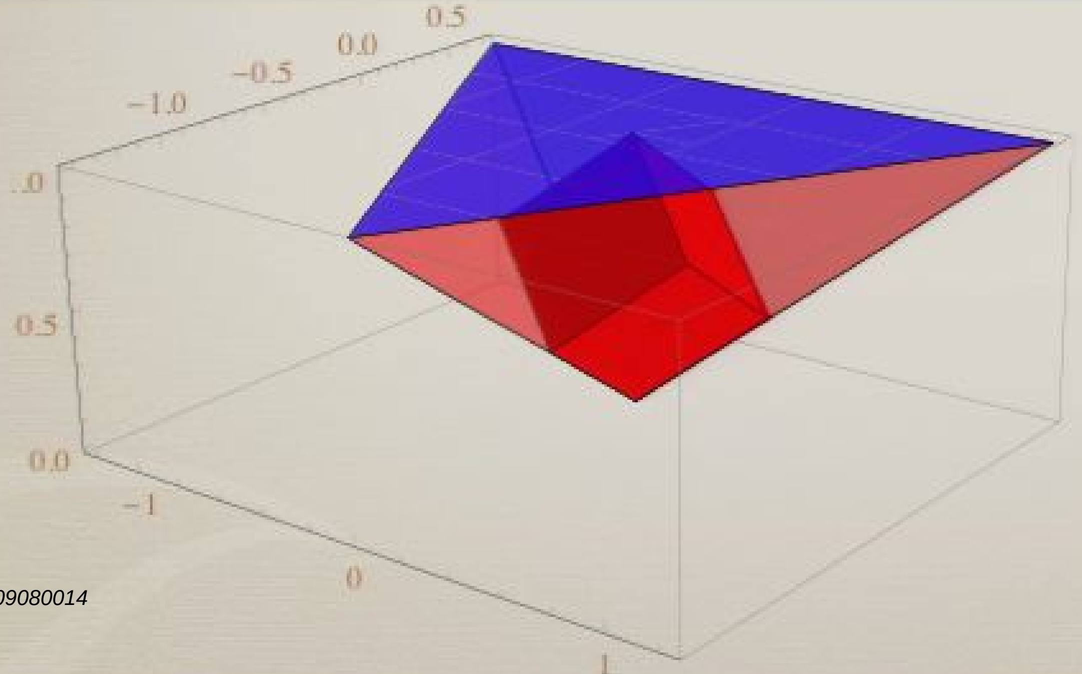
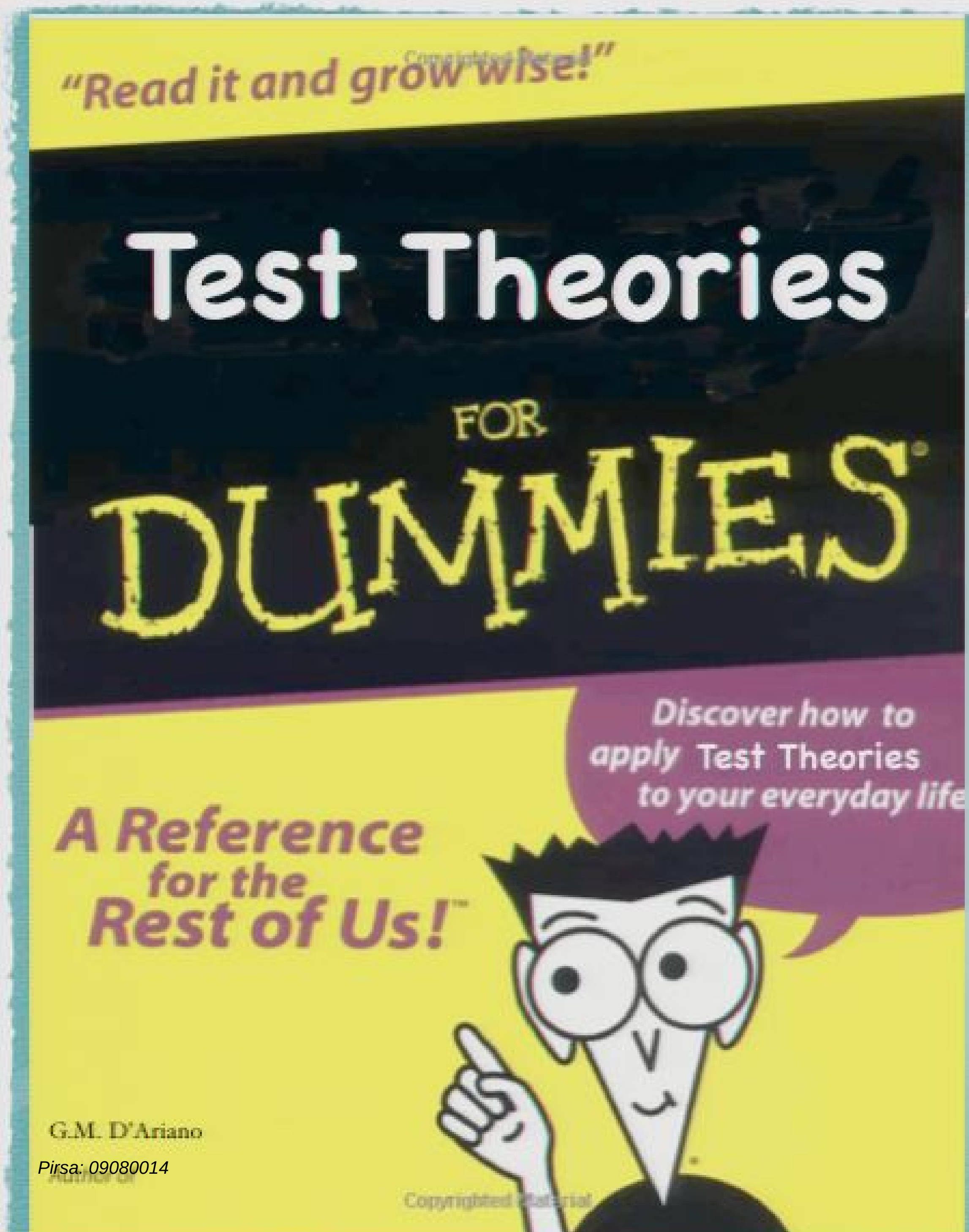
- A prep. faithful state is pure iff $\mathscr{I}$ is atomic (nonlocal feature from local one!)

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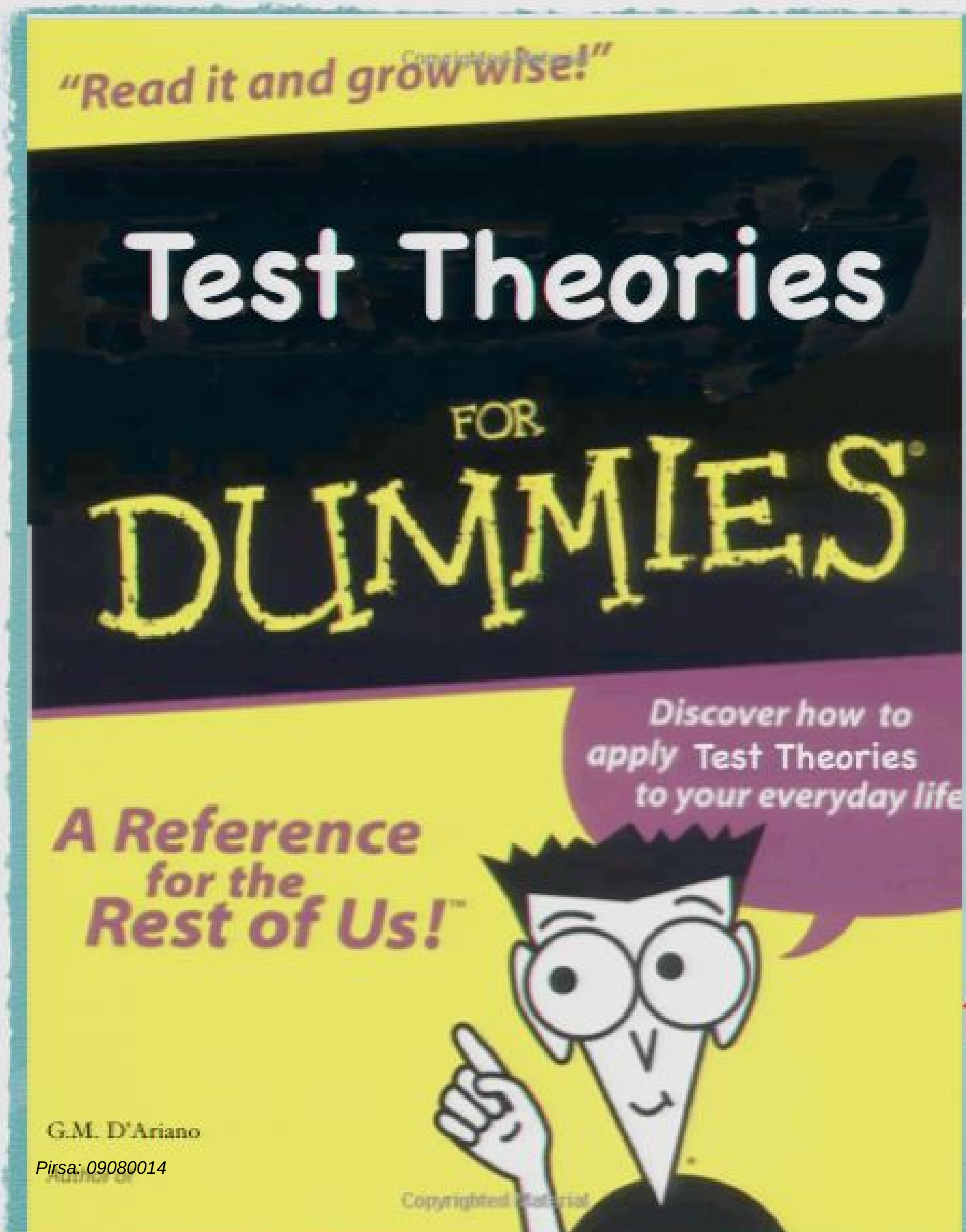
Test Theories
FOR
DUMMIES
Discover how to apply Test Theories to your everyday life
A Reference for the Rest of Us!

THE END



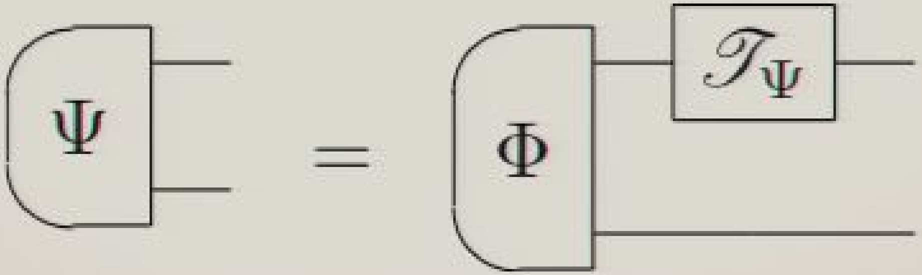
THE END

Lesson learnt:
causal loc-discr.
test-theories have a
nice matrix
representation

EXPLORING POSSIBLE PRINCIPLES OF THE QUANTUMNESS

Postulate PFAITH

PFAITH: For any couple of identical systems, there exist a symmetric pure state Φ that is preparationally faithful.



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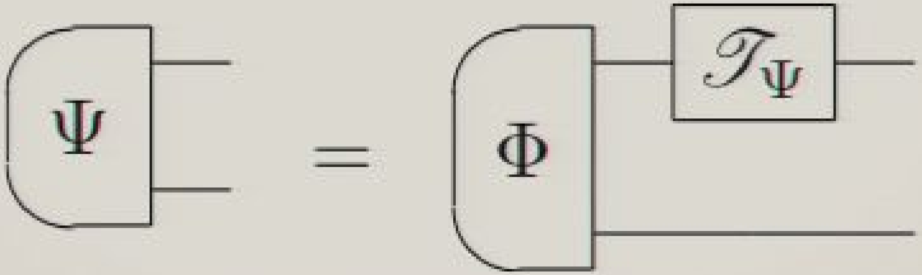

- Calibrability & Preparability by just a single preparation

PRINCIPLES OF QUANTUMNESS

QM: probabilistic theory satisfying:

1. Causality
2. Local observability
3. Conservation of information

arXiv:0807.4383: in *Philosophy of Quantum Information and Entanglement*, Eds A. Bokulich and G. Jaeger (CUP, Cambridge UK, 2009)

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THANK YOU FOR YOUR ATTENTION

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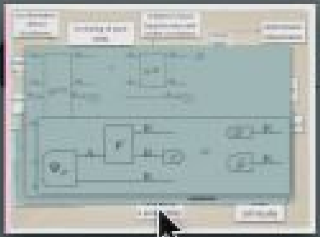

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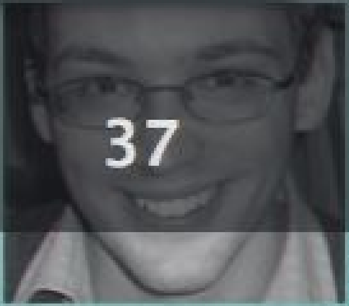

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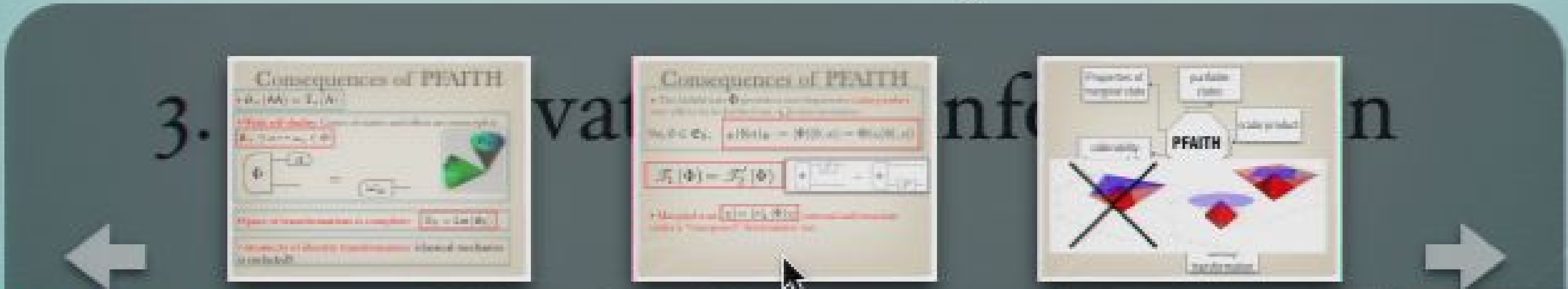

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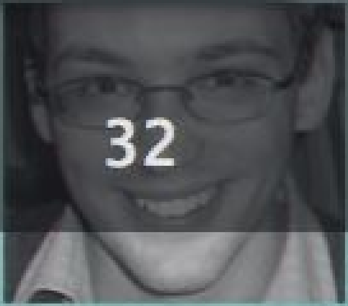

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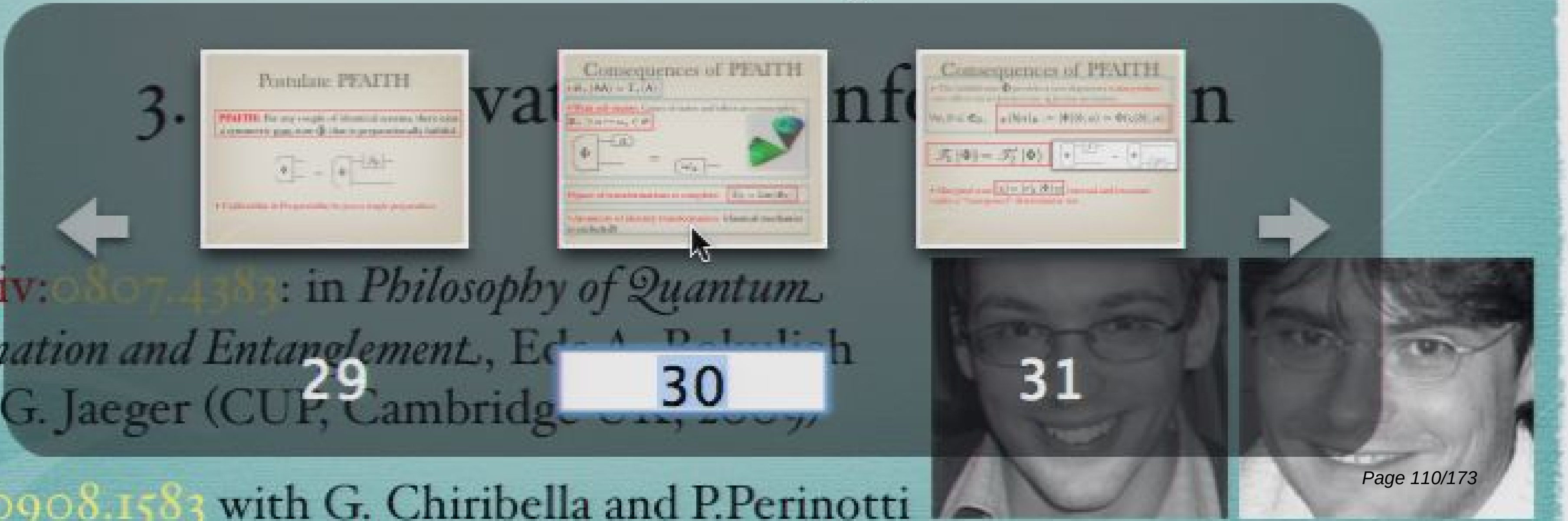

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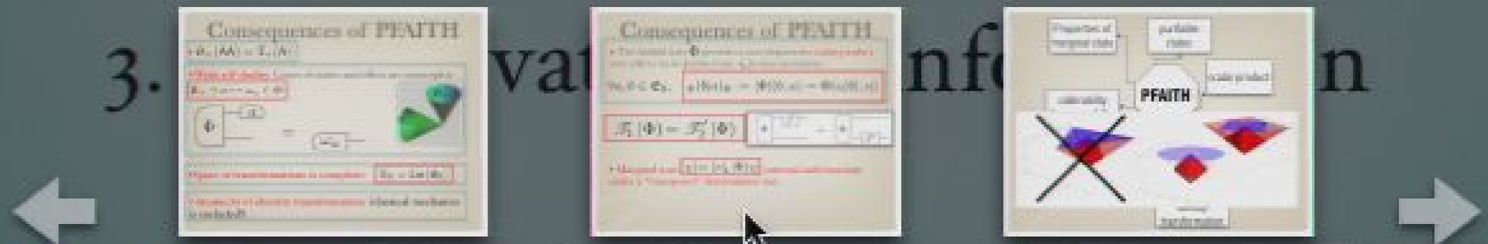

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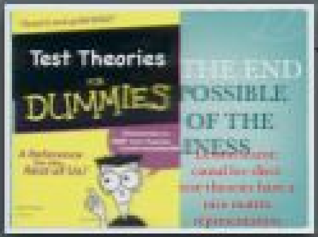

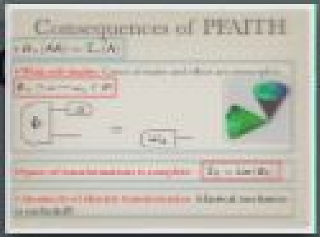

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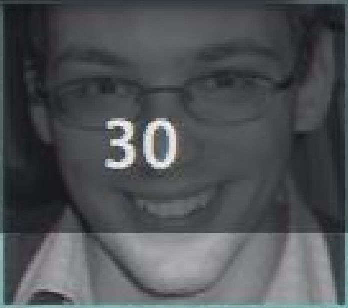

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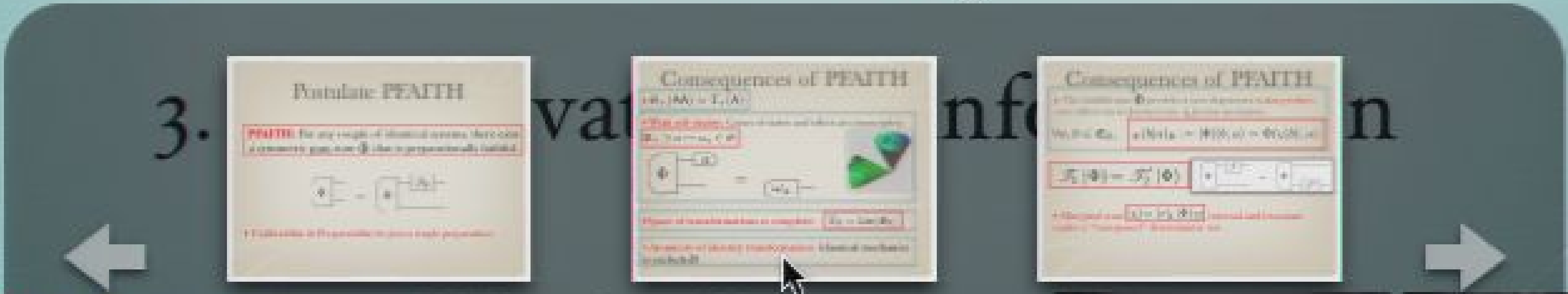

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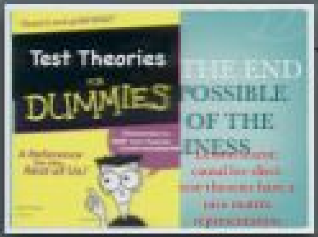

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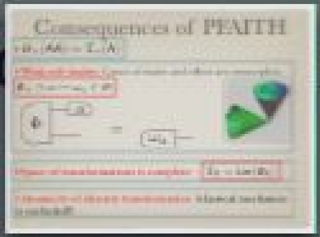

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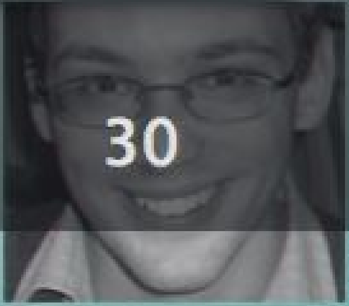

