Title: Gauge Duals of some Singularities

Date: Jul 15, 2009 11:30 AM

URL: http://pirsa.org/09070018

Abstract: Certain classes of cosmological backgrounds in asymptotically anti-de-Sitter space-times have hologropically dual descriptions in terms of gauge theories. We analyse such backgrounds with smooth initial conditions, where curvatures become strong in a finite time and the gravity equations break down. We will show how the dual gauge theory can be used to continue time evolution in this singular region and speculate on the nature of space-time at late times.

Gauge Duals of some Singularities

Sumit R. Das

A.Awad, A. Ghosh, J. H. Oh, S. Trivedi
arXiv:0906.3275
Awad, S.R.D., K. Narayan, S. Nampuri, S. Trivedi
arXiv:0807.1517

The Setup

- Near singularities, the equations of General Relativity or their cousins – supergravity – break down.
- What replaces them ?
- Holographic correspondences which arise from String Theory provide a possible clue.
- Here, gravity in the bulk is an approximate description of a more fundamental non-gravitational theory in lower number of dimensions.
- Question : can we use this description to ask what happens near singularities ?
- This talk : use the AdS/CFT correspondence.

- In this talk I will discuss one approach to understand this problem.
- There are several other approaches – 2 of them

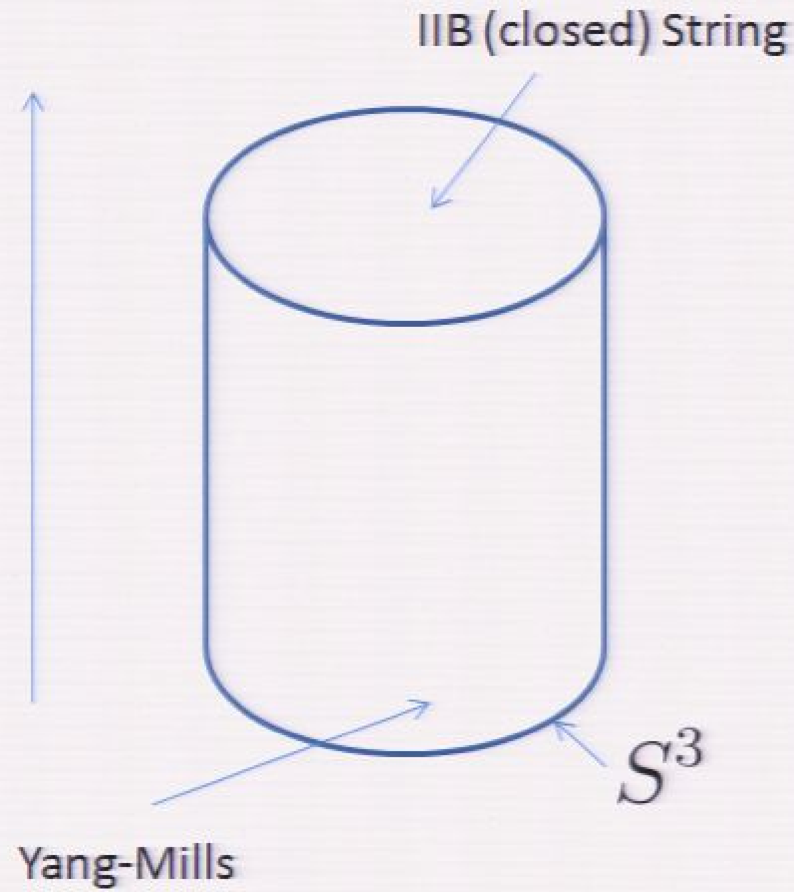
Craps, Hertog, Turok

Horowitz, Lawrence, Silverstein

will be discussed later in the conference.

- Simplest setting – IIB string theory in $AdS_5 \times S^5$ dual to a $\mathcal{N} = 4 \;\; SU(N)$ Yang-Mills theory living on the boundary.

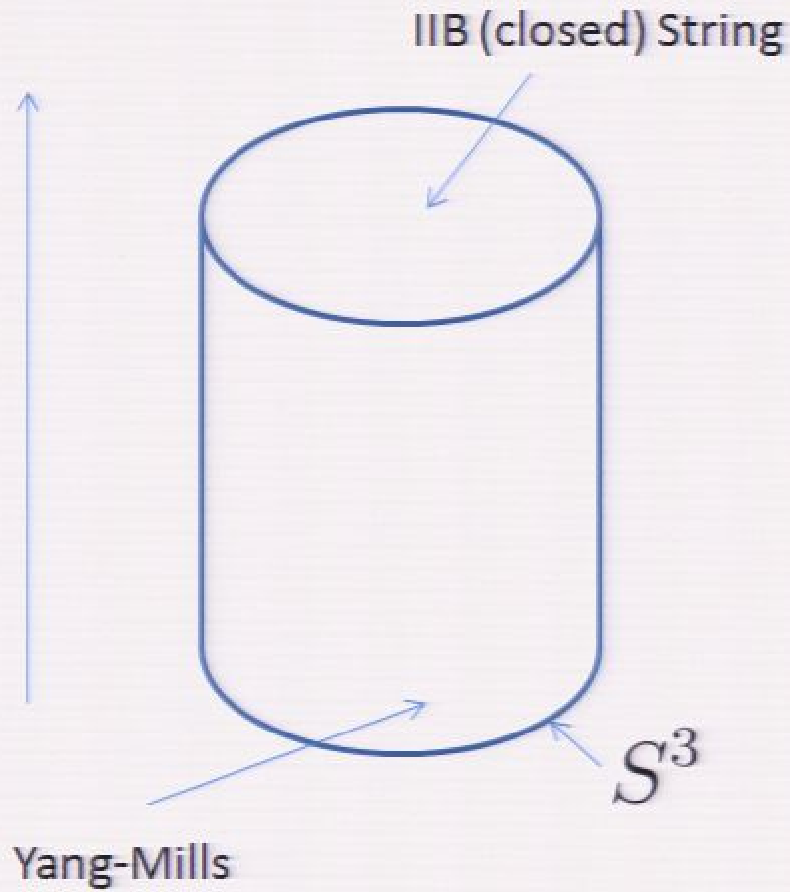
$$\left(\frac{R}{l_s}\right)^4 = 4\pi\lambda$$



- Simplest setting – IIB string theory in $AdS_5 \times S^5$ dual to a $\mathcal{N}=4 \ SU(N)$ Yang-Mills theory living on the boundary.

$$\left(\frac{R}{l_s}\right)^4 = 4\pi\lambda$$

- When $N \gg 1$ and the 't Hooft coupling is large, $\lambda = g_{YM}^2 N \gg 1$ the bulk theory may be approximated by supergravity and usual notions of space and time apply.

