Title: The No-Boundary Measure and Eternal Inflation

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Abstract: TBA

# The No-Boundary Measure in the Regime of Eternal Inflation

Perimeter Institute

July 2009

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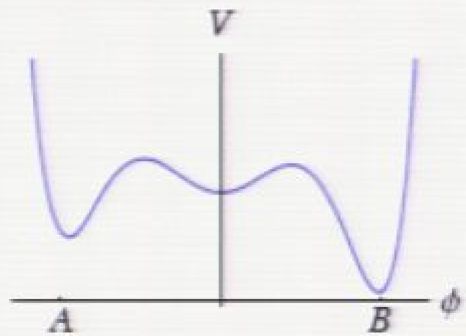
# Measure problem




The usual approach: $\frac{p_A}{p_B} = \frac{\langle N_A \rangle}{\langle N_B \rangle}$

By contrast, in quantum cosmology such relative probabilities are calculated from the wave function.

*Generally, quantum cosmology provides well-defined "bottom-up" probabilities for different "histories".*

# Probabilities for Observation

- Observations are restricted to part of a light cone extending over a Hubble volume located somewhere in spacetime.
- Probabilities for observations are conditioned on part of our data $D$ that describe the local observational situation.
- In quantum cosmology, there is a probability that $D$ occurs in any spacetime volume.
- In (very) large universes the probability may become significant that our local observational situation is replicated elsewhere.
- All we know is that the universe exhibits at least one region with data $D$ somewhere in spacetime.

# Probabilities for Observation

Probabilities for observation therefore involve "top-down" probabilities conditioned on $D^{\geq 1}$.

$$p(\mathcal{F}|D^{\geq 1})$$

Top-down probabilities are calculated by summing the bottom-up probabilities of different histories, weighted by the probability that $D$ occurs at least once somewhere in spacetime.

The observable $\mathcal{F}$ can be a local or global property of the universe.

# Probabilities for Observation

$$p(\mathcal{F}|D^{\geq 1}) = \frac{p(\mathcal{F}, D^{\geq 1})}{p(D^{\geq 1})}$$

Let $\phi_0$ label the different possible histories. Then

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where $p(\mathcal{F}, \phi_0)$ is the bottom-up probability of $\mathcal{F}$ in the history labeled by $\phi_0$.

Focus on $D$ that specify the observational situation to be somewhere on (possibly many) spacelike surfaces.

Let $p_E$ be the probability that $D$ occurs in any one of the Hubble volume on these surfaces. Then

$$p(D^{\geq 1}|\mathcal{F}, \phi_0) = 1 - [1 - p_E]^{N_h(\mathcal{F}, \phi_0)}$$

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# Volume Weighting

$$p(\mathcal{F}|D^{\geq 1}) = \frac{\int d\phi_0\ p(\mathcal{F},\phi_0)\{1-[1-p_E]^{N_h}\}}{\int d\mathcal{F} d\phi_0\ \{1-[1-p_E]^{N_h}\} p(\mathcal{F},\phi_0)}$$

Top-down weighting simplifies if data $D$ are rare in all histories predicted with any significant probability by the wave function.

If

$$p_E(D) << 1/N_h(\phi_0) \qquad \text{for all } \phi_0$$

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*In histories where our data are rare, volume weighting connects bottom-up to top-down probabilities.*

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# Replication

Volume weighting only applies when $p_E < 1/N_h$.

In histories where $D$ specifies very large or infinite spacelike surfaces the more general weighting applies:

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*This takes in account the probability that our data are replicated. It provides a well behaved, normalizable measure for prediction even in infinite universes.*

In models where $D$ is common in all histories:

$$p(\mathcal{F}|D^{\geq 1}) \propto \int d\phi_0\ p(\mathcal{F},\phi_0)$$

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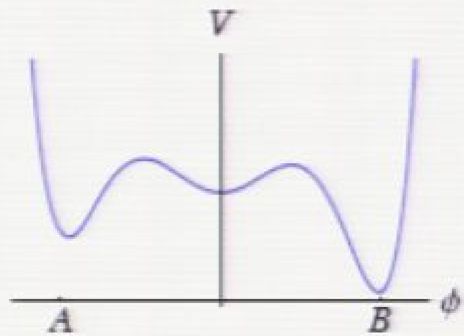


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# "No-Boundary" wave function

$$\Psi[h,\chi] = \int_0^{\Sigma} \delta g \delta \phi \, \exp(-\mathrm{I_E}[\mathrm{g}, \phi])$$

*" The integral is over all metrics $g$ and matter fields $\phi$ which are regular on a disk and match $(h,\chi)$ on its boundary."* [Hartle & Hawking '83]

