

Title: Cosmology inside the brane and holography

Date: Jul 02, 2009 11:00 AM

URL: <http://pirsa.org/09070016>

Abstract: TBA

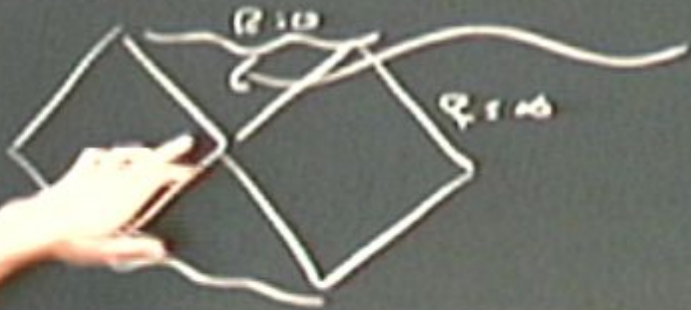
Cosmology inside the brane + holography



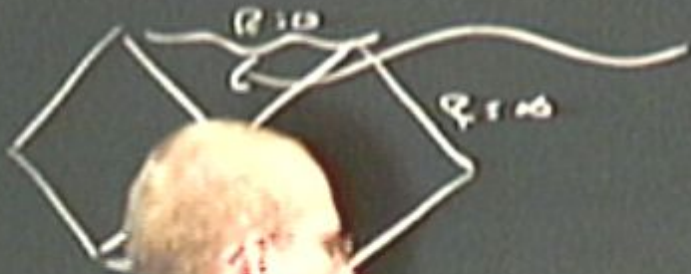
Cosmology inside the brane + holography

Cosmology inside the brane + holography

- ① black holes + branes as cosmologies
- ② holographic cosmology + wave fun of the universe
- ③ differences between Λ and w/out Λ

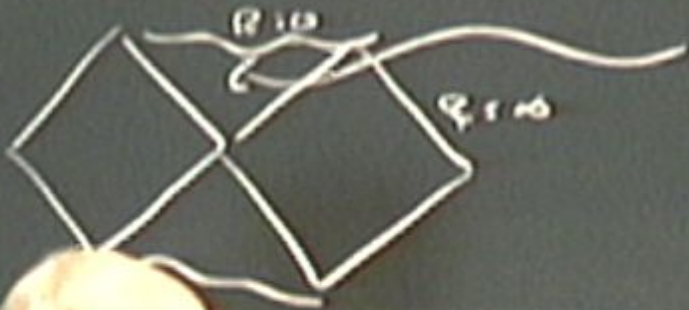


$$\partial s^2 = -\left(\frac{2m}{r} - 1\right) dt^2 + \left(\frac{2m}{r} - 1\right) dr^2 + r^2 d\Omega^2$$



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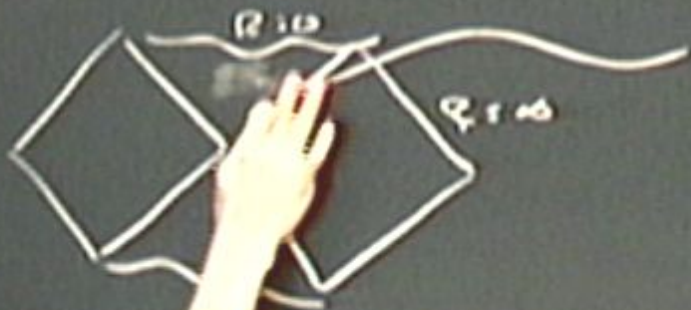
$$X = \frac{1}{4M}, \quad r = \sqrt{16M^2 - 8MR}$$



$$\partial s^2 = -\left(\frac{2M}{r} - 1\right) dt^2 + \left(\frac{2M}{r} - 1\right) dr^2 + r^2 d\Omega_2^2$$

$$x = \frac{r}{4M}, \quad \tau = \sqrt{16M^2 - 8Mr}$$

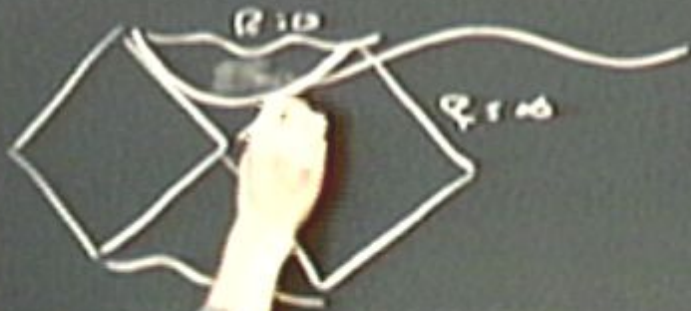
$$\partial s^2 = -\partial \tau^2 + \tau^2 \partial x^2 + 4M^2 \partial \Omega_2^2$$



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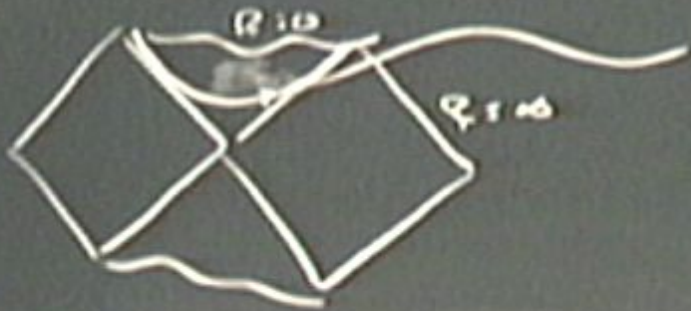
$$\partial s^2 = -\partial \tau^2 + \tau^2 \partial x^2 + 4M^2 \partial \Omega_2^2$$



$$ds^2 = -\left(\frac{2M}{r} - 1\right)^2 dt^2 + \left(\frac{2M}{r} - 1\right) dr^2 + r^2 d\Omega^2$$

$$x = \frac{1}{4M}, \quad r = \sqrt{16M^2 - 8MR}$$

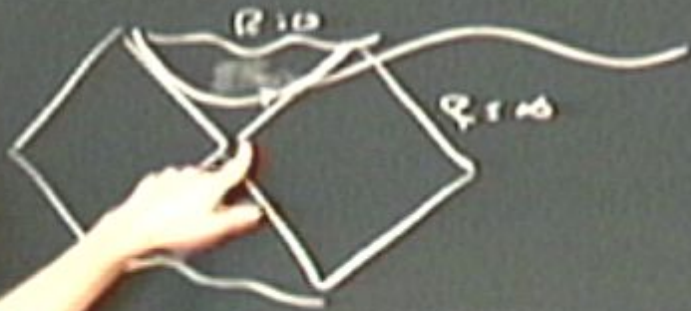
$$ds^2 = -dr^2 + r^2 dx^2 + 4M^2 d\Omega^2$$



$$\partial s^2 = -\left(\frac{2M}{r} - 1\right) \partial r^2 + \left(\frac{2M}{r} - 1\right) \partial t^2 + 4r^2 \partial \Omega_2^2$$

$$x = \frac{1}{4M}, \quad \tau = \sqrt{16M^4 - 8MR}$$

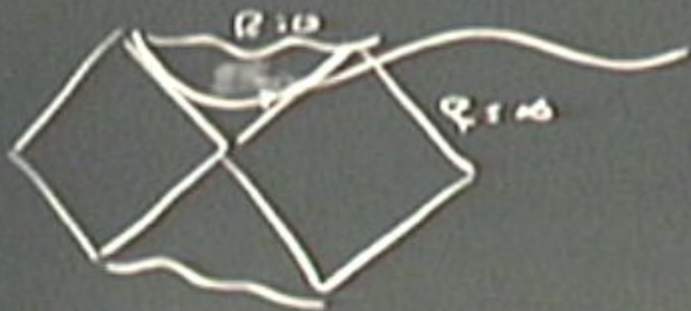
$$\partial s^2 = -\partial \tau^2 + \tau^2 \partial x^2 + 4M^2 \partial \Omega_2^2$$



$$ds^2 = -\left(\frac{2M}{r} - 1\right)^2 dt^2 + \left(\frac{2M}{r} - 1\right) dr^2 + r^2 d\Omega_2^2$$

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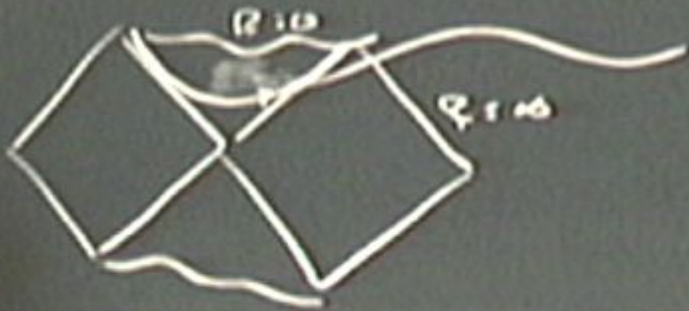
$$ds^2 = \underbrace{-d\tau^2 + \tau^2 dx^2}_{\text{on FRW}} + \underbrace{4M^2 d\Omega_2^2}_{\text{"unperturbed } S^2"}$$



$$ds^2 = -\left(\frac{2M}{r} - 1\right) dt^2 + \left(\frac{2M}{r} - 1\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

$$x = \frac{r}{4M}, \quad \tau = \sqrt{16M^2 - 8Mr}$$

$$ds^2 = \underbrace{-d\tau^2 + \tau^2 dx^2}_{\text{on FRW}} + \underbrace{4M^2 d\Omega_2^2}_{\text{"unperturbed } S^2"}$$

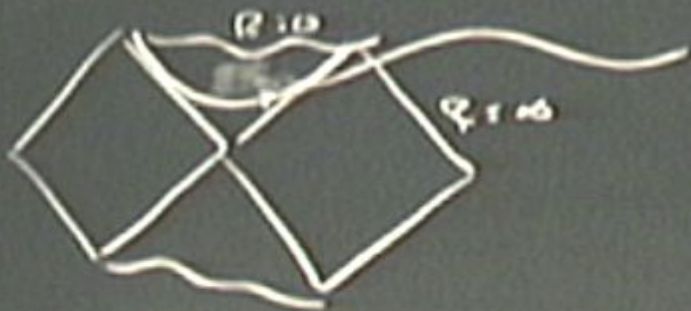


$$ds^2 = -\left(\frac{2M}{r} - 1\right)^2 dt^2 + \left(\frac{2M}{r} - 1\right) dr^2 + r^2 d\Omega_2^2$$

$$X = \frac{t}{4M} \quad (new \quad \tau \rightarrow 0) \quad \tau = \sqrt{16M^2 - 8MR}$$

$\tau \rightarrow 0$ is the big bang

$$ds^2 = \underbrace{-d\tau^2 + \tau^2 dx^2}_{\text{on FRW}} + \underbrace{4M^2 d\Omega_2^2}_{\text{"unperturbed } S^2"}$$



$$ds^2 = -\left(\frac{2M}{r} - 1\right) dt^2 + \left(\frac{2M}{r} - 1\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$X = \frac{1}{4M} \quad (\text{near } r \rightarrow 0), \quad \tau = \sqrt{16M^2 - 8MR}$$

$\tau \rightarrow 0$ is the big bang

$$ds^2 = \underbrace{-d\tau^2 + \tau^2 dx^2}_{\text{FRW}} + \underbrace{4M^2 d\Omega^2}_{\text{"unperturbed } S^2"}$$

Move terms from $\mathcal{L}_0$

$$\delta \mathcal{L} = -\partial \mathcal{L}^2 + 2\epsilon \eta \partial \mathcal{L}^2 + \epsilon^2 \mathcal{L} \partial \mathcal{L}^2$$

