Title: Holographic Resolution of Cosmological Singularities

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Abstract: An update is given on AdS/CFT models of cosmological singularities, in particular on models in which a big crunch instability of the bulk theory is induced by an unstable multi-trace deformation of the dual field theory.

Holographic Resolution of Cosmological Singularities

BC, Herzog, Turok 0712..., 0905..., in progress

Berenstein, BC 0907..

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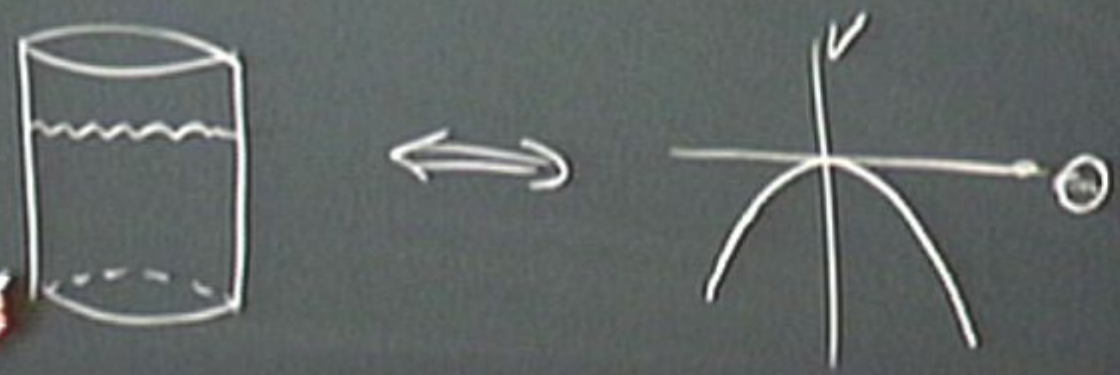

Holographic Resolution of Cosmological Singularities

BC, Herzog, Turok 0712..., 0905..., in progress

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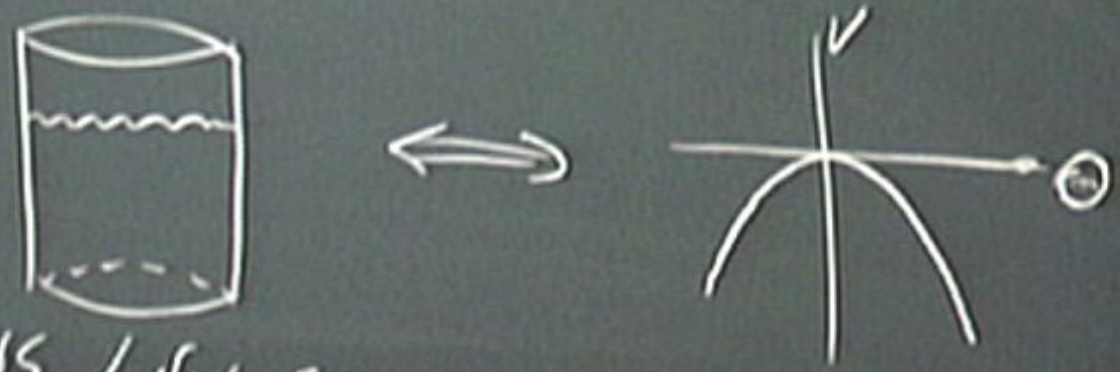
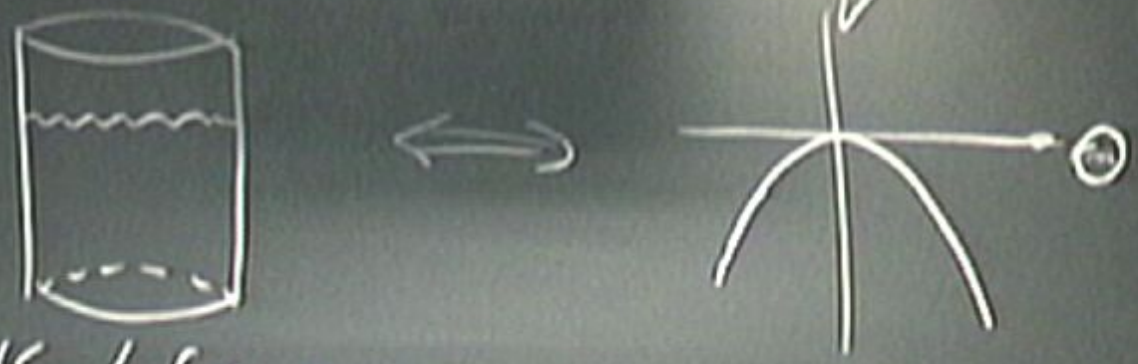


1) $AdS_5 / \mathcal{N}=4$ SYM

Bananaiti, BC



1) AdS_5 / $\mathcal{N}=4$ SYM

2) AdS_4 / ABJM

$AdS_5 \times \mathcal{N}=4$ SYM

IIB supgra on S^5

$\downarrow$

5d grav + scalar φ

$AdS_5 \times \mathcal{N}=4$ SYM

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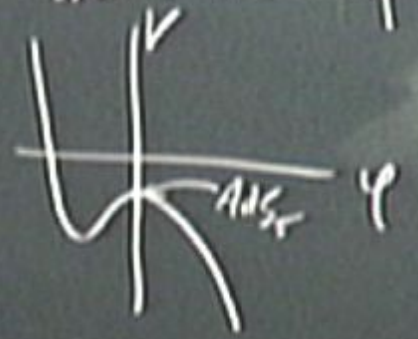
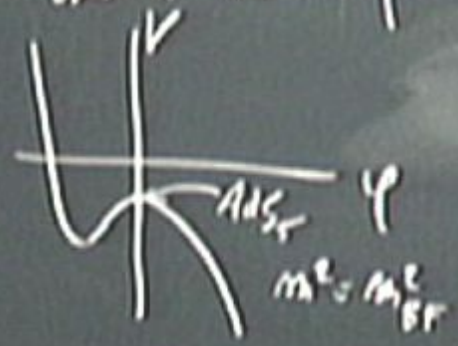


IIB supgra on S^5

$\downarrow$

Sd $g_{\mu\nu}$ + scalar φ



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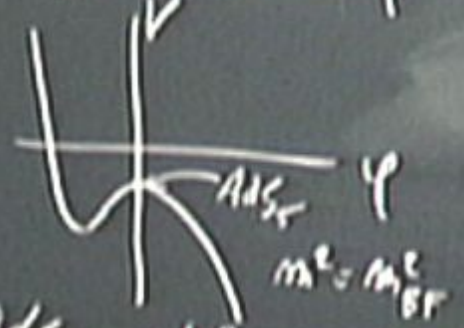
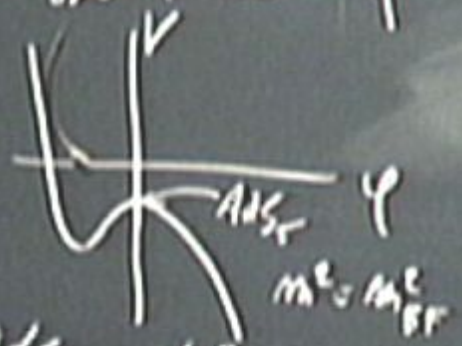


$AAdS_5$: $ds^2 \underset{r\to\infty}{\sim} -(r/\ell^2)dt^2 + \frac{dr^2}{r/\ell^2} + r^2 d\Omega^2_3$

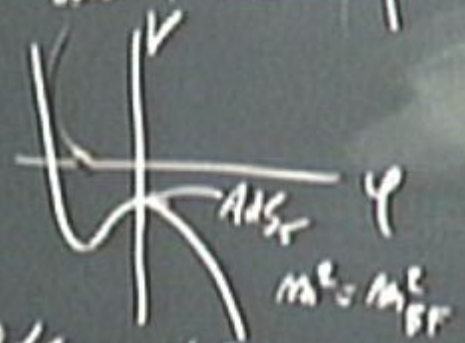
SU $g_{\mu\nu}$ + scalar φ



$$AAdS_5: \quad ds^2 \underset{r\to\infty}{\sim} -(r+r^2)dt^2 + \frac{dr^2}{r+r^2} + r^2 d\Omega_3^2$$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha(t,\Omega)\ln r}{r^2} + \frac{\beta(t,\Omega)}{r^2}$$

SI grav + scalar φ



$AAdS_5$: $$ds^2 \underset{r\to\infty}{\sim} -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega_3^2$$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha(t,\Omega)\ln r}{r^2} + \frac{\beta(t,\Omega)}{r^2}$$

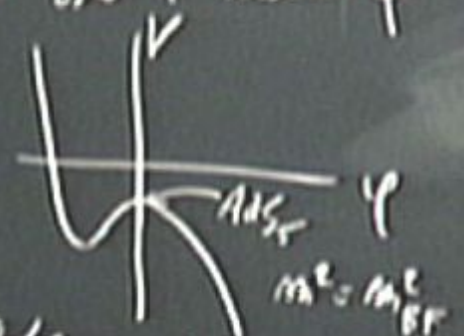
b.c $\alpha = f\beta$ $\quad f>0$

$AdS_5 / \mathcal{N}=4$ SYM

IIB supgra on S^5 $\longleftrightarrow$ $\mathcal{N}=4$ SYM on $\mathbb{R}\times S^3$

$\downarrow$

5d grav + scalar φ $\longleftrightarrow$



$AAdS_5$: $ds^2 \underset{r\to\infty}{\sim} -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2$

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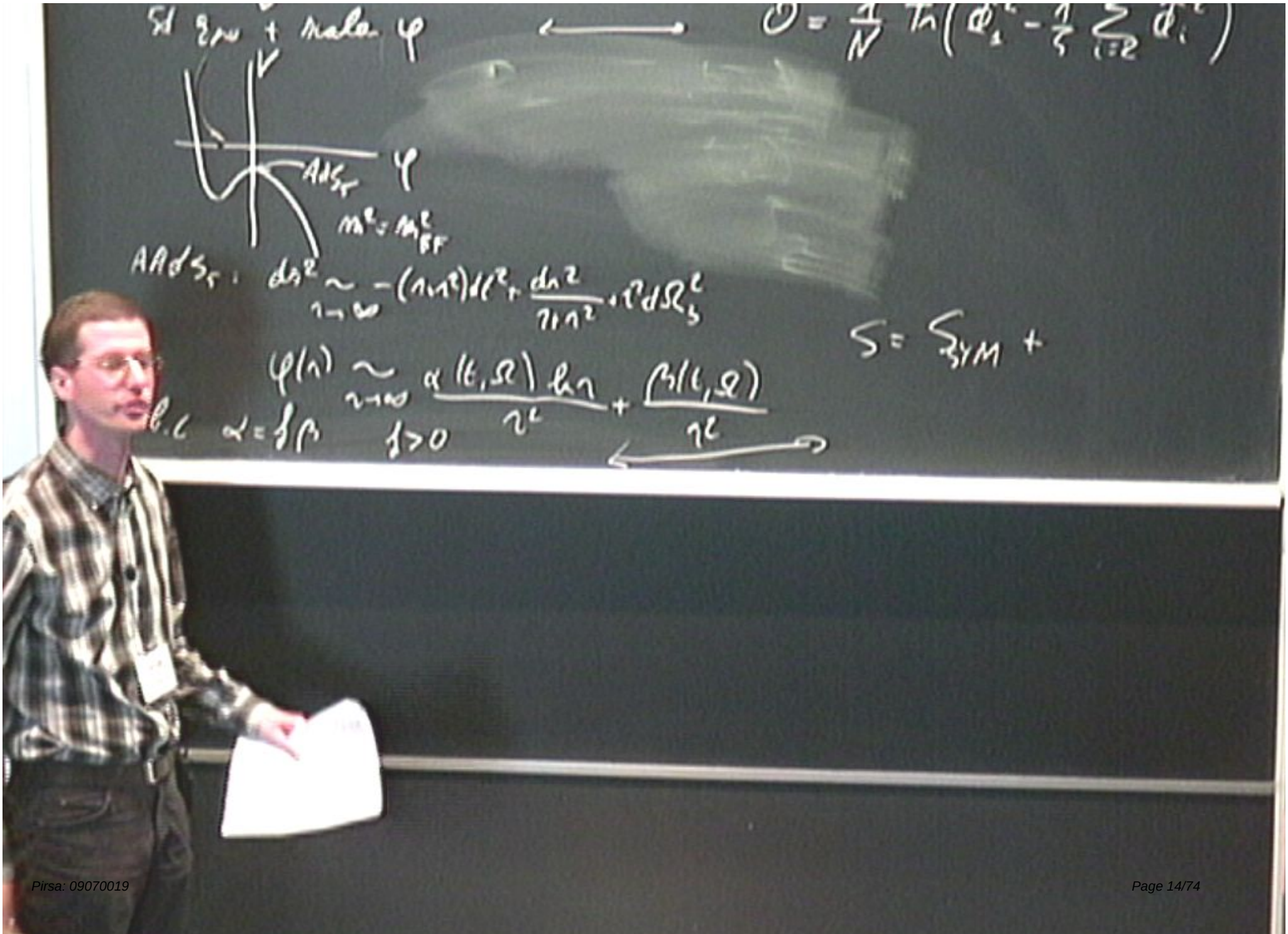
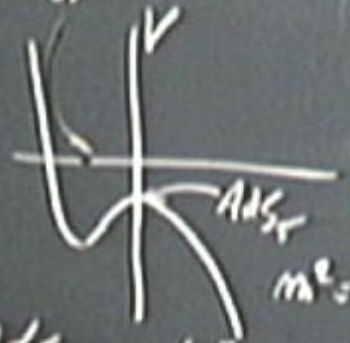

