Title: Some hidden symmetry in M-theory

Date: Jul 07, 2009 02:00 PM

URL: http://pirsa.org/09070007

Abstract: TBA

Canonical GR

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

α – lapse

β^i – shift

γ_{ij} – spatial metric

Canonical GR

(d+1) spacetime dimensions

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γ_{ij} – spatial metric – d dimensional.

$\alpha = 1 \qquad \beta^i = 0 \qquad$ gauge choice

Global well-defined t

Diffeomorphism constraint $\chi^i = -2D_j\pi^{ij}$

$$H = \gamma^{-1/2}\left(\pi^{ij}\pi_{ij} - \frac{1}{d-1}\pi^2 + \gamma R^{(d)}(\gamma)\right)$$

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Einstein eq^{n}s

$$\ddot{\gamma}_{ij} + \Gamma_{ij}{}^{k\ell,mn}\,\dot{\gamma}_{k\ell}\,\dot{\gamma}_{mn} = -2\left(R_{ij} - \frac{1}{2(d-1)}\gamma_{ij}R\right)$$

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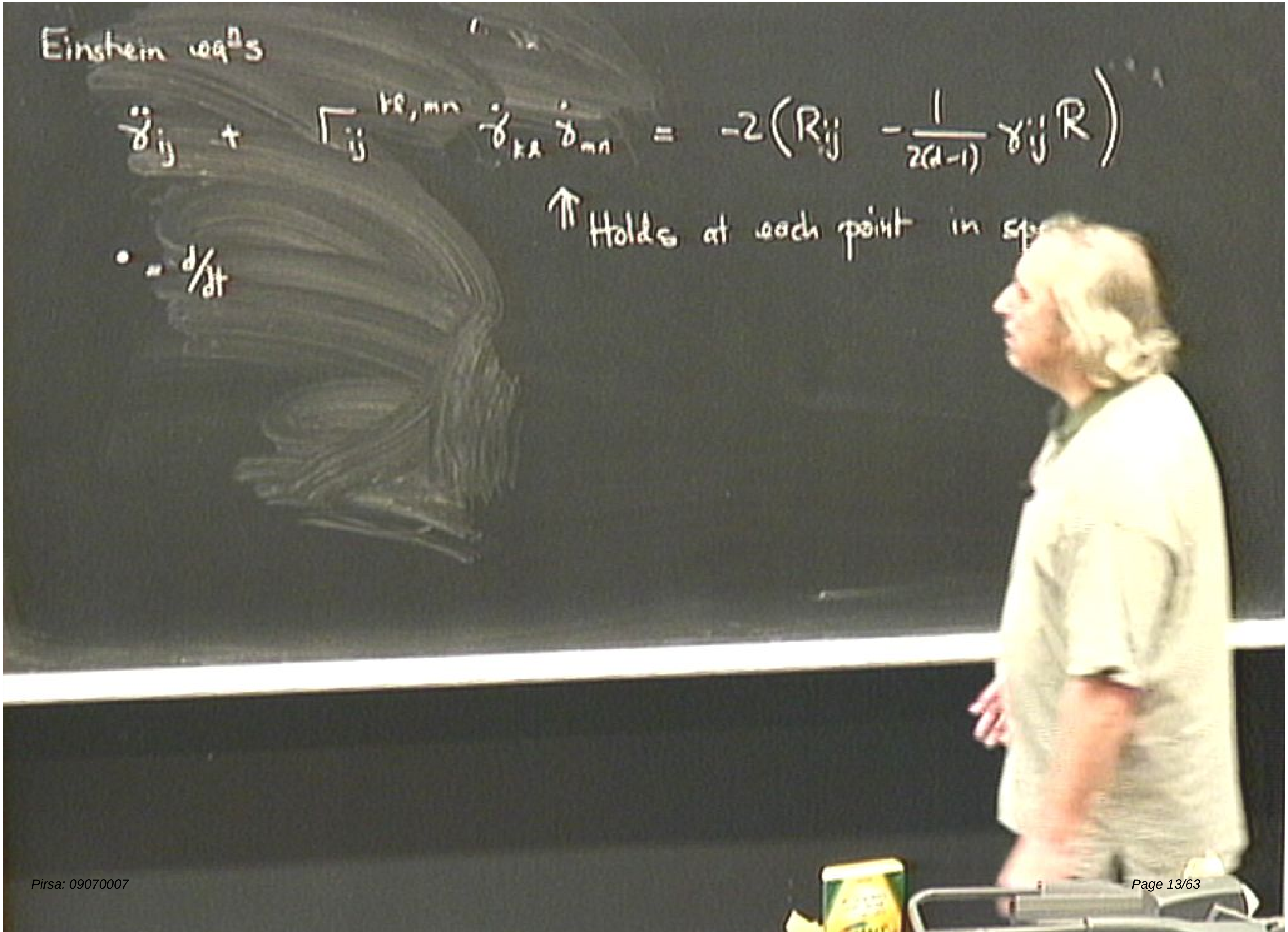
⇑ Holds at each point in space.

$\bullet = d/dt$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In the absence of rhs, this is the geodesic equation
for $\gamma_{ij}(x)$

