Title: Galactic Dynamics: an overview for physicists

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Abstract: The standard theorist's model for the dynamics of galaxies is the limit of a Newtonian N-body system at fixed mass as the number of particles goes to infinity - i.e a phase space fluid After going over conventional wisdom, some interesting open issues which remain will be highlighted, and their relation to real galaxies explored.

# Galactic dynamics: an overview for physicists

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# GMRT: The talk I am *not* giving

# The “dwarf” NGC 3741 with the GMRT:

Begum, Chengalur, Karachantsev

A&A 433, L1, (2005)

Flat rotation curve out to 38 optical scale lengths meas- -ured with neutral atomic hydrogen gas

# The model – a collisionless phase space fluid

The Liouville Poisson equation aka "collisonless Boltzmann" (Binney / Tremaine) or "gravitational Vlasov" "Hartree" "mean field" (introduced by Jeans)

$$\partial f/\partial t + v\,\partial f/\partial x + a\,\partial f/\partial v = 0$$

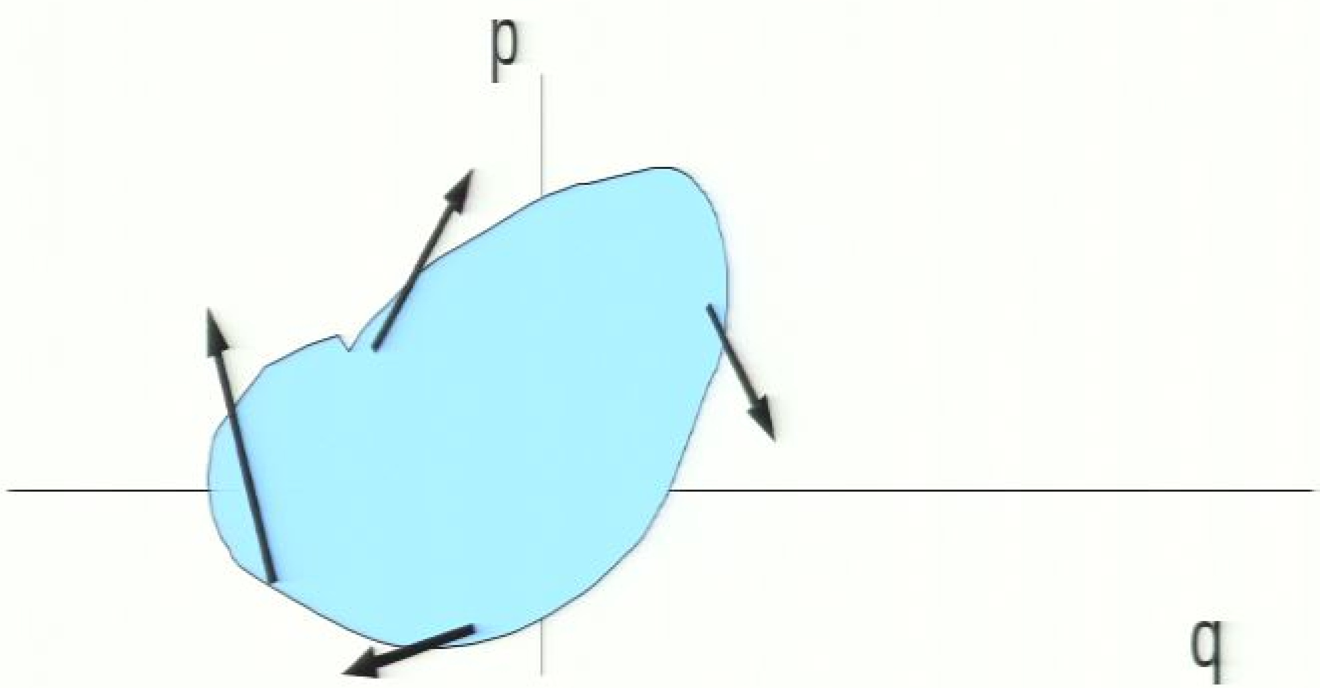
$$a = -\nabla\phi$$

$$\nabla^2\phi = 4\pi G\rho$$

f normalised to mass per Unit volume in xv space

$$\rho = \int f(x,v)\,dv$$

# In words and pictures...




Phase space flow with a self consistently determined potential Even in a fixed potential such flows in 2+2 and higher are nontrivial