Sd g_μν + scalar φ
V
φ
AdS_5
m² = m²_BF
AAdS_5 : ds² ~ -(r/r²)dt² + dr²/(r/r²) + r²dΩ²_3
r→∞
φ(r) ~ α(t,Ω) ln r / r² + β(t,Ω) / r²
r→∞
b.c α = δβ δ>0
S = S_YM +

Sol $g_{\mu\nu}$ + scalar φ $\longleftrightarrow$ $\mathcal{O} = \frac{1}{N}\,\mathrm{Tr}\left(\Phi_1^2 - \frac{1}{5}\sum_{i=2}^{5}\Phi_i^2\right)$



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b.c $\alpha = f\beta$ $\quad f>0$

$S = S_{SYM}$ + conf coupl $+ \frac{1}{2}\int \mathcal{O}^2$

AdS_5 / $\mathcal{N}=4$ SYM

IIB supergr. on S^5 $\longleftrightarrow$ $\mathcal{N}=4$ SYM on $\mathbb{R}\times S^3$

$\downarrow$

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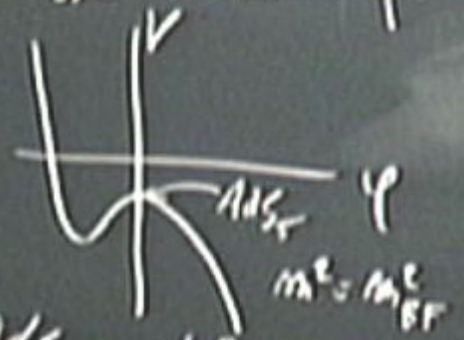

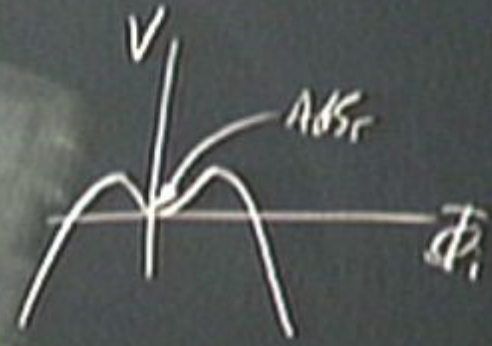

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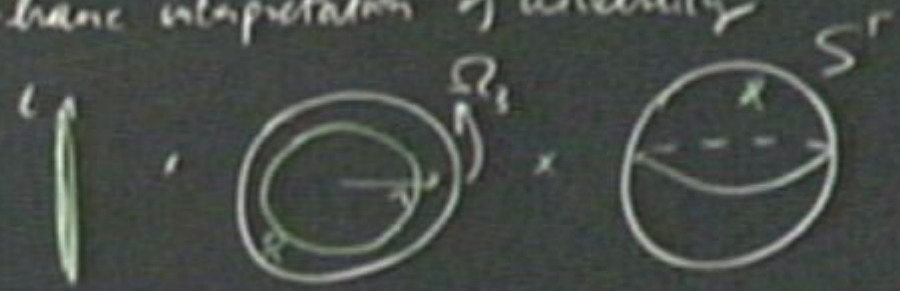
• D-brane interpretation of instability



AdS_5 / $\mathcal{N}=4$ SYM

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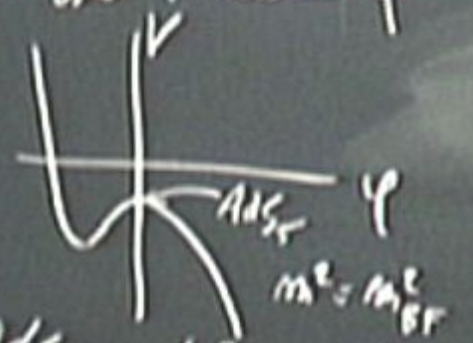

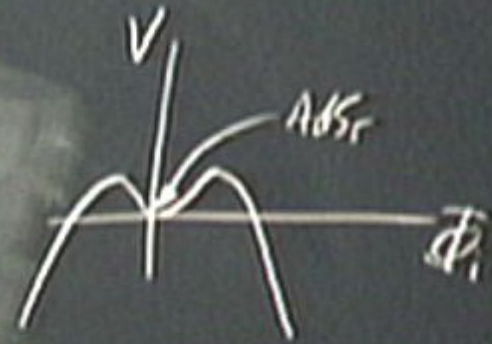

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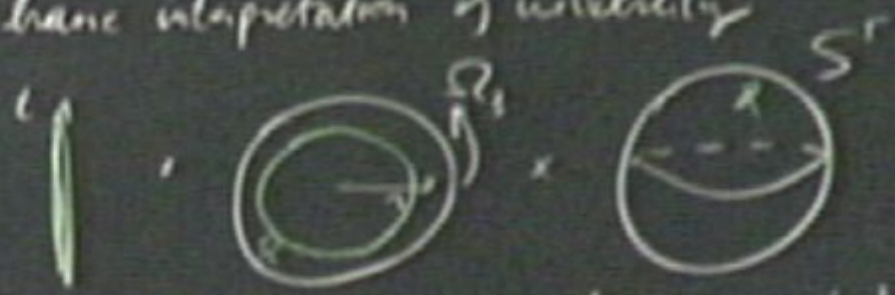
- D-brane interpretation of instability

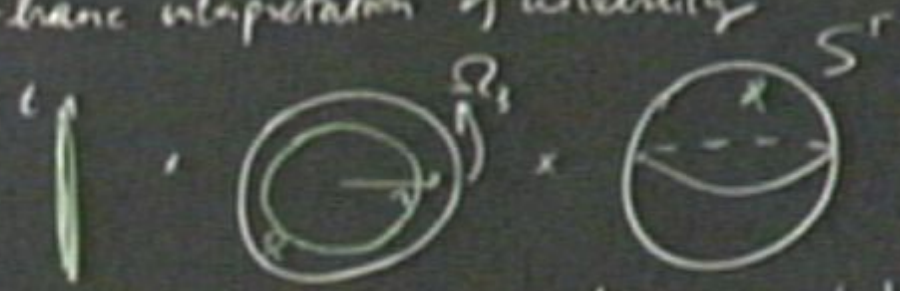


- $\int$ runs at leading order in $\frac{1}{N}$, asymptotic free

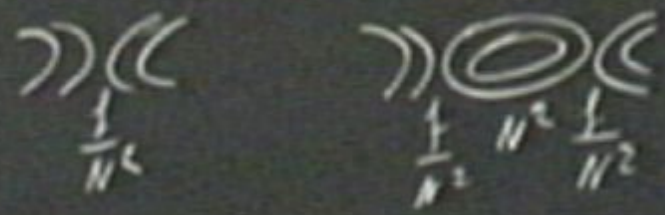
$$\gg \ll$$

$$\frac{1}{N^2}$$

- D-brane interpretation of instability



- f runs at leading order in $\frac{1}{N}$, asymptotic free



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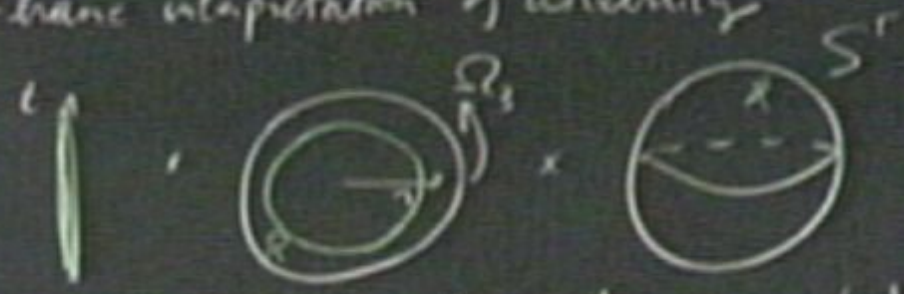
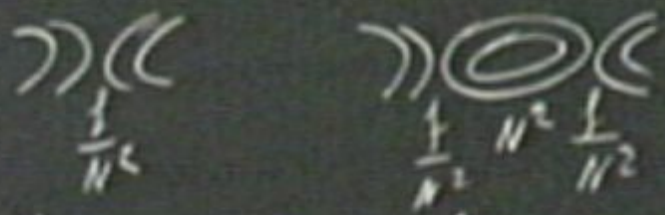
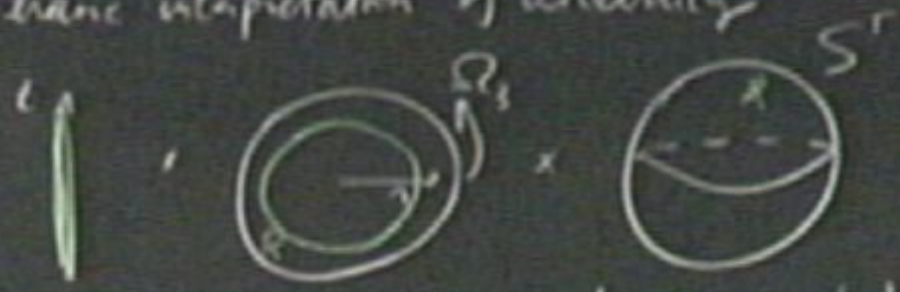


