

Title: The effective theory of quintessence: the $w < -1$ side unveiled

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URL: <http://pirsa.org/09070027>

Abstract: We study generic single-field dark energy models, by a parametrization of the most general theory of their perturbations around a given background, including higher derivative terms. In appropriate limits this approach reproduces standard quintessence, k-essence and ghost condensation. We find no general pathology associated to an equation of state $w_Q < -1$ or in crossing the phantom divide $w_Q = -1$. Stability requires that the $w_Q < -1$ side of dark energy behaves, on cosmological scales, as a k-essence fluid with a virtually zero speed of sound. This implies that one should set the speed of sound to zero when comparing with data models with $w_Q < -1$ or crossing the phantom divide. We summarize the theoretical and stability constraints on the quintessential plane $(1+w_Q)$ vs. speed of sound squared

Paolo Creminelli (ICTP, Trieste)

The effective theory of quintessence: the $w < -1$ side unveiled

arXiv:0811.0827 [astro-ph]

with G. D'Amico, J. Noreña and F. Vernizzi



hep-th/0606090

with M. Luty, A. Nicolis and L. Senatore

arXiv:0709.0293

with C. Cheung, L. Fitzpatrick, J. Kaplan and L. Senatore

The Universe accelerates

In 1998 the Universe started accelerating...

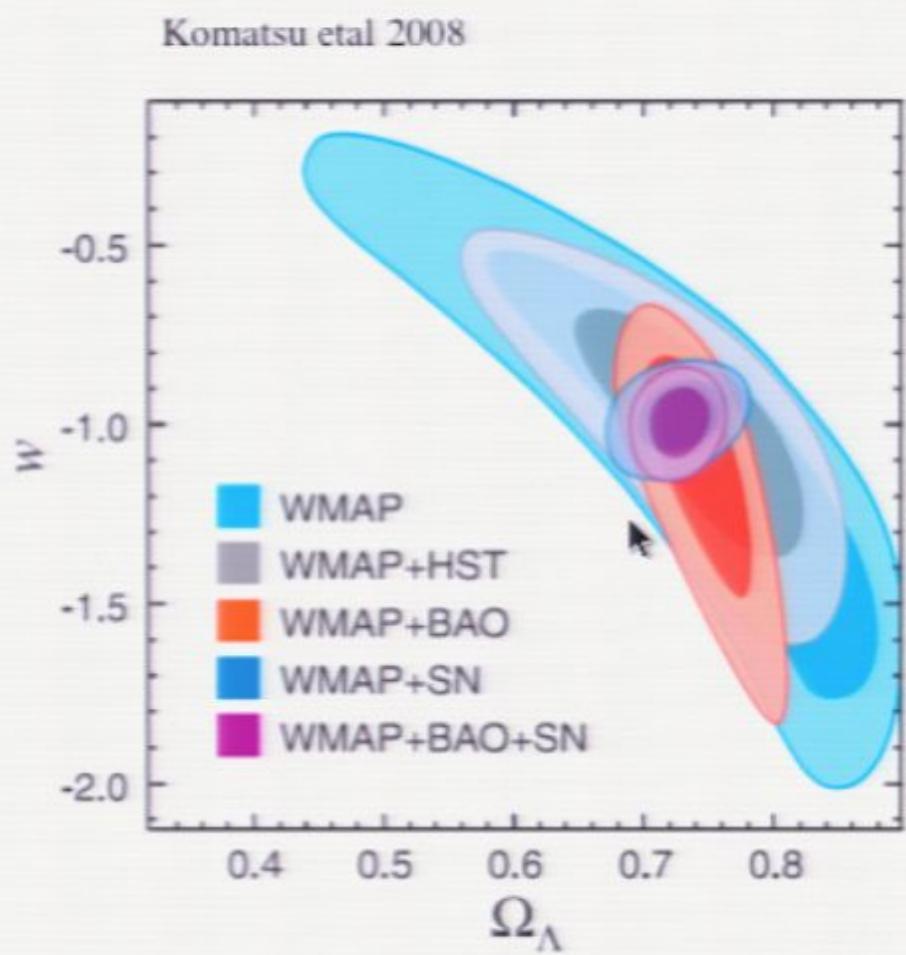
Compelling evidence from supernovae
+ other observations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad w \equiv p/\rho$$

- Data are converging towards $w \approx -1$
- Λ is the simplest explanation: $w = -1$
- **Quintessence**
(here a general single field dark energy)

$$w_Q(z) \neq -1$$

Not spatially homogeneous



Outline

- Study the most general theory of single field quintessence
- Quintessence **perturbations** with a given background evolution $w_Q(z)$
- $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$ gives $w_Q > -1$. Is there life for $w_Q < -1$?
- Can we cross $w_Q = -1$ (**phantom divide**)?
- Any relation with the **speed of sound c_s** ?
- Phenomenology of $c_s=0$
- Theoretical constraints on the **quintessential plane**: $(1+w_Q) \Omega_Q$ vs c_s^2



Building up the action

K-essence: $S = \int d^4x \sqrt{-g} P(\phi, X) , \quad X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

Let us expand around: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad \phi = \phi_0(t)$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \quad \rho_Q = 2X_0 P_X - P_0 , \quad p_Q = P_0$$

Action for perturbations, making explicit the **background dependence**

Convenient parametrization:

$$\phi(t, \vec{x}) = \phi_0(t + \pi(t, \vec{x}))$$

$$\phi(t, \vec{x}) = \phi_0 + \dot{\phi}_0 \pi + \frac{1}{2} \ddot{\phi}_0 \pi^2 + \dots$$

$$X(t, \vec{x}) = X_0 + \dot{X}_0 \pi + \frac{1}{2} \ddot{X}_0 \pi^2 + 2X_0 \dot{\pi} + 2\dot{X}_0 \pi \dot{\pi} + X_0 \left(\dot{\pi}^2 - \frac{(\nabla \pi)^2}{a^2} \right) + \dots$$

$$S = \int d^4x a^3 \left[P_0 + \dot{P}_0 \pi + \frac{1}{2} \ddot{P}_0 \pi^2 + 2P_X X_0 \dot{\pi} + 2(P_X X_0) \dot{\pi} \dot{\pi} + P_X X_0 \left(\dot{\pi}^2 - \frac{(\nabla \pi)^2}{a^2} \right) + 2P_{XX} X_0^2 \dot{\pi}^2 \right]$$

The action for perturbations

... integrating by parts + using background EOM

Metric perturbations in synchronous gauge: $ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$

$$S = \int d^4x a^3 \left[\frac{1}{2} (\rho_Q + p_Q + 4M^4) \dot{\pi}^2 - \frac{1}{2} (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} + \frac{3}{2} \dot{H}(\rho_Q + p_Q) \pi^2 - \frac{1}{2} (\rho_Q + p_Q) \dot{h} \pi \right]$$

$$M^4 \equiv P_{XX} X_0^2$$

$(\rho_Q + p_Q)(t)$ and $M^4(t)$ are completely unconstrained

Perturbations cannot be switched off if $\rho_Q + p_Q \neq 0$

One can always find $P(\phi, X)$:

$$P(\phi, X) = \frac{1}{2}(p_Q - \rho_Q)(\phi) + \frac{1}{2}(\rho_Q + p_Q)(\phi)X + \frac{1}{2}M^4(\phi)(X - 1)^2$$

$\phi=t$ and the correct $\rho_Q(t)$ and $p_Q(t)$

No field redefinition ambiguities: $\phi \rightarrow \tilde{\phi}(\phi)$



No ghost!



We require a positive definite time kinetic term

$$\frac{1}{2}(\rho_Q + p_Q + 4M^4)\dot{\pi}^2 > 0$$

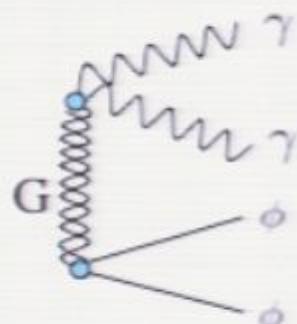
E.g. a minimal ghost field: $\mathcal{L} = +\frac{1}{2}(\partial\phi)^2 + V(\phi)$ w_Q<-1 !!

- **Classically**. Hamiltonian not bounded. Possibility of exchanging energy between positive and negative energy sectors.

No pathology until linear theory remains valid.

- **Quantum mechanically**. Vacuum is unstable.

