Title: Conformal invariance and the multiverse

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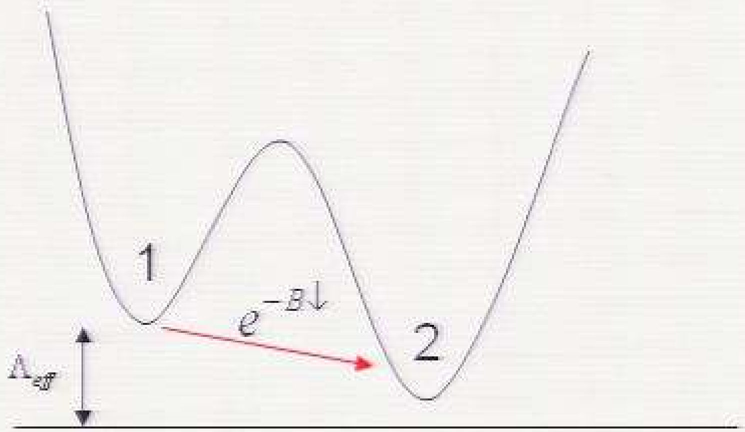
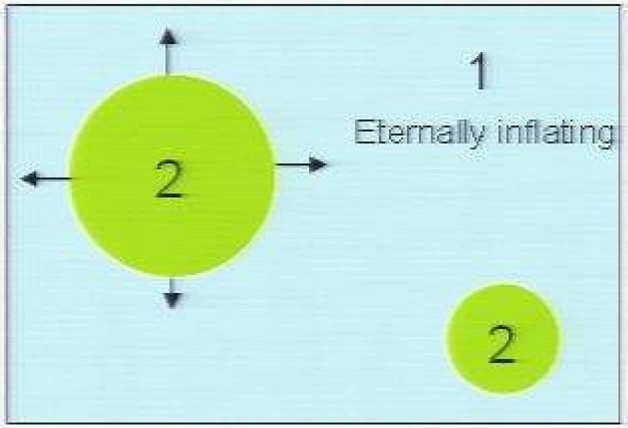
Abstract: TBA

Conformal invariance and the multiverse

Jaume Garriga,
(U. Barcelona)

Metastable dS vacua





$\lambda \equiv$ dimensionless decay rate ~ $e^{-B} \ll 1$

$$\frac{dV_1}{dt} = 3HV_1 - \lambda H V_1$$

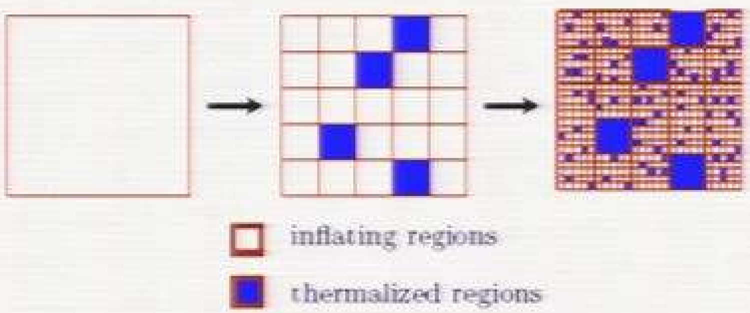
$$V_1 = C\, e^{(3-\lambda)Ht}$$

Average volume grows unbounded for $\lambda \ll 1$,

Eternal inflation

Finite probability that transition is never complete

The eternally inflating fractal (Vilenkin 87, Winitzki 02,05)



$b \equiv e^3$ branching factor

$p \equiv e^{-\lambda}$ survival prob.

$N \equiv Ht$ number of steps

Sierpinsky carpet

$\langle V_1 \rangle = H^{-3}(bp)^N$ grows for $bp > 1$

$X \equiv$ **Probability that a Hubble sized region contains eternal points**

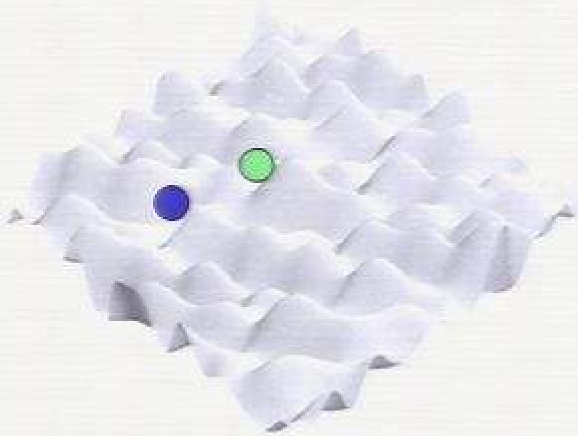
$$\boxed{X = 1 - (1 - Xp)^b}$$

For $bp < 1$ (ie. $\lambda < 3$) only trivial solution $X = 0$,
but for $bp > 1$, nontrivial solution $0 < X \le 1$.

$E =$ **Set of eternal points (non-empty with finite probability for $\lambda < 3$)**