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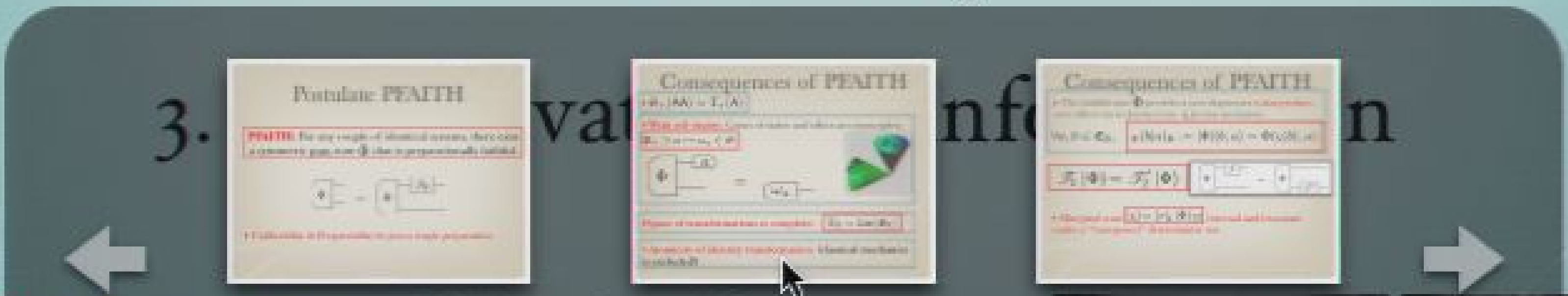

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Consequences of PFAITH

- $\mathfrak{S}_+(\mathsf{AA}) \simeq \mathfrak{T}_+(\mathsf{A})$

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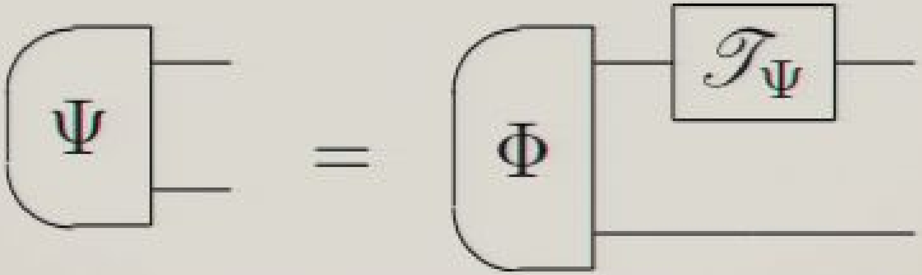

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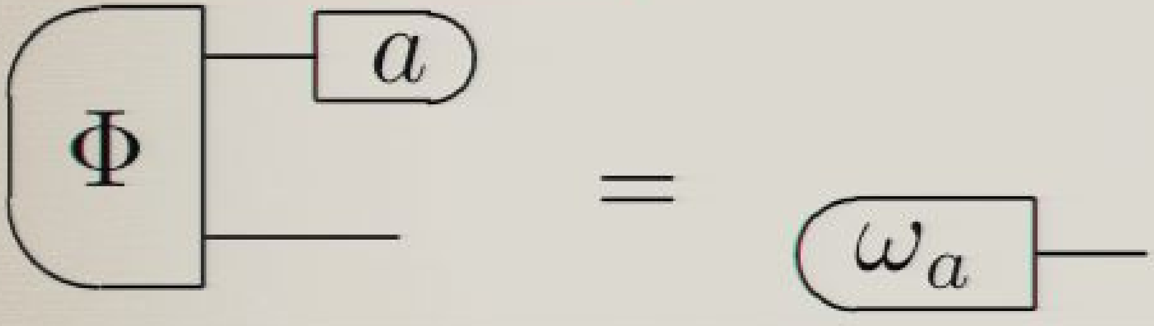

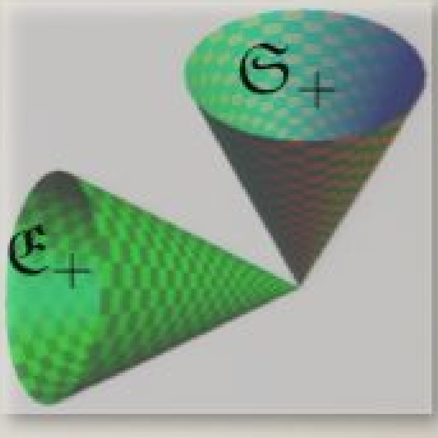

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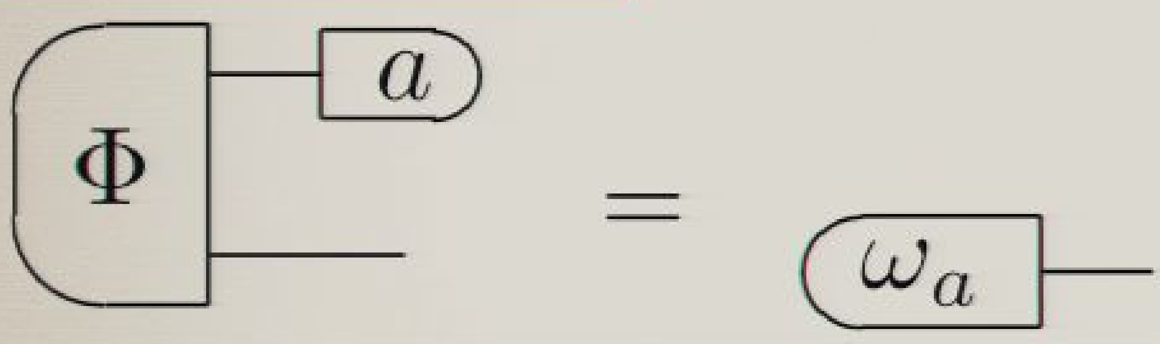

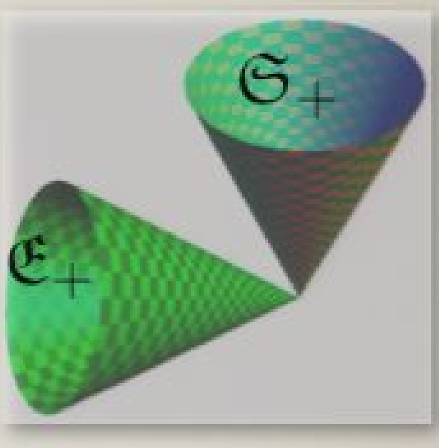

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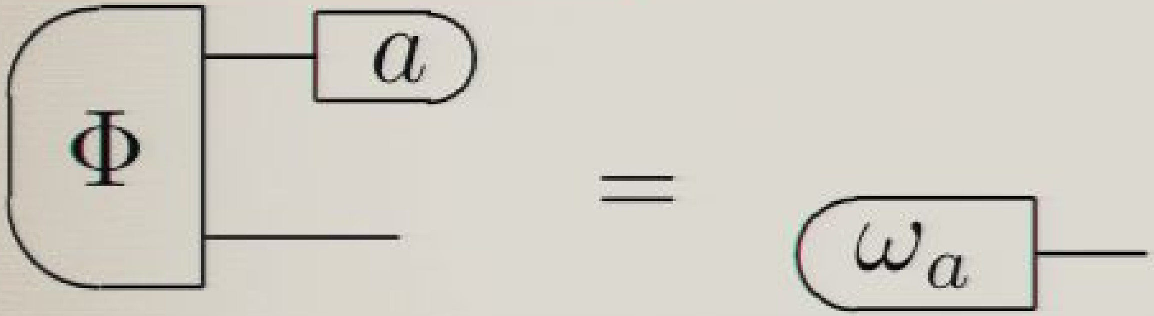
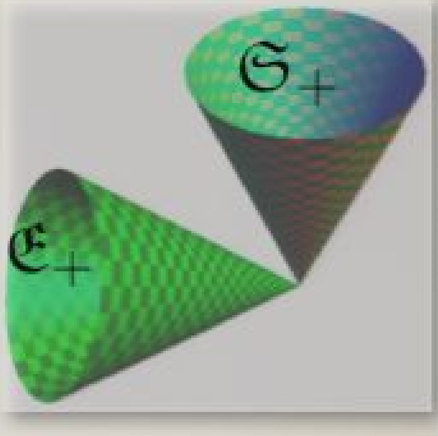




- **Space of transformations is complete:** $\boxed{\mathfrak{T}_\mathbb{R} = \mathsf{Lin}(\mathfrak{E}_\mathbb{R})}$

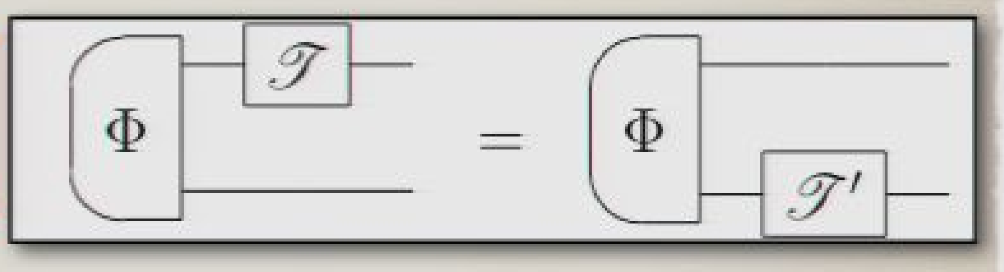
- **Atomicity of identity transformation** (classical mechanics is excluded!)

Consequences of PFAITH

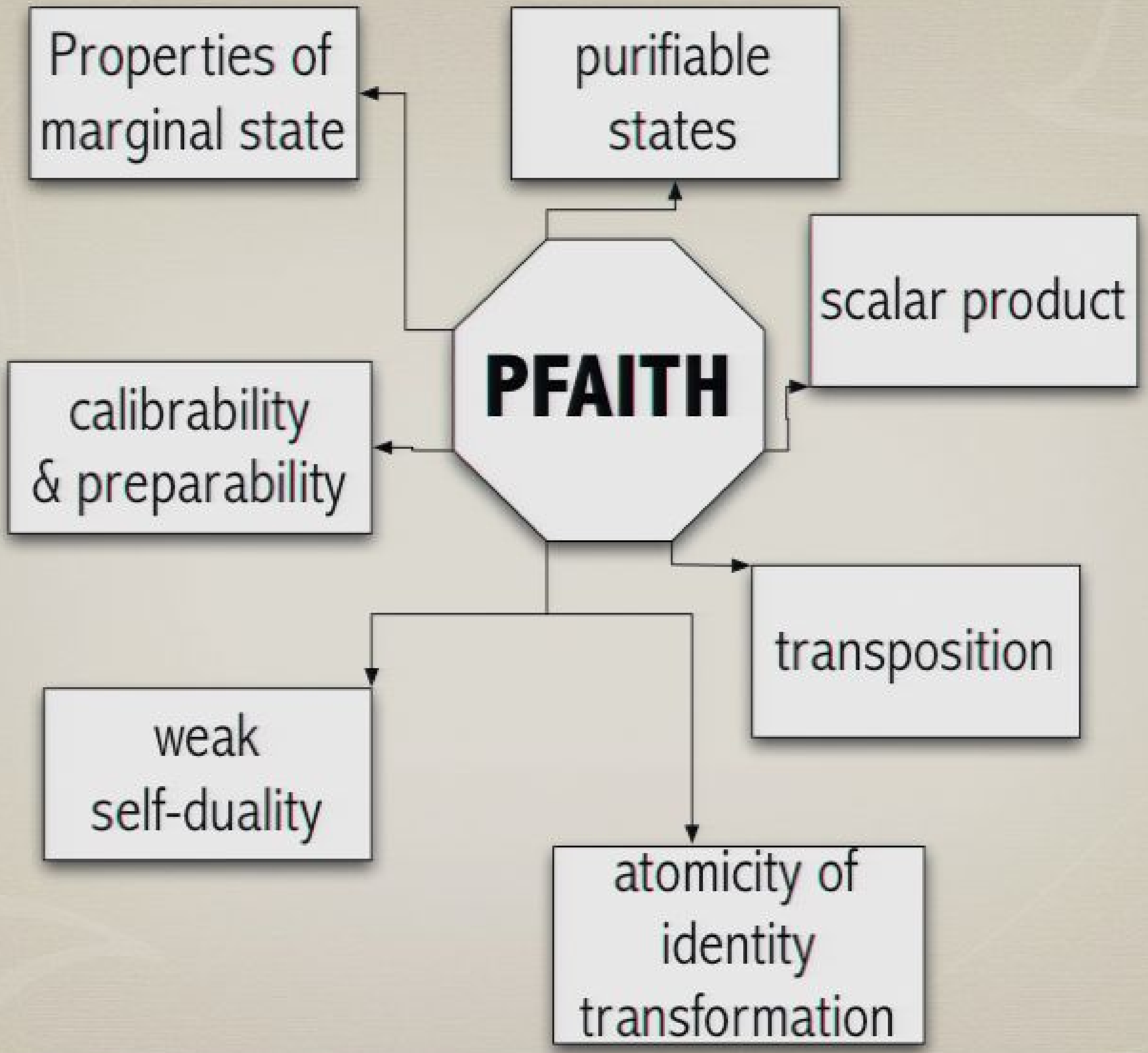
- The faithful state Φ provides a non-degenerate scalar product over effects via its Jordan form (ς Jordan involution):

$$\forall a, b \in \mathfrak{E}_{\mathbb{R}}, \quad \boxed{{}_{\Phi}(b|a)_{\Phi} := |\Phi|(b,a) = \Phi(\varsigma(b),a)}$$

$$\boxed{\mathscr{T}_1 \, |\Phi) = \mathscr{T}'_2 \, |\Phi)}$$



- Marginal state $\boxed{|\chi) = (e|_1 \, |\Phi)_{12}}$ internal and invariant under a “transposed” deterministic test



PFAITH

Properties of marginal state

purifiable states

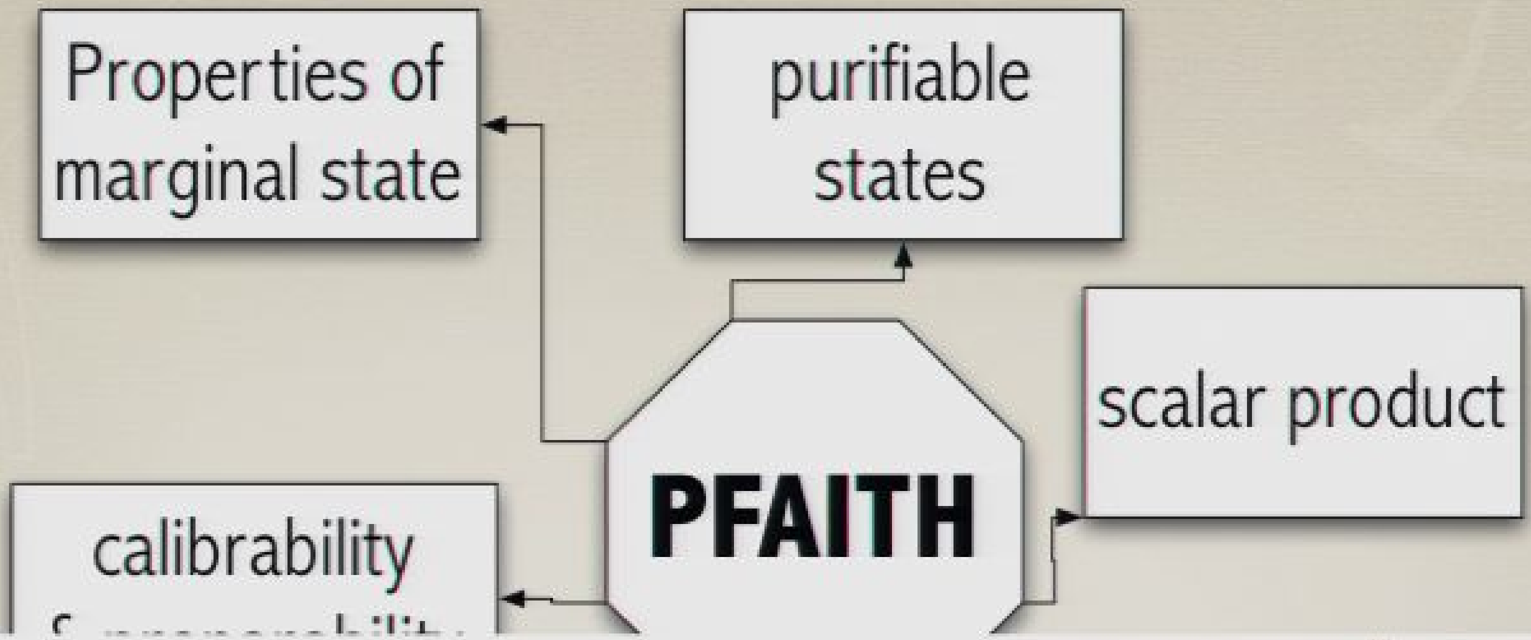
scalar product

calibrability & preparability

transposition

weak self-duality

atomicity of identity transformation



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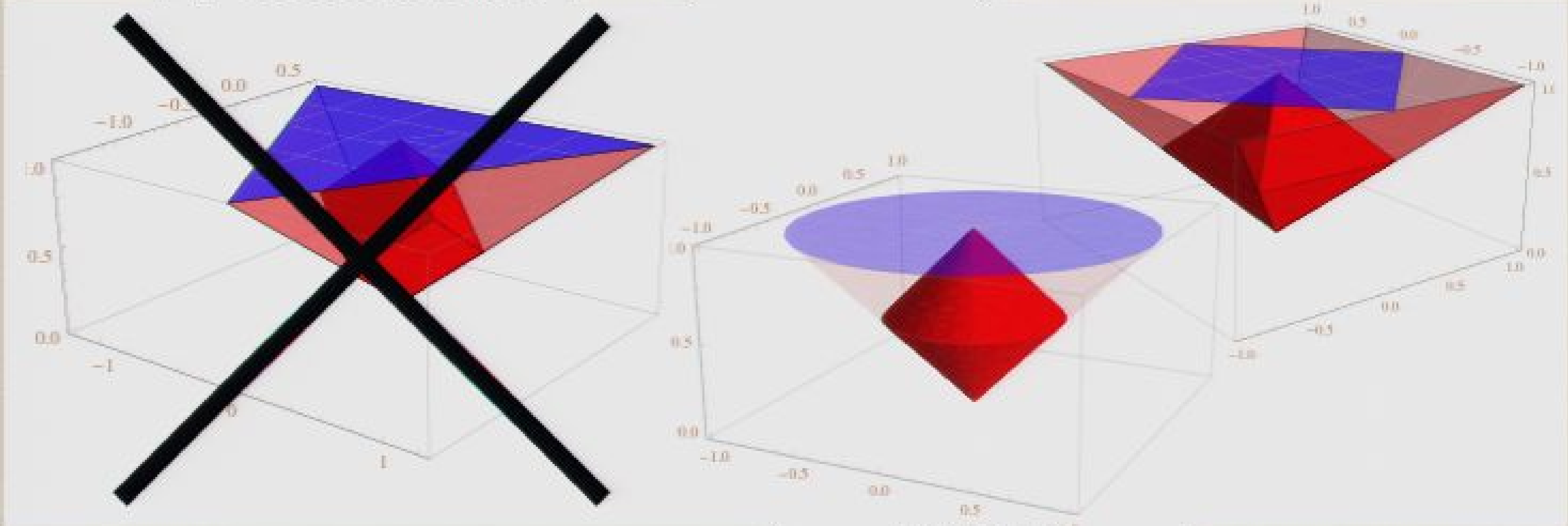
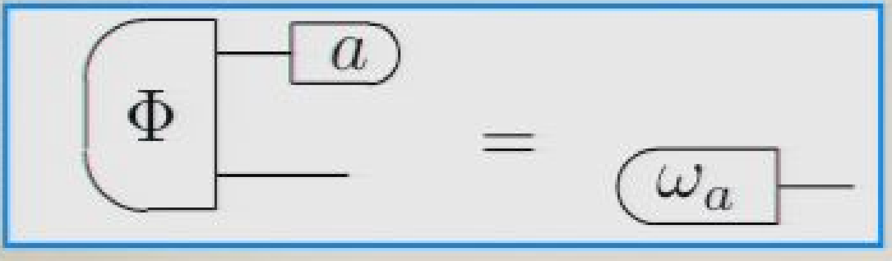


identity transformation

Postulate: FAITHE

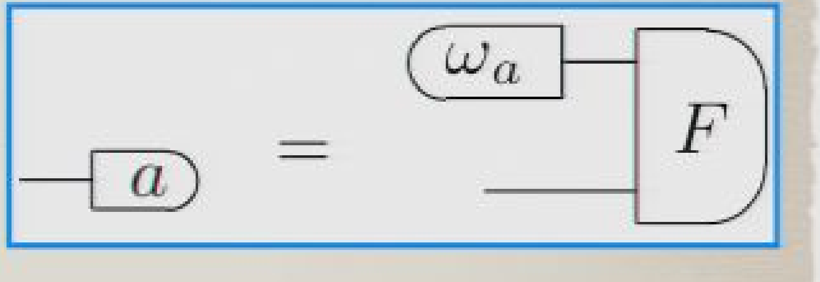


Cone-isomorphism from the faithful state Φ

Postulate FAITHE: (faithful effect)

$$F := \alpha \left(\Phi^{-1}\right| \in \mathfrak{E}(\mathsf{SS}),\ 0 < \alpha \leqslant 1$$

proportional to a joint effect.



