Title: World sheets for world sheets revisited

Date: Jul 09, 2009 10:00 AM

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Abstract: TBA

WORLD SHEETS

FOR

WORLD SHEETS

REVISITED.

* WORLD SHEET FOR WORLD SHEETS

GREEN

* SYMMETRIES, DUALITIES, SURPRISES

THE EFFECTIVE TARGET SPACE

I.E.

REPRODUCES ALL OBSERVABLES.

$c=26$ STRING THEORY

$$\mathcal{L}_{eff} = \int d^D x \sqrt{g^{(26)}} \left( R^{(26)} + \ldots\ldots ? \right)$$

$\sigma$ MODEL WAS FLAT SPACE

WHAT IS ?

$c=26 \qquad M^{24,1} \times S^1_R$

$$\int$$

$M^{24,1} \times S^1_R$ ?

$M^{24,1} \times S^1_{1/R}$ ?

DUALITIES MANIFEST

T

MORE TOPOLOGIES ?

SOMETHING ELSE ? $(AdS)_5 \times S^5 \leftrightarrow$ $N=4$ $D=4$ SUSY

MORE UNIVERSES ?

NON PERTURBATIVE SYMMETRIES ?

S DUALITY

# CLOSED STRING SECTOR

* S. ELITZUR, A. FORGE, E.R. ([redacted])
NPB 388 (1992)

WITTEN, BAULIEU-SINGER.

MONTANO-SONNENSCHEIN

OPEN + CLOSED

* S. ELITZUR, Y. OZ, E.R, J. WALCHER
SOON

HORAVA

SHATHASVILI

WITTEN

HORI

EFFECTIVE THEORY

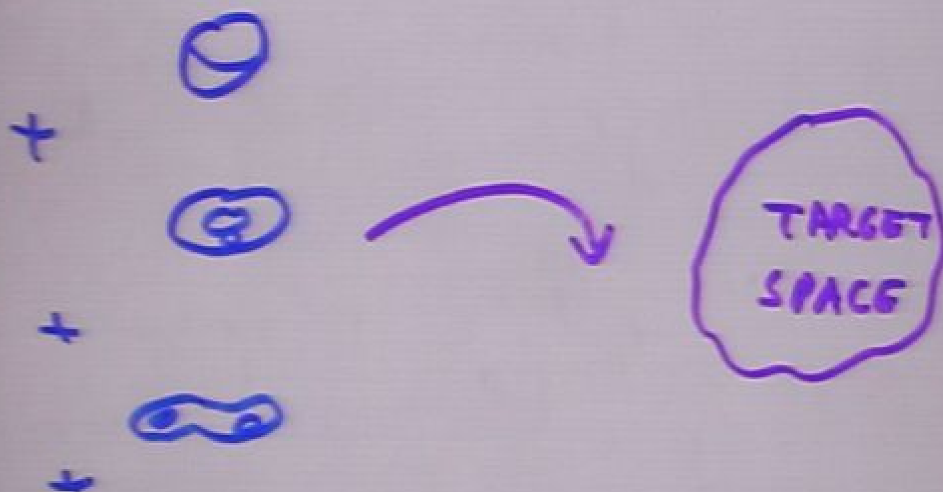
* DOES TFT LEAD TO EFFECTIVE TFT ?

