Title: Insightful D-branes

Date: Jul 17, 2009 02:30 PM

URL: http://pirsa.org/09070035

Abstract: We will describe black holes in AdS with hyperbolic horizons, and obtain a holographic description of the region inside the horizon, focusing on the dynamics of the scalar fields in the dual gauge theory. This leads to a proposal for a dual description of D-branes falling through the horizon of any AdS black hole. The proposal uses a field-dependent time reparameterization in the field theory. We relate this reparametrization to various gauge invariances of the theory. Finally, we speculate on information loss and the black hole singularity in this context. This talk will be based on arxiv:0904.3922, and ongoing work with the same authors.

Outline

I. Introduction

II. A holographic cosmology

III. Gauge theory vs. spacetime coordinate transformations

IV. Gauge theory dynamics and hyperbolic black holes

V. Conclusions

Based on work with G. Horowitz and E. Silverstein

arxiv:0904.3922
and work in progress

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Albion Lawrence
Brandeis University

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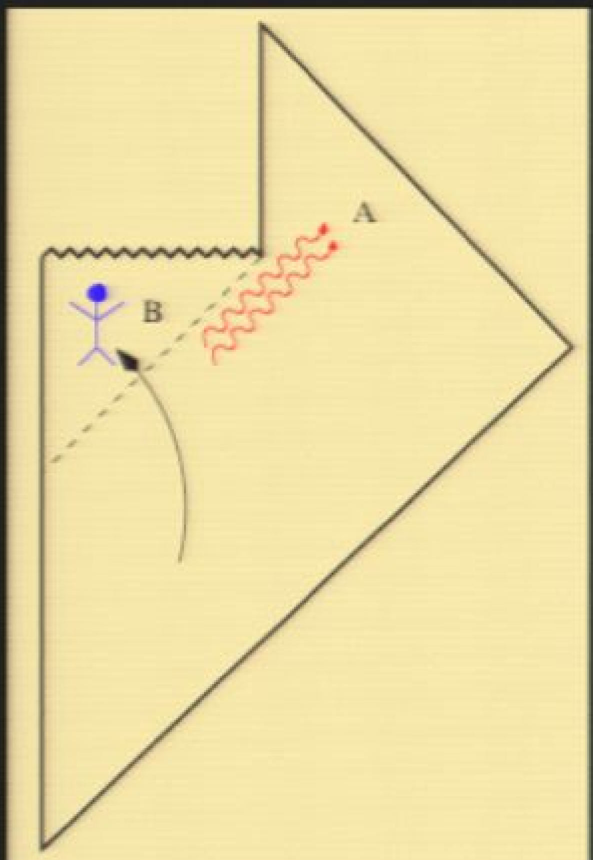
I. Introduction

Black hole complementarity

't Hooft, Susskind



Large black hole: curvature remains weak well inside the horizon.

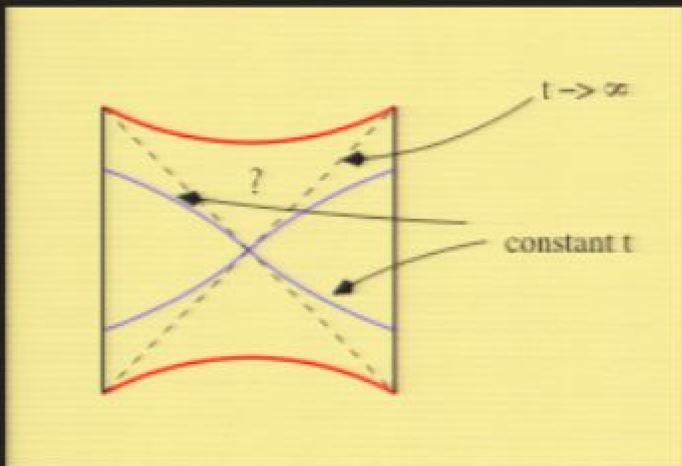
1. Infalling observer (B) remains semiclassical until it reaches the singularity.

2. External observer (A) sees black hole evaporate via long-wavelength, thermal, Hawking radiation. Infalling observer is "cooked" near the horizon and re-emitted as Hawking radiation.

Unitarity of BH evaporation implies that these two pictures are equivalent (dual).

What is the map?

Black holes in AdS/CFT



AdS_5 black hole $\sim$ 4d gauge theory at temperature $T = T(M)$

Gauge theory time $\sim$ Schwarzschild time

(Time experienced by observer at fixed distance from BH).

Bulk: infalling objects approach horizon as $t \to \infty$

Gauge theory: excitations spread and thermalize

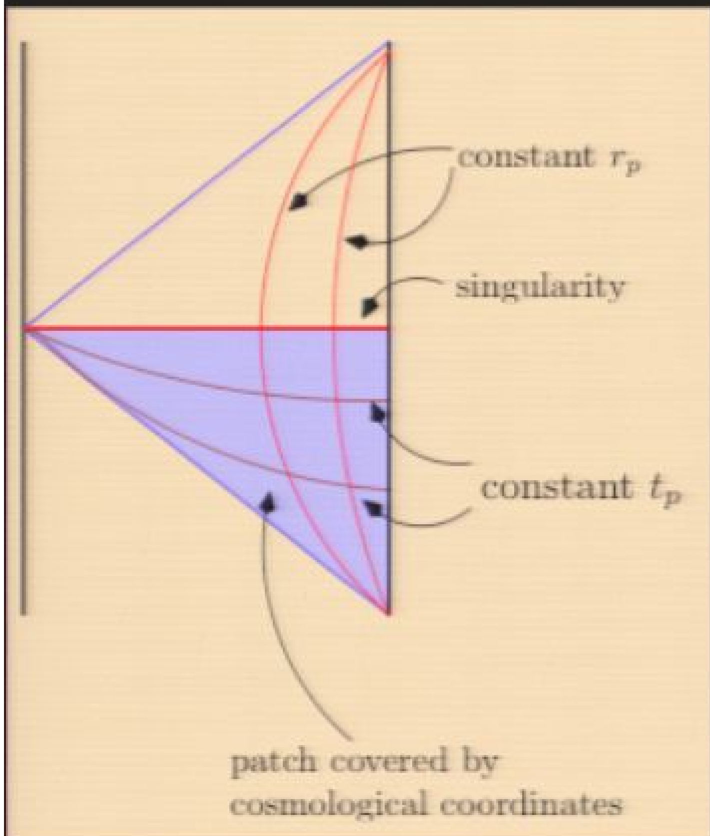
Gauge theory description of semiclassical physics behind horizon?

Gauge theory description of singularity?

II. A holographic cosmology

Green, Lawrence, McGreevy, Morrison, and Silverstein

HLS



Poincare coordinates:

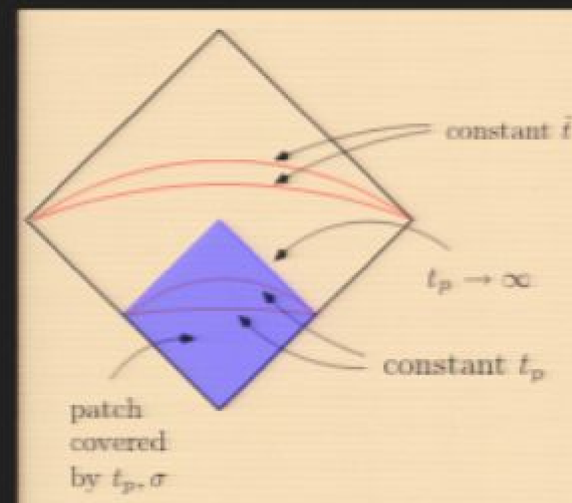
$$ds_5^2 = \frac{r_p^2}{\ell^2}\left(-d\tilde{t}^2 + d\vec{x}_3^2\right) + \ell^2 \frac{dr_p^2}{r_p^2} \qquad (\ell \text{ is AdS radius})$$

Patch of $\mathbb{R}^{3,1}$:

$$ds_4^2 = -dt_p^2 + t_p^2 d\sigma_{\mathbb{H}_3}^2$$

Orbifold: $\Sigma = \mathbb{H}_3/\Gamma$

$t_p \to 0^-$ becomes singular



Final bulk metric:

$$ds_5^2 = \frac{r_p^2}{\ell^2}\left(-dt_p^2 + t_p^2 d\sigma_\Sigma^2\right) + \ell^2 \frac{dr_p^2}{r_p^2}$$

Dual gauge theory lives on "collapsing cone" metric $\quad ds_4^2 = -dt_p^2 + t_p^2 d\sigma_\Sigma^2$

1. Similar in spirit to other time-dependent QFTs

Das *et. al.*
Awad *et. al.*
Craps, Sethi, *et. al.*
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2. Distinct from example of unstable QFTs

