Title: Holography in String Gas Cosmology

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Abstract: I will review the string gas scenario of structure formation, stressing the role which holography plays. I will also discuss another way of obtaining a scale-invariant spectrum of cosmological perturbations (with specific signatures in the bispectrum) which may be realizable in scenarios based on the AdS/CFT correspondence which can resolve the cosmological singularity.

Holography in String Gas Cosmology

Robert Brandenberger
McGill University & CERN

July 16, 2009

Outline

1. Introduction
2. String Gas Cosmology
   - Principles
   - Features of String Gas Cosmology
   - Moduli stabilization in SGC
3. Holography in String Thermodynamics
   - Formalism
   - Application to a String Gas
   - Specific Heat
4. String Gas Cosmology and Structure Formation
   - Review of the Theory of Cosmological Perturbations
   - Fluctuations in String Gas Cosmology vs. Inflation
   - Analysis
5. Discussion
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Goal

Goal: Alternative to inflation based on holographic thermodynamics.

$$C_V(R) \sim R^2 \tag{1}$$

Realization: String gas cosmology on a compact space (Nayeri, R.B. & Vafa, *Phys. Rev. Lett.* 2006).

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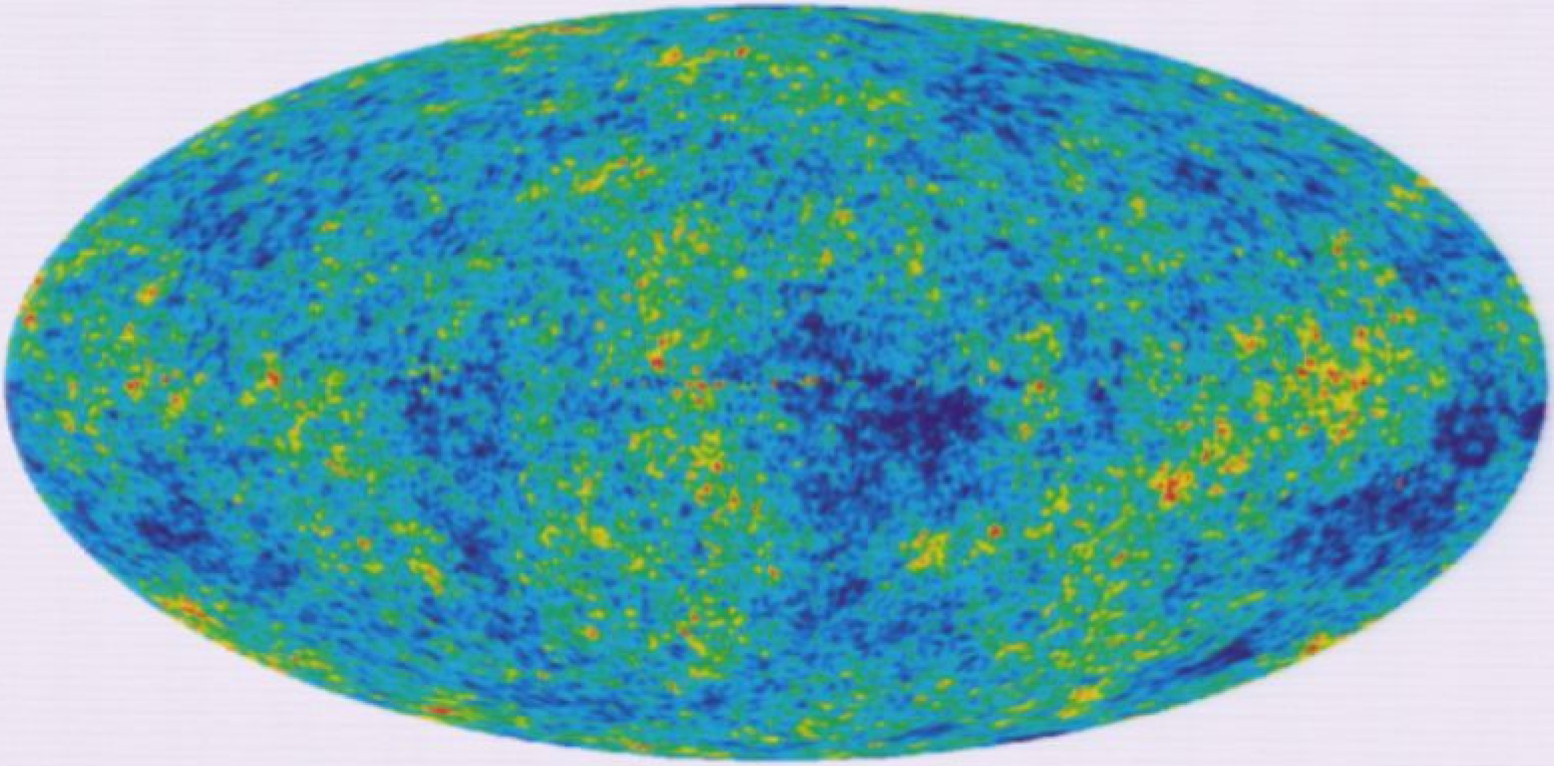
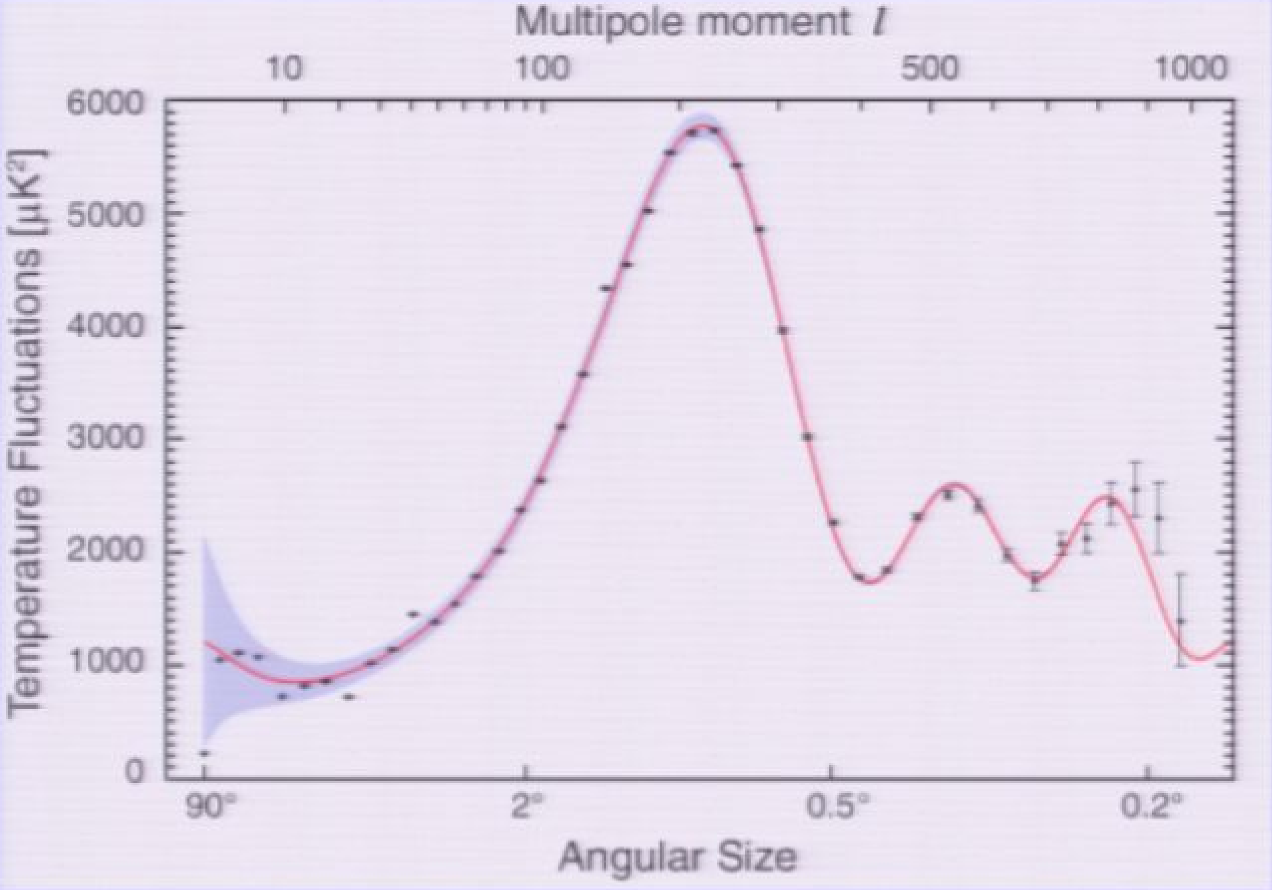


Credit: NASA/WMAP Science Team



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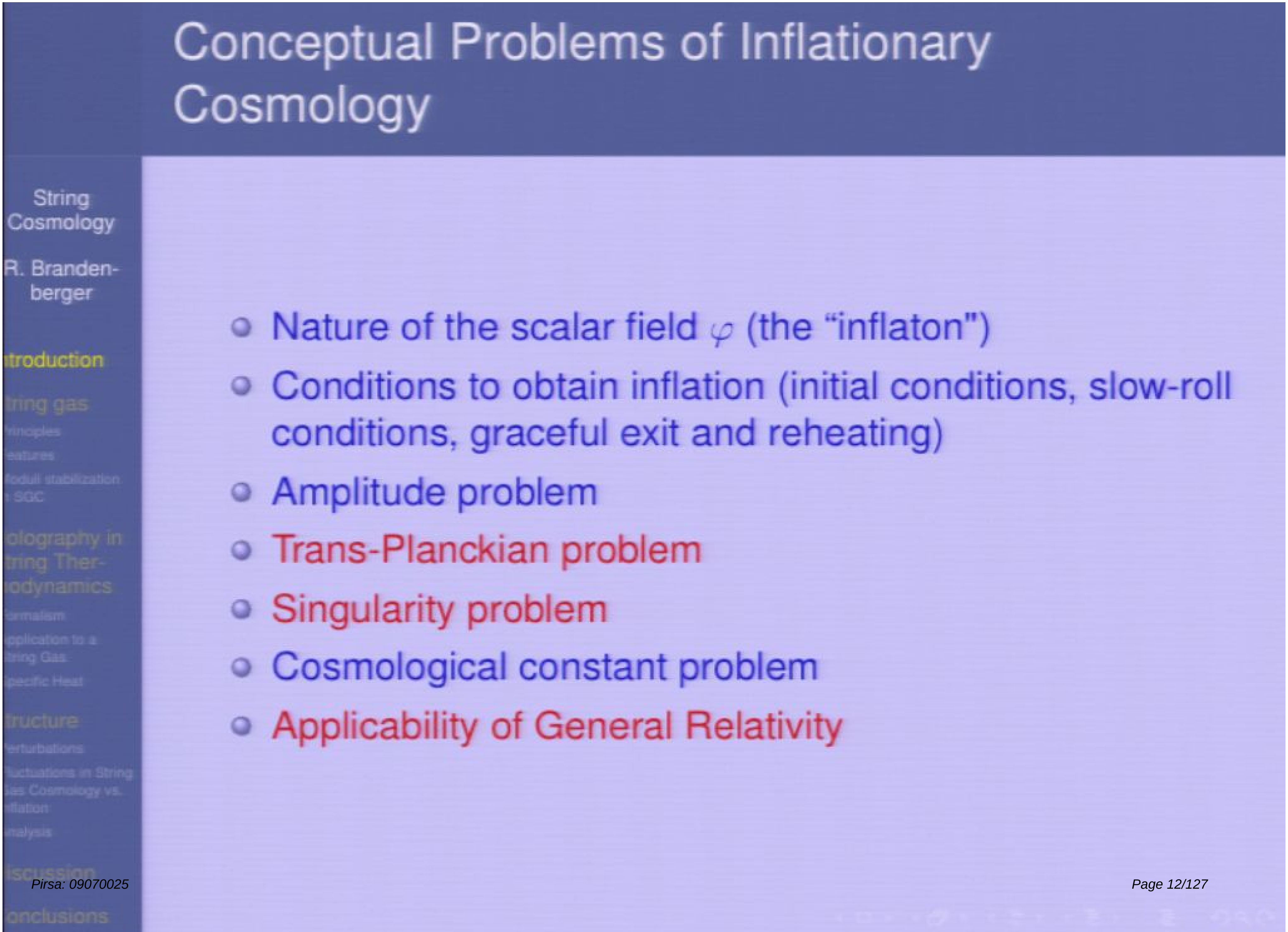
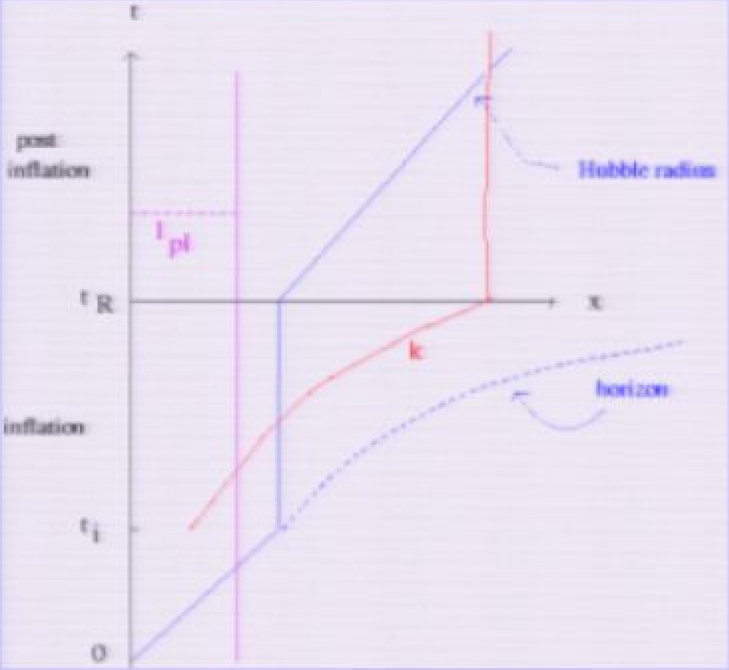
- Inflation has been phenomenologically successful:
- However, inflation faces conceptual problems:



Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field φ (the “inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- → new physics MUST enter into the calculation of the fluctuations.