Decay rate is infinite in any Lorentz invariant theory.



$$\Gamma \sim \frac{\Lambda^8}{M_P^4}$$

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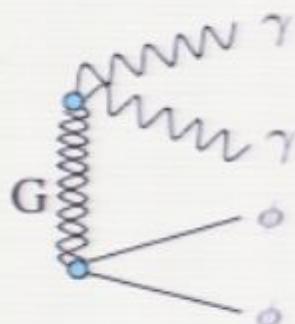
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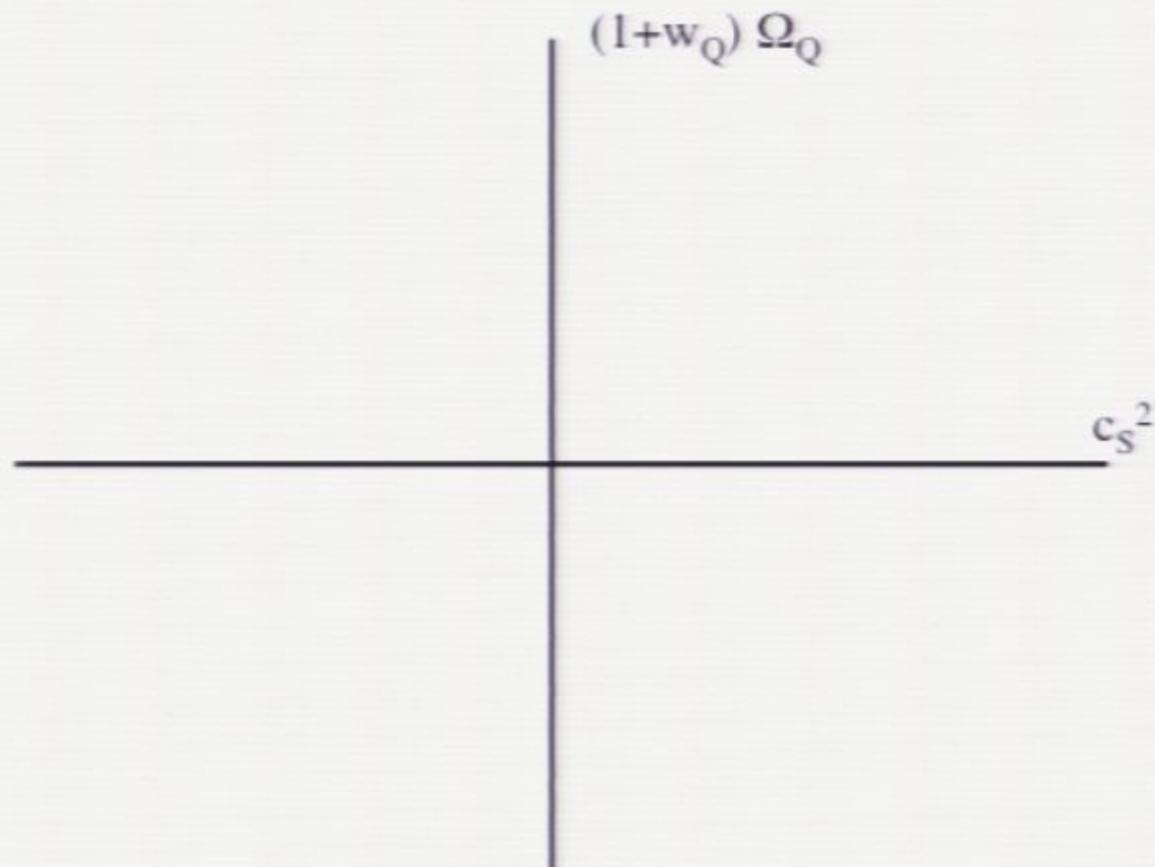


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Quintessential plane



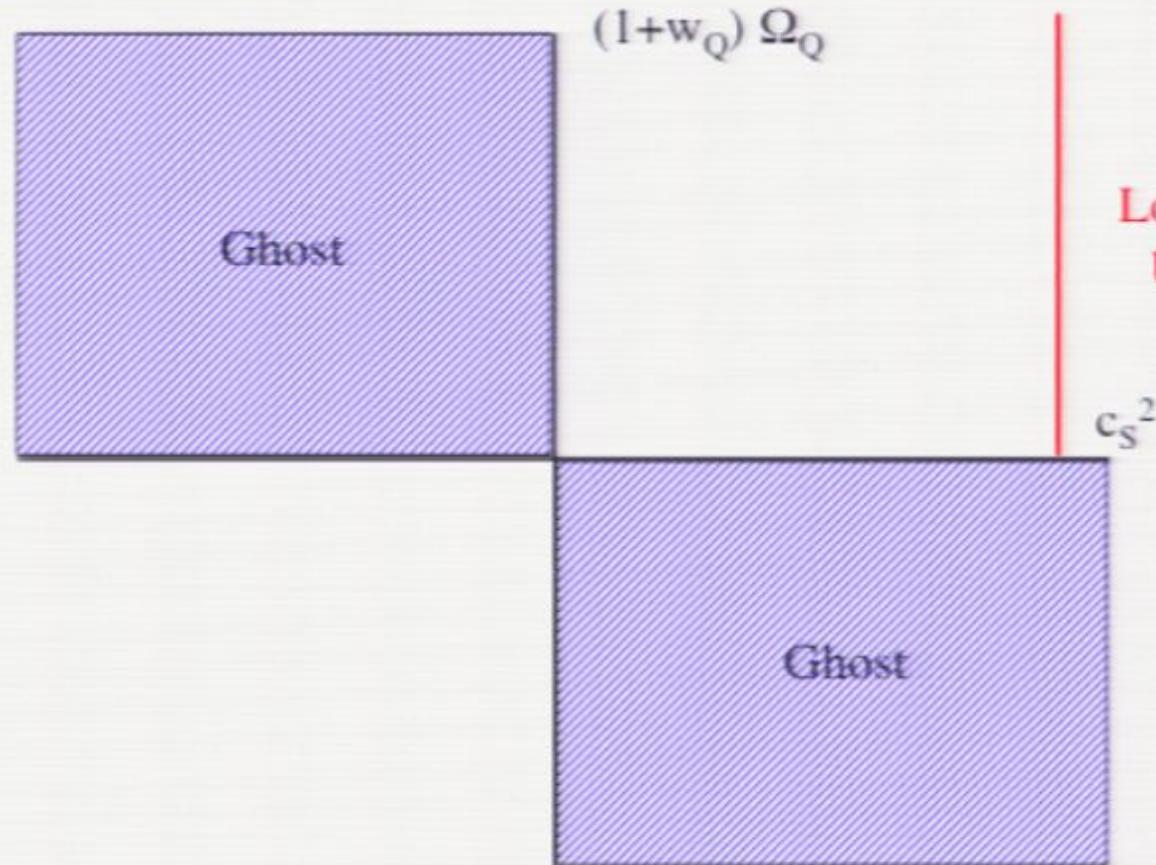
Let us study the different theoretical constraints on quintessence

No ghost and c_s^2



$$\frac{1}{2} (\rho_Q + p_Q + 4M^4) \dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} \quad \longrightarrow \quad c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

Faster than light?

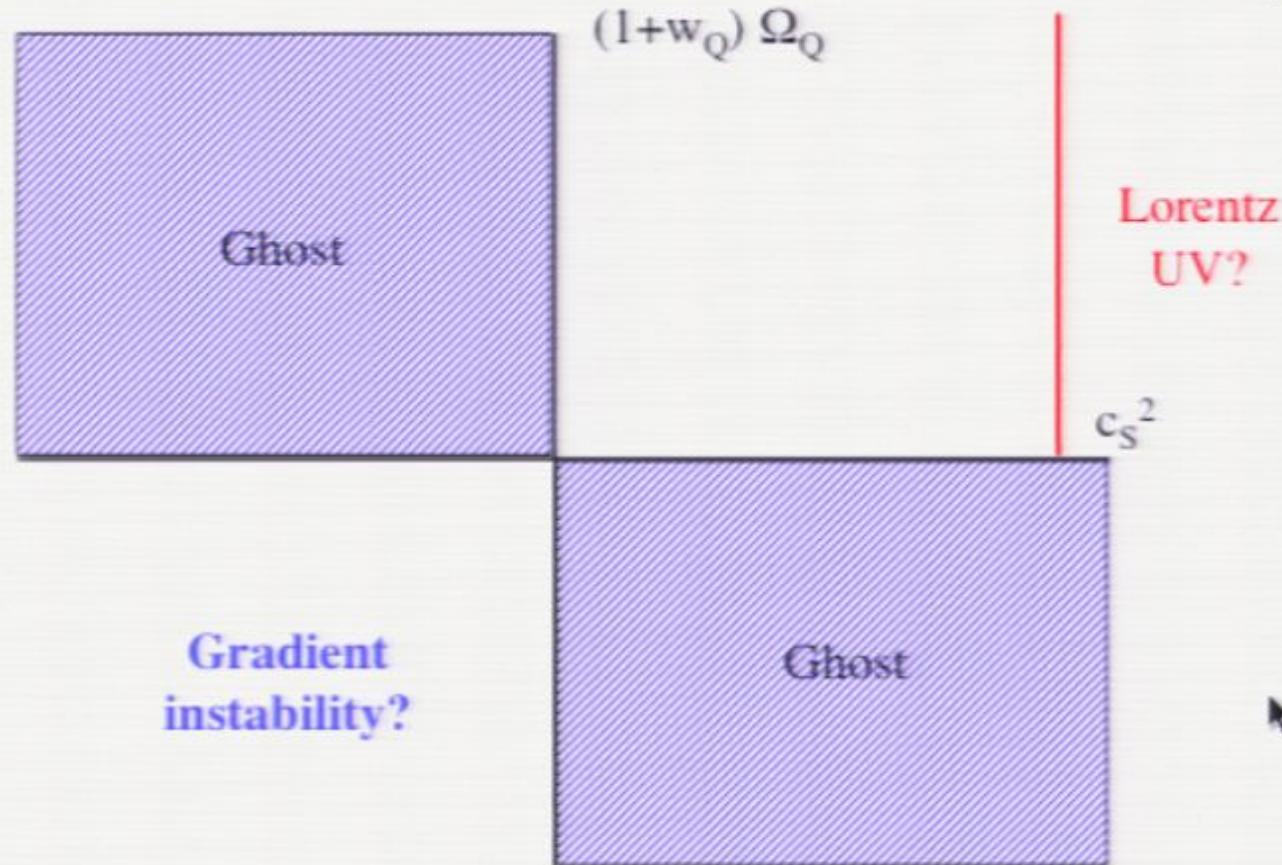


Lorentz
UV?

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$w < -1$ and gradient instabilities

Wise et al 04
Rattazzi et al 05



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Small c_s^2 limit

Instability rate: $\omega = i c_s k$. What happens when c_s^2 is very small?

