Title: Quantum Theory from Entropic Inference

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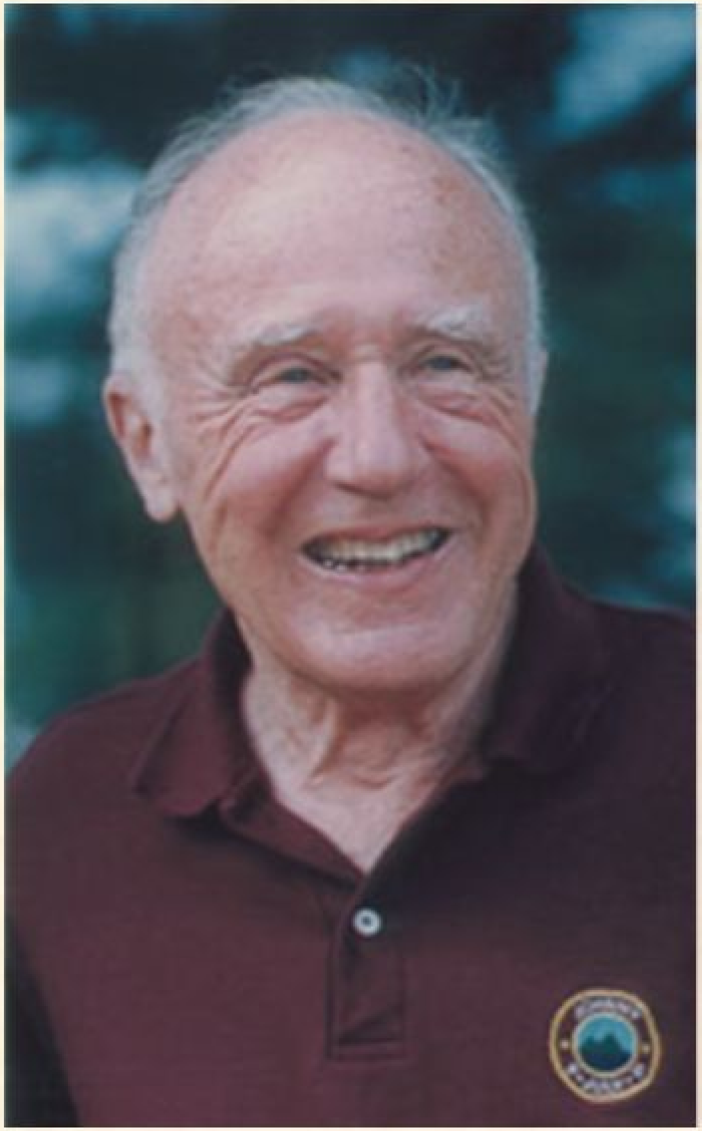
Abstract: Non-relativistic quantum theory is derived from information codified into an appropriate statistical model. The basic assumption is that there is an irreducible uncertainty in the location of particles so that the configuration space is a statistical manifold with a natural information metric. The dynamics then follows from a principle of inference, the method of Maximum Entropy: entropic dynamics is an instance of law without law. The concept of time is introduced as a convenient device to keep track of the accumulation of changes. The resulting formalism is close to Nelson's stochastic mechanics. The statistical manifold is a dynamical entity: its (information) geometry determines the evolution of the probability distribution which, in its turn, reacts back and determines the evolution of the geometry. As in General Relativity there is a kind of equivalence principle in that “fictitious” forces – in this case diffusive “osmotic” forces – turn out to be “real”. This equivalence of quantum and statistical fluctuations – or of quantum and classical probabilities – leads to a natural explanation of the equality of inertial and “osmotic” masses and allows explaining quantum theory as a sophisticated example of entropic inference. Mass and the phase of the wave function are explained as features of purely statistical origin. Recommended Reading: arXiv:0907.4335 "From Entropic Dynamics to Quantum Theory" (2009)

Quantum Theory from Entropic Inference

Ariel Caticha
Department of Physics
University at Albany - SUNY

Reconstructing Quantum Theory
PI 2009



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No test of these views looks like being someday doable, nor more interesting and more instructive, than a derivation of the structure of quantum theory... No prediction lends itself to a more critical test than this, that every law of physics, pushed to the extreme, will be found statistical and approximate, not mathematically perfect and precise.

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The Challenge of "Law without Law":

We can ask ourselves if it is not absolutely preposterous to put into a formula anything at first sight so vague as *law without law* and *substance without substance*. How can we hope to move forward with no solid ground at all under our feet?

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"...is often called Bayesian inference"
"...supplemented by the notion of information entropy"

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What would the laws of physics look like if they were rules for processing information?

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General Relativity

Entropic Inference: What is Information?

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To update from old beliefs to new beliefs when new information becomes available.

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new beliefs → the posterior $p(x)$

information → ??

Information is what induces a change in beliefs.

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the prior $q(x)$ (old beliefs)

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constraints (information)

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Entropic Inference: What is Information?

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Information is what induces a change in beliefs.

Information is what constrains rational beliefs.

An analogy from mechanics:

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initial state
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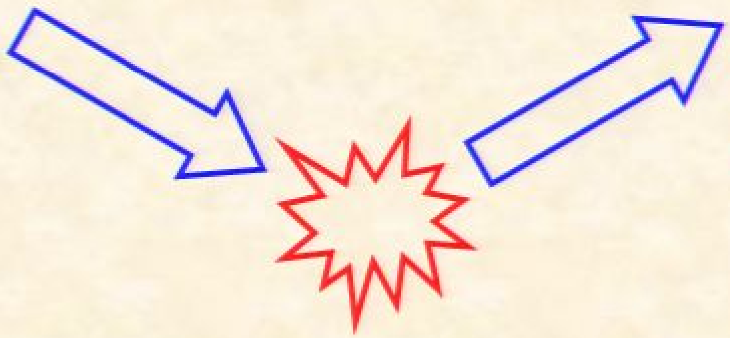

final state
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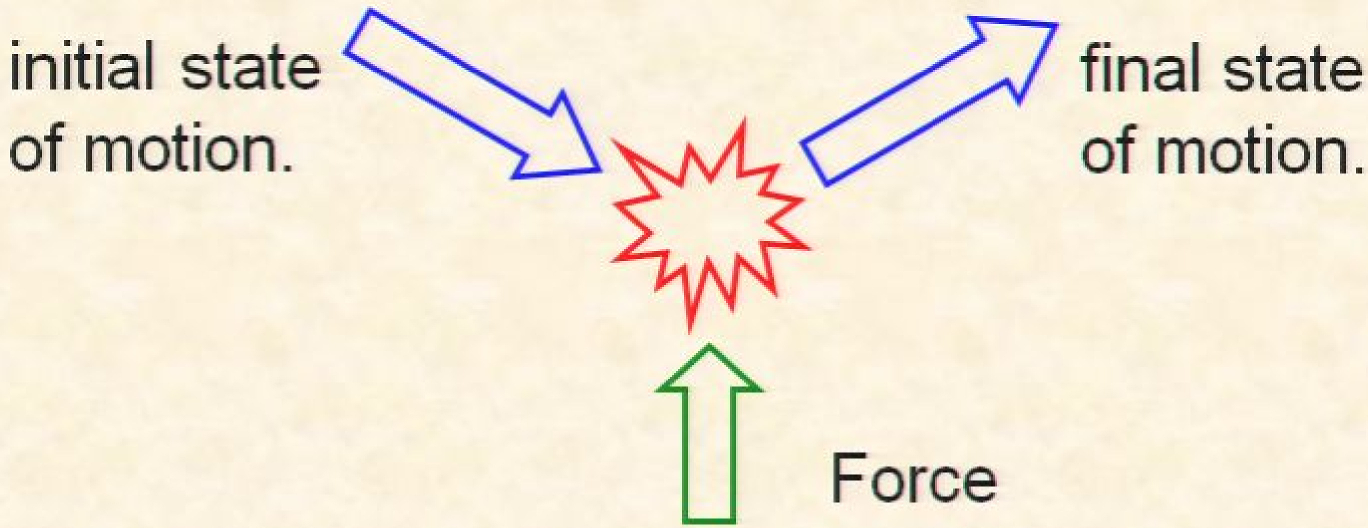