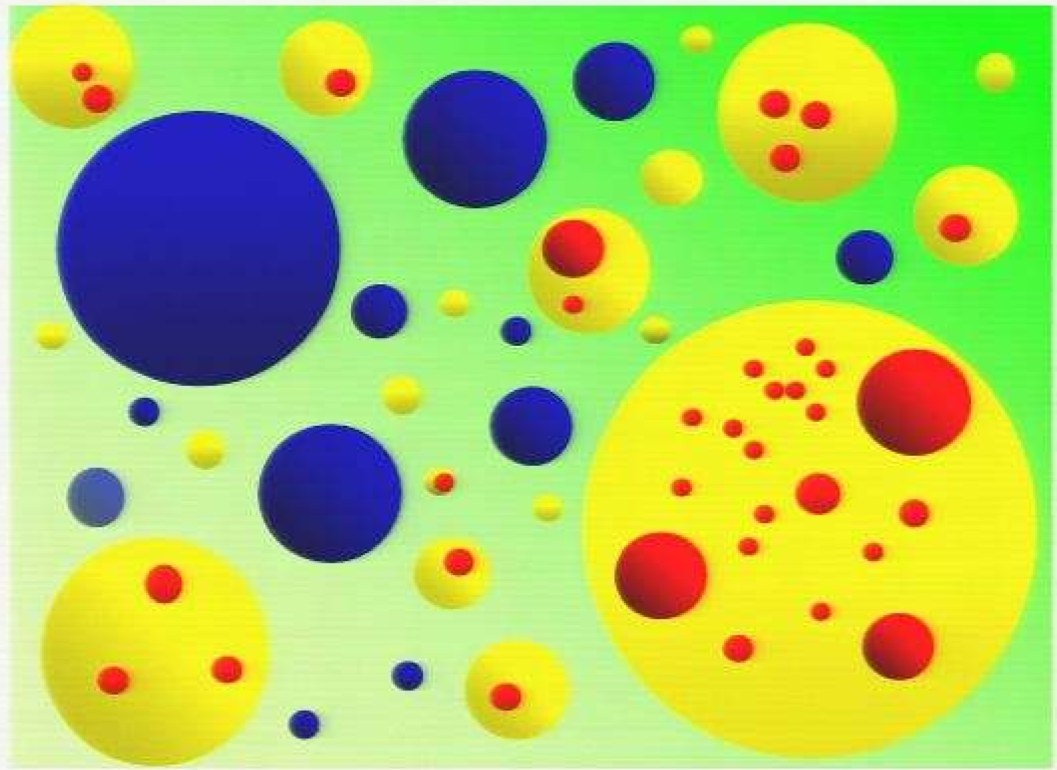
$$\boxed{\dim E = 3 - \lambda}$$

Eternally inflating multiverse



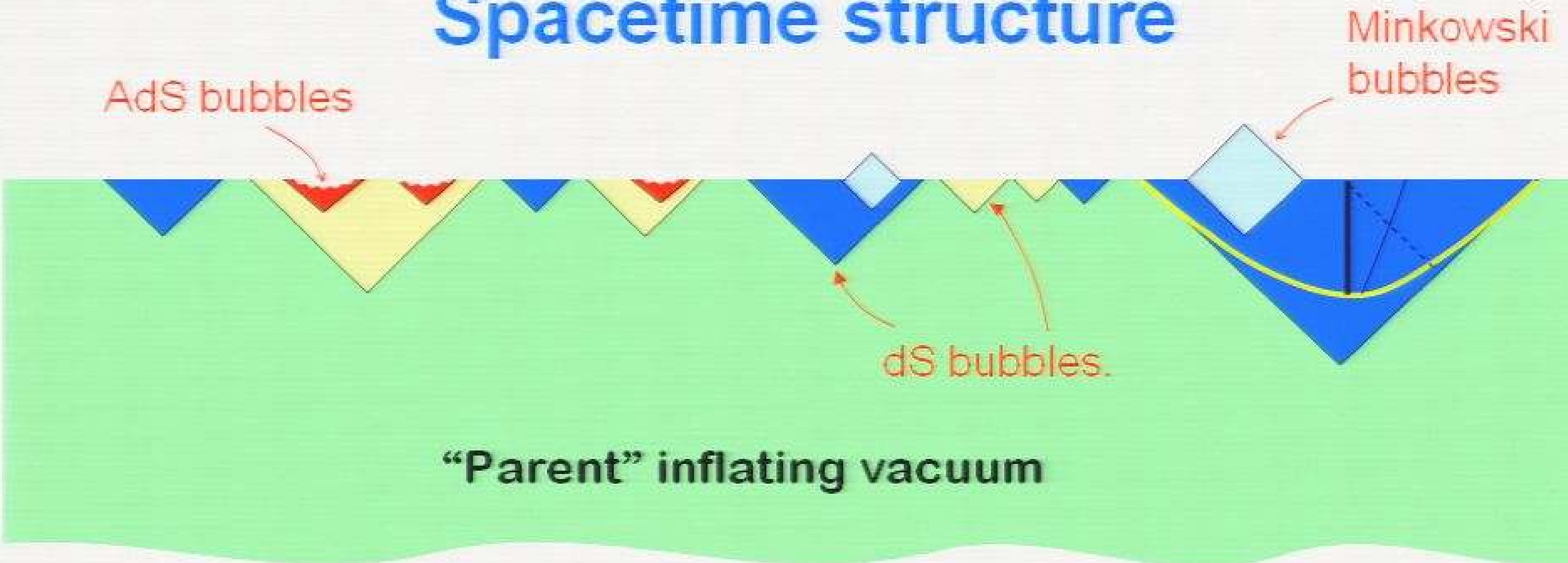
Field space
(Landscape of vacua)

$N_{vac} \sim 10^{500}$



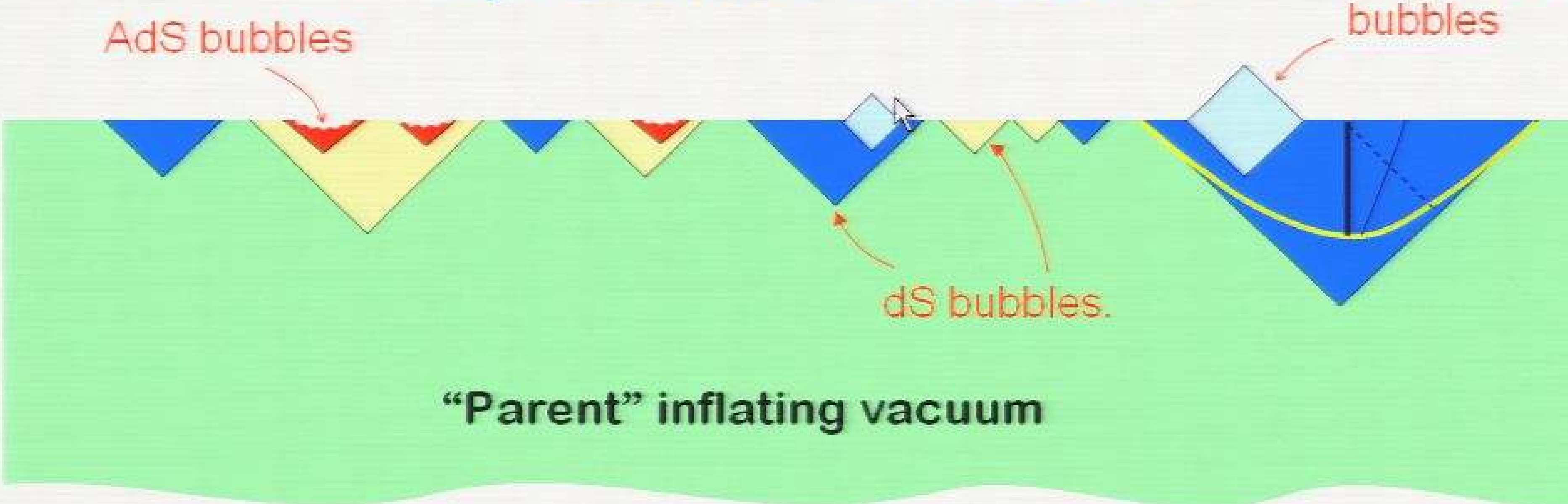
Physical space

Spacetime structure



- Bubbles nucleate and expand at nearly the speed of light.
- dS (**Inflating)**
  AdS
  Minkowski } **(Terminal bubbles)**

Spacetime structure



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Attractor behaviour of volume distribution:

Fraction of volume $V_i(t)$ in inflating vacuum of type

Scale factor gauge $t = \log a$

$$\frac{dV_i}{dt} = 3V_i + M_{ij}V_j$$

rate equation

$$M_{ij} = \underbrace{\lambda_{ij}}_{\text{Gained from other vacua}} - \underbrace{\delta_{ij}\sum_r \lambda_{ri}}_{\text{Lost to other vacua}}$$

$$\lambda_{ij} = \frac{4\pi}{3} H_j^{-4} \Gamma_{ij}$$

From bubbles of type "i" in vacuum "j".

