Title: A surprising relation between N=2 gauge theory in 4 dimensions and pure quantum gravity in 3 and 2 dimensions

Date: Jul 21, 2009 03:00 PM

URL: http://pirsa.org/09070038

Abstract: TBA

Electric - Magnetic duality

Electric - Magnetic duality . $\mathcal{N} = 2$

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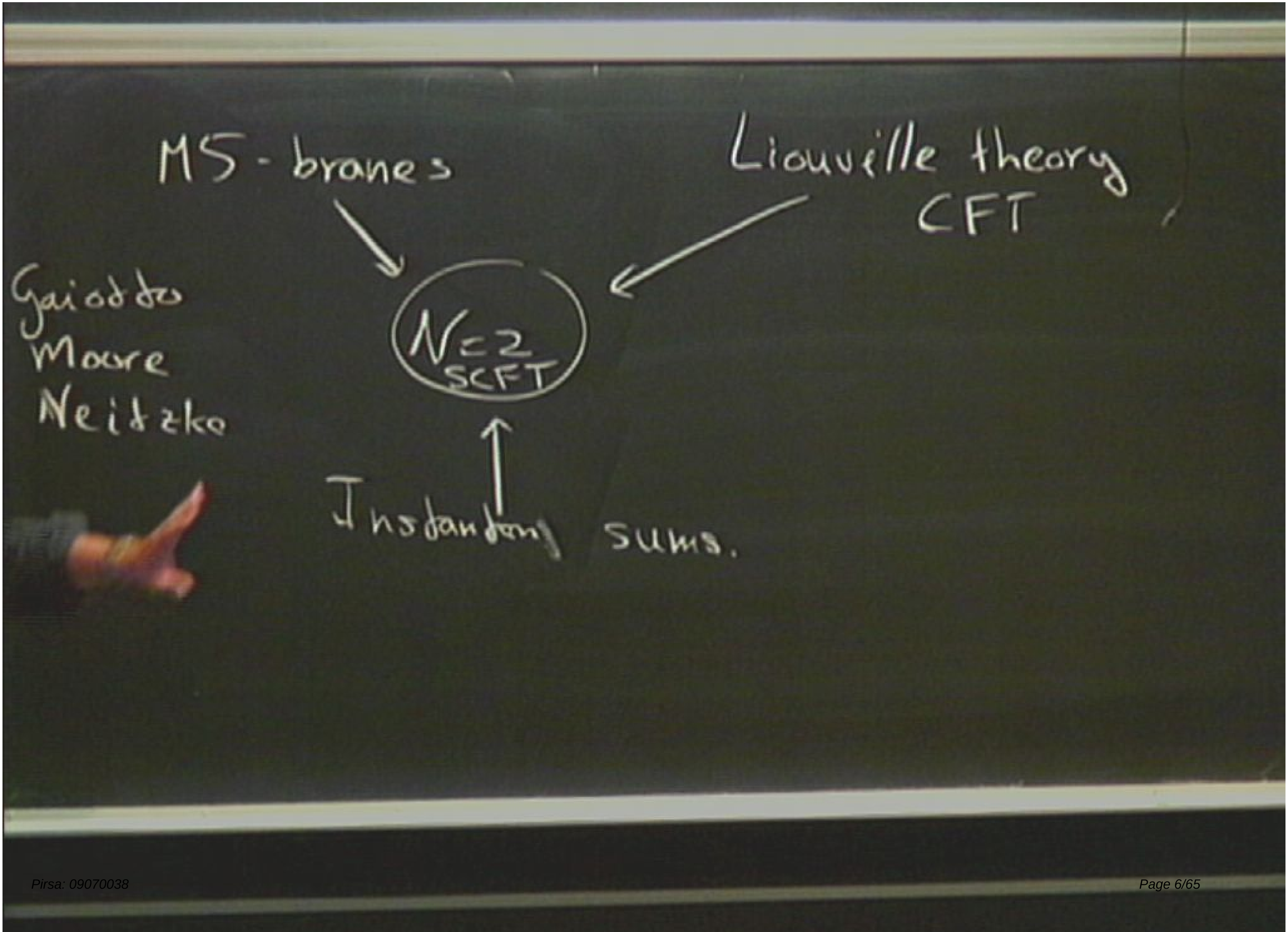
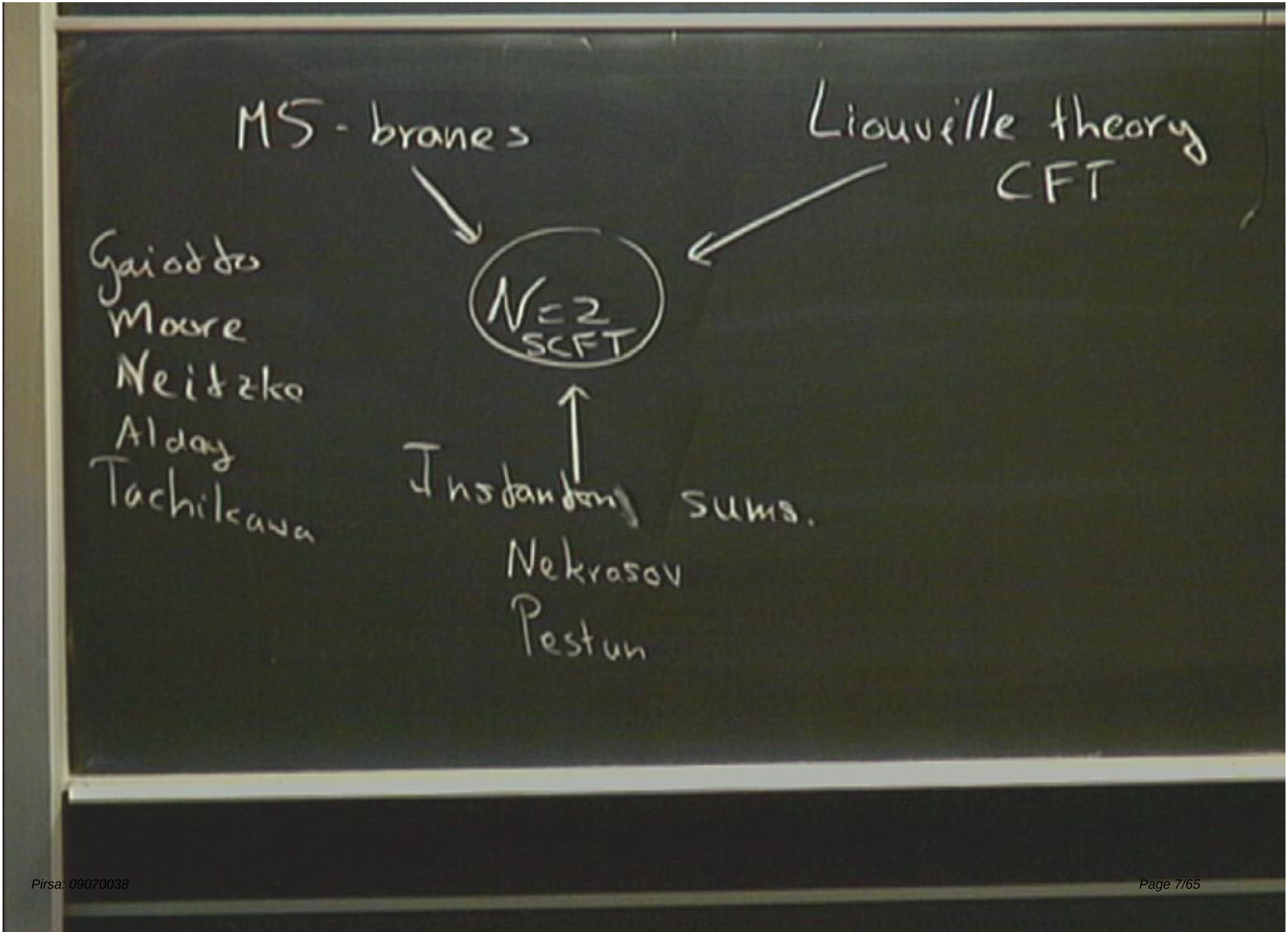