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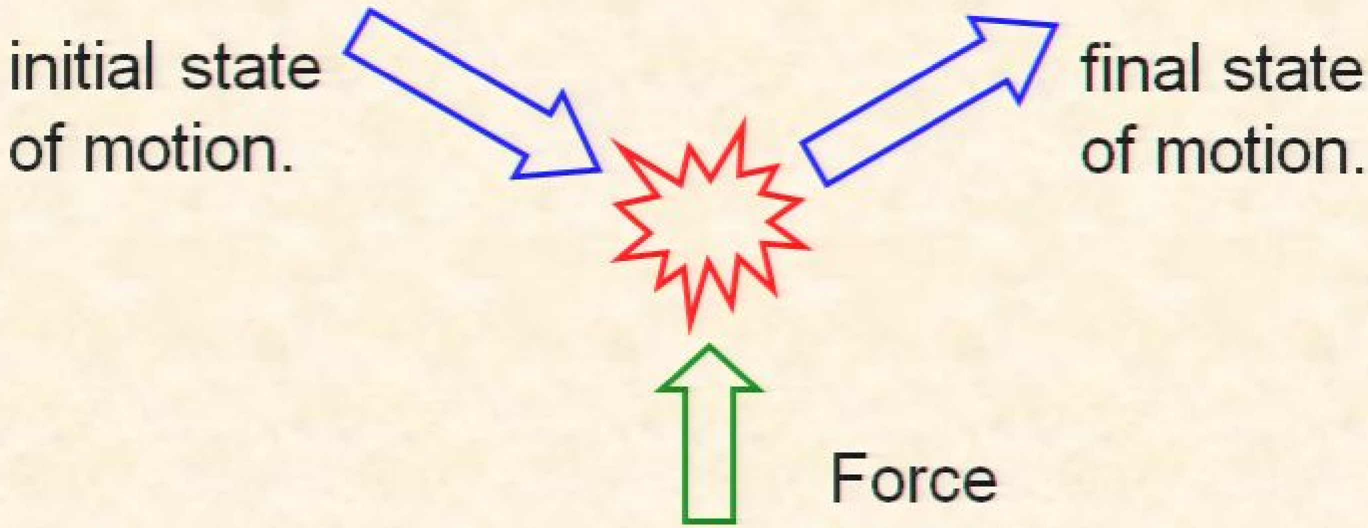

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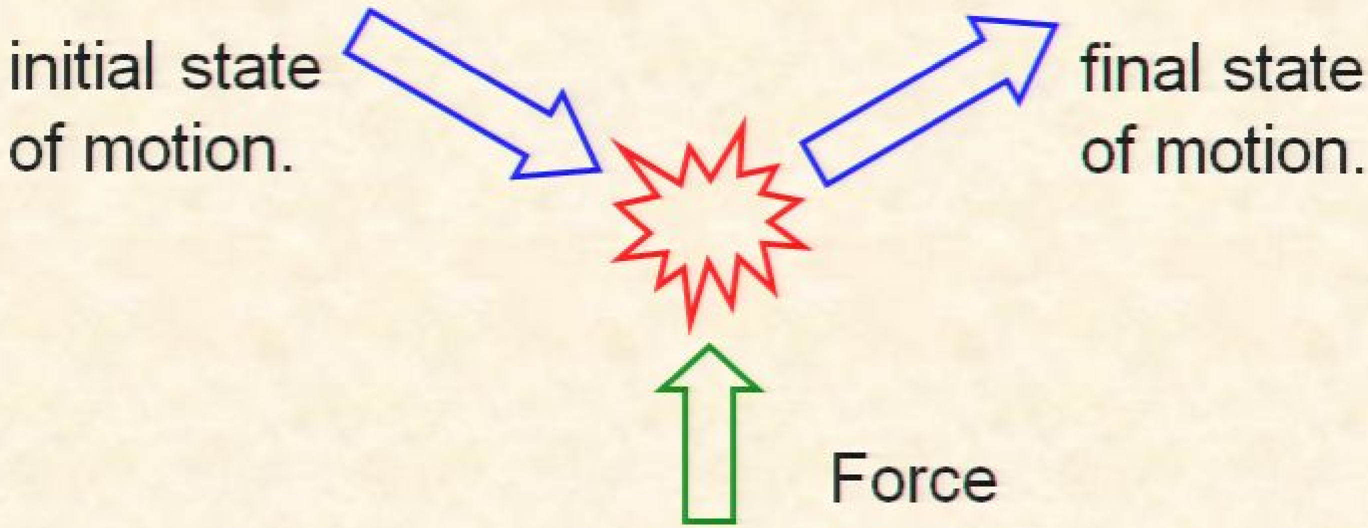

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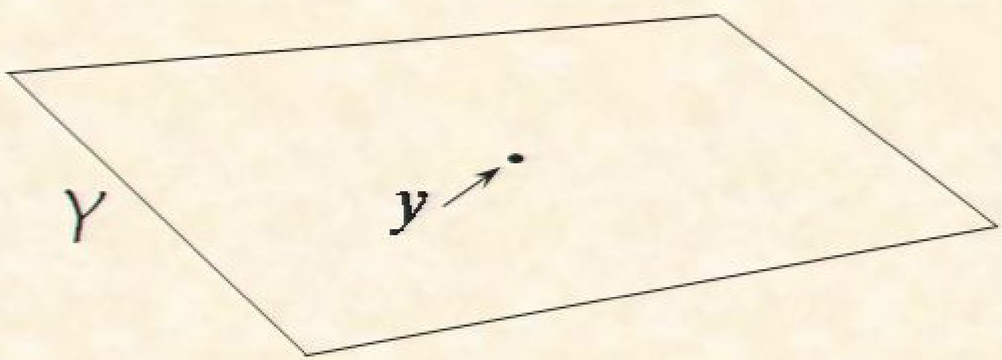

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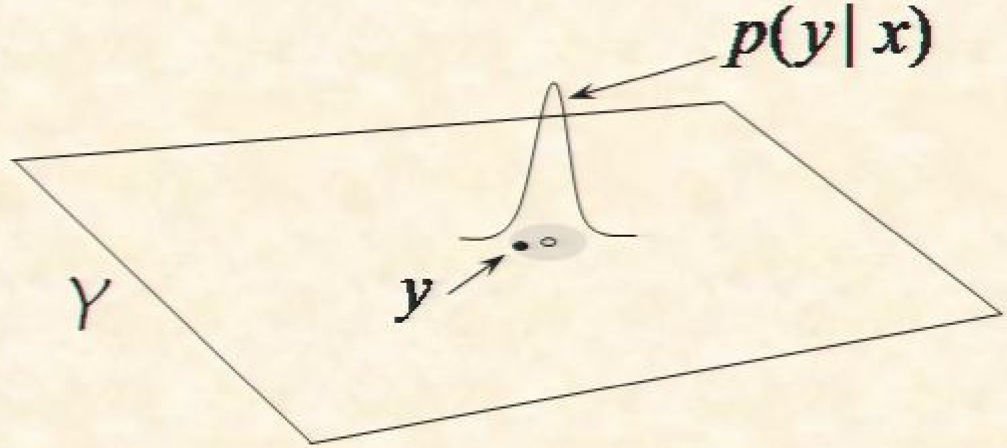