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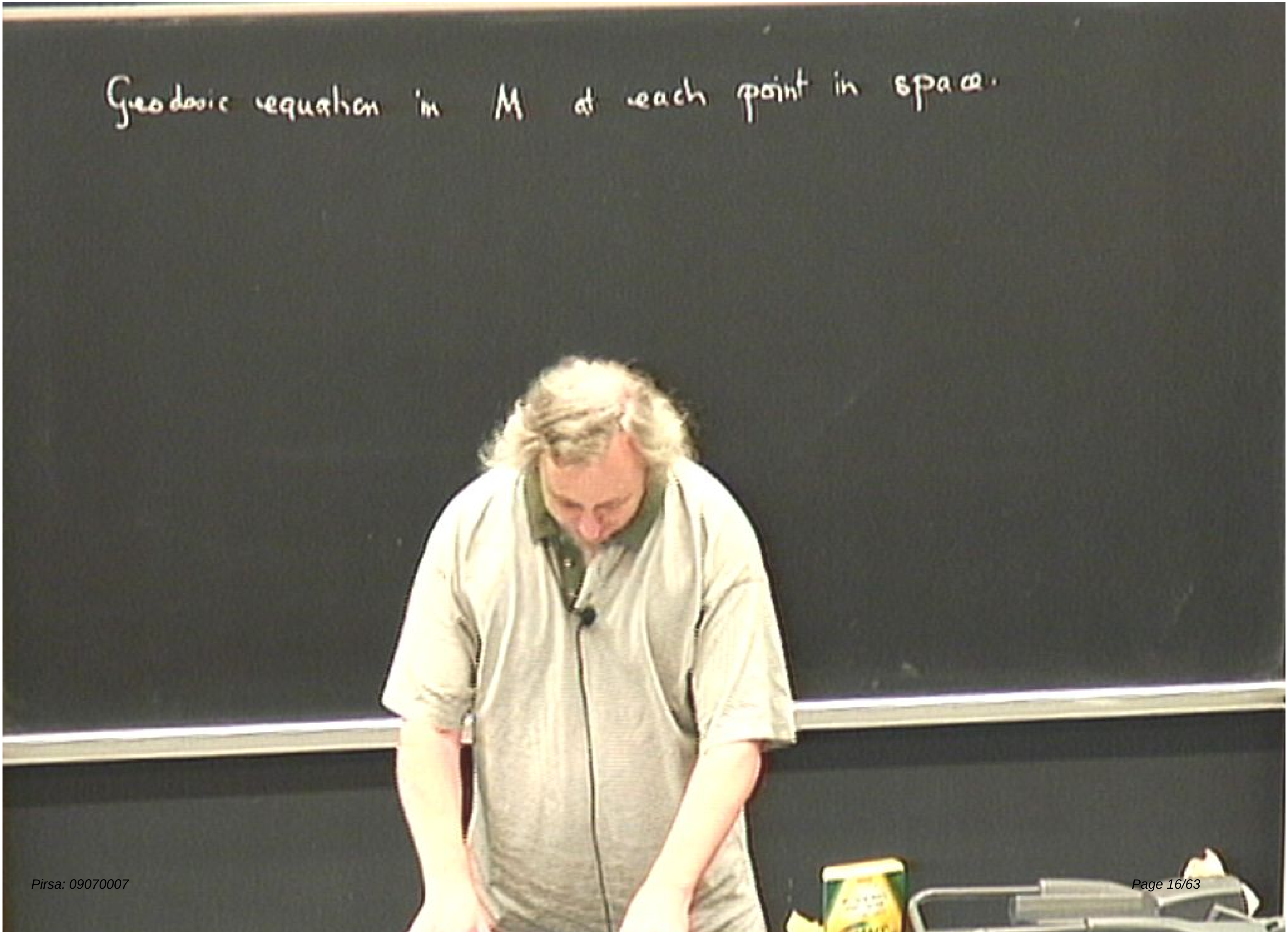
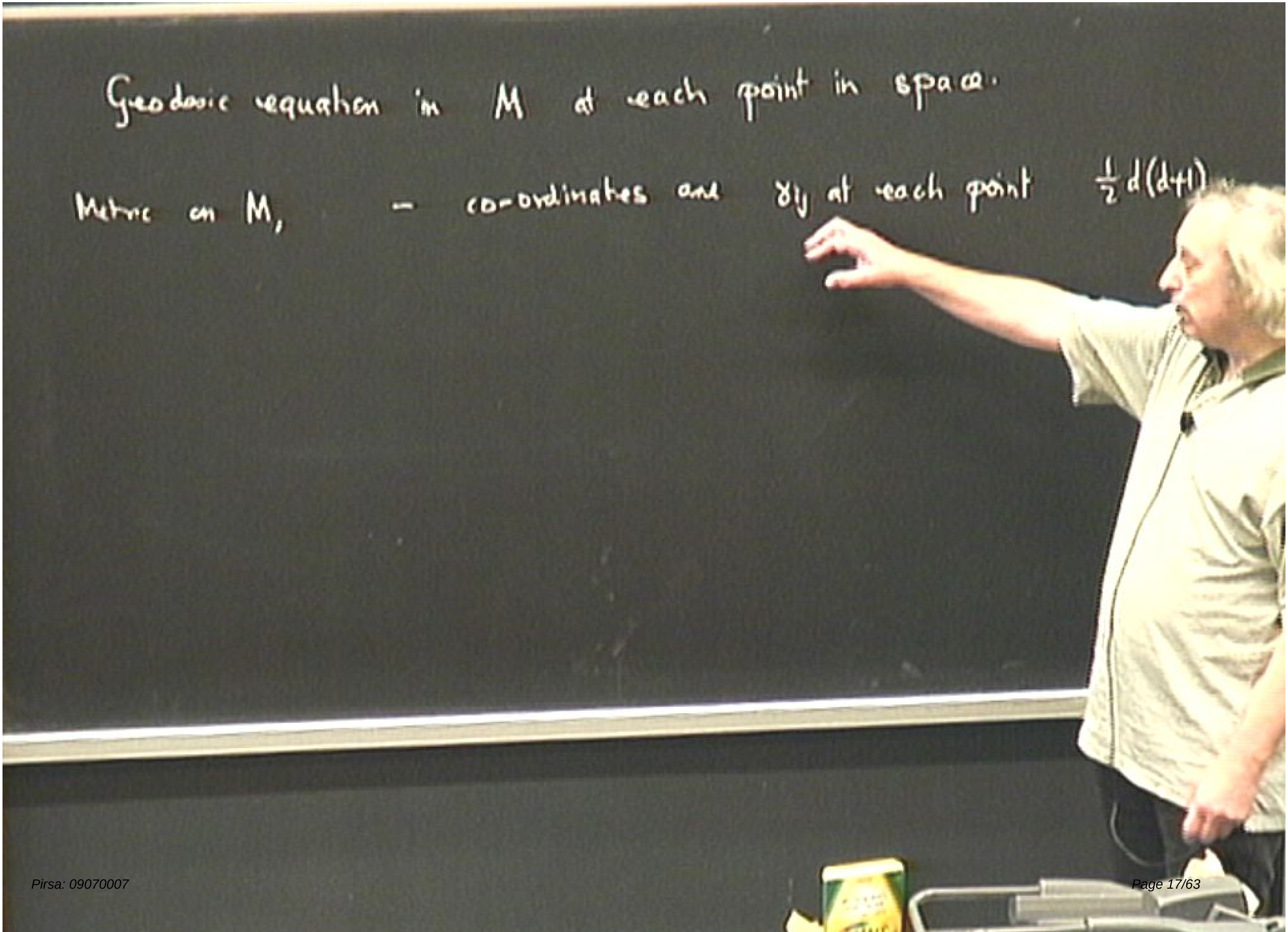

Geodesic equation in M at each point in space.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$$\frac{1}{2}\gamma^{1/2}\left(\gamma^{ik}\gamma^{jl}+\gamma^{il}\gamma^{jk}-2\gamma^{ij}\gamma^{kl}\right)$$

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$$\delta\Sigma^2 = \underbrace{G^{ijkl}}_{\text{metric}}\delta\gamma_{ij}\delta\gamma_{kl}$$

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de Witt

Metric on M, — co-ordinates and γ_{ij} at each point $\frac{1}{2}d(d+1)$

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de Witt — metric, superspace metric — metric on the space of all metrics.

Signature, $\left(-, +^{\frac{1}{2}d(d+1)-1}\right)$

Corresponds to an overall scaling.

Signature, $\left(-, +^{\frac{1}{2}d(d+1)-1}\right)$ Metric on the symmetric space $SL(d)/SO(d)$

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Same as 'wrong' sign of the conformal factor.

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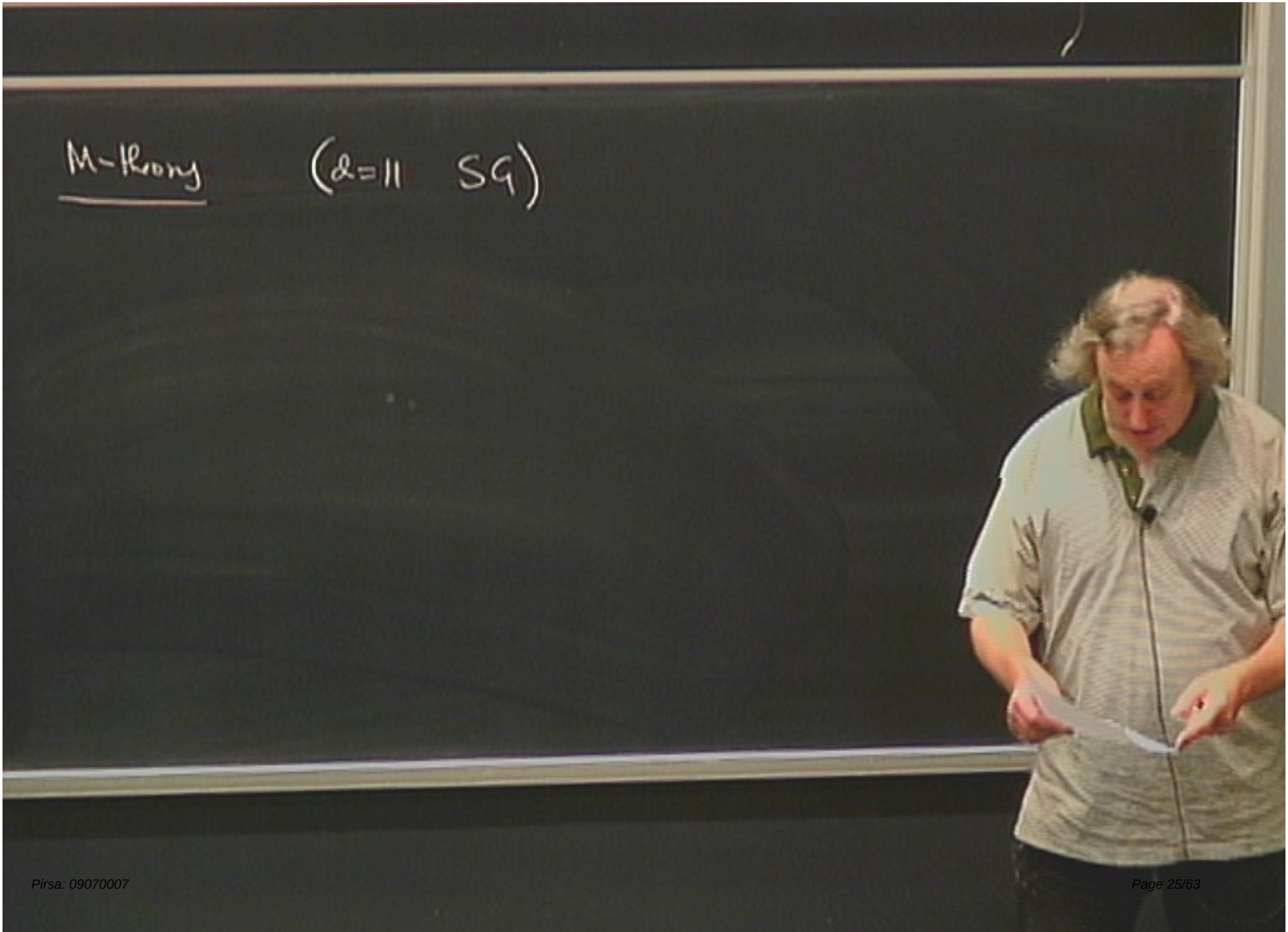
Metric on the symmetric space