M5 - branes
Liouville theory
CFT
Gaiotto
Moore
Neitzke
N=2
SCFT
Instanton sums.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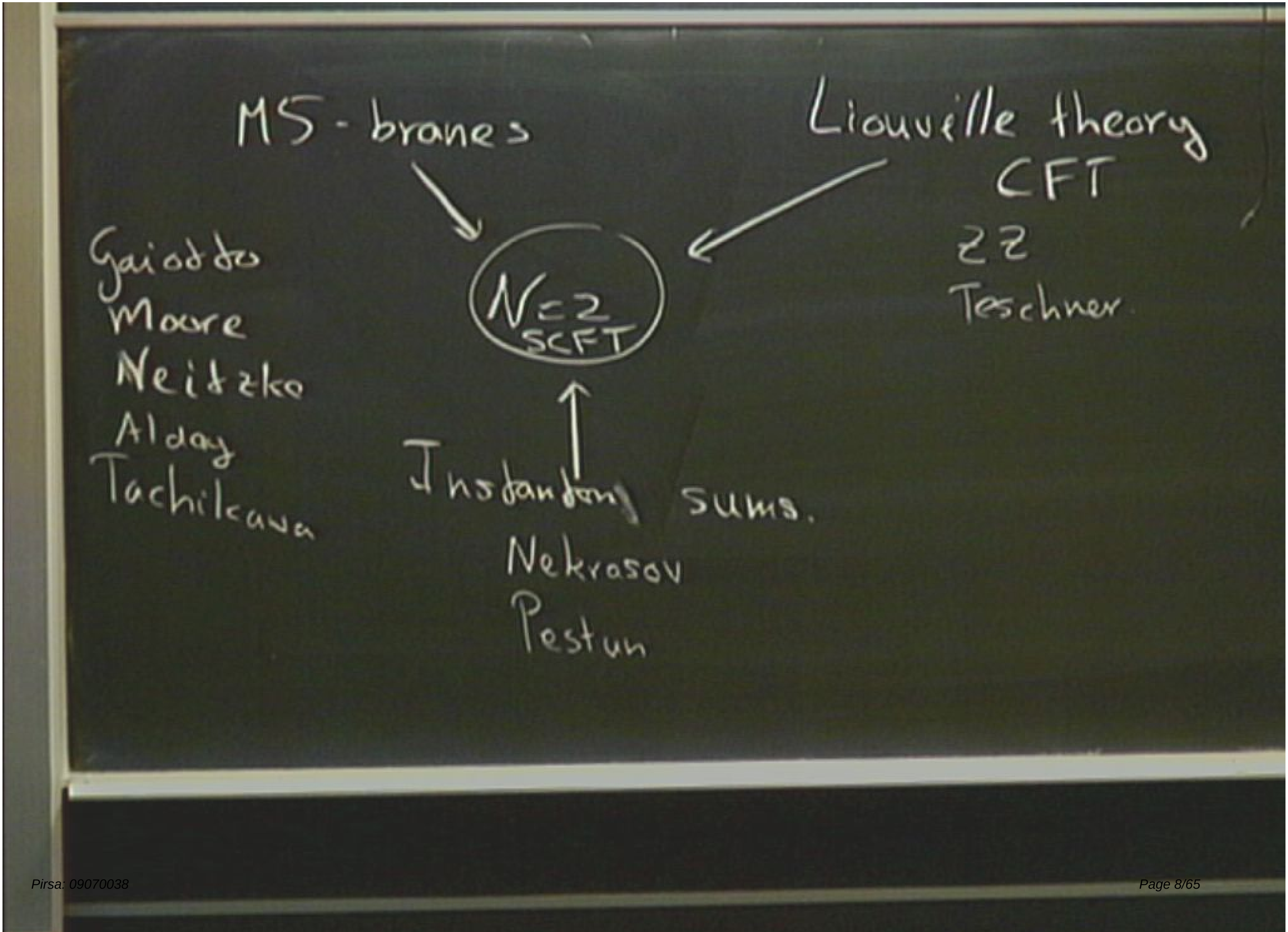
Nekrasov
Pestun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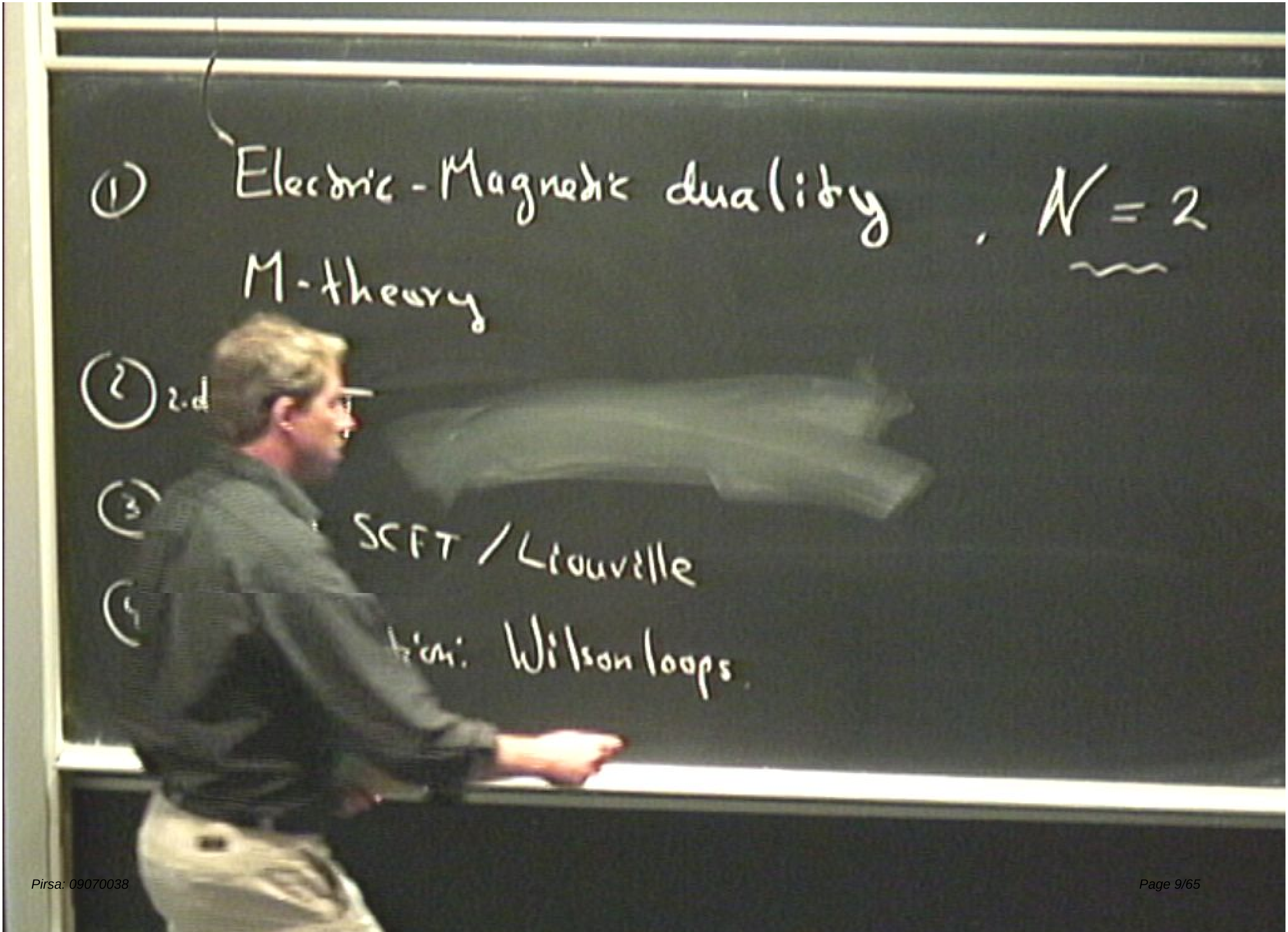
① Electric - Magnetic duality . N = 2
M-theory
② 2.d
③
SCFT / Liouville
④
Wilson loops.

① Electric - Magnetic duality , $\mathcal{N} = 2$

M-theory

② 2d CFT

③ $\mathcal{N}=2$ SCFT / Liouville

④ Computation: Wilson loops. 't Hooft loops.

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⑤

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⑤

$$\tau = \theta + \frac{4\pi i}{g_{ym}}$$

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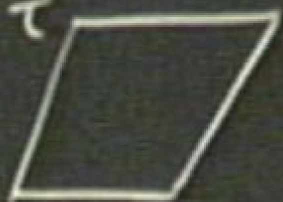
τ



$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\tau = \theta + \frac{4\pi i}{g_{ym}^2}$$

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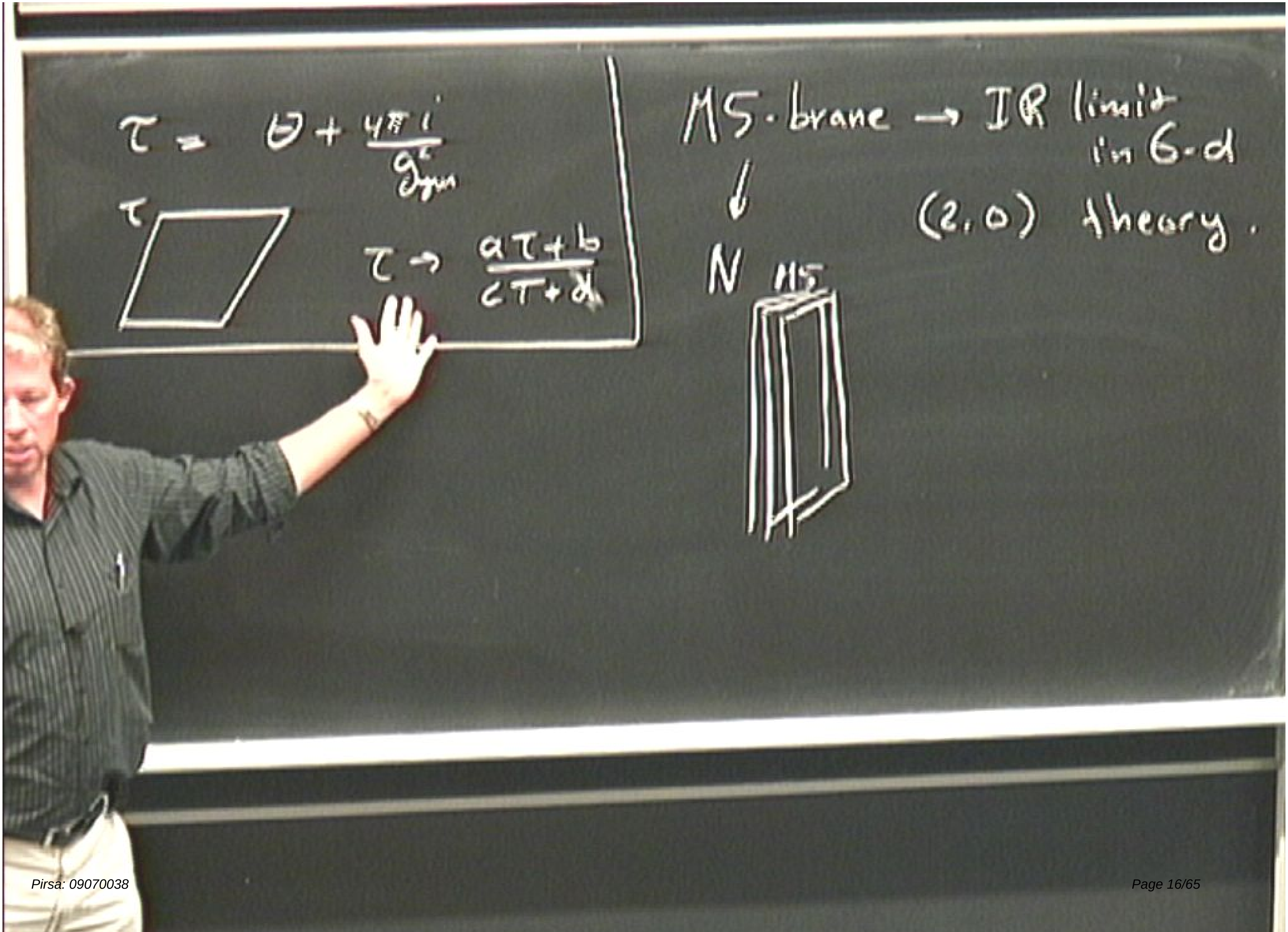
$\tau = \theta + \frac{4\pi i}{g_{ym}^2}$
τ
$\tau \to \frac{a\tau + b}{c\tau + d}$
M5-brane → IR limit in 6-d
(2,0) theory.
N
M5

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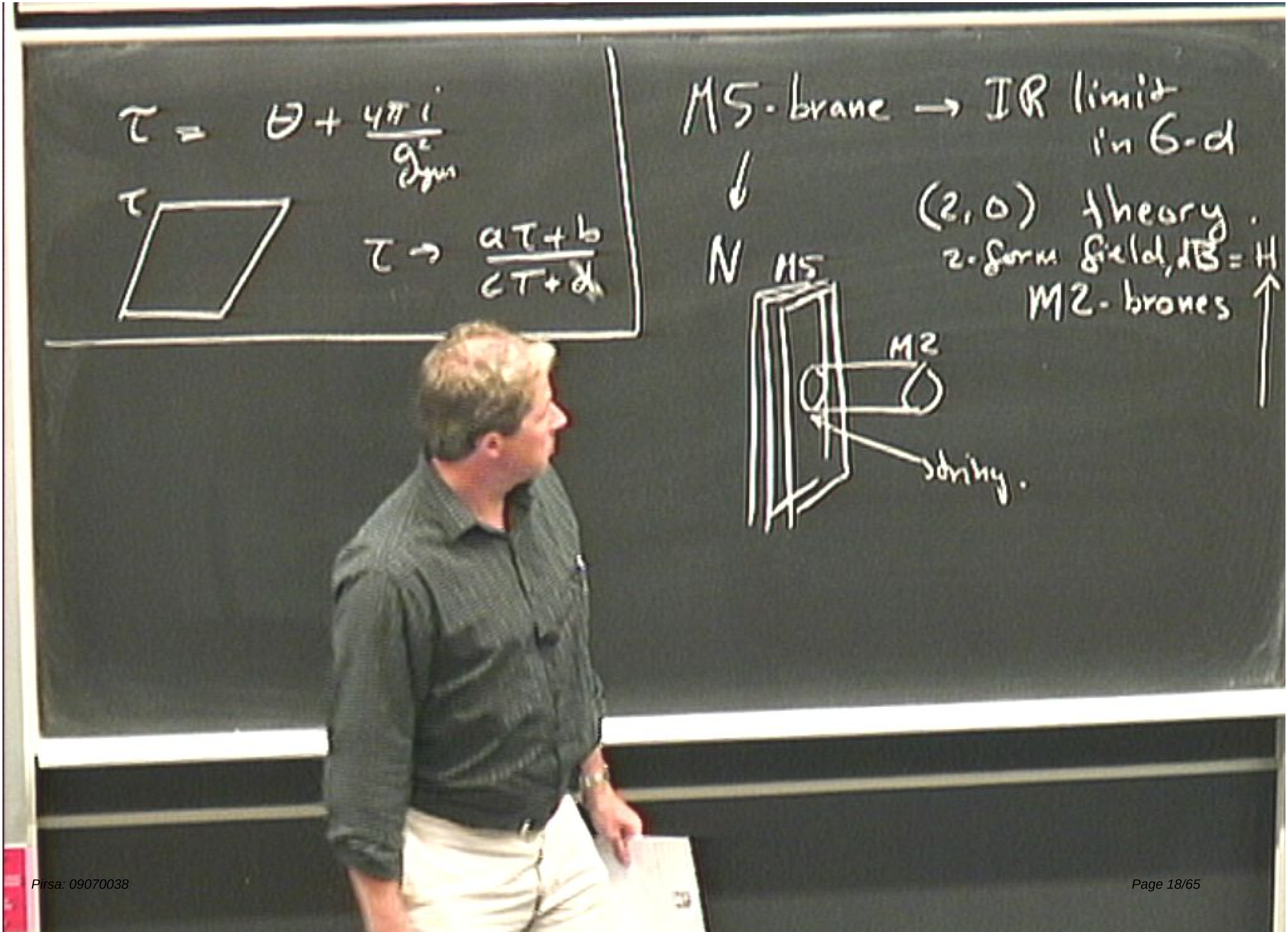
M5-brane → IR limit in 6-d

↓ (2,0) theory.

