Title: Holographic Multiverse

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Abstract: TBA

HOLOGRAPHIC MULTIVERSE

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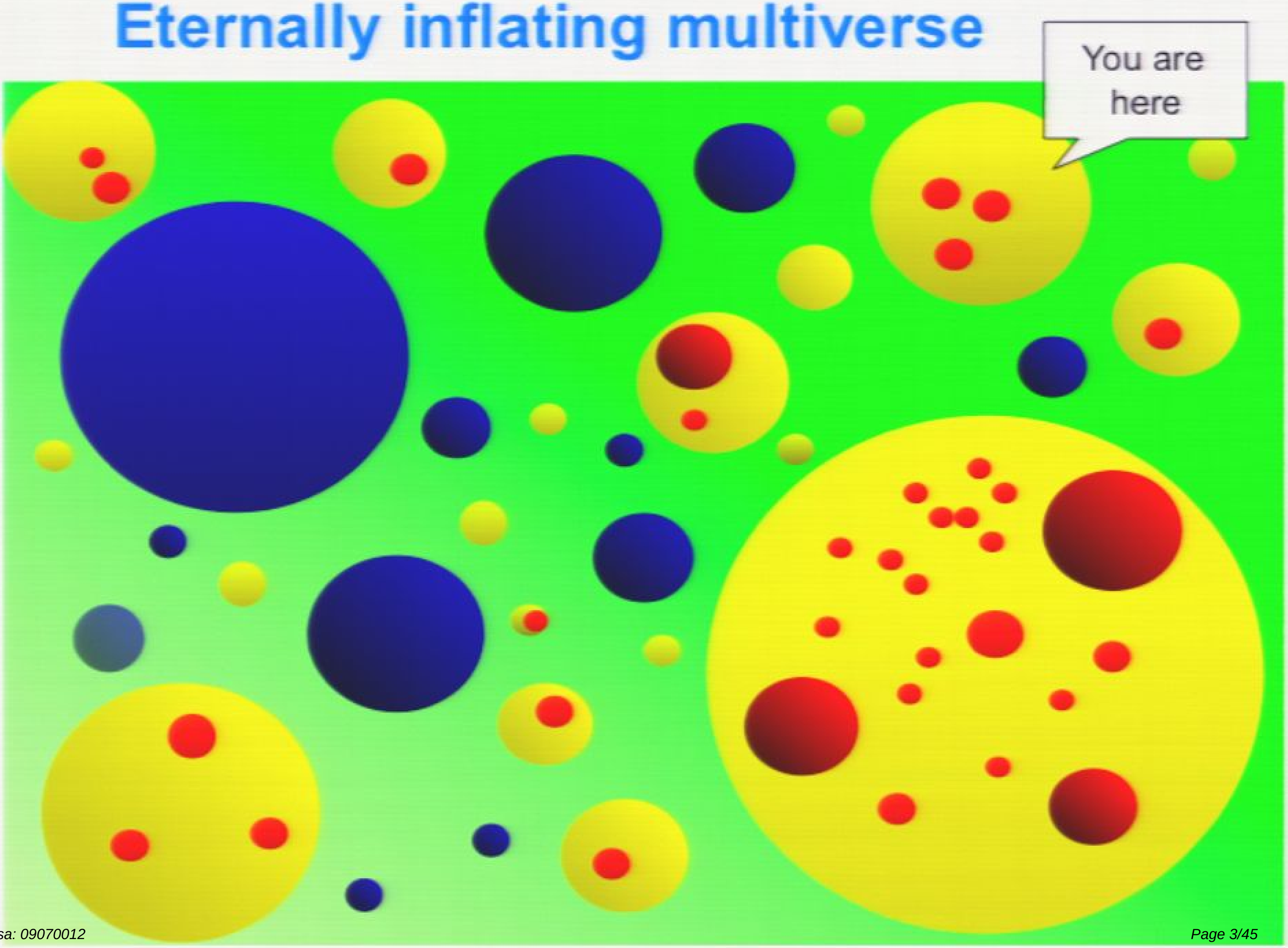
Perimeter Institute, July 2009

Eternally inflating multiverse



You are here

Spacetime structure

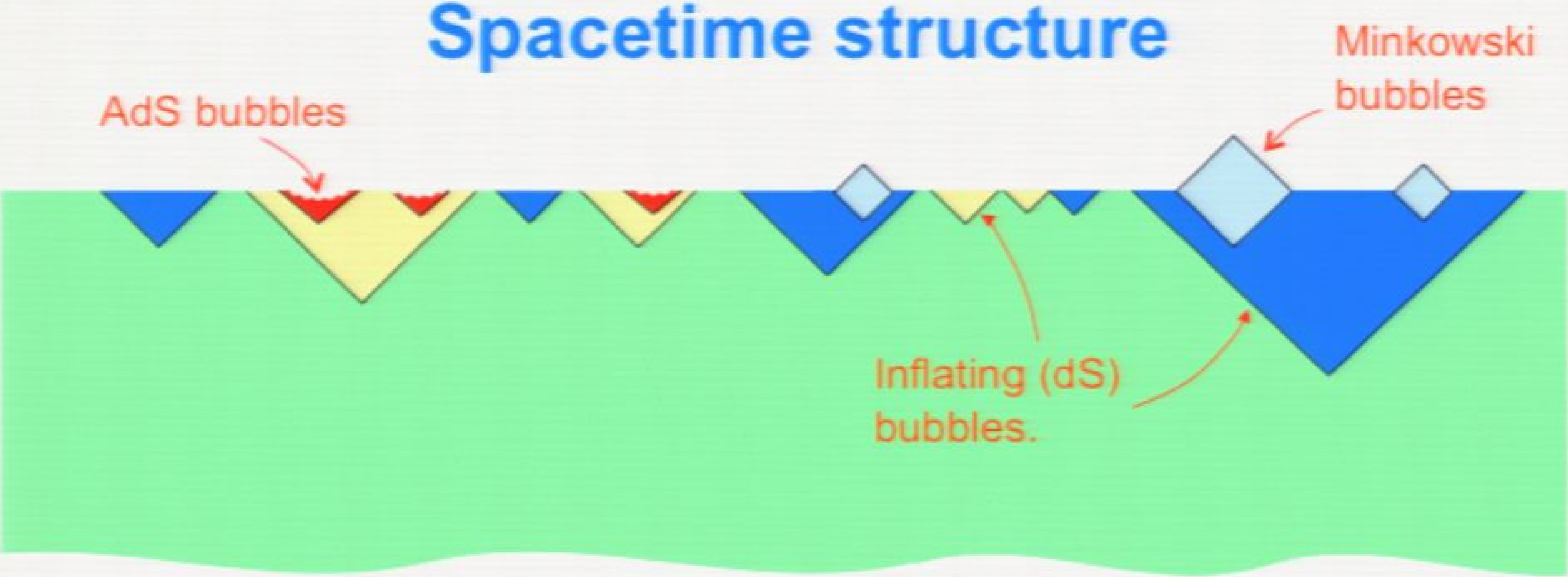
AdS bubbles

Minkowski bubbles

Inflating (dS) bubbles.

