Title: NEC Violations in de Sitter Space & Cosmology

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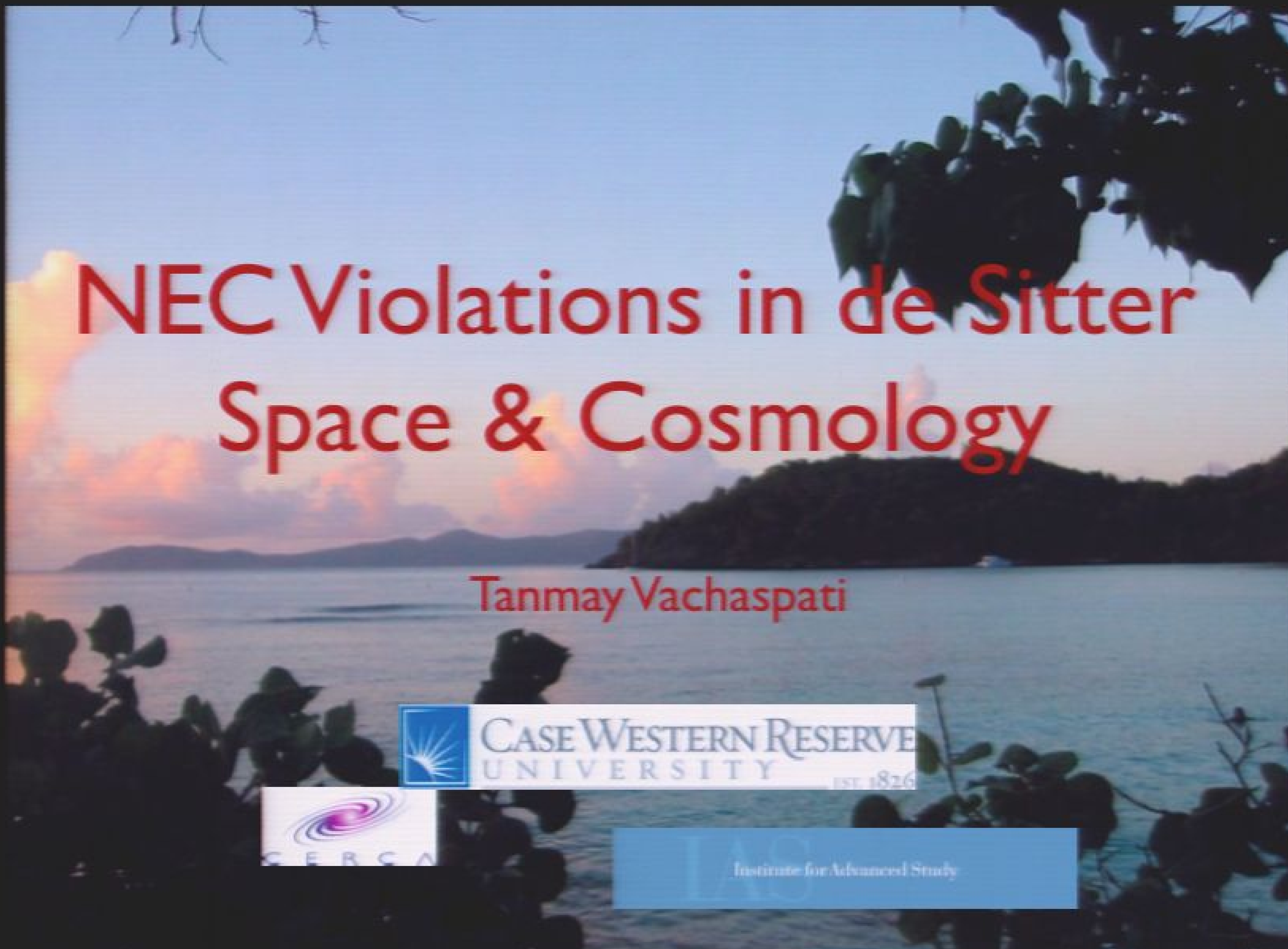
Abstract: TBA


NEC Violations in de Sitter Space & Cosmology
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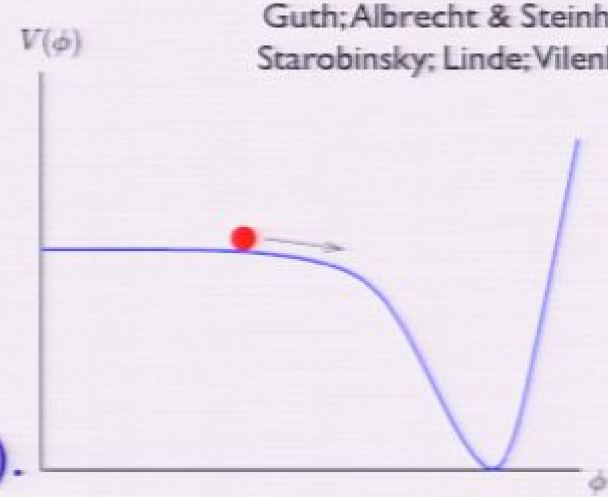
Inflation

Guth; Albrecht & Steinhardt; Starobinsky; Linde; Vilenkin; ...

Scalar field rolls down a flat potential.

Vacuum energy dominates cosmological dynamics.

Exponential expansion (inflation).



Inflation ends when field rolls off the cliff (reheating).

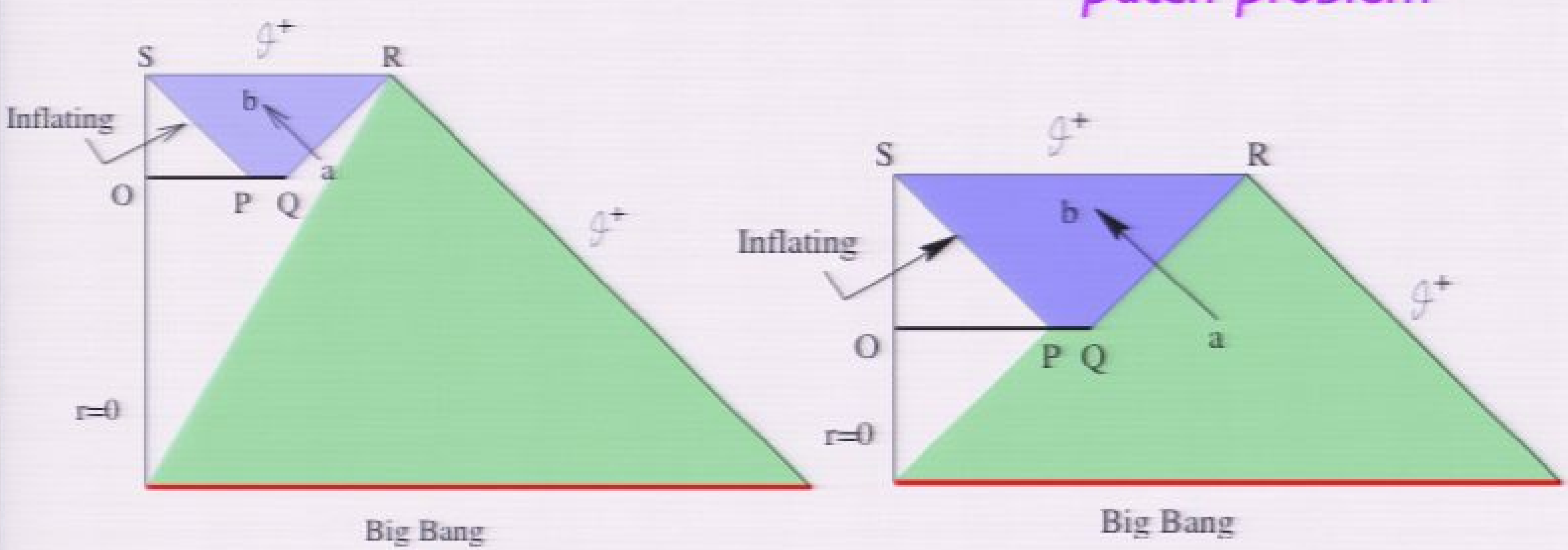
Implicit: the pre-inflationary state of the universe (e.g. "scalar field is at top of potential") is known.
What can we say more generally about the initial state?