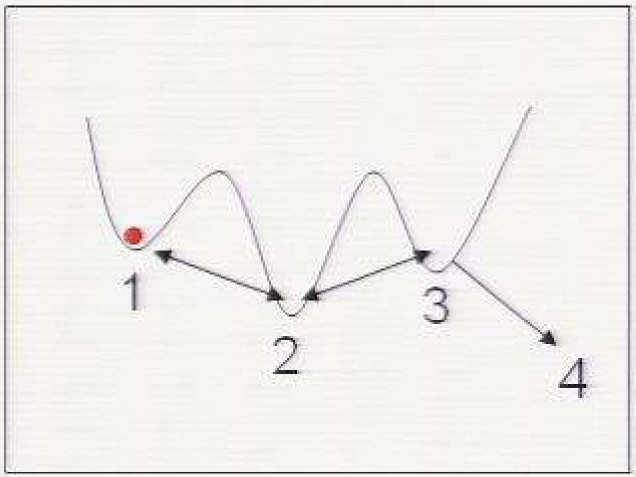
To each irreducible ``landscape'' there corresponds a unique attractor volume distribution.

$$V_i(t) \rightarrow V_i^{(0)}\, e^{(3-q)t} \qquad 0<q\ll 1$$

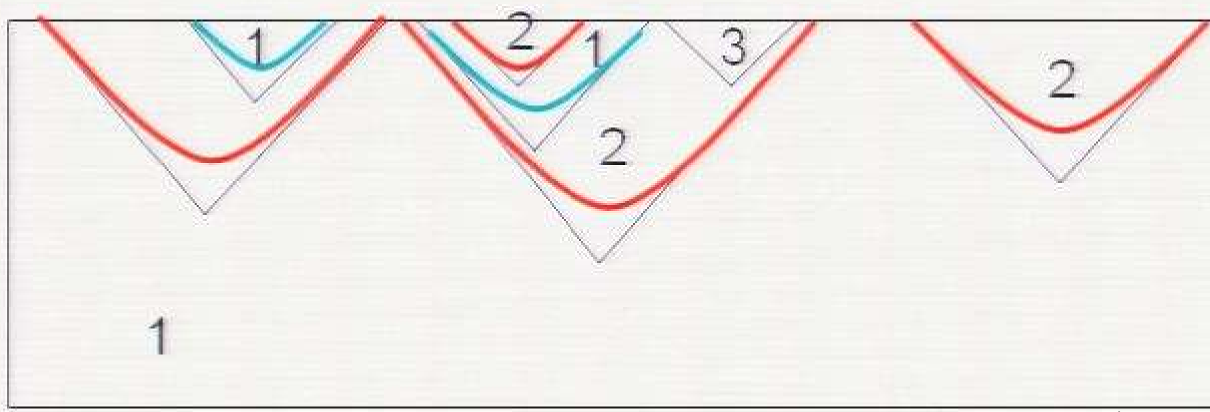
In this sense, initial conditions do not play a role.

J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)

THEORY



(Self-similar fractal)



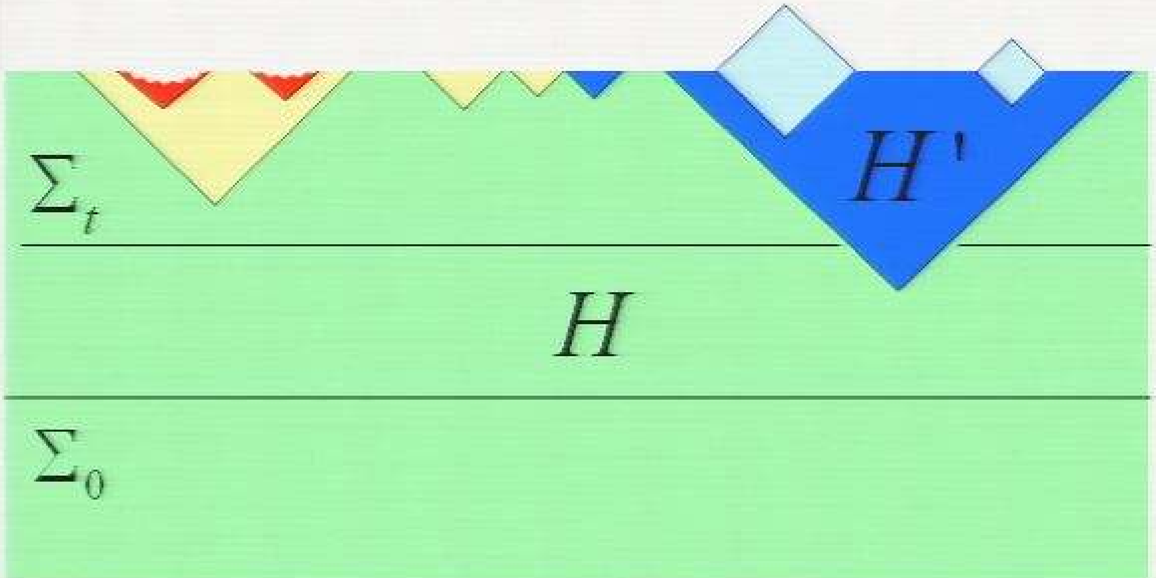
MULTIVERSE

Proposal:

The dynamics of the multiverse admits a holographic description in terms of a CFT at the future boundary.

(With Alex Vilenkin)

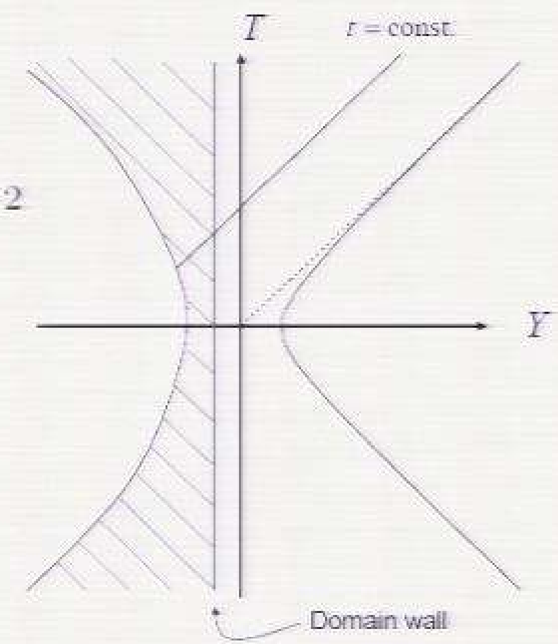
Simple model: dS bubbles separated by thin walls



- Nested bubbles
  plus linearized fluctuations.
- Inflating part of spacetime can be foliated by flat surfaces. (They are very close to constant-a surfaces.)

