Title: Exact uncertainty, quantum mechanics and beyond

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Abstract: The fact that quantum mechanics admits exact uncertainty relations is used to motivate an ‘exact uncertainty’ approach to obtaining the Schrödinger equation. In this approach it is assumed that an ensemble of classical particles is subject to momentum fluctuations, with the strength of the fluctuations determined by the classical probability density [1]. The approach may be applied to any classical system for which the Hamiltonian is quadratic with respect to the momentum, including all physical particles and fields [2]. The approach is based on a general formalism that describes physical ensembles via a probability density P on configuration space, together with a canonically conjugate quantity S [3]. Quantum and classical ensembles are particular cases of interest, but one can also ask more general questions within this formalism, such as (i) Can one consistently describe interactions between quantum and classical systems? and (ii) Can one obtain local nonlinear modifications of quantum mechanics? These questions will be briefly discussed, with respect to measurement interactions and spin-1/2 systems respectively. 1. M.J.W. Hall and M. Reginatto, “Schroedinger equation from an exact uncertainty principle”, J. Phys. A 35 (2002) 3289 (http://lanl.arxiv.org/abs/quant-ph/0102069). 2. M.J.W. Hall, “Exact uncertainty approach in quantum mechanics and quantum gravity”, Gen. Relativ. Gravit. 37 (2005) 1505 (http://lanl.arxiv.org/abs/gr-qc/0408098). 3. M.J.W. Hall and M. Reginatto, “Interacting classical and quantum systems”, Phys. Rev. A 72 (2005) 062109 (http://lanl.arxiv.org/abs/quant-ph/0509134).

Exact uncertainty, quantum mechanics … and beyond

Michael Hall (Australian National University)
Marcel Reginatto (PTB, Braunschweig)

OUTLINE

- Exact uncertainty relations from QM
- QM from exact uncertainty principle

Intermission: ensembles on configuration space

- Quantum-classical interactions
- Local non-linear modifications of QM
- Future work / Open problems

Exact uncertainty from QM

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For $\psi = P^{1/2} e^{iS/\hbar}$ (particle moving in one dimension)

- define 'classical' and 'nonclassical' momentum components by

$$p_{cl} := dS/dx \, , \qquad p = p_{cl} + p_{nc} \, .$$

Then,

$$<p^2> = < p_{cl}{}^2> + < p_{nc}{}^2> \, , \qquad (\Delta p)^2 = (\Delta p_{cl})^2 + (\Delta p_{nc})^2 \, .$$

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- define the 'Fisher length' measure of uncertainty by

$$(\delta x)^2 := 1/F\,,$$

where F is the Fisher information $(F = \int dx\, (dP/dx)^2 / P)$.

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One then has the 'exact uncertainty relation'

$$\delta x\, \Delta p_{nc} = \hbar/2\,.$$

(moreover, noting $\Delta p \geq \Delta p_{nc}$ and $\Delta x \geq \delta x$, one also has $\Delta x\, \Delta p \geq \hbar/2$)

Going the other way ?

Can one add 'nonclassical' momentum fluctuations to a classical ensemble to obtain QM ? Making uncertainty the core distinguishing feature ?

A classical ensemble on configuration space is described by a probability distribution *P(x,t)* and a Hamilton-Jacobi function *S(x,t)*:

$$\frac{\partial P}{\partial t} + \nabla \cdot \left(P \frac{\nabla S}{m} \right) = 0$$ (continuity equation)

$$\frac{\partial S}{\partial t} + \frac{|\nabla S|^2}{2m} + V(x) = 0$$ (H-J equation)

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Defining the 'ensemble Hamiltonian' by

$$H_c[P,S] := \int dx\, P\, [\, |\nabla S|^2/(2m) + V(x)\,] = \langle E \rangle ,$$

then P and S evolve as canonical conjugates (Bohm, 1952):

$$\frac{\partial P}{\partial t} = \frac{\delta H_c}{\delta S}, \qquad \frac{\partial S}{\partial t} = -\frac{\delta H_c}{\delta P}.$$

Adding nonclassical fluctuations

- Classically one has $p = \nabla S$. But what if this actually only holds on average? If position is statistical, why not momentum too?

