Title: Non-Gaussianity from non-equilibrium physics

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Abstract: Non-equilibrium processes such as inflationary preheating or the ekpyrotic bounce can turn fluctuations of light scalar fields into potentially highly non-Gaussian curvature perturbations. I show how these perturbations can be calculated at fully non-linear level using lattice field theory simulations. As concrete examples, I present results for preheating in chaotic inflation and resonant curvaton decay.

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Non-Gaussianity from Non-Equilibrium Physics

Arttu Rajantie (with Alex Chambers and Sami Nurmi)

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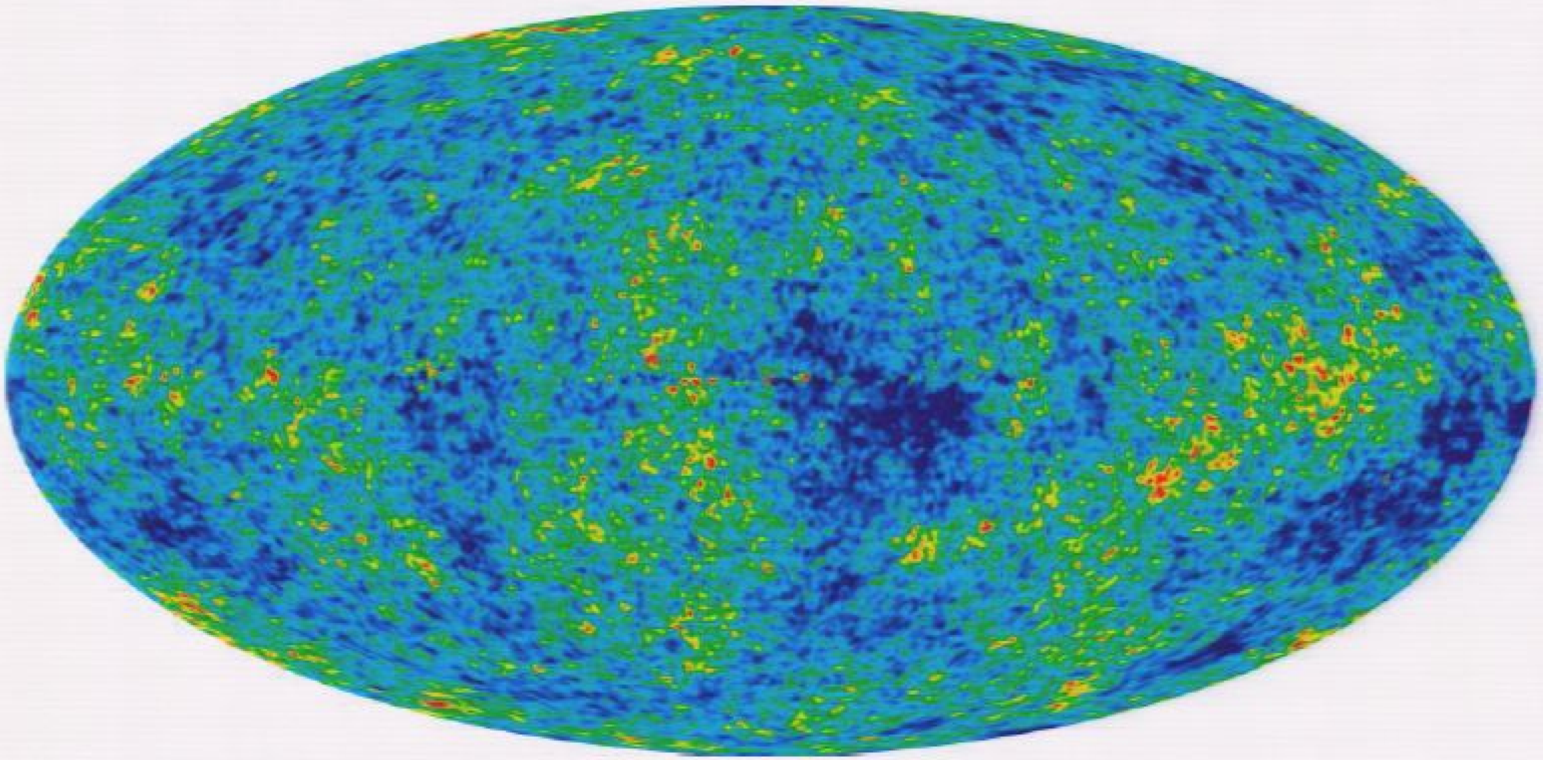
Curvature Perturbations



- $\delta T/T \approx \zeta \sim 10^{-5}$
- Nearly scale invariant
- Gaussian

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Non-Gaussianity

- Simplest assumption: Local non-Gaussianity(Komatsu&Spergel 2000)

$$\zeta(x) = \zeta_0(x) - \frac{3}{5} f_{\rm NL} \left(\zeta_0(x)^2 - \langle \zeta_0(x)^2 \rangle \right)$$

- Three-point function

$$\begin{aligned} \langle \zeta(k_1)\zeta(k_2)\zeta(k_2) \rangle &= -\frac{5}{6} f_{\rm NL} \left[P(k_1)P(k_2) + \text{cyclic} \right] \\ &\quad \times (2\pi)^3 \delta(k_1 + k_2 + k_3) \end{aligned}$$

- Use this as a *definition* of $f_{\rm NL}$
  - Generally $f_{\rm NL} = f_{\rm NL}(k_1, k_2, k_3)$
- Single field inflation: $f_{\rm NL} \sim \epsilon \ll 1$ (Maldacena 2003)
- Observations show a hint for $f_{\rm NL} \sim 50$:
  - $-9 < f_{\rm NL} < 111$ (WMAP 2008)
  - $-4 < f_{\rm NL} < 80$ (Smith et al 2009)

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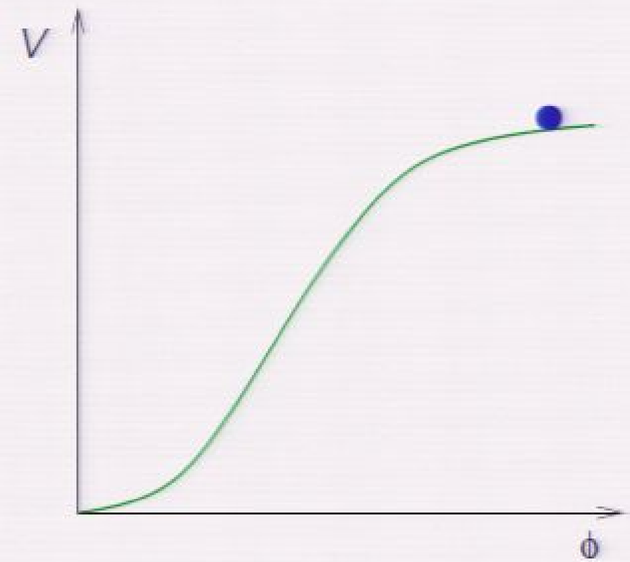
Inflation

- Scalar inflaton field ϕ, with $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$
- Potential $V(\phi)$
  - Dominates $\rho \approx V(\phi)$
  - Flat $\epsilon = \frac{1}{2}M_{\rm Pl}^2(V'/V)^2 \ll 1$, $|\eta| = M_{\rm Pl}^2|V''/V| \ll 1$

$\Rightarrow$ Slow roll



$$H^2 = \frac{\rho}{3M_{\rm Pl}^2} \sim \text{constant} \quad \Rightarrow \quad a(t) \sim e^{Ht}$$

- Solves horizon, flatness problems

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Field Perturbations

- Inhomogeneous modes (comoving wave number k):

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + V''\right)\delta\phi_k \approx 0$$

- Overdamped outside horizon $k \ll Ha$: Freeze out

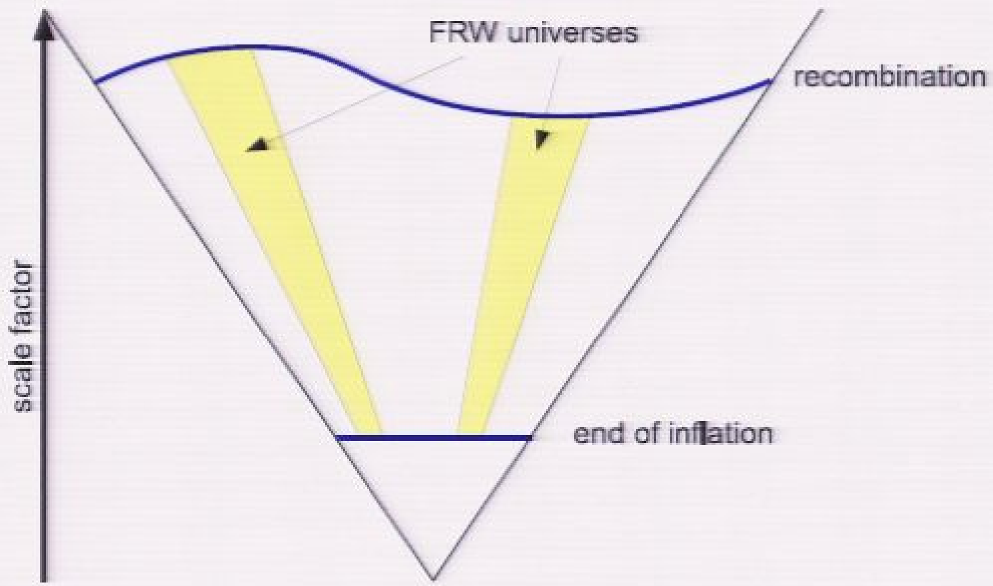
$$P(k) = \frac{H^2}{2k^3} \text{for} \quad k \ll aH$$

where $\langle\delta\phi_{k_1}\delta\phi_{k_2}\rangle \equiv P(k_1)(2\pi)^3\delta(k_1 + k_2)$

- Power spectrum (contribution from log scale)

$$\mathcal{P}_\phi(k) = \frac{k^3}{2\pi^2}P(k) = \frac{H^2}{4\pi^2}$$

Separate Universes



- Each Hubble patch described by a separate FRW universe (Salopek&Bond 1990)
- Curvature perturbation $\zeta = \delta \ln a|_{H=H_*}$
- Valid at distances $d \gg 1/H$

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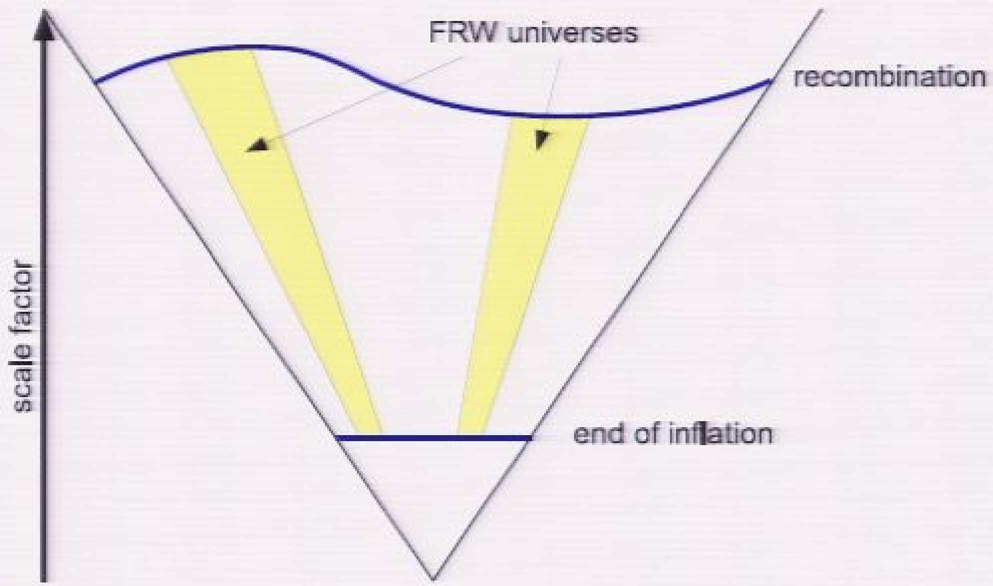

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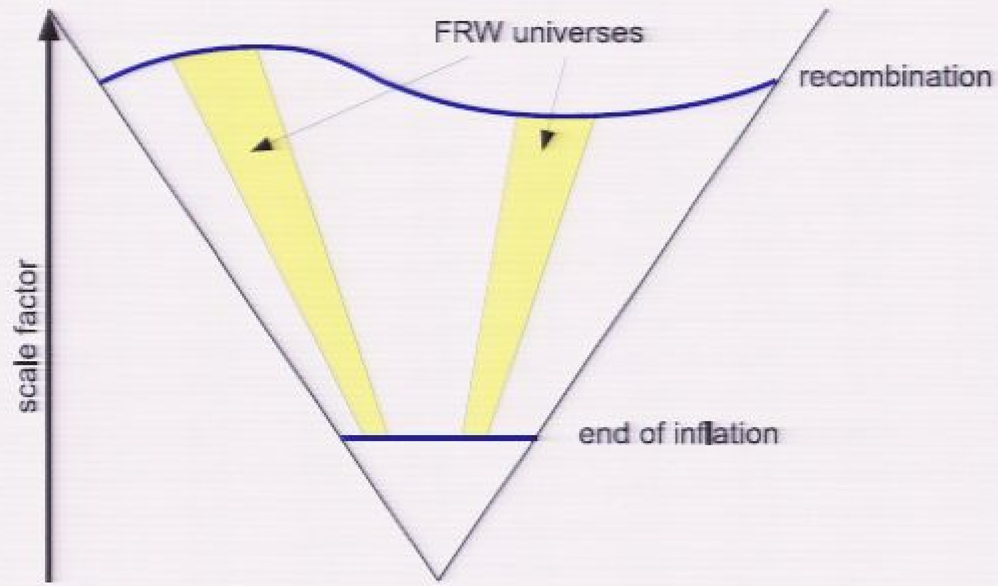

- Curvature perturbation $\zeta = \delta \ln a|_{H=H_*}$
- Conserved if $w = w(\rho)$:

$$\frac{d \ln a}{dH} = -\frac{2}{3(1+w)}$$

Single-Field Inflation

- All universes identical after inflation: $H_* = H_{\rm end} \Rightarrow \phi = \phi_{\rm end}$

$$\frac{\delta T}{T} = \zeta = \delta \ln a|_{\phi=\phi_{\rm end}} = -\frac{H}{\dot\phi}\delta\phi$$

- Temperature power spectrum, observed by COBE, WMAP, ...

