Title: Operational structures and Natural Postulates for Quantum Theory

Date: Aug 14, 2009 09:00 AM

URL: http://pirsa.org/09080011

Abstract: <div id="Cleaner">In this talk we provide four postulates that are more natural than the usual postulates of QT. The postulates require the predefinition of two integers, K and N, for a system. K is the number of probabilities that must be listed to specify the state. N is the maximum number of states that can be distinguished in a single shot measurement and consequently log N is the information carrying capacity. The postulates are<div id="Cleaner"><div id="Cleaner">P1 Information: Systems having, or constrained to have, a given information carrying capacity have the same properties.
P2 Composites: For a composite system, AB, we have N_AB=N_A N_B and K_AB=K_A K_B.
P3 Continuity: There exists a continuous reversible transformation between any two pure states.
P4 Simplicity: For each N, K takes the smallest value consistent with the other postulates.
Note that P2 is equivalent to requiring that information carrying capacity be additive and that the state of a composite system can be determined by measurements on the components alone (local tomography is possible). We can prove a reconstruction theorem: the standard formalism of QT (for finite N) follows from these postulates. This includes the properties that quantum states can be represented by density operators on a complex Hilbert space, evolution is given by completely positive maps (of which unitary evolution is a special case), and that composite systems are formed using the tensor product. We derive the Born rule (or, equivalently, the trace rule) for calculating probabilities. If the single word â€œcontinuousâ€• is dropped from P3 the postulates are consistent with both Classical Probability Theory and Quantum Theory. In this talk we will place particular emphasis on laying the operational foundations for such postulates. Then we will provide some highlights of the proof. Finally we will speculate on what needs to be changed for a theory of quantum gravity.

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Postulates of SR

1) The Principle of Relativity: "The same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good."

and "another postulate which is only apparently irreconcilable with the former"

2) Constancy of speed of light: "Light is always propogated in empty space with a definite velocity c which is independent of the state of motion of the emitting body."

or
1) Physics is same in all inertial frames
2) Speed of light independent of source.

We want something at least as compelling for QT

Note though that

a) A lot of background framework is assume
(global inertial ref frame x,y,z,t)

b) It is this background framework that needed to be changed in GR

$\boxed{\text{Einstein's Postulates}} + \boxed{\text{Background framework: global inertial frame } x, y, z, t} \Rightarrow \boxed{\text{Special Relativity}}$

Important lesson for QT + GR => QG

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Approach

$\underline{r}$, Z, p framework

Probability theory

Postulates + Foliable operational structure $\Rightarrow$ QT

finite dim

unsympathetic to GR and too restrictive for QG

P1 Information: Systems having, or constrained to have, a given information carrying capacity have the same properties.

P2 Composites: Information carrying capacity is additive and local tomography is possible. $(N_{ab} = N_a N_b\ ,\ K_{ab} = K_a K_b)$

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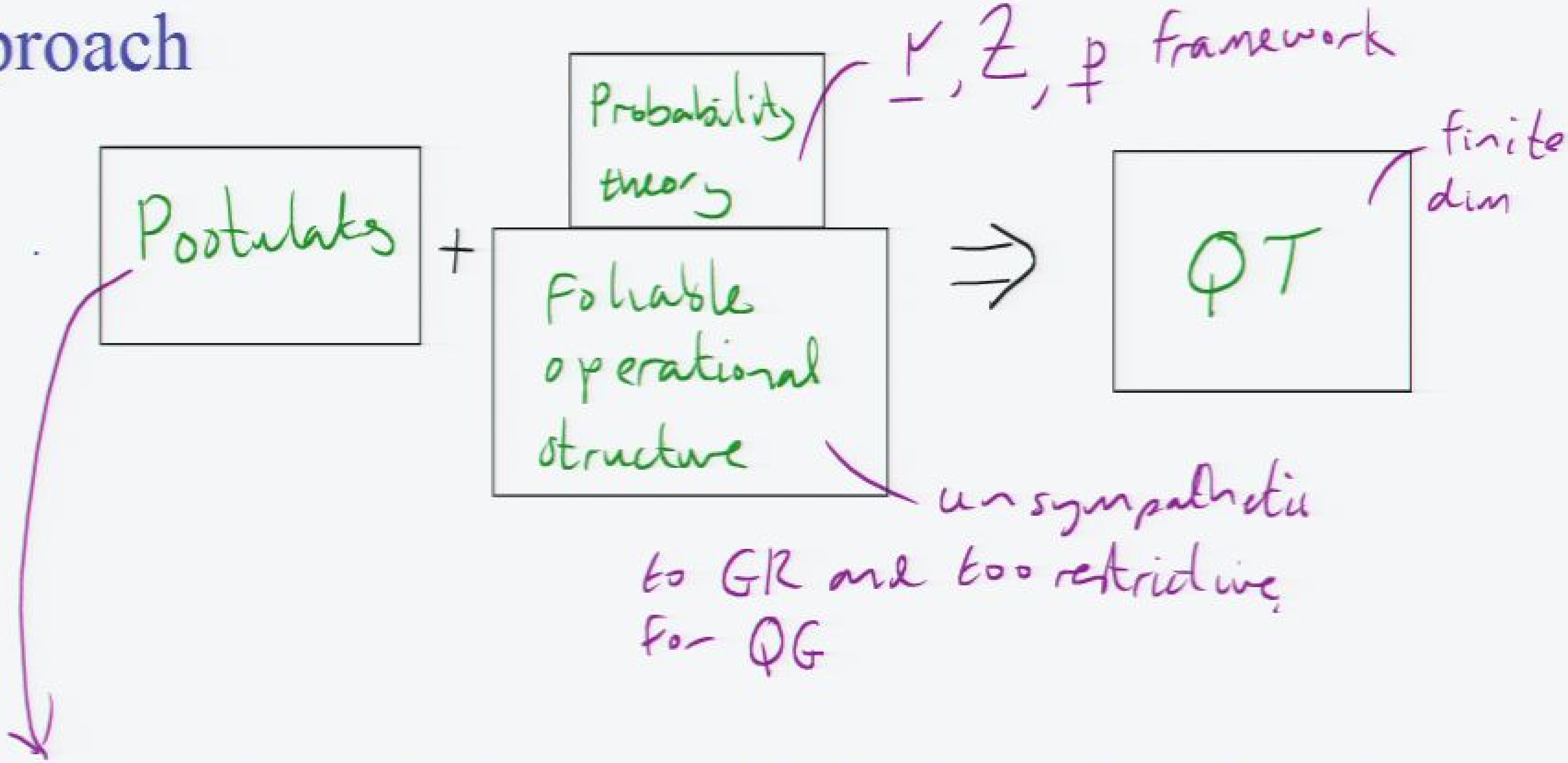
Approach



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Aside - COOOL approach to QG

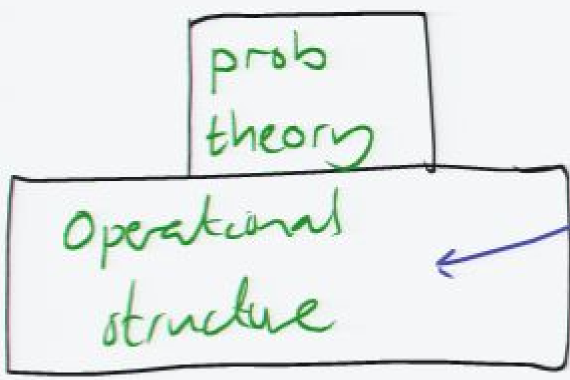
CProbT/QT/ProbGR/QG Postulates + Causaloid 2.0 / object oriented operationalism $\Rightarrow$ CProbT/QT/ProbGR/QG

not nec. foliable

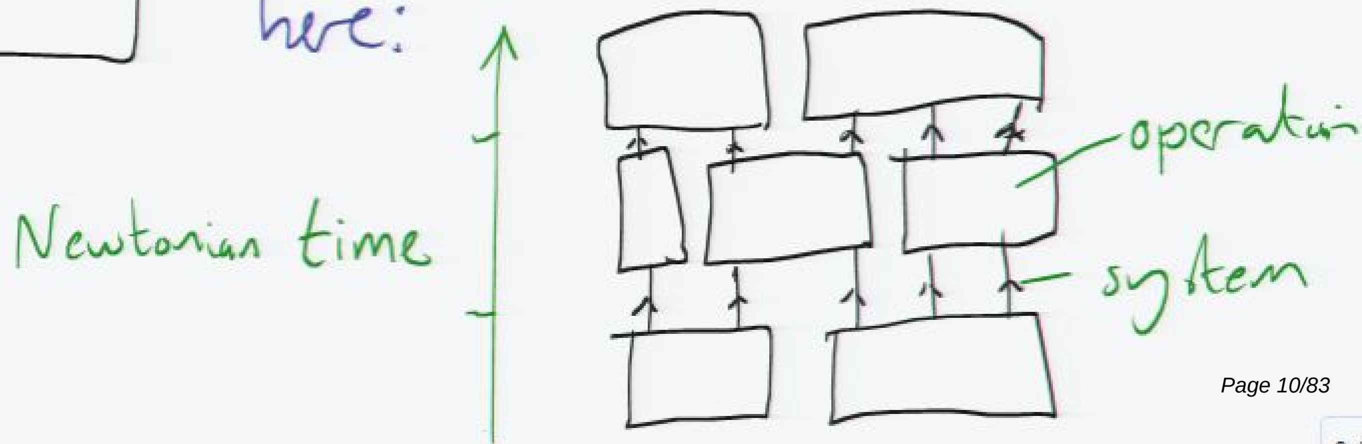
$$\begin{tikzcd} QT \arrow[r] & QG \\ CProbT \arrow[u, "\text{``quantization''}"] \arrow[r, "\text{``GRization''}"'] & ProbGR \arrow[u] \end{tikzcd}$$

In the recent literature, general probabilistic theories are usually formulated in an operational context

— so operational structures form a foundation for general probabilistic theories.