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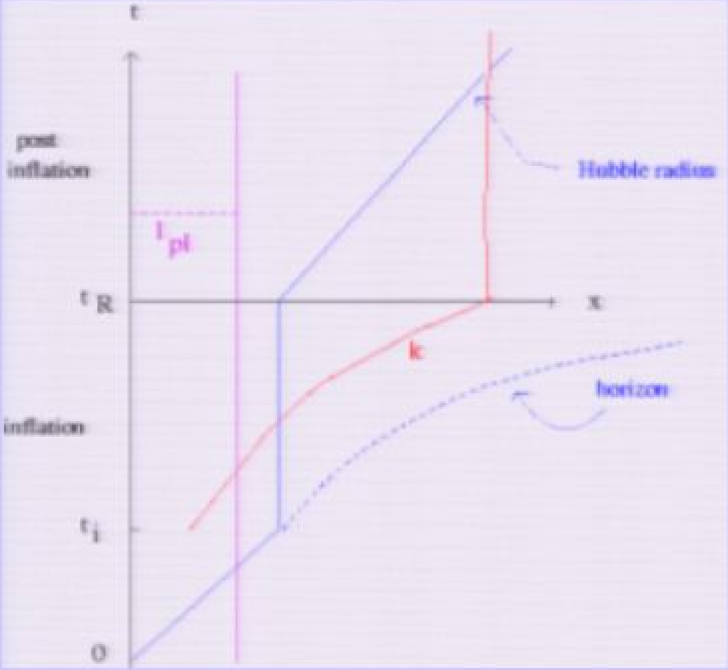

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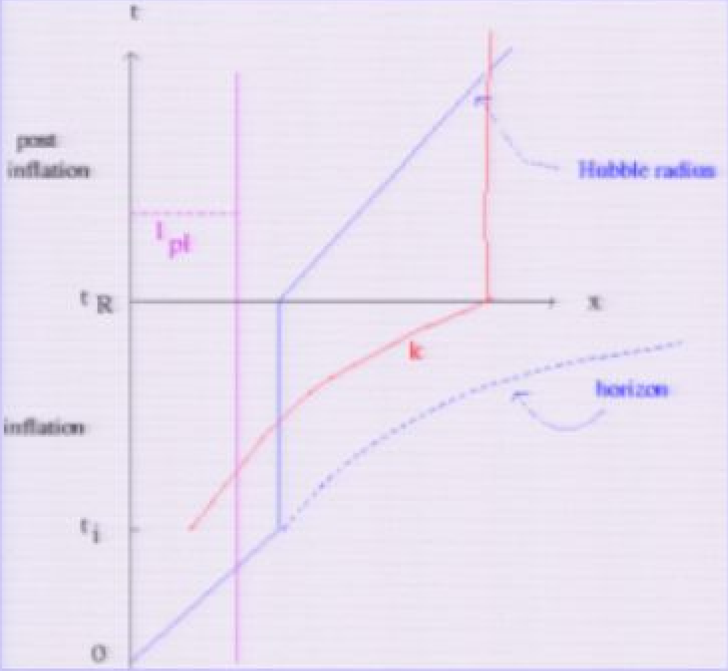

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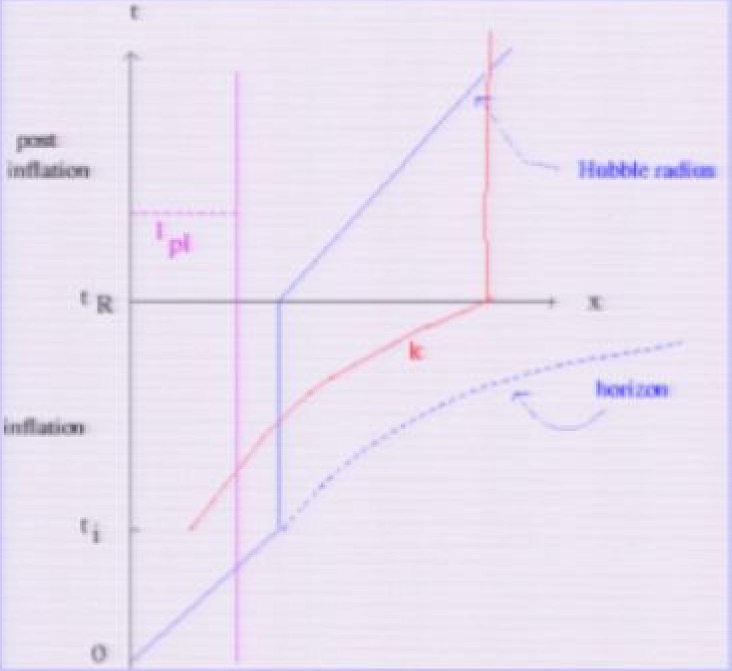

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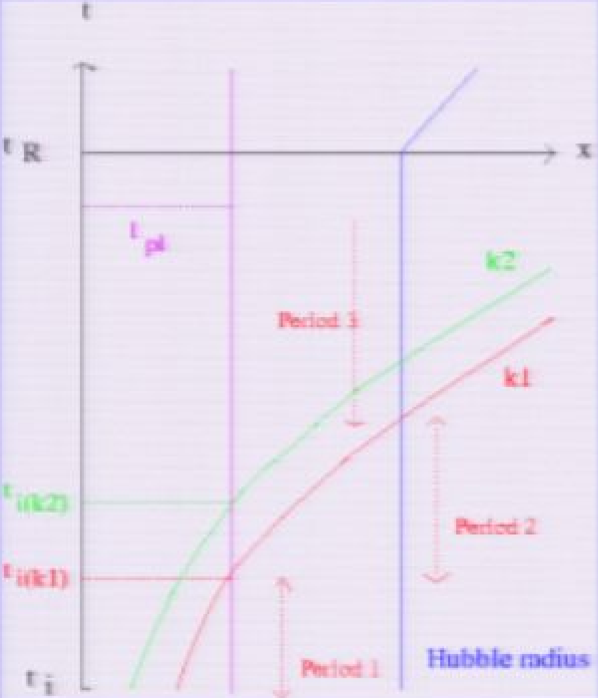
Trans-Planckian Window of Opportunity



- If evolution in Period I is non-adiabatic, then scale-invariance of the power spectrum will be lost [J. Martin and RB, 2000]
- → Planck scale physics testable with cosmological observations!

Singularity Problem

- Standard cosmology: Penrose-Hawking theorems → initial singularity → incompleteness of the theory.
- Inflationary cosmology: In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin] → incompleteness of the theory.

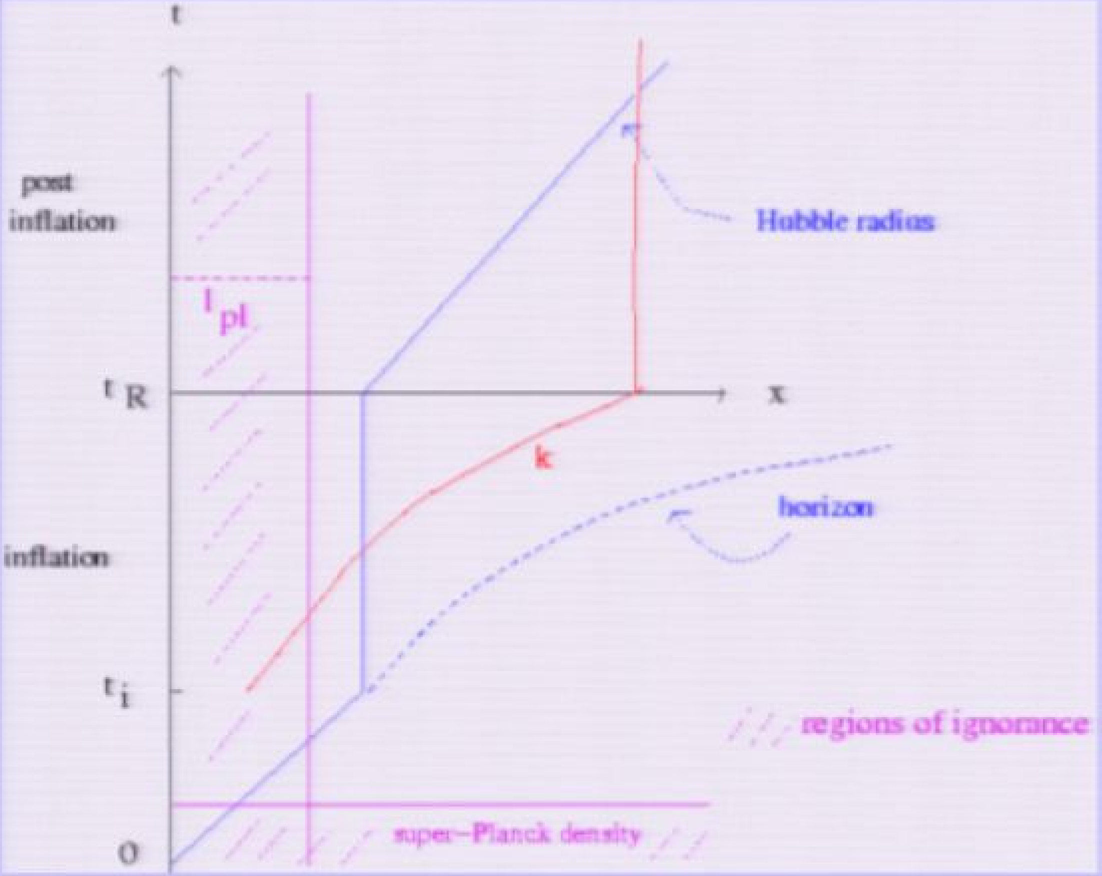
Penrose-Hawking theorems:

- Ass: i) Einstein action, 2) weak energy conditions $\rho > 0, \ \rho + 3p \geq 0$
- → space-time is geodesically incomplete.

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}\mathrm{GeV}$.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



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- Inflation has been phenomenologically successful:
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- We need a new paradigm of early universe cosmology based on new fundamental physics. Can such a paradigm be based on holographic principles?

Message

- Holographic scaling of correlation functions emerges from String Gas Cosmology (SGC) [R.B. and C. Vafa, 1989]
- New structure formation scenario emerges from SGC [A. Nayeri, R.B. and C. Vafa, 2006].
- String Gas Cosmology makes testable predictions for cosmological observations
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Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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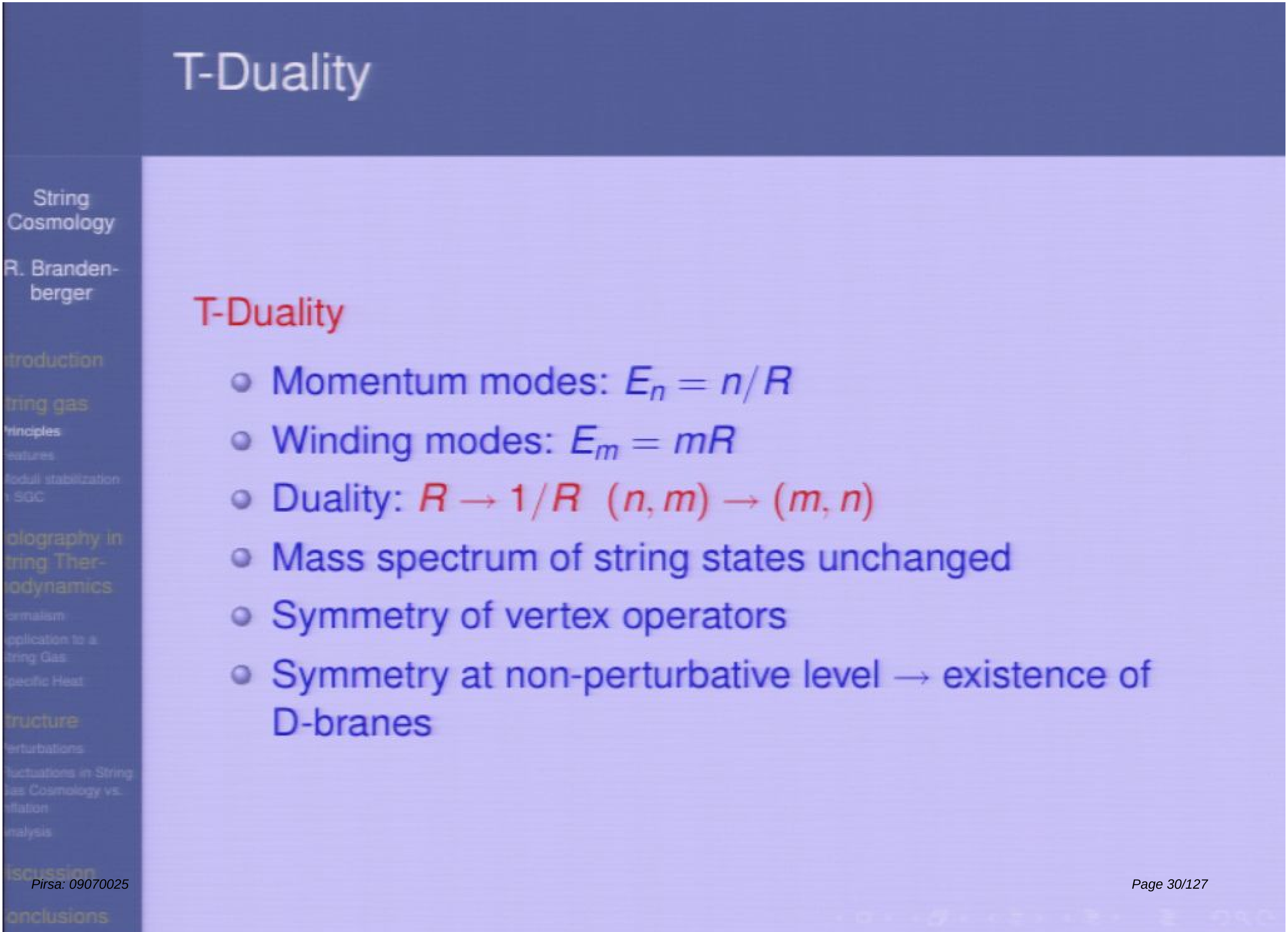