$$\mathcal{P}_T \approx (10^{-5})^2 = \frac{H^2}{\dot\phi^2}\mathcal{P}_\phi = \frac{H^4}{4\pi^2\dot\phi^2}$$

  - (Almost) Scale invariant (constant H)
  - (Almost) Perfectly Gaussian ($f_{\rm NL} \sim \epsilon$)

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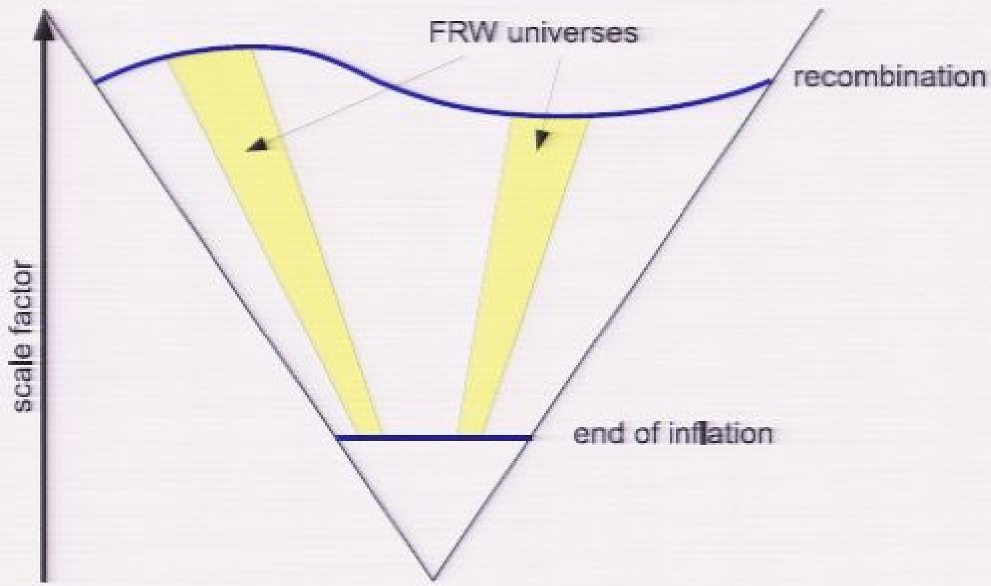

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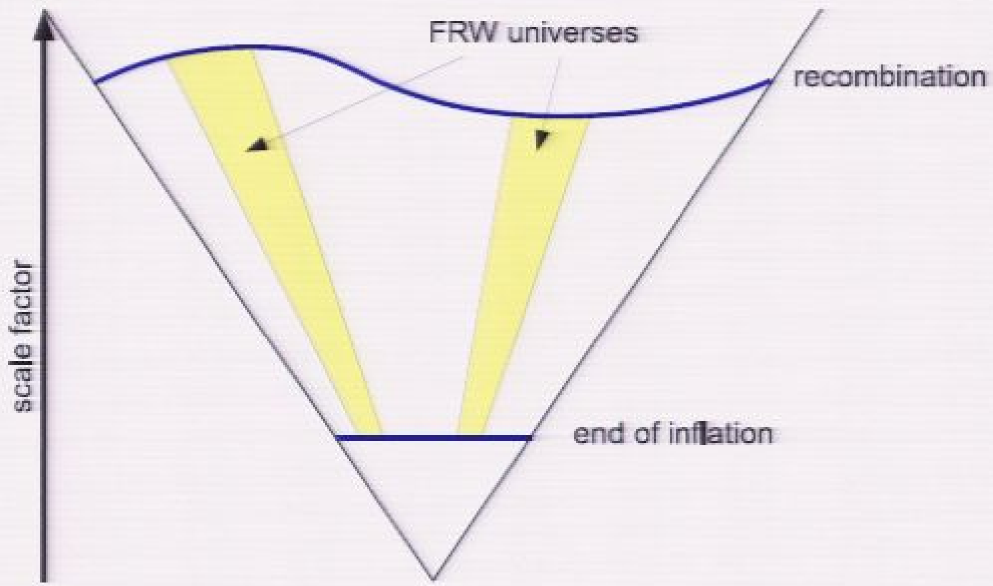

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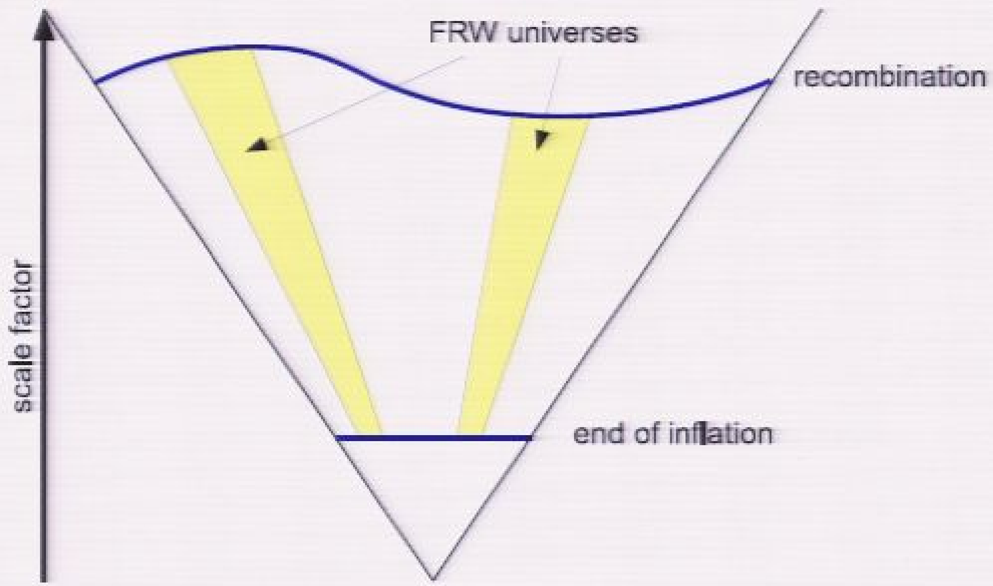

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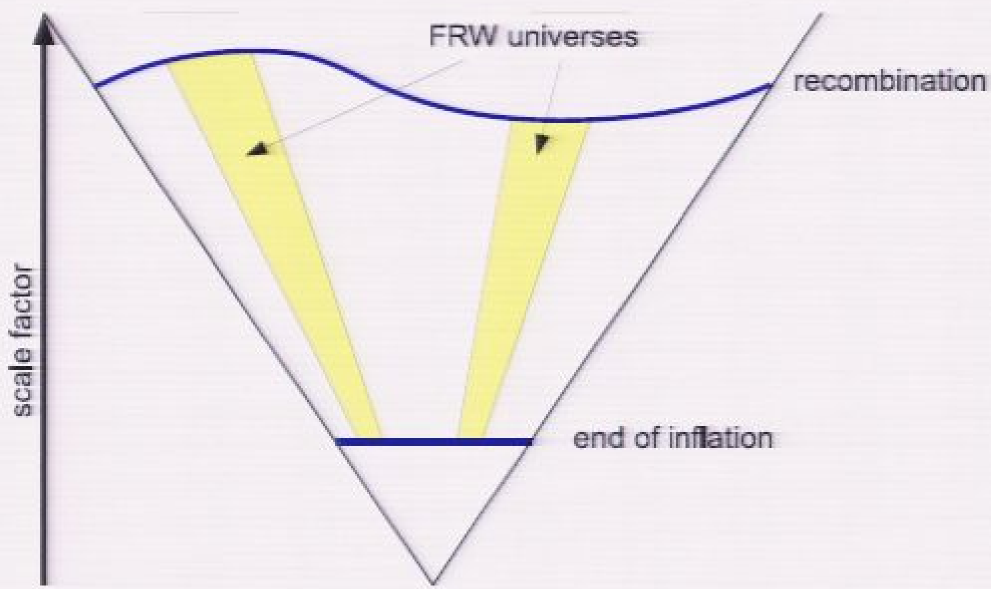
Light Scalar Field χ



- Gaussian scale-invariant perturbations at the end of inflation

$$\mathcal{P}_\chi \approx \mathcal{P}_\phi \approx \frac{H^2}{4\pi^2}$$

- Different universes: Different χ

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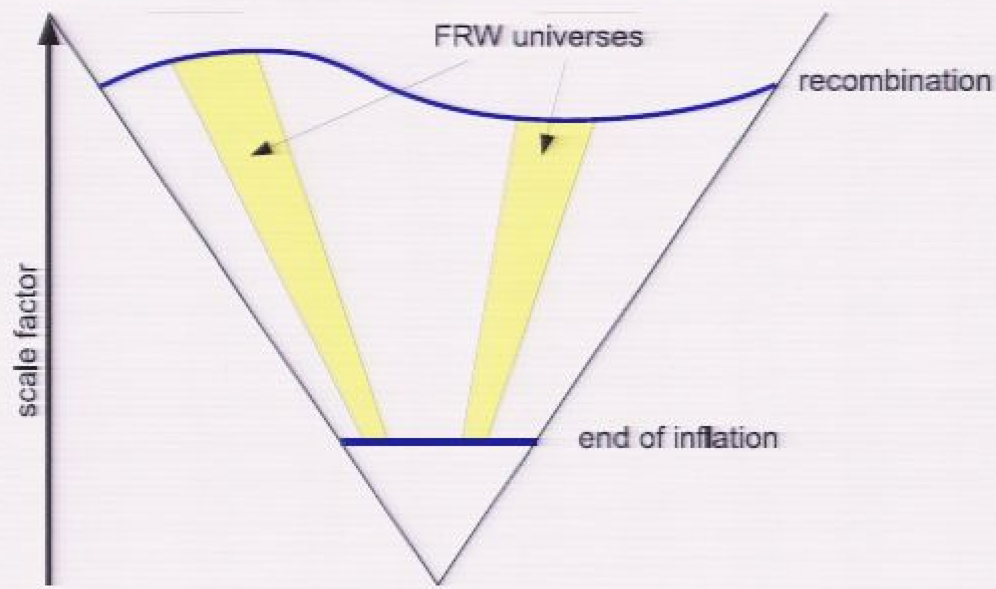

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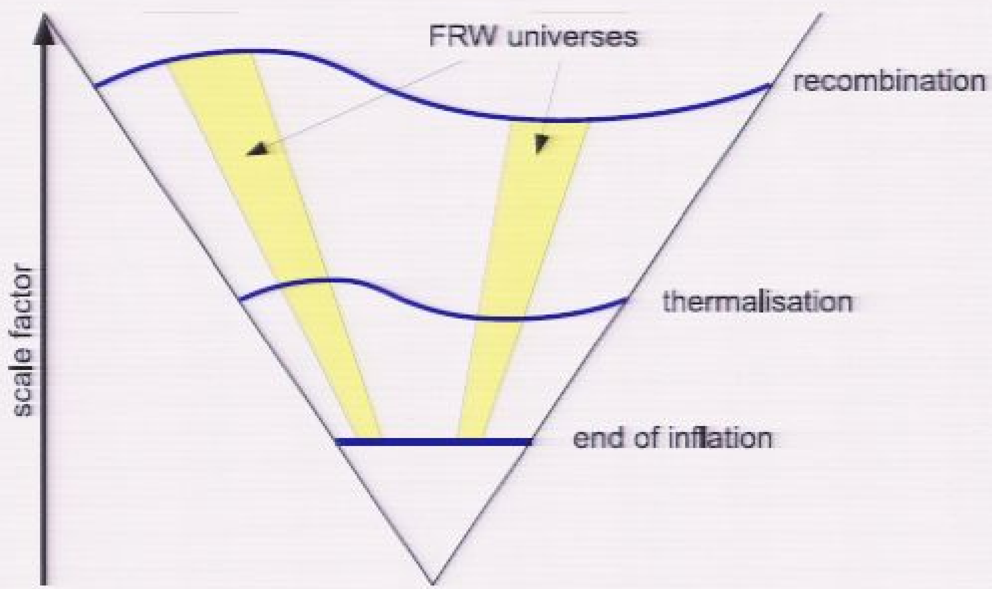

- Thermalisation erases memory of initial $\chi \Rightarrow w = w(\rho)$
- Curvature perturbation determined at thermalisation:

$$\zeta_{\rm rec} = \zeta_{\rm therm} = \delta \ln a|_{H=H_{\rm therm}}$$

Calculating ζ

- Solve Friedmann eq. for each separate universe $\Rightarrow a(t), H(t)$
  - Non-linear, includes gravity, valid at $d \gg H^{-1}$
- Pick $H_* < H_{\rm therm}$, and calculate $\zeta = \delta \ln a|_{H=H_*}$
  - Two fields: $\zeta = \zeta(\phi_{\rm ini}, \chi_{\rm ini})$
  - $\delta\phi_{\rm ini} \Leftrightarrow$ Shift in time:
    Usual inflationary perturbations
  - $\delta\chi_{\rm ini}$: New contribution
- Need to calculate $N(\chi_{\rm ini}) = \ln a(\chi_{\rm ini})$
  - Taylor expand:

$$\zeta = \zeta_\phi + \frac{\partial N}{\partial \chi_{\rm ini}} \delta\chi_{\rm ini} + \frac{1}{2}\frac{\partial^2 N}{\partial \chi_{\rm ini}^2} \delta\chi_{\rm ini}^2 + \ldots$$

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Calculating $f_{\rm NL}$

$$\zeta = \zeta_\phi + \frac{\partial N}{\partial \chi_{\rm ini}} \delta\chi_{\rm ini} + \frac{1}{2}\frac{\partial^2 N}{\partial \chi_{\rm ini}^2} \delta\chi_{\rm ini}^2 + \ldots$$

- If χ dominates, one has the usual $f_{\rm NL}$ form:

$$f_{\rm NL} = \frac{5N(\chi_{\rm ini})''}{6N'(\chi_{\rm ini})^2}$$

- If the inflaton ϕ dominates, one finds (Boubekeur&Lyth 2006)

$$f_{\rm NL} \approx -\frac{5}{48} N''(\chi_{\rm ini})^3 \frac{\mathcal{P}_\chi^3}{\mathcal{P}_\zeta^2} \ln\frac{k}{H_0} \approx -\frac{5\epsilon^2 V M_{\rm Pl}^2}{144\pi^2} N''(\chi_{\rm ini})^3 \ln\frac{k}{H_0}$$

