Title: Holography for cosmology

Date: Jul 18, 2009 10:00 AM

URL: http://pirsa.org/09070032

Abstract: We propose a holographic description of four dimensional single scalar inflationary universes, in particular asymptotically de Sitter cosmologies and power-law inflation. We show how cosmological observables such as the primordial power spectrum and non-gaussianities are encoded in correlation functions of a three dimensional QFT.

Holography for cosmology

Kostas Skenderis

University of Amsterdam

Holographic Cosmology, Perimeter Institute
18 July 2009

Outline

1. Introduction
2. Cosmological Observables
3. The domain-wall/cosmology correspondence
4. Holography for Cosmology
   - Weakly coupled gravity
5. Beyond weak gravitational description
   - Holographic phenomenology for cosmology
6. Conclusions

Introduction

- Over the past decade striking new observations have transformed cosmology from a qualitative to a quantitative science.
- New observational data are expected over the next decade that will lead to an era of precision cosmology.
- This presents a unique window to physics at the Planck scale and a challenge for fundamental theory.

Holography

- During the same period new ideas have dominated fundamental theory: holographic dualities.

Definition

Holography states that a theory which includes gravity can be described by a theory with no gravity is one fewer dimension.

- It is natural to ask how cosmology fits into the framework of holography.
- The purpose of this work is to propose a concrete holographic framework for inflationary cosmology.

Holography for cosmology

Any holographic proposal for cosmology should specify

1. what the dual QFT is
2. how it can be used to compute cosmological observables

Having defined the duality,

- the new description should recover established results in the regime where the weakly coupled gravitational description is valid
- new results should follow by using the duality in the regime where gravity is strongly coupled (Planck scale physics).

References

The talk is based on

- Paul McFadden, KS,
  Holography for Cosmology,
  to appear

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Cosmological Perturbations

We start by reviewing standard inflationary cosmology and the cosmological observables we would like to compute holographically.

- We will discuss single field (for simplicity) four dimensional inflationary models,

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(\frac{1}{\kappa^2}R - (\partial\Phi)^2 - 2V(\Phi)\right)$$

- We assume a spatially flat background (for simplicity) and perturb

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)[\delta_{ij} + h_{ij}(t,\vec{x})]dx^i dx^j \\ \Phi &= \varphi(t) + \delta\varphi(t,\vec{x}) \end{aligned}$$

where $h_{ij} = \psi(z,\vec{x})\delta_{ij} + \partial_i\partial_j\chi(z,\vec{x}) + \gamma_{ij}(z,\vec{x})$

- γ_{ij} is transverse traceless and we form the gauge invariant combination $\zeta = -\psi/2 + (H/\dot{\varphi})\delta\varphi$.

First order formalism

For background solutions with scalar field $\varphi(t)$ having only isolated zeros one can show that:

- the background equations of motion are equivalent to first order equations [Bond, Salopek (1990)] [SK, Townsend (2006)].

$$\dot{a}/a = H(\kappa\varphi), \quad \kappa\dot{\varphi} = -\frac{1}{2}H', \quad 2\kappa^2 V = \frac{1}{4}\left(\frac{3}{2}H^2 - (H')^2\right)$$

- The equations for perturbations take the form:

$$\begin{aligned} 0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} + a^{-2}q^2\zeta \\ 0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + a^{-2}q^2\gamma_{ij} \end{aligned}$$

where $\epsilon = 2(H'/H)^2$ is the slow-roll parameter. We are not assuming that ϵ is small.

Power spectrum

In the inflationary paradigm, cosmological perturbations are assumed to originate at sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\langle \zeta(t,\vec{q})\zeta(t,-\vec{q})\rangle = |\zeta_q(t)|^2$$
$$\langle \gamma_{ij}(t,\vec{q})\gamma_{kl}(t,-\vec{q})\rangle = 2|\gamma_q(t)|^2\Pi_{ijkl},$$

  where Π_{ijkl} is the transverse traceless projection operator and $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are obtained by

$$P_s(q) = \frac{q^3}{2\pi^2}|\zeta_{q(0)}|^2, \quad P_t(q) = \frac{2q^3}{\pi^2}|\gamma_{q(0)}|^2,$$

  where $\gamma_{q(0)}$ and $\zeta_{q(0)}$ are the constant late-time values of the cosmological mode functions. Initial conditions are set by the Bunch-Davies vacuum.