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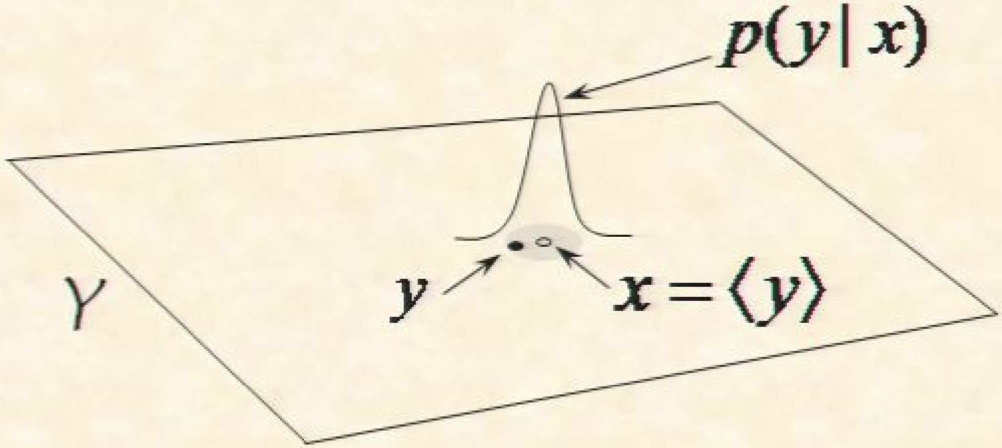

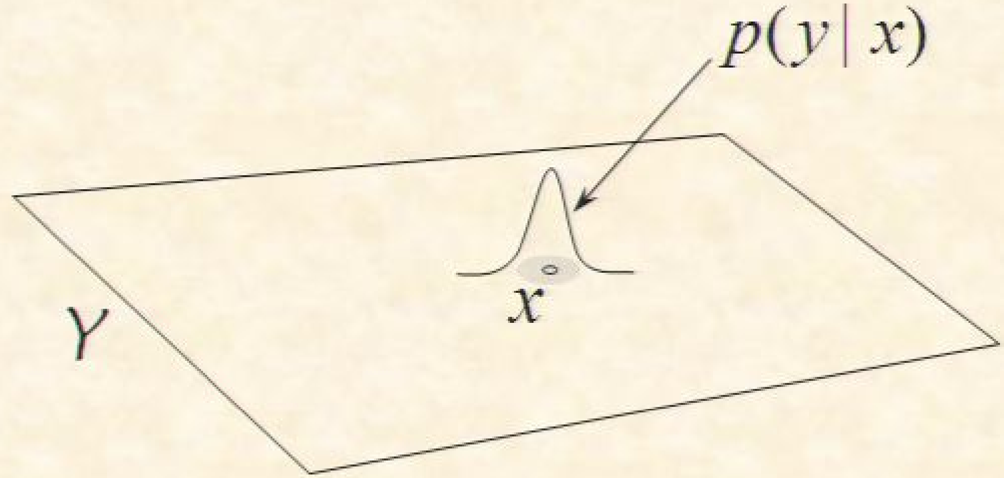
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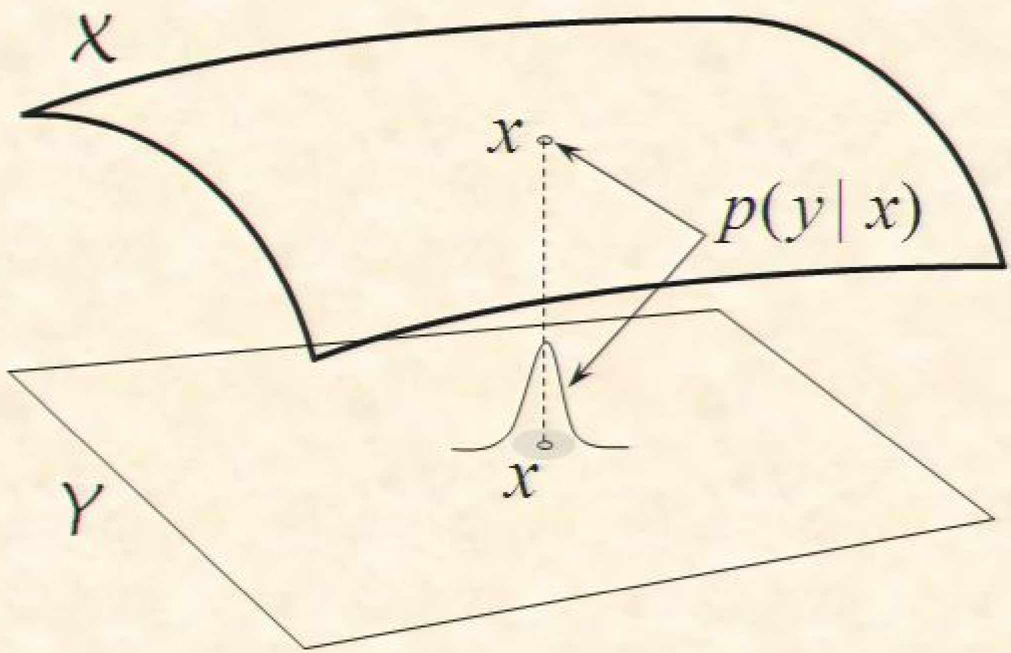
X
x
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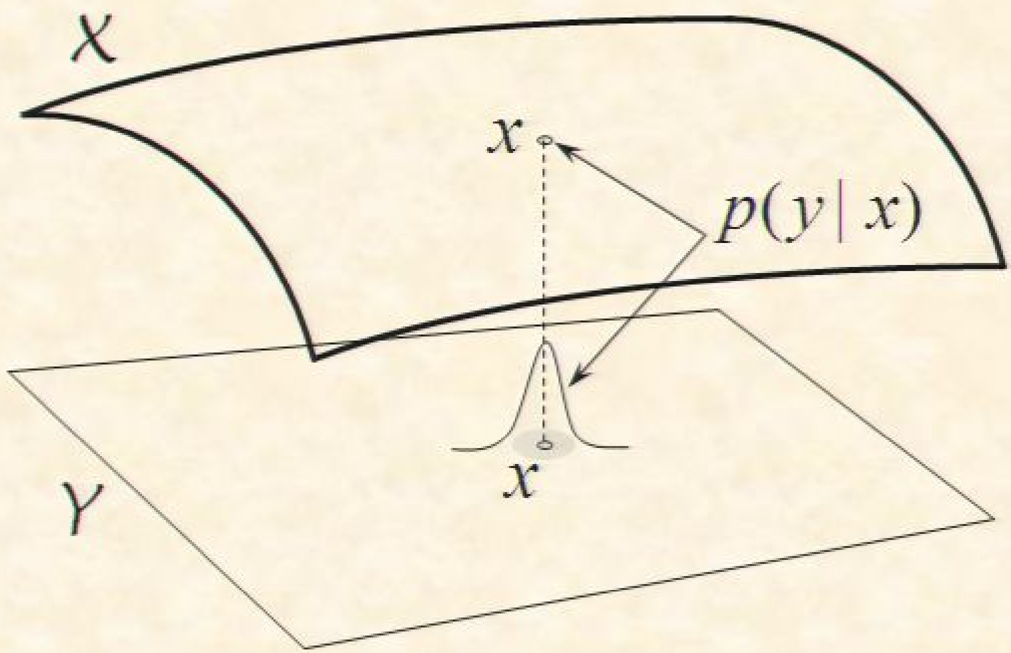