Curvaton

- Simplest example: Perturbative curvaton model (Mollerach 1990; Lyth&Wands 2002)

$$V(\phi,\chi) = V(\phi) + \frac{1}{2}m^2\chi^2 + h\chi\bar{\psi}\psi$$

- Curvaton χ becomes massive after inflation when $H < m$
  - Density affects expansion: $w = \frac{1}{3}(1-r)$ where $r = \rho_\chi/\rho_{\rm tot}$
- Decays perturbatively to ψ particles at time $t_{\rm decay} = 1/\Gamma \sim 8\pi/h^2 m$
  - Derivatives of N:

$$N'(\chi_{\rm ini}) = \frac{r_{\rm decay}}{2\chi_{\rm ini}}, \quad N''(\chi_{\rm ini}) = \frac{r_{\rm decay}}{2\chi_{\rm ini}^2}$$

$\Rightarrow$ $f_{\rm NL} = \frac{5}{3}r_{\rm decay}^{-1}$ if the curvaton dominates perturbations

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Non-Equilibrium Physics

- Preheating: Reheating by parametric resonance (=preheating) (Kofman et al 1994)
- Resonant curvaton decay (Enqvist et al. 2009)
- Phase transitions
- Brane collisions
- Ekpyrotic bounce

$\Rightarrow$ Non-linear non-equilibrium processes:

  - Non-linear $N(\chi_{\rm ini}) \Rightarrow$ High non-Gaussianity?
  - Normal perturbative methods fail:
    Instead, solve the coupled Friedmann and field equations numerically (Bassett&Tanaka 2003)

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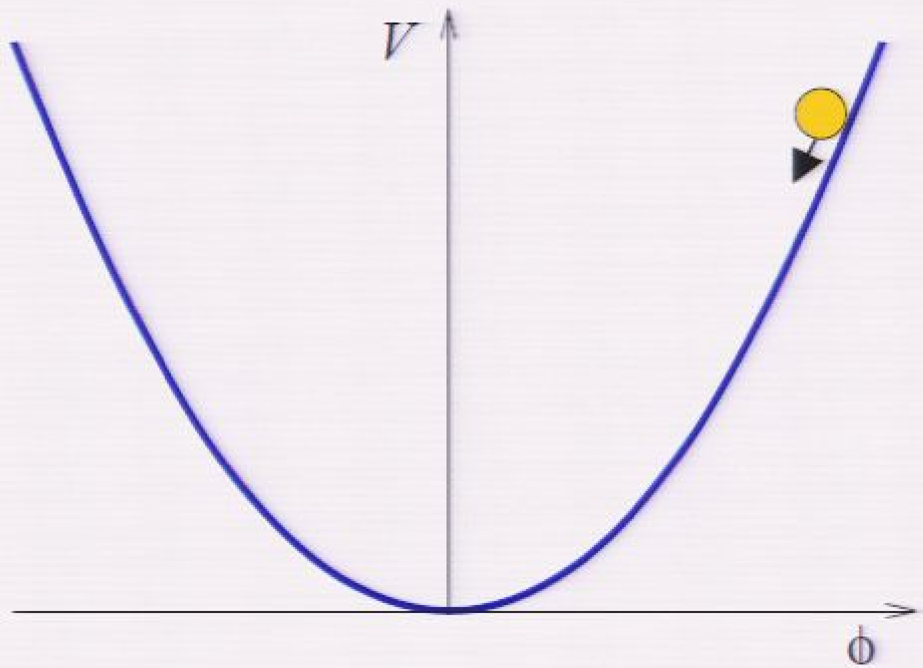
Massless Preheating

- Chaotic inflation + massless scalar χ (Prokopec&Roos 1997)

$$V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2, \quad \lambda \sim 7\times 10^{-14}$$

- Inflation: $\phi > 2\sqrt{3}M_{\rm Pl}$



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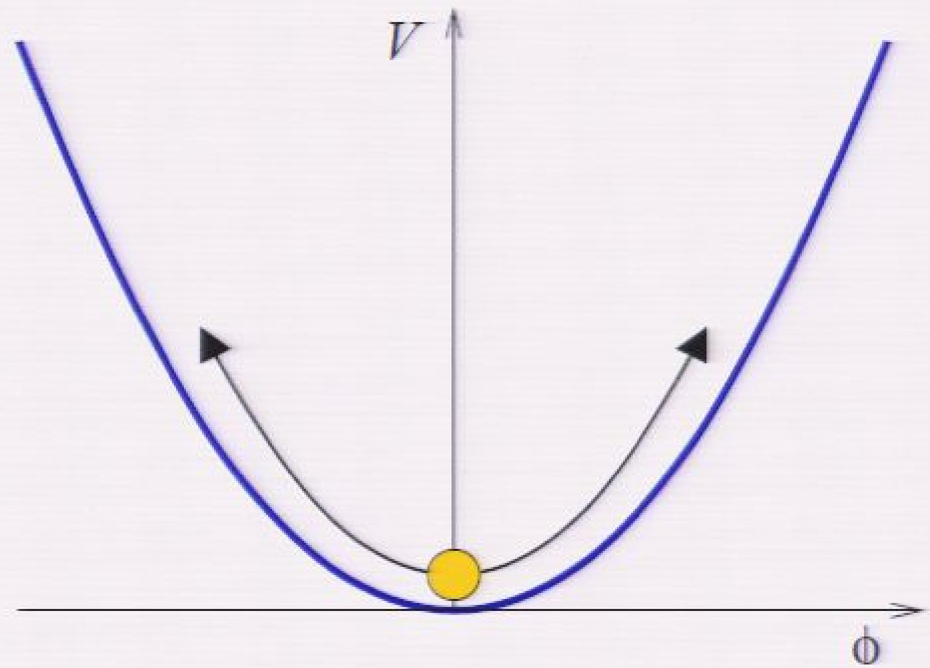

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- Inflaton zero mode $\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 = 0$
  - Rescale the field $\phi = a^{-1}\tilde{\phi}$
  - Rescale time $d\tau = a^{-1}\lambda^{1/2}\tilde{\phi}_{\rm ini}dt$

$\Rightarrow$ $\tilde{\phi}'' + \lambda\tilde{\phi}^3 = 0$ $\Rightarrow$ $\tilde{\phi}(\tau) = \tilde{\phi}_{\rm ini}{\rm cn}(\tau; 1/\sqrt{2})$ (Jacobi cosine)

- Inhomogeneous χ modes $\chi_k = a^{-1}\tilde{\chi}_k$

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}{\rm cn}^2(\tau; 1/\sqrt{2})\right]\tilde{\chi}_k = 0, \quad \kappa^2 = \frac{k^2}{\lambda\tilde{\phi}^2_{\rm ini}}$$

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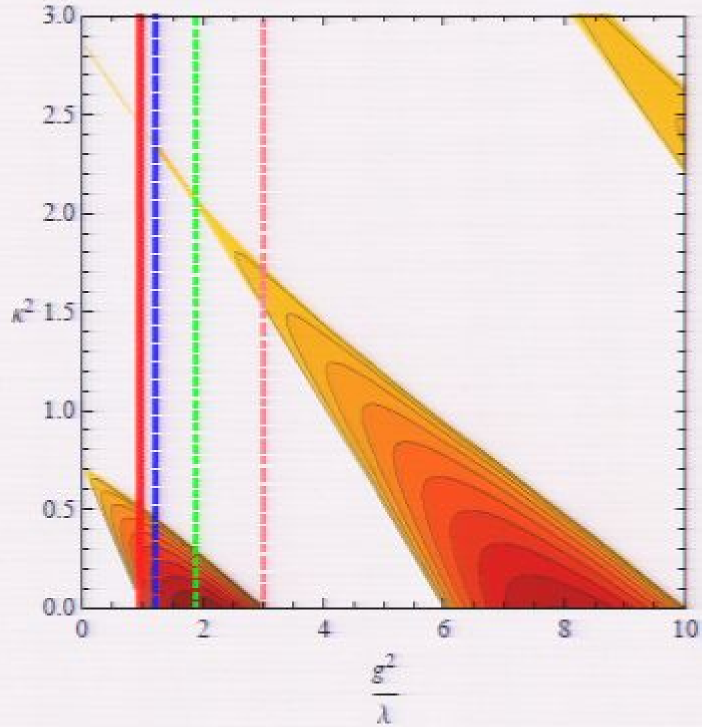
Resonance Bands

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(\tau; 1/\sqrt{2})\right]\tilde{\chi}_k = 0$$



- $\tilde{\chi}_k(\tau) = e^{\mu x} f(\tau)$ with periodic $f(\tau)$

Homogeneous Fields

- Field equations for homogeneous ϕ and χ:

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} + g^2\phi^2\chi &= 0\end{aligned}$$

- Couple to Friedmann eq

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right)$$

- No sub-horizon dynamics

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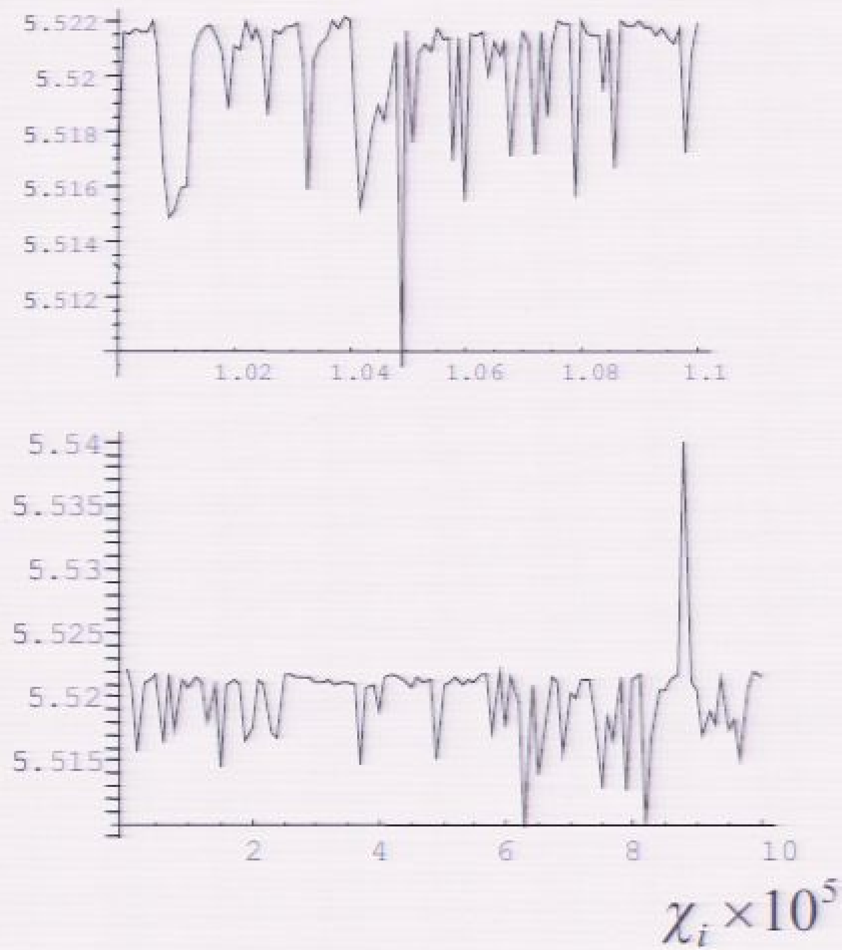
Homogeneous Fields



(Bassett and Tanaka 2003)

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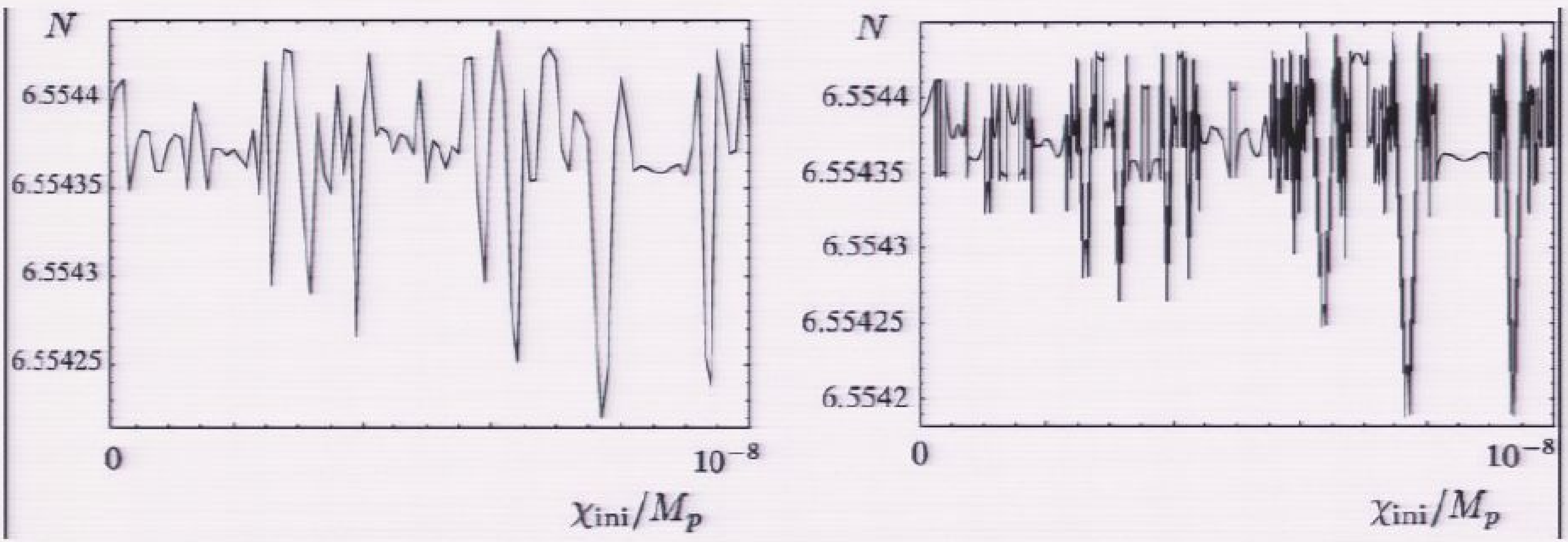

(Suyama and Yokoyama 2006)