Move terms from (20)

$$\delta \xi^2 = -\partial \xi^2 + 2\cos \partial \pi^2 + \xi^2 \partial \xi^2$$

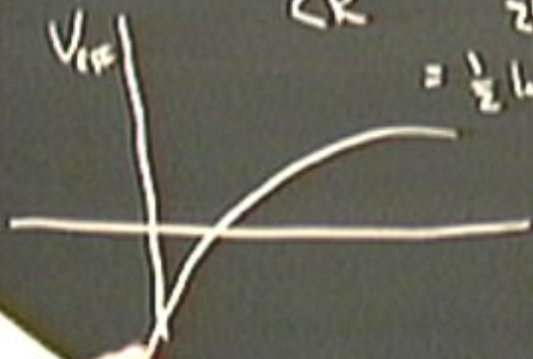
$$\Rightarrow R'' + \frac{R'^2}{2R} = -\frac{1}{2R} ; \quad \alpha = R'$$

Move away from $\tau=0$

$$\delta s^2 = -dt^2 + 2\alpha dt dx^2 + R^2(\tau) d\Omega_2^2$$

$$\Rightarrow R'' + \frac{R'^2}{2R} = -\frac{1}{2R} \quad ; \quad \alpha = R'$$

$$= \frac{1}{2} \log R$$

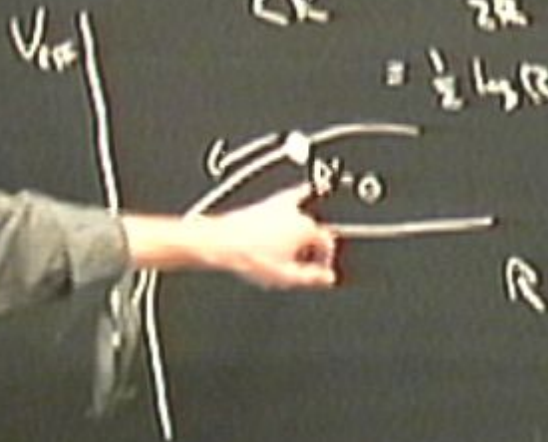


Move away from $\tau=0$

$$\delta s^2 = -dt^2 + 2\alpha dt d\tau + R^2 d\tau^2$$

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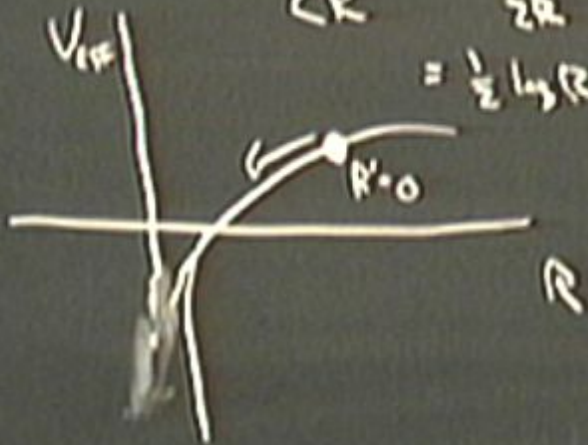


Move away from $\tau=0$

$$\delta s^2 = -\delta \tau^2 + 2\tau \delta x^2 + R^2(\tau) \delta \sigma_2^2$$

$$\Rightarrow R'' + \frac{R'^2}{2R} = -\frac{1}{2R} \quad ; \quad \alpha = R'$$

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What about 4-D FRW in localized region
embedded in higher D?

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 embedded in higher D?

$$S = \frac{M_{\text{Pl}4}^{4+2}}{2} \int d^4x \sqrt{-g} \left[R^{(4+2)} - \frac{1}{2g_5} F^2 - 2\Lambda \right]$$

embedded in higher D?

$$S = \frac{M^{9+2}_{10}}{2} \int d^{10}x \sqrt{-g} \left[\tilde{R}^{(10)} - \frac{1}{2\ell_s^2} F^2 - 2\Lambda \right]$$

$$\begin{aligned} dS^2 &= \tilde{g}_{\mu\nu} dx^\mu dx^\nu + R^2(x) d\Omega_7^2 \\ &= Q \sin^4 \theta \end{aligned}$$

embedded in higher D?

$$S = \frac{M_{\text{Pl}}^{4+2}}{2} \int d^{4+n}x \sqrt{-g} \left[\hat{R}^{(4+n)} - \frac{1}{2(n!)} F_n^2 - 2\Lambda \right]$$

$$\partial S^2 = \sum_{\mu\nu} g_{\mu\nu} \partial x^\mu \partial x^\nu + R^2(x) \partial \Omega_1^2$$

$$F_6 = Q \sin^4 \theta_1 \dots \sin \theta_{k+1} \partial \theta_1 \wedge \dots \wedge \partial \theta_k$$

What about 4-D FRW in localized region
 embedded in higher D?

$$S = \frac{M_{4+2}^{4+2}}{2} \int d^{4+2}x \sqrt{-g} \left[\tilde{R}^{(4+2)} - \frac{1}{2g_5} \tilde{F}^2 - 2\Lambda \right]$$

$$ds^2 = \sum_{\mu, \nu}^{4+1} dx^\mu dx^\nu + R^2(x) d\Omega_1^2$$

$$F_5 = Q \sin^4 \theta_1 \dots \sin \theta_{4+1} d\theta_1 \wedge \dots \wedge d\theta_{4+1}$$

1) Assume $\rightarrow$ Sph. Sym.

2) 4-D action $\rightarrow$ Einstein frame

$$dS^2 = (M_{\text{Pl}}(R))^2 \sum_{\mu\nu} dy^\mu dy^\nu + e^{2\sigma} d\Omega_3^2$$

Can. norm. function

M_{Pl}

1) Assume $\rightarrow$ Sph. Sym.

2) 4-D action $\rightarrow$ Einstein frame

$$dS^2 = (M_{\text{pl}} R)^2 \sum_{\mu\nu} dy^\mu dy^\nu + e^{2\sigma} d\Omega_5^2$$

3) Can. norm. radiation

$$M_{\text{pl}} R = \exp \left[\sqrt{\frac{2}{9(2+\epsilon)}} \frac{\phi}{M_{\text{pl}}} \right]$$

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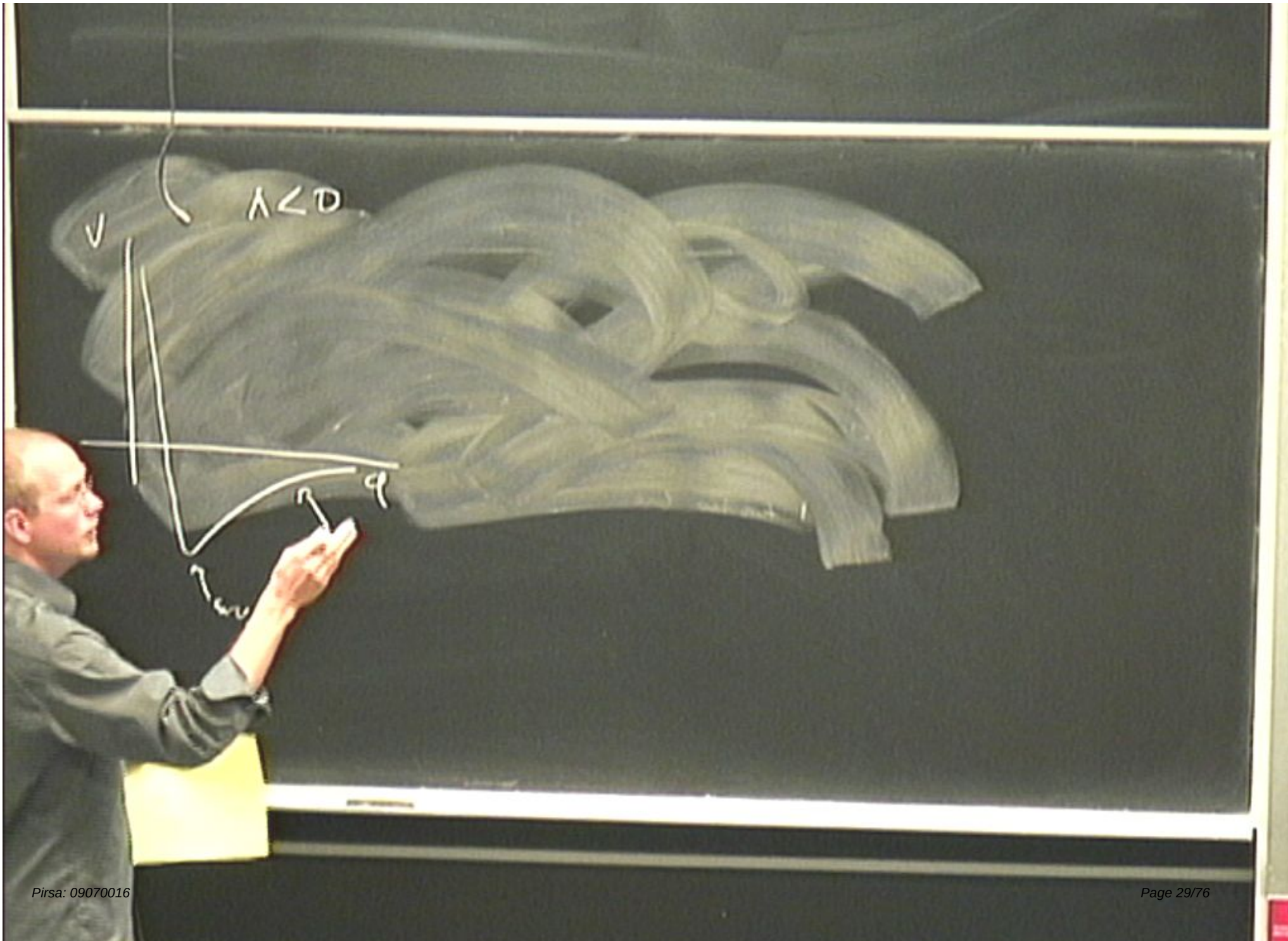
2) 4-D action $\rightarrow$ Einstein frame

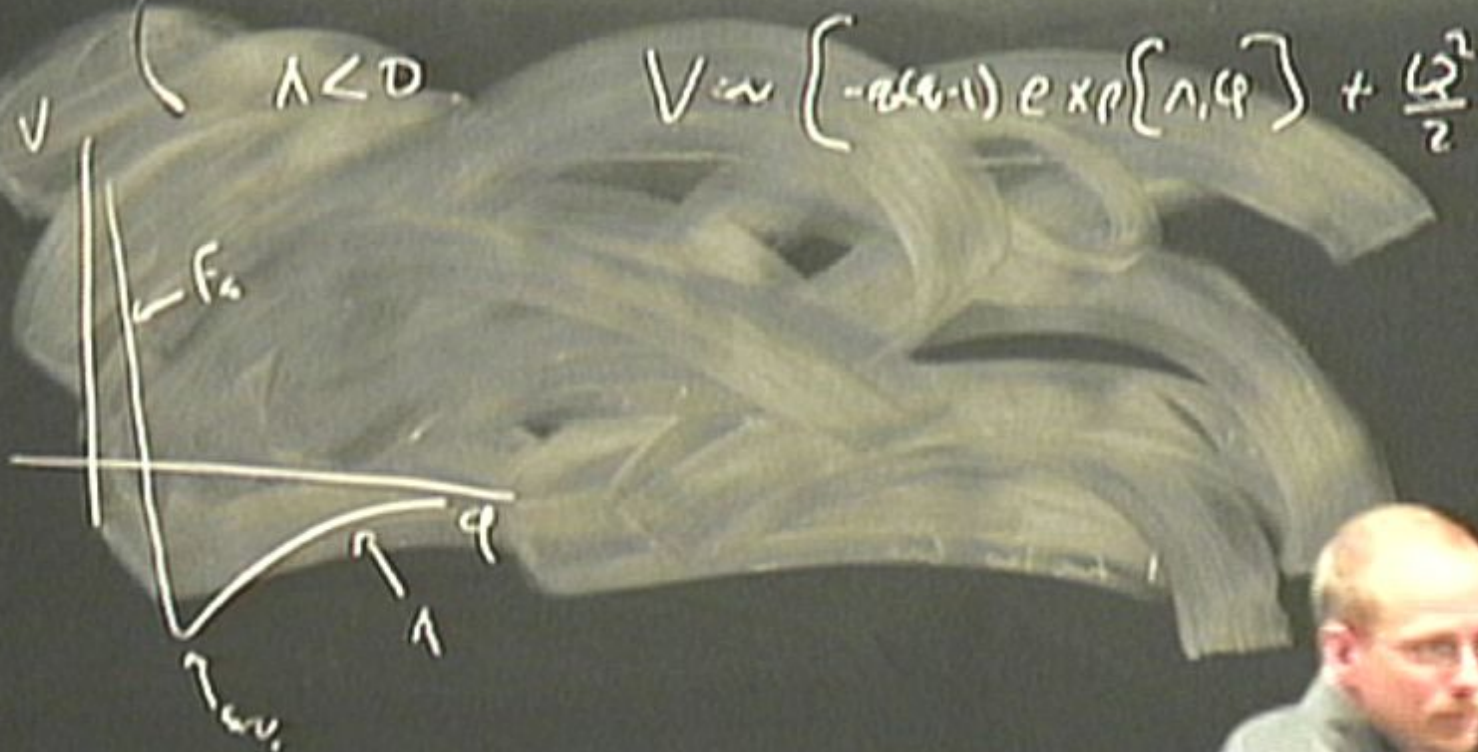
$$dS^2 = (M_{pl} R)^2 \sum_{\mu\nu} dy^\mu dy^\nu + e^2 d\Sigma_3^2$$