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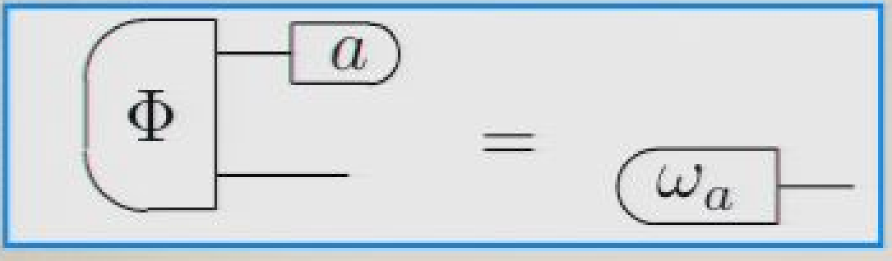

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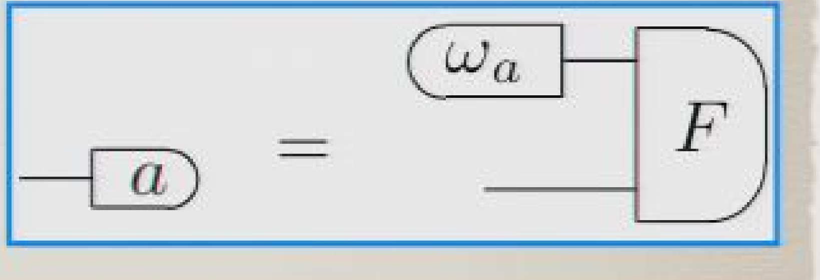
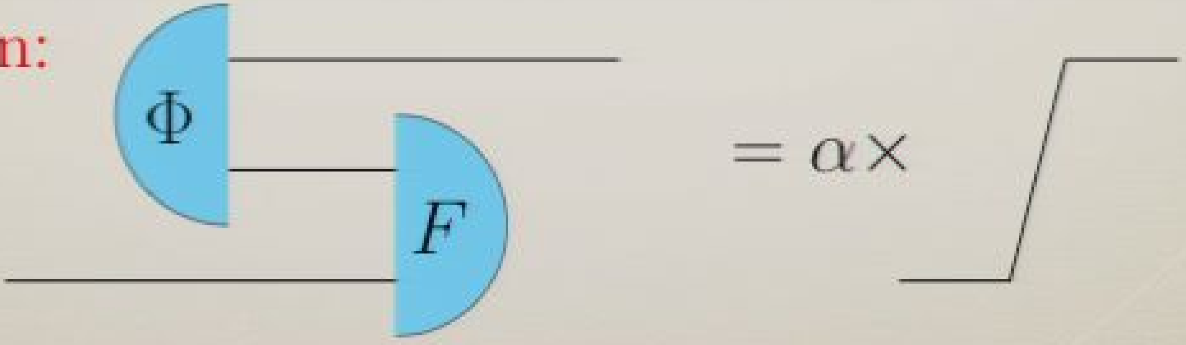


- Teleportation:



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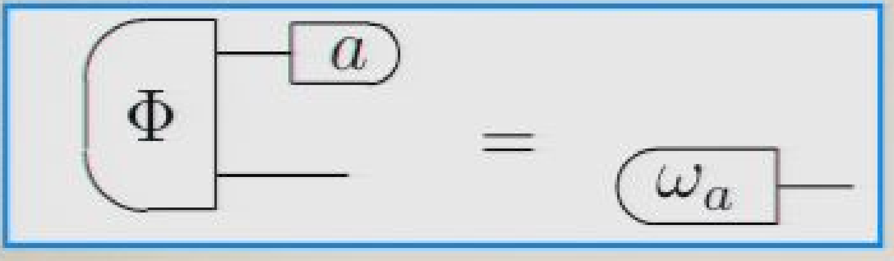
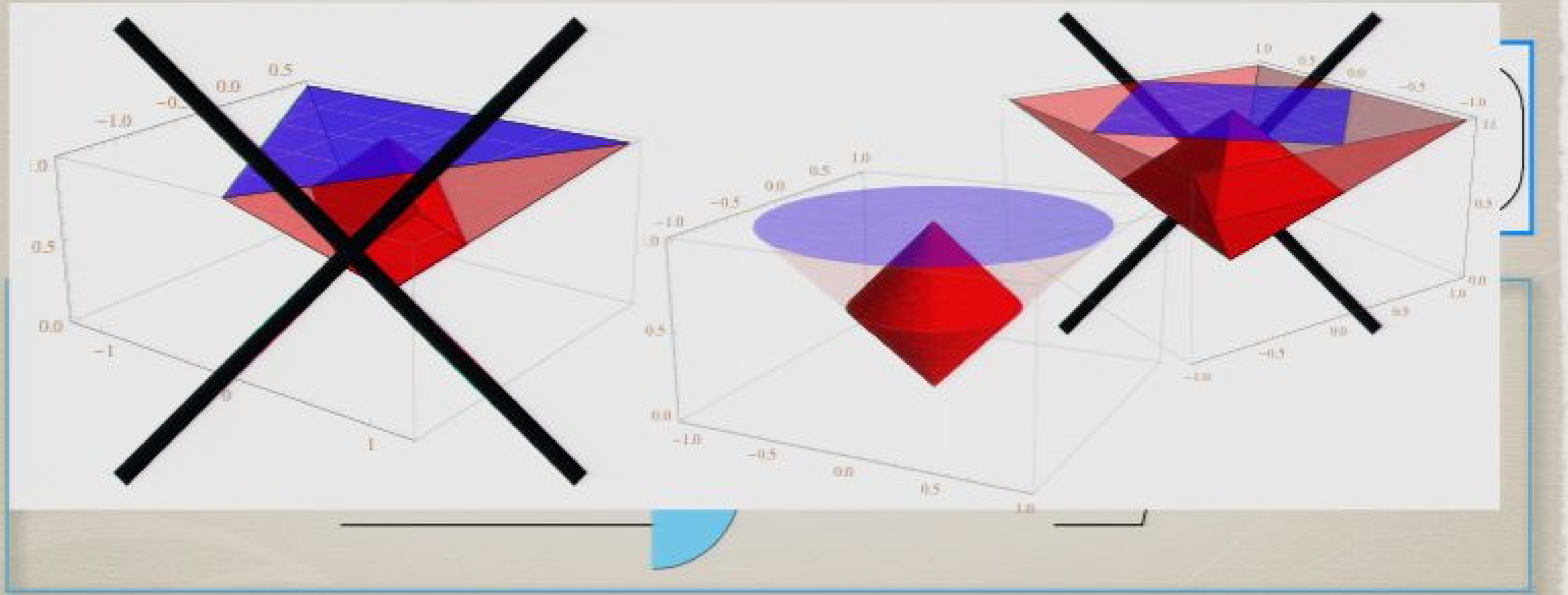
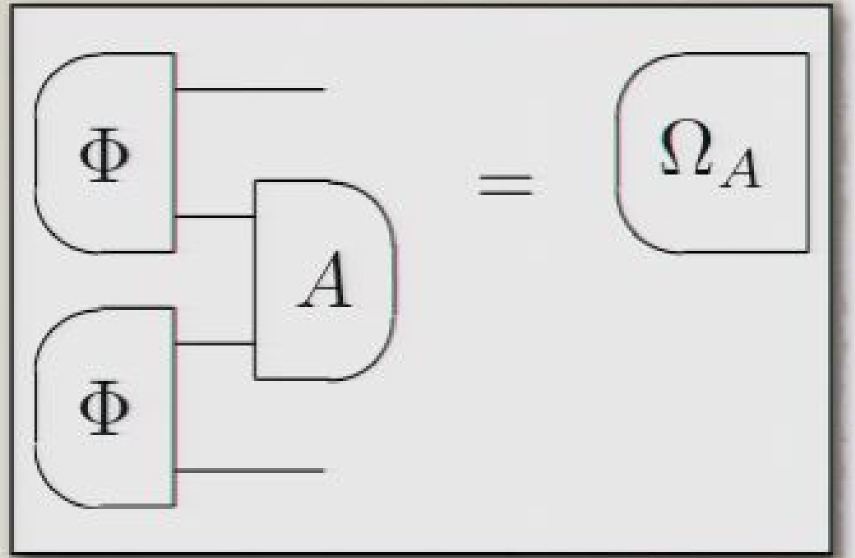


Cone-isomorphism from the faithful state Φ



SUPERFAITH

- $\mathfrak{E}_+(\mathsf{SS}) \ni (A| \mapsto |\Omega_A) := (A|_{23}\,|\Phi)_{12}\,|\Phi)_{34} \in \mathfrak{S}_+(\mathsf{SS})$



is a bijective map between
$\mathfrak{S}_{\mathbb{R}}(\mathsf{SS})$ and $\mathfrak{E}_{\mathbb{R}}(\mathsf{SS})$

SUPERFAITH

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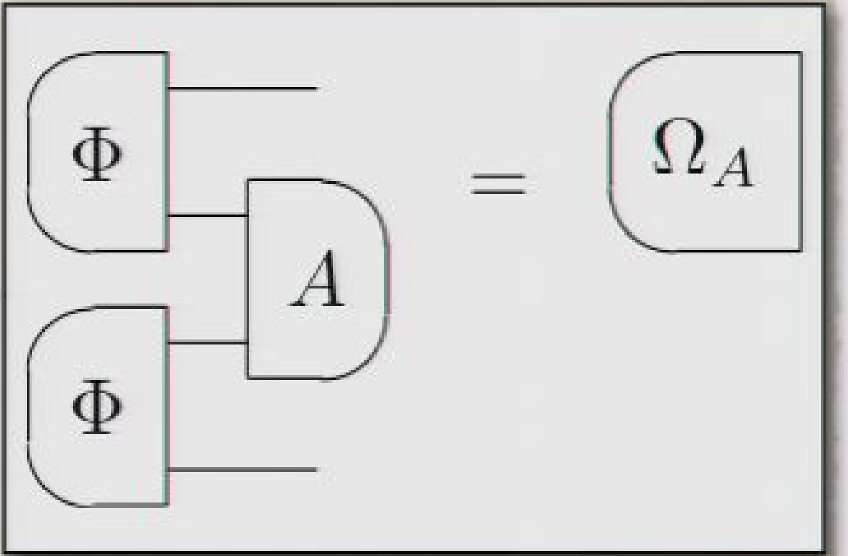
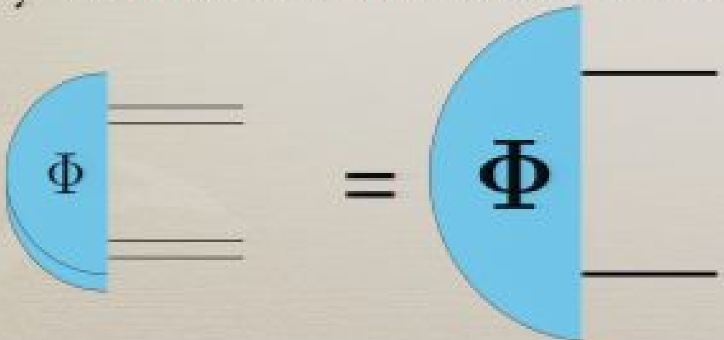


is a bijective map between
$\mathfrak{S}_{\mathbb{R}}(\mathsf{SS})$ and $\mathfrak{E}_{\mathbb{R}}(\mathsf{SS})$

However, it does not necessarily realize the cone-isomorphism:

$$\mathfrak{E}_+(\mathsf{SS}) \simeq \mathfrak{S}_+(\mathsf{SS})$$

namely we don't have SUPERFAITH



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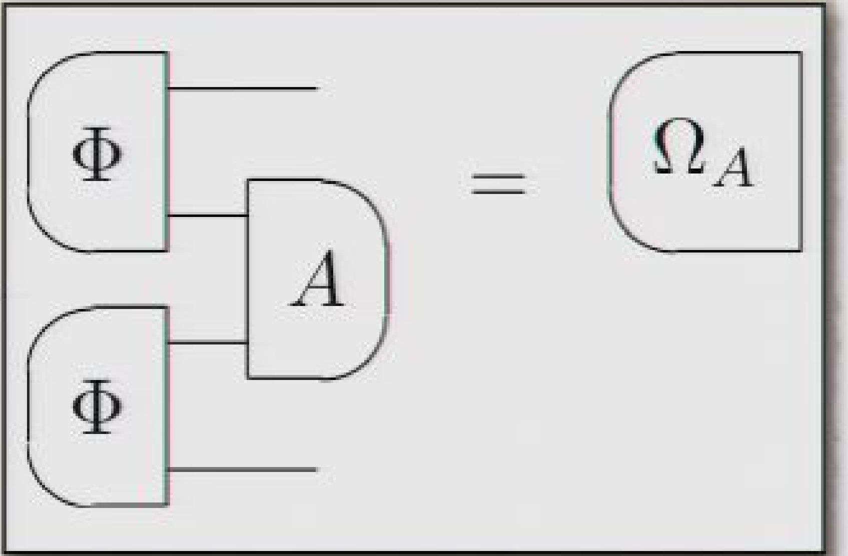

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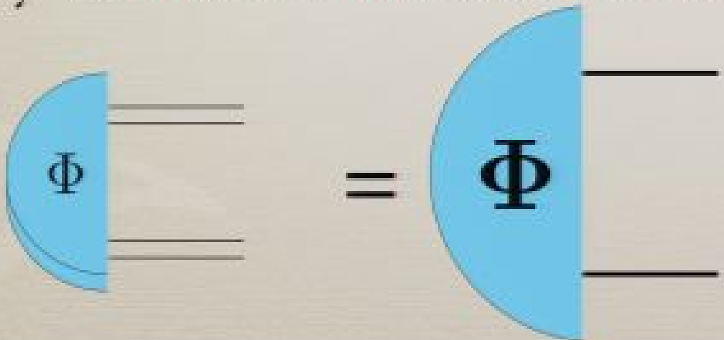


Viceversa SUPERFAITH → PFAITH+FAITHE

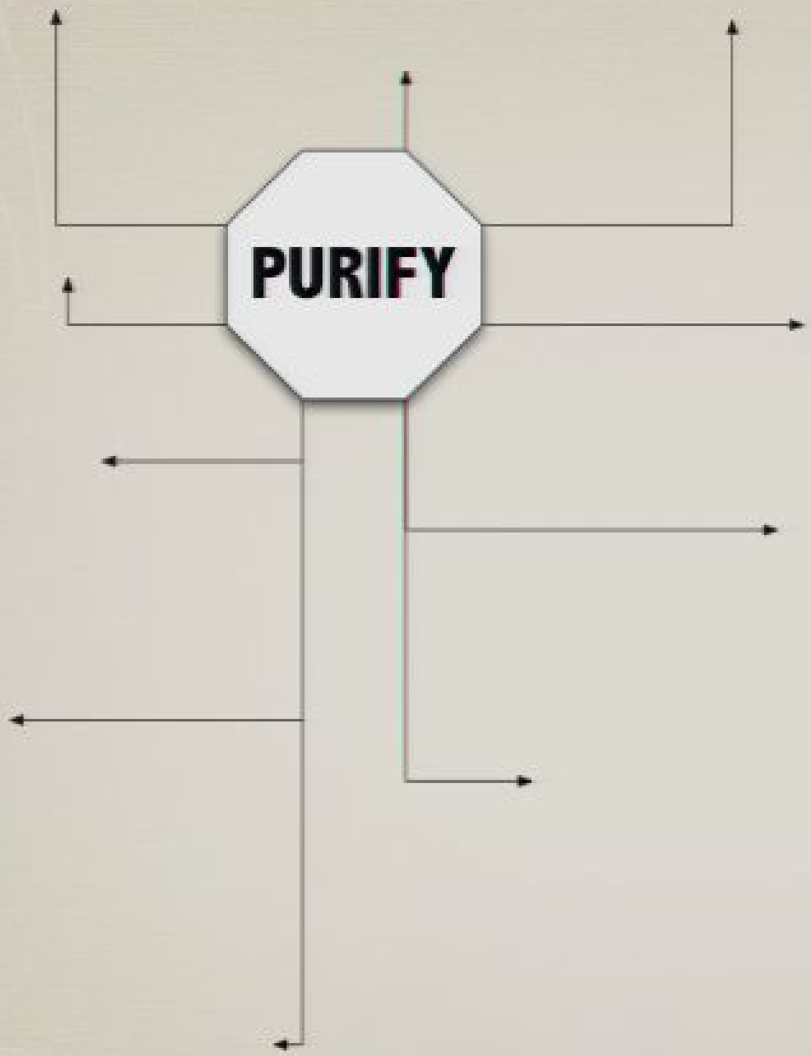
Postulate: Purification

Postulate PURIFY: Every state has a purification. The purification is unique modulo local reversible transformations of the purifying system.

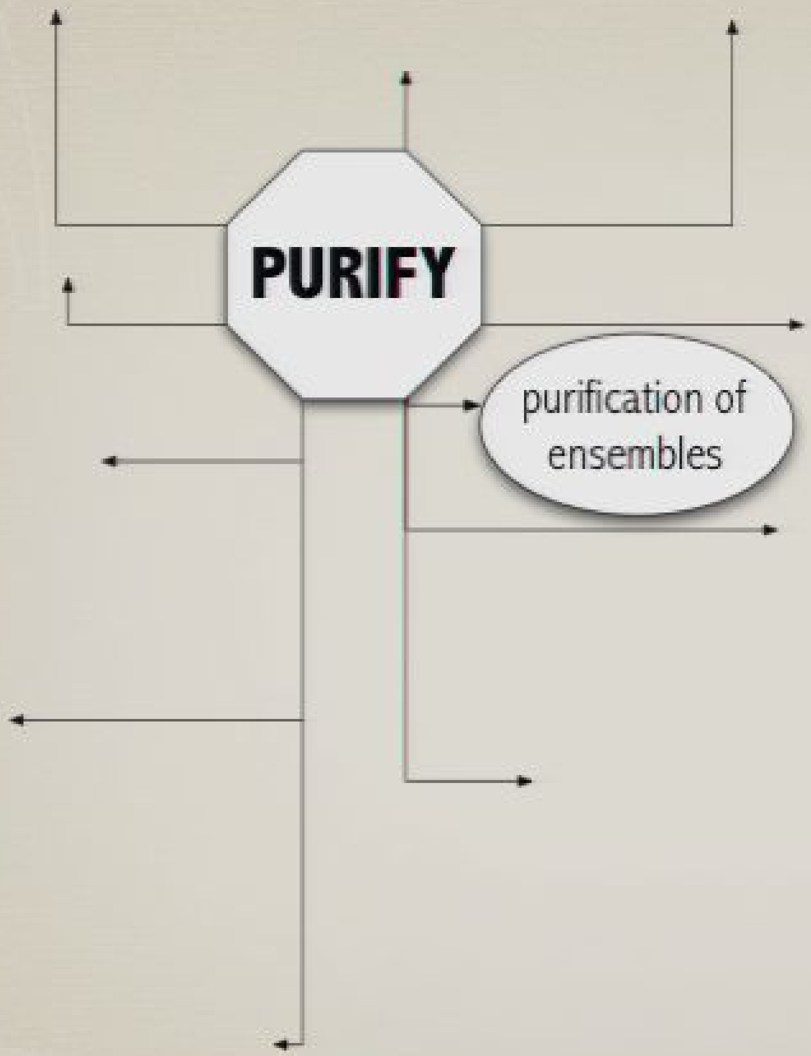
Postulate STRONG PURIFY:

There exists a system $\tilde{\mathrm{A}}$ that purifies all states of A s.t.

