Title: The quantum logical reconstruction from Rovelli's axioms and its limits

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Abstract: What belongs to quantum theory is no more than what is needed for its derivation. Keeping to this maxim, we record a paradigmatic shift in the foundations of quantum mechanics, where the focus has recently moved from interpreting to reconstructing quantum theory. We present a quantum logical derivation based on Rovelli's information-theoretic axioms. Its strengths and weaknesses will be studied in the light of recent developments, focusing on the subsystems rule, continuity assumptions, and the definition of observer. Publications: * "Reconstruction of quantum theory," British Journal for the Philosophy of Science, 58, 2007, pp. 387-408. * "Information-theoretic principle entails orthomodularity of a lattice," Foundations of Physics Letters 18 (6), 2005, pp. 563-572. * "Elements of information-theoretic derivation of the formalism of quantum theory", International Journal of Quantum Information 1(3), 2003, pp. 289-300.

The quantum logical reconstruction from Rovelli's axioms and its limits

Alexei Grinbaum
LARSIM, CEA-Saclay, France

Found. Phys. Lett. **18,** 563-572, 2005

BJPS, **58**, 387-408, 2007

Thesis

- Quantum theory is a general theory of information constrained by the information-theoretic principles.
- It can be reconstructed from an information-theoretic axiomatic system.

Historical context

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- **Information-theoretic approach:**

  Wheeler (1978, 1988), Rovelli (1996), Steane (1998), Fuchs (2001), Brukner and Zeilinger (2002 & 2009), Clifton, Bub and Halverson (2003), Jozsa (2004), Popescu (1996), Spekkens (2003) and others.

Mackey

M1 Function p is a probability measure. Mathematically, we have $p(x,f,\emptyset) = 0$, $p(x,f,\mathbb{R}) = 1$, and $p(x,f,M_1 \cup M_2 \cup M_3 \ldots) = \sum_{n=1}^{\infty} p(x,f,M_n)$ whenever the M_n are Borel sets that are disjoint in pairs.

M2 Two states, in order to be different, must assign different probability distributions to at least one observable; and two observables, in order to be different, must have different probability distributions in at least one state. Mathematically, if $p(x,f,M) = p(x',f,M)$ for all f in $\mathcal{S}$ and all M in $\mathfrak{B}$ then $x = x'$; and if $p(x,f,M) = p(x,f',M)$ for all x in $\mathcal{O}$ and all M in $\mathfrak{B}$ then $f = f'$.

M3 Let x be any member of $\mathcal{O}$ and let u be any real bounded Borel function on the real line. Then there exists y in $\mathcal{O}$ such that $p(y,f,M) = p(x,f,u^{-1}(M))$ for all f in $\mathcal{S}$ and all M in $\mathfrak{B}$.

M4 If $f_1, f_2, \ldots$ are members of $\mathcal{S}$ and $\lambda_1 + \lambda_2 + \ldots = 1$ where $0 \leq \lambda_n \leq 1$, then there exists f in $\mathcal{S}$ such that $p(x,f,M) = \sum_{n=1}^{\infty} \lambda_n p(x,f_n,M)$ for all x in $\mathcal{O}$ and M in $\mathfrak{B}$.

M5 Call *question* an observable e in $\mathcal{O}$ such that $p(e,f,\{0,1\}) = 1$ for all f in $\mathcal{S}$. Questions e and e' are disjoint if $e \leq 1 - e'$. Then a question $\sum_{n=1}^{\infty} e_n$ exists for any sequence (e_n) of questions such that e_m and e_n are disjoint whenever $n \neq m$.

M6 If E is any compact, question-valued measure then there exists an observable x in $\mathcal{O}$ such that $\chi_M(E) = E(M)$ for all M in $\mathfrak{B}$, where χ_M is a characteristic function of M.

M7 The partially ordered set of all questions in quantum mechanics is isomorphic to the partially ordered set of all closed subspaces of a separable, infinite-dimensional Hilbert space.

M8 If e is any question different from 0 then there exists a state f in $\mathcal{S}$ such that $m_f(e) = 1$.

M9 For each sequence (f_n) of members of $\mathcal{S}$ and each sequence (λ_n) of non-negative real numbers whose sum is 1, one-parameter time evolution group $V_t : \mathcal{S} \mapsto \mathcal{S}$ acts as follows: $V_t\left(\sum_{n=1}^{\infty} \lambda_n f_n\right) = \sum_{n=1}^{\infty} \lambda_n V_t(f_n)$ for all $t \geq 0$;

Rovelli

Quantum mechanics will cease to look puzzling only when we will be able to *derive* the formalism of the theory from a set of simple physical assertions ("postulates", "principles") about the world. Therefore, we should not try to append a reasonable interpretation to the quantum mechanical formalism, but rather to *derive* the formalism from a set of experimentally motivated postulates.

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II. **Lattice orthomodularity from an information-theoretic principle**

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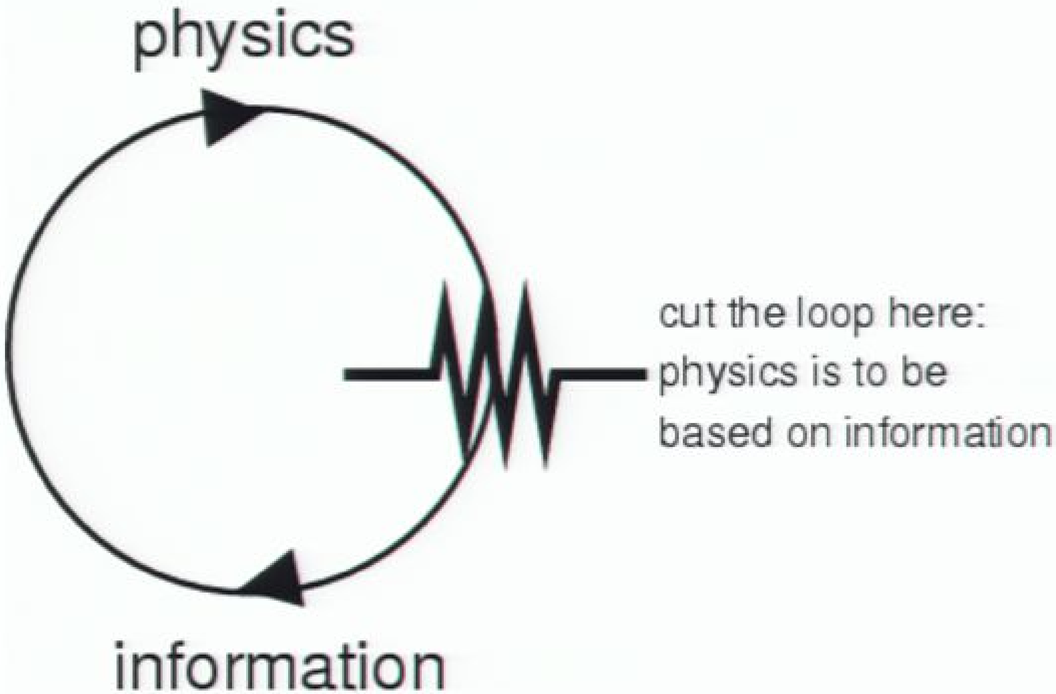
II. Lattice orthomodularity from an information-theoretic principle