* $c = 26$

$\vdots$

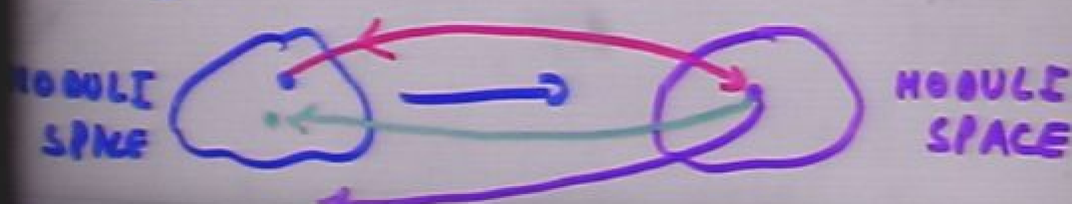
CLOSED TOPOLOGICAL

MATTER STRING

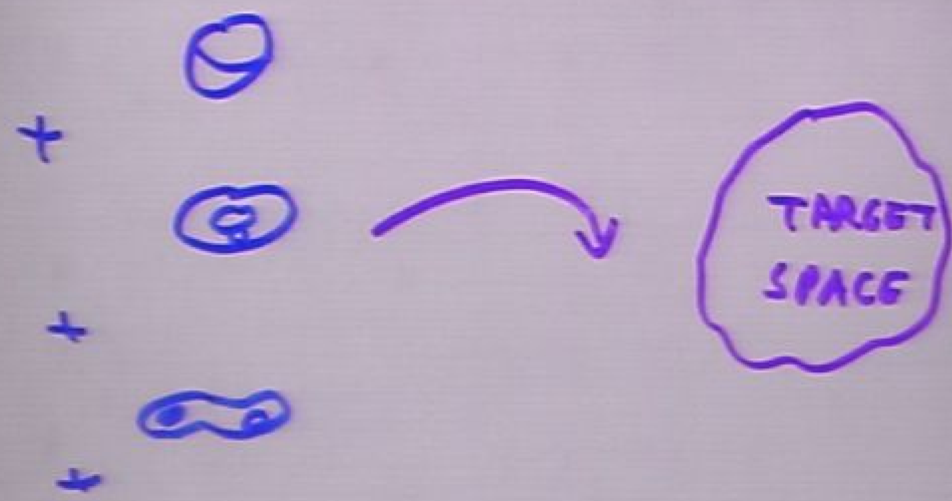



SYMMETRIES PERT., NON-PERT.

$$\sum_{g} TFT \xrightarrow{?} TFT$$



MATTER STRING

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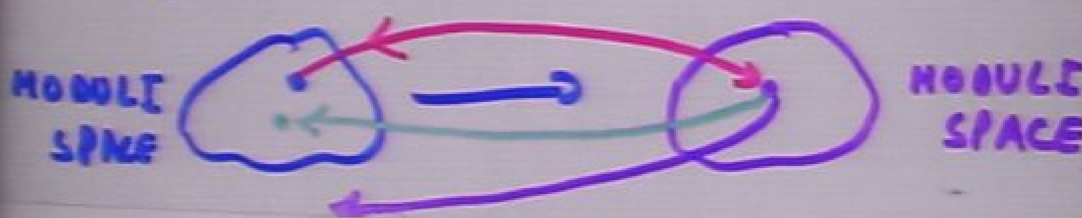
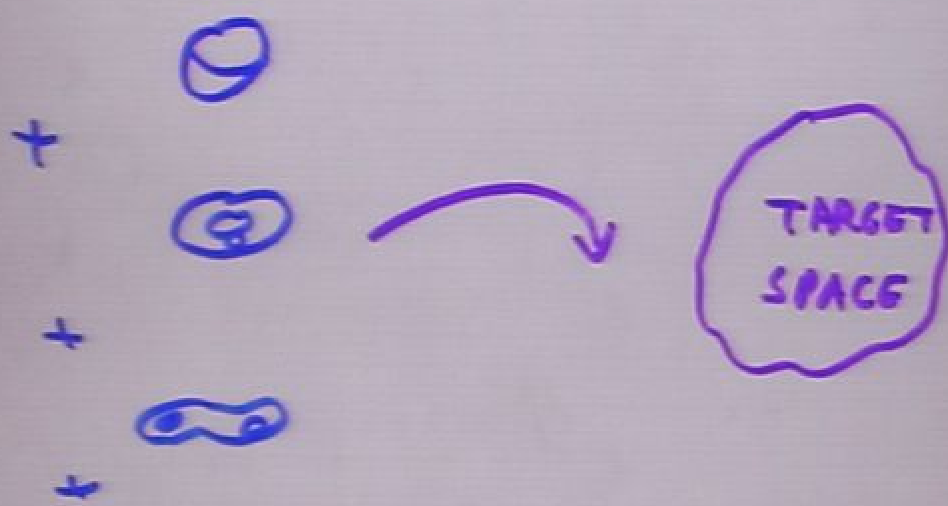


