Title: Black Holes localized on the brane

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Abstract: In Randall-Sundrum single-brane (RS-II) model, it was conjectured that there is no static large black hole localized on the brane based on adS/CFT correspondence. Here we consider the phase diagram of black objects in the models extended from the RS-II model. We propose a scenario for the phase diagram consistent with the classical black hole evaporation conjecture. The proposed scenario indicates the existence of a rich variety of the families of black objects. We present several side evidences that support the whole picture.

Black Holes localized on the brane

Classical BH evaporation conjecture
and
floating black holes

Takahiro Tanaka (YITP, Kyoto university)

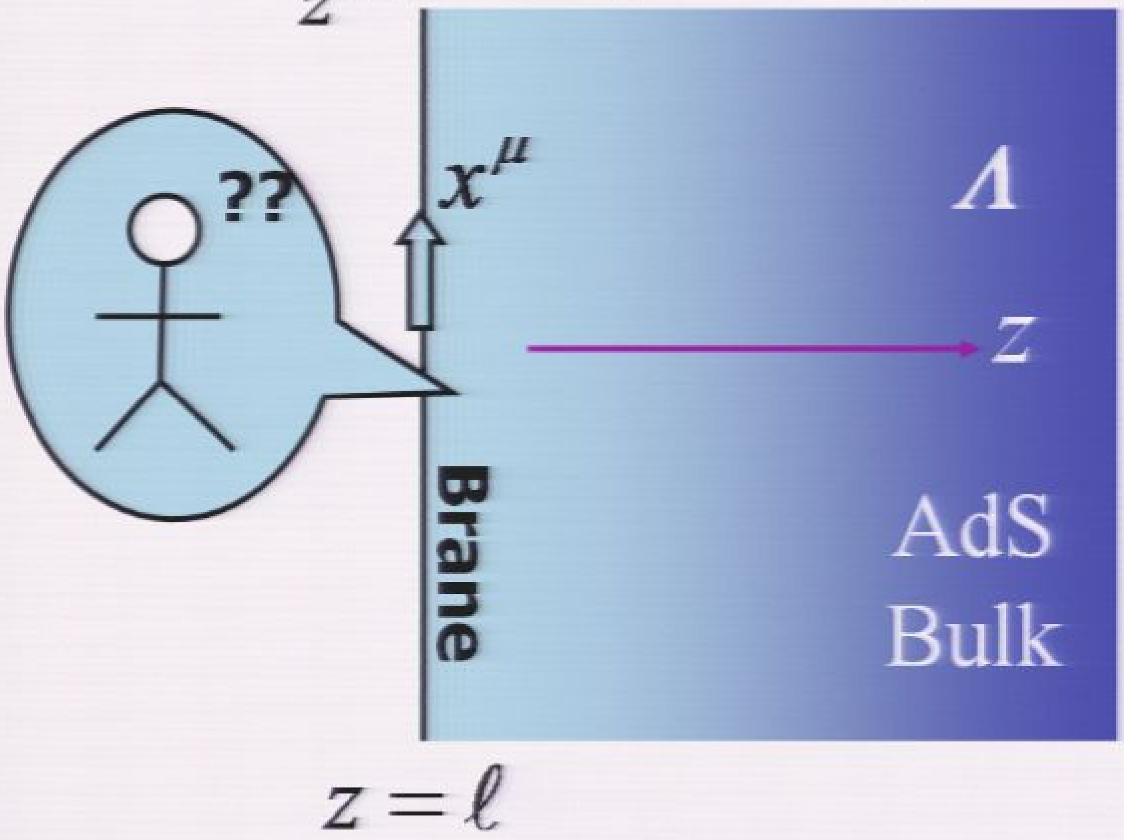
Prog. Theor. Phys. (2009) arXiv:0709.3674

$+\alpha$ (work in collaboration with
N. Tanahashi, K. Kashiyama, A. Flachi)

Infinite extra-dimension: Randall-Sundrum II model

Volume of the bulk is finite due to warped geometry although its extension is infinite.

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$



ℓ : AdS curvature radius

$\Lambda = -\frac{6}{\ell^2}$ Negative cosmological constant

$\sigma = \frac{3}{4\pi G_5 \ell}$ Brane tension

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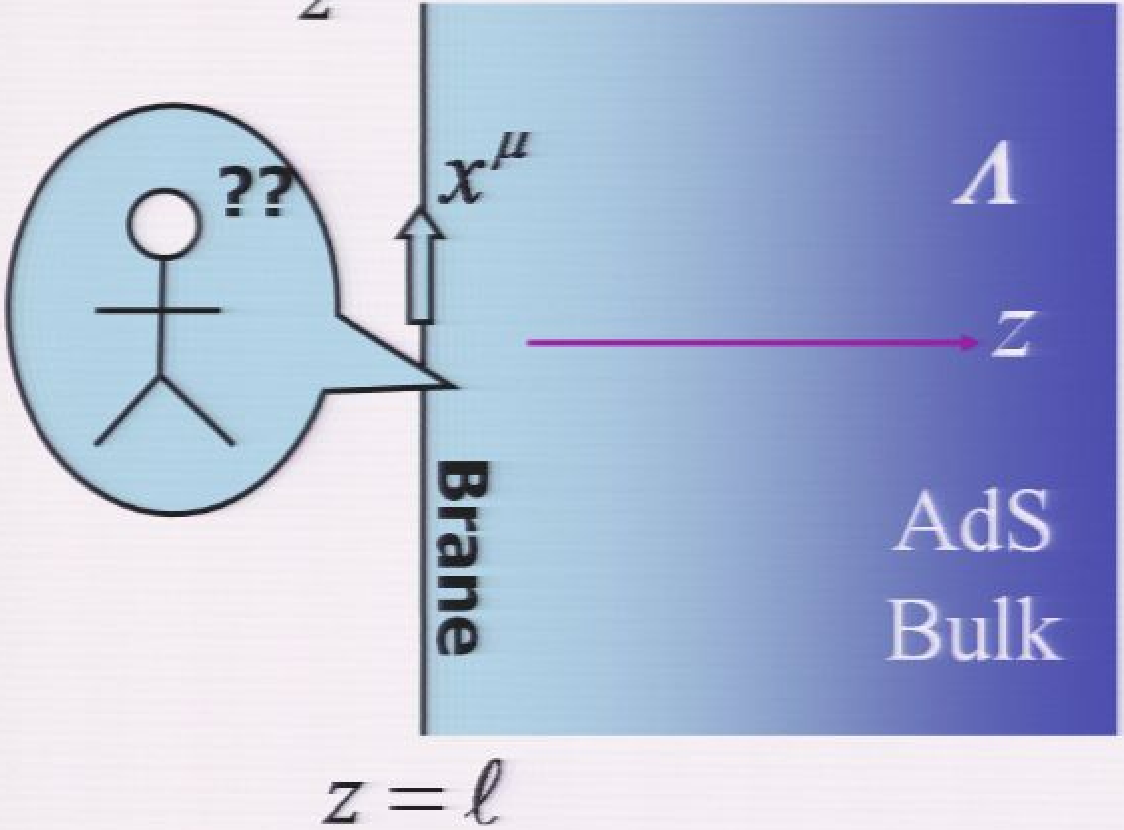

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- Extension is infinite, but 4-D GR seems to be recovered!

Gravity on the brane looks like 4D GR approximately,

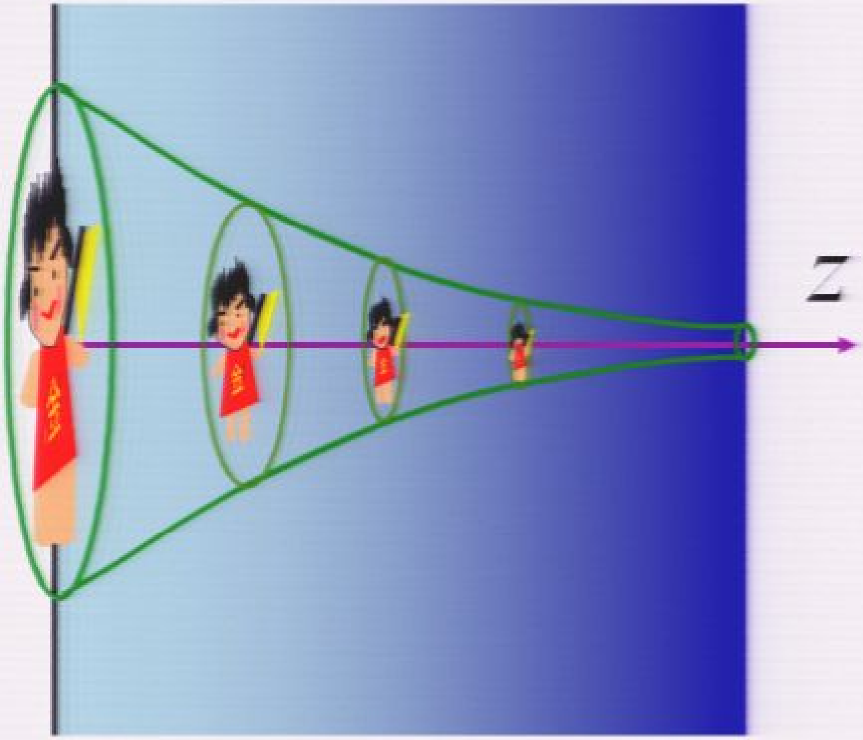
BUT

- No Schwarzshild-like BH solution????

Black string solution

(Chamblin, Hawking, Reall ('00))

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \bar{g}^{(Sch)}_{\mu\nu} dx^\mu dx^\nu\right)$$



Metric induced on the brane $\bar{g}_{\mu\nu}(x)$ is exactly Schwarzschild solution.

However, this solution is singular.

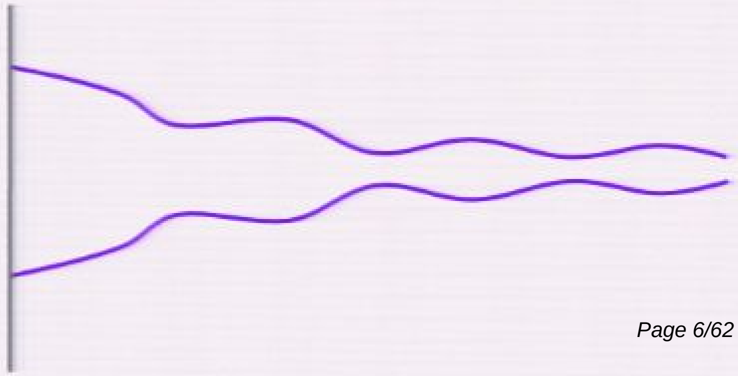
- $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \propto z^4$

  behavior of zero mode

Moreover, this solution is unstable.

- *Gregory Laflamme instability*

"length $\gtrsim$ width"



AdS/CFT correspondence

(Maldacena ('98))
(Gubser ('01))
(Hawking, Hertog, Reall ('00))

- $$Z[\underbrace{q}_{\text{Boundary metric}}]=\int d[\phi]\,\exp(-S_{CFT}[\phi,q])$$
$$=\int d[g_{bulk}]\,\exp(-S_{HE}-S_{GH}+\underbrace{S_1+S_2+S_3}_{\text{Counter terms}})\equiv\exp(-W_{CFT}[q])$$

$$S_{EH}=-\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{-g}\left({}^{(5)}R+\frac{12}{\ell^2}\right)$$
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$$S_2=-\frac{\ell}{4\kappa_5^2}\int d^4x\,\sqrt{-q}\;{}^{(4)}R$$
$$S_3=\cdots$$

- $\underbrace{z_0}_{\text{Brane position}}\rightarrow 0$ limit is well defined with the counter terms.

z_0 ⇔ cutoff scale parameter

- $$\int d[g]\,\exp(-S_{RS})=\int d[g]\,\exp(-2(S_{EH}+S_{GH})+\underbrace{2S_1}_{\text{brane tension}}-S_{matter})$$
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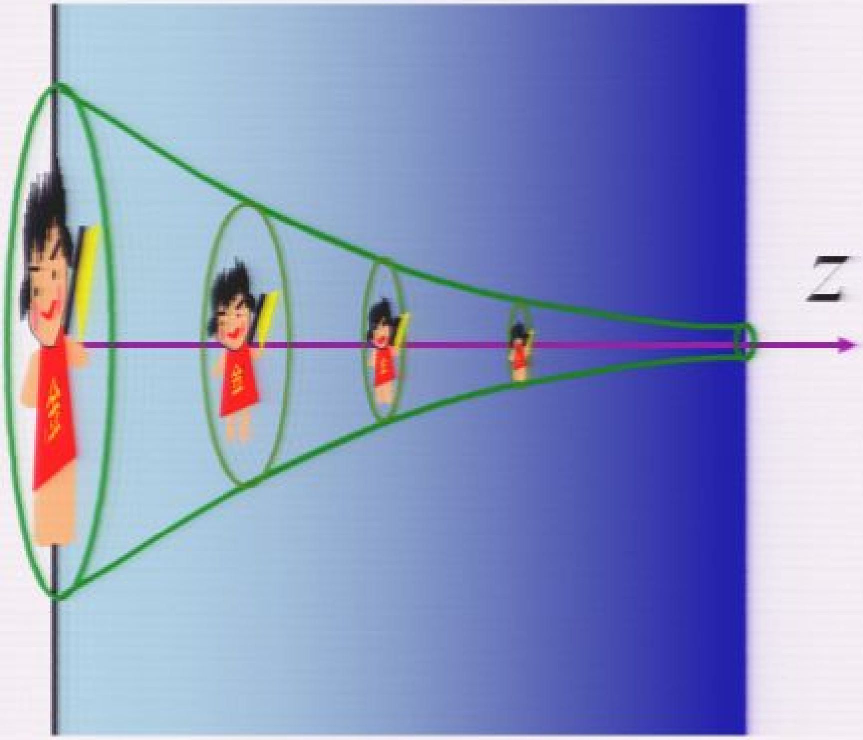

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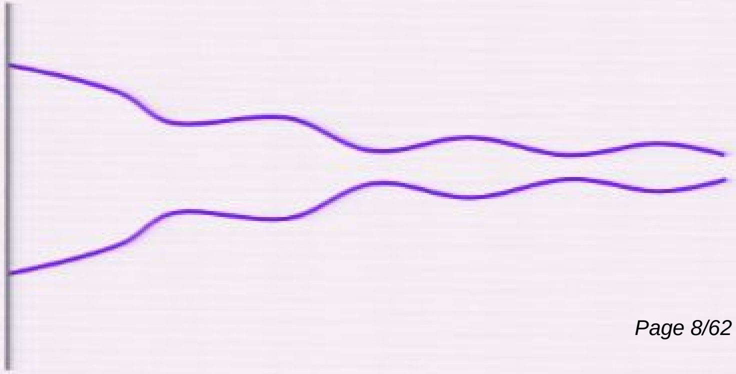