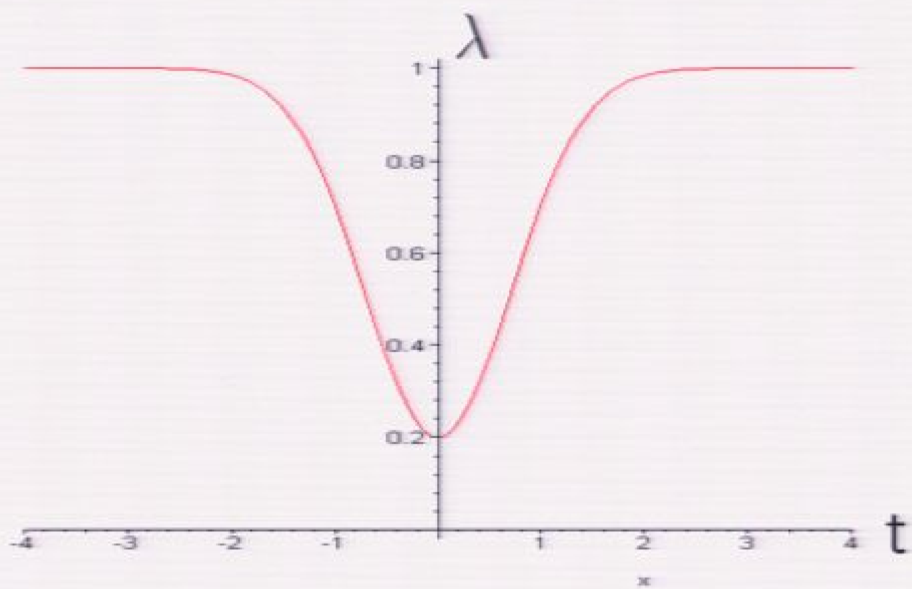


- When $\lambda \leq O(1)$ the gauge theory is still well-formulated.
- However, the curvatures are large compared to the string scale. Supergravity breaks down and there is no meaningful interpretation in terms of a 10 dimensional local gravitational theory.

$$\mathcal{R}_{string} = e^{-\Phi/2}\left[\mathcal{R}_{einstein} - \frac{9}{2}\partial^2\Phi - \frac{9}{2}(\partial\Phi)^2\right]$$

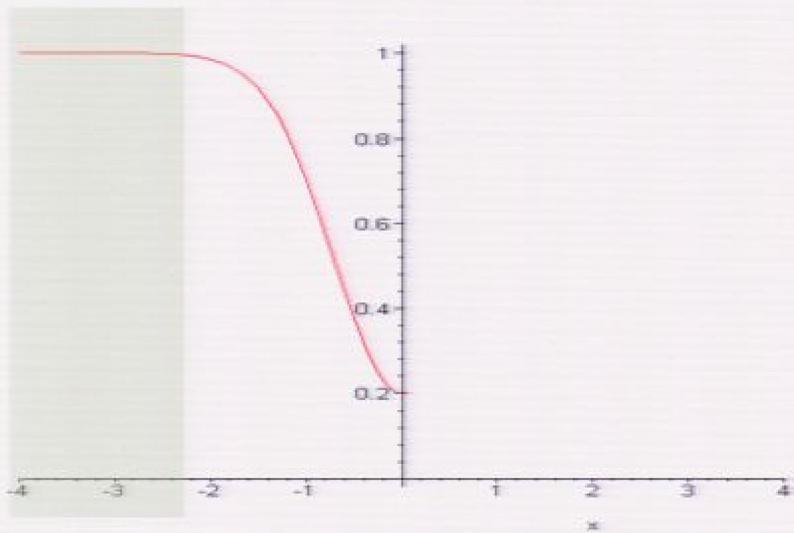
- This is physically a singularity.
- If $N \gg 1$ we can still ignore quantum effects in this string theory, but we need to understand stringy effects
- Very little is known about even *classical* string theory in such backgrounds.
- The hope is that we can use the Yang-Mills description in this regime.

- We will investigate toy models of cosmological singularities by considering Yang-Mills theory with a time-dependent 't Hooft coupling .



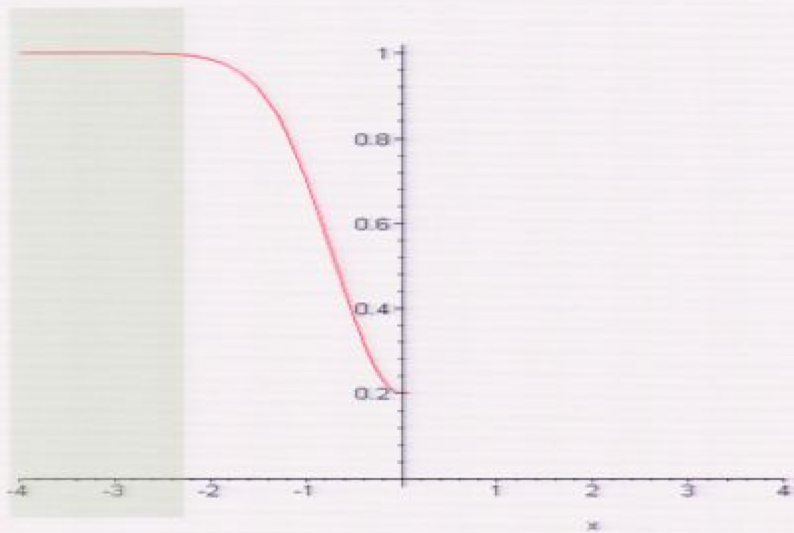
In the bulk this corresponds to a time dependent dilaton Φ,
and $N\ e^{\Phi}$ becomes small at some intermediate time ,
making $\mathcal{R}_{string} l_s^2$ large.

- We will start the system in the vacuum of the gauge theory, with a large value of the ‘t Hooft coupling. The dual space-time is now pure $AdS_5 \times S^5$



Once we turn on the time dependent source, the gauge theory evolves according to the deformed hamiltonian.