# What can one do with this nonlinear equation?

Exact solutions (stationary)

Exact solutions (time dependent)
Numerical solution (stationary)
"Statistical mechanics" (equilibrium? Non-equilibrium? )
Numerical solutions (time dependent)

# Jeans theorem for steady state systems

Best way to make $\partial f/\partial t=0$ is to make the distribution function a function of conserved quantities so that RHS of P-L equation vanishes.

For example in spherical case, case,choose a function of $E=(v_x^2+v_y^2+v_z^2)/2+\varphi(x,y,z)$ (for example exponential)

We then get $\rho=\chi(\varphi)$ (also exponential )

We can then solve a nonlinear elliptic problem for the potential, $\nabla^2\varphi=4\pi G\chi(\varphi)$

Leads to the 'isothermal sphere' in exponential case

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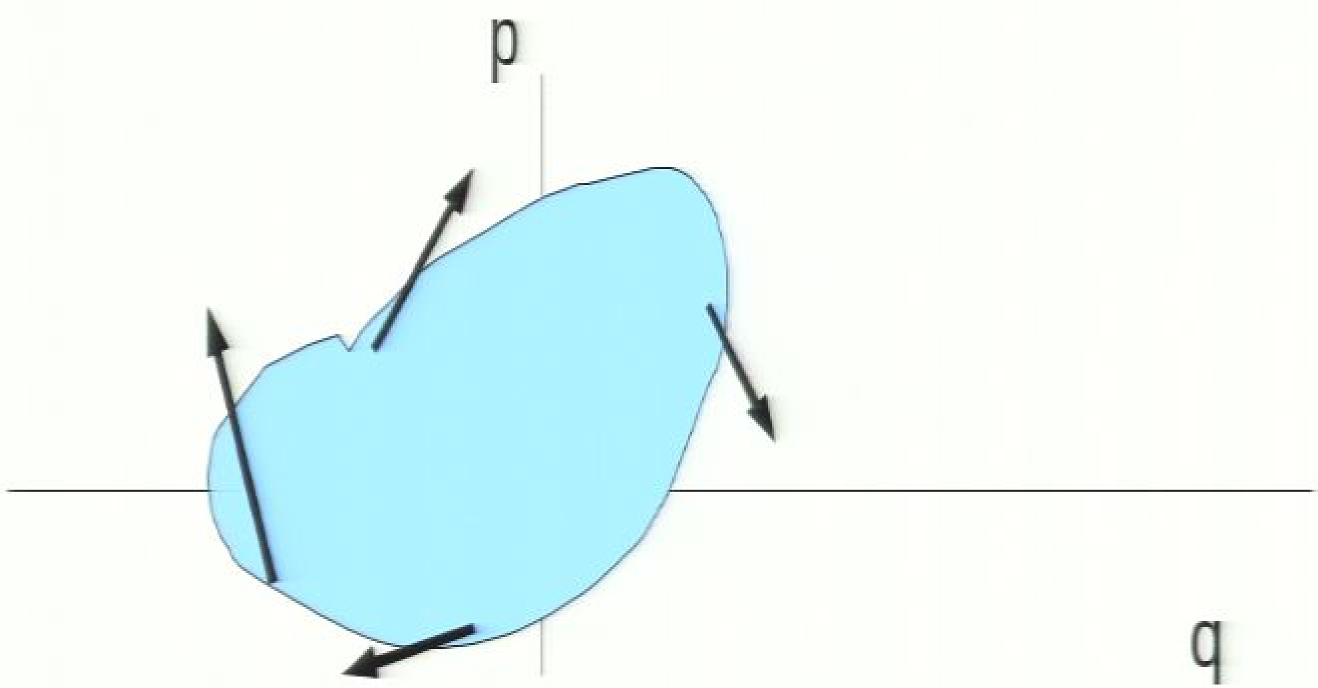


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# Jeans theorem for steady state axisymmetric systems