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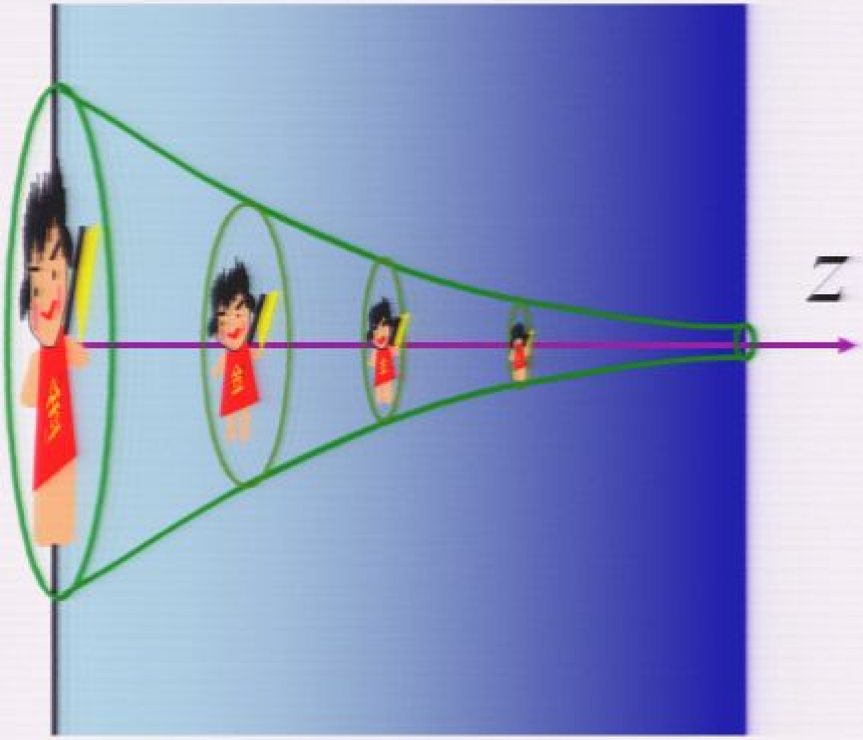

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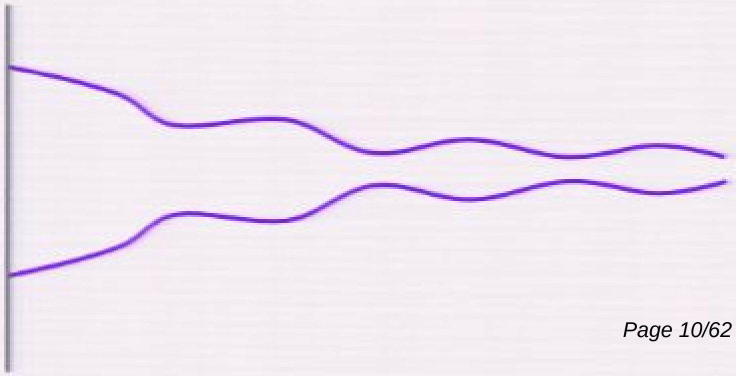

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Classical black hole evaporation conjecture

(T.T. ('02), Emparan et al ('02))

4D Einstein+CFT with the lowest order quantum correction $\left(\text{number of field of CFT}\right) \approx \frac{\ell^2}{\kappa_4^2}$

AdS/CFT correspondence

⟺ equivalent

Classical 5D dynamics in RS II model

4D BH with CFT ⟺ equivalent 5D BH on brane

Hawking radiation in 4D Einstein+CFT picture ⟺ equivalent Classical evaporation of 5D BH

Time scale of BH evaporation

$$\boxed{\tau = \left(\frac{M}{M_{Solar}}\right)^3 \left(\frac{1\text{mm}}{\ell}\right)^2 \times 120\text{year}}$$

$$\frac{\dot{M}}{M} \approx \left(\begin{matrix}\text{Number of}\\ \text{species}\end{matrix}\right) \times \frac{1}{G_N^2 M^3} \approx \frac{\ell^2}{(G_N M)^3}$$

$10 \pm 5 M_{\odot}$ BH+K-type star X-ray binary A0620-00

$\ell < 0.132$mm ($10 M_{\odot}$ BH is assumed)

(Johansen, Psaltis, McClintock arXiv:0803.1835)

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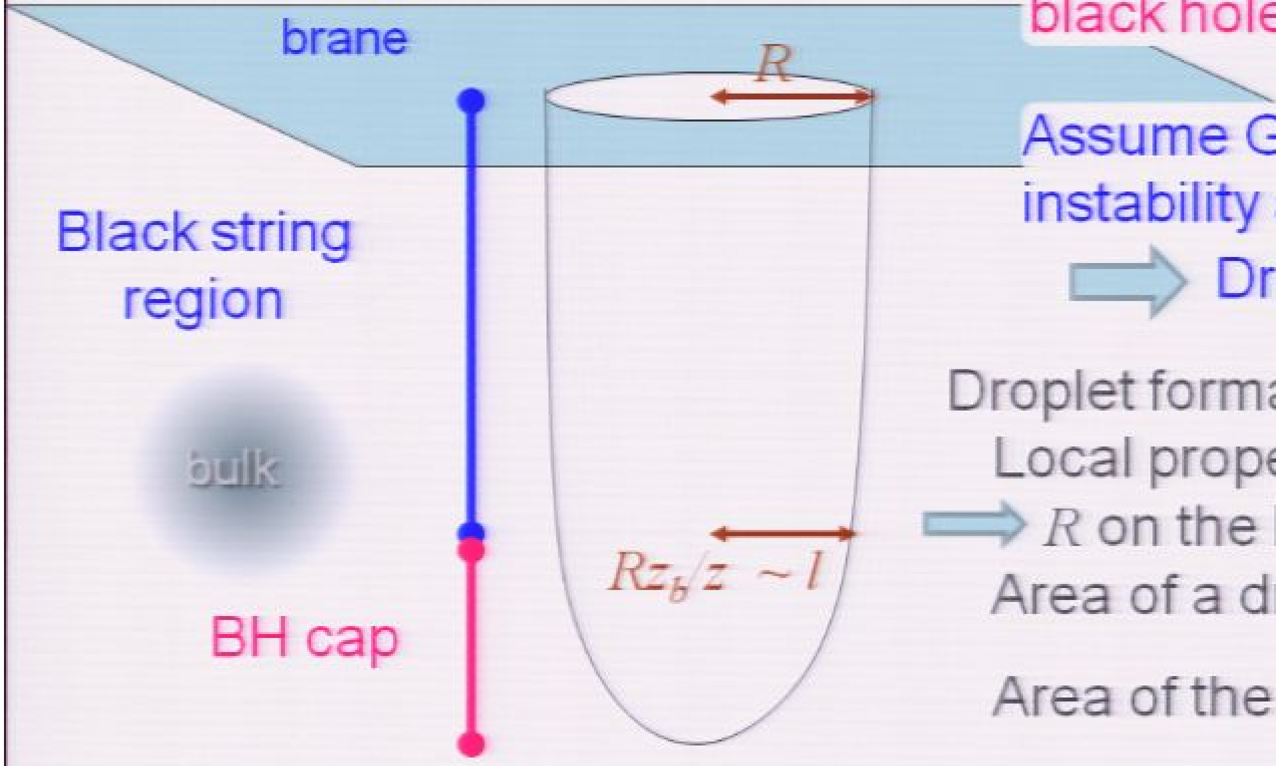

Structure near the cap region will be almost independent of the size of the black hole. ~discrete self-similarity

Assume Gregory-Laflamme instability at the cap region

⇒ Droplet escaping to the bulk

Droplet formation

Local proper time scale: l

⇒ R on the brane due to redshift factor

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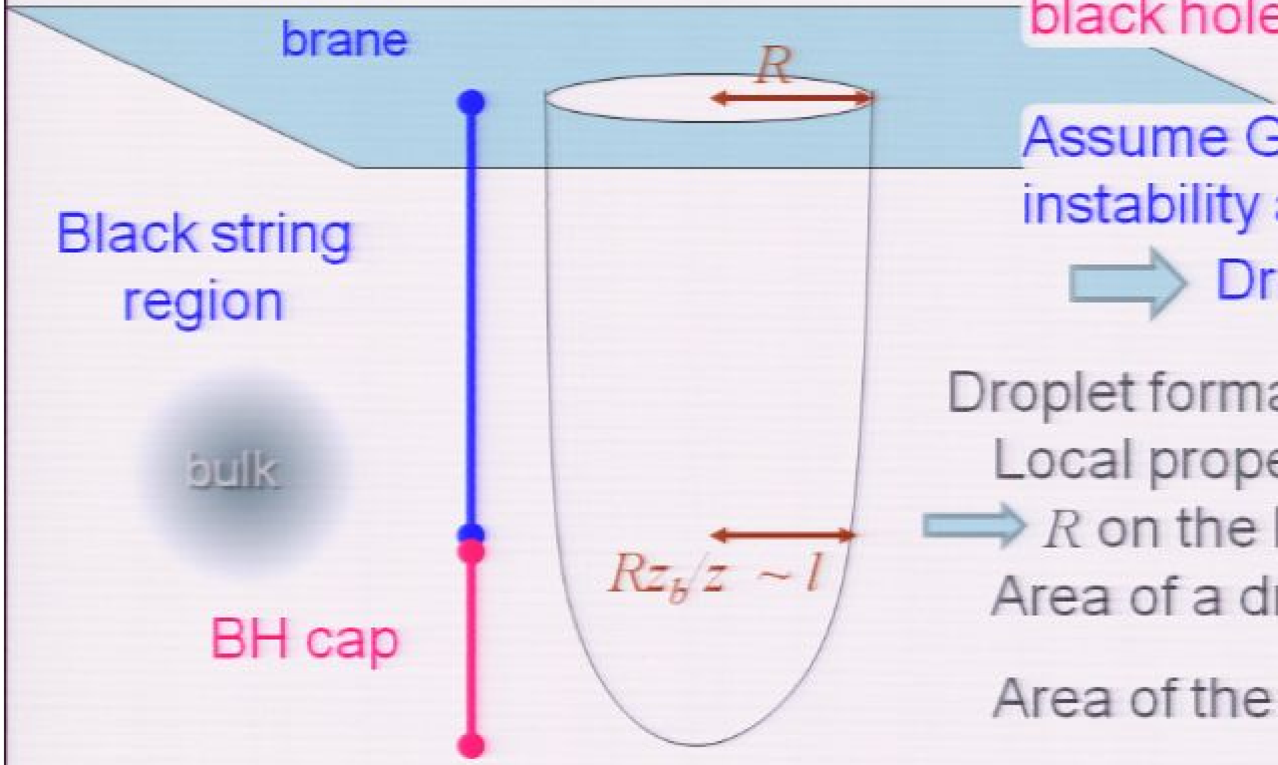

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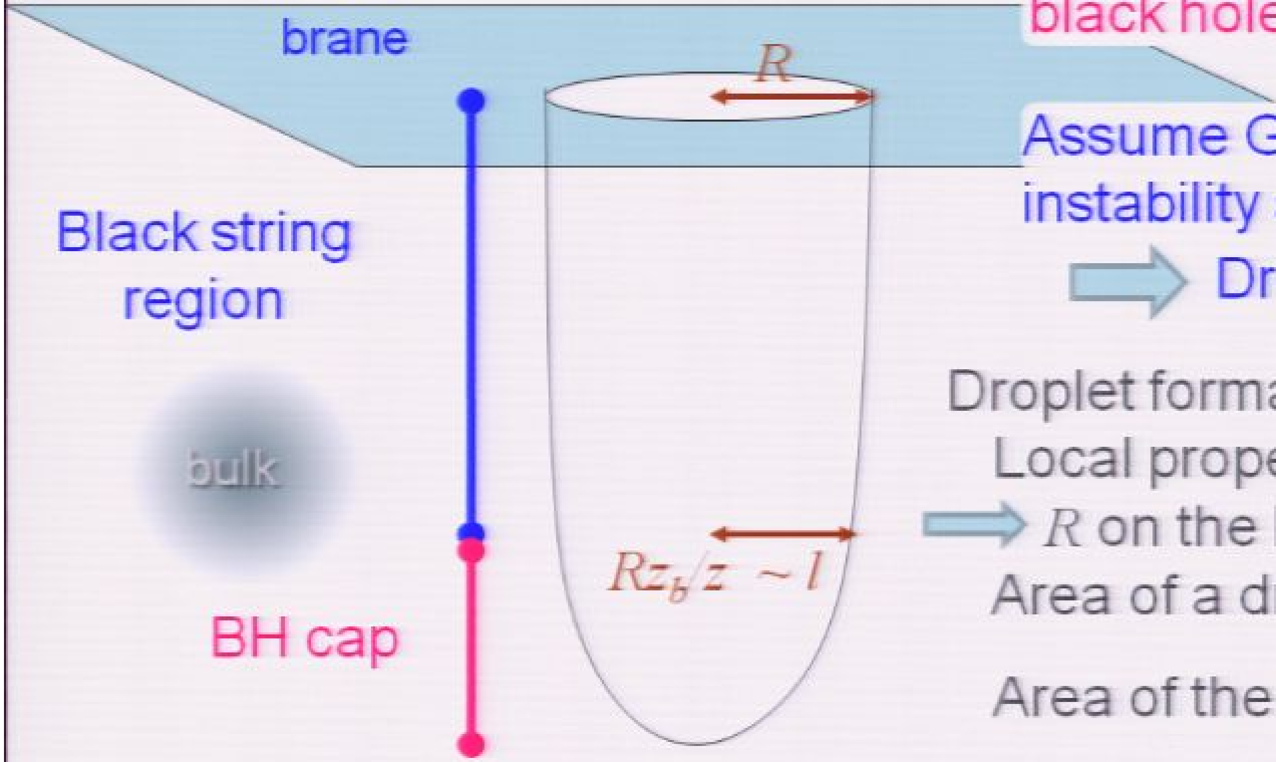

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Numerical brane BH

Kudoh, Nakamura & T.T. ('03)
Kudoh ('04)

- Static and spherical symmetric configuration

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Comparison of 4D areas with 4D and 5D Schwarzschild sols.