T-Duality

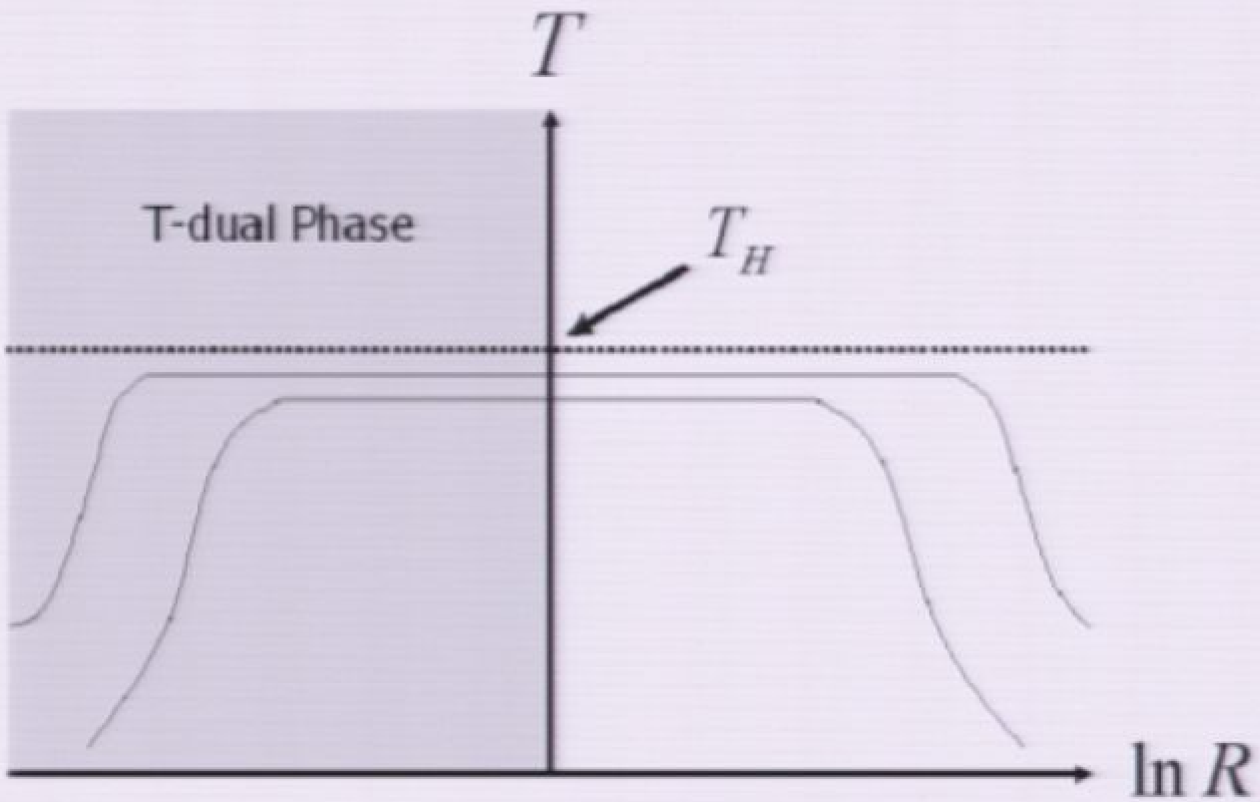
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level → existence of D-branes

Adiabatic Considerations

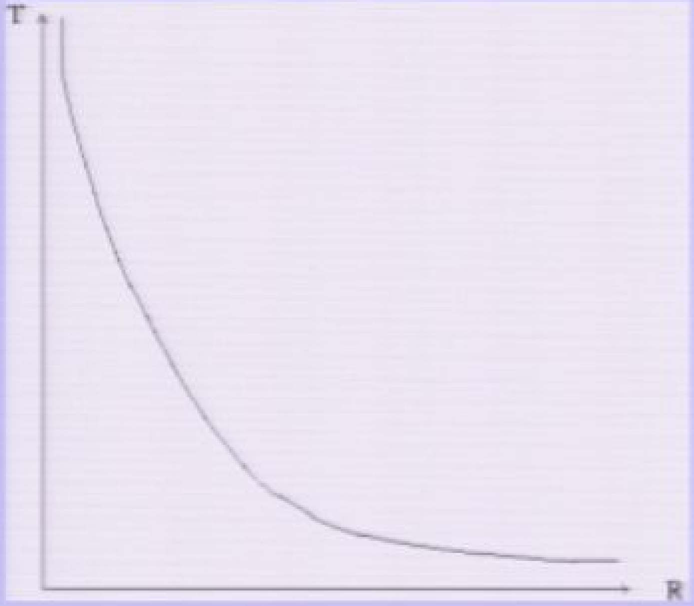
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



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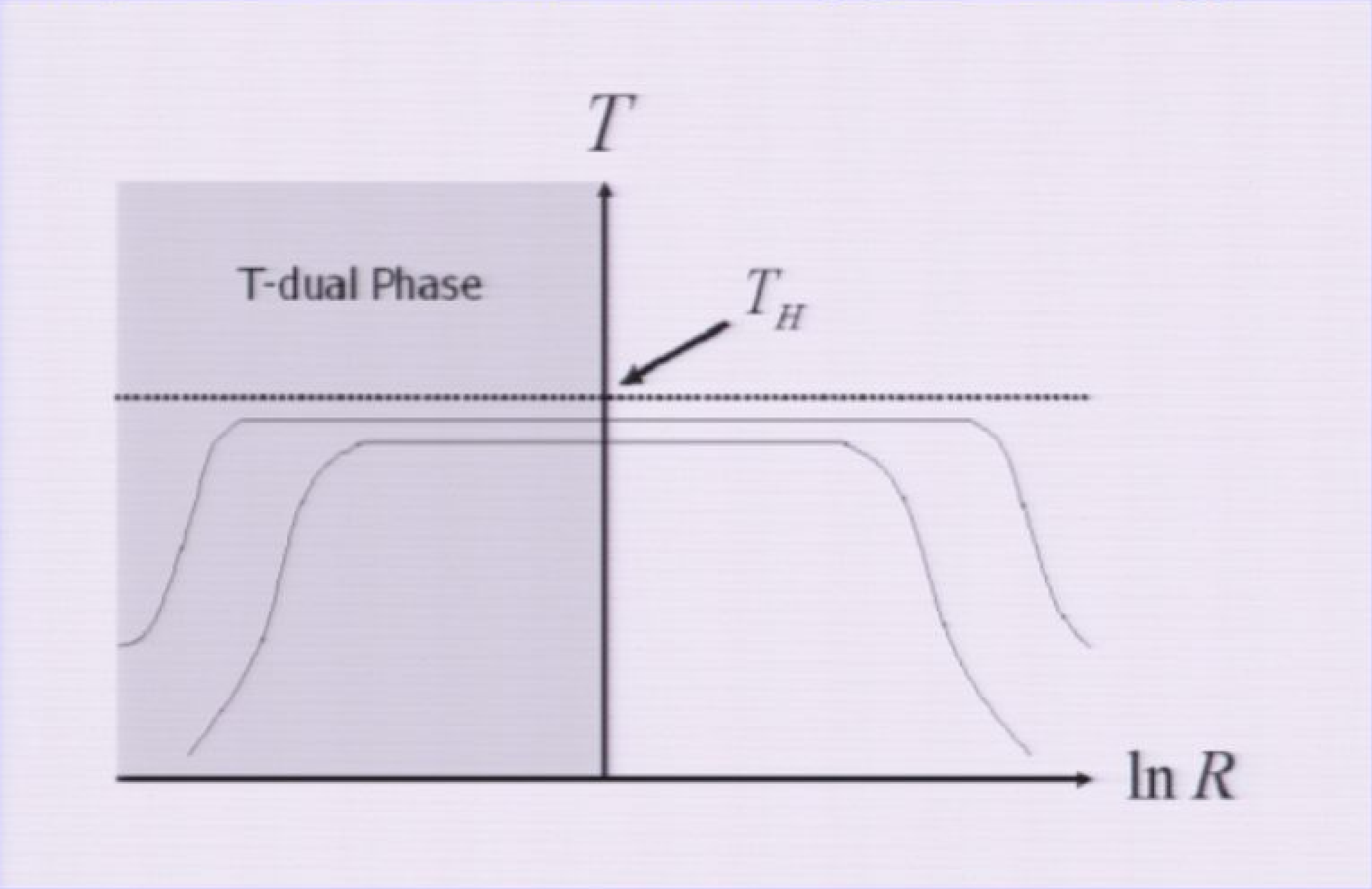

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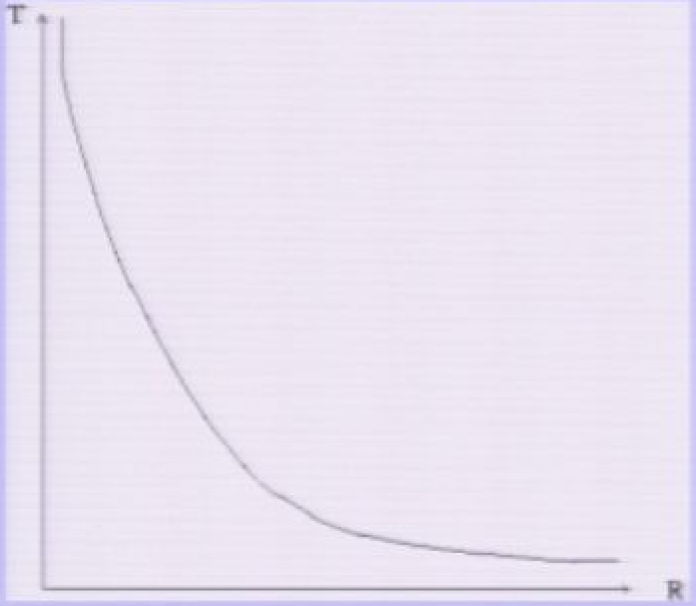

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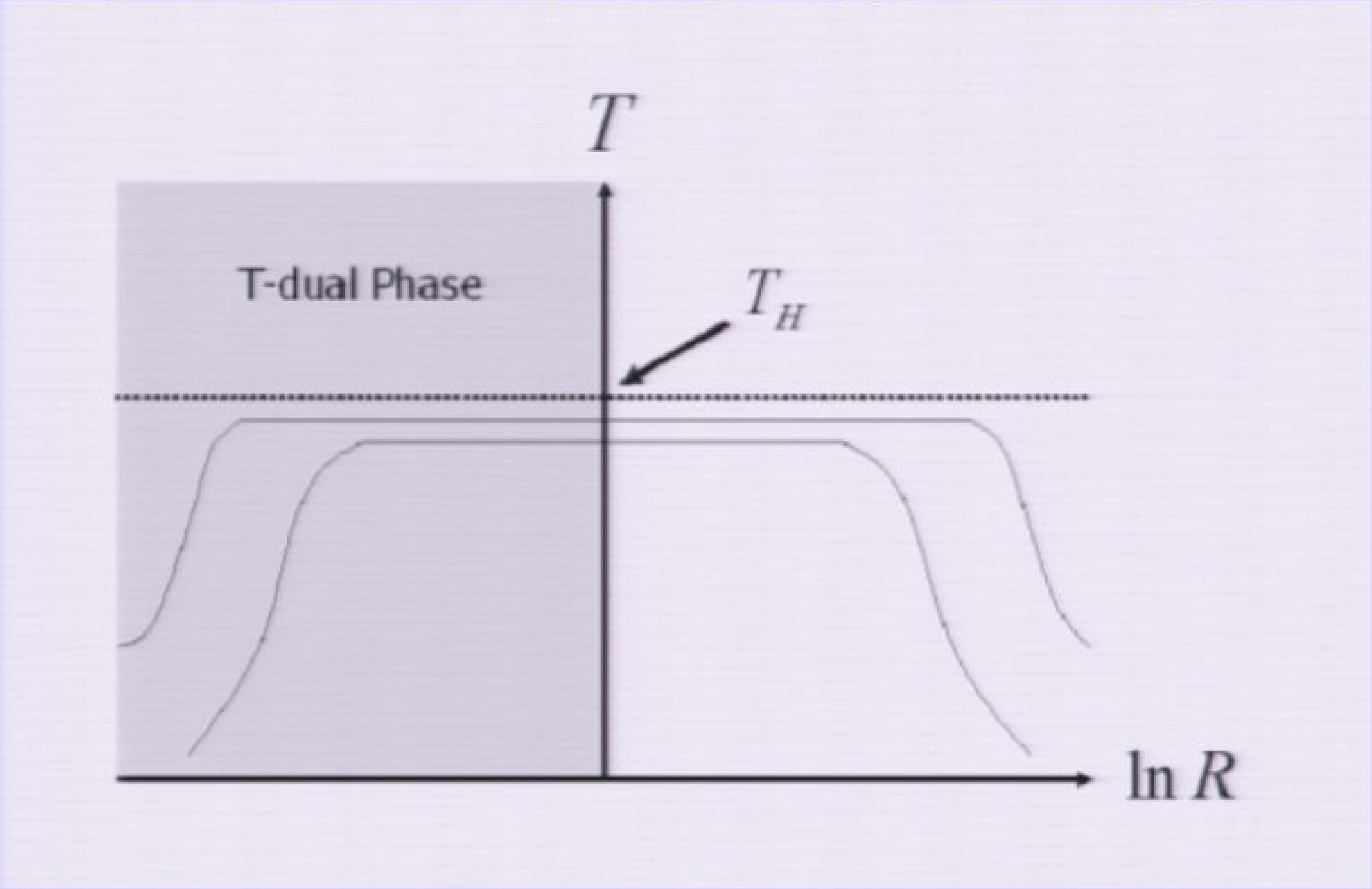