Motivation – our galaxy and other disc galaxies are approximately axisymmetric

Choose a function of

$$L_z = r v_\phi, E = v_\phi^2 + v_r^2 + v_z^2 + \varphi(x, y, z)$$

We then get $\rho = X(r, \varphi)$

We can then solve a nonlinear elliptic problem for the potential,

$$\nabla^2 \varphi = 4\pi G X(r, \varphi)$$

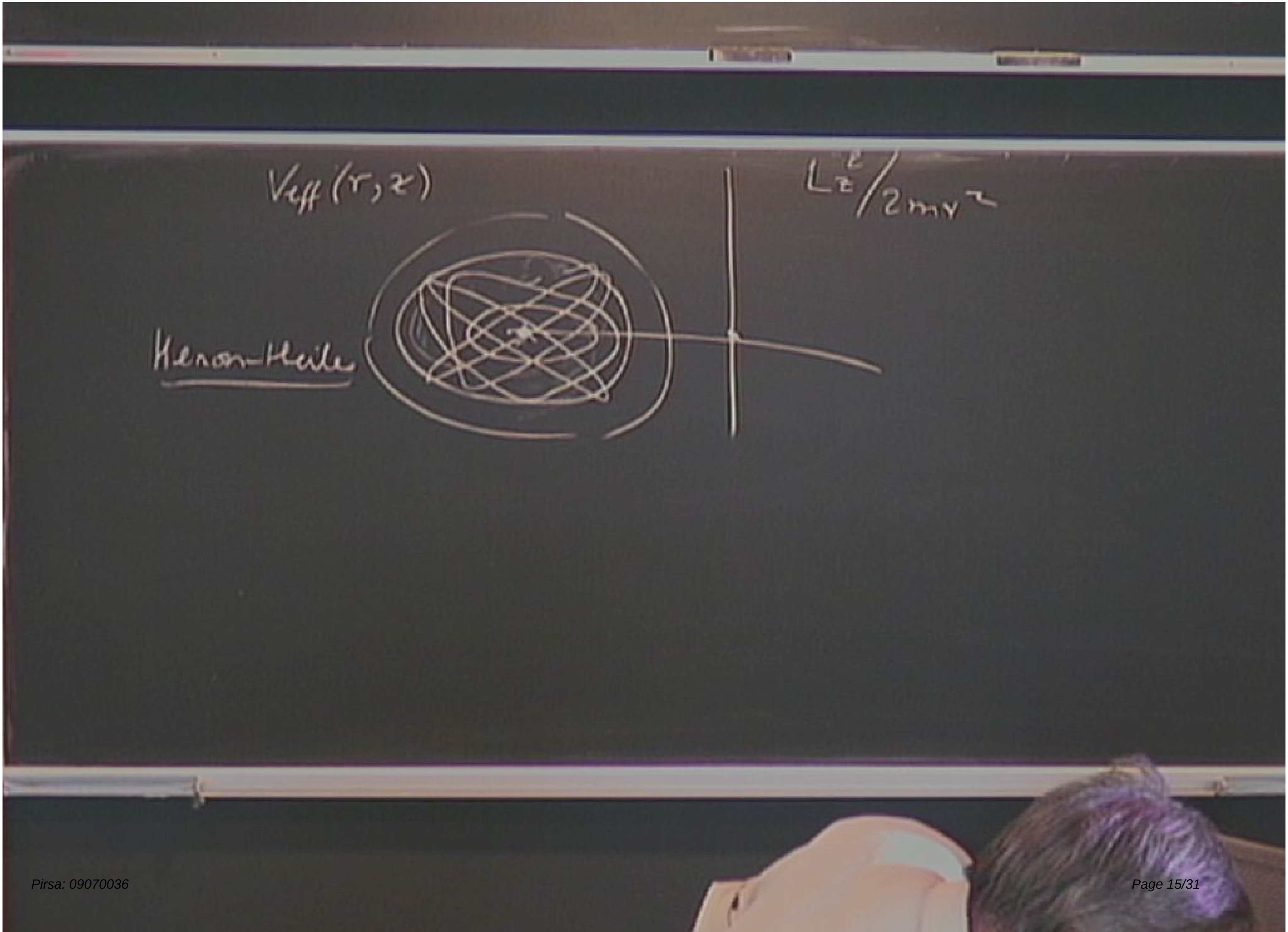
# Our galaxy needs a ‘third integral’