1. $\tilde{\tilde{\mathrm{A}}} = \mathrm{A}$
2. $\widetilde{\mathrm{AB}} = \tilde{\mathrm{A}}\tilde{\mathrm{B}}$

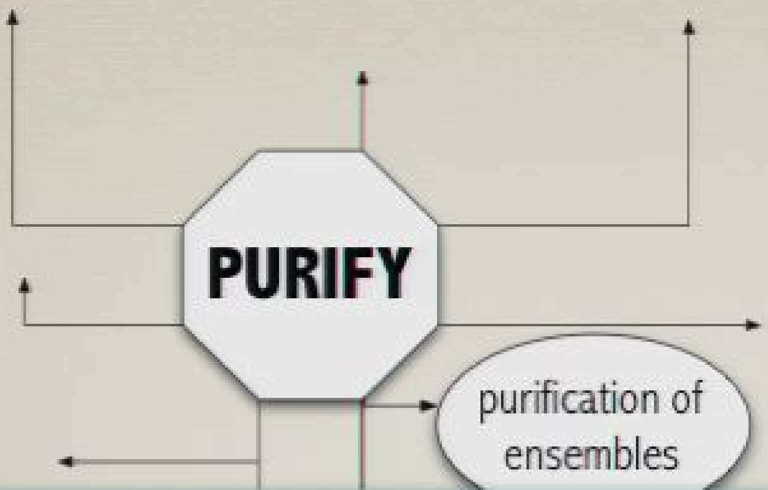


PURIFY



PURIFY

purification of ensembles



PURIFY

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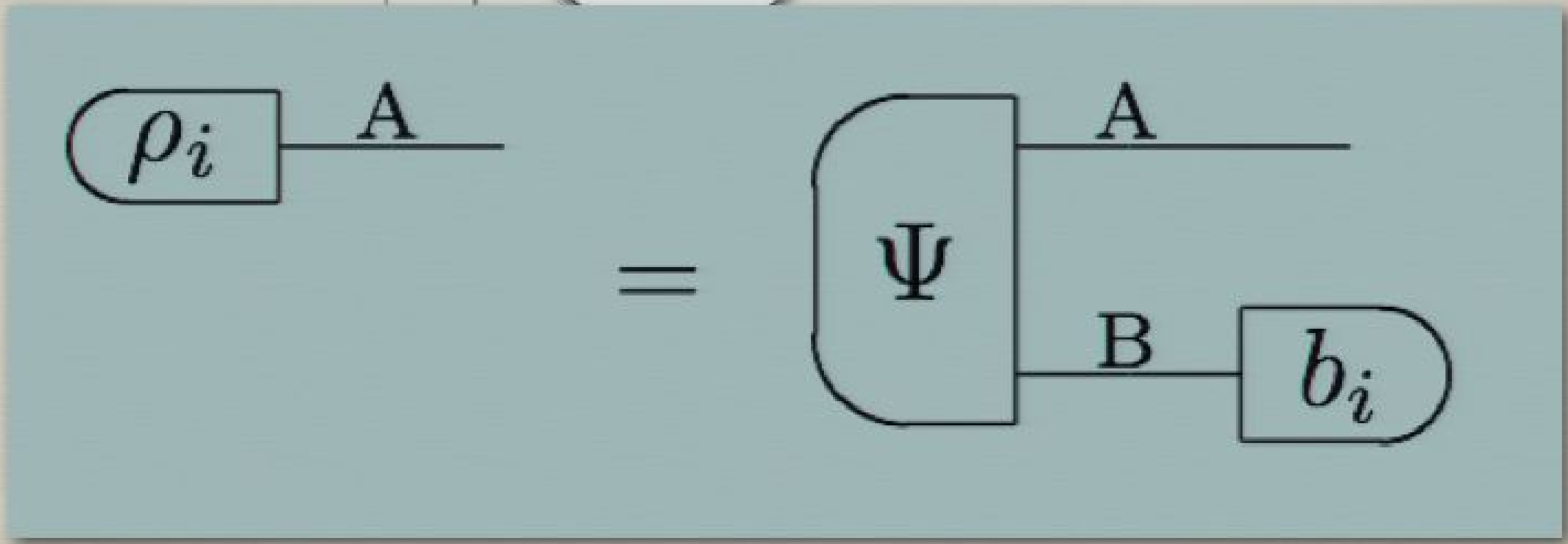

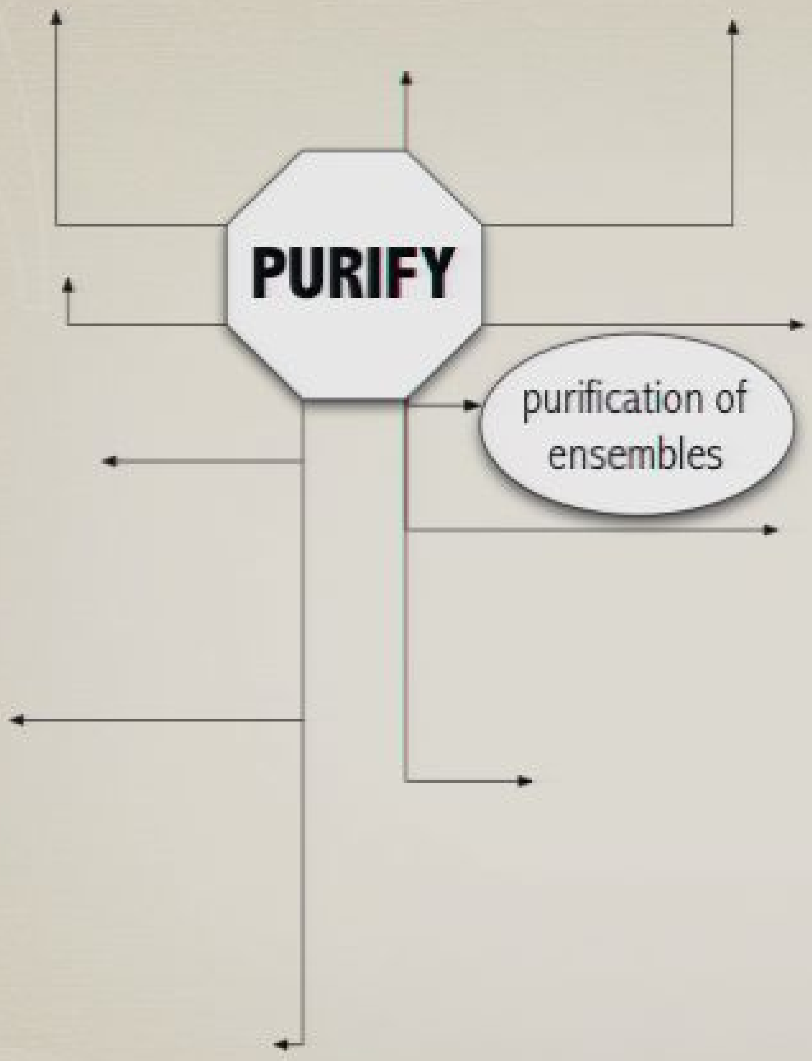

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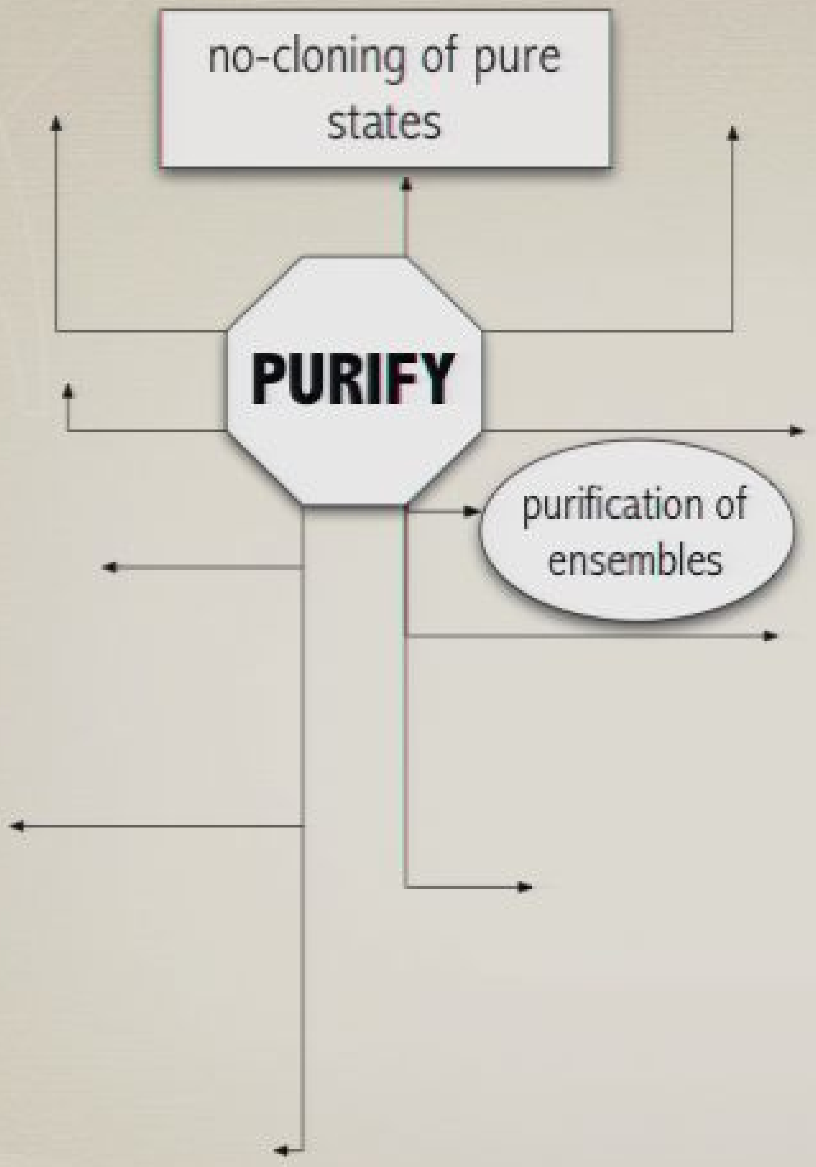
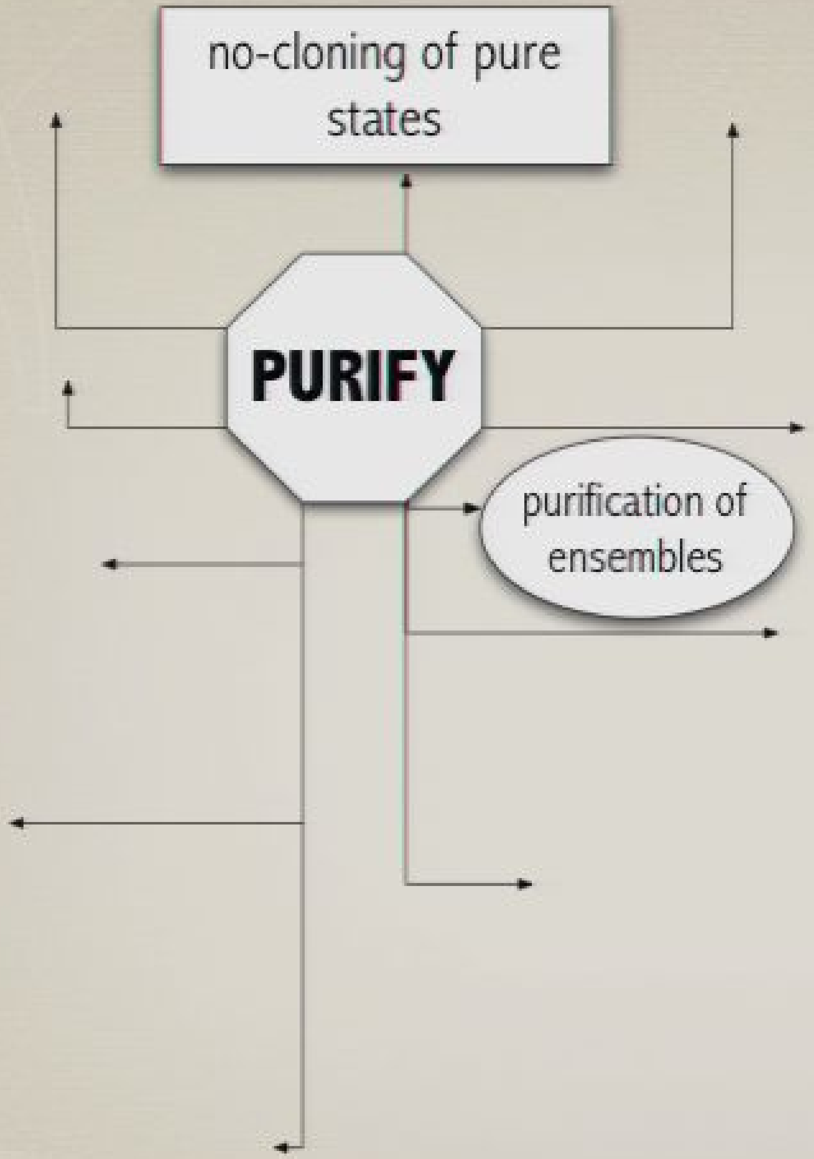


no-cloning of pure states

PURIFY

purification of ensembles



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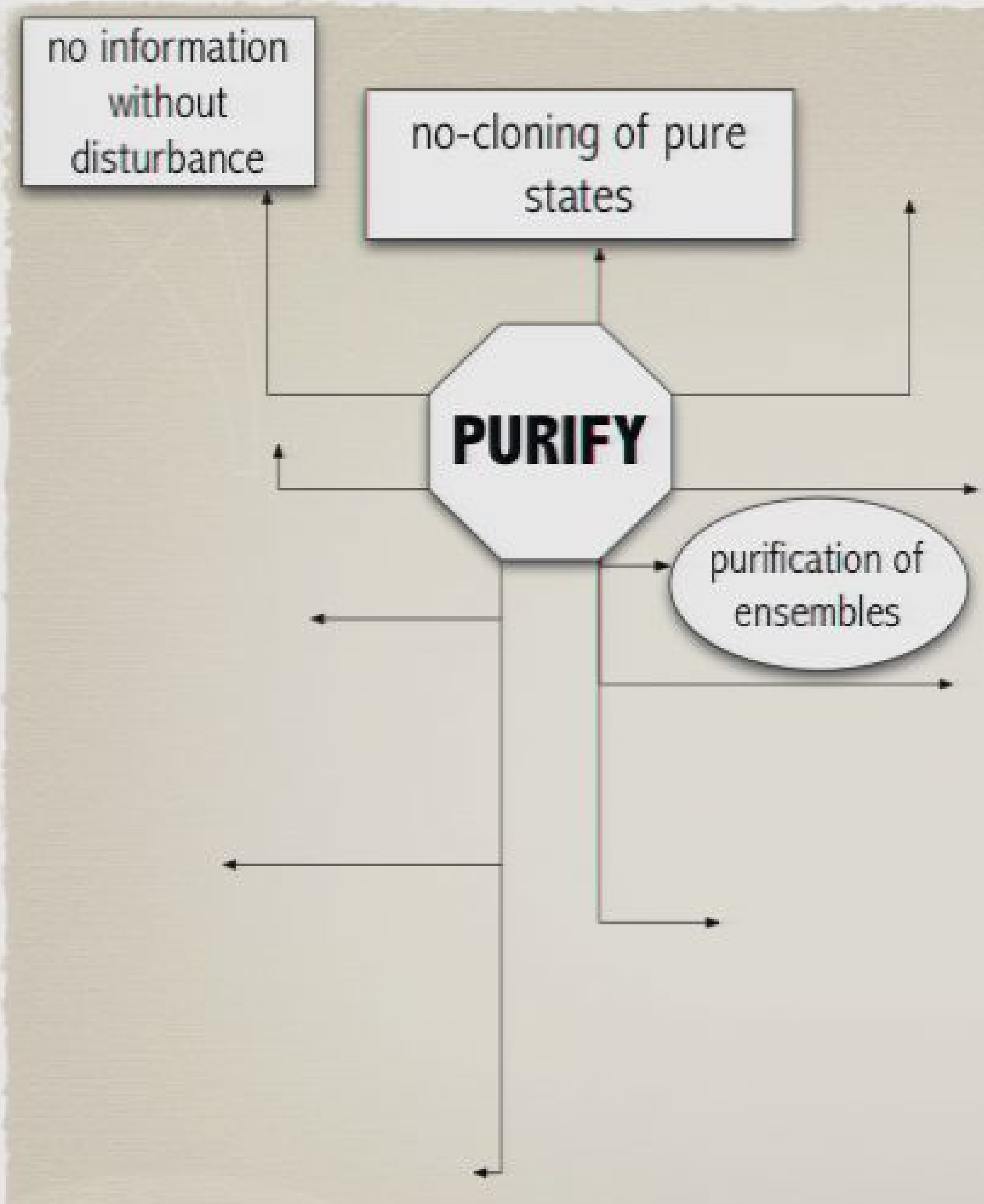
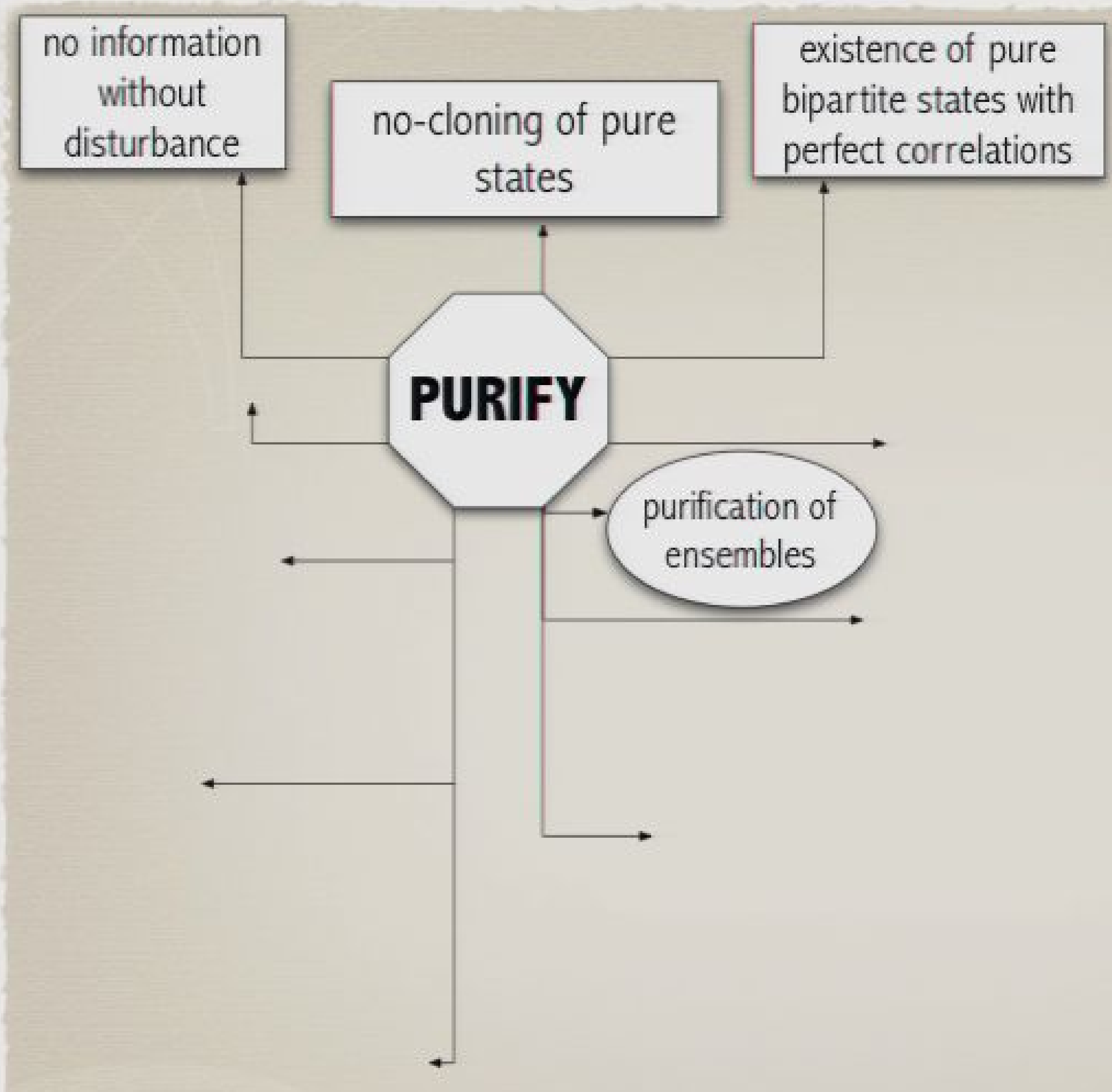


no information without disturbance

no-cloning of pure states

PURIFY

purification of ensembles



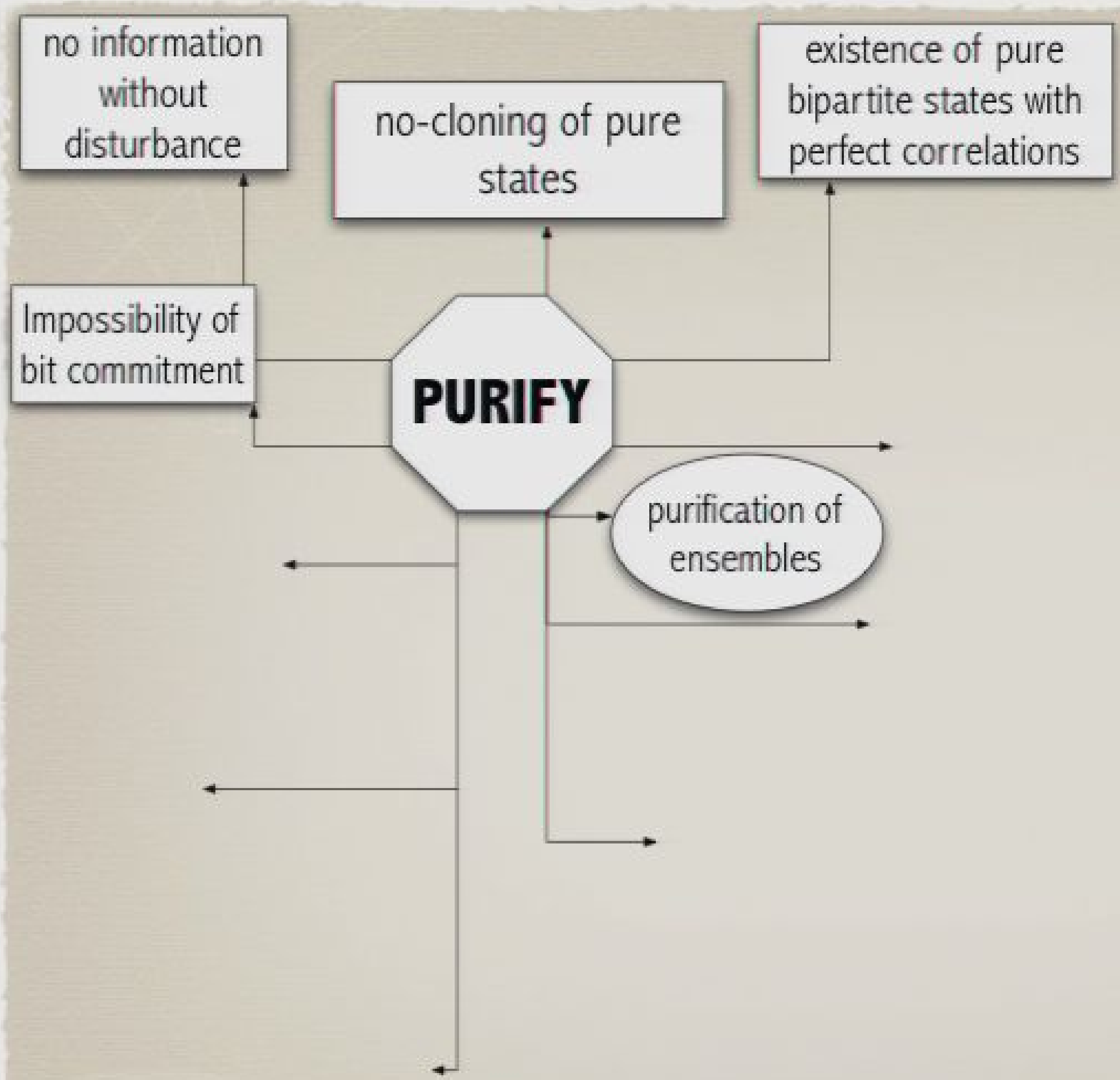
no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

