Title: Dynamical compactification from higher dimensional de Sitter space

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Abstract: TBA

Dynamical compactification from higher dimensional de Sitter space

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0904.3115

Landscapes and extra dimensions

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? Why are some dimensions small and others large ?

- Eternal inflation - transitions within 4D EFT between vacua.

? What about the extra dimensions ?

- Do extra dimensions play a direct role in dynamics, or just provide the possibility of different 4D physics?

Dynamical Compactification

$$S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}^{(D)}} \left(\tilde{\mathcal{R}}^{(D)} - 2\Lambda - \frac{1}{2q!} \tilde{F}_q^2 \right)$$

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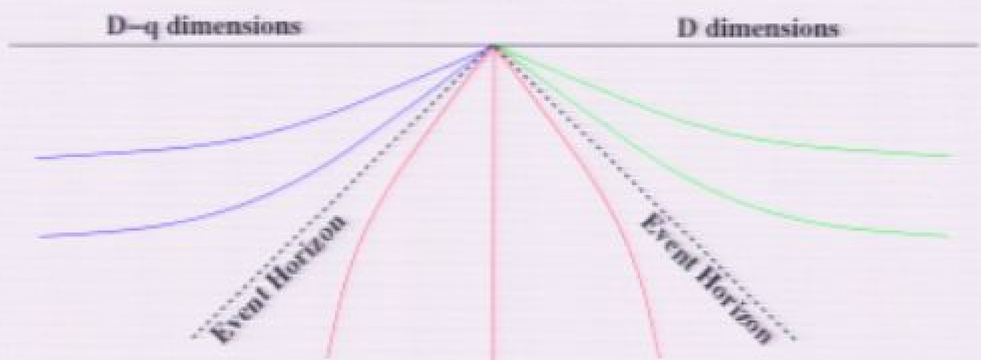
- We will find non-singular black brane solutions that interpolate across event horizons between a D dimensional de Sitter space and a D-q dimensional open FRW universe with a stabilized q-sphere.



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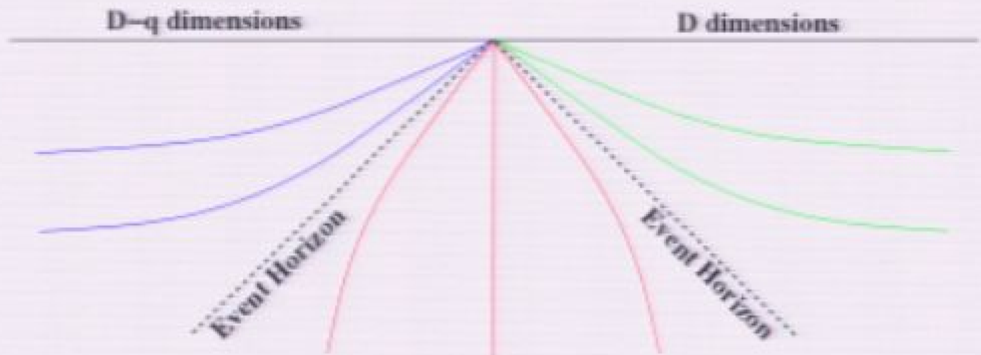

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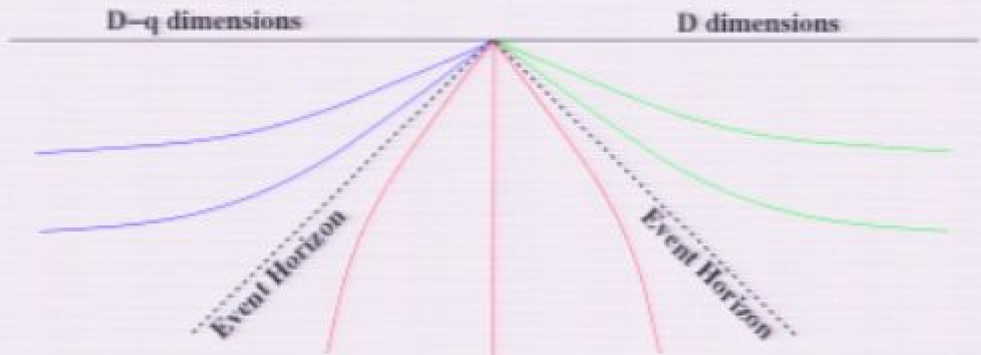

- These solutions can be nucleated out of D-dimensional dS space, explaining how extra dimensions became compact.
- Many types of lower-dimensional vacua exist and can be populated.

Previous work

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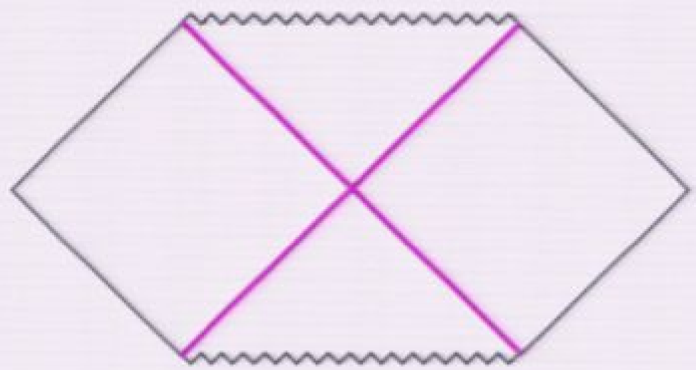
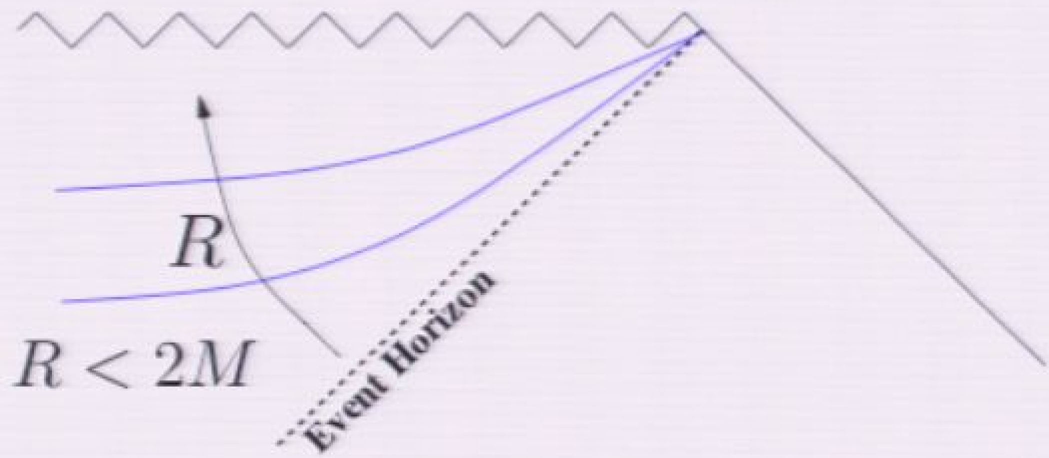
Cosmology inside a black hole

Each element of this picture can be understood from completely vanilla black holes in 4 dimensions.



Cosmology inside a black hole

$$ds^2 = -\frac{dR^2}{\left(\frac{2M}{R} - 1\right)} + \left(\frac{2M}{R} - 1\right) dt^2 + R^2 d\Omega_2^2$$



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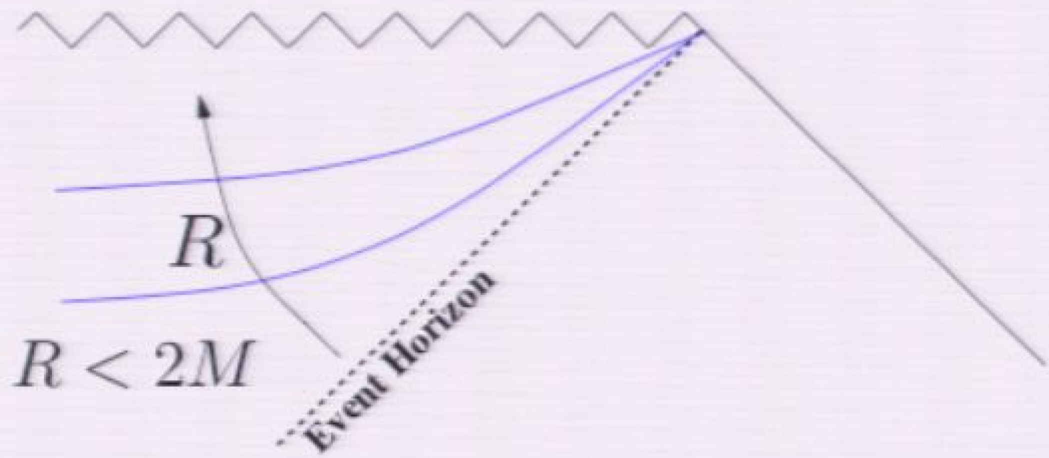

Near the horizon:

$$x = \frac{t}{4M}, \tau = \sqrt{16M^2 - 8MR} \qquad ds^2 = -d\tau^2 + \tau^2 dx^2 + 4M^2 d\Omega_2^2$$

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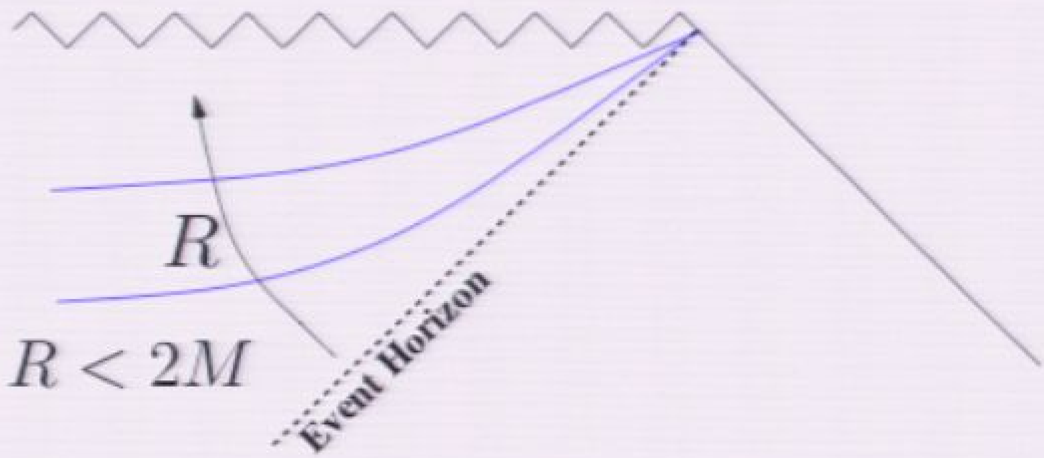

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2D open FRW

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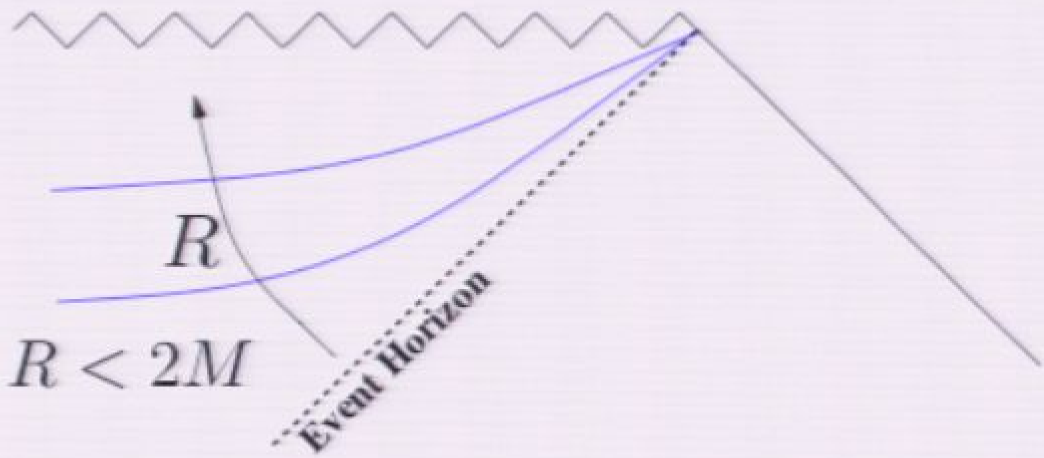

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"compactified" 2-sphere

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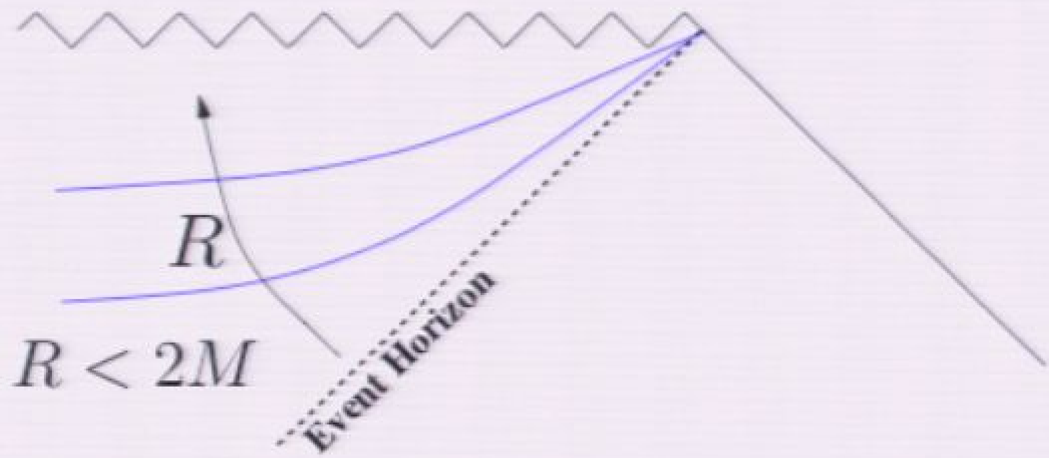

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2D open FRW "compactified" 2-sphere

$\tau = 0$ 2D "Big-bang" is non-singular - just the event horizon.

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Can continue across the horizon by taking $\tau \longrightarrow i\tau$, R is spacelike.

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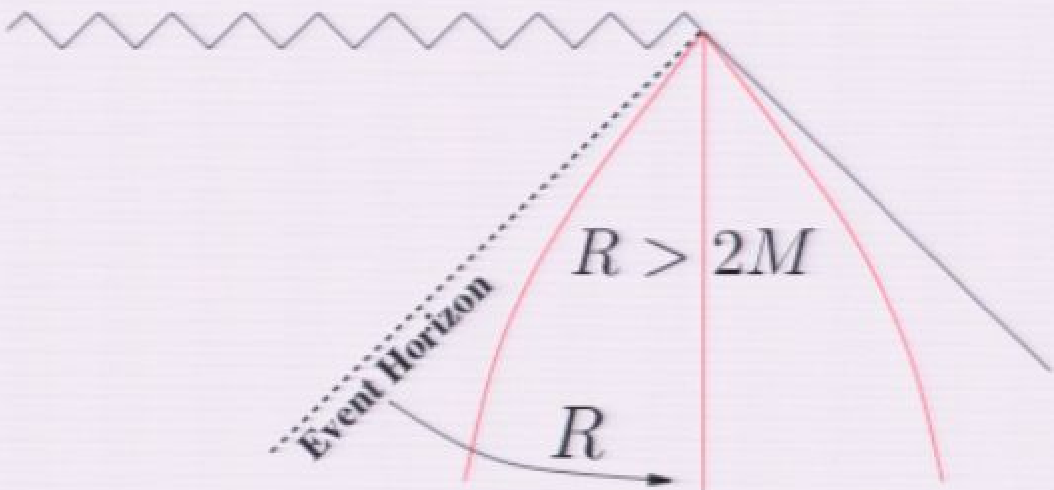

- Event horizon separates 2D big-crunch cosmology from asymptotically flat 4D space.
- Can study in more detail.....