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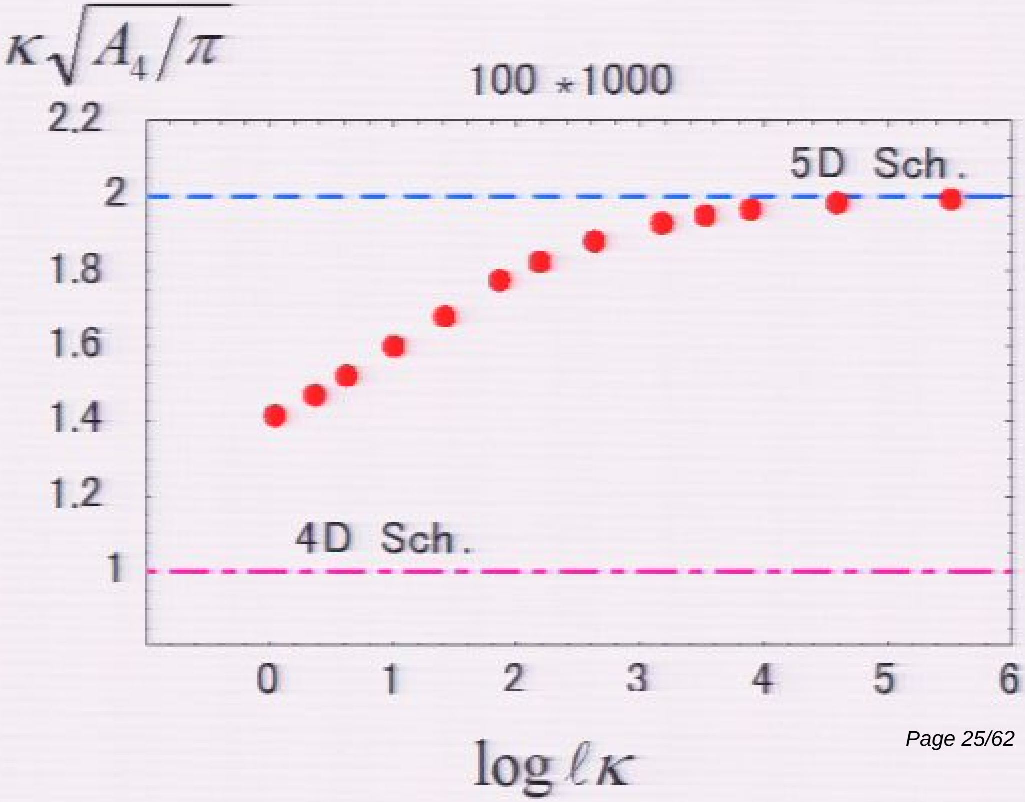

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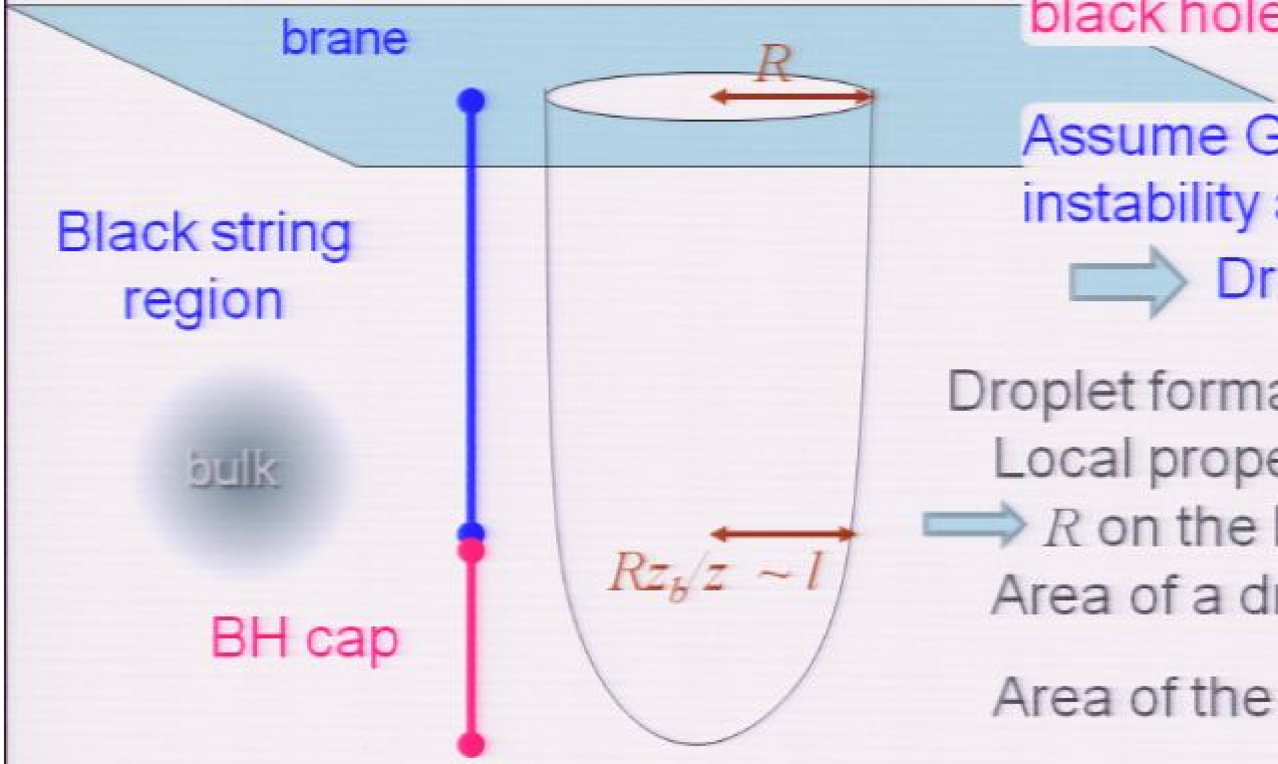

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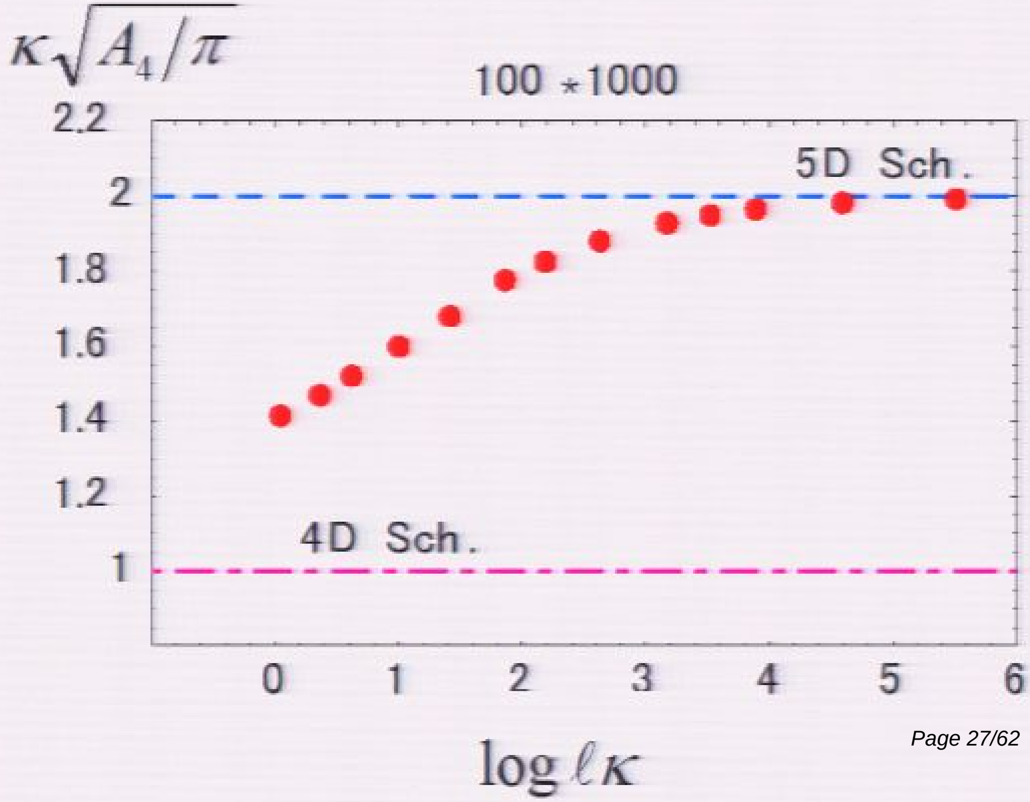

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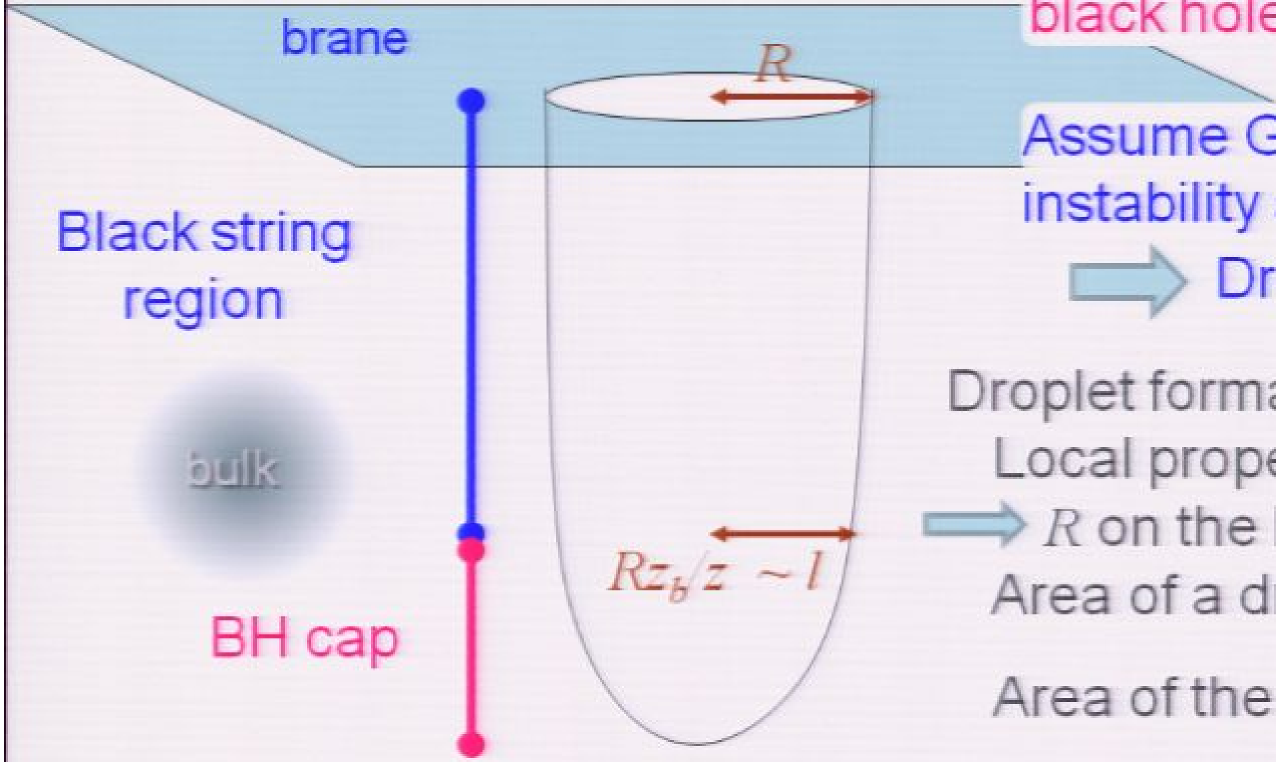

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  $$=\int d[g_{bulk}]\,\exp(-S_{HE}-S_{GH}+\underbrace{S_1+S_2+S_3}_{\text{Counter terms}})\equiv\exp(-W_{CFT}[q])$$

$$S_{EH}=-\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left({}^{(5)}R+\frac{12}{\ell^2}\right)$$

$$S_{GH}=-\frac{1}{\kappa_5^2}\int d^4x\sqrt{-q}\;K$$

$$S_1=-\frac{3}{\kappa_5^2\ell}\int d^4x\sqrt{-q}$$

$$S_2=-\frac{\ell}{4\kappa_5^2}\int d^4x\sqrt{-q}\;{}^{(4)}R$$

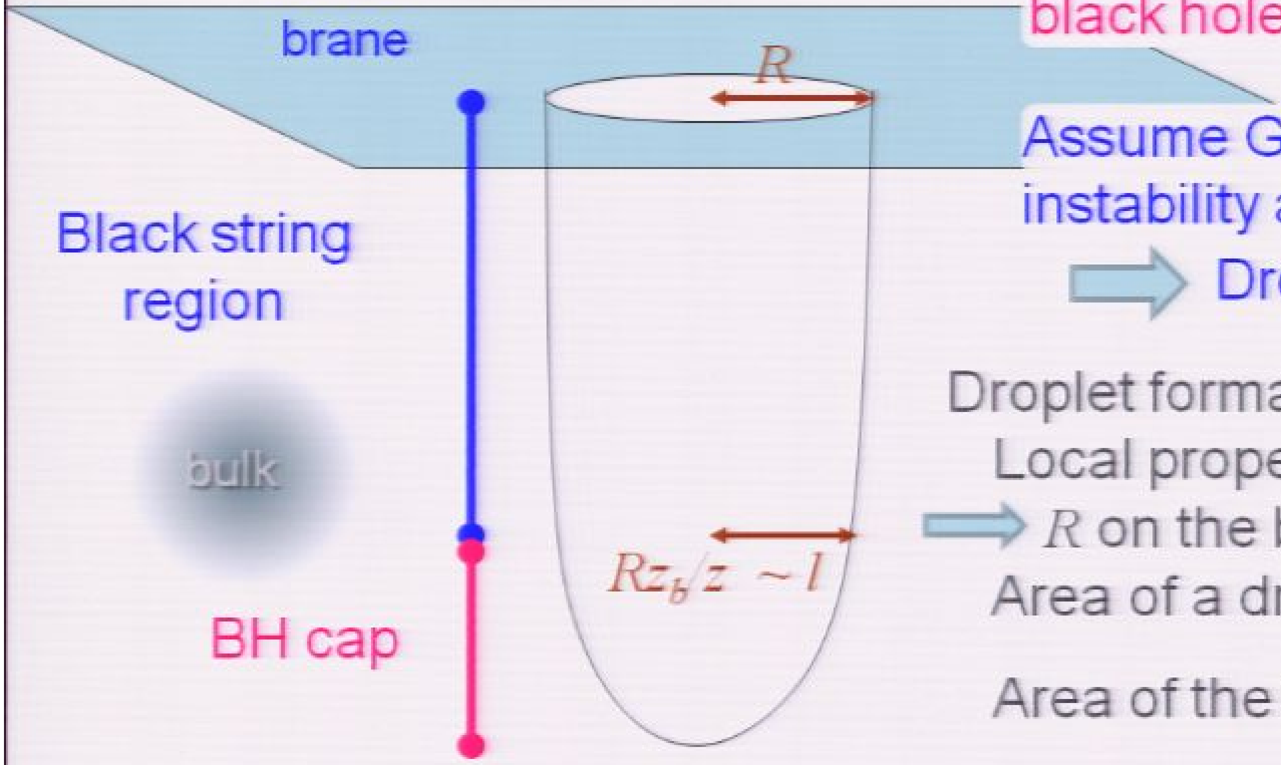
$$S_3=\cdots$$

- $\underbrace{z_0}_{\text{Brane position}}\to 0$ limit is well defined with the counter terms.