Homogeneity

TV & Trodden

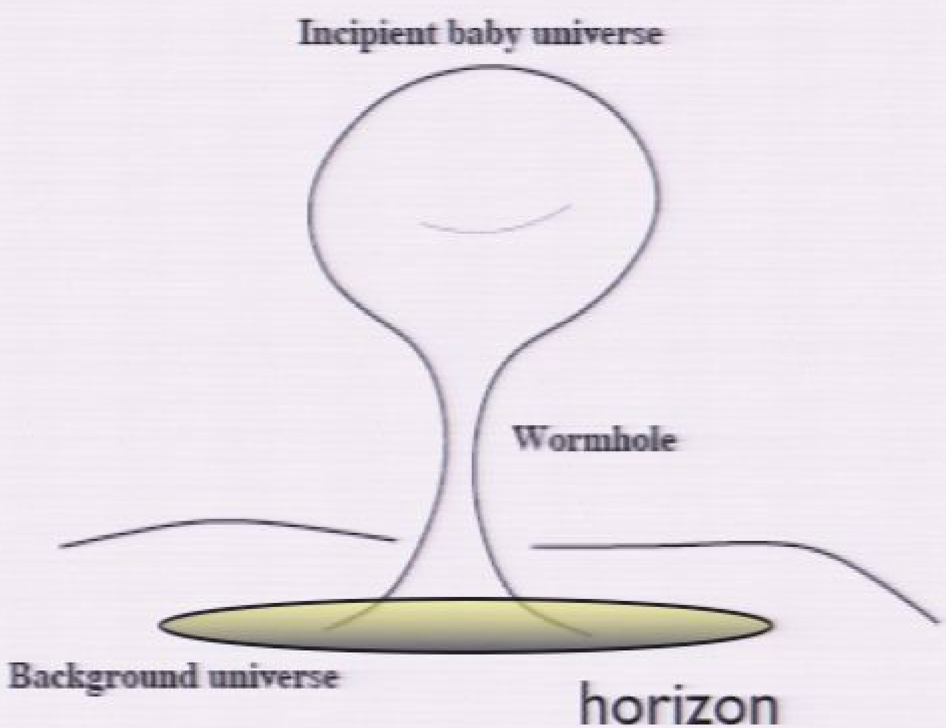
Initial inflationary patch has to be homogeneous on *superhorizon* scales.

"patch problem"



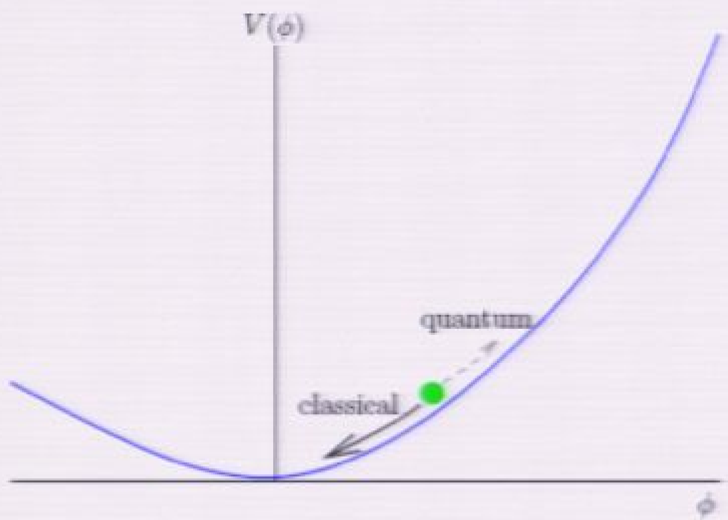
If inflation is to start from a sub-horizon patch, Null Energy Condition (NEC) violation must persist for an *infinite* duration. Or else, we must start with a *superhorizon* patch that is suitable for inflation.

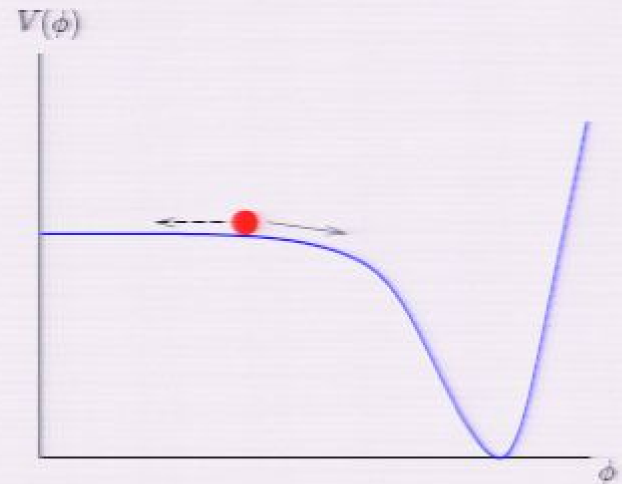
Embedding Picture



Eternal inflation

Steinhardt; Vilenkin; Linde; Starobinsky...

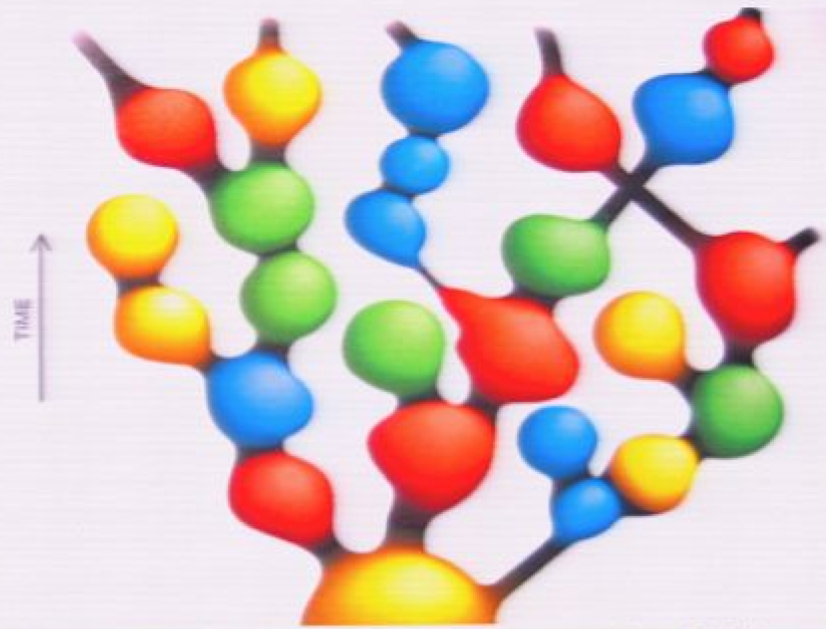




Dynamics: classical roll down, quantum jump up.

"Inflation goes on forever"
and the patch problem is less of a concern.

Multiverse



SELF-REPRODUCING COSMOS appears as an extended branching of inflationary bubbles. Changes in color represent "mutations" in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.

Image: Andrei Linde

NEC in Eternal Inflation

Decrease in Hubble length scale is possible only if the NEC is violated.

$$\dot{H} \propto -(\rho + p)$$

$\text{Therefore } (\rho + \text{p}) < 0 \text{ if } \dot{\text{H}} > 0.$ Borde & Vilenkin

$\text{Or, in covariant form, we need } \text{N}^\mu \text{N}^\nu \text{T}_{\mu\nu} < 0,$
$\text{where } \text{N}^\mu \text{ is a null vector.}$

There are no *classical* violations of NEC for known matter.

Therefore investigate NEC violations in de Sitter space due to quantum effects.

Naive Argument

$g_{\mu\nu}$ is the only available tensor in de Sitter space.

$$\langle 0|\hat{T}^{\rm ren}_{\mu\nu}|0\rangle \propto g_{\mu\nu}$$

$$\langle 0|\hat{T}^{\rm ren}_{\mu\nu}\hat{T}^{\rm ren}_{\lambda\sigma}|0\rangle \propto g_{\mu\nu}g_{\lambda\sigma} + \ldots$$

Then, contractions with null vectors vanish, and there seem to be no NEC violations in de Sitter space.

On the other hand --

$$\begin{aligned} N^\mu N^\nu \hat{T}_{\mu\nu}|0\rangle &= \sum[(\ldots)a_l a_k^\dagger + (\ldots)a_l^\dagger a_k^\dagger]|0\rangle \\ &= \sum[(\ldots)|0\rangle + (\ldots)|2;k,l\rangle] \end{aligned}$$

So the vacuum is not an eigenstate of NNT, and there must be NEC violating fluctuations.