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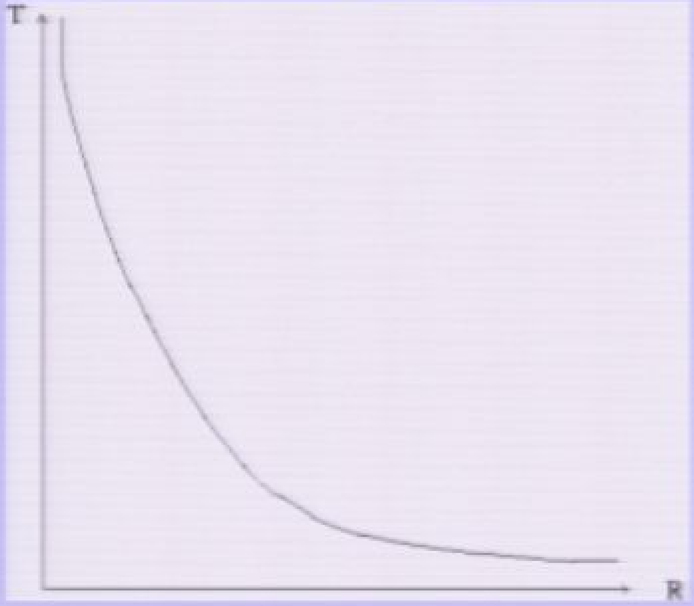
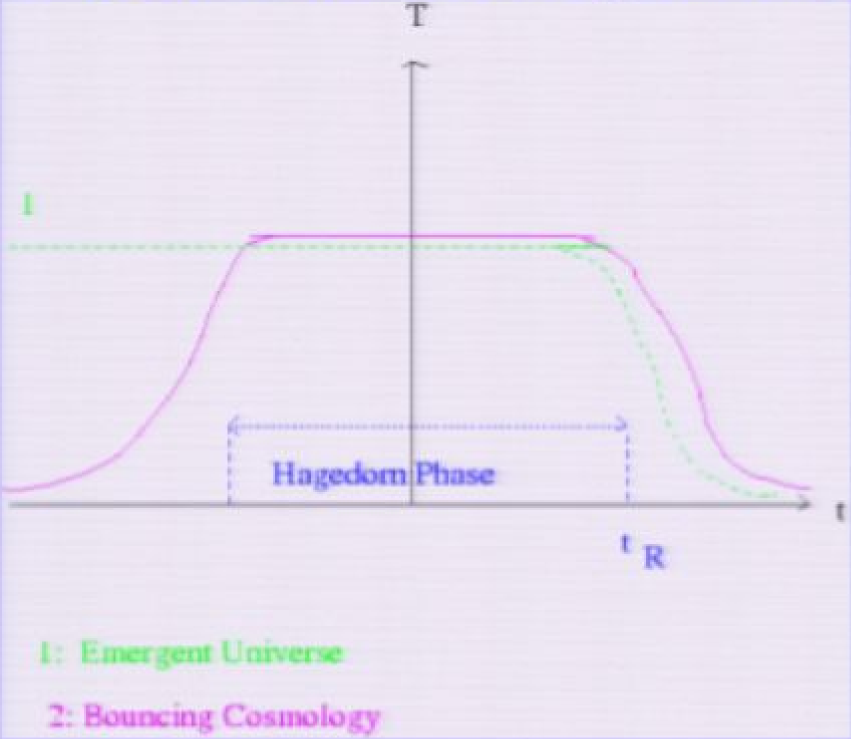


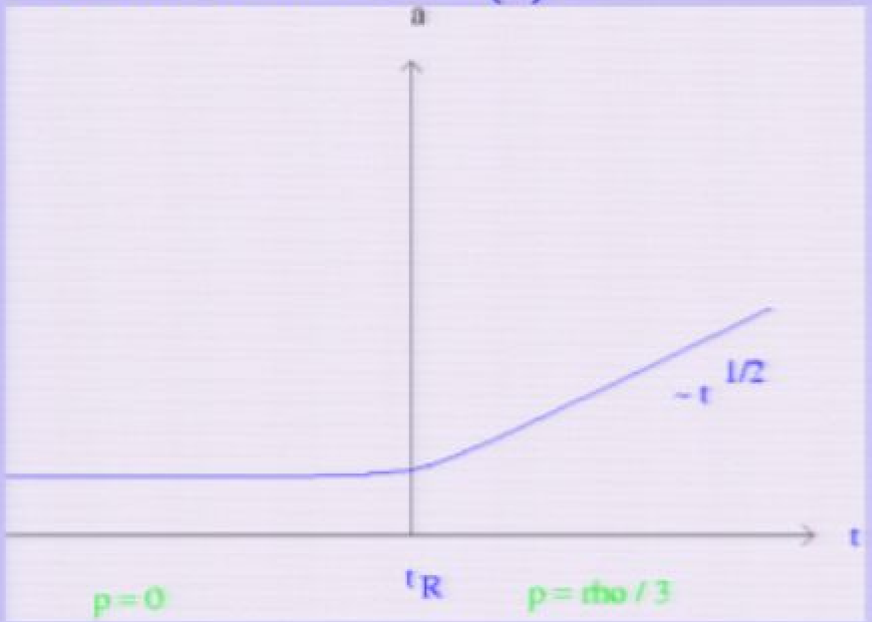
Dynamics

Assume some action gives us $R(t)$



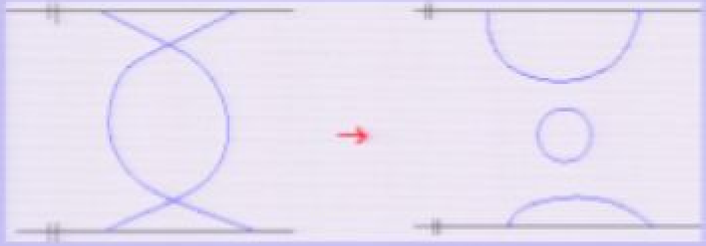
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H → winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- → dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

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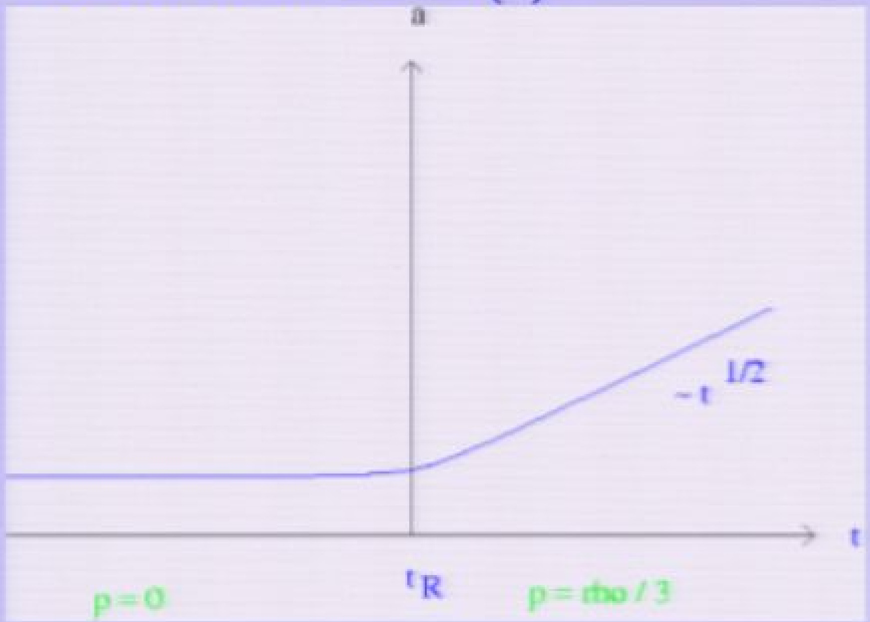

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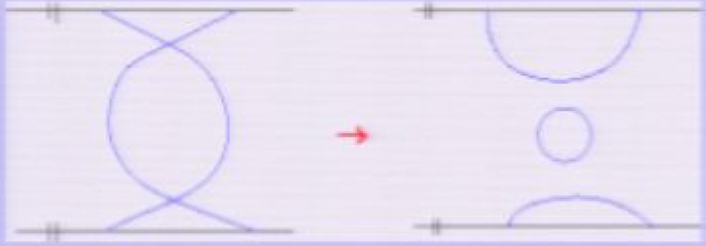

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Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
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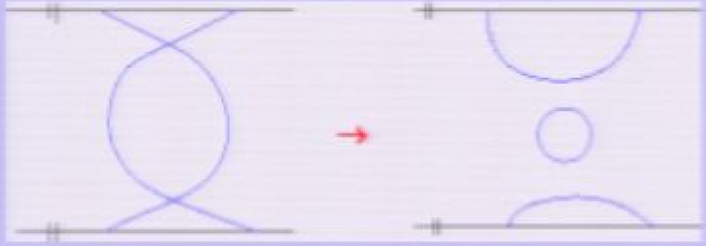

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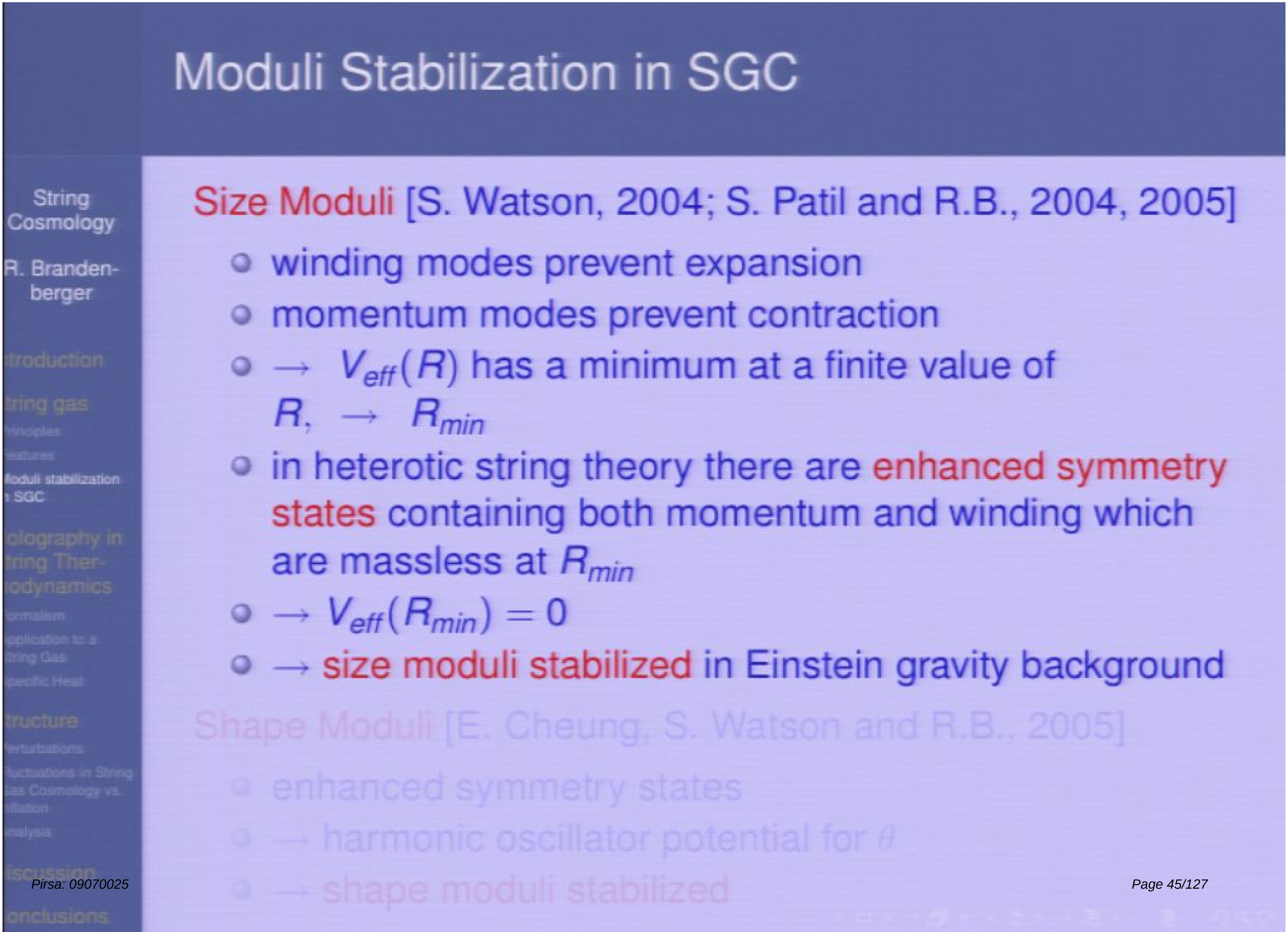

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Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
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Partition Function

Starting point: partition function of a string gas:

$$Z = \sum_s e^{-\beta\sqrt{-g_{00}}H(s)} . \tag{2}$$

The free energy follows:

$$F = -\frac{1}{\beta} lnZ . \tag{3}$$

The action follows from the free energy:

$$S = \int dt\sqrt{-g_{00}}F[g_{ij}, \beta] . \tag{4}$$

The action is used to determine the energy-momentum tensor.