III. Open problems

Four points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cut the loop here:
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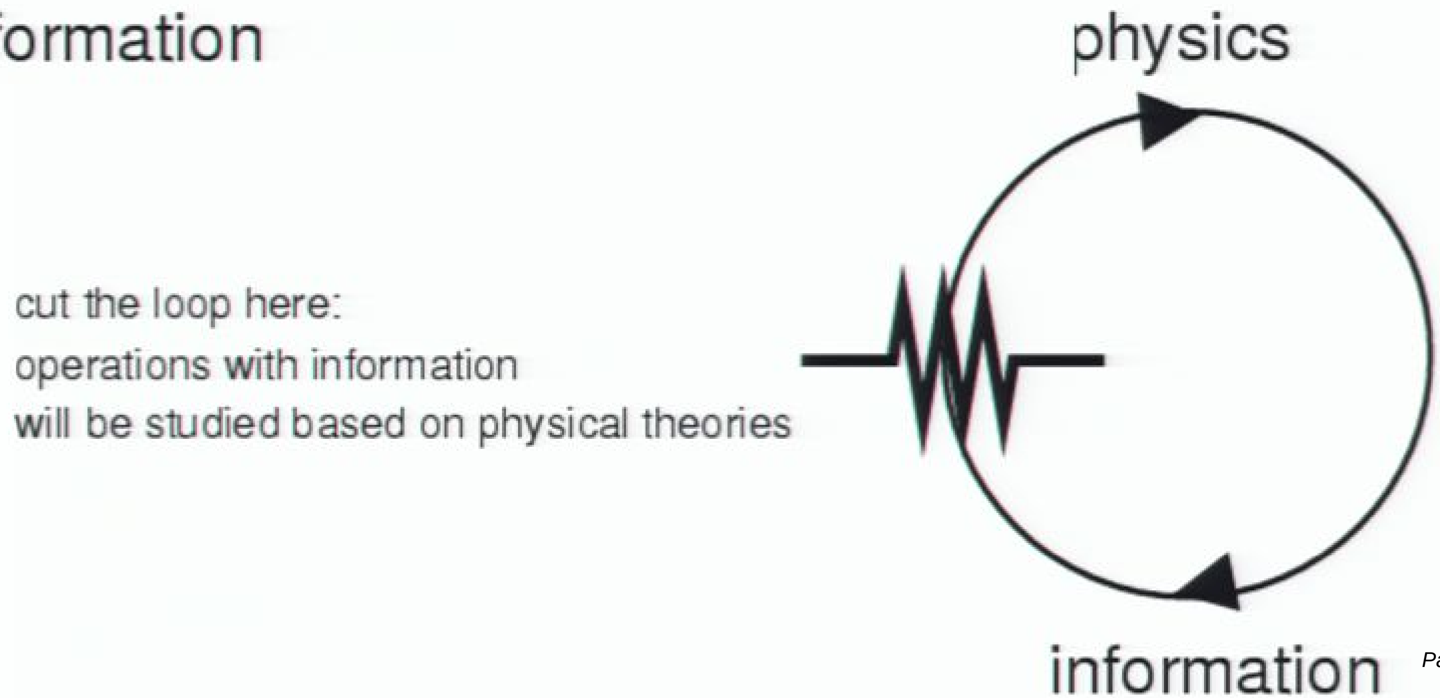
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Quantum logical reconstruction

Language

Fundamental notions

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Fact (act of bringing about information)

Language

Fundamental notions	Formal representation
System	Systems S, O, P…
Information	Yes-no questions
Fact (act of bringing about information)	Answer to a yes-no question (given at time *t*)

Cf. Beltrametti and Casinelli

Axioms

Axiom I: There is a maximum amount of **relevant** information that can be extracted from a system.

Axiom II: It is always possible to acquire new information about a system.

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Quantum logical reconstruction of the Hilbert space

1. Definition of the lattice of yes-no questions.
2. Definition of orthogonal complement.
3. Definition of relevance and proof of orthomodularity.
4. Introduction of the space structure.
5. Lemmas about properties of the space.
6. Definition of the numeric field.
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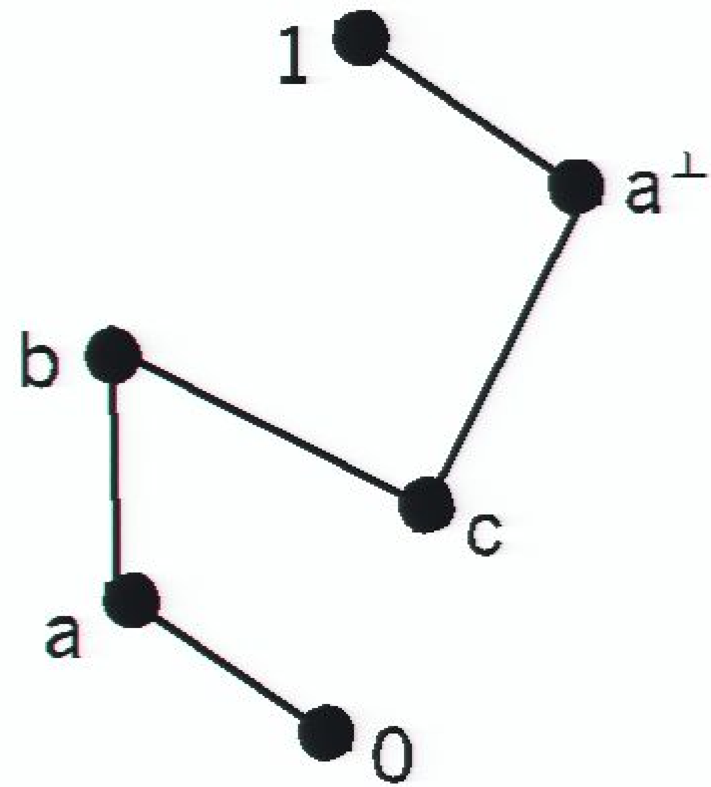

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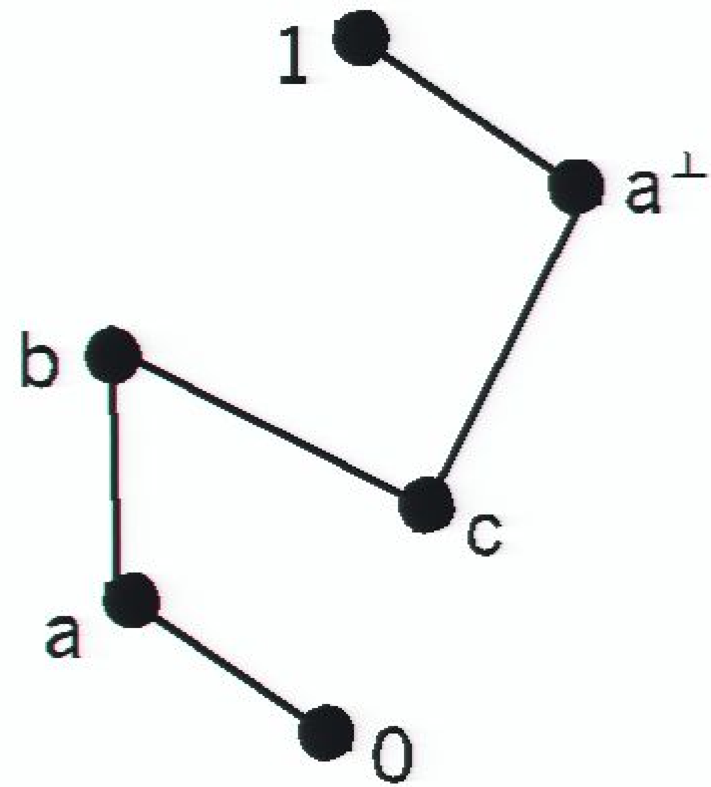