Conclusions

- NEC violations on super-horizon scales are necessary in current cosmological models.
- NEC is violated by 50% of quantum fluctuations in de Sitter space.
- Island Cosmology.

* Singularity free, eternal/semi-eternal, time symmetric.
* Works with any field content.
* Scale invariant fluctuations for some degrees of freedom.
* Open issues (e.g. backreaction, measure) closely parallel those in inflationary cosmology.

Perturbation spectrum in sudden approximation

de Sitter: $\frac{a''}{a} = \frac{2}{\eta^2}$ $\qquad v_k = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right), \; \eta < \eta_f$

Radiation FRW:

$a'' = 0, \; \eta > \eta_f \qquad v_k = \alpha_k e^{-ik(\eta-\eta_f)} + \beta_k e^{+ik(\eta-\eta_f)}$

Technical segue:

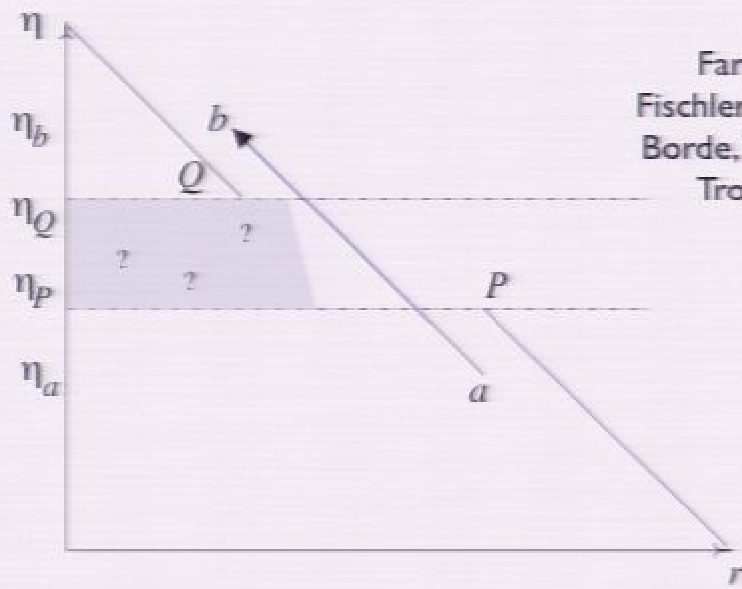
Write $Q = -\partial + f, \;\; Q^+ = \partial + f, \;\; f = a'_{\mathrm{dS}}/a_{\mathrm{dS}}$

$$H_{\mathrm{dS}} = -\partial^2 + \frac{a''}{a} = Q^+Q \;\;\underset{\text{partners}}{\longleftrightarrow}\;\; H_{\mathrm{M}} = -\partial^2 = QQ^+$$

$$v_M(k) = \frac{1}{\sqrt{2k}}e^{-ik\eta} \;\;\longleftrightarrow\;\; v_{\mathrm{dS}}(k) = Q^+ v_M(k) = -ik\frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)$$

"de Sitter-Minkowski duality"

Extent and Duration for Cosmology



Context
Farhi and Guth;
Farhi, Guth and Guven;
Fischler, Morgan and Polchinski;
Borde, Trodden and Vachaspati;
Trodden and Vachaspati

$R > H_i^{-1}$ from minimum size of island.

$T \to 0$ from largest fluctuation amplitude.

"sudden approximation"

Quantum Operators and NEC

Guth, Vachaspati & Winitzki
Winitzki, gr-qc/0111109
TV, astro-ph/0305439

Quantum operators are *distributions* that act on test functions.
So first smear the operator with a test/window function.

Streater & Wightman

Space-time smeared NEC operator:

$$\hat{O}_W^{\rm ren} \equiv \int d^4x \sqrt{-g}\, W(x; R, T) N^\mu N^\nu \hat{T}_{\mu\nu}^{\rm ren}$$

$$O_{\rm av} = \langle 0 | \hat{O}_W^{\rm ren} | 0 \rangle = 0$$

$$O_{\rm rms}^2 \equiv \langle 0 | (\hat{O}_W^{\rm ren})^2 | 0 \rangle = ?$$

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Evaluation of NEC violation

$$O^2_{\rm rms} = \langle 0| \int d^4x \sqrt{-g}\, W(x;R,T) N^\mu N^\nu \hat{T}^{\rm ren}_{\mu\nu}(x) \times \int d^4y \sqrt{-g}\, W(y;R,T) N^\lambda N^\sigma \hat{T}_{\lambda\sigma}(y)|0\rangle$$

In general, this is complicated to evaluate and leads to opaque expressions. However, on dimensional grounds:

$$O^2_{\rm rms} \sim H^8_\Lambda \qquad \text{if } R \sim H^{-1}_\Lambda,\ T \sim H^{-1}_\Lambda$$

This implies that NEC is violated by fluctuations on super-horizon scales, on super-Hubble time scales.

Note: de Sitter symmetry broken by choice of smearing function.

NEC Violation During Slow Roll

$$R = T = (\epsilon H)^{-1}$$

$$W(\eta, \mathbf{r}) = \frac{1}{\sqrt{-g}} \frac{a_0^4}{R^3 \tau} W_\eta \left(\frac{|\eta - \eta_0|}{a_0^{-1} \tau} \right) W_r \left(\frac{|\mathbf{r} - \mathbf{r}_0|}{a_0^{-1} R} \right)$$

$$O_{\text{rms}}^2 \sim \frac{H^4 \dot{\phi}_0^2 \max\left(c_1^2 \varepsilon^2, c_1'^2 \varepsilon^4\right)}{(2\pi)^2} + \frac{c_2^2 H^8 \varepsilon^8}{(2\pi)^4}$$

where c_1 and c_2 are numerical values of certain integrals.

$$\frac{O_{\text{rms}}^2}{O_{\text{av}}^2} \sim \frac{H^4 \max\left(c_1^2 \varepsilon^2, c_1'^2 \varepsilon^4\right)}{\left(2\pi \dot{\phi}_0\right)^2} + \frac{c_2^2 H^8 \varepsilon^8}{\left(2\pi \dot{\phi}_0\right)^4}.$$

Limits

NEC violation *diverges* as the duration (T) or the spatial extent (R) are taken to zero.

(Reflection of Heisenberg uncertainty.)

NEC violation *vanishes* as the spatial extent is taken to infinity.

(Total Hamiltonian is conserved and hence no fluctuations in the infinite volume. In particular, any fluctuation inside a finite volume must be compensated outside.)

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What have we learnt?

- NEC violations on super-horizon scales are present due to quantum field fluctuations in de Sitter space.
- The fluctuations are large on very short time and length scales.
- Assuming symmetric distributions, NEC is violated by a fluctuation with 50% probability.

What we still don't understand.

- Choice and role of window function.
- Backreaction of NEC violation on spacetime.
- Probability measure on fluctuations.

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Cosmology

We are entering a de Sitter epoch.

The future is an empty de Sitter universe...

Cosmology

We are entering a de Sitter epoch.

The future is an empty de Sitter universe...

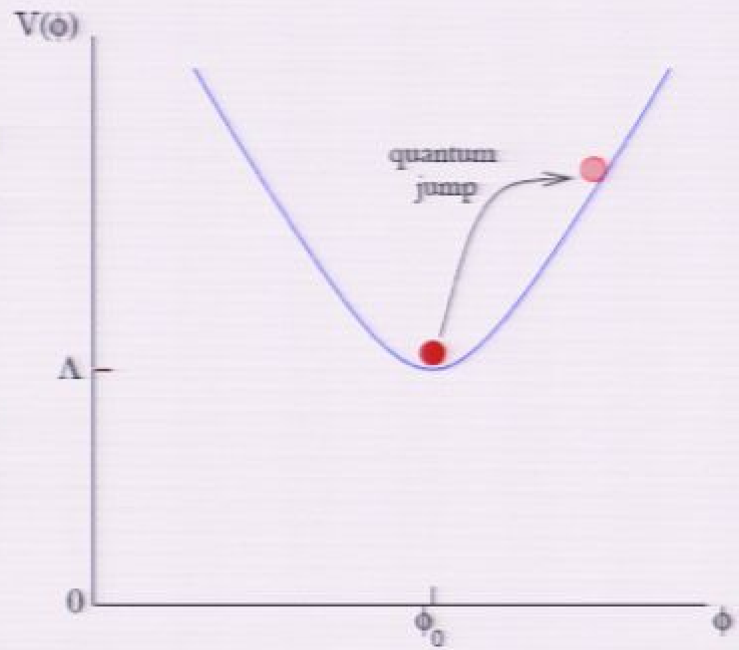
... except for NEC violating fluctuations on superhorizon scales that produce faster expanding FRW universes.