  $\therefore$ assume: $p = \nabla S + f,$ $E_x[f] = 0\,.$

- Hence, $E_x[p] = \nabla S$, and the average energy becomes

$$< E > = \int dx\, P\, E_x[\, |\nabla S + f|^2/(2m) + V(x)\,]$$

$$= H_c[P,S] + (2m)^{-1} \int dx\, P\, E_x[f.f] \quad \leftarrow \textit{kinetic energy term}$$

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- To fix the ensemble Hamiltonian, assume the fluctuations:

  * are uncorrelated for independent systems

  $$E_{xy}[f_i^{(1)} f_k^{(2)}] = 0 \;\; for \;\; P(x,y) = P^{(1)}(x)\, P^{(2)}(y)$$

  * scale correctly under linear canonical transformations

  $$f \rightarrow \lambda f \;\; for \;\; x \rightarrow x/\lambda$$

  * are completely determined by the position indeterminacy

  $$E_x[f.f] = \alpha(x, P, \nabla P, \ldots) \qquad \leftarrow \textit{exact uncertainty principle}$$

Obtaining the Schrödinger equation

- The independence, scaling and exact uncertainty assumptions lead directly to a local fluctuation strength of the form

$$E_x[f.f] = C\,|\nabla P|^2/P^2 + \textit{higher order derivative terms},$$

where C is a positive universal constant, i.e., $C \geq 0$, with the same value for all systems.

- Thus, the simplest ensemble Hamiltonian obtainable via the addition of nonclassical momentum fluctuations is of the form

$$H_q[P,S] = H_c[P,S] + C/(2m) \int dx\, |\nabla P|^2/P\ .$$

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- Defining $\hbar := 2\sqrt{C}$, $\quad \psi := P^{1/2}\, e^{iS/\hbar}$,

  these can be rewritten in the 'normal mode' form

  $$i\,\hbar\,\partial\,\psi/\partial t = [\,-(\hbar^2/2m)\,\nabla^2 + V\,]\,\psi\ .$$

Observables

- Observables are suitable generators of motion, $G[P,S]$:

  $$P \rightarrow P + \varepsilon\, \delta G/\delta S, \qquad S \rightarrow S - \varepsilon\, \delta G/\delta P.$$

- Eg, the generator of translations, inducing the variations $P \rightarrow P(x\text{-}\varepsilon)$, $S \rightarrow S(x\text{-}\varepsilon)$, is the 'ensemble momentum' observable

  $$\Pi := \int dx\, P\ \nabla S\,.$$

  Defining the 'position' observable $X := \int dx\, P\, x$, one finds

  $$\{X_j, \Pi_k\} = \int dx\, [(\delta X_j/\delta P)\ (\delta \Pi_k/\delta S) - (\delta X_j/\delta S)\ (\delta \Pi_k/\delta P)] = \delta_{jk}\,.$$

  These act linearly with respect to ψ - as does angular momentum – thus obtain the observables representing the Galilean group.

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- Generators that preserve linearity/'normal modes', have the form

  $$Q_A[P,S] = \int dx\, \psi^* A\, \psi = < A >,$$

  with $\{Q_A, Q_B\} = Q_{[A,B]/i\hbar} = (1/i\hbar) < [A, B] >$.

  Thus, the Poisson bracket algebra of such observables is isomorphic to the usual quantum commutator algebra.

Some pluses and minuses

- No a priori assumptions concerning the existence of a universal constant $\hbar$ with units of action, nor a complex wave function ψ satisfying a linear equation, nor some algebra of observables. These are all consequences.
- A single key concept distinguishing QM from CM: *exact uncertainty* - an intrinsic uncertainty in the configuration is assumed to determine a corresponding degree of uncertainty in the momentum.
- Can be generalised straightforwardly to any classical system with Hamiltonian *quadratic* in the momentum terms (including fields).
- Gives a unique operator ordering in the case of position-dependent kinetic terms. For example, for a position-dependent mass $m(x)$ one obtains the corresponding unique 'sandwich' ordering

$$\frac{p^2}{2m(x)} \rightarrow \hat{p}\,\frac{1}{2m(x)}\,\hat{p}$$

- Gives a new 'configuration ensemble' way of thinking about physical systems.
- Can be applied to rigid rotators to obtain angular momentum quantization, but requires representation theory to restrict to finite Hilbert spaces – 'special pleading')
- Starts from a classical system – one can motivate the form of $H_c[P,S]$ but, even so, quantum mechanics should not depend on the assumption of classical systems.

INTERMISSION (not really)

What are the ingredients of 'configuration ensembles'?

- A 'configuration space' of measurement outcomes (discrete or continuous or mixed)
- The fundamental physical quantity describing a physical system is a probability density P over the space of outcomes [eg, P_j, $P(x)$, $P(x,j)$].
- This quantity evolves according to an action principle

$\therefore$ Have a canonically conjugate quantity S, an ensemble Hamiltonian $H[P,S]$, and a Poisson bracket algebra $\{ , \}$.