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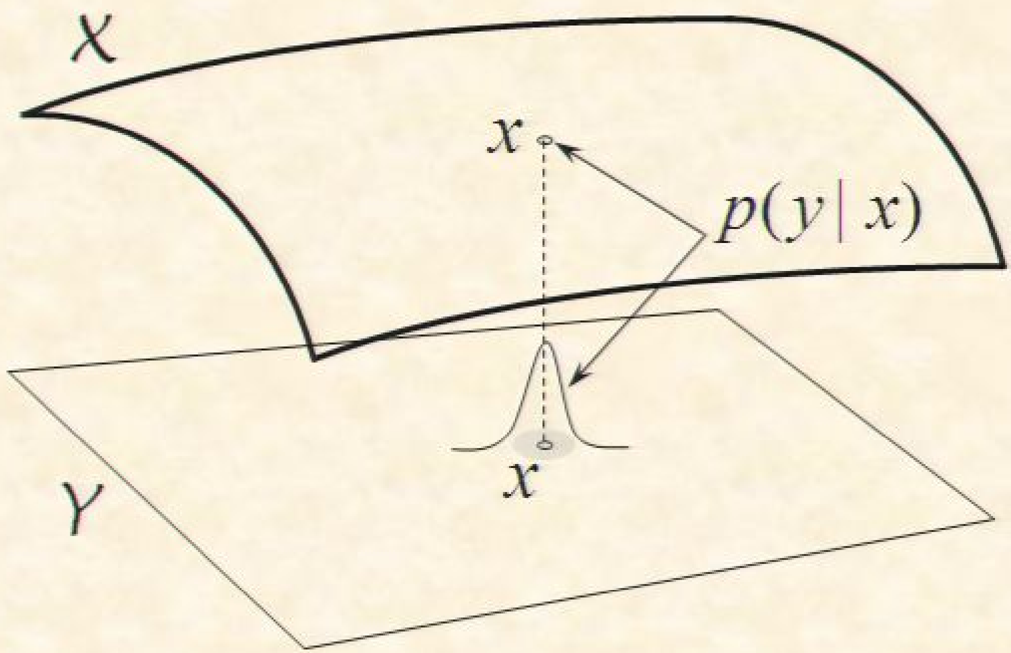

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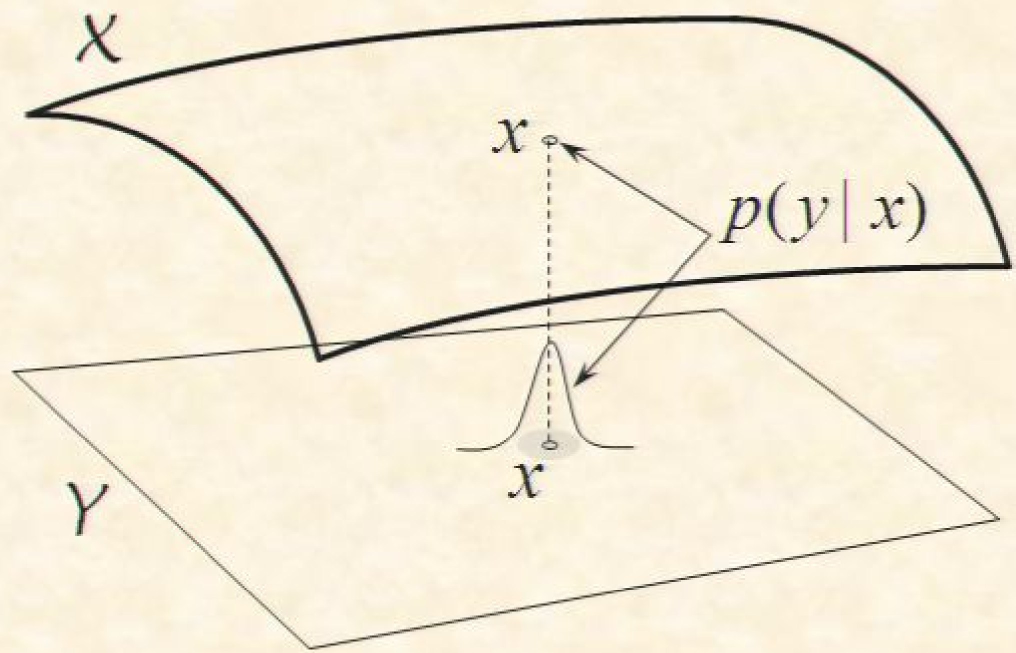
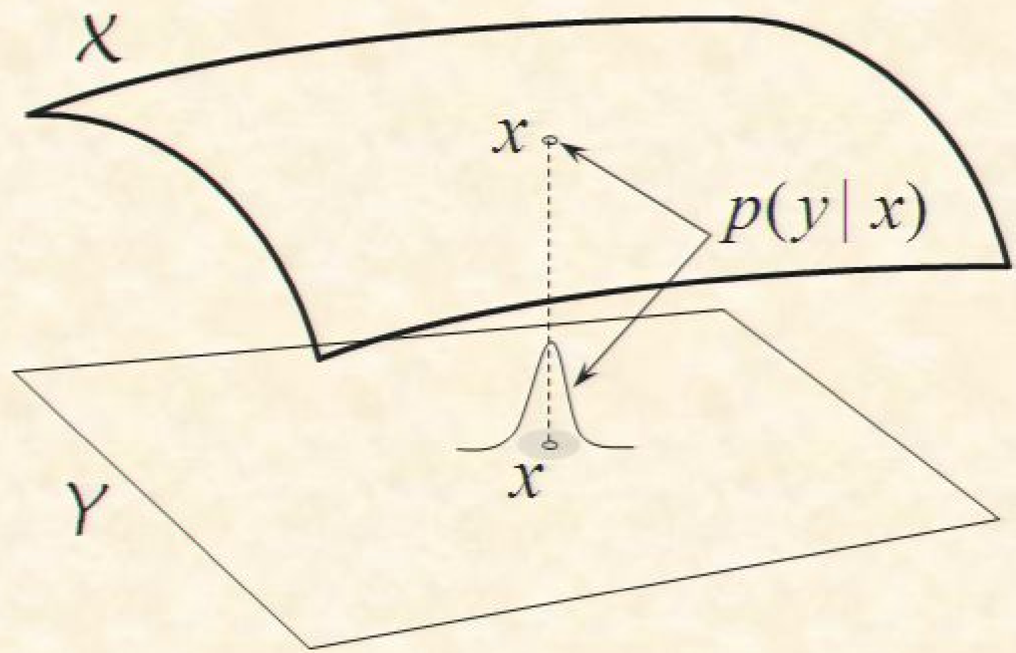
$\mathcal{X}$ configuration space



Y "physical" space "microstates"

X configuration space



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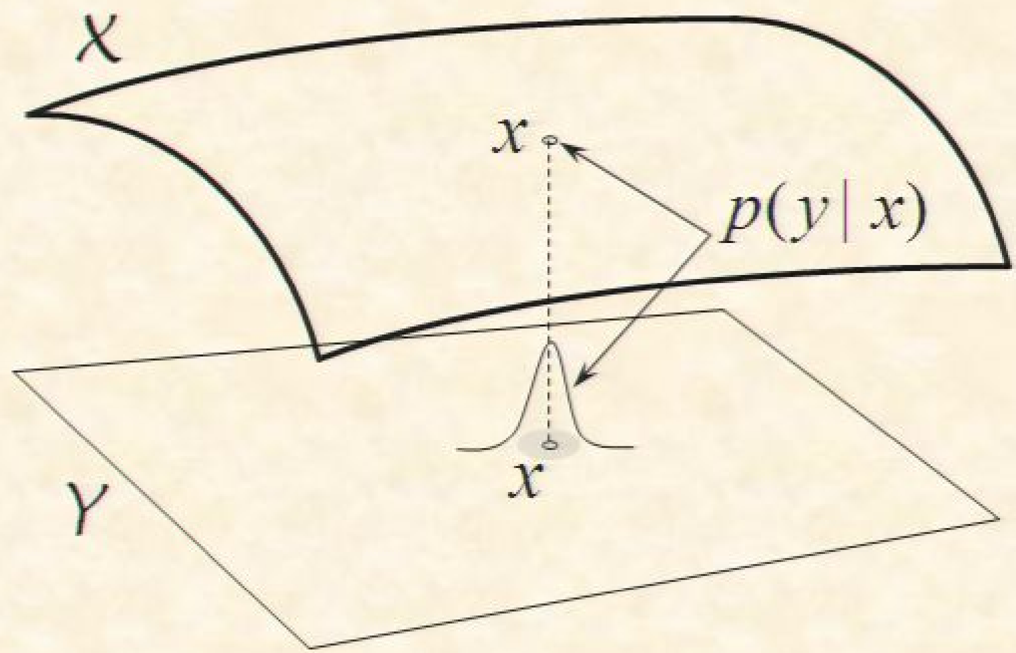

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A "point" is not just a dot:

Configuration space is a statistical manifold.