Horowitz and Hertog
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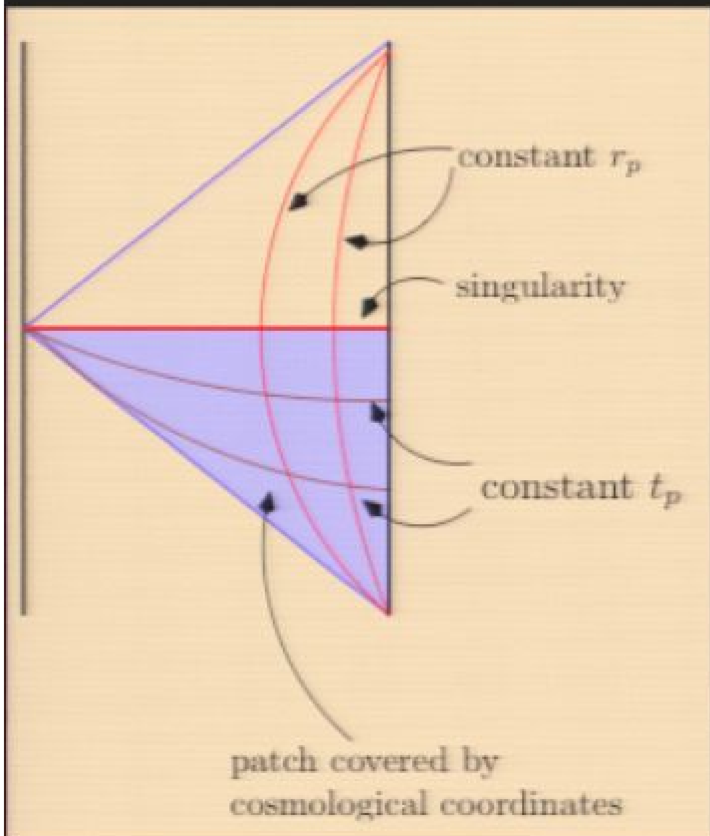

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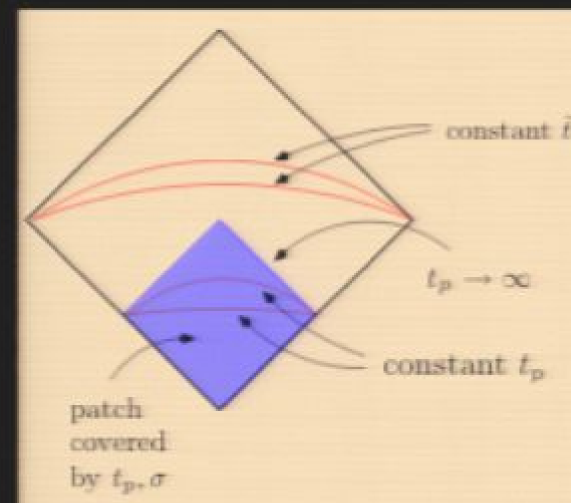

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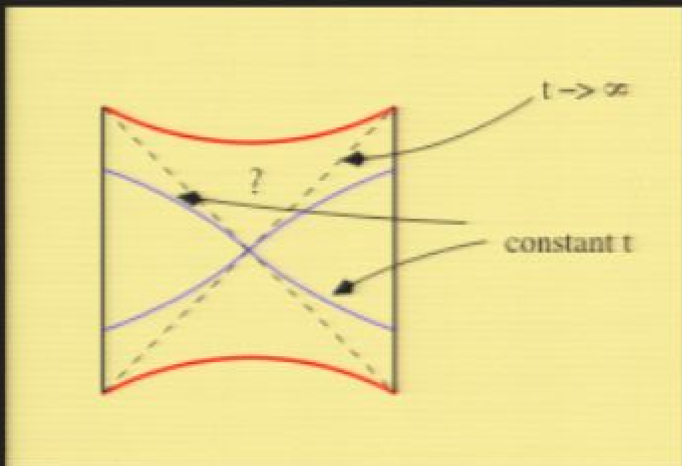

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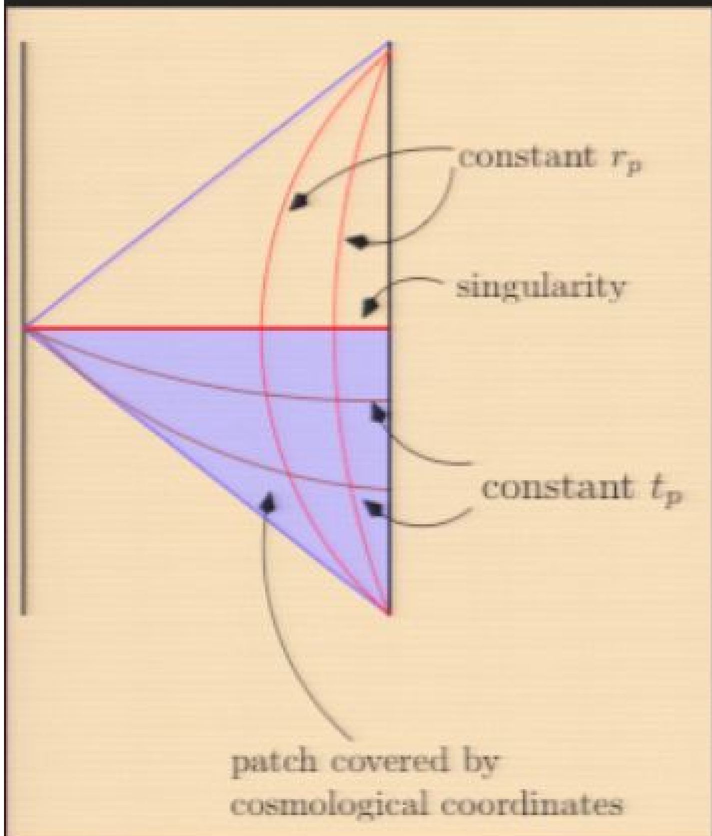

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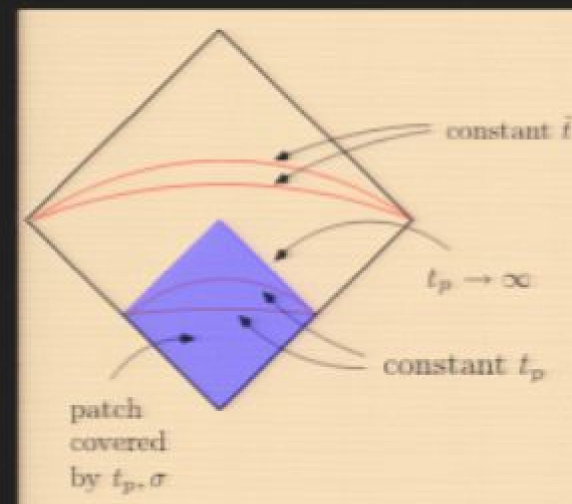

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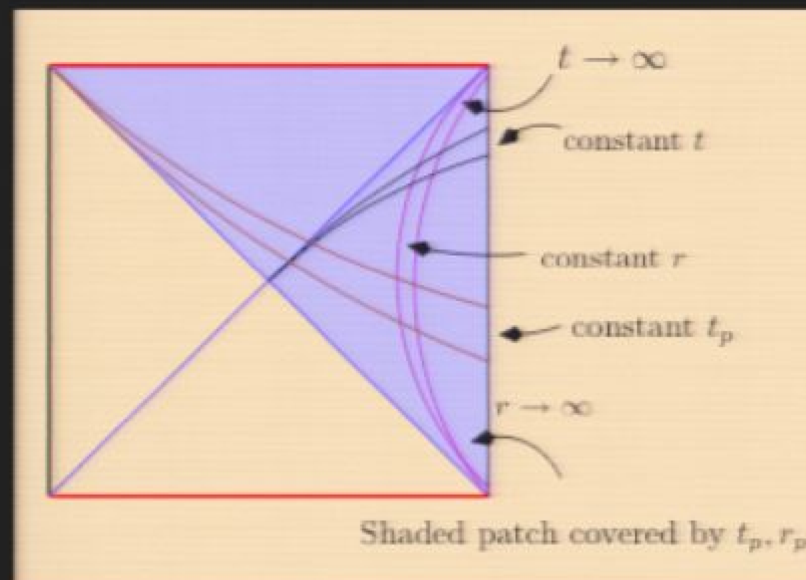
Static coordinate system

$$r = -\frac{r_p t_p}{\ell} \qquad t = -\frac{\ell}{2} \ln\left(\frac{t_p^2 r_p^2 - \ell^2}{\ell^2 r_p^2}\right)$$

$$ds_5^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\sigma_\Sigma^2$$

$$f(r) = \frac{r^2}{\ell^2} - 1$$



Shaded patch covered by t_p, r_p

Topological black hole: Emparan

AdS_5 version of BTZ black hole

- Negatively curved horizon at $r = \ell$.
- Temperature $T \sim 1/\ell$.
- Horizon area $\sim \ell^3$; entropy $\sim N^2$.

Dual: gauge theory on $\Sigma \times \mathbb{R}$ at $T = 1/R_\Sigma$

7/39

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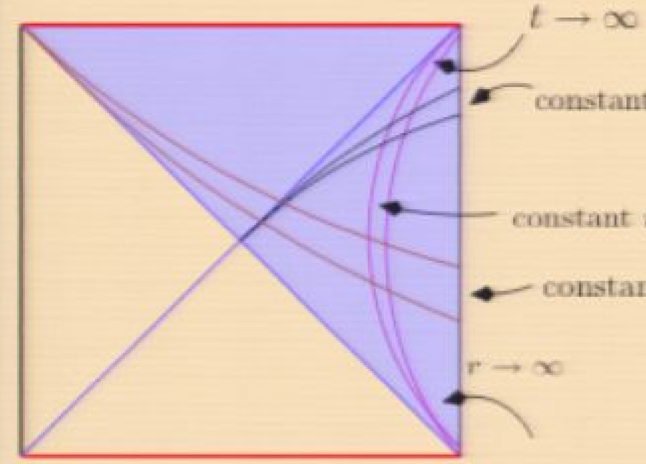

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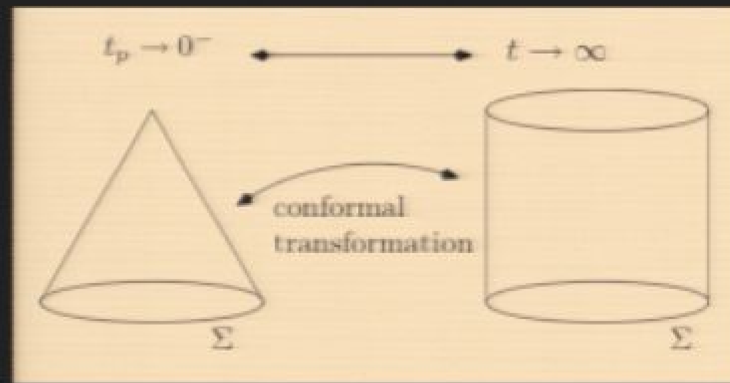
Conformal transformation:



$$t_p = -\ell e^{-t/\ell}$$

$$ds^2_{cone} \to ds^2_{cyl} = e^{2t/\ell} ds^2_{cone}$$

Seems to map QFT variables describing Scwharzschild observers to QFT variables describing infalling observers

- How does the map act on bulk probes?
- Can this be generalized to other BHs?