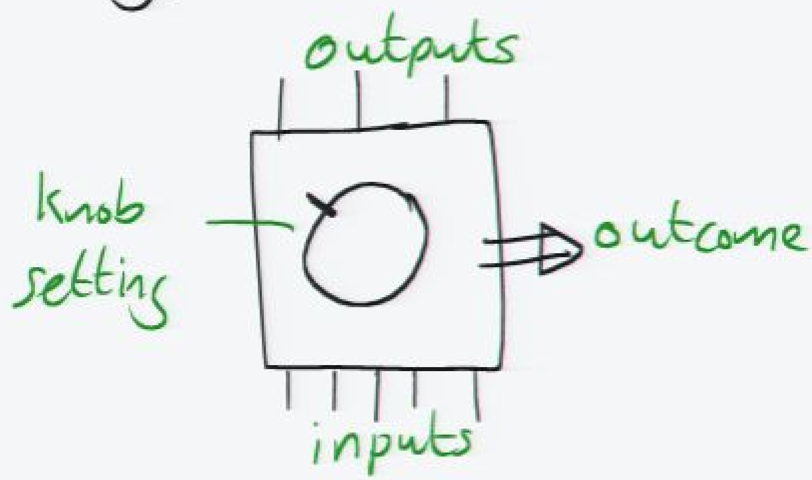


researches into gen prob structures usually take naive approach here:



We can try to be more serious. Here's one way to proceed:

Basic building block is the Operation. An operation is one use of an apparatus having:

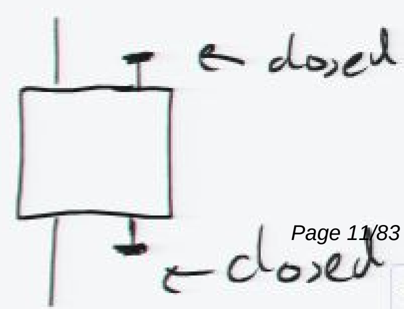


knob settings: can adjust

Outcomes: can read off

Inputs and Outputs come in different types a, b, c, ...

and can be open or closed.



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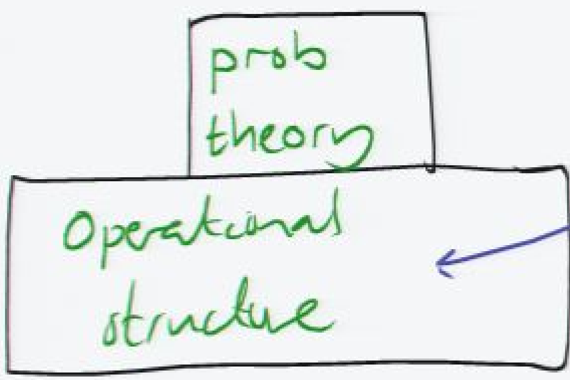

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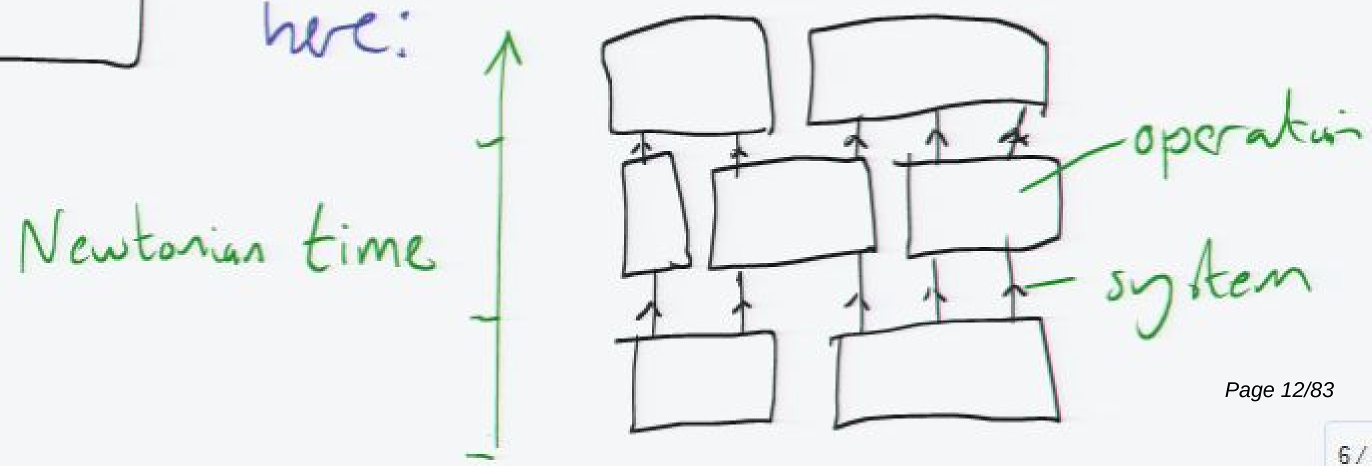

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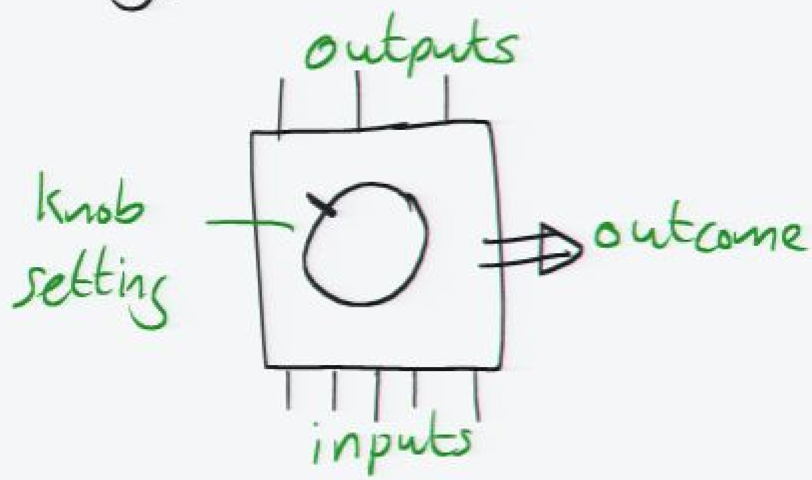

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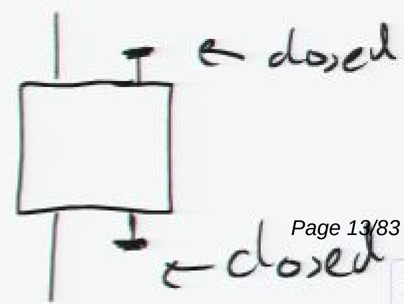

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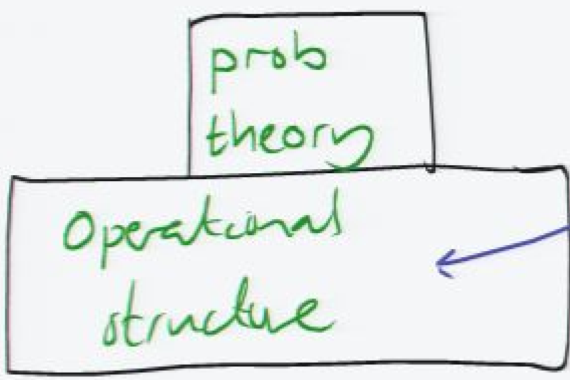

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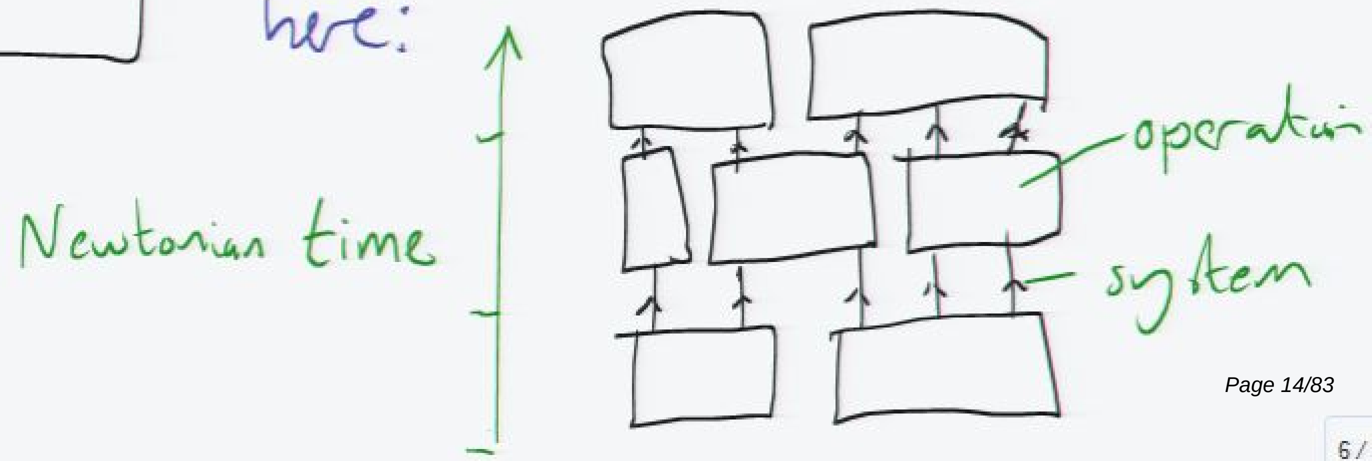

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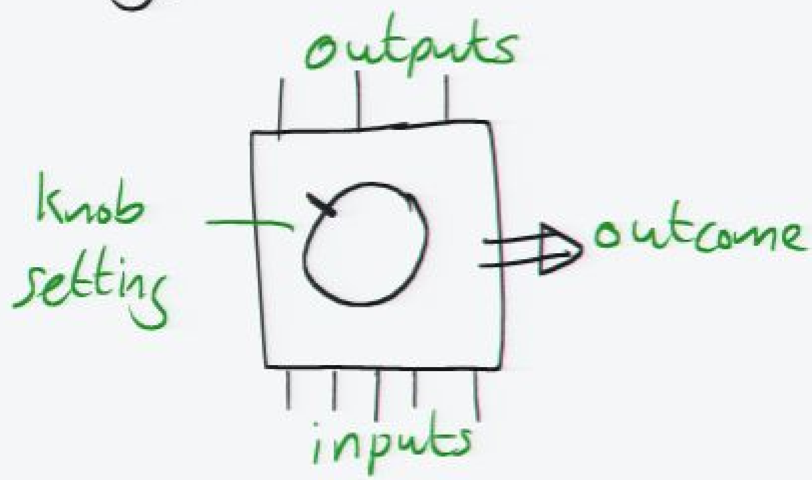

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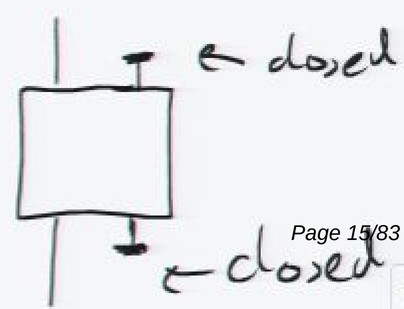