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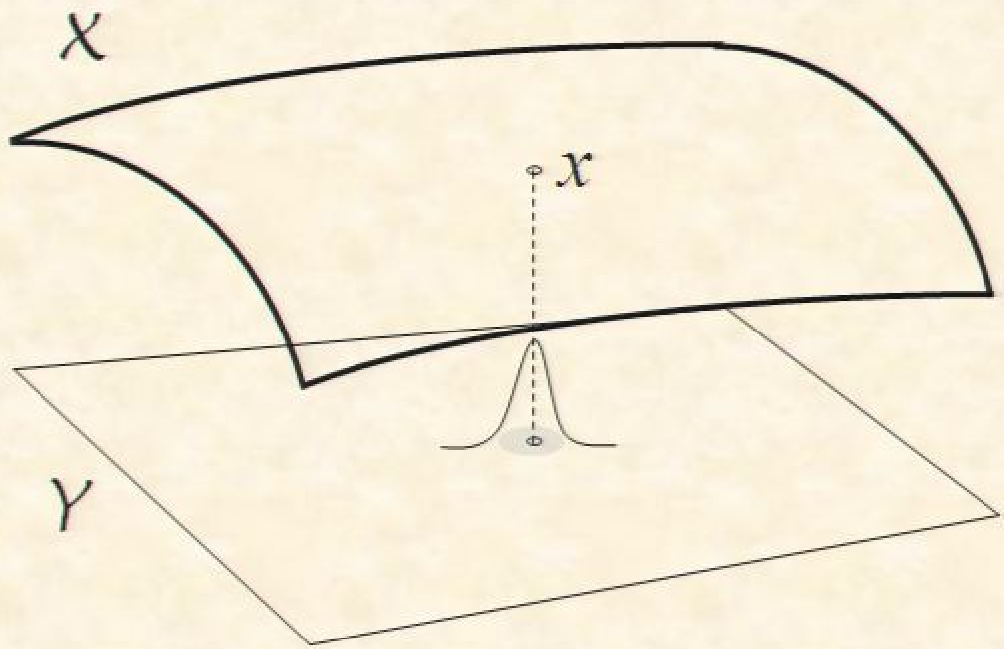
X
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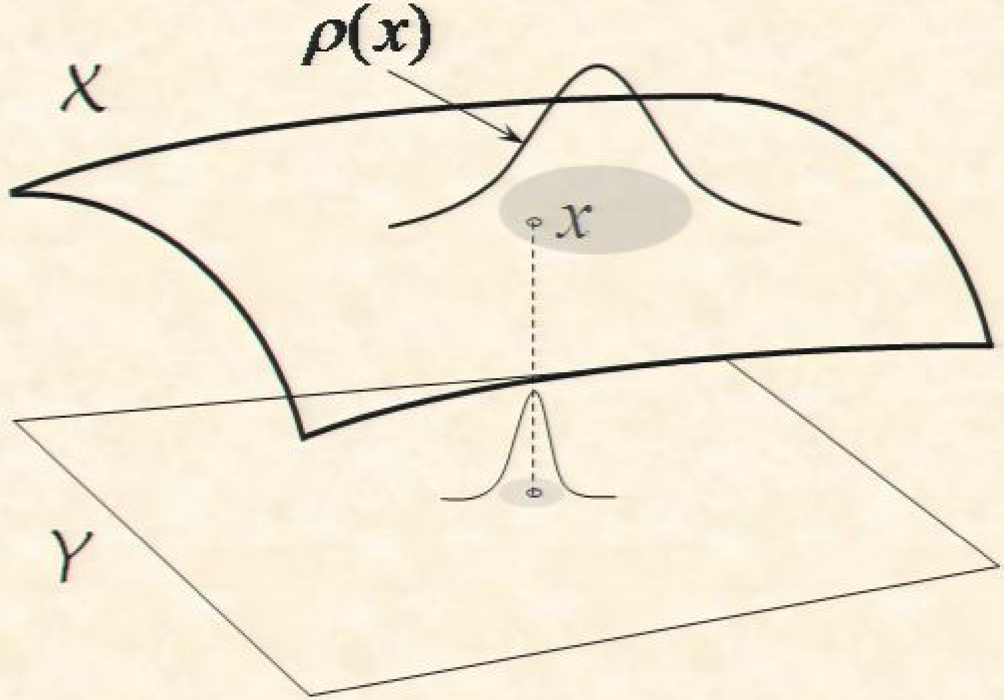

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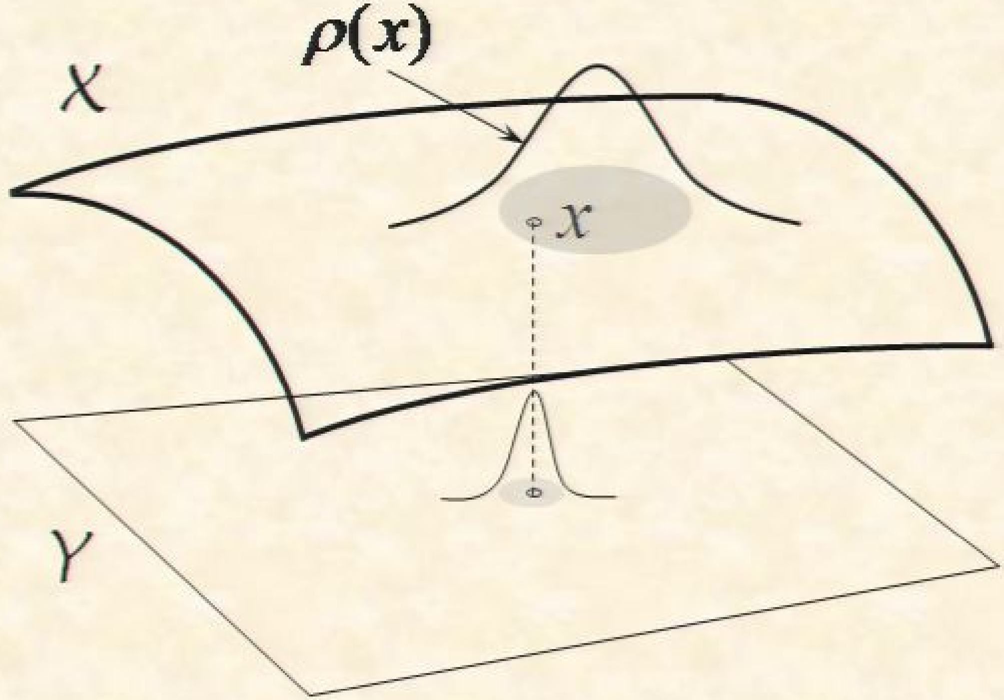
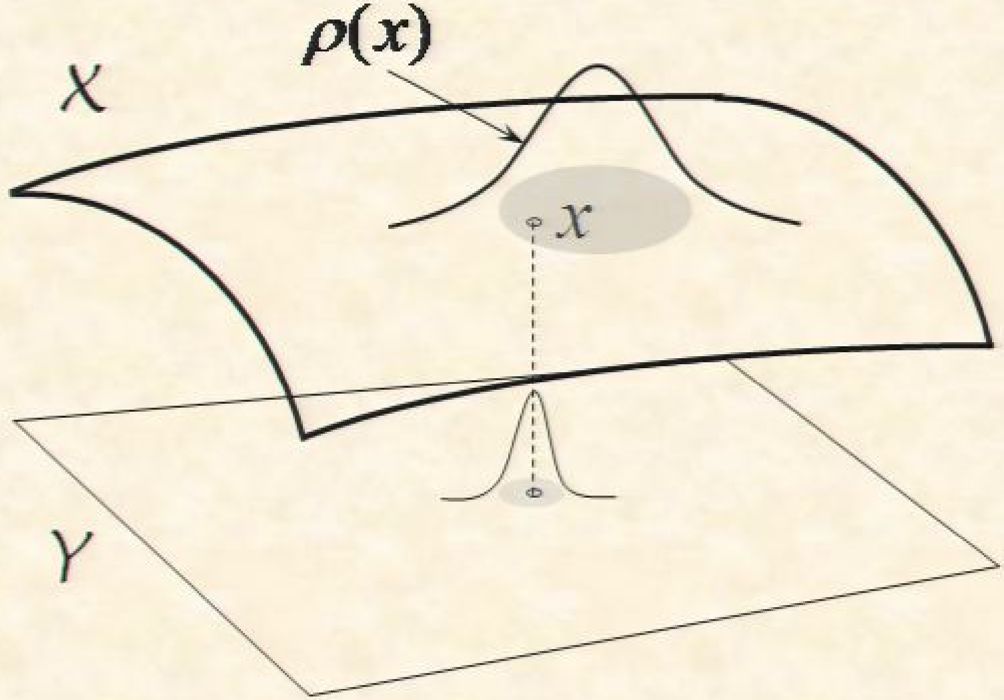