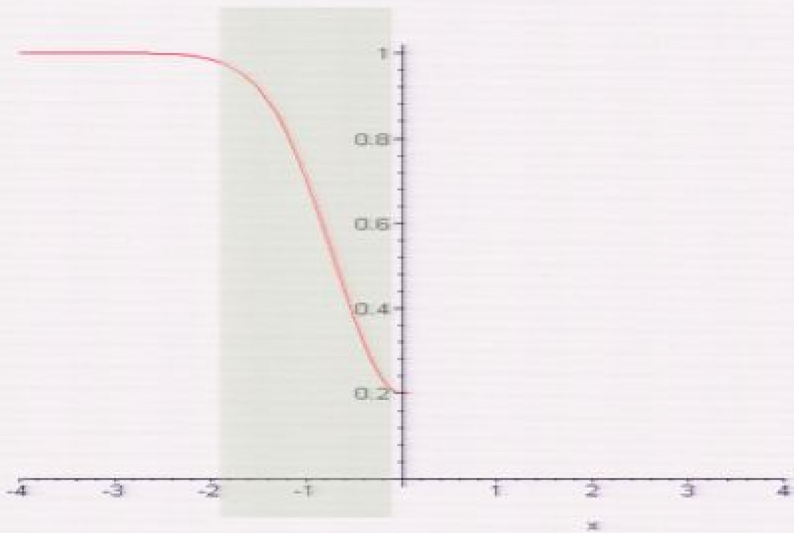
- We will start the system in the vacuum of the gauge theory, with a large value of the ‘t Hooft coupling. The dual space-time is now pure $AdS_5 \times S^5$



Once we turn on the time dependent source, the gauge theory evolves according to the deformed hamiltonian.

In this regime, the bulk is described by a **non-normalizable** dilaton mode . This evolves via the supergravity equations of motion – and produces a non-trivial metric by back-reaction.

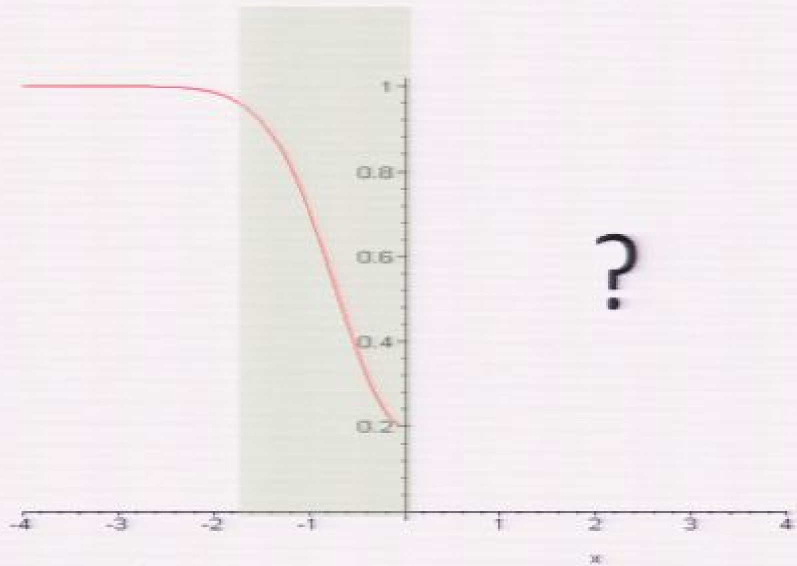
- Once the gauge theory becomes weakly coupled, the supergravity description is not valid any more.



Now we take recourse to the gauge theory.

In particular, when the coupling really hits a zero, and there is a curvature *singularity* in the bulk, we will ask if the gauge theory can meaningfully describe time evolution beyond this time.

- Once the gauge theory becomes weakly coupled, the supergravity description is not valid any more.



Now we take recourse to the gauge theory.

In particular, when the coupling really hits a zero, and there is a curvature *singularity* in the bulk, we will ask if the gauge theory can meaningfully describe time evolution beyond this time.

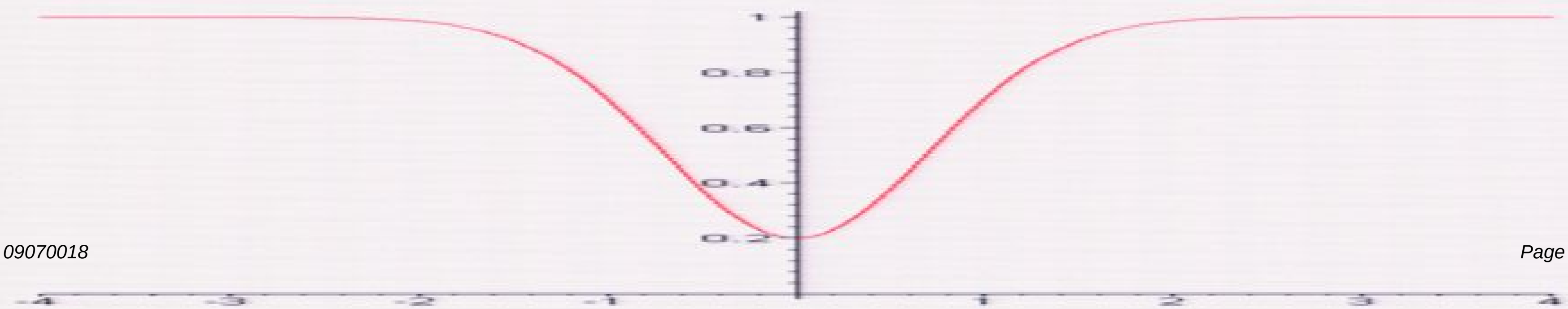
If not, we would like to learn – what precisely is the problem ?

Slowly varying dilatons

- A breakdown of super-gravity can be achieved even by a coupling which is slowly varying, starting with a large λ in the past.
- Since the gauge theory is defined on a S^3 whose radius can be taken to be R, slow variation means

$$R\partial_t = \epsilon \ll 1$$

- Therefore, if such a variation takes place over a timescale $t \sim \dfrac{R}{\epsilon}$ one can reach $\lambda \leq O(1)$



Supergravity Solutions

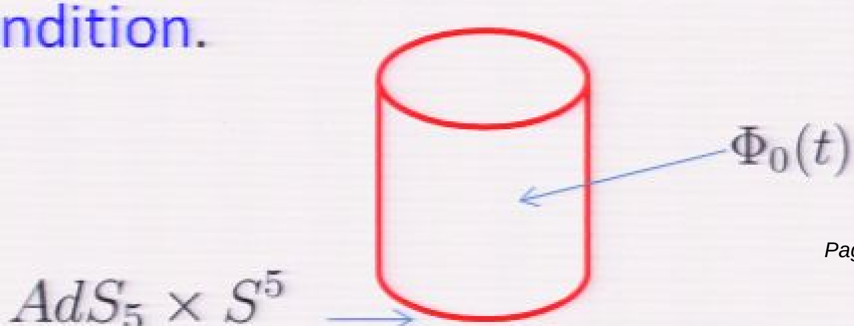
- In the infinite past (in terms of global time), the space-time is pure $AdS_5 \times S^5$ with a constant dilaton $\Phi_{-\infty}$ such that the string frame curvature is small in string units. The Einstein frame metric is

$$ds_0^2 = -(1 + \frac{r^2}{R^2})dt^2 + \frac{dr^2}{1 + \frac{r^2}{R^2}} + r^2 d\Omega_3^2$$

This provides the initial condition.