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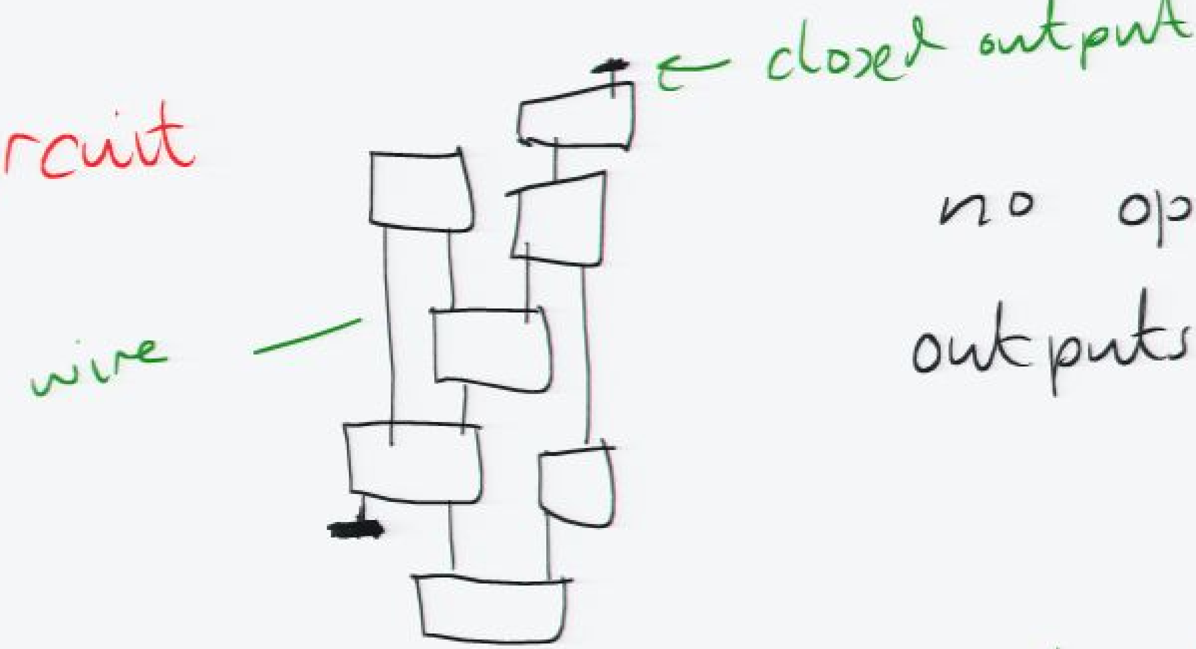
Wires show how inputs connected to outputs of same type (Ikea idea)

A circuit



no open inputs or outputs left over.

no closed loops

Markopoulou.
Blute, Ivanov, Panangaden

Abramsky Coecke
categorical approach

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Wiring constraints

i) one wire at most may be attached to input/output

ii) wires connect output to input of matching type

iii) no closed loops

iv) may be additional constraints

Wiring Freedoms

i) no constraints between disconnected parts

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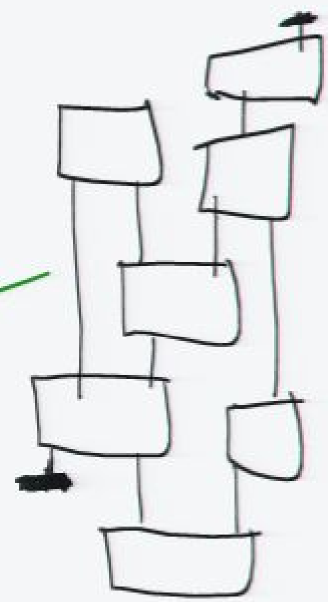

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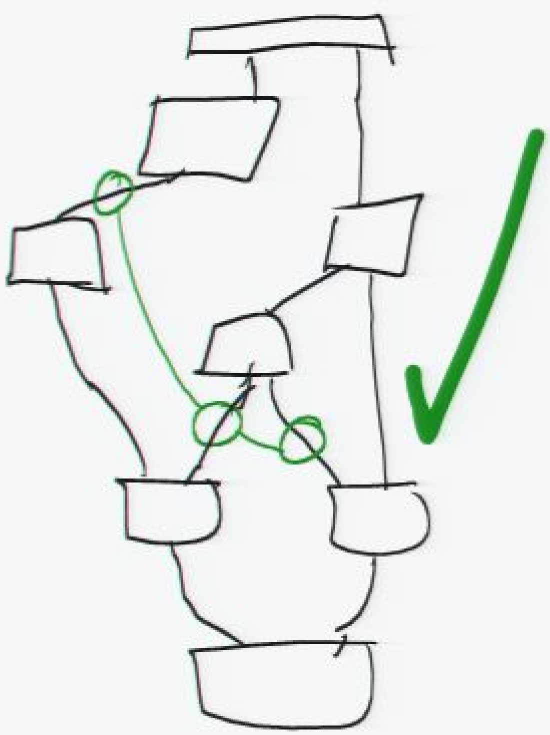
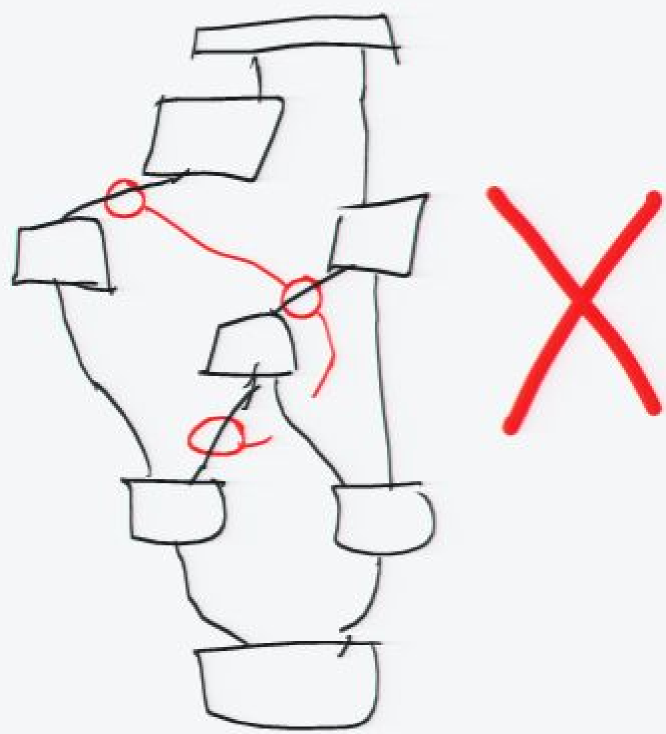
Time

A synchronous set of wires in a circuit is a set for which there exists no path from one wire to another in the set by tracing forward.





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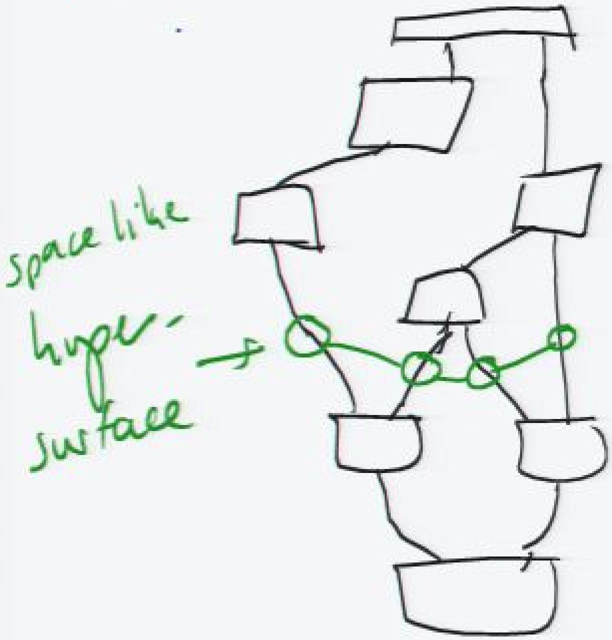
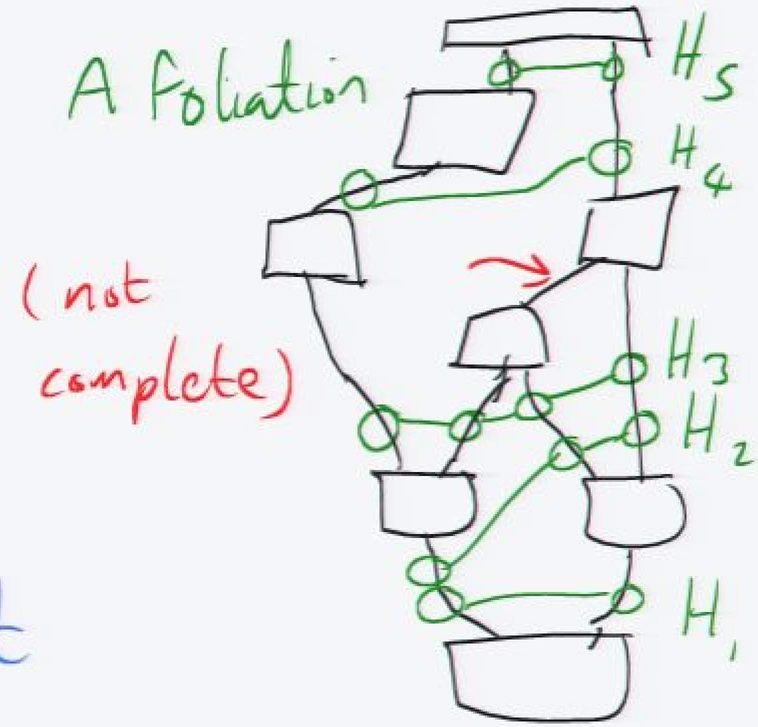
A space like hypersurface is a synchronous set that partitions the ciricuit into two parts