Fluctuations of the Energy-Momentum Tensor

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} . \tag{5}$$

The thermal expectation value is

$$\langle T^{\mu}{}_{\nu} \rangle = 2\frac{g^{\mu\lambda}}{\sqrt{-g}}\frac{\delta \ln Z}{\delta g^{\nu\lambda}} \tag{6}$$

The fluctuations of the energy-momentum tensor are

$$\begin{aligned} \langle T^{\mu}{}_{\nu} T^{\sigma}{}_{\lambda} \rangle \;&-\; \langle T^{\mu}{}_{\nu} \rangle \langle T^{\sigma}{}_{\lambda} \rangle = 2\frac{g^{\mu\alpha}}{\sqrt{-g}}\frac{\partial}{\partial g^{\alpha\nu}}\left(\frac{g^{\sigma\delta}}{\sqrt{-g}}\frac{\partial \ln Z}{\partial g^{\delta\lambda}}\right) \\ &+2\frac{g^{\sigma\alpha}}{\sqrt{-g}}\frac{\partial}{\partial g^{\alpha\lambda}}\left(\frac{G^{\mu\delta}}{\sqrt{-g}}\frac{\partial \ln Z}{\partial g^{\delta\nu}}\right) . \end{aligned} \tag{7}$$

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Correlation Functions

The scalar metric fluctuations are determined by the energy density correlation function

$$\begin{aligned} \langle\delta\rho^2\rangle &= \langle\rho^2\rangle - \langle\rho\rangle^2 && (8) \\ &= -\frac{1}{R^6}\frac{\partial}{\partial\beta}\left(F + \beta\frac{\partial F}{\partial\beta}\right) = \frac{T^2}{R^6}C_V\,. && (9) \end{aligned}$$

The Specific heat capacity is given by

$$C_V = (\partial E/\partial T)|_V\,, \qquad (10)$$

where

$$E \equiv F + \beta\left(\frac{\partial F}{\partial\beta}\right)\,, \qquad (11)$$

Fluctuations of the Energy-Momentum Tensor

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} . \tag{5}$$

The thermal expectation value is

$$\langle T^{\mu}{}_{\nu}\rangle = 2\frac{g^{\mu\lambda}}{\sqrt{-g}}\frac{\delta \ln Z}{\delta g^{\nu\lambda}} \tag{6}$$

The fluctuations of the energy-momentum tensor are

$$\begin{aligned}\langle T^{\mu}{}_{\nu} T^{\sigma}{}_{\lambda}\rangle \quad - \quad \langle T^{\mu}{}_{\nu}\rangle\langle T^{\sigma}{}_{\lambda}\rangle &= 2\frac{g^{\mu\alpha}}{\sqrt{-g}}\frac{\partial}{\partial g^{\alpha\nu}}\left(\frac{g^{\sigma\delta}}{\sqrt{-g}}\frac{\partial \ln Z}{\partial g^{\delta\lambda}}\right) \\ &+2\frac{g^{\sigma\alpha}}{\sqrt{-g}}\frac{\partial}{\partial g^{\alpha\lambda}}\left(\frac{G^{\mu\delta}}{\sqrt{-g}}\frac{\partial \ln Z}{\partial g^{\delta\nu}}\right) .\end{aligned} \tag{7}$$

Correlation Functions

The scalar metric fluctuations are determined by the energy density correlation function

$$\langle \delta\rho^2 \rangle = \langle \rho^2 \rangle - \langle \rho \rangle^2 \tag{8}$$

$$= -\frac{1}{R^6}\frac{\partial}{\partial\beta}\left(F + \beta\frac{\partial F}{\partial\beta}\right) = \frac{T^2}{R^6}C_V\,. \tag{9}$$

The Specific heat capacity is given by

$$C_V = (\partial E/\partial T)|_V\,, \tag{10}$$

where

$$E \equiv F + \beta\left(\frac{\partial F}{\partial\beta}\right)\,, \tag{11}$$

Gravitational Waves

The spectrum of gravitational waves is determined by the off-diagonal pressure fluctuations:

$$\langle \delta {T^i}_j^{\,2} \rangle = \langle {T^i}_j^{\,2} \rangle - \langle {T^i}_j \rangle^2 , \qquad (12)$$

with $i \neq j$.

$$\langle \delta {T^i}_j^{\,2} \rangle = \frac{1}{\beta R^3} \frac{\partial}{\partial \ln R} \left(- \frac{1}{R^3} \frac{\partial F}{\partial \ln R} \right) = \frac{1}{\beta R^2} \frac{\partial p}{\partial R} , \qquad (13)$$

The pressure is given by

$$p \equiv - \frac{1}{V} \left(\frac{\partial F}{\partial \ln R} \right) = T \left(\frac{\partial S}{\partial V} \right)_E . \qquad (14)$$

String Gas In the Hagedorn Phase I

Starting point: the following expression for the entropy:

$$S(E,R) = \ln \Omega(E,R) \tag{15}$$

in terms of the density of states $\Omega(E,R)$.

Close to the Hagedorn temperature (see Deo, Jain and Tan, 1992)

$$\Omega(E,R) \simeq \beta_H e^{\beta_H E + n_H V}[1 + \delta\Omega_{(1)}(E,R)], \tag{16}$$

where

$$\delta\Omega_{(1)}(E,R) = -\frac{(\beta_H E)^5}{5!} e^{-(\beta_H - \beta_1)(E - \rho_H V)} \ll 1 \tag{17}$$

deep in the Hagedorn phase and

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$$\rightarrow S(E,R) \simeq \beta_H E + n_H V + \ln\left[1+\delta\Omega_{(1)}\right], \qquad (19)$$

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Specific Heat and Correlation Functions

The correlation functions of interest are:

$$C_V \approx \frac{R^2 / l_s^3}{T\,(1 - T/T_H)}\,, \tag{21}$$

and

$$\langle \delta {T^i}_j{}^2 \rangle \simeq \frac{T(1 - T/T_H)}{l_s^3 R^4} \ln^2 \left[\frac{R^2}{l_s^2} (1 - T/T_H) \right]. \tag{22}$$

Note:

- holographic scaling of the correlation functions!
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2. String Gas Cosmology
   - Principles
   - Features of String Gas Cosmology
   - Moduli stabilization in SGC
3. Holography in String Thermodynamics
   - Formalism
   - Application to a String Gas
   - Specific Heat
4. String Gas Cosmology and Structure Formation
   - **Review of the Theory of Cosmological Perturbations**
   - **Fluctuations in String Gas Cosmology vs. Inflation**
   - **Analysis**
5. Discussion
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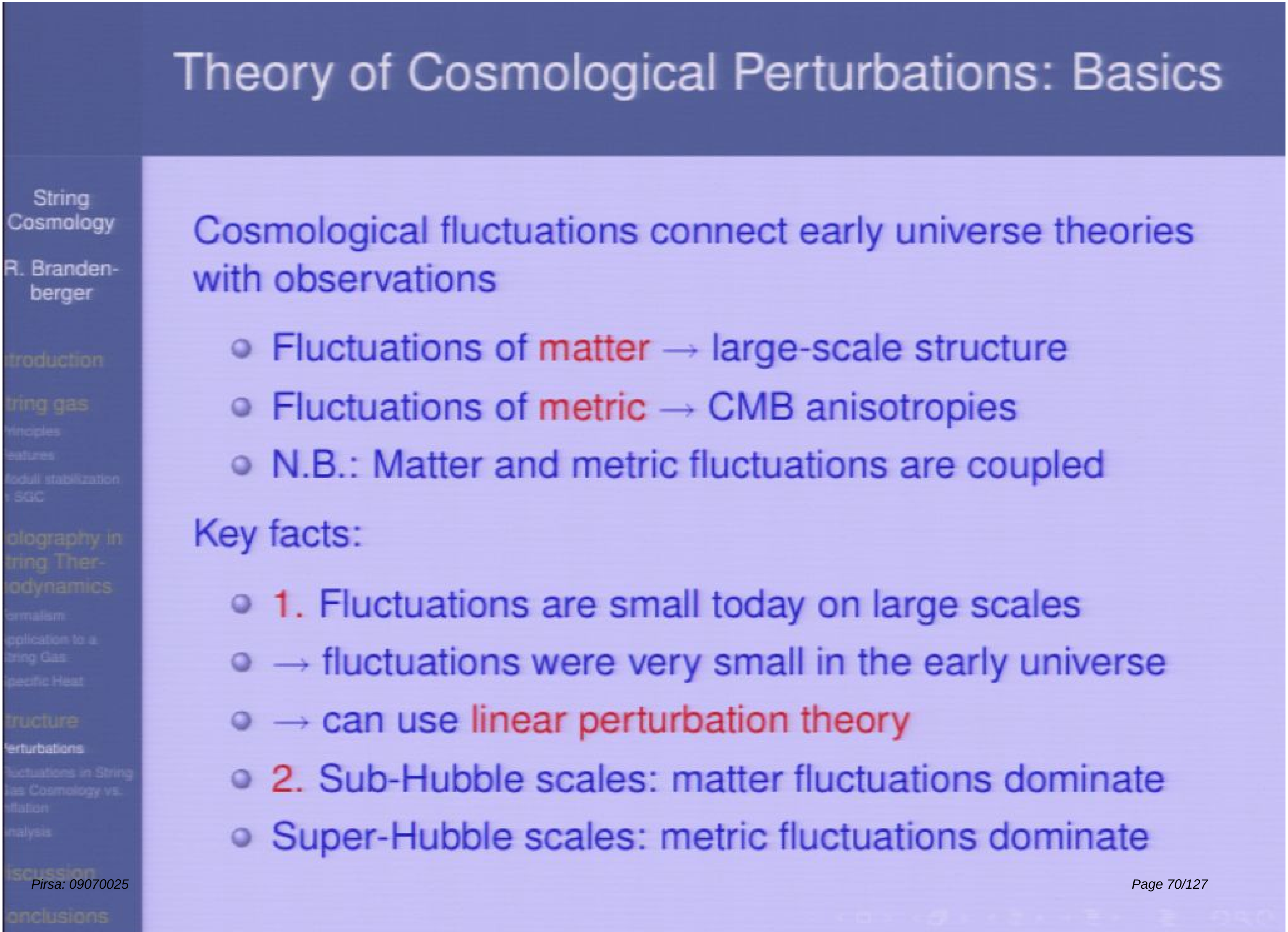
Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter → large-scale structure
- Fluctuations of metric → CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- → fluctuations were very small in the early universe
- → can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate



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Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$ds^2 = a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \qquad (23)$$

$$\varphi = \varphi_0 + \delta\varphi \qquad (24)$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \qquad (25)$$

$$v = a\left(\delta\varphi + \frac{z}{a}\Phi\right) \qquad (26)$$

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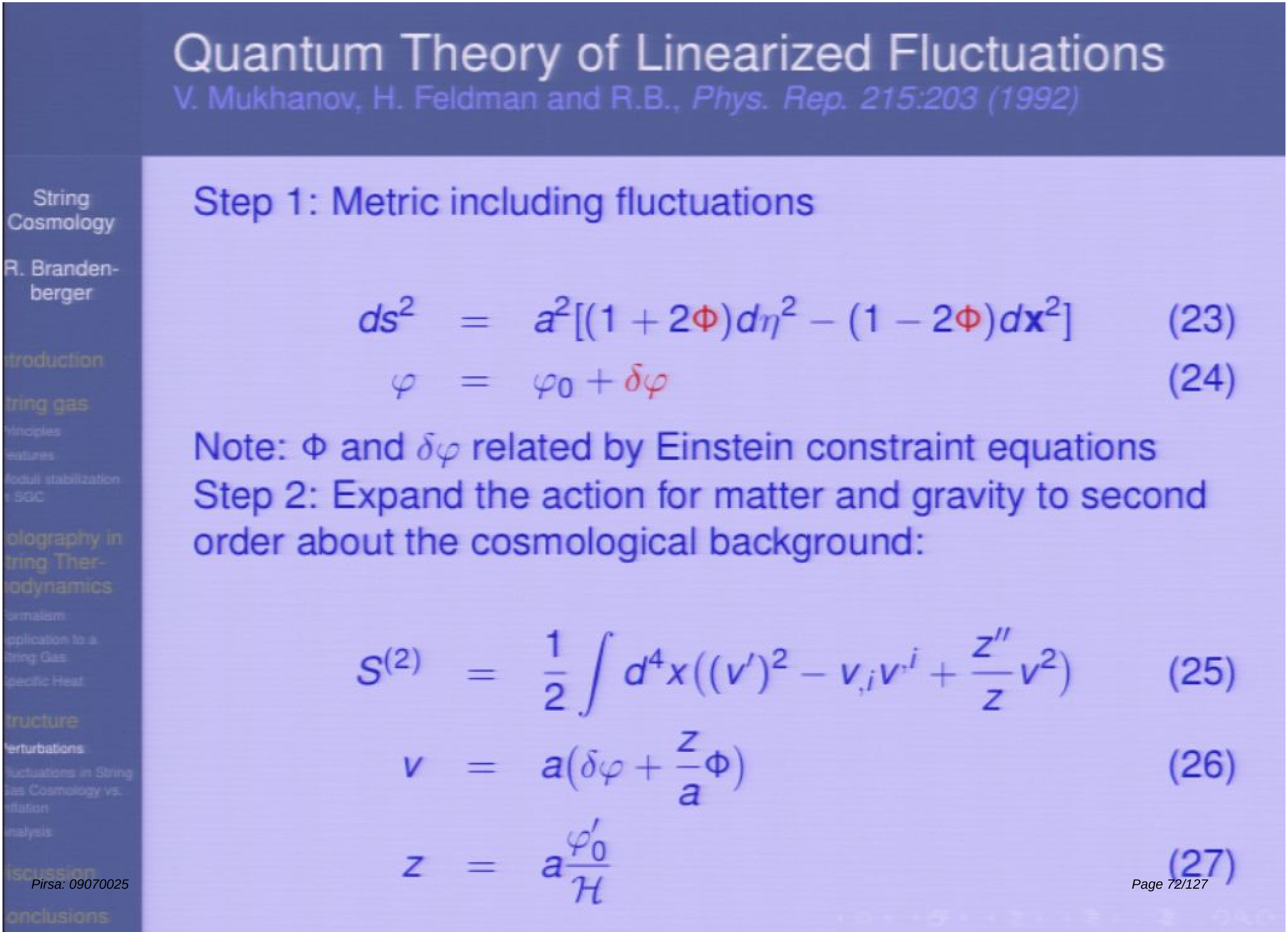

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R. Brandenberger

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$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0 \tag{28}$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1} \tag{29}$$

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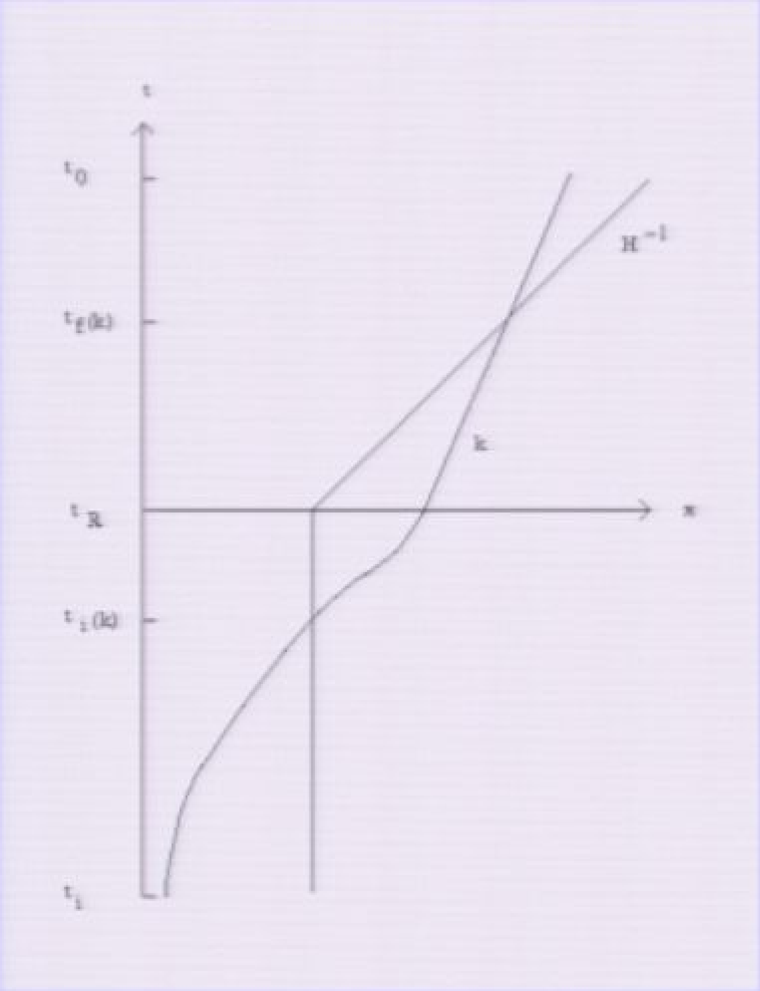

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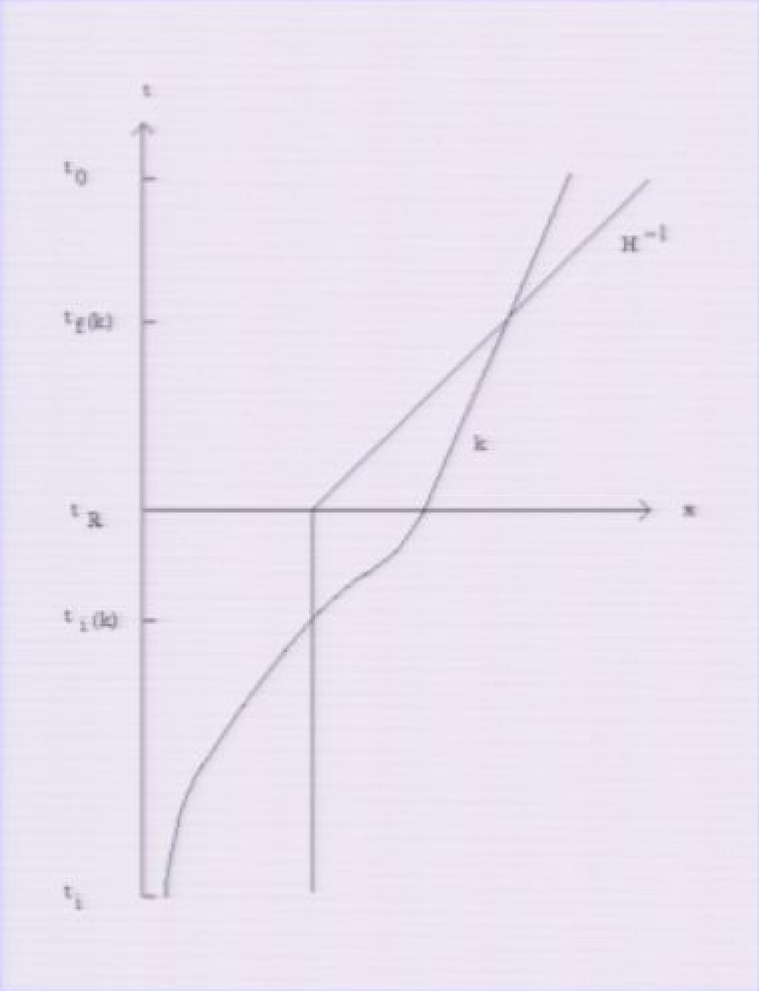

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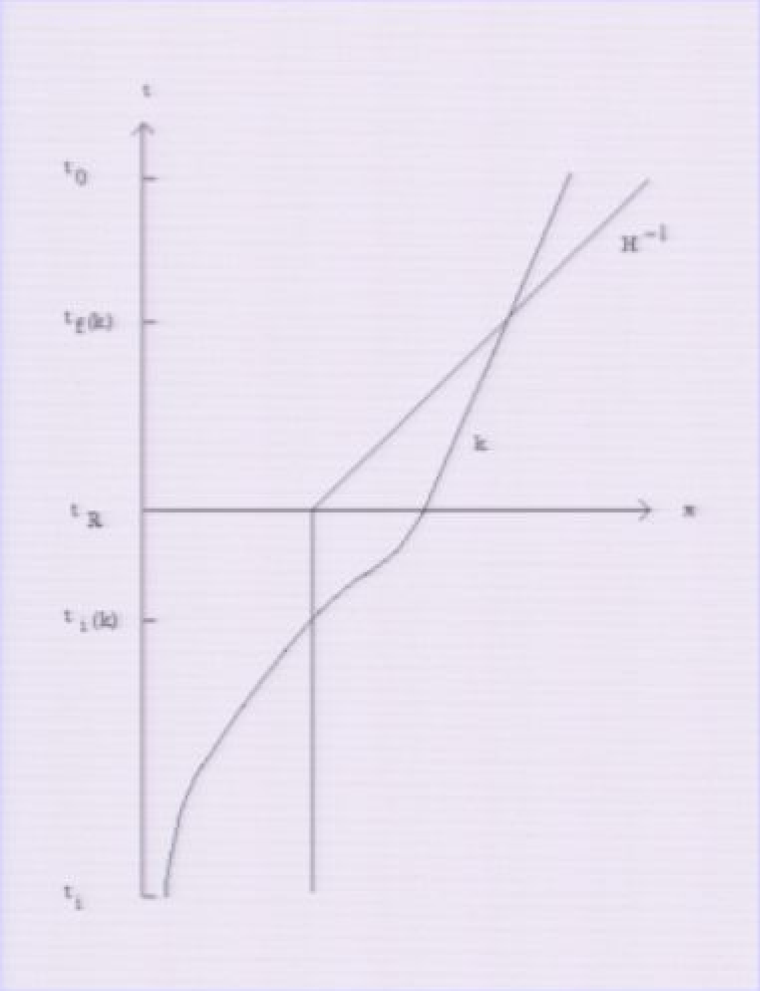

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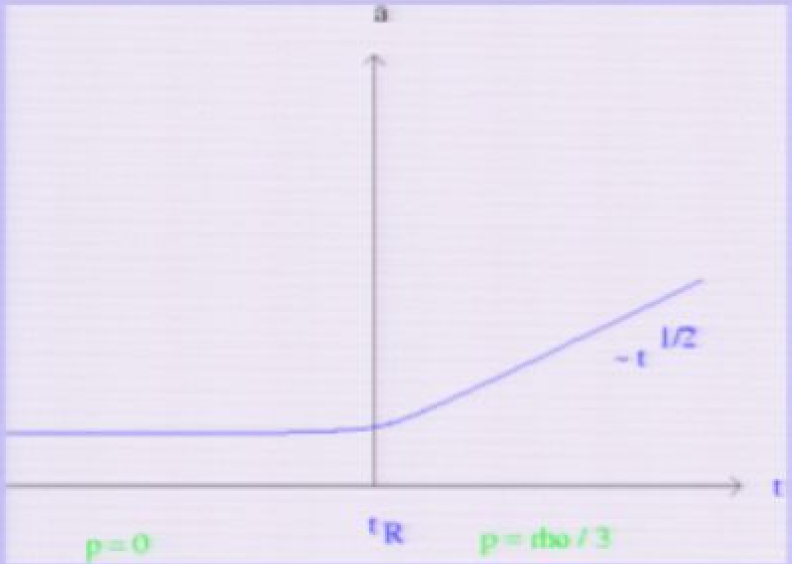
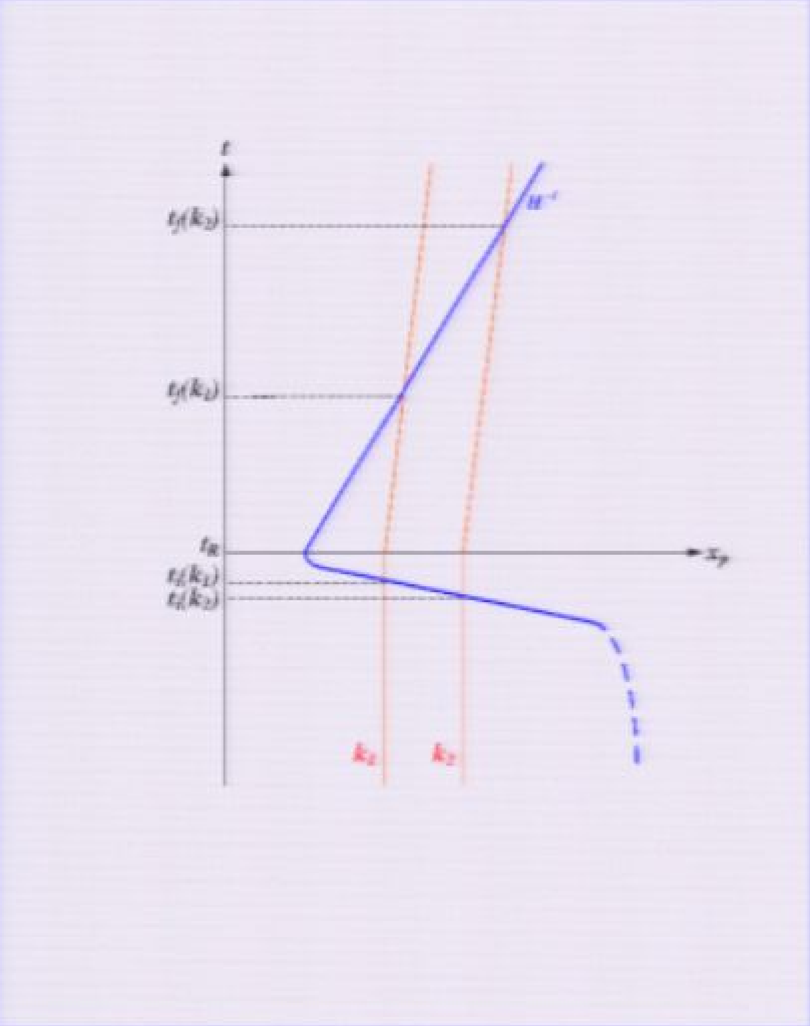
Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