- The source on the boundary is a boundary value of the dilaton, which has been chosen as we described above - $\Phi_0(t)$

This provides the boundary condition.



- We need to solve the equations of motion in a power series expansion in ϵ

$$R_{AB} = -\frac{4}{R^2} g_{AB} + \frac{1}{2} \partial_A \Phi \partial_B \Phi \qquad \nabla^2 \Phi = 0$$

- Expand the fields $O(\epsilon)$

$$\Phi(t) = \Phi_0(t) + \Phi_1(r,t) + \Phi_2(r,t) \cdots$$

$$g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)} + g_{ab}^{(2)} + \cdots$$

- Derivatives with respect to r are not small.
- To the lowest nontrivial order in ϵ the solution is very simple. Since every time derivative comes with a factor of ϵ it is straightforward to see that

$$\Phi_1 = g_{ab}^{(1)} = 0$$

- We need to solve the equations of motion in a power series expansion in ϵ

$$R_{AB} = -\frac{4}{R^2} g_{AB} + \frac{1}{2} \partial_A \Phi \partial_B \Phi \qquad \nabla^2 \Phi = 0$$

- Expand the fields

$$O(\epsilon^2)$$

$$\Phi(t) = \Phi_0(t) + \Phi_1(r,t) + \Phi_2(r,t) \cdots$$

$$g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)} + g_{ab}^{(2)} + \cdots$$

- Derivatives with respect to r are not small.
- To the lowest nontrivial order in ϵ the solution is very simple. Since every time derivative comes with a factor of ϵ it is straightforward to see that

$$\Phi_1 = g_{ab}^{(1)} = 0$$

- In fact, the equations for Φ_2 and $g_{ab}^{(2)}$ decouple – they are simply equations in the $AdS_5 \times S^5$ background with a source term provided by the boundary value of the dilaton.
- The solution to lowest order in ϵ is smooth everywhere – there are no horizons - no black holes are formed.

$$\Phi_2(r,t) = \frac{1}{4}\ddot{\Phi}_0(t)\left[\frac{1}{r^2}\log(1+r^2) - \frac{1}{2}(\log(1+r^2))^2 - \mathrm{dilog}(1+r^2) - \frac{\pi^2}{6}\right]$$

- We have used – and will keep using - $R = 1$ units.

- We need to solve the equations of motion in a power series expansion in ϵ

$$R_{AB} = -\frac{4}{R^2} g_{AB} + \frac{1}{2}\partial_A \Phi \partial_B \Phi \qquad \nabla^2 \Phi = 0$$

- Expand the fields

$O(\epsilon^2)$

$$\Phi(t) = \Phi_0(t) + \Phi_1(r,t) + \Phi_2(r,t) \cdots$$

$$g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)} + g_{ab}^{(2)} + \cdots$$

- Derivatives with respect to r are not small.
- To the lowest nontrivial order in ϵ the solution is very simple. Since every time derivative comes with a factor of ϵ it is straightforward to see that

$$\Phi_1 = g_{ab}^{(1)} = 0$$

- In fact, the equations for Φ_2 and $g_{ab}^{(2)}$ decouple – they are simply equations in the $AdS_5 \times S^5$ background with a source term provided by the boundary value of the dilaton.
- The solution to lowest order in ϵ is smooth everywhere – there are no horizons - no black holes are formed.

$$\Phi_2(r,t) = \frac{1}{4}\ddot{\Phi}_0(t)\left[\frac{1}{r^2}\log(1+r^2) - \frac{1}{2}(\log(1+r^2))^2 - \text{dilog}(1+r^2) - \frac{\pi^2}{6}\right]$$

- We have used – and will keep using - $R = 1$ units.

- In fact, the equations for Φ_2 and $g_{ab}^{(2)}$ decouple – they are simply equations in the $AdS_5 \times S^5$ background with a source term provided by the boundary value of the dilaton.
- The solution to lowest order in ϵ is smooth everywhere – there are no horizons - no black holes are formed.

$$\Phi_2(r,t) = \frac{1}{4}\ddot{\Phi}_0(t)\left[\frac{1}{r^2}\log(1+r^2) - \frac{1}{2}(\log(1+r^2))^2 - \mathrm{dilog}(1+r^2) - \frac{\pi^2}{6}\right]$$

$$g_{tt} = 1 + r^2 - \frac{1}{4}\dot{\Phi}_0^2 + \frac{1}{12}\dot{\Phi}_0^2\frac{\ln(1+r^2)}{r^2}$$

$$\frac{1}{g_{rr}} = 1 + r^2 - \frac{1}{12}\dot{\Phi}_0^2[1 - \frac{1}{r^2}\ln(1+r^2)]$$

- We have used – and will keep using - $R=1$ units.

- The fact that black holes are not formed for slowly varying dilaton driving is not surprising.

- The fact that black holes are not formed for slowly varying dilaton driving is not surprising.
- When the variation becomes fast enough we expect black holes to form – staying in the supergravity approximation. (see e.g. *Bhattacharya and Minwalla* for an analysis with small amplitude dilaton).

- The fact that black holes are not formed for slowly varying dilaton driving is not surprising.
- When the variation becomes fast enough we expect black holes to form – staying in the supergravity approximation. (see e.g. *Bhattacharya and Minwalla* for an analysis with small amplitude dilaton).
- In the gauge theory this means that so long as the 't Hooft coupling is large – so that the supergravity calculation can be trusted – thermalization does not happen and energy is not dissipated.

- Holographic RG calculation of the energy yields

$$E = - < T_t^t > V_{S^3} = \frac{3N^2}{16} + \frac{N^2 \dot{\Phi}_0^2}{32}$$

- While the expectation value of the operator dual to the dilaton is

$$< \hat{\mathcal{O}}_{l=0} >= - \frac{N^2}{16} \ddot{\Phi}_0$$

- The Noether relation is satisfied

$$\frac{dE}{dt} = -\dot{\Phi}_0 < \hat{\mathcal{O}}_{l=0} >$$

- Therefore, if we always stay in the supergravity regime – nothing dramatic happens : when the coupling gets back to a constant value – all the energy which was pumped into the system is extracted out and we have a perfect bounce.