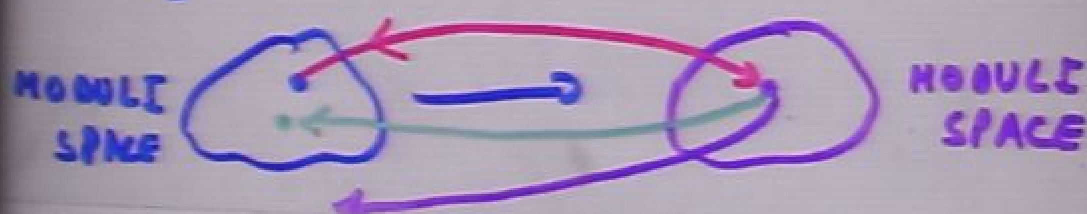
# CLOSED TOPOLOGICAL

MATTER STRING




SYMMETRIES PERT., NON-PERT.

$$\sum_{g} \text{TFT} \xrightarrow{?} \text{TFT}$$



# DATA

* LIST OF BRST INVARIANT OPERATORS

$$O_\alpha(x) \qquad \alpha = 1, \ldots, N$$

* THE TWO POINT FUNCTIONS

$$\langle O_\alpha(x_1)\, O_\beta(x_2) \rangle_{g=0} = \eta_{\alpha\beta}$$

NO $x_1, x_2$ DEPENDENCE

* OPERATOR PRODUCT EXPANSIONS

$$O_\alpha(x)\, O_\beta(y) = \sum C^{\gamma}_{\alpha\beta}\, O_\gamma(y)$$

ENOUGH $\quad N, \eta_{\alpha\beta}, C^{\gamma}_{\alpha\beta}$

* FOR A GIVEN $g$

$$\underbrace{\langle \theta_1 \cdots\cdots \theta_n \rangle_g}_{n} \longrightarrow \sum \underbrace{\langle \theta_1 \cdots\cdots \theta'_{n-1} \rangle_g}_{n-1} \#$$

$$n \longrightarrow n-1$$

** $g \to g-1$

$$\langle \theta_1 \cdots \theta_n \rangle_g \longrightarrow \sum \langle \theta_1 \cdots\cdots \theta_n \theta_{n+1}' \rangle_{g-1} \#$$

*

$g \to g$

$n \to n-1$

$$\langle \Theta_{\alpha_1}(x_1) \cdots \Theta_{\alpha_{n-1}}(x_{n-1}) \Theta_{\alpha_n}(x_n) \rangle_g$$

$$C^{\alpha'_{n-1}}_{\alpha_{n-1}\,\alpha_n} \Theta_{\alpha'_{n-1}}(x'_n)$$

NO $x_i$ DEPENDENCE.

$g \to g \qquad n \to n-1$

GO ALL THE WAY $g \to g$ $\langle\ \rangle \to$ NUMBERS

I.E.

$$\langle \Theta_\alpha(x_1) \Theta_\beta(x_2) \Theta_\gamma(x_3) \rangle_{g=0} = C^{\delta}_{\alpha\beta} \underbrace{\langle \Theta_\delta(x_2) \Theta_\gamma(x_3) \rangle_{g=0}}_{\eta_{\delta\gamma}}$$

$$= C^{\delta}_{\alpha\beta} \eta_{\delta\alpha} = C_{\alpha\beta\gamma}$$

$n \to n-1$

$$\langle \theta_{\alpha_1}(x_1) \cdots \underbrace{\theta_{\alpha_{n-1}}(x_{n-1})\, \theta_{\alpha_n}(x_n)}_{C_{\alpha_{n-1}\alpha_n}^{\alpha'_{n-1}}\, \theta_{\alpha'_{n-1}}(x_n')} \rangle_g$$

NO $X_i$ DEPENDENCE.