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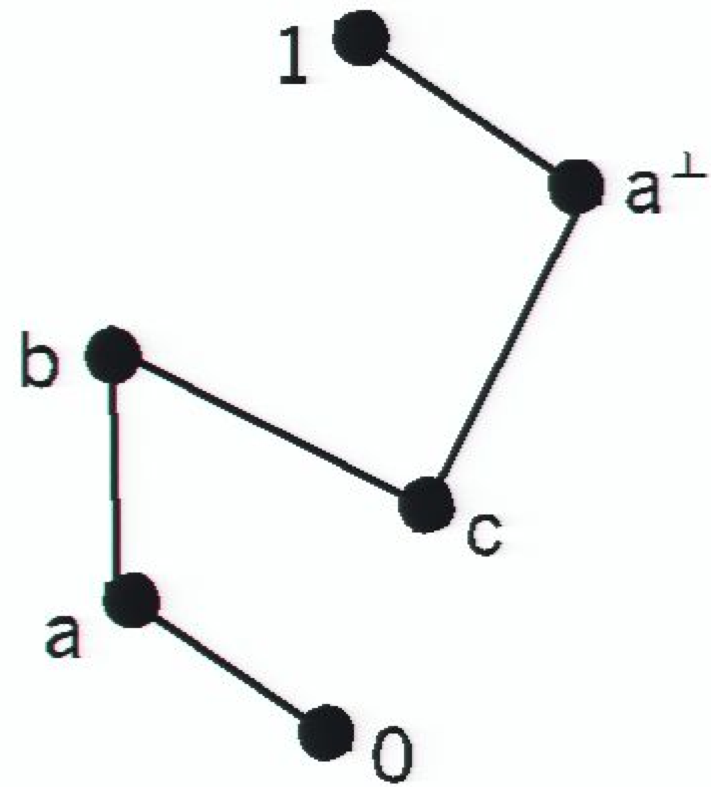

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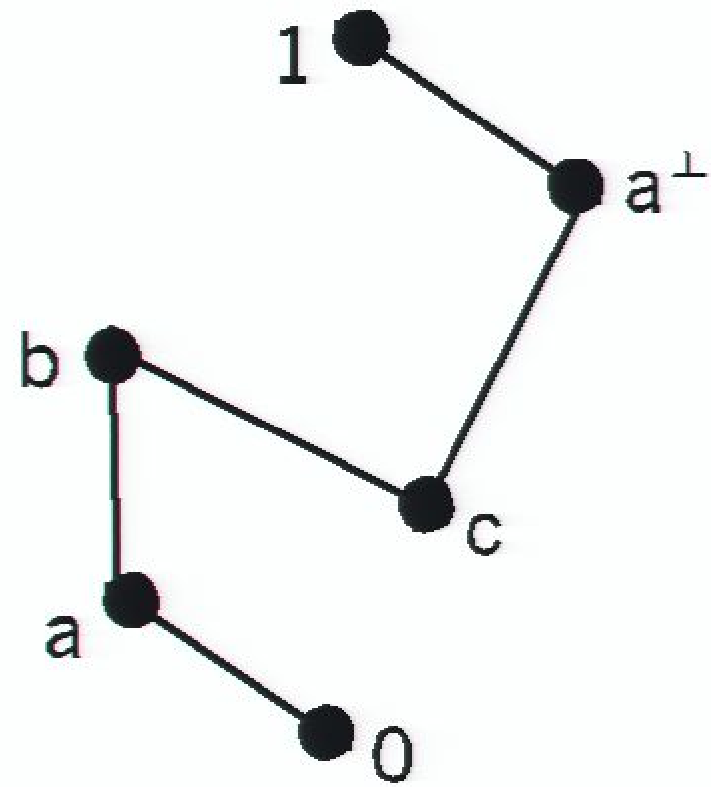

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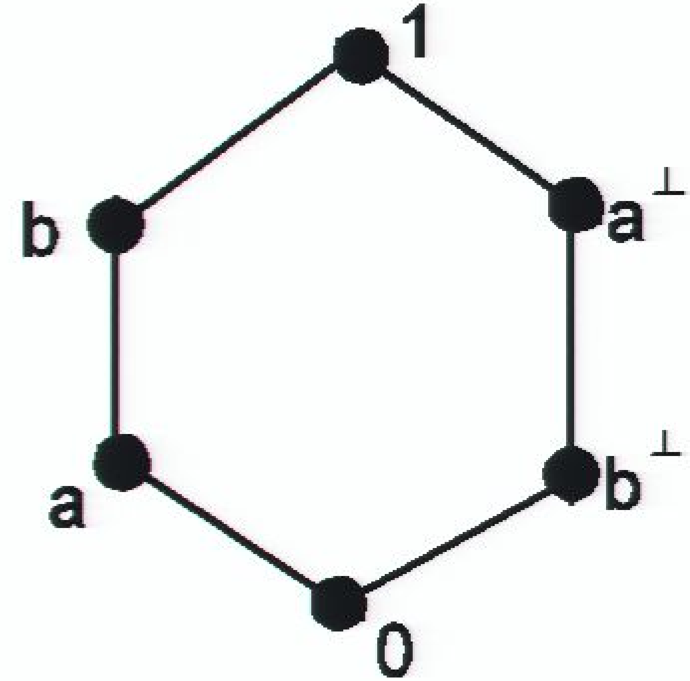
Non-trivial notion of relevance

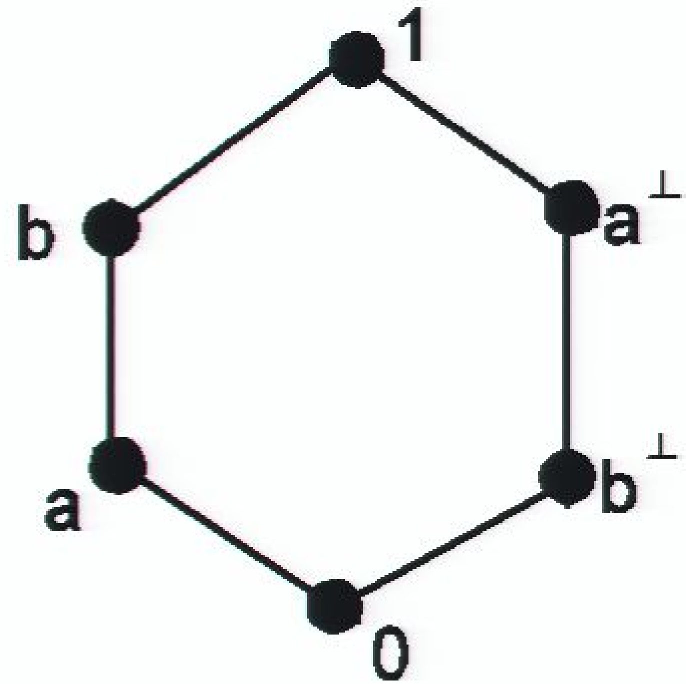


Non-trivial notion of relevance

- Question *b* is relevant with respect to question *a*

  and

- $b \geq a$



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  1. If relevance is not lost, the amount of information grows monotonously as new information comes in.
  2. The lattice contains all possible information (yes-no questions). Thus, there are sufficiently many questions as to bring about any *a priori* allowed amount of information.