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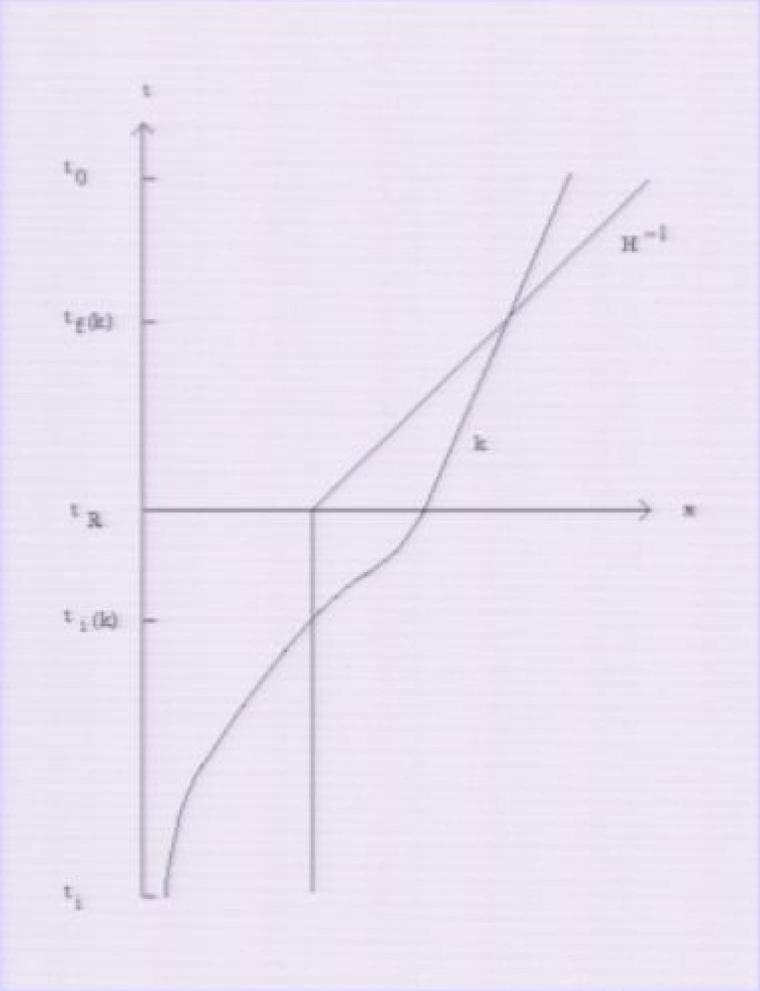

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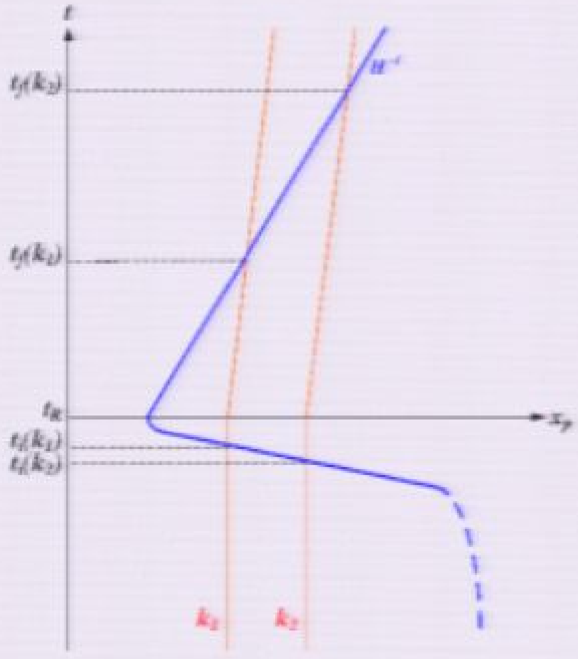

N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) . \quad (30)$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle , \quad (31)$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle . \quad (32)$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, . \qquad (33)$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \, . \qquad (34)$$

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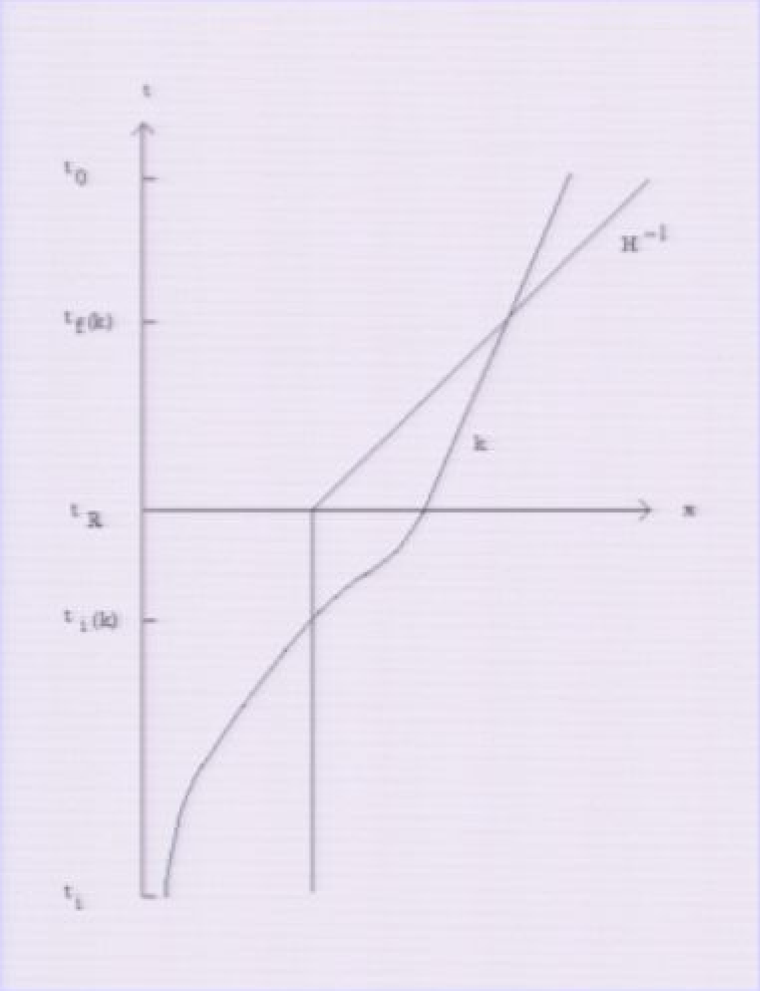

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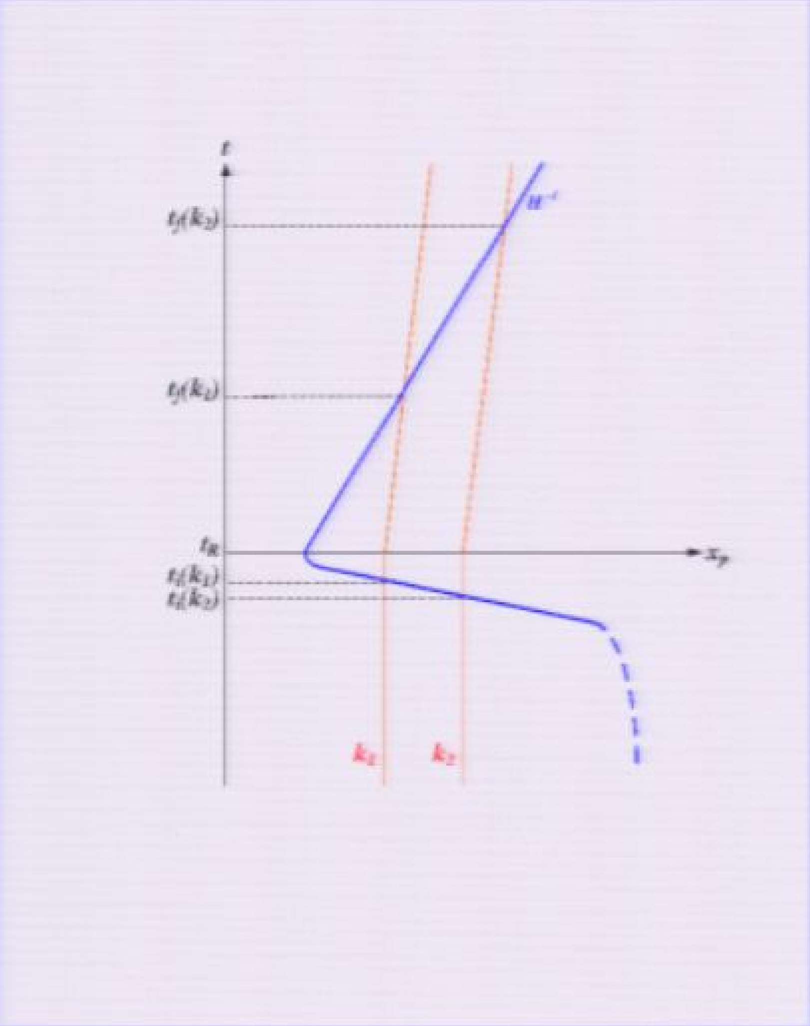

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$$P_\Phi(k) = 8G^2 k^{-1} < |\delta\rho(k)|^2 > \quad (35)$$

$$= 8G^2 k^2 < (\delta M)^2 >_R \quad (36)$$

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Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology → acoustic oscillations in the CMB angular power spectrum
- In a dilaton gravity background the dilaton fluctuations dominate → different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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Requirements

- static Hagedorn phase (including static dilaton) → new physics required.
- holographic scaling $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

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Weak Points

In its present form, the string gas cosmology structure formation scenario faces challenges:

- No consistent effective field theory description of the dynamics (for a toy model, see however R.B., A. Frey and S. Kanno, 2007).
- Keeping the metric flat during the Hagedorn phase is unrealistic.

Question: Is there an improved description of the Hagedorn phase?

Holographic Cosmology V

G. Veneziano, hep-th/0312182

- Hagedorn phase: $p = 0$ gas of string holes with $r_s = H^{-1}$.
- string hole: black hole on the string correspondence curve $M = M_s g_s^{-2}$
- satisfies cosmological entropy bound: $\sigma = \sigma_{max} \equiv HM_p^2$
- string holes decay into radiative string states
- $\rightarrow$ transition between $p = 0$ and $p = 1/3\rho$ state

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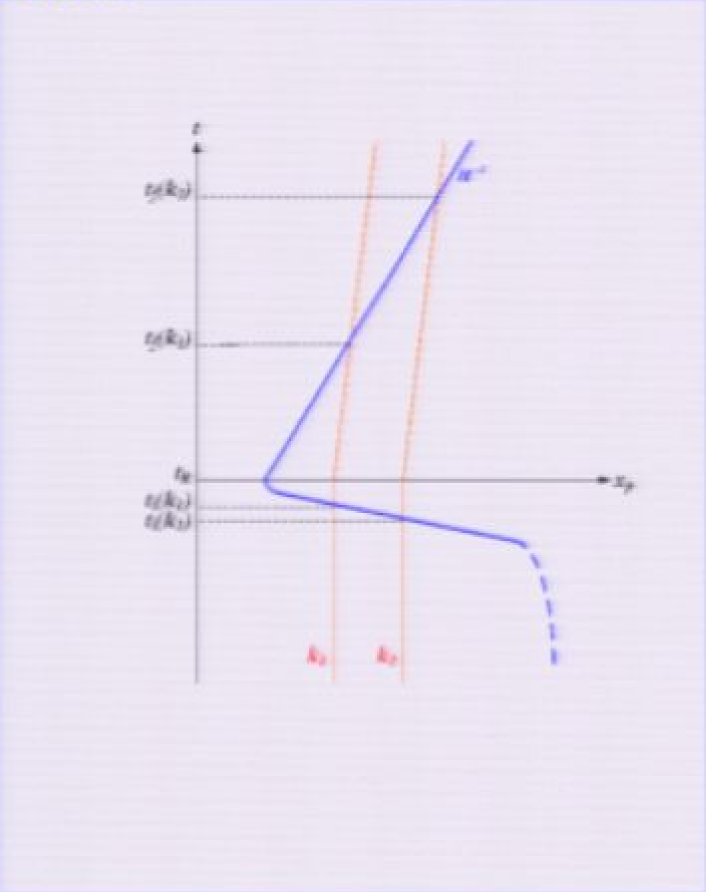
Assume Deep in the Hagedorn phase the universe is almost static