In terms of scalar fields...

Start with empty de Sitter.

Large NEC violating quantum fluctuation up the hill dumps energy locally.

Matter excitations produced while rolling down give FRW cosmology.



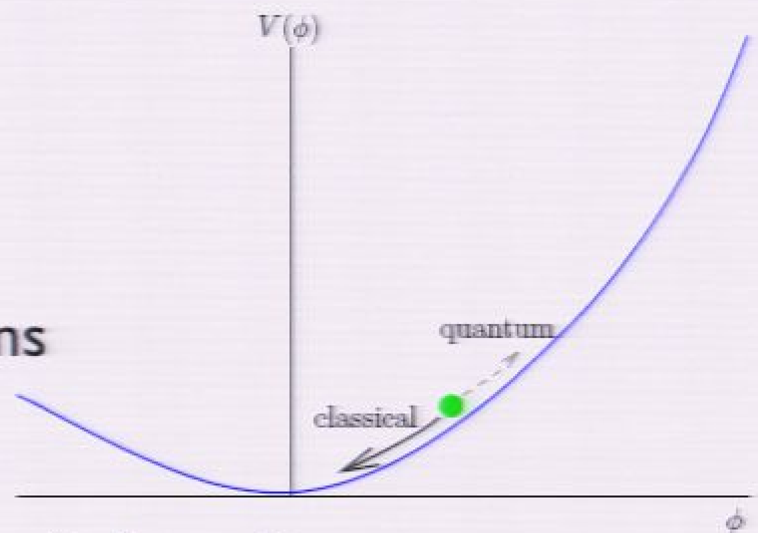
Eventually FRW goes back to empty de Sitter.

Backreaction

Semiclassical gravity:

$$G_{\mu\nu} = 8\pi G \langle \hat{T}^{\text{ren}}_{\mu\nu} \rangle$$

... blind to rms NEC violations in de Sitter space.



Need to go beyond semiclassical gravity during fluctuation.

FRW equations can be assumed before and after the fluctuation.

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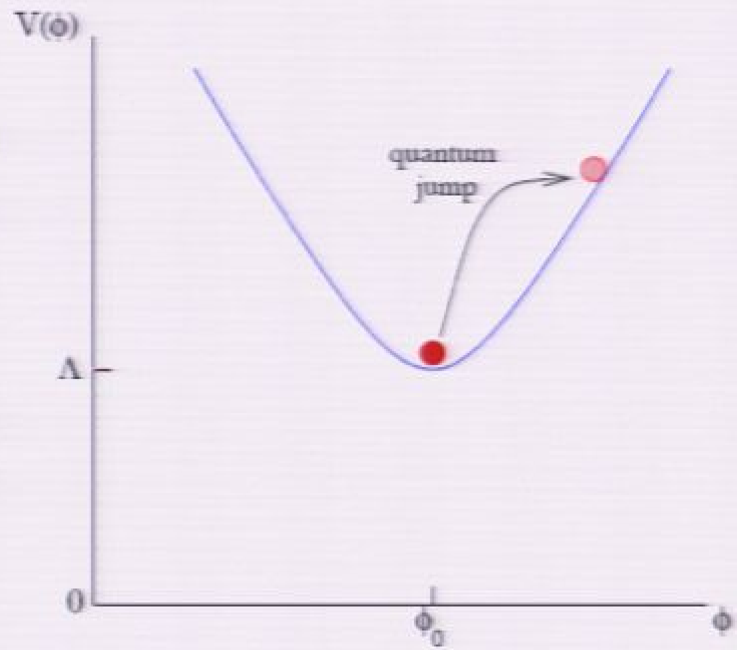

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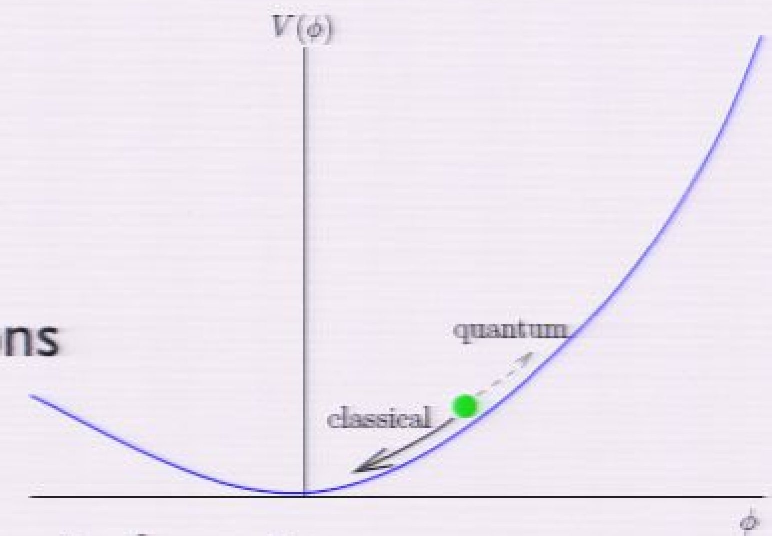