Relevant instabilities $\omega > H$ only at short scales

$$S = \int d^4x a^3 \left[\frac{1}{2} (\rho_Q + p_Q + 4M^4) \dot{\pi}^2 - \frac{1}{2} (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} + \frac{3}{2} \dot{H} (\rho_Q + p_Q) \pi^2 - \frac{1}{2} (\rho_Q + p_Q) \dot{h} \pi \right]$$

Consider the limit $\rho_Q + p_Q = 0$, no spatial kinetic term. **Enhanced symmetry:** $\pi \rightarrow \pi + c$

Scalar with shift symmetry: $\mathcal{L} = \sqrt{-g} M^4 P(X)$, $X \equiv -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

In an expanding Universe, we expect $\dot{\phi} \rightarrow 0$.

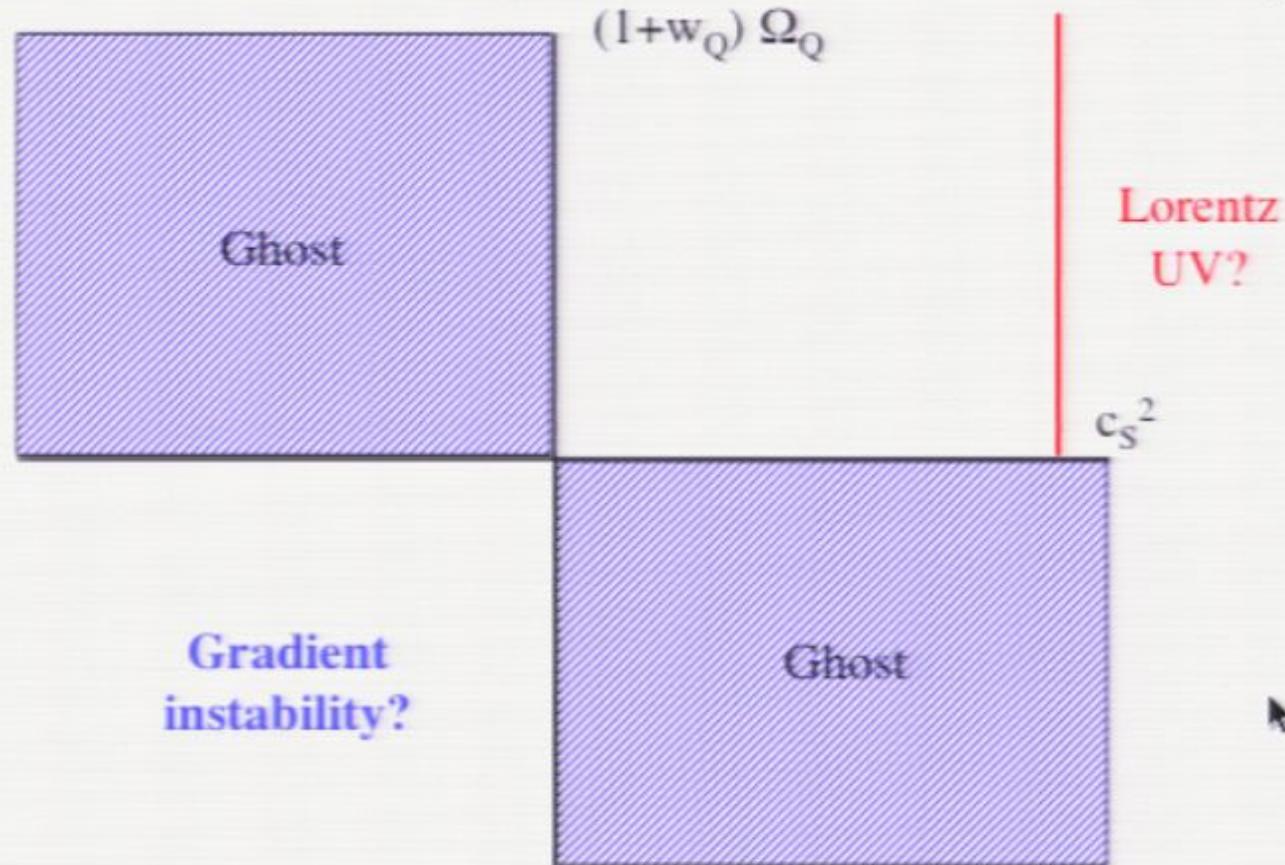
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Ghost Condensate limit: time evolving scalar in exact de Sitter

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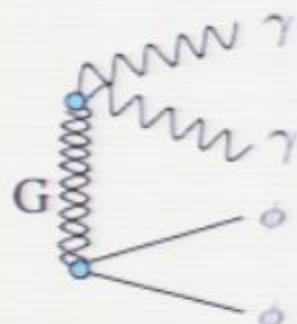
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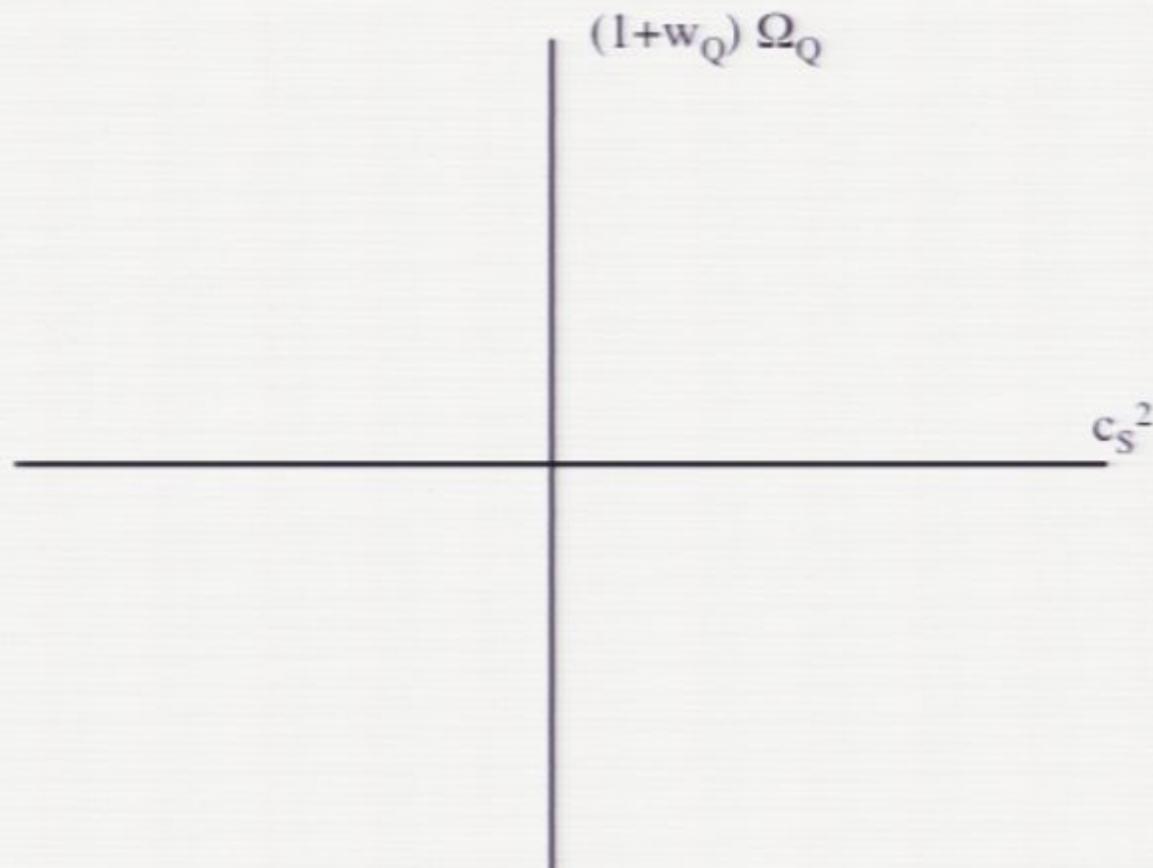
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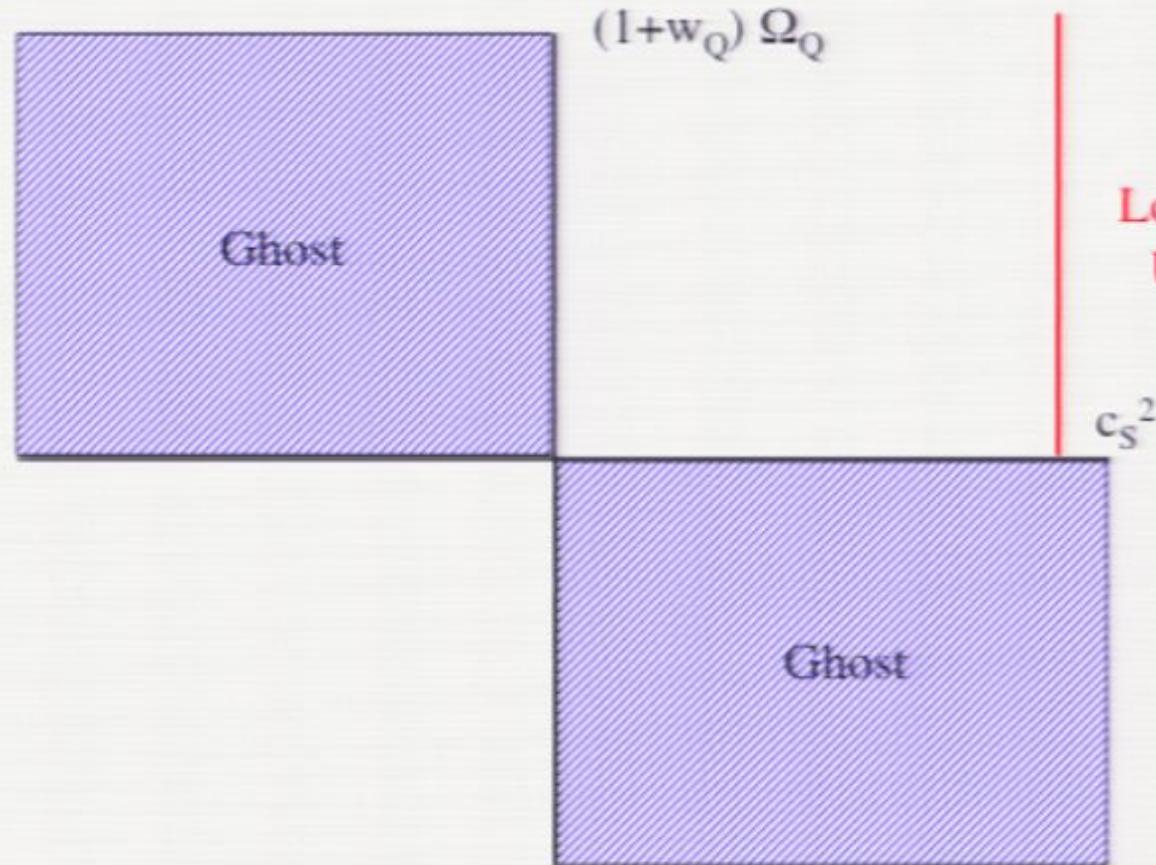
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Higher derivative