- What else could have happened ?
- One could repeat the same exercise for a black 3-brane with a dilaton varying slowly compared to the temperature. This is like changing the coupling of the gauge theory on flat space at a finite temperature. In that case

$$< \hat{\mathcal{O}}_{l=0} >= c_1 \dot{\Phi}_0$$

- The rate of change of the temperature is

$$\frac{dT}{dt} = \frac{1}{12\pi}\dot{\Phi}_0^2$$

- The temperature therefore keeps increasing – the energy pumped into the system by a time-varying coupling gets dissipated.

[Bhattacharya, Loganayagam, Minwalla, Nampuri, Trivedi ,Wadia (2008)]

- A key feature of the above calculation is the decoupling of the various modes at leading order in ϵ
- This happens because we have started with the vacuum and driving the system by banging on the boundary with a force with frequency ϵ .
- The source on the boundary couples directly to the dilaton – other modes are excited by nonlinear coupling s of the dilaton to these modes.
- Since these couplings necessarily involve derivatives of the dilaton, they are suppressed at small ϵ .

- A key feature of the above calculation is the decoupling of the s modes at leading order in ϵ
- ppens because we have started with the vacuum and g the system by banging on the boundary with a force with ncy ϵ .
- urce on the boundary couples directly to the dilaton – other modes are excited by nonlinear coupling s of the dilaton to these modes.
- Since these couplings necessarily involve derivatives of the dilaton, they are suppressed at small ϵ .

- A key feature of the above calculation is the decoupling of the various modes at leading order in ϵ
- This happens because we have started with the vacuum and driving the system by banging on the boundary with a force with frequency ϵ .
- The source on the boundary couples directly to the dilaton – other modes are excited by nonlinear coupling s of the dilaton to these modes.
- Since these couplings necessarily involve derivatives of the dilaton, they are suppressed at small ϵ .

- What else could have happened ?
- One could repeat the same exercise for a black 3-brane with a dilaton varying slowly compared to the temperature. This is like changing the coupling of the gauge theory on flat space at a finite temperature. In that case

$$< \hat{\mathcal{O}}_{l=0} >= c_1 \dot{\Phi}_0$$

- The rate of change of the temperature is

$$\frac{dT}{dt} = \frac{1}{12\pi}\dot{\Phi}_0^2$$

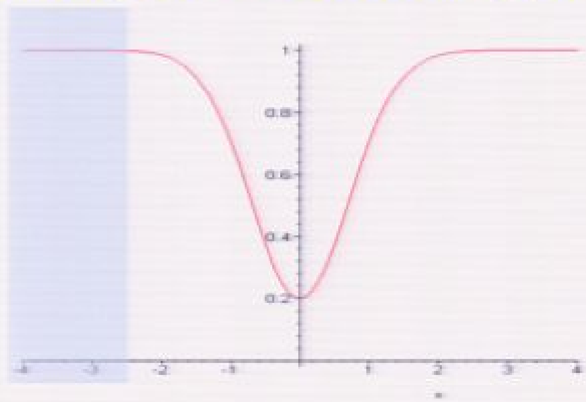
- The temperature therefore keeps increasing – the energy pumped into the system by a time-varying coupling gets dissipated.

[Bhattacharya, Loganayagam, Minwalla, Nampuri, Trivedi ,Wadia (2008)]

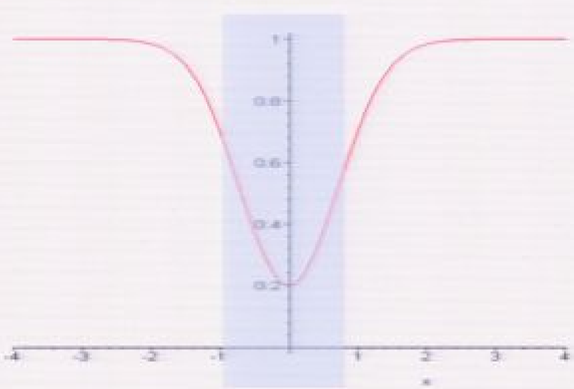
- A key feature of the above calculation is the decoupling of the various modes at leading order in ϵ
- This happens because we have started with the vacuum and driving the system by banging on the boundary with a force with frequency ϵ .
- The source on the boundary couples directly to the dilaton – other modes are excited by nonlinear coupling s of the dilaton to these modes.
- Since these couplings necessarily involve derivatives of the dilaton, they are suppressed at small ϵ .

The Stringy Regime

- We have been talking about the regime of large ‘t Hooft coupling,



- But we really want to know what happens when the coupling has become small



- In this regime of weak coupling in the gauge theory, the string frame curvature is large, and stringy effects cannot be ignored any more

$$\begin{aligned}\mathcal{R}_{string} l_s^2 &= e^{-\Phi/2} \frac{l_s^2}{R^2} \\ &\sim (g_{YM}\, e^{\Phi(t)/2} \sqrt{N})^{-1} = [\lambda(t)]^{-1/2}\end{aligned}$$

We want to turn to the gauge theory to see what happens.

Adiabatic Approximation

- The boundary gauge theory is a standard quantum mechanical system with

  (1) A slowly varying time dependent parameter – the coupling

  (2) The instantaneous Hamiltonian has a discrete spectrum with a gap above the ground state.

- This latter follows from the fact that the theory lives on S^3 and that the states form a unitary representation of the conformal algebra for any value of the coupling.
- **The appropriate approximation scheme is the Adiabatic Approximation.**

- In this regime of weak coupling in the gauge theory, the string frame curvature is large, and stringy effects cannot be ignored any more

$$
\begin{aligned}
\mathcal{R}_{string} l_s^2 &= e^{-\Phi/2} \frac{l_s^2}{R^2} \\
&\sim (g_{YM}\, e^{\Phi(t)/2} \sqrt{N})^{-1} = [\lambda(t)]^{-1/2}
\end{aligned}
$$

We want to turn to the gauge theory to see what happens.

Adiabatic Approximation

- The boundary gauge theory is a standard quantum mechanical system with

(1) A slowly varying time dependent parameter – the coupling

(2) The instantaneous Hamiltonian has a discrete spectrum with a gap above the ground state.

- This latter follows from the fact that the theory lives on S^3 and that the states form a unitary representation of the conformal algebra for any value of the coupling.
- **The appropriate approximation scheme is the Adiabatic Approximation**.