Power spectrum through response functions

In preparation to the holographic discussion, we rewrite the power spectrum as follows.

- We define the response functions as

$$\Pi^{\zeta} = \Omega \zeta, \quad \Pi^{\gamma}_{ij} = E \gamma_{ij},$$

where Π^{ζ} and Π^{γ}_{ij} are the canonical momentum densities.

- We use the the Wronskian relations

$$\begin{aligned} i\kappa^2 &= 2\epsilon a^3(\zeta_q \dot{\zeta}^*_q - \zeta^*_q \dot{\zeta}_q) \\ 2i\kappa^2 &= a^3(\gamma_q \dot{\gamma}^*_q - \gamma^*_q \dot{\gamma}_q) \end{aligned}$$

- to obtain

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E(q)].$$

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Cosmological observables: scalar power spectrum

$$P_s(q) = A_s(q_*)\,(q/q_*)^{n_s-1+\frac{1}{2}\alpha_s(q_*)\ln(q/q_*)}$$

- Scalar amplitude A_s

$$A_s = (2.445 \pm 0.096) \times 10^{-9}$$

  $q_* = 0.002 Mpc^{-1}$ is the pivot scale.
- Scalar index n_s. A scale invariant spectrum corresponds to $n_s = 1$. Observationally,

$$n_s = 0.960 \pm 0.013$$

- Scalar running $\alpha_s \equiv dn_s/d\ln q$. Observationally

$$-0.068 < \alpha_s < 0.012$$

(Data from combined 5-year WMAP + Type Ia Supernovae (SN) + Baryon Acoustic Oscillations (BAO), [Komatsu et al 0803.0547])

Cosmological observables

- Tensor power spectrum P_t:

$$P_t(q) = A_t(q_*)\,(q/q_*)^{n_t(q_*)}$$

Only upper limits on A_t and n_t.

- Tensor-to-scalar ratio $r = P_t/P_s$. Observationally,

$$r < 0.22(95\%C.L.)$$

- Non-gaussianity. These are related to higher-point functions, e.g.

$$\langle \zeta_{\vec{q}_1}\zeta_{\vec{q}_2}\zeta_{\vec{q}_3}\rangle = (2\pi)^3\delta(\vec{q}_1+\vec{q}_2+\vec{q}_3) f_{NL} F(q_1, q_2, q_3)$$

Observations impose constraints on f_{NL}.

Outline

Domain-wall/cosmology correspondence

The springboard for our discussion is a correspondence between cosmologies and domain-wall spacetimes.

- Domain-wall spacetime:

$$\begin{aligned} ds^2 &= dr^2 + e^{2A(r)} dx^i dx^i \\ \bar{\Phi} &= \bar{\Phi}(r) \end{aligned}$$

- This solves the field equations that follow from

$$S_{DW} = \frac{1}{2} \int d^4x \sqrt{g} \, [-\frac{1}{\bar{\kappa}^2} R + (\partial\bar{\Phi})^2 + 2\bar{V}(\bar{\Phi})],$$

Domain-wall/cosmology correspondence

- One can prove the following:

Domain-wall/Cosmology correspondence

For **every** domain-wall solution of a model with potential $\bar{V}$ there is a FRW solution for a model with potential ($V = -\bar{V}$). [Cvetic, Soleng (1994)], [KS, Townsend (2006)]

- The correspondence also applies to open and closed FRW universes which correspond to curved domain-walls.
- The correspondence can be understood as analytic continuation for the metric. The flip in the sign of V guarantees that the scalar field remains real.
- An equivalent way to state the correspondence is

$$\bar{\kappa} = \pm i\kappa, \qquad \kappa\Phi = \bar{\kappa}\bar{\Phi}$$

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Fake supersymmetry [Freedman, Nunez, Schnabl, KS (2003)]

Domain-wall spacetimes have remarkable properties. Provided the scalar field $\bar{\Phi}(r)$ has only isolated zeroes, the following properties hold [KS, Townsend (2006)]:

1. The spacetime admits a covariantly constant spinor,

$$\mathcal{D}_\mu \epsilon = 0, \qquad \mathcal{D}_\mu = D_\mu + W(\bar{\Phi})\Gamma_\mu$$

   where $W(\bar{\Phi})$, the fake superpotential, is determined by the solution. The spinor ϵ is called fake Killing spinor.
2. The existence of fake Killing spinors guarantees perturbative and non-perturbative stability of all non-singular domain-wall spacetimes.
3. All domain-wall spacetimes solve first order "BPS" equations. These follow from the fake Killing spinor equation.