A Foliation an ordered set

A complete Foliation includes all wires

Thrm: complete foliations exist for all circuits

Important definition – now introduce probability

A closable set of operations, $\mathcal{O}$, is one such that

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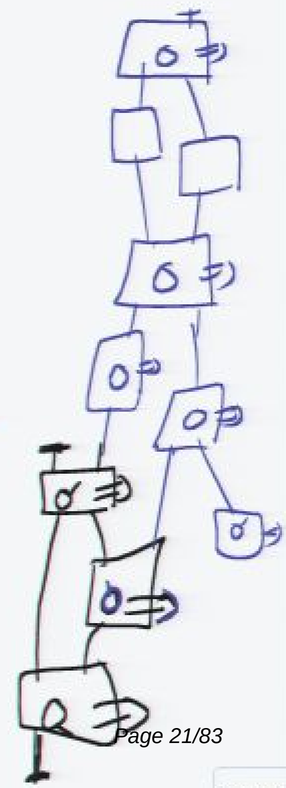
well defined prob

same prob



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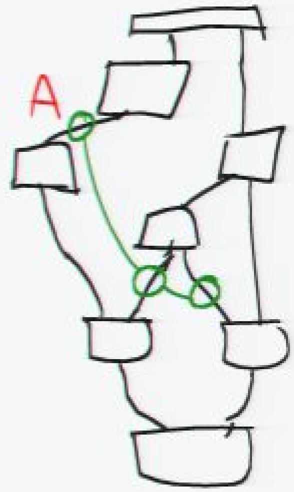
Operational definition of system



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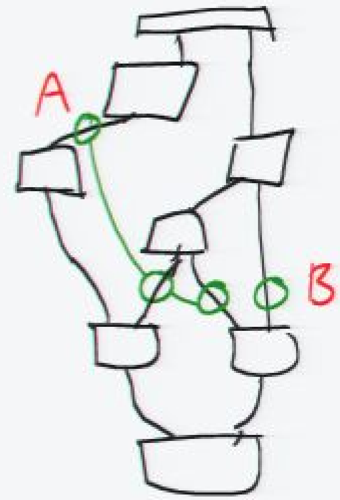

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Notation: will sometimes denote a composite system type with a single letter e.g. $d \equiv aac$

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← closed output

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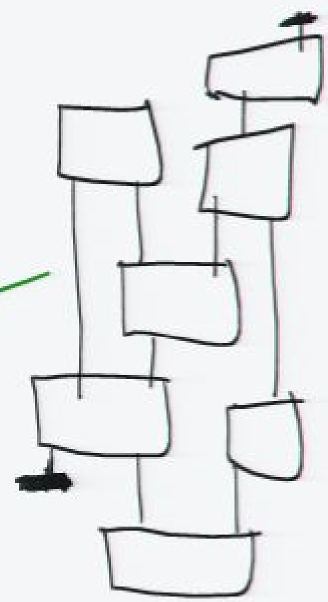

wire

no open inputs or
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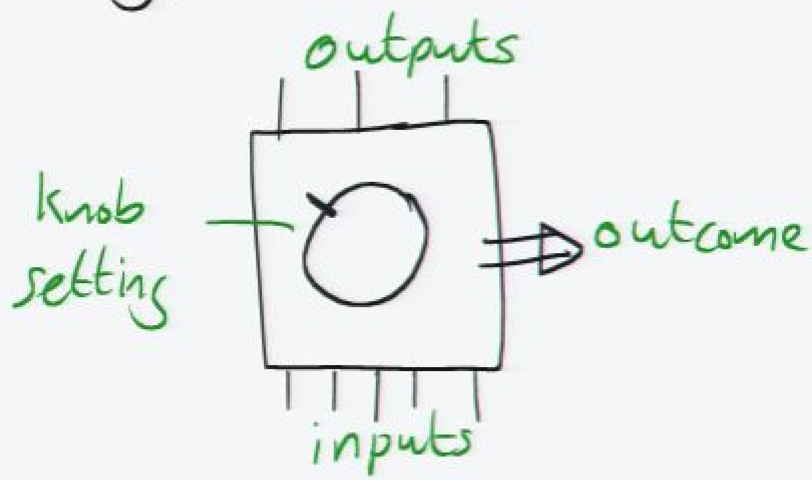

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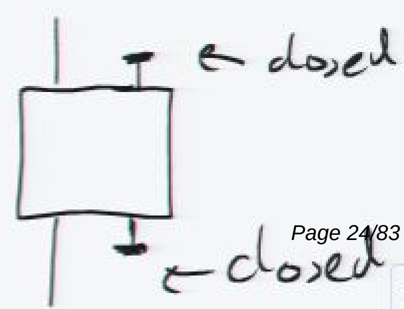

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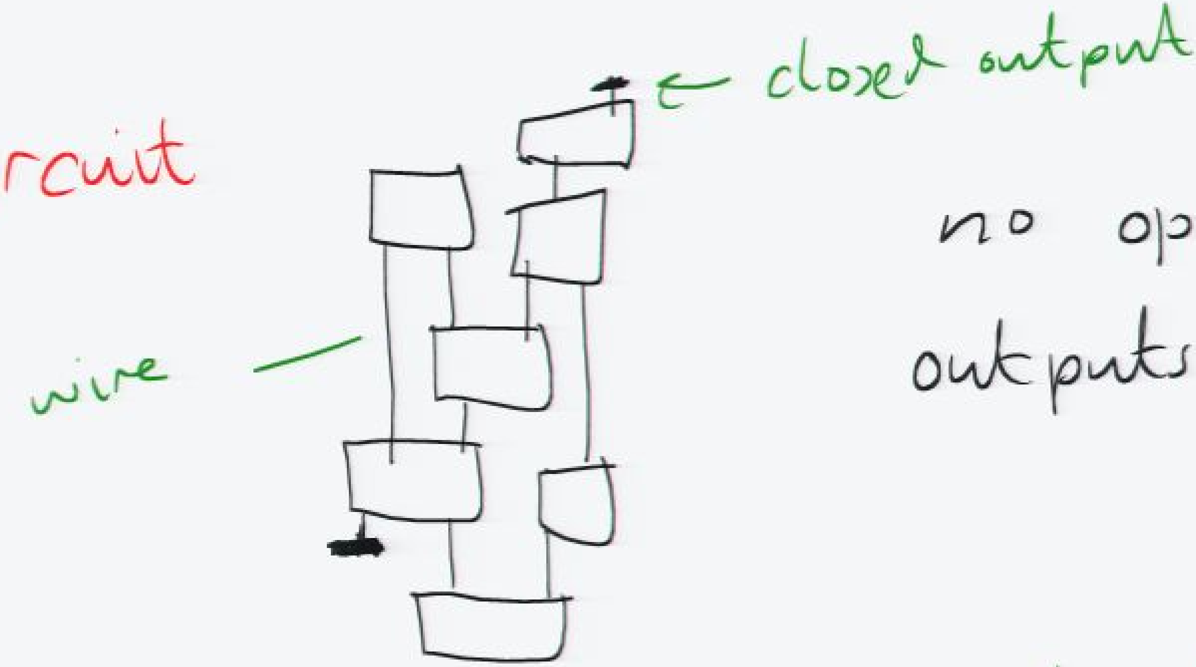

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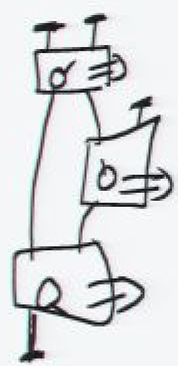
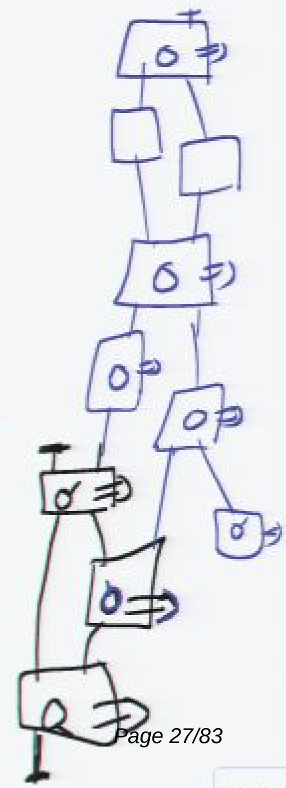
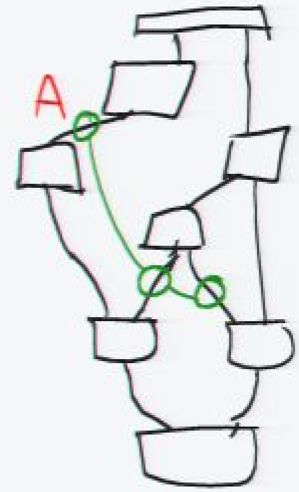


same prob



Operational definition of system



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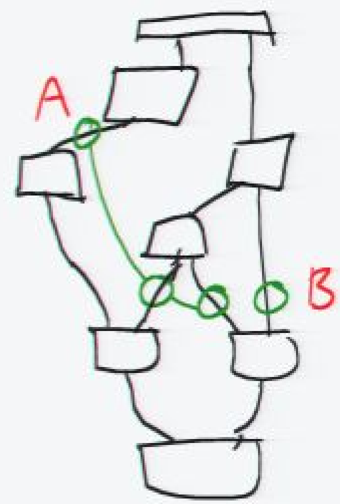

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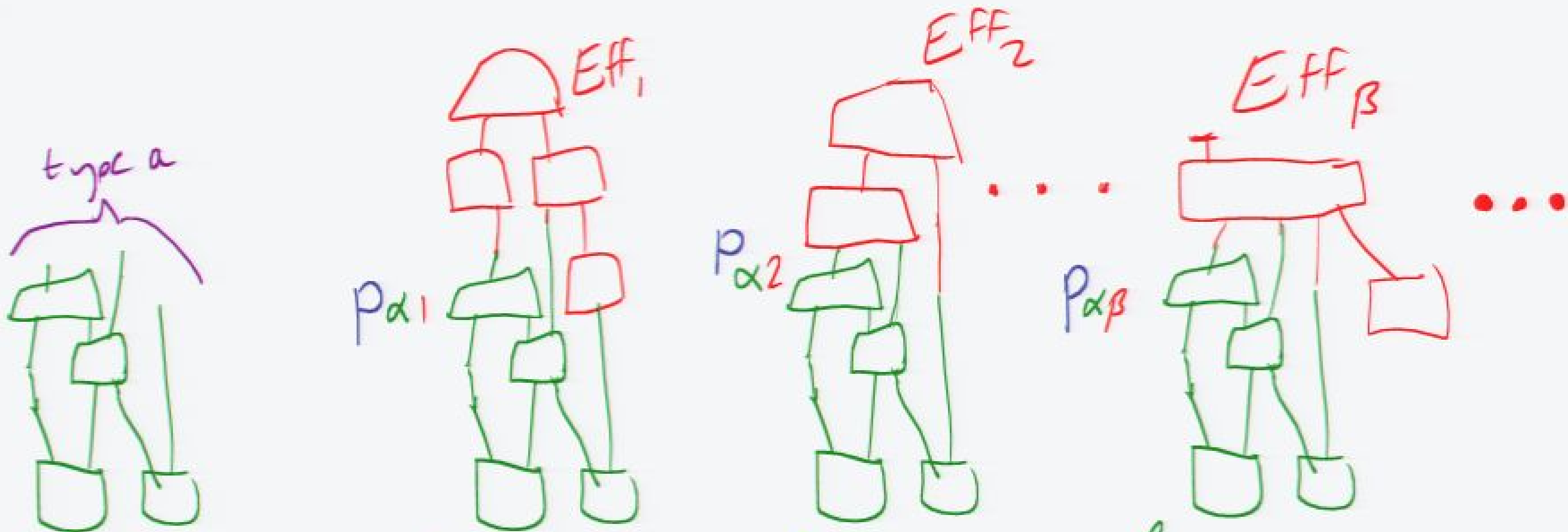
Notation: will sometimes denote a composite system type with a single letter e.g. $d \equiv aac$