III. Gauge theory vs. spacetime coordinate transformations

A. $\mathcal{N} = 4$ SYM on $\Sigma \times \mathbb{R}$, $T = 1/R_\Sigma$

Consider D3-brane wrapping Σ and moving in r, t

String theory: D3-brane probe dynamics described by DBI action

$$S_{DBI} = \frac{1}{g_s(\alpha')^2} \int d\tau d^3\sigma \left(\sqrt{\det \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)} - A^{(4)}_{RR} \right)$$

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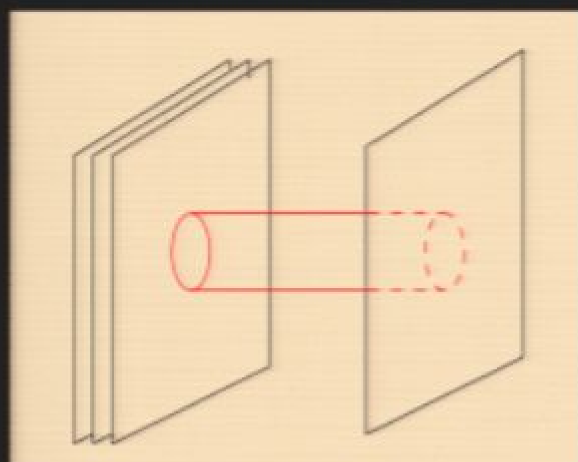

- S_{static} is resulting effective action for ϕ

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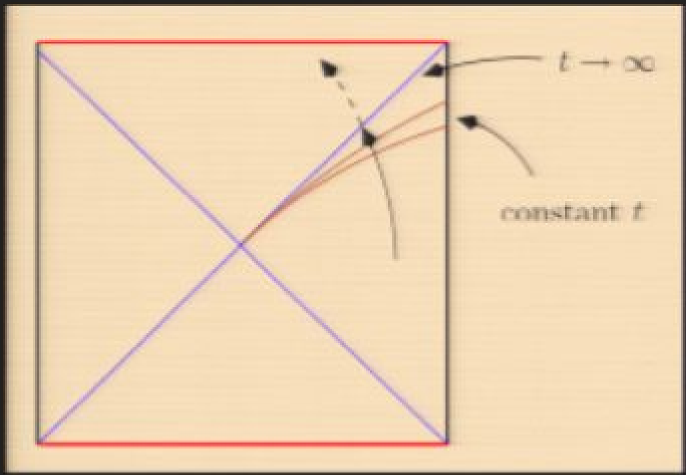

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2. Let $r = r_0(t) + \delta r(t)$:

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- Expand S_{static} in δr

Expansion in δr breaks down as $f \to 0$.

Semiclassical physics in r, t breaks down at horizon

B. $\mathcal{N} = 4$ SYM on collapsing cone

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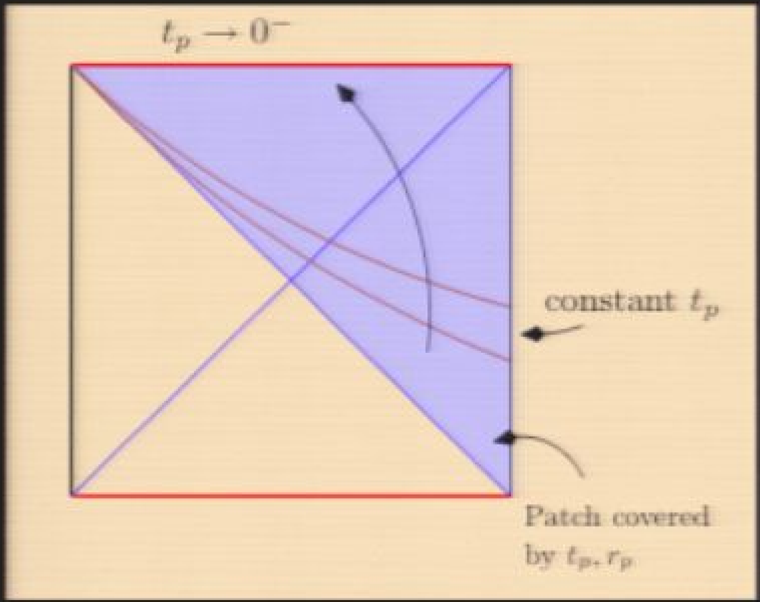
1. Horizon at $r_p t_p = \ell^2$, singularity as $t_p \to 0^-$.

   Horizon reached in finite time



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   Expansion in δr_p

   - regular at horizon
   - breaks down at singularity

C. Transformation of QM variables of probes

1. Conformal transformation of QFT

$$t_p = -\ell e^{-\frac{\tilde{t}}{\ell}}$$

maps collapsing cone to $\Sigma \times \mathbb{R}$

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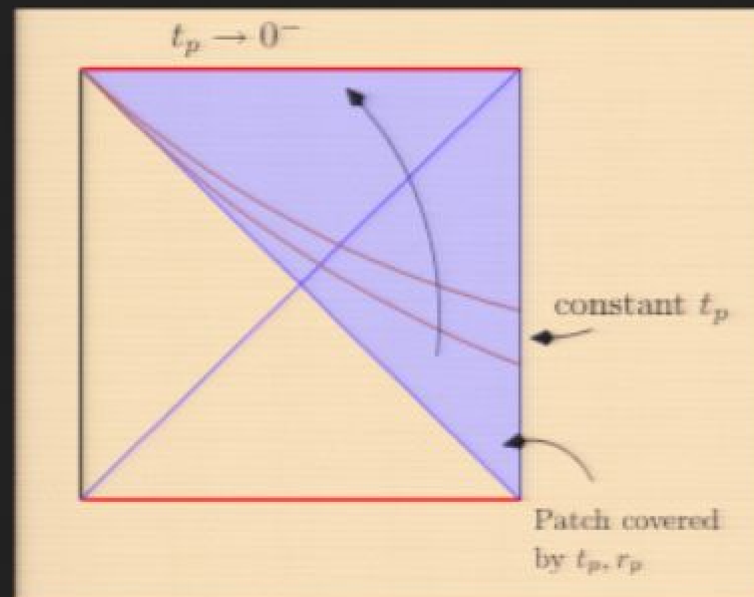

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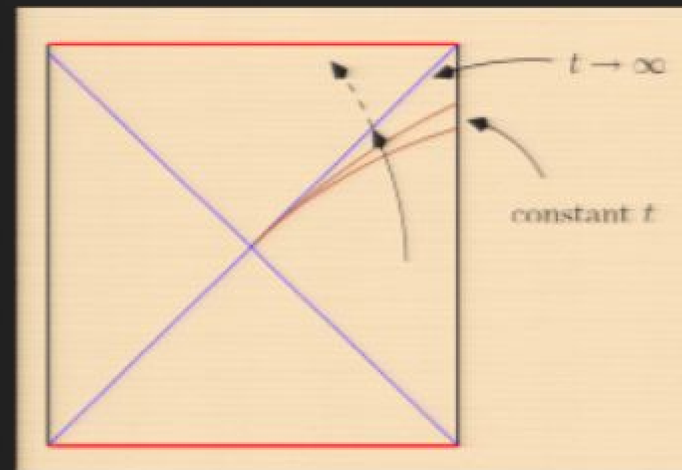

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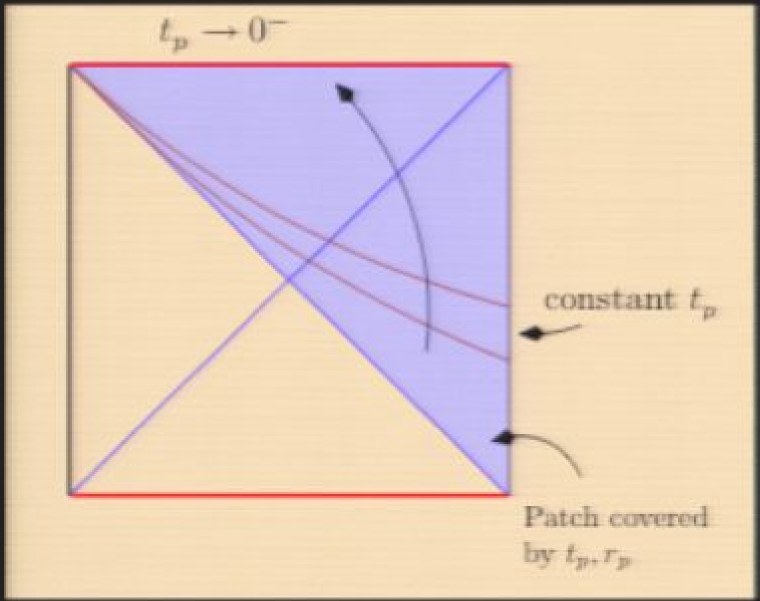
This is not surprising:

(a) Coordinate transformations

$$t_p = -\ell e^{-\bar{t}/\ell}$$

$$r_p = \tilde{r} e^{\bar{t}/\ell}$$

do not change equal-time slices in bulk



(b) Bulk metric:

$$ds_5^2 = -f(\tilde{r})d\bar{t}^2 + \tilde{r}^2 d\sigma_\Sigma^2 + \frac{2\ell}{\tilde{r}} d\bar{t} d\tilde{r} + \frac{\ell^2}{\tilde{r}^2} d\tilde{r}^2 \qquad f(\tilde{r}) = \frac{\tilde{r}^2}{\ell^2} - 1$$

$$S_{DBI} = \frac{1}{g_s(\alpha')^2} \int d\tau d^3\sigma \left(\sqrt{\det \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)} - A_{RR}^{(4)} \right)$$

$$\downarrow \qquad \begin{aligned} \bar{t} &= \tau \\ \tilde{r} &= \tilde{r}(\bar{t}) \\ \sigma^i &: \Sigma \to \Sigma \end{aligned}$$

$$\bar{S} = -\frac{\hat{V}}{g_s(\alpha')^2} \int d\bar{t} \left(\tilde{r}^3 \sqrt{\frac{\tilde{r}^2}{\ell^2} - \ell^2 \frac{(\dot{\tilde{r}} + \tilde{r}/\ell)^2}{\tilde{r}^2}} - \frac{\tilde{r}^4}{\ell} \right)$$

1. Similar in spirit to other time-dependent QFTs

Das *et. al.*
Awad *et. al.*
Craps, Sethi, *et. al.*
Martinec *et. al.*

2. Distinct from example of unstable QFTs

Horowitz and Hertog
Craps, Hertog, and Turok
Bernamonti and Craps

- $D3$ branes at constant r_p are solutions of e.o.m.
- Stretched W bosons have mass m_W constant in time.
- Momentum m_{KK} along Σ grows with $t_p \to 0^-$.
- Dimensionless ratio $m_W/m_{KK} \to 0$.

Singularity associated with IR of QFT

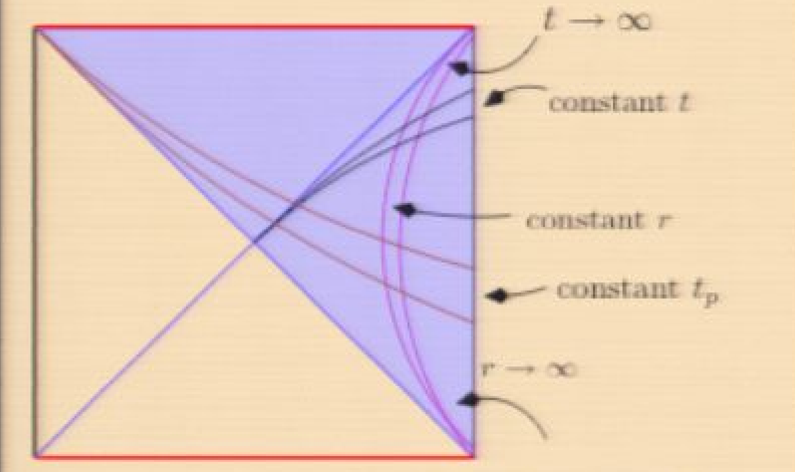
3. Unclear if QFT is well defined as $t_p \to 0^-$

Static coordinate system

$$r = -\frac{r_p t_p}{\ell} \qquad t = -\frac{\ell}{2}\ln\left(\frac{t_p^2 r_p^2 - \ell^2}{\ell^2 r_p^2}\right)$$

$$ds_5^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\sigma_\Sigma^2$$

$$f(r) = \frac{r^2}{\ell^2} - 1$$



Shaded patch covered by t_p, r_p

Topological black hole: Emparan

AdS_5 version of BTZ black hole

- Negatively curved horizon at $r = \ell$.
- Temperature $T \sim 1/\ell$.
- Horizon area $\sim \ell^3$; entropy $\sim N^2$.

Dual: gauge theory on $\Sigma \times \mathbb{R}$ at $T = 1/R_\Sigma$

C. Transformation of QM variables of probes

1. Conformal transformation of QFT

$$t_p = -\ell e^{-\frac{\tilde{t}}{\ell}}$$

maps collapsing cone to $\Sigma \times \mathbb{R}$

$$ds^2_{cone} \to ds^2_{cyl} = e^{2\tilde{t}/\ell} ds^2_{cone}$$

$\phi = r/\alpha'$ dimension-1 field:

$$r_p = e^{\tilde{t}/\ell}\tilde{r}$$

$$\tilde{t}, \tilde{r} \neq t, r$$

Conformal transformation: $S_{cosmo} \not\to S_{static}$

$$S_{cosmo} = \tilde{S} = -\frac{\hat{V}}{g_s(\alpha')^2}\int d\tilde{t}\left(\tilde{r}^3\sqrt{\frac{\tilde{r}^2}{\ell^2} - \ell^2\frac{(\dot{\tilde{r}}+\tilde{r}/\ell)^2}{\tilde{r}^2}} - \frac{\tilde{r}^4}{\ell}\right)$$

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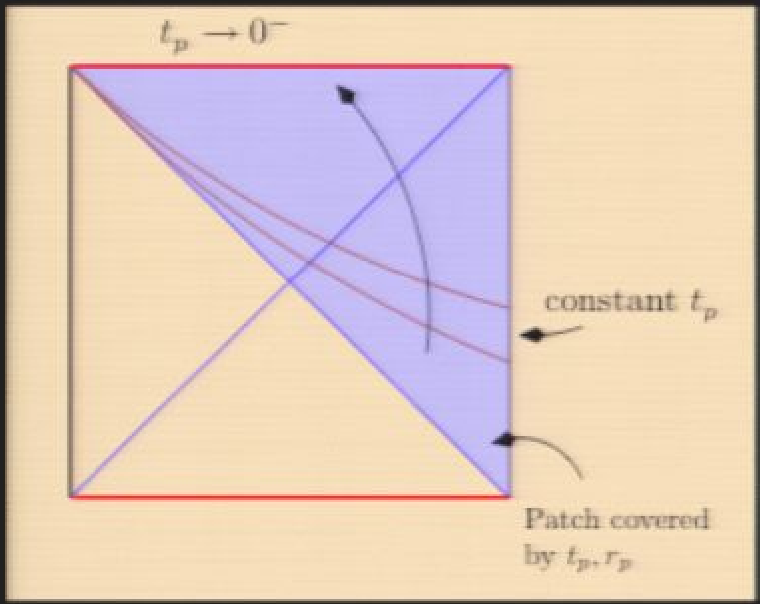

(b) Bulk metric:

$$ds_5^2 = -f(\tilde{r})d\bar{t}^2 + \tilde{r}^2 d\sigma_\Sigma^2 + \frac{2\ell}{\tilde{r}} d\bar{t}d\tilde{r} + \frac{\ell^2}{\tilde{r}^2} d\tilde{r}^2 \qquad f(\tilde{r}) = \frac{\tilde{r}^2}{\ell^2} - 1$$

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2. Bulk coordinate transformations in dual QFT

$$r = -\frac{r_p t_p}{\ell}$$
$$t = -\frac{\ell}{2} \ln\left(\frac{t_p^2 r_p^2 - \ell^2}{\ell^2 r_p^2}\right)$$

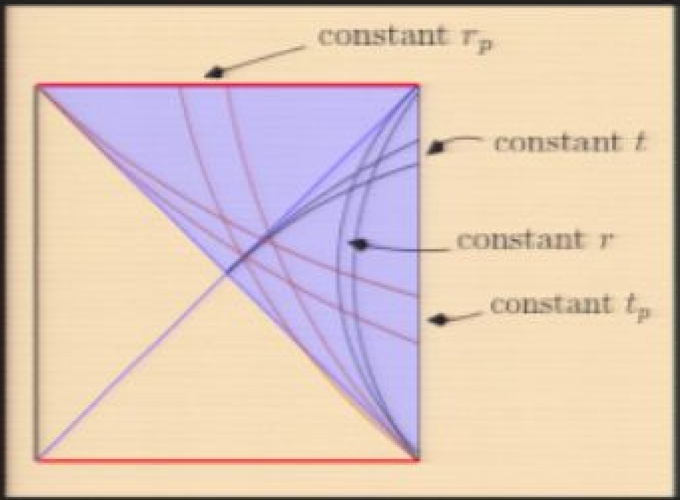
map "cosmological" to "static" coordinates

In QFT:

- t, t_p are QFT times
- r, r_p are quantum fields

Map $(r_p, t_p) \to (r, t)$ includes a field-dependent time reparametrization

Generic to any nontrivial change of bulk equal-time slicings



Seems exotic in QFT

(but remember gauge transformations for quantum inflaton fluctuations)

Solution: add gauge invariance

Let $t_p = t_p(\tau), r_p = r_p(\tau)$.