PURIFY

purification of ensembles



no information without disturbance

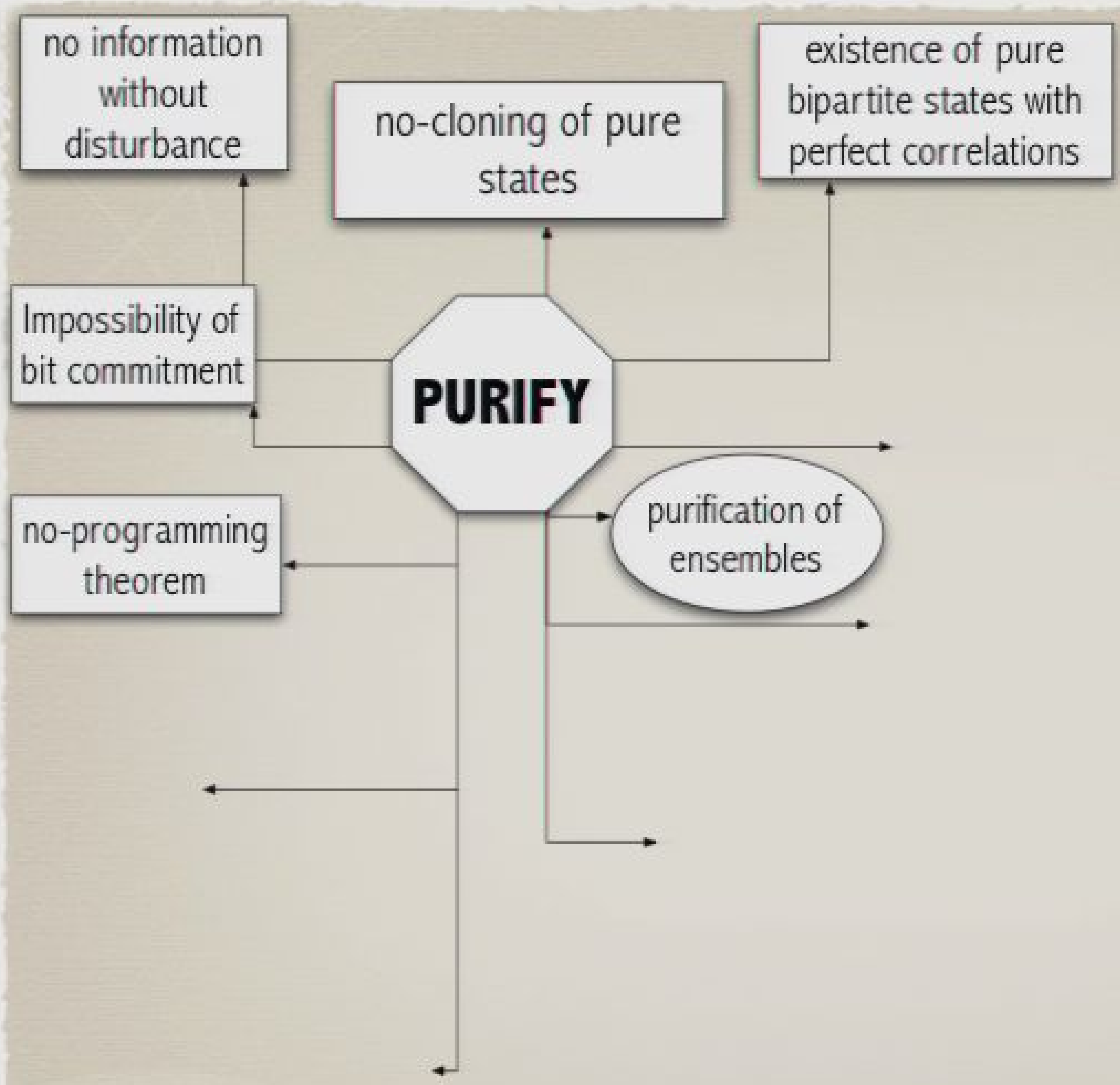
no-cloning of pure states

existence of pure bipartite states with perfect correlations

Impossibility of bit commitment

PURIFY

purification of ensembles



no information without disturbance

no-cloning of pure states

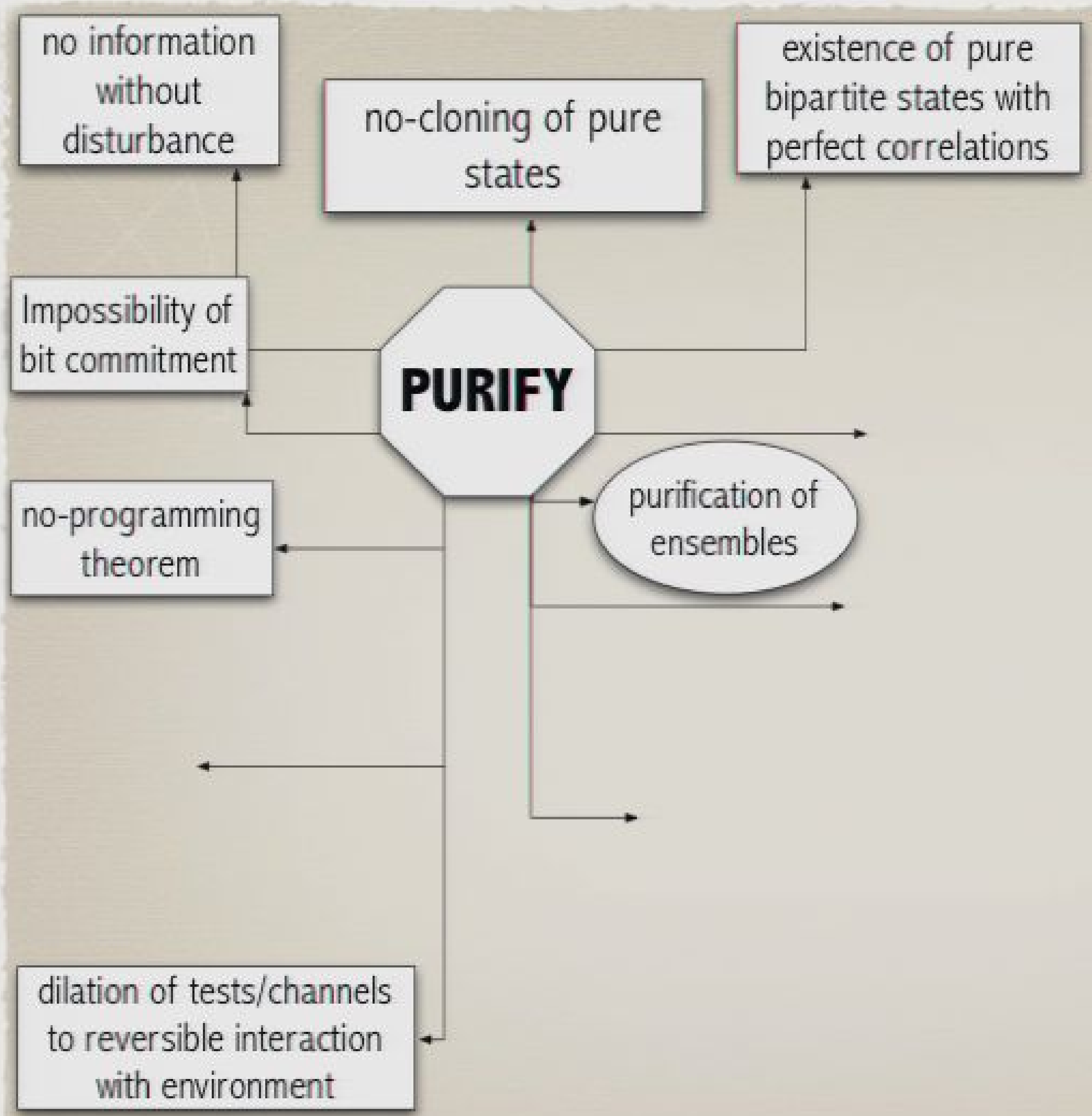
existence of pure bipartite states with perfect correlations

Impossibility of bit commitment

PURIFY

purification of ensembles

no-programming theorem



no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

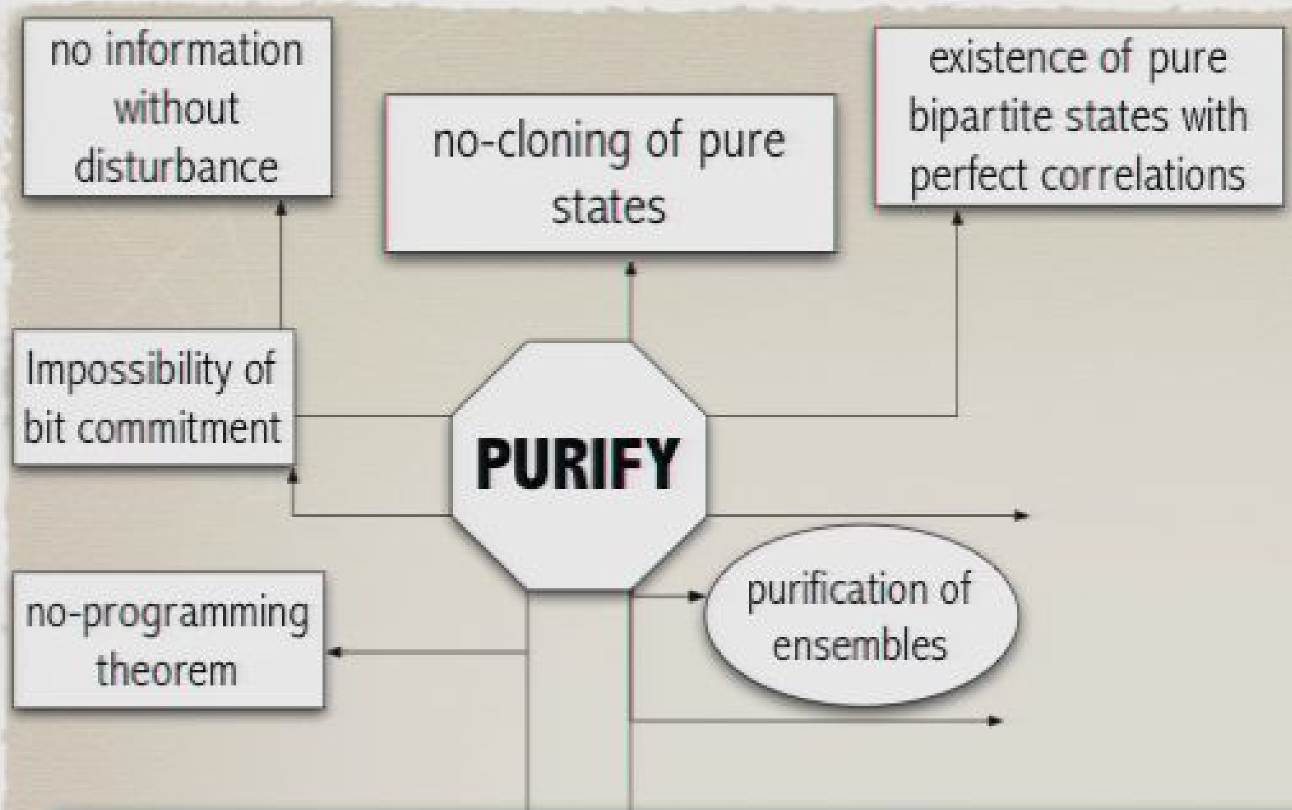
Impossibility of bit commitment

PURIFY

purification of ensembles

no-programming theorem

dilation of tests/channels to reversible interaction with environment



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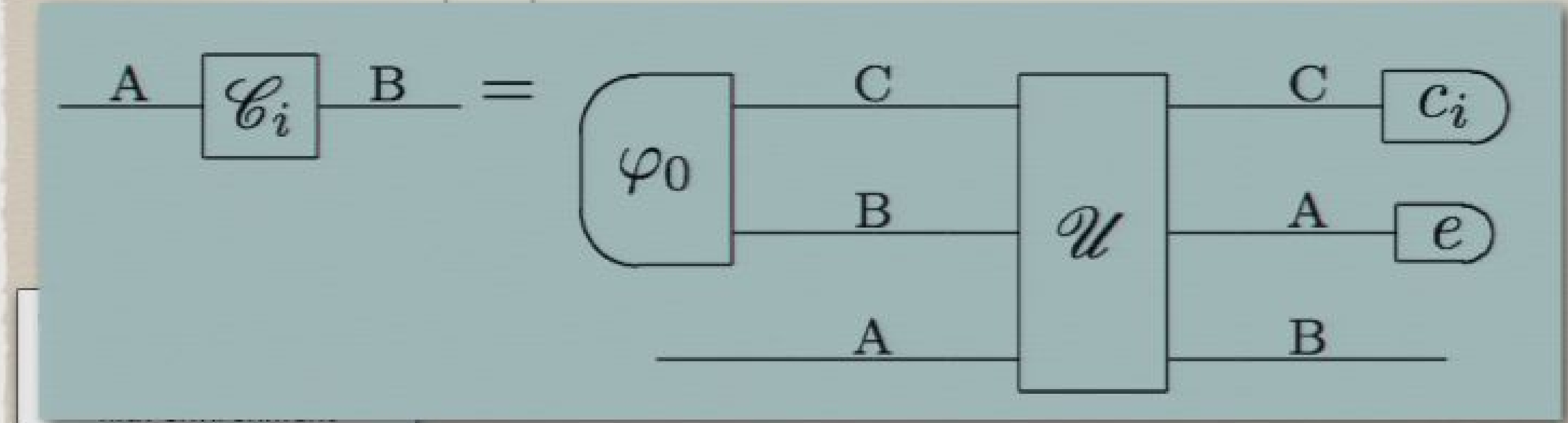

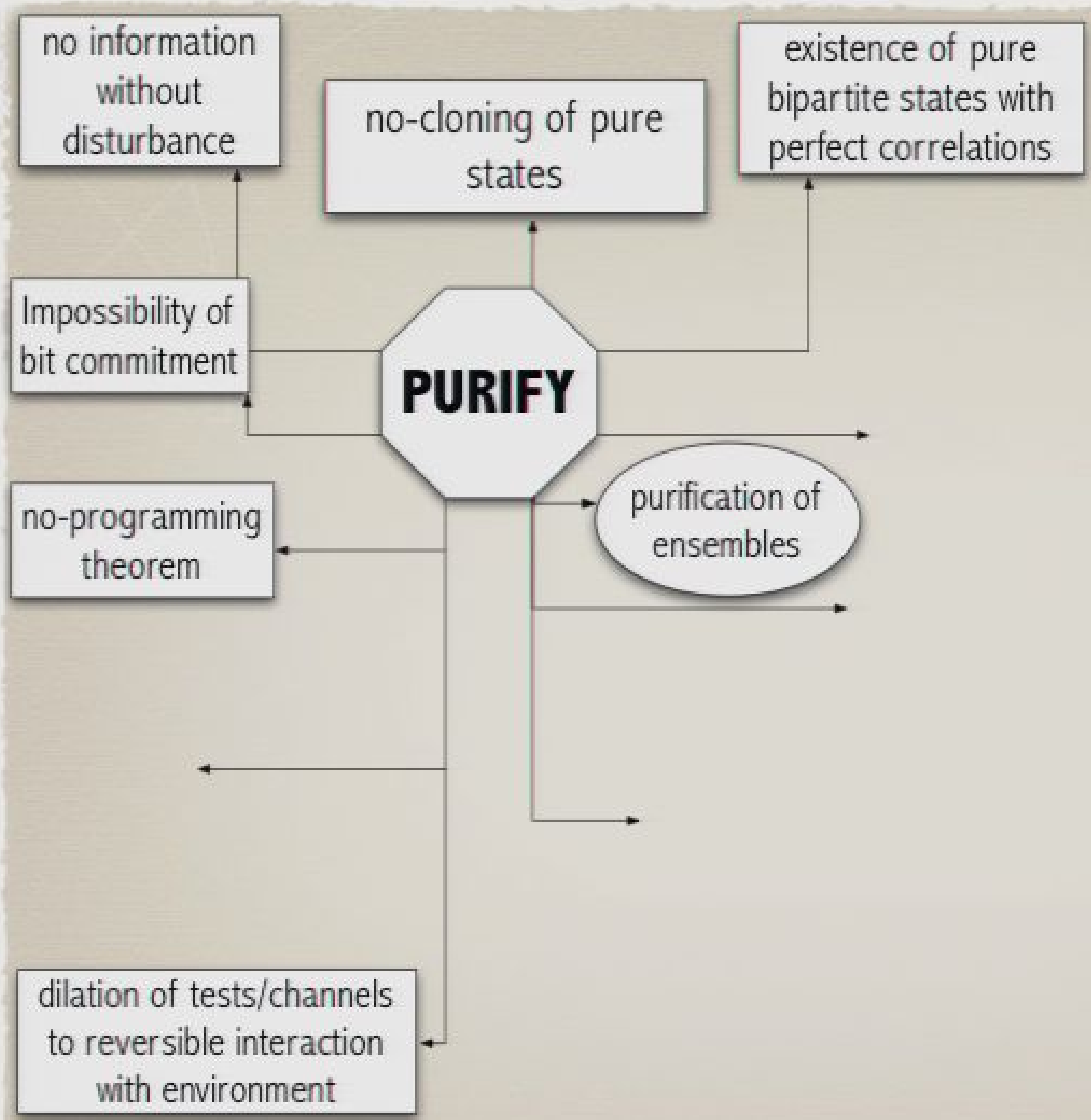

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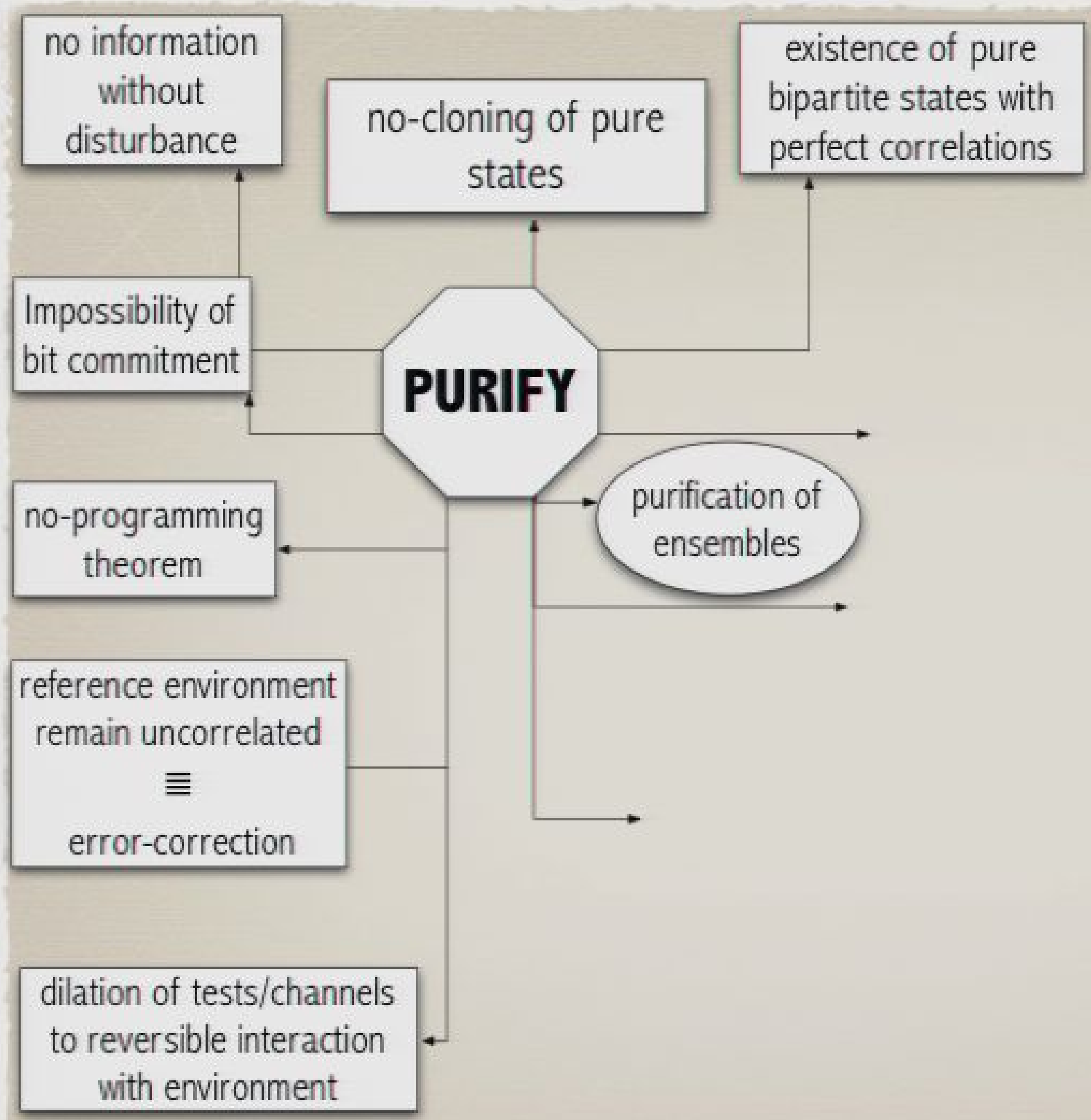