N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(2,0) theory.
2-form field, dB = H
N
M5
M2-branes
M2
string.

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M5-brane → IR limit in 6-d

↓

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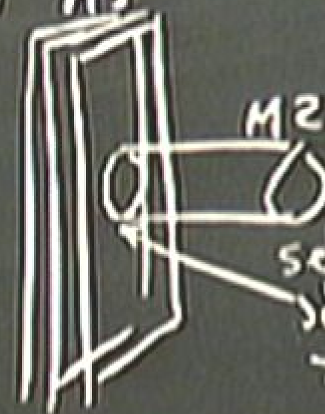
N

M5

2-form field, $dB = H$

M2-branes



M2

self-dual string.

$*_6 H = H$

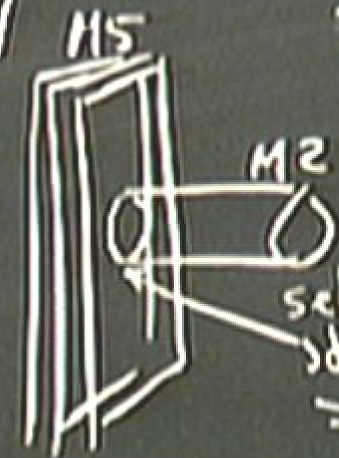
$$\tau = \theta + \frac{4\pi i}{g_{ym}^2}$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

M5-brane $\to$ IR limit in 6-d

$\downarrow$ (2,0) theory.

N



2-form field, $dB = H$

M2-branes $\uparrow$

$*_6 H = H$

<u>Claim:</u>

$$\tau = \theta + \frac{4\pi i}{g^2_{ym}}$$

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

<u>Claim</u>: (2,0)-theory on $T^2 \times \mathbb{R}^4$

$\Leftrightarrow$ $N=4$ SYM in $\mathbb{R}^4$

M5-brane $\to$ IR limit in 6-d

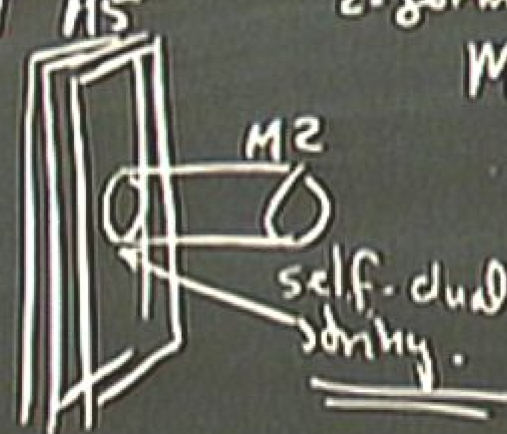
$\downarrow$

N

(2,0) theory.

2-form field, $dB = H$

M2-branes



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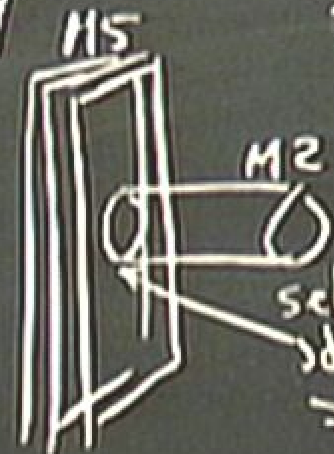

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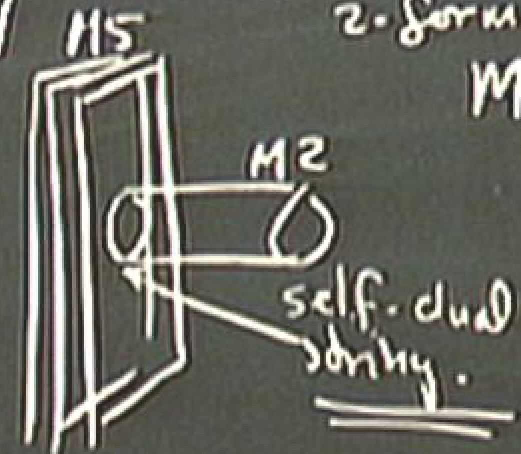
M5-brane → IR limit in 6-d

↓

N (2,0) theory.

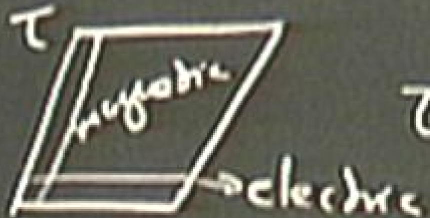
2-form field, $dB = H$

M2-branes



$*H = H$

$$\tau = \theta + \frac{4\pi i}{g^2_{ym}}$$



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M5-brane $\to$ IR limit in 6-d

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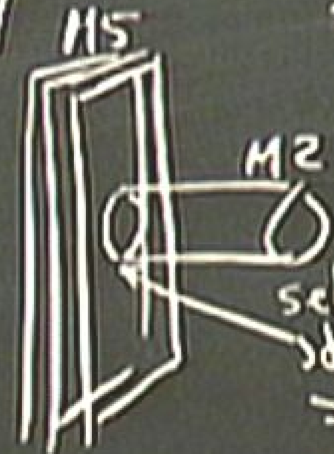
2-form field, $dB = H$

M2-branes $\uparrow$

$*_6 H = H$



self-dual string.

Generalize to (2,0) on $\Sigma \times \mathbb{R}^4$

Σ ↑ punctured Riemann surface.

Generalize to (2,0) on $\Sigma \times \mathbb{R}^4$

Σ $\uparrow$ punctured Riemann surface

Generalize to (2,0) on $\Sigma \times \mathbb{R}^4$

Σ: punctured Riemann surface.

Generalize to (2,0) on $\Sigma \times \mathbb{R}^4$

$\uparrow$ punctured Riemann surface.

Generalize to (2,0) on $\Sigma \times \mathbb{R}^4$

$\uparrow$

punctured Riemann surface

thin tubes $\leftrightarrow$ $SU(N)$

Generalize to (2,0) on $\Sigma \times \mathbb{R}^4$

Σ ↑ punctured Riemann surface.

thin tubes ⟷ $SU(N)$

"pair of pants" ⟷

Coupling constants of 4-d SCFT

$$\Updownarrow$$

Moduli space of complex structures on Σ.

Coupling constants of 4-d SCFT

$\Updownarrow$

Moduli space of complex structures on Σ.

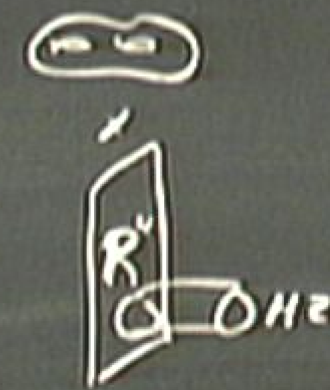
$N = 2^*$

$N_F = 4$

Observables in 4d

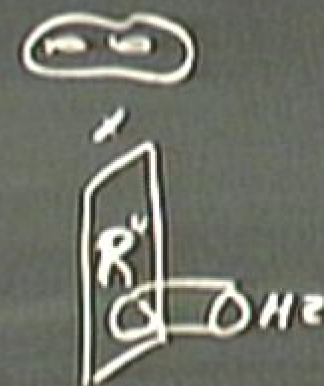
Observables in 4d

* surface operators



Observables in 4d

* surface operators.



observables in 4d

* surface operators.

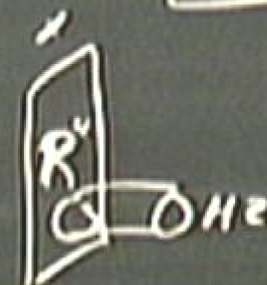
Observables in 4d

$x \in \Sigma$

$\pi_1(\Sigma)$

* Surface operators.

magnetic flux



* line operators.

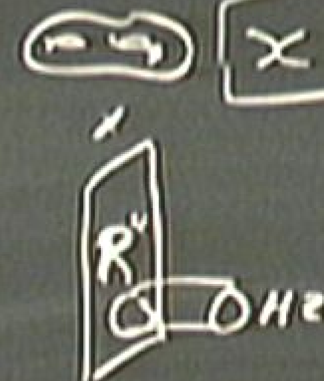
dyonic point particles

$\gamma \in \pi_1(\Sigma)$

Observables in 4d

$\boxed{x \in \Sigma}$ $\pi_1(\Sigma)$

* surface operators
  magnetic flux

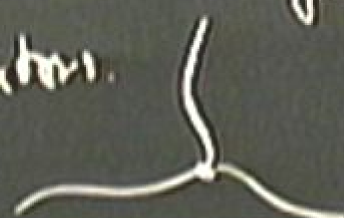


* line operators
  in $\mathbb{R}^4$

$\gamma \in \pi_1(\Sigma)$

dyonic point particles

* point operators



Observables in 4d

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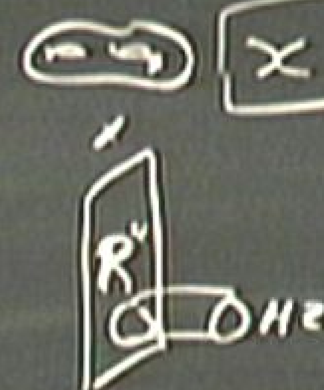