$SL(d)/SO(d)$

Remains of the Lorentz gp.


M-theory
(d=11 SG)

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3-form potential $A_{abc} \longrightarrow F_{abcd} = 4\,\partial_{[a}A_{bcd]}$

$$\int -\frac{1}{48} F_{abcd}^{2}$$

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$$\int -\frac{1}{48} F_{abcd}^2 + \frac{2}{(12)^4}\,\eta_{abc\ldots\ldots} A^{\cdots} F^{\cdots\cdots} F^{\cdots\cdots}$$

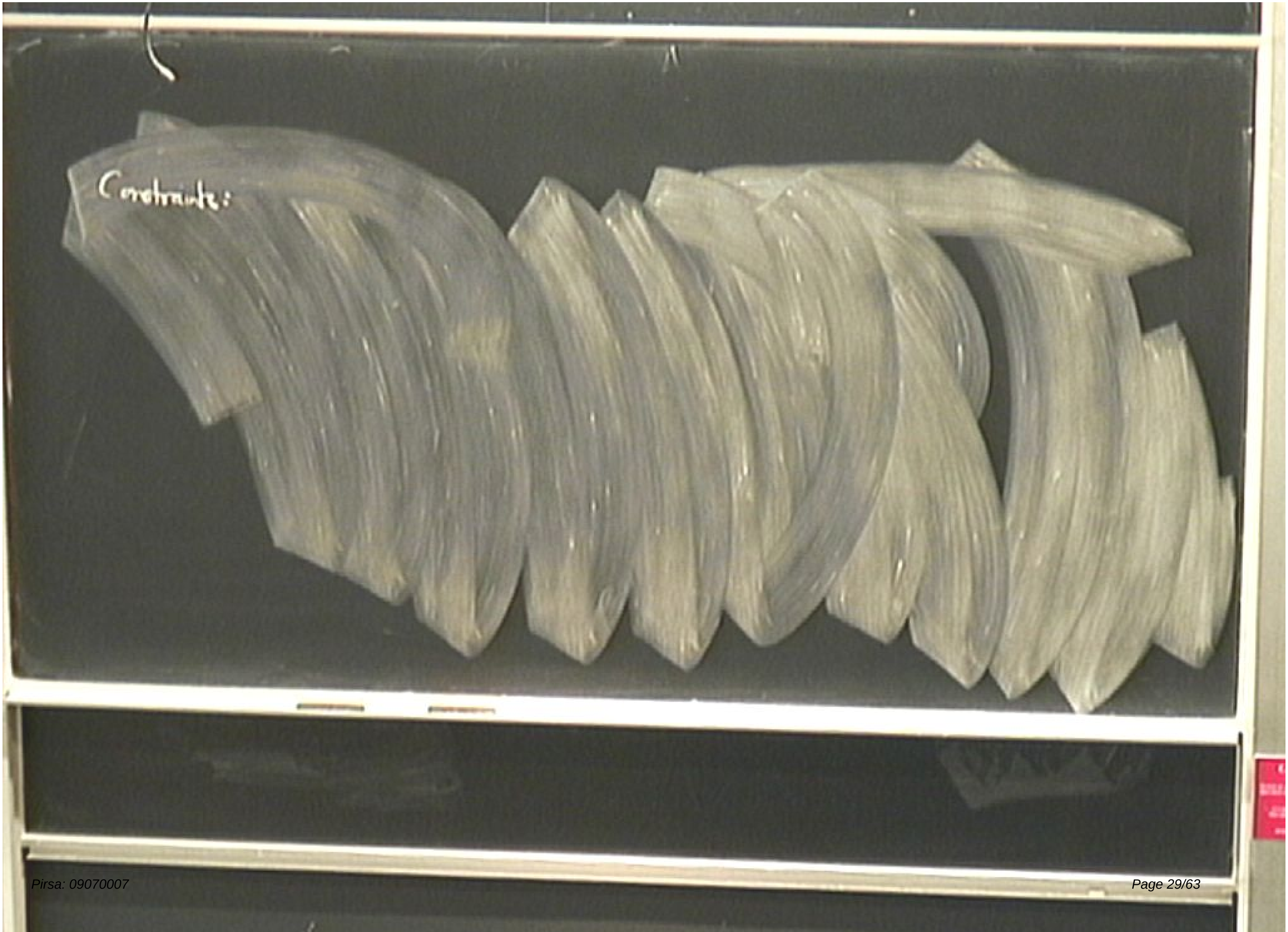
M-theory ($d=11$ SG)

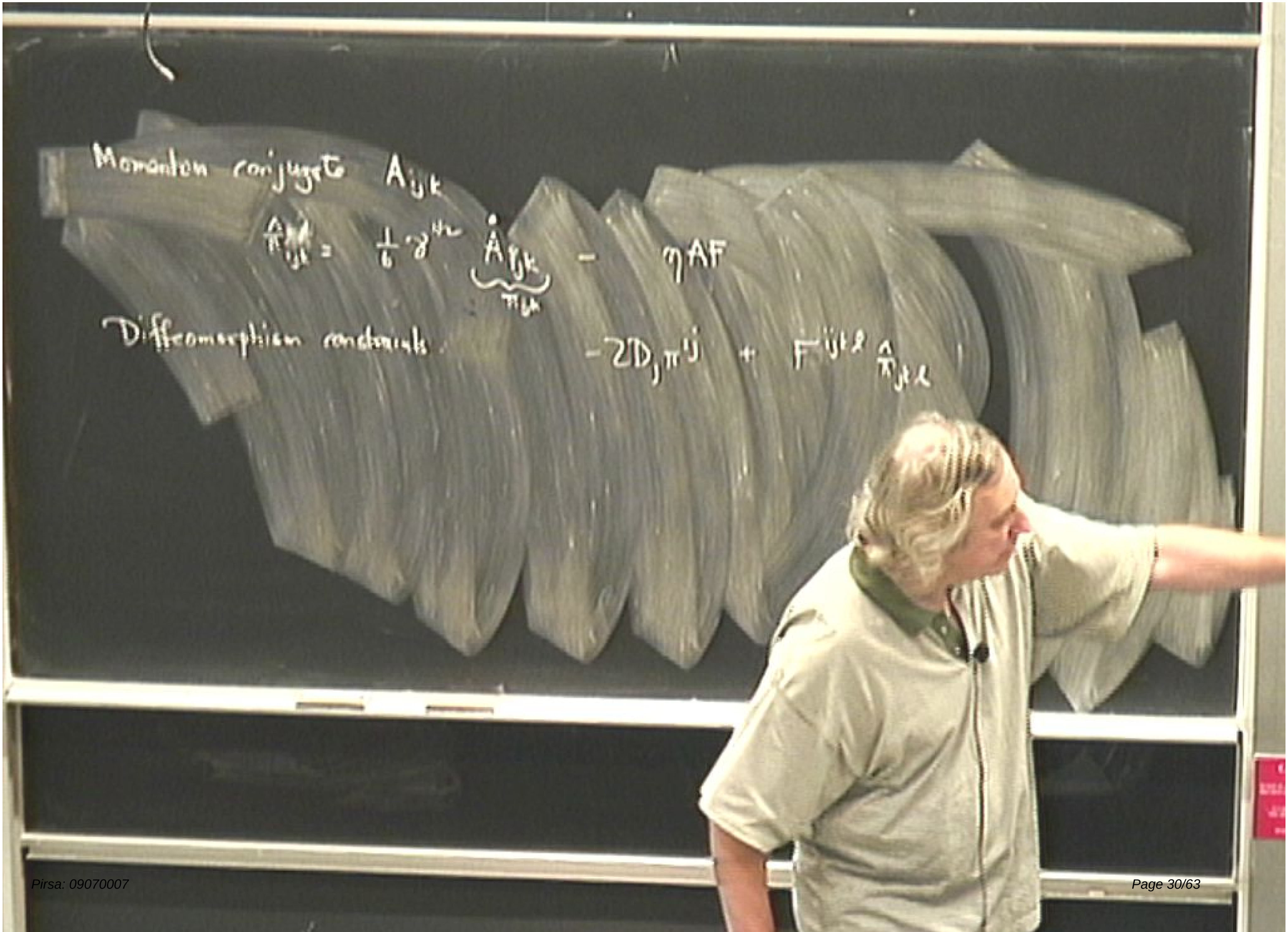
3-form potential $\underline{A_{abc}} \longrightarrow F_{abcd} = 4\,\partial_{[a}A_{bcd]}$