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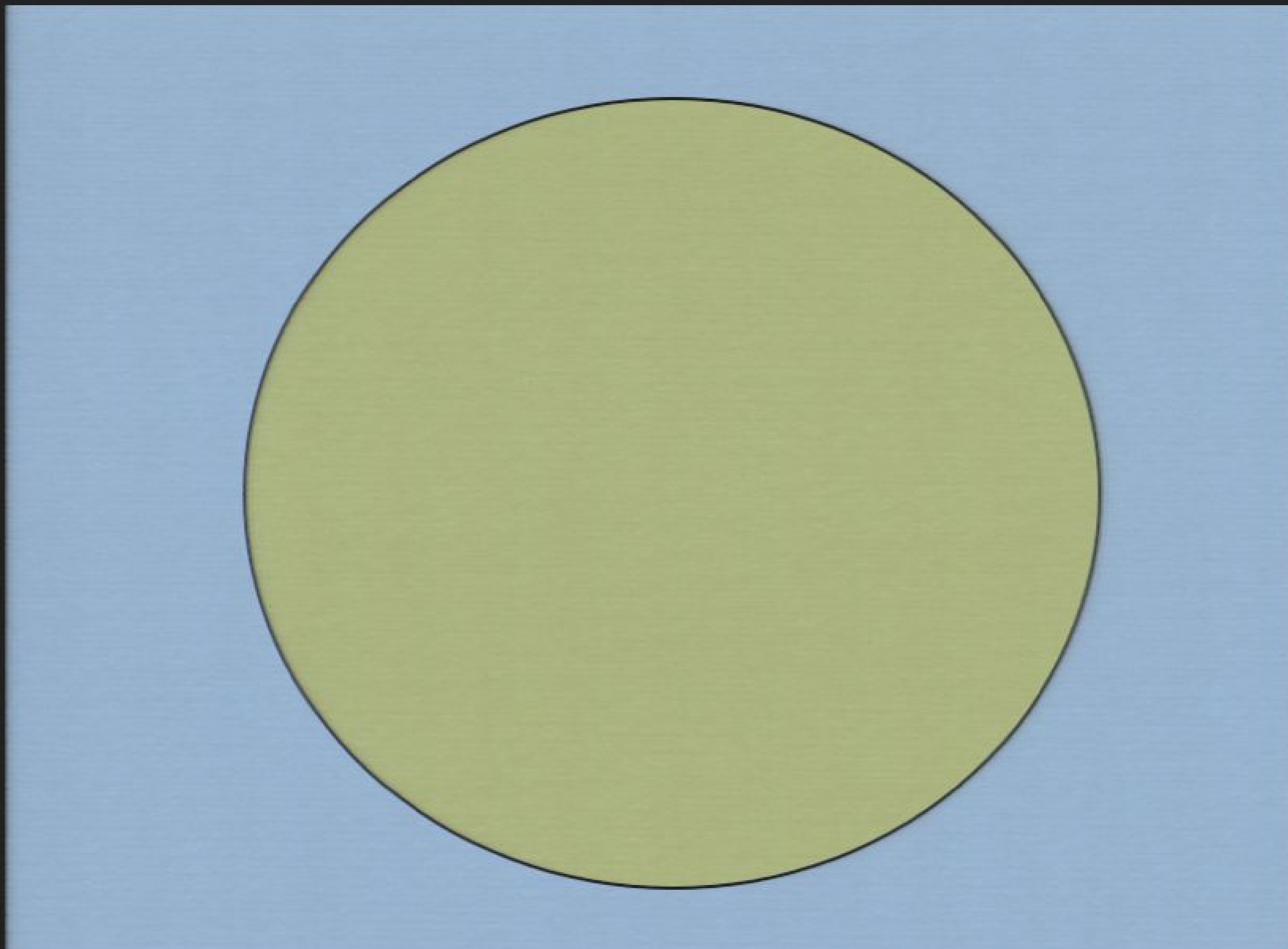
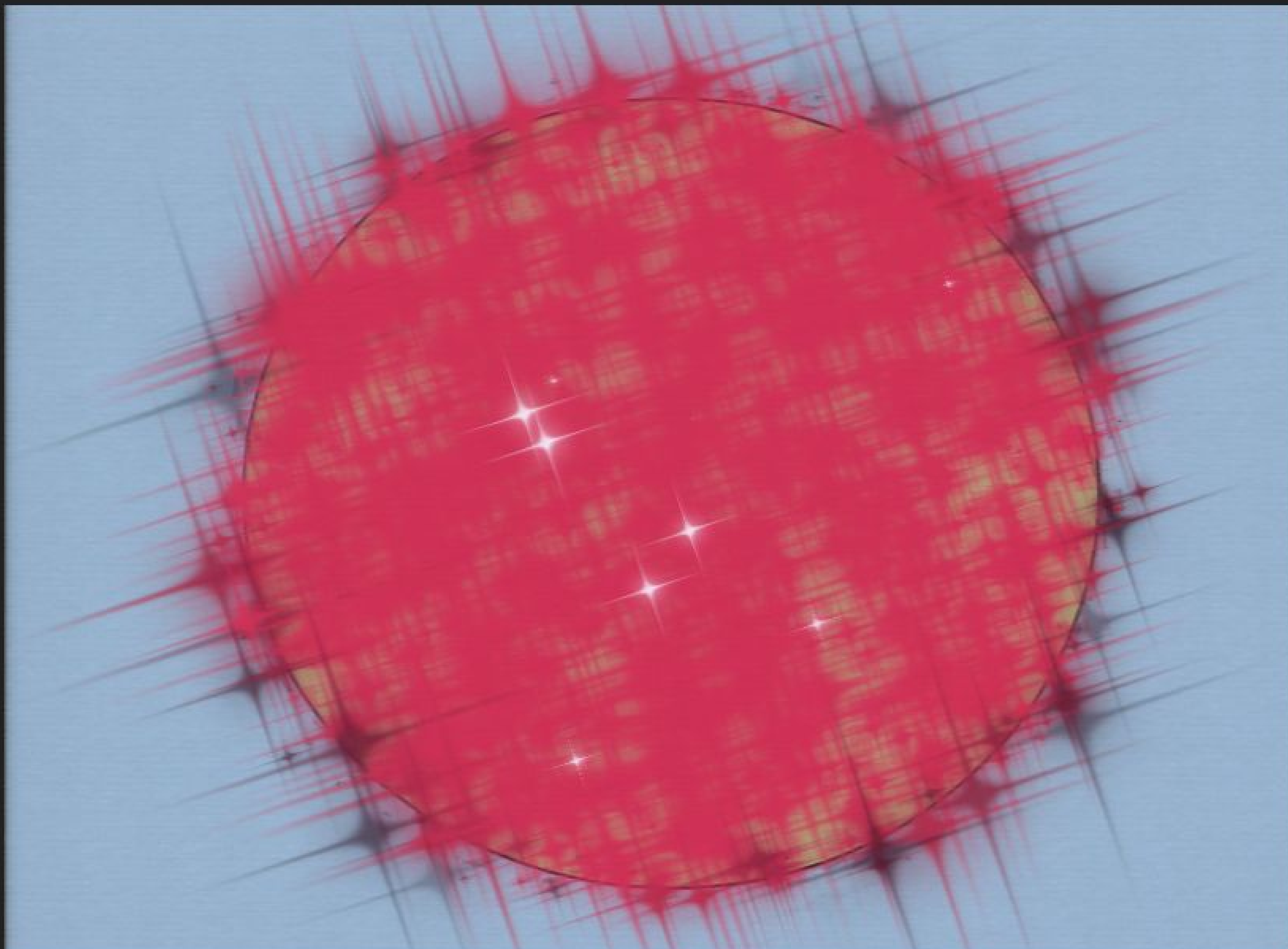
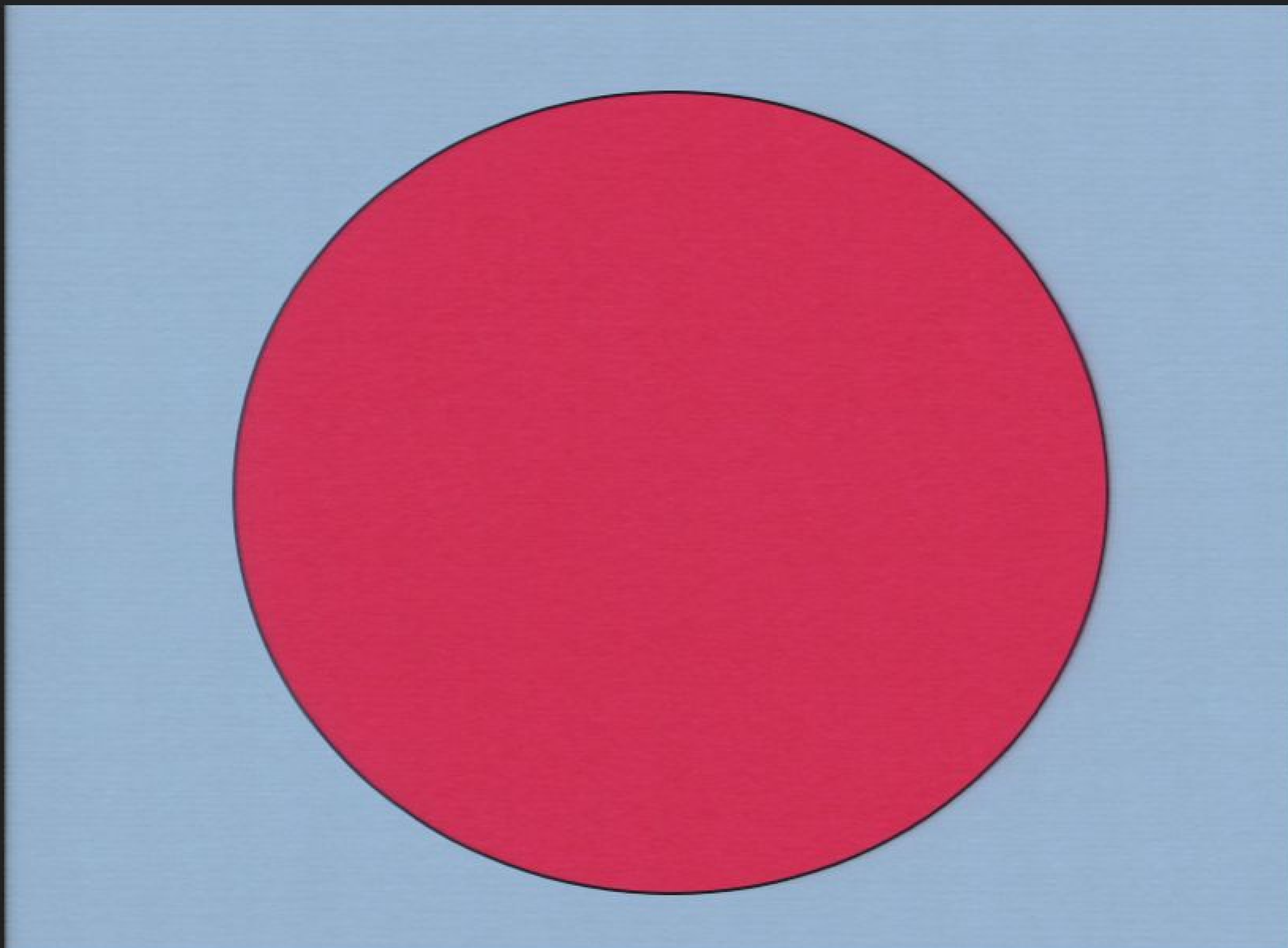
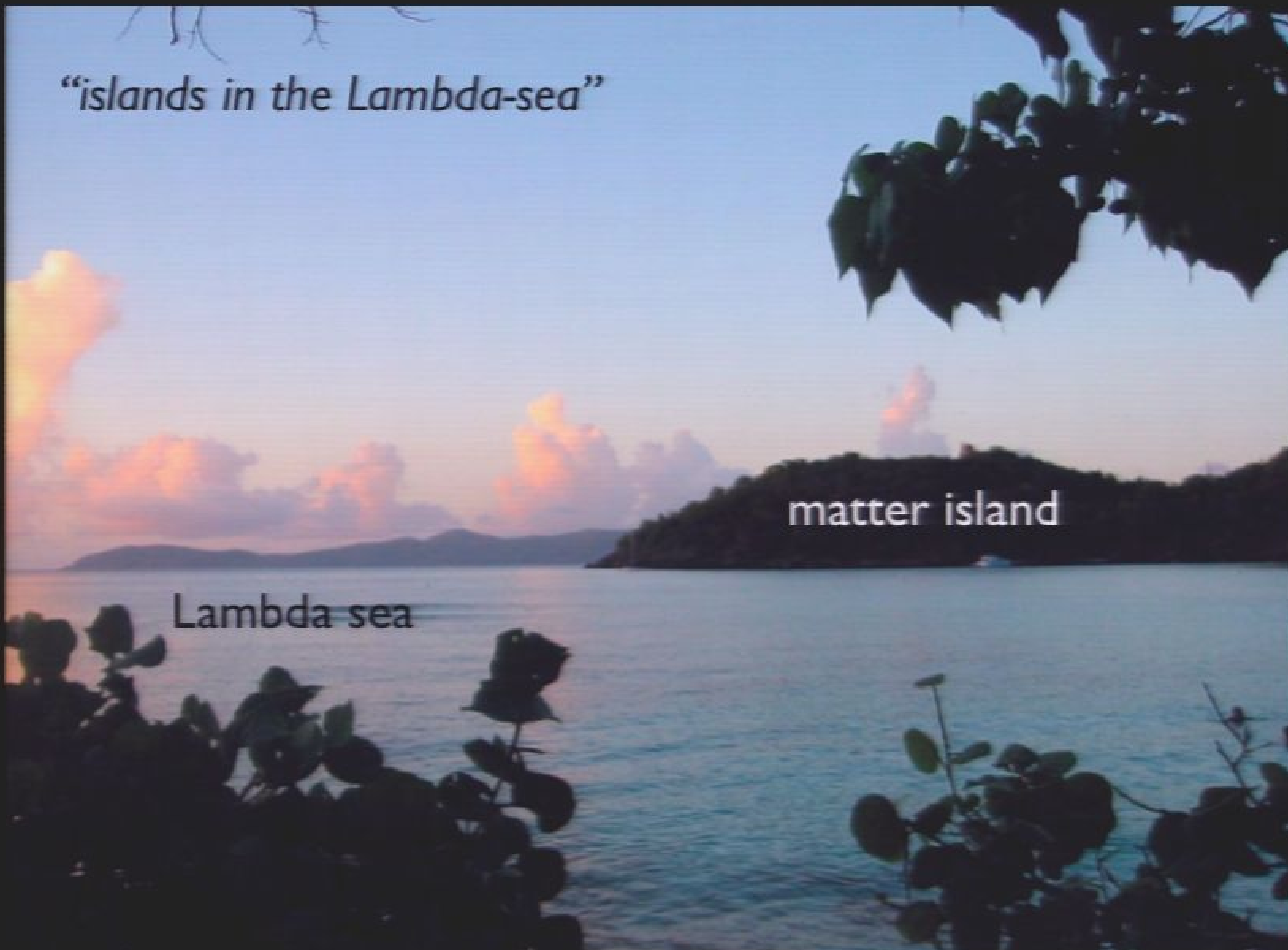
FRW equations can be assumed before and after the fluctuation.