Dimensional reduction

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- R evolves in the potential $V_{eff} = \frac{1}{2}\log R \Rightarrow R'' + \frac{R'^2}{2R} = -\frac{dV_{eff}}{dR}$

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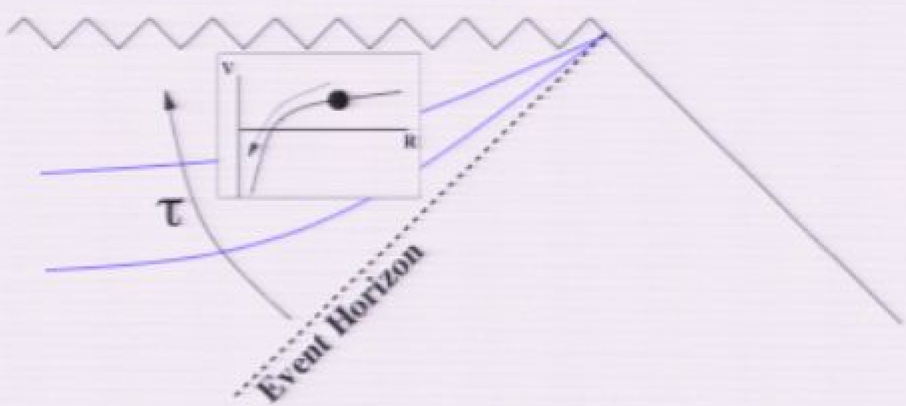

Going outside the horizon

- Continuing across the horizon:

$$\tau \rightarrow i\tau$$
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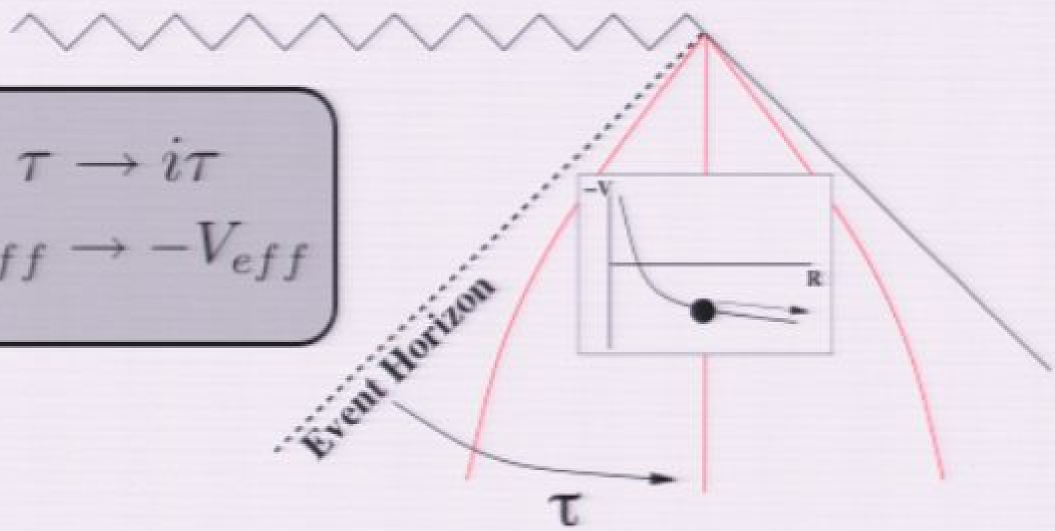

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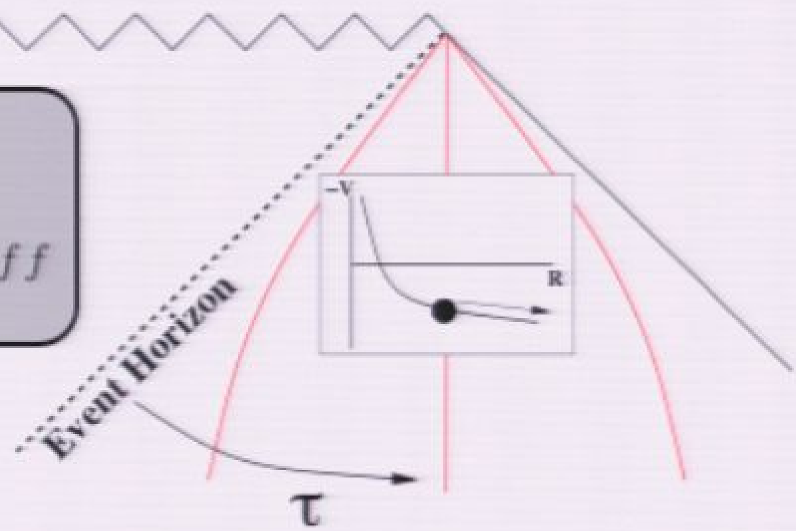

- This method of dimensionally reducing to a "radion" R living in lower dimensions (the open FRW) can be used to classify a wide variety of solutions.

Adding matter

- Add a 2-form: charge the black hole.

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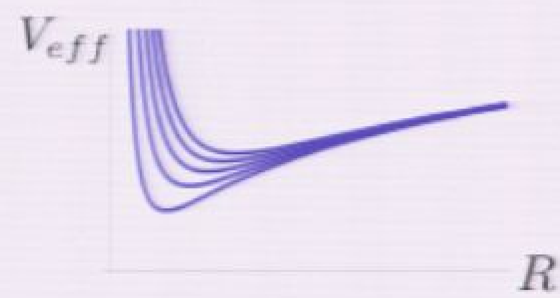

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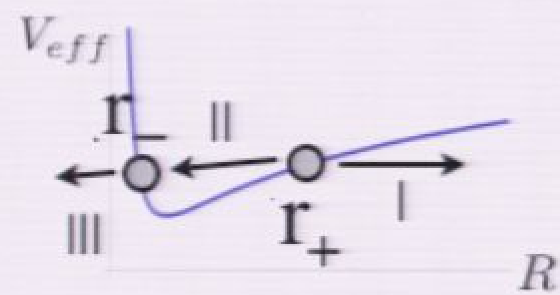

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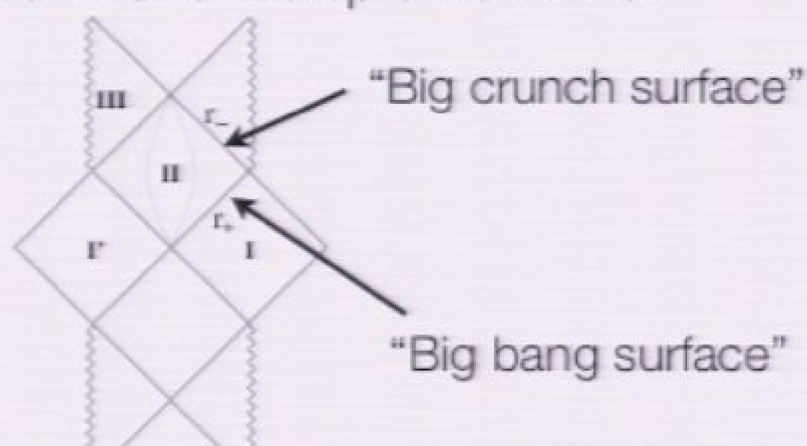
- There is a "landscape" of vacua, one for each Q.
- The black hole solutions can have multiple horizons.



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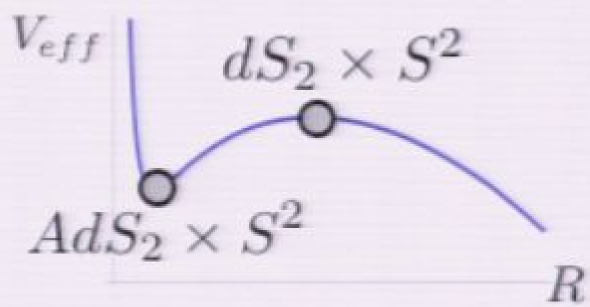

- There are new “compactification” solutions.

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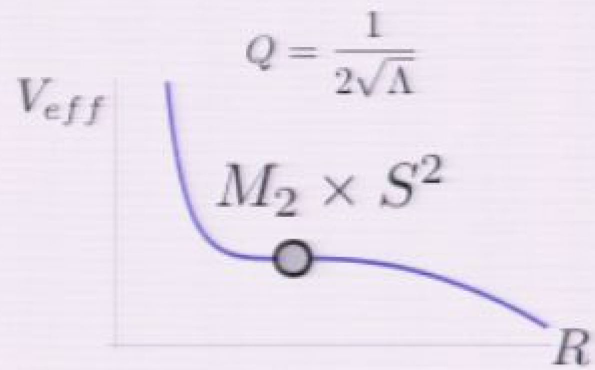

- Q is bounded.

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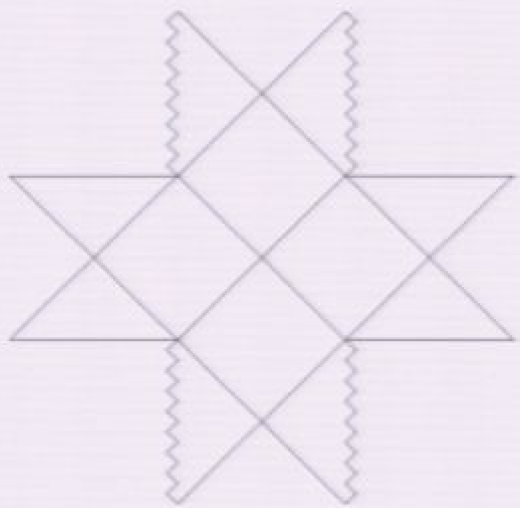

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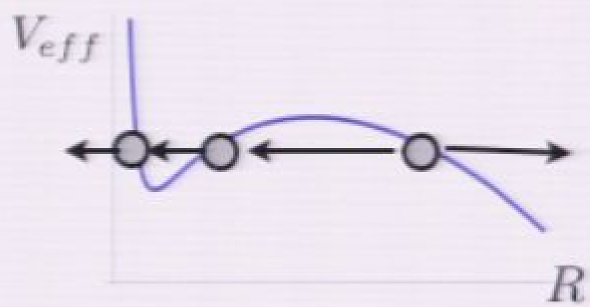

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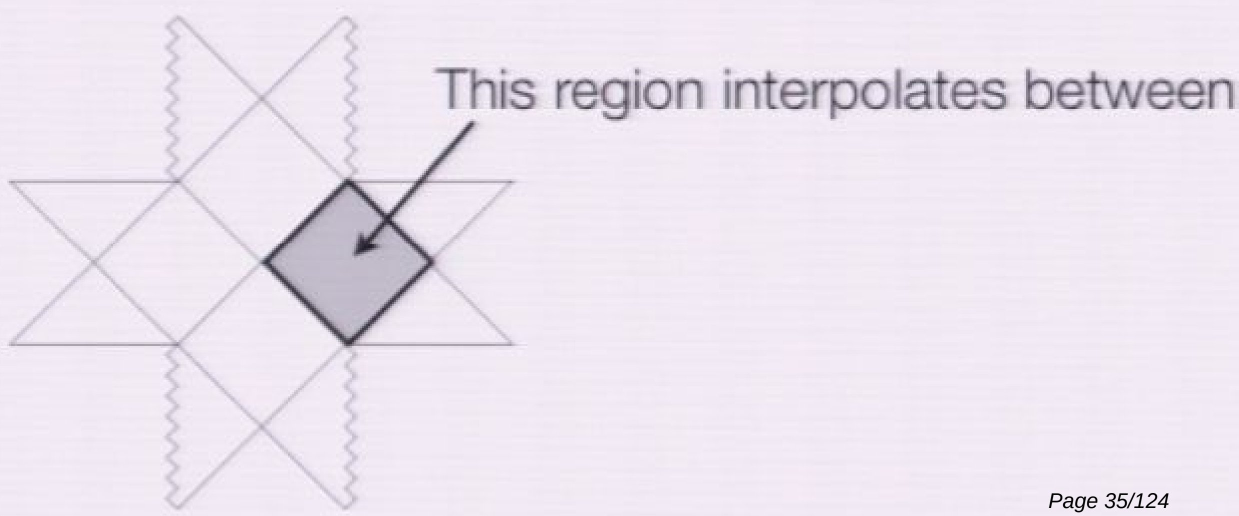

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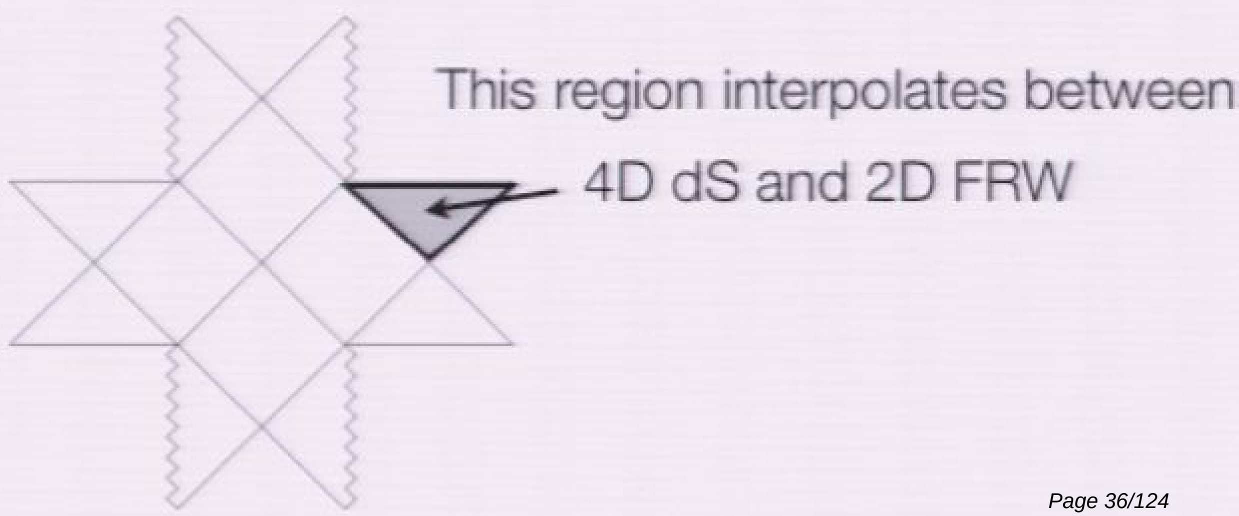

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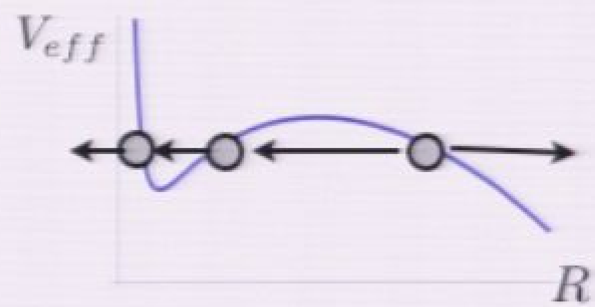

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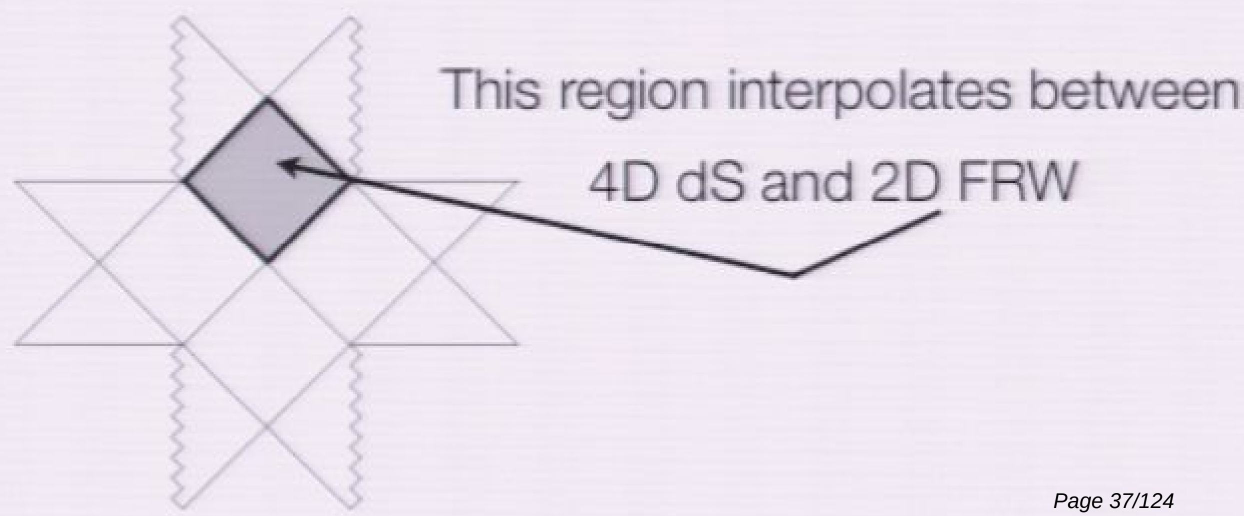

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- Can have up to three horizons: 2 BH and 1 cosmological
- Charged black holes in de Sitter are "interpolating solutions."
- The thermal properties of de Sitter space add interesting dynamics......

Black hole nucleation

- de Sitter space is semi-classically unstable to the nucleation of charged black holes.

$$\Gamma = A \exp\left[-(S_{inst} - S_{dS})\right]$$

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- Future infinity of the dS space is split into many disconnected regions.

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What if the lower dimensional FRW was 4D and didn’t end in a crunch?

Now enters the magic of higher dimensional GR....