$g \to g \qquad n \to n-1$

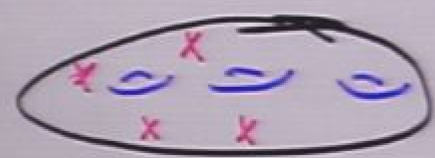
GO ALL THE WAY $g \to g$ $\langle\ \rangle \to$ NUMBERS

I.E.

$$\langle \theta_\alpha(x_1)\, \theta_\beta(x_2)\, \theta_\gamma(x_3) \rangle_{g=0} = C_{\alpha\beta}^{\delta} \underbrace{\langle \theta_\delta(x_2)\, \theta_\gamma(x_3) \rangle}_{\eta_{\delta\gamma}}$$

$$= C_{\alpha\beta}^{\delta}\, \eta_{\delta\gamma} = C_{\alpha\beta\gamma}$$

$* \quad g \longrightarrow g-1$



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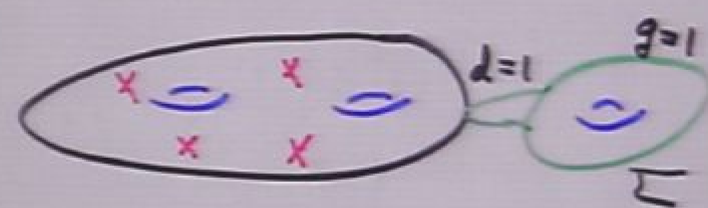
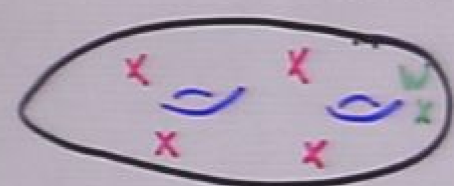



$\|$ INTEGRATE OVER $\Sigma$

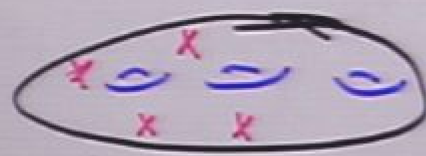



x "FAR" FROM x

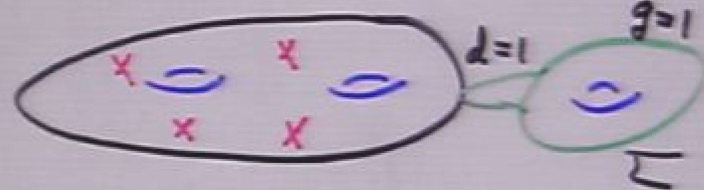
FIND w

w IS UNIVERSAL DOES NOT DEPEND ON $\theta_i$ OR ON $g$ IN

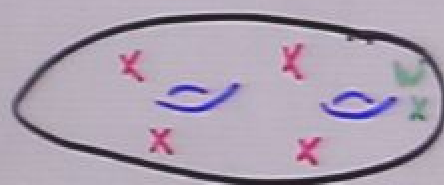
* $g \to g-1$



$\parallel$




$\parallel$ INTEGRATE OVER $\Sigma$




x "FAR" FROM x

FIND W

W IS UNIVERSAL DOES NOT DEPEND ON $\theta_i$ OR ON $g$ IN

$\langle \theta_1 \cdots \theta_n \rangle_g$

$$W = \sum_{\sigma=1}^{N} w^{\sigma} \theta_{\sigma}$$

FIND THEM

ASSUME FOUND

$$\langle \overbrace{\theta_1 \cdots \theta_n}^{A} \rangle_g = \langle \theta_1 \cdots \theta_n W \rangle_{g-1}$$

$$= \sum_{\sigma=1}^{N} w^{\sigma} \langle \theta_1 \cdots \theta_n \theta_\sigma \rangle_{g-1} =$$

$$= \langle A w^{?} \rangle_0$$

FUNCTIONS OF $N$, $\eta_{\alpha\beta}$, $C^{\gamma}_{\alpha\beta}$, $w^{\sigma}$

$w^{\sigma}$ ARE FUNCTIONS OF $\eta_{\alpha\beta}$, $C^{\gamma}_{\alpha\beta}$.