The State

The state of a system of type a associated with a preparation is that thing represented by any mathematical object that can be used to calculate the probability for any circuit with that preparation



preparation α Prep α Prep α Prep α

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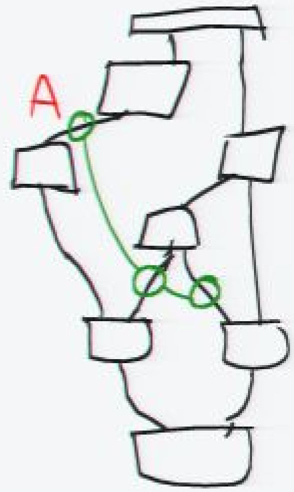

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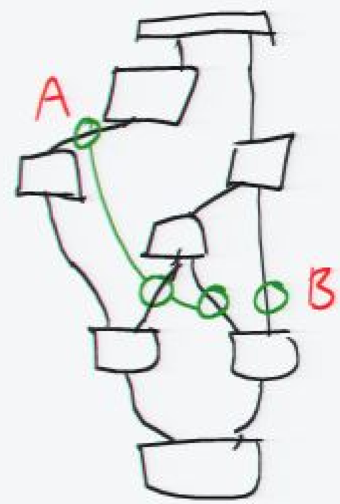

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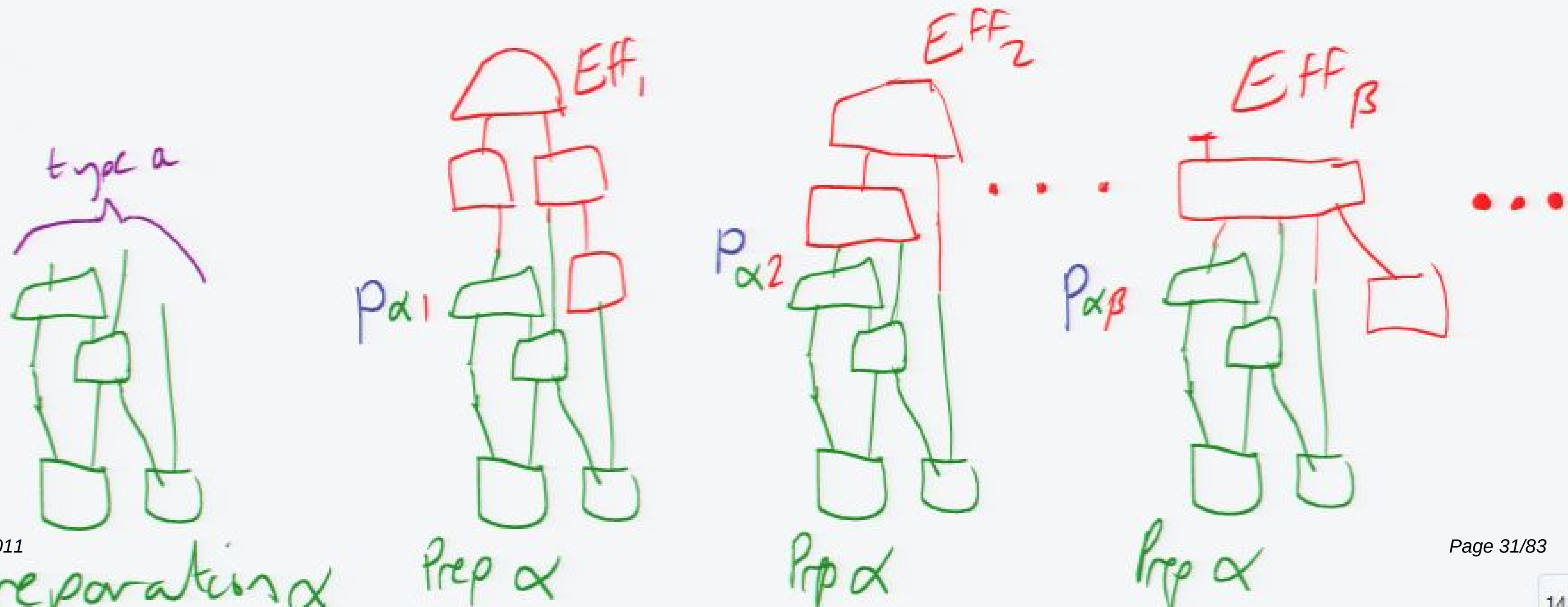

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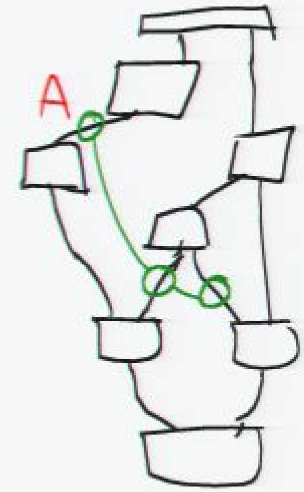

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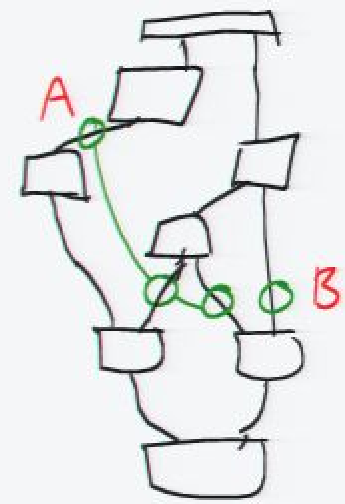

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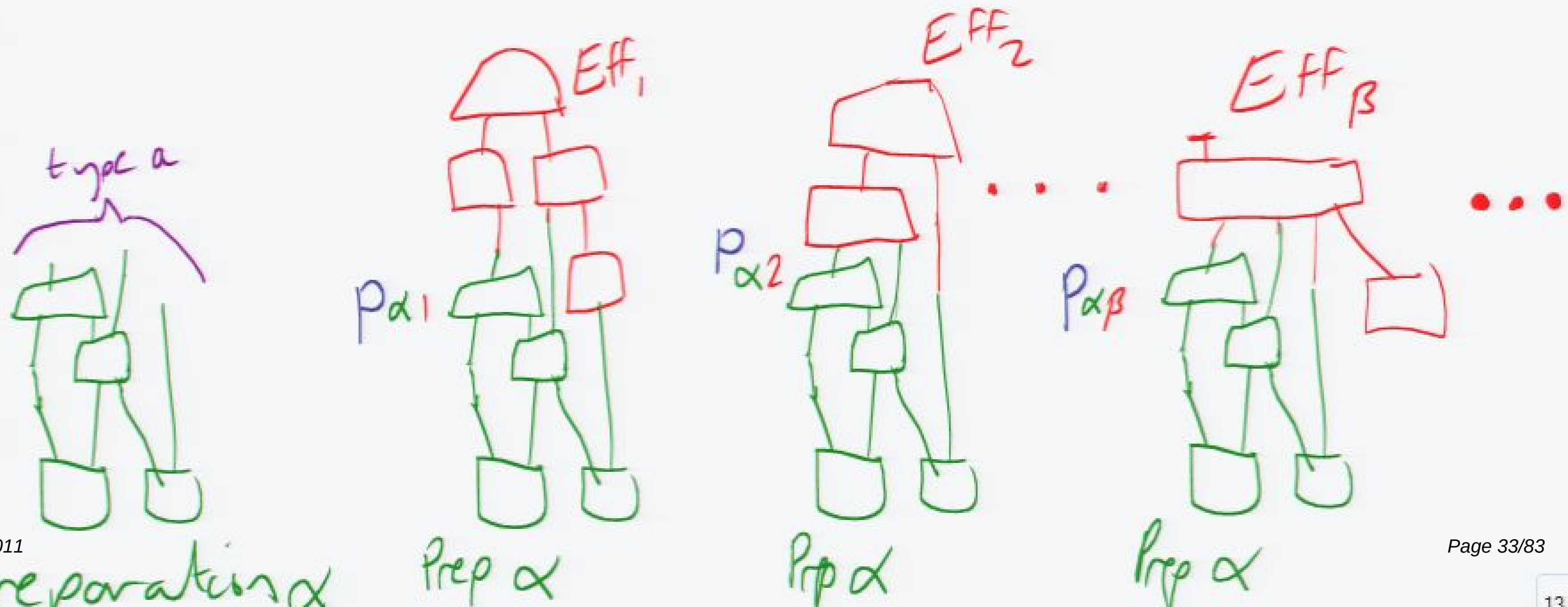

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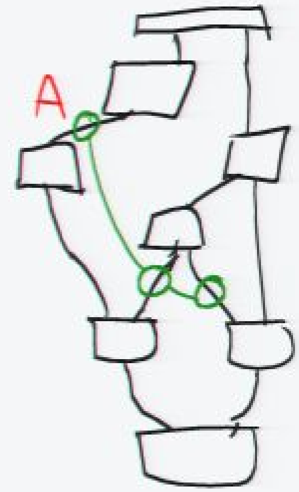