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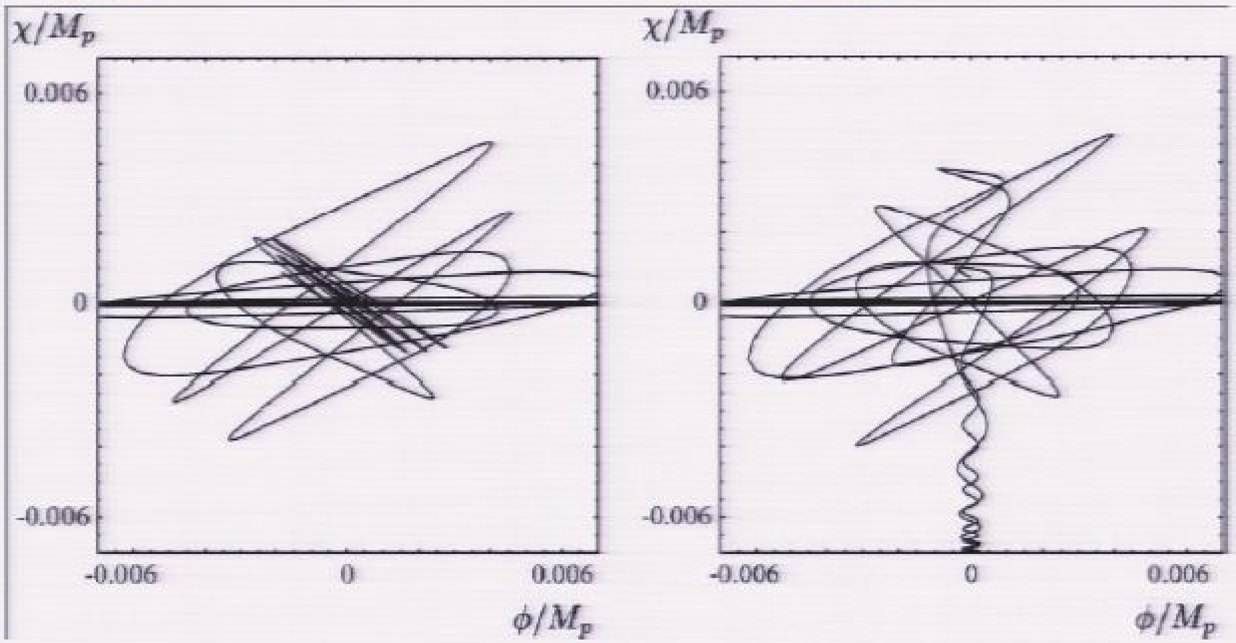
Homogeneous Fields

$\chi_{\rm ini} = 5.0 \times 10^{-9}$ $\chi_{\rm ini} = 5.2 \times 10^{-9}$



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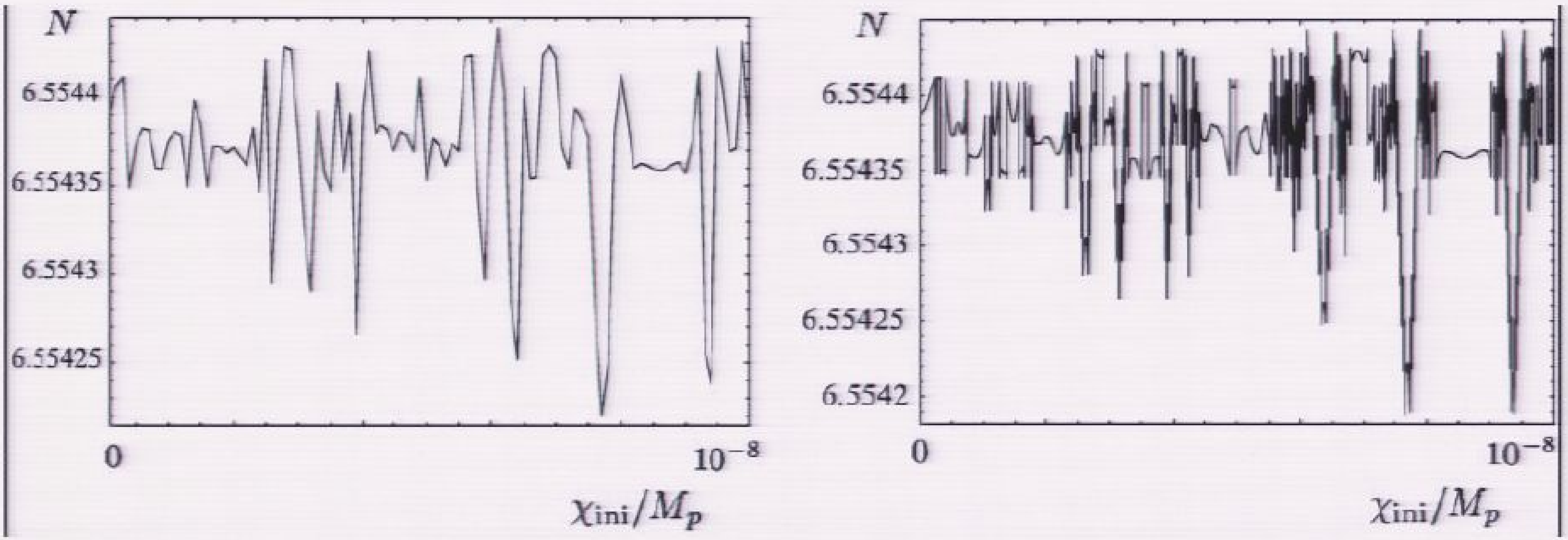

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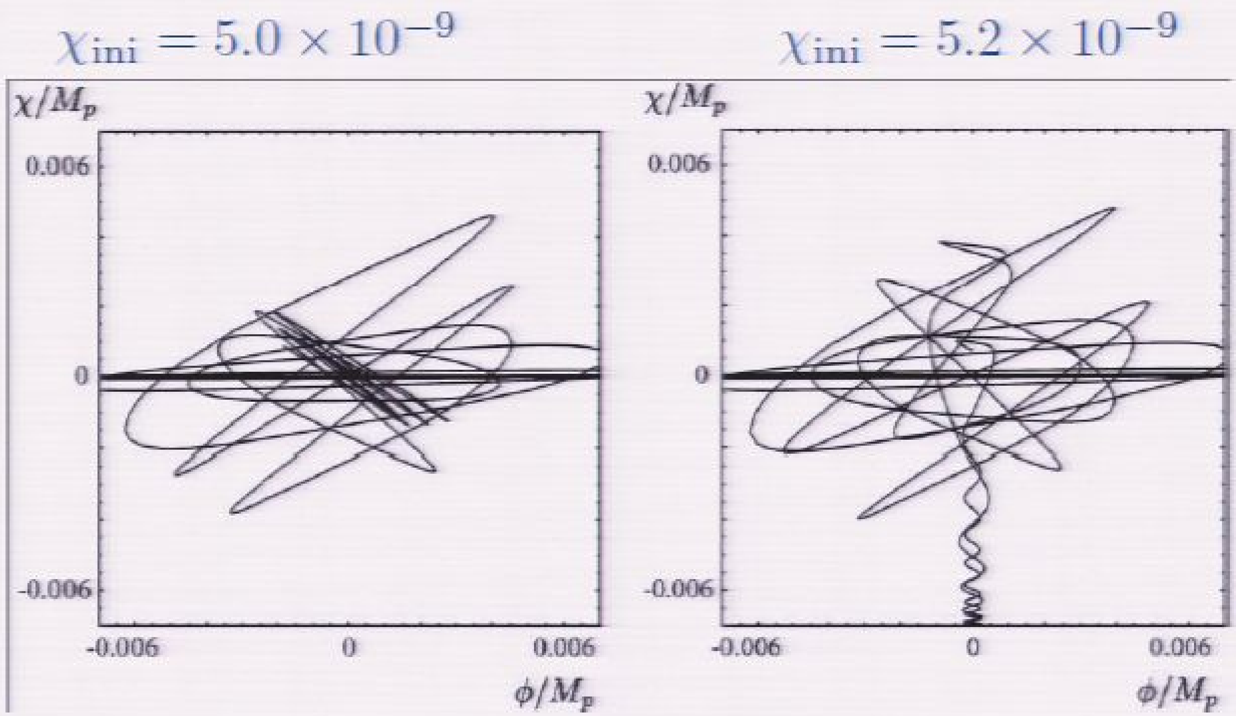

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Inhomogeneous Fields

- Non-equilibrium field dynamics:

  Inhomogeneous modes $k > H$ important

- Make ϕ and χ position-dependent (Khlebnikov&Tkachev 1996)

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0\end{aligned}$$

- Couple to Friedmann eq with average energy density

$$\begin{aligned}H^2 &= \frac{1}{3M_{\rm Pl}^2 V}\int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}(\vec{\nabla}\chi)^2\right. \\ &\left. + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right)\end{aligned}$$

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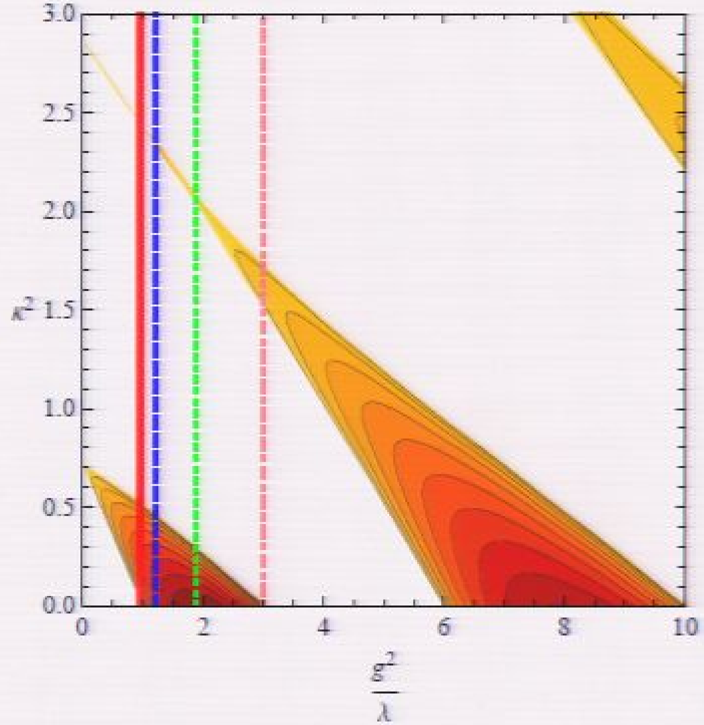

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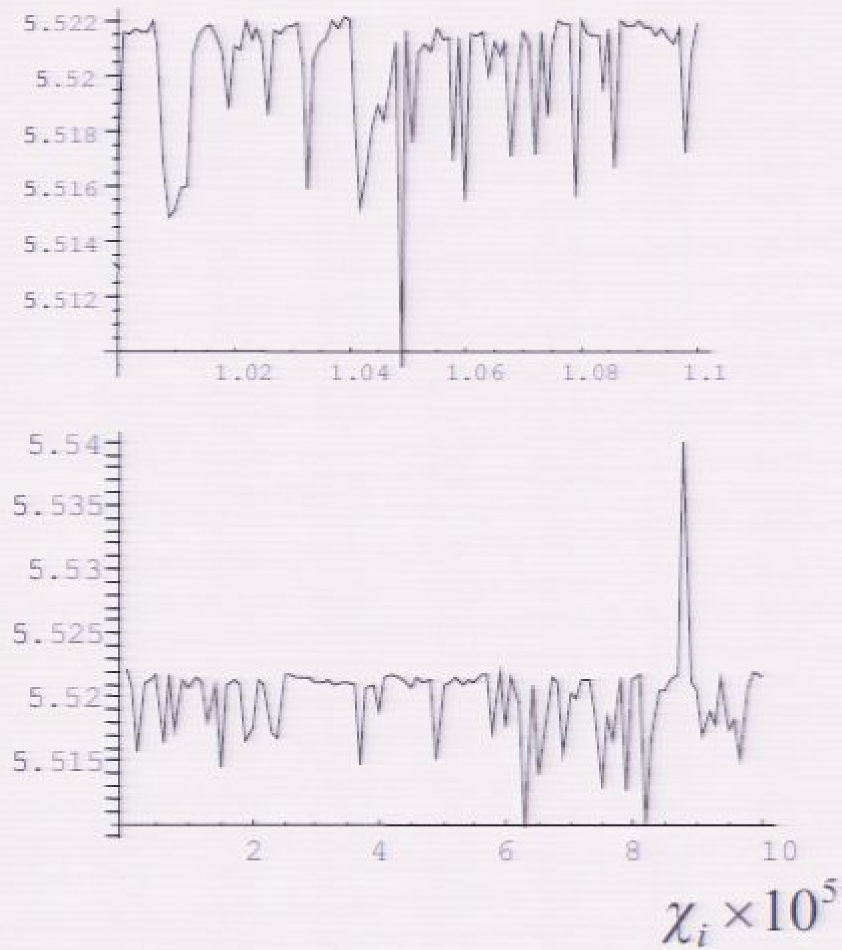

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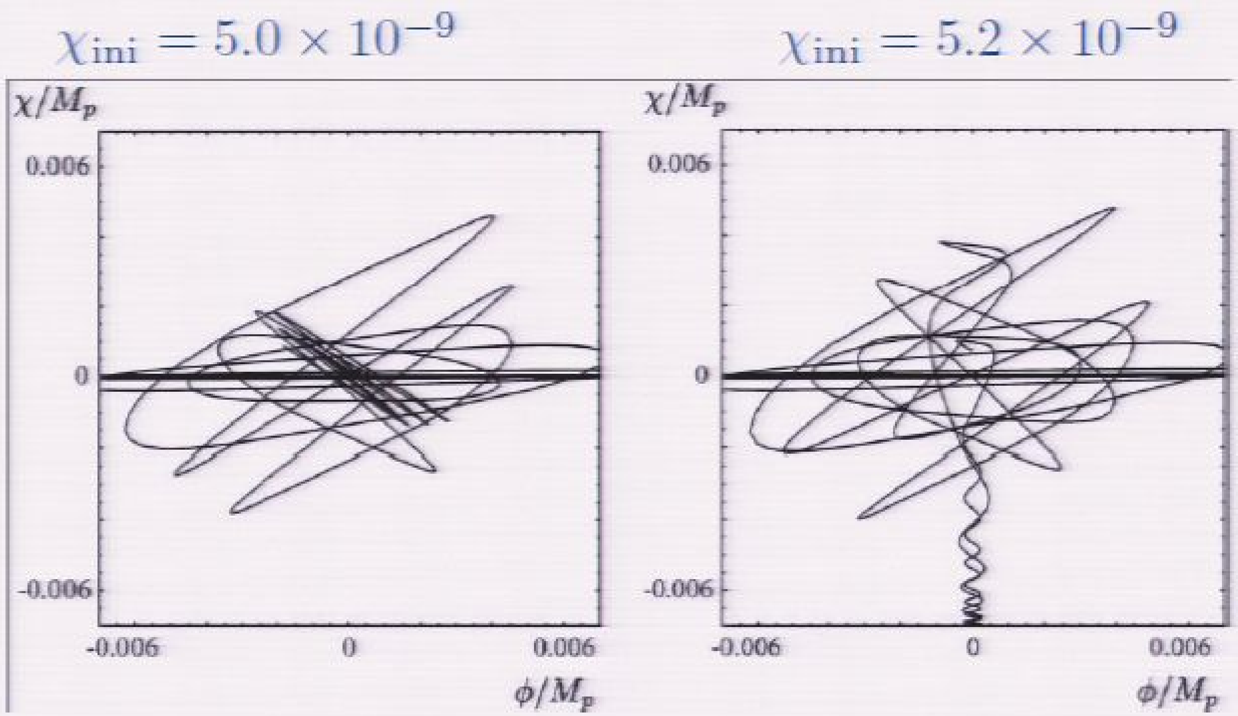