A movie of NEC violation in de Sitter space.



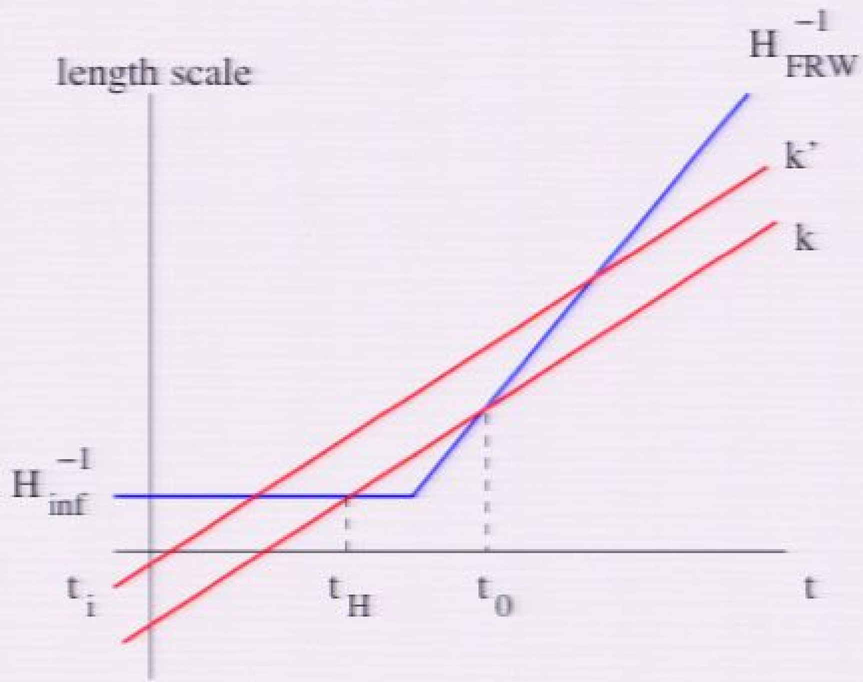





"islands in the Lambda-sea"
matter island
Lambda sea

What is the spectrum of density fluctuations on an island?

Recall: Inflationary Cosmology

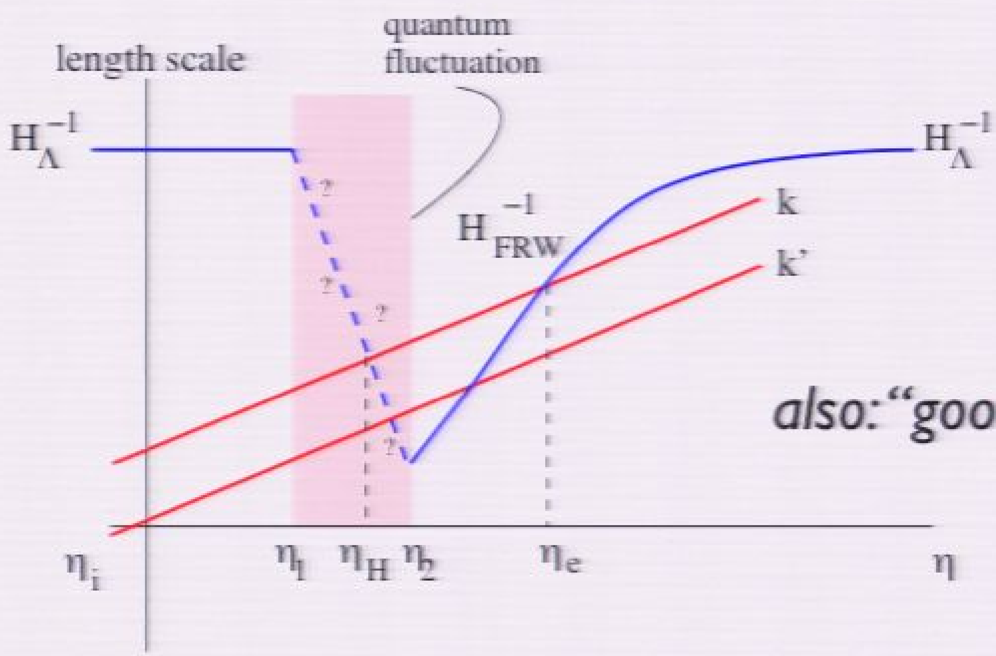


"good-bye, hello again"