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  $\therefore$ Have a canonically conjugate quantity S, an ensemble Hamiltonian $H[P,S]$, and a Poisson bracket algebra $\{\,,\,\}$.
- Generators of canonical transformations are *suitable* functions of P and S, $G[P,S]$, which form a set closed with respect to $\{\,,\,\}$.
- Two fundamental restrictions: P must remain positive and normalised under *physical* canonical transformations ($P_i \geq 0$, $\Sigma_i P_i = 1$):

$$\partial G/\partial S_i = 0 \text{ when } P_i=0, \qquad G[P_i, S_i+c] = G[P_i,S_i].$$

Examples

- Quantum mechanics:

  * Start with Hilbert space indexed by complete basis set $\{|a>\}$, $\Sigma_a |a><a| = 1$; define $P(a)$ and $S(a)$ via $<a|\psi> = P^{1/2} e^{iS/\hbar}$.

  * Generators $Q_A[P,S] := <\psi|A|\psi>$ for Hermitian A.

  * One has $\{Q_A, Q_B\} = Q_{[A,B]/i\hbar}$.

- Classical mechanics

  * Start with configuration space indexed by x.

  * Generators $C_f[P,S] := \int dx\, P(x)\, f(x, \nabla S)$ for phase space function $f(x,p)$.

  * One has $\{C_f, C_g\} = C_{\{f,g\}}$.

- Mixtures of ensembles give density operators, or general phase space distributions, thermodynamic mixtures, etc.

What is the value of the 'configuration ensemble' formalism?

- A simple basic structure including both QM and CM, based on (i) statistical indeterminacy, and (ii) an action principle

- Useful for thinking about QM and other physical systems, without bringing in an unconscious interpretational bias (eg, why assume that generators also yield *expectation values*? Why assume generators have a *multiplicative* structure $A*B$? Why assume superselection rules are *linear* constraints?) – 'deconstructing' quantum theory

- Can more easily think outside the box – classical-quantum interactions, nonlinear QM – perhaps get around no-go theorems

- May lead somewhere new …

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END OF INTERMISSION

Classical-quantum interactions

- Of interest (i) as a possible approach to measurement (making the 'unanalyzable interaction' analyzable); (ii) as a consistent approximation for treating some physical aspects classically and others quantum mechanically (eg, nuclei vs electrons, spacetime vs matter)

Classical-quantum interactions

- Of interest (i) as a possible approach to measurement (making the 'unanalyzable interaction' analyzable); (ii) as a consistent approximation for treating some physical aspects classically and others quantum mechanically (eg, nuclei vs electrons, spacetime vs matter)
- Supposedly not possible – various suggested approaches do not, eg, conserve normalisation or positivity; have correct classical limit; have back reaction; or only allow limited set of interactions – supported by several 'no-go' theorems

"*The standard paradigm 'quantum system is measured by a classical apparatus' is untenable, while a quantum matter can be consistently coupled only with a quantum gravity*"

Daniel Terno, 2004

"Nevertheless, it should be kept in mind that our conclusions only apply under the assumptions made, and so more general forms of mixed quantum-classical systems cannot be ruled out"

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- The formulation of configuration ensembles **is** sufficiently general to allow the consistent description of quantum-classical interactions! (due to 'algebraic' assumptions of no-go theorems not being applicable)

- The observables include both Q_A and C_f, for ensembles on a joint configuration space – and a generalised Ehrenfest's theorem holds for particles coupled by a potential $V(x_c,x_q)$: $(d/dt)\langle x_c\rangle = m_c^{-1}\langle p_c\rangle$, $(d/dt)\langle p_c\rangle = -\langle \partial V/\partial x_c\rangle$, etc.

- Measurement example here. Marcel will treat gravity example

Measurement of spin

- Classical 1-d particle coupled to quantum spin-1/2 system
- Configuration (x, α) describes position at x and spin $\alpha = \pm 1$ in z-direction.
- Measurement coupling analogous to $\hat{H}_{int} = \kappa(t)\, p\, \sigma_z$:

$$H[P,S] = \Sigma_\alpha \int dx\, P(x,\alpha) \left[(\partial S(x,\alpha)/\partial x)^2/(2m) + V(x) \right]$$
$$+ \kappa(t)\, \Sigma_\alpha \int dx\, \alpha\, P(x,\alpha)\, (\partial S(x,\alpha)/\partial x)\,.$$

- For short interaction can ignore first term, and integrate to give

$$P(x,\alpha,T) = P(x-\alpha K,\alpha,0), \quad S(x,\alpha,T) = S(x-\alpha K,\alpha,0).$$

- If initially independent, i.e., $P(x,\pm,0) = P_C(x)\, w_\pm$, then

$$P(x,T) = w_+ P_C(x-K) + w_- P_C(x+K)\,.$$

- For observed 'pointer position' $x=a$, can update the ensemble via Bayes theorem, with 'collapse' to correlated position-spin.