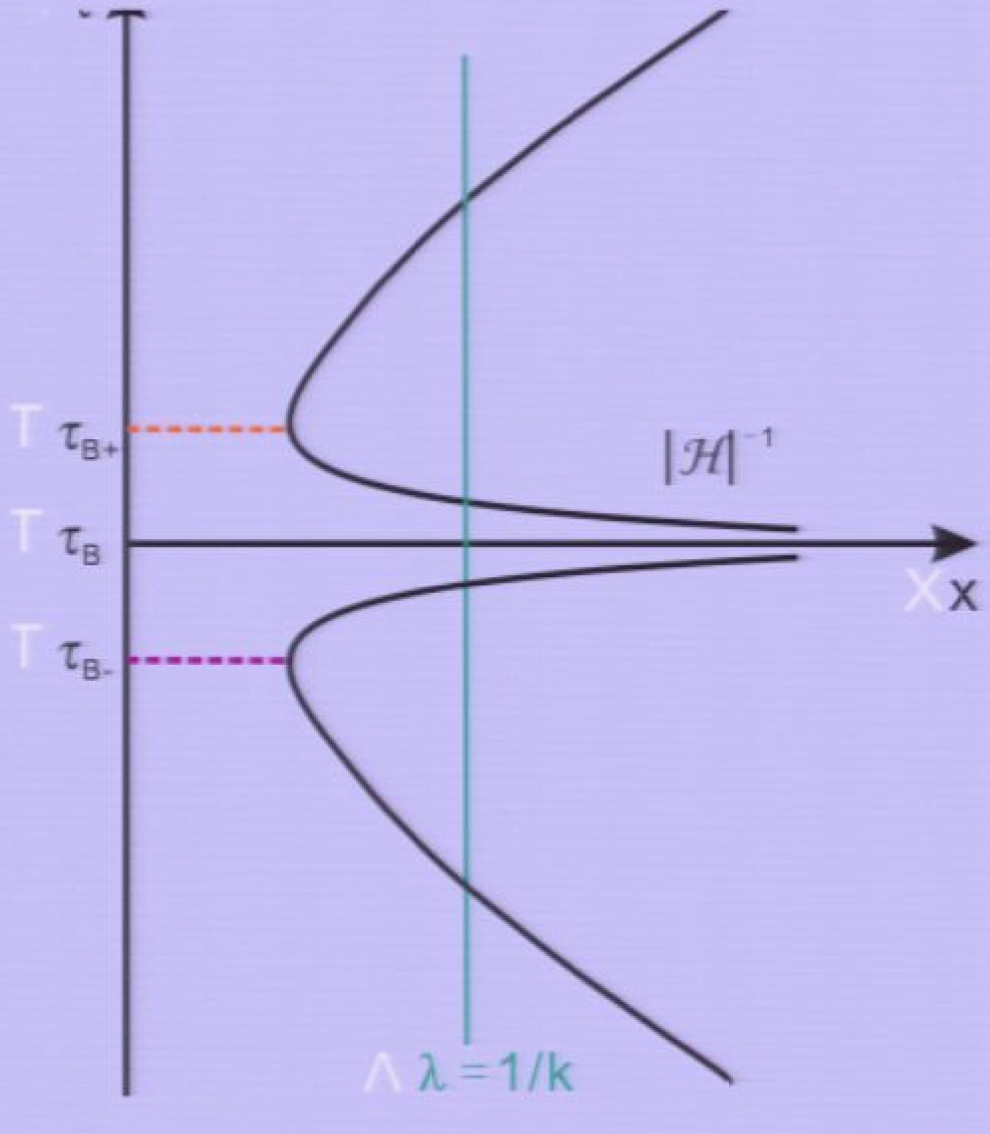
Also: holographic scaling $C_V(R) \sim R^2$.

Matter Bounce

F. Finelli and R.B., Phys. Rev. **D65** (2002); Y. Cai et al, arXiv:0810.4677

- Message: In the context of a bouncing cosmology (possibly obtained by AdS/CFT approaches to singularity resolution) there is another way to obtain a scale-invariant spectrum of cosmological perturbations: a matter bounce.
- Matter bounce: bouncing cosmology with a matter-dominated phase of contraction when the relevant scales exit the Hubble radius.
- Statement: In a matter bounce setup, fluctuations which are in their vacuum state on sub-Hubble scales early in the contracting phase evolve into a scale-invariant spectrum of curvature fluctuations on super-Hubble scales at late times.

Space-Time Sketch



Key Points

- Vacuum spectrum: $P_\zeta(k) \sim k^2$
- → need a boosting of IR modes relative to UV modes.
- In a contracting universe the dominant mode of ζ is growing on super-Hubble scales (whereas it is constant in an expanding phase).
- Long wavelengths are super-Hubble for a longer time → preferential growth of IR modes.
- For a matter-dominated phase of contraction the boost is exactly right to convert a vacuum spectrum into a scale-invariant one.

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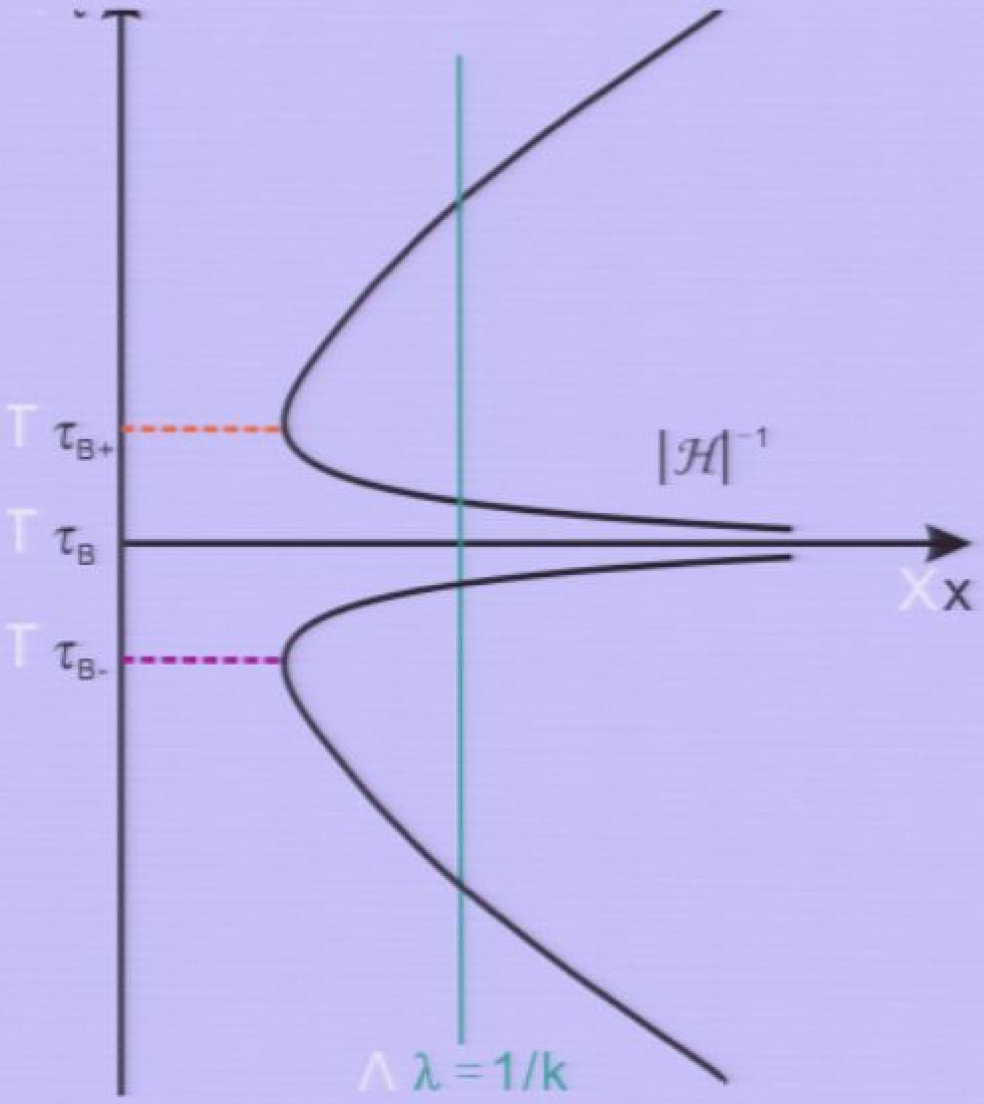

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   - Principles
   - Features of String Gas Cosmology
   - Moduli stabilization in SGC
3. Holography in String Thermodynamics
   - Formalism
   - Application to a String Gas
   - Specific Heat
4. String Gas Cosmology and Structure Formation
   - Review of the Theory of Cosmological Perturbations
   - Fluctuations in String Gas Cosmology vs. Inflation
   - Analysis
5. Discussion
6. **Conclusions**

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- SGC → nonsingular cosmology
- SGC → natural explanation of the number of large spatial dimensions.
- Holographic scaling of SGC correlation functions → new scenario of structure formation
- Scale invariant spectrum of cosmological fluctuations (like in inflationary cosmology).
- Spectrum of gravitational waves has a small blue tilt (unlike in inflationary cosmology).
- But we need a better model of the Hagedorn phase.

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Holographic Cosmology V

G. Veneziano, hep-th/0312182

- Hagedorn phase: $p = 0$ gas of string holes with $r_s = H^{-1}$.
- string hole: black hole on the string correspondence curve $M = M_s g_s^{-2}$
- satisfies cosmological entropy bound: $\sigma = \sigma_{max} \equiv HM_P^2$
- string holes decay into radiative string states
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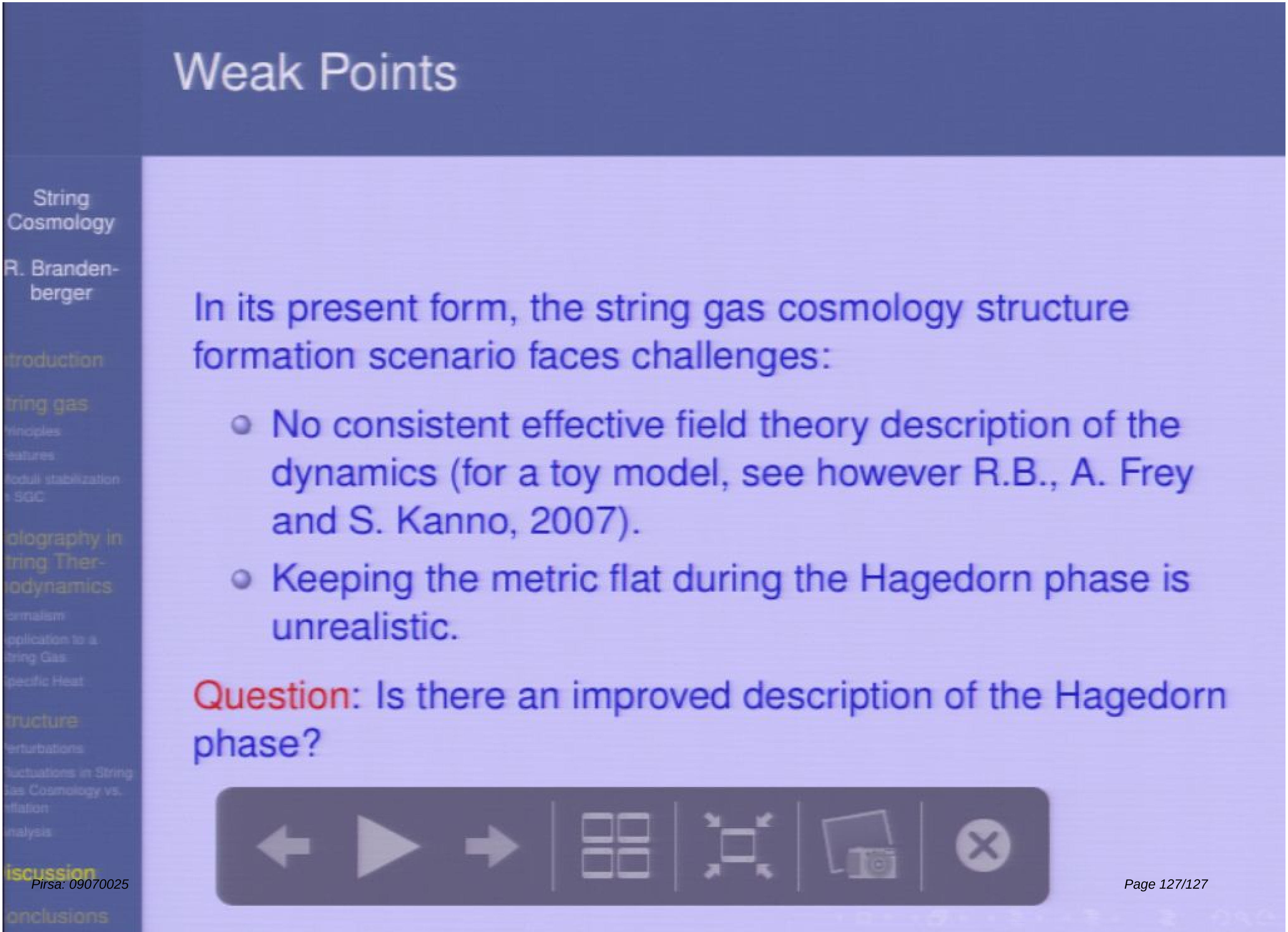
In its present form, the string gas cosmology structure formation scenario faces challenges:

- No consistent effective field theory description of the dynamics (for a toy model, see however R.B., A. Frey and S. Kanno, 2007).
- Keeping the metric flat during the Hagedorn phase is unrealistic.

Question: Is there an improved description of the Hagedorn phase?



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