  $z_0\Leftrightarrow$ cutoff scale parameter

- $$\int d[g]\,\exp(-S_{RS})=\int d[g]\,\exp(-2(S_{EH}+S_{GH})+\overset{\text{brane tension}}{2S_1}-S_{matter})$$
  $$=\exp(-\underset{\text{4D Einstein-Hilbert action}}{2S_2}-S_{matter}-2(W_{CFT}+S_3))$$

most-probable shape of a large BH



Structure near the cap region will be almost independent of the size of the black hole. ~discrete self-similarity

Assume Gregory-Laflamme instability at the cap region

⇒ Droplet escaping to the bulk

Droplet formation

Local proper time scale: l

⇒ R on the brane due to redshift factor

Area of a droplet: l^3

Area of the black hole: $A \sim lR^2$

$$\frac{dA}{dt} \approx \frac{l^3}{R} \Rightarrow \frac{1}{M}\frac{dM}{dt} \approx \frac{1}{A}\frac{dA}{dt} \approx \frac{l^2}{R^3}$$

Numerical brane BH

Kudoh, Nakamura & T.T. ('03)
Kudoh ('04)

- Static and spherical symmetric configuration

$$ds^2 = \frac{\ell^2}{z^2}\left(-T^2dt^2 + e^{2R}(dr^2 + dz^2) + r^2e^{2C}d\Omega^2\right)$$

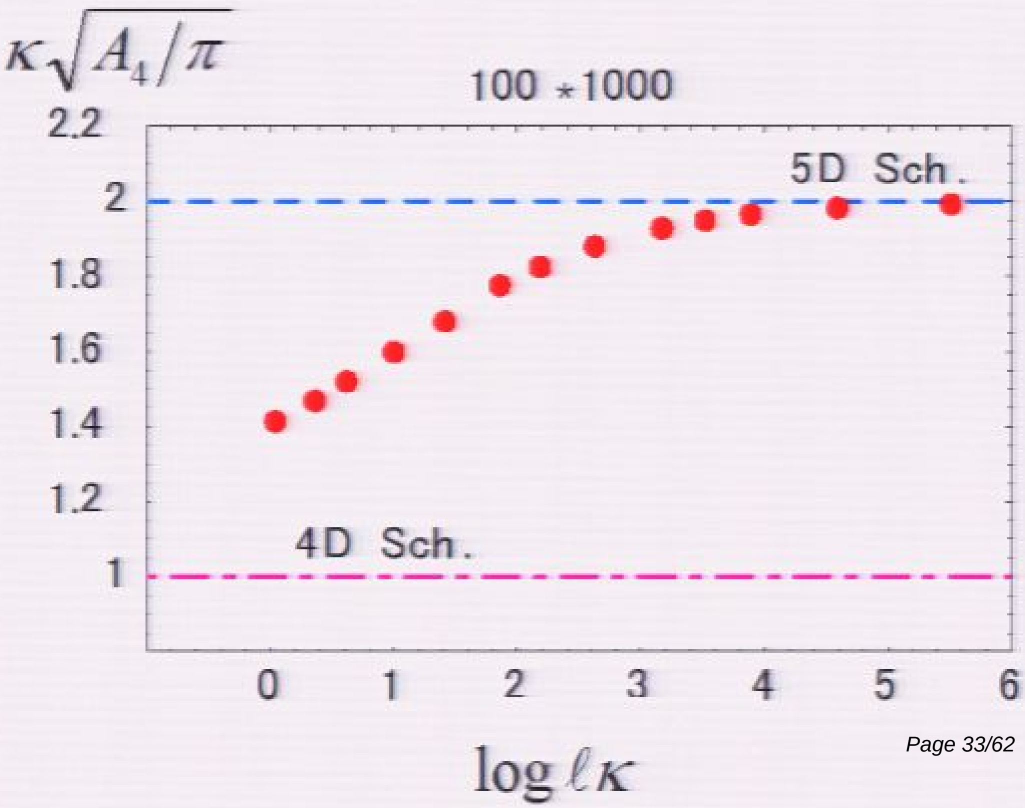
T, R and C are functions of z and r.

Comparison of 4D areas with 4D and 5D Schwarzschild sols.

5D Sch. $A_4 = 4\pi\kappa^{-2}$

4D Sch. $A_4 = 4\pi(2\kappa)^{-2}$

κ is surface gravity



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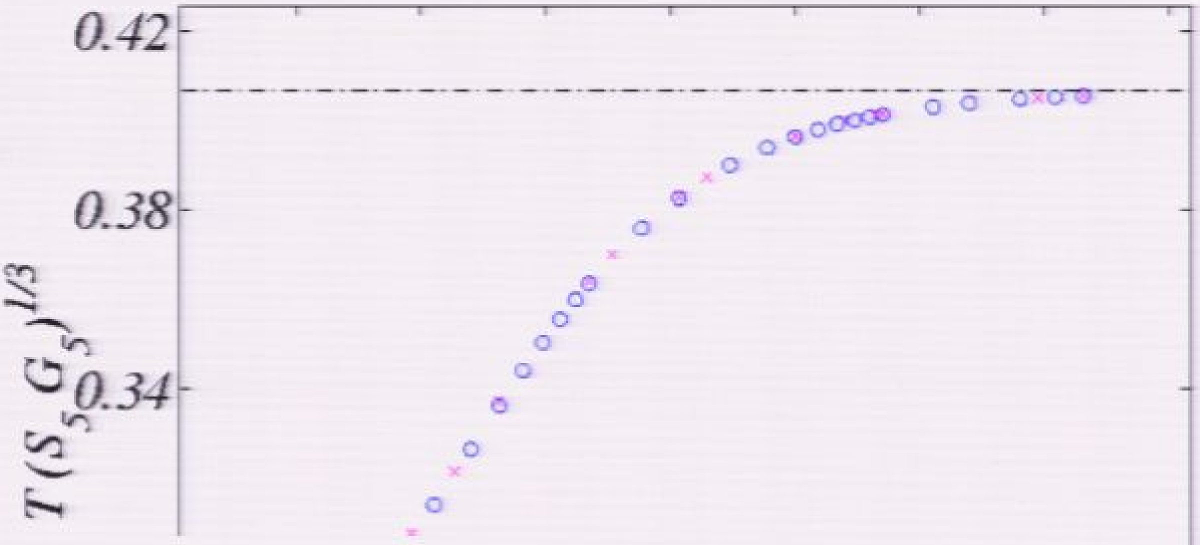

- It becomes more and more difficult to construct brane BH solutions numerically for larger BHs.
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Classical black hole evaporation conjecture

(T.T. ('02), Emparan et al ('02))

4D Einstein+CFT with the lowest order quantum correction $\begin{pmatrix}\text{number of}\\ \text{field of CFT}\end{pmatrix} \approx \frac{\ell^2}{\kappa_4^2}$

AdS/CFT correspondence

equivalent

Classical 5D dynamics in RS II model

4D BH with CFT

equivalent

5D BH on brane

Hawking radiation in 4D Einstein+CFT picture

equivalent

Classical evaporation of 5D BH

Time scale of BH evaporation

$$\boxed{\tau = \left(\frac{M}{M_{Solar}}\right)^3 \left(\frac{1\text{mm}}{\ell}\right)^2 \times 120\text{year}}$$

$$\frac{\dot{M}}{M} \approx \begin{pmatrix}\text{Number of}\\ \text{species}\end{pmatrix} \times \frac{1}{G_N^2 M^3} \approx \frac{\ell^2}{(G_N M)^3}$$

$10 \pm 5 M_\odot$ BH+K-type star X-ray binary A0620-00

$\ell < 0.132$mm ($10 M_\odot$ BH is assumed)

(Johansen, Psaltis, McClintock arXiv:0803.1835)

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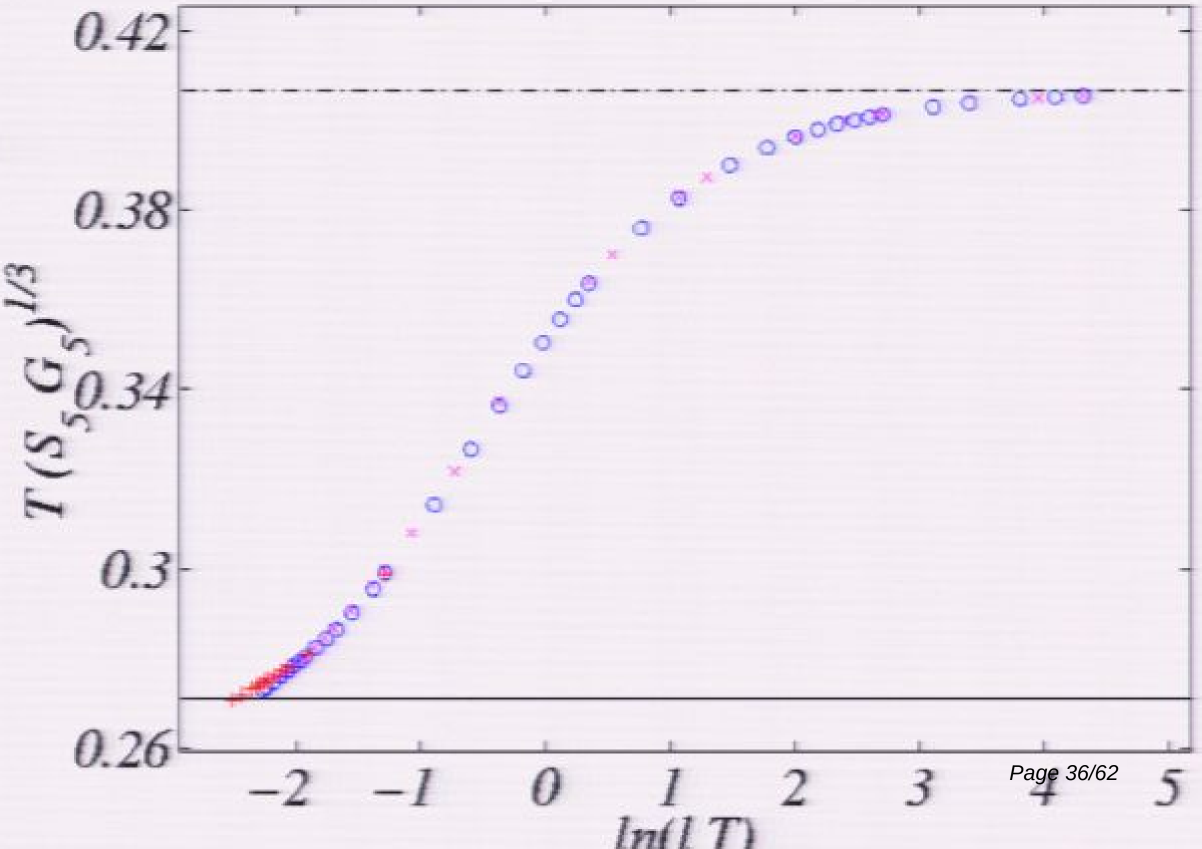

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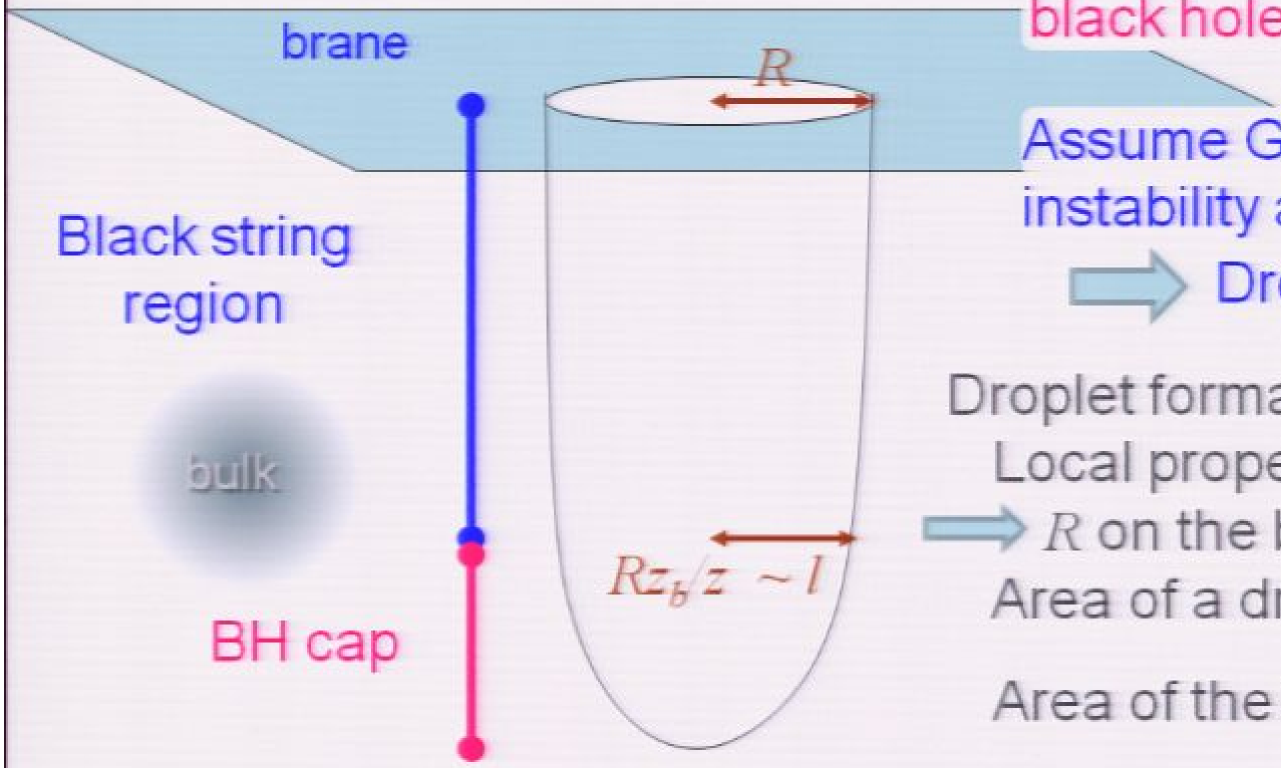

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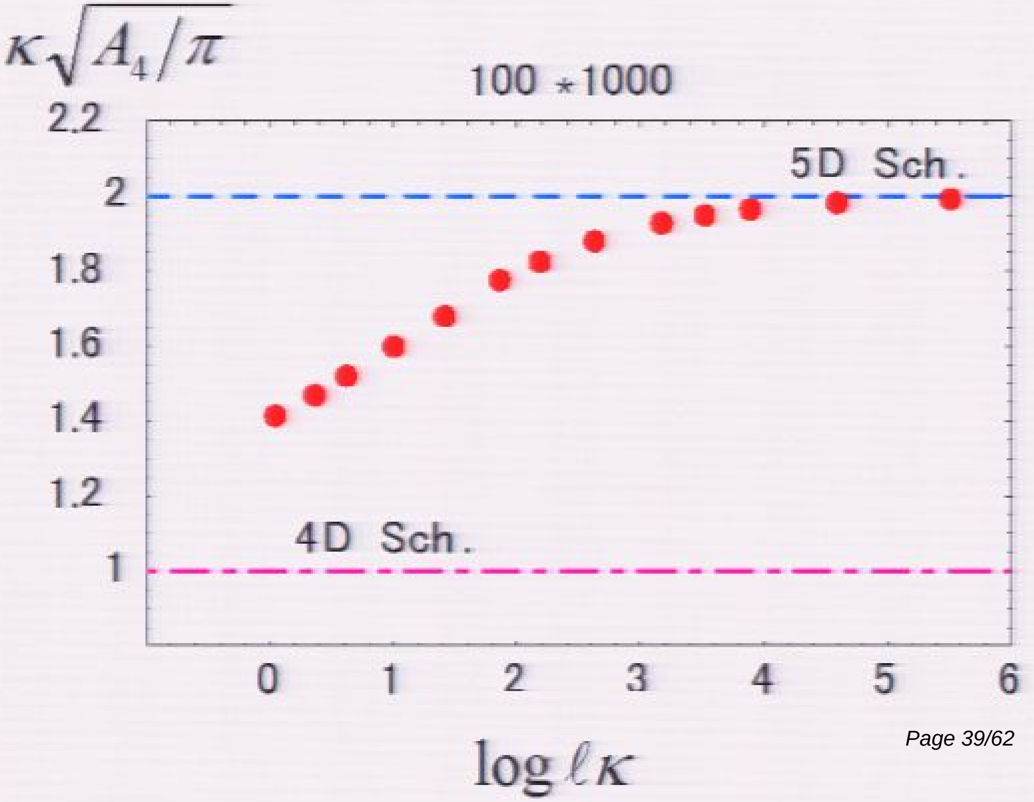