The general recipe $f = F(L_z, E)$ implies

$$\langle v_z^2 \rangle = \langle v_r^2 \rangle \text{ but not } \langle v_\phi^2 \rangle$$

Observationally, random velocities in the vertical direction are not equal to those in the radial direction (known for 100 years! - (K)Schwarzschild ellipsoid)

In practice, numerical work on galactic like potentials showed an extra conservation law


Veff (r, z)
Henon-Heile
Lz²/2mr²

# Back to spherical systems

Can have $f = F(E, L^2, L_z)$

Might appear to break spherical symmetry becaue of the term, $L_z$

But one can construct an example where there is net rotation but a spherically symmetric density, which is all that is required for self consistency. f is underdetermined by the density

Rotation does not imply flattening (anisotropic stresses)

People found that quite flattened elliptical galaxies rotate quite slowly

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# .. and triaxial systems

Can get evidence for triaxiality even from two dimensional galaxy images. “isophote twists”

Modeling faces a difficulty if we have only the energy as a constant of the motion

How can we fit a function of three variables with one of one variable ?

How can we get triaxiality with isotropic stresses implied by F(E) ?

# (M) Schwarzschild models

Integrate orbits in a triaxial potential

They come in families or libraries

Find the contribution of each orbit to the density

Find positive weights for the orbits to represent the original density as closely as possible

Iterate

# Analytic triaxial models

Constructed by G.G.Kuzmin and later in generality by de Zeeuw

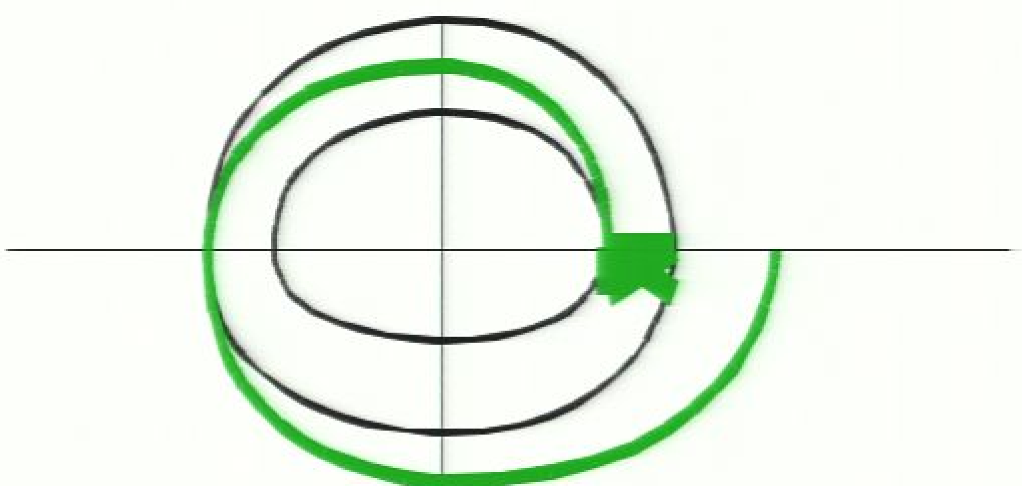
Used potentials separable in ellipsoidal co-ordinates

Orbit families similar to Schwarzschild
A case of 'self organised integrability' !

Has a magneto fluid analogue...

# How is the steady state with f=f(I) attained?

- Fundamentally reversible dynamics so its all to do with initial conditions
- Landau damping an example 'phase mixing' (due to a continuum of frequencies)
- After 'coarse graining', f is a function of the action variables alone



# H-theorem? (Tremaine, Henon. Lynden-Bell)

Start with an initial state with coarse grained distribution function equal to fine grained (CGDF=FGDF)

The “entropy” associated with the FGDF remains the same, but with CGDF increases over its initial value (not monotonically! e.g exact oscillating models exist – AN, SS)

True for $H[f]=\int C(f)\,d\Gamma$ where C need not be the standard -f ln f but any convex function

# Link to astronomy

Simulations show approach to a steady state

Fine structure in phase space of stars in our galaxy an observed fact! (Hiipparcos satellite) – not possible until distances and transverse velocities could be measured.