A very simple theory

$$S = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-\tilde{g}^{(D)}} \left(\tilde{\mathcal{R}}^{(D)} - 2\Lambda - \frac{1}{2q!} \tilde{F}_q^2 \right)$$

Dimensional reduction

- Assume q-dimensional spherical symmetry (D=q+4):

$$ds^2 = \exp\left[-\sqrt{\frac{2q}{q+2}}\frac{\phi(\mathbf{x})}{M_4}\right] g^{(4)}_{\mu\nu}(\mathbf{x})dx^\mu dx^\nu + M_D^{-2}\exp\left[\sqrt{\frac{8}{q(q+2)}}\frac{\phi(\mathbf{x})}{M_4}\right] d\Omega_q^2$$

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- For magnetic flux, Maxwell equations satisfied for:

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- Can integrate over the angular coordinates on the q-sphere and go to the Einstein frame of a 4-dimensional theory:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_4^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right]$$

$$M_4 \equiv M_D\sqrt{\mathrm{Vol}(S^q)} \qquad M_D R = \exp\left[\sqrt{\frac{2}{q(q+2)}}\frac{\phi}{M_4}\right]$$

A landscape of lower-dimensional vacua

- The potential is given by:

$$V(\phi) = \frac{M_4^4}{2\mathrm{Vol}(\mathrm{S}^q)}\left[-q(q-1)\exp\left(-\sqrt{\frac{2(q+2)}{q}}\frac{\phi}{M_4}\right) + \frac{2\Lambda}{M_D^2}\exp\left(-\sqrt{\frac{2q}{(q+2)}}\frac{\phi}{M_4}\right) + \frac{Q^2}{2}\exp\left(-3\sqrt{\frac{2q}{(q+2)}}\frac{\phi}{M_4}\right)\right],$$

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curvature cosmological constant

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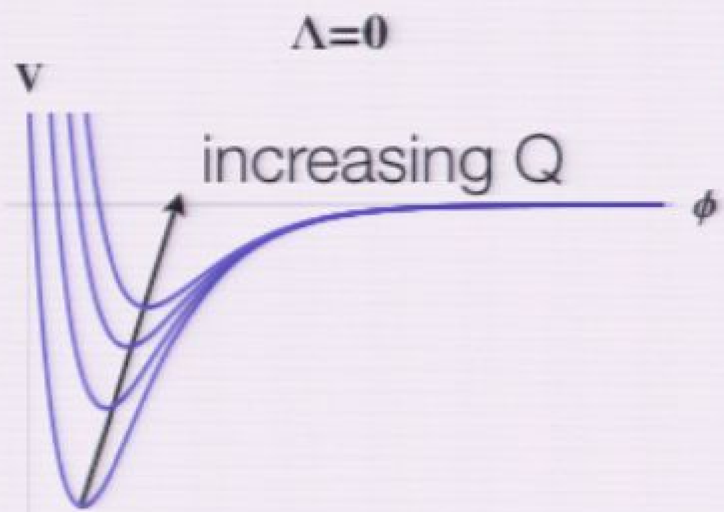

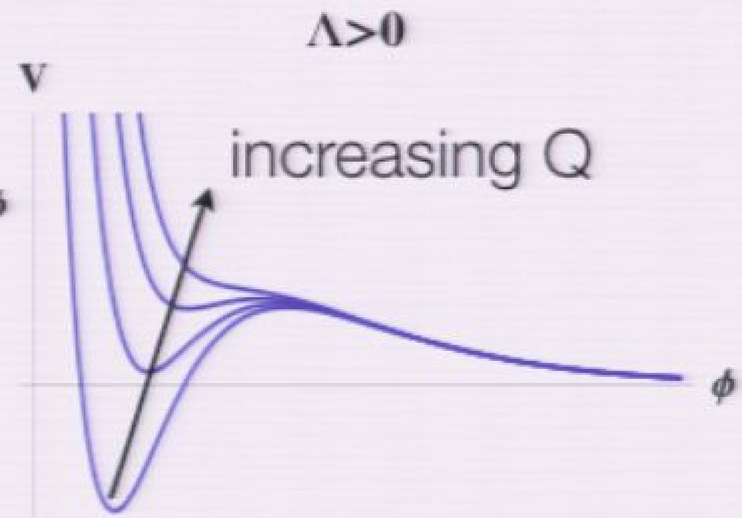

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- Possible to have 4D vacua (if $q = D-4$) with a small vacuum energy.

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- The radius of the stabilized sphere is always less than $R \sim \Lambda^{-1/2}$
- The sphere can be small, so this is a true compactification.
- If there are multiple q-forms, there can be vacua with various numbers of compact and non-compact dimensions.

$$\frac{F_q^2}{2q!} \rightarrow \sum_{i=2}^{D-2} \frac{F_{q_i}^2}{2q_i!}$$

Solutions with a dynamical radion.

$$S = \int d^4x \sqrt{-g} \left[\frac{M_4^2}{2} \mathcal{R} - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

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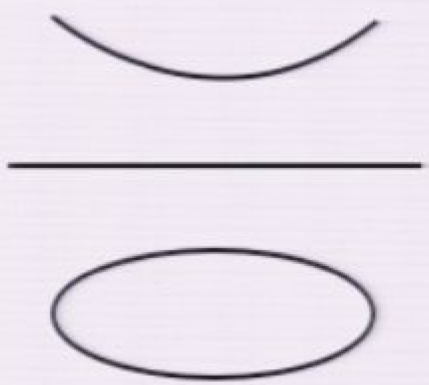
- spacelike



negative curvature (open)

no curvature (flat)

positive curvature (closed)

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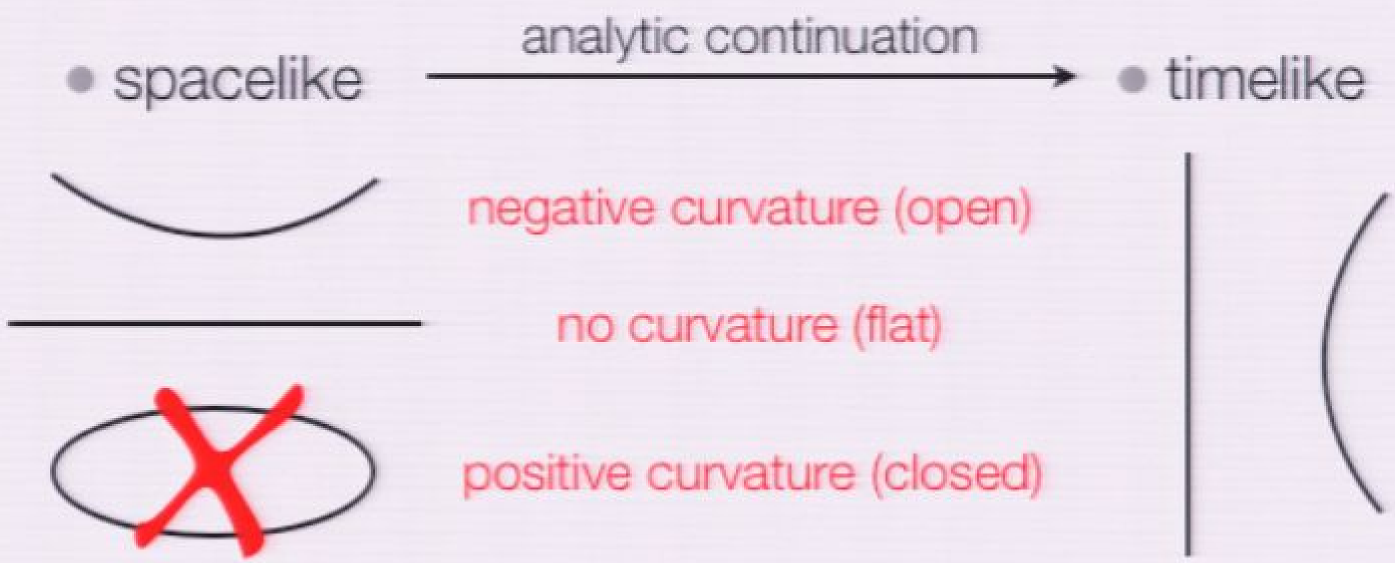

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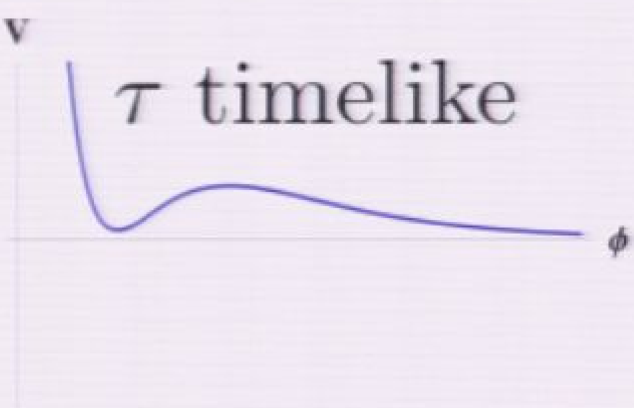

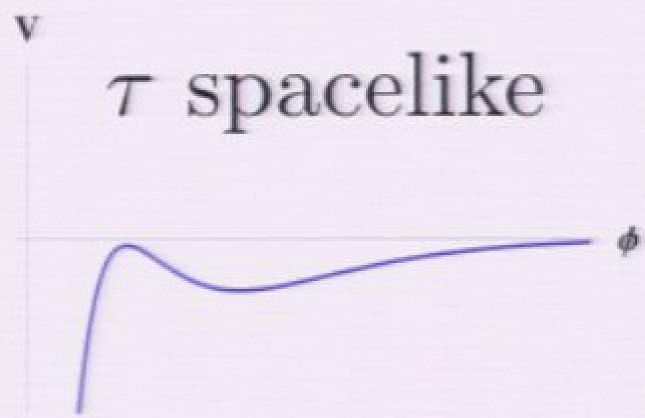

Non-singular big-bang and big-crunch

- What about big-bang and big-crunch singularities (where a=0)?

$$\mathcal{R} = -\frac{\dot{\phi}^2}{M_4^2} + 4\frac{V(\phi)}{M_4^2}$$

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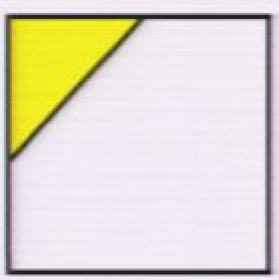

Classifying solutions

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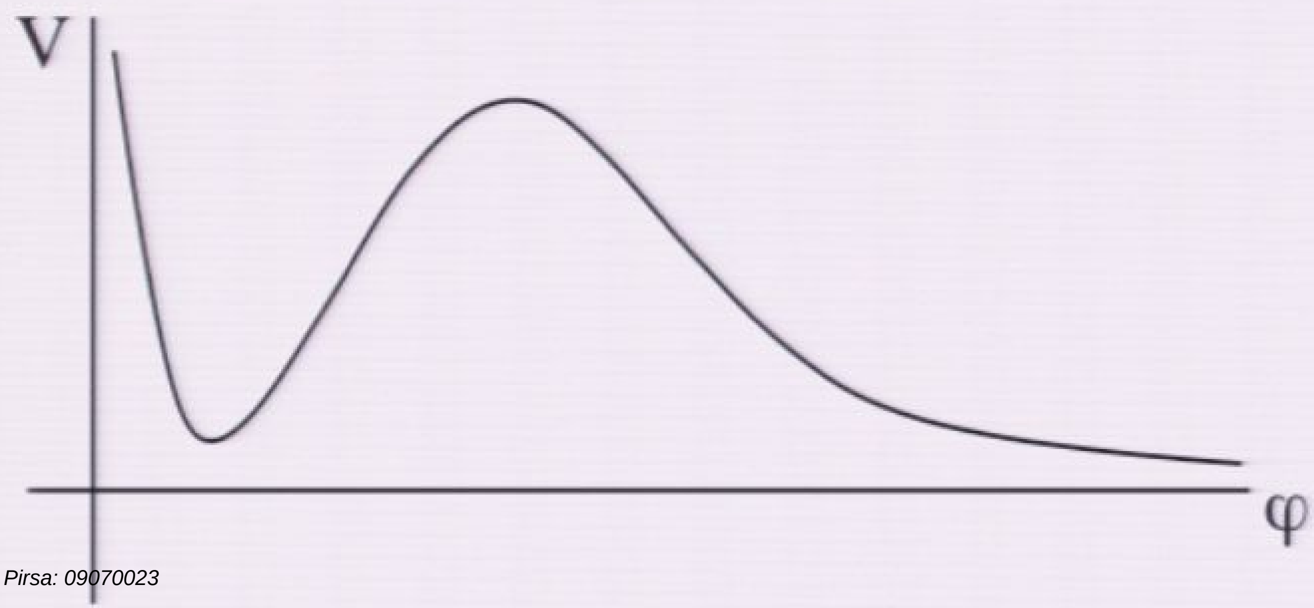

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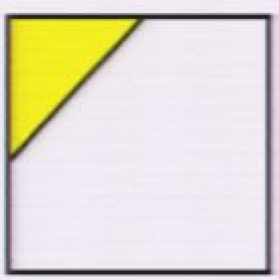

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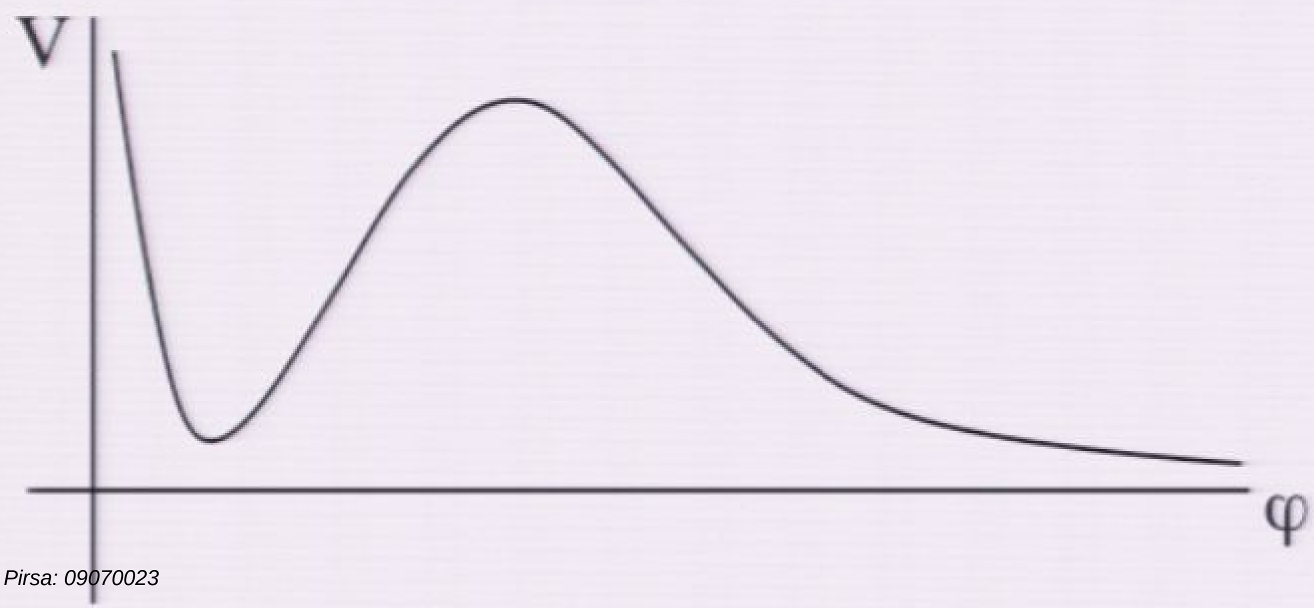

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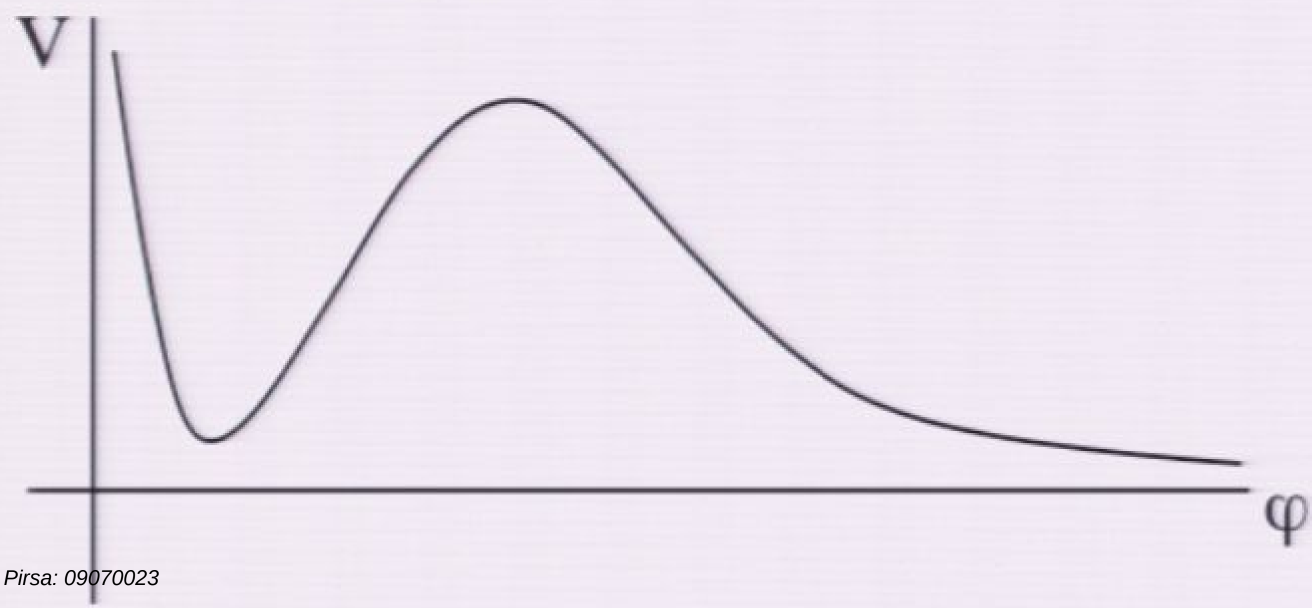
open FRW:



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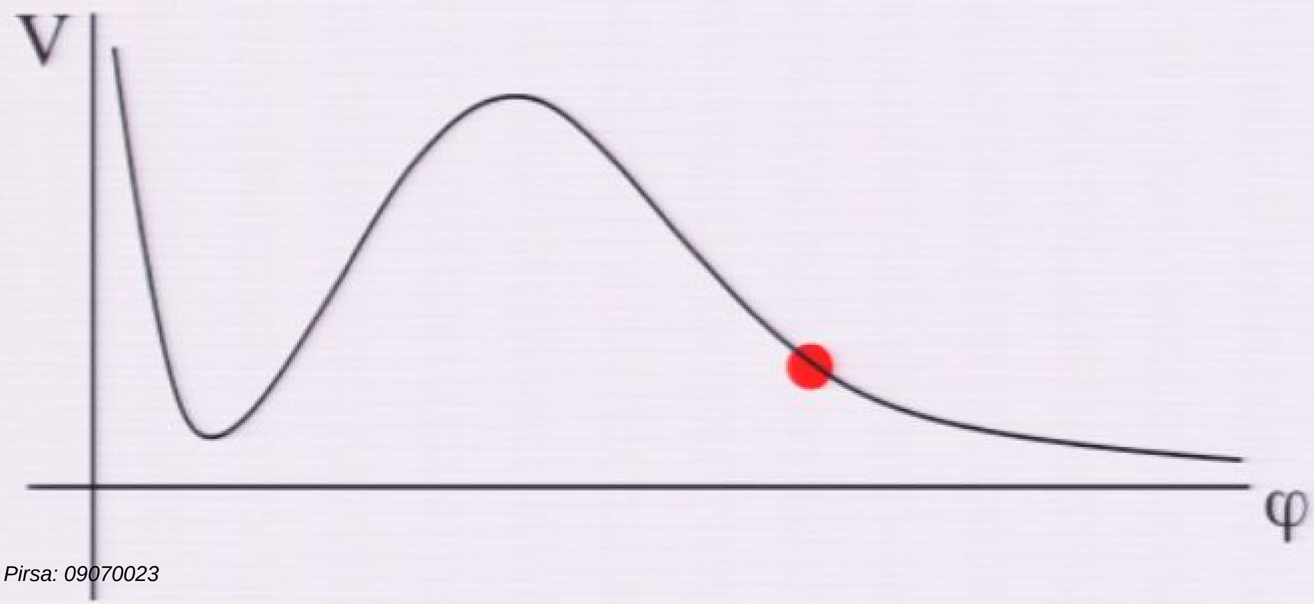

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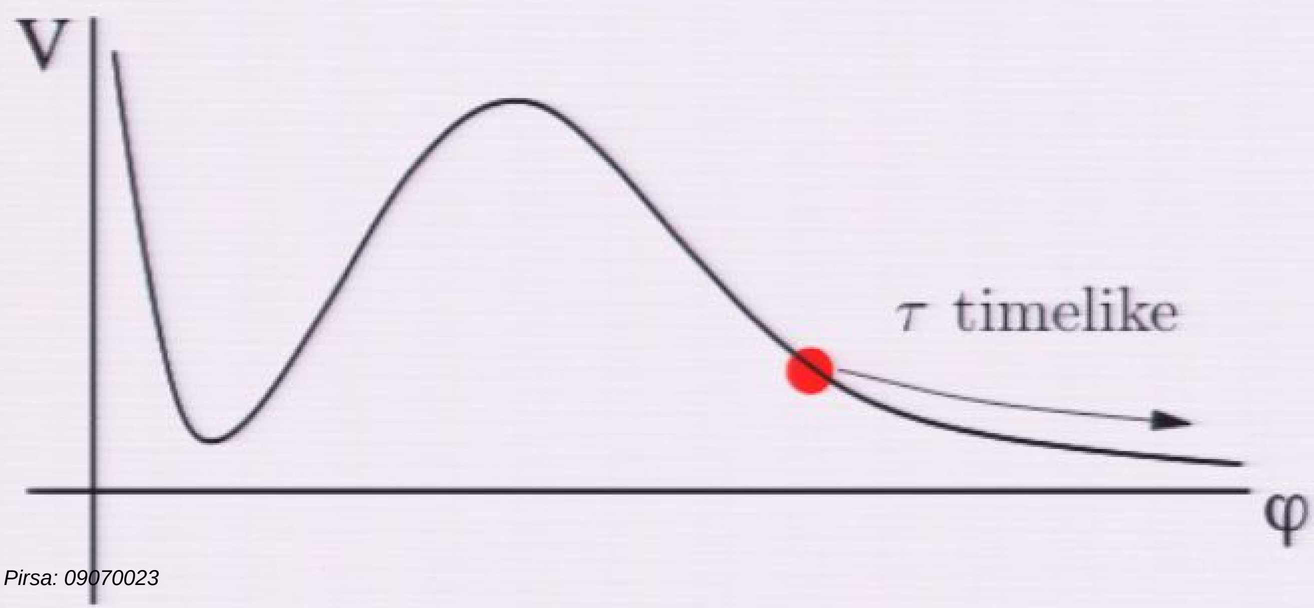

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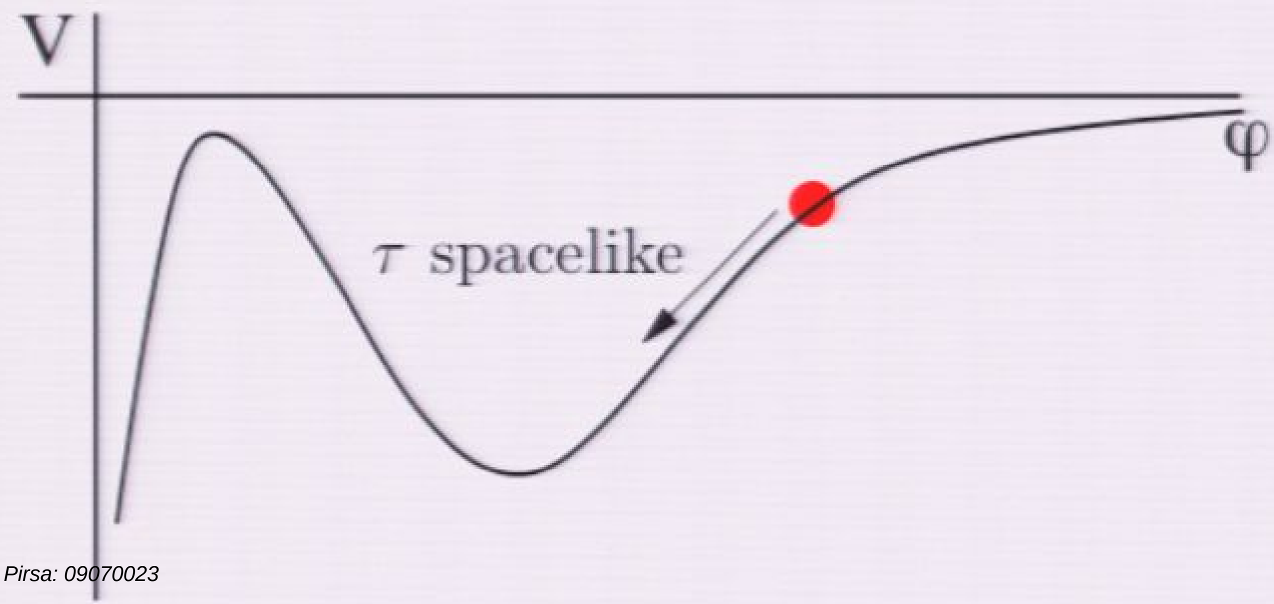
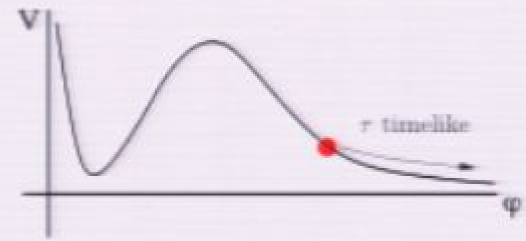


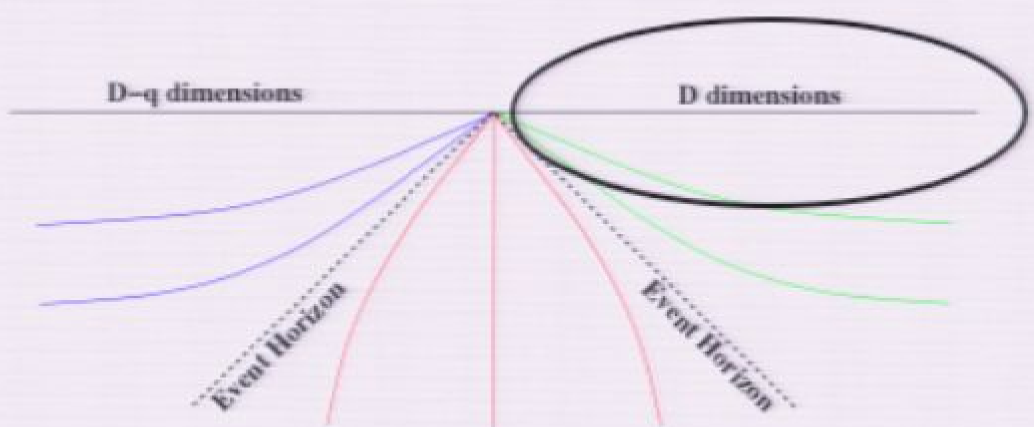
Timelike τ ①



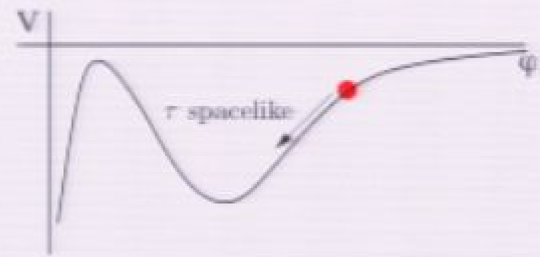
- At large ϕ the dominant term in the potential is

$$V \simeq M_4^2 \Lambda \exp\left(-2\sqrt{\frac{q}{2(2+q)}}\frac{\phi}{M_4}\right)$$

- Exponential potentials admit attractor solutions.
- The metric describes the approach to D-dimensional de Sitter space as the radius of the q-sphere goes to infinity.



Spacelike τ ②



$$\frac{\ddot{a}}{a} = -\frac{1}{3M_4^2}\left(\dot{\phi}^2 + |V|\right)$$

- Scale factor is bounded. Generic choices of initial conditions lead to a singularity:

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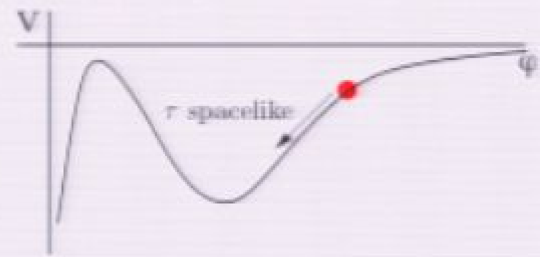

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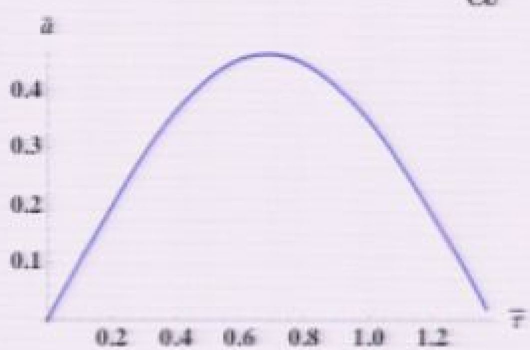

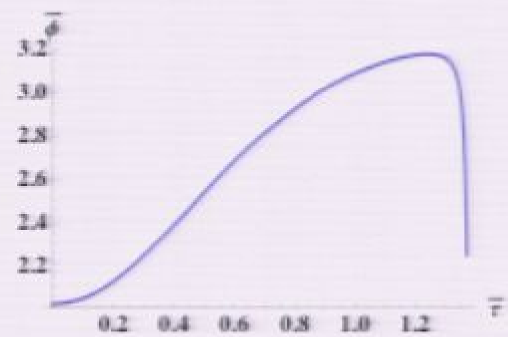

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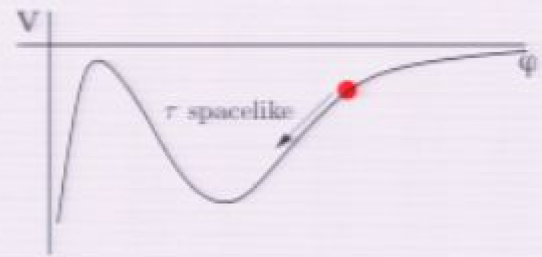

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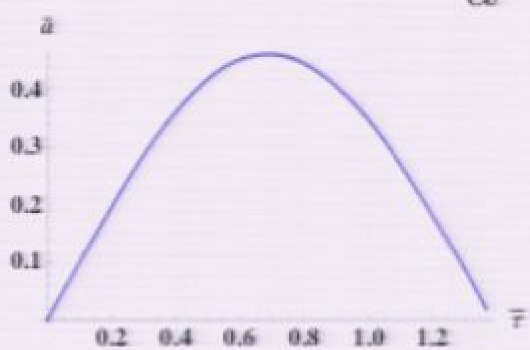
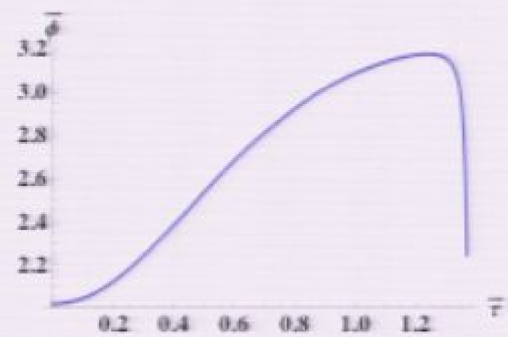
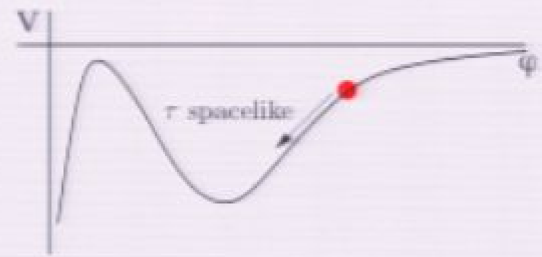




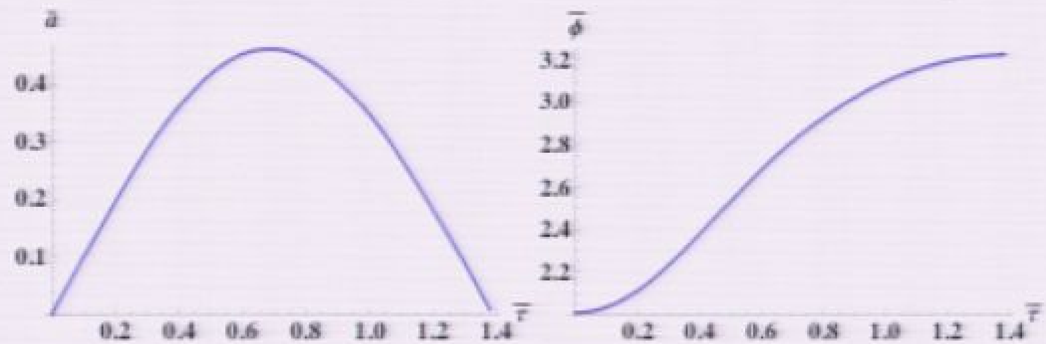
- Need to match the period of the scale factor to the barrier crossing time:

$$\Delta\tau_\phi \sim \frac{1}{\sqrt{|V''(\phi_{max})|}} \qquad \Delta\tau_a \sim \frac{M_4}{\sqrt{|V''(\phi_{max})|}}$$

Spacelike τ ②

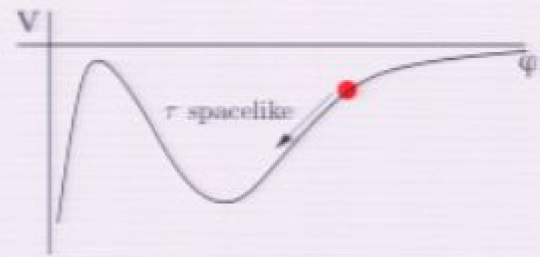


- For small enough Q, the periods can be adjusted by moving the endpoints. For each potential there can exist one set of non-singular endpoints:

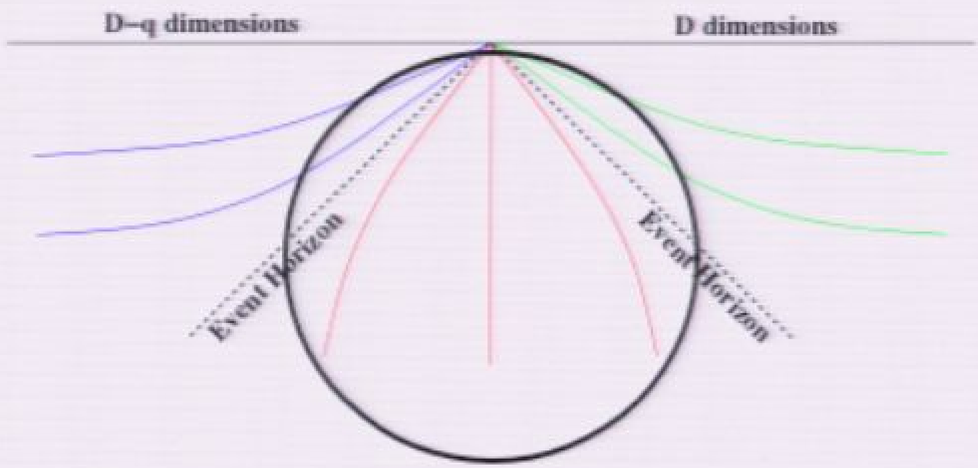


- There are two non-singular a=0 endpoints, and so two event horizons.