$$\int -\frac{1}{48} F_{abcd}^{2} + \frac{2}{(12)^4}\,\eta_{abc\ldots}\, A^{\ldots} F^{\ldots\ldots} F^{\ldots\ldots}$$

$B_{ij} = A_{0ij} \implies$ Gauge away.

$A_{ijk} \longrightarrow F_{ijkl}$


Constraints:


Momentum conjugate A_{ijk}
$\hat{\pi}^{ijk} = \frac{1}{6} g^{ik} \underbrace{\dot{A}_{ijk}}_{\pi^{ijk}} - \eta AF$
Diffeomorphism constraints: $-2D_j \pi^{ij} + F^{ijkl} \hat{\pi}_{jkl}$

Momentum conjugate A_{ijk}

$$\hat{\pi}_{ijk} = \frac{1}{6}\gamma^{i\ldots}\underbrace{\dot{A}_{ijk}}_{\pi_{ijk}} - \eta AF$$

Diffeomorphism constraints: $-2D_j\pi^{ij} + F^{ijkl}\hat{\pi}_{jkl}$

Gauge field " $-3D_k\hat{\pi}^{ijk} - \eta F F$

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Gauge field " $-3D_k \hat{\pi}^{ijk} - \eta F F$

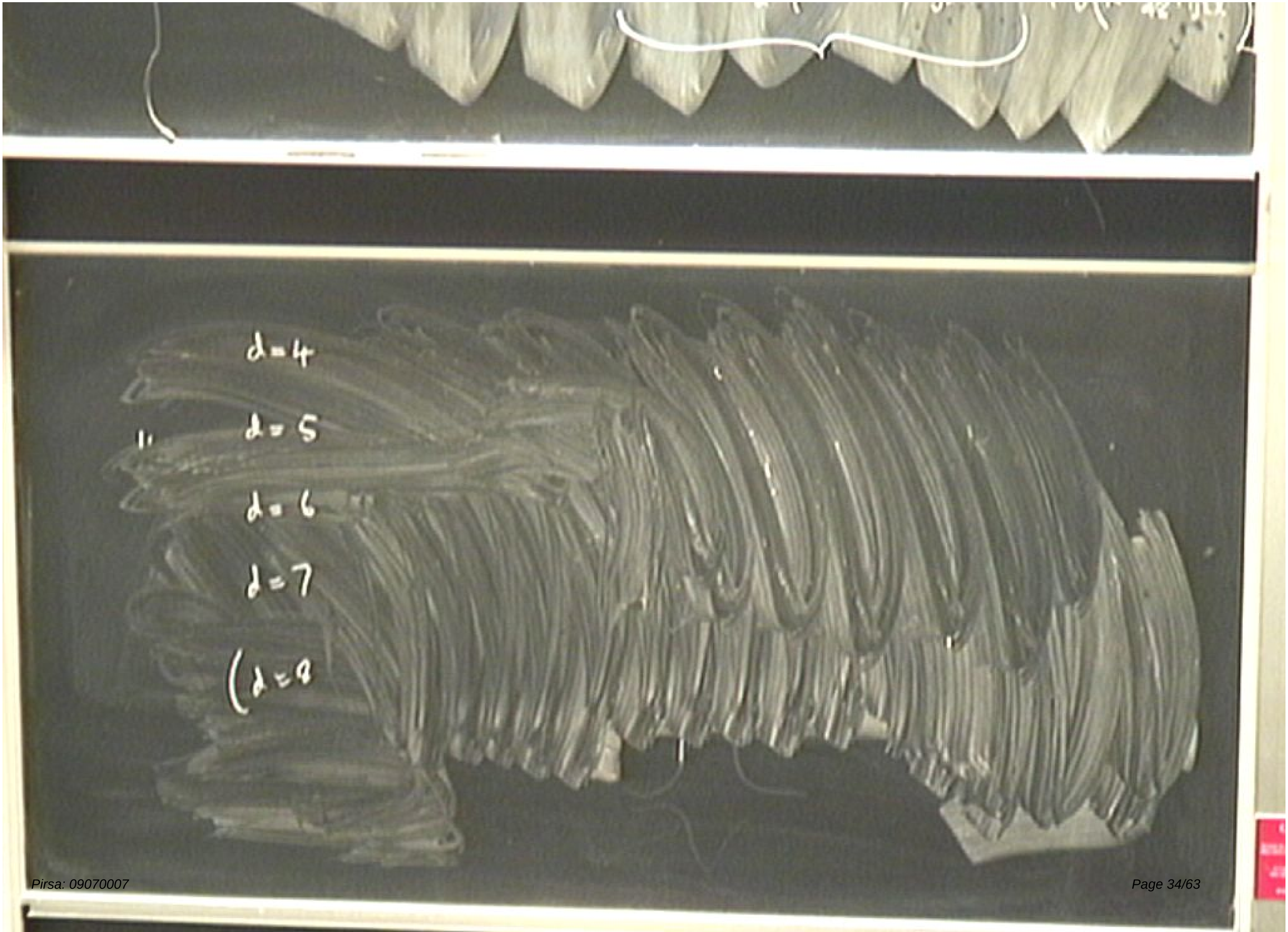
Hamiltonian constraints $\pi^{ij}\pi_{ij} - \frac{1}{d-1}\pi^2 + \hat{\pi}_{ijk}\hat{\pi}^{ijk} +$

for a particle

Hamiltonian constraints $\gamma^{-1/2}\left[\pi^{ij}\pi_{ij} - \frac{1}{d-1}\pi^2 + \hat{\pi}_{ijk}\hat{\pi}^{ijk} + \gamma\left(R + \frac{1}{12}\Gamma^2_{ijk}\right)\right]$

Diffeomorphism constraint $\chi^i = -2D_j\pi^{ij}$

Hamiltonian constraints $H = \gamma^{-1/2}\left(\underbrace{\underbrace{\pi^{ij}\pi_{ij} - \frac{1}{d-1}\pi^2}_{} + \gamma R^{(d)}(\gamma)}_{\text{Mass-shell constraint for a particle}}\right)$


d=4
d=5
d=6
d=7
(d=8

$d = 4$ $E_4 = SL(5)/SO(5)$

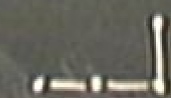
$d = 5$ $E_5 = SO(5,5)/SO(5) \times SO(5)$

$d = 6$ $E_6 / USp(8)$

$d = 7$ $E_7 / SU(8)$

$(d = 8$

$d=4$ $\quad E_4 = SL(5)/SO(5)$



$d=5$ $\quad E_5 = SO(5,5)/SO(5)\times SO(5)$

$d=6$ $\quad E_6 / USp(8)$

$d=7$ $\quad E_7 / SU(8)$

($d=8$ $\quad E_8 / SO(16)$?)