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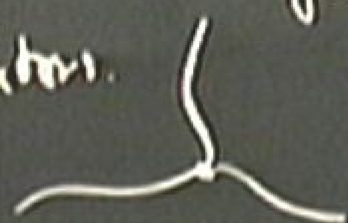
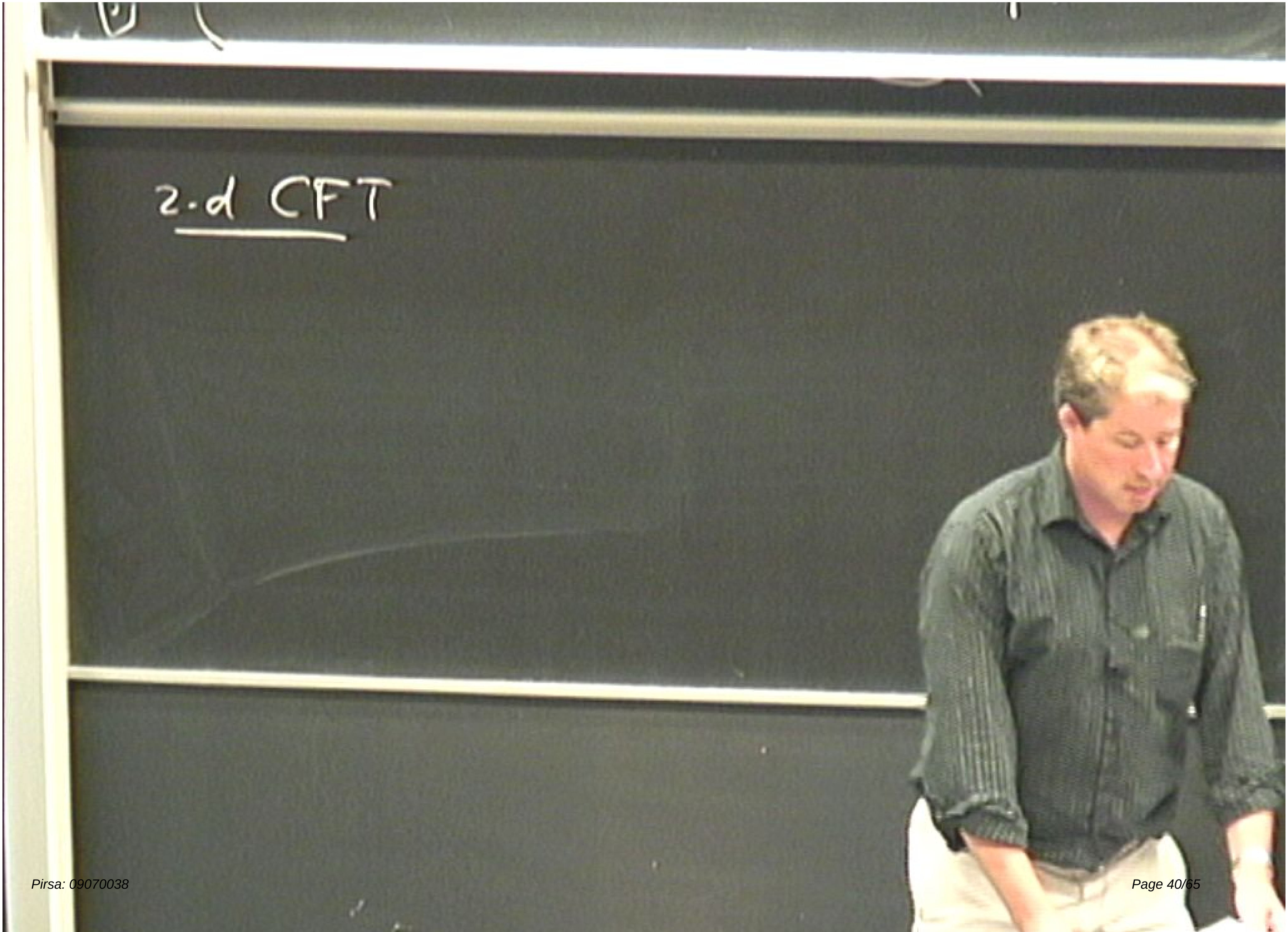
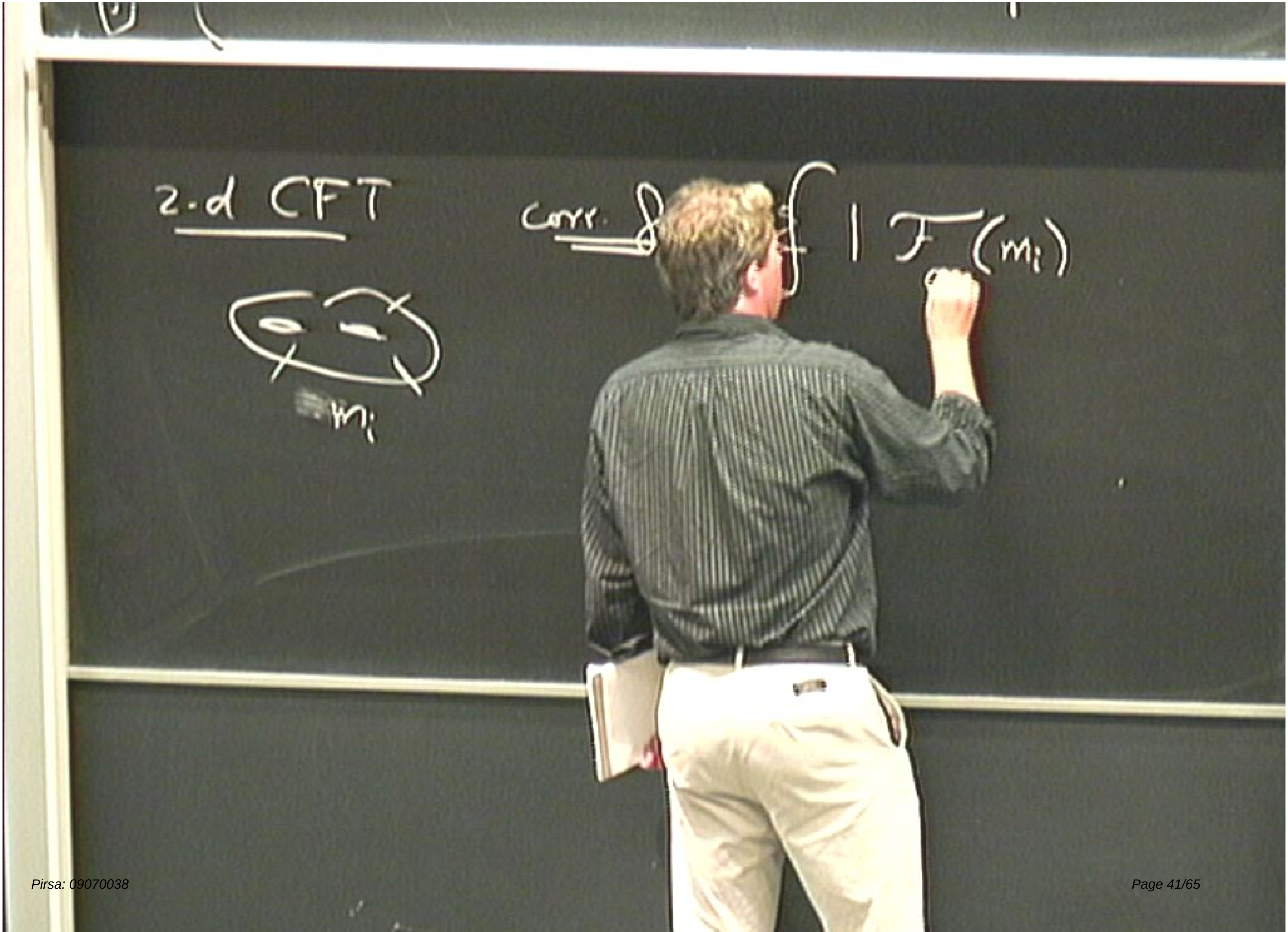



2-d CFT


2-d CFT
Corr. fn = ∫ | F(m_i)
m_i

2-d CFT

$$\text{corr. fn} = \int da \, \left| \mathcal{F}_a(m_i) \right|^2$$

m_i

2-d CFT

$(m_i, \bar{m}_i)$

$$\text{Corr. fn} = \int da \, |\mathcal{F}_a(m_i)|^2$$

conformal blocks.

$$\mathcal{F}_a = M_{ab} \tilde{\mathcal{F}}_b$$

fusion & braiding matrices.

2-d CFT

Corr. fn $= \oint da \, |\mathcal{F}_a(m_i)|^2$

conformal blocks.

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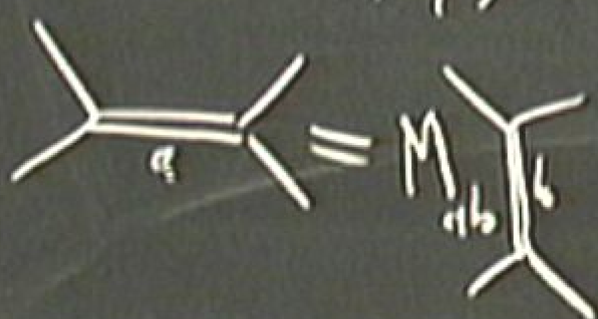

$$\mathcal{F}_a = M_{ab} \, \tilde{\mathcal{F}}_b$$

fusion & braiding matrices.

Nekrasov on $\mathbb{R}^4$

Pestun on S^4

$$\mathcal{F}(m;a) = \sum_{\text{inst.}} e^{-S}$$

$$\int da\, |F(a)|^2$$

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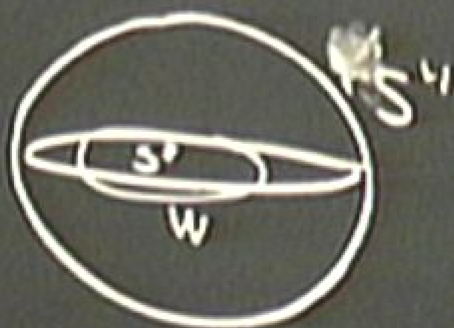
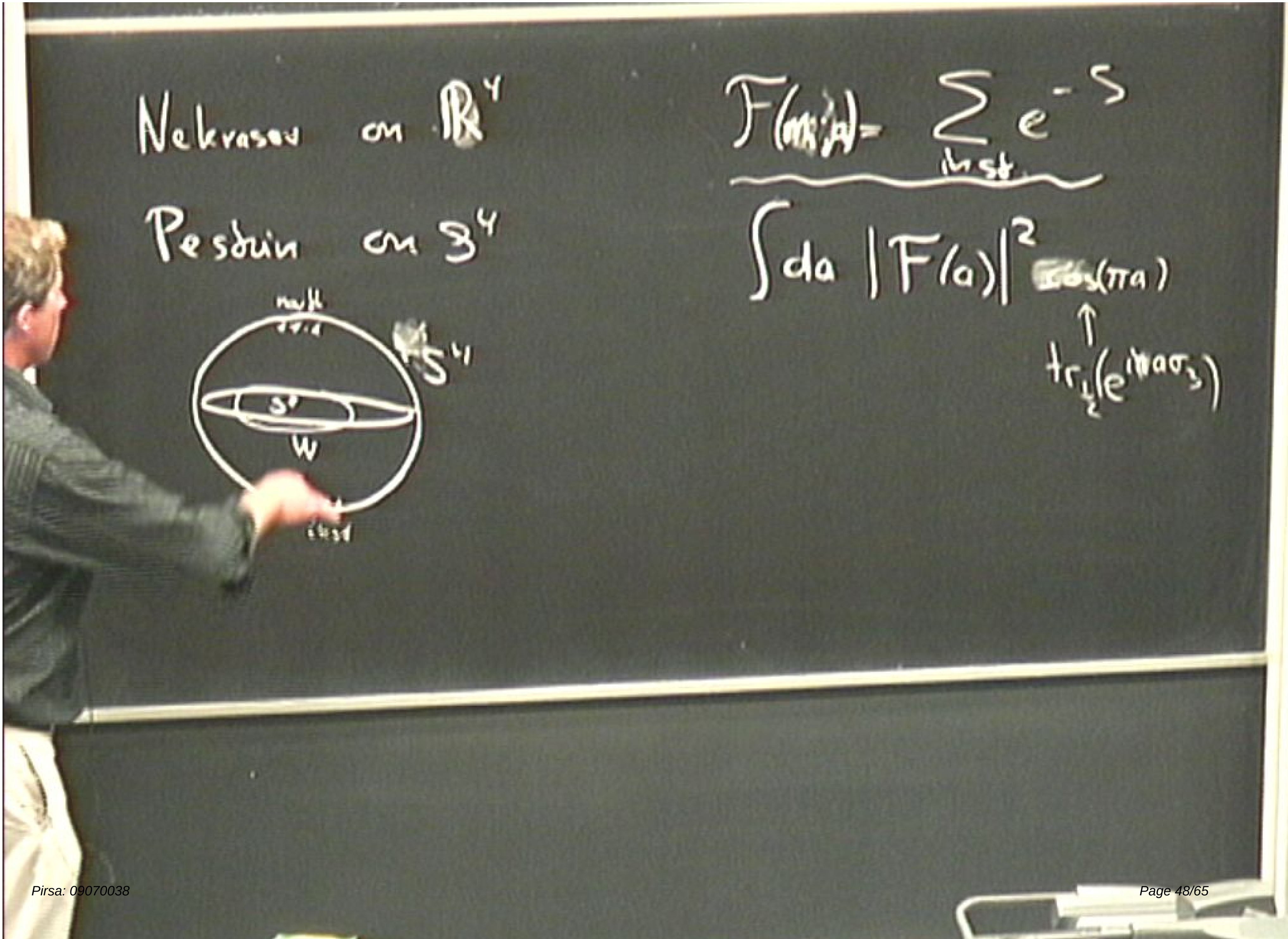


$$\mathcal{F}(a;\hbar) = \sum_{\text{inst.}} e^{-S}$$

$$\int da\, |F(a)|^2 \cos(\pi a)$$


Nekrasov on R^4
Pestun on S^4
north
S^4
S^3
W
= Σ_inst e^-S
∫ da |F(a)|^2 cos(πa)
tr_1/2 (e^iπaσ_3)

Nekrasov on $\mathbb{R}^4$

Pestun on S^4

$$\mathcal{F}(m;\hbar) = \sum_{\text{inst.}} e^{-S}$$

$$\int da \, |F(a)|^2 \cos(\pi a)$$

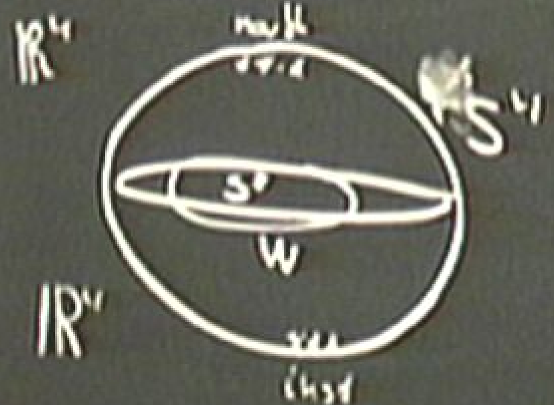
$$\uparrow$$

$$\mathrm{tr}_{\frac{1}{2}}(e^{i\pi a \sigma_3})$$

Liouville theory!