- f runs at leading order in $\frac{1}{N}$, asymptotic free

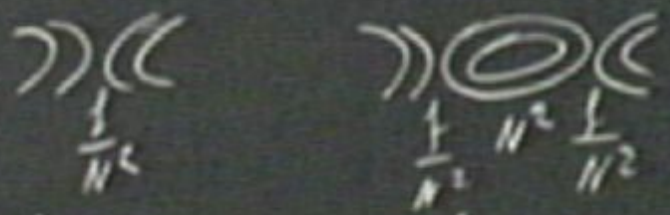


- self-adjoint extension in QM

- D-brane interpretation of instability



- f runs at leading order in $\frac{1}{N}$; asymptotic free



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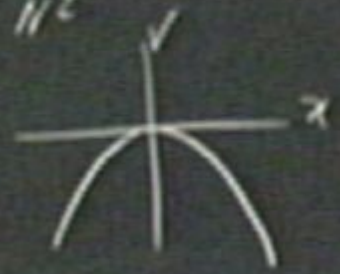
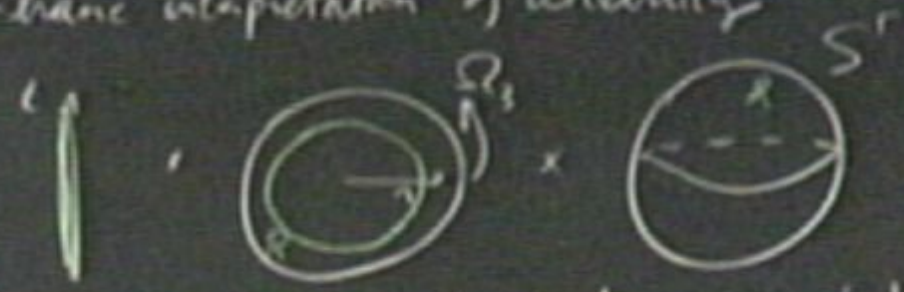
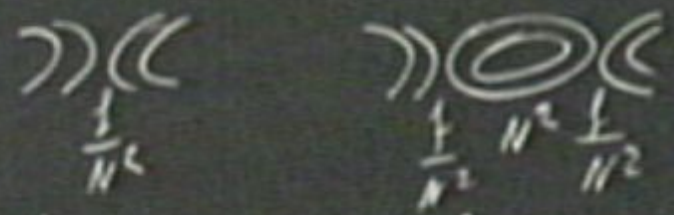


$$H = \frac{p^2}{2} - \frac{\lambda}{4} x^4$$

- D-brane interpretation of instability



- f runs at leading order in $\frac{1}{N}$, asymptotic free



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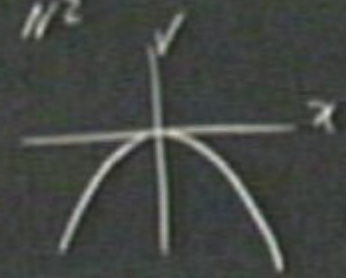
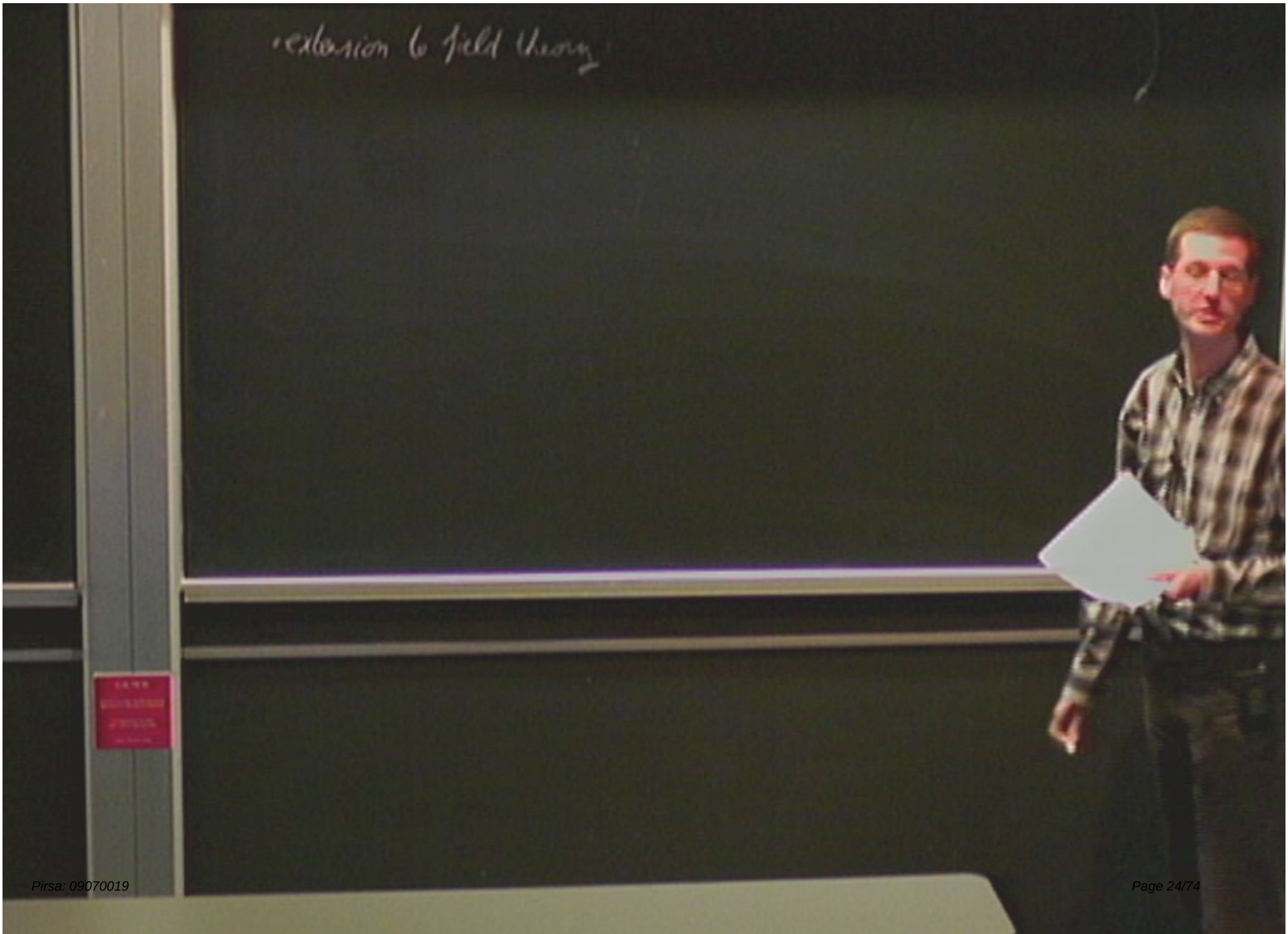
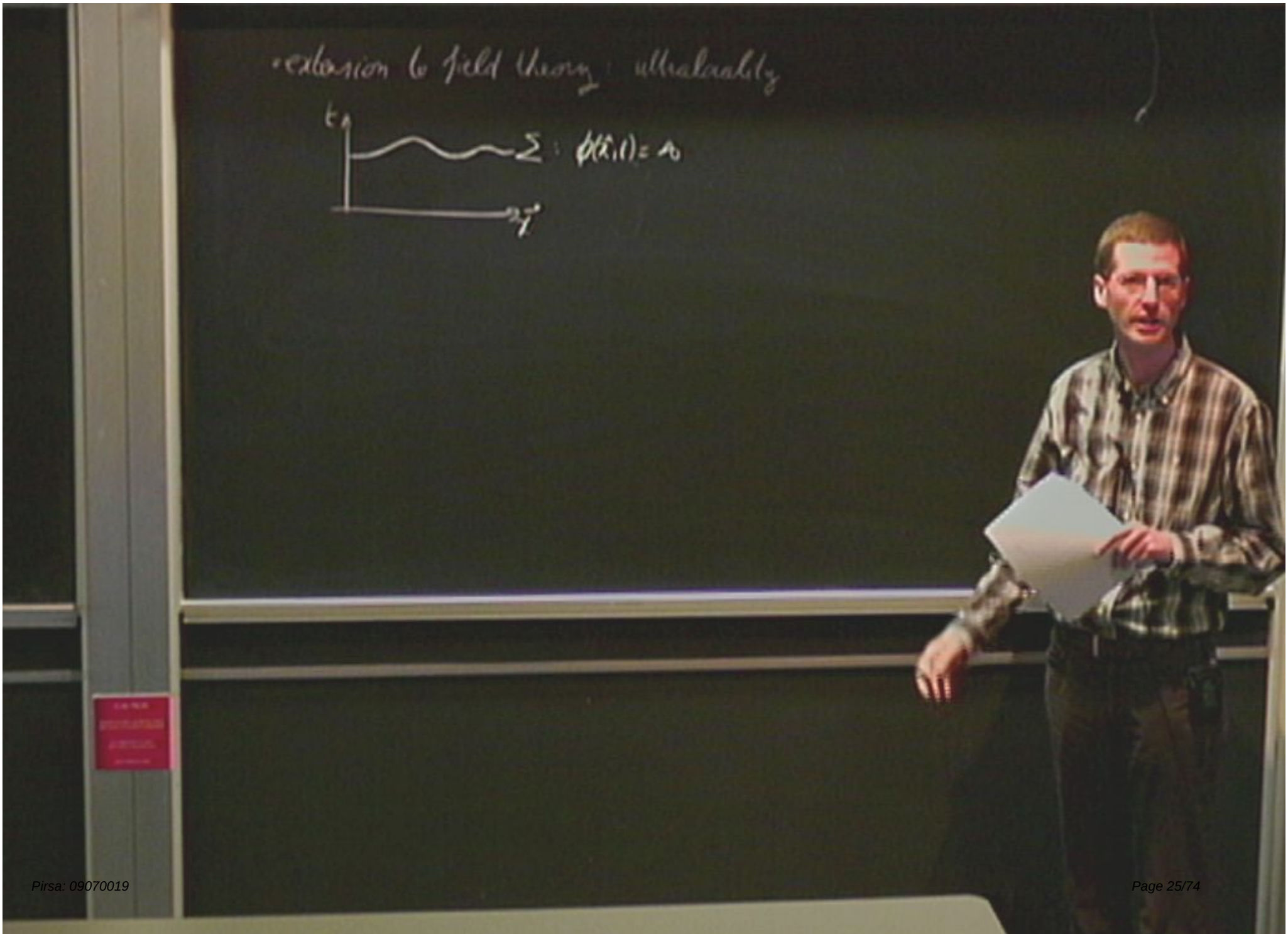
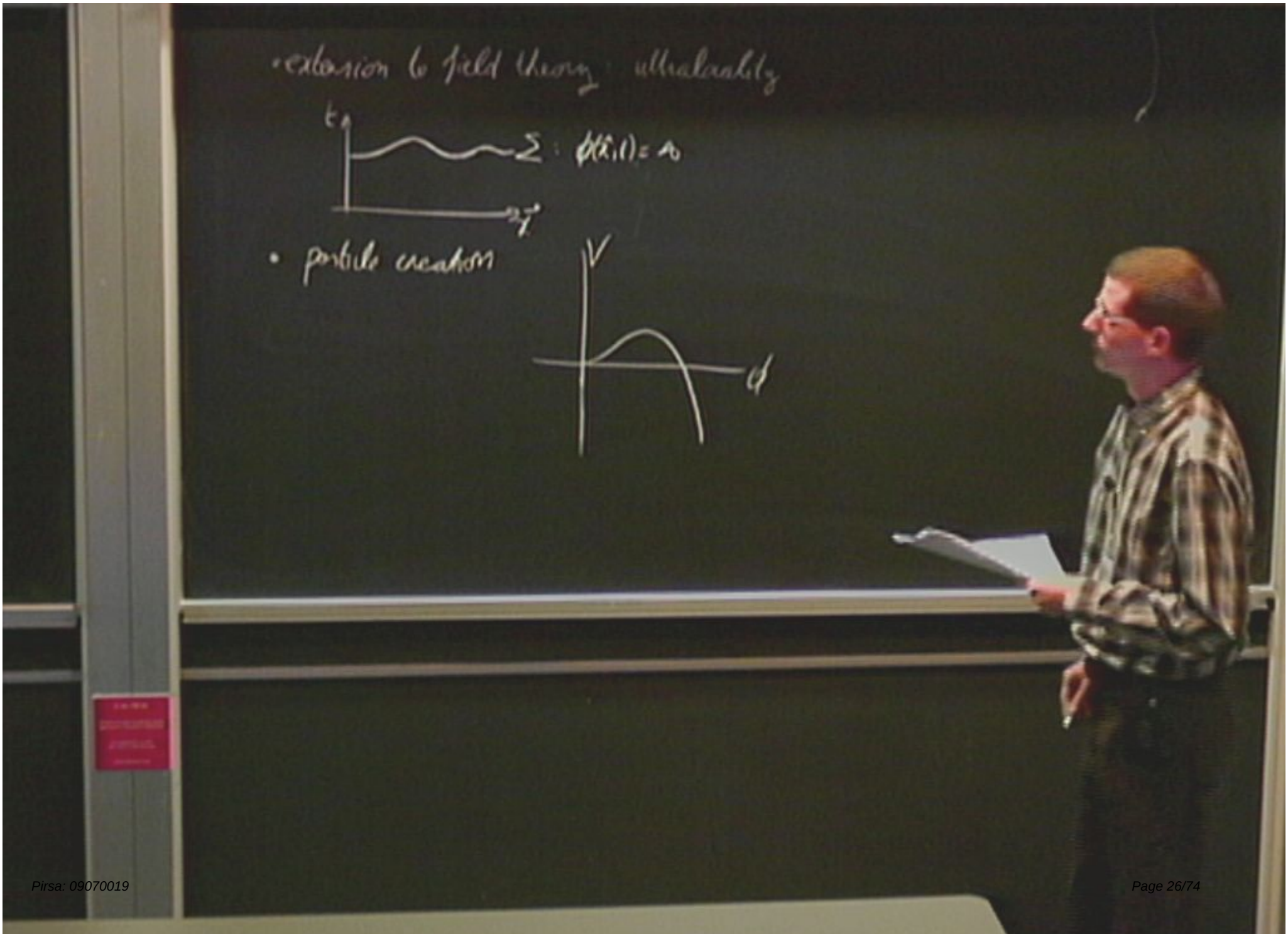