3) Can. norm. radiation

$$M_{pl} R = \exp\left[\sqrt{\frac{2}{3(2+\epsilon)}} \frac{\phi}{M_{pl}}\right]$$

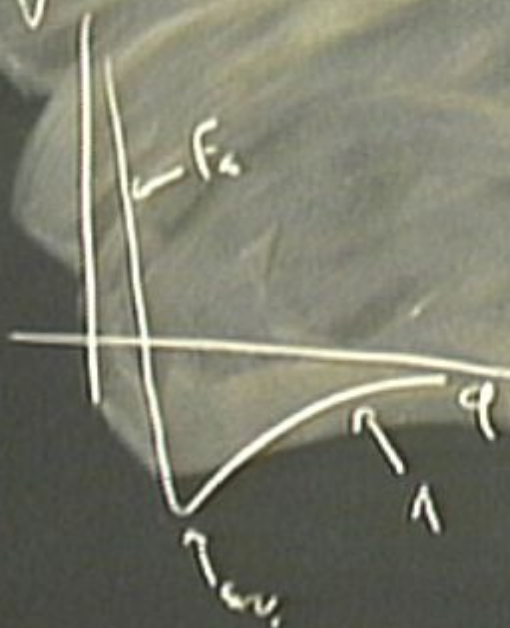
$$4) \quad S = \int d^4y \sqrt{-g} \left[\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right]$$



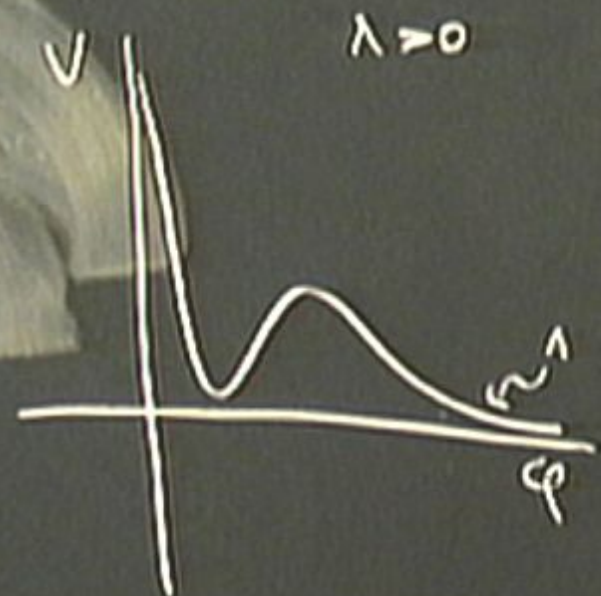
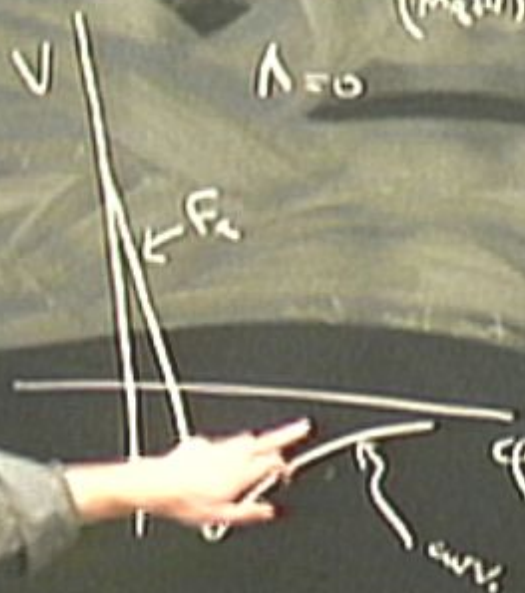
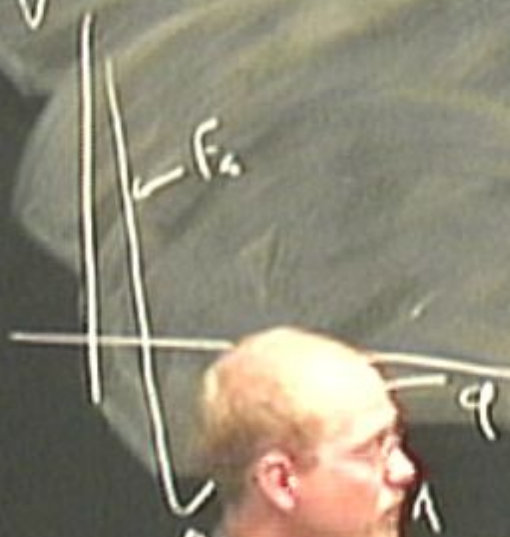


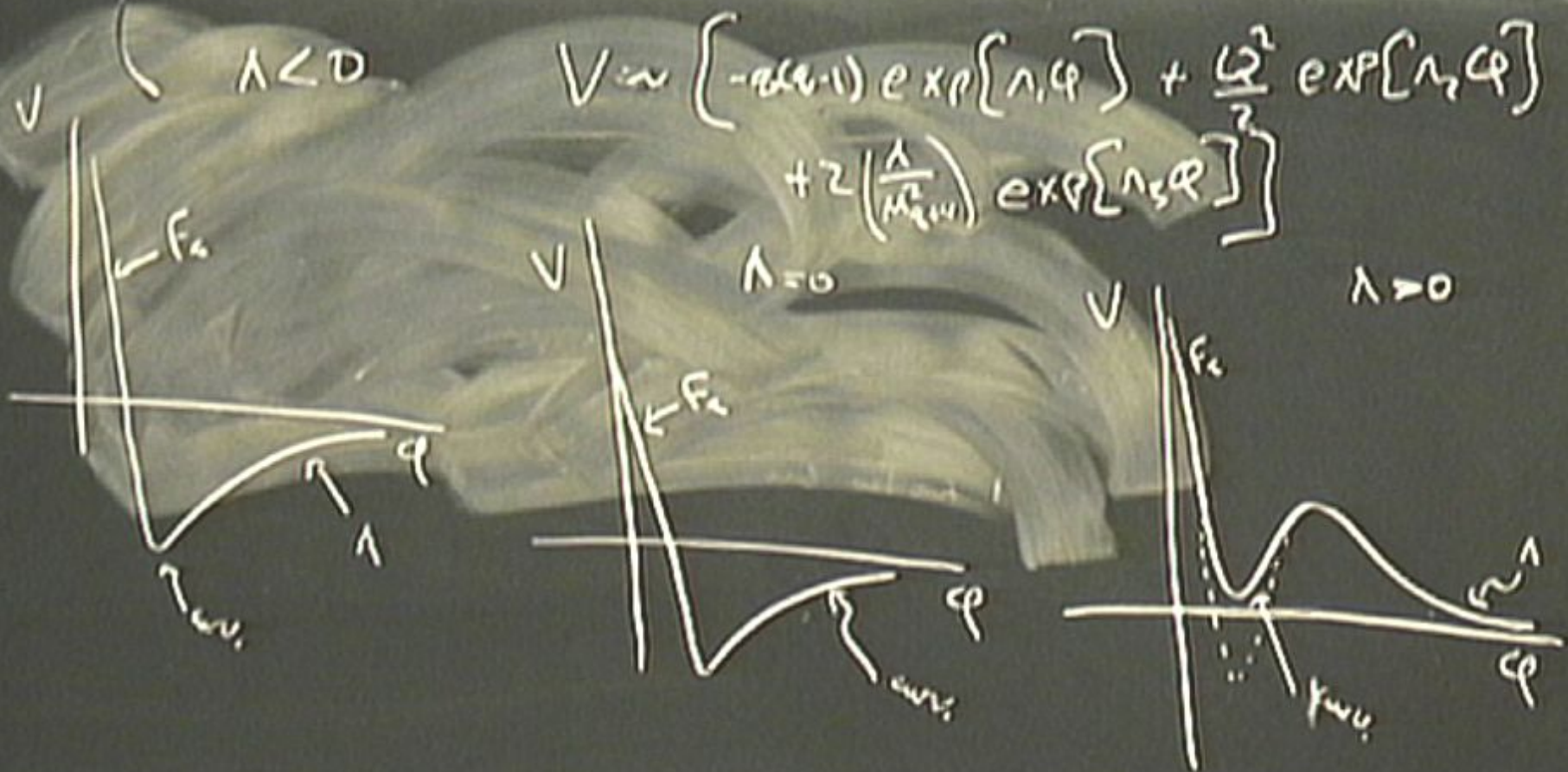
$$V \sim \left\{ -\alpha(\alpha-1) \exp[\Lambda_1 \varphi] + \frac{\omega^2}{2} \exp[\Lambda_2 \varphi] + 2 \left(\frac{\Lambda}{M_{\text{pl}}^2} \right) \exp[\Lambda_3 \varphi] \right\}$$

$\Lambda = 0$



$$V \sim \left\{ -\alpha(\alpha+1) \exp[\lambda_1 \varphi] + \frac{\omega^2}{2} \exp[\lambda_2 \varphi] + 2 \left(\frac{\lambda}{M_{\text{pl}}^2} \right) \exp[\lambda_3 \varphi] \right\}$$





Open FRW 4-d metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dx^2 + \sinh^2 \chi d\Omega^2]$$

Open FRW 4-d metric

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$$R^{(4)} = -\frac{\dot{\phi}^2}{M_{\text{pl}}^2} + \frac{4V(\phi)}{M_{\text{pl}}^2}$$

Open FRW 4-d metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dx^2 + \sinh^2 x d\Omega_2^2]$$

$$R^{(4)} = -\frac{\dot{\phi}^2}{M_{\text{pl}}^2} + \frac{4V(\phi)}{M_{\text{pl}}^2}$$

want $R^{(4)} \neq \infty$
as $a \rightarrow 0$

Open FRW 4-d metric

$$g_{\mu\nu} \partial x^\mu \partial x^\nu = -dt^2 + a^2(t) \left[d\chi^2 + \sinh^2 \chi d\Omega_2^2 \right]$$

$$\ddot{a} = -\frac{\dot{\phi}^2}{M_{\text{pl}}^2} + \frac{4V(\phi)}{M_{\text{pl}}^2}$$