Proof of Orthomodularity

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- By Axiom I there exists a finite upper bound of the amount of relevant information, call it *N*. Select an arbitrary question *a* and consider a question $\tilde{a}$ such that $\{a,\tilde{a}\}$ bring N bits of information. Then $a^{\perp} \wedge \tilde{a} = 0$.

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- Lemma: An orthocomplemented lattice is orthomodular if and only if $a \leq b$ and $a^{\perp} \wedge b = 0$ imply $a = b$.
- Question *b* is relevant with respect to *a*; and question $\tilde{a}$ is relevant with respect to *b*.
- Consider {a,b,ã}. If $b > a$, this sequence preserves relevance and brings about strictly more than N bits of relevant information.

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From the contradiction follows $a = b$.

Step 7: Construction of the Hilbert space

- **Theorem**:

  Let W(P) be an ensemble of yes-no questions that can be asked to a physical system and V a vector space over real or complex numbers or quaternions such that a lattice of its subspaces L is isomorphic to W(P).

  Then there exists an inner product f on V such that V together with f form a Hilbert space.

Kalmbach's theorem

Infinite-dimensional Hilbert space characterization theorem:

Let H be an infinite-dimensional vector space over real or complex numbers or quaternions. Let L be a complete orthomodular lattice of subspaces of H which satisfies:

(i) Every finite-dimensional subspace of H belongs to L.

(ii) For every element U of L and for every finite-dimensional subspace V of H, linear sum $U+V$ belongs to L.

Then there exists an inner product f on H such that (H,f) is a Hilbert space with L as its lattice of closed subspaces.

Structure of Hilbert space

- Why does a quantum system live in a complex Hilbert space?
- When we reconstruct the Hilbert space, an assumption of continuity is responsible for supplying the structure.

Step 6: Definition of the numeric field

- Axiom VII: The underlying numeric field of *V* is one of the real or complex numbers or quaternions, and the involutory anti-automorphism (conjugation) is continuous.

- Substitutes: Solèr's theorem assuming the existence of an infinite orthonormal sequence of vectors. Cf. Zieler, Holland, Landsman

Zieler

(C′), (C) For every finite $a \in \mathcal{L}$ and for each i, $0 \le i \le \dim a$, the set of elements $\{x \in \mathcal{L} \,:\, x \le a \text{ and } \dim x = i\}$ is compact in the topology provided by the metric

$$f(x, y) = \sup\{|m(x) - m(y)| \,:\, m \in \mathcal{L}\}.$$

For each $i = 0, 1, \ldots$ the set of finite elements in $\mathcal{L}$ of dimension i is complete with respect to the same metric.

(Co) For some finite b and real interval I there exists a nonconstant function from I to $\mathcal{L}(0, b)$.

Topology of the Hilbert space

- Algebraic argument for a topological result
- Involves infinities

Solèr's theorem

Assume that exists an infinite orthonormal sequence of vectors.

Then Solèr's theorem provides for an *infinite-dimensional* Hilbert space.

Theorem 6.16 (Solèr). *Let $\mathbb{D}$ be a field with involution, V a left vector space over $\mathbb{D}$, and f an orthomodular form on V that has an infinite orthonormal sequence. Then $\mathbb{D} = \mathbb{R}, \mathbb{C}$ or $\mathbb{H}$, and $\{V, \mathbb{D}, f\}$ is the corresponding Hilbert space.*

Quantumness

Axiom II: It is always possible to acquire new information about a system.

Criterion: An orthomodular lattice, in order to describe a quantum mechanical system, must be nondistributive.

- Lemma: all Boolean subalgebras of L(V) are proper.
- Corollary: W(P) is non-Boolean.

State space and the Born rule

➢ An attempt to use Gleason's theorem to build the state space.

State space and the Born rule

- Axiom III

  "No metainformation":

  If information *I* about a system was obtained, then there is no further information *J* available to the observer *about* information *I*. The observer has no metaknowledge of the circumstances of bringing about information *I*.

- An attempt to use Gleason's theorem to build the state space.

Superposition principle

- Via Gleason’s theorem
- Rovelli’s solution
- Brukner’s and Zeilinger’s solution
- Holland’s solution
- Landsman’s solution
- Hardy’s solution

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Rovelli

From intersubjective agreement:

- Try to use consistency between the descriptions given by different observers

Brukner and Zeilinger

- Total information content of the system is invariant under a change of the set of mutually complementary propositions.
- Postulate "homogeneity of parameter space" (2002) or that mutually complementary propositions can be changed continuously (2009).

S.S. Holland

(C) Superposition principle for pure states:

1. Given two different pure states (atoms) *a* and *b*, there is at least one other pure state *c*, $c \neq a$ and $c \neq b$, that is a superposition of *a* and *b*.
2. If the pure state *c* is a superposition of the distinct pure states *a* and *b*, then *a* is a superposition of *b* and *c*.

(D) Ample unitary group: Given any two orthogonal pure states *a*, *b* in *L*, there is a unitary operator *U* such that $U(a)=b$.

Landsman

- "Two-sphere property":

Some algebraic structure is required to be isomorphic to a topological object (a sphere).

Lucien Hardy

- Axiom H5:
  There exists a continuous reversible transformation on a system between any two pure states of that system.
- Motivation: “there are generally no discontinuities in physics.”

Time and unitary dynamics

- Assume an isomorphism between the sets of yes-no questions at different time moments. In other words, time evolution commutes with orthogonal complementation, hence with relevance.
- Wigner's theorem: unitary or anti-unitary transformation $U(t_1,t_2)$: $W_{t_1}(P) \rightarrow W_{t_2}(P)$.
  Select unitary transformation only in virtue of the condition of continuity in the limit $t_2 \rightarrow t_1$.
- Stone's theorem: Hamiltonian description $U(t_2 - t_1) = \exp[-i(t_2 - t_1)H]$.

List of axioms

Information-theoretic axioms:

I. There is a maximum amount of relevant information that can be extracted from a system.

II. It is always possible to acquire new information about a system.

III. If information I about a system was obtained, then there is no further information J about the fact of bringing about information I.