$t - \ln a$ – scale factor time

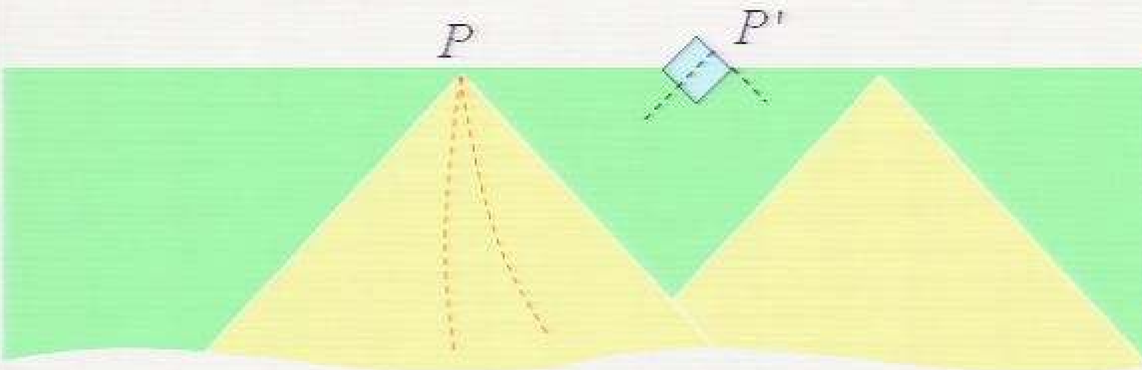
$$\mathbf{X}^2 + (Y - Y_0)^2 - T^2 = H'^{-2}$$



$$\mathbf{X}^2 + Y^2 - T^2 = H^{-2}$$

$$ds^2 = -H^{-2}dt^2 + e^{2t}d\mathbf{x}^2$$

Formal definition of the future boundary



What is a "point" P at the future boundary?

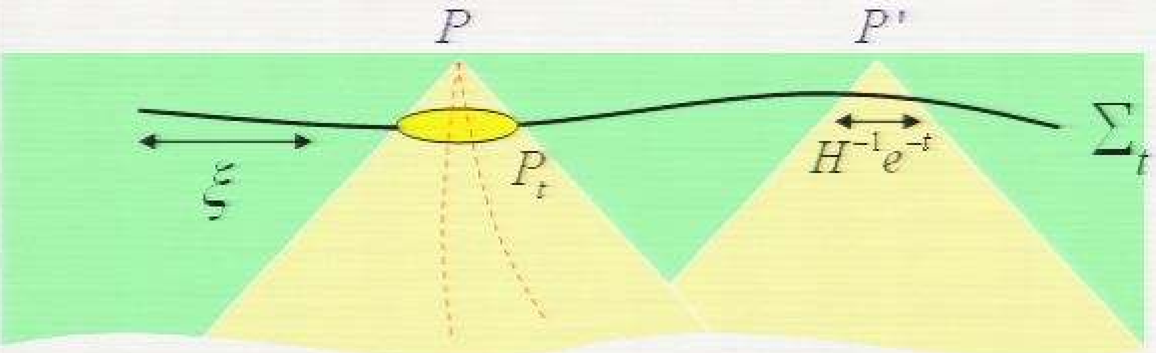
It is the past of an inextendible timelike curve.

Hawking & Ellis (1973)

Curves having the same past define the same point P

Points in the eternal set $\mathcal{E}$ are the "endpoints" of eternally inflating curves.

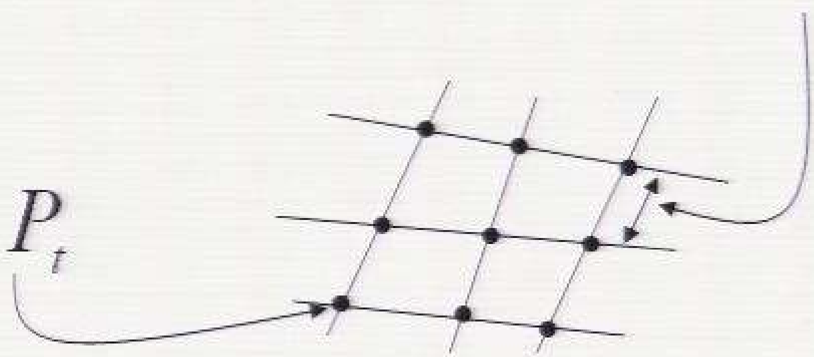
The auxiliary surface Σ_t and the discrete boundary theory



Each point $P \in \mathcal{E}$ is represented on Σ_t as a "blob" P_t of co-moving size $H^{-1}e^{-t}$.

This gives a representation of $\mathcal{E}$ with position dependent resolution $l_c \sim H^{-1}(x)e^{-t}$