- Bubbles nucleate and expand at nearly the speed of light.
- dS, AdS, and Minkowski bubbles

Spacetime structure



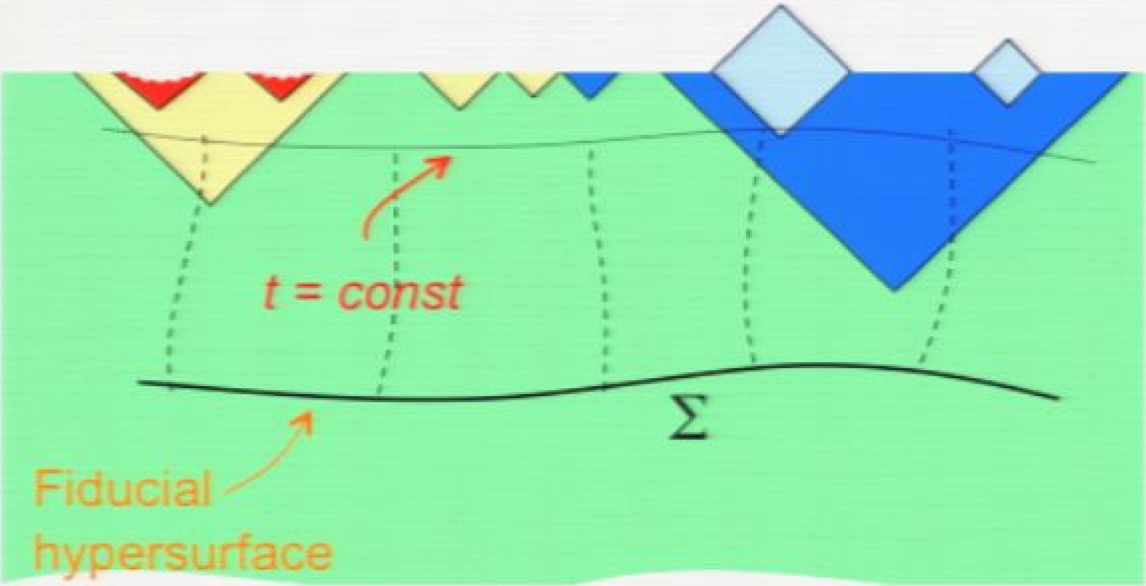
Everything that can happen will happen an infinite number of times. We have to learn how to compare these infinities. (Otherwise we cannot distinguish probable events from highly improbable & cannot make any predictions.)

Need a cutoff. Results are strongly cutoff-dependent.

(The measure problem)

Global time cutoff:

Count only observations that were made before some time *t*.



Garcia-Bellido, Linde
& Linde (1994); A.V. (1995)

Possible choices of t :

(i) proper time $t = \tau$ along geodesics orthogonal to Σ ;

(ii) scale-factor time, $t = \ln a$.

$t \to \infty$ ➡ steady-state evolution.

The distribution does not depend on the choice of Σ
-- but depends on what we use as *t*.

Measure proposals:

● Proper time cutoff	*Garcia-Bellido, Linde & Linde (1994)* *A.V. (1995)*
● Scale factor cutoff	*Garcia-Bellido, Linde & Linde (1994)* *De Simone, Guth, Salem & A.V. (2008)*
● Stationary	*Linde (2007)*
● Pocket based	*Garriga, Schwartz-Perlov, A.V. & Winitzki (2005)* *Easther, Lim & Martin (2005)*
● Causal patch	*Bousso (2006)* *Freivogel, Sekino, Susskind & Yeh (2006)*

Empirical approach:

Investigate different measure proposals and discard those which suffer from internal inconsistencies or strongly disagree with observations.

	Youngness paradox	Q catastrophe	Dependence on initial state	Freak observers
Proper time cutoff	☹	☹		
★ **Scale factor cutoff**				
Pocket-based measure		☹		☹
Stationary measure		☹		
Causal patch measure			☹	

This talk:

A measure from fundamental theory

Based on work with Jaume Garriga

- The dynamics of the multiverse may be encoded in its future boundary (suitably defined).

  Inspired by holographic ideas: *Quantum dynamics of a spacetime region is describable by a boundary theory.*

- The measure can be obtained by imposing a UV cutoff in the boundary theory.
  Related to scale-factor cutoff.

Holographic ideas

AdS/CFT	*Maldacena (1998)*
DS/CFT	*Strominger (2001)*
CdL/CFT	*Freivogel, Sekino, Susskind & Yeh (2006)*

$\mathrm{AdS}_{D+1}/\mathrm{CFT}_D$ correspondence

Maldacena (1998)
Susskind & Witten (1998)

String theory in asymptotically AdS space
is equivalent (dual) to a CFT on the boundary.

Euclidean AdS:

$$ds^2 = dr^2 + \sinh^2 r \, d\Omega_D^2$$

Regulate boundary theory:

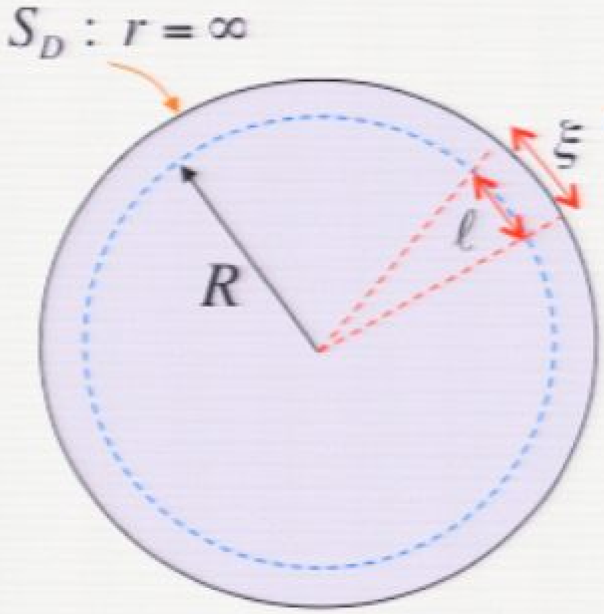
Integrate out short-wavelength modes of wavelength up to ξ .