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Fake pseudo-susy for cosmologies

The DW/cosmology correspondence implies that there is an analogue of these properties for cosmologies [KS, Townsend (2006)]:

1. Cosmologies admit a covariantly constant spinor,

$$\mathcal{D}_\mu \epsilon = 0, \qquad \mathcal{D}_\mu = D_\mu + iH(\Phi)\Gamma_\mu$$

   where $H(\Phi)$ is the Hubble function. The spinor ϵ is called fake pseudo-Killing spinor.
2. The first order equations discussed earlier are the "BPS" equations that follow from fake pseudo-Killing spinors.
3. Implications of this new fermionic symmetry are to a large extent unexplored.

Domain-walls and holography

Domain-wall spacetimes enter prominently in holography. They describe holographic RG flows.

- The AdS_{d+1} metric is the unique metric whose isometry group is the same as the conformal group in d dimensions. This is the main reason why the bulk dual of a *CFT* is *AdS*.
- The domain-wall spacetimes are the most general solutions whose isometry group is the Poincaré group in d dimensions. Thus, if a QFT has a holographic dual the bulk solution must be of the domain-wall type.

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Holographic RG flows

There are two different types of domain-wall spacetimes whose holographic interpretation is fully understood.

1. The domain-wall is asymptotically AdS_{d+1},

$$A(r) \to r, \qquad \bar{\Phi}(r) \to 0, \qquad \text{as} \quad r \to \infty$$

This corresponds to a QFT that in the UV approaches a fixed point. The fixed point is the CFT which is dual to the *AdS* spacetime approached as $r \to \infty$.

→ The rate at which $\bar{\Phi}(r)$ approaches zero signifies whether the *QFT* is a relevant deformation of the CFT or the *CFT* in a non-conformal vacuum.

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Holographic RG flows

2 The domain-wall has the following asymptotics

$$A(r) \to n \log r, \qquad \bar{\Phi}(r) \to \sqrt{2n} \log r, \qquad \text{as} \quad r \to \infty$$

This case has only been understood very recently [Kanitscheider, KS, Taylor (2008)] [Kanitscheider, KS (2009)].

→ Specific cases of such spacetimes are ones obtained by taking the near-horizon limit of the non-conformal branes (D0, D1, F1, D2, D4).

→ These solutions describe QFTs with a dimensionful coupling constant in the regime where the dimensionality of the coupling constant drives the dynamics.

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Domain-wall/cosmology correspondence

Let us see how the correspondence acts on the domain-walls describing holographic RG flows.

1. Asymptotically AdS domain-walls are mapped to inflationary cosmologies that approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

2. The second type of domain-walls is mapped to solutions that approach power-law scaling solutions at late times,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

Holography: a primer

The holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

1. There is 1-1 correspondence between local gauge invariant operators $\mathcal{O}$ of the boundary QFT and bulk supergravity modes Φ.
   - → The bulk metric corresponds to the energy momentum tensor of the boundary theory.
   - → Bulk scalar fields correspond to boundary scalar operators, i.e. $F_{\mu\nu}F^{\mu\nu}$, $\bar{\psi}\psi$, etc.
2. Correlation functions of gauge invariant operators can be extracted from the asymptotics of bulk solutions.

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Asymptotic solutions

To understand the holographic computations we need to know a few things about the structure of solutions of Einstein's theory with a negative cosmological constant.

- For the metric, the most general asymptotic form (in 4 bulk dimensions) looks like [Fefferman, Graham (1985)]

$$ds^2 = dr^2 + e^{2r} g_{ij}(x,r) dx^i dx^j$$

$$g_{ij}(x,r) = \mathbf{g}_{(0)\mathbf{ij}}(\mathbf{x}) + e^{-2r} g_{(2)ij}(x) + e^{-3r} g_{(3)ij}(x) + \ldots$$

  - The metric with $g_{ij}(x,r) = \eta_{ij}$ is the AdS_{d+1} metric.
  - The metric with $g_{(0)ij}(x) = \eta_{ij}$ is an Asymptotically AdS_{d+1} metric.
  - The metric with general $g_{(0)}(x)$ is an Asymptotically locally AdS_{d+1} metric.