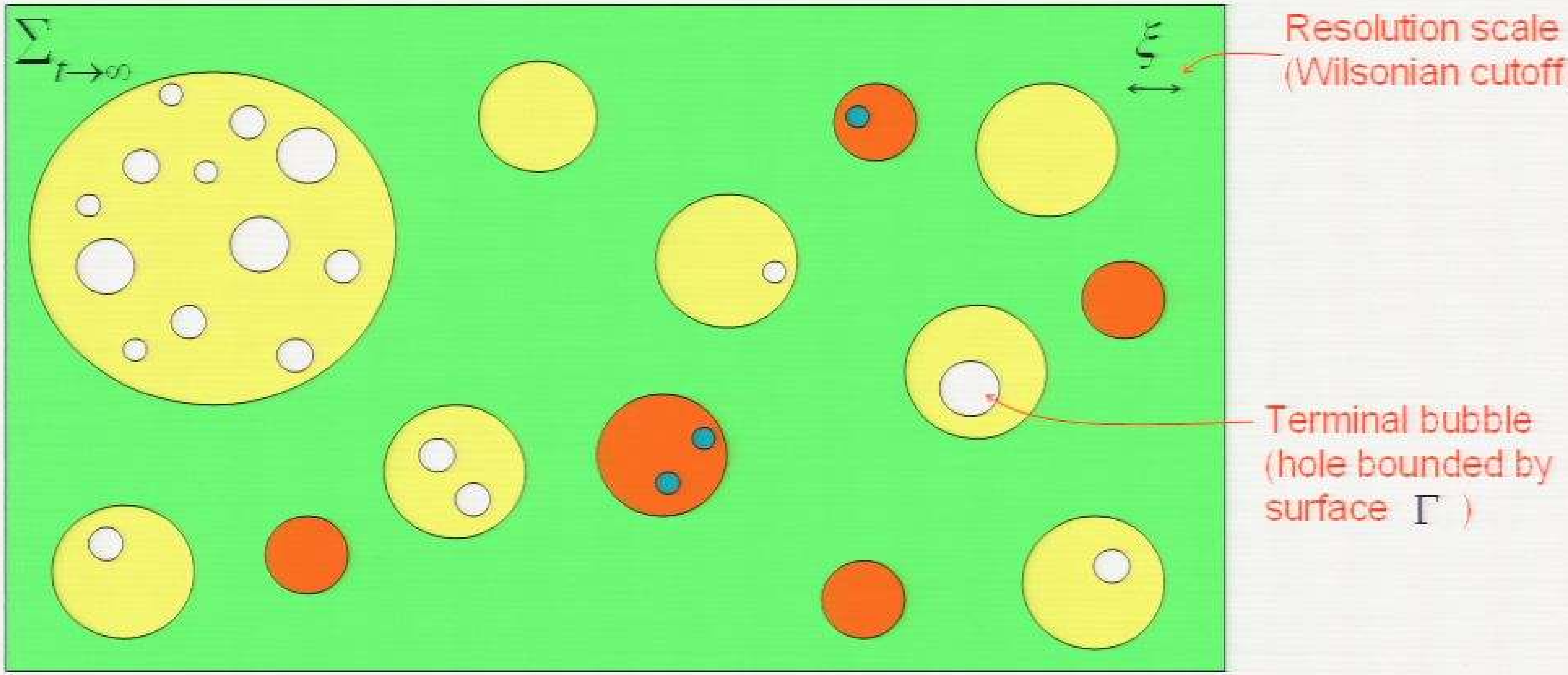
Σ_t **is a "fishnet" of "points"** P_t



Each point carries $N_{bulk} \sim S_{GH} \sim H^{-2}(x)$ degrees of freedom (discrete theory)

Continuum description on scales larger than a Wilsonian cut-off $\xi \gg \max[l_c(x)]$

Representation of the eternal set $\mathcal{E}$ at finite resolution ξ



- Each bubble becomes a fractal "sponge" in the limit $\xi \to 0$.
- Terminal bubbles correspond to holes (with 2D CFTs on their boundaries).
- Bubbles correspond to boundary surfaces in the 3D theory on $\mathcal{E}$.

Statement of the duality:

similar to: **Maldacena (2002)**
Larsen, McNees (2003)

$$\Psi[\overline{\varphi}(\vec{x})] = \int_{\Sigma_0}^{\Sigma_{t\to\infty}} D\varphi \; e^{iS_{bulk}[\varphi]} = e^{iW_{CFT}[\overline{\varphi}]}$$

Wave function of the universe

Effective action for the boundary theory with `sources' $\overline{\varphi}(\vec{x})$.

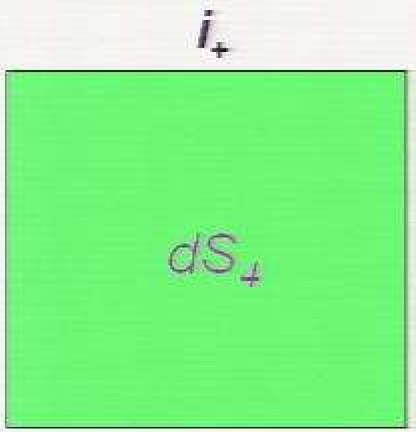
For any fixed Wilsonian cutoff ξ the continuum limit is achieved as $t \to \infty$

Short-distance physics at `late time' surface $\Sigma_{t\to\infty}$ should not depend on the initial state on Σ_0

dS/CFT correspondence

Strominger (2001)

Is the 4D theory describing an asymptotically de Sitter space equivalent to a 3D Euclidean CFT at future infinity ***i₊?***



$$ds^2 = -dt^2 + H^{-2}\cosh^2(Ht)\, d\Omega_3^2$$

Future infinity is $S_3: \ t \to \infty.$

Some potential problems:

1- Scaling dimensions are complex for bulk fields with

$$m^2 > \left(\frac{d}{2}\right)^2 H^2$$ (these fields oscillate in time)

2- In String Theory, dS space is metastable, so there is no such thing as asymptotically dS space.

The wave function and conformal invariance

$$\Psi[\bar{\varphi}] = e^{iW_{CFT}[\bar{\varphi}]}$$

- Linearized tensor modes in de Sitter
- Linearized bubble fluctuations

Maldacena 02
J.G., Vilenkin 08,09

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Gaussian wave functional

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$$h(\mathbf{x}) = \int d^d\mathbf{k} \frac{e^{i\mathbf{k}\mathbf{x}}}{(2\pi)^{d/2}} h_\mathbf{k}.$$

$$v^*_\mathbf{k} v'_\mathbf{k} - v_\mathbf{k} v'^*_\mathbf{k} = i a^{1-d}$$

Choice of vacuum (for m=0)

$$v_\mathbf{k}(\eta) = \frac{\pi^{1/2}}{2} a^{-d/2} H^{(1)}_{d/2}(k\eta)$$

Bunch-Davies

(d+1=5)

$$W[\bar{h}(\mathbf{x})] = \frac{1}{2} \int d^4\mathbf{k} \left(\frac{-k^2 a^2}{2H} + \frac{k^4}{8H^3} [\ln(k^2/H^2 a^2) + \boxed{i\pi} + 2\gamma] + O(a^{-2}) \right) |h_\mathbf{k}|^2 + \ldots$$

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8H^3}k^4\right)|h_\mathbf{k}|^2\right]$$

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Imaginary part of W determines the amplitude of cosmological perturbations.