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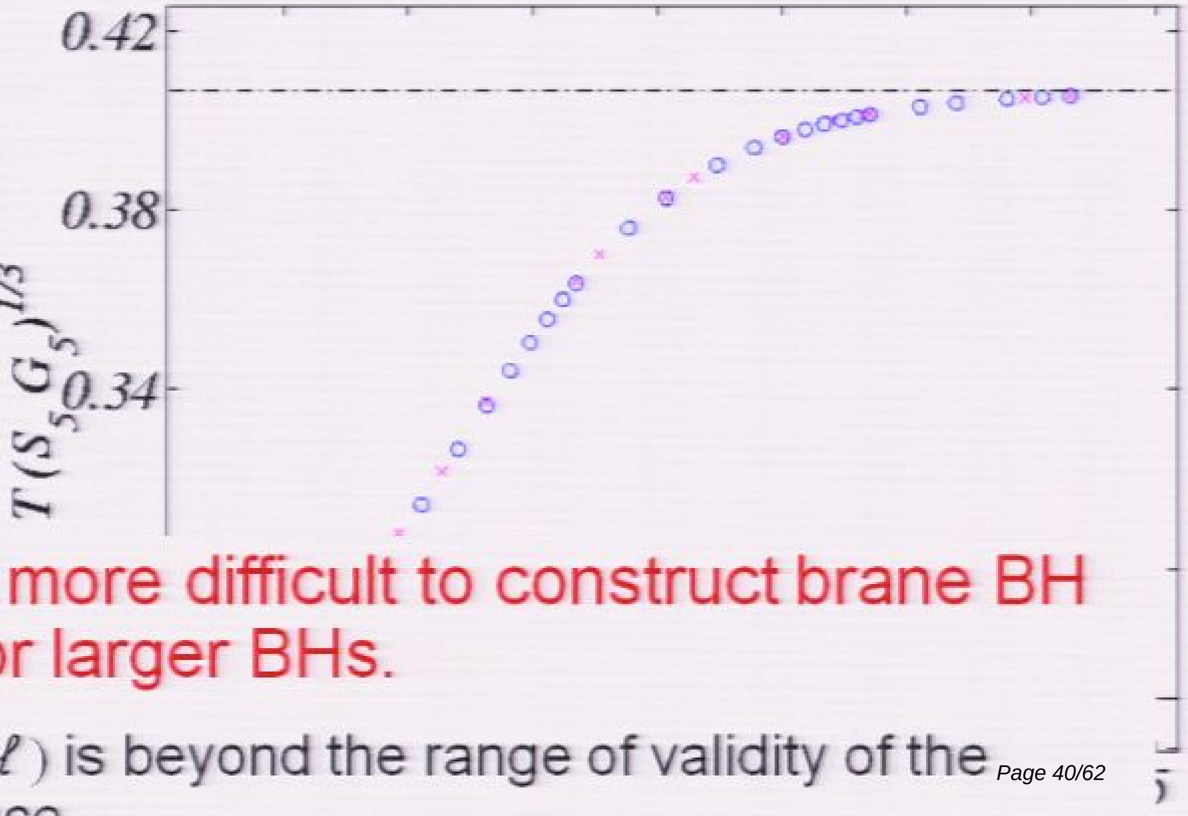


■ It becomes more and more difficult to construct brane BH solutions numerically for larger BHs.

■ Small BH case ($\kappa^{-1} < \ell$) is beyond the range of validity of the AdS/CFT correspondence.

Let’s assume that the followings are all true,

1) Classical BH evaporation conjecture is correct.

Namely, there is no static large localized BH solution.

2) Static small localized BH solutions exist.

3) A sequence of solutions does not disappear suddenly.

then, what kind of scenario is possible?

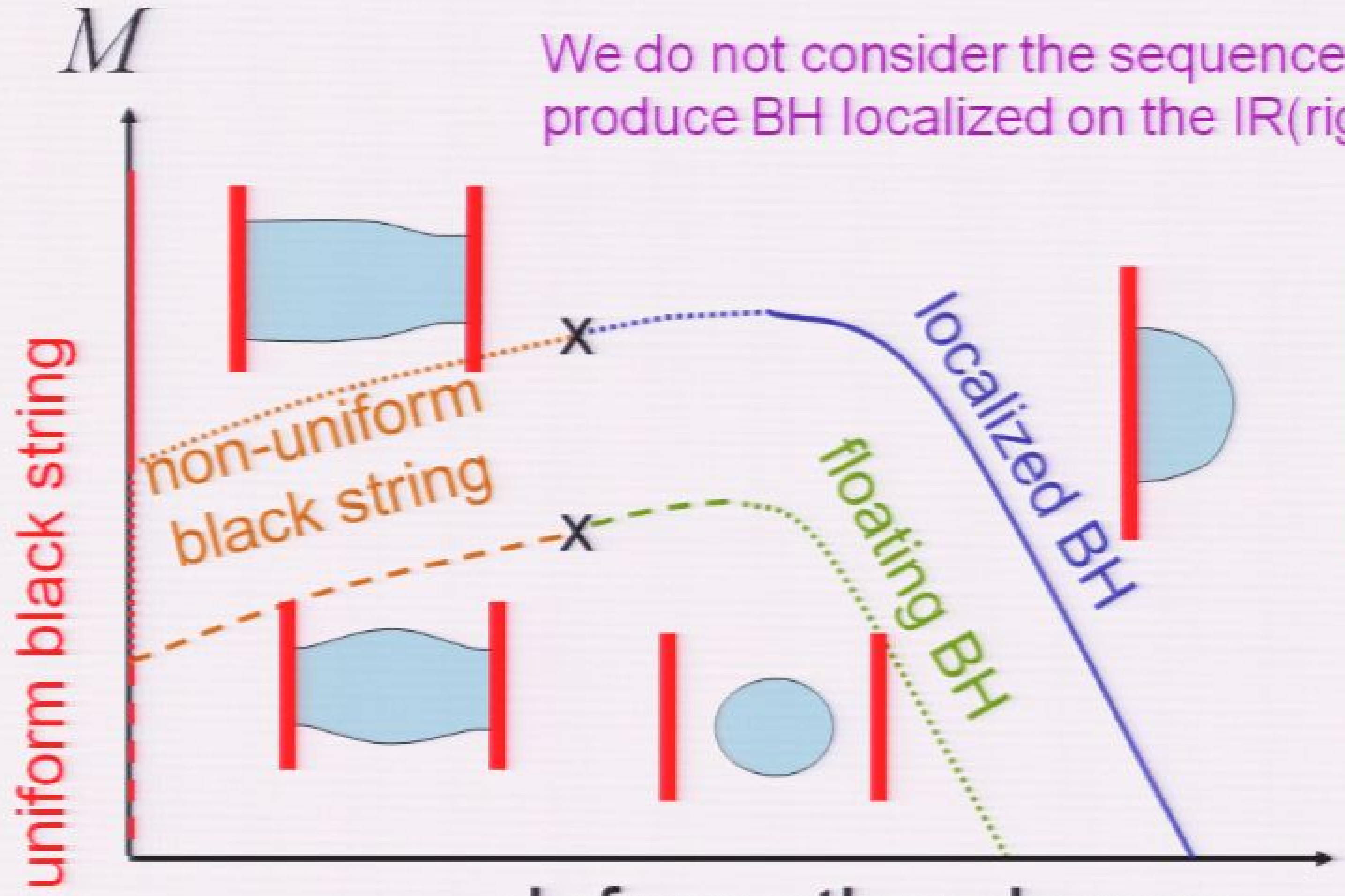
In generalized framework, we seek for consistent phase diagram of sequences of static black objects.

RS-I (two branes)
Karch-Randall (AdS-brane)

$\sigma = 0$ & $\Lambda = 0$

Un-warped two-brane model

We do not consider the sequences which produce BH localized on the IR(right) brane.



deformation degree

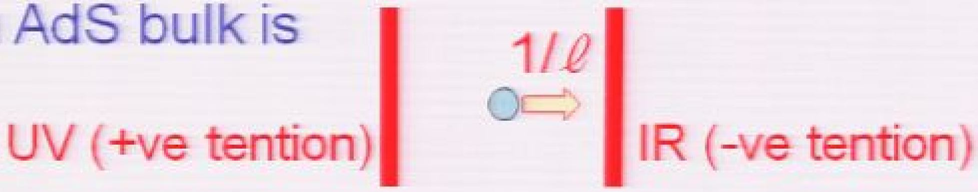
(Kudoh & Wiseman (2005))

Warped two-brane model (RS-I)

$\Lambda \neq 0$ & σ is fine-tuned

In the warped case the stable position of a floating black hole shifts toward the UV (+ve tention) brane.

Acceleration acting on a test particle in AdS bulk is

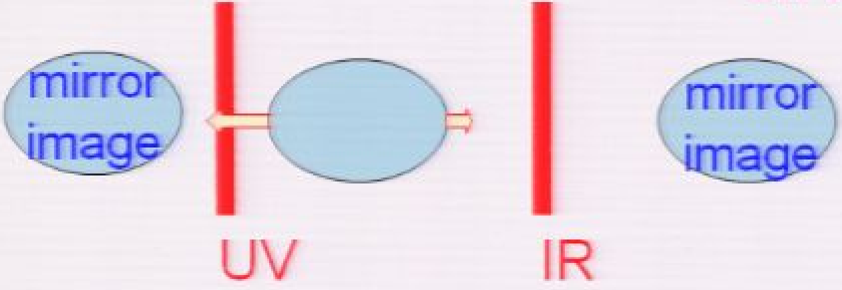


$$ds^2 = dy^2 + e^{-2y/\ell}\left(-dt^2 + d\mathbf{x}^2\right)$$

$$a = \frac{\left(\log g_{tt}\right)_{,y}}{\sqrt{g_{yy}}} = -\frac{1}{\ell}$$

Compensating force toward the UV brane is necessary.

Self-gravity due to the mirror images on the other side of the branes

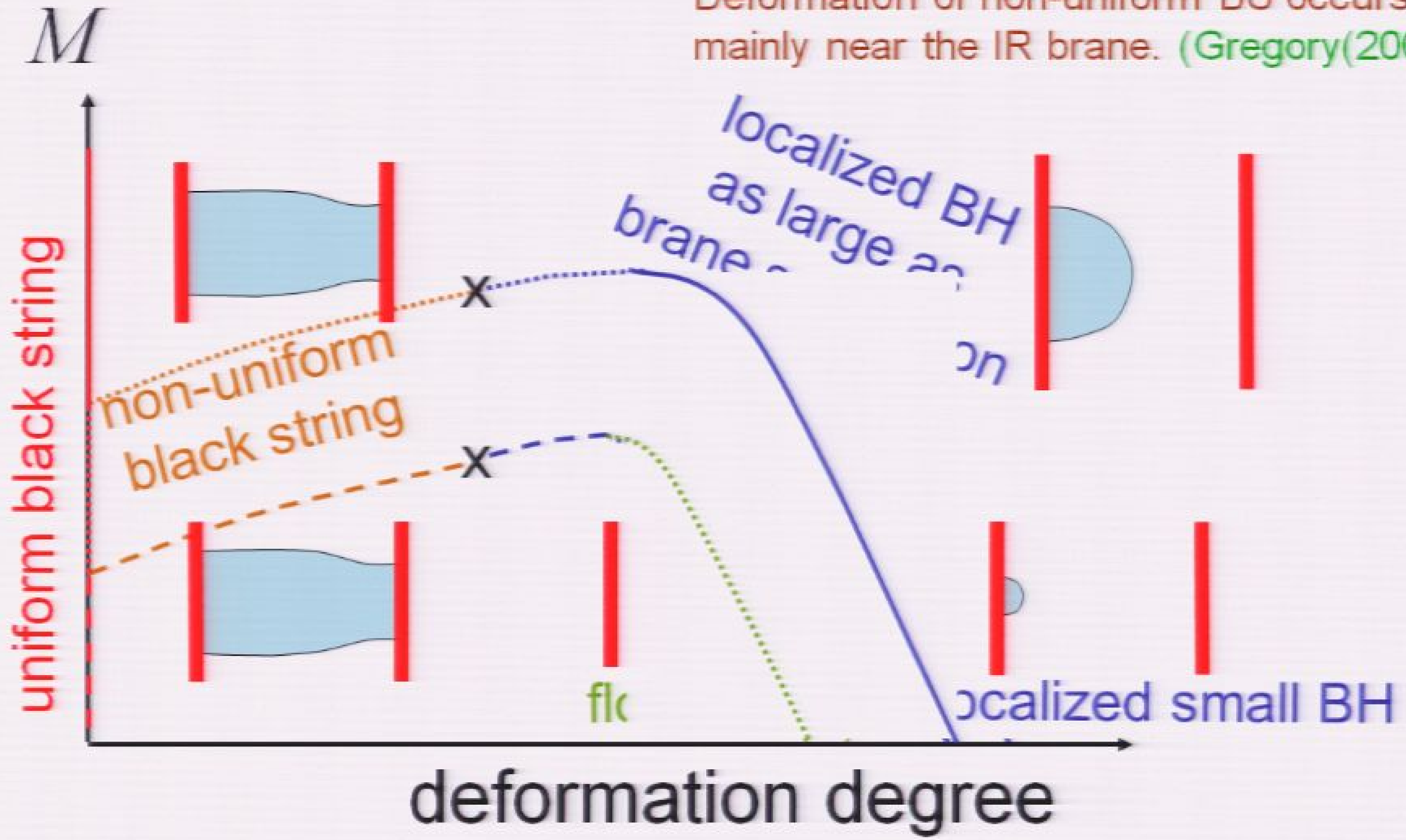


When $R_{\rm BH} > \ell$, self-gravity (of $O(1/R_{\rm BH})$ at most), cannot be as large as $1/\ell$.

Large floating BHs become large localized BHs. → Pair annihilation of two sequences of localized BH, which is necessary to be consistent with AdS/CFT.