Want $R^{(4)} \neq \infty$
as $a \rightarrow 0$

$\dot{\phi} \rightarrow 0$ as $a \rightarrow 0$

Open FRW 4-d metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dx^2 + \sinh^2 x d\Omega_2^2]$$

$$R^{(3)} = -\frac{\dot{\phi}^2}{M_{\text{pl}}^2} + \frac{4V(\phi)}{M_{\text{pl}}^2}$$

Want $R^{(3)} \neq \infty$
as $a \rightarrow 0$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -V' \Rightarrow \dot{\phi} \rightarrow 0 \text{ as } a \rightarrow 0$$

Open FRW 4-d metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dx^2 + \sinh^2 x d\Omega_2^2]$$

$$R^{(4)} = -\frac{\dot{\phi}^2}{M_{\text{pl}}^2} + \frac{4V(\phi)}{M_{\text{pl}}^2} \quad \text{want } R^{(4)} \neq \infty \text{ as } a \rightarrow 0$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -V' \Rightarrow \dot{\phi} \rightarrow 0 \text{ as } a \rightarrow 0$$

$$\left(\frac{\dot{a}}{a}\right)' = \epsilon + \frac{1}{2\eta} \Rightarrow a \rightarrow 0, \eta = \tau$$

Open FRW 4-d metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dr^2 + \sin^2 r d\Omega^2]$$

$$R^{(4)} = -\frac{6\dot{a}^2}{a^2} + \frac{4V(a)}{M_{\text{Pl}}^2}$$

$$\text{want } R^{(4)} \neq \infty \text{ as } a \rightarrow 0$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -V' \Rightarrow \dot{\phi} \rightarrow 0 \text{ as } a \rightarrow 0$$

$$\left(\frac{\dot{a}}{a}\right)' = \epsilon + \frac{1}{a^2} \Rightarrow a \rightarrow 0, \mu = r$$

Open FRW 4-d metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dx^2 + \sinh^2 x d\Omega_2^2]$$

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$$\left(\frac{\dot{a}}{a}\right)' = \epsilon + \frac{1}{2\eta} \Rightarrow a \rightarrow 0, \quad \eta = \tau$$

$$\Lambda = 0, \Lambda < 0$$

V

ϕ

$$\phi = 0, a = 0$$

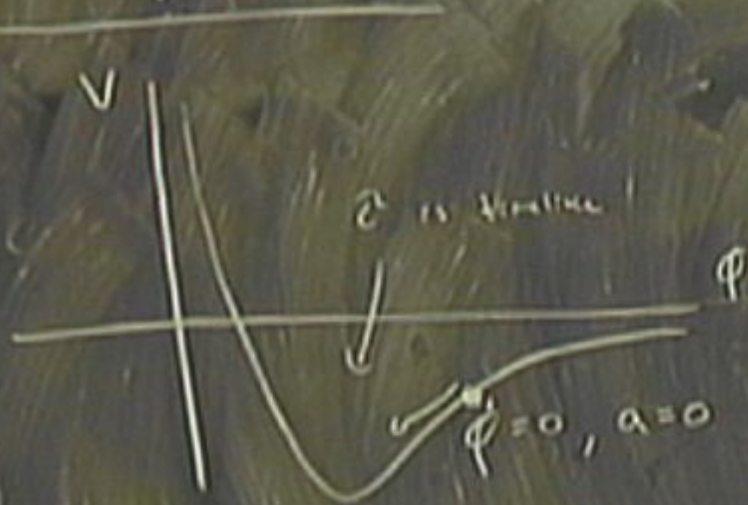
$$\Lambda = 0, \Lambda < 0$$



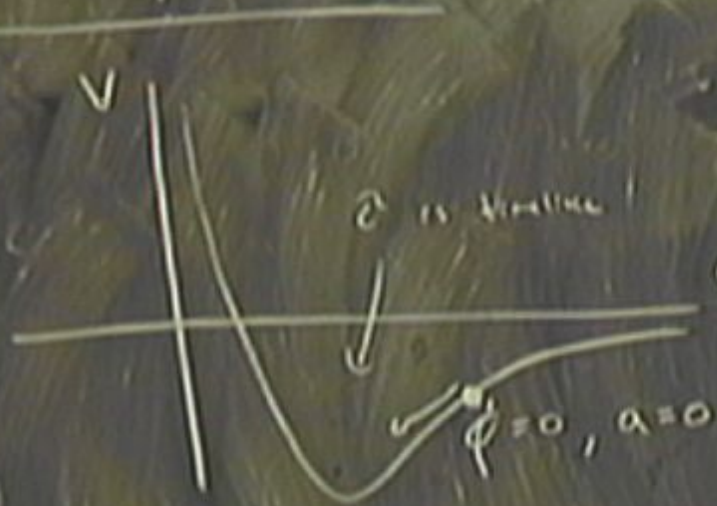
$$\Lambda = 0, \Lambda < 0$$



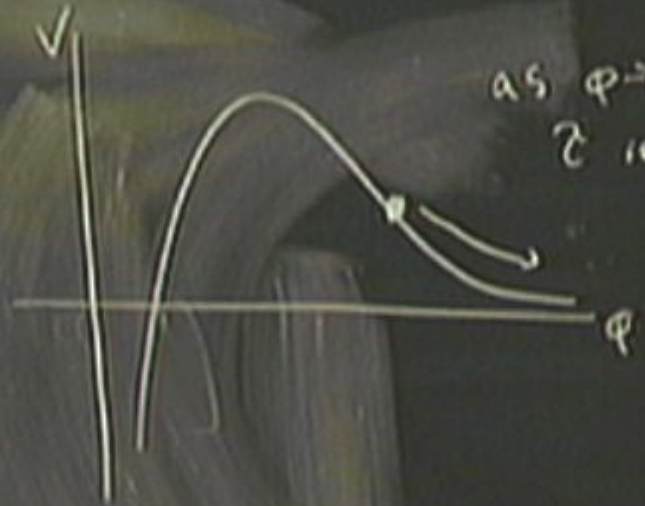
$$\Lambda = 0, \Lambda < 0$$



$$\Lambda = 0, \Lambda < 0$$



$$\frac{d\phi}{dt} = -(\dot{\phi}^2 + 4V)$$



As $\phi \rightarrow \infty$,
 τ is spacelike

$$\Lambda = 0, \Lambda < 0$$

V

τ is timelike

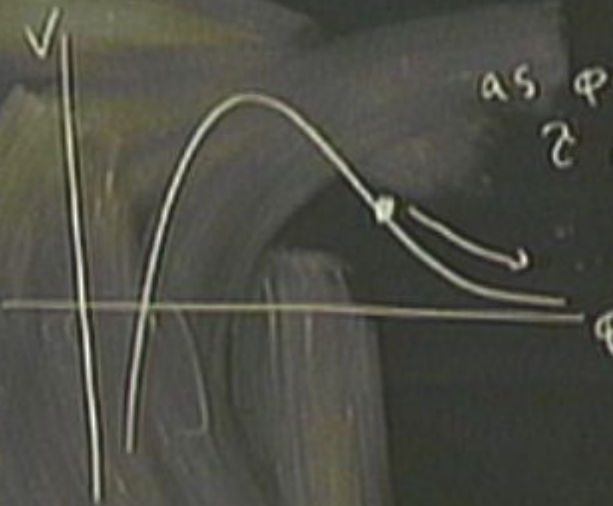
ϕ

$\phi = 0, a = 0$

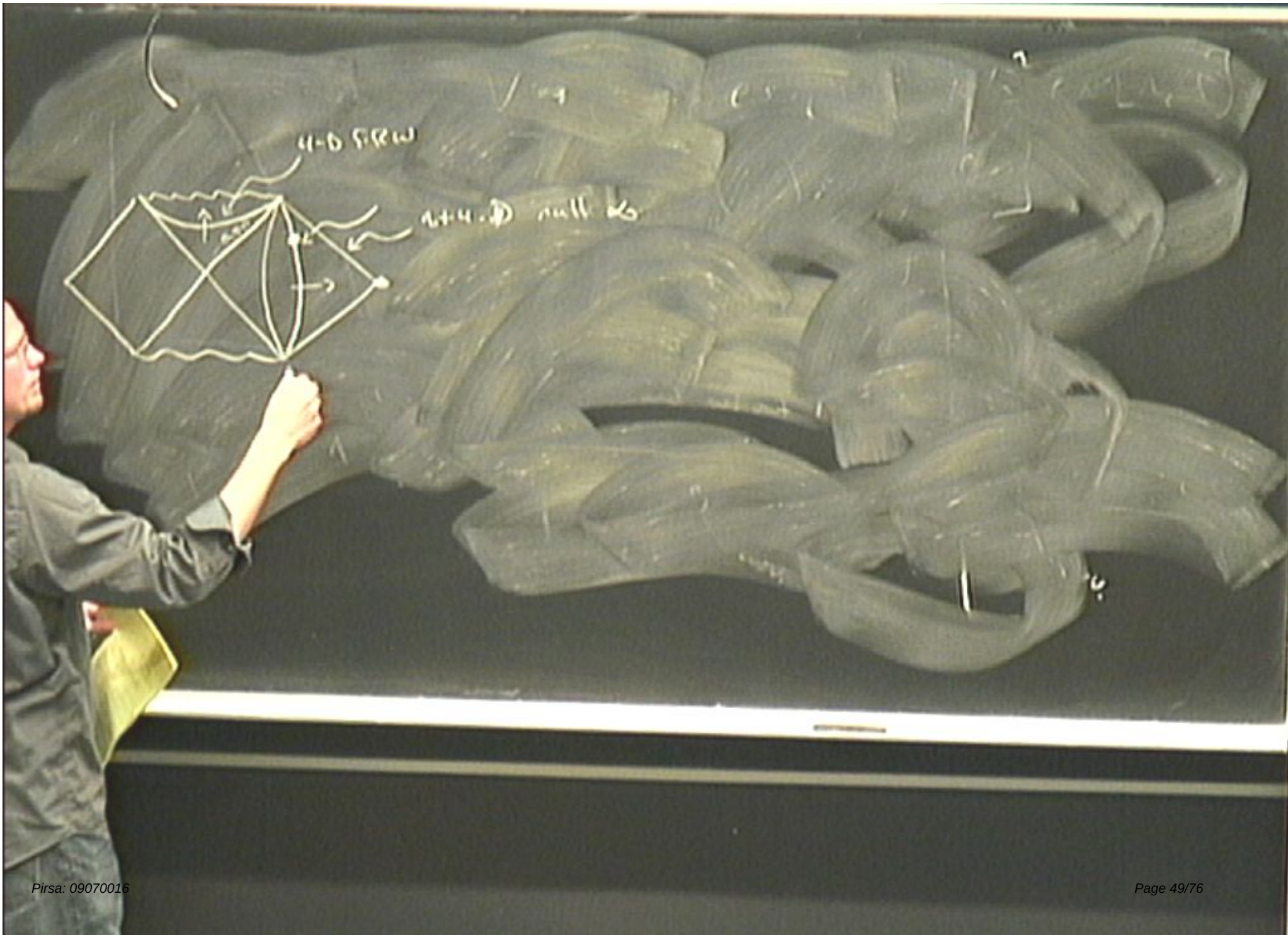
$$\frac{d\phi}{d\tau} = -(\dot{\phi}^2 + V)$$