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Inhomogeneous Fields

- Non-equilibrium field dynamics:
  Inhomogeneous modes $k > H$ important
- Make ϕ and χ position-dependent (Khlebnikov&Tkachev 1996)

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\vec{\nabla}^2\phi + \lambda\phi^3 + g^2\chi^2\phi &= 0 \\ \ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\vec{\nabla}^2\chi + g^2\phi^2\chi &= 0 \end{aligned}$$

- Couple to Friedmann eq with average energy density

$$\begin{aligned} H^2 &= \frac{1}{3M_{\rm Pl}^2 V}\int d^3x \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}(\vec{\nabla}\chi)^2 \right. \\ & \left. + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2\right) \end{aligned}$$

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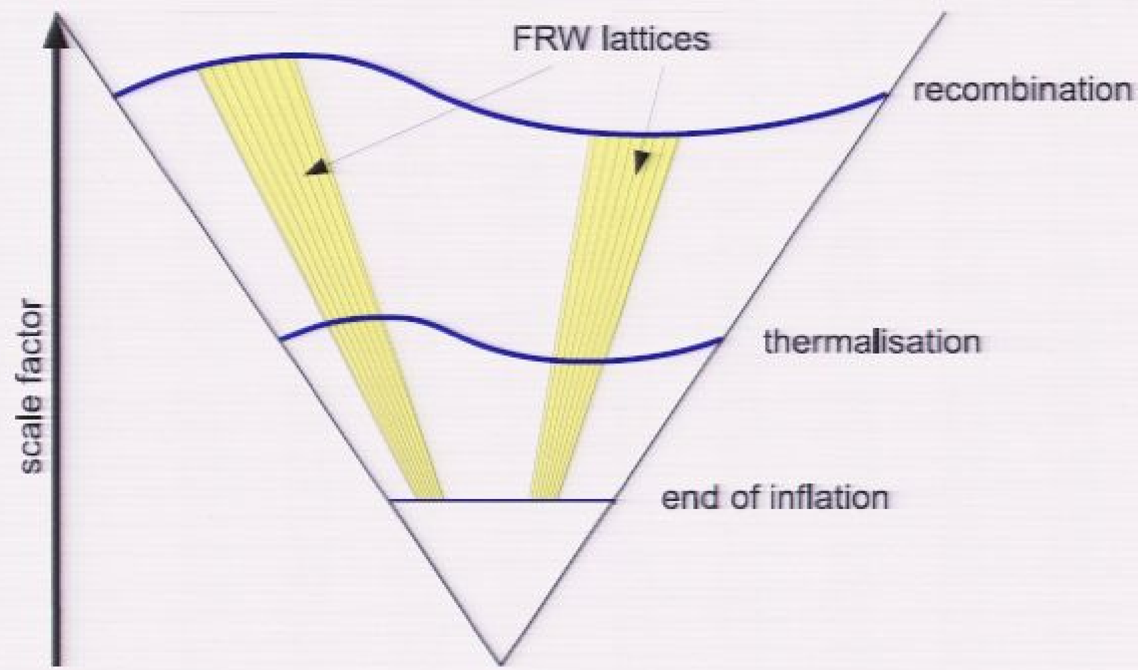
Inhomogeneous Fields



- Calculate dependence on the initial super-horizon modes

  $\approx$ Averages over the lattice $\chi_{\rm ini} \equiv \overline{\chi}(t=0)$
- Frozen modes at the end of inflation

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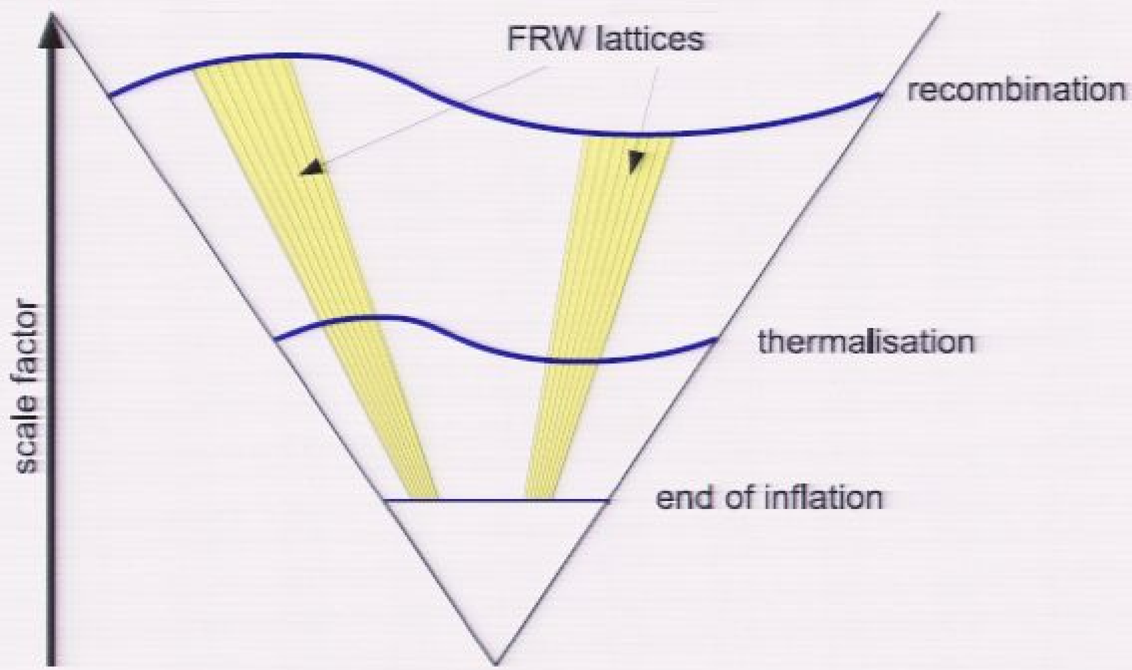
Inhomogeneous Fields



- Dynamical scales must be smaller than the lattice $m^{-1} \ll L$
- Lattice must be smaller than the horizon $L \ll H^{-1}$
- Wavelength of perturbations must be longer than the horizon $H^{-1} \ll k^{-1}$

Quantum Initial Conditions

- Start time: Early enough during inflation so that $m_\chi \ll H$
- Gaussian fluctuations with the same two-point function as the quantum vacuum (Khlebnikov&Tkachev 1996)

$$\overline{|\chi_k|^2} = \frac{1}{V}\frac{1}{2\omega_k}, \quad \overline{|\dot{\chi}_k|^2} = \frac{1}{V}\frac{\omega_k}{2}$$

- Linear dynamics: quantum = classical
- Non-linear dynamics: quantum $\approx$ classical

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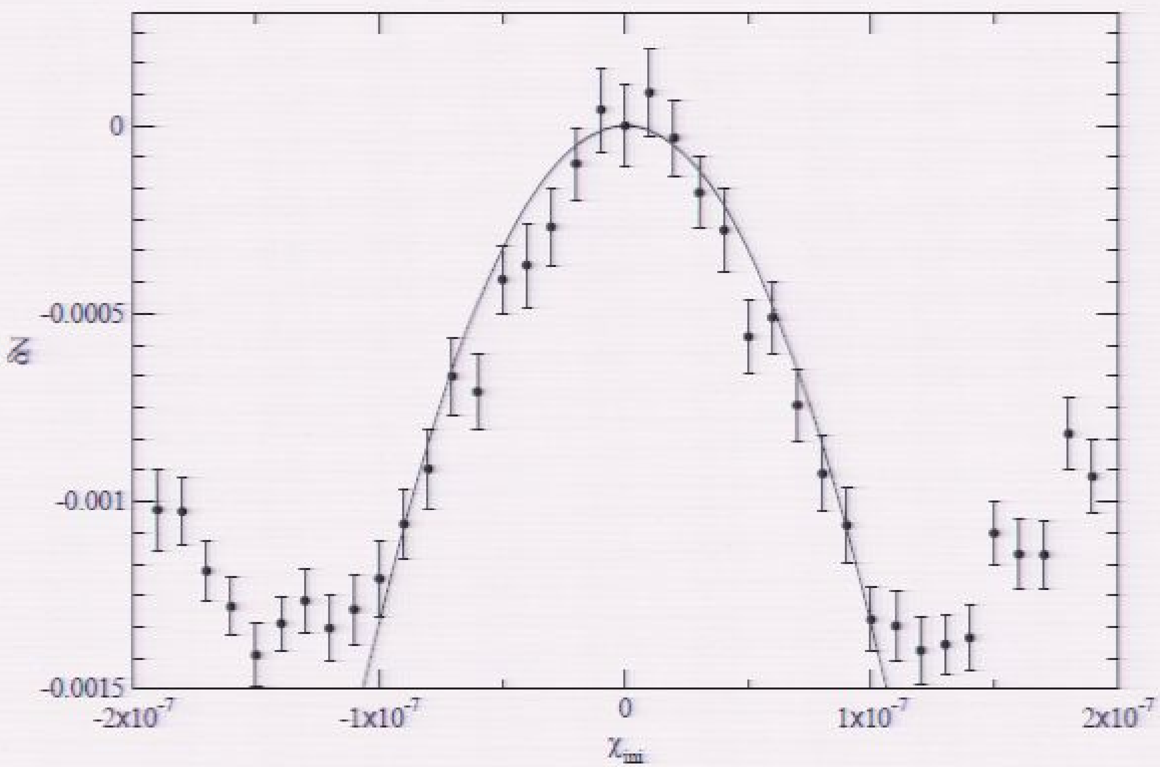
First Data



- $g^2/\lambda = 1.875$, measured at $H = 5.53 \times 10^{-12} M_{\rm Pl}$ (Chambers&Rajantie 2007)

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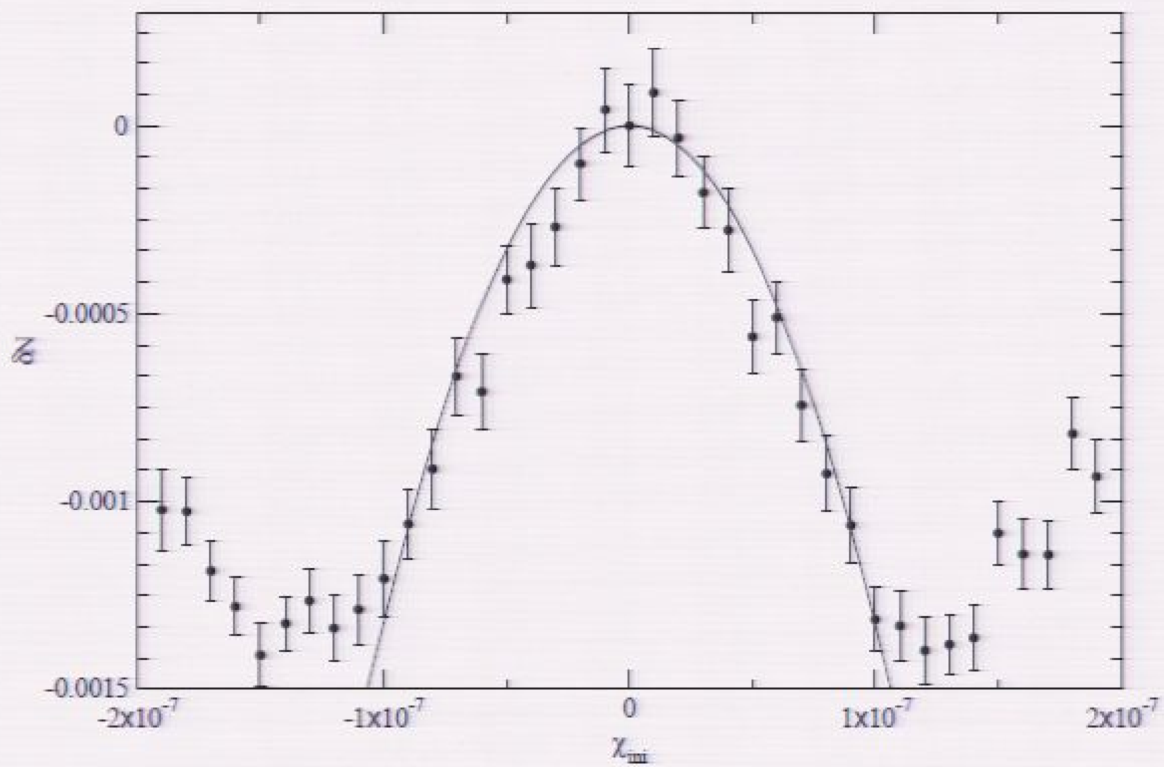