- $\mathbf{g}_{(0)}(\mathbf{x})$ is the metric of the spacetime where the boundary theory lives and (as such) it is also the source of the boundary energy momentum tensor.

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1-point functions

- Matter fields, e.g. scalar fields, have a similar asymptotic expansion

$$\bar{\Phi}(x,r) = e^{-(3-\Delta)r}\left(\phi_{(0)} + ... + e^{(2\Delta-3)r}\left(r\psi_{(2\Delta-3)} + \phi_{(2\Delta-3)}\right) + ...\right)$$

where Δ is the dimension of the dual operator, related to the mass of $\bar{\Phi}$ via $m^2 = \Delta(\Delta-3)$.

- Using the formalism of holographic renormalization, we then find a precise relation between correlation functions and asymptotics [de Haro, Solodukhin, KS (2000)]

$$\langle T_{ij}\rangle = \frac{3}{2\kappa^2} g_{(3)ij}, \qquad \langle O\rangle = -(2\Delta-3)\phi_{(2\Delta-3)}$$

– Correlators satisfy the expected Ward identities,

$$\nabla^i\langle T_{ij}\rangle = -\langle O\rangle\partial_j\phi_{(0)}, \qquad \langle T^i_i\rangle = (\Delta-3)\phi_{(0)}\langle O\rangle$$

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Outline

Holography for cosmology

Applying the analytic continuation,

$$\bar{\kappa} = \pm i\kappa, \qquad \bar{q} = -iq$$

one finds a direct relation between:

- power spectra and holographic 2-point functions,

$$P_s(q) = \frac{q^3}{2\pi^2}\left(\frac{-1}{8\mathrm{Im}B(-iq)}\right), \quad P_t(q) = \frac{2q^3}{\pi^2}\left(\frac{-1}{\mathrm{Im}A(-iq)}\right),$$

- non-Gausianities and holographic higher-point functions.

Example 1: power-law cosmology

- Consider the potential

$$V(\varphi) = V_0 \exp(-\sqrt{2/n}\kappa\varphi)$$

- The corresponding solution is

$$ds^2 = -dt^2 + (t/t_0)^n dx^i dx^i, \qquad \kappa\varphi = \sqrt{2n} \ln t/t_0$$

- When $n = 7$ this solution is related via the DW/cosmology correspondence to the near-horizon limit of a stack of D2 branes.

Example 1: power-law cosmology

- The holographic 2-point functions have been computed for any n [Kanitscheider, KS, Taylor (2008)]

$$A(\bar{q}) = 2nB(\bar{q}) = -\frac{2\pi}{4^{\sigma}\Gamma^2(\sigma)\sin\pi\sigma}\kappa^{-2}\bar{q}^{2\sigma}.$$

where $\sigma = (3n-1)/(n-1) > 3/2$.

- Using the analytic continuation one obtains

$$P_t(q) = \frac{16}{n}P_s(q) = \frac{4^{\sigma}\Gamma^2(\sigma)}{\pi^3}\kappa^2 q^{3-2\sigma},$$

which is the correct answer.

Example 2: Asymptotically dS cosmologies

- These results essentially follow from earlier work [Maldacena (2002)] .....
- The corresponding domain-walls are asymptotically AdS and the boundary theory is either a deformation of the CFT or the CFT in a non-trivial state.
- The slow-roll parameter is related to the beta function of the boundary theory.

Analytic continuation in QFT variables

- The analytic continuation

$$\bar{\kappa} = \pm i\kappa, \qquad \bar{q} = -iq, \qquad \bar{\kappa}\bar{\Phi} = \kappa\Phi$$