Supplementary assumptions:

IV. For any two yes-no questions there exists a yes-no question to which the answer is positive if and only if the answer to at least one of the initial question is positive.

V. For any two yes-no questions there exists a yes-no question to which the answer is positive if and only if the answer to both initial questions is positive.

VI. The lattice of questions is complete.

VII. The underlying field of the space of the theory is one of the numeric fields R, C or D and the involutory anti-automorphism in this field is continuous.

Open questions

1. Meaning of the lattice structure.
2. Meaning of the numeric field.
3. Interpretation of probability.
4. Origin of assumptions concerning time evolution.
5. Dimension of the Hilbert space.
6. Superselection rules.

The observer in information-theoretic derivations

Observers 1

- Rovelli in RQM:

  "Hypothesis 1: All systems are equivalent: Nothing distinguishes a priori macroscopic systems from quantum systems. If the observer *O* can give a quantum description of the system *S*, then it is also legitimate for an observer *P* to give a quantum description of the system formed by the observer *O*."

- Is any physical system going to have a sufficient number of the degrees of freedom to qualify as observer system?

The Landauer principle

- Erasure of information leads to entropy increase.

Define observer as system capable of obtaining and erasing information.

1) There is a bound on the accuracy with which the values of noncommuting observables can be simultaneously prepared. When we prepare the system in a state characterized by position and momentum, the preparation must obey $S_x + S_p \geq 1 + \ln \pi$.

Uffink and Maassen, Phys. Rev. Lett. 60, 1103-1106 (1988)

2) When we prepare a quantum system, there is a minimal number of bits that we must erase from our memory. For example, we have measured position and now we now prepare the system to be in a momentum eigenstate (or similarly with polarizations). This minimal number of bits to erase is $(1+\ln \pi) / \ln 2 = 3.094$

3) Landauer's principle stipulates that erasure involves generating heat at $(k \ln 2)$ per bit. Then there is a minimal entropy of this state preparation equal to $k\,(1+ \ln \pi)$.

State preparation in itself has an entropy cost which is $(k \ln 2)$ times the lower bound of entropy uncertainty.

Observers 2

- Human observers all have approximately the same low entropy.

- Imagine two observers with radically different entropies: a low-entropy observer O and a high-entropy observer P.
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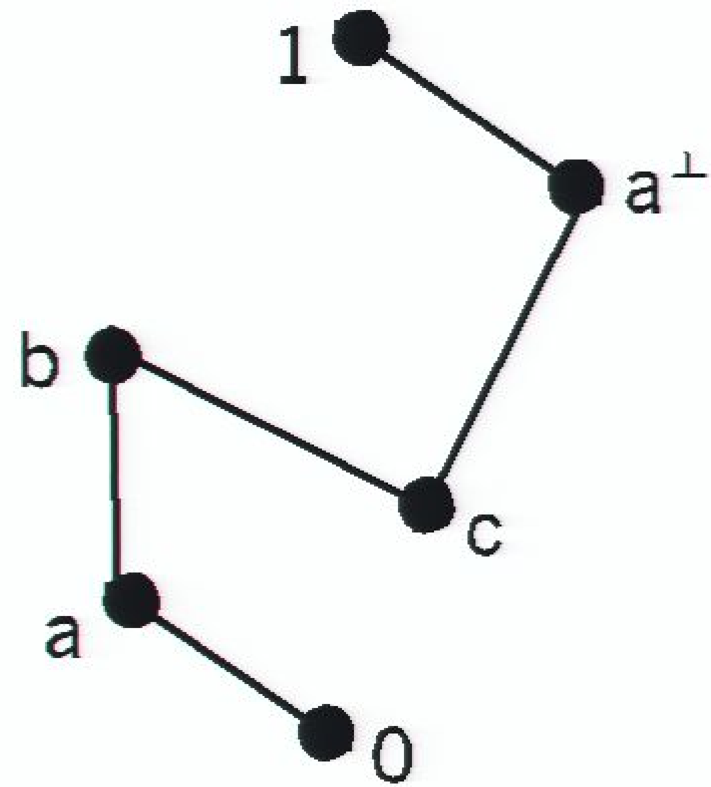
- Corollary: W(P) is non-Boolean.

Proof of Orthomodularity

- By Axiom I there exists a finite upper bound of the amount of relevant information, call it *N*. Select an arbitrary question *a* and consider a question $\tilde{a}$ such that $\{a,\tilde{a}\}$ bring N bits of information. Then $a^{\perp} \wedge \tilde{a} = 0$.
- Lemma: An orthocomplemented lattice is orthomodular if and only if $a \leq b$ and $a^{\perp} \wedge b = 0$ imply $a = b$.

Definition of Relevance

- Question *b* is called irrelevant with respect to question *a* if $b \wedge a^{\perp} \neq 0$.

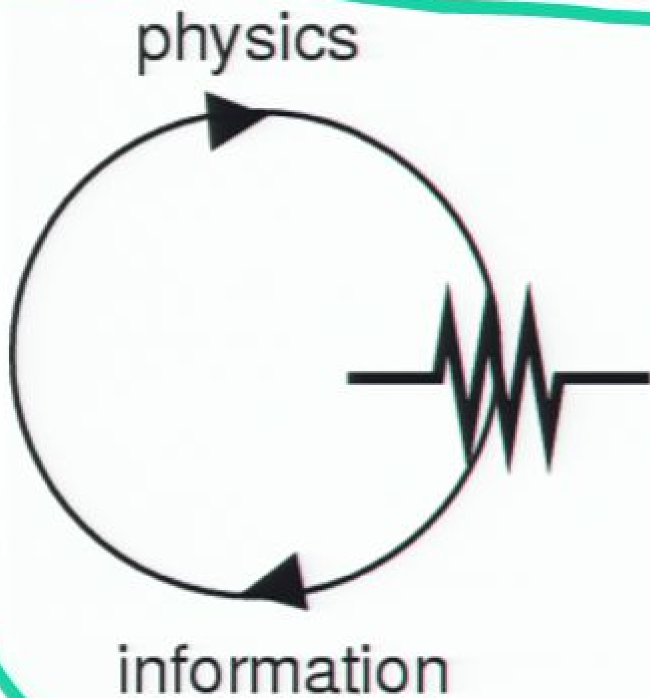


- Trivial in Hilbert lattices: $x \leq y$ are relevant with respect to y, all others irrelevant.
- Non-trivial if used to *derive* what in Hilbert lattices is *assumed*

Axioms

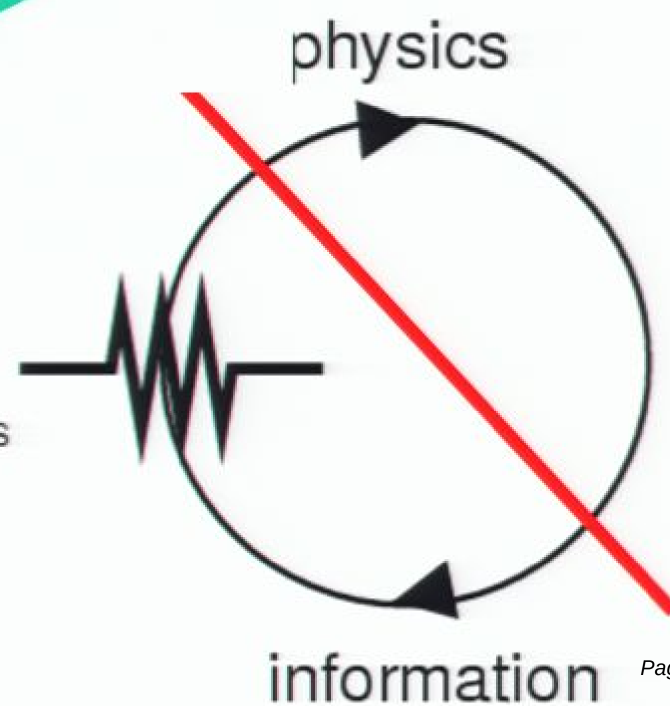
Axiom I: There is a maximum amount of **relevant** information that can be extracted from a system.