→ toy model quantum cosmology

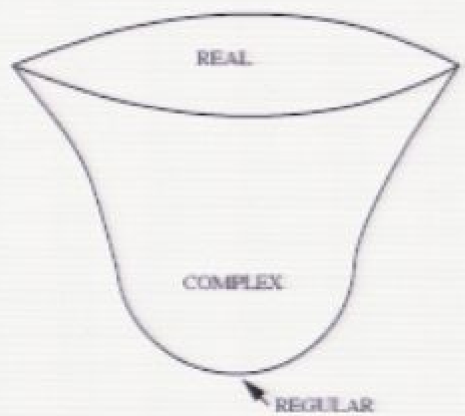
## Semiclassical Approximation: Fuzzy Instantons

In some regions of (mini)superspace the wave function may be evaluated in the steepest descents approximation.

To leading order in $\hbar$ the NBWF will then have the semiclassical form,

$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}$$

In general the extremal geometries will be complex:



# Lorentzian histories in Quantum Cosmology

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The semiclassical wave function specifies Lorentzian cosmologies if at the boundary

$$|\nabla_A I_R| \ll |\nabla_A S_L|$$

[Hawking '84, Grischuk & Rozhansky '90]

The predicted cosmologies are then the integral curves of $S_L$:

$$p_A = \nabla_A S_L$$

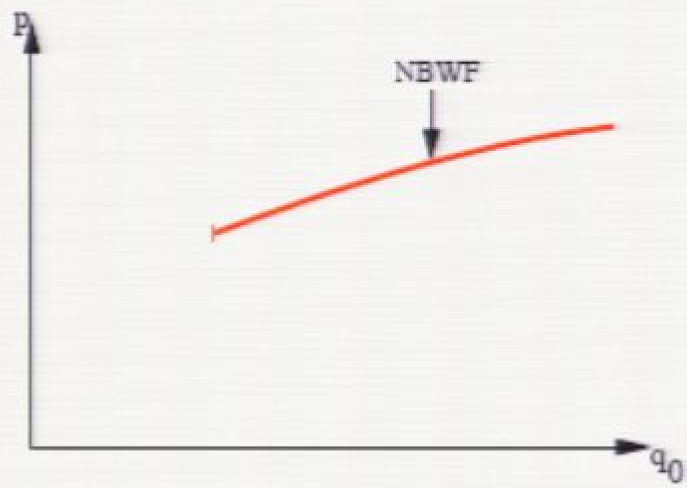
and have probability

$$P_{history} \propto \exp[-2\mathrm{I_R}/\hbar]$$

# Measure on Classical Phase Space

A wave function predicts an ensemble of universes that can be labeled by points in phase space.

→ provides measure on classical phase space.




Regularity on disk → slice through phase space

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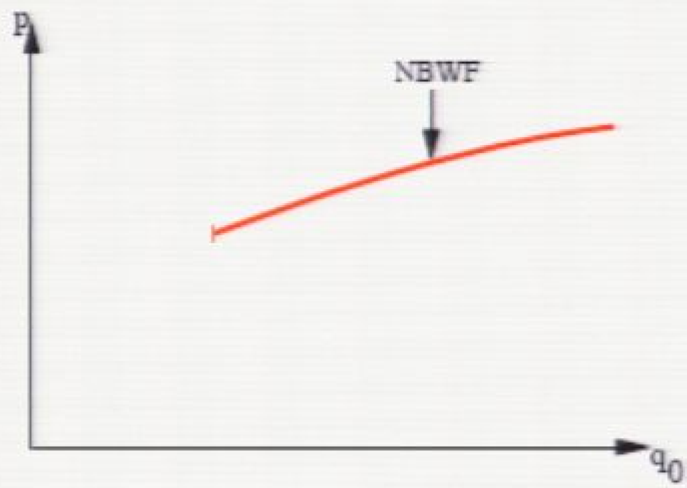
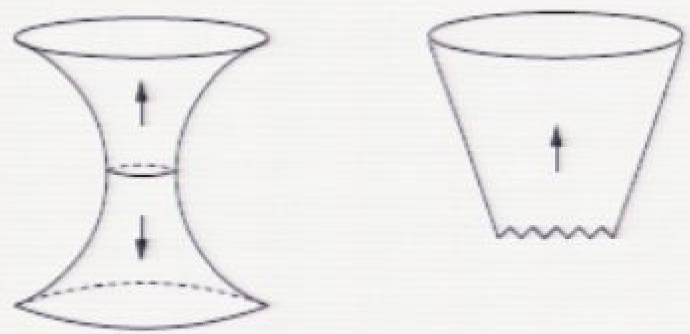



Regularity on disk → slice through phase space

# Classical Histories are Real!

Histories on slice are integral curves of $S$:

$$p_A = \nabla_A S$$

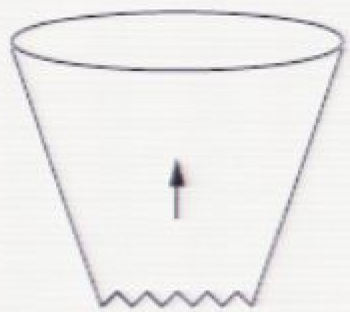


*The Lorentzian histories (universes) predicted by the NBWF are distinct from the complex extrema that provide the semiclassical approximation to the wave function.*

The role of the complex extrema is just to assign probabilities to all possible cosmologies.