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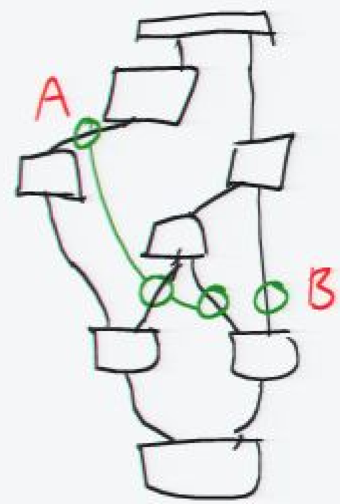

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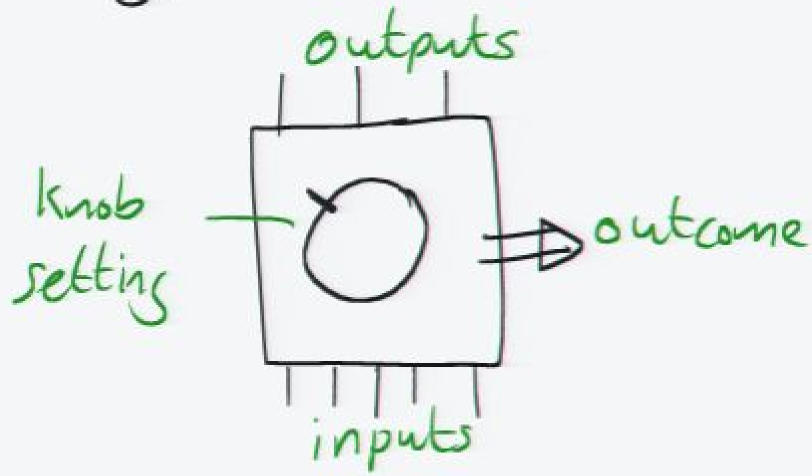

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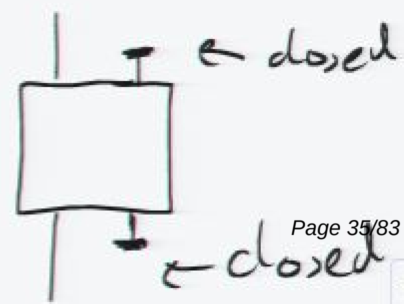

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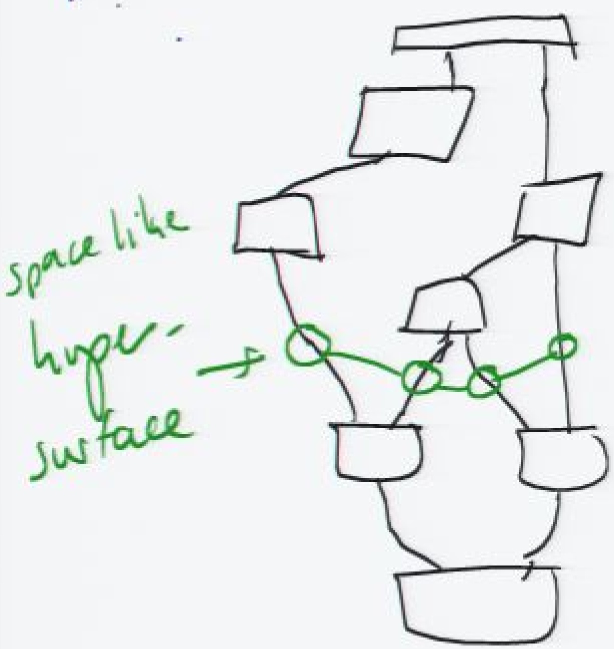

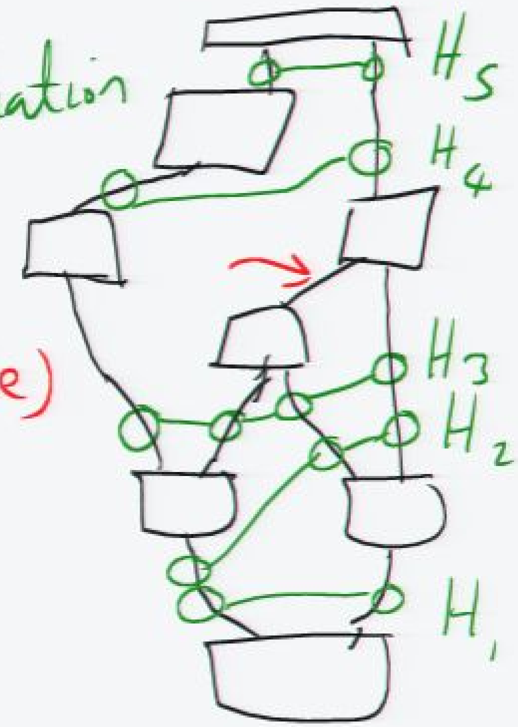

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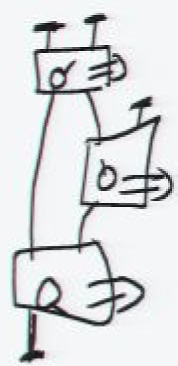
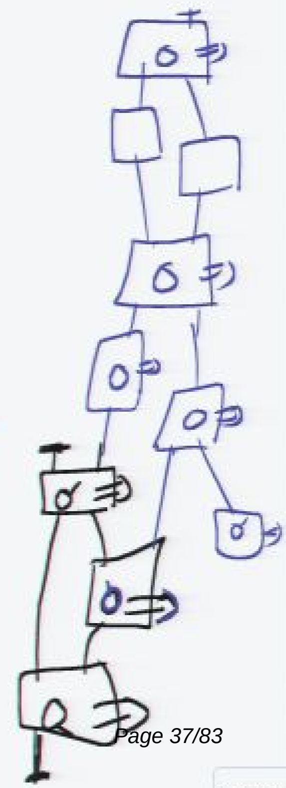
well defined prob



same prob



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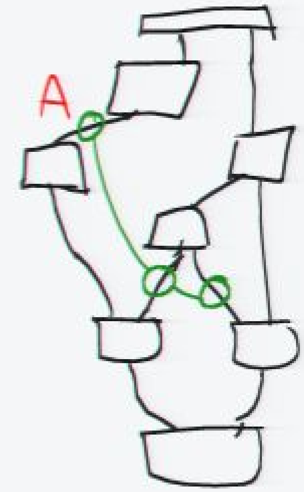
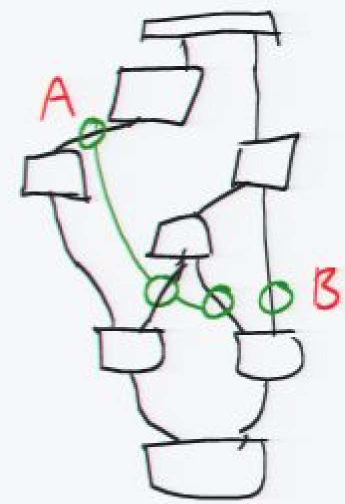
Operational definition of system



A system of type abc... is, by definition, associated with any synchronous set of wires of type a, b, c, .. in any circuit formed from a closable set of operations.



A composite system AB is associated with two systems A and B if the intersection of the associated wires is null and the union is a synchronous set.

Notation: will sometimes denote a composite system type with a single letter e.g. $d \equiv aac$

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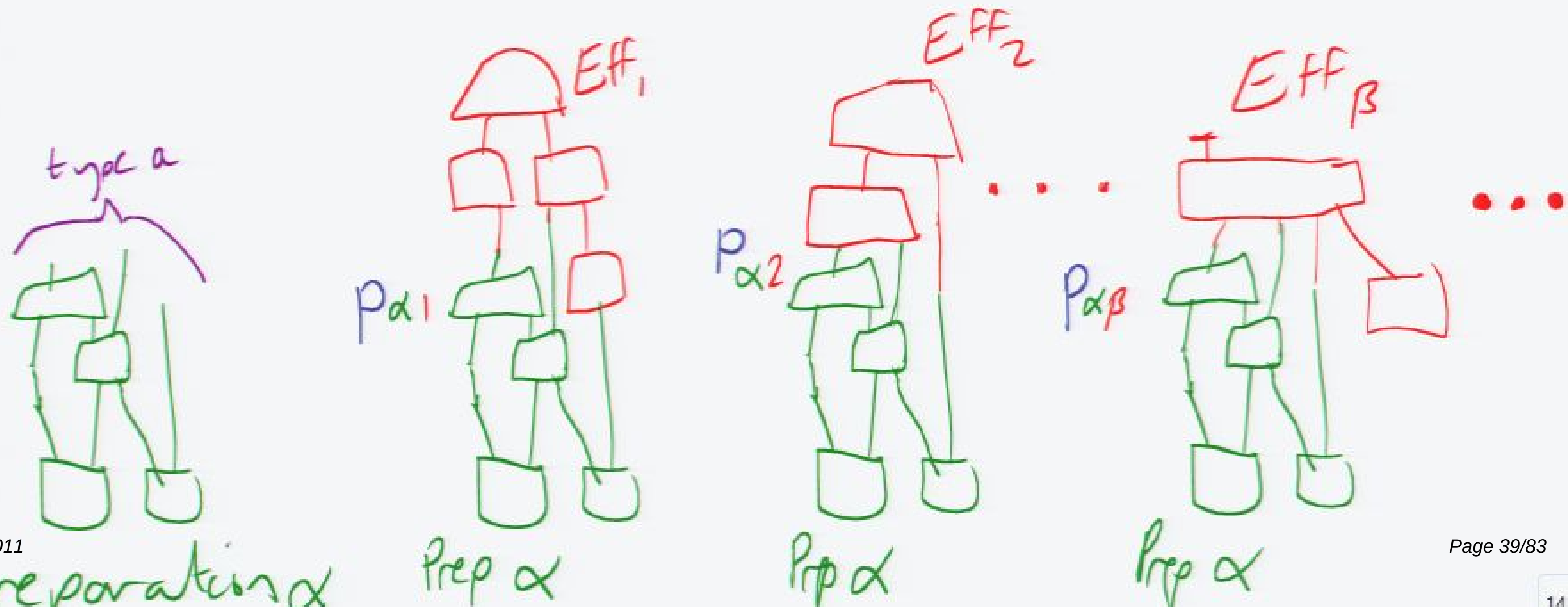
The State

The state of a system of type a associated with a preparation is that thing represented by any mathematical object that can be used to calculate the probability for any circuit with that preparation