Geodesic equation in M at each point in space.

$SL(5)/SO(5)$,

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Enlarge the remains of the Lorentz group

Exceptional Geometric (Hitchin, Hull ....)

Take each point in space — construct a vector

— vierbein maps into the

$SL(5)/SO(5)$, ← Enlarge the remains of the Lorentz group.

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— vierbein maps into the space.

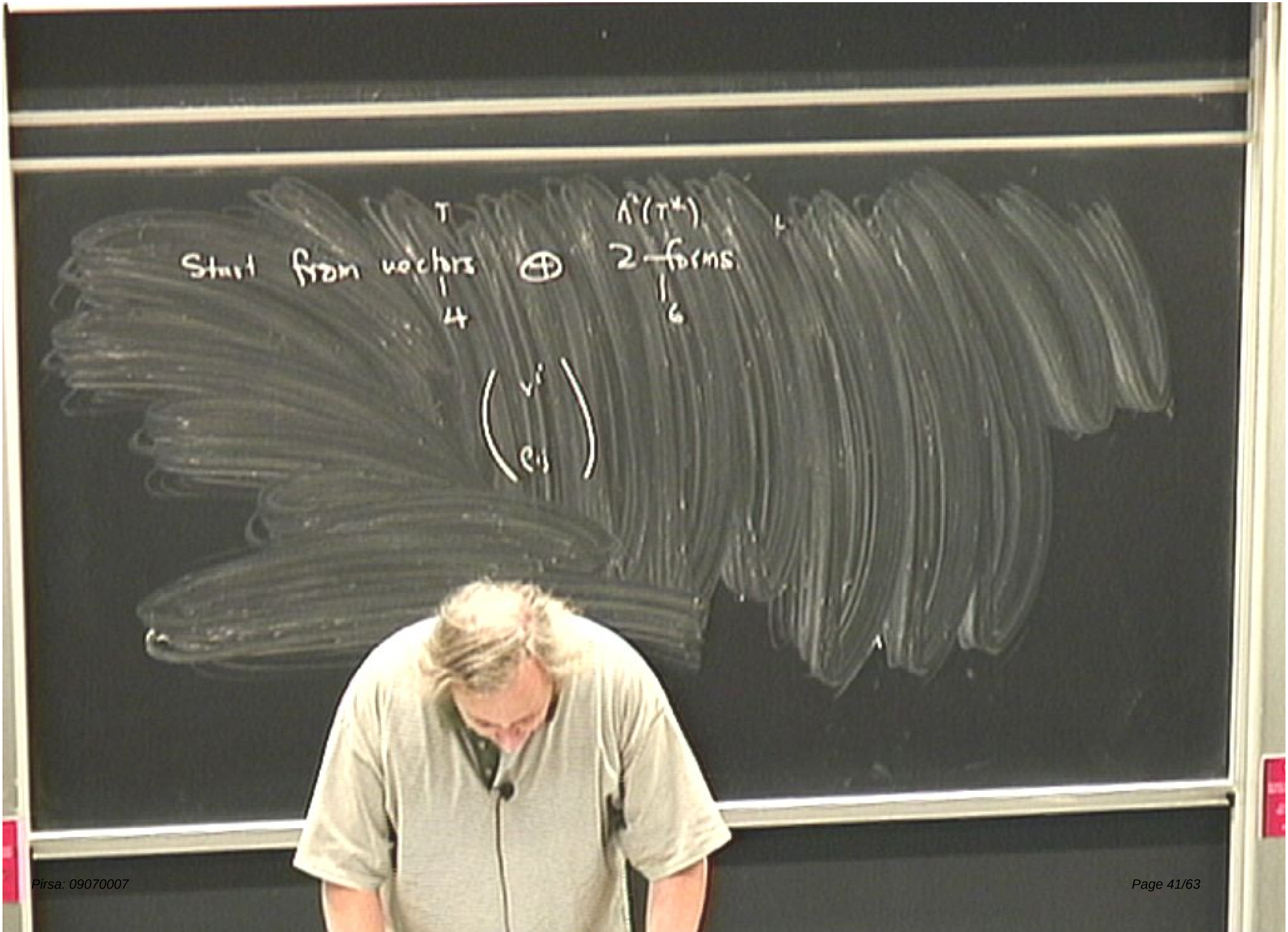
$SL(5)/SO(5)$, — Enlarge the remains of the Lorentz group.

Exceptional Geometric (Hitchin, Hull ....) $SL(4)/SO(4)$

Take each point in space — construct a vector v^i — $SL(4)$

— vierbein e^a_i maps into the space. $e^a_i v^i = v^a$

— metric $e^a_i e^b_j \delta_{ab} = \gamma_{ij}$ ← $SO(4)$ metric


T $\Lambda^2(T^*)$
Start from vectors $\oplus$ 2-forms
4 6
$\begin{pmatrix} v^i \\ e_{ij} \end{pmatrix}$

$$\begin{array}{ccc} T & & \Lambda^2(T^*) \\ \text{Start from vectors} & \oplus & \text{2-forms} \\ | & & | \\ 4 & & 6 \end{array}$$

$$\begin{pmatrix} v^i \\ \rho_{ij} \end{pmatrix}$$

How does $SL(5)$ act..