Kolb & Turner

Island Cosmology

Dutta & TV

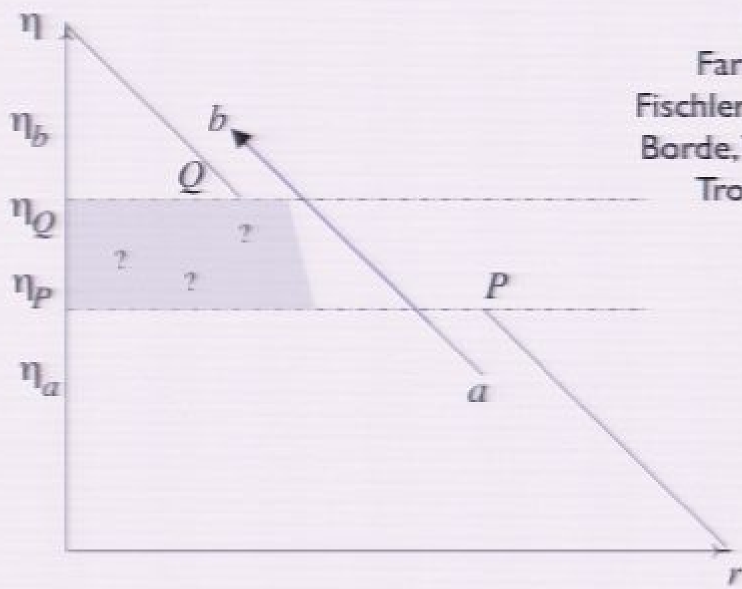


also: "good-bye, hello again"

shares features with

Steady State: Bondi & Gold; Hoyle. *Recycling universe:* Garriga & Vilenkin.
Entropy: Dyson, Kleban & Susskind; Albrecht & Sorbo; Albrecht.
Ekpyrotic: Khoury, Ovrut, Steinhardt and Turok. *String Gas Cosmology:* Brandenberger et al.
Time symmetry: Carroll & Chen.

Extent and Duration for Cosmology



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$R > H_i^{-1}$ from minimum size of island.

$T \to 0$ from largest fluctuation amplitude.

"sudden approximation"

Backreaction: working hypothesis

$$\rho = \Lambda\,, \quad w = -1\,, \quad \eta < \eta_f$$

$$\rho = \rho_{\rm FRW}\,, \quad w = +\frac{1}{3}\,, \quad \eta > \eta_f$$

$$H^2 = \frac{8\pi G}{3}\rho$$ as in eternal inflation.

Note: density needs to be positive even when NEC is violated.

No spatial curvature due to early de Sitter phase, and also, (i) fluctuations are not de Sitter invariant, and (ii) there is continual expansion in FRW equation.

NEC violation today: sources?

Scalar

$$\hat{\rho} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

$$\hat{p} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla\phi)^2 - V(\phi)$$

$$\hat{\rho} + \hat{p} = \dot{\phi}^2 + \frac{(\nabla\phi)^2}{3}$$

Photon

$$\hat{\rho} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$$

$$\hat{p} = \frac{1}{6}(\mathbf{E}^2 + \mathbf{B}^2) = \frac{1}{3}\hat{\rho}$$

$$\hat{\rho} + \hat{p} = \frac{4}{3}\hat{\rho}$$

Quantum fluctuations of any field can give NEC violation.

e.g. Higgs, W, Z, etc.

Scalar field not essential.

Perturbation spectrum

Sudden approximation $R \sim H_\Lambda^{-1},\ T \to 0$

Consider a scalar field that *only* couples to the geometry.

$$\chi(x) = \sum_k \chi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

Mukhanov variable $v_k(\eta) \equiv a(\eta)\chi_k(\eta)$

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\mathcal{P}_\chi(k,\eta) = \frac{k^3}{2\pi^2}\left|\frac{v_k}{a}\right|^2$$

Perturbation spectrum in sudden approximation

de Sitter: $\frac{a''}{a} = \frac{2}{\eta^2}$ $\qquad v_k = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right), \ \eta < \eta_f$

Radiation FRW:

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Technical segue: Write $Q = -\partial + f, \ Q^+ = \partial + f, \ f = a'_{\rm dS}/a_{\rm dS}$

$$H_{\rm dS} = -\partial^2 + \frac{a''}{a} = Q^+Q \ \underset{\text{partners}}{\longleftrightarrow} \ H_{\rm M} = -\partial^2 = QQ^+$$

$$v_M(k) = \frac{1}{\sqrt{2k}}e^{-ik\eta} \longleftrightarrow v_{\rm dS}(k) = Q^+ v_M(k) = -ik\frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)$$

"de Sitter-Minkowski duality"

Matching across fluctuation

<u>Note:</u> scale factor is continuous at jump but derivative is not i.e. second derivative has a delta function contribution.

Junction conditions: $v_k(\eta_f+) = v_k(\eta_f-)$

$$v'_k(\eta_f+) = v'_k(\eta_f-) + a_f(H_f - H_\Lambda)v_k(\eta_f)$$

where the last term is due to the delta function contribution.

$$\alpha_k \approx +\frac{1}{2\sqrt{2k}}\frac{1}{(k\eta_f)^2}\frac{H_f}{H_\Lambda}, \qquad \beta_k \approx -\frac{1}{2\sqrt{2k}}\frac{1}{(k\eta_f)^2}\frac{H_f}{H_\Lambda}$$

$$v_k(\eta) \approx \frac{-i}{\sqrt{2k}}\frac{1}{(k\eta_f)^2}\frac{H_f}{H_\Lambda}\sin(k\eta)$$

Power spectrum

$$\mathcal{P}_\chi(k,\eta) = \frac{k^3}{2\pi^2}\left|\frac{v_k}{a}\right|^2 \approx \frac{H_\Lambda^2}{4\pi^2}$$

Scale invariant; small amplitude.

This is for a field that only experiences the background spacetime (e.g. gravitational waves) and does not directly interact with the NEC violating field.

Density fluctuations

Density fluctuations due to NEC violating scalar field.

Assume a stage of *classical* phantom cosmology. Y. Piao S. Dutta

$$L = \lambda(\partial_\mu \phi)^2 - V(\phi)\ , \quad \lambda = \pm 1$$

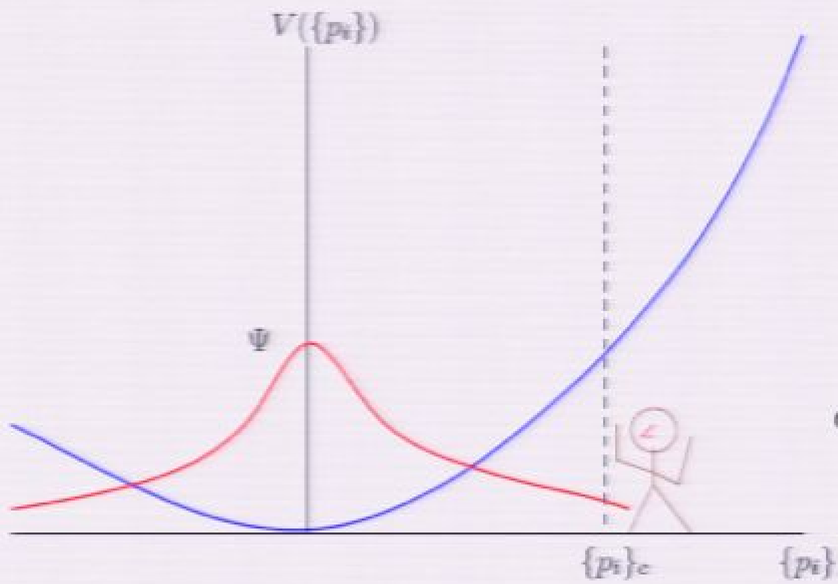
Result: Density fluctuations are scale invariant with amplitude given by the Hubble scale at the end of the NEC violating fluctuation.

$$\mathcal{P}_{\rm density}(k,\eta_k) \approx \frac{H_f^2}{4\pi^2}, \quad \mathcal{P}_{\rm grav}(k,\eta_k) \approx \frac{H_\Lambda^2}{4\pi^2}$$

Density fluctuations: quantum calculation?