## Singularity Resolution

A subset of the predicted Lorentzian histories may be singular in the past,



but probabilities for late time observables like CMB fluctuations are calculated directly from the NBWF.

$\longrightarrow$ singularity no longer an obstacle to prediction.

# Model

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What is ensemble of homogeneous isotropic universes?

$$ds^2 = (3/\Lambda)\left[d\tau^2 + a^2(\tau)d\Omega_3^2\right]$$

$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}$$

Dimensionless parameter: $\mu = (3/\Lambda)^{1/2} m$

# Saddle points

Field equations:

$$\dot{a}^2 - 1 + a^2 + a^2\left(-\dot{\phi}^2 + \mu^2\phi^2\right) = 0$$

$$\ddot{\phi} + 3(\dot{a}/a)\dot{\phi} - \mu^2\phi = 0$$

Regularity at SP: $a(0) = 0, \quad \dot{a}(0) = 1, \quad \dot{\phi}(0) = 0$

Free parameter at SP: $\phi(0) = \phi_0 e^{i\gamma}$

At boundary $\tau_f = X + iY$:

$$a(\tau_f) = b, \quad \phi(\tau_f) = \chi$$

$\rightarrow$ 4 real parameters at SP to meet 4 real conditions:

$$(\phi_0, \gamma, X, Y) \rightarrow (b, \chi, 0, 0)$$

$\rightarrow$ expect countable set of solutions for each $\phi_0$.

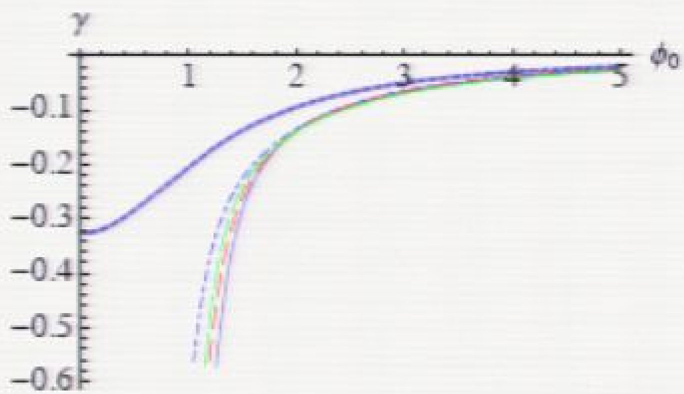
# Saddle points

For each $\phi_0$, tune remaining parameters $(\gamma, X, Y)$ to find curves in $(b, \chi)$ plane along which $I_R$ approaches a constant at large $b$.

This ensures universe obeys Lorentzian Einstein eqs at boundary

$$|\nabla_A I_R| \ll |\nabla_A S|$$

$\rightarrow$ (at most) a unique complex solution for each $\phi_0$

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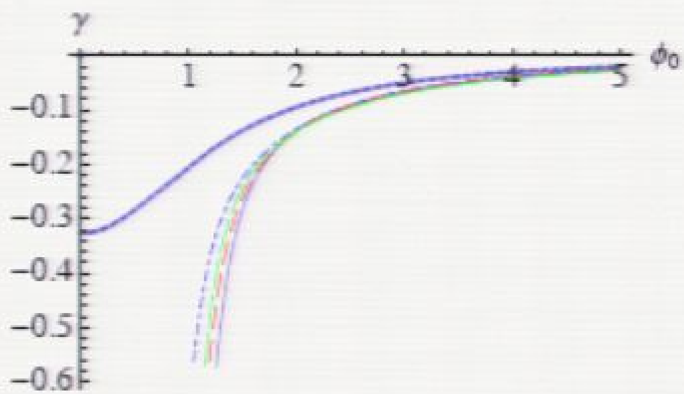