We have to consider higher derivative operators

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}(\square\phi + 3H(\phi))^2$$

It does not change the background evolution.
Only perturbations.

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2} \left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} \right)^2$$

$\ll M^4\dot{\pi}^2$ Higher time derivative terms can be neglected for $\omega < M$
No additional degrees of freedom

Leading spatial derivative term

In the ghost condensate limit:
 $\omega \propto k^2$

The Ghost Condensate is a point of enhanced symmetry.

A small breaking of the shift symmetry (and thus a small c_s^{-2}) is **technically natural**

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Stability analysis

Gradient instability: $(\rho_Q + p_Q + 4M^4) \omega^2 - (\rho_Q + p_Q) \frac{k^2}{a^2} - \bar{M}^2 \frac{k^4}{a^4} = 0$

$$\omega_{\text{grad}}^2 \simeq -\frac{(\rho_Q + p_Q)^2}{\bar{M}^2 M^4} \longrightarrow -\frac{\rho_Q + p_Q}{\bar{M} M^2} \lesssim H$$

Jeans instability: taking into account the mixing with gravity gives rise to a sort of Jeans like instability

$$S = \int d^4x \left[2M^4 \dot{\pi}^2 - \frac{\bar{M}^2}{2} \left(\frac{\dot{h}}{2} - \nabla^2 \pi \right)^2 \right] \longrightarrow \ddot{\pi} + \frac{\bar{M}^2}{4M^4} \nabla^4 \pi = \frac{\bar{M}^2}{8M^4} \nabla^2 \dot{h}$$

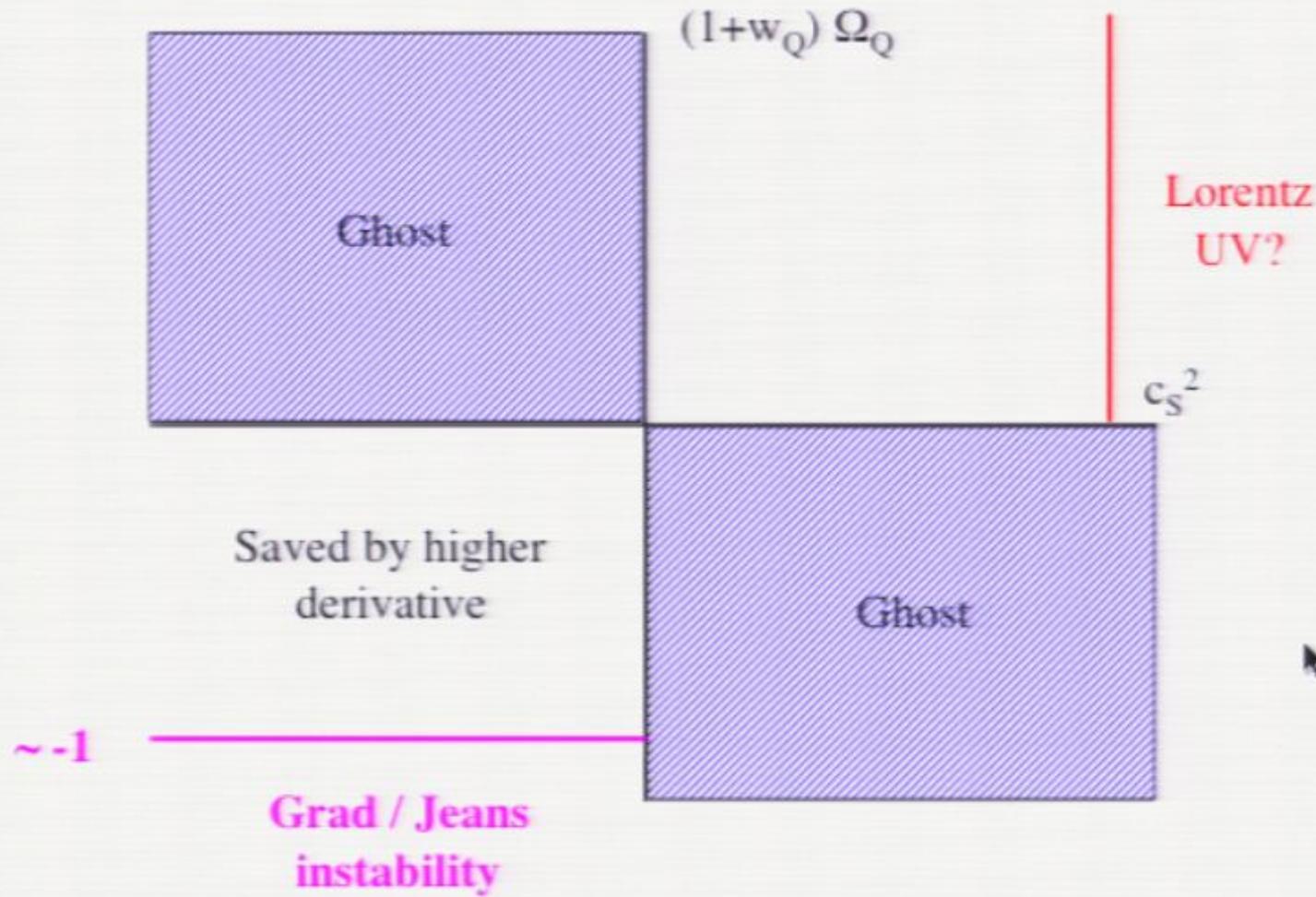
Solving for h : $\ddot{\pi} + \frac{\bar{M}^2}{4M^4} \nabla^4 \pi = -\frac{\bar{M}^2}{2M_{\text{Pl}}^2} \nabla^2 \pi \longrightarrow \omega_{\text{Jeans}}^2 \simeq -\left(\frac{\bar{M} M^2}{M_{\text{Pl}}^2}\right)^2$

Stability window

$$-(1 + w_Q) \Omega_Q \lesssim \frac{\bar{M} M^2}{H M_{\text{Pl}}^2} \lesssim 1$$

Very conservative...