Axiom II: It is always possible to acquire new information about a system.



cut the loop here:
physics is to be
based on information

cut the loop here:
operations with information
will be studied based on physical theories



Four points

Mackey

M1 Function p is a probability measure. Mathematically, we have $p(x, f, \emptyset) = 0$, $p(x, f, \mathbb{R}) = 1$, and $p(x, f, M_1 \cup M_2 \cup M_3 \ldots) = \sum_{n=1}^{\infty} p(x, f, M_n)$ whenever the M_n are Borel sets that are disjoint in pairs.

M2 Two states, in order to be different, must assign different probability distributions to at least one observable; and two observables, in order to be different, must have different probability distributions in at least one state. Mathematically, if $p(x, f, M) = p(x', f, M)$ for all f in $\mathcal{S}$ and all M in $\mathfrak{B}$ then $x = x'$; and if $p(x, f, M) = p(x, f', M)$ for all x in $\mathcal{O}$ and all M in $\mathfrak{B}$ then $f = f'$.

M3 Let x be any member of $\mathcal{O}$ and let u be any real bounded Borel function on the real line. Then there exists y in $\mathcal{O}$ such that $p(y, f, M) = p(x, f, u^{-1}(M))$ for all f in $\mathcal{S}$ and all M in $\mathfrak{B}$.

M4 If $f_1, f_2, \ldots$ are members of $\mathcal{S}$ and $\lambda_1 + \lambda_2 + \ldots = 1$ where $0 \leq \lambda_n \leq 1$, then there exists f in $\mathcal{S}$ such that $p(x, f, M) = \sum_{n=1}^{\infty} \lambda_n p(x, f_n, M)$ for all x in $\mathcal{O}$ and M in $\mathfrak{B}$.

M5 Call *question* an observable e in $\mathcal{O}$ such that $p(e, f, \{0, 1\}) = 1$ for all f in $\mathcal{S}$. Questions e and e' are disjoint if $e \leq 1 - e'$. Then a question $\sum_{n=1}^{\infty} e_n$ exists for any sequence (e_n) of questions such that e_m and e_n are disjoint whenever $n \neq m$.

M6 If E is any compact, question-valued measure then there exists an observable x in $\mathcal{O}$ such that $\chi_M(E) = E(M)$ for all M in $\mathfrak{B}$, where χ_M is a characteristic function of M.

M7 The partially ordered set of all questions in quantum mechanics is isomorphic to the partially ordered set of all closed subspaces of a separable, infinite-dimensional Hilbert space.

M8 If e is any question different from 0 then there exists a state f in $\mathcal{S}$ such that $m_f(e) = 1$.

M9 For each sequence (f_n) of members of $\mathcal{S}$ and each sequence (λ_n) of non-negative real numbers whose sum is 1, one-parameter time evolution group $V_t : \mathcal{S} \mapsto \mathcal{S}$ acts as follows: $V_t\left(\sum_{n=1}^{\infty} \lambda_n f_n\right) = \sum_{n=1}^{\infty} \lambda_n V_t(f_n)$ for all $t \geq 0$;

Four points

1. Epistemological attitude: one is concerned with *theories*.

Science is the construction of theories. A theory is an *objective description of certain phenomena*, while selection criteria for phenomena and the understanding of objectivity of the description vary depending on the particular theory in question. All such phenomena, however, have *repeatable* traits. A theory is a description of repeatable traits of the observed phenomena with the goal of predicting these traits in unobserved (unknown, future) phenomena.

Four points

3. Various theories can be depicted in the loop form.

Quantum logical reconstruction

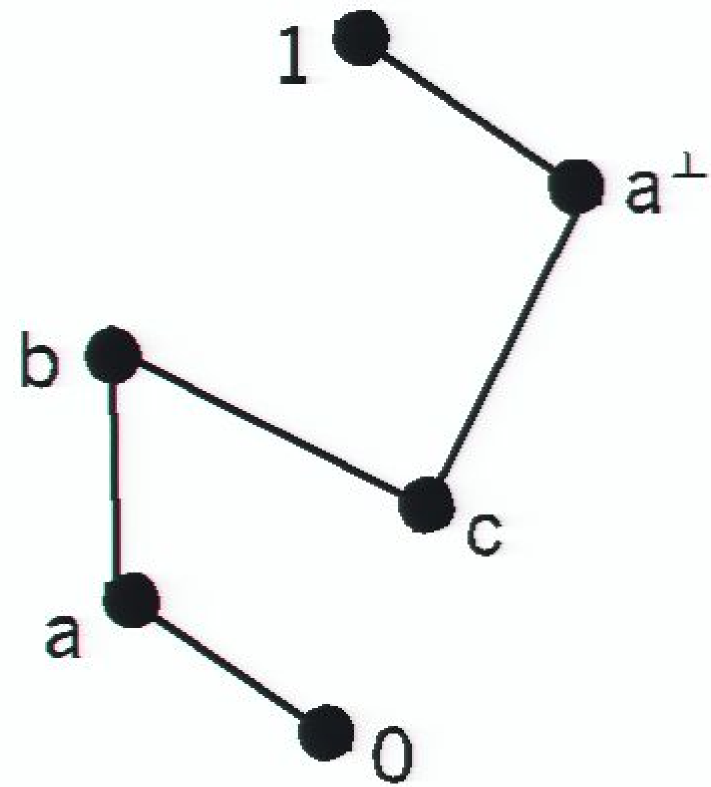
Relevance: Motivation

Quantum logical reconstruction of the Hilbert space

1. Definition of the lattice of yes-no questions.
2. Definition of orthogonal complement.
3. Definition of relevance and proof of orthomodularity.
4. Introduction of the space structure.
5. Lemmas about properties of the space.
6. Definition of the numeric field.
7. Construction of the Hilbert space.

Definition of Relevance

- Question *b* is called irrelevant with respect to question *a* if $b \wedge a^{\perp} \neq 0$.



Amount of information

- Assumptions:

  1. If relevance is not lost, the amount of information grows monotonously as new information comes in.

  2. The lattice contains all possible information (yes-no questions). Thus, there are sufficiently many questions as to bring about any *a priori* allowed amount of information.