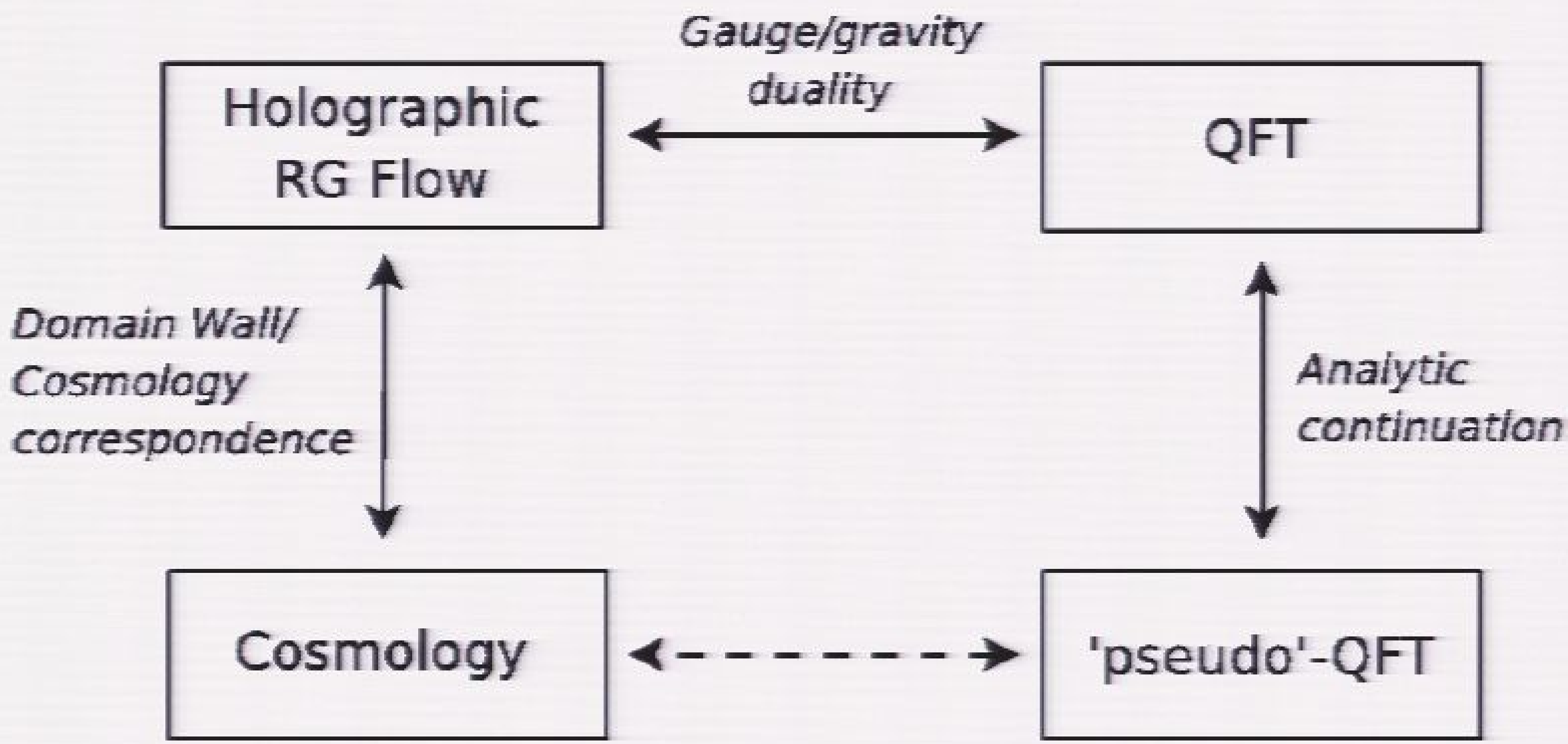
translates in QFT language to

$$N \to iN, \qquad \bar{q} \to -iq$$

The proposal

1. A given inflationary model, based on a single scalar model, can be mapped to a domain-wall via the domain-wall/cosmology correspondence.
2. As we discussed, these domain-walls are the ones with operational gauge/gravity duality, i.e. there is a dual QFT via the usual gauge/gravity duality.
3. The analytic continuation that enters in the DW/cosmology correspondence can be expressed entirely in terms of QFT variables.
4. We now apply this analytic continuation to the QFT dual of the domain-wall to obtain the QFT dual of the inflationary model.

The proposal



Pseudo-QFT

We operationally define the pseudo-QFT as follows:

- we do the computation in the QFT dual to the domain-wall and then analytically continue parameters and momenta appropriately.

Perhaps a more fundamental perspective is to consider the QFT action with complex parameters as the fundamental object.

- Then the results on different real domains will be applicable to DW/cosmology as appropriate.

$\rightarrow$ The supergravity realization of the DW/cosmology correspondence works this way.

Domain-wall/Cosmology correspondence in SUGRA

- In some cases, one can embed the DW/cosmology correspondence in supergravity [Bergshoeff et al, (2007)] [KS, Townsend, Van Proeyen (2007)]:

→ In these cases, there is a common supergravity action with complex-valued fields, which becomes AdS supergravity or dS supergravity, depending on the reality conditions imposed on the fields.