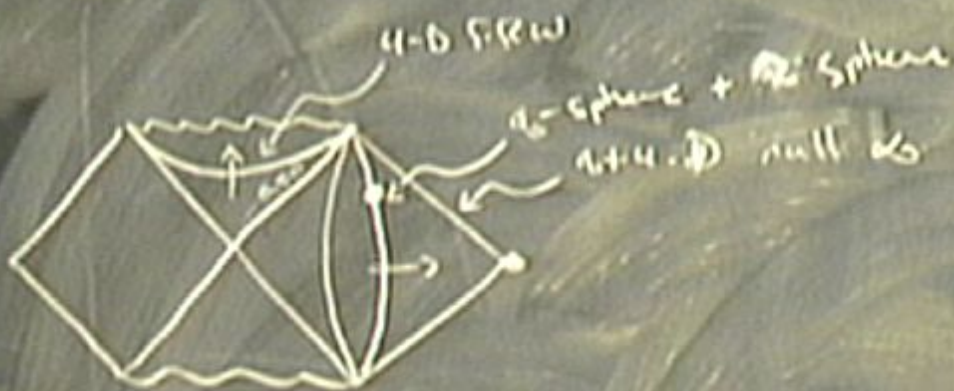
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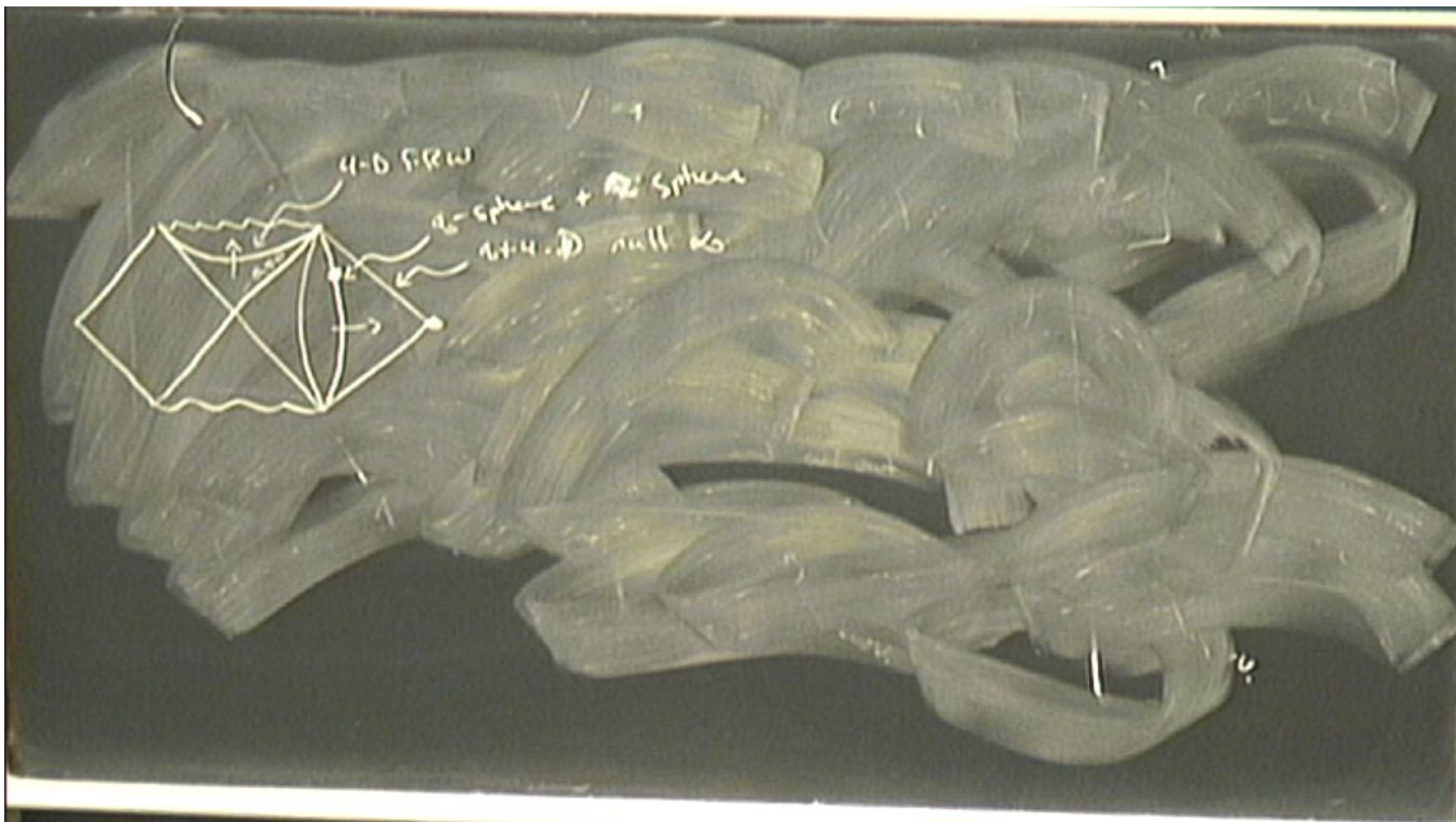
ϕ











$$\Lambda = 0$$

4-D flow

d_0 -sphere + d_1 -sphere
 $n+4$ null d_0



$$\Lambda < 0$$



$$\Lambda = 0$$

4-D F.R.W

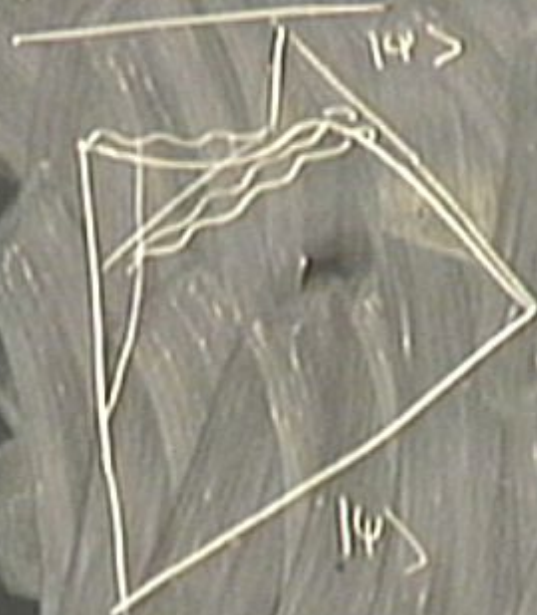
d_0 -sphere + d_2 -sphere
 $d_2 + 4-D$ null d_0