$$S_{cosmo} \to S_{DBI} = -\frac{\hat{V}}{g_s(\alpha')^2\ell^3}\int d\tau \left(t_p^3 r_p^3 \sqrt{\frac{r_p^2 \dot{t}_p^2}{\ell^2} - \ell^2 \frac{\dot{r}_p^2}{r_p^2}} - \frac{\dot{t}_p t_p^3 r_p^4}{\ell} \right)$$

Invariant under reparametrizations of τ

Reduces to S_{cosmo} if we gauge fix $t_p = \tau$

$$r = -\frac{r_p t_p}{\ell}$$
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is now a simple field redefinition

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(Up to boundary term/RR gauge transformation)

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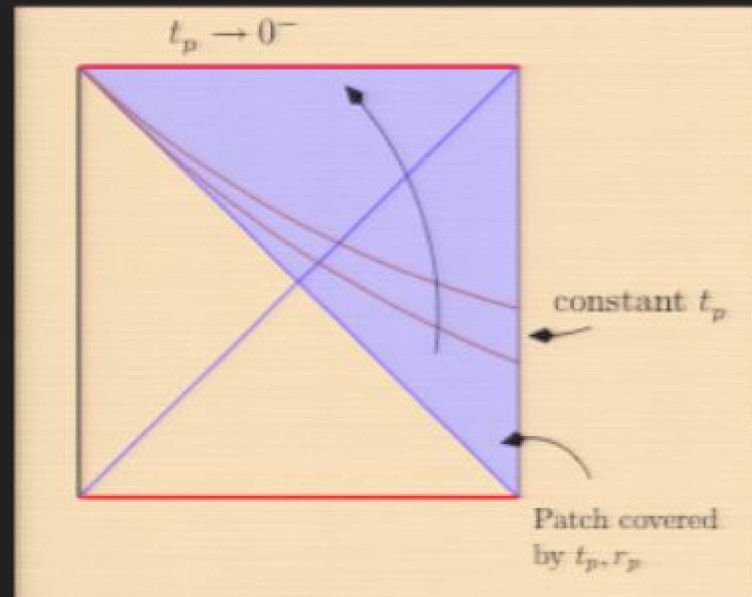

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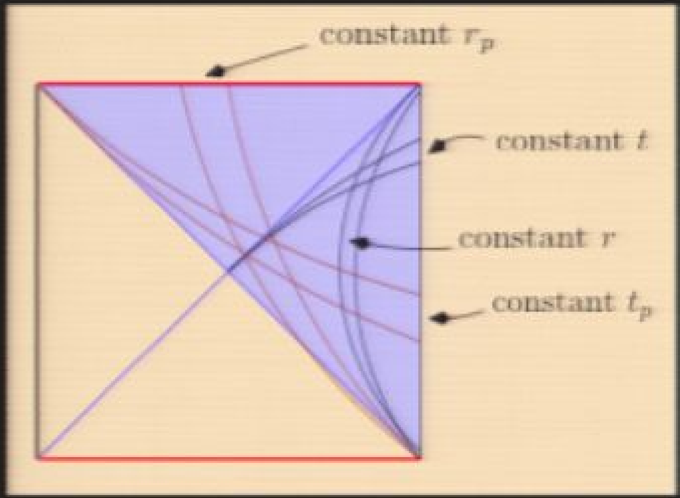

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Conformal transformation to $\Sigma \times \mathbb{R}$

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$$\downarrow \quad \tilde{r} \to \infty$$

$$ds_5^2 \sim \frac{\tilde{r}^2}{\ell^2}\left(-d\tilde{t}^2 + d\sigma_\Sigma^2\right) + \ell^2 \frac{d\tilde{r}^2}{\tilde{r}^2}$$

Field theory dual seems to live on $\Sigma \times \mathbb{R}$

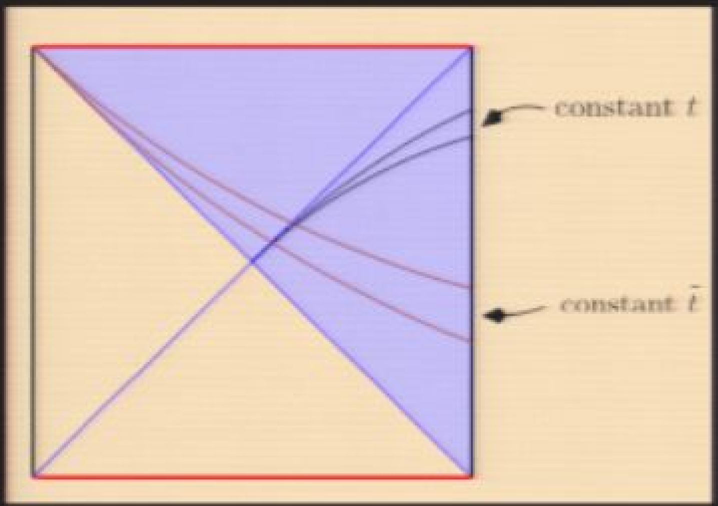
Nonsingular field theory on nonsingular space: $t_p \to 0^-$ mapped to $\tilde{t} \to \infty$.

$\tilde{t}, \tilde{r}$ extend behind horizon.

$\tilde{S}[\tilde{r}]$ well behaved at horizon

Better variables to see behind horizon

Which arises from QFT effective action?



Transformation of quantum observables

Conjugate momenta:

$$p_{\tilde{r}} = p_r - \frac{\ell}{r f(r)} p_t + \frac{r^4 - \ell^4}{\ell^4 r f(r)} \hat{V} N$$

$$p_{\tilde{t}} = p_t + \frac{\hat{V} N}{\ell}$$

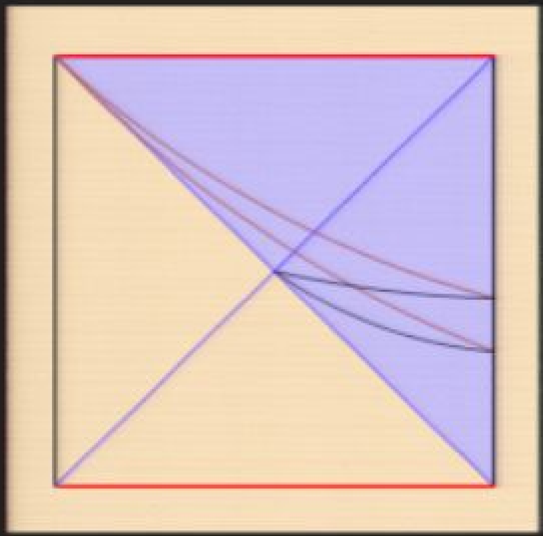
Hamiltonians:

$$H_t = -p_t \; ; \; H_{\tilde{t}} = -p_{\tilde{t}}$$

Equal up to a constant

$r \to \infty$: $\tilde{r} \to \infty$

Equal-t slices asymptotically identical to equal-$\tilde{t}$ slices



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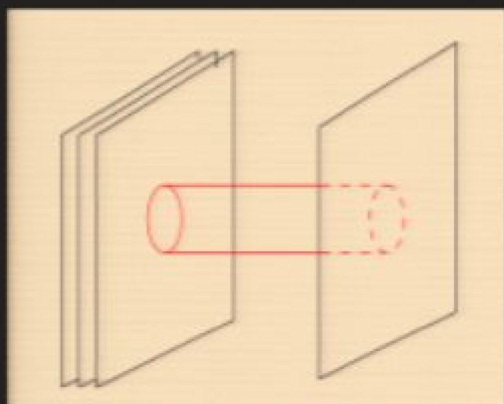
D. Transformation of full QFT

Puzzles:

1. Yang-Mills action invariant under conformal transformation
2. $S_{cosmo} \not\rightarrow S_{static}$ under conformal transformation
3. S_{cosmo}, S_{static} effective actions for SYM?
4. Is S_{static} or $\tilde{S}$ effective action for SYM on $\Sigma \times \mathbb{R}$?

Gauge theory:

- $t =$ gauge theory time.
- Take adjoint scalar out on Coulomb branch, $\phi = \alpha' r$.
- Integrate out W-bosons charged under $U(1) \times U(N-1)$.



- S_{static} is resulting effective action for ϕ

Hidden step: *must fix gauge* before integrating out W bosons.

Functional form of effective action depends on gauge choice.

Transformation of quantum observables

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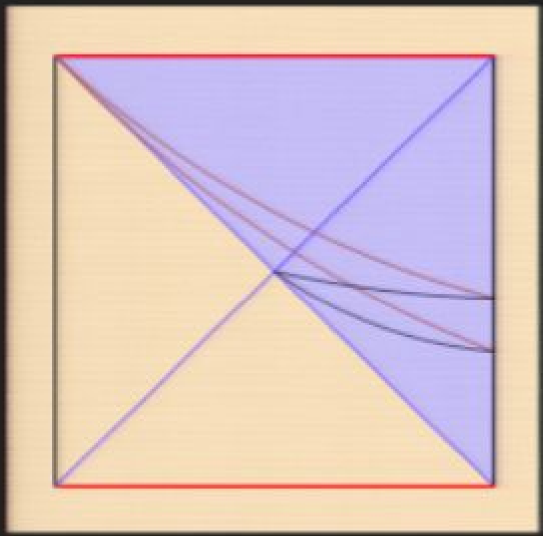

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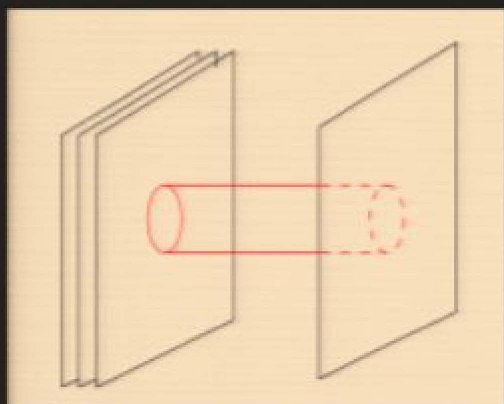


- S_{static} is resulting effective action for ϕ

Hidden step: *must fix gauge* before integrating out W bosons.