no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

Impossibility of bit commitment

PURIFY

purification of ensembles

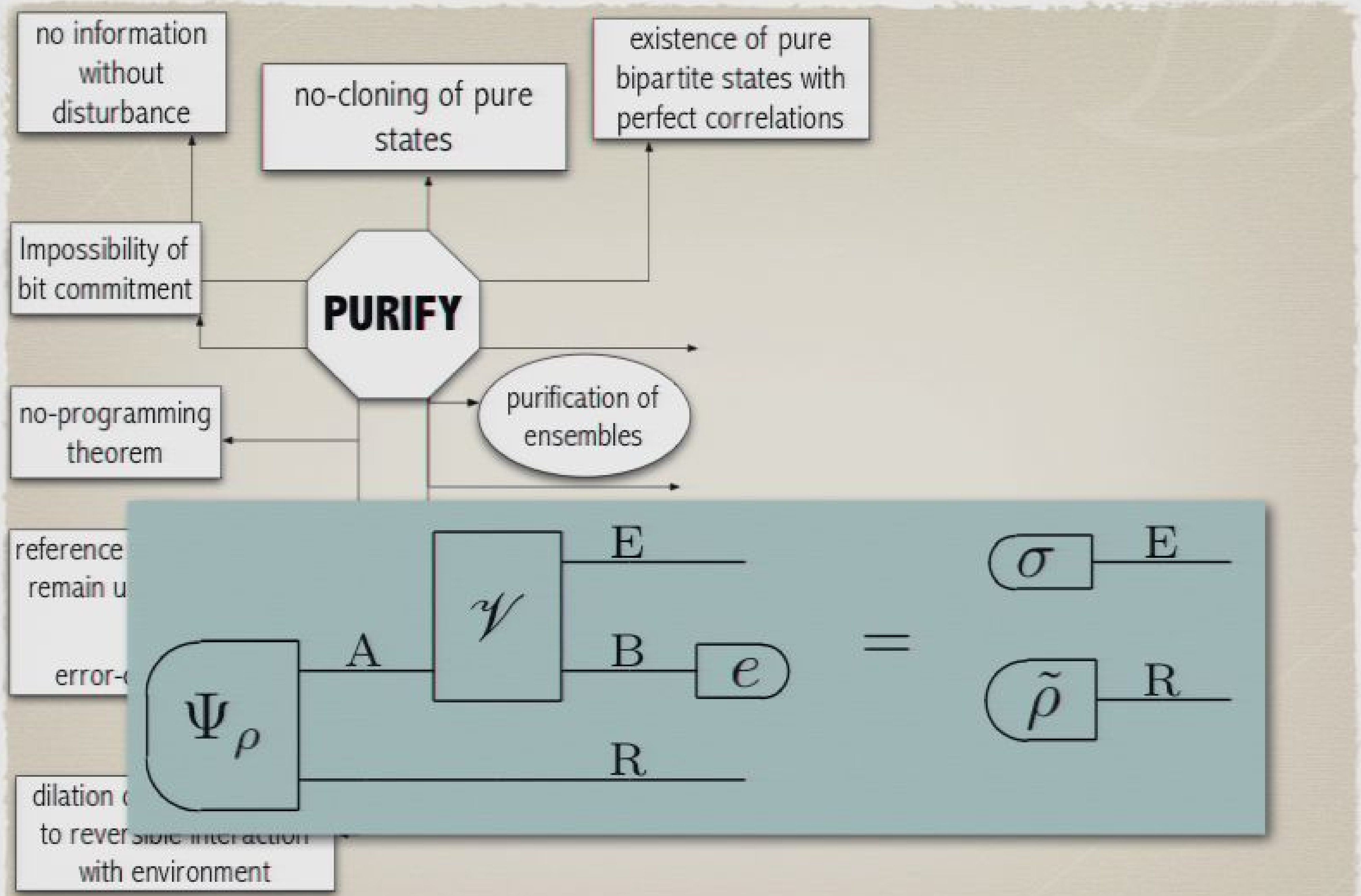
no-programming theorem

reference environment remain uncorrelated

≣

error-correction

dilation of tests/channels to reversible interaction with environment



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no-cloning of pure states

existence of pure bipartite states with perfect correlations

Impossibility of bit commitment

PURIFY

purification of ensembles

no-programming theorem

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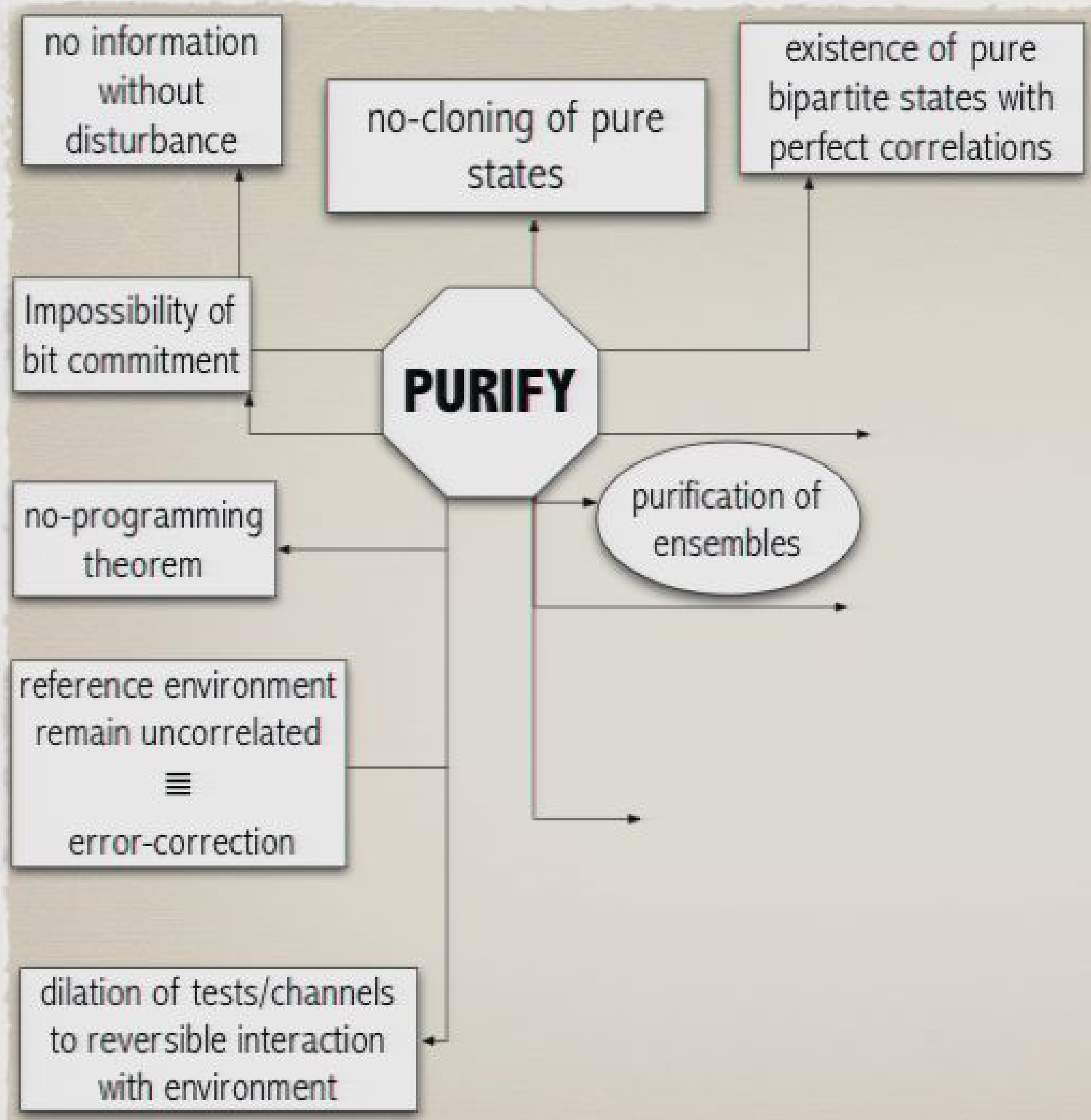
dilation c to reversible interaction with environment



no information without disturbance

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purification of ensembles

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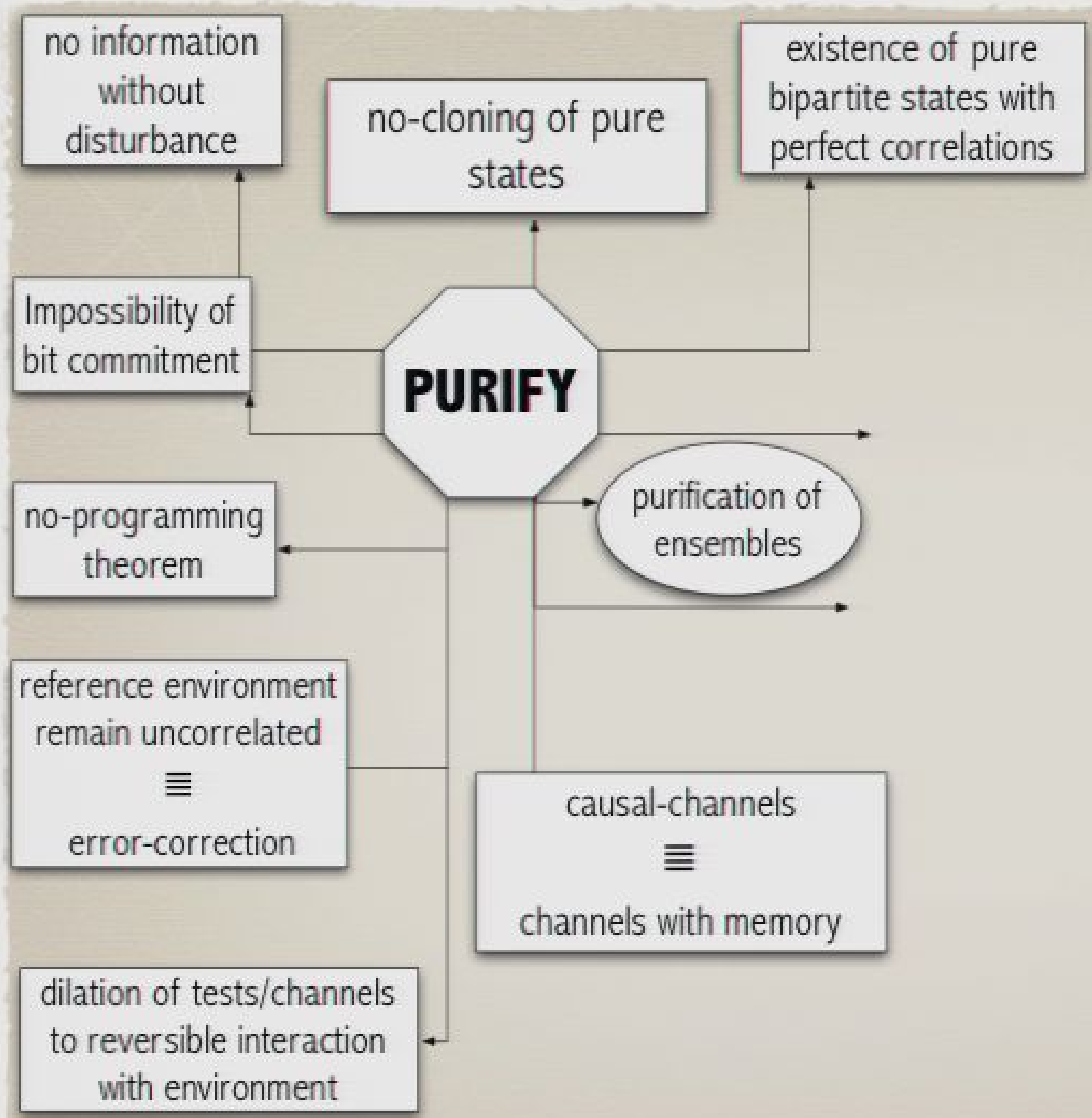

no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

Impossibility of bit commitment

PURIFY

purification of ensembles

no-programming theorem

reference environment remain uncorrelated

≣

error-correction

causal-channels

≣

channels with memory

dilation of tests/channels to reversible interaction with environment

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existence of pure
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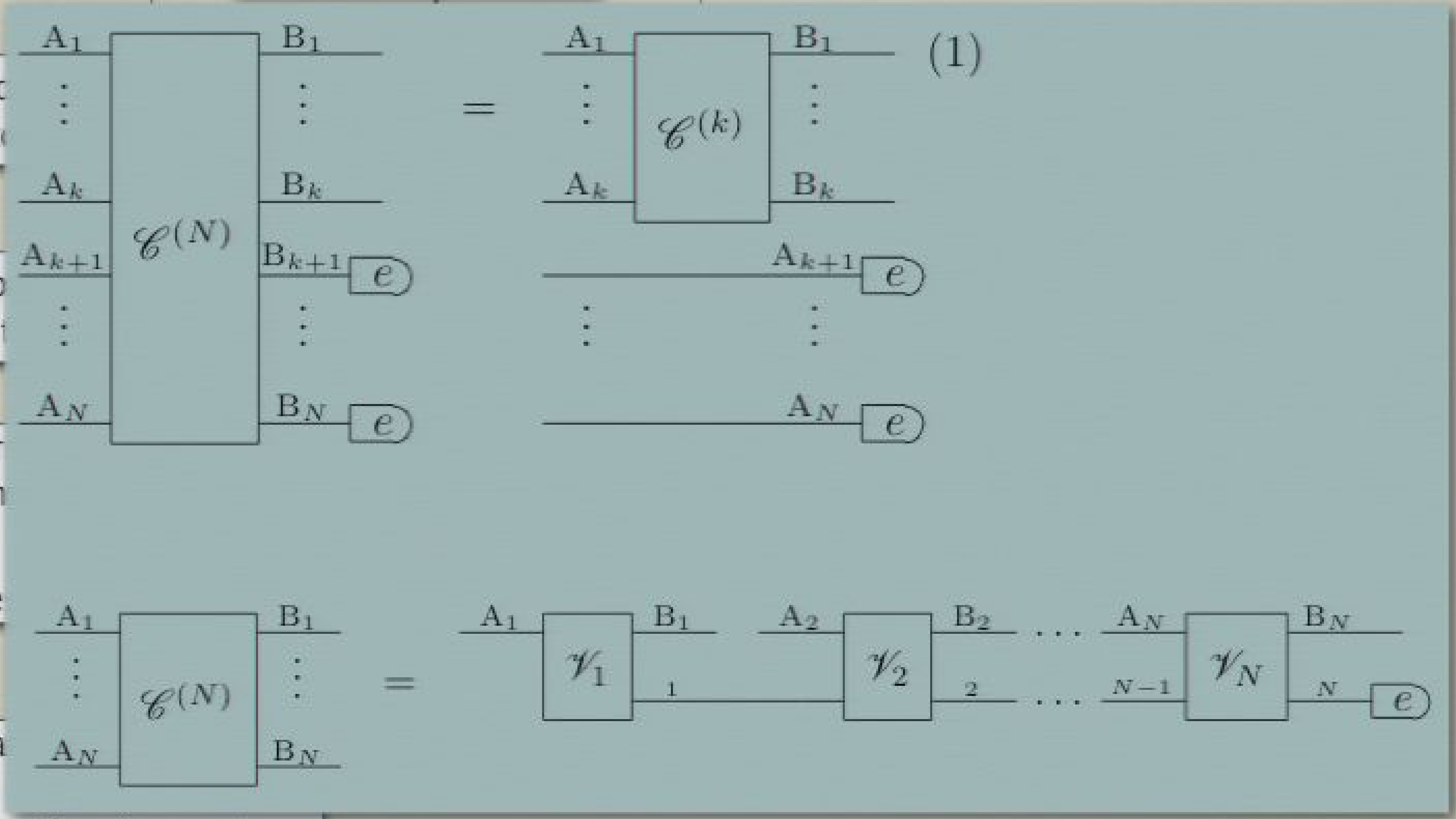

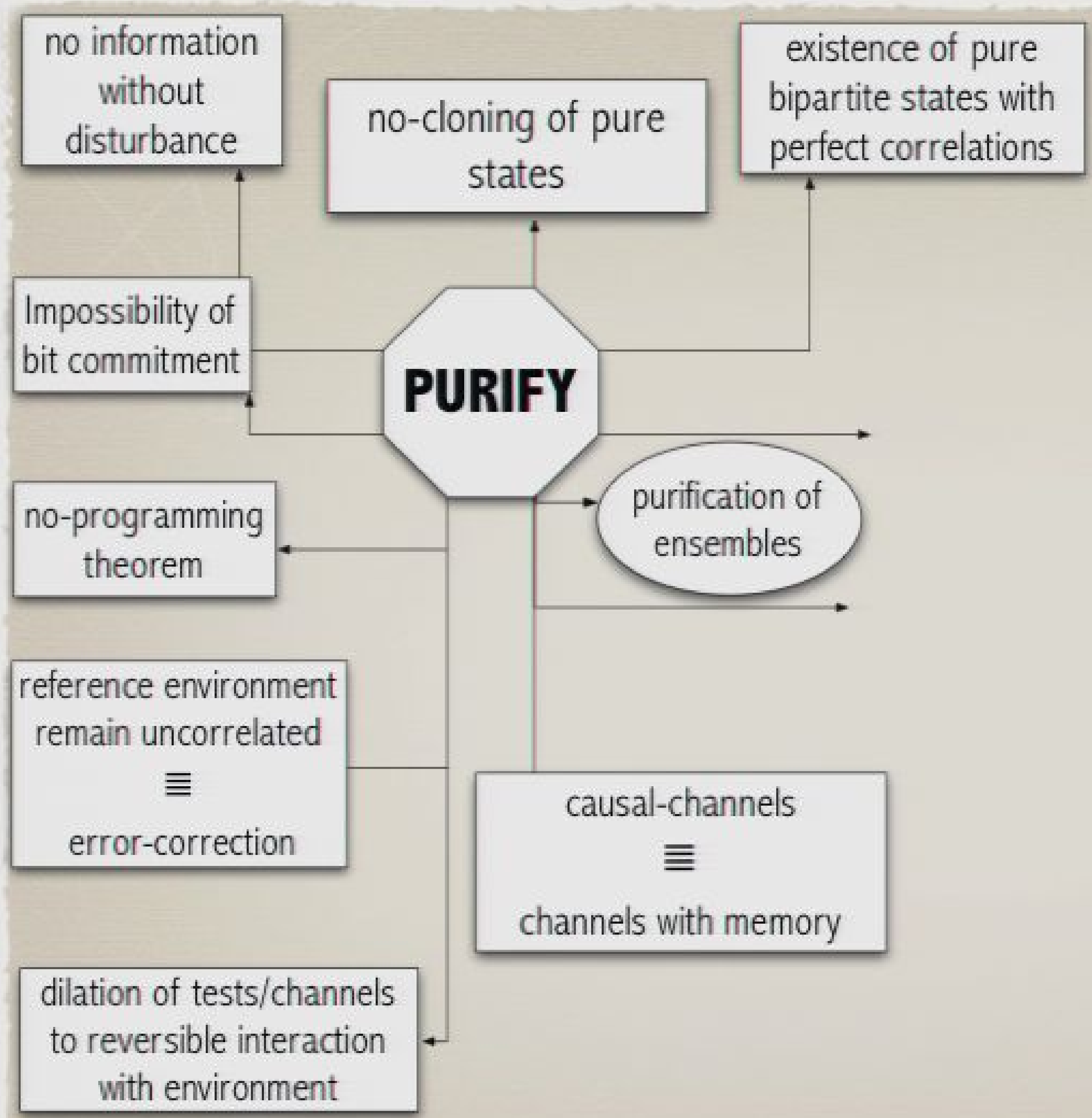

no information without disturbance

no-cloning of pure states

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Impossibility of bit commitment

PURIFY

purification of ensembles

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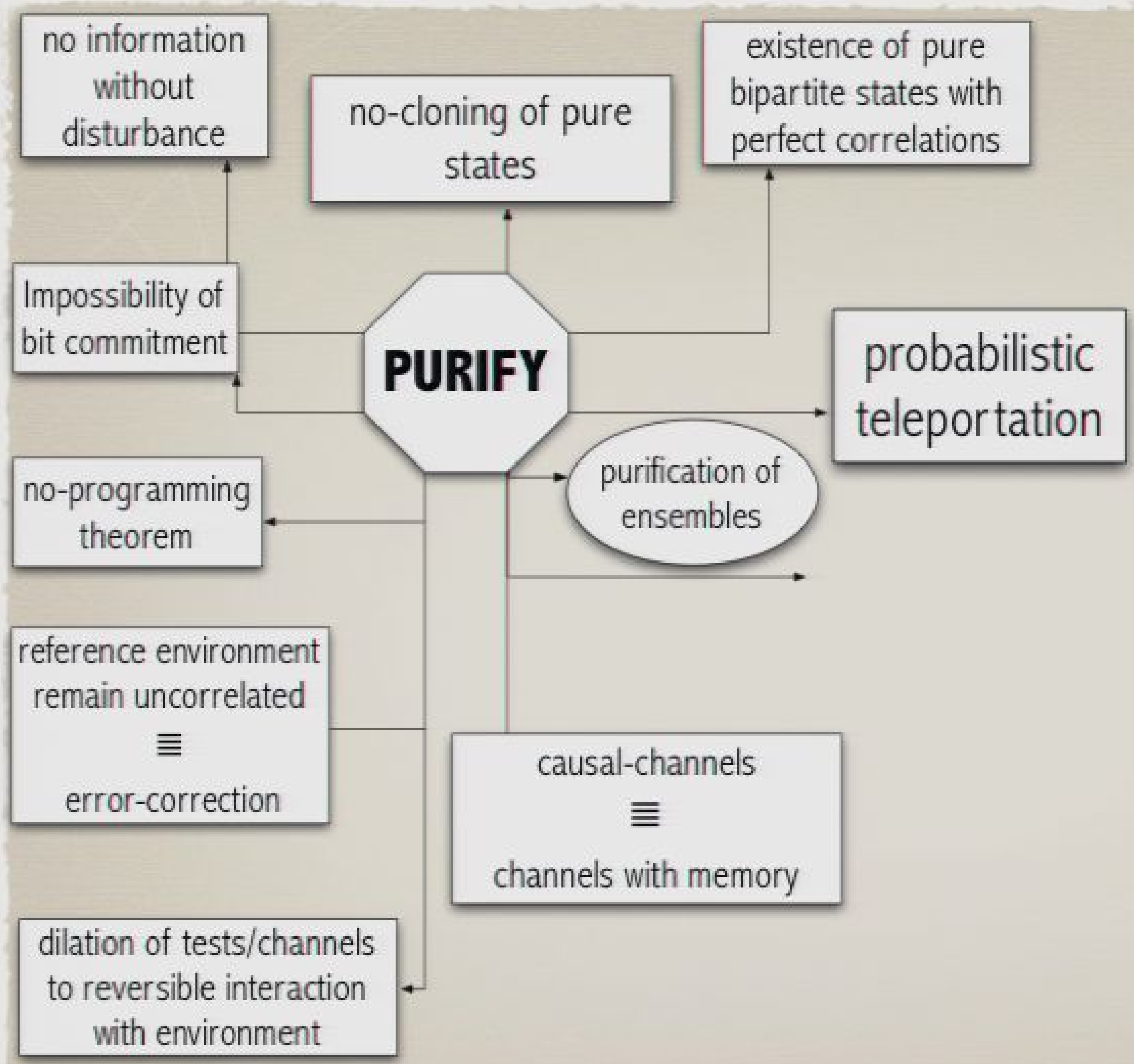
dilation of tests/channels to reversible interaction with environment