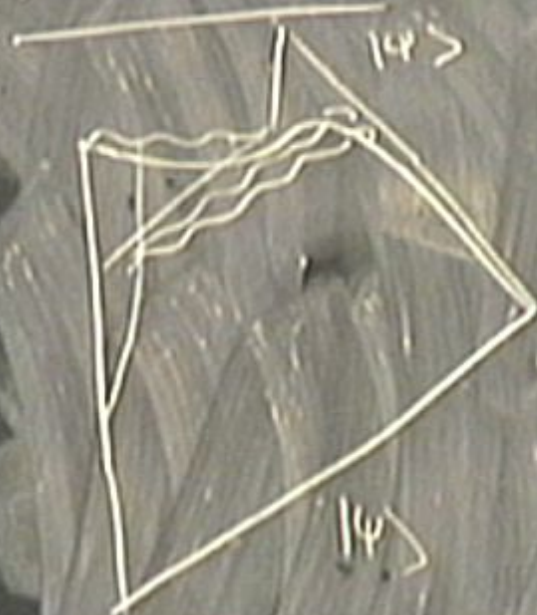
$$\Lambda < 0$$



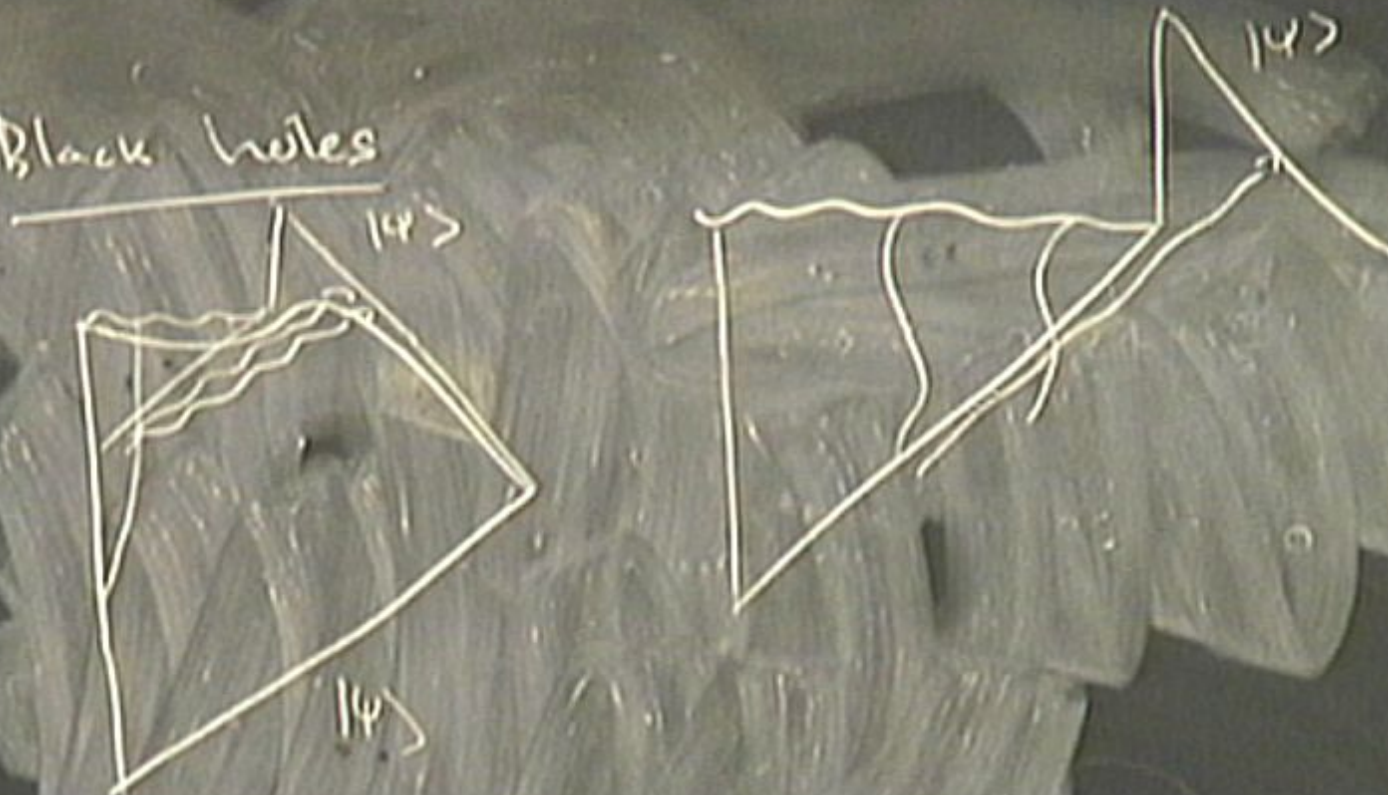
Black holes



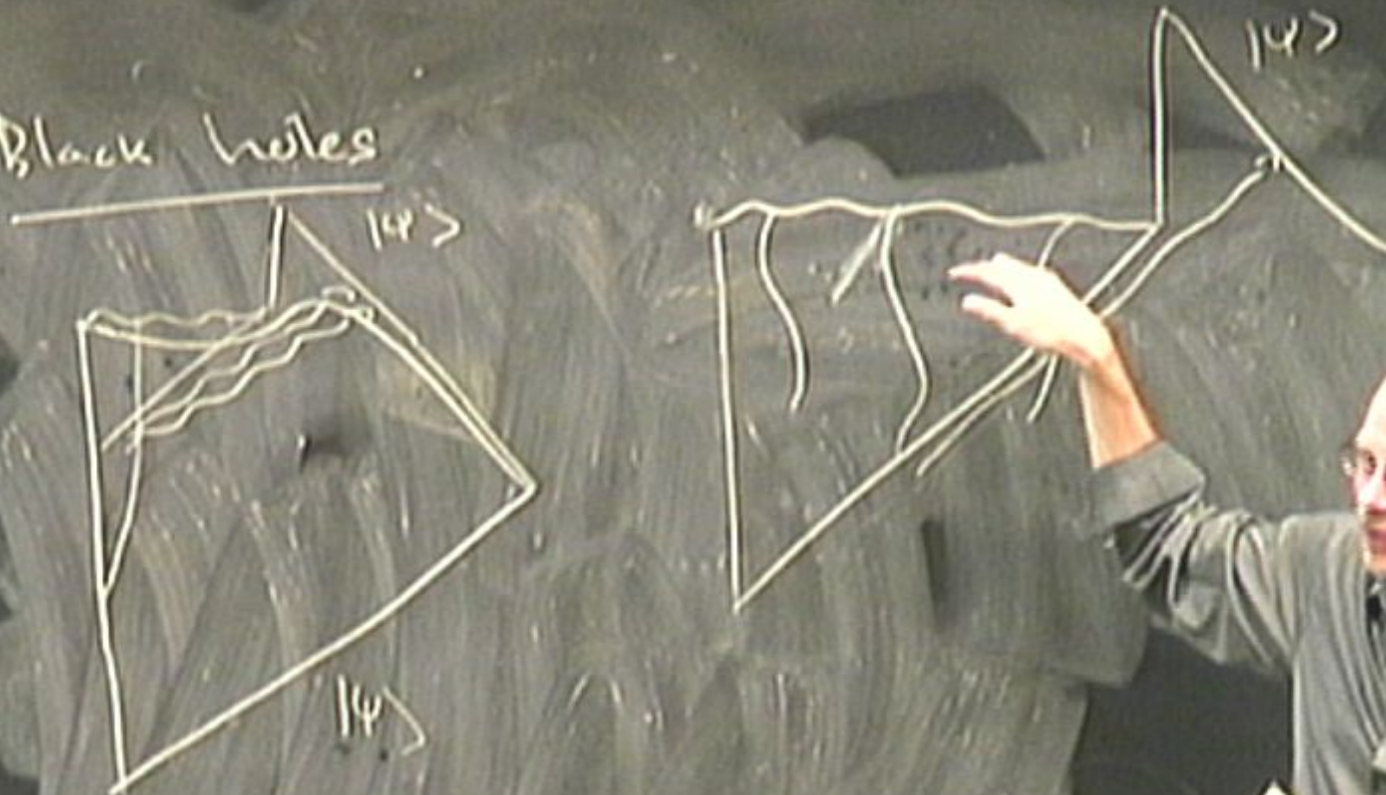
Black holes



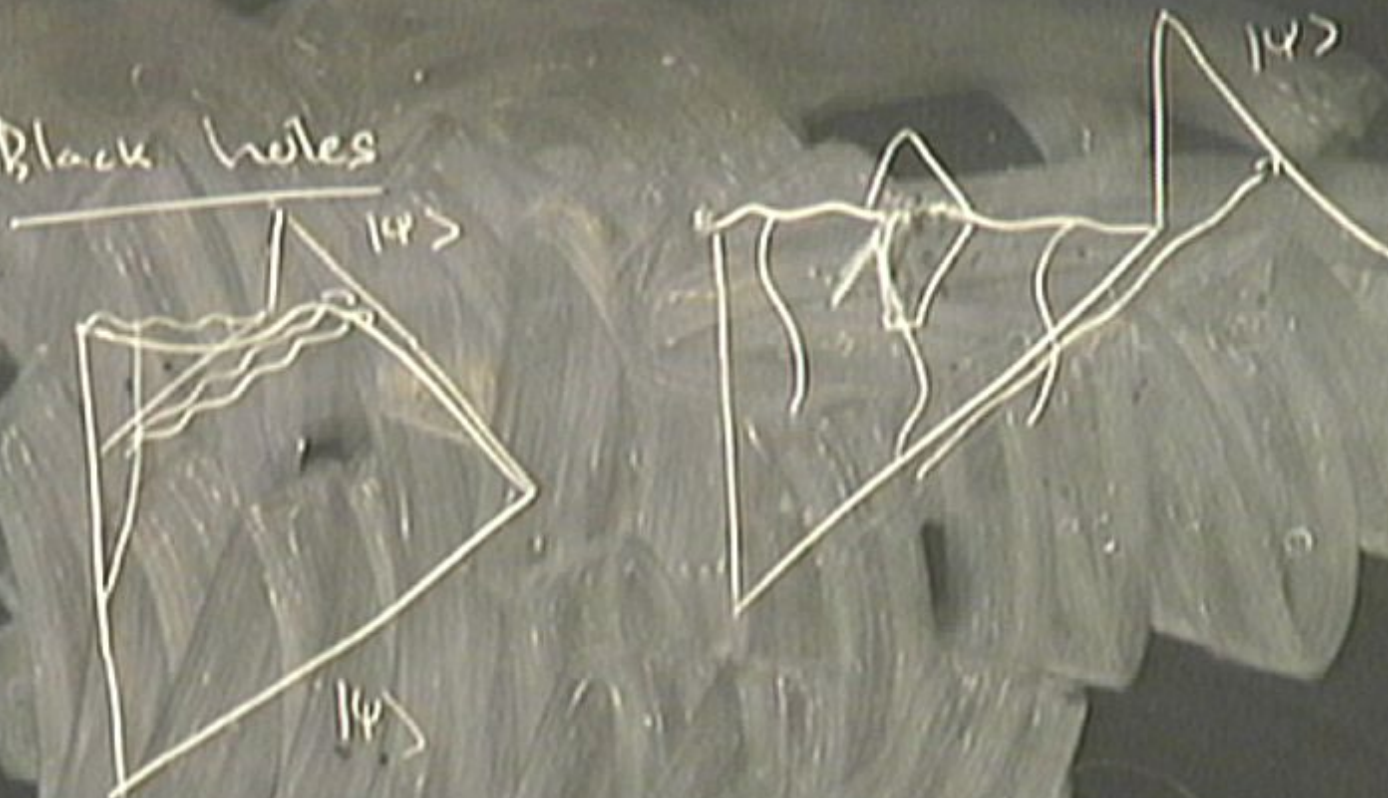
Black holes



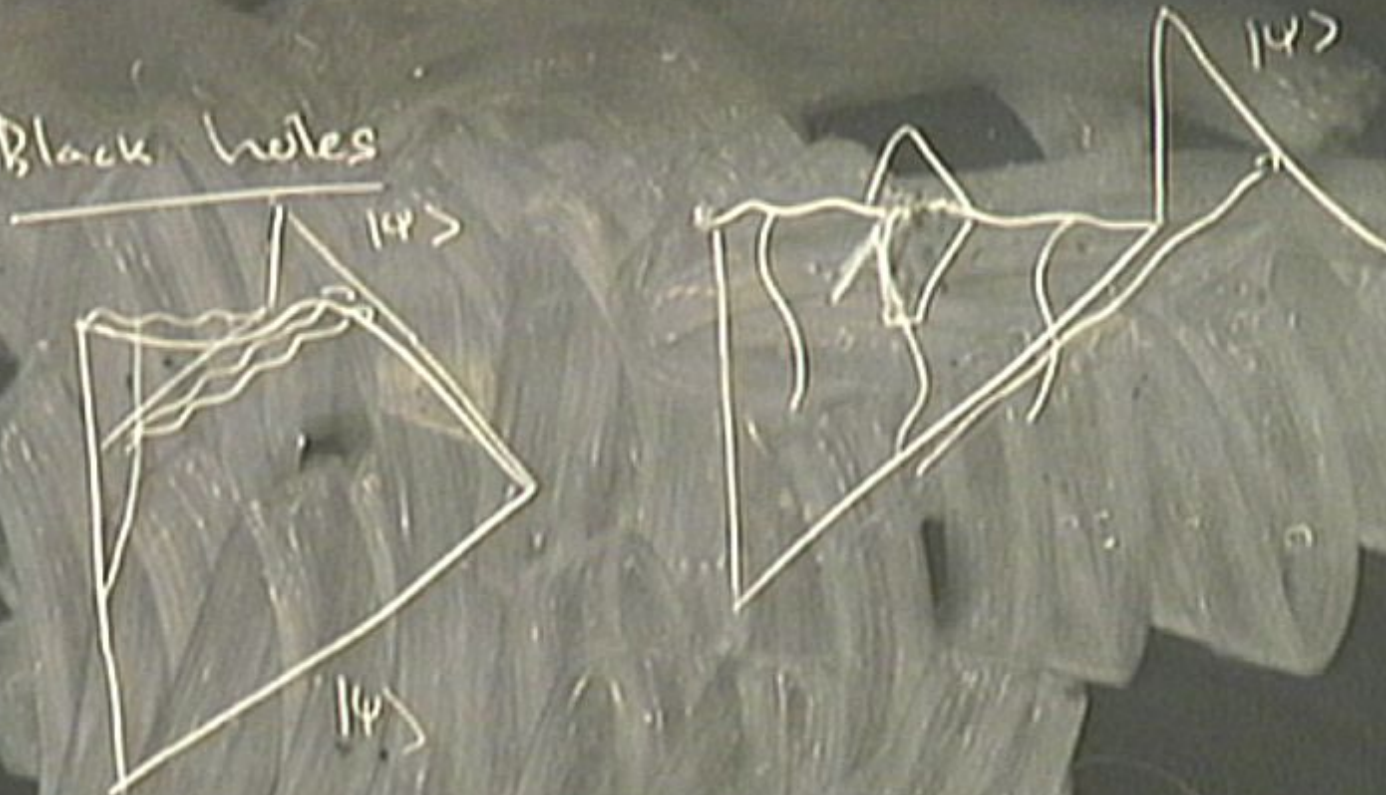
Black holes



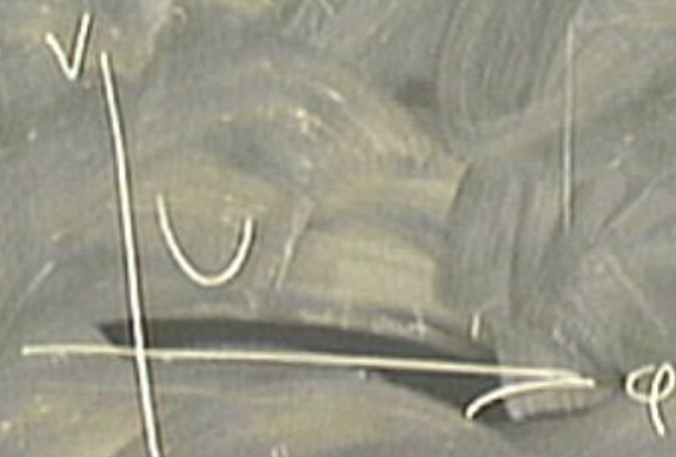
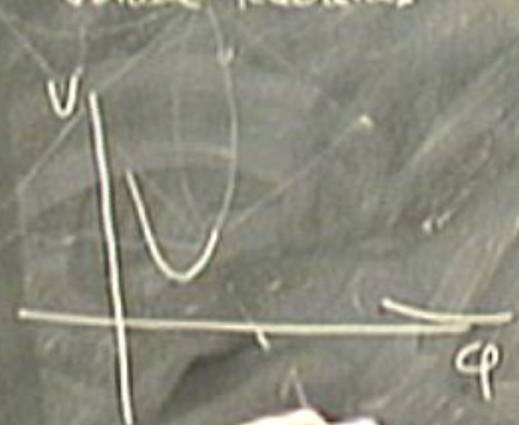
Black holes