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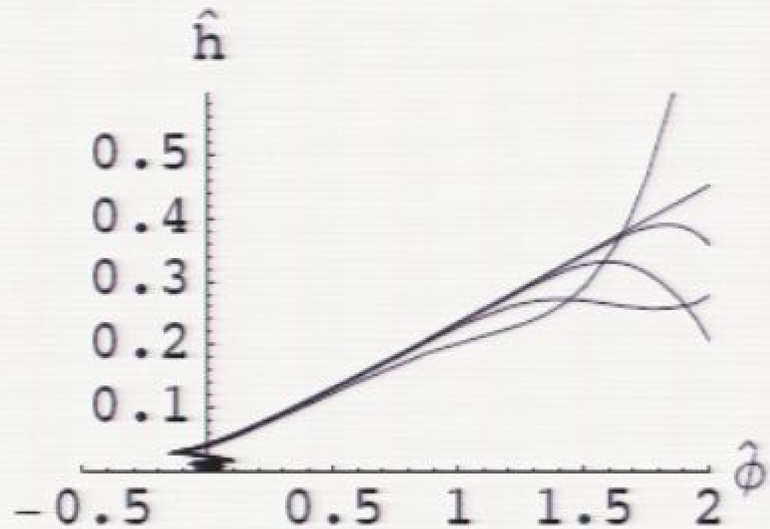
# Inflation

The complex saddle points provide Cauchy data for Lorentzian histories at the boundary $a = b, \phi = \chi$.

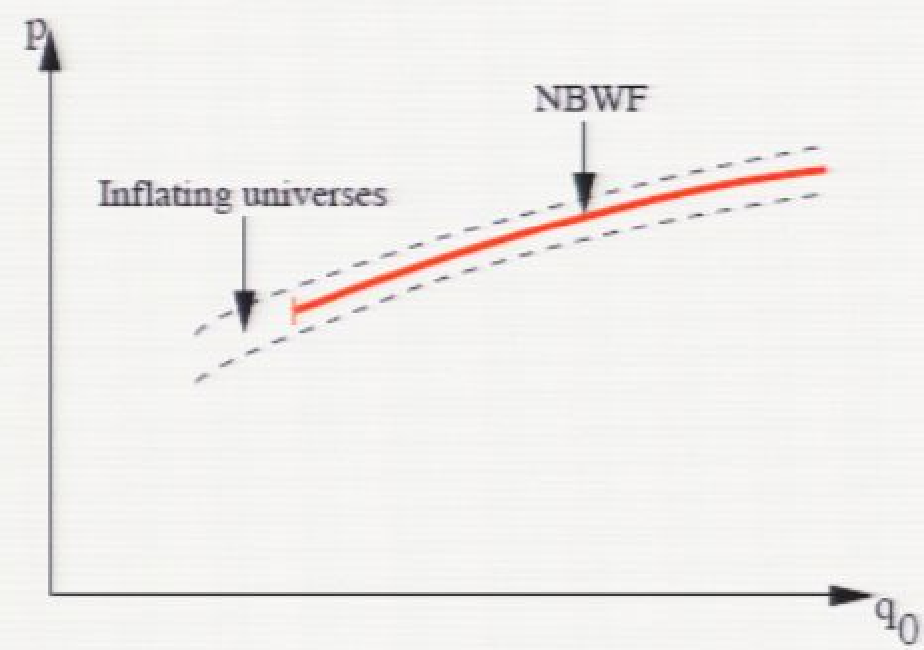
Extrapolate backward/forward using the *Lorentzian* equations to find behavior at early/late times.




*All Lorentzian universes predicted by NBWF inflate at early times:* $\hat{h} = m\hat{\phi}$

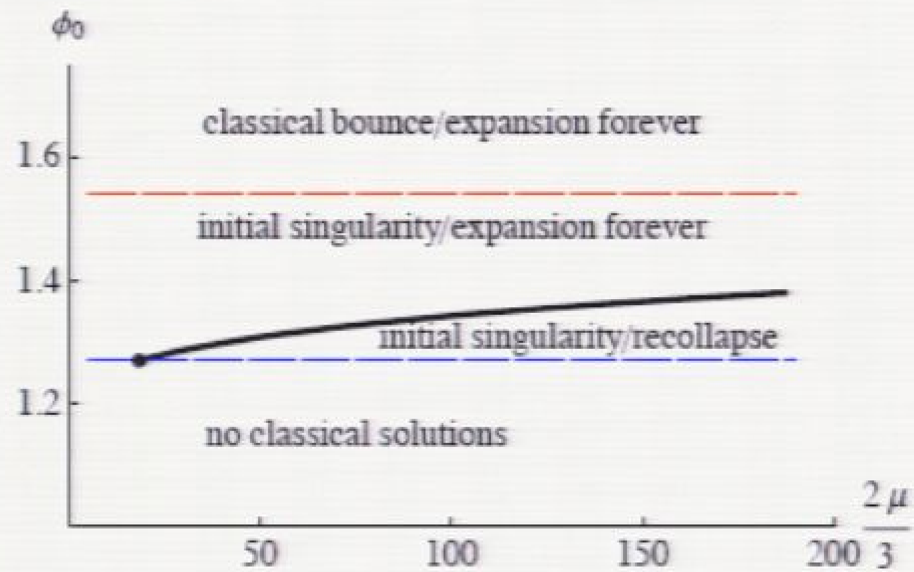
# Inflation

→ The NBWF selects inflating histories




which are exponentially improbable $\Delta \sim e^{-3N}$ with a flat measure on phase space [Gibbons & Turok '06].