Phase diagram for warped two-brane model

Deformation of non-uniform BS occurs mainly near the IR brane. (Gregory(2000))



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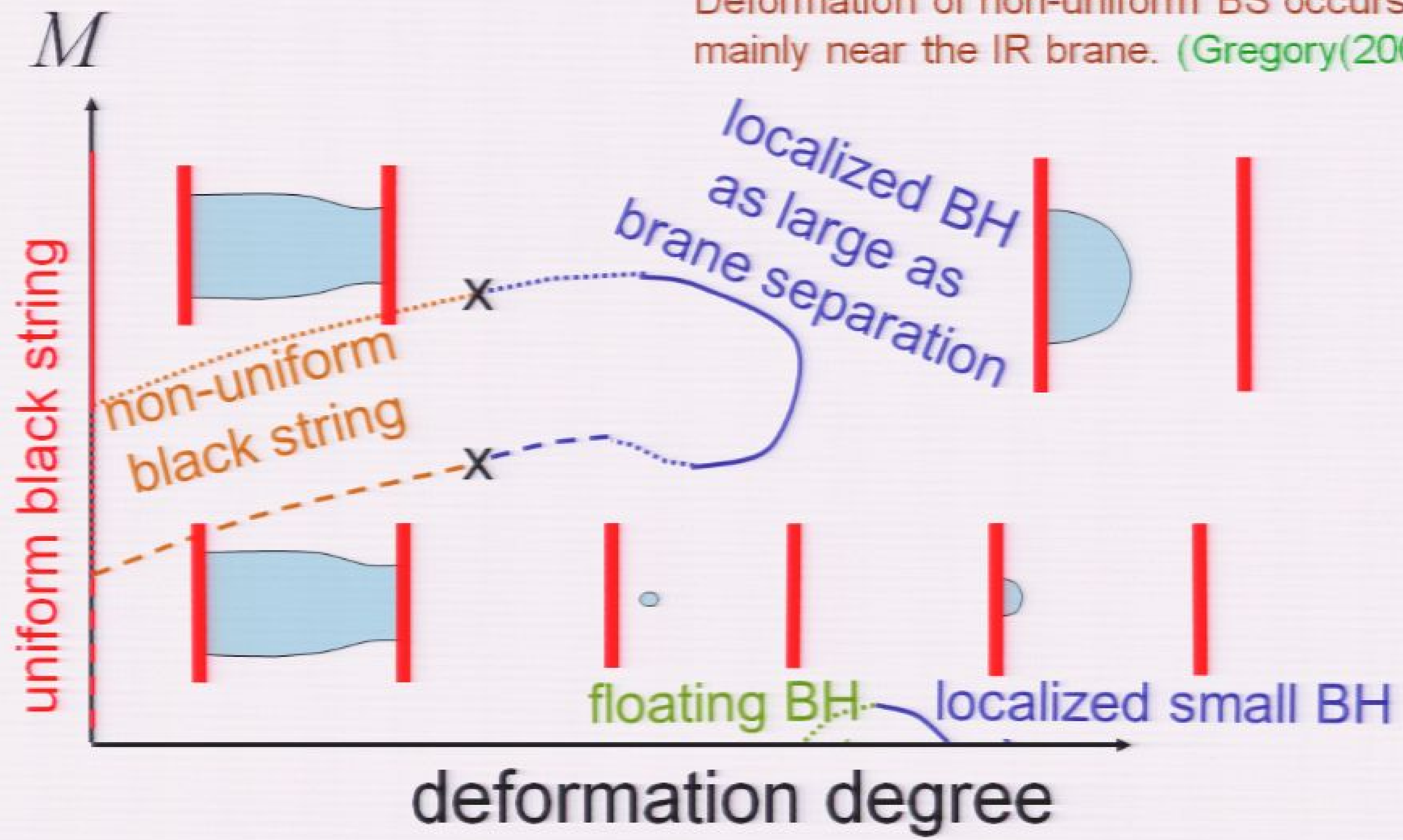

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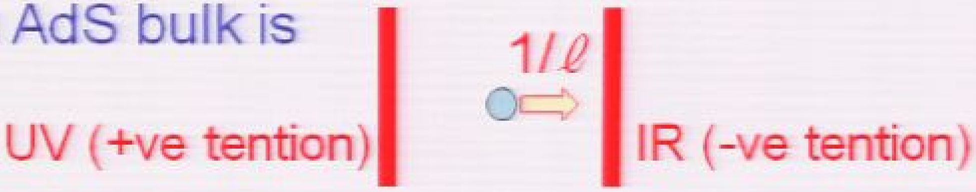

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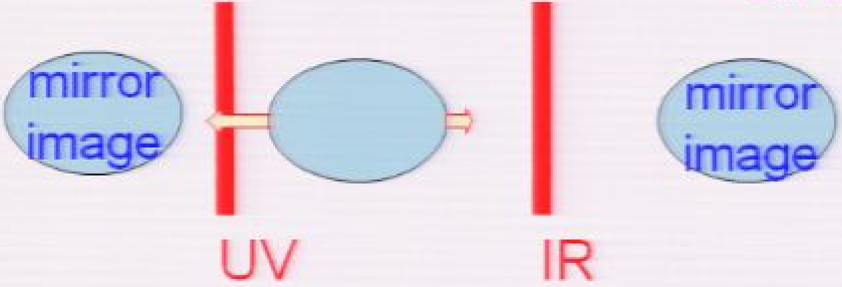

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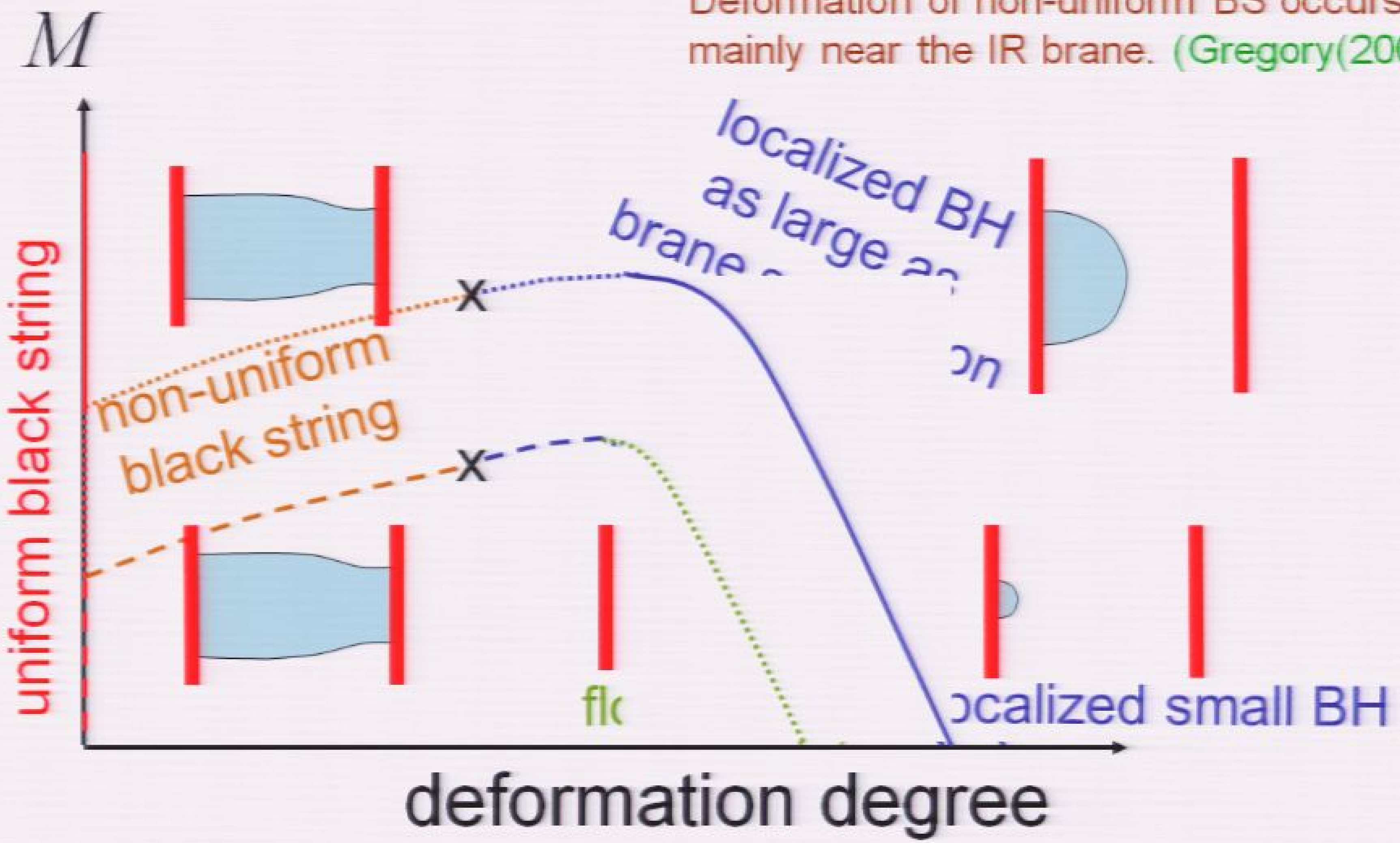

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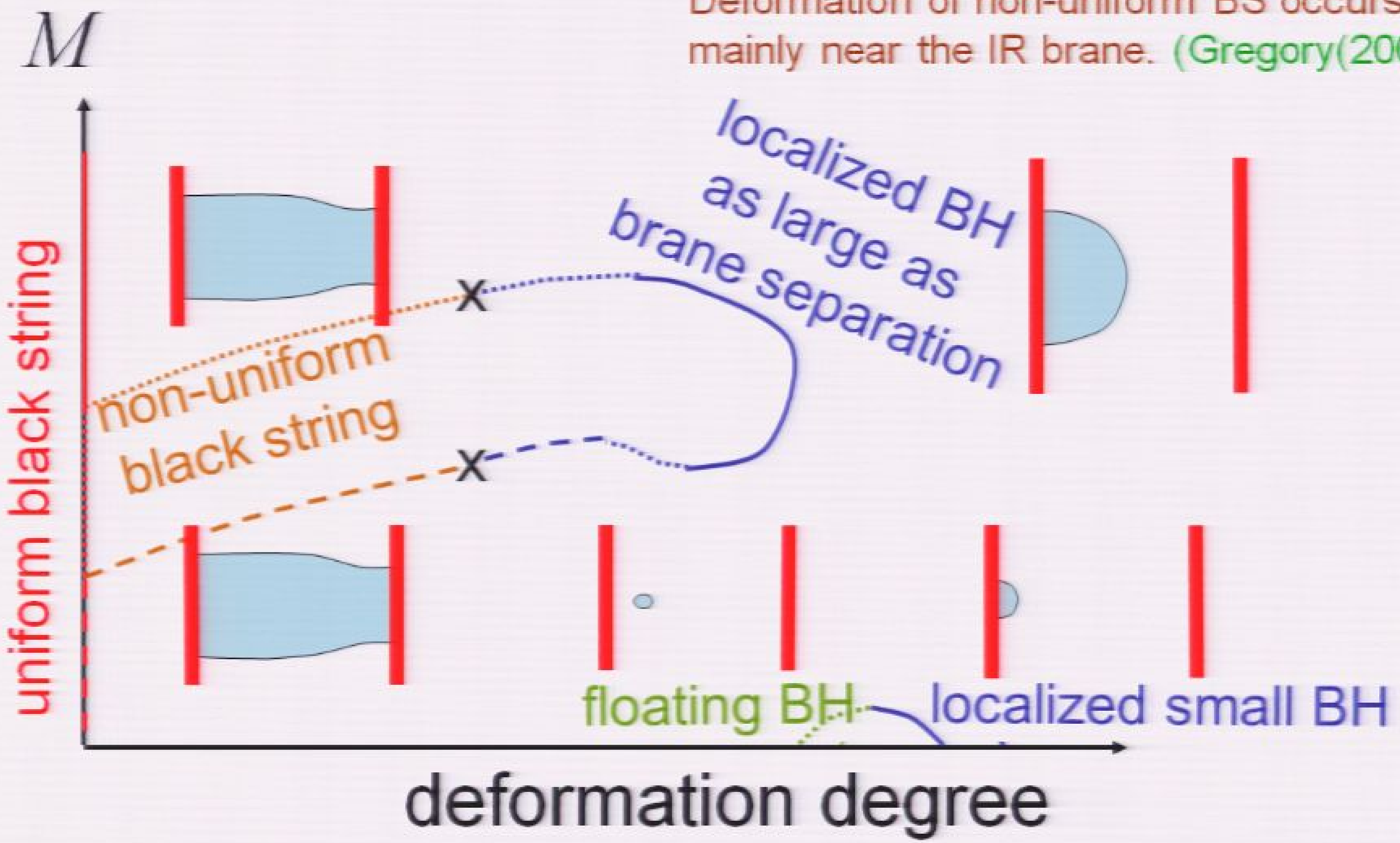


Model with detuned brane tension

Karch-Randall model JHEP0105.008(2001)

$$S=-\int d^5x\frac{\sqrt{-g}}{2\kappa_5}[R-2\Lambda]-\sigma\int\sqrt{-g^{(4)}}d^4x$$

Background configuration:

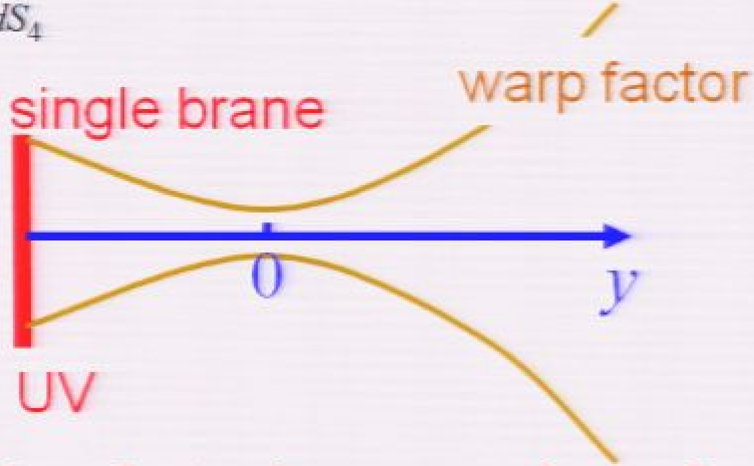
$$ds^2=dy^2+\ell^2\cosh^2(y/\ell)ds^2_{AdS_4}$$

Brane placed at a fixed y.

$$\kappa_5\sigma=-\frac{6}{\ell}\tanh\frac{y}{\ell}$$

$y\to-\infty \Longleftrightarrow \sigma\to 6/\ell$ (RS limit)

$y\to 0 \Longleftrightarrow \sigma\to 0$



Warp factor increases for y>0

⇒ Zero-mode graviton is absent since it is not normalizable

(Karch & Randall(2001), Porrati(2002))