Y "physical" space "microstates"



Y	"physical" space	"microstates"
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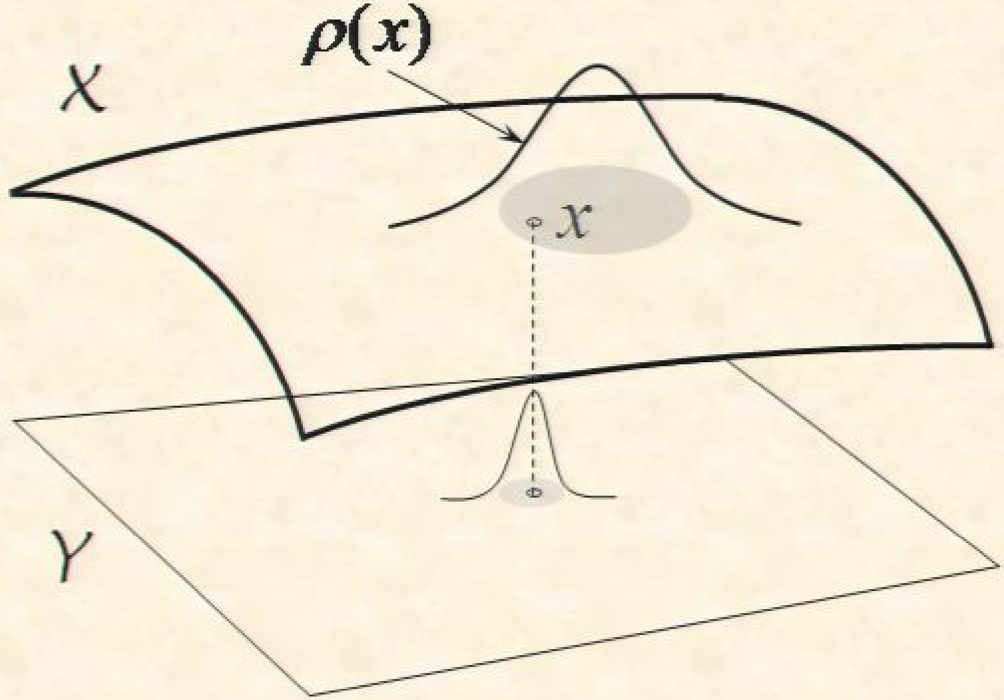

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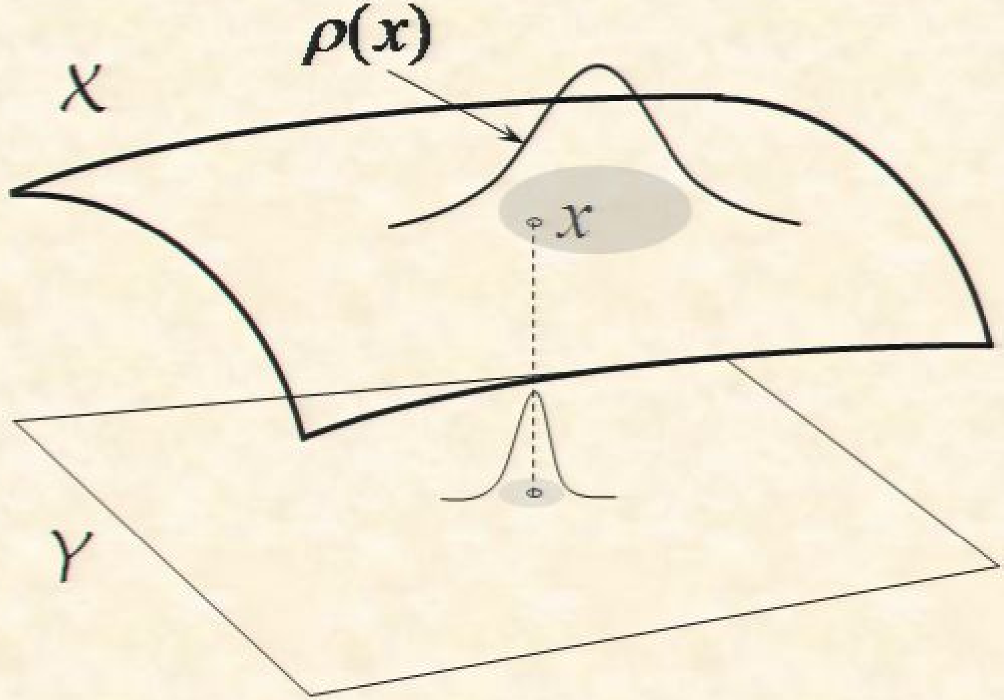

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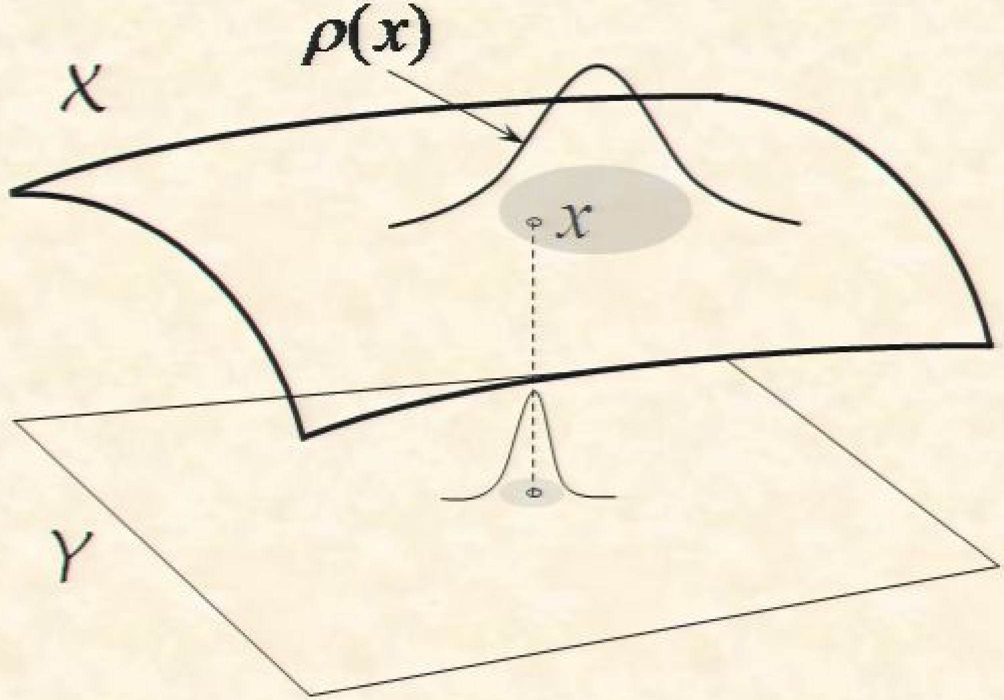