# Origin and Future




→ large class of predicted inflationary universes are regular in the past

# Probabilities of Histories

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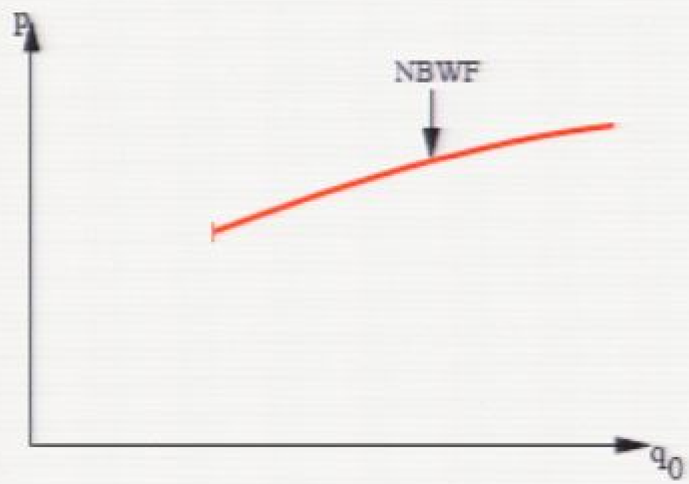
$$p(\phi_0) \sim \exp[-2\mathrm{I}_\mathrm{R}/\hbar]$$



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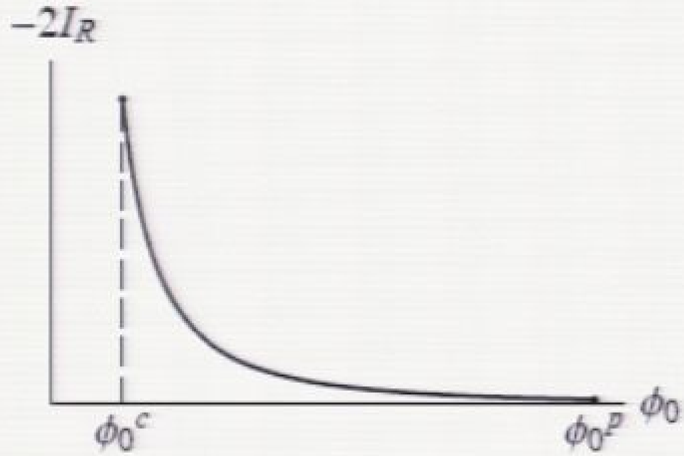


$$I_R \approx -\frac{\pi}{2(m\phi_0)^2} \approx -\frac{\pi}{3m^2N}$$

The "bottom-up" probabilities favor histories with a small number of e-folds.

# E-folds of Inflation

Top-down probabilities:

$$p(N|D^{\geq 1}) \propto \{1-[1-p_E(D)]^{\exp[3N]}\}e^{2\pi/m^2N}$$

For sufficiently small $p_E$:

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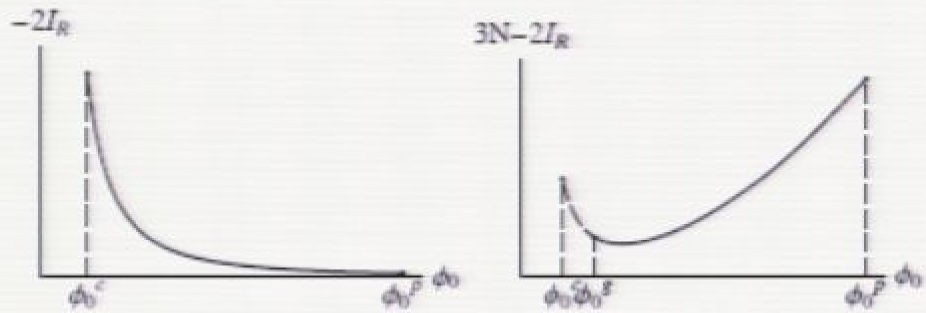