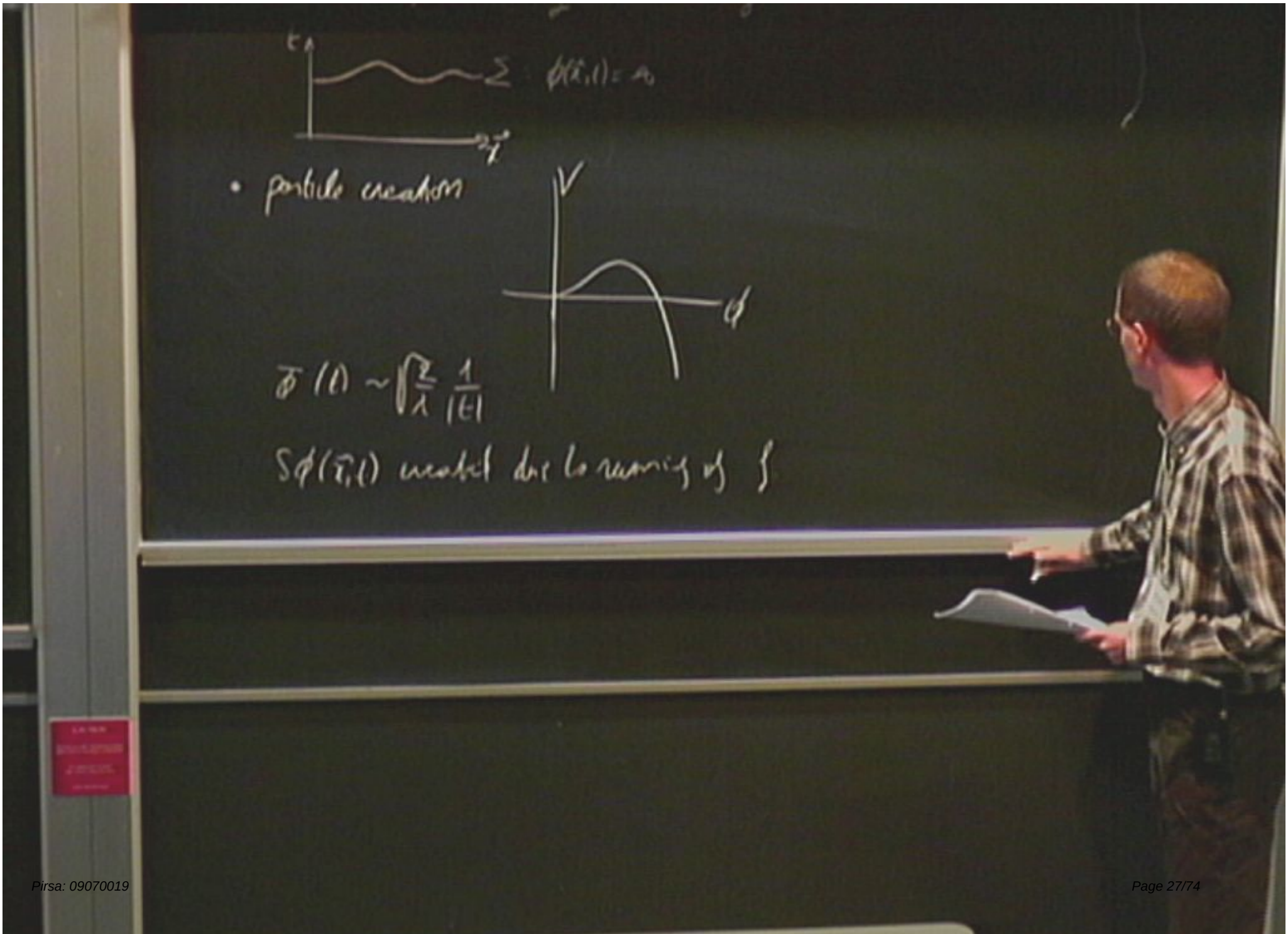
$H = \frac{p^2}{2} - \frac{\lambda}{4} x^4$

4 param.

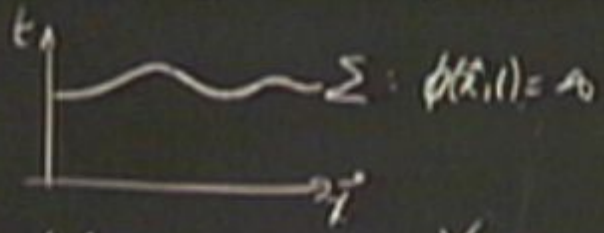

extension to field theory


• extension to field theory: ultralocality
t
Σ : φ(x,t) = φ0
x

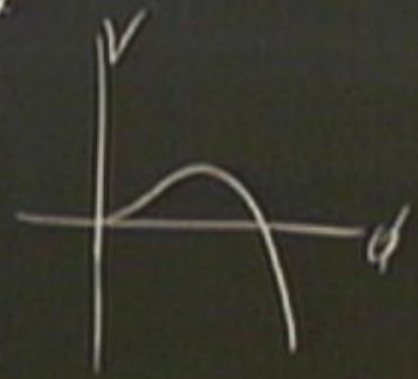

• extension to field theory: ultralocality
t
Σ : φ(x,t) = φ0
x
• particle creation
V
φ


t
Σ
$\phi(\vec{x},t) = \phi_0$
$\vec{x}$
• particle creation
V
ϕ
$\phi(t) \sim \sqrt{\frac{2}{\lambda}} \frac{1}{|t|}$
$S\phi(\vec{x},t)$ created due to running of ξ

• extension to field theory: ultralocality



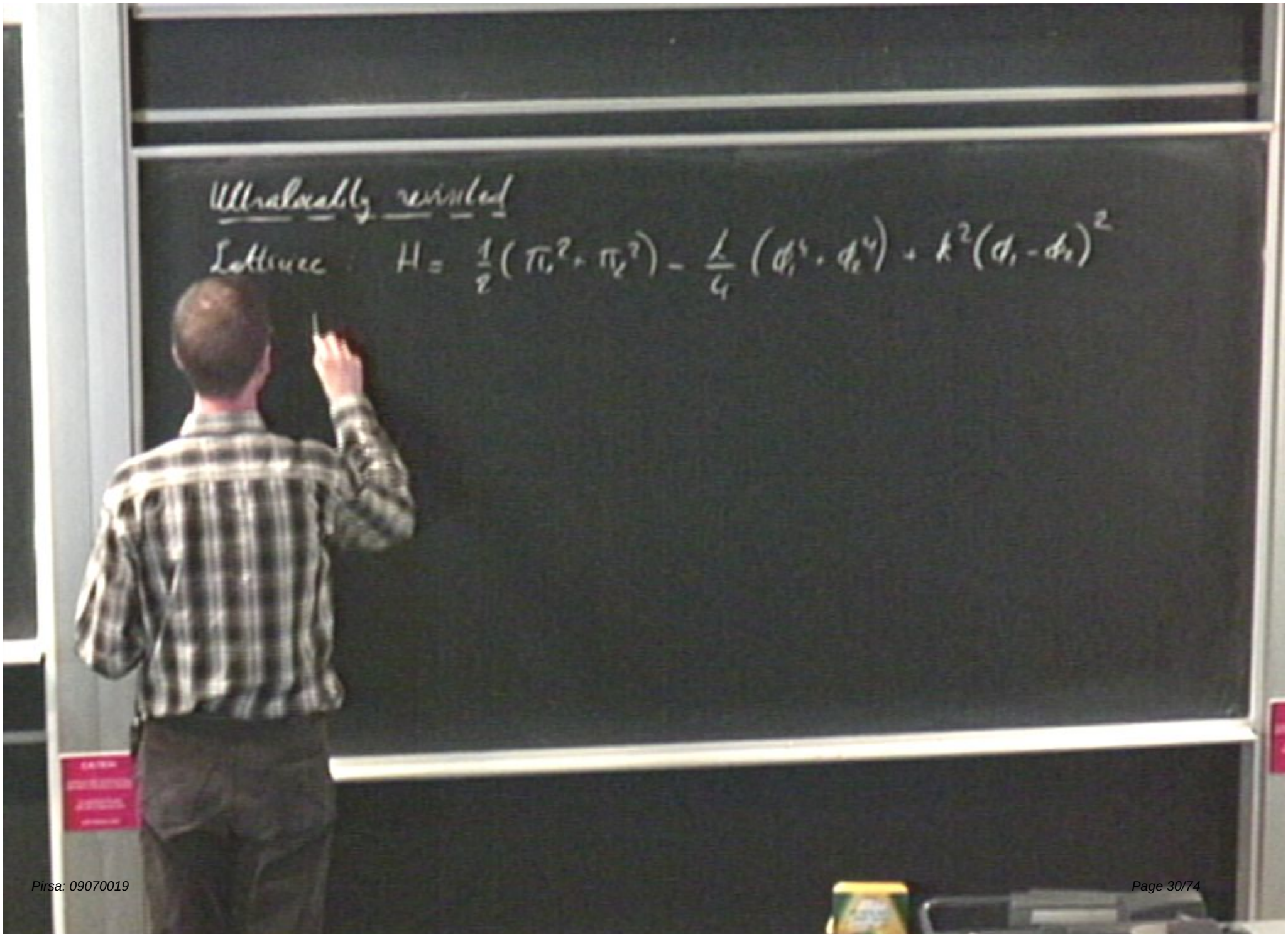
• particle creation



$\bar{\phi}(t) \sim \sqrt{\frac{2}{\lambda}} \frac{1}{|t|}$

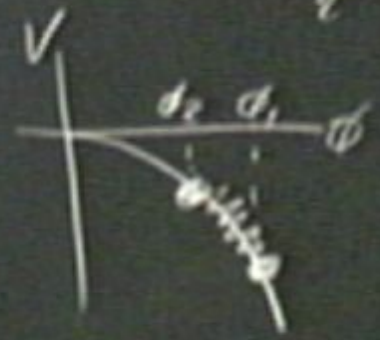
$\delta\phi(\vec{r},t)$ created due to rearrangement of $\bar{\phi}$

Ultrabreadth revisited


Ultralocally revisited
Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + k^2(\phi_1 - \phi_2)^2$