PURIFY

no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

Impossibility of bit commitment

probabilistic teleportation

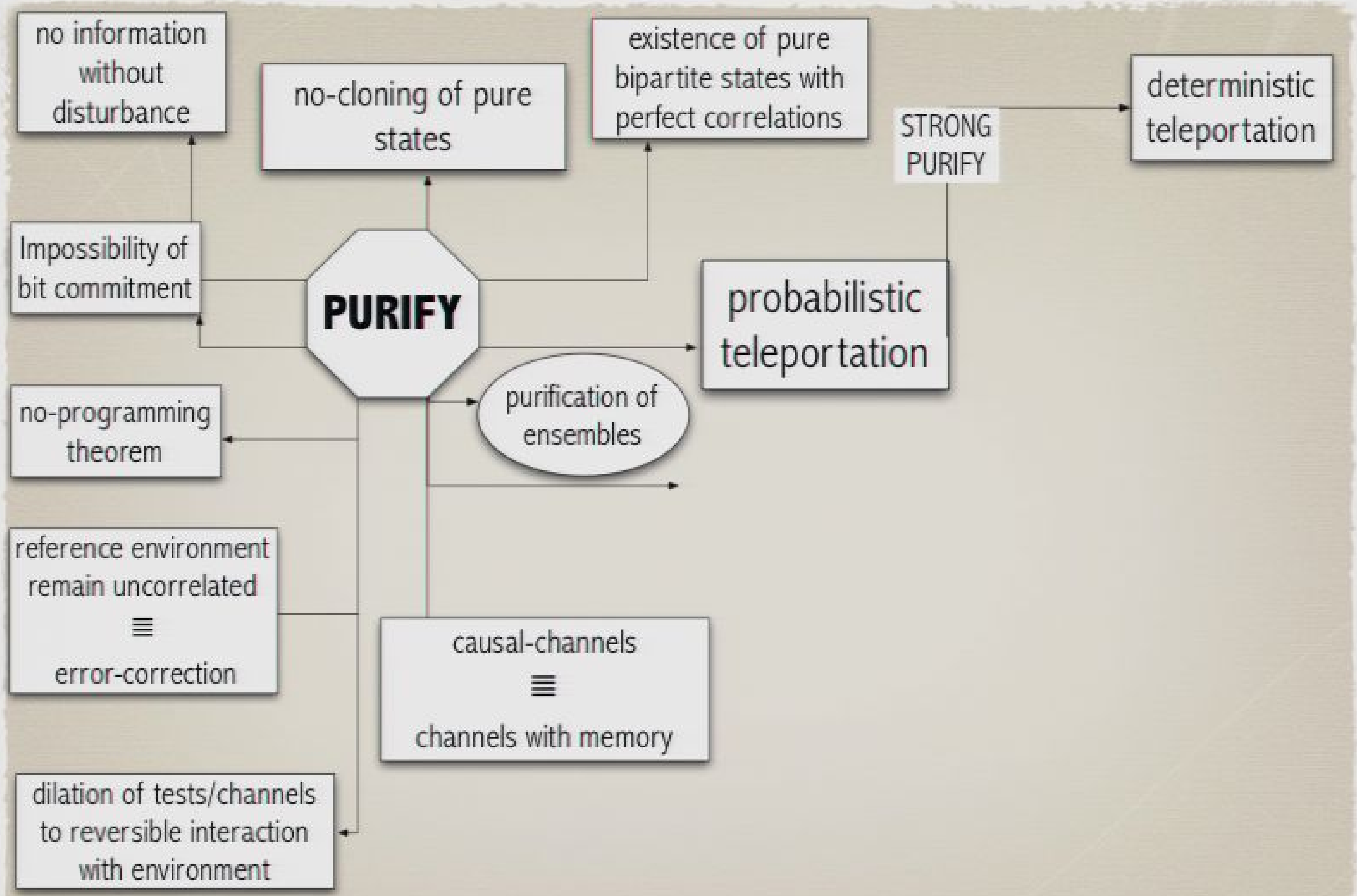
purification of ensembles

no-programming theorem

reference environment remain uncorrelated ≣ error-correction

causal-channels ≣ channels with memory

dilation of tests/channels to reversible interaction with environment



no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

deterministic teleportation

STRONG PURIFY

Impossibility of bit commitment

PURIFY

probabilistic teleportation

purification of ensembles

no-programming theorem

reference environment remain uncorrelated

≣

error-correction

causal-channels

≣

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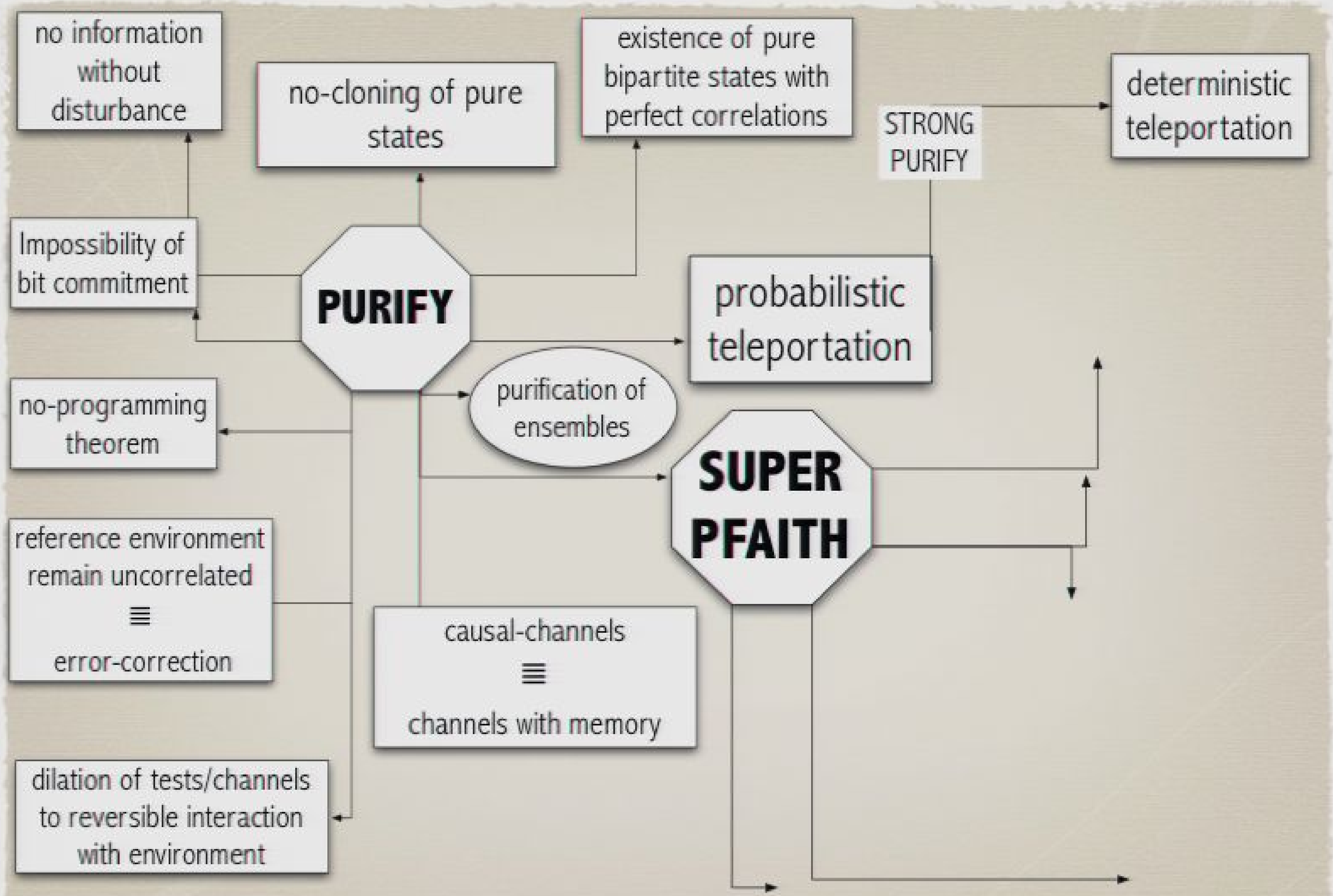
dilation of tests/channels to reversible interaction with environment



no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

STRONG PURIFY

deterministic teleportation

Impossibility of bit commitment

PURIFY

probabilistic teleportation

purification of ensembles

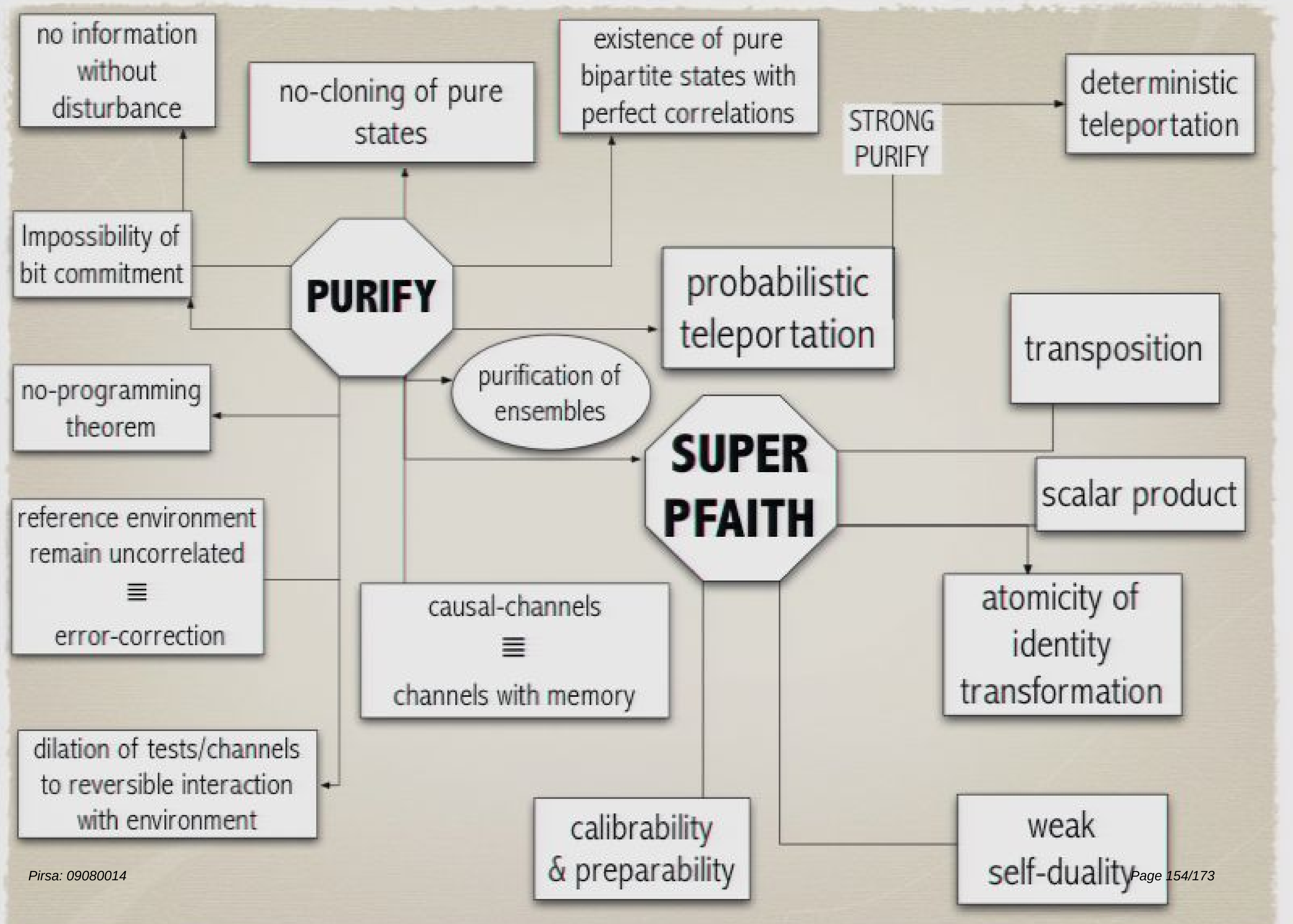
no-programming theorem

SUPER PFAITH

reference environment remain uncorrelated
≡
error-correction

causal-channels
≡
channels with memory

dilation of tests/channels to reversible interaction with environment



no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

STRONG PURIFY

deterministic teleportation

Impossibility of bit commitment

PURIFY

probabilistic teleportation

transposition

purification of ensembles

no-programming theorem

SUPER PFAITH

scalar product

reference environment remain uncorrelated
≡
error-correction

causal-channels
≡
channels with memory

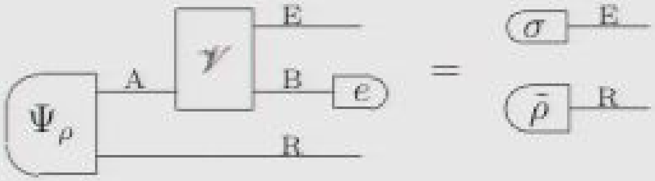
atomicity of identity transformation

dilation of tests/channels to reversible interaction with environment

calibrability & preparability

weak self-duality

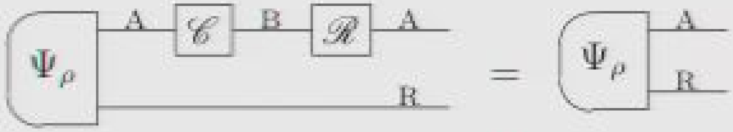
Theorem 17 *A channel* $\mathscr{C} \in \mathfrak{T}(\mathrm{A},\mathrm{B})$ *is correctable upon input of* ρ *if and only if threre are a reversible dilation* $\mathscr{V} \in \mathfrak{T}(\mathrm{A},\mathrm{BE})$ *of* $\mathscr{C}$ *and a purification* $|\Psi_\rho)_{\mathrm{AR}}$ *of* ρ *such that the marginal of the output state on system* RE *is factorized. Diagrammatically,*



(111)

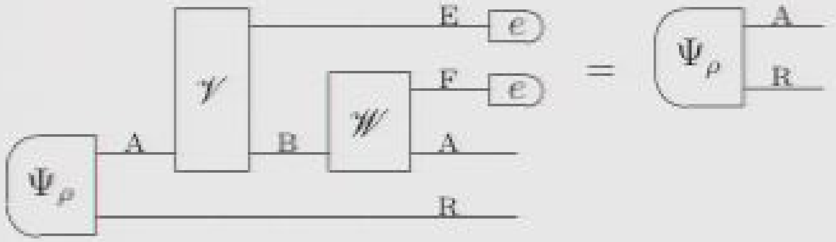
where σ *is some state of* E *and* $\tilde{\rho}$ *is the complementary state of* ρ *on system* R.

Proof. Suppose that $\mathscr{C}$ is correctable upon input of ρ with some recovery channel $\mathscr{R}$. Then, by Theorem 6 we have



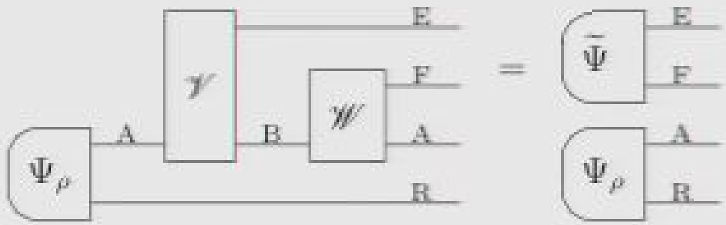
(112)

and, inserting two arbitrary dilation schemes for $\mathscr{C}$ and $\mathscr{R}$,



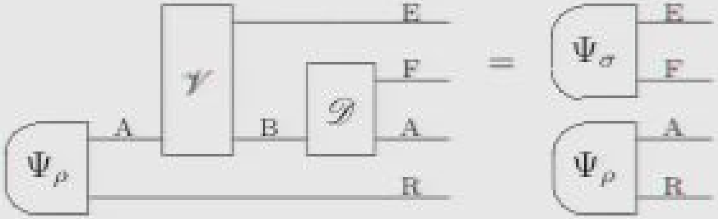
(113)

This means that $\mathscr{W}\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ is a purification of $|\Psi_\rho)_{\mathrm{AR}}$. Then, Lemma 8 ensures that $\mathscr{W}\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ is of the form



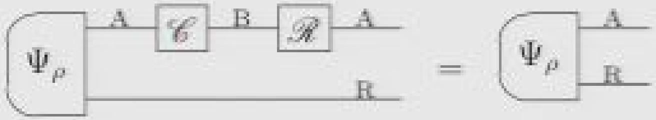
(114)

where $\tilde{\Psi}$ is some pure state on EF. Applying the deterministic effect on FA and using the fact that $\mathscr{W}$ is a channel, we then obtain Eq. (111). Conversely, suppose that Eq. (111) holds for some dilation $\mathscr{V}$ and some purification $|\Psi_\rho)_{\mathrm{AR}}$. Then take a purification of σ, say $\Psi_\sigma \in \mathfrak{S}(\mathrm{EF})$. Since $\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ and $|\Psi_\rho)_{\mathrm{AR}}\,|\Psi_\sigma)_{\mathrm{EF}}$ are both purifications of $|\sigma)_{\mathrm{E}}\,|\tilde{\rho})_{\mathrm{R}}$, by Lemma 10 we have



(115)

for some channel $\mathscr{D} \in \mathfrak{T}(\mathrm{B},\mathrm{FA})$. Applying the deterministic effect on E and F and defining $\mathscr{R} := (e|_{\mathrm{F}}\,\mathscr{D}$ we then obtain



(116)

By Theorem 6, this implies $\mathscr{R} \circ \mathscr{C} =_\rho \mathscr{I}_{\mathrm{A}}$, namely $\mathscr{C}$ is correctable upon input of ρ. ■

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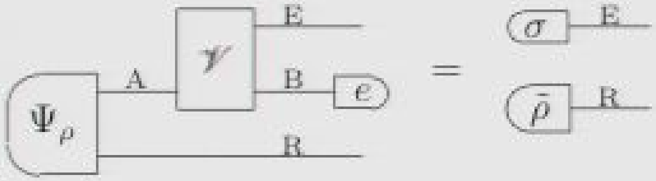

(111)

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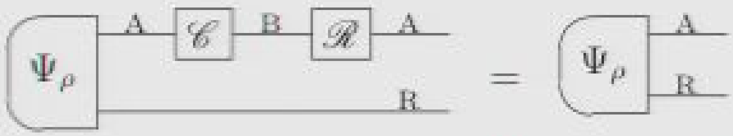

(112)

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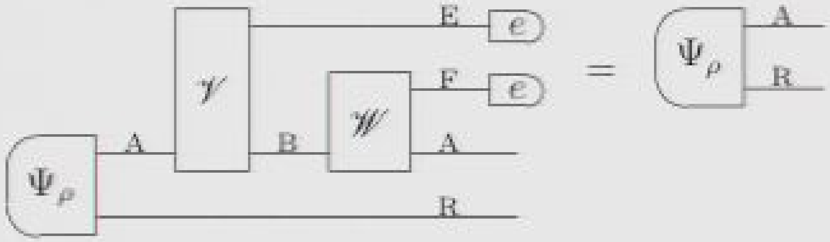

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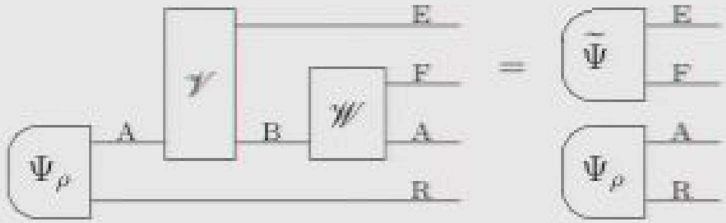

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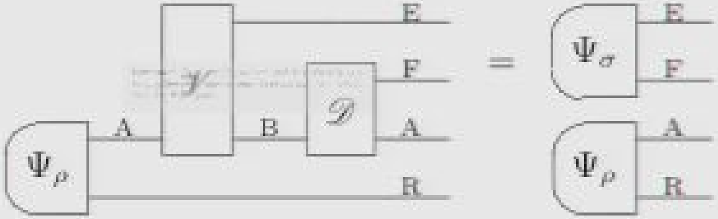

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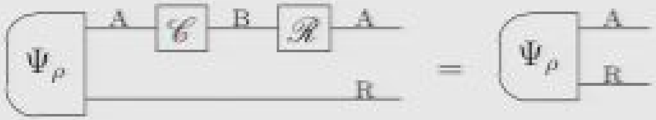

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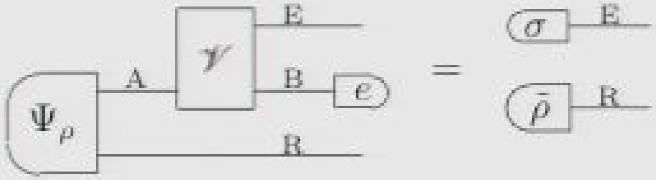

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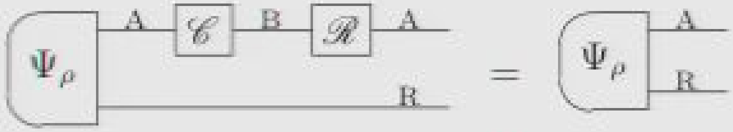

(112)

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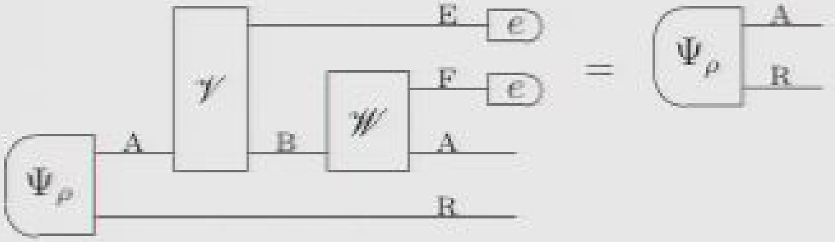