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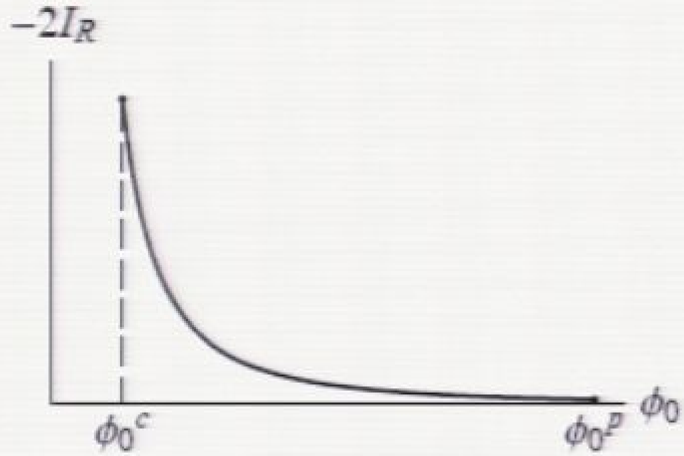


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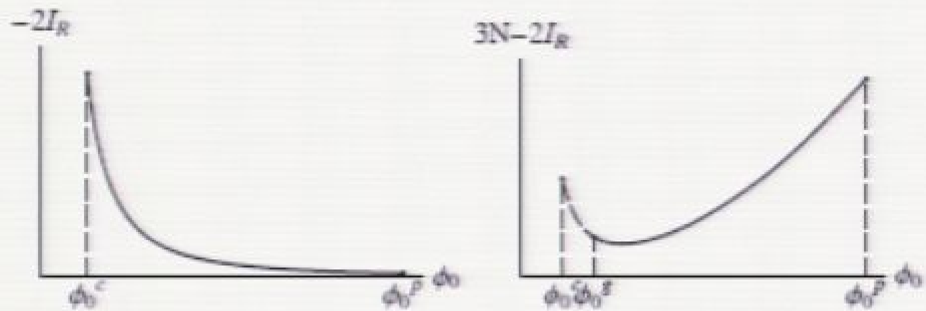


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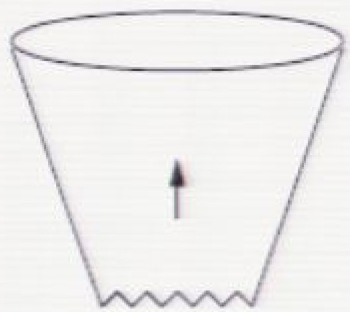

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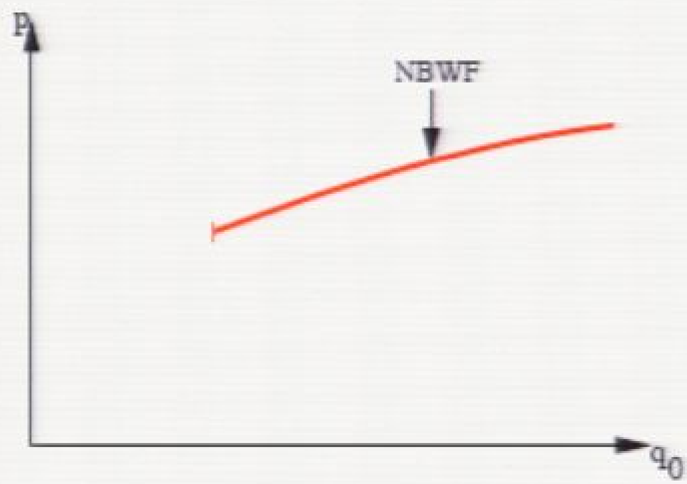

# Objectivity

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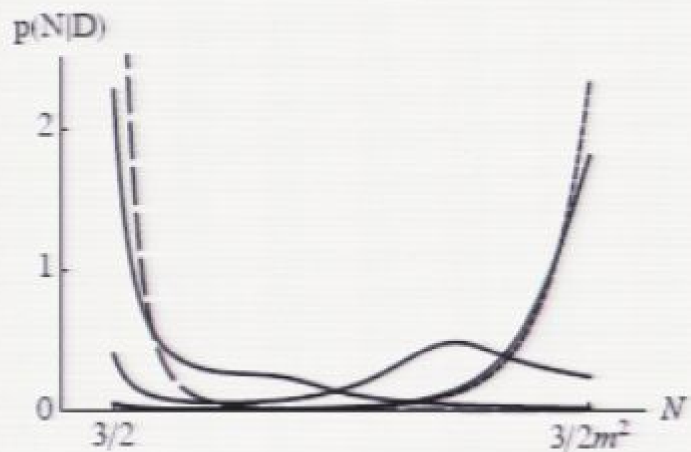



For realistic values of $p_E$ and $m^2$ volume weighting applies in the homog/isotr ensemble.

The top-down probabilities are then independent of the precise value of $p_E$ and the data that determine it.