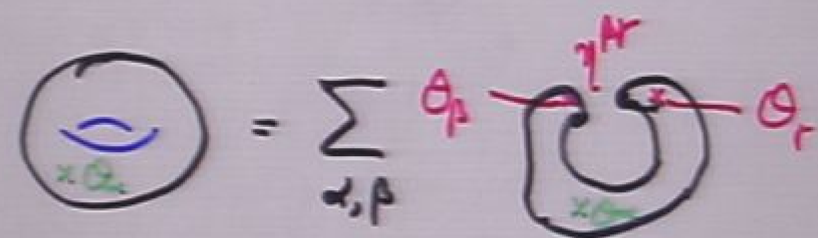
FIND $w^\sigma$

*

$$\langle \theta_\alpha \rangle_1 = \langle \theta_\alpha W \rangle_0 = \sum_{\sigma=1}^{N} w^\sigma \langle \theta_\alpha \theta_\sigma \rangle_0 =$$

$$= \sum_{\sigma=1}^{N} w^\sigma \eta_{\alpha\sigma} = w_\alpha$$

**




FACTORIZATION ON NON-CONTRACTABLE LINE

$$\langle \theta_\alpha \rangle_1 = \eta^{\beta\gamma} \langle \theta_\alpha \theta_\beta \theta_\gamma \rangle_0 = \eta^{\beta\gamma} C_{\alpha\beta}^{\delta} \eta_{\delta\gamma} =$$

$$= C_{\alpha\beta}^{\delta} \eta_\gamma{}^{\beta} = C_{\alpha\beta}^{\beta} \equiv Tr(\theta_\alpha)$$

$$\Rightarrow \quad \boxed{w_\alpha = C_{\alpha\beta}^{\beta} \equiv Tr(\theta_\alpha)}$$

FIND $w^\sigma$

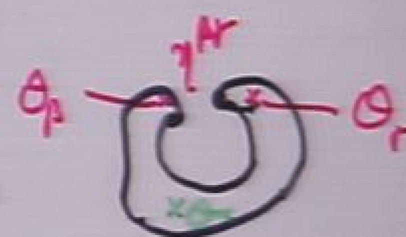
*

$$\langle\theta_\alpha\rangle_1 = \langle\theta_\alpha W\rangle_0 = \sum_{\sigma=1}^{N} w^\sigma \langle\theta_\alpha\theta_\sigma\rangle_0$$

$$= \sum_{\sigma=1}^{N} w^\sigma \eta_{\alpha\sigma} = w_\alpha$$

**


$$= \sum_{\alpha,\beta}$$




FACTORIZATION ON NON-CONTRACTABLE LINE

$$\langle\theta_\alpha\rangle_1 = \eta^{\beta\gamma}\langle\theta_\alpha\theta_\beta\theta_\gamma\rangle_0 = \eta^{\beta\gamma} C_{\alpha\beta}^{\delta}\,\eta_{\delta\gamma} =$$

$$= C_{\alpha\beta}^{\delta}\,\eta_{\delta}{}^{\beta} = C_{\alpha\beta}{}^{\beta} \equiv Tr(\theta_\alpha)$$

$$\Rightarrow \quad \boxed{w_\alpha = C_{\alpha\beta}{}^{\beta} \equiv Tr(\theta_\alpha)}$$

$$\langle A \rangle_g = \mathrm{Tr}\left( A W^{g-1} \right)$$

$$\langle A \rangle_0 = \langle A W^{-1} W \rangle_0 = \langle A W^{-1} \rangle_1 = \mathrm{Tr}\left( A W^{-1} \right)$$

IF W INVERTIBLE

$$\langle A \rangle_{\mathrm{EXACT}} = \sum_{g=0}^{\infty} \lambda^{g-1} \mathrm{Tr}\left( A W^{g-1} \right) =$$

$$* \quad = \mathrm{Tr}\left( A \, (\lambda W)^{-1} \frac{1}{1 - \lambda W} \right)$$

N OPERATORS $\Longrightarrow$ N MODULI

DATA: $N$, $C_{\alpha\beta}^{\gamma}$, $\eta_{\alpha\beta}$ DETERMINE THEORY

THEORY HAS MANY EQUIVALENT DATA.

* CONSTRAINTS ON $C_{\alpha\beta}^{\gamma}$

s-t DUALITY $C_{\alpha\beta}^{\sigma} C_{\sigma\gamma}^{\delta} = C_{\alpha\gamma}^{\sigma} C_{\sigma\beta}^{\delta}$

* FOR BOSONS: SYMMETRY

$$\langle \theta_\alpha \theta_\beta \theta_\gamma \rangle_0 = C_{\alpha\beta\gamma} = C_{\alpha\beta}^{\delta} \eta_{\delta\gamma}$$

SYMMETRIC IN $\alpha, \beta, \gamma$.