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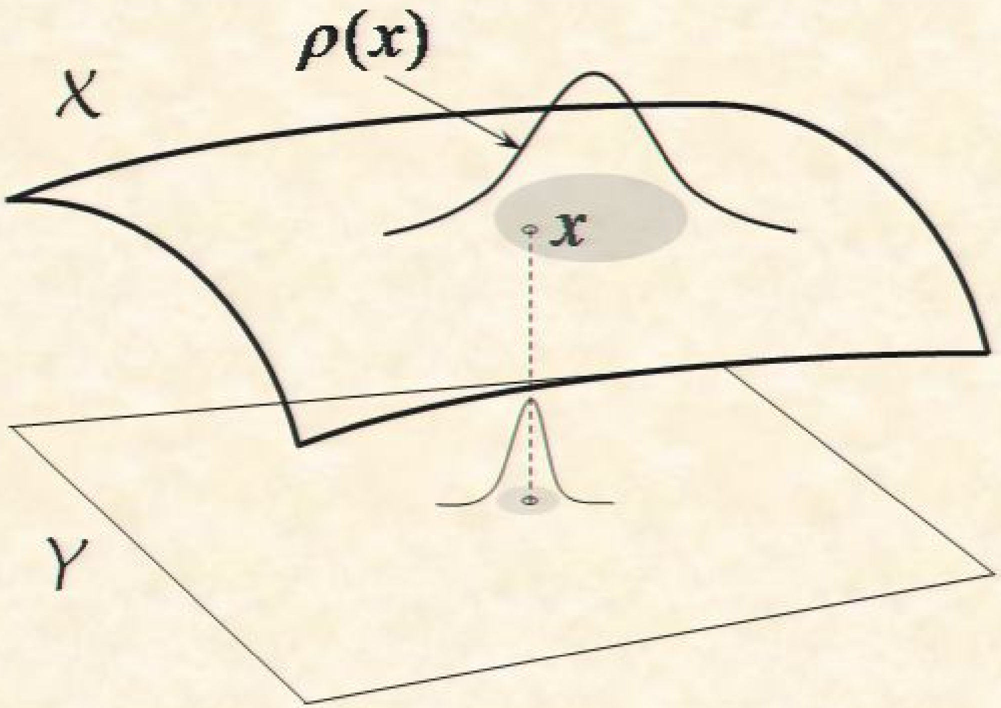

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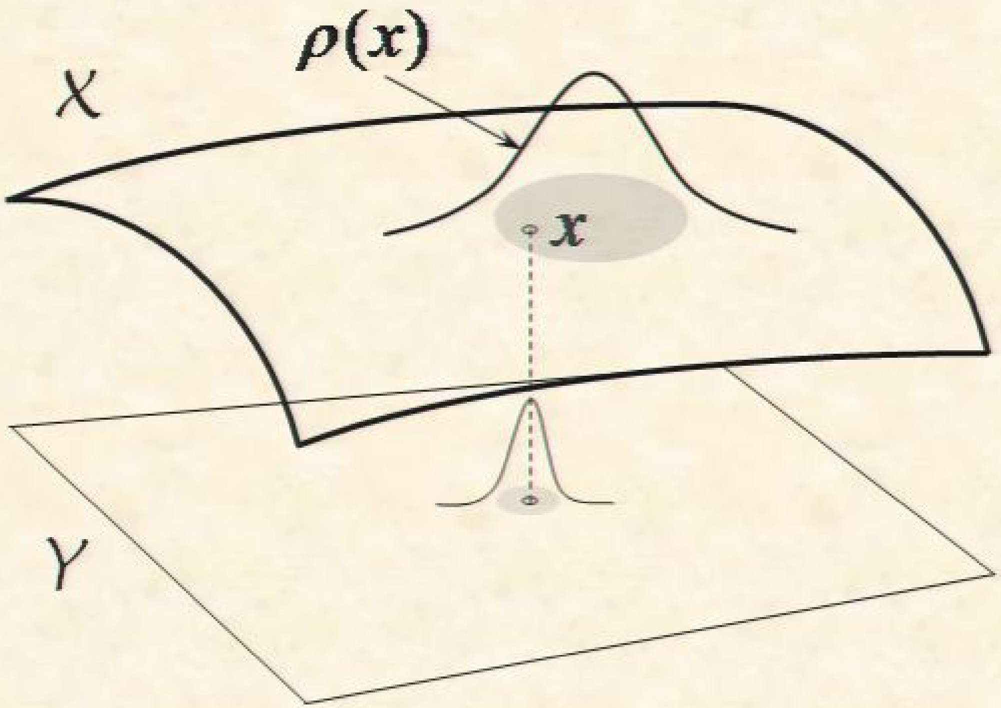

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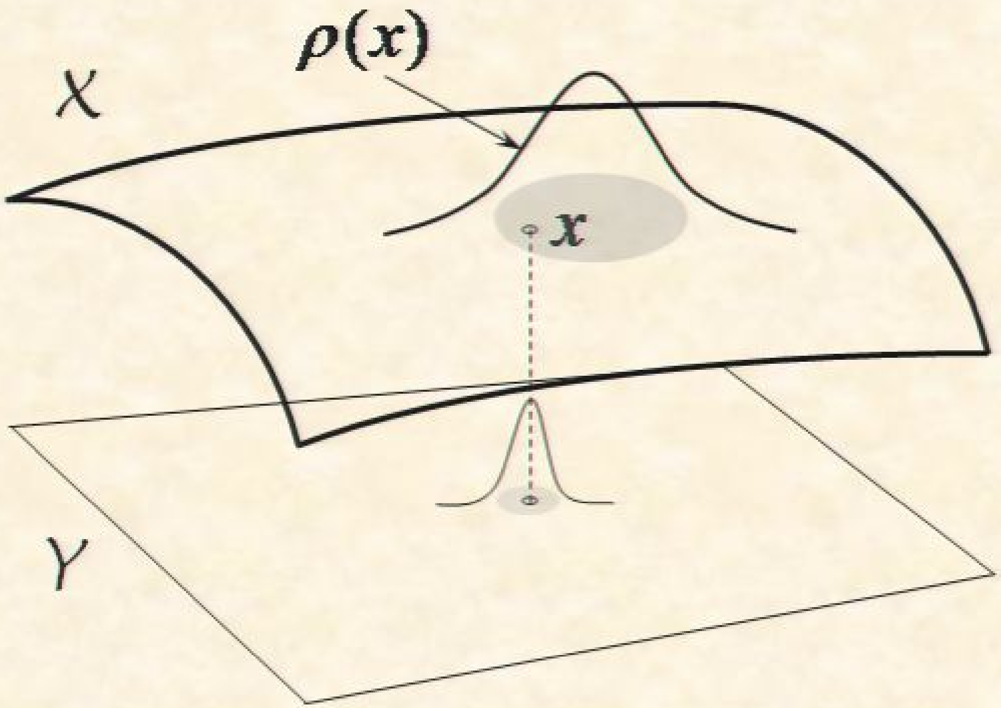