# Inhomogeneities

[Hawking, LaFlamme, Lyons '93]

Perturbed metric:

$$ds^2 = (1 + 2\varphi)d\tau^2 + 2a(\tau)B_{|i}dx^i d\tau$$

$$+a(\tau)^2[(1 - 2\psi)\gamma_{ij} + 2E_{|ij}]dx^i dx^j$$

Expansion in modes on $S^3$:

$$\varphi = \sum_n g_n \frac{Q^n}{\sqrt{6}}, \qquad \psi = \sum_n -(a_n + b_n)\frac{Q^n}{\sqrt{6}},$$

$$B = \sum_n k_n \frac{Q^n}{(n^2-1)\sqrt{6}}, \quad E = \sum_n b_n \frac{3Q^n}{(n^2-1)\sqrt{6}}$$

and the scalar field perturbation

$$\delta\phi(\tau, x) = \sum_n f_n \frac{Q^n}{\sqrt{6}}$$

$\rightarrow$ five scalar degrees of freedom.

## Complex Perturbations

Constraints: [Shirai & Wada '88]

$$\Psi_n(b,\chi,a_n,b_n,f_n) \to \Psi_n(b,\chi,z_n)$$

where $z_n$ is the real boundary value of

$$\zeta_n = (a_n + b_n) - \frac{H}{\dot\phi} f_n$$

Semiclassical approximation:

$$\Psi(b,\chi,z) = \exp[-I(b,\chi,z)/\hbar]$$

where

$$I(b,\chi,z) = I^{(0)}(b,\chi) + \sum_n I^{(n)}(b,\chi,z_n)$$

is the action of perturbed complex saddle-points.

# Complex Perturbations

Extremum equations (in $b_n = k_n = 0$ gauge):

$$\ddot{a}_n + 4H\dot{a}_n - (3m^2\phi^2 - 2/a^2)a_n = -3\dot{\phi}\dot{f}_n - 3m^2\phi f_n$$

$$\ddot{f}_n + 3H\dot{f}_n - (m^2 + (n^2-1)/a^2)f_n = -4\dot{\phi}\dot{a}_n - 2m^2\phi a_n$$

$$\dot{a}_n + Ha_n = -3\dot{\phi}f_n$$

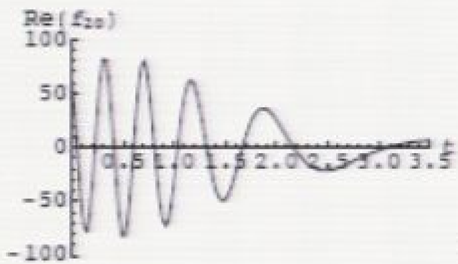
Regularity at South Pole: $a_n, f_n \to 0$

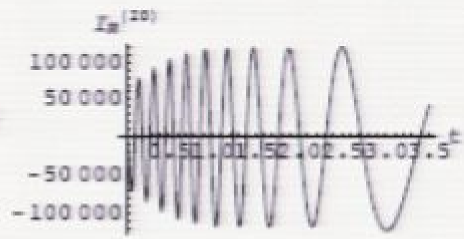
At boundary: tune phases of $\zeta_n(0)$ so that $z$ real.

$\to$ $\zeta_{n0} \equiv |\zeta_n(0)|$ and $\phi_0$ label ensemble of perturbed histories.

# Evolution Inside Horizon

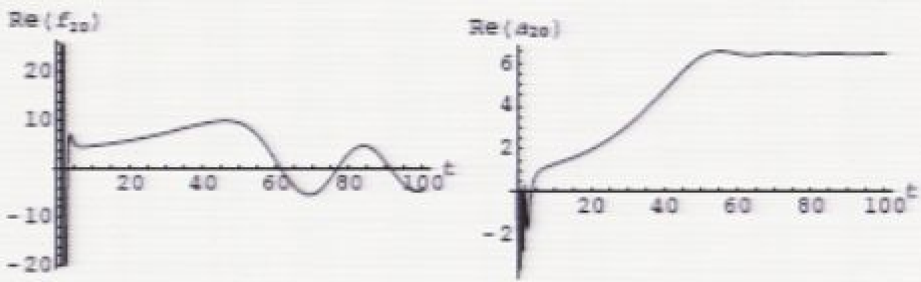
Inside horizon where $n/a >> H$, matter perturbation decouples and oscillates:



# Outside Horizon

At horizon crossing $n/a \sim H$ the nature of the solutions changes:



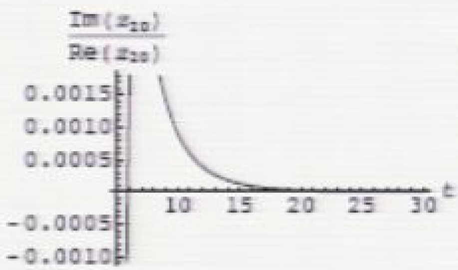

→ slowly growing matter/metric perturbations.