→ Domain-wall solutions of AdS SUGRA are mapped to cosmological solutions of dS SUGRA.

→ Cosmologies can be supersymmetric solutions of dS SUGRA and fake susy is genuine susy in this context.

→ dS supergravities are known to be contain fields with "wrong sign kinetic terms". None of these "ghost fields" however participate in the cosmological solutions.

Outline

Beyond weak gravitational description

- So far the discussion was on the gravitational side.
- We inferred a QFT description using the AdS/CFT correspondence and analytic continuation, but all computations were done on the gravitational side.
- When gravity is strong coupled the QFT description is weakly coupled, so one may use the duality.
- This allows us to compute the late time behavior of the response functions and therefore the power spectra etc when the early time behavior is strongly coupled/stringy.

Holographic phenomenology for cosmology

- The boundary theory will be a combination of gauge fields, fermions and scalars and it should admit a large N expansion.
- To extract predictions we need to compute the coefficients A and B,

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

  analytically continue the result and insert in the formulae for the power spectra.
- One can then look for a holographic theory that models well the observations.

Beyond weak gravitational description

- So far the discussion was on the gravitational side.
- We inferred a QFT description using the AdS/CFT correspondence and analytic continuation, but all computations were done on the gravitational side.
- When gravity is strong coupled the QFT description is weakly coupled, so one may use the duality.
- This allows us to compute the late time behavior of the response functions and therefore the power spectra etc when the early time behavior is strongly coupled/stringy.

Holographic phenomenology for cosmology

- The boundary theory will be a combination of gauge fields, fermions and scalars and it should admit a large N expansion.
- To extract predictions we need to compute the coefficients A and B,

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Holographic phenomenology for cosmology

- As a starting point one can consider the strong coupling version of asymptotically dS cosmologies and power-law cosmology.
- In this talk we focus on QFTs dual to the latter. These are super-renormalizable QFTs that depend on a single dimensionful coupling. For example, the g^2_{YM} coupling constant.
- The leading contribution to the 2-point function of the energy momentum tensor is at 1-loop. Since T_{ij} has dimension 3,

$$A(\bar{q}) \sim N^2 \bar{q}^3, \qquad B(\bar{q}) \sim N^2 \bar{q}^3$$

$\Rightarrow$ A generic such holographic model has a scale invariant spectrum!

Fixing the parameters of the holographic model

- N is fixed by comparing the amplitude of the power spectra with the holographic value. Recall that it is A^{-1}, B^{-1} that enter in the spectra.

→ Smallness of the amplitude implies $N >> 1$, so the large N expansion is justified.

- g^2_{YM} is fixed by the tilt of the spectrum. More precisely, the form of the leading correction is determined by dimensional analysis

$$n_s - 1 = \# g^2_{eff} = \# g^2_{YM} N/q$$

where $\#$ is a model depended constant.

→ In these theories the scalar index runs

$$\alpha_s = \frac{dn_s}{d \ln q} = -(n_s - 1) \sim 0.04$$

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Other cosmological observables

- The tensor-to-scalar ratio is given by

$$r = 32 \frac{\mathrm{Im} B(-iq)}{\mathrm{Im} A(-iq)}$$

  In these models, vectors and scalars have A = B and conformally coupled scalars and fermions have B=0 to leading order. It follows that with appropriately chosen field content one can achieve

$$r < 0.22$$

- Once N and g^2_{YM} (at some scale) and the field content are fixed, all other cosmological observables such as non-Gaussianities etc uniquely follow by straightforward computations.

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$\Rightarrow$ These models are extremely predictive!

Outline

1. Introduction
2. Cosmological Observables
3. The domain-wall/cosmology correspondence
4. Holography for Cosmology
   - Weakly coupled gravity
5. Beyond weak gravitational description
   - Holographic phenomenology for cosmology
6. Conclusions

Conclusions

- I have presented a concrete proposal for holography for cosmology.
- When gravity is weakly coupled, holography correctly reproduces standard results for cosmological observables.
- When gravity is strongly coupled, one finds new models that have a QFT description.
- We initiated a holographic phenomenological approach to cosmology.

Holographic phenomenology

- Generic holographic models lead to a scale invariant spectrum.
- One can find models that fit all current observations. This fixes the parameters of the model, N, g^2_{YM}, and constrains the field content.
- Further cosmological observables are computable, essentially with no further adjustable parameters.