CONSIDER

$$\mathcal{O}'_\alpha = \sum_\beta S_\alpha^{\ \beta}\, \mathcal{O}_\beta$$

$S$ NON SINGULAR

SAME TFT DIFFERENT $C_{\alpha\beta}^{\ \ \gamma}, \eta_{\alpha\beta}$

BACK TO COUNTING MODULI

DEFINING N MATRICES

$$(C_\alpha)^\gamma_{\ \beta} = C_{\alpha\beta}^{\ \ \gamma}$$

s-t DUALITY MEANS THEY COMMUTE

IN GENERIC CASE

THEY CAN BE SIMULTANEOUSLY

DIAGONALIZED BY SOME S

IN THIS BASIS

$$C_{\alpha\beta}^{\gamma} = \lambda_{\alpha\beta}\,\delta_{\beta}^{\gamma} = \lambda_{\beta\alpha}\,\delta_{\alpha}^{\gamma}$$

BY SYMMETRY OF $\alpha, \beta, \gamma$

$$\lambda_{\alpha\beta} = \lambda_{\alpha}\,\delta_{\alpha\beta}$$

$$\theta_{\alpha}\, O_{\beta} = \lambda_{\alpha}\,\delta_{\alpha\beta}\,\theta_{\alpha}$$

USE S'

* $\theta_{\alpha}\,\theta_{\beta} = \delta_{\alpha\beta}\,\theta_{\alpha}$

* $\eta_{\alpha\beta} = \langle \theta_{\alpha}\theta_{\beta} \rangle_{0} = \delta_{\alpha\beta}\langle \theta_{\alpha} \rangle_{0} = \delta_{\alpha\beta}\,\eta_{\alpha}$

THEY CAN BE SIMULTANEOUSLY

DIAGONALIZED BY SOME S

IN THIS BASIS

$$C_{\alpha\beta}^{\gamma} = \lambda_{\alpha\beta}\delta^{\gamma}_{\beta} = \lambda_{\beta\alpha}\delta^{\gamma}_{\alpha}$$

BY SYMMETRY OF $\alpha,\beta,\gamma$

$$\lambda_{\alpha\beta} = \lambda_{\alpha}\delta_{\alpha\beta}$$

$$\Theta_{\alpha}\Theta_{\beta} = \lambda_{\alpha}\delta_{\alpha\beta}\Theta_{\alpha}$$

USE S'

* $$\Theta_{\alpha}\Theta_{\beta} = \delta_{\alpha\beta}\Theta_{\alpha}$$

* $$\eta_{\alpha\beta} = \langle\Theta_{\alpha}\Theta_{\beta}\rangle_{0} = \delta_{\alpha\beta}\langle\Theta_{\alpha}\rangle_{0} = \delta$$

THUS WE HAVE N PARAMETER

$\langle \Theta_\alpha \rangle_0 \equiv \eta_\alpha$  N MODULI

* $W = \sum_\alpha \frac{1}{\eta_\alpha} \Theta_\alpha$

* $\langle \Theta_\alpha^n \rangle_g = \langle \Theta_\alpha \rangle_g = \langle \Theta_\alpha W^g \rangle_0 = \eta_\alpha^{1-}$

$\langle \Theta \text{ OTHER} \rangle_g = 0$

$\mathcal{L}' = \mathcal{L}_0 + \frac{1}{8\pi} \sum_\alpha \mu_\alpha \Theta_\alpha(x) \sqrt{g(x)}\, R^{(2)}(x$

$\mathcal{L}_0$: TFT

$\Theta_\alpha(x)$: 0 FORM

TOPOLOGICAL $\sigma$ MODEL ON $CP^1$

TWO PARAMETERS:

* STRING COUPLING - $\lambda$ $\left(\sum \lambda^n \langle\ \rangle\right)$

WINDING NUMBER WEIGHT - $\beta$

* FOR $S^2 - CP^1$

TWO COHOMOLOGIES

1 IDENTITY (CONSTANT MAP)

0 VOLUME (WINDING)

$$w^{-1} = (2\beta)^{-1}\, O \qquad \beta \neq 0$$

$$\langle 1 \rangle_{exact} = \frac{1}{\lambda} \mathrm{Tr}\left( w^{-1} \frac{1}{1-\lambda w} \right)$$

$$(C_2)_\alpha^{\ \beta} = O = \begin{pmatrix} 0 & 1 \\ \beta & 0 \end{pmatrix}$$