Kalmbach's theorem

Infinite-dimensional Hilbert space characterization theorem:

Let H be an infinite-dimensional vector space over real or complex numbers or quaternions. Let L be a complete orthomodular lattice of subspaces of H which satisfies:

(i) Every finite-dimensional subspace of H belongs to L.

(ii) For every element U of L and for every finite-dimensional subspace V of H, linear sum $U+V$ belongs to L.

Then there exists an inner product f on H such that (H,f) is a Hilbert space with L as its lattice of closed subspaces.

Structure of Hilbert space

- Why does a quantum system live in a complex Hilbert space?
- When we reconstruct the Hilbert space, an assumption of continuity is responsible for supplying the structure.

Step 6: Definition of the numeric field

- Axiom VII: The underlying numeric field of *V* is one of the real or complex numbers or quaternions, and the involutory anti-automorphism (conjugation) is continuous.

- Substitutes: Solèr's theorem assuming the existence of an infinite orthonormal sequence of vectors. Cf. Zieler, Holland, Landsman

Step 7: Construction of the Hilbert space

- **Theorem**:

  Let W(P) be an ensemble of yes-no questions that can be asked to a physical system and V a vector space over real or complex numbers or quaternions such that a lattice of its subspaces L is isomorphic to W(P).

  Then there exists an inner product f on V such that V together with f form a Hilbert space.

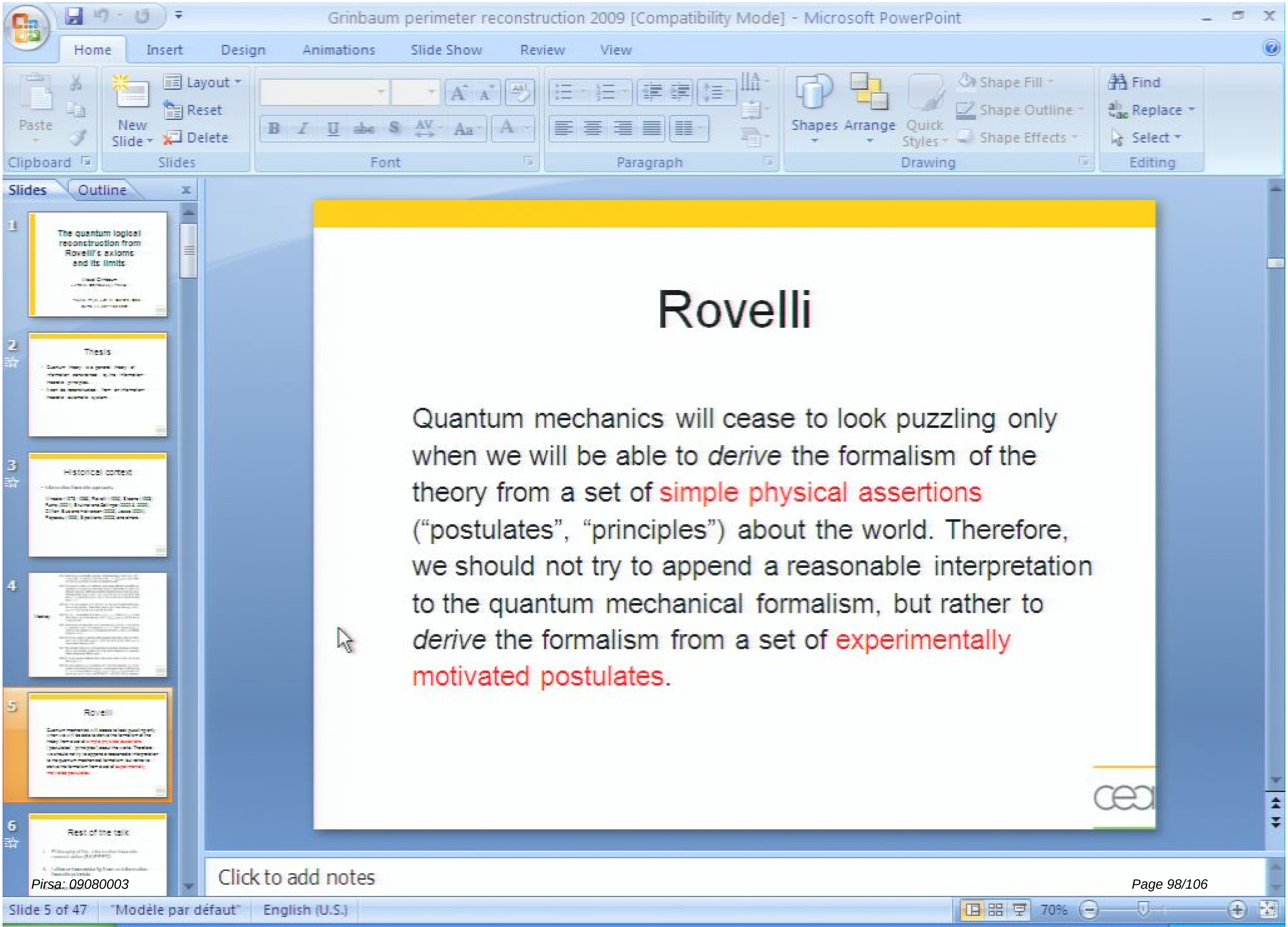
Historical context

- **Information-theoretic approach:**

Wheeler (1978-1988), Rovelli (1996), Steane (1998),
Clifton, Bub and Halverson (2003), Jozsa (2004)