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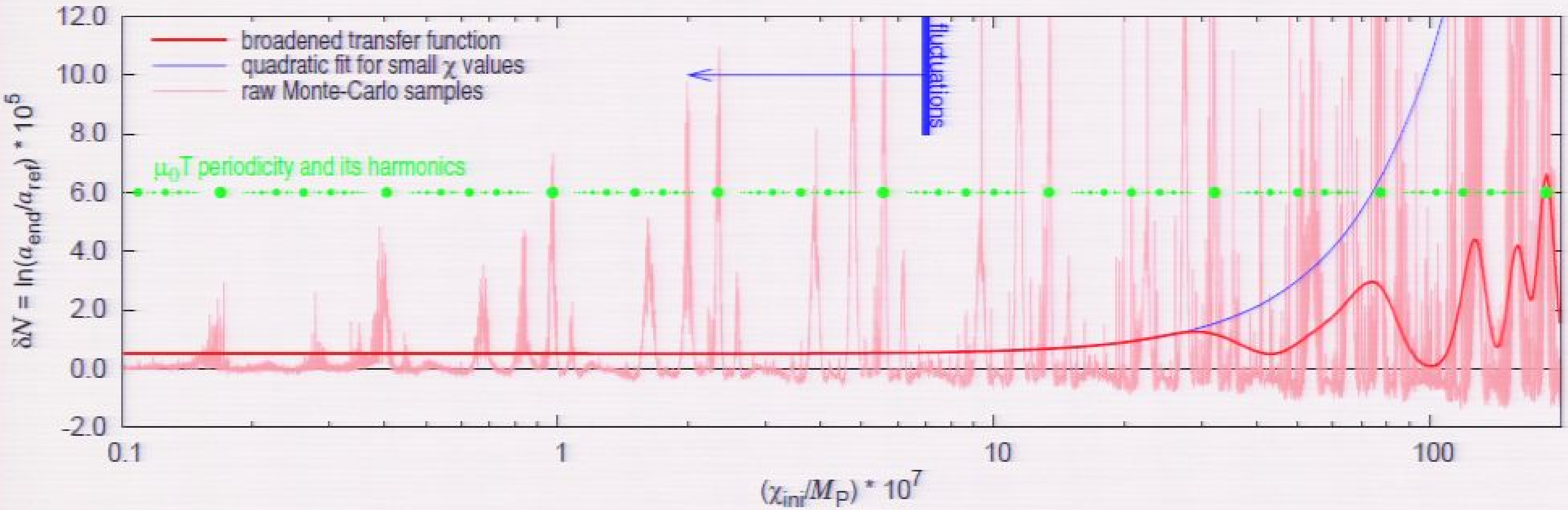
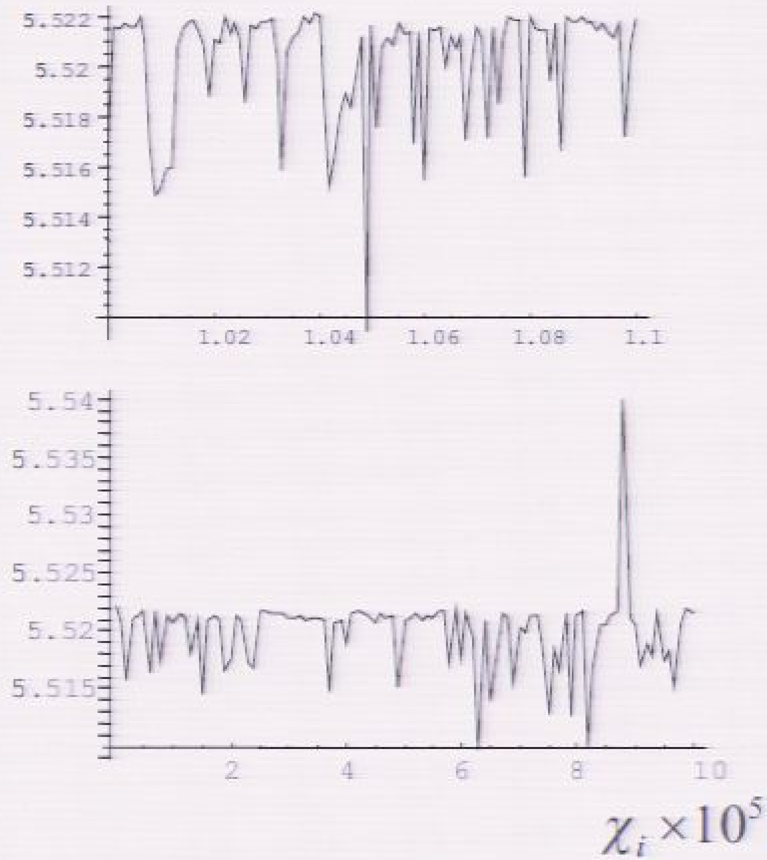
Numerical Error



(Bond et al 2009)

- Bond et al 2009: Corrected a numerical error
- Now three different algorithms agree
- Chaotic behaviour

Earlier Results



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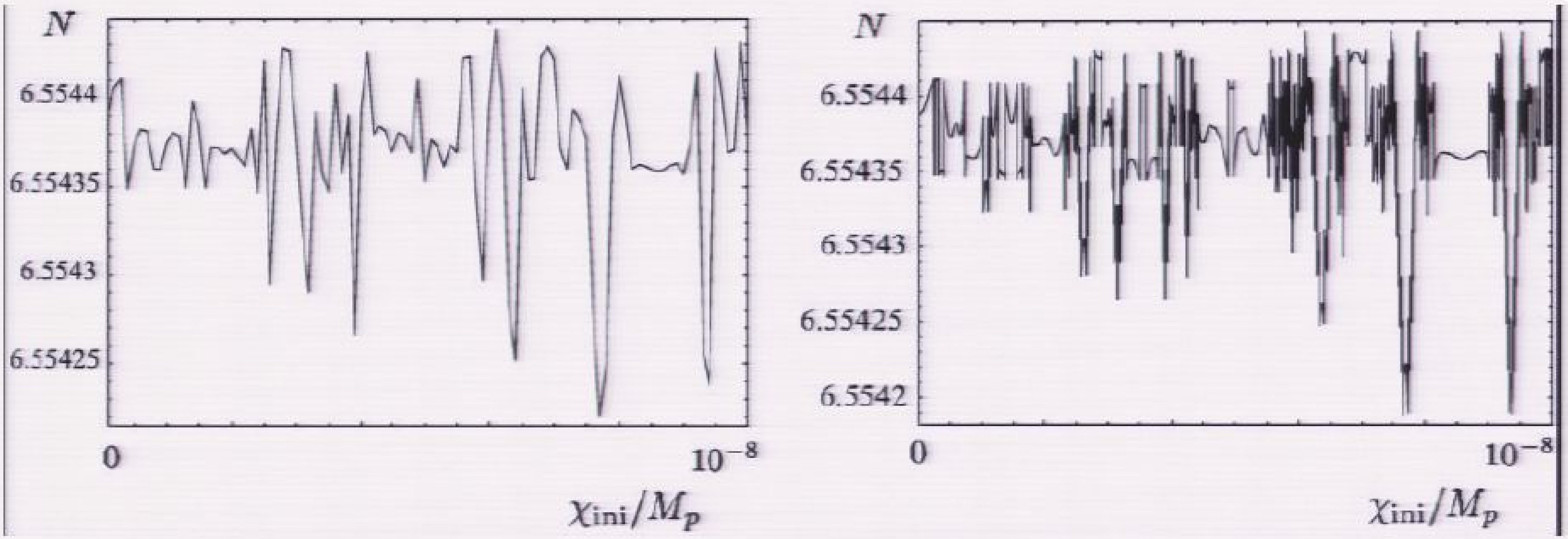

(Suyama and Yokoyama 2006)

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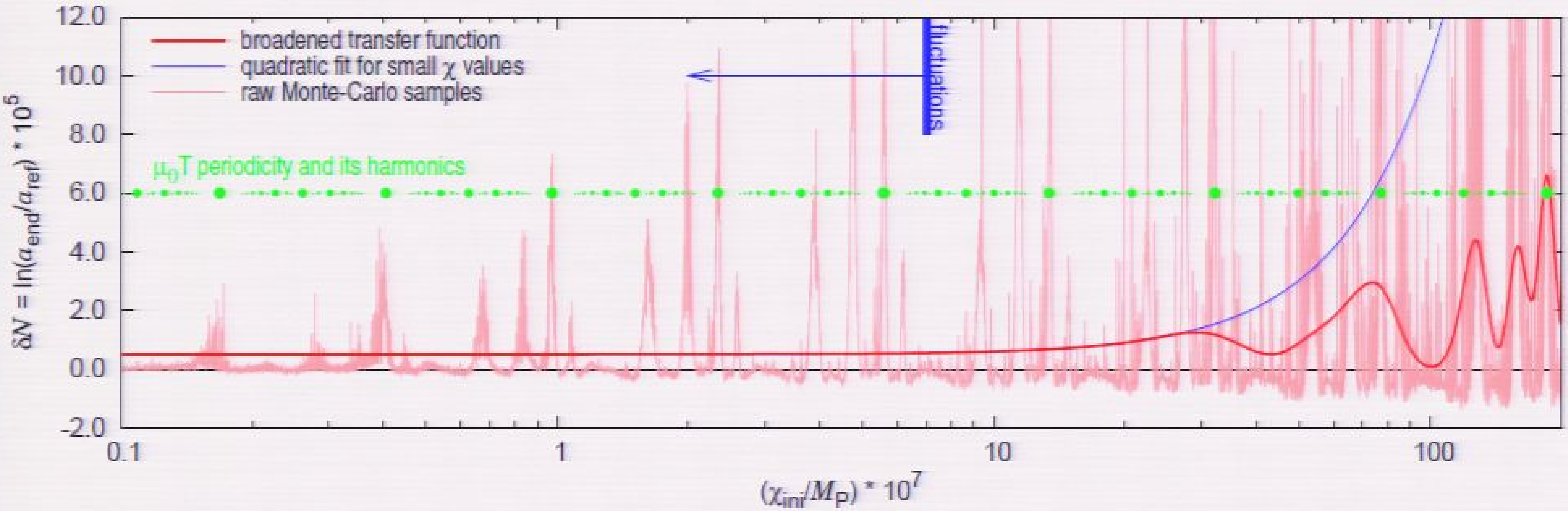
Interpreting the Data



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- Smear the peaks (Bond et al 2009)
- Quadratic fit $N''(\chi_{\rm ini}) \approx 4\times 10^6 M_{\rm Pl}^{-2}$ for $g^2/\lambda = 2$

  $\Rightarrow f_{\rm NL} \sim 10000$, well above observational bounds

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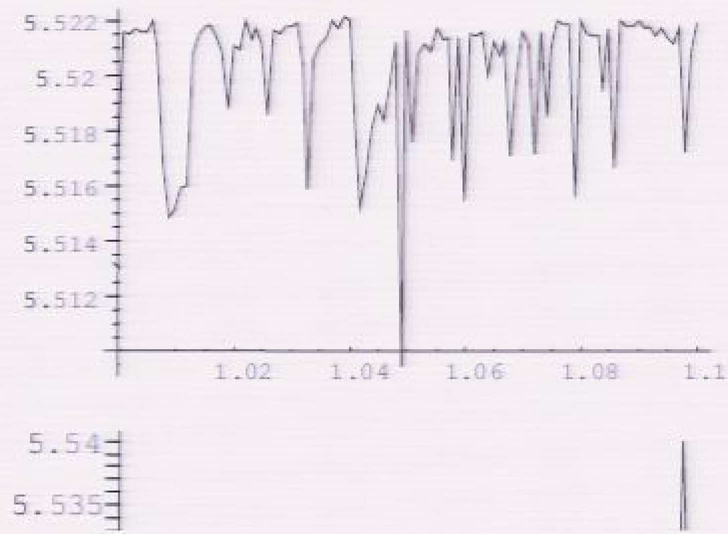
Homogeneous Fields



- No sub-horizon dynamics

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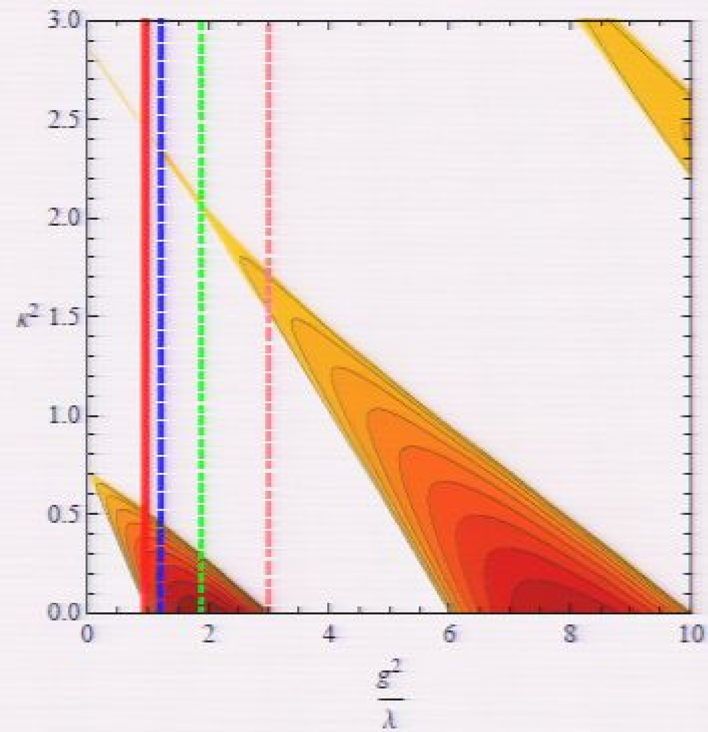
Resonance Bands

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(\tau; 1/\sqrt{2})\right]\tilde{\chi}_k = 0$$



- $\tilde{\chi}_k(\tau) = e^{\mu x} f(\tau)$ with periodic $f(\tau)$

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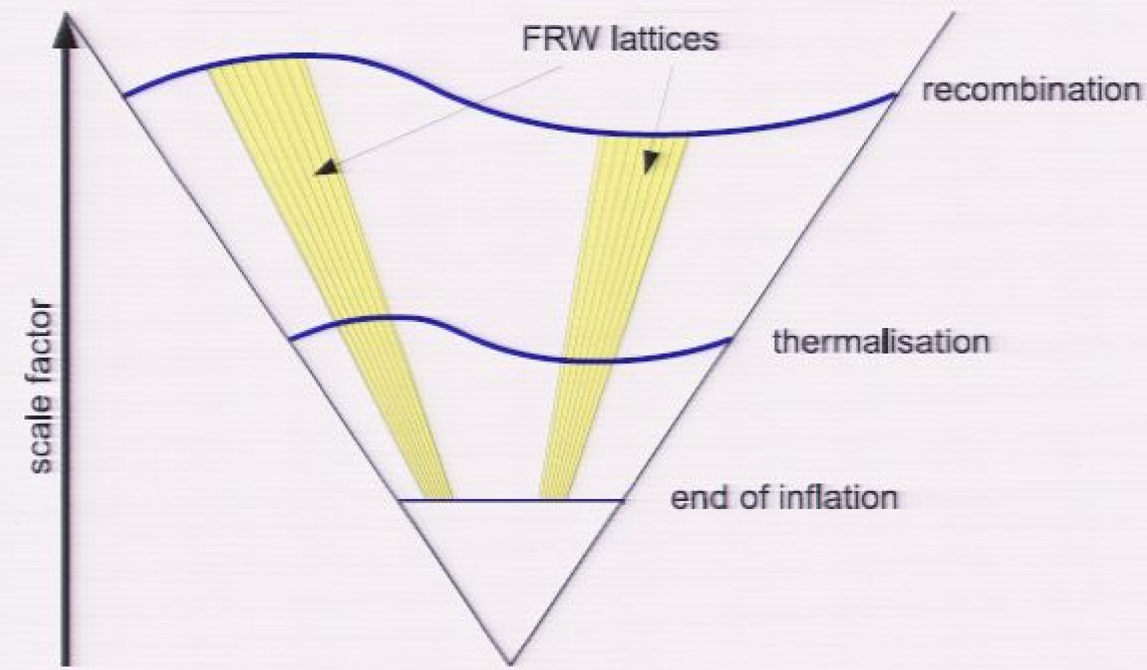
Inhomogeneous Fields