Nekrasov on $\mathbb{R}^4$

Pestun on S^4



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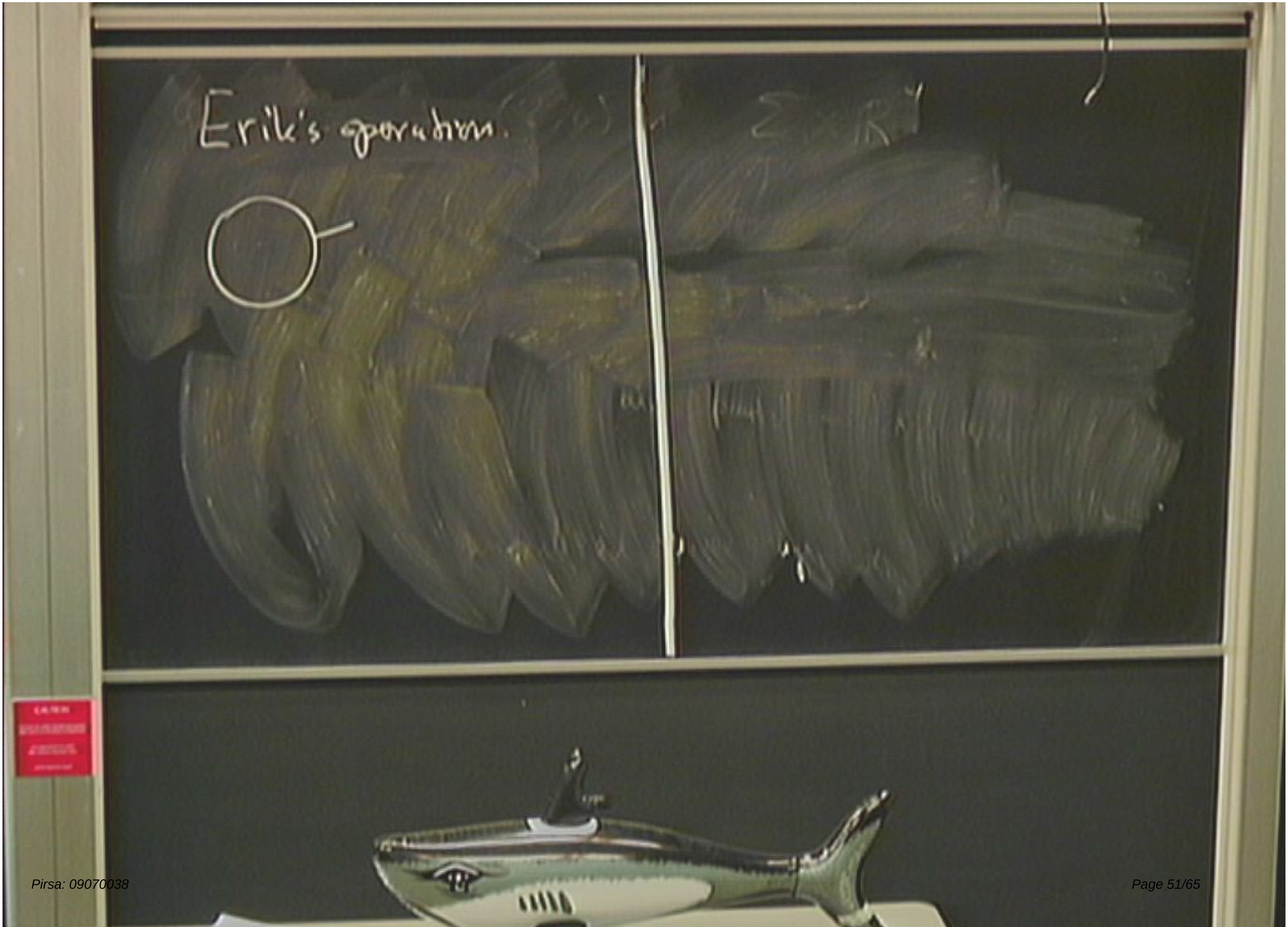
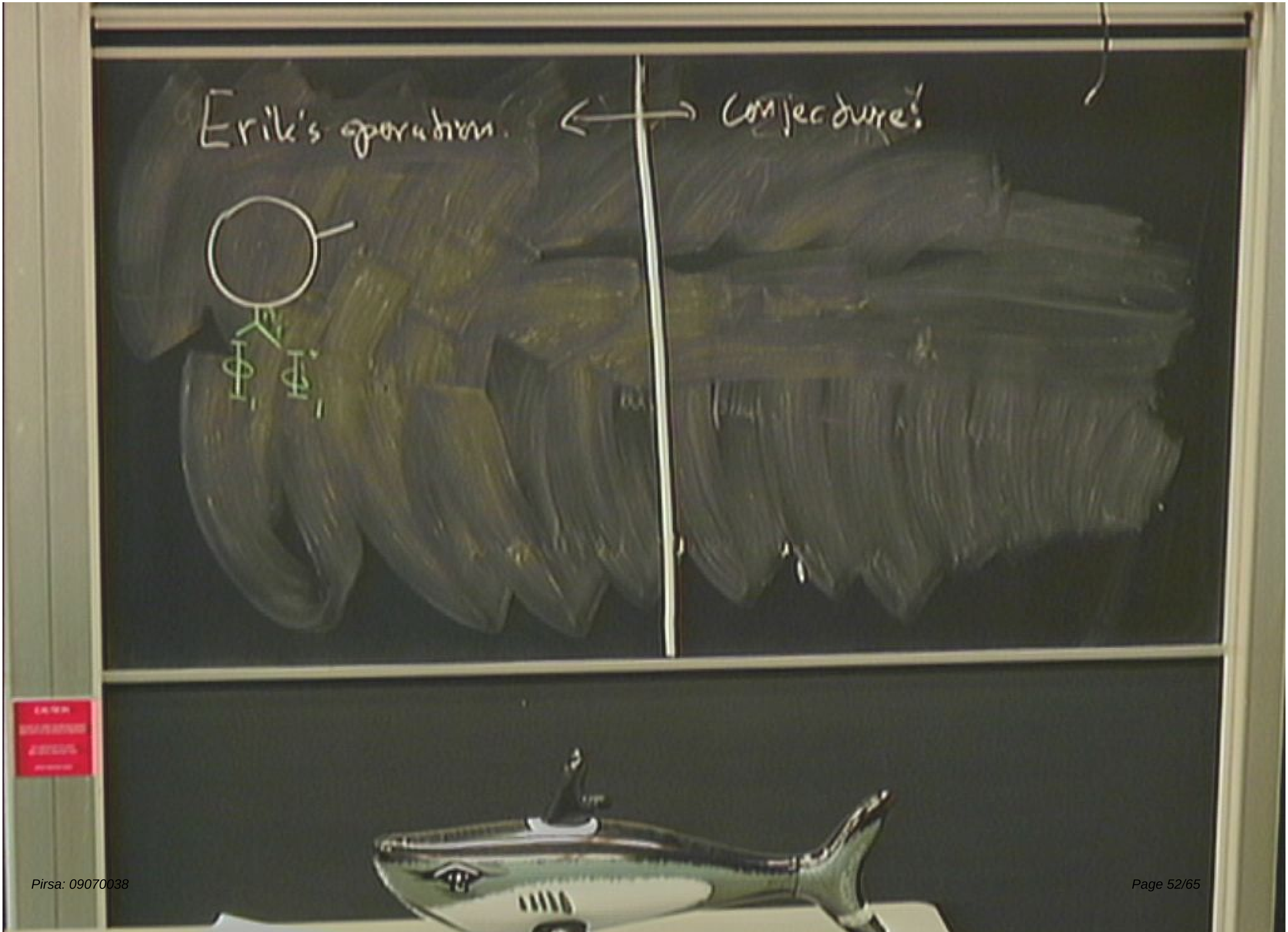