But there is also the non-local part

$$Re[W] = \frac{H^{-3}}{16}\int d^4\mathbf{k}\; k^4 \ln(k^2/\mu^2)\; |h_\mathbf{k}|^2 + \text{analytic.}$$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\, k^4 \ln k^2$$

Expected form in a CFT

(As in Gubser, Klebanov, Polyakov 98)

1- Linearized tensor modes in de Sitter

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In general, the coefficient of the logarithmically divergent term is the trace anomaly

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Numerology: At maximum resolution, when the Wilsonian cutoff is comparable to the "fishnet" spacing

$$\xi \sim l_c \sim H^{-1}(x)e^{-t}$$

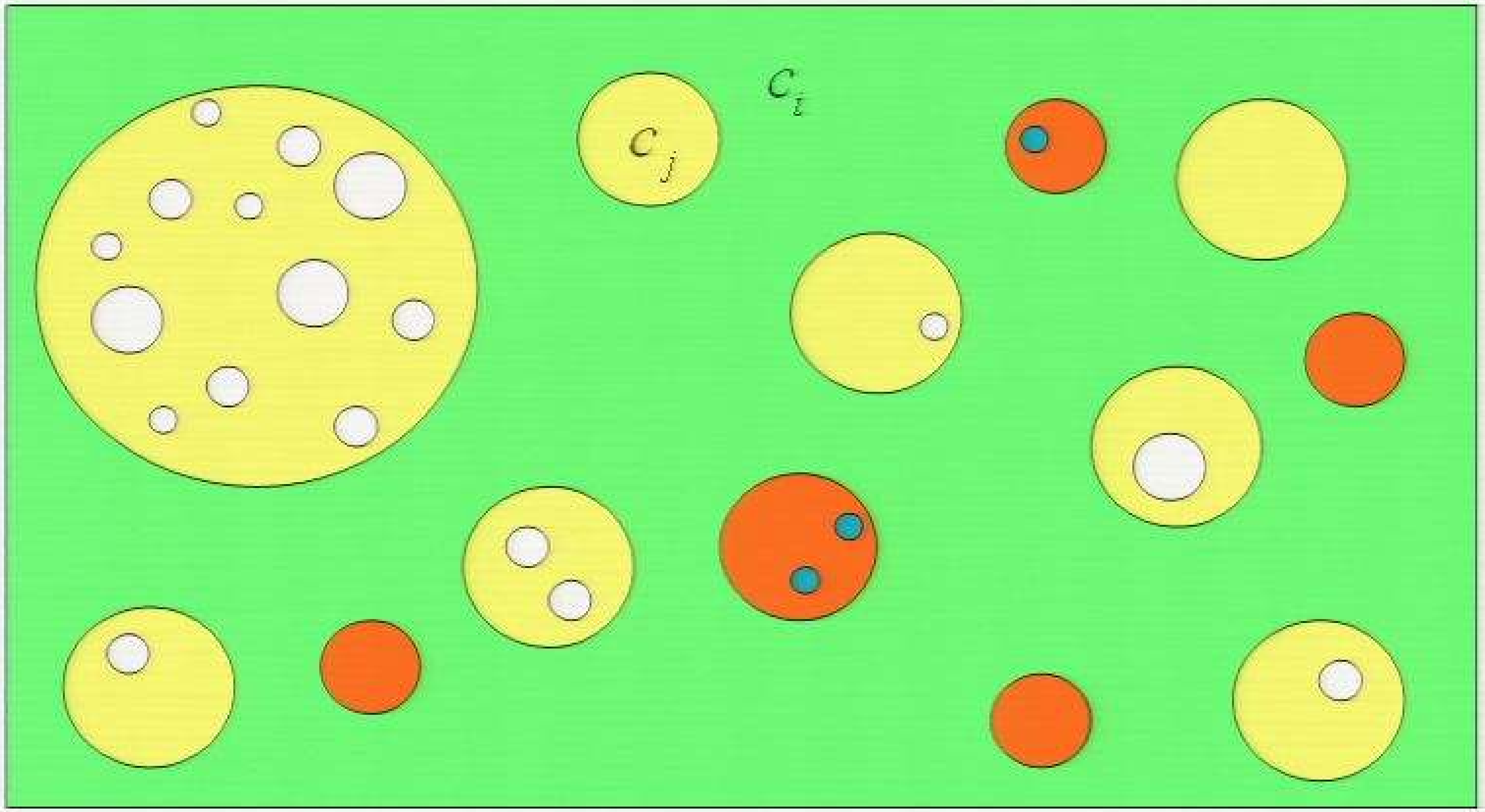
there should be as many degrees of freedom per fishnet point as the number of CFT fields in the boundary.

$$N_{boundary} \sim c \sim H^{-2}$$

This is in agreement with the bulk estimate:

$$N_{bulk} \sim S_{dS} \sim H^{-2}$$

2- Linearized bubble fluctuations



- The boundary effective action should depend on the shape of the surfaces which separate regions with different central charge.

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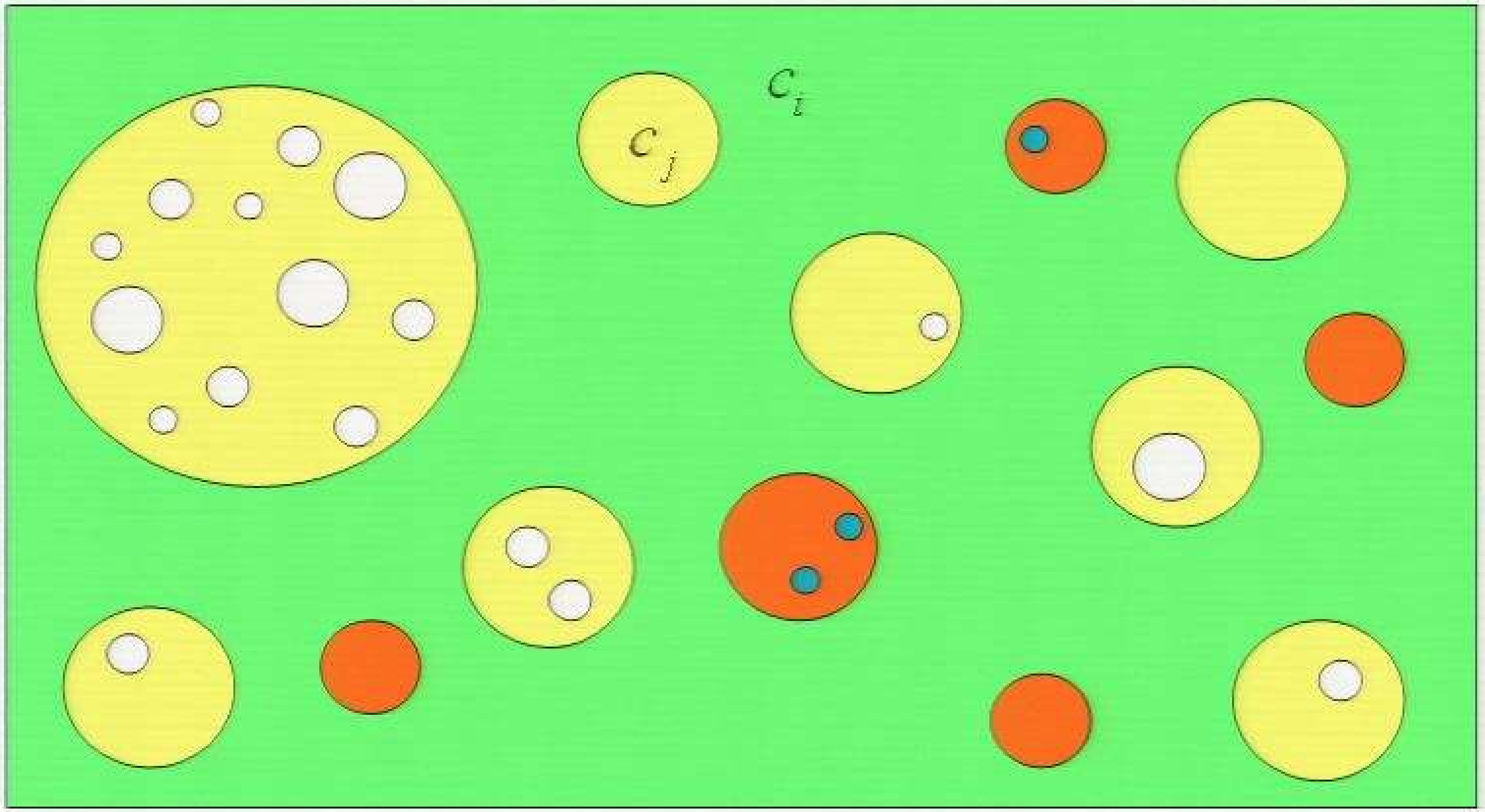
2- Linearized bubble fluctuations