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Use linear compression

Fiducial set of effects

$$\text{state}_a^{\alpha} \Longleftrightarrow \begin{pmatrix} \vdots \\ p_{\alpha\beta} \\ \vdots \end{pmatrix} \searrow \quad \nearrow \underline{p}_a^{\alpha} = \begin{pmatrix} \vdots \\ p_{\alpha\beta} \\ \vdots \end{pmatrix} \quad \beta \in \Omega_a$$

$$\boxed{|\Omega_a| \equiv K_a}$$

such that

$$p_{\alpha\beta} = \underline{r}_a^{\beta} \cdot \underline{p}_a^{\alpha}$$

$$\left[\text{c.f.} \qquad p_{\alpha\beta} = tr(\hat{A}_{\beta} \hat{\rho}_{\alpha}) \right]$$

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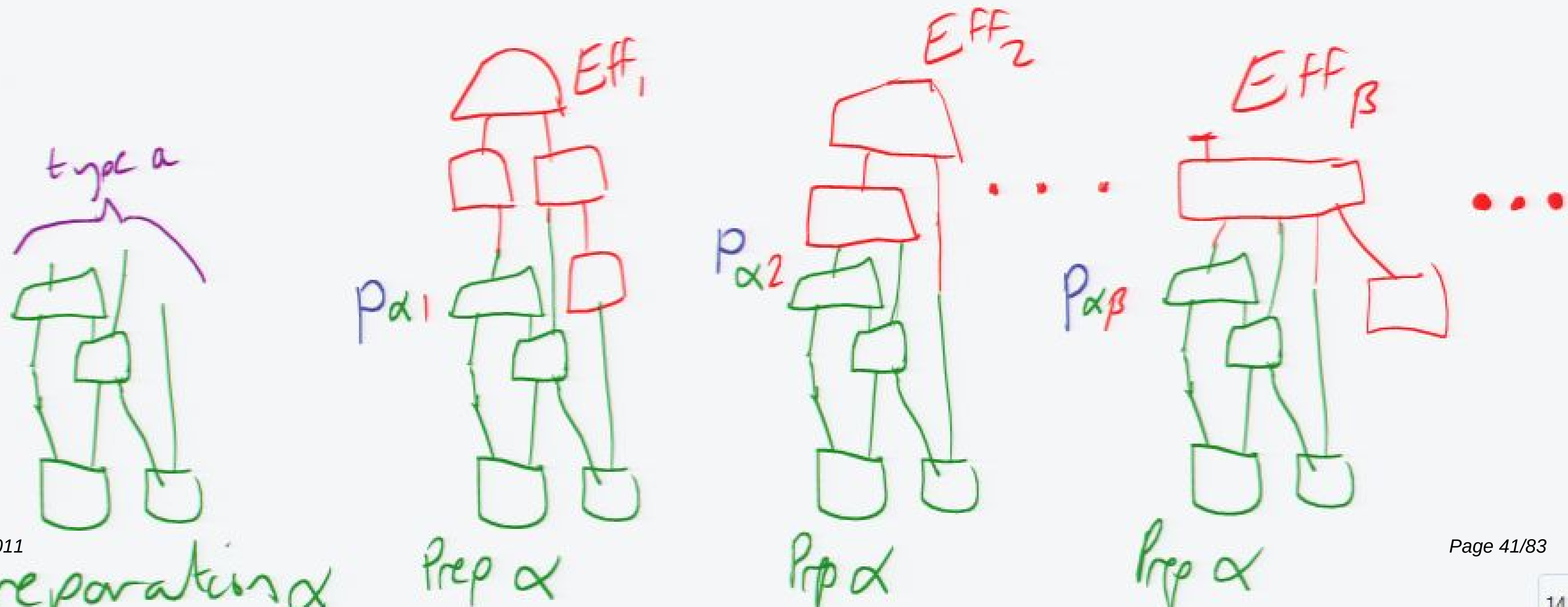

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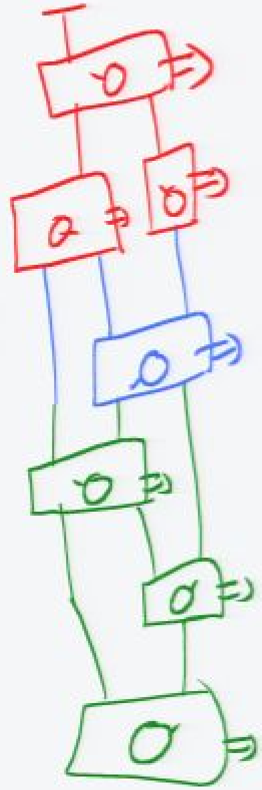
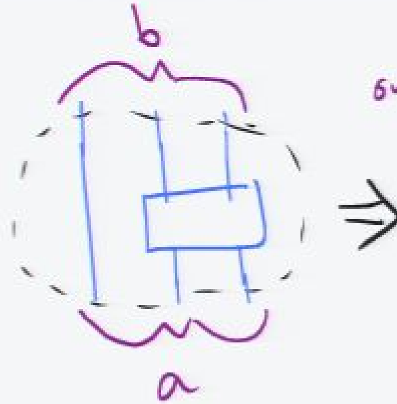
effect γ

transformation β

preparation α





$\Rightarrow$ output ${}^{b}Z^{\beta}_{a}$ setting, outcome; input

$K_b \times K_a$ matrix

$$p_{\alpha\beta\gamma} = r^{\gamma}_{b} \; {}^{b}Z^{\beta}_{a} \; p^{\alpha}_{a}$$

Can write:

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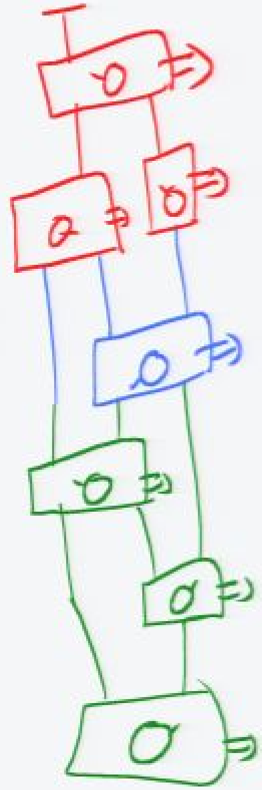

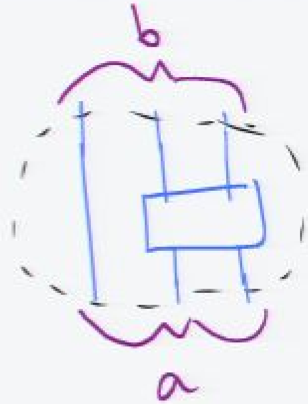

$\Rightarrow$ output $\rightarrow {}^{b}Z^{\beta}_{a}$ ← setting, outcome; ↑ input

$K_b \times K_a$ matrix

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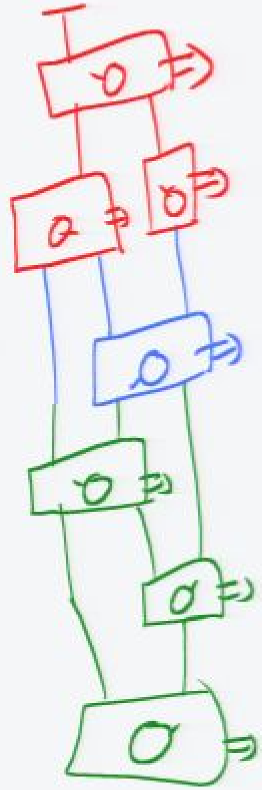

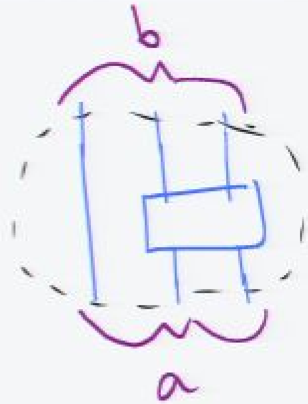

$\Rightarrow \; {}^{b}Z^{\beta}_{a}$

output $\rightarrow b$

setting / outcome $\rightarrow \beta$

input $\rightarrow a$

$K_b \times K_a$ matrix

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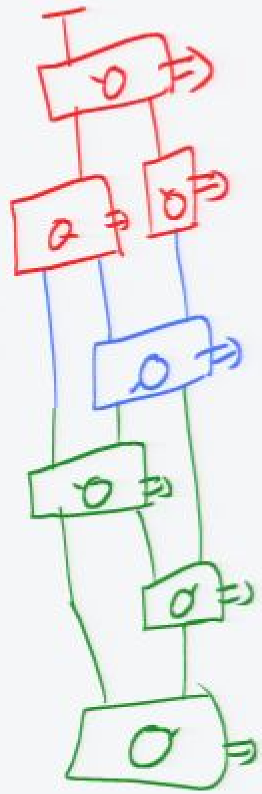

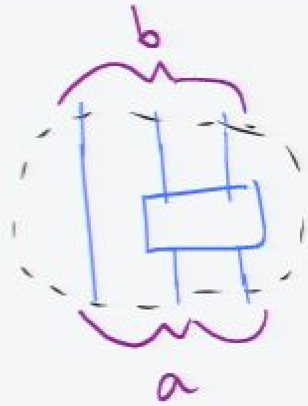

$\Rightarrow$ output $\to$ ${}^{b}Z^{\beta}_{a}$ $\leftarrow$ setting + outcome, input

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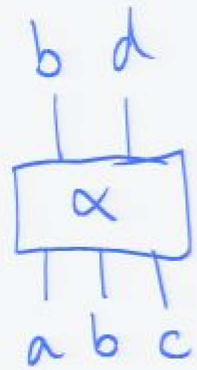
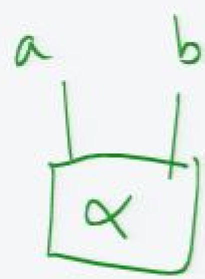
Composites

transformation



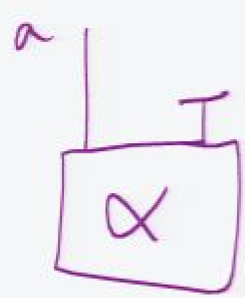
$\ni \; {}^{bd}Z^{\alpha}_{abc}$

preparation



${}^{ab}Z^{\alpha}$ (or p^{α}_{ab})

reduced state



${}^{a}Z^{0-}_{ab}\;{}^{ab}Z^{\alpha}$ (or ${}^{a}Z^{0-}_{ab}\;p^{\alpha}_{ab}$)

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Theorems for composites

Theorem C1: If one system of a composite system is pure then joint probabilities factorise.