Erik's operation.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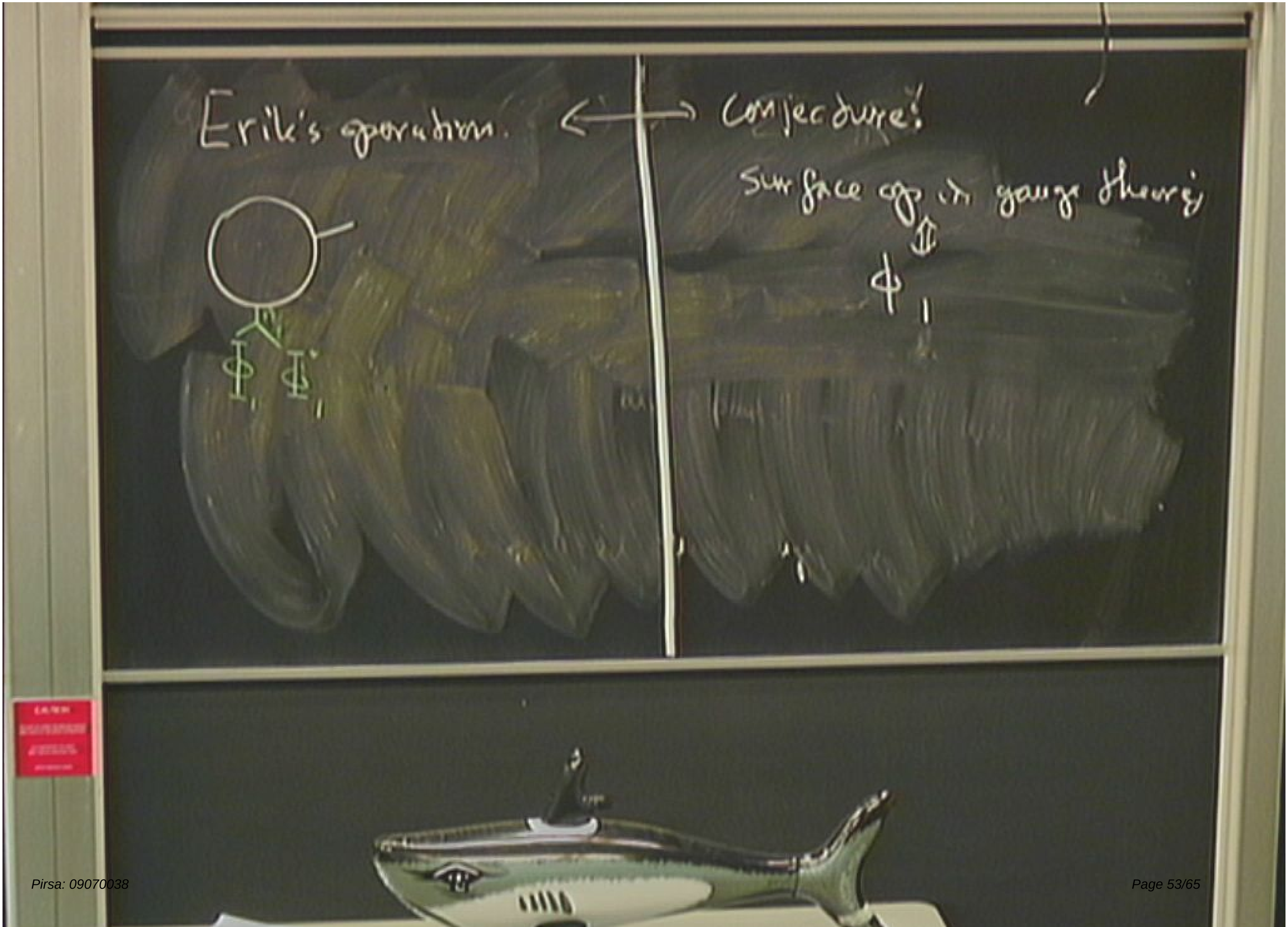
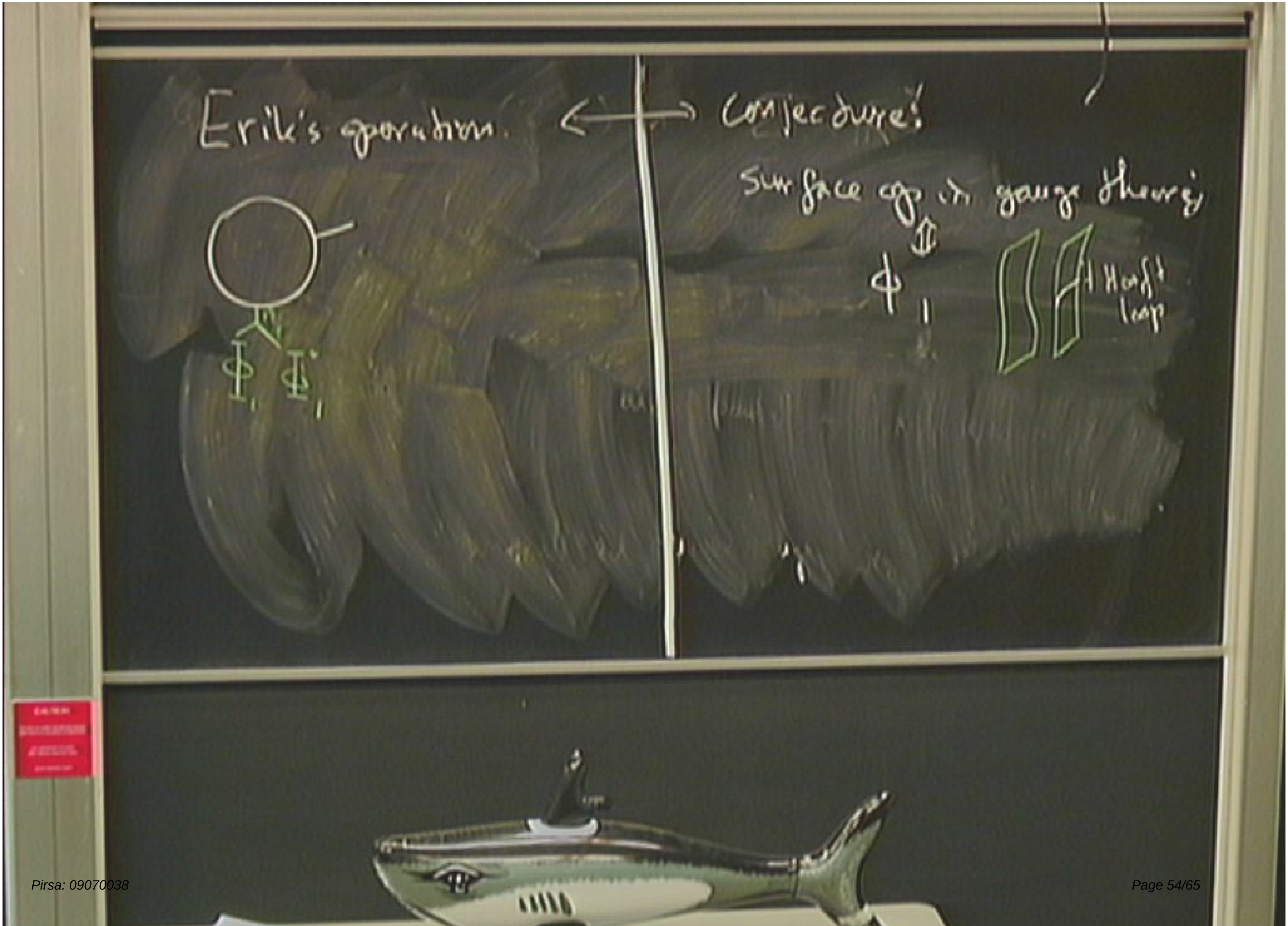
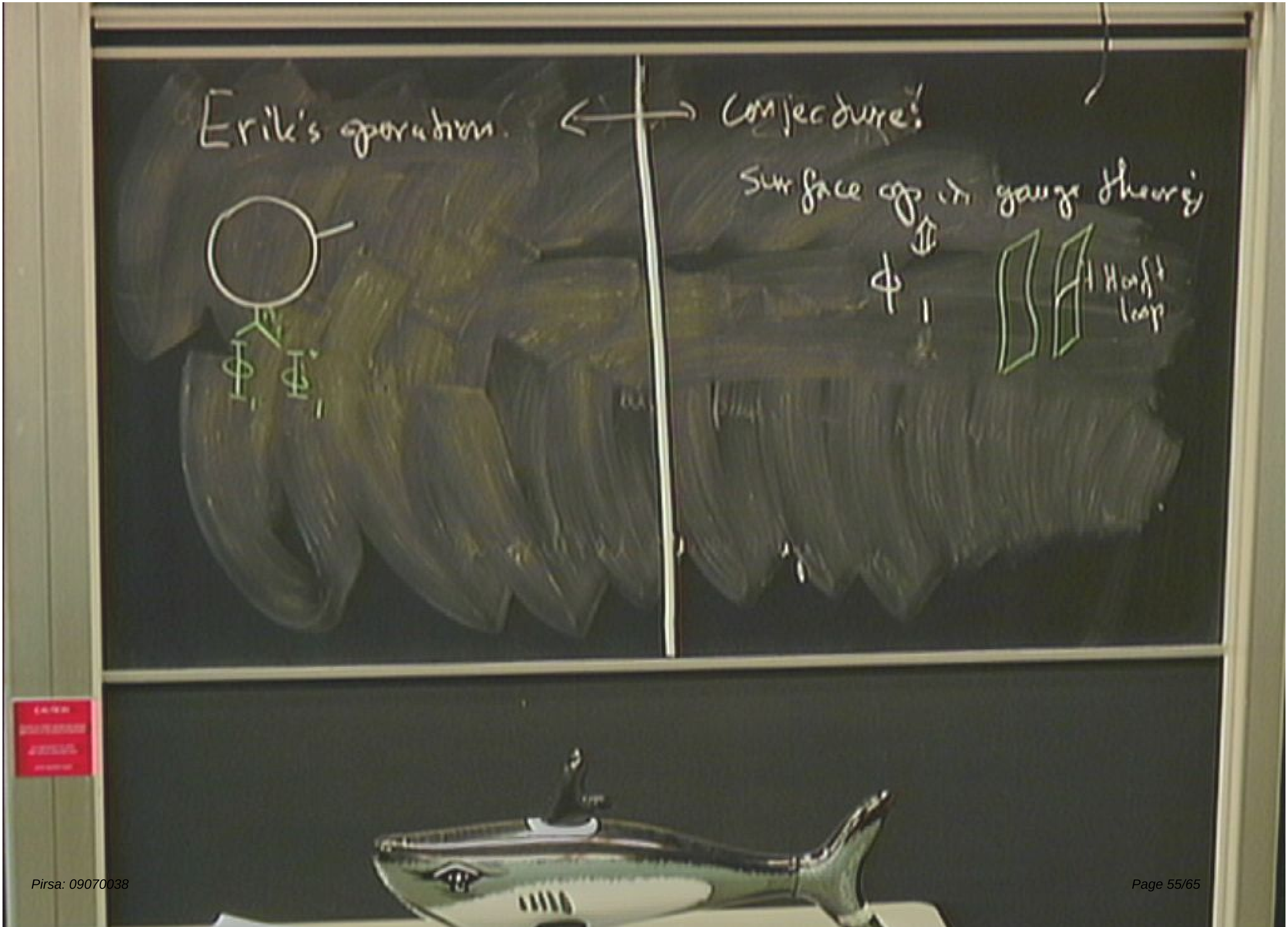
conjecture!