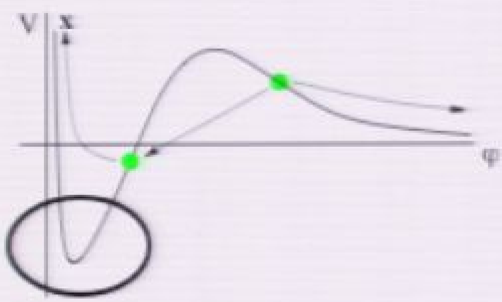
Spacelike τ ②



- The metric interpolates between the two event horizons.



Timelike τ ③



- For a negative minimum, there is always a spacelike singularity as perturbations are re-focused.

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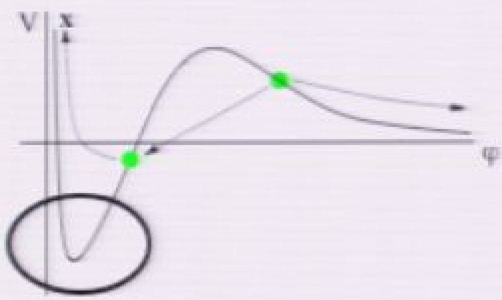

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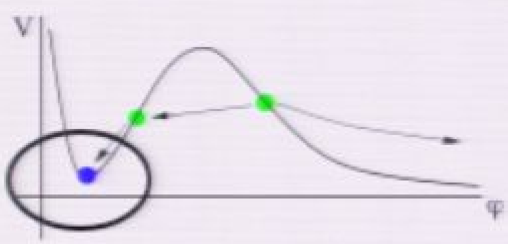
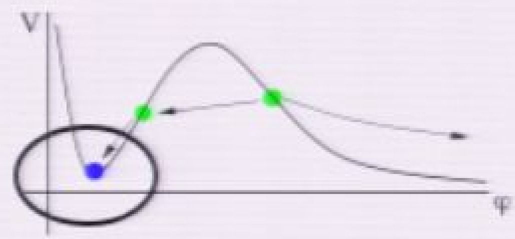
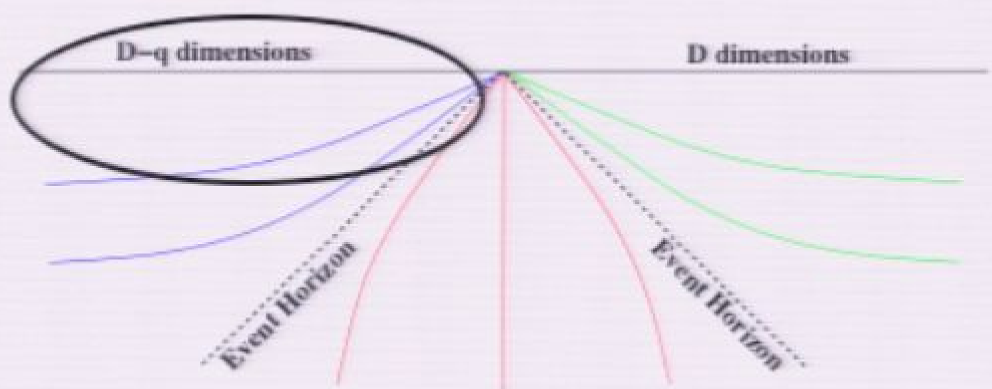


- For a zero or positive minimum,the field settles into the vacuum. There is no singularity.

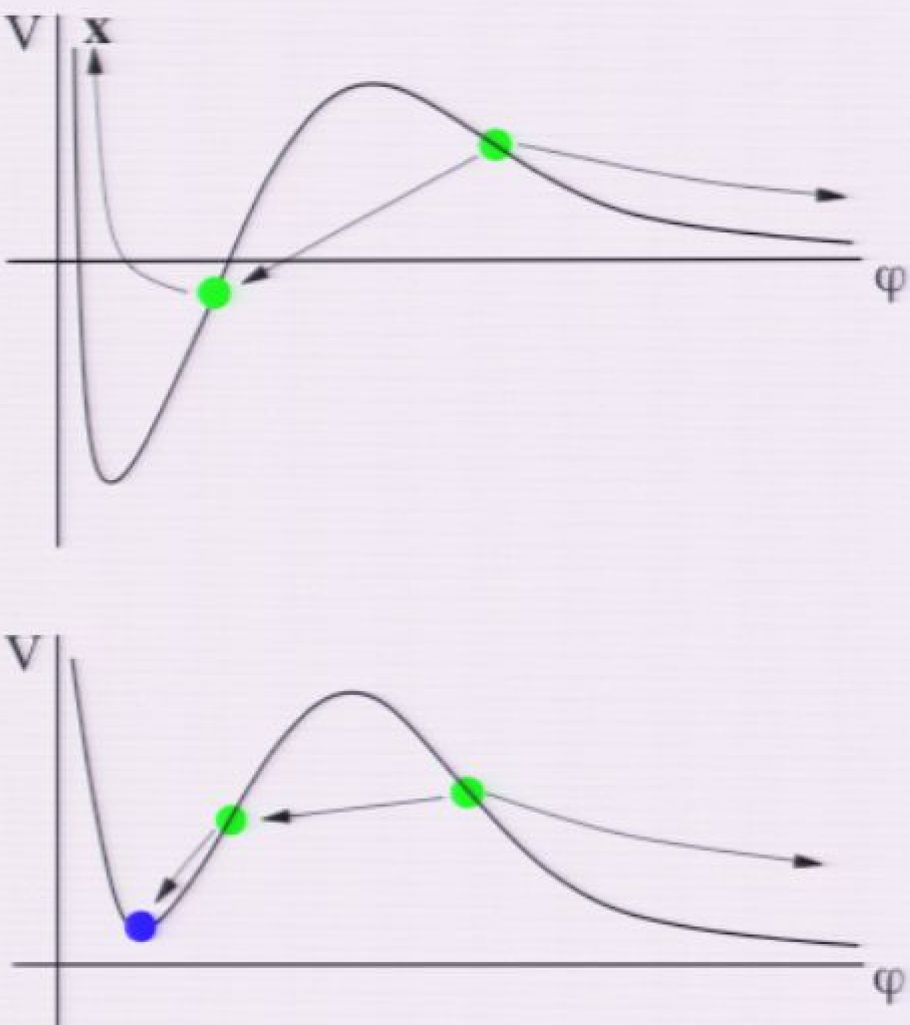
Timelike τ ③

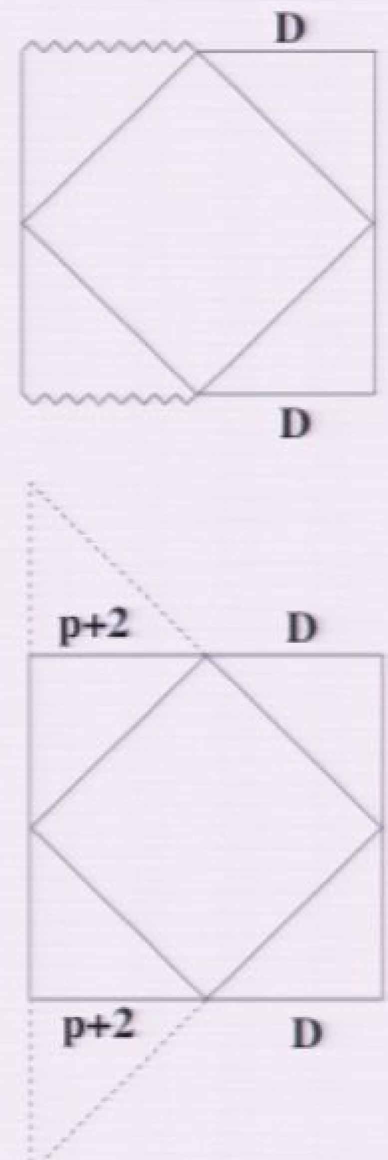


- In this region there is a 4 dimensional open FRW universe that evolves at late times to de Sitter: This could be how our universe began!



Interpolating solutions: open FRW ansatz





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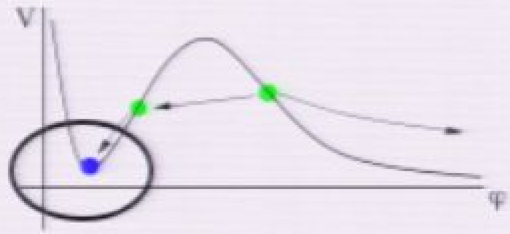

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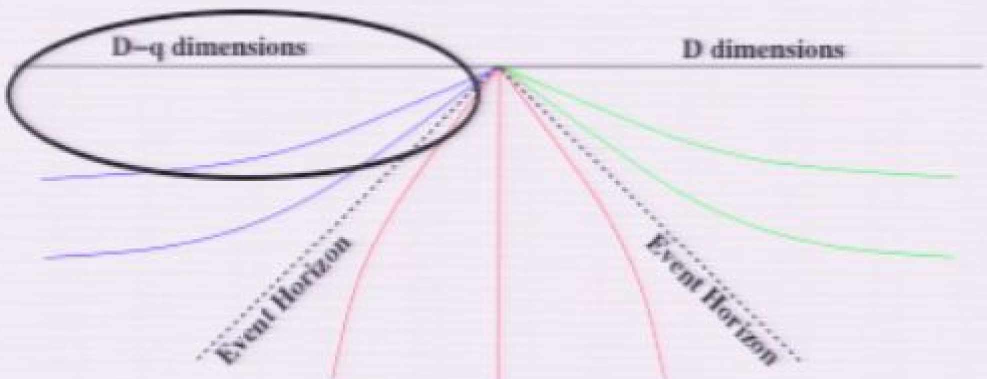

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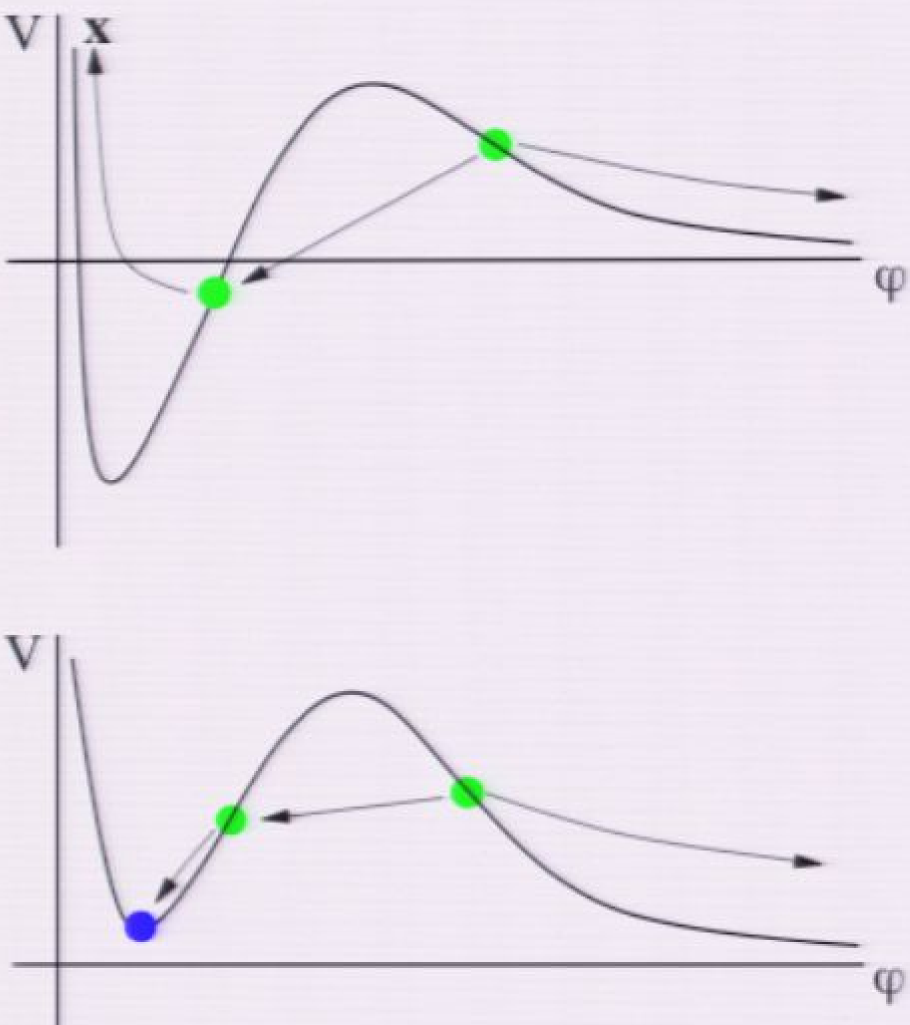

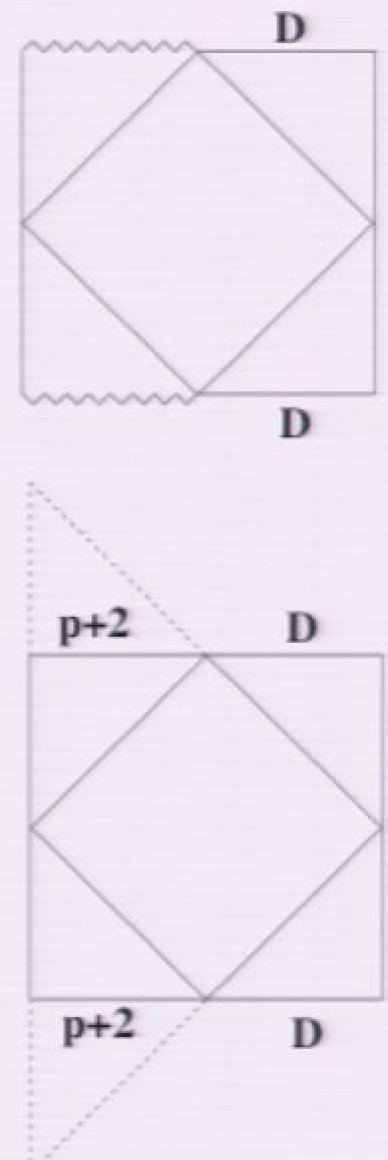

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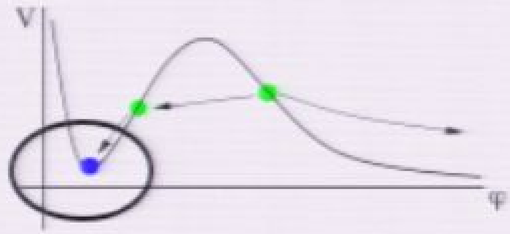

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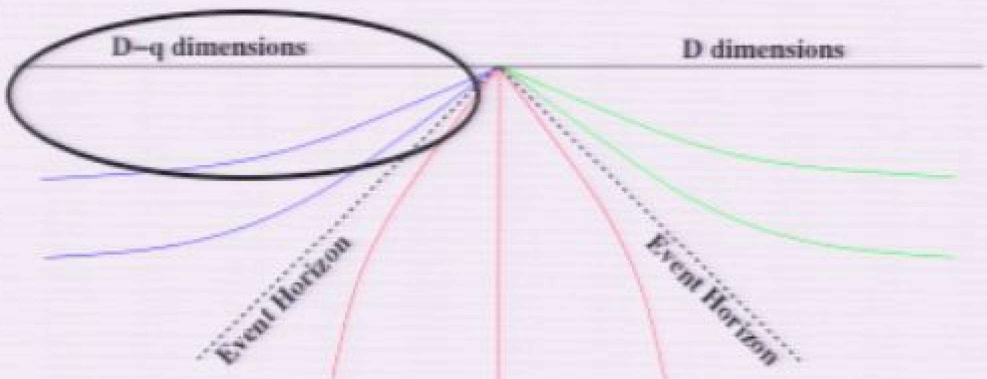

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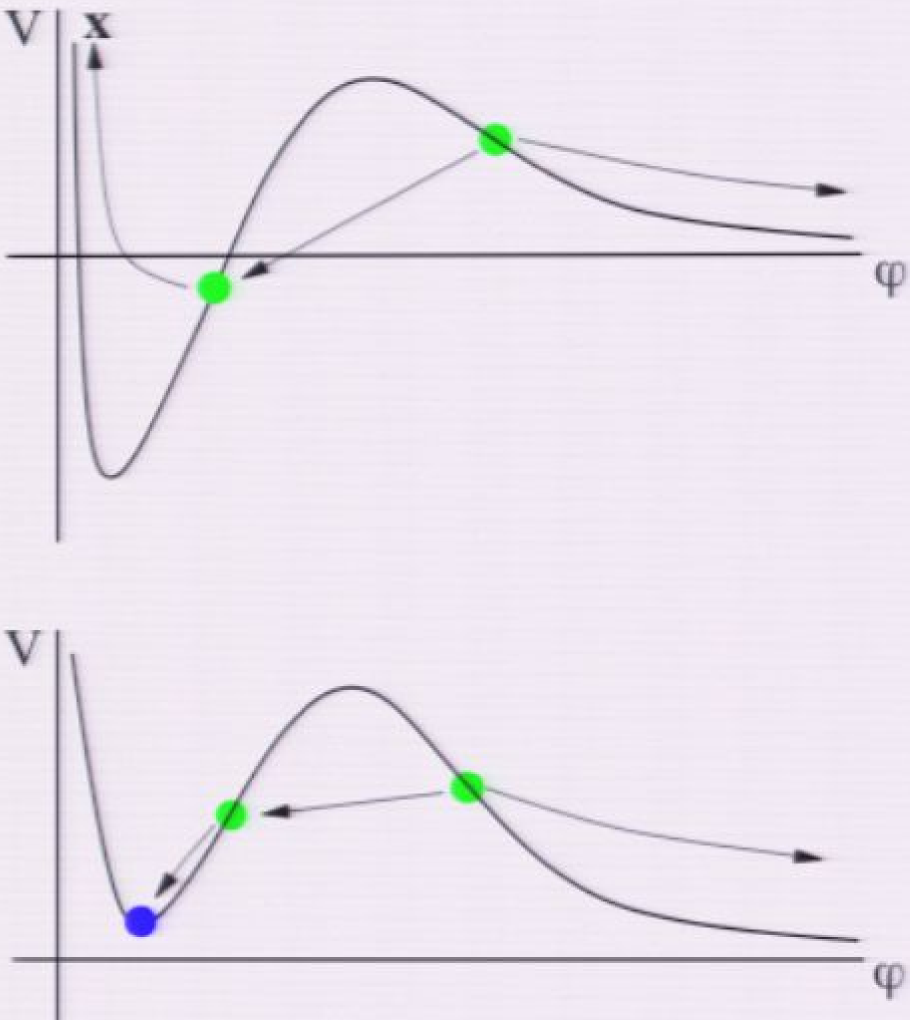