Ultralocality revisited

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + k^2(\phi_1 - \phi_2)^2$



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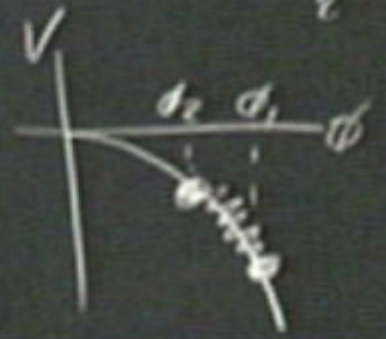
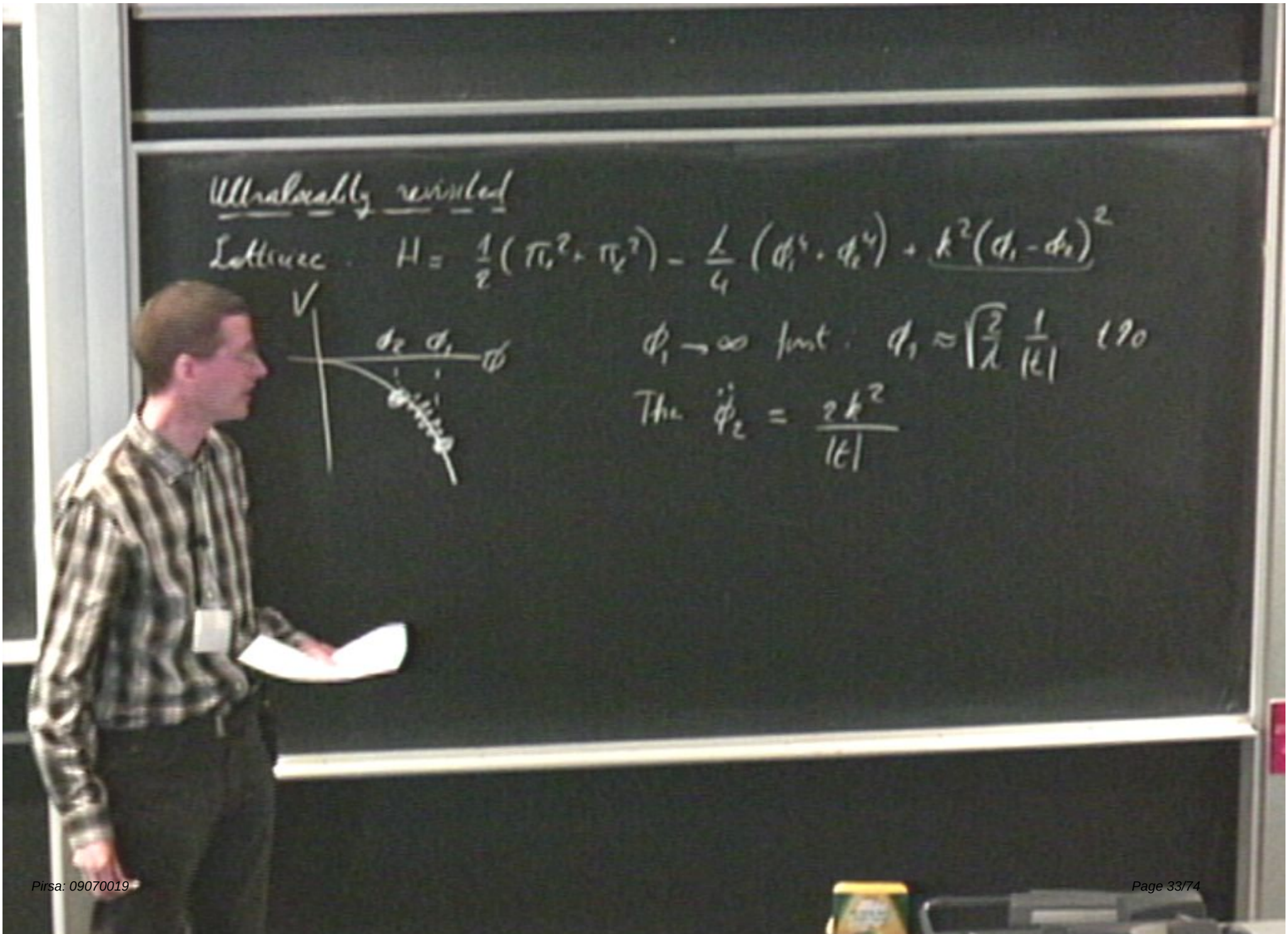


$\phi_1 \to \infty$ first: $\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|}$ 190

Then $\ddot{\phi}_2 = \frac{2k^2}{|t|}$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V
ϕ_2 ϕ_1
ϕ
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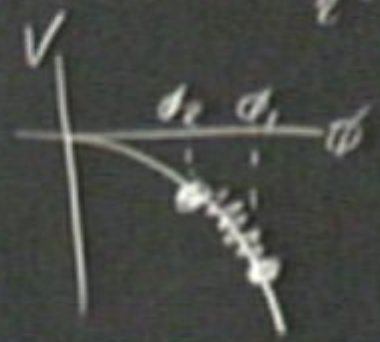
Ultrahorably revisited

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + \underline{k^2(\phi_1 - \phi_2)^2}$

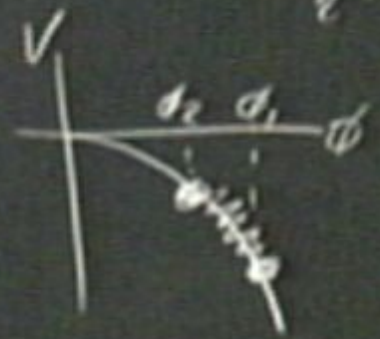


$\phi_1 \to \infty$ fast. $\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|}$ $t \gg 0$

Then $\ddot{\phi}_2 = \frac{2k^2}{|t|} \rightsquigarrow \phi_2 \approx$ const $(t \gg 0)$

Ultralocally revisited

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + \underbrace{k^2(\phi_1 - \phi_2)^2}$



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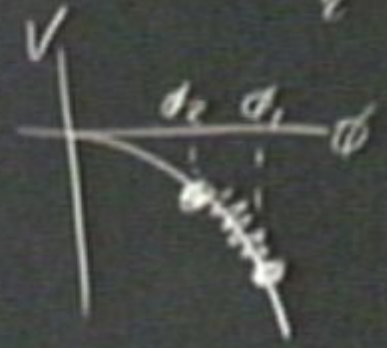
$\Rightarrow$ ultralocality

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + \boxed{k^2(\phi_1 - \phi_2)}$



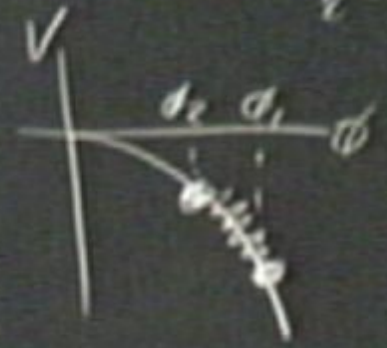
$\phi_1 \to \infty$ first. $\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|}$ $(t \gg 0)$

Then $\ddot{\phi}_2 = \frac{2k^2}{|t|}$ $\rightsquigarrow$ $\phi_2 \approx$ const $(t \gg 0)$

$\Rightarrow$ stabilizability

However: model degenerate at $t > 0$

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + \boxed{k^2(\phi_1 - \phi_2)}$



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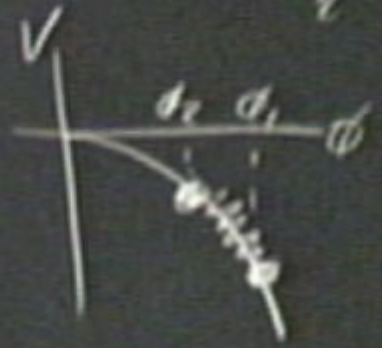
Then $\ddot{\phi}_2 = \frac{2k^2}{|t|}$ $\rightsquigarrow$ $\phi_2 \approx$ const $(t \gg 0)$

$\Rightarrow$ attractivity

However: model degenerate at $t > 0$

Possible cure:

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + \underline{k^2(\phi_1 - \phi_2)}$



$\phi_1 \to \infty$ first: $\phi_1 \approx \sqrt{\frac{2}{\lambda}}\frac{1}{|t|}$ (190)

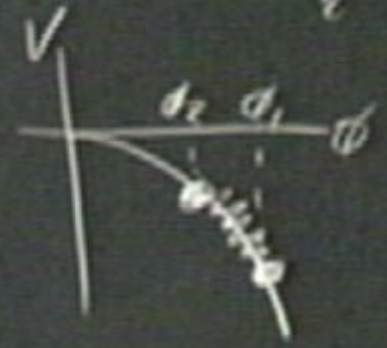
Then $\ddot{\phi}_2 = \frac{2k^2}{|t|}$ $\rightsquigarrow$ $\phi_2 \approx$ const (190)

$\Rightarrow$ adiabaticity

However: model degenerate at $t > 0$

Possible cure: $\bar{\phi}(t) \sim \frac{1}{t - i\varepsilon}$

Lattice: $H = \frac{1}{2}(\pi_1^2 + \pi_2^2) - \frac{\lambda}{4}(\phi_1^4 + \phi_2^4) + \boxed{k^2(\phi_1 - \phi_2)}$



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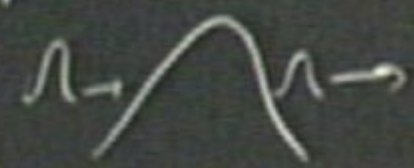
Then $\ddot{\phi}_2 = \frac{2k^2}{|t|} \rightsquigarrow \phi_2 \approx$ const $(t \gg 0)$