Erik's operators.
Conjecture!
Surface ops in gauge theory


Erik's operation.
conjecture:
Surface op in gauge theory
't Hooft loop


Erik's operation.
Conjecture!
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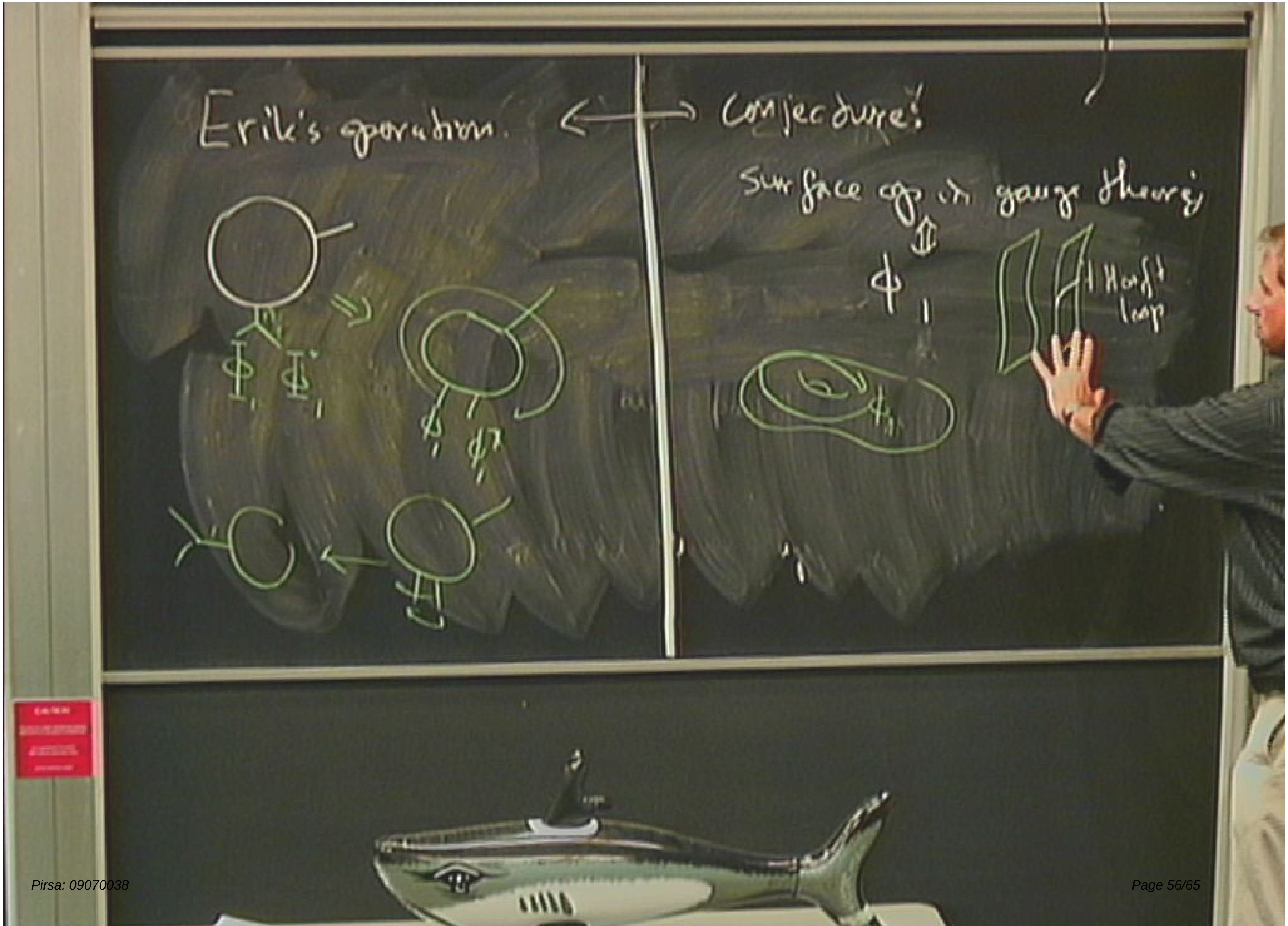
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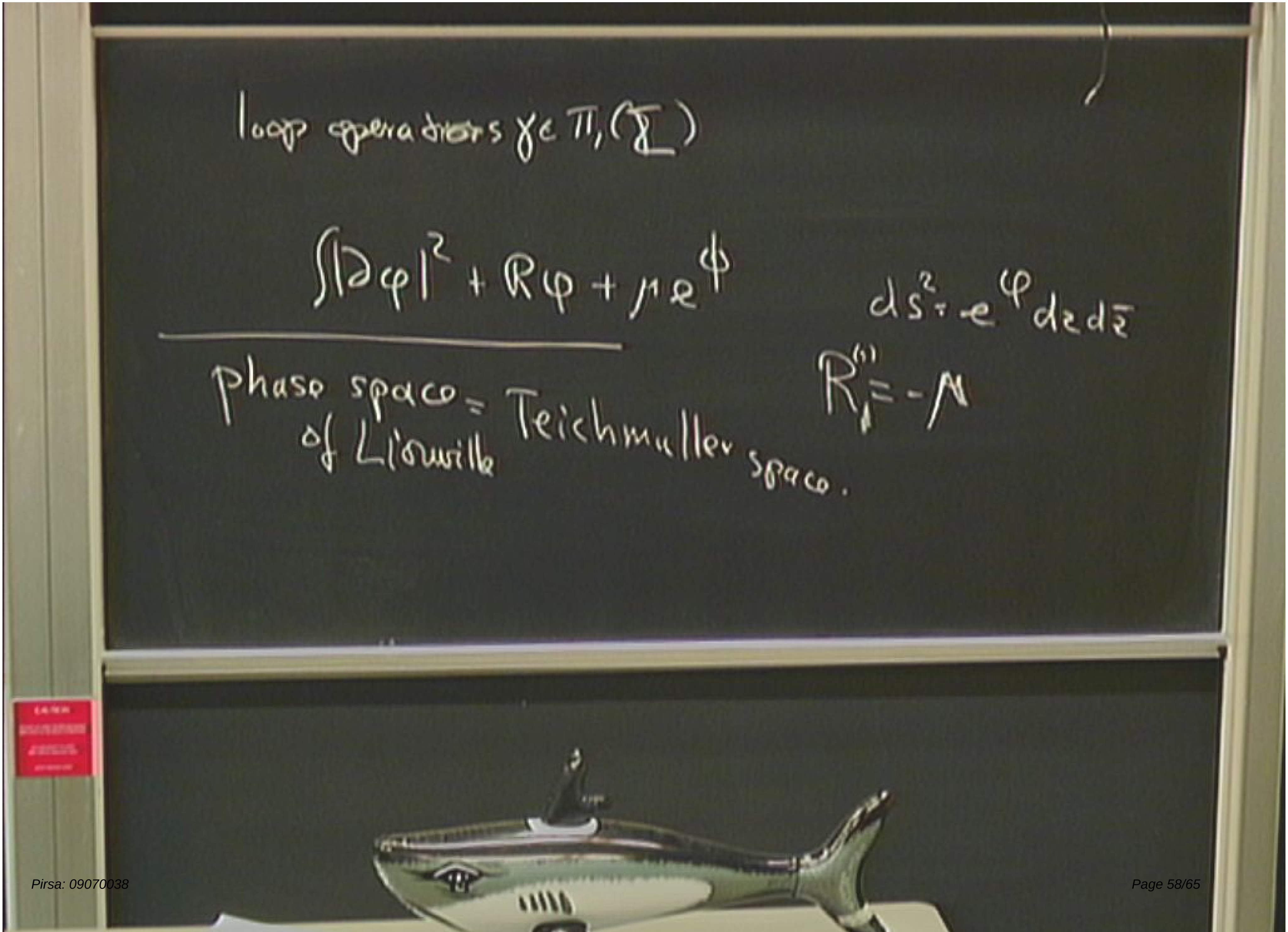
loop operators $\gamma \in \pi_1(\Sigma)$

$$\int |\partial\varphi|^2 + R\varphi + \mu e^{\phi}$$

$$ds^2 = e^{\varphi}\, dz\, d\bar{z}$$

$$R^{(1)}_{1} = -\Lambda$$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phase space of Liouville = Teichmuller space.

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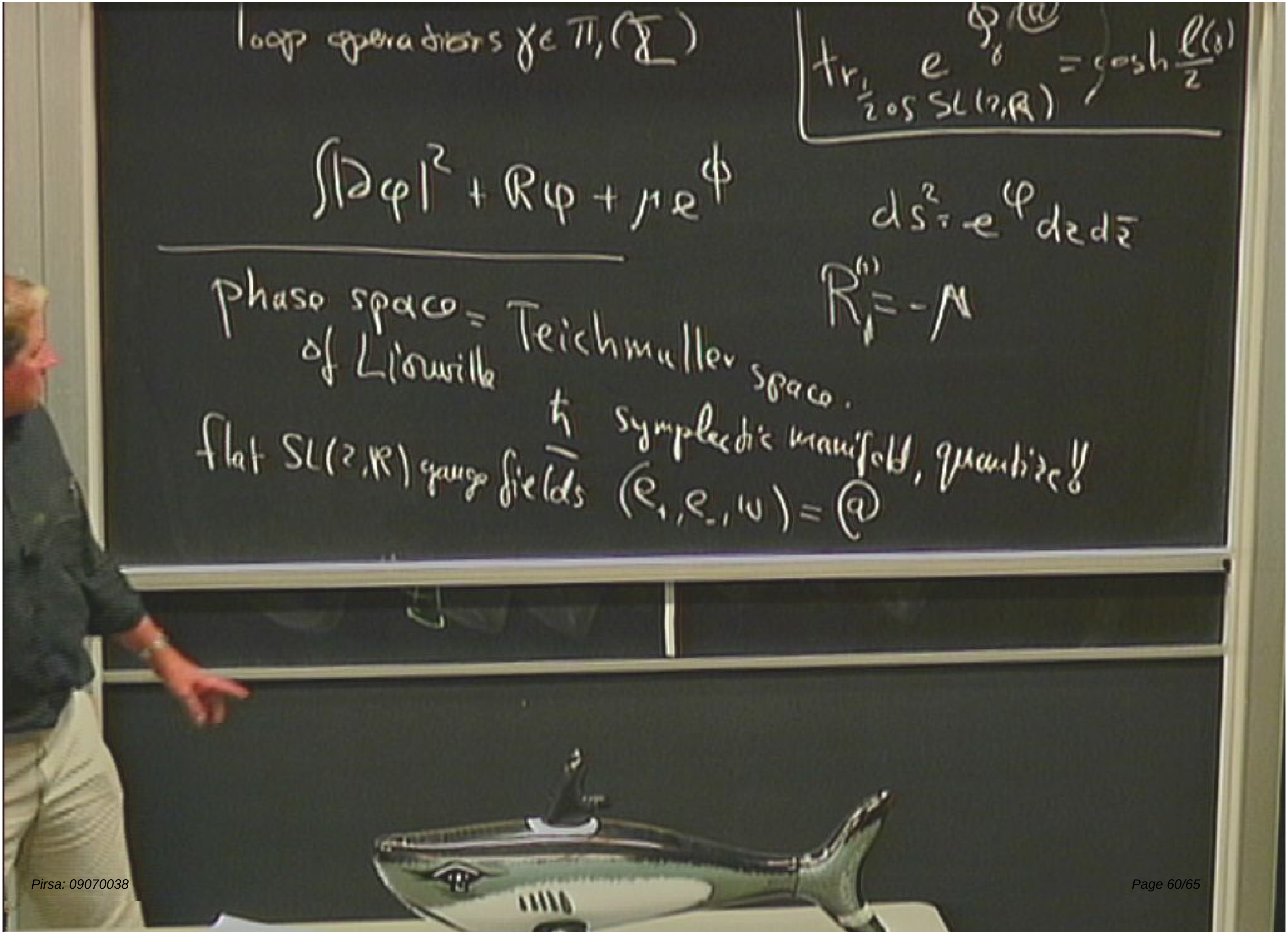
symplectic manifold, quantize!

flat $SL(2,\mathbb{R})$ gauge fields (e_+, e_-, ω)


loop operators γ ∈ π1(Σ)
tr 1/2 of SL(2,R) e^φγ(a) = cosh ℓ(γ)/2
∫|∂φ|^2 + Rφ + μe^φ
ds^2 = e^φ dz dz̄
phase space of Liouville = Teichmuller space.
R = -Λ
symplectic manifold, quantize!!
flat SL(2,R) gauge fields (e+, e-, ω) = a

loop operators $\gamma \in \pi_1(\Sigma)$

$$\boxed{\mathrm{tr}_{\frac{1}{2} \text{ os } SL(2,\mathbb{R})}\, e^{\varphi_\gamma(@)} = \cosh \frac{\ell(\gamma)}{2}}$$