The corresponding 4D modes have minimum wavelength

$$\lambda_{\min}(r) = \xi \sinh r.$$

$$\Rightarrow \quad \sinh R < \ell / \xi$$

$$R \to \infty \quad \Leftrightarrow \quad \xi \to 0.$$



Boundary UV cutoff

R - bulk IR cutoff

Bulk resolution scale

Require $\lambda_{\min} < \ell$

Variation of R $\Leftrightarrow$ RG flow in the boundary theory.

(on scales > AdS curvature scale)

dS/CFT correspondence

Strominger (2001)

The 4D theory describing an asymptotically de Sitter space is equivalent to a 3D Euclidean CFT at the future infinity i_+.

$$ds^2 = -dt^2 + H^{-2}\cosh^2(Ht)\,d\Omega_3^2$$

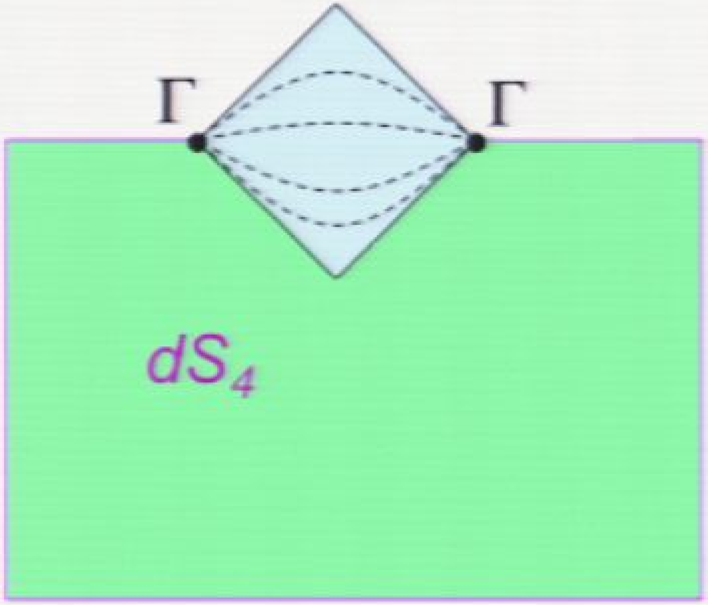
Future infinity is $S_3:\ t \to \infty.$

Potential problem:

In String Theory, *dS space is metastable*, so there is no such thing as asymptotically dS space.

CdL/CFT correspondence

Freivogel, Sekino,
Susskind & Yeh (2006)
(FSSY)



Bubble interior:

$$ds^2 = -dt^2 + a^2(t)(dr^2 + \sinh^2 r\, d\Omega_2^2)$$

AdS$_3$

FSSY: *The 4D theory inside the bubble is equivalent to a Euclidean 2D field theory on Γ.*

The boundary theory includes a Liouville field $L(\Omega)$, which describes fluctuations of the boundary geometry: $e^{2L(\Omega)} d\Omega_2^2$

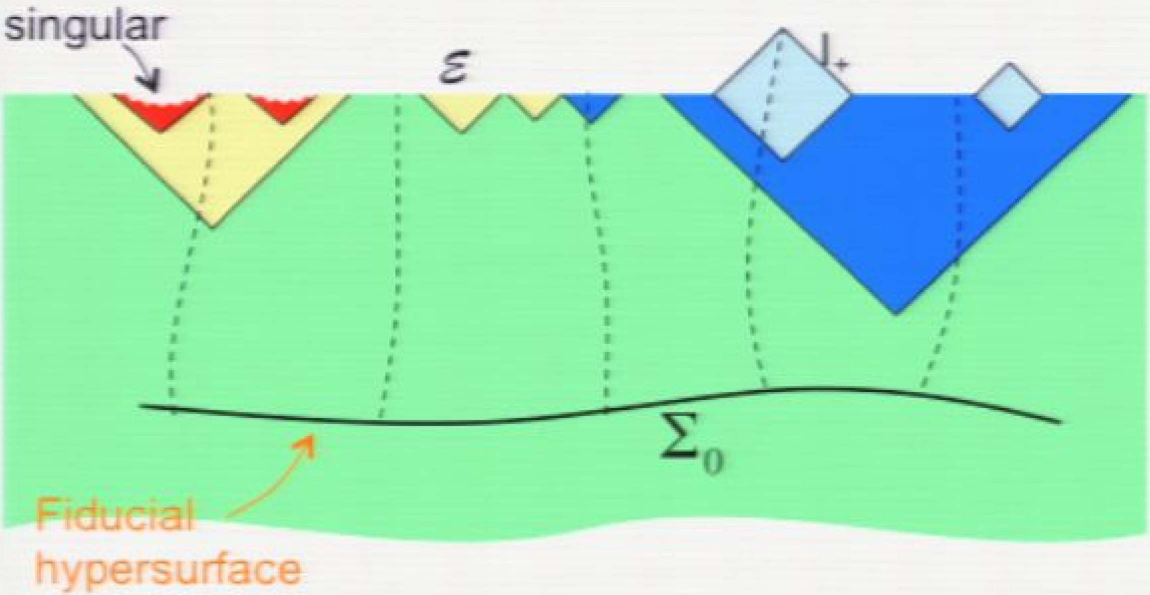
This additional field plays the role of time variable ***t***, as in Wheeler-DeWitt equation, while ***r*** is recovered from RG flow.

FSSY go further: *The 2D boundary is affected by collisions with other bubbles, and so it may represent a larger part of the multiverse.*

The proposal:

The boundary theory lives at the future boundary of the multiverse (suitably defined).