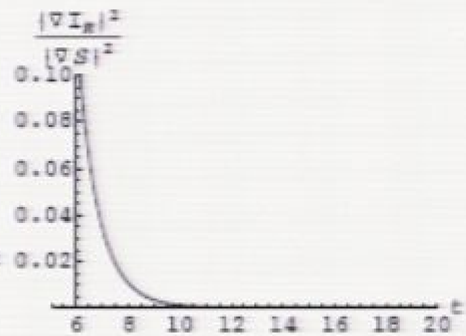
Gauge-invariant variable $\zeta_n$ tends to constant value

# Outside Horizon

Real boundary value $z$ means $\mathrm{Im}[\zeta_n] \to 0$ outside horizon.

$\longrightarrow$ classicality condition automatically holds:




$$p(z_n^2|\phi_0) \propto \exp[-(\epsilon_*/V_*)n^3 z_n^2]$$

$$\langle(\Delta T/T)^2\rangle \approx \langle z_n^2\rangle n^3 = V_*/\epsilon_*$$

# Probabilities for Observation

1. Probabilities for observing different values of one particular fluctuation mode in an otherwise homogeneous/isotropic ensemble, given a local observational situation $D$:

$$p(z_n|D^{\geq 1}) \sim \int d\phi_0 \; N_h(z_n, \phi_0) \; p(z_n, \phi_0)$$

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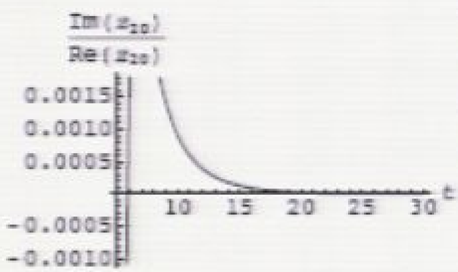

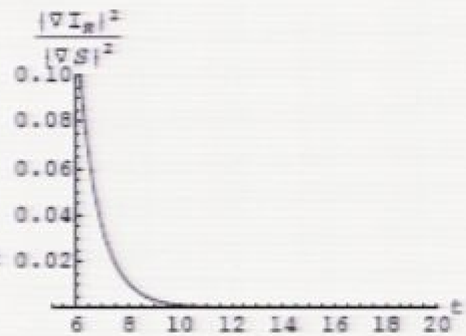

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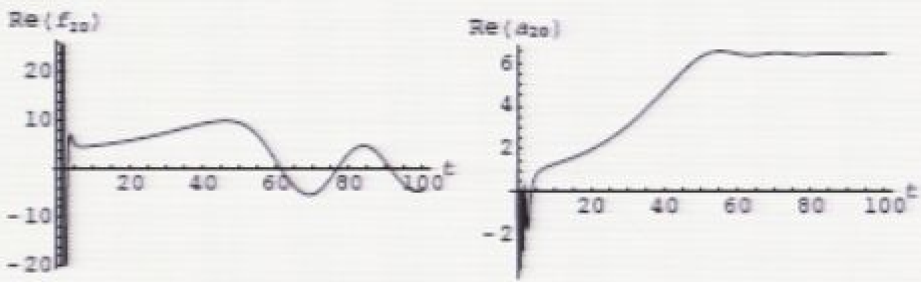


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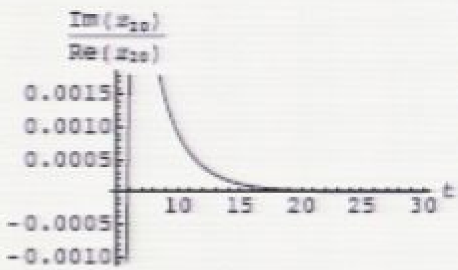


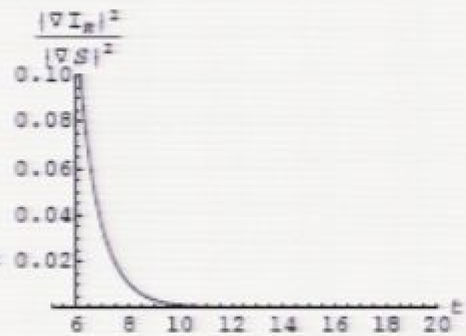


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# Probabilities for Observation

*In models of eternal inflation the observational situation is common in the dominant histories.*

*Top-down probabilities for observing different $C_l$'s are then determined by the relative frequency with which different values occur.*

*This is given by the bottom-up probabilities calculated from the quantum state.*

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# Outlook

Prediction in extensions of the classical ensemble that include bifurcations from bubble nucleation, Boltzmann brains etc