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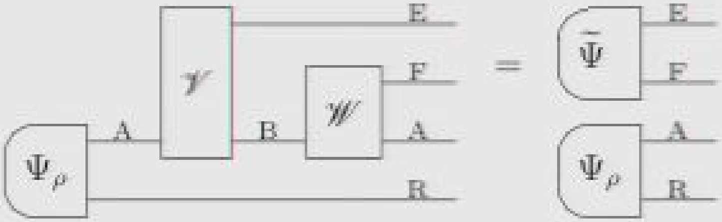
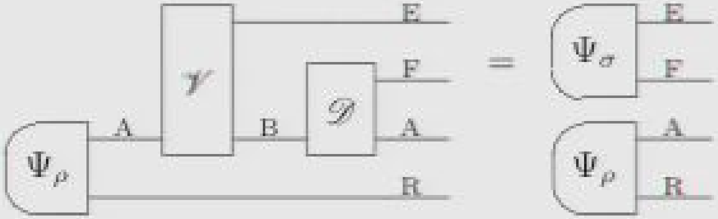
This means that $\mathscr{W}\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ is a purification of $|\Psi_\rho)_{\mathrm{AR}}$. Then, Lemma 8 ensures that $\mathscr{W}\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ is of the form



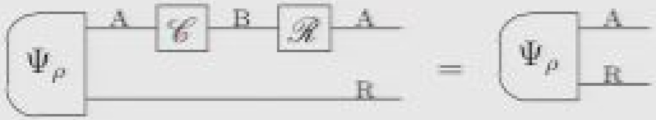
(114)

where $\tilde{\Psi}$ is some pure state on EF. Applying the deterministic effect on FA and using the fact that $\mathscr{W}$ is a channel, we then obtain Eq. (111). Conversely, suppose that Eq. (111) holds for some dilation $\mathscr{V}$ and some purification $|\Psi_\rho)_{\mathrm{AR}}$. Then take a purification of σ, say $\Psi_\sigma \in \mathfrak{S}(\mathrm{EF})$. Since $\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ and $|\Psi_\rho)_{\mathrm{AR}}\,|\Psi_\sigma)_{\mathrm{EF}}$ are both purifications of $|\sigma)_{\mathrm{E}}\,|\tilde{\rho})_{\mathrm{R}}$, by Lemma 10 we have



(115)

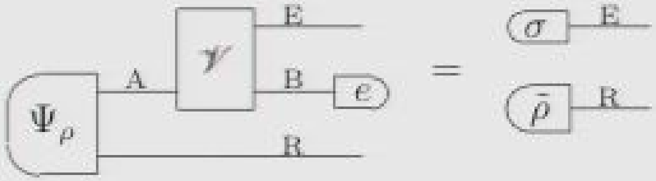
for some channel $\mathscr{D} \in \mathfrak{T}(\mathrm{B},\mathrm{FA})$. Applying the deterministic effect on E and F and defining $\mathscr{R} := (e|_{\mathrm{F}}\,\mathscr{D}$ we then obtain



(116)

By Theorem 6, this implies $\mathscr{R} \circ \mathscr{C} =_\rho \mathscr{I}_{\mathrm{A}}$, namely $\mathscr{C}$ is correctable upon input of ρ. ■

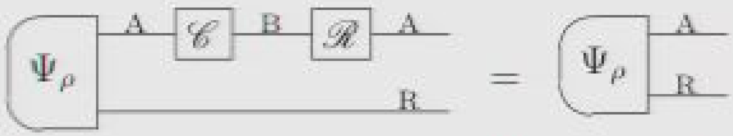
Theorem 17 *A channel $\mathscr{C} \in \mathfrak{T}(\mathrm{A},\mathrm{B})$ is correctable upon input of ρ if and only if threre are a reversible dilation $\mathscr{V} \in \mathfrak{T}(\mathrm{A},\mathrm{BE})$ of $\mathscr{C}$ and a purification $|\Psi_\rho)_{\mathrm{AR}}$ of ρ such that the marginal of the output state on system* RE *is factorized. Diagrammatically,*



(111)

where σ is some state of E *and $\tilde\rho$ is the complementary state of ρ on system* R.

Proof. Suppose that $\mathscr{C}$ is correctable upon input of ρ with some recovery channel $\mathscr{R}$. Then, by Theorem 6 we have



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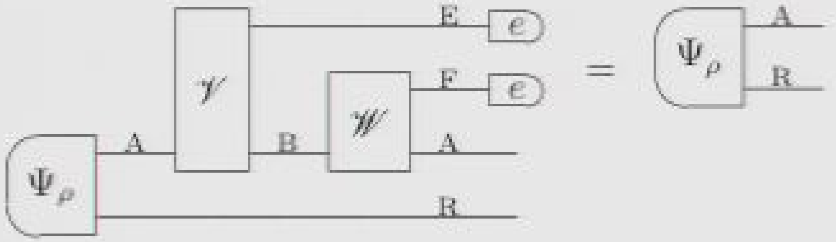
and, inserting two arbitrary dilation schemes for $\mathscr{C}$ and $\mathscr{R}$,



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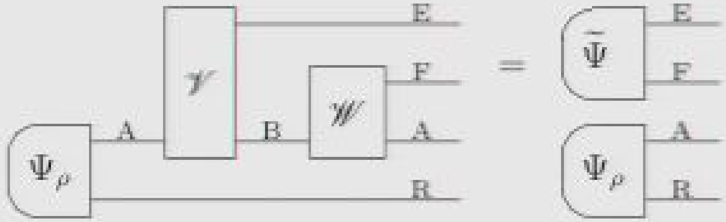

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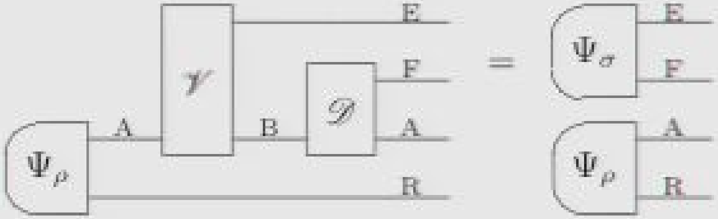

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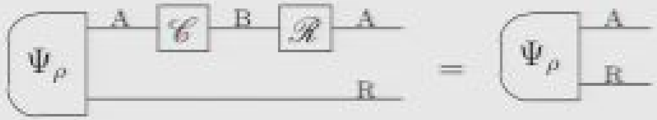

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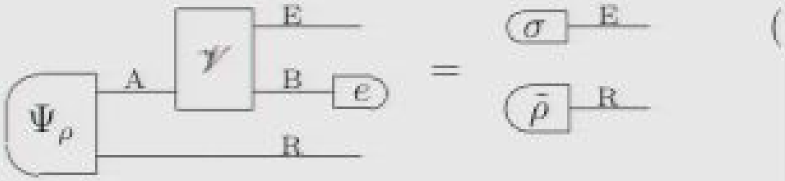

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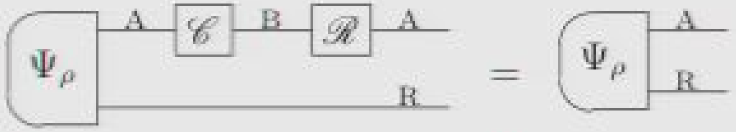

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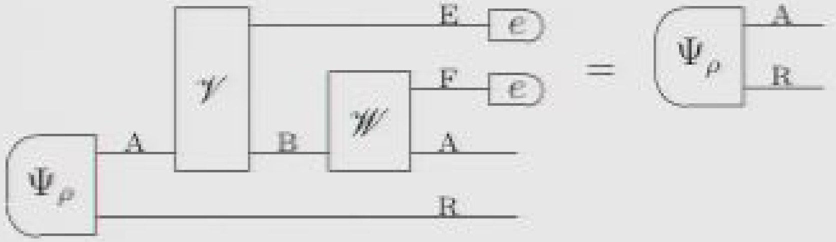

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Theorem 6 (Equality upon input of ρ) *Let $\Psi \in \mathfrak{S}(\mathrm{AC})$ be a purification of $\rho \in \mathfrak{S}(\mathrm{A})$, and let $\mathscr{A}, \mathscr{A}' \in \mathfrak{T}(\mathrm{A},\mathrm{B})$ be two transformations. Then one has*

$$\mathscr{A}\,|\Psi)_{\mathrm{AC}} = \mathscr{A}'\,|\Psi)_{\mathrm{AC}} \qquad \Longrightarrow \qquad \mathscr{A} =_\rho \mathscr{A}' \, . \tag{58}$$

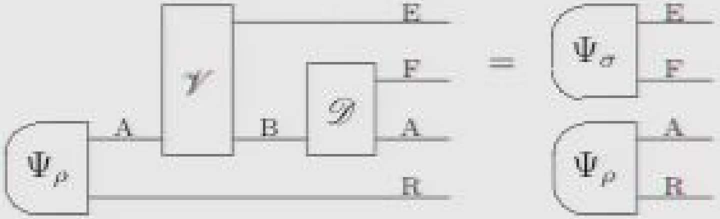
If local discriminability holds, one has the equivalence

$$\mathscr{A}\,|\Psi)_{\mathrm{AC}} = \mathscr{A}'\,|\Psi)_{\mathrm{AC}} \qquad \Longleftrightarrow \qquad \mathscr{A} =_\rho \mathscr{A}' \, . \tag{59}$$

When one of the two transformations is proportional to the identity Eq. (59) holds.

---

a channel, we then obtain Eq. (111). Conversely, suppose that Eq. (111) holds for some dilation $\mathscr{V}$ and some purification $|\Psi_\rho)_{\mathrm{AR}}$. Then take a purification of σ, say $\Psi_\sigma \in \mathfrak{S}(\mathrm{EF})$. Since $\mathscr{V}\,|\Psi_\rho)_{\mathrm{AR}}$ and $|\Psi_\rho)_{\mathrm{AR}}\,|\Psi_\sigma)_{\mathrm{EF}}$ are both purifications of $|\sigma)_{\mathrm{E}}\,|\tilde\rho)_{\mathrm{R}}$, by Lemma 10 we have



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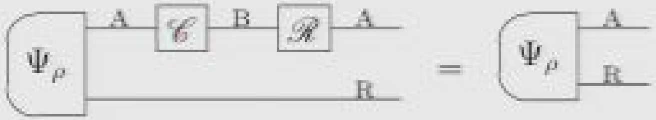

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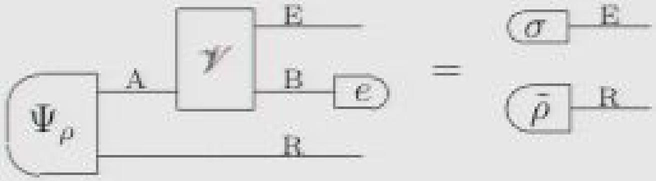

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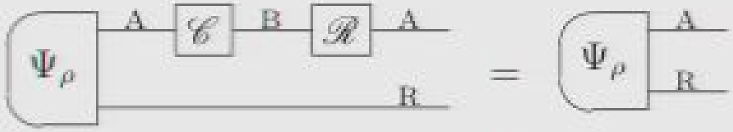

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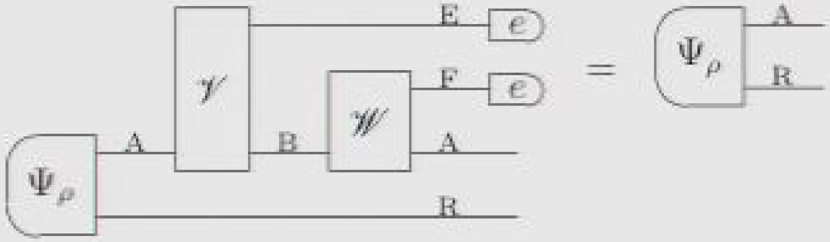

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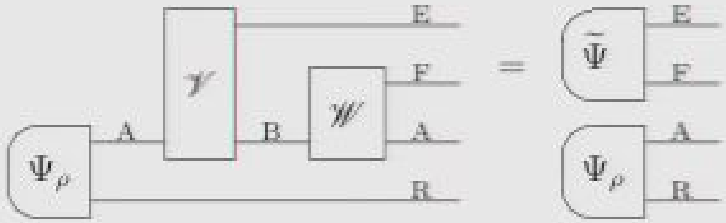

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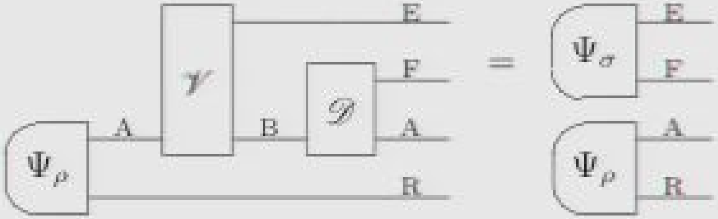

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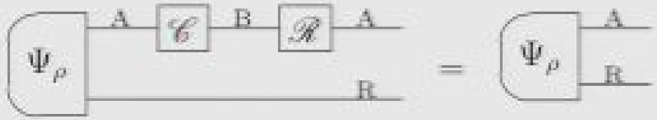

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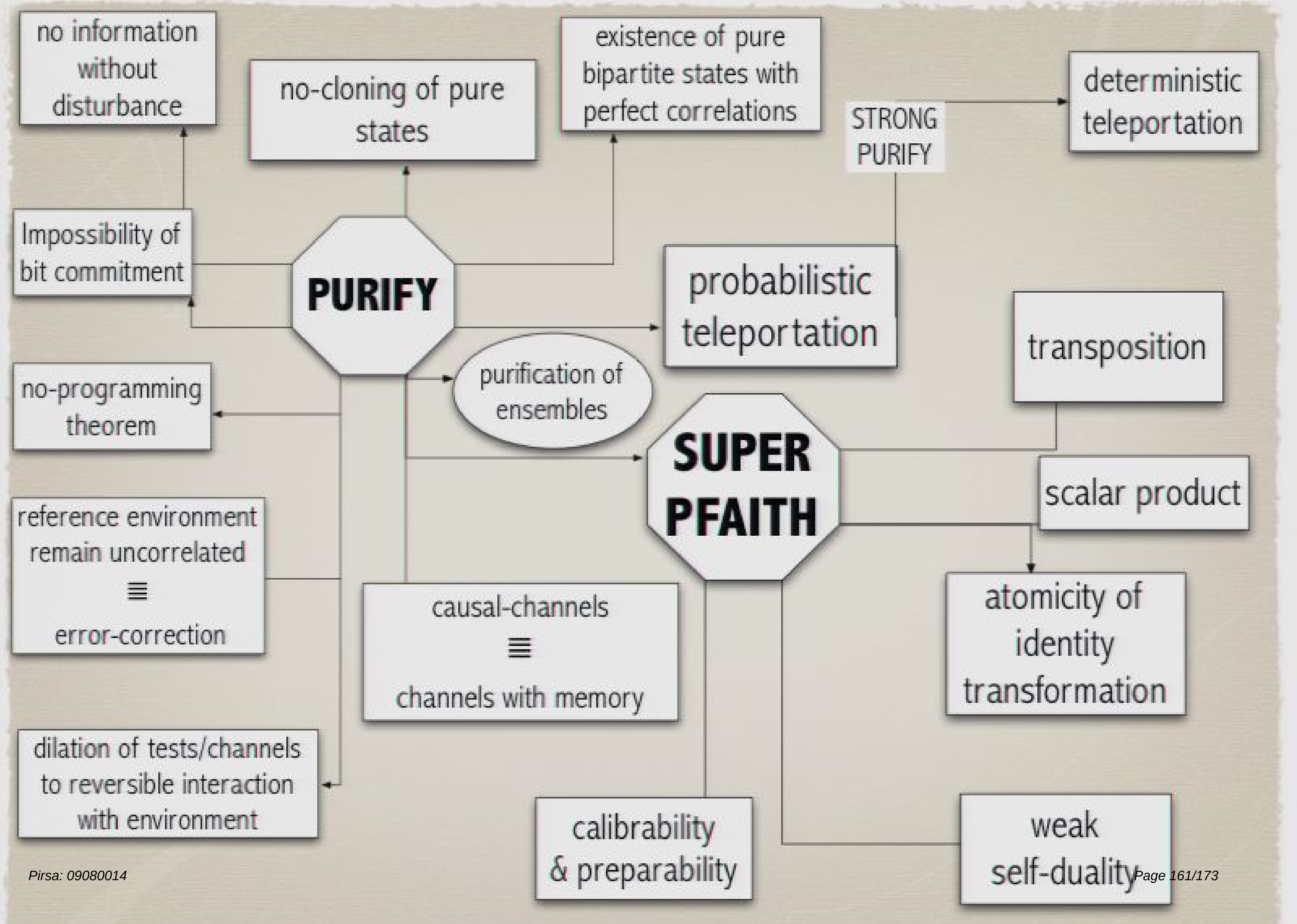

no information without disturbance

no-cloning of pure states

existence of pure bipartite states with perfect correlations

STRONG PURIFY

deterministic teleportation

Impossibility of bit commitment

PURIFY

probabilistic teleportation

transposition

no-programming theorem

purification of ensembles

SUPER PFAITH

scalar product

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≣

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atomicity of identity transformation

dilation of tests/channels to reversible interaction with environment

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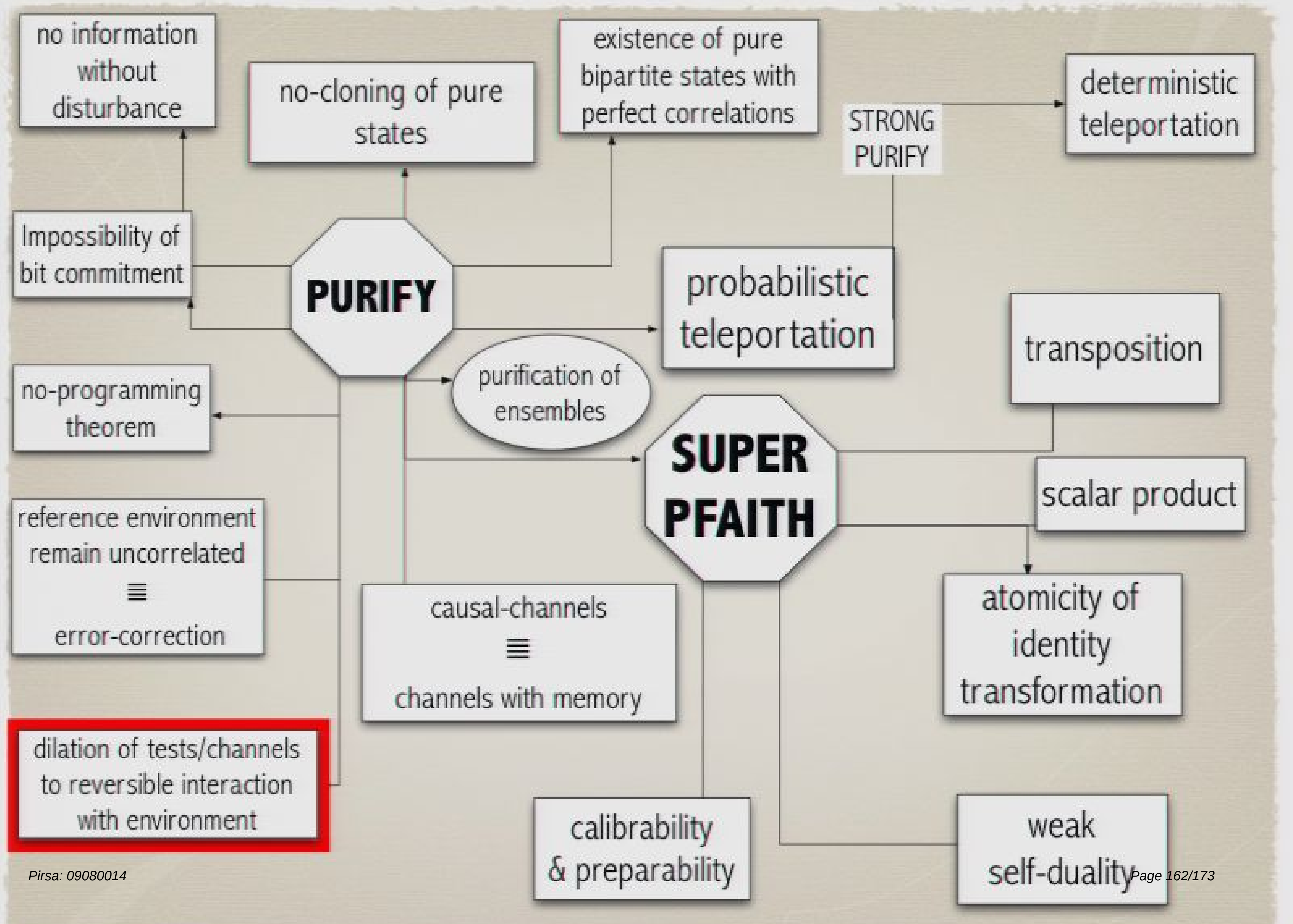
weak self-duality



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Reformulation of PURIFY

Conservation of information

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Conservation of information

Every irreversible process arises from a reversible interaction with an environment that is eventually lost. Information cannot be erased, it can only be "discarded".

PRINCIPLES OF QUANTUMNESS

QM: probabilistic theory satisfying:

1. Causality
2. Local observability
3. Conservation of information

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still a conjecture ...

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3. Showing that the group of reversible transformations is isomorphic to SU(d)

FUTURE LINES OF RESEARCH

1. Prove reconstruction of QM
2. Derive the math. structure of the non causal theory
3. Work-out the notion of subsystem and the theory of state-compression

THANK YOU FOR YOUR ATTENTION

PRINCIPLES OF QUANTUMNESS

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arXiv:0807.4383: in *Philosophy of Quantum Information and Entanglement*, Eds A. Bokulich and G. Jaeger (CUP, Cambridge UK, 2009)

arXiv:0908.1583 with G. Chiribella and P.Perinotti