Functional form of effective action depends on gauge choice.

Standard gauge for computing DBI actions:

Background field gauge: expand around

$$A_\mu = \bar{A}_{cl,\mu} + \delta A_\mu,\ \Phi = \bar{\phi}_{cl} + \delta\phi$$

$$G = D_\mu^{\bar{A}} \delta A_\mu + i[\bar{\phi}, \delta\phi]$$

(This is the gauge implicit in string theory computations)

Under conformal transformation:

$$G \to \tilde{G} = \tilde{D}_{\tilde{\mu}}^{\bar{A}} \delta A_{\tilde{\mu}} + i[\bar{\phi}, \delta\phi] + \frac{2}{\ell} A_{\tilde{t}}$$

- Background field gauge not conformally invariant
- $\tilde{G}$ breaks $\tilde{t}$-reversal invariance
- $\tilde{S} \propto \int d\tilde{t}\tilde{r}^3 \sqrt{\frac{\tilde{r}^2}{\ell^2} - \frac{\ell^2}{\tilde{r}^2}(\dot{r} + \frac{\tilde{r}}{\ell})^2} + S_{RR}$ is not $\tilde{t}$-reversal invariant

Proposal:

1. S_{static} is effective action for SYM on $\Sigma \times \mathbb{R}$ in background field gauge.
2. $\tilde{S}$ is effective action for SYM on $\Sigma \times \mathbb{R}$ in gauge $\tilde{G} = 0$.
3. Full 5d coordinate transformations $\leftrightarrow$ Yang-Mills gauge transformations.

Related story: special conformal
transformations in SYM vs. DBI

Jevicki, Kazama, and Yoneya

IV. Gauge theory dynamics and black hole formation

A. Topological black holes

Emparan

Solutions to 5d SUGRA with negative c.c.:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\sigma^2_{\Sigma_k}$$

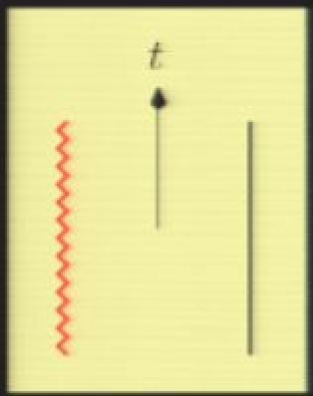
$$f(r) = \frac{r^2}{\ell^2} + k - \frac{\mu}{r^2}\ ;\ k = 0, \pm 1 \qquad \mu = G_N M$$

Σ_k 3-manifold of constant curvature:

- $\Sigma_1 = S^3$: AdS-Schwarzschild
- $\Sigma_0 = \mathbb{R}^3$: near horizon limit of black D3-brane
- $\Sigma_{-1} = \mathbb{H}_3/\Gamma$: "topological" black hole

Causal structure for $k = -1$ changes with μ:

(a) $\mu < \mu_{ext} = -\frac{\ell^2}{4}$: naked singularity

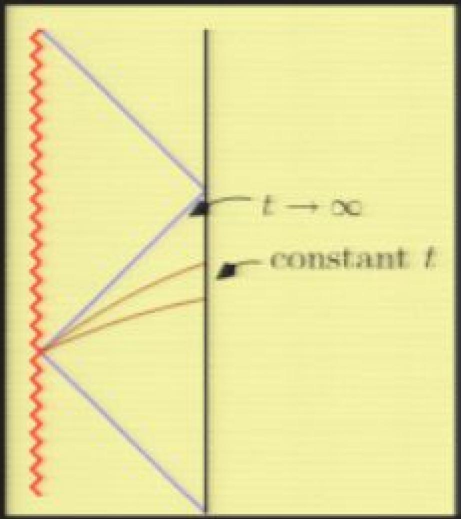


"bad" type: same structure as negative mass Schwarschild

should be forbidden if flat space is stable

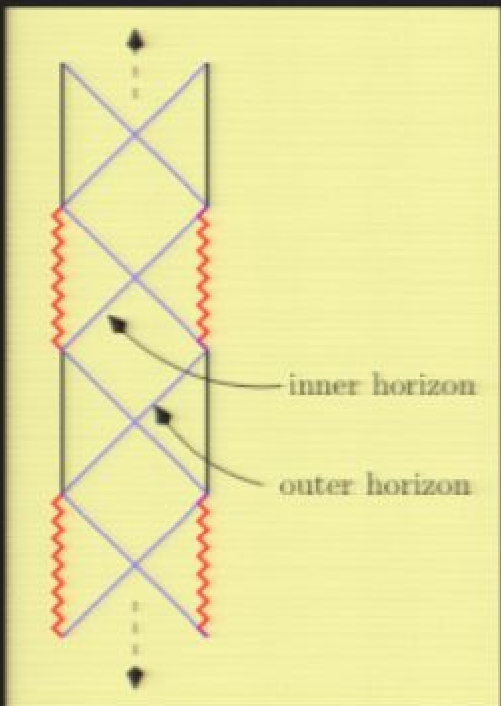
Horowitz and Myers

(b) $\mu = \mu_{ext} = \frac{\ell^2}{4}$: similar to extremal RN



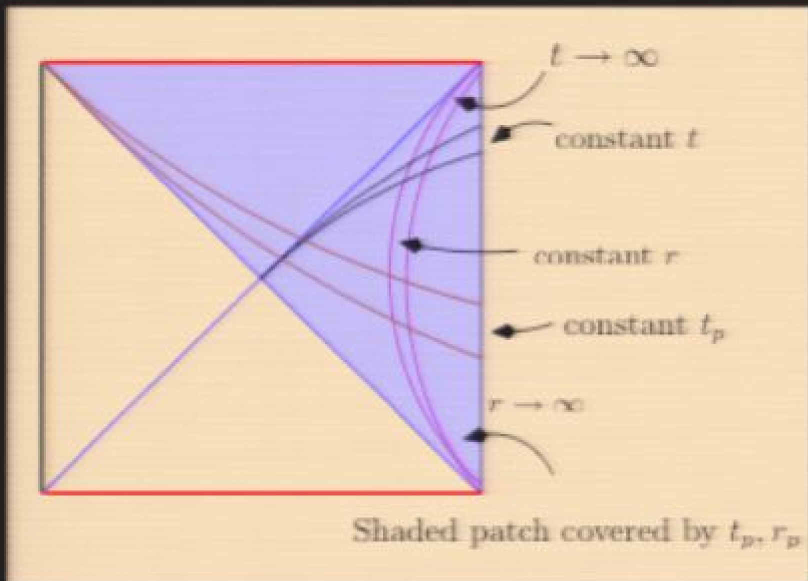
- $f(r)$ has double zero at horizon: horizon at infinite distance.
- Solution has zero temperature.
- Solution has finite entropy $S \propto \frac{\hat{V}\ell^3}{G_N}$.

(c) $0 > \mu > \mu_{ext}$: similar to RN



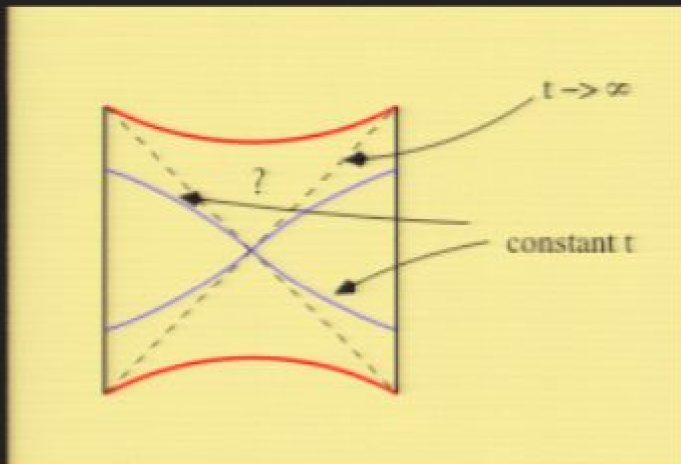
- Finite temperature $T \sim \mathcal{O}(1/\ell)$.
- Inner horizon unstable to forming singularity.

(d) $\mu = 0$: higher-dimensional analog of BTZ (same as in sec II)



- Locally equivalent to AdS.
- Singularity of orbifold type.
- $M = 0$ but $T \sim 1/\ell$.