Rovelli

Quantum mechanics will cease to look puzzling only when we will be able to *derive* the formalism of the theory from a set of simple physical assertions ("postulates", "principles") about the world. Therefore, we should not try to append a reasonable interpretation to the quantum mechanical formalism, but rather to *derive* the formalism from a set of experimentally motivated postulates.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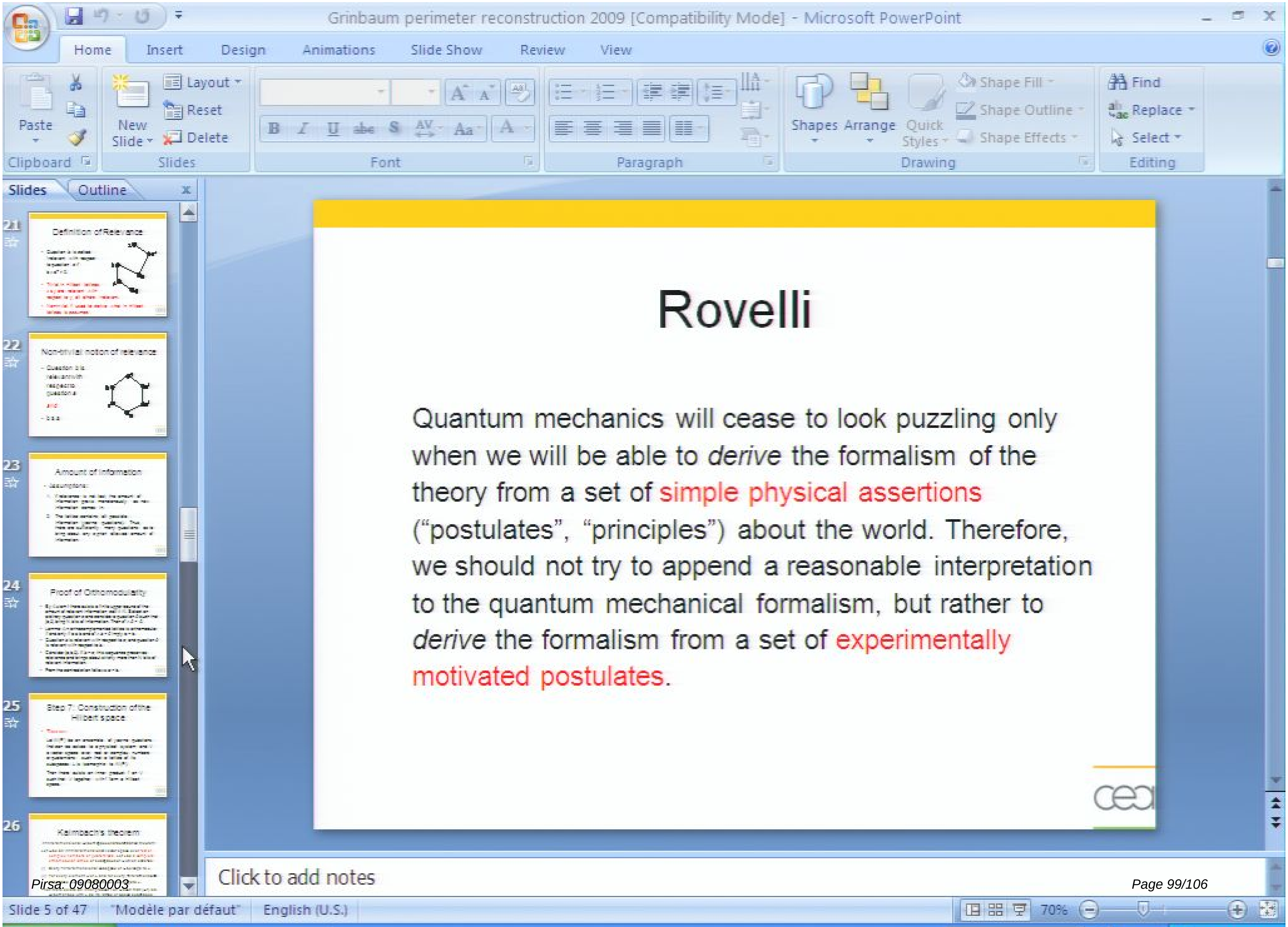
"Modèle par défaut"
English (U.S.)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70%

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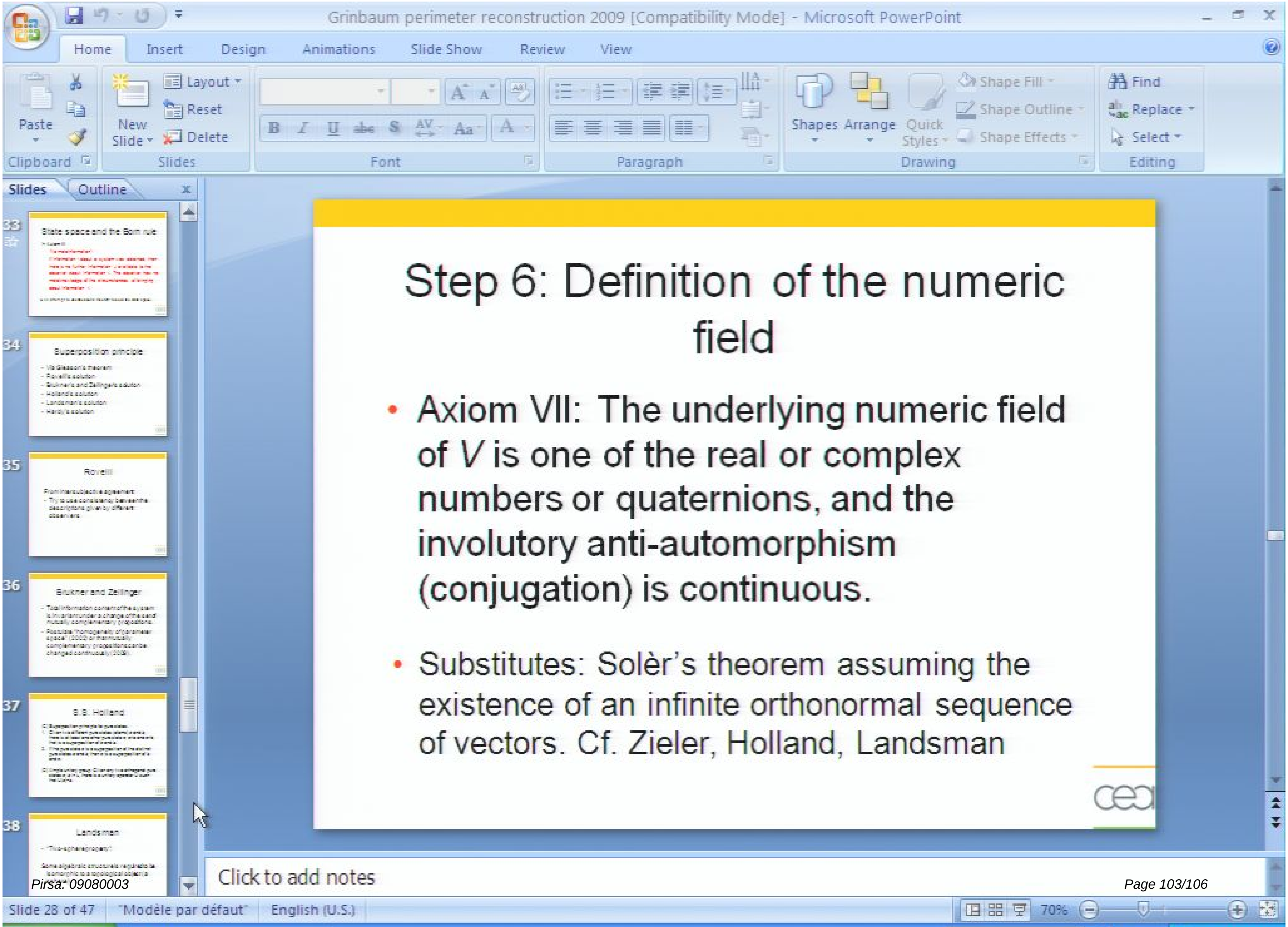
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State space and the Born rule
Superposition principle
Rovelli
Bruckner and Zeilinger
S. S. Holland
Landsman
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Time and unitary dynamics

- Assume an isomorphism between the sets of yes-no questions at different time moments. In other words, time evolution commutes with orthogonal complementation, hence with relevance.
- Wigner's theorem: unitary or anti-unitary transformation $U(t_1,t_2)$: $W_{t_1}(P) \rightarrow W_{t_2}(P)$.
  Select unitary transformation only in virtue of the condition of continuity in the limit $t_2 \rightarrow t_1$.
- Stone's theorem: Hamiltonian description $U(t_2 - t_1) = \exp[-i(t_2 - t_1)H]$.

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