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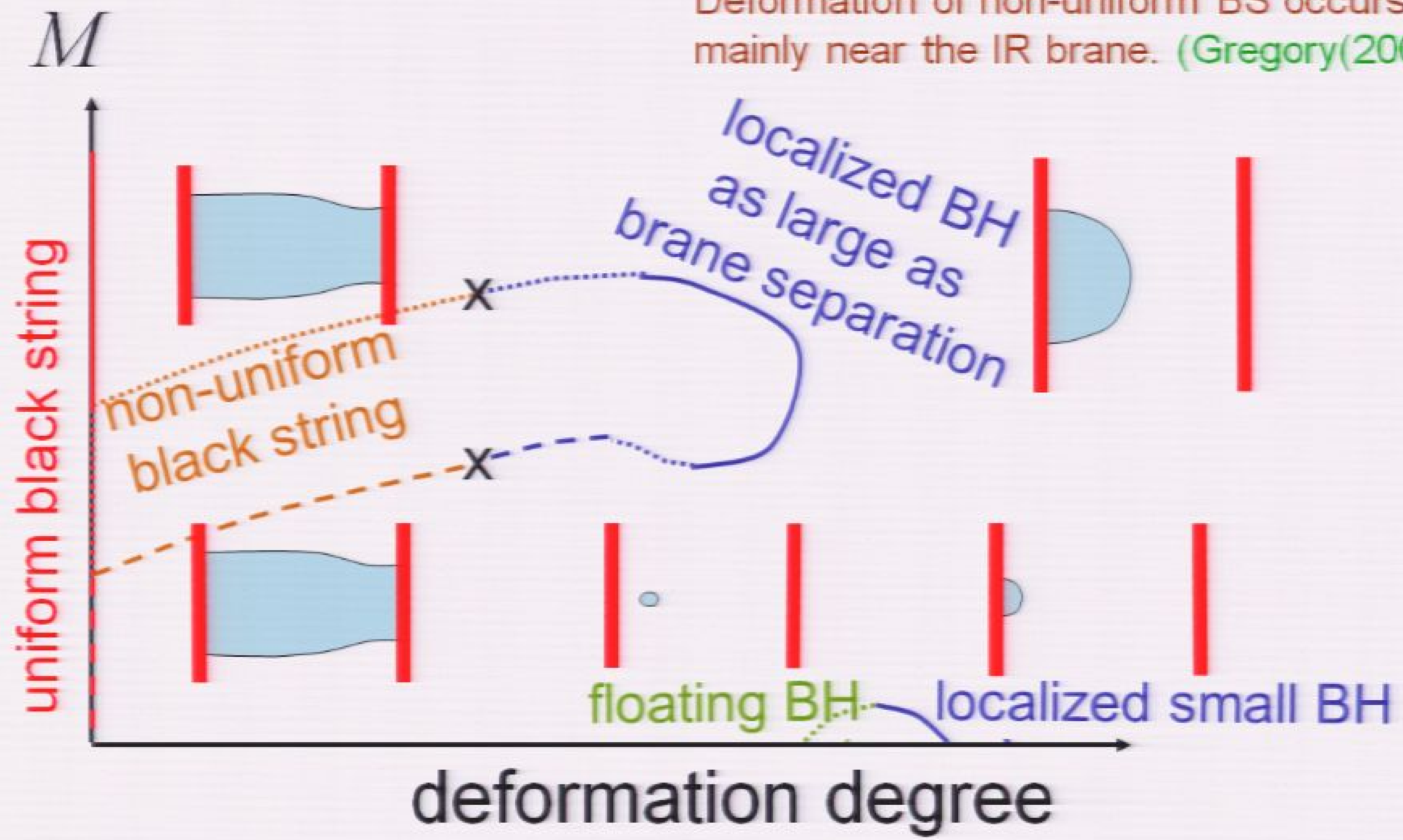

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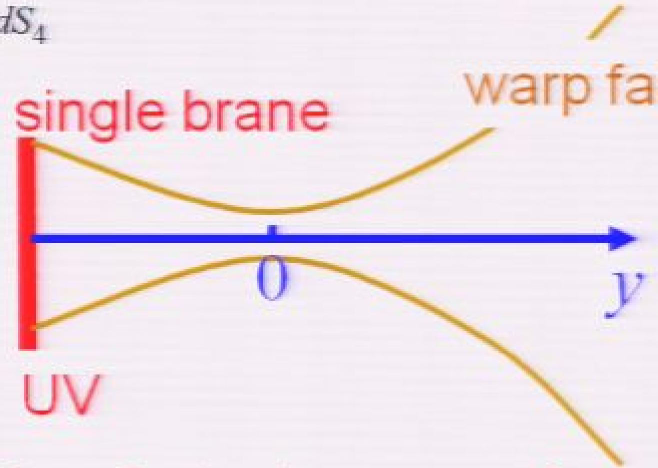

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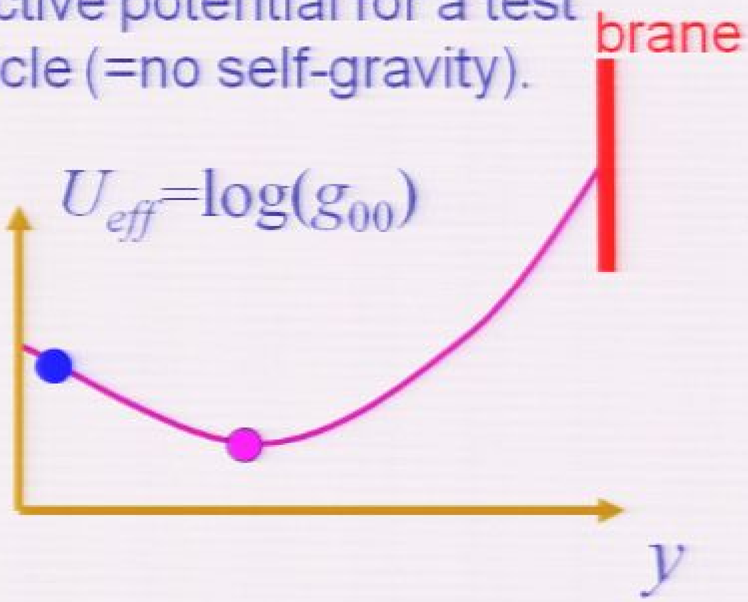
Effective potential for a test particle (=no self-gravity).



There are stable and unstable floating positions.

1) When $\delta\sigma = \sigma - 6/\ell$ is very small, stable floating BH is very far from the brane.

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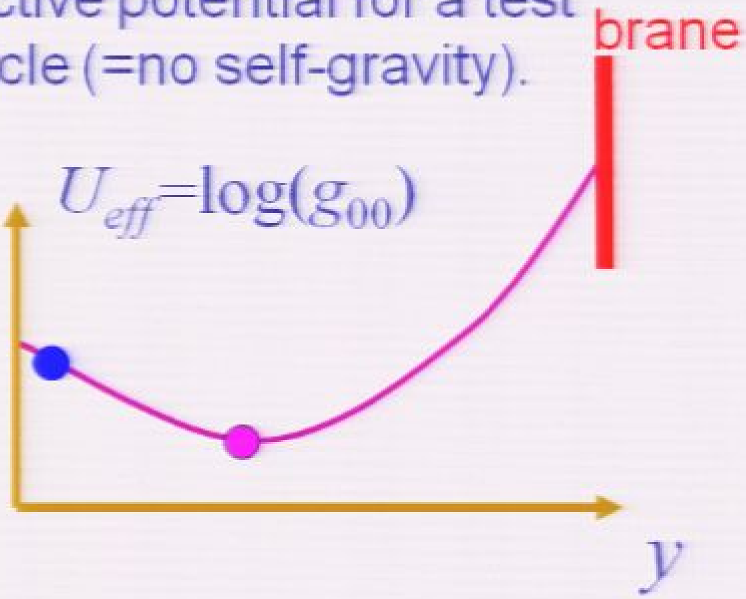

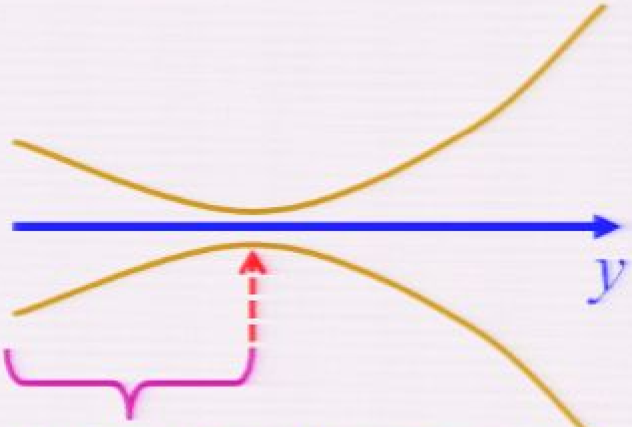

Since this distance is finite, very large BHs cannot float.

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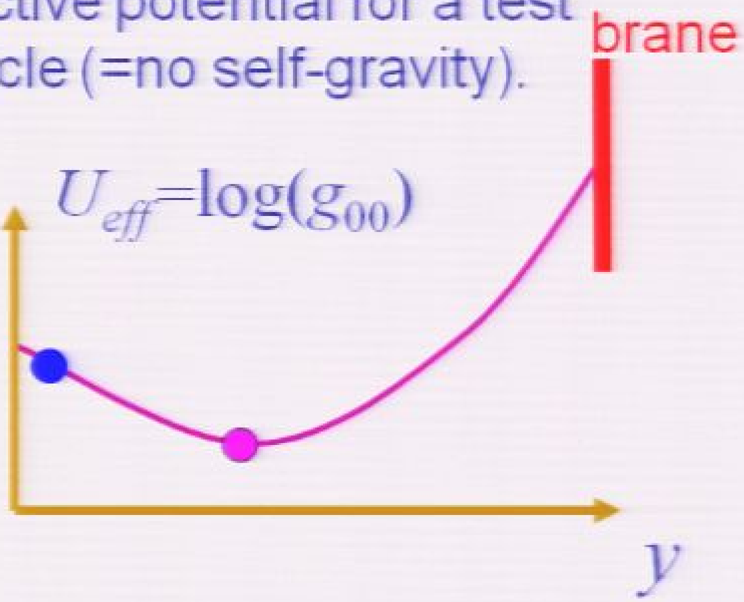

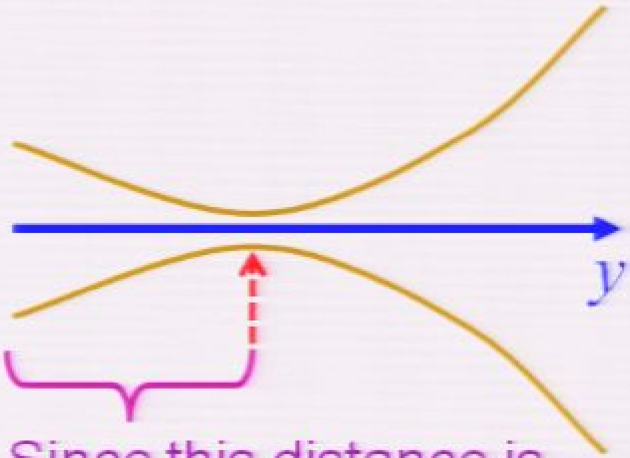

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➡ Large BHs necessarily touch the brane.

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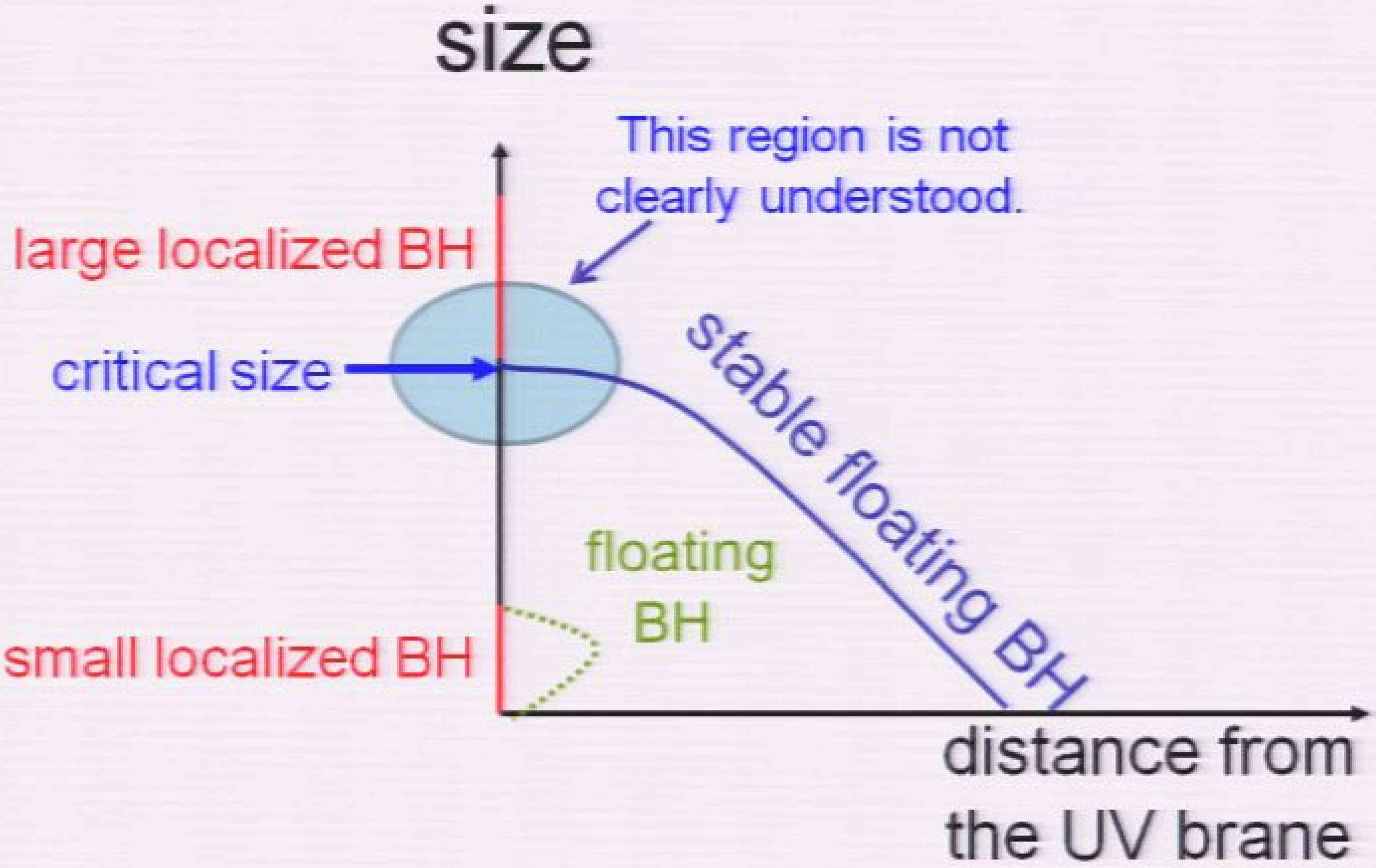
Phase diagram for detuned tension model



From the continuity of sequence of solutions,
large localized BHs are expected to exist above the critical size.

Showing presence will be easier than showing absence.

Large localized BHs above the critical size are consistent with AdS/CFT?

Why does static BHs not exist in asymptotically flat spacetime? ⇨ Hartle-Hawking (finite temperature) state has regular $T_{\mu\nu}$ on the BH horizon, but its fall-off at large distance is too slow.