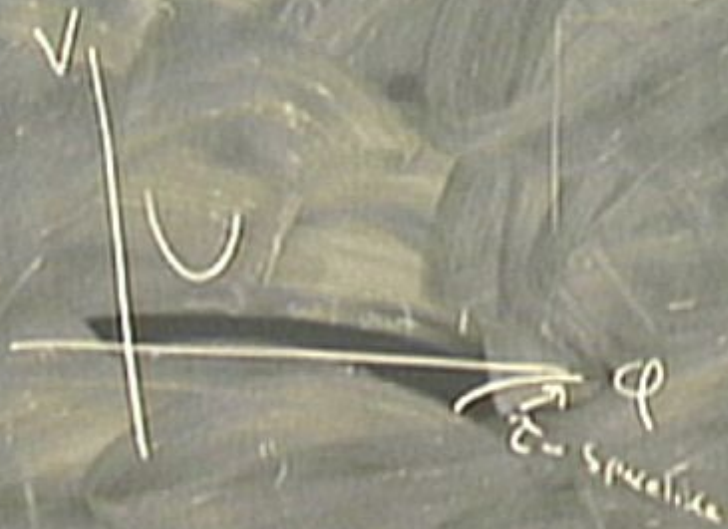
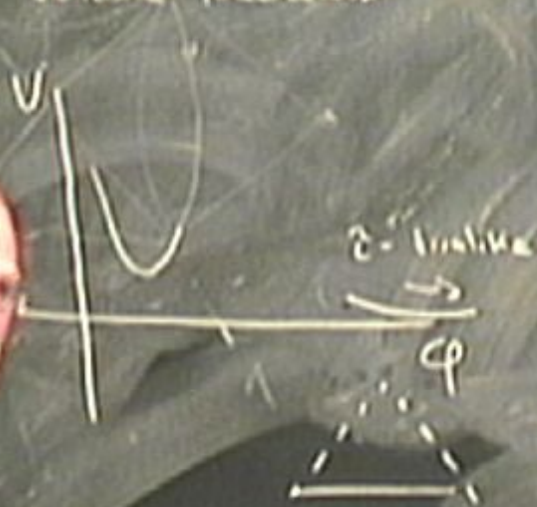
Black holes



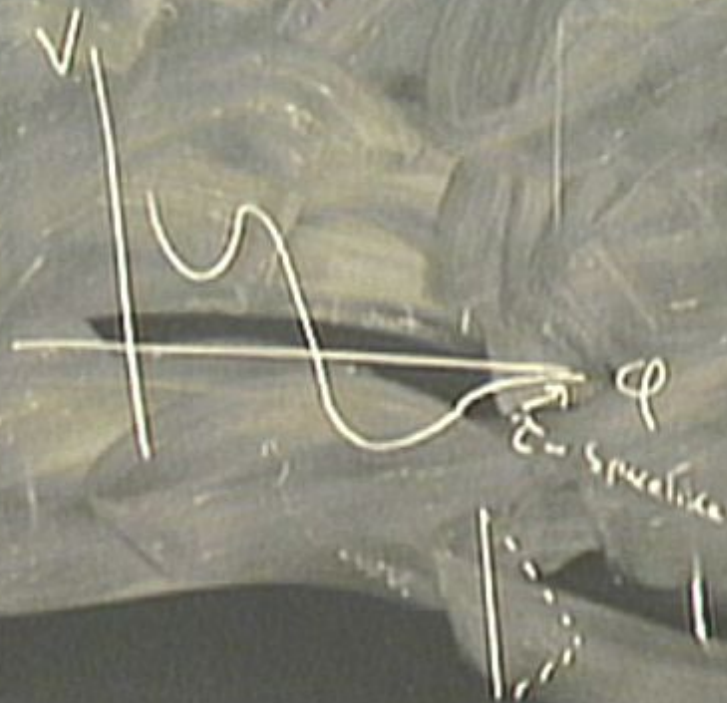
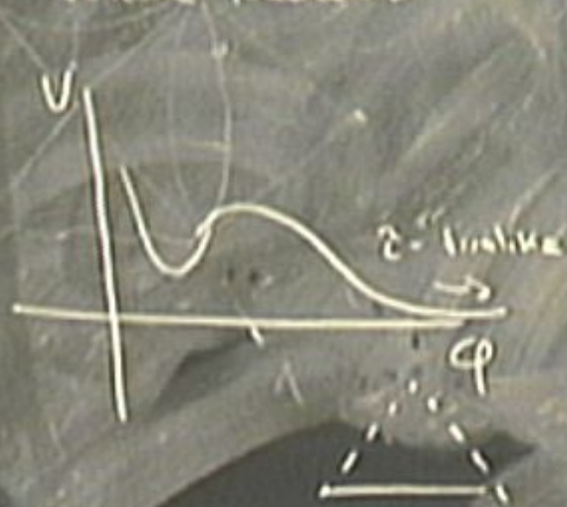
- 4-D flow slices
- Volume modulus

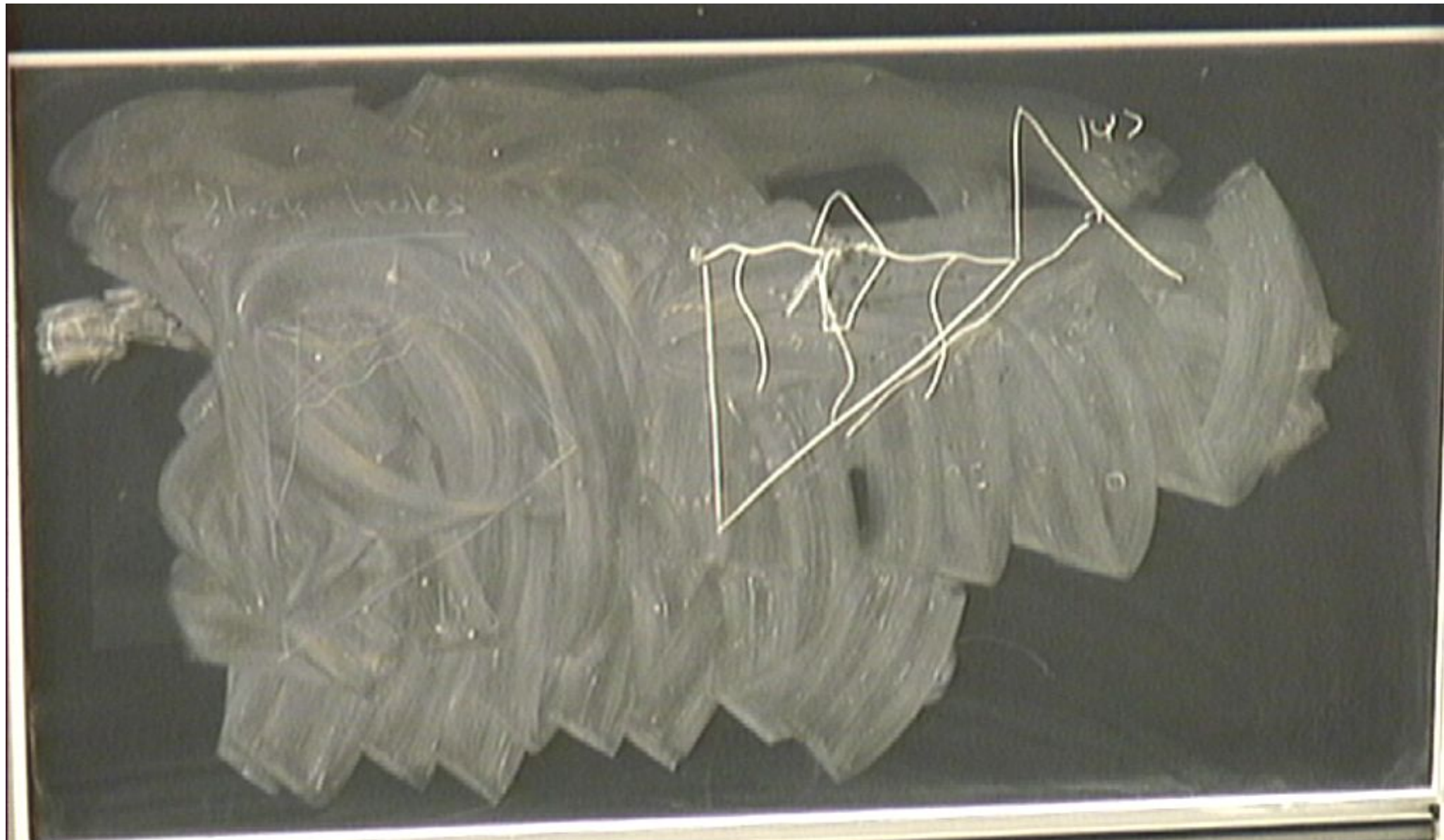


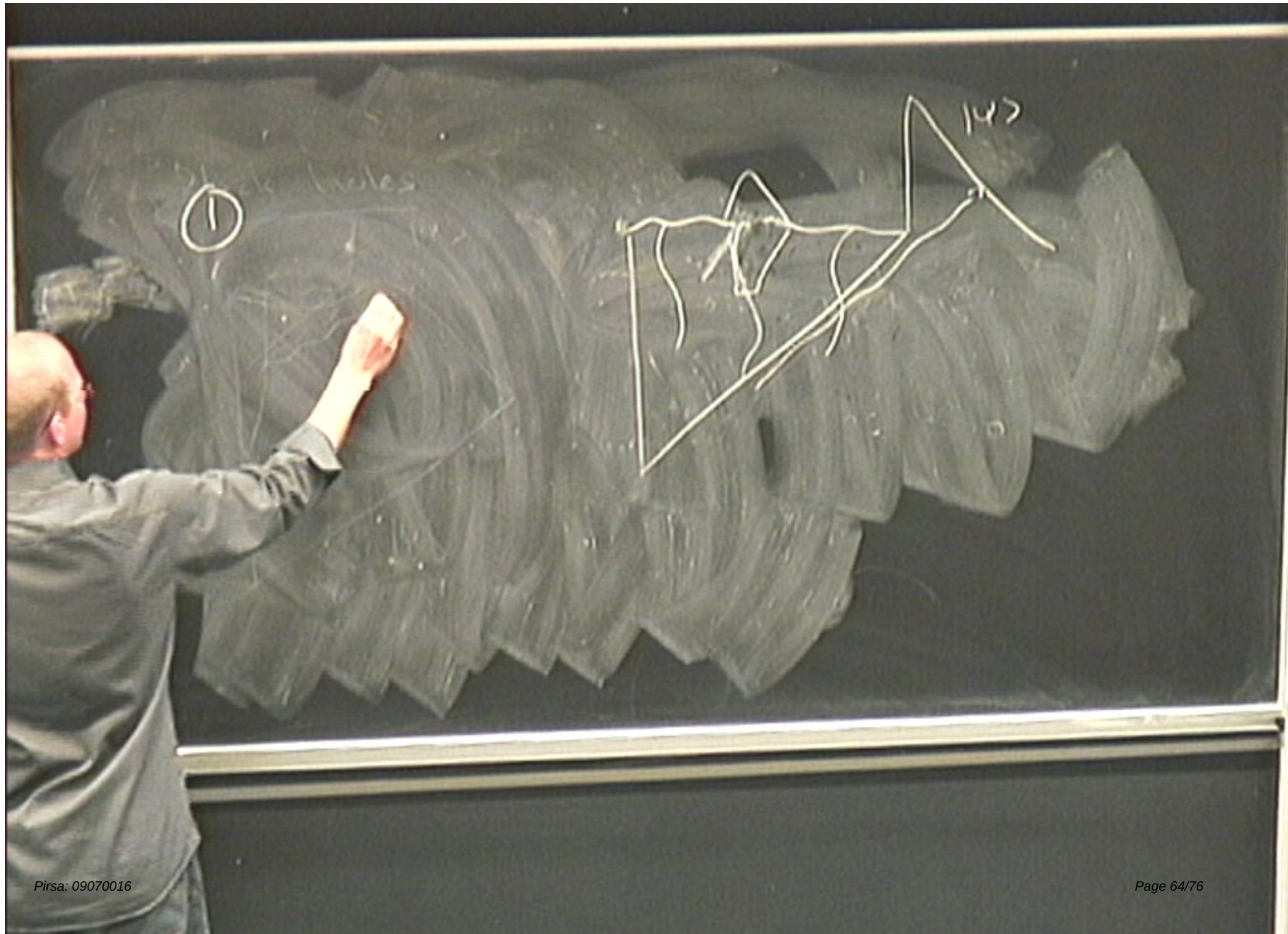
- $\Lambda = 0$
- 4-D flow slices
- Volume modulus