- The boundary effective action should depend on the shape of the surfaces which separate regions with different central charge.

Bubble fluctuations

Restrict attention to the case where gravity of the bubble is unimportant

$$TR_0 \ll 1, \quad (\Delta\rho_V)R_0^2 \ll 1.$$

↑ Bubble wall tension

$$R_0^2 \approx \frac{(p+1)^2T^2}{(p+1)^2H^2T^2 + (\Delta\rho_V)^2}.$$

↑ Intrinsic curvature radius of the worldsheet of bubble wall (which is a p+1 dS space)

$$\delta x^\mu = T^{-1/2}\phi\, n^\mu.$$

Normal displacement of the worldsheet

ϕ - Canonical world-sheet scalar field with a tachyonic mass

$$m_\phi^2 = -(p+1)R_0^{-2}.$$

$$ds_w^2 = \tilde{a}^2\left(-d\tilde{\eta}^2 + d\Omega_p^2\right).$$

Unperturbed w.s. metric

$$\tilde{a} = R_0/\cos\tilde{\eta},$$

Bubble radius

$$\phi = \sum_{LM} \phi_{LM} Y_{LM}(\Omega),$$

Worldsheet normal displacement

$$\Psi[\phi] = e^{iW[\phi]}$$

Gaussian wave function

$$W = \sum_{LM}\left(\frac{\tilde{a}^{d-1}}{2}\,\frac{v_L'}{v_L}|\phi_{LM}|^2 + i\ln v_L\right).$$

Bunch-Davies vacuum

$$v_L = A_L(\cos\tilde{\eta})^{p/2}\left(P^{\nu}_{L-1+p/2}(\sin\tilde{\eta}) + \frac{2i}{\pi}Q^{\nu}_{L-1+p/2}(\sin\tilde{\eta})\right)$$

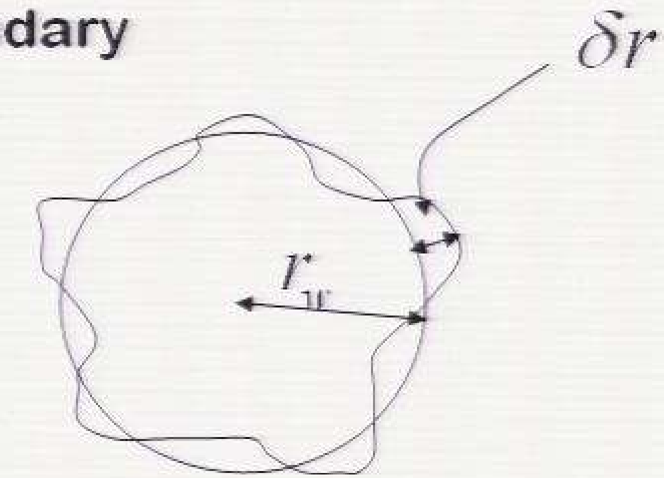
$$\nu = \left(\frac{p^2}{4} - m_\phi^2 R_0^2\right)^{1/2} = \frac{p+2}{2}$$

p=2

$$W=\sum_{LM}\left(\frac{\tilde a^2}{2R_0}+\frac{R_0\Delta}{4}+\frac{R_0^3\Delta(\Delta+2)}{16\,\tilde a^2}\left[\ln\left(\frac{R_0^2}{4\tilde a^2}\right)+2\left(\psi(L)+\frac{1}{L}\right)+i\pi+2\gamma\right]\right)|\phi_{LM}|^2.$$

$\downarrow$ $0\ (a\to\infty)$

Relative displacement at the future boundary



$$\delta\equiv\frac{\delta r}{r_w}=\frac{1}{\gamma}\frac{T^{-1/2}\phi}{\tilde a(t)}.$$

$\gamma\sim 1/(R_0H)$ **Lorentz factor**

$$W[\bar\delta]=\frac{TR_0}{H^2}\sum_{LM}\frac{\Delta(\Delta+2)}{16}\left[\ln\left(\frac{R_0^2}{4\tilde a^2}\right)+2\left(\psi(L)+\frac{1}{L}\right)+i\pi+2\gamma\right]|\delta_{LM}|^2+\ldots$$

Effective action for the deformations of the separation surface in the boundary theory

We just saw that bulk calculation leads to:

$$W[\bar{\delta}] = \frac{TR_0}{H^2} \sum_{LM} \frac{\Delta(\Delta+2)}{16} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right) + 2\left(\psi(L) + \frac{1}{L}\right) + i\pi + 2\gamma \right] |\delta_{LM}|^2 + \ldots$$

For $L \gg 1$ $\qquad W \sim d \int d^2k \; k^4 \log(k^2/\mu^2) \qquad d \sim \frac{TR_0}{H^2} \ll c$

What do we expect from the CFT side? $\quad W_{CFT} \sim a_{d/2} \ln \mu^2 + \ldots$

$$a_{3/2} = \int d\Sigma_2 \left[d_1 \left(K_{ab}K^{ab} - \frac{1}{2}K^2 \right) + d_2 \hat{R} \right] \propto \boxed{\int d\Omega \; \delta\Delta(\Delta+2)\delta.} \quad !$$