Outlook

- Further develop holographic phenomenology.
- Utilize connection of cosmological observables to QFT correlators to find more efficient ways to perform bulk computations (e.g. computations of non-gausianities).
- Understand better the analytic continuation on the QFT side. Do "pseudo-QFT"s exist?
- Understand better the analytic continuation in the bulk. What is the meaning of the relation with dS supergravities and the M^* and II^* theories? What are the implications of pseudo-supersymmetry?

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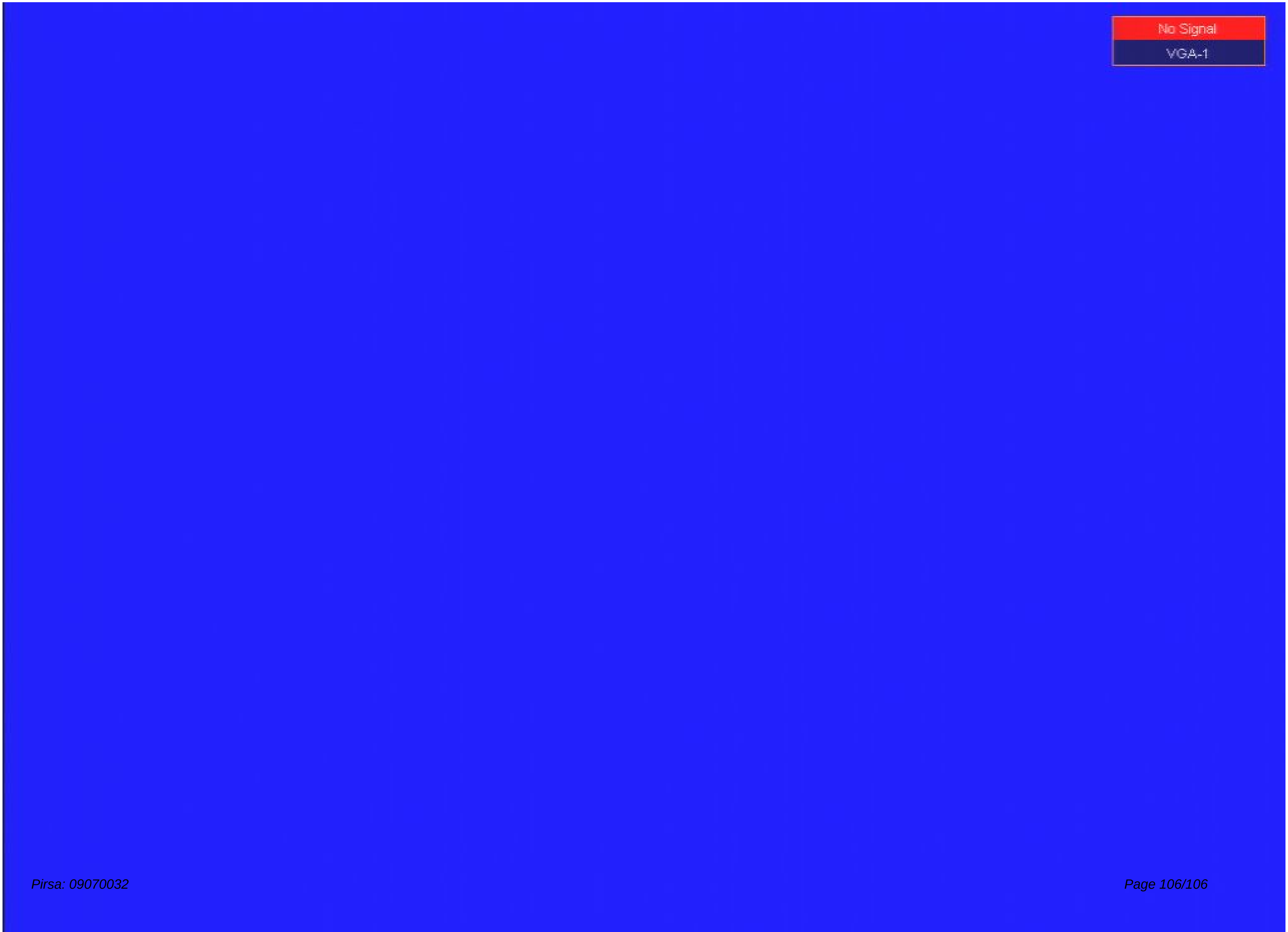
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