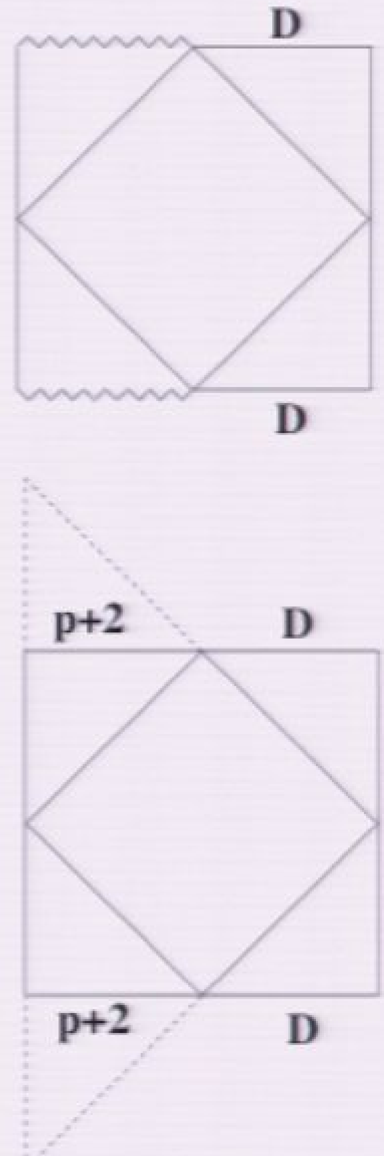

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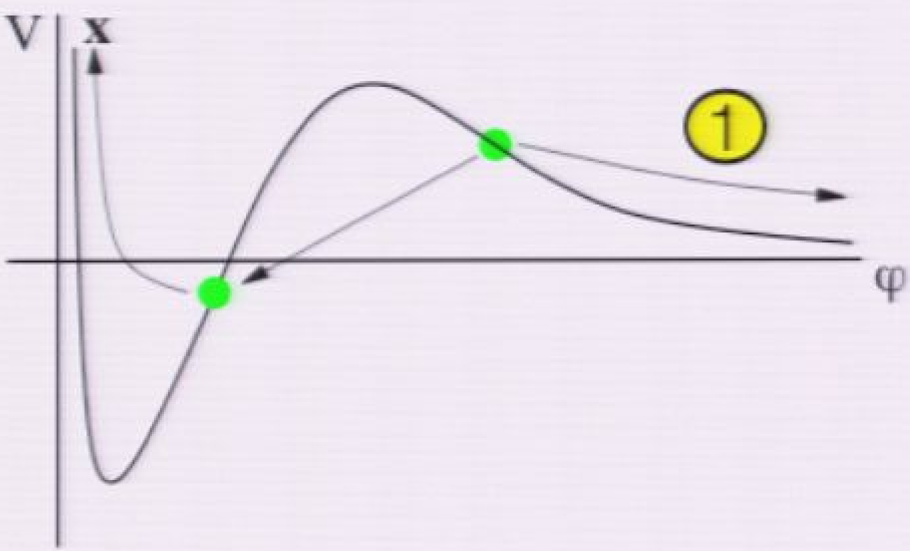

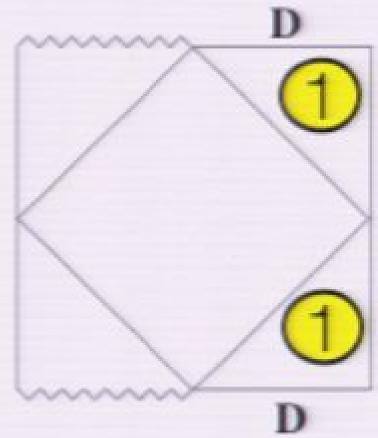

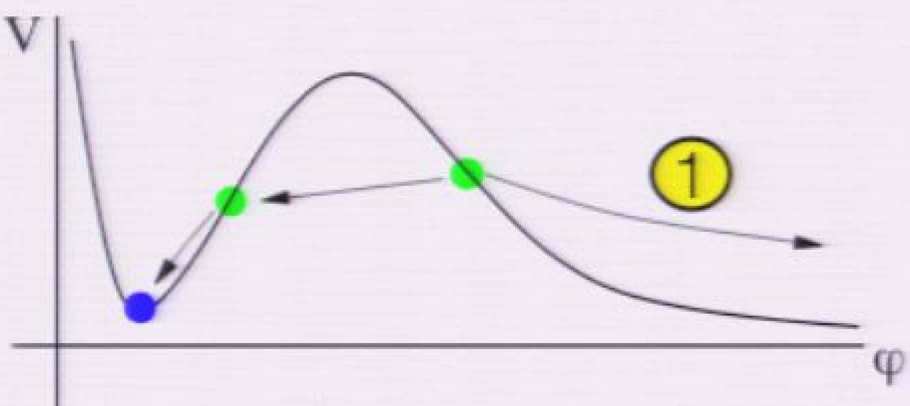

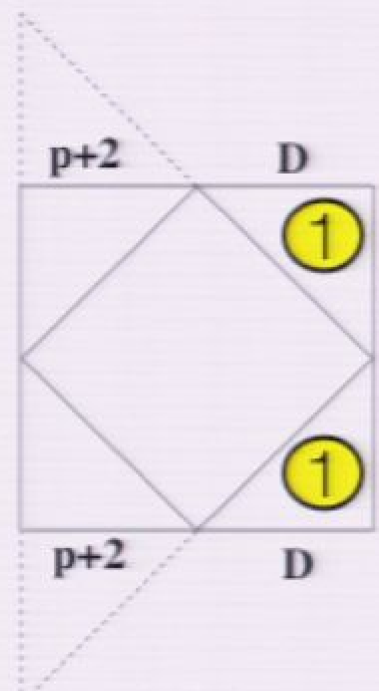

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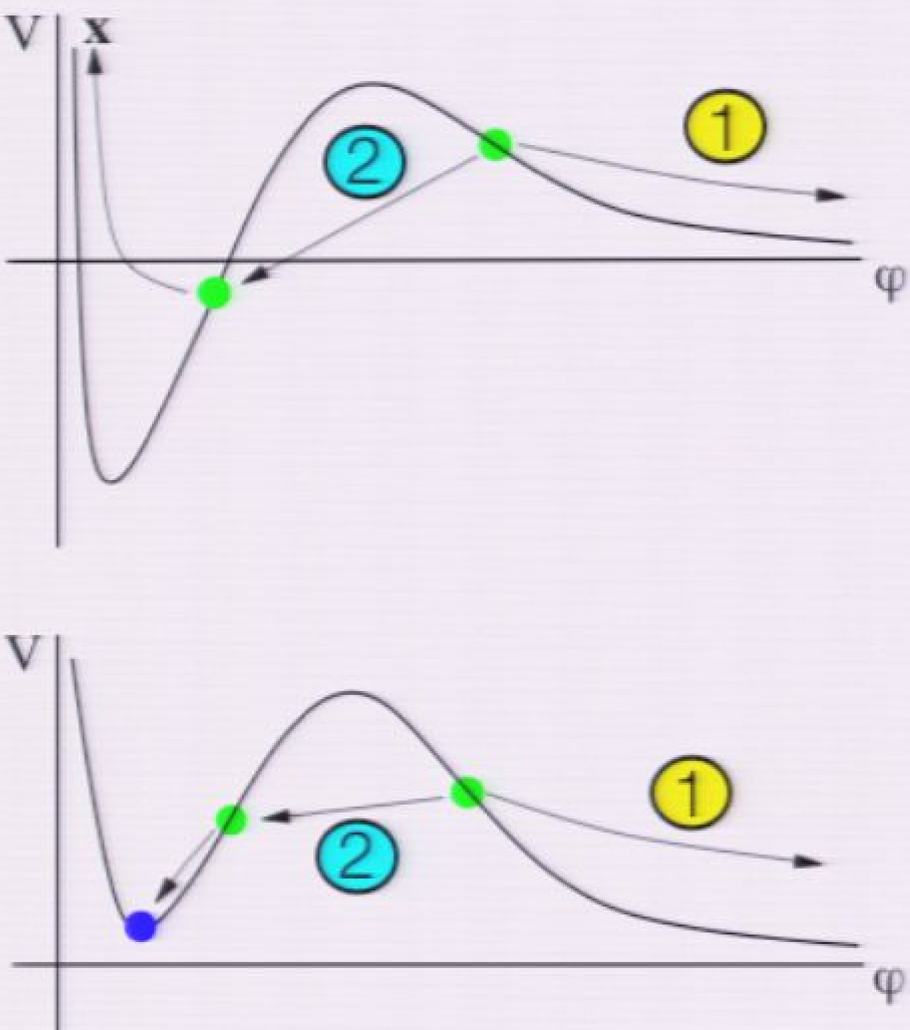
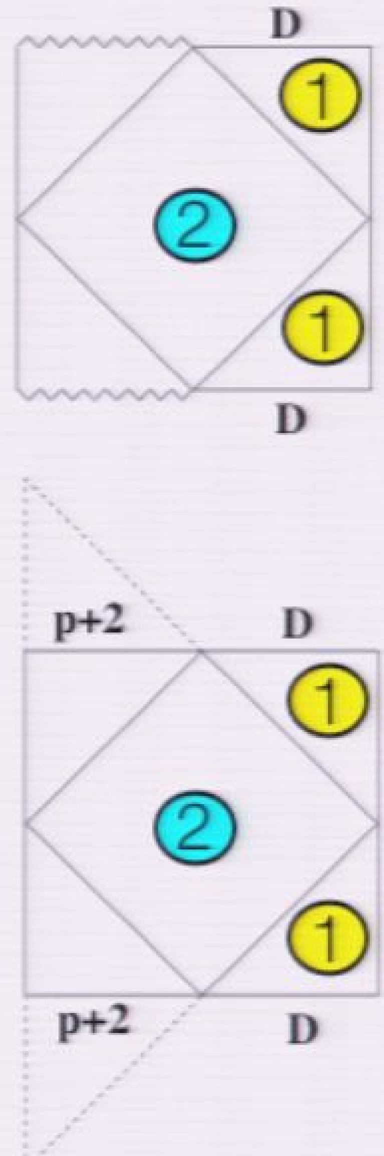
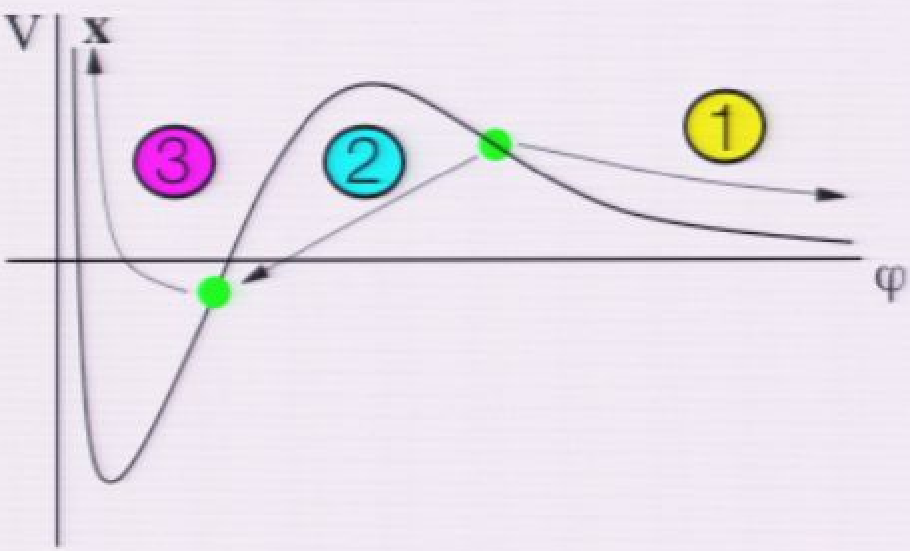
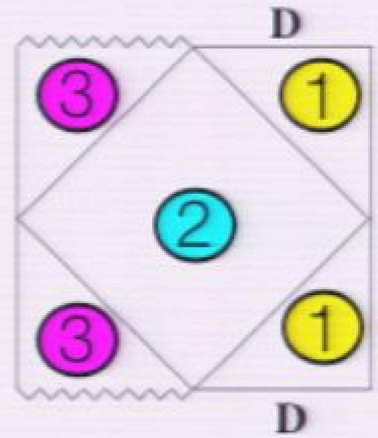
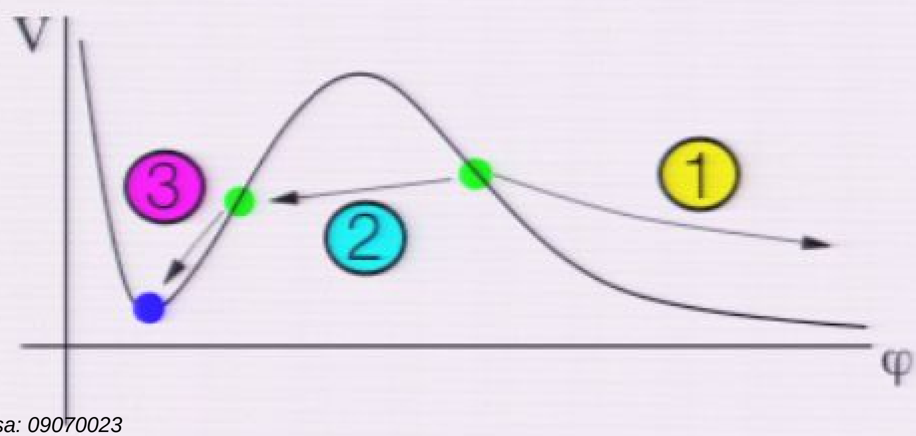


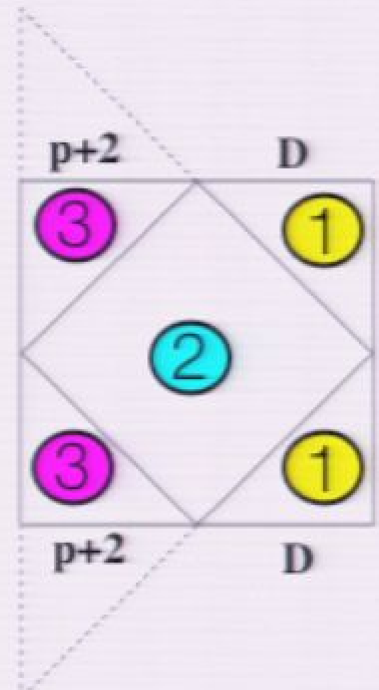


Interpolating solutions: open FRW ansatz









Classifying solutions

Many other solutions can be generated from other choices of the metric ansatz.

0904.3115

An aside: embedding Inflation

- Add a scalar:

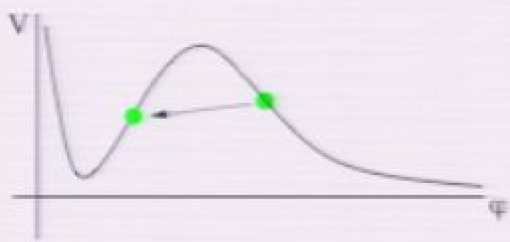
$$S = \frac{M_D^{q+2}}{2}\int d^{q+4}x\sqrt{-\tilde{g}^{(q+4)}}\left(f(\psi)\tilde{R}^{(q+4)} - 2\Lambda - \frac{h(\psi)}{2q!}\tilde{F}_q^2\right) + \int d^{q+4}x\sqrt{-\tilde{g}^{(q+4)}}\left(-M_\psi^q k(\psi)\tilde{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V(\psi)\right)$$

- The coupling to curvature and flux induces a negative mass squared for the scalar inside an event horizon:

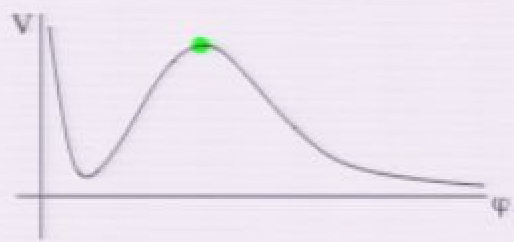
- This can drive an epoch of inflation.