- Consider a hamiltonian $H(\zeta(t))$ which depends on a time dependent parameter $\zeta(t)$ Consider the eigenstates of the instantaneous hamiltonian $H(\zeta)$

$$H(\zeta)|\phi_m(\zeta)> = E_m(\zeta)|\phi_m(\zeta)>$$

- The Adiabatic Theorem implies that if $\zeta \to \zeta_0$ in the far past, and we start with the ground state $|\phi_0>$ of $H(\zeta_0)$ in the far past, the state at any time t is well approximated by

$$|\psi^0(t)> \simeq |\phi_0(\zeta)> e^{-i\int_{-\infty}^{t} E_0(\zeta)dt}$$

where $|\phi_0(\zeta)>$ is the ground state of the instantaneous hamiltonian corresponding to $\zeta = \zeta(t)$. $E_0(\zeta)$ is the value of the ground state energy for $\zeta = \zeta(t)$.

Adiabatic Approximation

- The boundary gauge theory is a standard quantum mechanical system with

  (1) A slowly varying time dependent parameter – the coupling

  (2) The instantaneous Hamiltonian has a discrete spectrum with a gap above the ground state.

- This latter follows from the fact that the theory lives on S^3 and that the states form a unitary representation of the conformal algebra for any value of the coupling.
- **The appropriate approximation scheme is the Adiabatic Approximation**.

- Consider a hamiltonian $H(\zeta(t))$ which depends on a time dependent parameter $\zeta(t)$ Consider the eigenstates of the instantaneous hamiltonian $H(\zeta)$

$$H(\zeta)|\phi_m(\zeta)>= E_m(\zeta)|\phi_m(\zeta)>$$

- The Adiabatic Theorem implies that if $\zeta \to \zeta_0$ in the far past, and we start with the ground state $|\phi_0>$ of $H(\zeta_0)$ in the far past, the state at any time t is well approximated by

$$|\psi^0(t)>\simeq |\phi_0(\zeta)> e^{-i\int_{-\infty}^{t} E_0(\zeta)dt}$$

where $|\phi_0(\zeta)>$ is the ground state of the instantaneous hamiltonian corresponding to $\zeta = \zeta(t)$. $E_0(\zeta)$ is the value of the ground state energy for $\zeta = \zeta(t)$.

- The leading corrections are given by

$$|\psi^1(t)>=\sum_{n\neq 0} a_n(t)|\phi_n(\zeta)>e^{-i\int_{-\infty}^{t} E_n dt}$$

- Where

$$a_n(t)=-\int_{-\infty}^{t} dt' \frac{<\phi_n(\zeta)|\frac{\partial H}{\partial \zeta}|\phi_0(\zeta)>}{E_0-E_n}\,\dot{\zeta}\, e^{-i\int_{-\infty}^{t'}(E_0-E_n)dt'}$$

- This correction is small provided

$$|<\phi_n|\frac{\partial H}{\partial \zeta}|\phi_0>\dot{\zeta}| \ll (E_1-E_0)^2$$

- Note that the quantity (E_1-E_0) is the energy gap between the ground state and the first excited state.

- In our case - Yang-Mills theory on S^3 of unit radius, and a time dependent coupling ,

$$\zeta(t) = \Phi_0(t)$$

- Furthermore $$\frac{\partial H}{\partial \Phi_0} \sim \hat{\mathcal{O}}_{l=0}$$

- Where $\hat{\mathcal{O}}_{l=0}$ is the operator dual to the spherically symmetric modes of the bulk dilaton. Thus the condition for validity of the adiabatic approximation is

$$| < \phi_n | \hat{\mathcal{O}}_{l=0} | \phi_0 > \dot{\Phi}_0 | \ll (E_1 - E_0)^2$$

- It may be easily seen that $< \phi_n | \hat{\mathcal{O}}_{l=0} | \phi_0 > \sim O(N)$. and since we are using $R = 1$ units, $(E_1 - E_0) \sim O(1)$ this condition becomes

$$N\epsilon \ll 1$$

- The adiabatic approximation of course has nothing to do with the value of the coupling constant – so this holds for weak 't Hooft coupling as well.
- If ϵ is so small that this condition holds, the adiabatic theorem ensures that at late times – when we again have a space-time interpretation of the gauge theory – we get back $AdS_5 \times S^5$ with exponentially small corrections.
- However this condition $N\epsilon \ll 1$ is much stronger than the condition $\epsilon \ll 1$ which we used in performing the supergravity analysis.
- We need to find a scheme which has an overlapping regime of validity with supergravity.

Coherent States and Adiabaticity

- The adiabatic approximation described above is good for description of the system in terms of states of the gauge theory which are obtained by a finite number of operators on the vacuum – these are states containing a finite number of particles in the bulk.
- Classical solutions in the bulk are, however, described by coherent states.
- In the boundary theory these are coherent states of gauge invariant operators.

- A general coherent state has the form

$$|\Psi(t)> = \exp\left[i\chi(t) + \sum_I \lambda^I(t)\hat{\mathcal{O}}^I_{(+)}\right]|0>_A$$

- Where $\hat{\mathcal{O}}^I_{(+)}$ are the creation parts of gauge invariant operators in the theory. For example, the operator dual to the spherically symmetric dilaton is

$$\hat{\mathcal{O}}_{l=0} = \int d\Omega_3 \, \mathrm{Tr} F_{\mu\nu}F^{\mu\nu}$$

- The adiabatic vacuum is denoted by $|0>_A$
- The algebra of the operators $\hat{\mathcal{O}}^I$, together with the Schrodinger equation determines the evolution of the coherent state parameters $\chi(t)$ and $\lambda^I(t)$.
- At $N = \infty$ these states which go over to classical configurations – which have a good description in terms of local fields in the large 't Hooft coupling regime.

- Usually this is an impossible plan to implement. The algebra of the operators is complicated and they all couple to each other.
- In our situation, however, the dynamics of these modes are driven entirely by a time dependent coupling constant. This directly drives the dynamics of the mode which comes with the operator $\hat{\mathcal{O}}_{l=0}$.
- Other modes are excited due to non-trivial 3-point functions

$$< \hat{O}_1 \hat{O}_2 \hat{O}_3 > \sim \; 1/N$$

- So that the corresponding probability goes as $1/N^2$. However, as we will see soon, a coherent state produced by this slow driving has roughly $O(N^2\epsilon^2)$ quanta, so that the effective 3 point coupling in such states is ϵ
- Thus for $\epsilon \ll 1$ these operators can be considered to be independent of each other with small non-linearities.