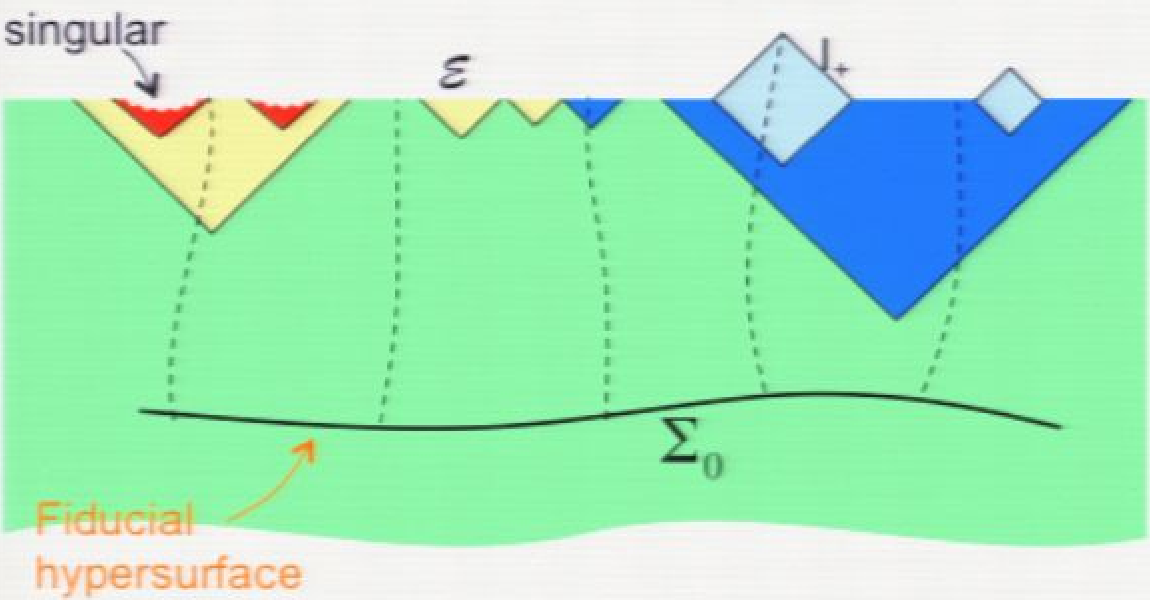
Future infinity



- Geodesic congruence projects bubbles onto Σ_0.
  $\Rightarrow$ Map of future infinity.
- Excise images of Minkowski bubbles. (They are described by the *2D* boundary degrees of freedom. *(FSSY)*
- AdS bubbles can be excised in a similar way (?).
  Hertog & Horowitz (2005)

What remains is the eternal set $\mathcal{E}$ *('scree').*

Future infinity



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Webster dictionary: scree \ˈ*skrē*\

An accumulation of loose stones or rocky debris at the base of a hill.

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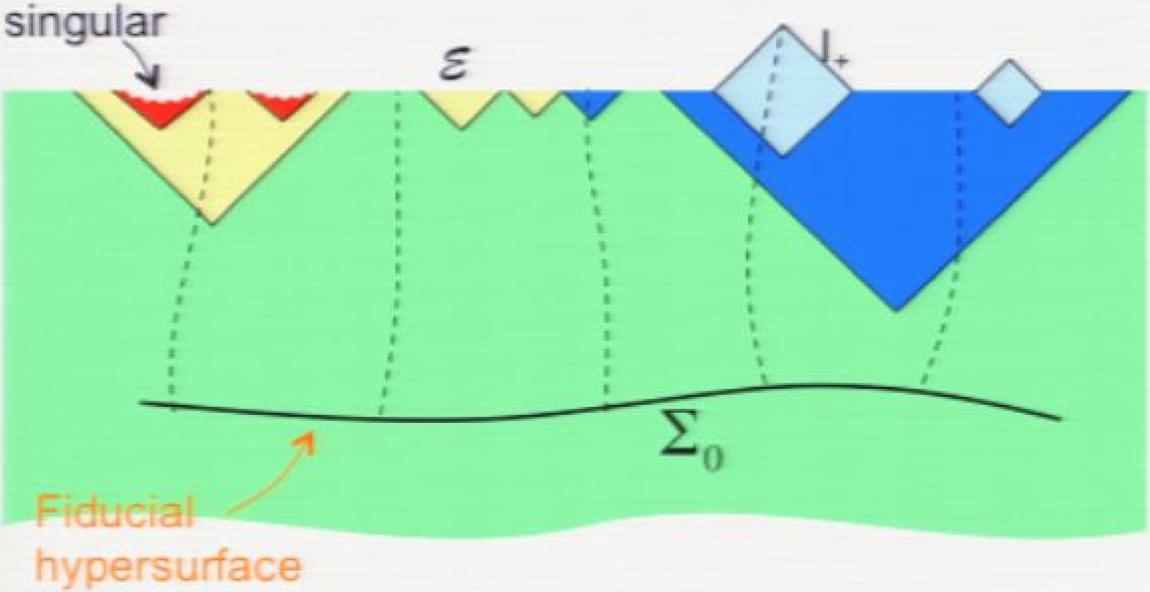

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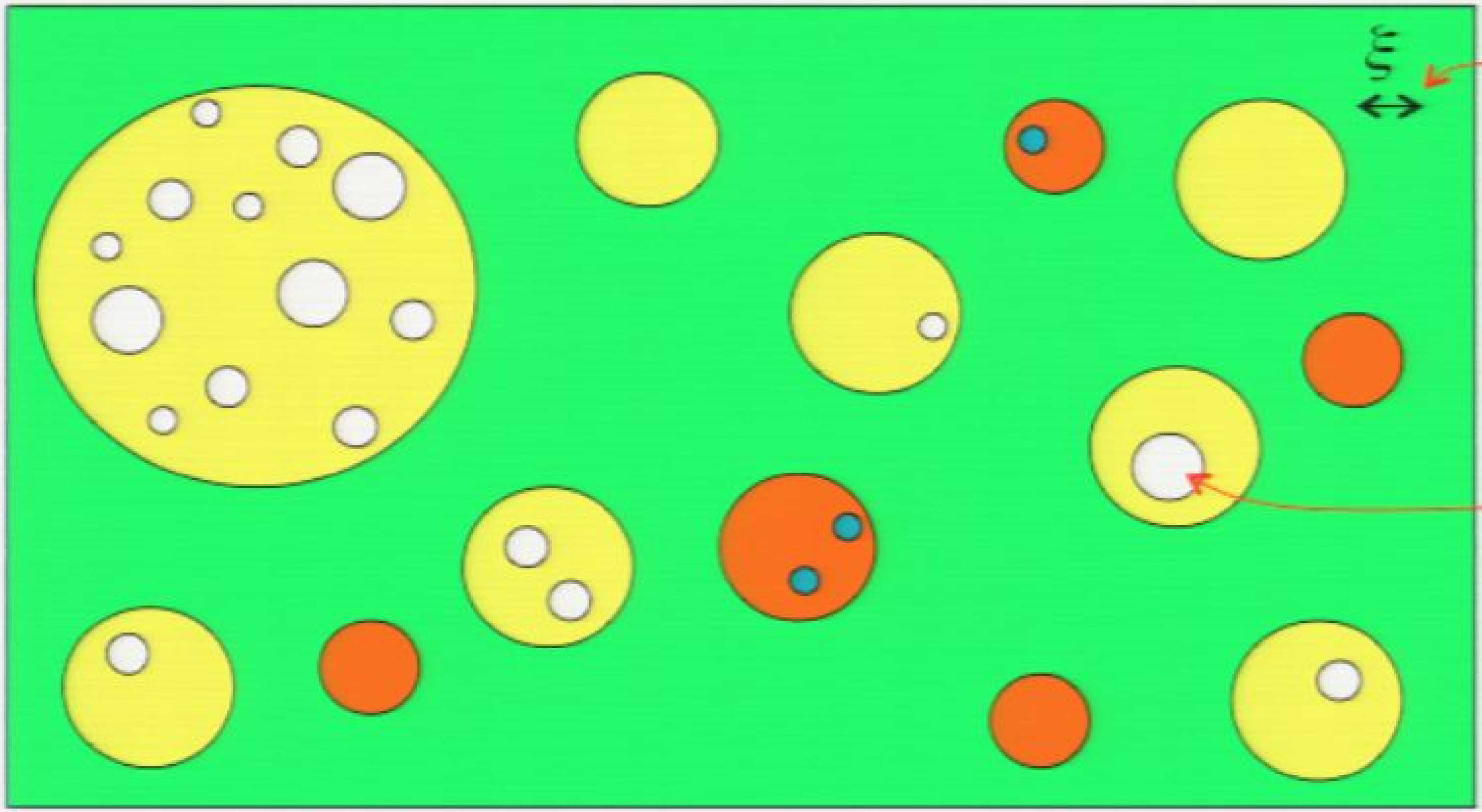
What remains is the eternal set $\mathcal{E}$ *('scree').*