Back to the plane



This limit is very conservative and anyway pheno irrelevant

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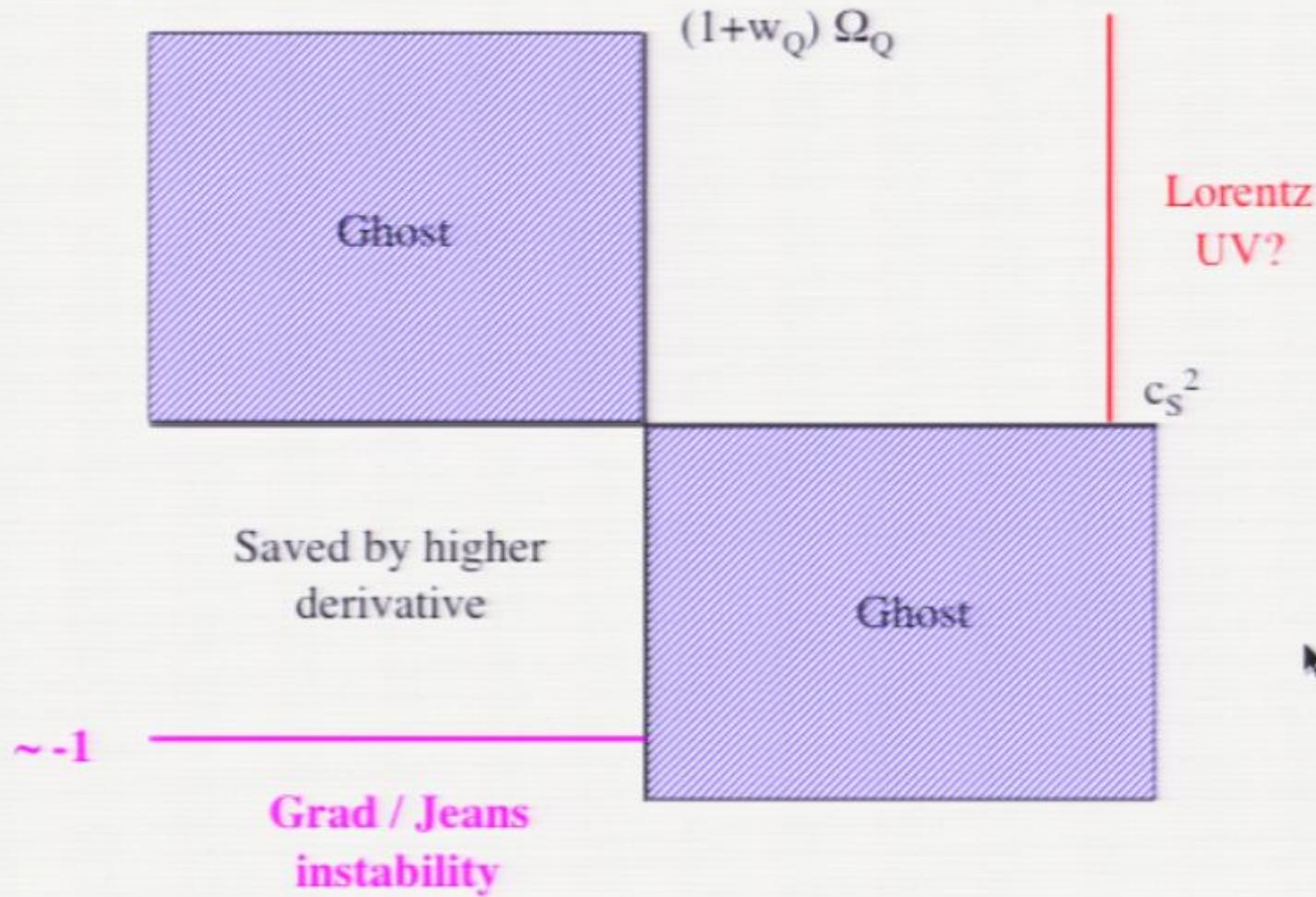
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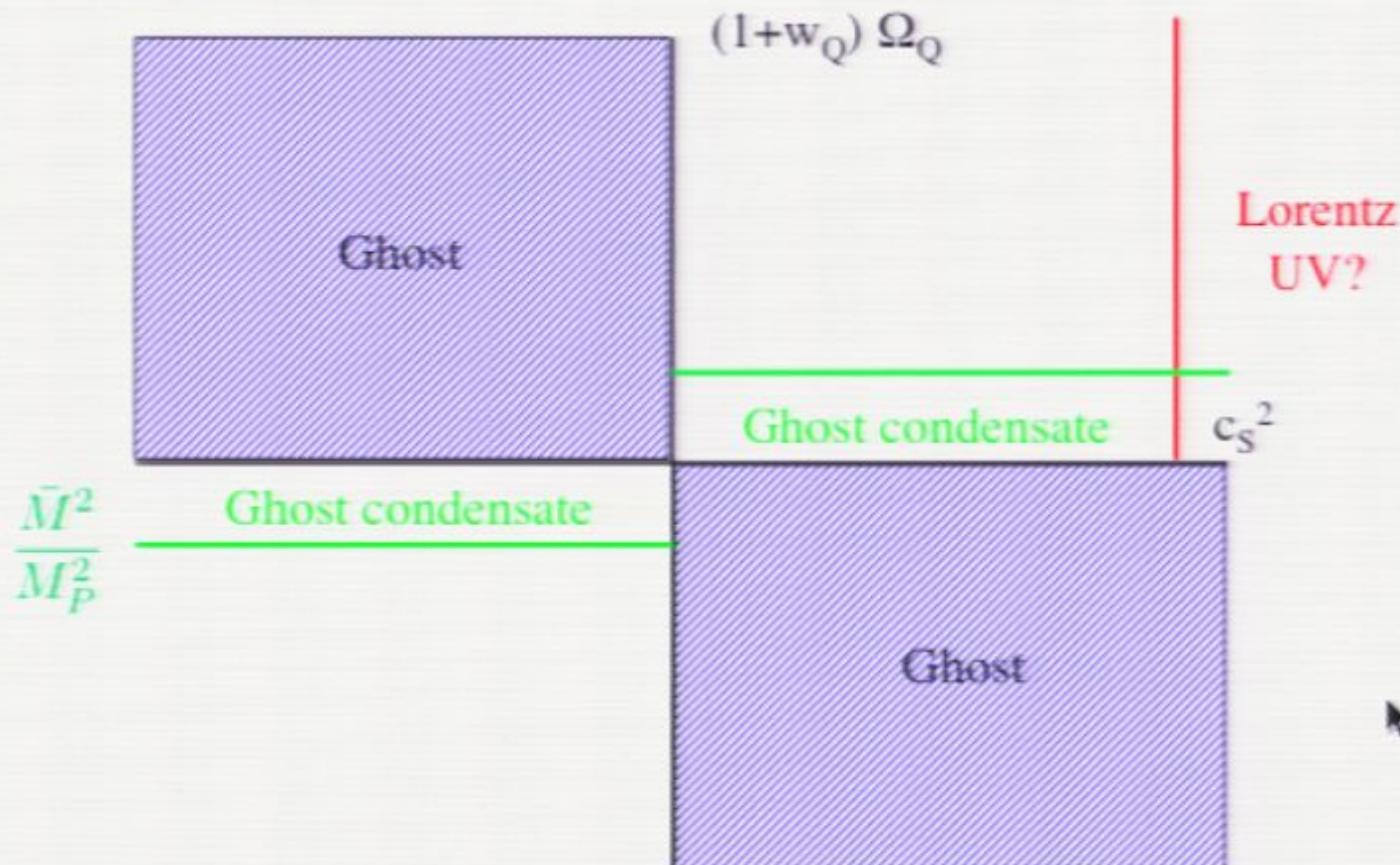
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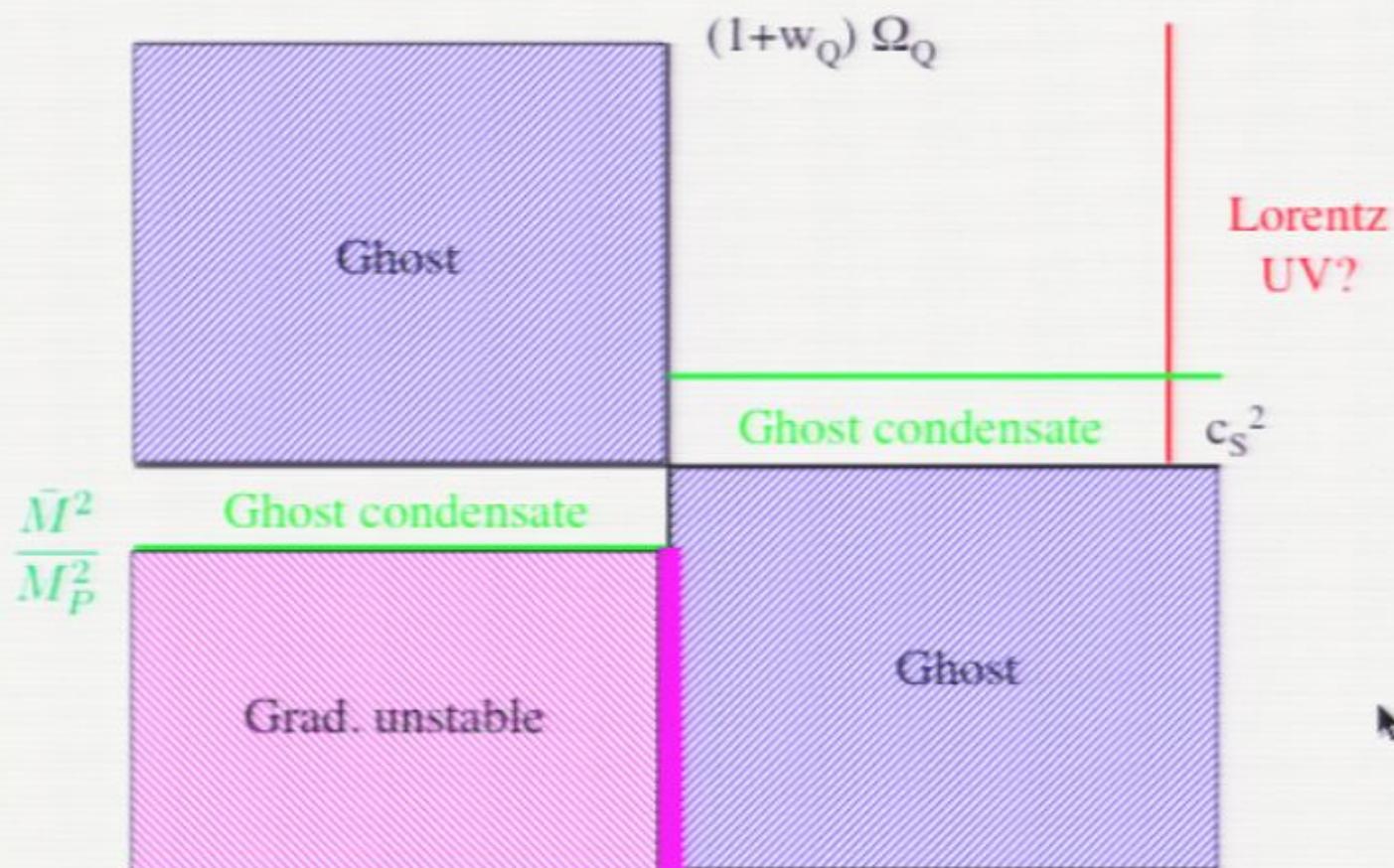
Higher derivative in the codes?



$$S_Q \supset \frac{1}{2} \int d^3x dt a^3 \left[-(\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} - \bar{M}^2 \left(\frac{\nabla^2 \pi}{a^2} \right)^2 \right]$$

Cosmo modes $k/a \sim H$ are dominated by $\omega = c_s k$ for: $|(1 + w_Q)\Omega_Q| \gg \frac{\bar{M}^2}{M_P^2}$

Small c_s^2 : how small?