- To leading order in ϵ these operators can be written as a sum of harmonic oscillators

$$\hat{\mathcal{O}}_{l=0} = N \sum_{n=1}^{\infty} F(2n)[A_{2n} e^{-i2nt} + A_{2n}^{\dagger} e^{i2nt}]$$

$$[A_m, A_n] = [A_m^{\dagger}, A_n^{\dagger}] = 0 \qquad [A_m, A_n^{\dagger}] = \delta_{m,n}$$

$$[H, A_{2n}^{\dagger}] = (2n) A_{2n}^{\dagger} \qquad [H, A_{2n}] = -(2n) A_{2n}$$

- In the strong 't Hooft coupling regime $A_{2n}^{\dagger}$ creates a single particle dilaton state in the bulk with zero S^3 angular momentum
- The integer n is a "radial" quantum number, conjugate to the extra dimension. The energy of this single particle state is $2n$.
- The factor $F(2n)$ can be determined by requiring that the above expansion leads to the correct 2 point function

$$|F(2n)|^2 = \frac{A\pi^4}{3} n^2(n^2 - 1)$$

- Finally, if Φ_0 denotes the boundary value of the dilaton, we have

$$\frac{\partial H}{\partial t} = -\hat{\mathcal{O}}_{l=0}\dot{\Phi}_0 = -N\sum_n F(2n)[A_{2n} + A_{2n}^\dagger]\dot{\Phi}_0$$

- Consider now a coherent state of the form

$$|\psi> = \hat{N}(t)e^{(\sum_n \lambda_n A^\dagger_{2n})}|\phi_0>$$

- Where $\hat{N}(t)$ is a normalization factor.
- The equation satisfied by λ_n is

$$i\frac{d\lambda_n}{dt} = -i\frac{F(2n)}{2n}\dot{\Phi}_0 + 2n\ \lambda_n$$

- The initial conditions are $\lambda_n(-\infty) = 0$ and the boundary dilaton has the property that $\dot{\Phi}_0(-\infty) = 0$
- This equation can be of course solved exactly

$$\lambda_n(t) \ = \ -\frac{F(2n)\ e^{-2int}}{2n}\int_{-\infty}^{t}\dot{\Phi}_0(t')\ e^{2int'}\ dt'$$

- However, we want to write this solution somewhat differently – by successively integrating by parts

$$
\begin{aligned}
\lambda_n(t) &= -\frac{F(2n)\, e^{-2int}}{2n} \int_{-\infty}^{t} \dot{\Phi}_0(t')\, e^{2int'}\, dt' \\
&= \frac{F(2n)}{2n}\left[\frac{\dot{\Phi}_0}{(2in)} - \frac{e^{-2int}}{(2in)} \int_{-\infty}^{t} \ddot{\Phi}_0(t') e^{2int'}\right] \\
&= \frac{F(2n)}{2n}\left[\frac{\dot{\Phi}_0}{(2in)} + \frac{\ddot{\Phi}_0}{4n^2} + \cdots\right]
\end{aligned}
$$

- This is an expansion in time derivatives – the adiabatic approximation we are seeking.
- Note that we have assumed that the oscillators are independent – this is valid for small $\dot{\Phi}_0$, $\ddot{\Phi}_0$. This means that only the first two terms in this expansion are significant.

- This adiabatic approximation is valid provided

$$|\frac{\ddot{\Phi}_0}{n\dot{\Phi}_0}| \ll 1 \qquad \forall n$$

- It is clearly sufficient to have

$$|\frac{\ddot{\Phi}_0}{\dot{\Phi}_0}| \sim \epsilon \ll 1$$

- Note that n is the characteristic frequency, which is quantized since the theory lives on S^3 . If there was no gap in the spectrum, the frequency could be arbitrarily small and the adiabatic approximation would not hold.
- The condition for validity is exactly what we had in our supergravity analysis.

- However for this to be applicable to coherent states which behave classically, we must also have

$$\lambda_n \gg 1$$

- Recall
$$\lambda_n(t) = \frac{F(2n)}{2n}\left[\frac{\dot{\Phi}_0}{(2in)} + \frac{\ddot{\Phi}_0}{4n^2} + \cdots\right]$$

So we must have $|F(2n)\dot{\Phi}_0| \gg n^2$

Since for large n, $F(2n) \sim n^2$ so we have the condition

$$|N\dot{\Phi}_0| \sim N\epsilon \gg 1$$

In this regime we can compare our answers with supergravity.

They agree upto numerical factors -

$$< \hat{O}_{l=0} > \sim N^2\ddot{\Phi}$$

$$< E > \sim N^2(\dot{\Phi})^2$$

Small 't Hooft coupling

- The framework developed above applies to all values of the 't Hooft coupling – therefore can be extended to the regime of small couplings as well.
- Now, however, we have an infinite tower of string modes – whose duals are gauge invariant operators which become as important as the ones which are dual to supergravity modes.
- This is because for large λ the dimensions of higher stringy modes - and hence the frequencies of the corresponding oscillators - are $O(N)$ as opposed to supergravity modes whose frequencies are $O(1)$.
- For small λ, however, the dimensions of all these modes are comparable.

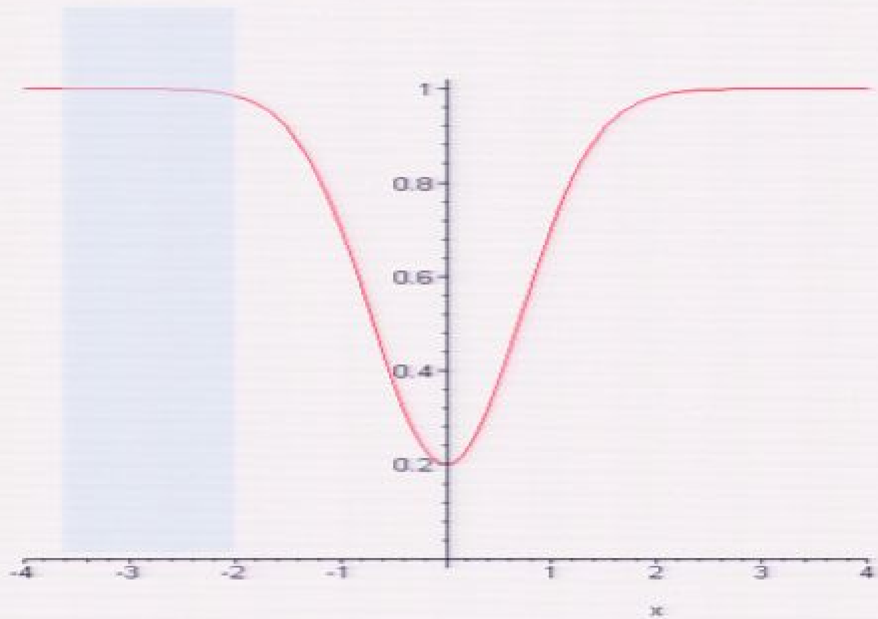
- Nevertheless, the basic ingredients which went into our coherent state adiabatic approximation are still in place

   (1) The couplings between different oscillators are still suppressed by ϵ.