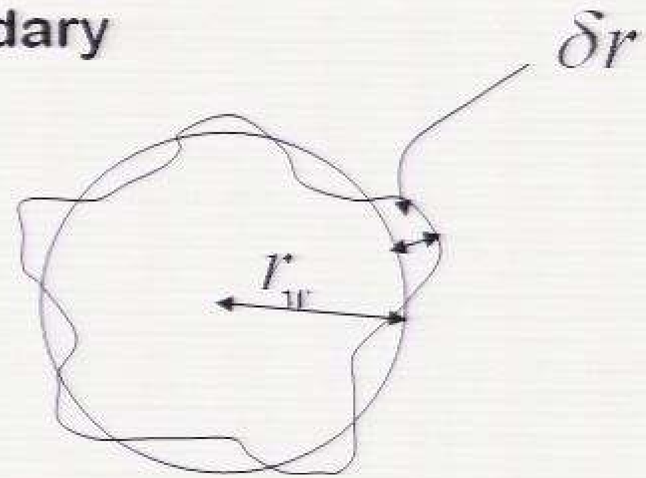
p=2

$$W = \sum_{LM} \left(\frac{\tilde{a}^2}{2R_0} + \frac{R_0 \Delta}{4} + \frac{R_0^3 \Delta(\Delta+2)}{16\, \tilde{a}^2} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right) + 2\left(\psi(L) + \frac{1}{L}\right) + i\pi + 2\gamma \right] \right) |\phi_{LM}|^2.$$

$0 \ (a \to \infty)$

Relative displacement at the future boundary

$$\delta \equiv \frac{\delta r}{r_w} = \frac{1}{\gamma} \frac{T^{-1/2}\phi}{\tilde{a}(t)}.$$



$\gamma \sim 1/(R_0 H)$ **Lorentz factor**

$$W[\bar{\delta}] = \frac{T R_0}{H^2} \sum_{LM} \frac{\Delta(\Delta+2)}{16} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right) + 2\left(\psi(L) + \frac{1}{L}\right) + i\pi + 2\gamma \right] |\delta_{LM}|^2 + \ldots$$

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Conformal invariance of the boundary theory

- $\langle TT \rangle$ correlator has the form expected in a CFT.
- Distribution of nested bubbles is invariant under the Euclidean conformal group in the UV (as Alex just showed)
- Correlator of bubble fluctuations also has the form expected in CFT.

$$ds_w^2 = \tilde{a}^2 \left(-d\tilde{\eta}^2 + d\Omega_p^2\right).$$

Unperturbed w.s. metric

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$$\nu = \left(\frac{p^2}{4} - m_\phi^2 R_0^2\right)^{1/2} = \frac{p+2}{2}$$

(d+1=4)

$$W[\bar{h}(\mathbf{x})] = \frac{1}{2}\int d^3\mathbf{k}\left(\frac{-k^2 a}{H} + \boxed{i\frac{k^3}{H^2}} + O(a^{-1})\right)|h_{\mathbf{k}}|^2 + \dots,$$

Non-analytic part

$c \sim H^{-2}$ **Number of fields in the CFT**

Numerology: **At maximum resolution, when the Wilsonian cutoff is comparable to the "fishnet" spacing**

$$\xi \sim l_c \sim H^{-1}(x)e^{-t}$$

there should be as many degrees of freedom per fishnet point as the number of CFT fields in the boundary.

$$N_{boundary} \sim c \sim H^{-2}$$

This is in agreement with the bulk estimate:

$$N_{bulk} \sim S_{GH} \sim H^{-2}$$

For a weakly coupled CFT (Vilkovisky et al.)

$$W[g_{ij}] = \int d^4x\sqrt{g}\,[c_1 R \ln(\Box/\mu^2) R + c_2 R_{ij} \ln(\Box/\mu^2) R^{ij} + c_3 R_{ijkl} \ln(\Box/\mu^2) R^{ijkl}] + \text{analytic}.$$

In general, the coefficient of the logarithmically divergent term is the trace anomaly

$$a_2 \sim \int d^4x\sqrt{g}\,[c_1 R^2 + c_2 R_{ij} R^{ij} + c_3 R_{ijkl} R^{ijkl}]\,.$$

$$\boxed{W_{ren} \sim a_{d/2} \ln \mu^2 + \ldots} \quad \sim \quad c \int d^4\mathbf{k}\; k^4 \ln(k^2/\mu^2)\; |h_{\mathbf{k}}|^2 + \text{analytic}.$$

$c \sim H^{-3}$ **Number of fields in the CFT (central charge)**

For a weakly coupled CFT (Vilkovisky et al.)

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$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8H^3}k^4\right)|h_\mathbf{k}|^2\right]$$

$$\langle h^*_\mathbf{k} h_{\mathbf{k}'}\rangle = (8H^3/\pi k^4)\delta(\mathbf{k}'-\mathbf{k})$$

Imaginary part of W determines the amplitude of cosmological perturbations.

But there is also the non-local part

$$Re[W] = \frac{H^{-3}}{16}\int d^4\mathbf{k}\; k^4 \ln(k^2/\mu^2)\;|h_\mathbf{k}|^2 + \text{analytic}.$$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\,k^4 \ln k^2$$

Expected form in a CFT

(As in Gubser, Klebanov, Polyakov 98)

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Expected form in a CFT

(As in Gubser, Klebanov, Polyakov 98)

1- Linearized tensor modes in de Sitter

$$ds^2 = a^2(\eta)[-d\eta^2 + d\mathbf{x}^2], \qquad a(\eta) = -1/H\eta$$

Gaussian wave functional

$$\Psi[h] = e^{iW[h]} \qquad W = \int d^d\mathbf{k} \left(\frac{a^{d-1}}{2} \frac{v'_{\mathbf{k}}}{v_{\mathbf{k}}} |h_{\mathbf{k}}|^2 + i \ln v_{\mathbf{k}} \right)$$

$$h(\mathbf{x}) = \int d^d\mathbf{k} \frac{e^{i\mathbf{kx}}}{(2\pi)^{d/2}} h_{\mathbf{k}}.$$

$$v^*_{\mathbf{k}} v'_{\mathbf{k}} - v_{\mathbf{k}} v'^*_{\mathbf{k}} = i a^{1-d}$$

Choice of vacuum (for m=0)

$$v_{\mathbf{k}}(\eta) = \frac{\pi^{1/2}}{2} a^{-d/2} H^{(1)}_{d/2}(k\eta)$$

Bunch-Davies

(d+1=5)

$$W[\bar{h}(\mathbf{x})] = \frac{1}{2} \int d^4\mathbf{k} \left(\frac{-k^2 a^2}{2H} + \frac{k^4}{8H^3} [\ln(k^2/H^2 a^2) + i\pi + 2\gamma] + O(a^{-2}) \right) |h_{\mathbf{k}}|^2 + \ldots$$

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Summary

- The dynamics of the multiverse may be encoded in its future boundary, in terms of a theory which is conformal in the UV.
- The measure can be obtained by imposing a UV cutoff in the boundary theory (closely related to the scale factor cutoff).
- The measure may perhaps be formulated even if there is no dual theory, based on the property of conformal invariance of the wave function.