The metric $g_{ij}(\boldsymbol{x})$ on Σ_0 defines a metric on $\mathcal{E}$.

Different choices of Σ_0 are related by Weyl rescalings.

The boundary theory lives on $\mathcal{E}$.

Structure of the eternal set $\mathcal{E}$

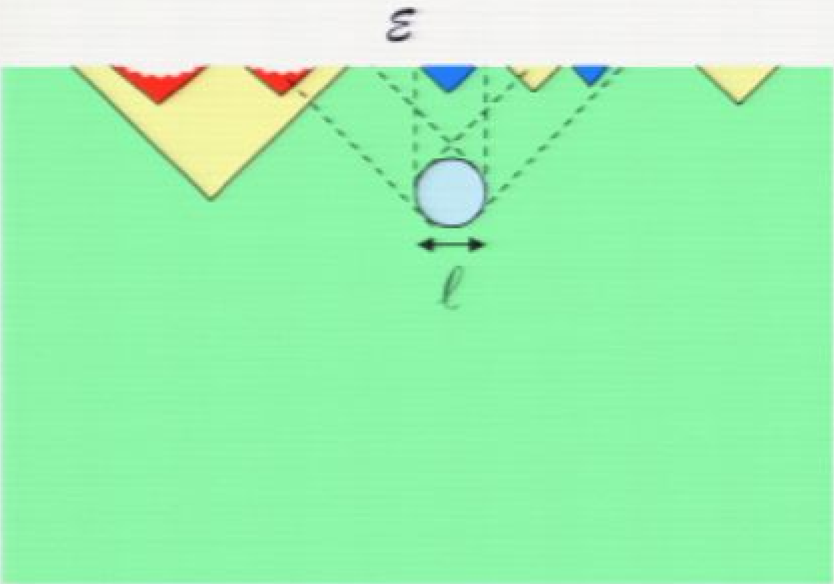


Resolution scale (Wilsonian cutoff)

Terminal bubble

- Each bubble becomes a fractal "sponge" in the limit $\xi \to 0$.
- Terminal bubbles correspond to holes (with 2D CFTs on their boundaries).

Information travels to $\mathcal{E}$ in the form of long-wavelength $(\lambda >> H^{-1})$ massless and very light fields (e.g., gravit. waves).



$$\lambda(\vec{x},t) \sim \ell \, a(\vec{x},t)/a_0 \ .$$

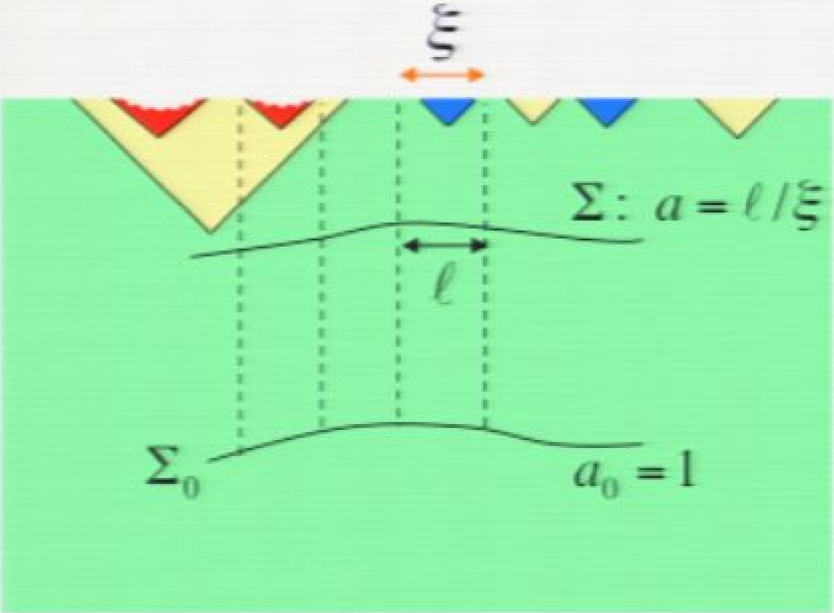
Modes with $\lambda >> H^{-1}$ are frozen

⟹ the information is indestructible.

Renormalization of the boundary theory

Integrate out boundary modes of wavelength up to ξ. ← Boundary UV cutoff

The corresponding 4D modes have minimum wavelength



$$\lambda_{\min}(\vec{x},t) = a(\vec{x},t)\ \xi.$$

Require $\lambda_{\min} < \ell$ ← Bulk resolution scale

➡ $a < \ell/\xi$ -- scale factor cutoff

$$\xi \to 0 \quad \Rightarrow \quad a \to \infty\ .$$

UV cutoff on the boundary ⟺ (IR) scale factor cutoff in 4D.

RG flow on the boundary ⟺ scale-factor time evolution.

(on super-horizon scales)

The boundary theory is conformally invariant in the UV.

Simple model: dS bubbles separated by thin walls

Inflating part of spacetime can be foliated by flat surfaces. (They are very close to constant-a surfaces.)

$$ds^2 = H^{-2}dt^2 - e^{2t}d\vec{x}^2$$

$t = \ln a$ – scale factor time

Size distribution of bubbles

$$dN_{ij} = \lambda_{ij} H_j^{-1} f_j e^{3t} dt$$

number of bubbles of type *i* formed in parent vacuum *j* in a unit comoving volume $V(t) = e^{3t}$ per time interval *dt*.