T $\Lambda^2(T^*)$

Start from vectors $\oplus$ 2-forms
4 6

$\begin{pmatrix} v^i \\ e_{ij} \end{pmatrix}$

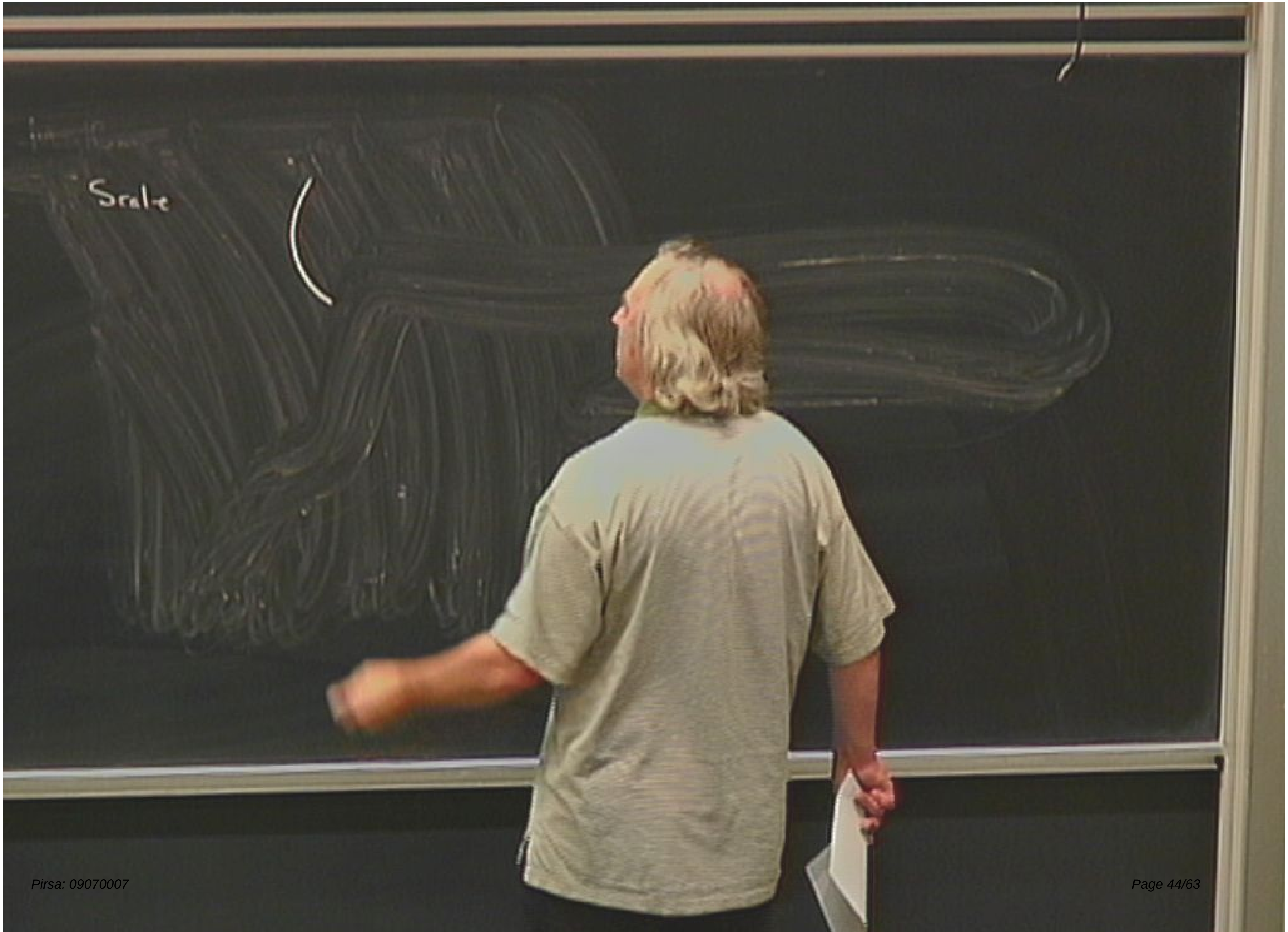
24

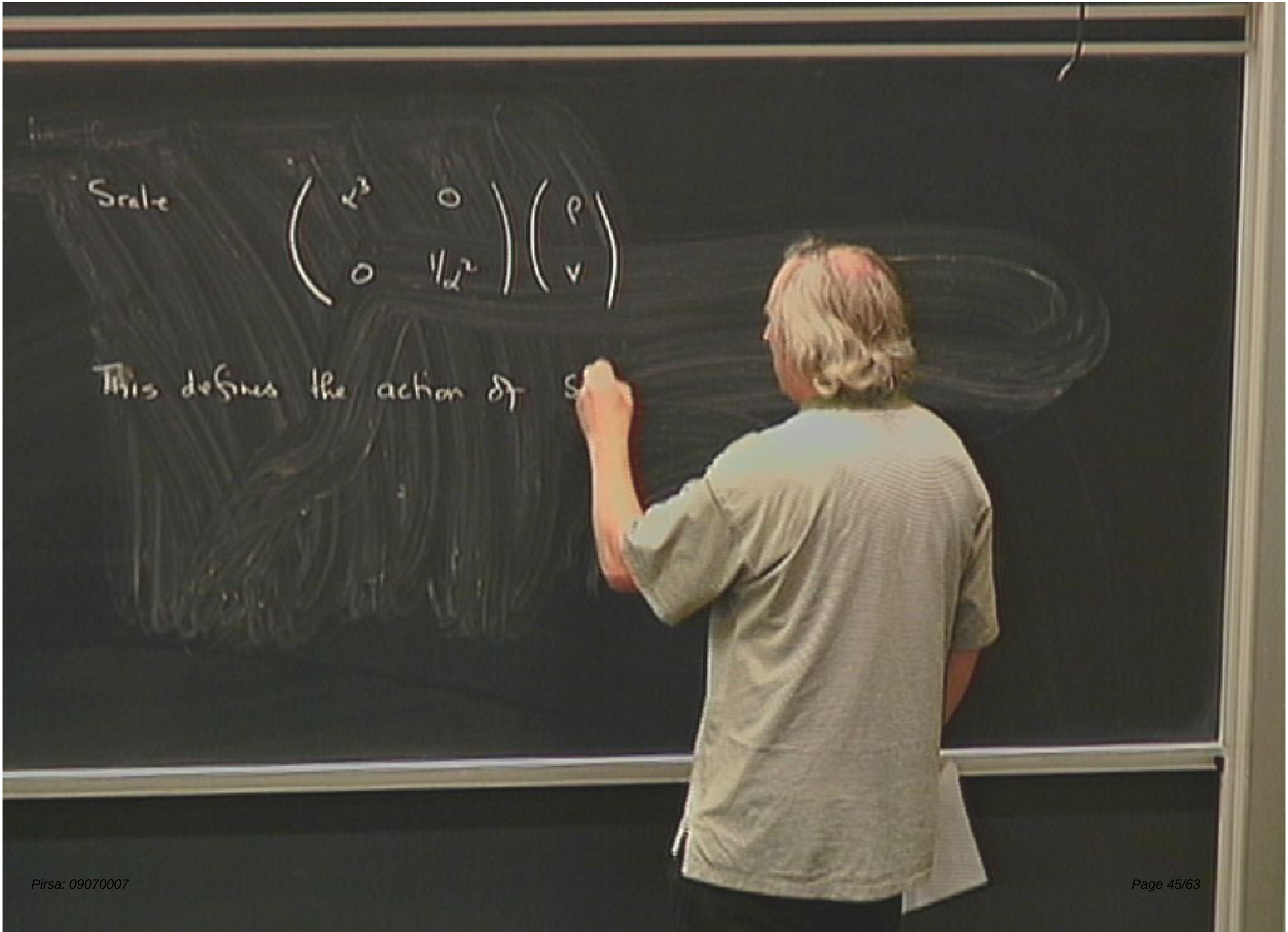
How does $SL(5)$ act.