Theorem C2: $K_{ab} \geq K_a K_b$

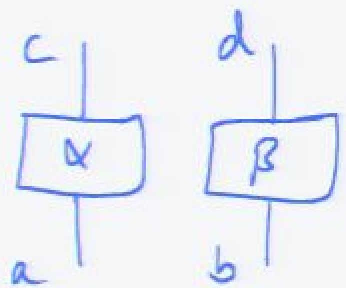
Corollary Quaternionic QT fails

Theorem C3 pure states $\Longleftrightarrow$ uncorrelatable states

Theorem C4 Local tomography possible $\Longleftrightarrow$ $K_{ab} = K_a K_b$

Theories with $K_{ab} = K_a K_b$

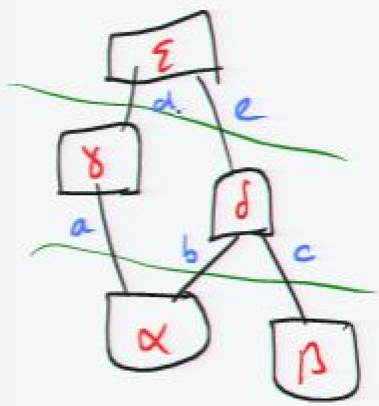
For disjoint transformations can show



$$ {}^{cd}Z^{\alpha\beta}_{ab} = {}^{c}Z^{\alpha}_{a} \otimes {}^{c}Z^{\beta}_{b} $$

This is very useful – can use it to calculate probability for circuit if we know a Z matrix for each operation.

Prob for a circuit.



any foliation

$$p_{\alpha\beta\gamma\delta\varepsilon} = Z^{\varepsilon}_{de} \left({}^{d}Z^{\gamma}_{a} \otimes {}^{e}Z^{\delta}_{bc} \right) \left({}^{ab}Z^{\alpha} \otimes {}^{c}Z^{\beta} \right)$$

(when $K_{ab} = K_a K_b$)

Have operation locality.

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Sketch of Reconstruction

Step 1 $K = N^2$

Proof

$P1 \Rightarrow K = K(N)$

$P1 \Rightarrow K(N+1) > K(N)$

$P2 \Rightarrow K(N_A N_B) = K(N_A) K(N_B)$

By defn $K \geq 1$

$\Rightarrow K = N^r \quad r = 1, 2, 3, \ldots$

$P3 + P4 \Rightarrow K = N^2$

P1 Information: Systems having, or constrained to have, a given information carrying capacity have the same properties.

P2 Composites: Information carrying capacity is additive and local tomography is possible.

P3 Continuity: There exists a continuous reversible transformation between any two pure states.

P4 Simplicity: Systems are described by the smallest number of probabilities consistent with the other axioms

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Step2 basis states pure

Proof if $N=1, K=1$

$P1 \Rightarrow$ basis states pure.

Step 3 The Bloch Sphere

$N=2 \quad K=4$

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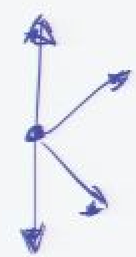
Normalized states in 3dim subspace



$\exists$ at least 3 linearly indep. pure states in subspace

P3 & Shur Auerbach theorem $\Rightarrow \exists$ representation in which vectors acted upon by $SO(3)$ .... Bloch Sphere.

Note no continuous subgroup of $SO(3)$ works so must be $SO(3)$

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Step 4 The qutrit

Consider $N_A = 2, N_B = 2 \quad N = 4$

(By P1 qutrit is given by constraining to $N=3$ case)

Have orthog set

$\underbrace{|11\rangle \ |12\rangle}_{\text{arbitary rotation}} \quad |21\rangle$

$|1\varphi\rangle \quad \underbrace{|1\varphi^\perp\rangle \ |21\rangle}_{\text{arbitary rotation}} \longrightarrow \text{general } |\psi\rangle$

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Step 5 General N

Iterate step 4 $\quad N_A = 2 \quad N_B = 3 \quad$ etc

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Step2 basis states pure

Proof if $N=1$, $K=1$

P1 $\Rightarrow$ basis states pure.

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Step 3 The Bloch Sphere

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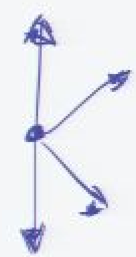

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Step 6 Details...

We get

states $\Leftrightarrow$ $\hat{\rho}$ +ve operator

effects $\Leftrightarrow$ $\hat{A}$ +ve operator

trace rule prob $= tr(\hat{A}\hat{\rho})$

CP maps $\hat{\rho}_b = {}^{b}\$_{a}^{\alpha}(\hat{\rho})$

composition $\mathcal{H}_{ab} = \mathcal{H}_a \otimes \mathcal{H}_b$

Can we get rid of simplicity axiom?

Step 1 $\quad K = N^r$

Put

$$K = x_1 N + x_2 \frac{N(N-1)}{2!} + x_3 \frac{N(N-1)(N-2)}{3!} + \cdots$$

Signature $\vec{x} = (x_1, x_2, x_3, \cdots)$

$x_N = (1, 0, 0, \cdots)$

$x_3 > 0 \Rightarrow x_2 > 0$.

$x_{N^2} = (1, 2, 0, \cdots)$

$x_{N^3} = (1, 6, 6, 0, \cdots)$

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Step 2 ✓

Step 3 (Bloch Sphere)

$N = 2 \quad K = 2^r$

normalised states in $2^r - 1$ dim sub space.

At least $2^r - 1$ lin. indep. pure states

in sub space.

Group must be subgroup of $SO(2^r - 1)$ by Shur Auerbach

> **Axiom 4'** For $N=2$ allow all orthogonal transformations.

$\Rightarrow$ Bloch hypersphere.

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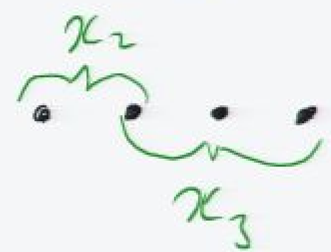

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$N_A = 2 \quad N_B = 2$ show that $x_2 \neq 0 \Rightarrow x_3 = 0$

(any postulated x_3 degree of freedom will "leak" into the x_2 degrees of freedom)

But $x_3 > 0 \Rightarrow x_2 > 0$ Hence $x_3 = 0$

$$\Rightarrow K = N^2$$

then continue as before.

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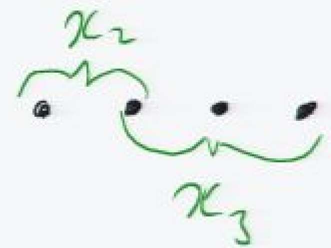

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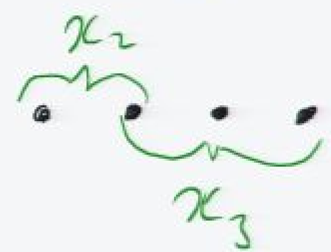

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$N = 2 \quad K = 2^r$

normalised states in $2^r - 1$ dim sub space.

At least $2^r - 1$ lin. indep. pure states in sub space.

Group must be subgroup of $SO(2^r - 1)$ by Shur Auerbach

> **Axiom 4'** For $N=2$ allow all orthogonal transformations.

$\Rightarrow$ Bloch hypersphere.

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Step 4 (qutrit)

$N_A = 2 \quad N_B = 2$ show that $x_2 \neq 0 \Rightarrow x_3 = 0$

(any postulated x_3 degree of freedom will "leak" into the x_2 degrees of freedom)

But $x_3 > 0 \Rightarrow x_2 > 0$ Hence $x_3 = 0$

$$\Rightarrow K = N^2$$

then continue as before.

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Conjecture: Only CProbT and QT consistent with 1.5 postulates:

P1 Information: Systems having, or constrained to have, a given information carrying capacity have the same properties.

P2 Composites: Local tomography is possible.

Then QT given by 3 (2.5) postulates where third postulate is anything not true in CProbT

Evidence.

Spekkens toy model violates P1

few groups in dim 3 other than SO(3)

$N_{AB} \neq N_A N_B$ very unnatural.

Open Problems

1) Get rid of simplicity axiom

2) Prove (or disprove) conjecture.

3) Formulate Prob GR operationally
   (reconstruct it from natural postulates?)

4) FIND A THEORY OF QUANTUM GRAVITY!!!

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Step 4 The qutrit

Consider $N_A = 2, N_B = 2 \quad N = 4$

(By P1 qutrit is given by constraining to $N = 3$ case)

Have ortho'g set

$\underbrace{|11\rangle \ |12\rangle}_{\text{arbitary rotation}} \quad |21\rangle$

$|1\varphi\rangle \quad \underbrace{|1\varphi^{\perp}\rangle \ |21\rangle}_{\text{arbitary rotation}} \longrightarrow \text{general } |\psi\rangle$

P1 Information: Systems having, or constrained to have, a given information carrying capacity have the same properties.

P2 Composites: Information carrying capacity is additive and local tomography is possible.

P3 Continuity: There exists a continuous reversible transformation between any two pure states.

P4 Simplicity: Systems are described by the smallest number of probabilities consistent with the other axioms.

Step 5 General N

Iterate step 4 $\quad N_A = 2 \quad N_B = 3 \quad$ etc

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Wiring constraints

i) one wire at most may be attached to input/output

ii) wires connect output to input of matching type

iii) no closed loops

iv) maybe additional constraints

Wiring Freedoms

i) no constraints between disconnected parts

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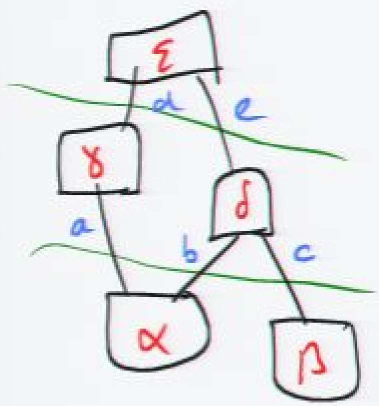
Prob for a circuit.



any foliation

$$p_{\alpha\beta\gamma\delta\epsilon} = Z^{\epsilon}_{de} \left({}^{d}Z^{\gamma}_{a} \otimes {}^{e}Z^{\delta}_{bc} \right) \left({}^{ab}Z^{\alpha} \otimes {}^{c}Z^{\beta} \right)$$

(when $K_{ab} = K_a K_b$)

Have operation locality.