$$\int |\partial \varphi|^2 + R\varphi + \mu e^{\phi}$$

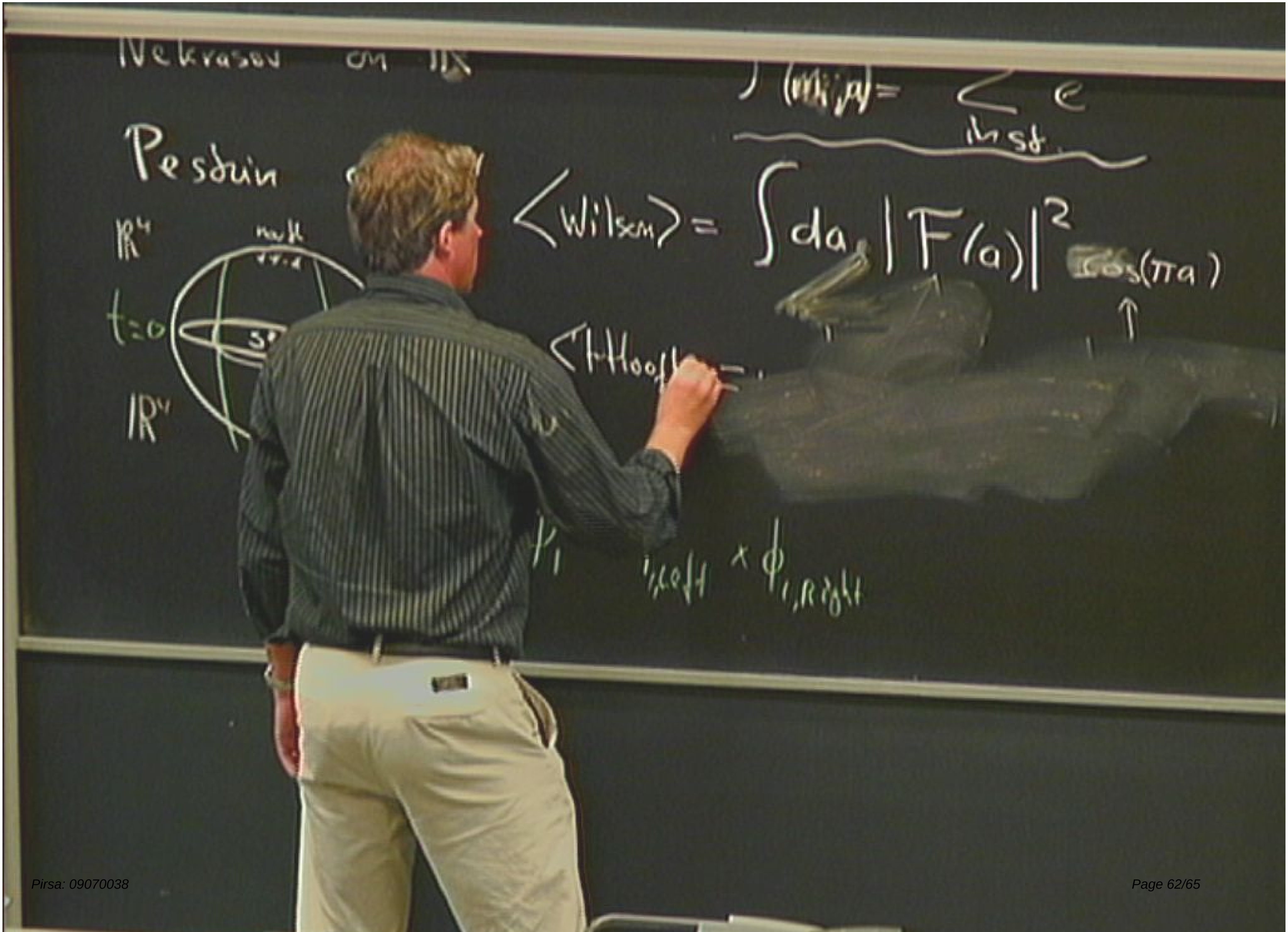
$$ds^2 = e^{\varphi}\, dz\, d\bar{z}$$

$$R^{(1)}_{\varphi} = -\Lambda$$

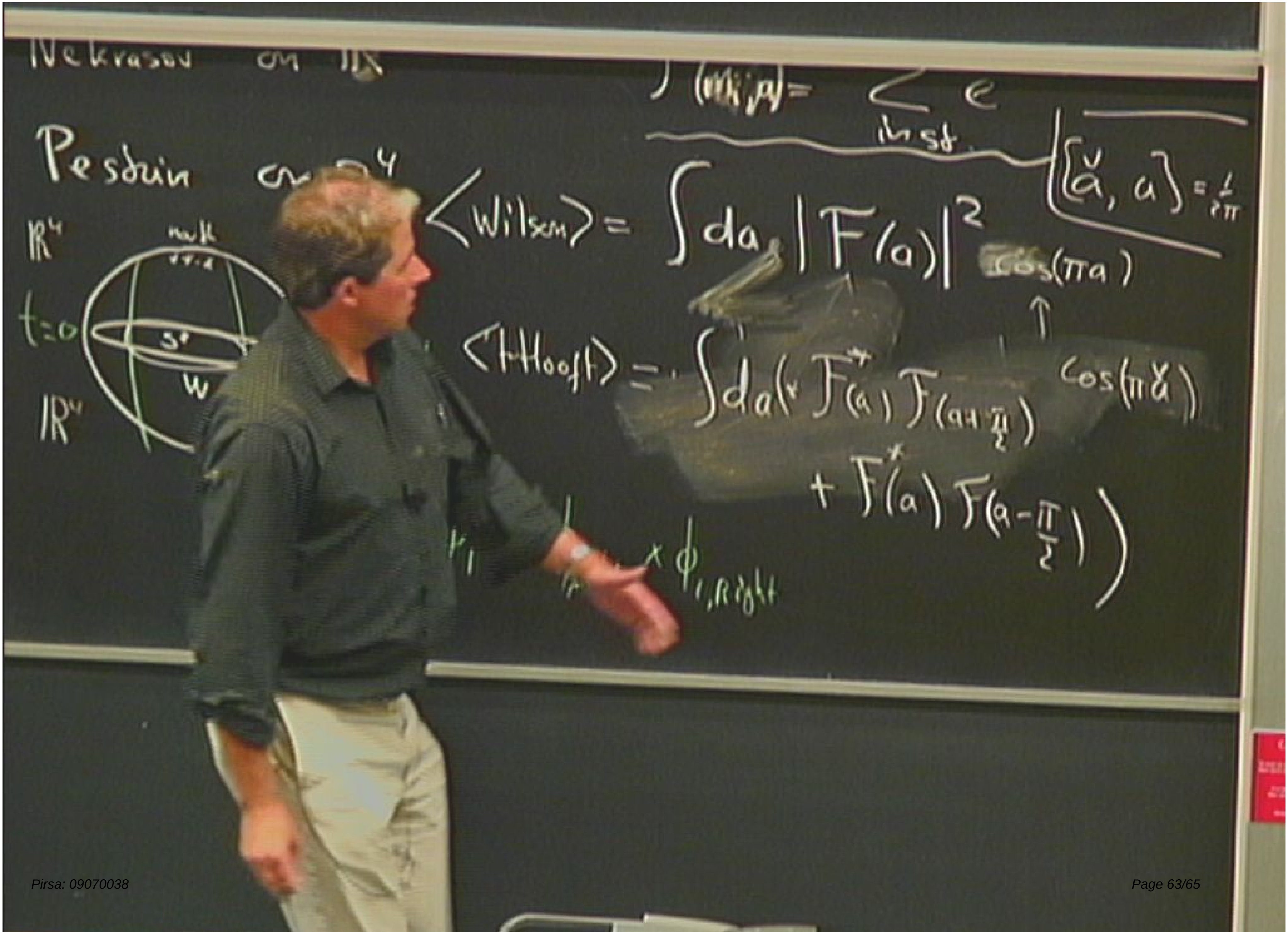
phase space of Liouville = Teichmuller space.

$\cong$ symplectic manifold, quantize!

flat $SL(2,\mathbb{R})$ gauge fields $(e_+, e_-, \omega) = @$



Nekrasov
Pestun
inst.
⟨Wilson⟩ = ∫ da |F(a)|²
t=0

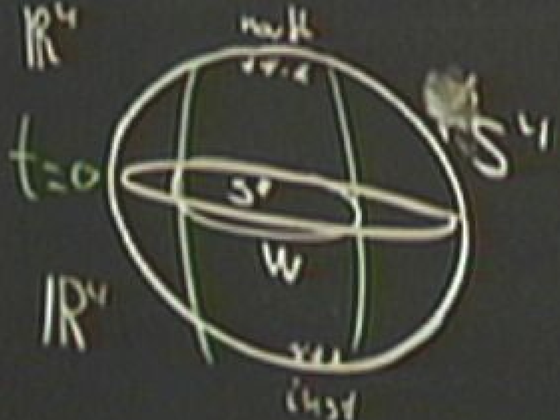

Nekrasov on
inst
Pestun on
$\langle$Wilson$\rangle$ = $\int da\, |F(a)|^2 \cos(\pi a)$
$[\check{a}, a] = \frac{i}{2\pi}$
$\langle$t Hooft$\rangle$ = $\int da \big(F^*(a)\, F(a+\frac{\pi}{2})$
$+ F^*(a)\, F(a-\frac{\pi}{2}) \big)$
$\cos(\pi\check{a})$
$\mathbb{R}^4$
$t=0$
S^3
W
$\phi_{1,Right}$

Nekrasov on $\mathbb{R}^4$

$Z(m_i, a) = \sum_{inst} e$

Pestun on S^4

$\mathbb{R}^4$



$\mathbb{R}^4$

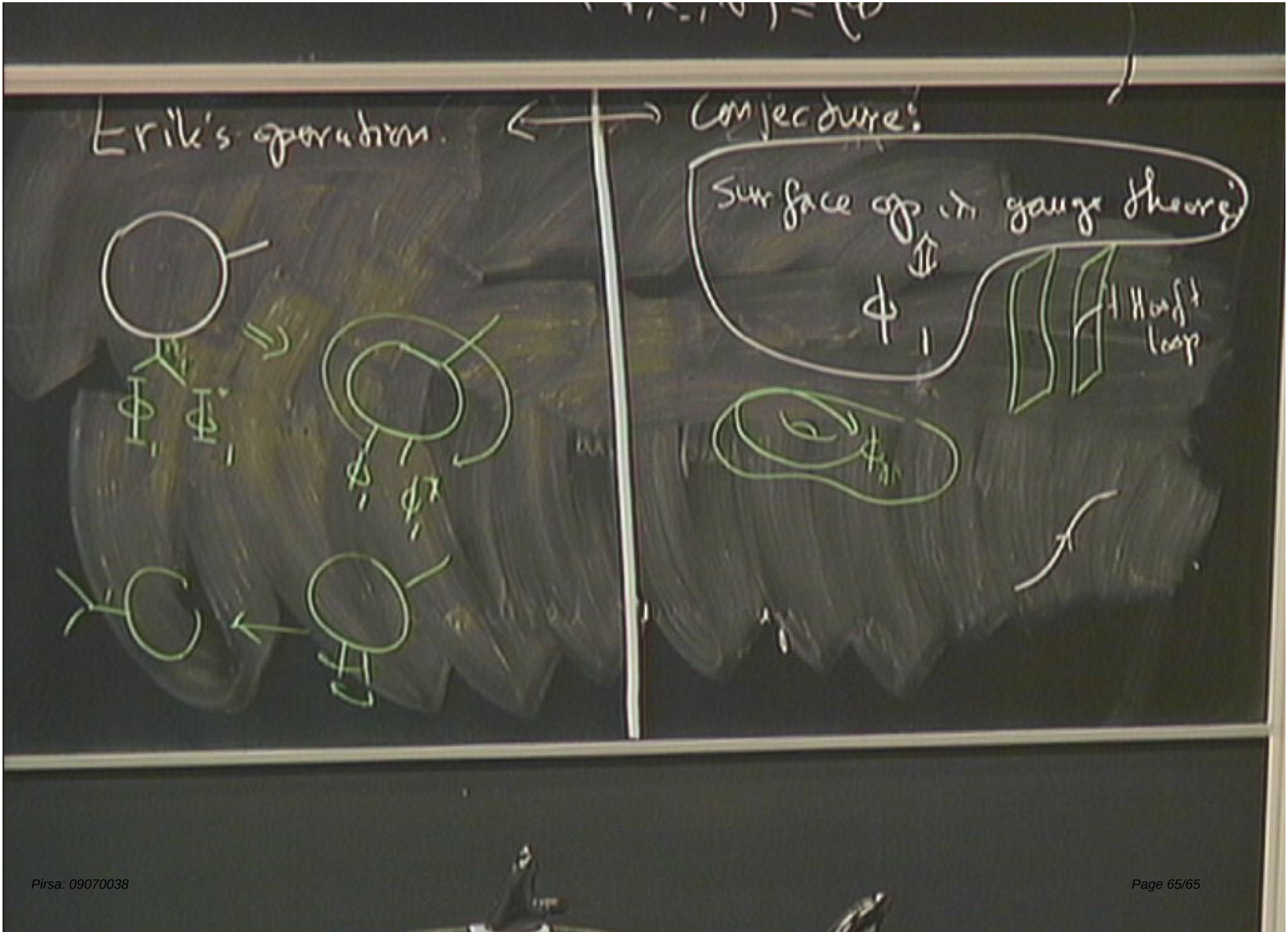
$$[\check{a}, a] = \frac{i}{2\pi}$$

$$\langle \text{Wilson} \rangle = \int da\, |F(a)|^2 \cos(\pi a)$$

$$\langle \text{'t Hooft} \rangle = \int da \left(F^*(a)\, F\!\left(a + \tfrac{\pi}{2}\right) + F^*(a)\, F\!\left(a - \tfrac{\pi}{2}\right) \right)$$

$\cos(\pi \check{a})$

$$\phi_1 - \phi_{1,\text{left}} \times \phi_{1,\text{Right}}$$


Erik's operation
Conjecture:
Surface op in gauge theory
't Hooft loop