Dynamical Compactification

- Two solutions that contain a non-singular 4 dimensional region:



Interpolating



Compactification

Dynamical Compactification

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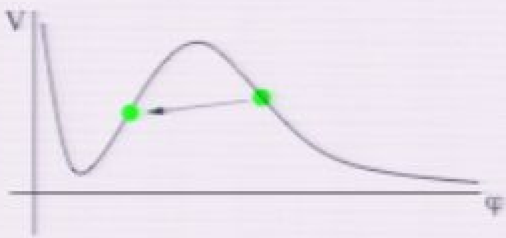
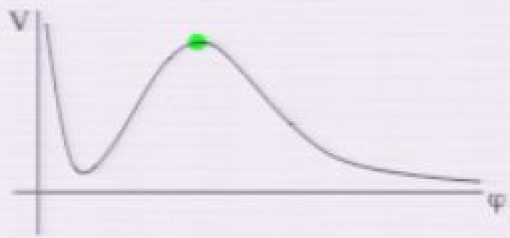


Coleman de Luccia



Hawking Moss

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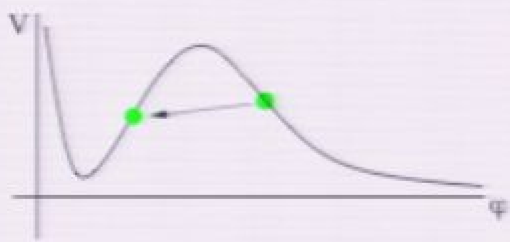

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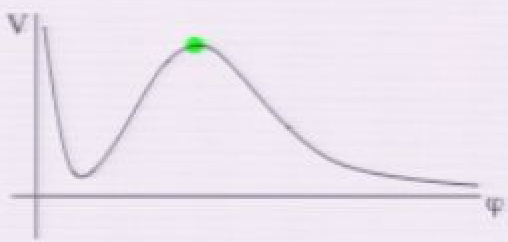

Hawking Moss

- These solutions are analogous to the charged dS black hole and compactification solution discussed earlier.
- Empty de Sitter space is unstable to the nucleation of these objects.

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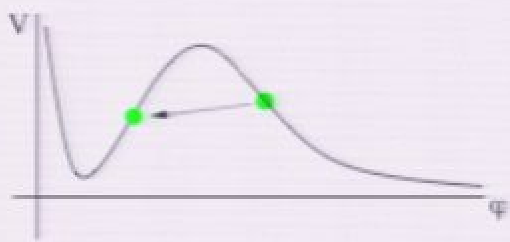

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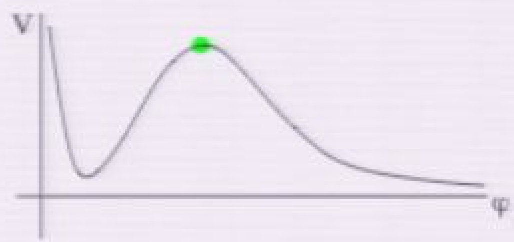


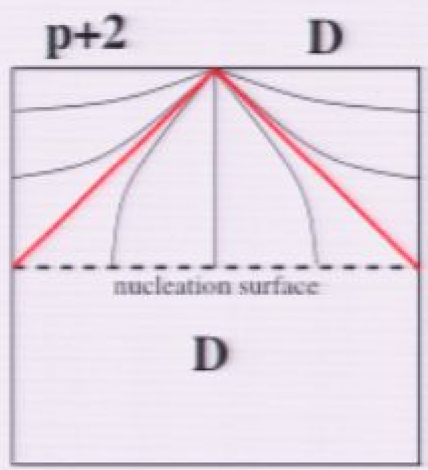
Hawking Moss

- These solutions are analogous to the charged dS black hole and compactification solution discussed earlier.
- Empty de Sitter space is unstable to the nucleation of these objects.
- We have answered our original question:

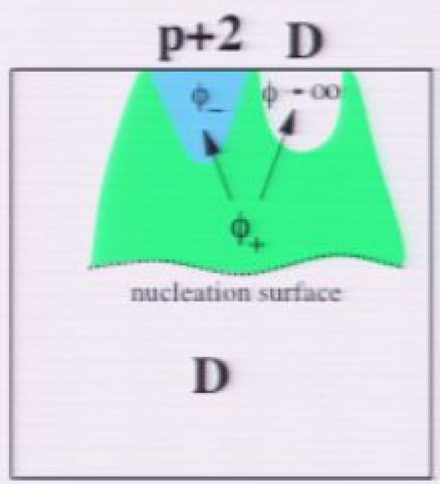
What if the lower dimensional FRW was 4D and didn't end in a crunch?

Dynamical compactification

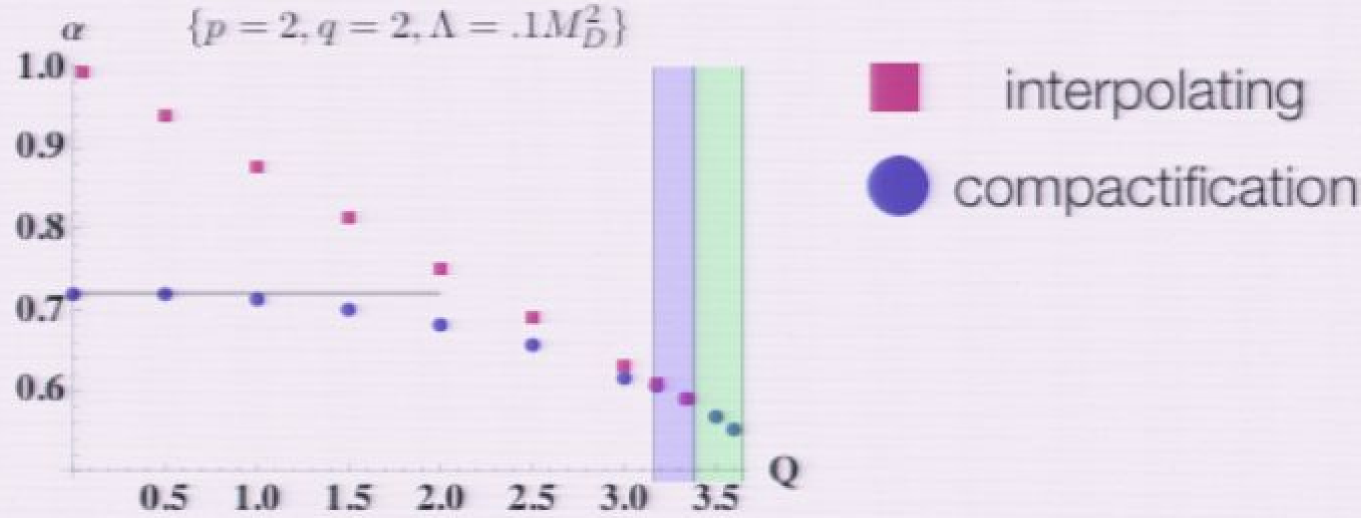
- Interpolating solution:



- Compactification solution:



Dynamical compactification: rates



$$\Gamma = A \exp\left[S_{ds}^{(D)}(1-\alpha)\right]$$

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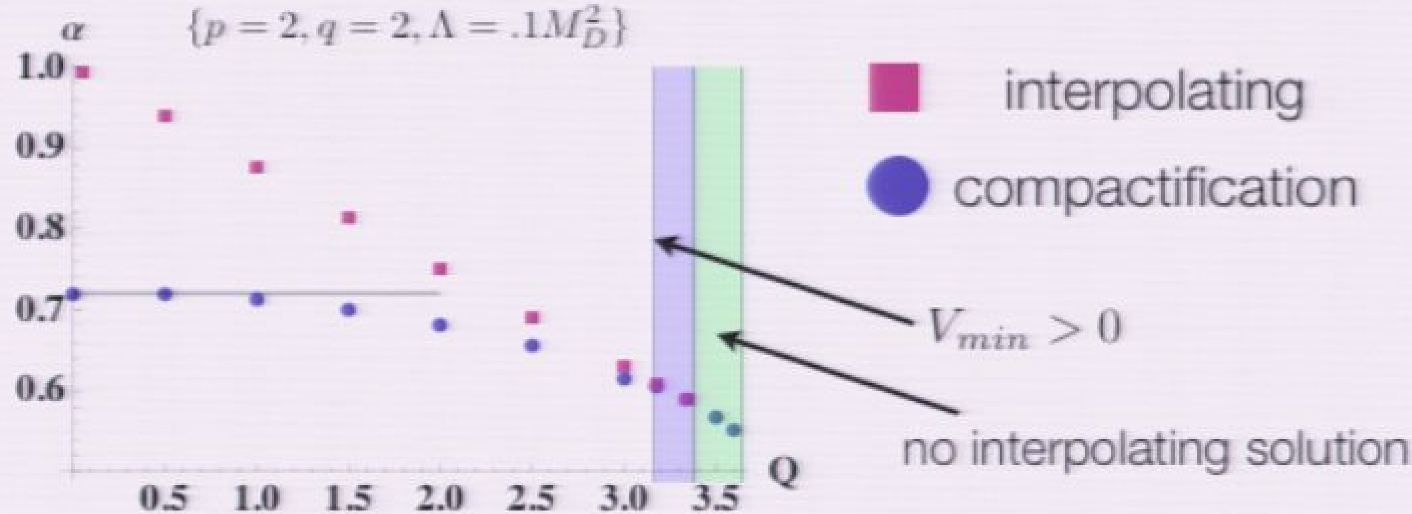

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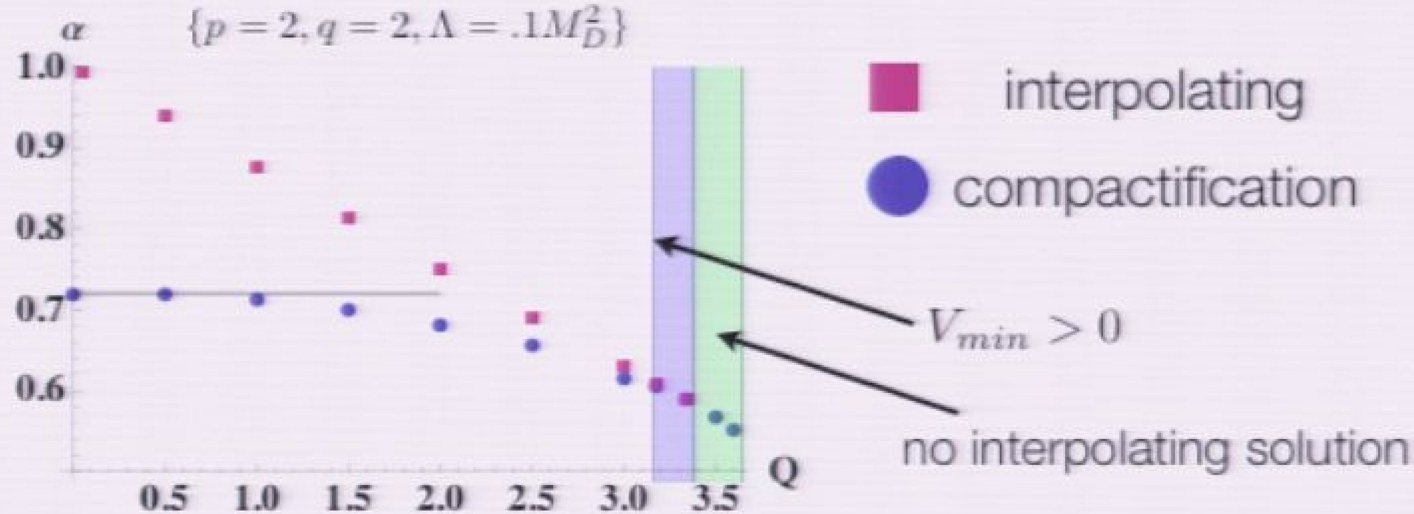
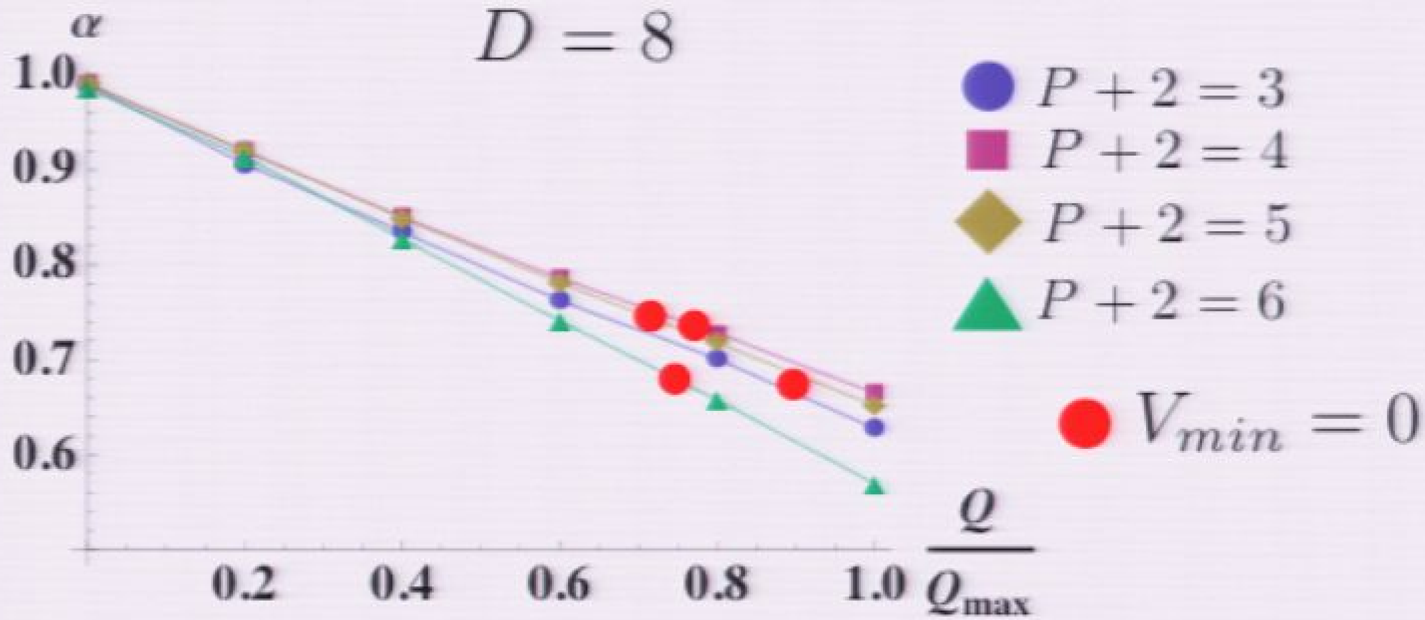


$$\Gamma = A \exp\left[S_{ds}^{(D)}(1-\alpha)\right]$$

- Rates are suppressed by the de Sitter action.
- The rate for the interpolating solutions is higher when it exists.
- The rate is highest for small Q = lowest vacuum energy.