Density profiles of products are reasonable

The phase space constraint is useful (dark matter, galaxy cores)

Interacting dark matter could violate that

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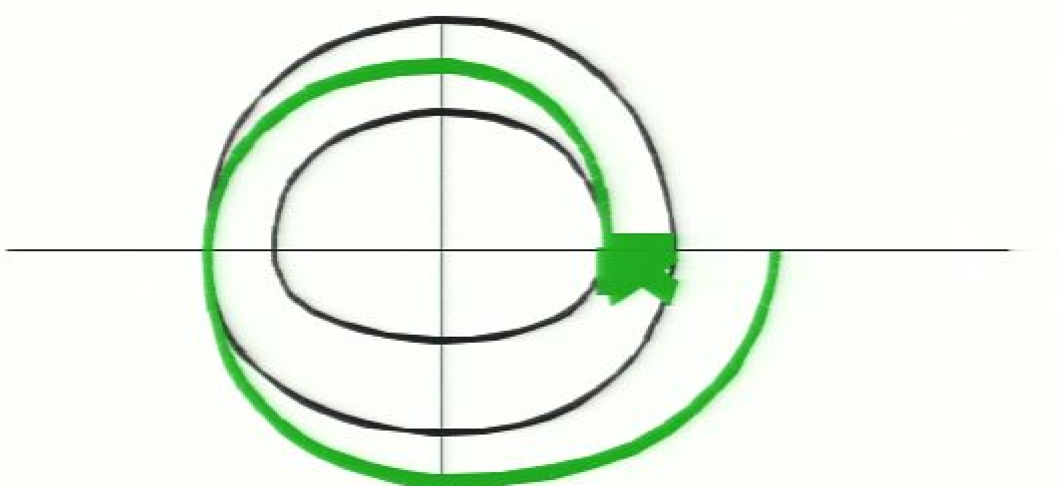

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# Collisional regime due to discreteness

- M=N*m, and we have been in the N->infinity limit
- Two particle interactions go as m^2, while one body with potential as M*m, so the scattering effects go as 1/N
- Better, use a Coulomb mean free path as in plasmas (actually, Jeans / Spitzer / Ambartsumian / Chandrasekhar did it in parallel to plasmas (Landau).

# Two body relaxation

For large angle scattering, impact parameter

$$b_{hard} \sim Gm/v^2; l \sim 1/(n b_{hard}^2);$$

$$\tau_{hard} \sim l/v \sim v^3/(Gm^2 n) \sim inverse\ of\ phase\ space\ density$$

When we use the virial theorem and the dynamical time $v^2 \sim GM/R \sim GNm/R; \tau_{dynamical} = R/v;$

$$we\ get\ \tau_{hard}/\tau_{dynamical} \sim N.$$

Actually, soft (small angle ) collisions dominate

– each is weaker by 1/b^2 but

the cross section goes up as b^2 so we get

– enhancement by a 'Coulomb logarithm'

# Globular star cluster Omega Centauri

Even with N ~ 10^6, two Body relaxation dominates over the Hubble time (they wouldnt be globular otherwise!)

# Consequences of relaxation

NOT going peacefully to an equilibrium – the isothermal sphere is unbounded and unstable!

Evapouration means “heating” (tighter binding plus virial theorem)

Mass gradients, velocity^2 (i.e temperature ) gradients all develop spontaneously

Angular momentum pushed out

# Instructive pathologies of stat mech with -1/r: beyond 2 bodies

Microcanonical ensemble fails because of infinite phase volume at finite energy even in a box

Phase volume below E_b for a binary ~ E_b^(-3/2), so density of states ~ E_b^(-5/2)

For N+2 particles, we then convolve E^(3(N+1)/2 -1) for the rest and E^(-5/2) for the binary). Diverges for N=1 and higher

Heggie's insight – hard binaries get harder and act as a heat source for the rest of the cluster