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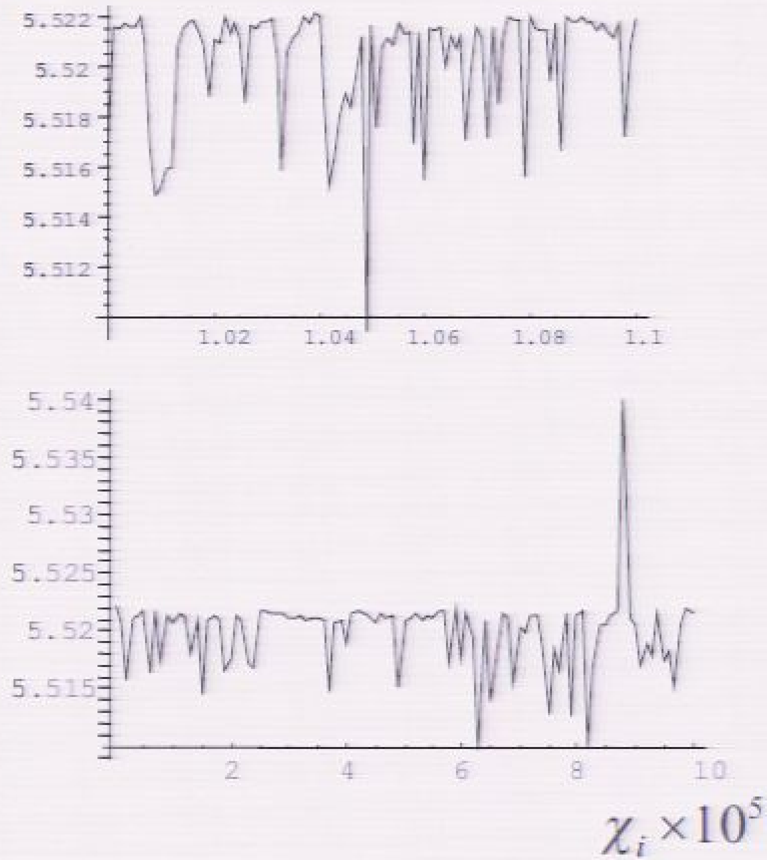

(Bassett and Tanaka 2003)

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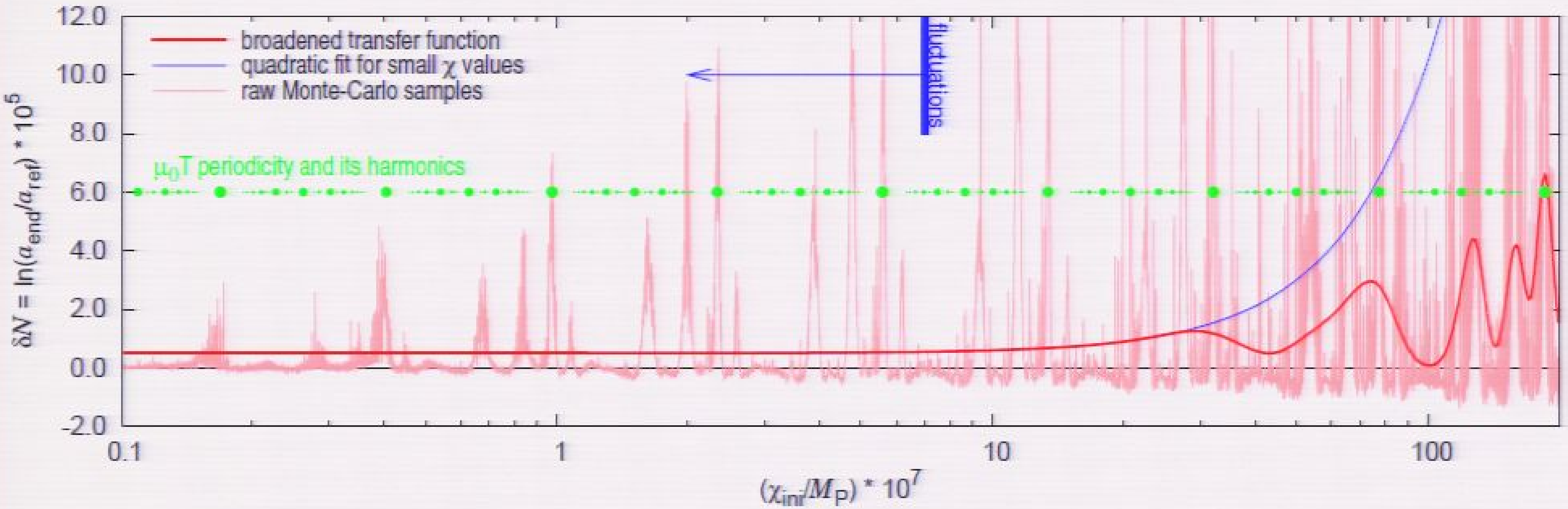

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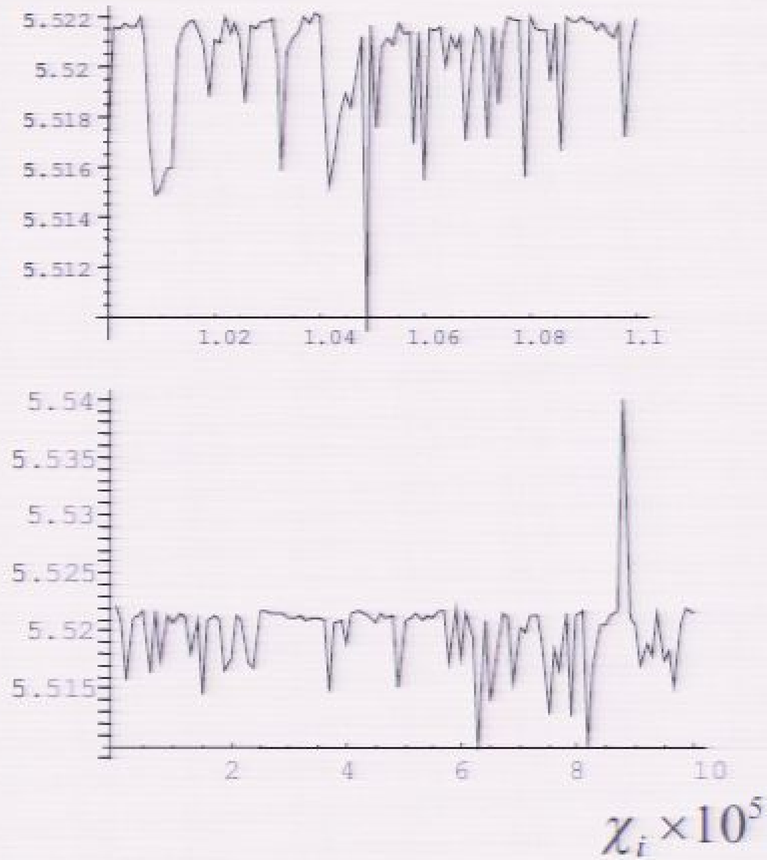

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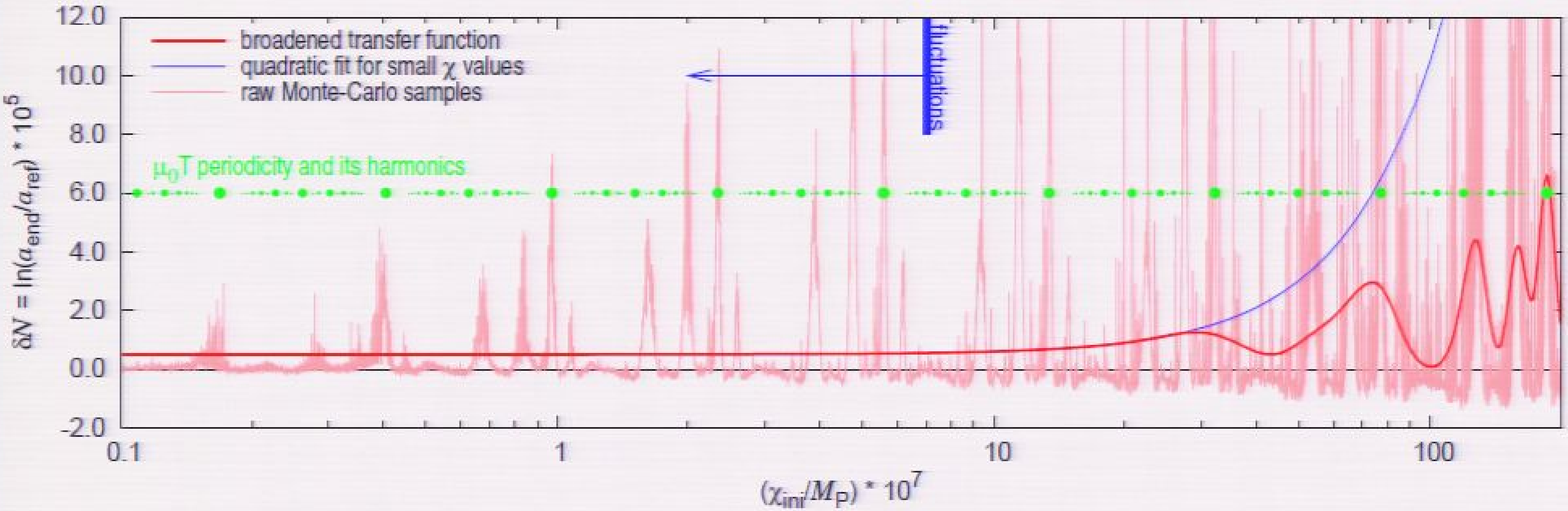

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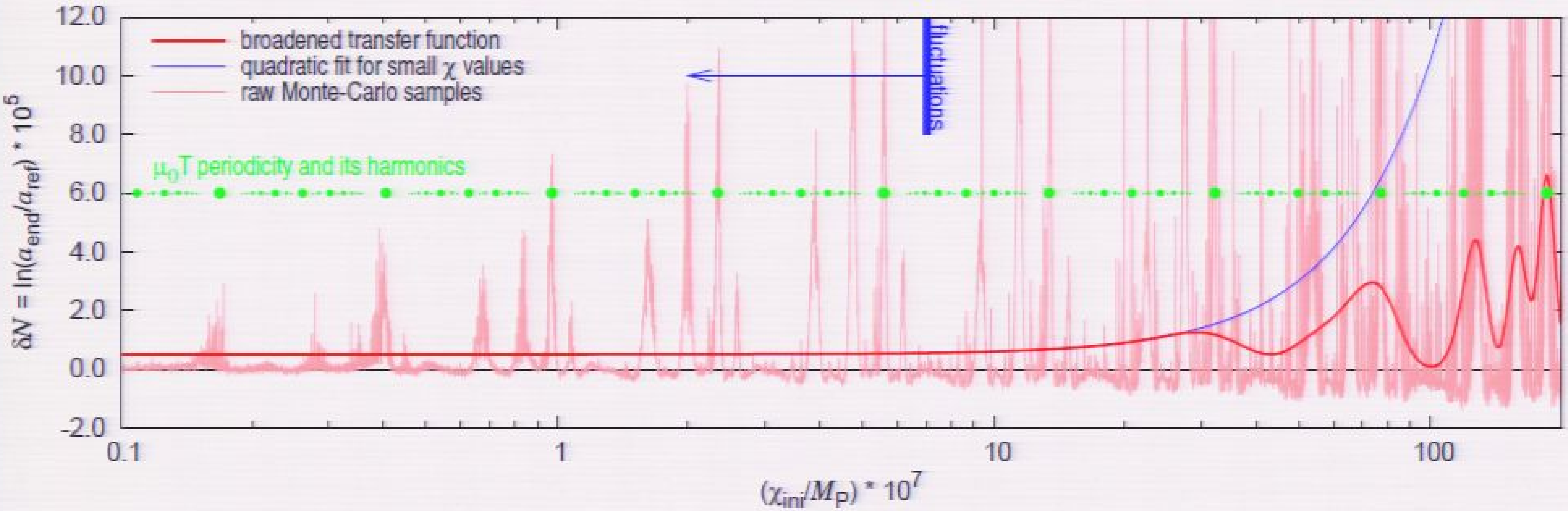

(Bond et al 2009)

- Non-Gaussian features: Cold spots

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Example 2: Resonant Curvaton Decay

$$V(\phi, \chi, \sigma) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}m^2\chi^2 + \frac{1}{2}g^2\chi^2\sigma^2$$

- Curvaton χ coupled to another scalar σ: Parametric resonance (Enqvist et al 2009)
- χ starts to oscillate when $H = m$

$$\chi(t) \sim \frac{\chi_{\rm ini}}{(mt)^{3/4}} \sin(mt + \pi/8)$$

- Resonates with σ

$$n_\sigma(t) \sim \frac{k_*^3}{a^3\sqrt{\mu m t}} e^{2m\mu t}, \quad \mu \approx 0.14$$

- Becomes non-linear at $a = a_{\rm br}$ when $n_\sigma \approx m^2\chi/g$
  - Some fraction $0 < \xi < 1$ of curvatons survive: Decay perturbatively later