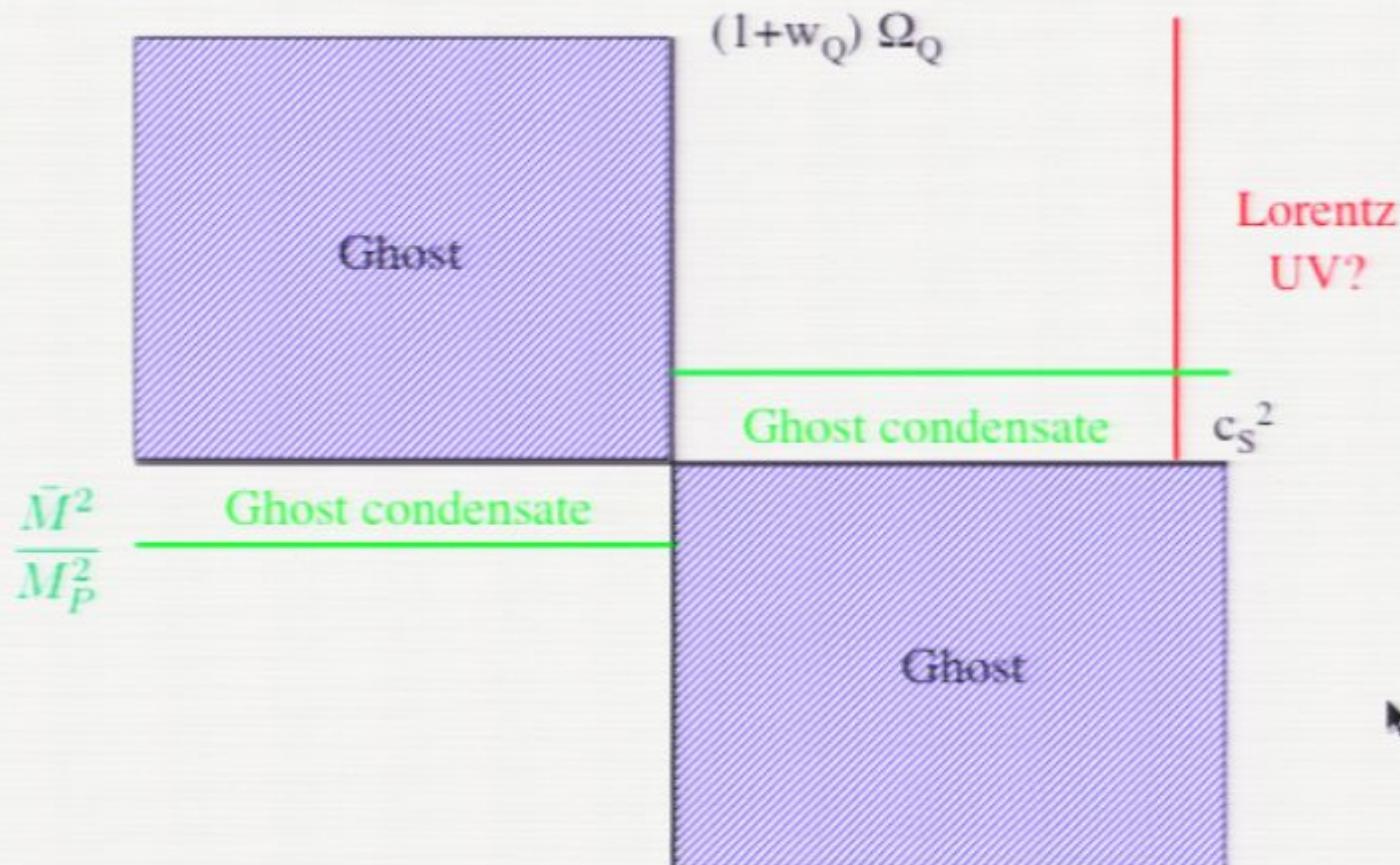
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$$\ll H \Rightarrow c_s^2 \ll \frac{H\bar{M}}{M^2}$$

The scales M are the cutoff of my theory

$$M > (.1 \text{ mm})^{-1} \rightarrow |c_s^2| < 10^{-30}!!$$

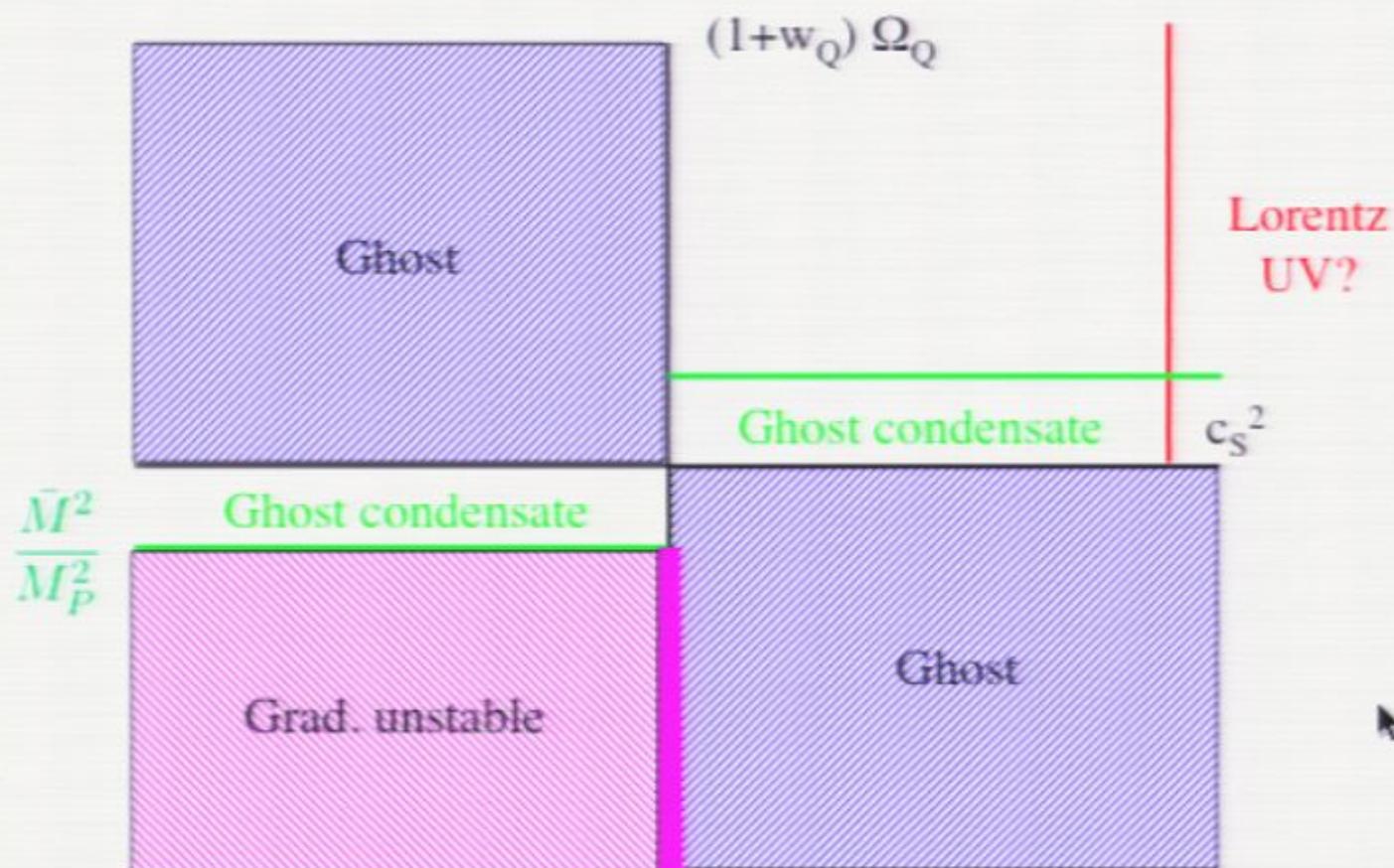
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The phantom divide

- What happens to perturbations when $w_Q = -1$?

Fluid equations:

$$\begin{aligned}\dot{\delta} &= -(1+w) \left\{ [k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)] \frac{\theta}{k^2} + \frac{\dot{h}}{2} \right\} - 3\mathcal{H}(c_s^2 - w)\delta \\ \frac{\dot{\theta}}{k^2} &= -\mathcal{H}(1-3c_s^2) \frac{\theta}{k^2} + \frac{c_s^2}{1+w} \delta.\end{aligned}$$

e.g. Bean, Doré 03

$$\theta \equiv ik^j v_j \quad c_a^2 \equiv \dot{p}/\dot{\rho} = w - \frac{1}{3H} \frac{\dot{w}}{1+w}$$
$$c_s^2 \equiv \delta \hat{p}/\delta \hat{\rho} \quad T_i^0 = 0$$

The one given by scalar kinetic term

- The phantom psychosis:

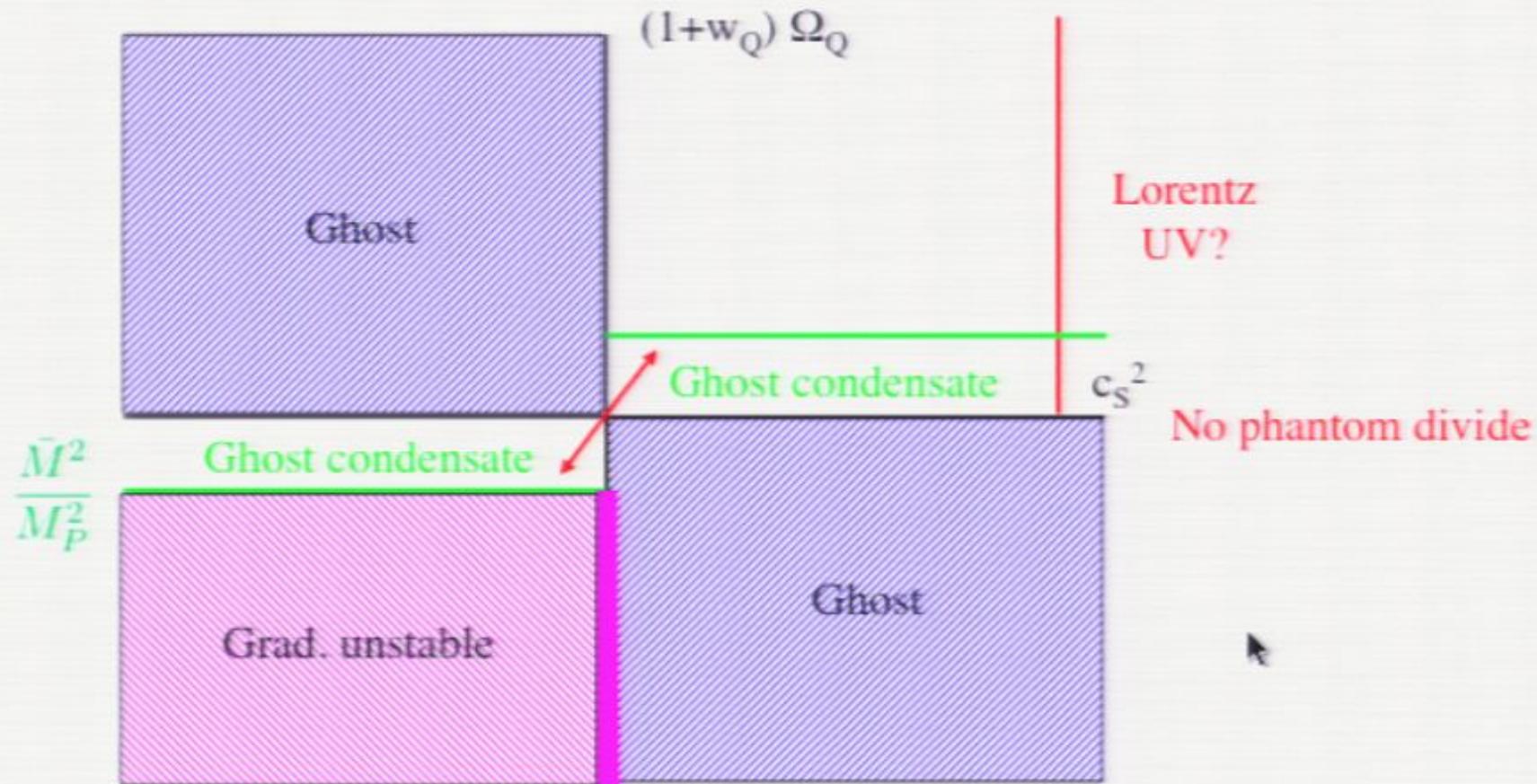
• 1st divergence: $c_a^2 \rightarrow \infty$ [Hu 04] So what?

• 2nd divergence: in θ equation [Caldwell, Doran 05] $c_s^2 \rightarrow 0$ at the crossing

• Instability: $c_s^2 \rightarrow 0 \Rightarrow c_s^2 < 0$

Higher derivative terms

The phantom divide is ... a phantom



$$S_Q = \frac{1}{2} \int d^3x dt a^3 \left[4M^4 \dot{\pi}^2 + (\rho_Q + p_Q) \dot{\pi}^2 - (\rho_Q + p_Q) \frac{(\nabla \pi)^2}{a^2} + 3\dot{H}(\rho_Q + p_Q) \pi^2 \right.$$

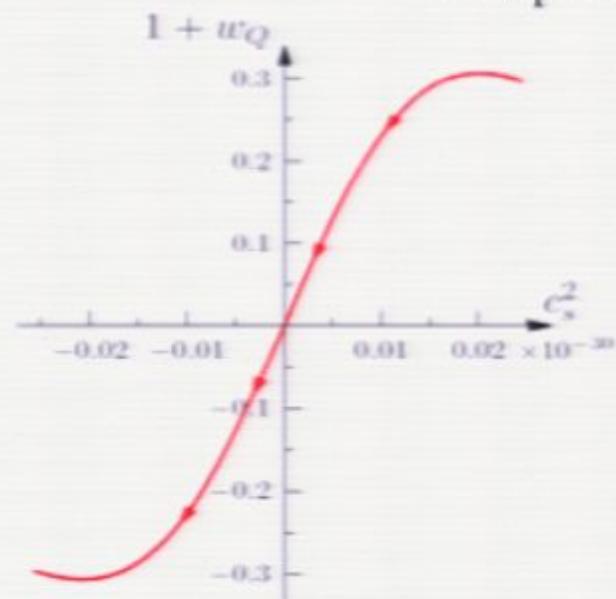
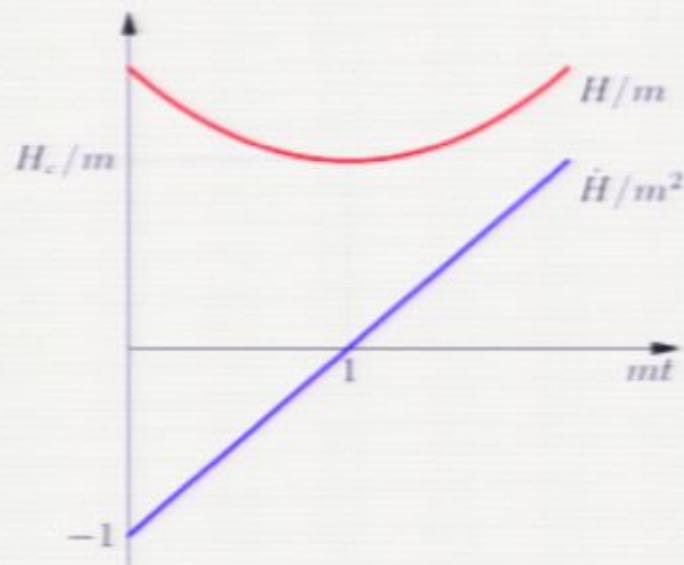
$$\left. - (\rho_Q + p_Q) \dot{h}\pi - \bar{M}^2 \left(\frac{\dot{h}}{2} - \frac{\nabla^2 \pi}{a^2} \right)^2 \right]$$

Nothing strange happens when you cross
 $w_Q = -1$

For example...

$$P(X, \phi) = -3M_{\text{Pl}}^2 H^2(\phi) - M_{\text{Pl}}^2 \dot{H}(\phi)(X + 1) + \frac{1}{2}M^4(\phi)(X - 1)^2$$

No other energy components



- The GC strip is very tiny. Effectively $w_Q = -1$ is crossed by a k-essence with $c_s^2 = 0$
- Numerical recipe. When comparing with data $w_Q(z)$ going through $w_Q = -1$, set $c_s^2 = 0$

w<-1: two interesting limits

$$\omega = c_s k \quad \text{with } c_s \sim 0$$

$$(1 + w_Q)\Omega_Q \gg \frac{\bar{M}^2}{M_P^2}$$

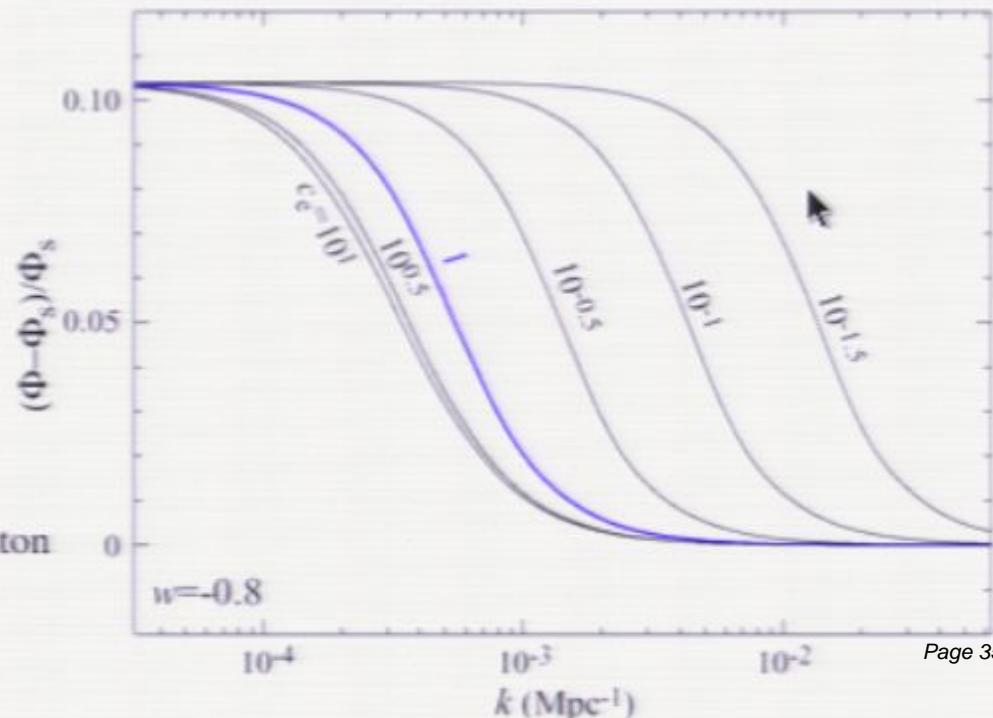
Dark energy escapes (!) from dark matter wells: $\delta_Q \simeq \frac{1+w}{1-3w}\delta_{\text{DM}}$