* $$\langle 1 \rangle_{exact} = \frac{2}{1-4\lambda^2\beta}$$

* $$\langle O^{2n} \rangle_{exact} = \frac{2\beta^n}{1-4\lambda^2\beta}$$

* $$\langle O^{2n+1} \rangle_{exact} = \frac{\beta^n}{\lambda(1-4\lambda^2\beta)}$$

NOTE:

$$\tilde{\Theta} \equiv 2\lambda\Theta$$

$$\langle \tilde{\Theta}^{2n+1} \rangle_{exact} = \langle \tilde{\Theta}^{2n} \rangle_{exact} =$$

$$= 2\frac{\tilde{\lambda}^{2n}}{1-\tilde{\lambda}^2} \qquad \tilde{\lambda} \equiv 2\lambda\rho^{1/2}$$

MORE GENERAL:

$$\mathcal{L} = \mathcal{L}_{CP^1} + (\mu_1 1 + \mu_2 \Theta)\sqrt{g}\, R$$

JUST RESCALES $\lambda$

$\mu_2 = \mu$

NOTE:

$$\tilde{\Theta} \equiv 2\lambda\Theta$$

$$\langle \tilde{\Theta}^{2n+1} \rangle_{exact} = \langle \tilde{\Theta}^{2n} \rangle_{exact} =$$

$$= 2\frac{\tilde{\lambda}^{2n}}{1-\tilde{\lambda}^2} \qquad \tilde{\lambda} \equiv 2\lambda\rho^{1/2}$$

MORE GENERAL:

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JUST RESCALES $\lambda$

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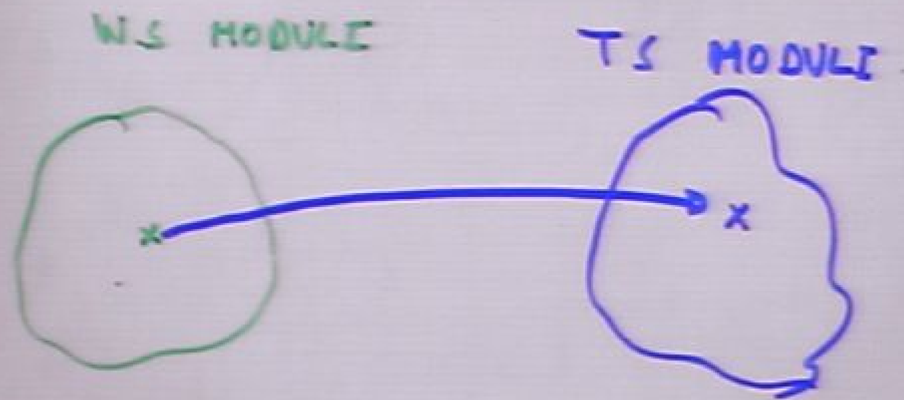
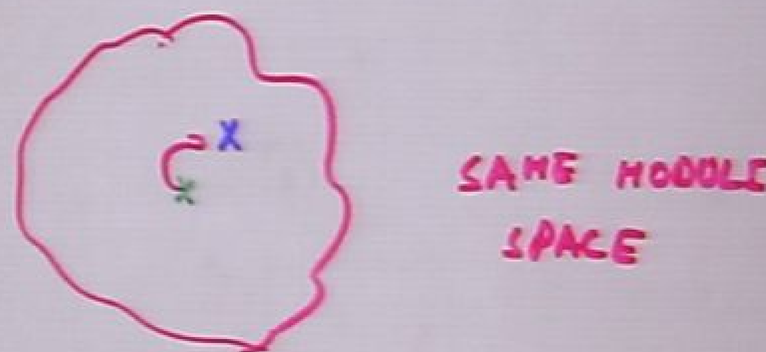