Similar to fluctuations produced during tunneling. TV & Vilenkin

"Vacuum bubbles need not be round". Garriga & Vilenkin



$$\phi = \phi_{\rm NEC} + \delta\hat{\phi}$$

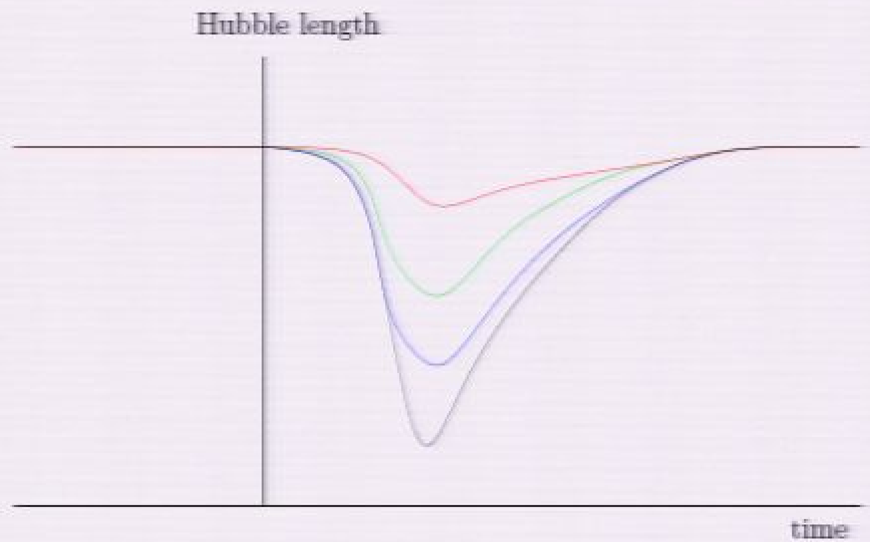
Hawking-Moss Process

Jump up to the maximum of a potential must be accompanied by some density fluctuations too.

What is the spectrum and amplitude of these fluctuations?

Batra & Kleban
(eternal+HM)

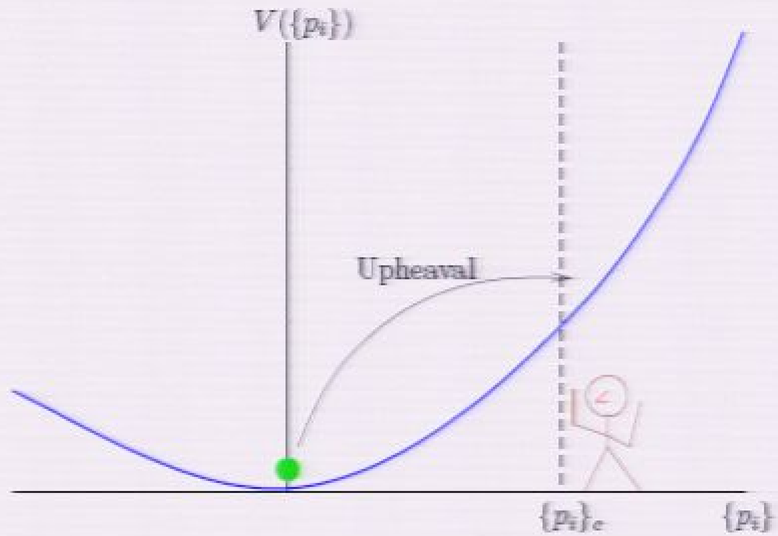
Amplitude of NEC Violating Fluctuation



Small fluctuations are more frequent than large fluctuations, but they shouldn't count...

Habitable islands

Assumption: a "critical" universe.

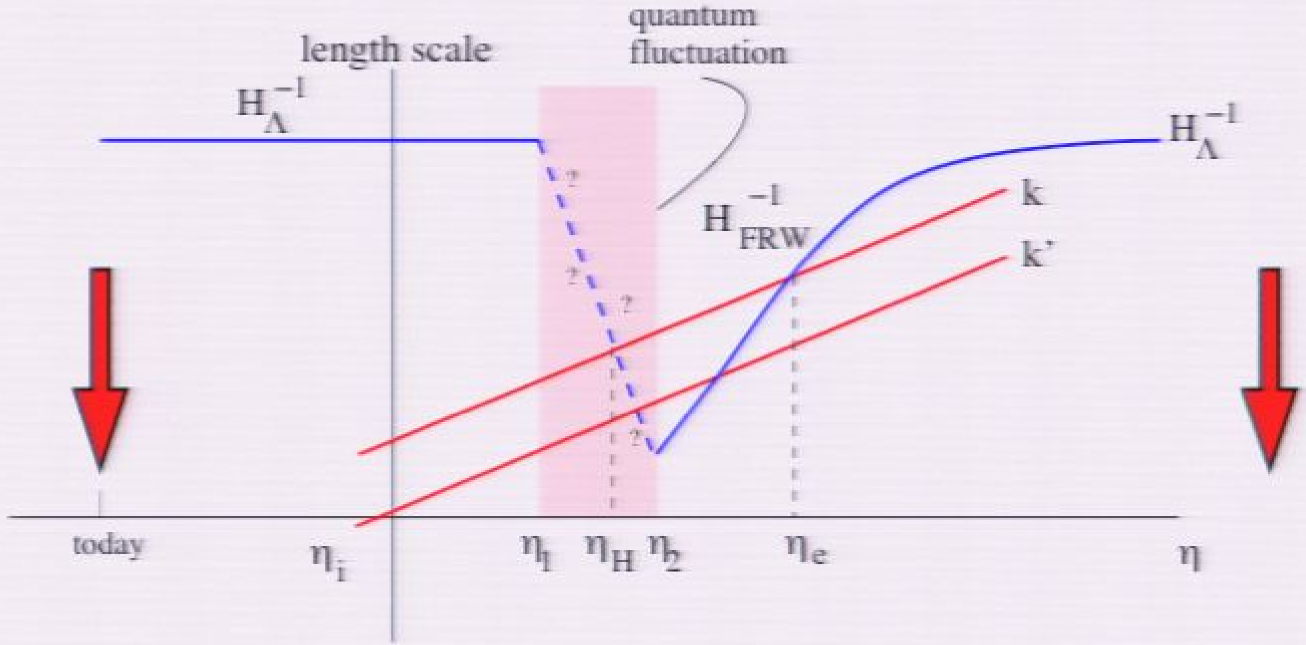


Critical parameter = baryon number?

Is baryogenesis critical?

The future

not with a crunch, nor with a whimper



the past is the future

Conclusions

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- NEC is violated by 50% of quantum fluctuations in de Sitter space.
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* Singularity free, eternal/semi-eternal, time symmetric.
* Works with any field content.
* Scale invariant fluctuations for some degrees of freedom.
* Open issues (e.g. backreaction, measure) closely parallel those in inflationary cosmology.

Perturbation spectrum in sudden approximation

de Sitter: $\frac{a''}{a} = \frac{2}{\eta^2}$ $\quad v_k = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right), \; \eta < \eta_f$

Radiation FRW:

$a'' = 0, \; \eta > \eta_f \quad v_k = \underline{\alpha_k} e^{-ik(\eta-\eta_f)} + \underline{\beta_k} e^{+ik(\eta-\eta_f)}$

Technical segue:

Write $Q = -\partial + f, \; Q^+ = \partial + f, \; f = a'_{\rm dS}/a_{\rm dS}$

$$H_{\rm dS} = -\partial^2 + \frac{a''}{a} = Q^+Q \;\underset{\text{partners}}{\longleftrightarrow}\; H_{\rm M} = -\partial^2 = QQ^+$$

$$v_M(k) = \frac{1}{\sqrt{2k}} e^{-ik\eta} \longleftrightarrow v_{\rm dS}(k) = Q^+ v_M(k) = -ik\frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)$$

"de Sitter-Minkowski duality"