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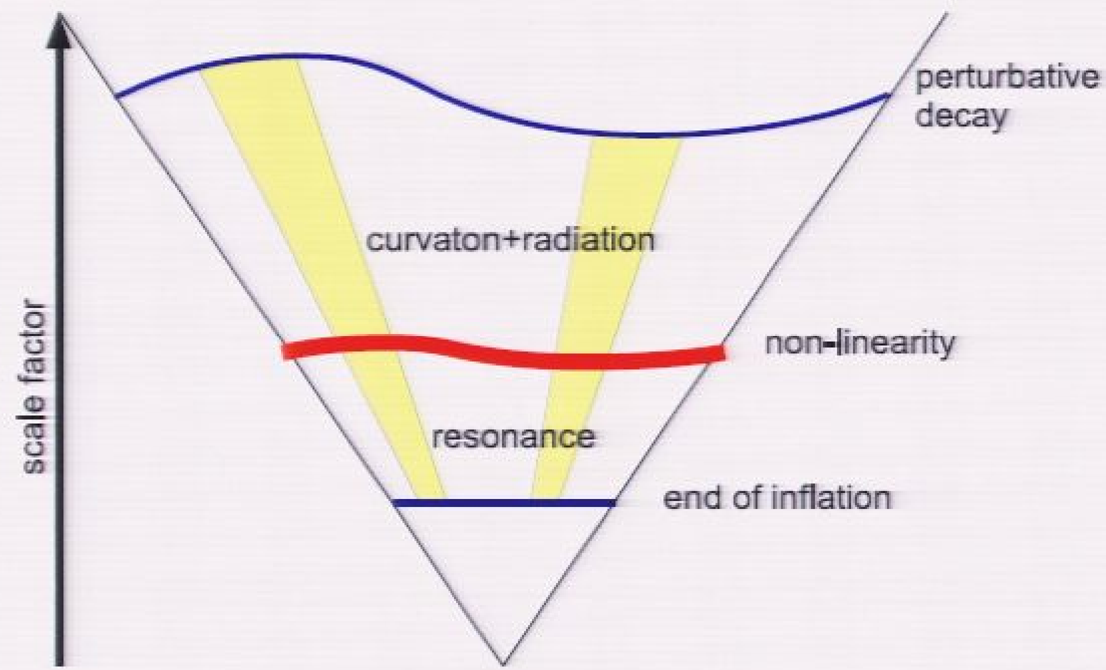
Example 2: Resonant Curvaton Decay



- Jump in curvaton density at $a_{\rm br}$
- Depends on $\chi \Rightarrow$ Curvature perturbation

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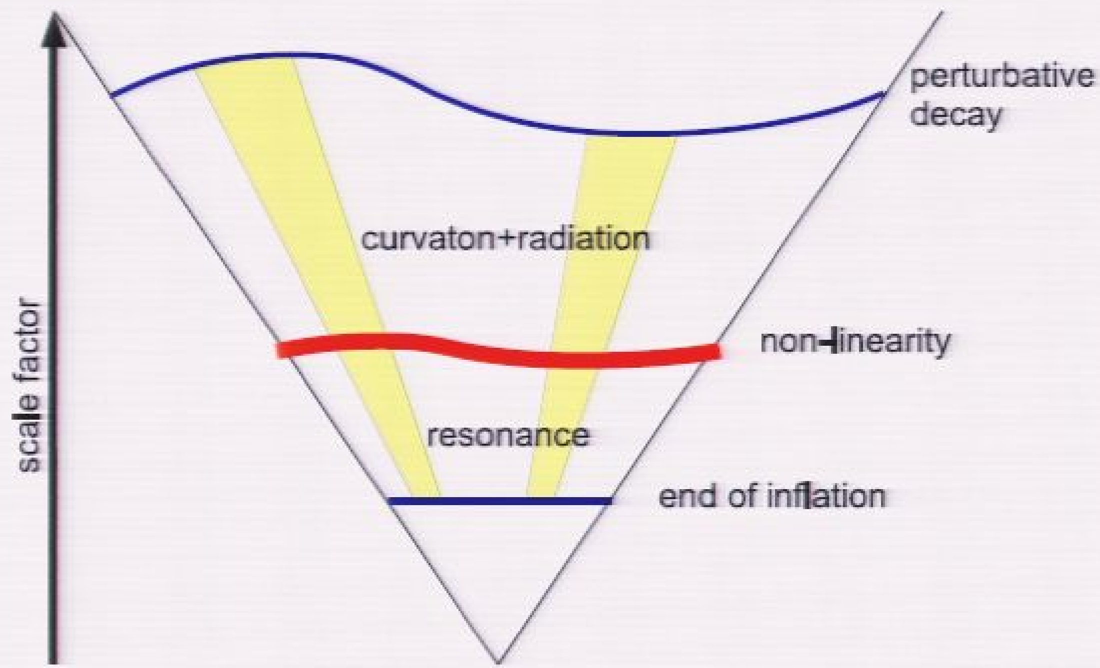
Numerical Calculation



- Too much expansion: Need a huge comoving lattice

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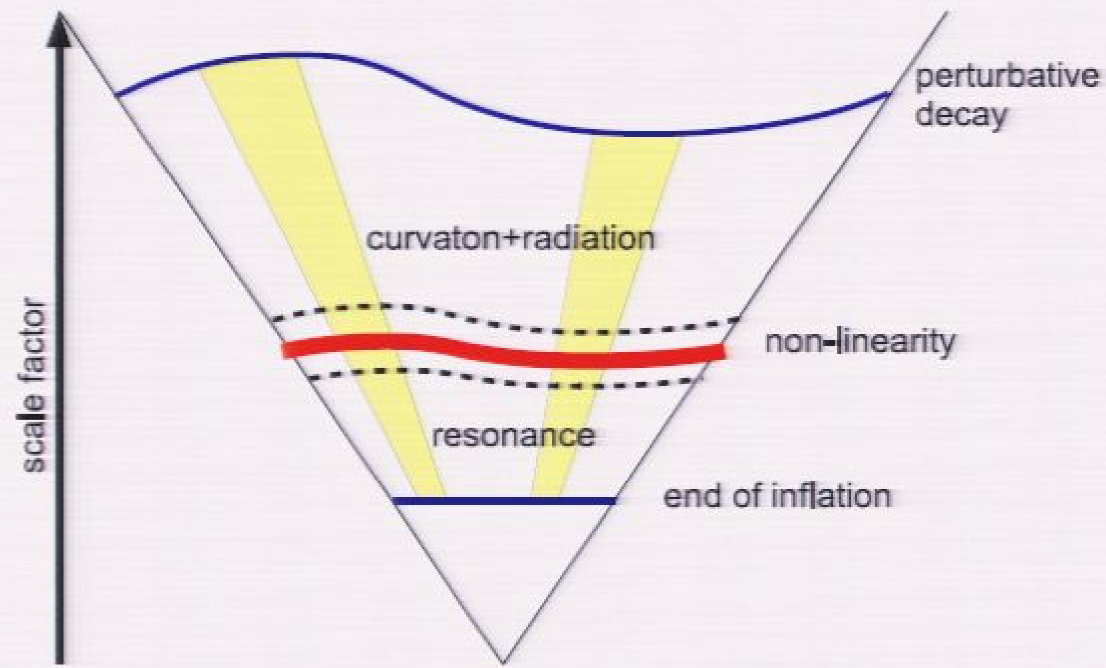
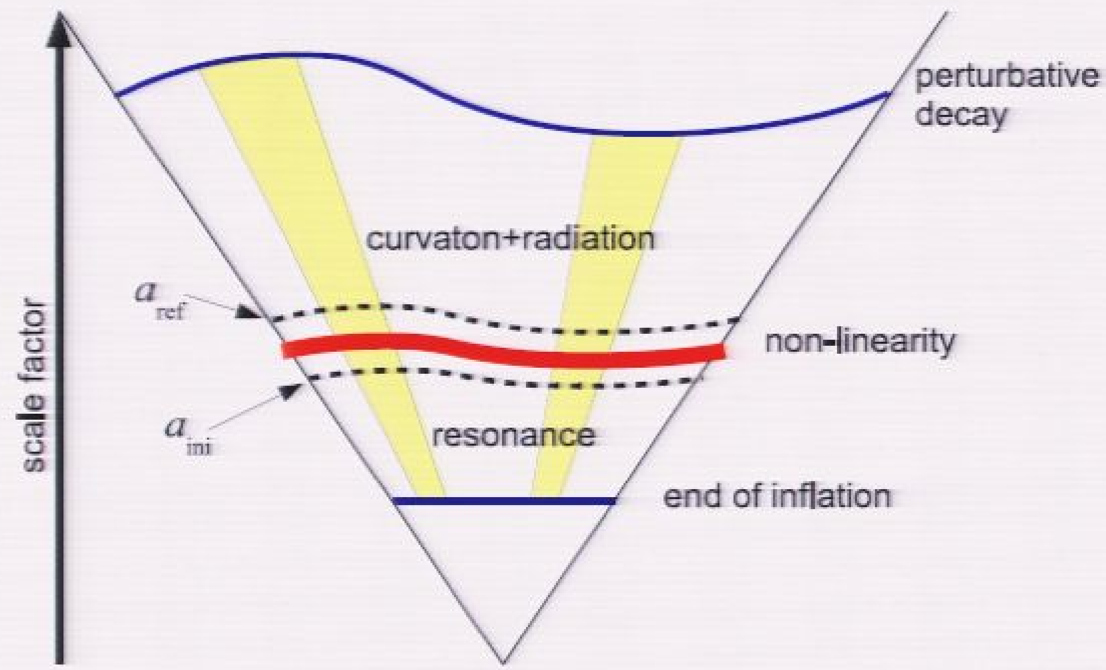
Numerical Calculation



- Too much expansion: Need a huge comoving lattice
- Stop simulation when equilibrated: Assume matter+radiation afterwards
- Start simulation shortly before non-linearity: Use linear theory until then

Numerical Calculation



- Measure a_{ref}, ρ_{ref} and r_{ref} at some arbitrary time after non-linearity
- Total expansion $a_{\text{ref}}/a_{\text{ini}} \sim 10$

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Calculating Perturbations

- Measure $a_{\rm ref}$, $\rho_{\rm ref}$ and $r_{\rm ref}$ at some arbitrary time after non-linearity
- Extrapolate to perturbative decay time:

$$
\begin{aligned}
N' &= (\ln a_{\rm ref})' + \frac{1}{4}\left[\frac{\rho'_{\rm ref}}{\rho_{\rm ref}} + \frac{r\rho'_{\rm ref}}{4\rho_{\rm ref}} + (r - r_{\rm ref})\frac{r'_{\rm ref}}{r_{\rm ref}}\right] \\
N'' &= (\ln a_{\rm ref})'' + \frac{1}{4}\left[\frac{\rho''_{\rm ref}}{\rho_{\rm ref}} - \left(\frac{\rho'_{\rm ref}}{\rho_{\rm ref}}\right)^2 + \frac{r}{4}\left(\frac{\rho''_{\rm ref}}{\rho_{\rm ref}} - \frac{3}{4}\left(\frac{\rho'_{\rm ref}}{\rho_{\rm ref}}\right)^2\right)\right. \\
&\quad \left. + \frac{r'_{\rm ref} r}{2 r_{\rm ref}}\frac{\rho'_{\rm ref}}{\rho_{\rm ref}} + (r - r_{\rm ref})\frac{r''_{\rm ref}}{r_{\rm ref}}\right]
\end{aligned}
$$

$$
\mathcal{P}_\zeta = (N')^2 \mathcal{P}_\chi, \quad f_{\rm NL} = \frac{5}{6}\frac{N''}{(N')^2}
$$

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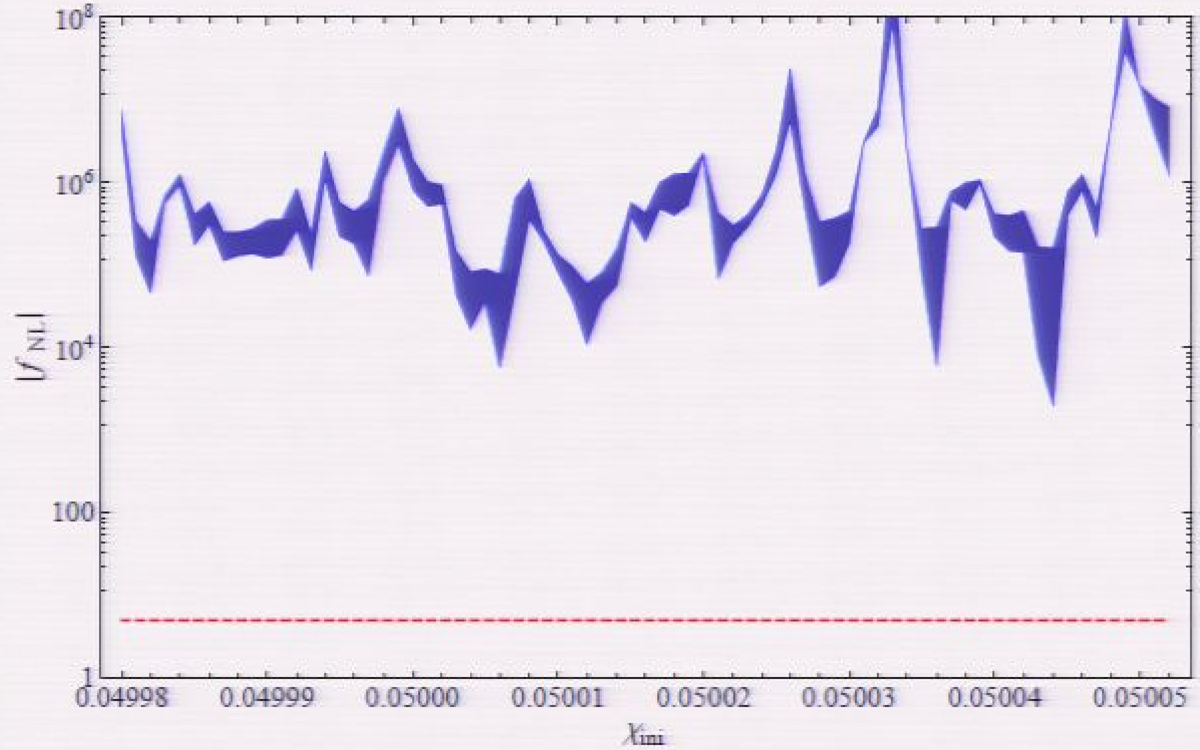
Simulations



- $f_{\rm NL}$ from simulations, compared with the perturbative result (red)
- Parameters $g = 5 \times 10^{-7}$, $\lambda = 10^{-15}$, $m = 10^{-11} M_{\rm Pl}$
- $r_{\rm decay}$ determined from the observed amplitude

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Conclusions

- Non-linear calculation of curvature perturbation due to non-equilibrium physics
  - Works with any (bosonic) field dynamics:
    - Other preheating models
    - Phase transitions (e.g. hybrid inflation)
    - Ekpyrotic bounce - Effective field theory?
- Massless preheating:
  - Possibly observable effect $\Delta\zeta \sim 10^{-5}$
  - Highly non-linear
    - How can we constrain it?
    - Perturbation theory could not work
- Curvaton resonance:
  - Amplifies perturbations
  - Enhances non-Gaussianity