(e) $\mu > 0$: similar to AdS-Schwarzschild



Singularity is curvature singularity as in Schwarzschild

Hyperbolic spatial slices go to zero volume:

Number of degrees of freedom may *increase*

Green, Lawrence, McGreevy, Morrison, and Silverstein

B. Phases of gauge theory dynamics

Lagrangian for adjoint scalars:

$$\mathcal{L} \sim \mathrm{Tr}\left[|D\Phi^I|^2 - ([\Phi^I, \Phi^J])^2 - \mathcal{R}^{(4)}(\Phi^I)^2 + \ldots\right]$$

Consider N D3-branes smeared over transverse S^5

Coincident radial position: dual to scalar zero mode $\phi(t)$

$\Sigma \times \mathbb{R}$ has negative curvature

- Zero modes of Φ are unstable
- Small number of momentum modes unstable
- $\Sigma = \mathbb{H}/\Gamma$ exist such that only zero modes unstable

Cornish, Spergel, and Starkman

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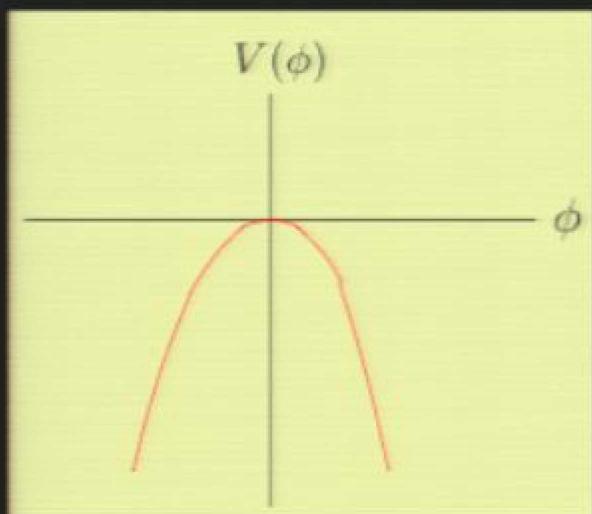
Quantum mechanics of $\phi(t)$

Lagrangian for large ϕ (no corrections from W loops):

$$L \sim (\partial_t \phi)^2 + \frac{1}{\ell^2}\phi^2$$

upside-down SHO



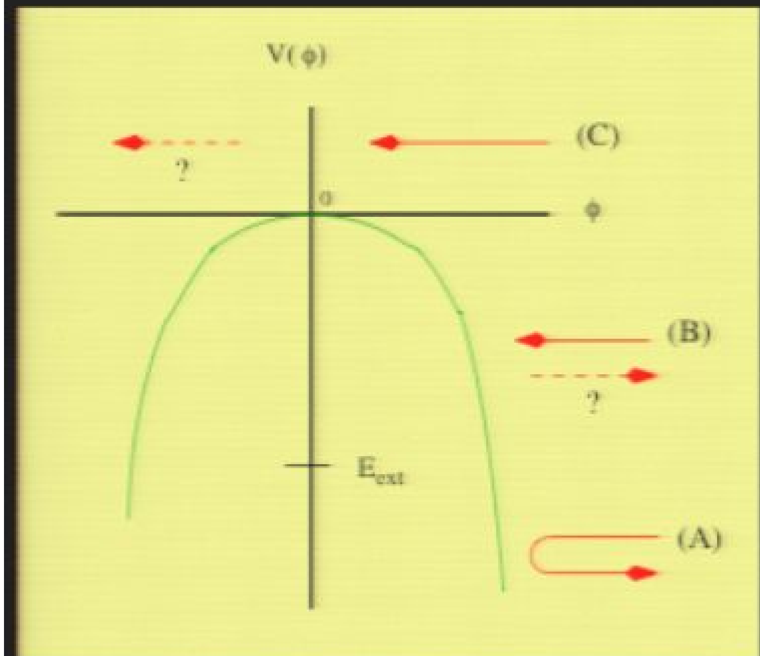
- Classically $\phi \to \infty$ in *infinite* time.
- Continuous spectrum, no ground state.
- Quantum mechanics nonsingular.

Quantum corrections to SHO action

1. Loops of W bosons when $\lambda \frac{\dot{\phi}}{\phi^2} \sim 1$.
2. W bosons produced when $\frac{\dot{\phi}}{\phi^2} \sim 1$.
3. Loops and production of
   - KK modes
   - Wilson lines on Σ
   - Flux tubes
   - ...

As ϕ evolves inwards, (1) becomes important first

Quantum corrections



$E_{ext} = \frac{\mu_{ext}}{G_N}$

(A) Scalar bounces before $\lambda\dot{\phi}/\phi^2 \sim 1$

(B) $\lambda\dot{\phi}/\phi^2 \sim 1$ before bounce

- $r = \alpha'\phi \sim r_{horizon}$ for $M = E$ black hole
- Expect W loops to slow down evolution (as with probe)

(C) $\lambda\dot{\phi}/\phi^2 \sim 1$ before $r \to 0$ reached

- Uncorrected motion describes a "bounce"
- Expect W loops to slow evolution near $r_{horizon} \sim \alpha'\phi$
- As $\phi \to 0$, production of QFT modes thermalizes system, traps branes

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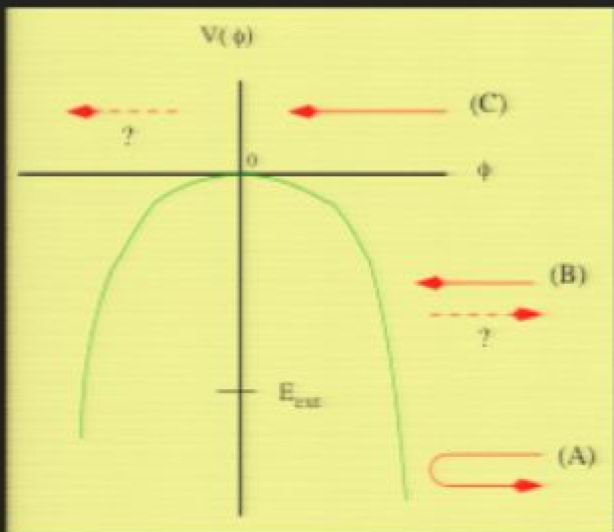
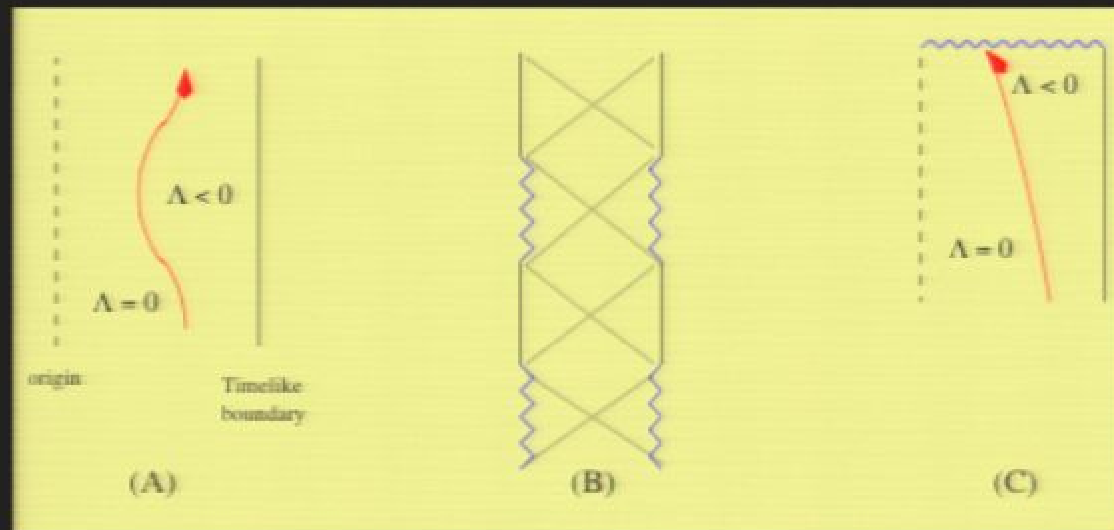
C. Spacetime causal structure





- Shell of D3-branes screens Λ.
- Outside of shell with energy E, spacetime is $M = E$ black hole.
- Trajectory (A) removes singularity a la enhancon mechanism.
- Trajectory (B) unknown: recall instability of inner horizon.
- Trajectory (C) stalls near origin.
  Thermal effective potential traps D3-branes.

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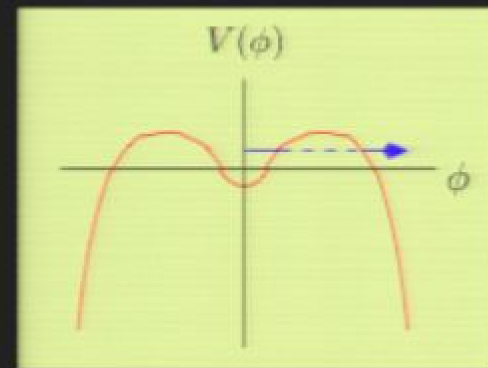
D. Late time behavio(u)r

Shell with $E \geq 0$ thermalizes gauge theory as $\phi \to 0$

Thermal effects modify effective potential for Φ^I:

- Eigenvalues trapped near origin by W-bosons
- W effects small for large ϕ: instability dominates



Nonperturbative instability to brane emission

DBI action $S \sim cN$ for single brane

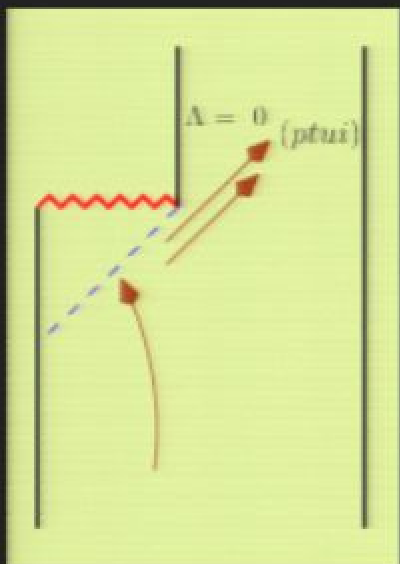
$$t_{emission} \sim e^{cN}$$

- Shorter than recurrence time for AdS-Schwarzschild $\sim e^{N^2}$
- Longer than lifetime of "small" BHs in AdS: $t_{evap} \sim M^{\alpha}$.