$$\beta_{TS} = \beta_{WS}$$

$$\tilde{\lambda}_{TS} = \tilde{\lambda}_{WS} \frac{\beta_{WS}}{\sqrt{\left(1-\tilde{\lambda}_{WS} \exp(-\tilde{\mu}_{WS})\right)\left(1+\tilde{\lambda}_{WS} \exp(\tilde{\mu}_{WS})\right)}}$$

$$\tilde{\mu}_{TS} = \tilde{\mu}_{WS} + \frac{1}{2} \ln \frac{1+\tilde{\lambda}_{WS} \exp(\tilde{\mu}_{WS})}{1-\tilde{\lambda}_{WS} \exp(-\tilde{\mu}_{WS})}$$

$$\eta_{\pm}^{TS} = \frac{\eta_{\pm\,WS}^{2}}{\eta_{\pm\,WS} - 1}$$

$$\eta_{\pm}^{TS} \geqslant \eta_{\pm}^{WS}$$

ONLY $g=0$ A FIXED POINT (CLASSICAL)

TRUE FOR ANY $N$.

SYMMETRIES.

$$\eta^{\pm} \equiv \frac{1}{\eta_{\pm}}$$

SYMMETRIES

$$\eta^{\pm}_{TS} = \eta^{\pm}_{WS}\left(1-\eta^{\pm}_{WS}\right)$$

* PERTURBATIVE (g by g)

$$\eta^{+}_{WS} \to \eta^{-}_{WS} \qquad \eta^{-}_{WS} \to \eta^{+}_{WS}$$

T DUALITY $\Theta^{+}_{WS} \leftrightarrow \Theta^{-}_{WS}$

LINE OF FIXED POINTS $\eta^{+}_{WS} = \eta^{-}_{WS}$

EXPLICIT T DUALITY

* NON PERTURBATIVE DUALITY

S DUALITY $\eta^{+}_{WS} \to 1-\eta^{+}_{WS} \qquad \eta^{-}_{WS} \to \eta^{-}_{WS}$

(OR BOTH) $\eta^{+}_{WS} \to \eta^{+}_{WS} \qquad \eta^{-}_{WS} = 1-\eta^{-}_{WS}$

THE SUMS DIVERGE FOR $\eta = 1$

SO $\eta \to 1-\eta$

IS LIKE WEAK $\leftrightarrow$ STRONG COUPLING DUALITY

DEFINE

$$\eta^{\pm} = \frac{1}{2(1-\rho_{\pm})}$$

T DUALITY $\rho_+ \to \rho_+ \quad \rho_- \to -\rho_-$

$$\langle (O_{\pm})^n \rangle^{TS} = \frac{4}{1-\rho_{\pm}^2} \qquad \rho_{\pm}^2 = 1$$

POLE

WHAT HAPPENS?

FOR $N>2$ MORE PERMUTATIONS.

* FOR $CP^1$ TS WE TOOK $CP^1$ AS THE WS OF THE EFFCTIVE THEORY. BY CHANGING PARAMETERS OF THE EFFCTIVE THEORY ANY $g$ IS ALLOWED ($g \to \infty$ $\lambda \to$ SINGULARITY).

AMBIGIOUS TOPOLOGIES !

* CONSIDER THE CASE WHERE CORRELATIONS ARE GIVEN BY SUMS OVER BOTH CONNECTED AND DISCONNECTED DIAGRAMS.

$$Z = \prod_{\alpha=1}^{n} \exp\left(\exp\langle \theta_\alpha \rangle \exp\left(I_\alpha\right)\right)$$

CLOSED + OPEN STRINGS

BRANES

TOPOLOGICAL MATTER

"REPEAT"

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_{g,h}$$

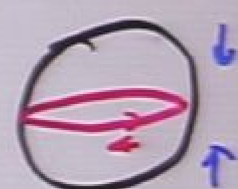
* THE SPHERE

$\mathcal{O}_1, \mathcal{O}_2$ $\quad 1 \quad H$

$$\langle 1 \rangle_{0,0} = 0$$
$$\langle H \rangle_{0,0} = 1$$
$$\langle H^2 \rangle_{0,0} = 0$$
$$\langle H^3 \rangle_{0,0} = \beta$$

* THE DISK

HORI.



* TWO BRANES

$\epsilon = \pm$ WILSON LINE

2 POINTS TS ?

GABERDIEL
ISRAEL, E.A.