Clusters on scales large than sound horizon:

$$1/k_{\text{DE s.h.}} = a \int_0^t \frac{c_s}{a} dt \simeq 2c_s H_0^{-1}$$

Fractional difference in Φ between smooth and clustered quintessence

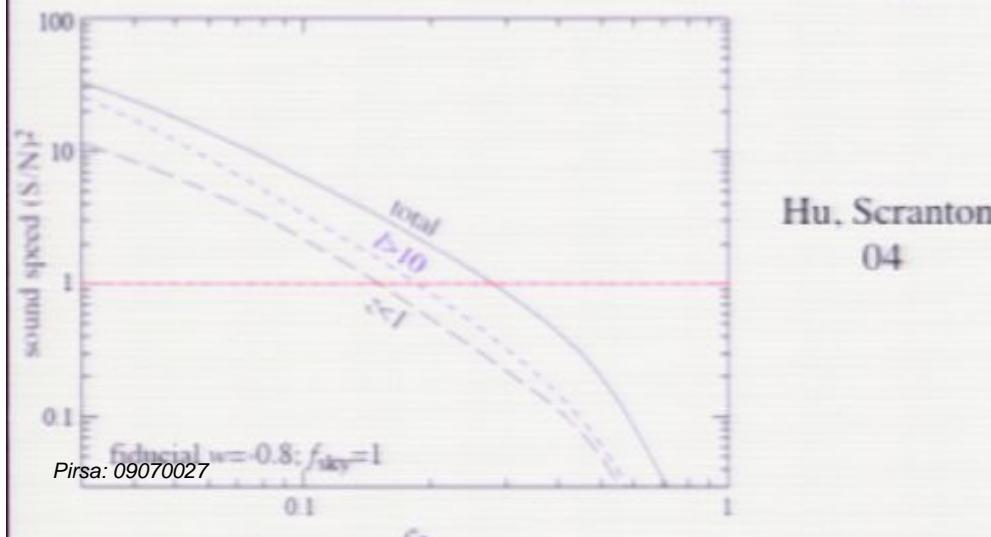
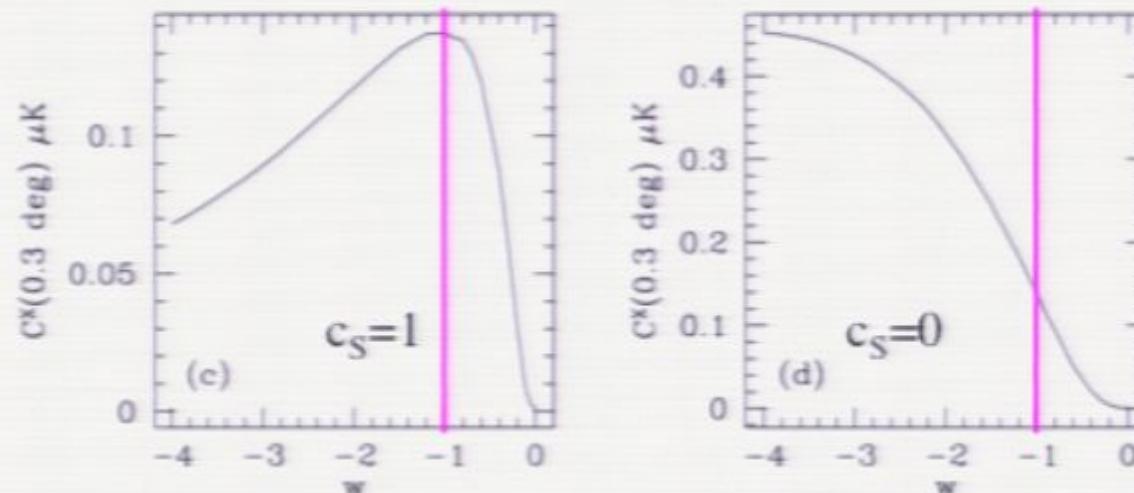
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ISW- galaxy correlation

Is it possible to exp distinguish $c_s=0$ from $c_s=1$? Until which value of $1+w_Q$?

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Melchiorri 05



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Distinction possible for $|1 + w_Q| \gtrsim 0.05$?

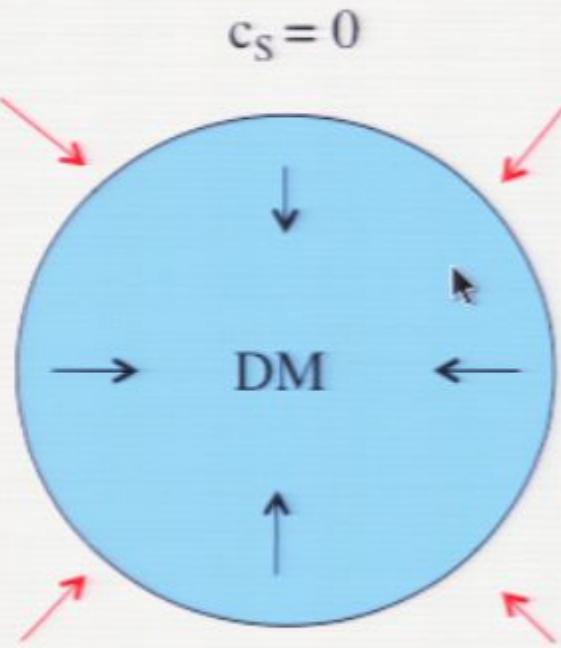
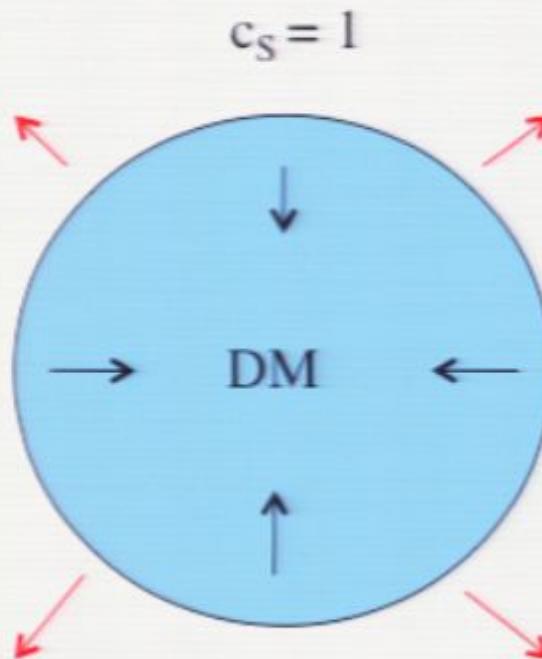
Forecasts done only for $w > -1$...

Non-linear clustering

What happens at very short scales? For $c_s^2 = 0$ quintessence clusters at all scales.

Effect on **non-linear structure formation**

Spherical collapse



Quintessence $\sim \Lambda$

Ghost condensate limit

For cosmo scales: $\omega \sim k^2$ $(1 + w_Q)\Omega_Q \ll \frac{\bar{M}^2}{M_P^2}$

$$S_Q = \frac{1}{2} \int d^3x dt a^3 \left[4M^4 \dot{\pi}^2 - \bar{M}^2 \left(\frac{\dot{h}}{2} - \frac{\nabla^2 \pi}{a^2} \right)^2 \right]$$

The scalar degree of freedom does not disappear even for $1+w_Q=0$

$$\ddot{\pi} + 3H\dot{\pi} = -\frac{\bar{M}^2}{12M^4 M_P^2} \frac{\nabla^2 \delta \rho_{DM}}{Ha^2}$$

The driving of DM is not suppressed by $1+w_Q$ in this limit

$$\delta \rho_Q = 4M^4 \dot{\pi} \sim \frac{\bar{M}^2}{M_P^2} \delta \rho_{DM} \ll \delta \rho_{DM}$$

No relevant perturbation!

The ghost condensate is a modification of gravity, but only on very short scales
Irrelevant cosmologically

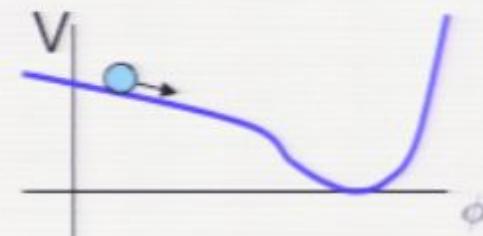
A more general approach

Usual approach to quintessence/inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu \phi, \square \phi \dots)$
2. Solve EOM of the scalar + FRW. Find an accelerating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \quad ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

3. Study perturbations around this solution to work out predictions



We want to **focus directly on the theory of perturbations** around the accelerating solution

- Time diffeomorphisms are broken: $t \rightarrow t + \xi^0(t, \vec{x})$ $\delta\phi \rightarrow \delta\phi + \dot{\phi}_0(t)\xi^0$
- In unitary gauge $\phi(t, \vec{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:
 - 3 degrees of freedom. Like in a broken gauge theory.
- The most generic action in unitary gauge

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R + \mathcal{L}_m + p_Q - \frac{1}{2}(\rho_Q + p_Q)(g^{00} + 1) + \frac{M^4(t)}{2}(g^{00} + 1)^2 - \frac{\bar{M}^2(t)}{2}\delta K^2 - \frac{\dot{M}(t)^3}{2}\delta K(g^{00} + 1) \right].$$

Conclusions

- General framework to study single field quintessence models
- Higher derivative terms can stabilize the $w_Q < -1$ region
Phenomenology is the same of **k-essence models with $c_s^2 = 0$**
- The **phantom divide can be smoothly crossed** if one sets $c_s^2 = 0$
- Phenomenology of models with $c_s^2 = 0$ VS $c_s^2 = 1$ must be further explored
- **Quintessential plane**



Quintessential plane