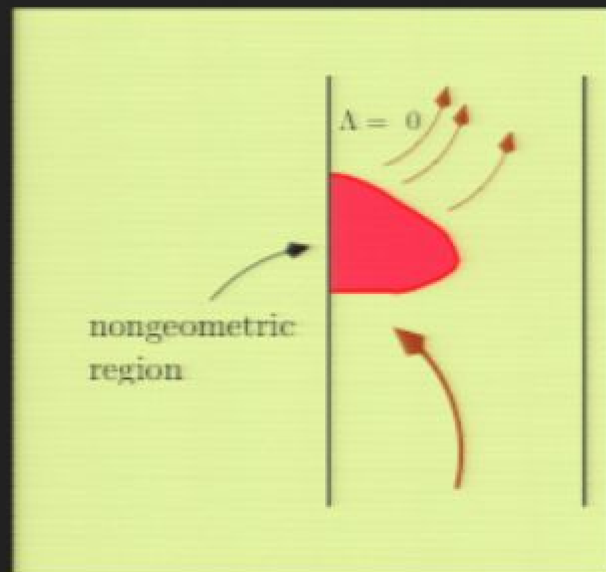
Branes emitted incoherently over time scale $\sim Ne^{cN}$.

Candidate spacetimes

(a)



(b)



(a) Unitarity: should not continue past singularity

(b) Not a simple bounce: branes re-emitted one by one quantum-mechanically

(Are (a) and (b) physically distinct?)

V. Conclusions

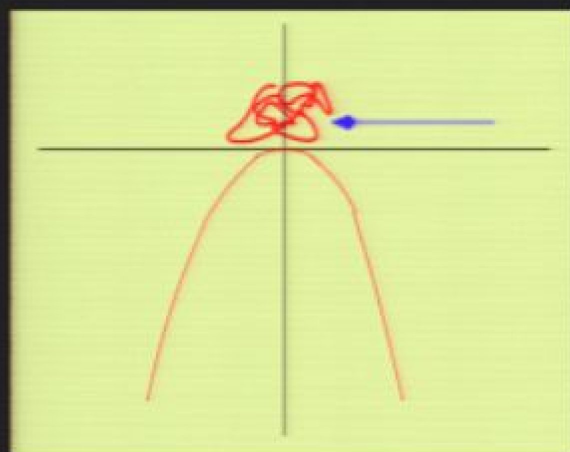
A. Lessons

1. Bulk coordinate transformations $\sim$ boundary gauge transformations.
2. Schwarzschild and infalling observers can be described by the same Hamiltonian, but with different variables: *dual* descriptions.
3. Physics well-defined but strongly interacting at singularities.
4. No sign of cosmological bounce

B. Differences from previous work

1. Singularity associated with *origin* of field space (IR).

vs. Horowitz and Hertog; Craps, Hertog, and Turok

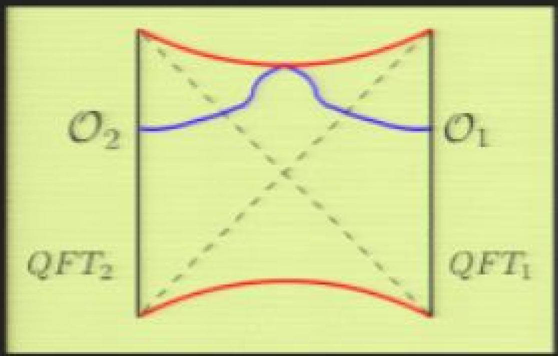


2. Singularity appears even in variables $\tilde{t}, \tilde{r}$ for which QFT is static.

vs. Das *et. al.*
Awad *et. al.*
Craps, Sethi, *et. al.*
Martinec *et. al.*

C. Future work

1. Relation to work using TFD correlators to probe singularity



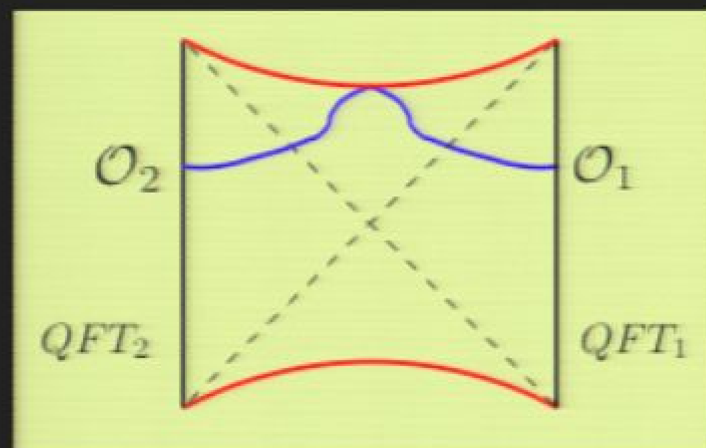
Kraus, Ooguri, and Shenker;
Fidkowski, Hubeny, Kleban, and Shenker
Liu and Festuccia

2. Distinction between horizon in (t, r) and singularity in $(\tilde{t}, \tilde{r})$
3. Understand transformation of full gauge theory (study other probes?)
4. Better understand $M < 0$ black holes
5. Source of $\mathcal{O}(N^2)$ ground state entropy?
6. Coordinate transformation for other black holes:
   - $\mu \neq 0$
   - $k = 0, 1$

Use ingoing Eddington-Finkelstein coordinates?

C. Future work

1. Relation to work using TFD correlators t



Kraus, Oogur
Fidkowski, H
Liu and Festu

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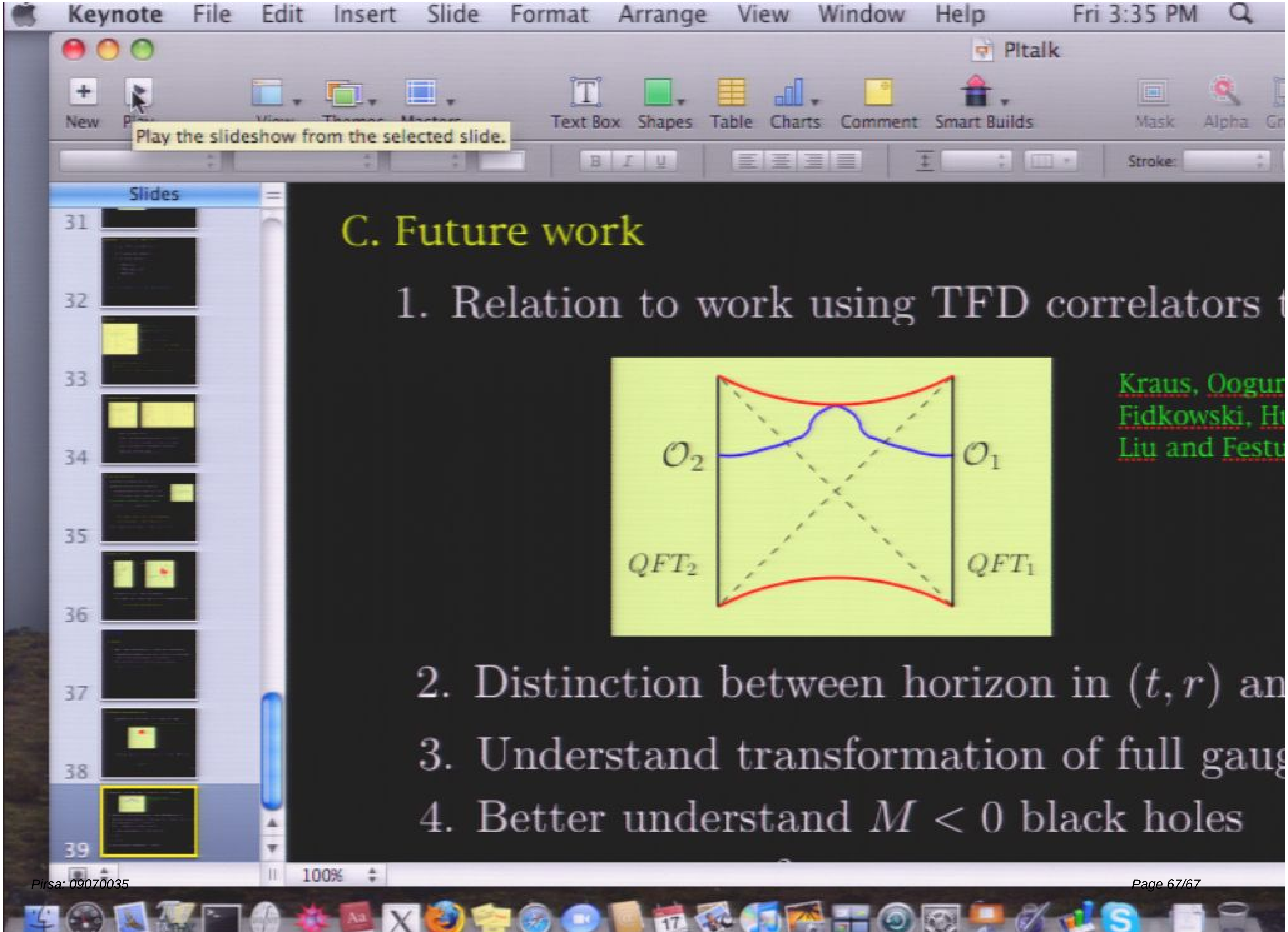
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