Dynamical compactification: rates

$\frac{F_q^2}{2q!} \rightarrow \sum_{i=2}^{D-2} \frac{F_{q_i}^2}{2q_i!}$ We can compare rates to vacua with different dimensionality:



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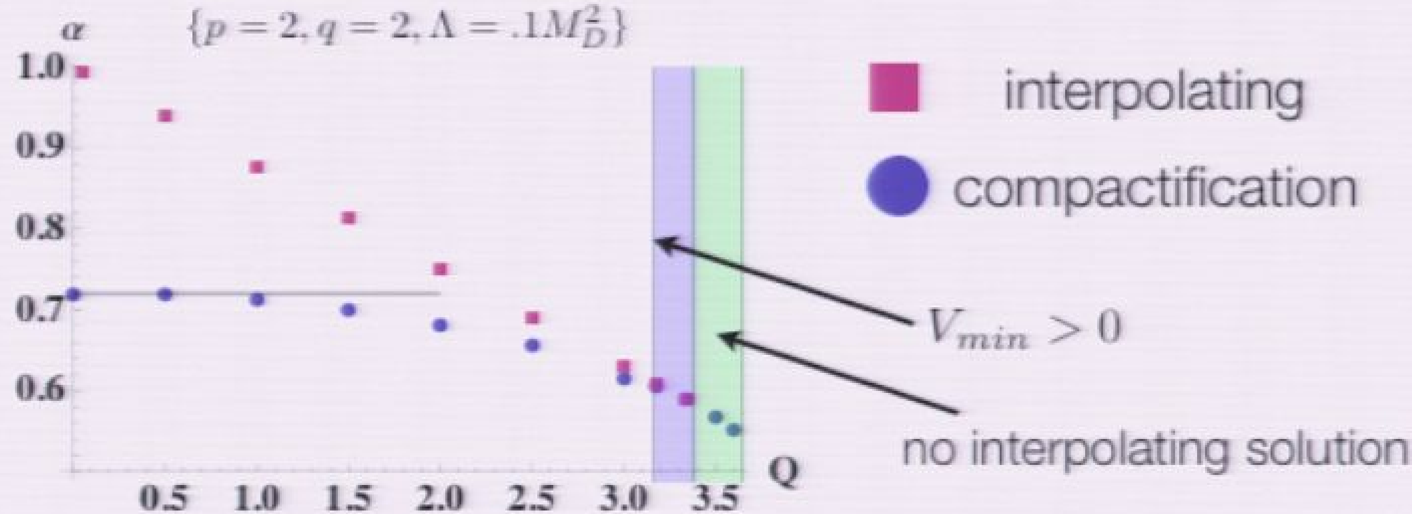

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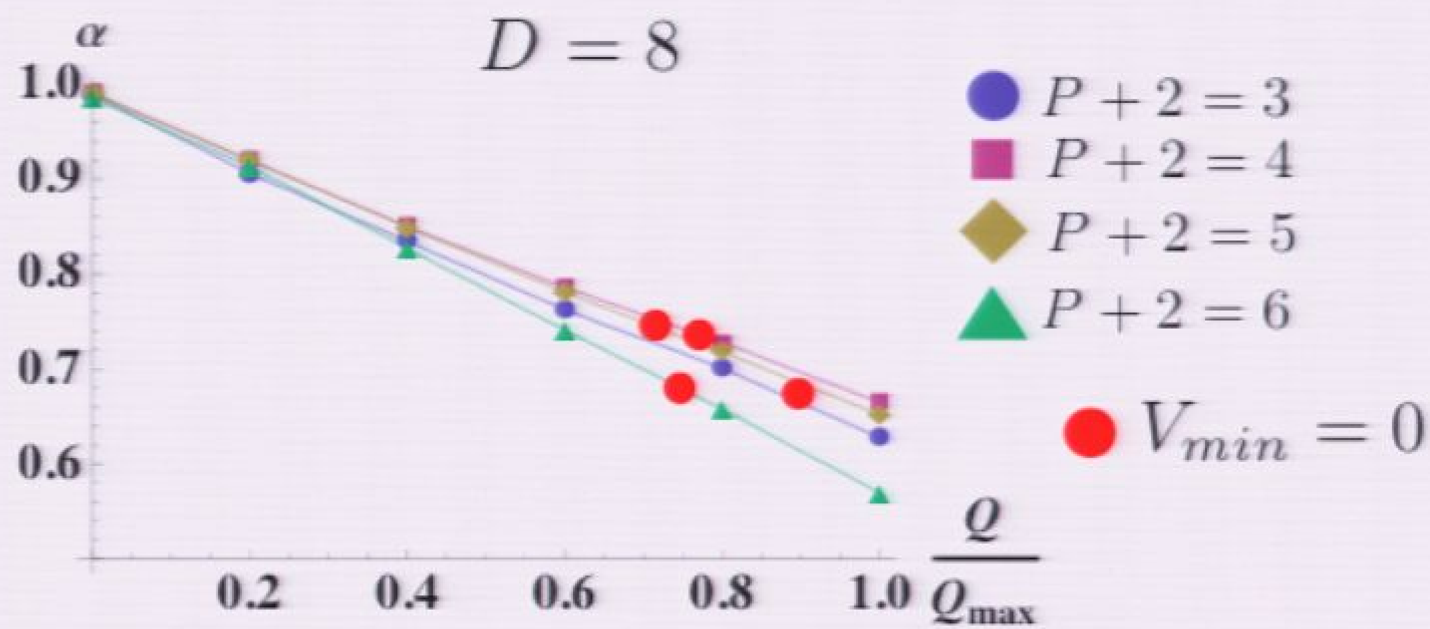

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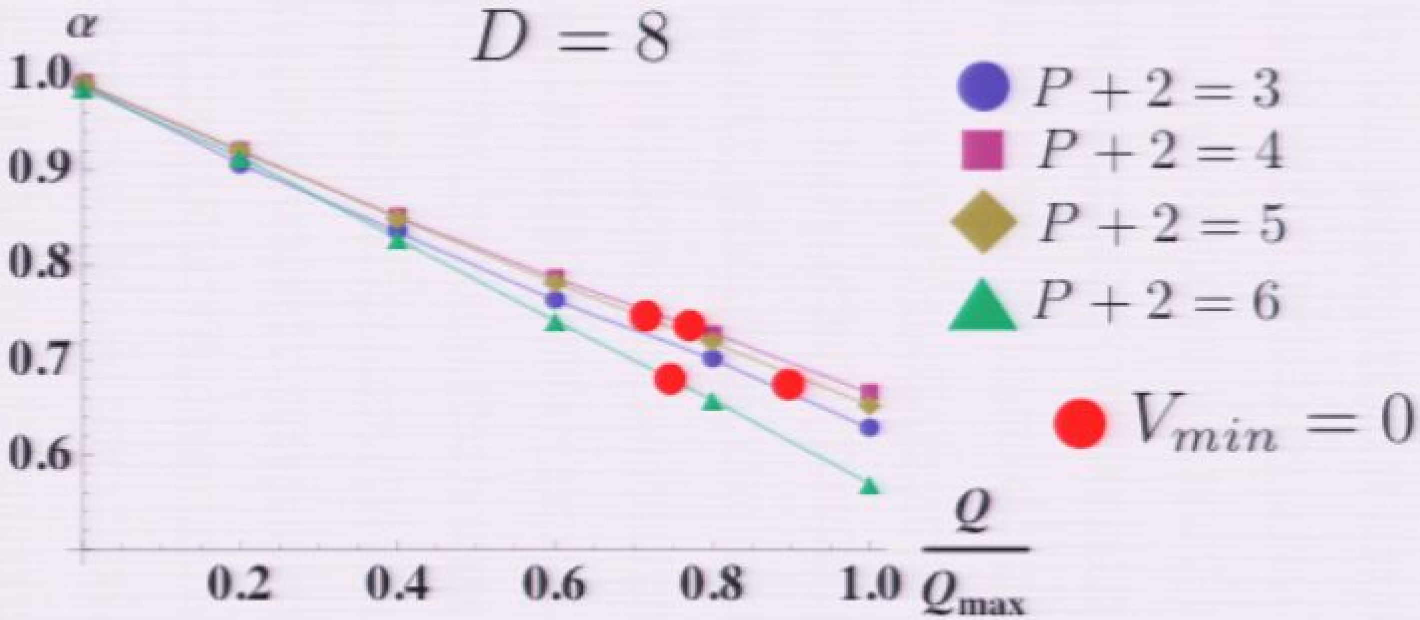

- No large disparity between different numbers of compactified dimensions.

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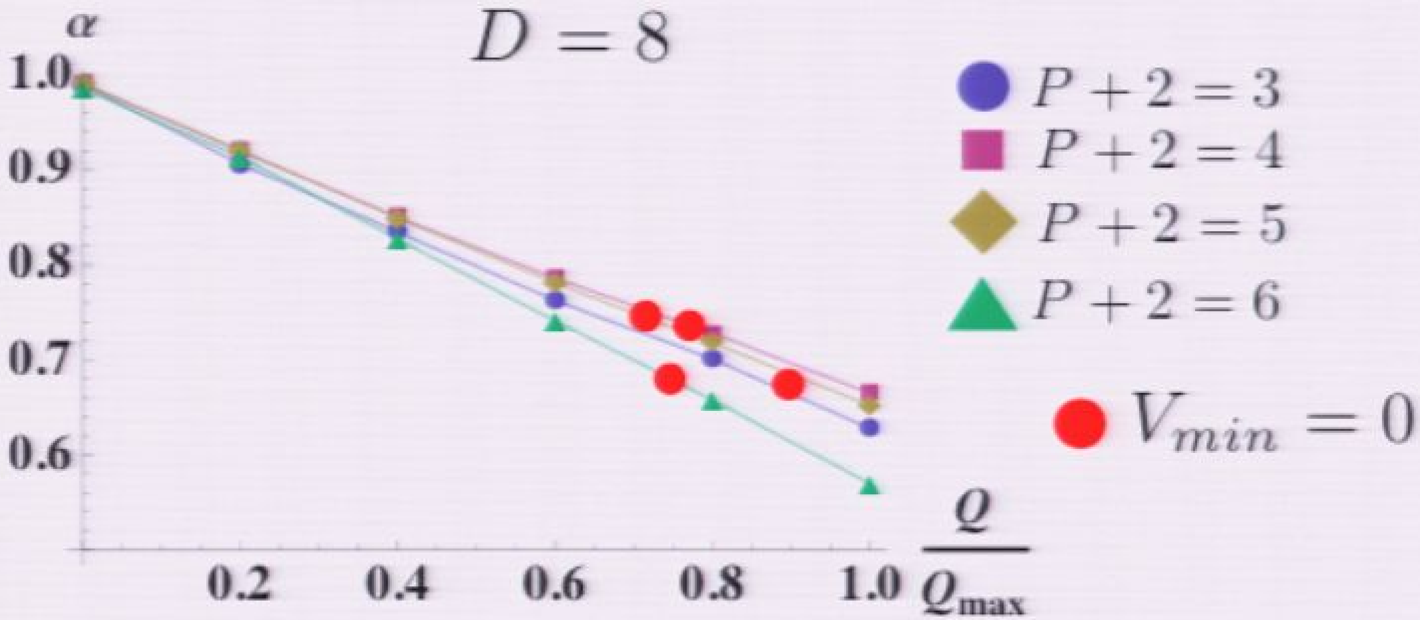
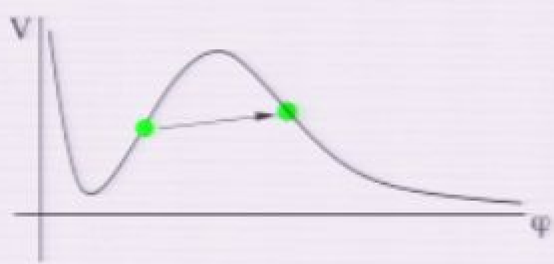


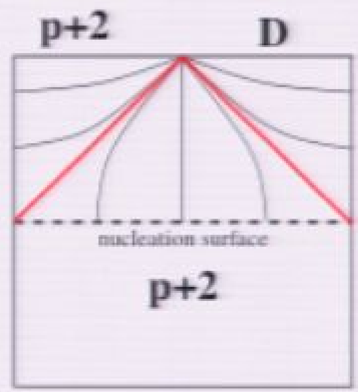
- No large disparity between different numbers of compactified dimensions.
- Unclear what to compare.....

Decompactification transitions (Giddings, Giddings+Myers)

- The p+2 dimensional de Sitter vacua decay back to D dimensional de Sitter space by the same instanton:

$$\Gamma = A \exp\left[-(S_{inst} - S_{dS}^{(4)})\right]$$





- The rate into a vacuum is always larger than the rate out

$$\frac{\Gamma_{in}}{\Gamma_{out}} = \exp\left[|S_{dS}^{(4)}| - |S_{dS}^{(D)}|\right] \qquad |S_{dS}^{(4)}| > |S_{dS}^{(D)}|$$

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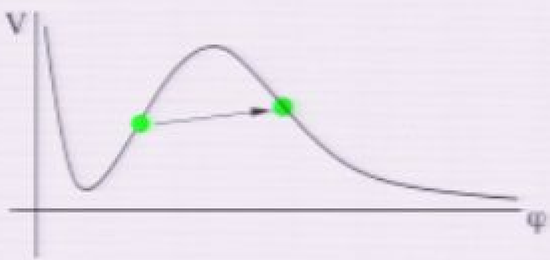

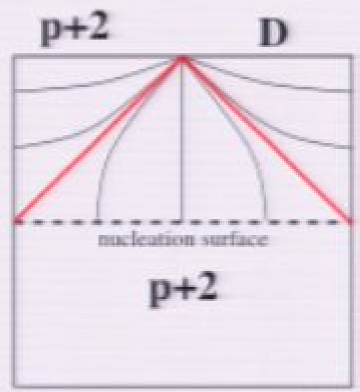

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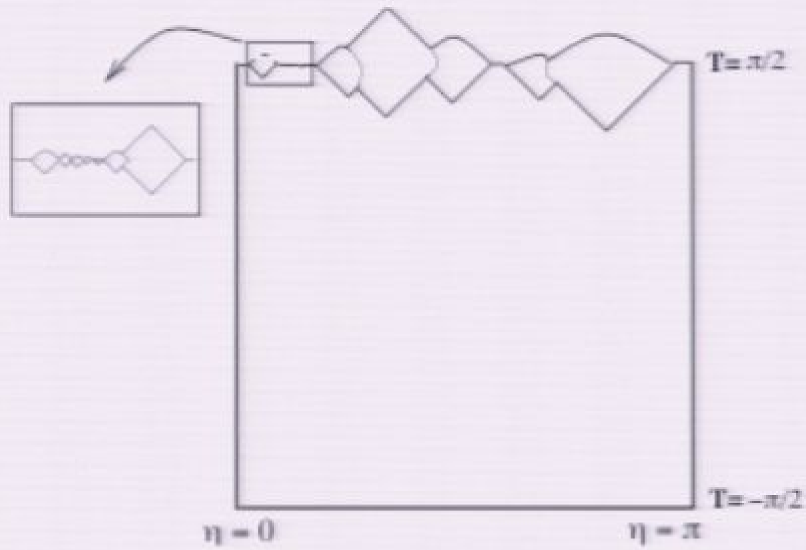
- Minkowski vacua are completely stable.

Global structure of the multiverse



- Future infinity is fractally distributed among vacua with different vacuum energy and numbers of non-compact dimensions.
- Transitions occur back and forth between p+2 and D dimensions.
- Higher-dimensional eternal inflation!
- Connecting to predictions is a very difficult measure issue.

Summary

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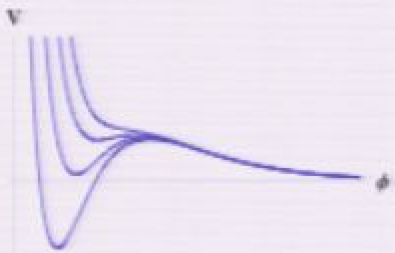

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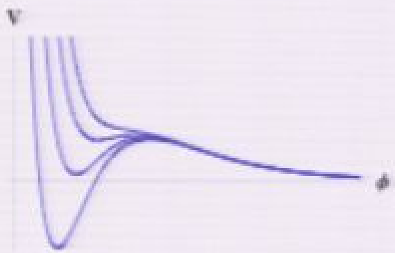

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Solutions interpolate between D-dimensional dS and p+2 dimensional FRW across event horizons.

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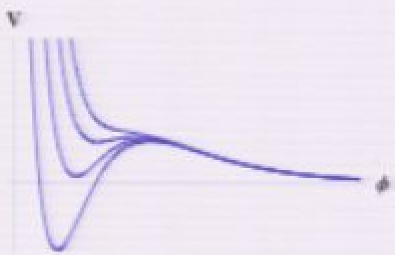

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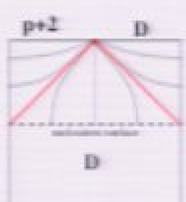

These solutions are nucleated from dS - Dynamical compactification.

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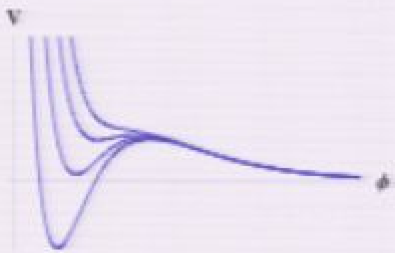

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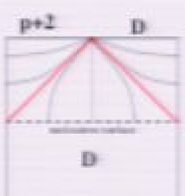

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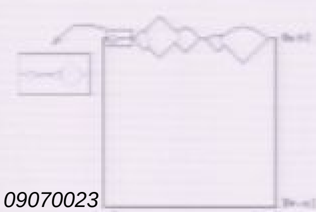

Transitions back and forth populate the landscape of vacua.

Future directions

- Stability analysis.

Future directions

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  - For D-q = 4, the compactification solutions have instabilities when q > 4 (Bousso, de Wolfe, Myers).
  - The endpoint of the instability may still be a compact manifold (warped sphere according to Kinoshita and Mukohyama).
  - What about the stability of the interpolating solutions?
  - What about thermodynamical stability? Can universes evaporate?

Future directions

- Stability analysis.
- Inhomogeneities.

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- Stability analysis.
- Inhomogeneities.
  - Inevitably "collisions" between interpolating solutions will occur.
  - Field outside of brane will cause stimulated emission of small-charge branes (similar to Schwinger pair production).
  - These are multi-centered black brane solutions.
  - This changes the geometry - what happens to the homogeneity of the 4 dimensional FRW inside the horizon?
  - Are there potentially observable effects?

Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
    - On the other side of the non-singular big-bang surface, extra dimensions become “large”.
    - Does this lead to any interesting effects?

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- Stability analysis.
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- Measure issues.

Future directions

- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
  - We have a catalog of nucleation rates. They have rather simple (and suggestive) properties.
  - Is it possible to go from this to statistical predictions for various fundamental parameters?
  - Requires an understanding of the measure - similar to eternal inflation.

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- What about standard 4D eternal inflation?

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- Stability analysis.
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- What about standard 4D eternal inflation?
  - Membrane nucleation can occur inside of the locally 4D region, leading to the standard picture of 4D eternal inflation.
  - Subtleties due to interaction of flux d.o.f. with radion.

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- Other solutions?

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- Stability analysis.
- Inhomogeneities.
- Pre-big bang effects.
- Measure issues.
- What about standard 4D eternal inflation?
- Other solutions?
  - Homogenous but anisotropic metric ansatz will generate different solutions.
  - A flat metric ansatz generates non-extremal black branes.
  - What about the other Bianchi types?
  - Bent branes?

The End.

Thanks!