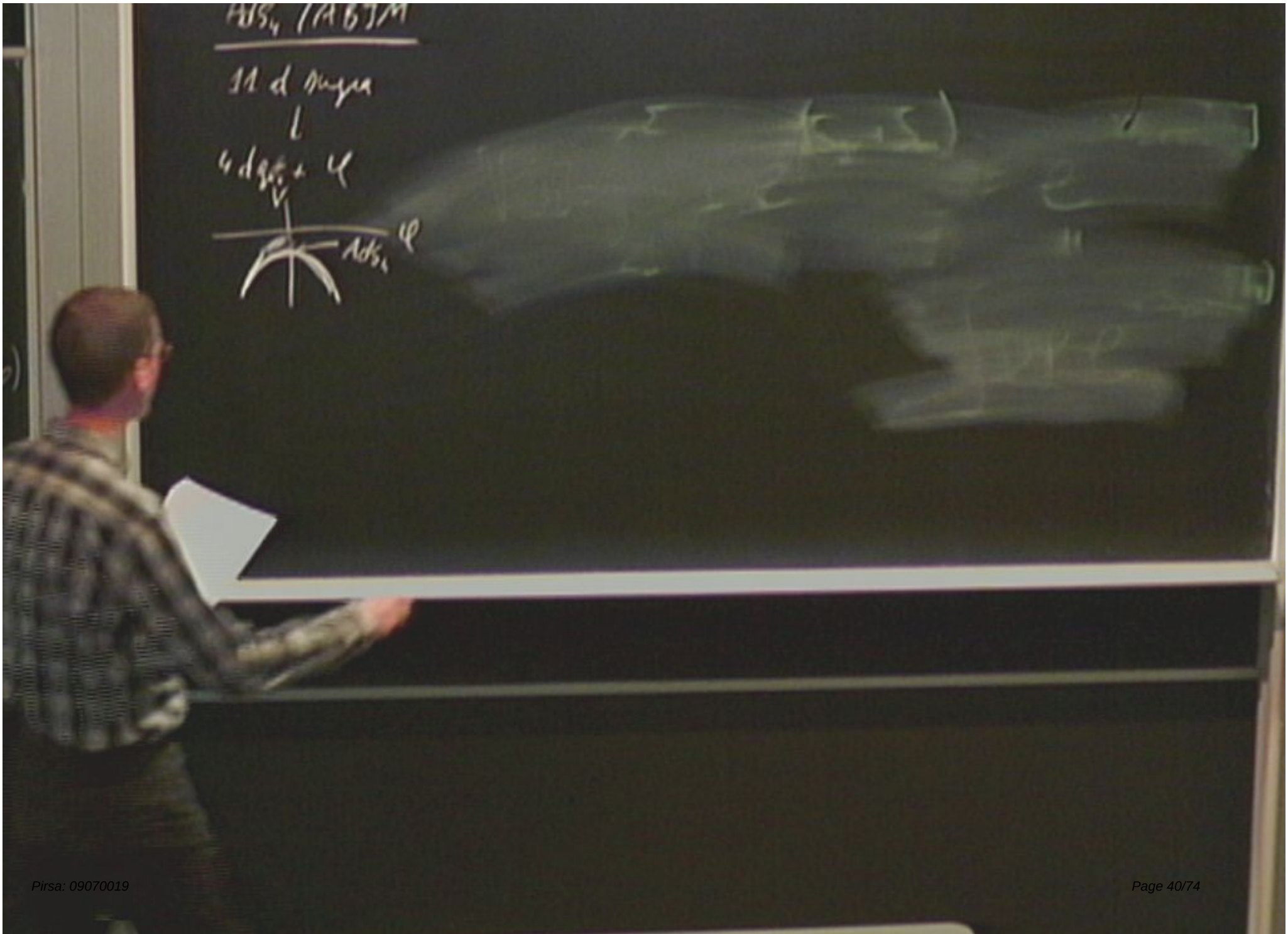
$\Rightarrow$ attractability

However: model degenerate at $t > 0$

Possible cure: $\bar{\phi}(t) \sim \frac{1}{t - i\varepsilon}$ analytic SA ext

with the $\frac{1}{|t|}$

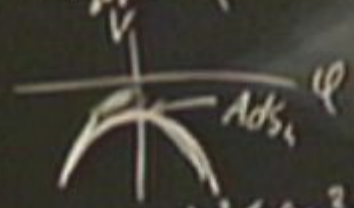



11 d supergra
4 dgree of φ
AdS4
φ

11 d supergr

$\downarrow$

4 d gauge + φ



$m_{BF}^2 < m^2 < m_B^2 + 1$

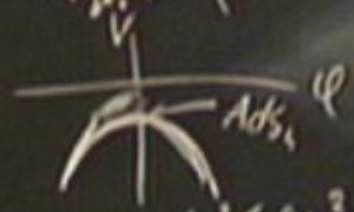
$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha}{r} + \frac{\beta}{r^2}$$

b.c. $\beta = -h\alpha^2$

11 d supra

$\downarrow$

4 d gauge + φ



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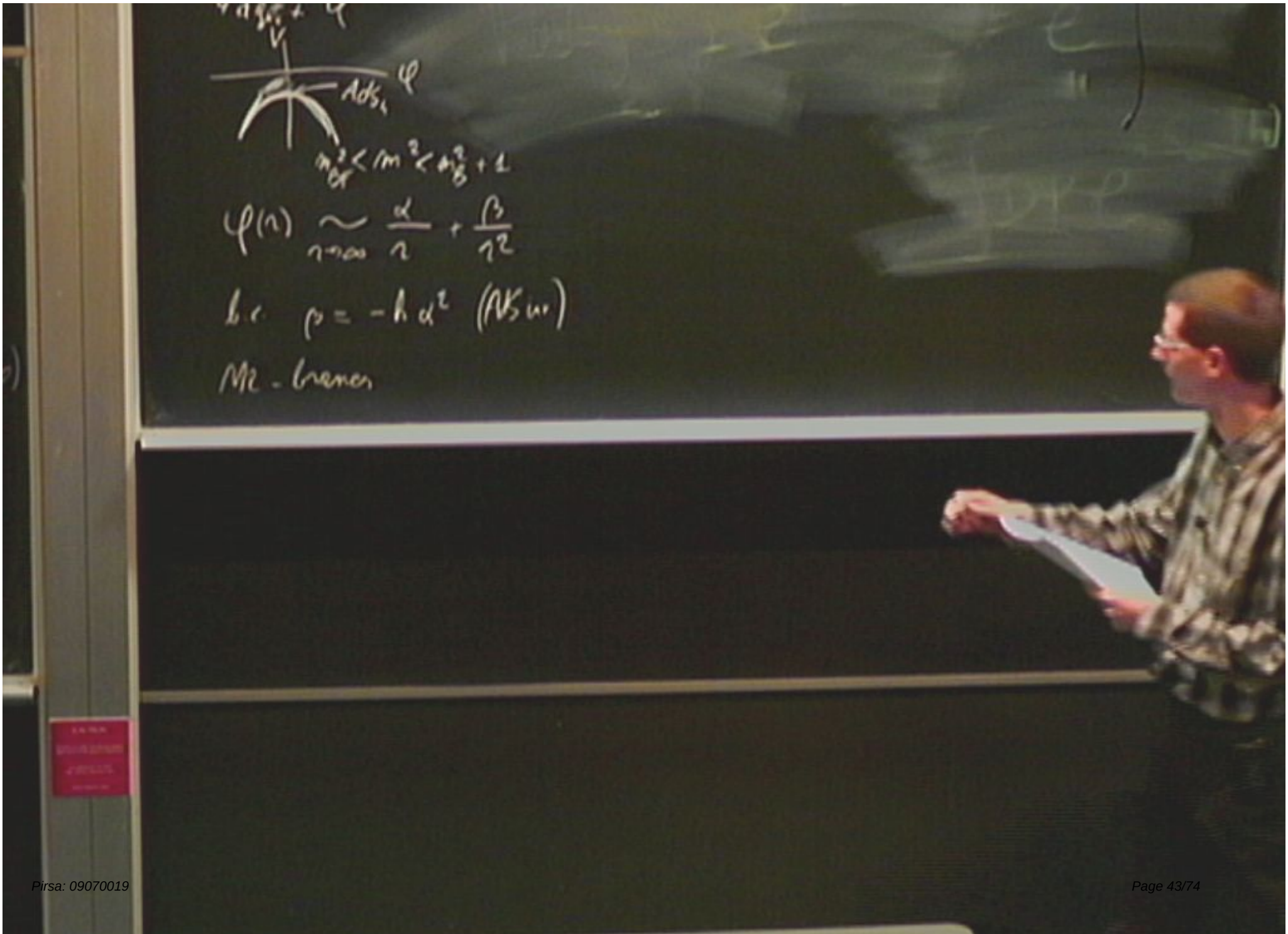
b.c. $\beta = -h\alpha^2$ (AdS_{inv})


V
φ
AdS4
m_BF^2 < m^2 < m_B^2 + 1
φ(r) ~ α/r + β/r^2
r→∞
b.c. β = −h α^2 (AdS/CFT)
Mz - Graner

AdS_4 / ABJM

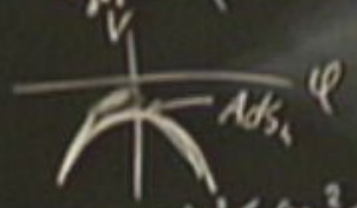
11 d supergravity $\longleftrightarrow$ ABJM theory

$\downarrow$

4 dgree + φ

$\mathcal{N}=6$ supersymmetric CS $U(N)\times U(N)$

levels k, $-k$



$m_{BF}^2 < m^2 < m_B^2 + 1$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha}{r} + \frac{\beta}{r^2}$$

b.c. $\beta = -h\alpha^2$ (AdS_{4+1})

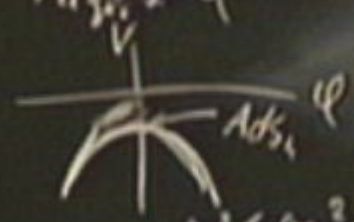
Mc-Gurren

AdS_4 / ABJM

11 d supergravity on S^7

$\downarrow$

4 d gravity + φ



$m_{BF}^2 < m^2 < m_B^2 + 1$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha}{r} + \frac{\beta}{r^2}$$

b.c. $\beta = -h\alpha^2$ (NS sector)

M2 - branes

ABJM theory

$\mathcal{N}=6$ superconf. CS $U(N)\times U(N)$

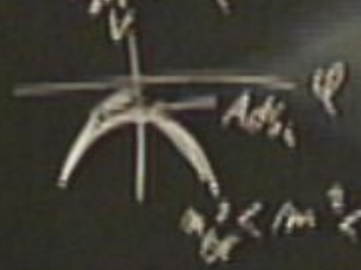
levels $k, -k$

- gauge fields $A_\mu, \hat{A}_\mu$
- scalars Y^A

11 d supergravity on S^7

l

4 degrees of freedom



$m_{BF}^2 < m^2 < m_{BF}^2 + 1$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha}{r} + \frac{\beta}{r^2}$$

i.e. $\rho = -h\,\alpha^2$ (Witten)

Mc-Guinness

ABJM theory

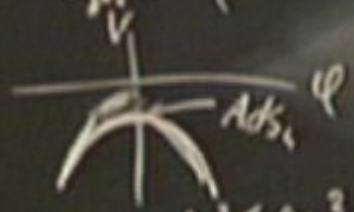
$\mathcal{N}=6$ supersymmetry $\subset$ $U(N)\times U(N)$

levels $k, -k$

- gauge fields $A_\mu, \hat{A}_\mu$
- scalars Y^A $A=1,\dots,4$

$\in \begin{cases} \text{fund of } SU(4)_R \\ (N,\bar{N}) \text{ of } U(N)\times U(N) \end{cases}$

4 dgs + φ



$m_{BF}^2 < m^2 < m_B^2 + 1$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha}{r} + \frac{\beta}{r^2}$$

b.c. $\beta = -h\alpha^2$ (NS w.)