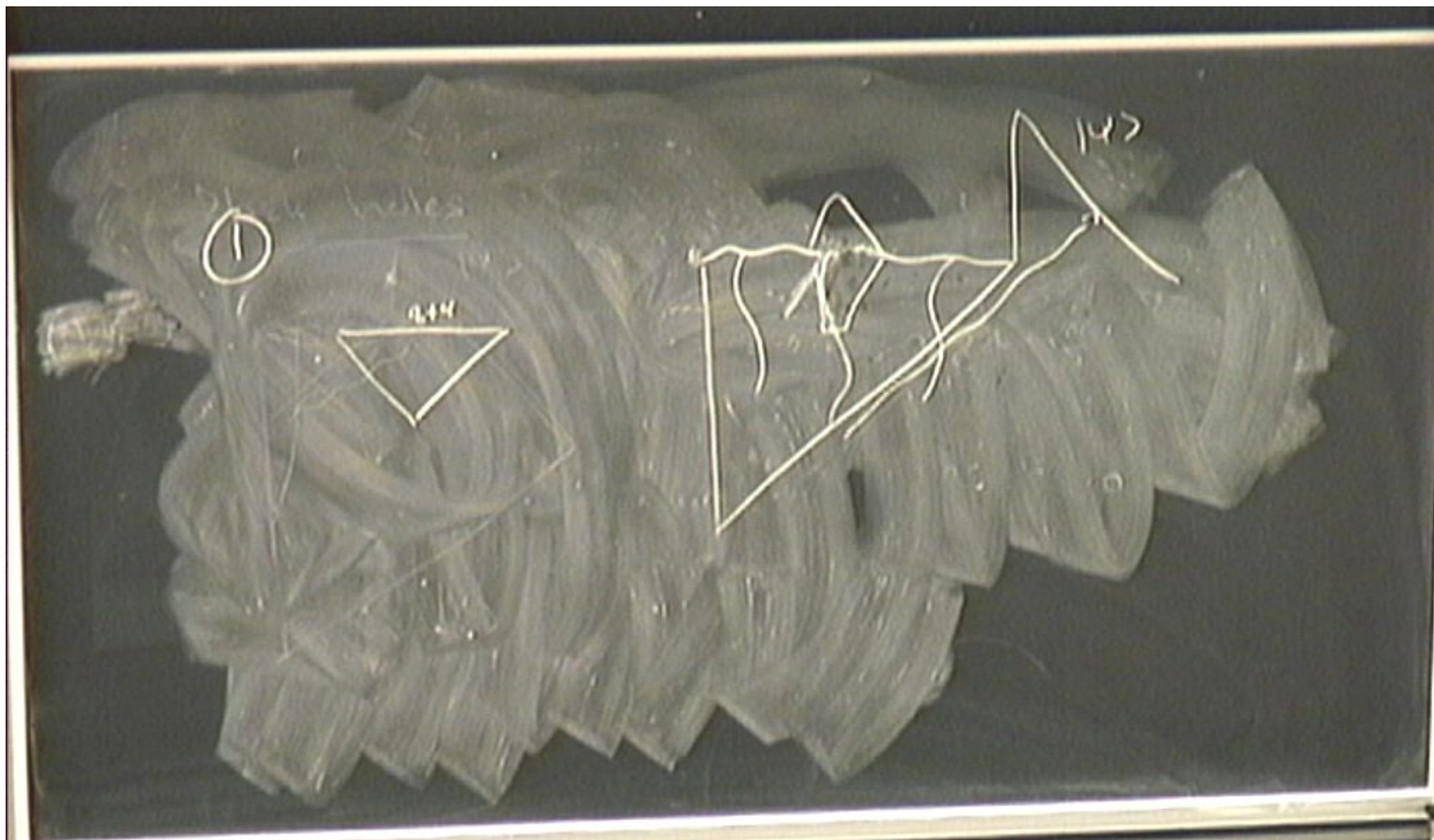


- $\lambda = 0$
- 4-D flow slices
 - Volume modulus









①

0.44

⑥

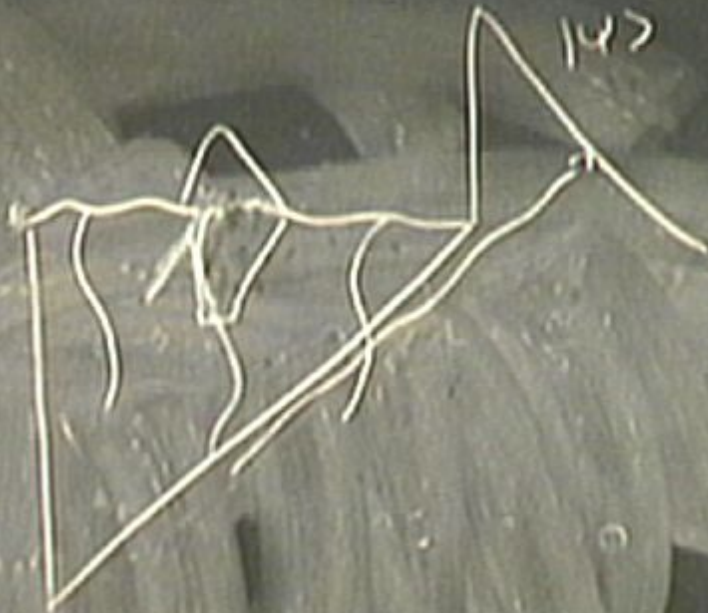
⑤

142

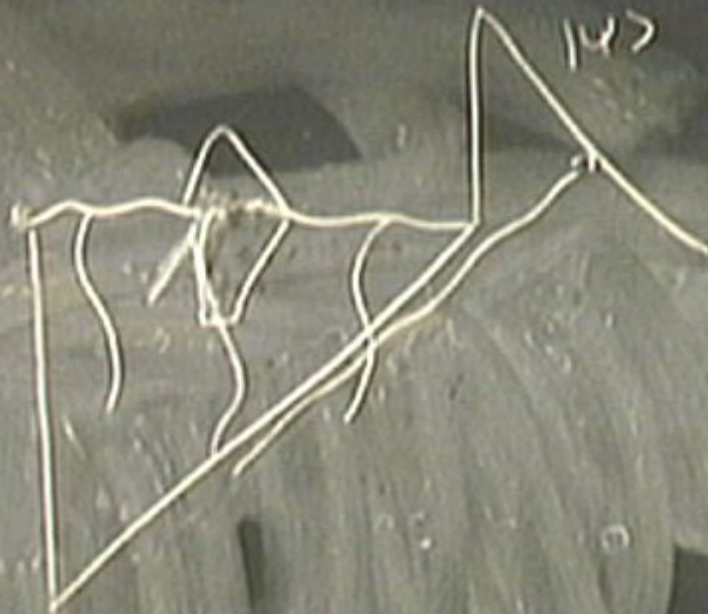
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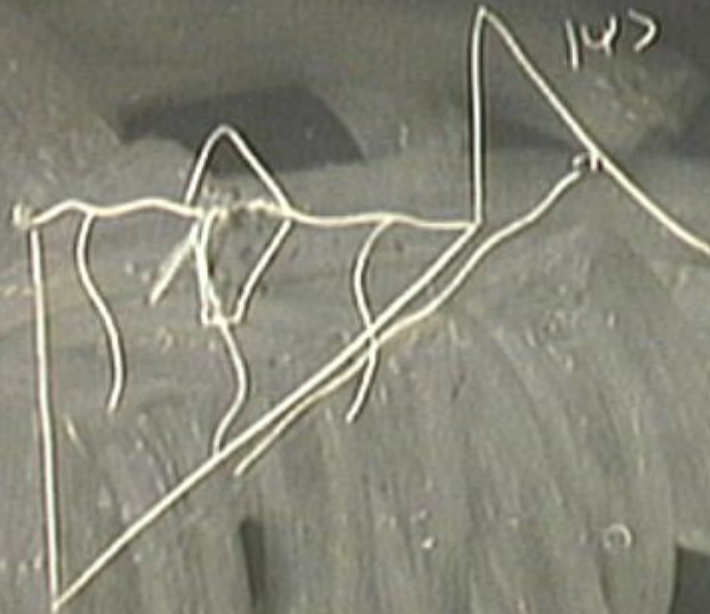
244



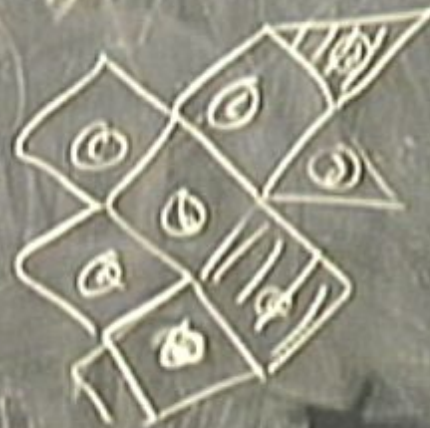
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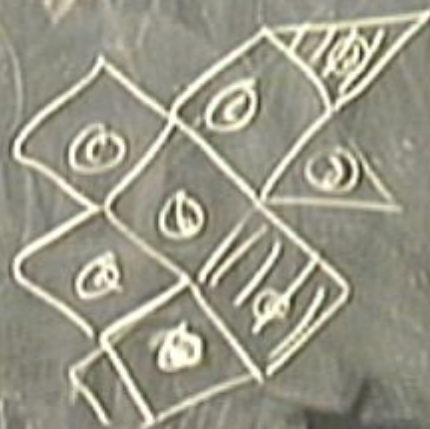
①



①



①



①



$$\frac{\Gamma_{u \rightarrow v}}{\Gamma_{u \rightarrow v, u \rightarrow u}} = \exp\left[|S_{u,v}| - |S_{u,v}^{(u,v)}|\right]$$

①



$\Gamma_{u \rightarrow u}$

$\Gamma_{u \rightarrow u}$

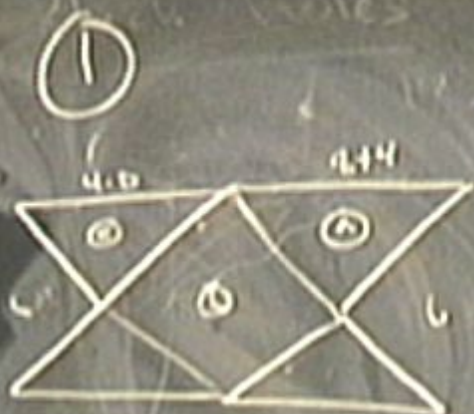
$$\exp[|S_{u'}| - |S_{u''}|]$$

①



$$\frac{\int_{q \rightarrow q} \mathcal{L}(q) \mathcal{L}(q)}{\int_{q \rightarrow q} \mathcal{L}(q)} = \exp\left[|S_{\text{eff}}^{(n)}| - |S_{\text{eff}}^{(n+1)}|\right]$$

$$|S_{\text{eff}}^{(n)}| \gg |S_{\text{eff}}^{(n+1)}|$$



$$\frac{\sqrt{q+1 \rightarrow q}}{\sqrt{q \rightarrow q+1}} = \exp\left[|S_{\text{eff}}^{(q)}| - |S_{\text{eff}}^{(q+1)}|\right]$$

$$|S_{\text{eff}}^{(q)}| \gg |S_{\text{eff}}^{(q+1)}|$$

①



$$\frac{\int_{q \rightarrow q'} \int_{q \rightarrow q''}}{\int_{q \rightarrow q'}} = \exp[|S_{q'}^{(u)}| - |S_{q''}^{(u)}|]$$

$$|S_{q'}^{(u)}| \geq |S_{q''}^{(u)}|$$