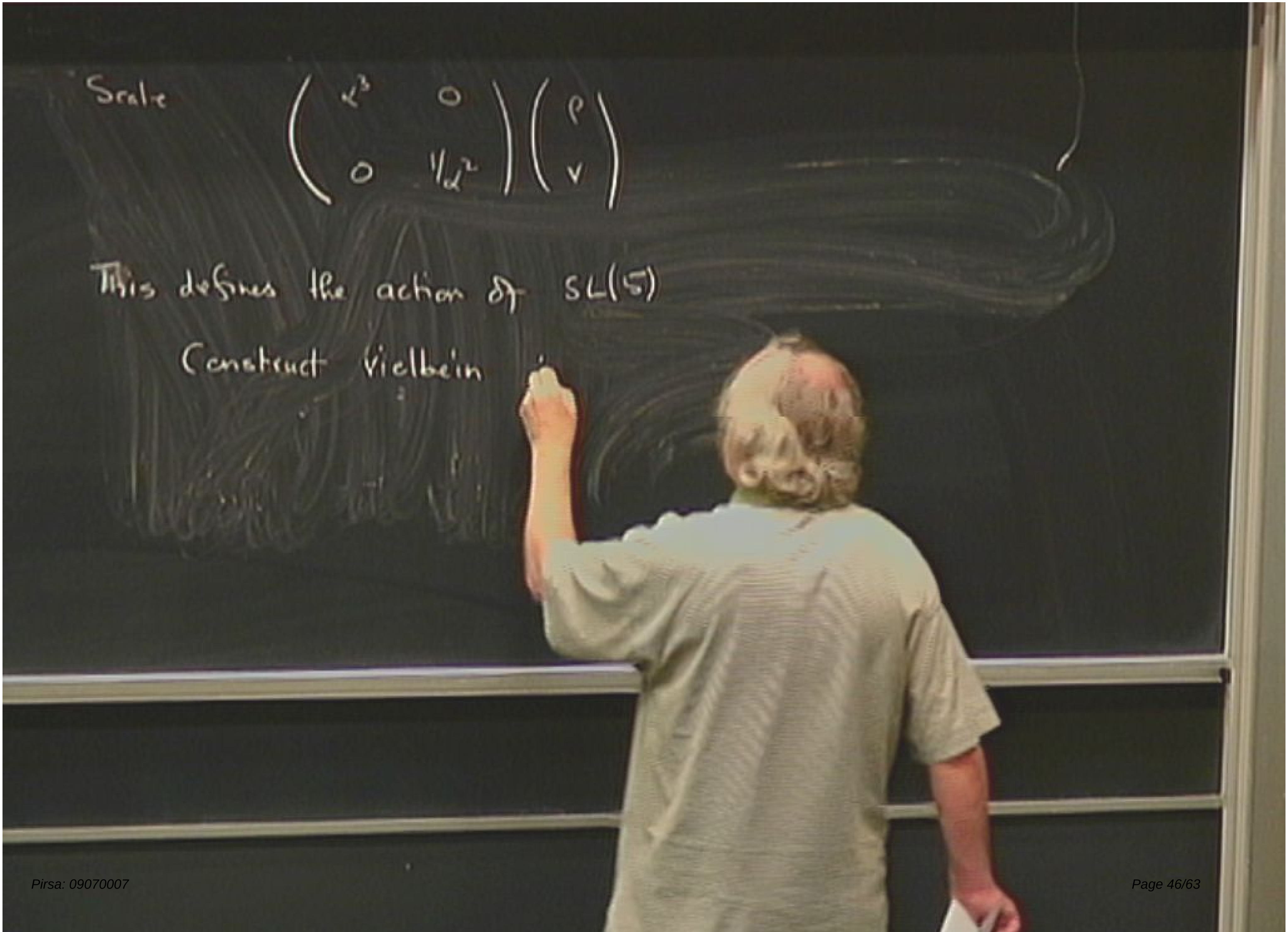
Mv

$SL(4)$ $M^T e M$

15


Scale


Scale
$\begin{pmatrix} \alpha^3 & 0 \\ 0 & 1/\alpha^2 \end{pmatrix} \begin{pmatrix} \rho \\ v \end{pmatrix}$
This defines the action of S


Scale
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This defines the action of SL(5)
Construct Vielbein

Scale $\begin{pmatrix} \alpha^3 & 0 \\ 0 & 1/\alpha^2 \end{pmatrix} \begin{pmatrix} \rho \\ v \end{pmatrix}$

This defines the action of $SL(5)$

Construct Vielbein in $SL(5)/SO(5)$

$$\underbrace{\mathcal{V}}_{\text{Vielbein}} \longrightarrow \underbrace{k(x)}_{\text{local } SO(5)} \mathcal{V}(x) \underbrace{g}_{\text{Rigid } SL(5)}$$

Dimension 14

Scale $\begin{pmatrix} \alpha^3 & 0 \\ 0 & 1/\alpha^2 \end{pmatrix} \begin{pmatrix} \rho \\ v \end{pmatrix}$

This defines the action of $SL(5)$

Construct Vielbein in $SL(5)/SO(5)$

$$\mathcal{V} \longmapsto k(x)\,\mathcal{V}(x)\,g$$

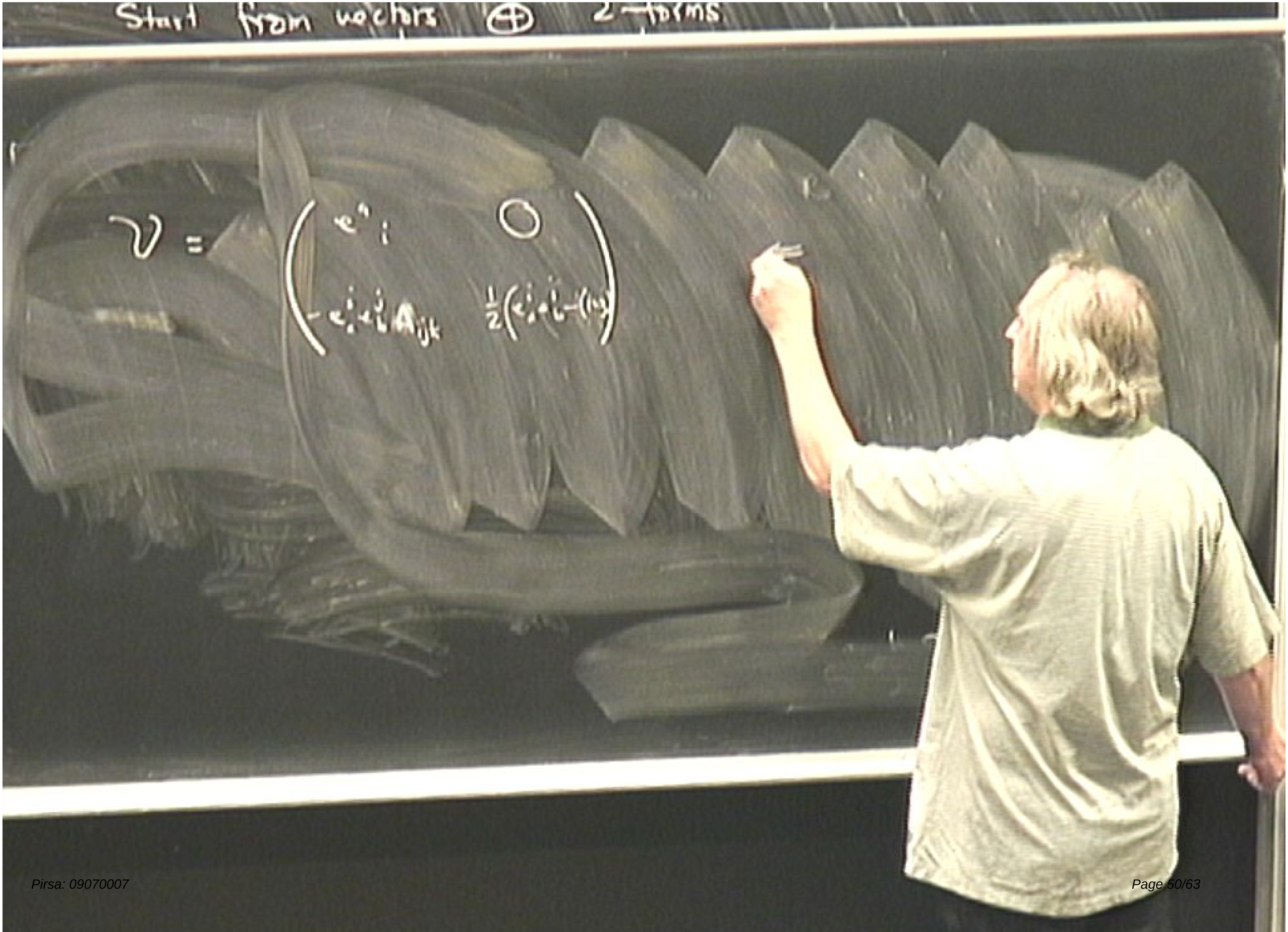
local $SO(5)$

Rigid $SL(5)$

Dimension 14

Start from vectors $\oplus$ 2-forms

$$\mathcal{V} = \begin{pmatrix} e^{a} \\ \vdots \\ \\ \end{pmatrix}$$


Start from vectors ⊕ 2-forms

$$\mathcal{V} = \begin{pmatrix} e^{a}{}_{i} & 0 \\ -e^{i}_{a} e^{j}_{b} A_{ijk} & \frac{1}{2}\left(e^{i}_{a} e^{j}_{b} - (\ldots)\right) \end{pmatrix} \Leftarrow \text{Parametrises } SL(5)/SO(5)$$

Metric

$$M = \mathcal{V}^T \mathcal{V}$$

$T \quad \Lambda^2(T^*)$

tgt space

spacetime

$$\mathcal{V} = \begin{pmatrix} e^{a}{}_{i} & 0 \\ -e^{i}_{a} e^{j}_{b} A_{ijk} & \frac{1}{2}\left(e^{i}_{a} e^{j}_{b} - (\cdots)\right) \end{pmatrix}$$

$\Leftarrow$ Parametrises $SL(5)/SO(5)$

Metric

$$M = \mathcal{V}^T \mathcal{V} = \begin{pmatrix} \gamma_{ij} + A_{ik\ell} A_{j}{}^{k\ell} & -A_{i}{}^{mn} \\ -A_{j}{}^{pq} & \frac{1}{2}\left(\gamma^{pm}\gamma^{qn} - \gamma^{pn}\gamma^{qm}\right) \end{pmatrix}$$

$\mathrm{Tr}\left(M^{-1}\delta M \; M^{-1}\delta M\right)$ gives the metric on the space of all M's.