Bubble nucleation rate

Fraction of volume in vacuum *j*

$t = \ln a$ – scale factor time

$$df_i / dt = M_{ij} f_j \,, \qquad M_{ij} = \kappa_{ij} - \delta_{ij} \sum_m \kappa_{mi} \,, \qquad \kappa_{ij} = \lambda_{ij} \frac{4\pi}{3} H_j^{-4}$$

$$f_i(t) \propto s_i e^{-qt} + \ldots \quad (t \to \infty)$$

Garriga, Schwartz-Perlov, A.V. & Winitzki (2005)

$-q < 0$ is the largest nonzero eigenvalue of *M*.
$|q| << 1$, nondegenerate.

$$dN_{ij} = \lambda_{ij} H_j^{-1} s_j e^{(3-q)t} dt$$

Comoving radius of a bubble formed at time t:

$$r = H_j^{-1} e^{-t}$$

$$\boxed{dN_{ij} = C_{ij} r^{-(4-q)} dr}$$

Independent of initial conditions.

Invariant under rescalings $r \to Br$.

Applies only approximately, becoming exact in the limit $r \to 0$.

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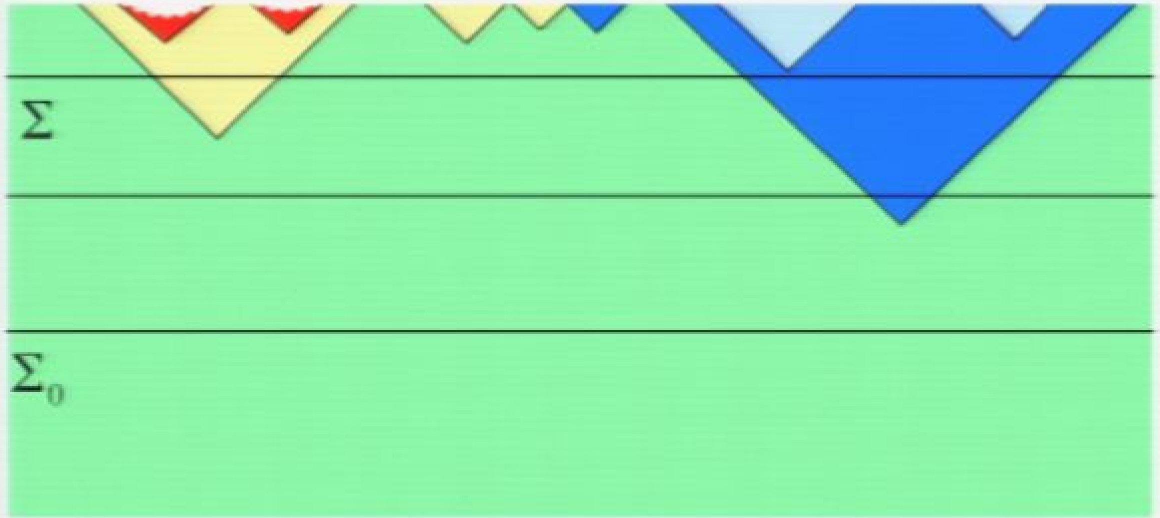

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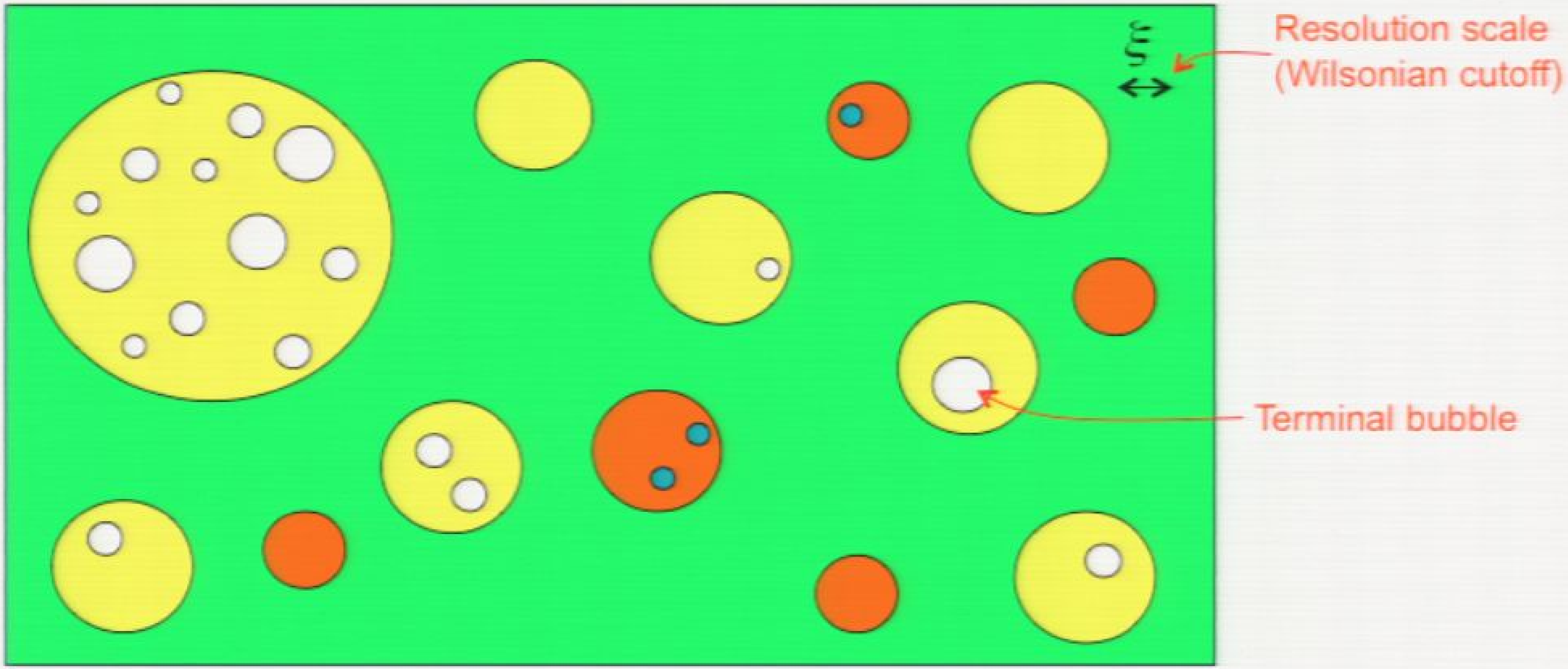