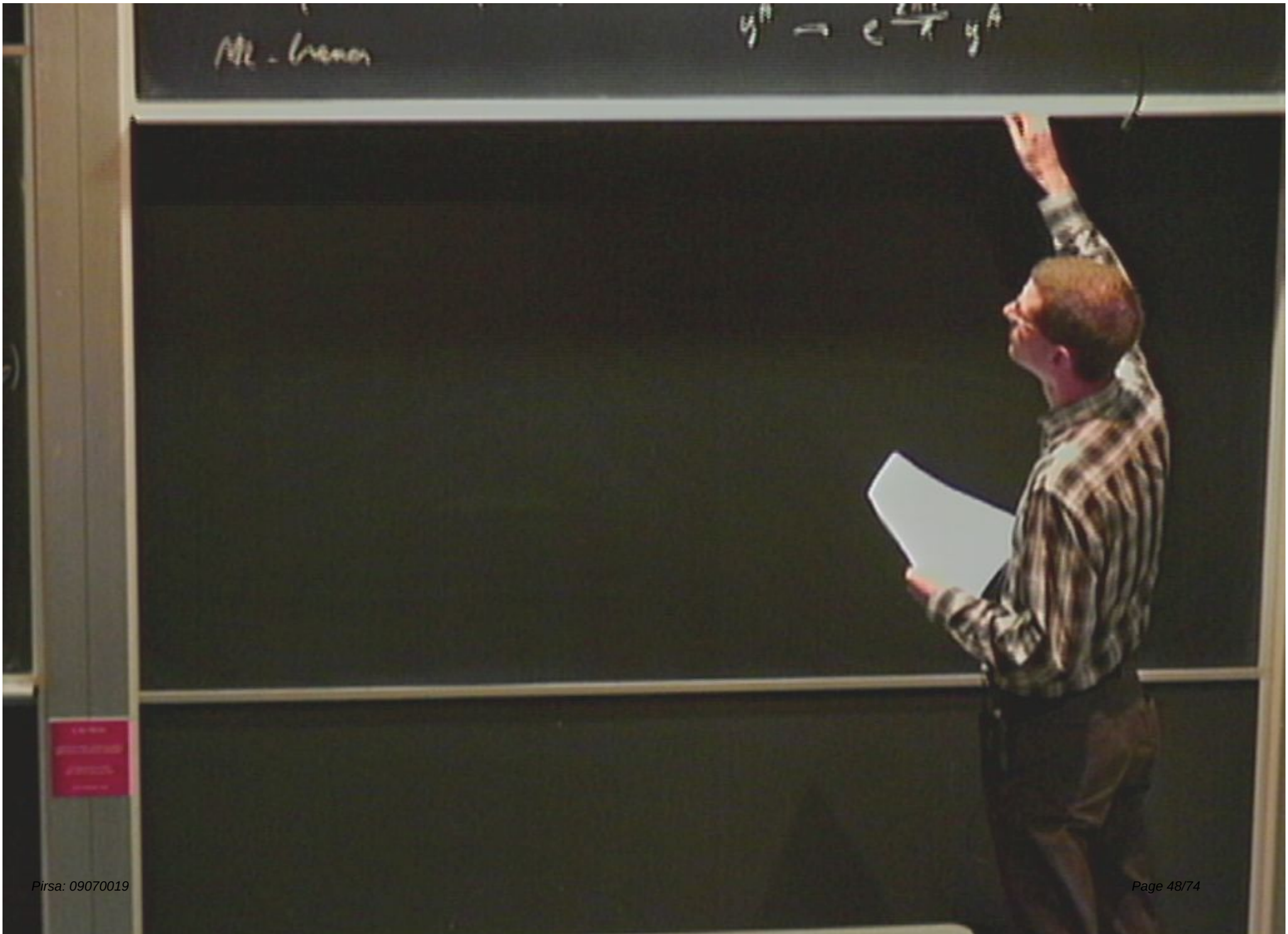
M2 - branes

levels $k, \ldots, k$

- gauge fields $A_\mu, \hat{A}_\mu$
- scalars Y^A $A = 1, \ldots, 4$

in $\begin{cases} \text{fund of } SU(4)_R \\ (N, \bar{N}) \text{ of } U(M) \times U(N) \end{cases}$

write single trace potential


Mr. Lemon

$m_{BF}^2 < m^2 < m_B^2 + \epsilon$

$$\varphi(r) \underset{r\to\infty}{\sim} \frac{\alpha}{r} + \frac{\beta}{r^2}$$

i.e. $\beta = -h\alpha^2$ (AVS inv.)

M2 - branes

$A = 1, \dots, 4$

in $\begin{cases} \text{fund of } SU(4)_R \\ (W, \bar{W}) \text{ of } U(M) \times U(N) \end{cases}$

write single trace potential

• describe N M2's on $\mathbb{C}^4/\mathbb{Z}_k$

$$y^A \to e^{\frac{2\pi i}{k}} y^A$$

't Hooft limit $N \to \infty$ $\frac{N}{k}$ fixed

- 't Hooft limit $N \to \infty$ $\frac{N}{k}$ fixed
- $\varphi \rightsquigarrow \mathcal{O} = \frac{1}{N^2} \mathrm{Tr}\left(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger\right)$
- $S = S_{ABJM} + \text{conf coupl} + \frac{h}{N^4}\left[\mathrm{Tr}\left(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger\right)\right]^3$

- 't Hooft limit $N \to \infty$ $\frac{N}{k}$ fixed
- $\varphi \rightsquigarrow \mathcal{O} = \frac{1}{N^2} \mathrm{Tr}\left(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger\right)$
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- 't Hooft limit $N \to \infty$ $\frac{N}{k}$ fixed
- $\varphi \leftrightarrow \mathcal{O} = \frac{1}{N^2} \mathrm{Tr}\left(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger\right)$
- $S = S_{ABJM} + \text{conf coupl} + \frac{h}{N^4}\left[\mathrm{Tr}\left(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger\right)\right]^3$
- conf in planar limit
- reduces to $O(2N^2) \times O(2N^2)$ vector model in weak coupling limit $N \to \infty$ $\frac{N}{k} \to 0$

$O(N)$ vector model

$$S = \int d^3x \left[-(\partial_\mu \vec{\phi})^2 - \frac{1}{6N^2} (\vec{\phi}^2)^3 \right]$$

$O(N)$ vector model

$$S = \int d^3x \left[-(\partial_\mu \vec{\phi})^2 - \frac{\lambda}{6N^2} (\vec{\phi}^2)^3 \right]$$

$$\beta_{gcc}(\lambda) = \frac{3}{16\pi^2 N} \left(\lambda^2 - \frac{\lambda^3}{192} \right) + \text{higher order in } \frac{1}{N}$$

O(N) vector model

$$S = \int d^3x \left[-(\partial_\mu \vec{\phi})^2 - \frac{\lambda}{6N^2} (\vec{\phi}^2)^3 \right]$$

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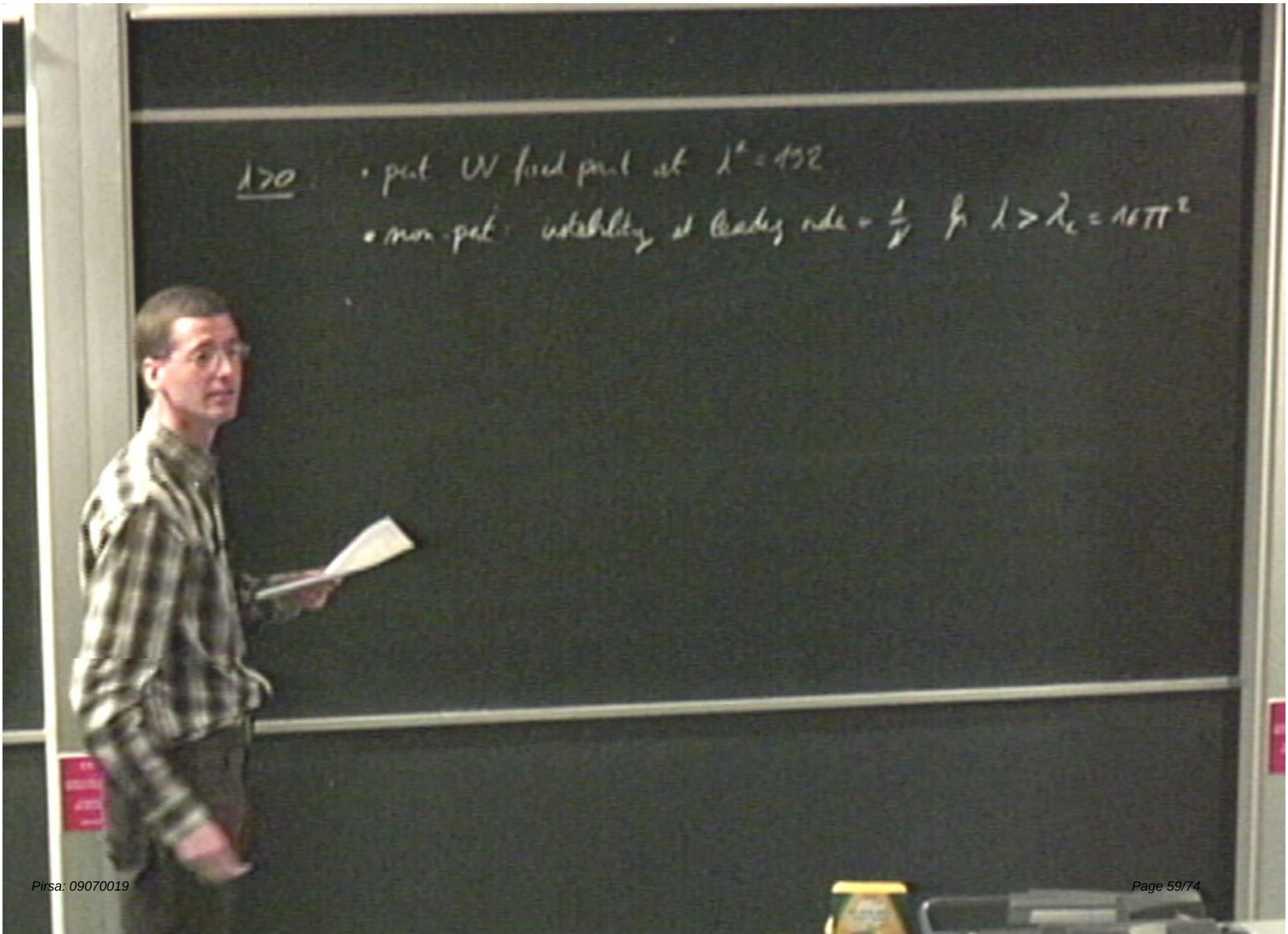
$O(N)$ vector model

$$S = \int d^3x \left[-(\partial_\mu \vec{\phi})^2 - \frac{\lambda}{6N^2} (\vec{\phi}^2)^3 \right]$$

$$\beta_{gcl}(\lambda) = \frac{3}{16\pi^2 N} \left(\lambda^2 - \frac{\lambda^3}{192} \right) + \text{higher order in } \frac{1}{N}$$

$\underline{\lambda < 0}$: UV fixed point at $\lambda = 0$

$\underline{\lambda > 0}$ · put UV fixed point at $\lambda^* = 132$


λ>0 : • pert UV fixed point at λ* = 132
• non-pert: instability at leading order = 1/N for λ > λc = 16π²

$\underline{\lambda > 0}$: • pert. UV fixed point at $\lambda^* = 152$

• non-pert. instability at leading order $= \frac{1}{N}$ for $\lambda > \lambda_c = 16\pi^2$

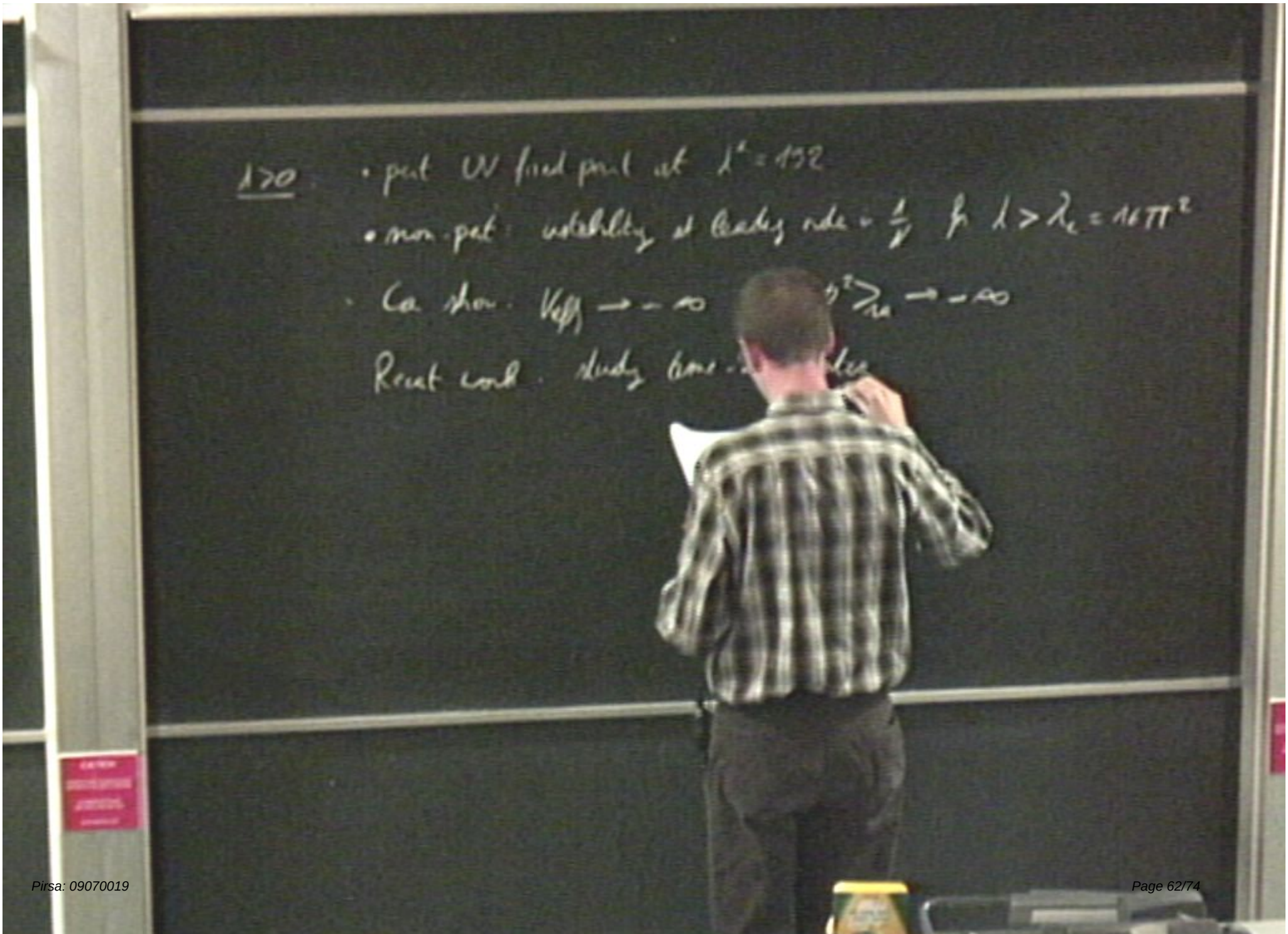
• Can show: $V_{eff} \to -\infty$ as $\langle \phi^2 \rangle_{ra} \to -\infty$