   (2) The frequencies are still $O(1)$ for any value of λ, so that the system is always far from resonance.

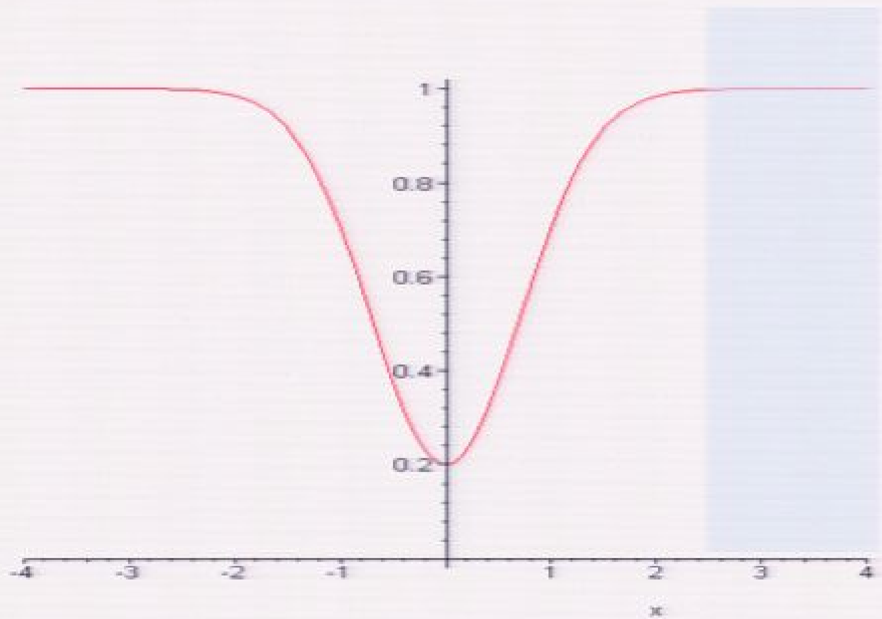
   (3) For $N\epsilon \gg 1$ the states are still classical.

- It would therefore appear that the adiabatic theorem still holds.



After passing through the stringy region of high curvature, one would essentially have $AdS_5 \times S^5$ with exponentially small corrections.

- **However, there is another possibility.**
- There are $O(N^2)$ stringy modes (non-chiral operators) , while there are only $O(1)$ supergravity modes.
- While individual couplings are suppressed by ϵ, there is a possibility that whatever energy is transferred to these modes may thermalize.
- If thermalization does happen – the energy is dissipated and cannot be extracted back when the coupling rises again to large values.
- At late times one would have $O(N^2\epsilon^2)$ thermalized energy in the system.



After passing through the stringy region of high curvature, one would essentially have $AdS_5 \times S^5$ with exponentially small corrections.

- **However, there is another possibility.**
- There are $O(N^2)$ stringy modes (non-chiral operators) , while there are only $O(1)$ supergravity modes.
- While individual couplings are suppressed by ϵ, there is a possibility that whatever energy is transferred to these modes may thermalize.
- If thermalization does happen – the energy is dissipated and cannot be extracted back when the coupling rises again to large values.
- At late times one would have $O(N^2\epsilon^2)$ thermalized energy in the system.

- At late times, the ‘t Hooft coupling is again large and we can use known results of AdS/CFT to guess the outcome.
- This depends on how small ϵ is.
- For $\epsilon^2 \ll \lambda^{\frac{1}{4}} N^{-2}$ the result would be a gas of supergravity modes.
- For $\lambda^{\frac{1}{4}} N^{-2} < \epsilon^2 \ll \lambda^{-\frac{7}{2}}$ one would have a gas of higher string modes.
- For $\lambda^{-\frac{7}{2}} < \epsilon^2 \ll 1$ one would have small black holes, i.e. Black holes whose size is much smaller than R_{AdS}
- This is the worst that can happen – large black holes require an energy $O(N^2)$ which is much larger than the energy we have.

  They will not form
- In any case most of space-time would be close to $AdS_5 \times S^5$

- **However, there is another possibility.**
- There are $O(N^2)$ stringy modes (non-chiral operators) , while there are only $O(1)$ supergravity modes.
- While individual couplings are suppressed by ϵ, there is a possibility that whatever energy is transferred to these modes may thermalize.
- If thermalization does happen – the energy is dissipated and cannot be extracted back when the coupling rises again to large values.
- At late times one would have $O(N^2\epsilon^2)$ thermalized energy in the system.

- At late times, the ‘t Hooft coupling is again large and we can use known results of AdS/CFT to guess the outcome.
- This depends on how small ϵ is.
- For $\epsilon^2 \ll \lambda^{\frac{1}{4}} N^{-2}$ the result would be a gas of supergravity modes.
- For $\lambda^{\frac{1}{4}} N^{-2} < \epsilon^2 \ll \lambda^{-\frac{7}{2}}$ one would have a gas of higher string modes.
- For $\lambda^{-\frac{7}{2}} < \epsilon^2 \ll 1$ one would have small black holes, i.e. Black holes whose size is much smaller than R_{AdS}
- This is the worst that can happen – large black holes require an energy $O(N^2)$ which is much larger than the energy we have. They will not form
- In any case most of space-time would be close to $AdS_5 \times S^5$

- It is difficult to determine whether thermalization would indeed occur – the time scale involved in interactions is the same as the time scale by which the system is driven – and there is no obvious answer to this question.

Rapidly varying Dilatons

- In previous work *(Awad, Das, Narayan, Nampuri & Trivedi)* we have explored a similar scenario for the gauge theory on flat space-time instead of S^3.
- Now of course the theory has no scale – so there is no sense in which the coupling is slowly varying – it in fact varies rapidly at some time.
- The AdS dual is now defined in the Poincare patch.

Epilogue

- Most of recent work in this area aims to arrive at toy models of cosmology where the meaning and physics of singularities can be studied in a controlled fashion.
- This is clearly a caricature of cosmology – and the investigation is in its early stages.
- The hope, however, is that this will eventually teach us something about real cosmology.

Epilogue

- Most of recent work in this area aims to arrive at toy models of cosmology where the meaning and physics of singularities can be studied in a controlled fashion.
- This is clearly a caricature of cosmology – and the investigation is in its early stages.
- The hope, however, is that this will eventually teach us something about real cosmology.

Next
Previous
Last Viewed
Go to Slide
Custom Show
Screen
Pointer Options
Help
Pause
End Show

THANK YOU