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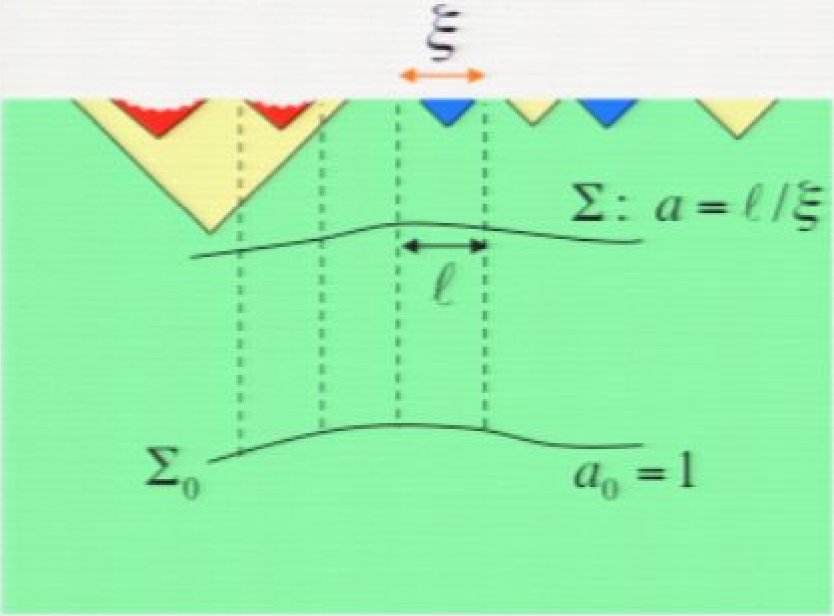

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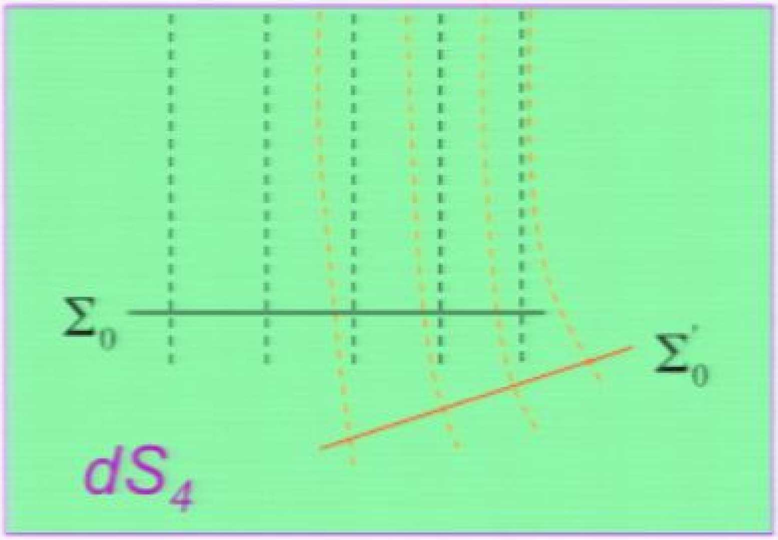
Symmetry related to dS boosts



Geodesic congruences orthogonal to Σ_0 and $\Sigma_0^{'}$ become asymptotically comoving.

This defines a transformation $\vec{x} \rightarrow \vec{x}'$ on $\mathcal{E}$.

For congruences related by a dS boost, the transformation is a *special conformal transformation* (SCT)

$$\frac{x'^i}{x'^2} = \frac{x^i}{x^2} - b^i$$

Maps spheres into spheres.

accompanied by a rotation.

Initial conditions corresponding to Σ_0 and $\Sigma_0^{'}$ should yield the same asymptotic bubble distribution. ⟹ The distribution should be invariant under SCT's (in the limit $r \rightarrow 0$).

Dilatations, translations, rotations, and SCT's comprise the Euclidean conformal group. ⟹ ***The boundary theory should be conformally invariant in the UV.***

Jaume will provide further evidence.

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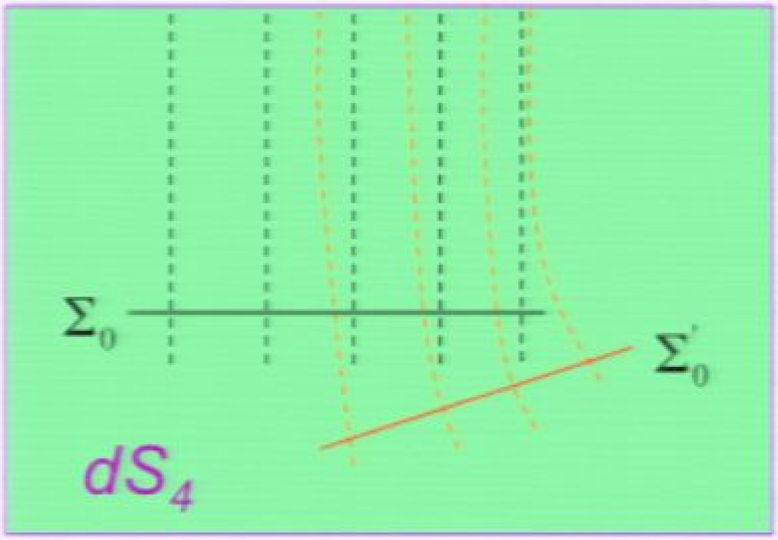

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Invariant under rescalings $r \to Br$.

Applies only approximately, becoming exact in the limit $r \to 0$.

Size distribution of bubbles

$$dN_{ij} = \lambda_{ij} H_j^{-1} f_j e^{3t} dt \leftarrow$$

number of bubbles of type *i* formed in parent vacuum *j* in a unit comoving volume $V(t) = e^{3t}$ per time interval *dt*.

Bubble nucleation rate

Fraction of volume in vacuum *j*

$t = \ln a$ – scale factor time

$$df_i / dt = M_{ij} f_j \,, \qquad M_{ij} = \kappa_{ij} - \delta_{ij} \sum_m \kappa_{mi} \,, \qquad \kappa_{ij} = \lambda_{ij} \frac{4\pi}{3} H_j^{-4}$$

$$f_i(t) \propto s_i e^{-qt} + \ldots \quad (t \to \infty)$$

Garriga, Schwartz-Perlov, A.V. & Winitzki (2005)

$-q < 0$ is the largest nonzero eigenvalue of *M*.