$\underline{\lambda > 0}$: • pert UV fixed point at $\lambda^* = 132$

• non-pert: instability at leading order $\sim \frac{1}{N}$ for $\lambda > \lambda_c = 16\pi^2$

• Can show: $V_{eff} \to -\infty$ as $\langle \phi^2 \rangle_{ra} \to -\infty$

Reset cont


λ>0
• pert UV fixed point at λ* = 152
• non-pert: instability at leading order ~ 1/N for λ > λc = 16π²
• Can show: Veff → −∞
Recent work: study

$\underline{\lambda > 0}$: • pert UV fixed point at $\lambda^* = 132$

• non-pert instability at leading order $\sim \frac{1}{\gamma}$ for $\lambda > \lambda_c = 16\pi^2$

• Can show: $V_{eff} \to -\infty$ as $\langle \phi^2 \rangle_{ra} \to -\infty$

Recent work: study time-dep states $\langle \phi^2 \rangle_{ra} = \frac{-CN}{t}$

$\therefore$

$\underline{\lambda > 0}$: • pert UV fixed point at $\lambda^* = 192$

• non-pert: instability at leading order $\sim \frac{1}{N}$ for $\lambda > \lambda_c = 16\pi^2$

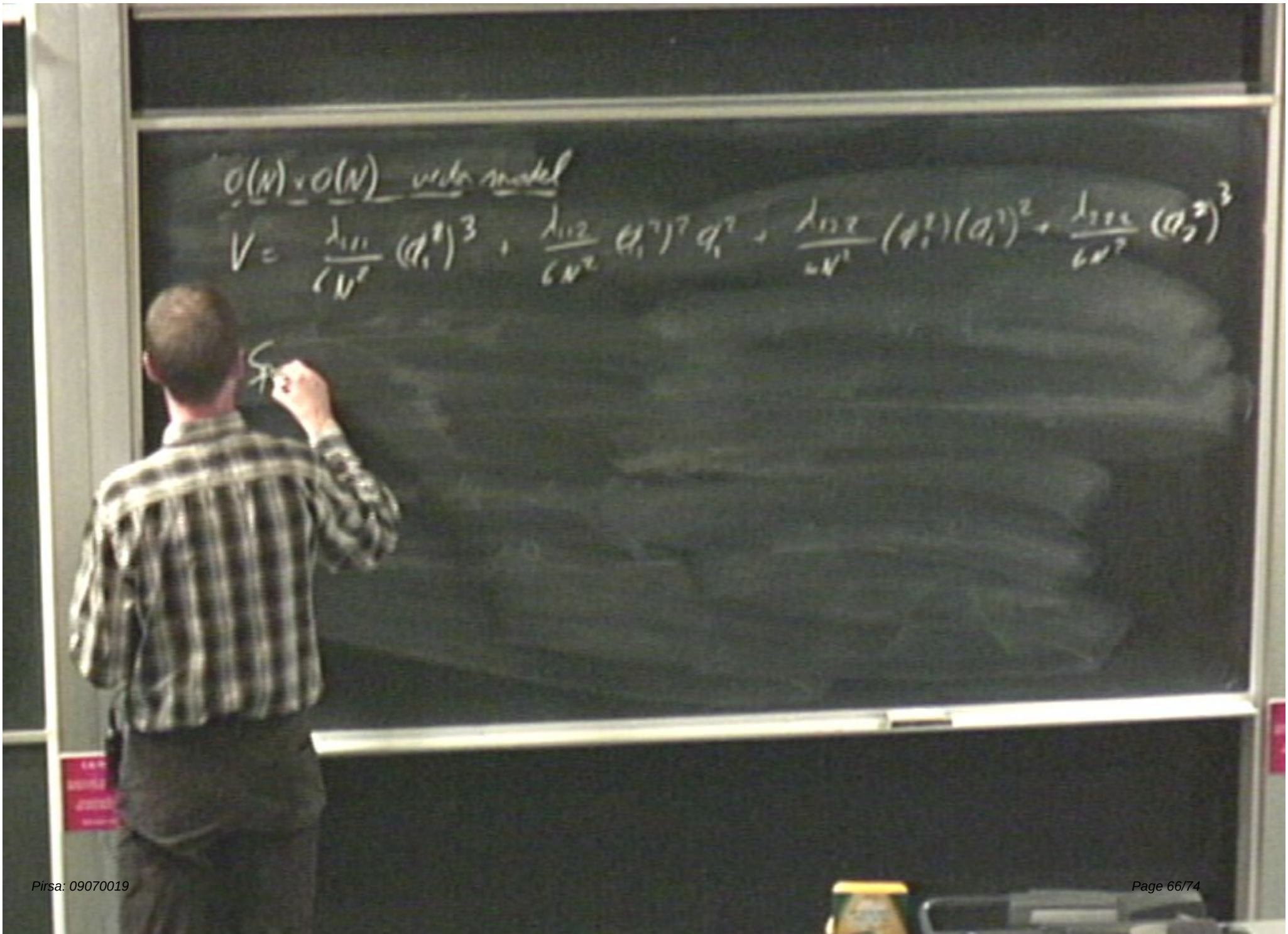
• Can show: $V_{eff} \to -\infty$ as $\langle \phi^2 \rangle_{ra} \to -\infty$

Recent work: study time-dep states $\langle \phi^2 \rangle_{ra} = \frac{-cN}{t}$

$\therefore$ • classical instability for $\lambda < 0$ $\quad \phi \sim \frac{1}{\sqrt{t}}$

• quantum instability for $\lambda > \lambda_c$ $\quad \langle \phi^2 \rangle \sim \frac{1}{t}$

$O(N) \times O(N)$ vector model


O(N) × O(N) vector model
V = λ111/6N² (a1²)³ + λ112/6N² (a1²)² a2² + λ122/4N² (a1²)(a2²)² + λ222/6N² (a2²)³

$O(N) \times O(N)$ vector model

$$V = \frac{\lambda_{111}}{6N^2}(\phi_1^2)^3 + \frac{\lambda_{112}}{6N^2}(\phi_1^2)^2\phi_2^2 + \frac{\lambda_{122}}{6N^2}(\phi_1^2)(\phi_2^2)^2 + \frac{\lambda_{222}}{6N^2}(\phi_2^2)^3$$

Special case $V = \frac{\lambda}{6N^2}(\phi_1^2 - \phi_2^2)^3$ $\lambda < 0$

$O(N) \times O(N)$ vector model

$$V = \frac{\lambda_{111}}{6N^2}(\phi_1^2)^3 + \frac{\lambda_{112}}{6N^2}(\phi_1^2)^2\phi_2^2 + \frac{\lambda_{122}}{6N^2}(\phi_1^2)(\phi_2^2)^2 + \frac{\lambda_{222}}{6N^2}(\phi_2^2)^3$$

Special case $V = \frac{\lambda}{6N^2}(\phi_1^2 - \phi_2^2)^3$ $\quad \lambda < 0$

Result • flow to UV fixed point $\begin{cases} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{cases}$

$O(N) \times O(N)$ vector model

$$V = \frac{\lambda_{111}}{6N^2}(\phi_1^2)^3 + \frac{\lambda_{112}}{6N^2}(\phi_1^2)^2\phi_2^2 + \frac{\lambda_{122}}{6N^2}(\phi_1^2)(\phi_2^2)^2 + \frac{\lambda_{222}}{6N^2}(\phi_2^2)^3$$

Special case $V = \frac{\lambda}{6N^2}(\phi_1^2 - \phi_2^2)^3$ $\quad \lambda < 0$

Result
- flow to UV fixed point under pert. RG flow
- approach

$$\begin{cases} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{cases}$$

Special case $V = \frac{\lambda}{6N^2}\left(\phi_1^2 - \phi_2^2\right)^3$ $\lambda < 0$

Result
- flows to UV fixed point under pert. RG flow
- approach

$$\begin{cases} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{cases}$$

Time evolution $\sim O(N) \times O(N)$ vector model

ϕ_2

Special case $V = \frac{\lambda}{6N^2}\left(\phi_1^2 - \phi_2^2\right)^3$ $\quad \lambda < 0$

Result
- flow to UV fixed point under pert. RG flow $\left\{ \begin{array}{l} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{array} \right.$
- approach

Time evolution in $O(N) \times O(N)$ vector model

ϕ_1 classically rolls to large values $\to$ UV $\Rightarrow \lambda_{222} > \lambda_c$

Special case $V = \frac{\lambda}{6N^2}\left(\phi_1^2 - \phi_2^2\right)^3$ $\quad \lambda < 0$

Result
- flow to UV fixed point
  under pert. RG flow
- approach

$$\begin{cases} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{cases}$$

Time evolution in $O(N) \times O(N)$ vector model

ϕ_2 classically rolls to large values $\to$ UV $\Rightarrow \lambda_{222} > \lambda_c$

$\to \phi_2$ quantum unstable $\to$ coupled system

Special case $V = \frac{\lambda}{6N^2}\left(\phi_1^2 - \phi_2^2\right)^3$ $\quad \lambda < 0$

Result
- flow to UV fixed point under pert. RG flow $\left\{ \begin{array}{l} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{array} \right.$
- approach

Time evolution in $O(N) \times O(N)$ vector model

ϕ_1 classically rolls to large values $\to$ UV $\Rightarrow \lambda_{222} > \lambda_c$

$\to \phi_2$ quantum unstable $\to$ coupled system (in progress)

$O(N) \times O(N)$ vector model

$$V = \frac{\lambda_{111}}{6N^2}(\phi_1^2)^3 + \frac{\lambda_{112}}{6N^2}(\phi_1^2)^2\phi_2^2 + \frac{\lambda_{122}}{4N^2}(\phi_1^2)(\phi_2^2)^2 + \frac{\lambda_{222}}{6N^2}(\phi_2^2)^3$$

Special case $V = \frac{\lambda}{6N^2}(\phi_1^2 - \phi_2^2)^3$ $\quad \lambda < 0$

Result
- flow to UV fixed point under pert. RG flow
- approach

$$\begin{cases} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{cases}$$

Time evolution in $O(N) \times O(N)$ vector model

ϕ_2 classically rolls to large values $\to$ UV $\Rightarrow \lambda_{222} > \lambda_c$

$\to \phi_2$ quantum unstable $\to$ coupled system (in progress)