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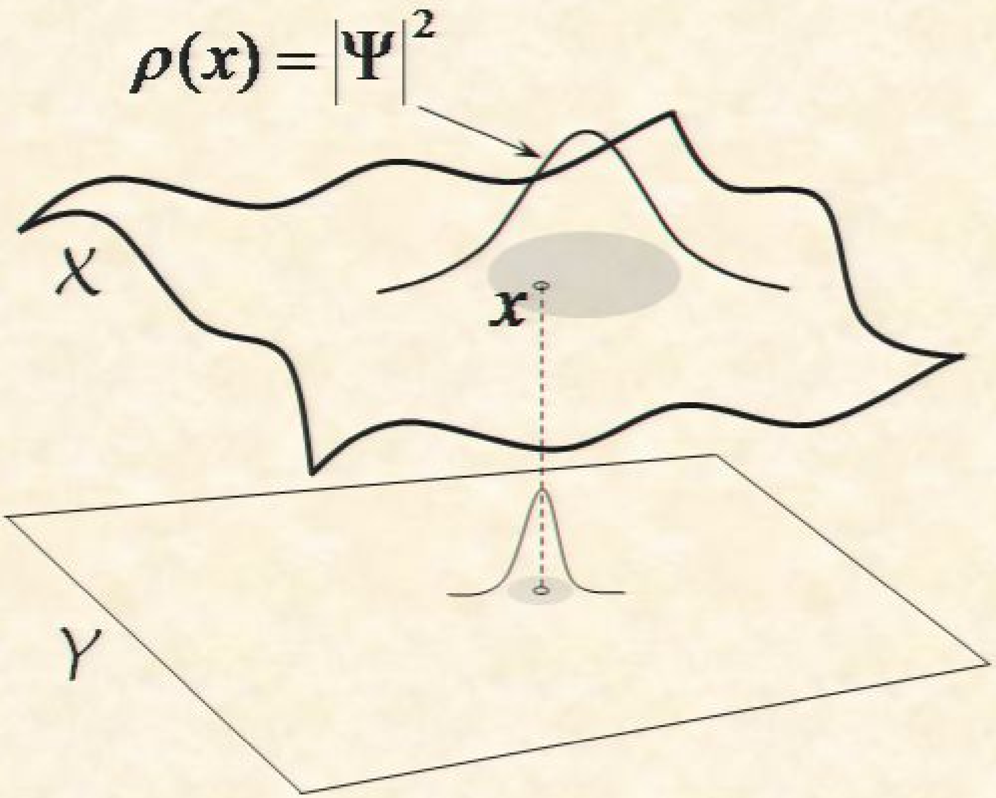

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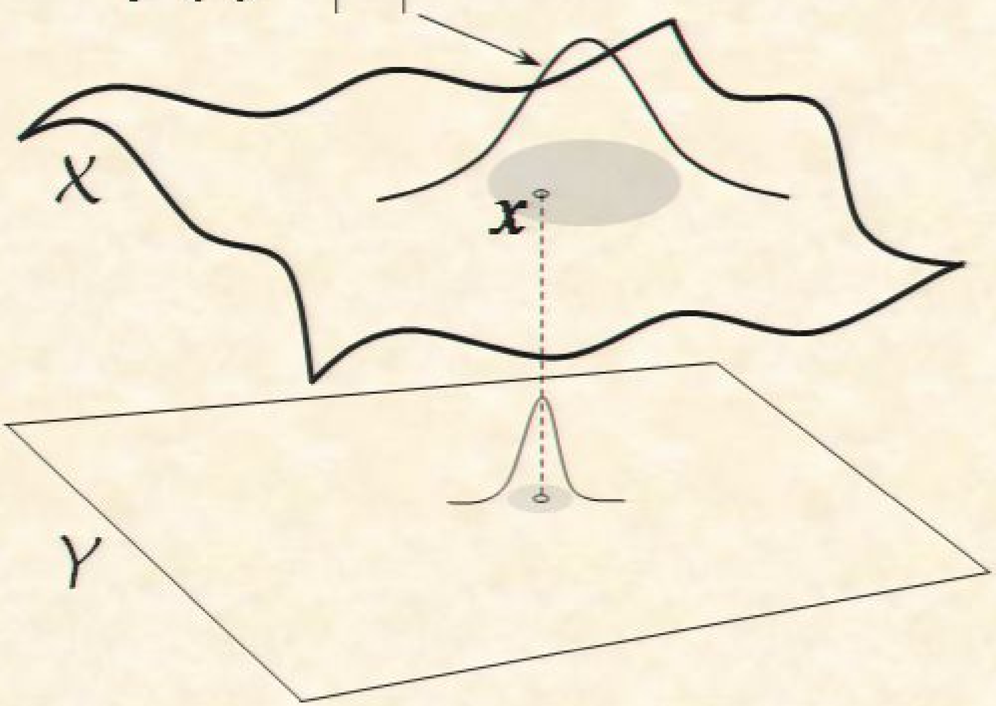

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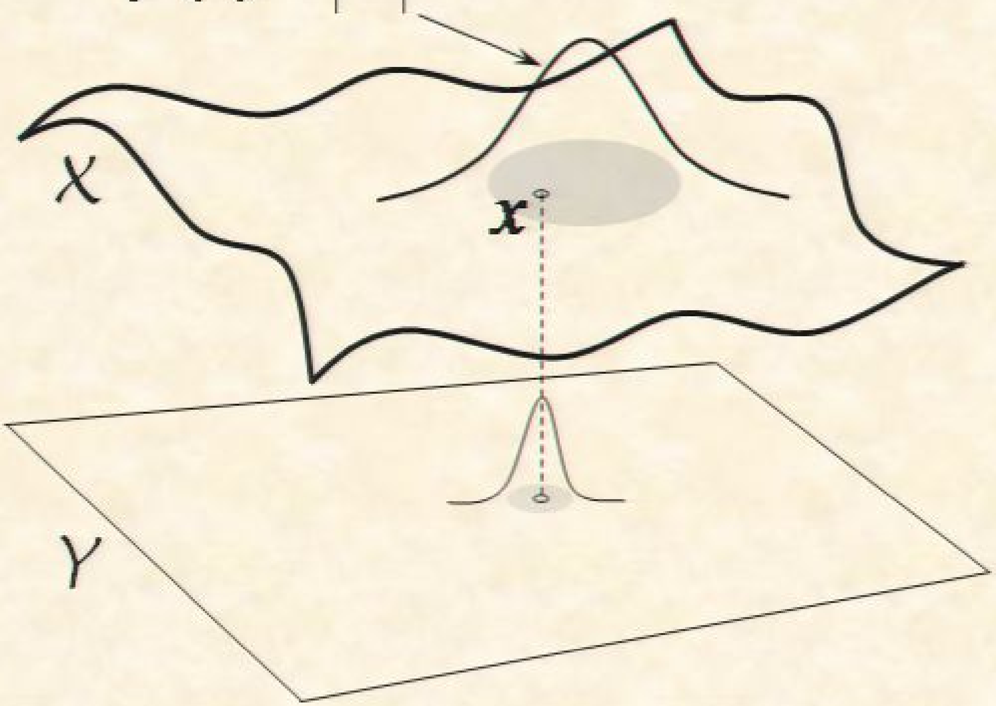

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it explains the equality of inertial and osmotic masses

The Gravitational Equivalence Principle:

We accept the equivalence of gravitational with the "fictitious" forces that arise in accelerated frames because

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and allows a geometrical explanation of gravity.

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On Mass and Phase:

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