$|q| << 1$, nondegenerate.

$$dN_{ij} = \lambda_{ij} H_j^{-1} s_j e^{(3-q)t} dt$$

Comoving radius of a bubble formed at time t:

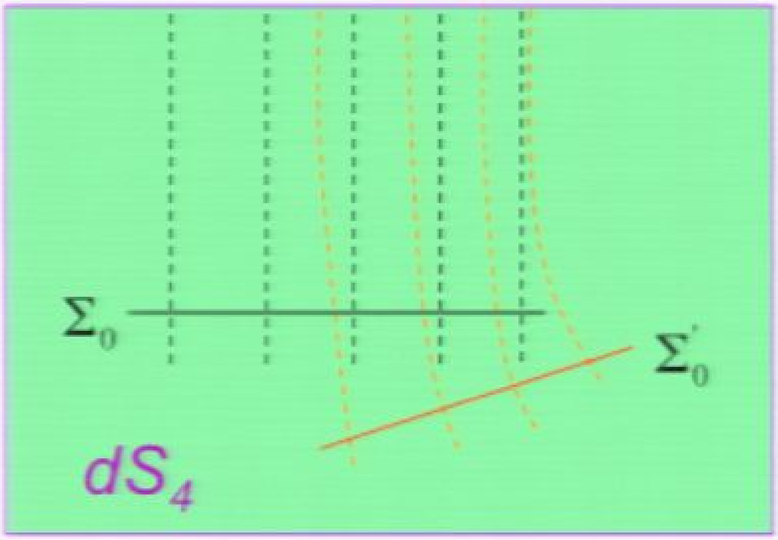
$$r = H_j^{-1} e^{-t}$$

$$\boxed{dN_{ij} = C_{ij} r^{-(4-q)} dr}$$

Note: $dN_{ij} \propto r^{-(D+1)} dr$

where $D = 3 - q$ is the fractal dimension of $\mathcal{E}$.

Symmetry related to dS boosts



Geodesic congruences orthogonal to Σ_0 and Σ_0' become asymptotically comoving.

This defines a transformation $\vec{x} \to \vec{x}'$ on $\mathcal{E}$.

For congruences related by a dS boost, the transformation is a *special conformal transformation* (SCT)

$$\frac{x'^i}{x'^2} = \frac{x^i}{x^2} - b^i$$

Maps spheres into spheres.

accompanied by a rotation.

Initial conditions corresponding to Σ_0 and Σ_0' should yield the same asymptotic bubble distribution. ⟹ The distribution should be invariant under SCT's (in the limit $r \to 0$).

Dilatations, translations, rotations, and SCT's comprise the Euclidean conformal group. ⟹ ***The boundary theory should be conformally invariant in the UV.***

Jaume will provide further evidence.

Simple model: dS bubbles separated by thin walls

Inflating part of spacetime
can be foliated by flat surfaces.
(They are very close
to constant-a surfaces.)

$$ds^2 = H^{-2}dt^2 - e^{2t}d\bar{x}^2$$

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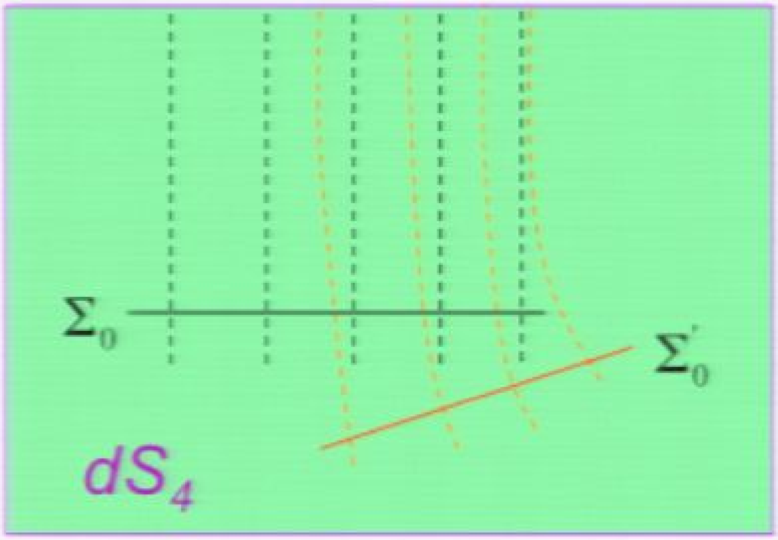

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Conclusions

- The dynamics of the multiverse may be encoded in its future boundary $\mathcal{E}$.
- The measure of the multiverse can be obtained by imposing a UV cutoff in the boundary theory.
- This measure is closely related to the scale factor cutoff.
- The boundary theory is expected to be conformally invariant in the UV.

Related recent work: *Freivogel & Kleban (2009); Bousso (2009).*

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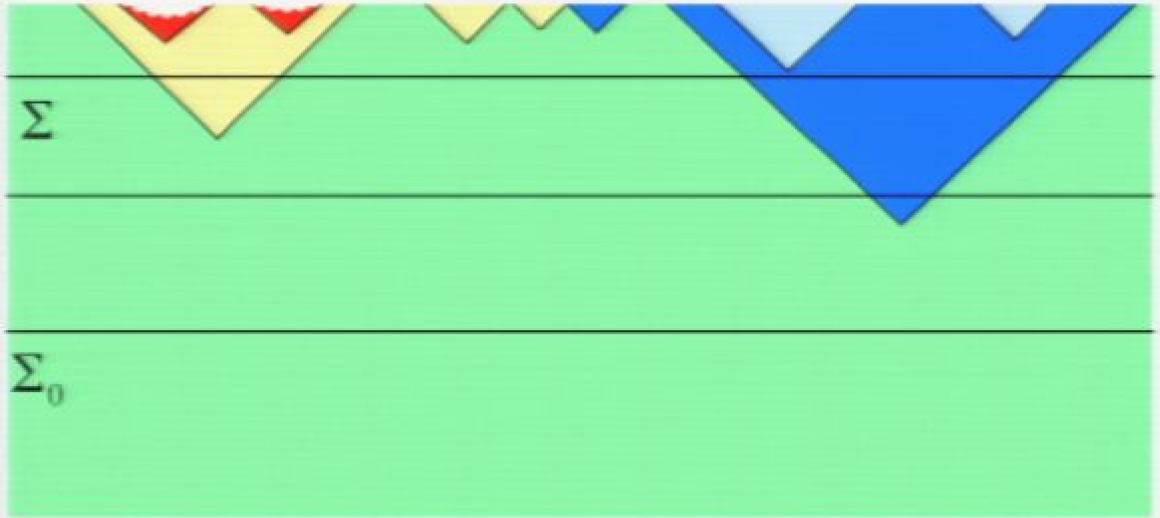

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