In AdS, temperature drops at infinity by the red-shift factor.

$$T \propto 1/\sqrt{g_{00}} = 1/\sqrt{1-\mu r^{-1}+(r/L)^2}$$

4D AdS curvature scale

Quantum state consistent with static BHs will exist if the BH mass is as large as $m_{pl}^2(\ell L)^{1/2}$. (Hawking & Page '83)

smaller $\delta\sigma$

In the RS-limit, stable floating BH disappears

tensionless limit ($\sigma \to 0$)

5d-AdS-Schwarzschild with brane on the equatorial plane

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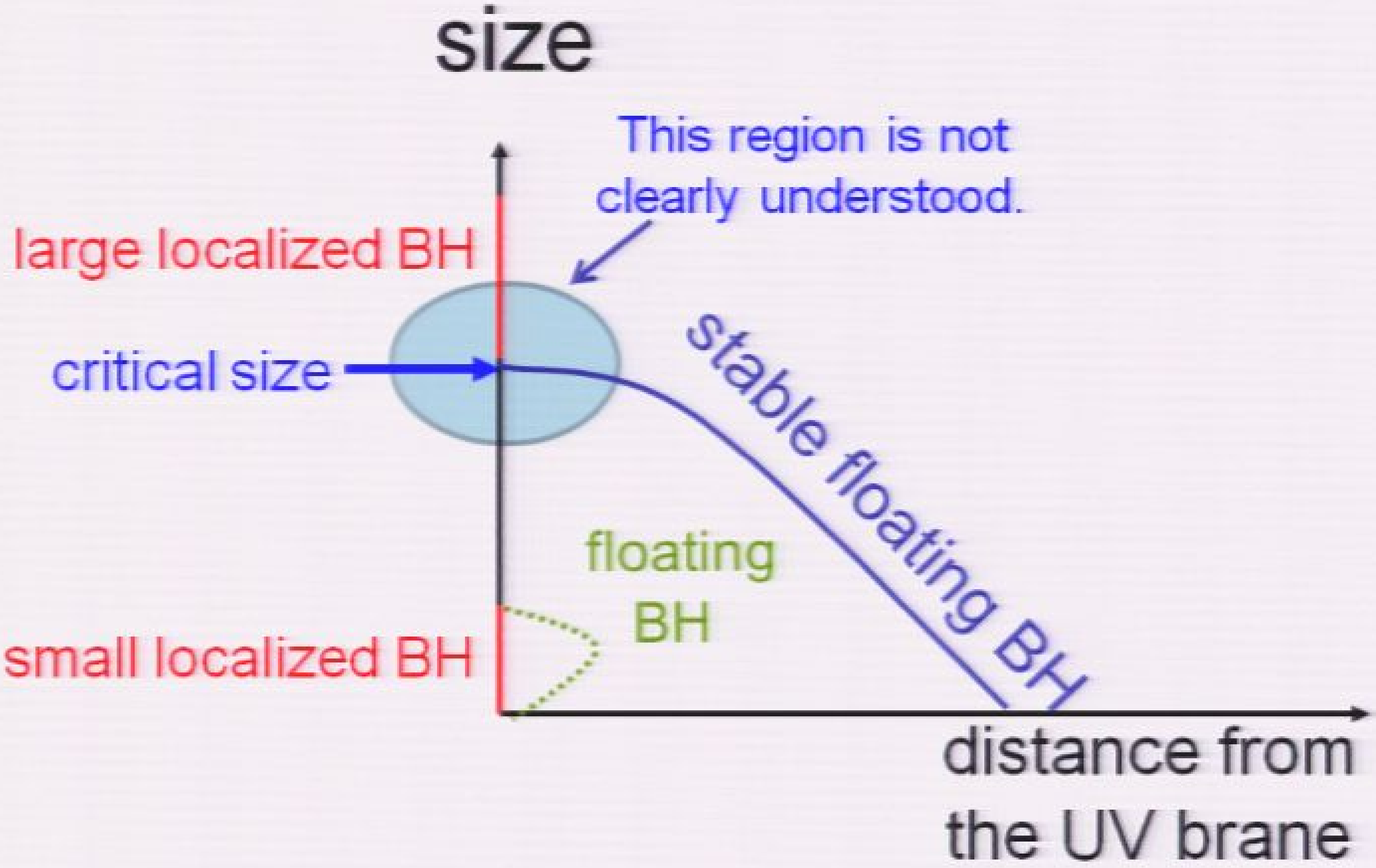

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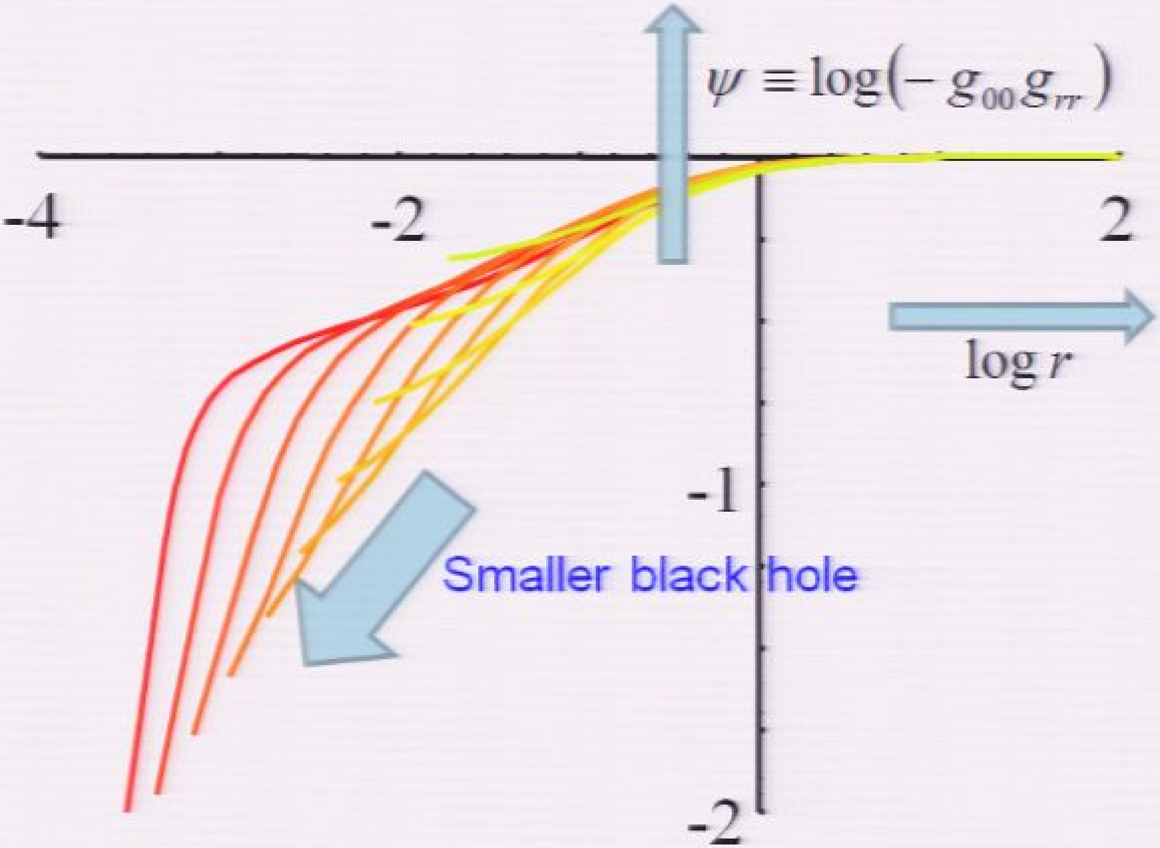
At the transition point, the temperature is finite at $T_{crit} \approx 1/\sqrt{lL}$,

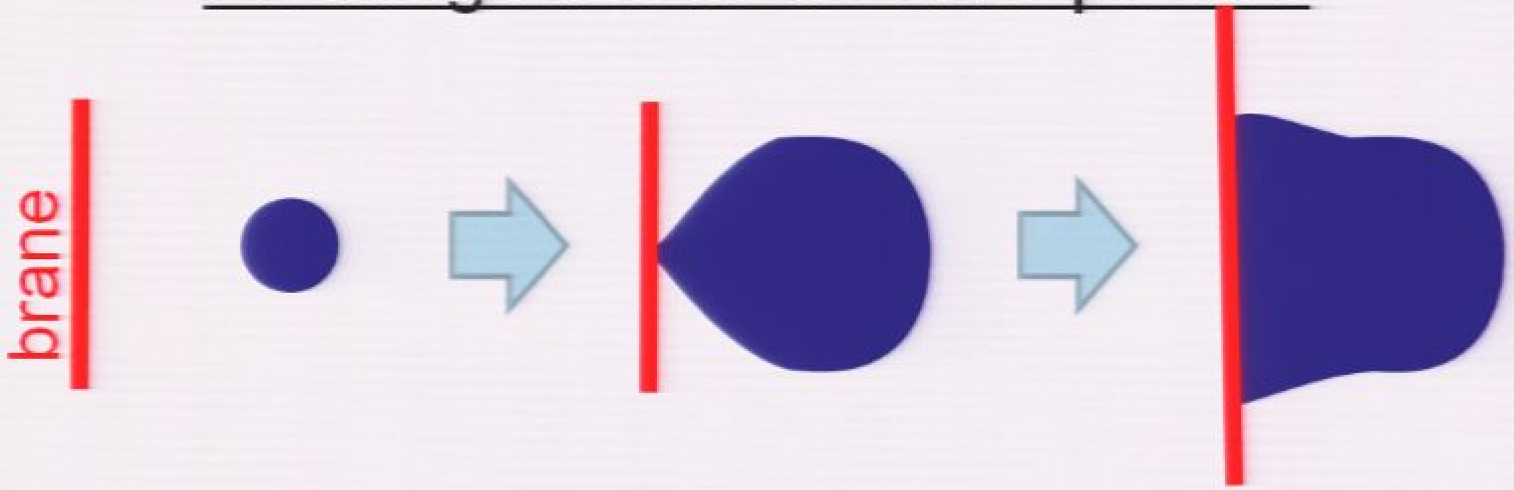
although the BH size goes to zero.

$$T_{BH} \approx \frac{1}{\sqrt{g_{rr}}}\frac{\partial\sqrt{g_{00}}}{\partial r} \approx e^{\psi}\frac{\partial g_{00}}{\partial r} \not\approx \frac{1}{r_{BH}}$$



Smaller black hole

Floating BHs in 5D AdS picture



Numerical construction of static BH solutions is necessary.

However, it seems difficult to resolve two different curvature scales l and L simultaneously. We are interested in the case with $l << L$.

We study time-symmetric initial data just solving the Hamiltonian constraint,

⮡ extrinsic curvature of t-const. surface $K_{\mu\nu}=0$.

$$R_{tt}-\frac{1}{2}Rg_{tt}+\Lambda g_{tt}=8\pi T_{tt}$$

We use 5-dimensional Schwarzschild AdS space as a bulk solution.

⟹ Hamiltonian constraint is automatically satisfied in the bulk.

Then, we just need to determine the brane trajectory to satisfy the Hamiltonian constraint across the brane.

Time-symmetric initial data for floating BHs

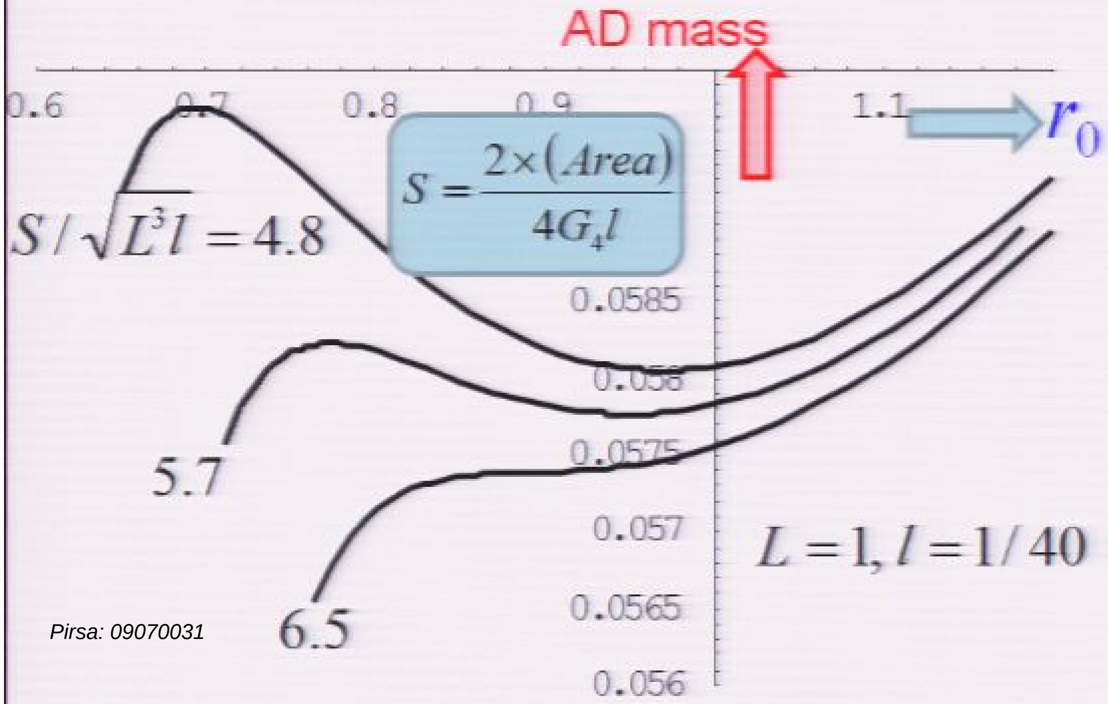
work in progress N. Tanahashi & T.T.

5D Schwarzschild AdS bulk:

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega^2$$

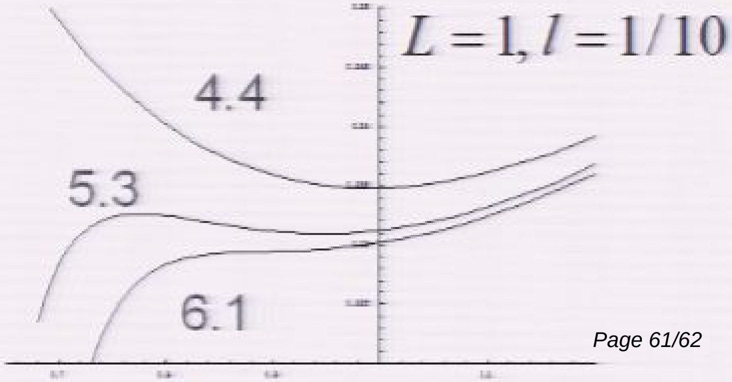
Brane:=3 surface in 4-dimensional space.
t=constant surface

Hamiltonian constraint on the brane ⇒ Trace of extrinsic curvature of this 3 surface $= 3\sqrt{\frac{1}{l^2} - \frac{1}{L^2}}$



$$S = \frac{2\times(Area)}{4G_4 l}$$

Critical value is close to the expected value $S_{crit}\sqrt{l} \approx 3.6$, and it is almost independent of l/L.



Summary

- AdS/CFT correspondence suggests that there is no static large $(\kappa^{-1} \gg \ell)$ brane BH solution in RS-II brane world.
- This correspondence has been tested in various cases.
- Small localized BHs were constructed numerically.
  - The sequence of solutions does not seem to terminate suddenly,
  - but bigger BH solutions are hard to obtain.
- Assuming

1) Classical BH evaporation conjecture is correct,
2) Static small localized BH solutions exist,
3) A sequence of solutions do not disappear suddenly,

we presented a consistent scenario for the phase diagram of black objects in wider class of models including cases with IR brane or detuned tension.

As a result, we predicted new sequences of black objects.

1) floating stable and unstable BHs
2) large BHs localized on AdS brane

Partial support for this picture is obtained from time-symmetric initial data.