Local nonlinear modifications of QM ?

"at least for the moment I have given up the problem; I simply do not know how to change quantum mechanics by a small amount without wrecking it altogether"

Steven Weinberg (1992)

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Problem 'Locality' – what is done over here shouldn't affect what happens over there – 'no go' theorems

Cause? 'Wedging' – trying to squeeze in extra generators of the motion while keeping everything else the same (classical mechanics is local and

Are all rotational bits qubits ?

Definition *A "robit" has a set of two-valued observables (eg, with values* $\pm\hbar/2$*), which generate rotations in 3-space:*

- ensemble described by $P \equiv (P_+, P_-)$, $S \equiv (S_+, S_-)$
- generators described by $L_j[P,S]$, $j=1,2,3$
- $\{L_j, L_k\} = \Sigma_\alpha [(\partial L_j/P_\alpha)(\partial L_k/S_\alpha) - (\partial L_j/S_\alpha)(\partial L_k/P_\alpha)]$
  $= \varepsilon_{jkl} L_l$. (and preserve positivity of P)

Question Is it possible to have two *local* robits that are not equivalent to two qubits ?

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Question Is it possible to have two *local* robits that are not equivalent to two qubits ?

Answer 1: No! – ∴ a derivation of qubit properties from locality assumptions is possible

Answer 2: Yes! – ∴ local nonlinear modifications of QM are possible

(∴ an interesting question!)

Remarks on what 'locality' means

- Rotation of one robit should not affect statistics of the other robit (evolution locality)
- Measurement on one robit should not affect statistics of the other robit (update locality – 'collapse')
- Initially uncorrelated robits should remain uncorrelated under local rotations

The first and third are not too difficult to satisfy - note the first corresponds to finding generators L_j, M_k satisfying

$$\{L_j, L_k\} = \varepsilon_{jkl} L_l,\ \{M_j, M_k\} = \varepsilon_{jkl} M_l,\ \{L_j, M_k\} = 0.$$

on the 8-dimensional 'phase space' $(P_{\pm\pm}, S_{\pm\pm})$.

⚒ The (non?) existence of nonquantum solutions, further satisfying update locality and positivity, is work-in-progress.

Future work / Connections

- Locality and nonlinearity – compatible?
- Classical-quantum gravity
- Classical-quantum scattering (eg, Coulomb scattering) – despite Galilean invariance, the centre of mass and relative motions do not decouple
- Motivation for the homogeneity property $G[\lambda P,S] = \lambda G[P,S]$? – allows definition of the 'weak value' of G , as $G_w := \delta G/\delta P$.

- For discrete configuration spaces, have a fixed constant N as per Hardy's approach, and his continuity axiom and $N_{AB}=N_A N_B$ are automatically satisfied for canonical transformations connected to the identity. However, the notion of a fixed constant K, corresponding to the number of parameters needed to specify a mixture, is not natural in the configuration space formalism – could one perhaps instead work with the number of parameters needed to specify an allowed canonical transformation?
- Canonical transformations connect different ensembles (P,S) and (P',S'), and hence different measurement statistics (generalising unitary transformations in QM and phase space transformations in CM). This means that the conditional probability $p(M'=j|M=k)$ may be calculated by assuming that $P_j=\delta_{jk}$, $S_j=0$ in the configuration space for measurement M, applying the canonical transformation linking this configuration space to the configuration space for M', and calculating the probability P'_j. When does this conditional probability satisfy Fivel's or Niestegge's axioms?

Conclusions

- The Schrödinger equation, and generalisations thereof, can be derived from an exact uncertainty principle
- The underlying formalism of ensembles on configuration space is of interest in its own right, providing a general description of physical systems based on the core concepts of (i) a probability density P on configuration space, which (ii) evolves according to an action principle
- Configuration ensembles have the advantage of a direct focus on empirical content (probabilities), and can help avoid interpretational confusion/bias (eg, physical relevance of various 'no go' theorems)
- Configuration ensembles can be used to consistently describe interacting classical and quantum systems
- Configuration ensembles provide the possibility of local nonlinear modifications of QM