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Generalised geometry of $E_4/SO(5)$.

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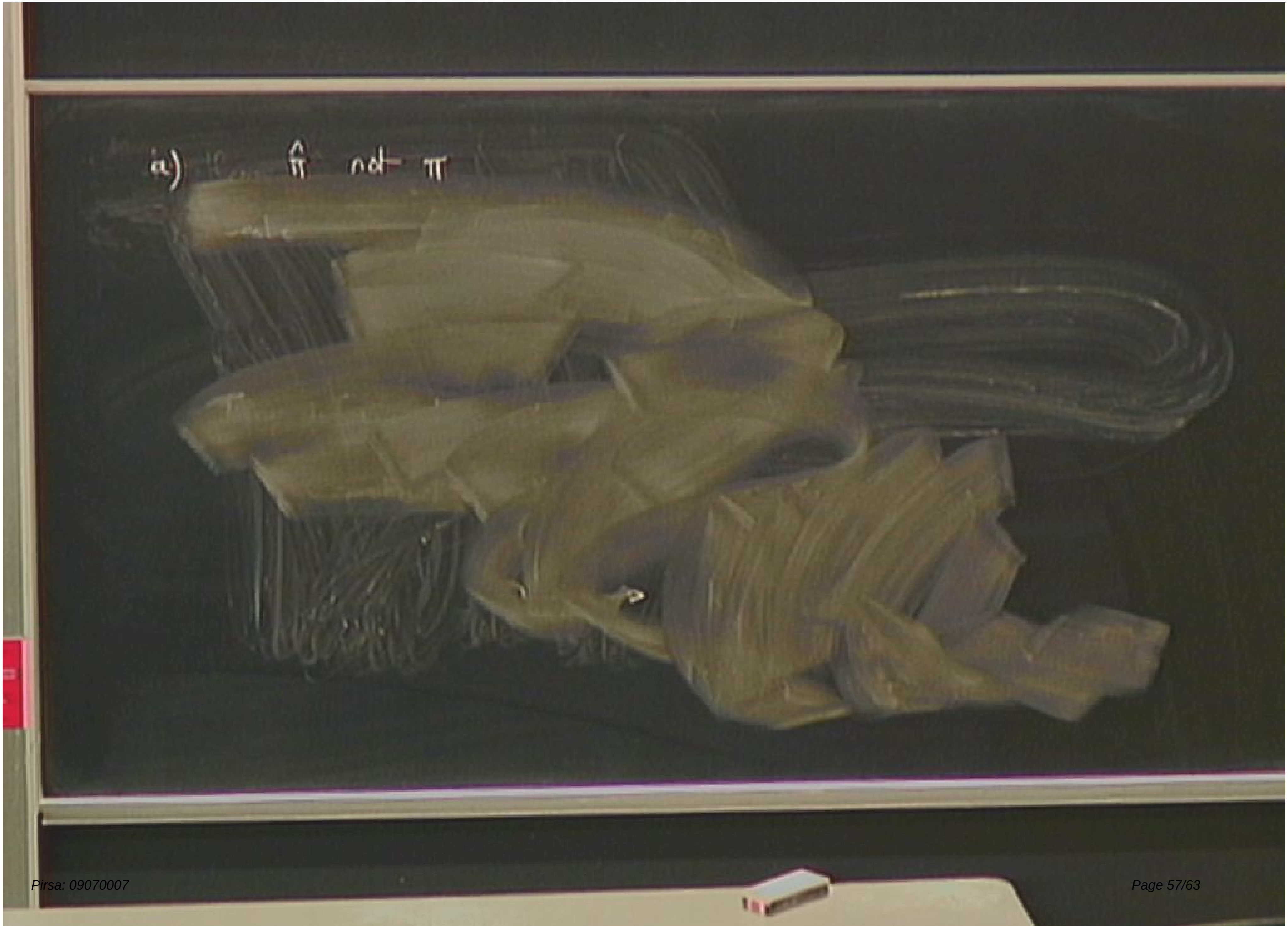
Generalised geometry of $E_4/SO(5)$.

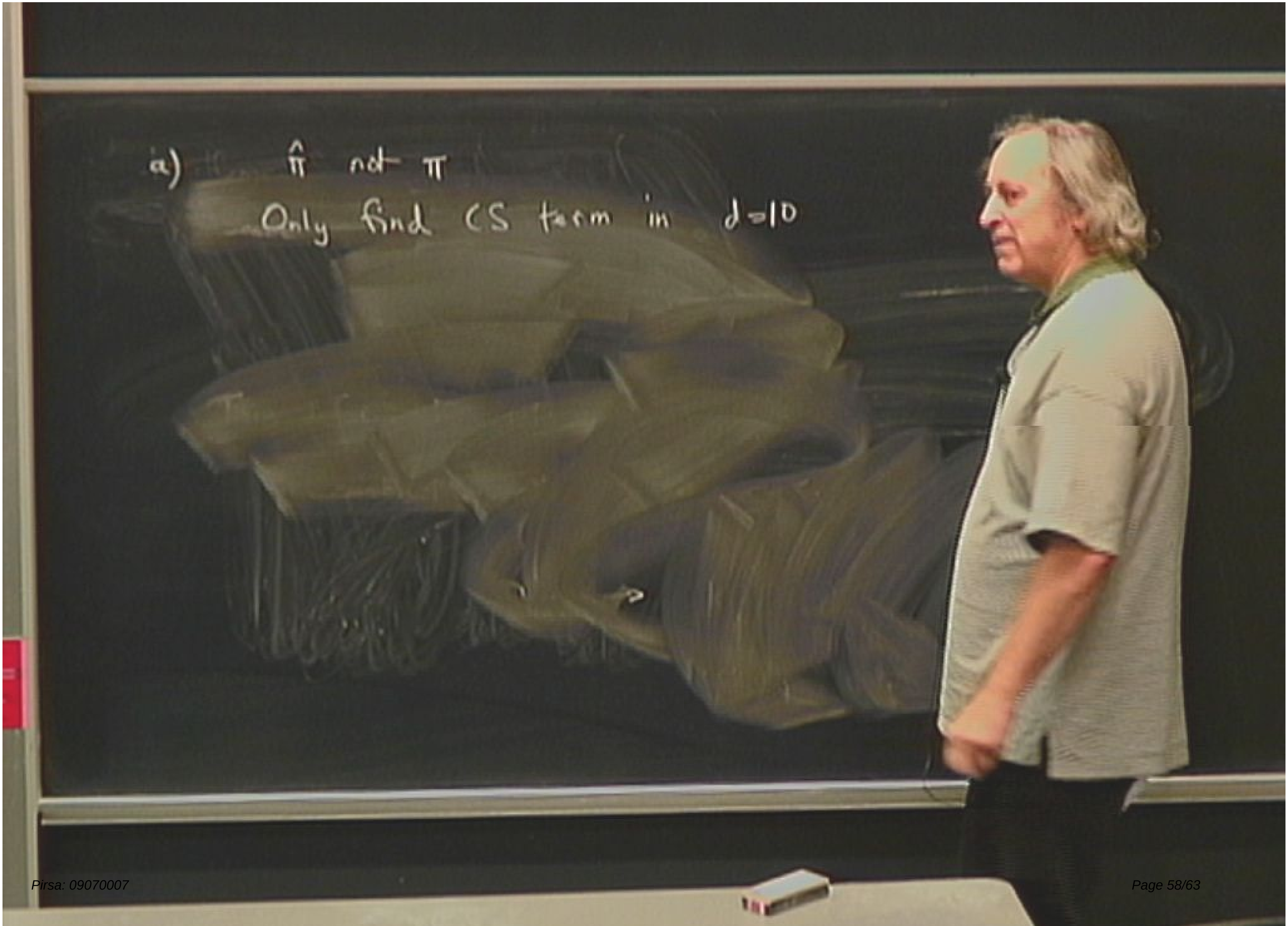
$\mathrm{Tr}(M^{-1}\delta M \; M^{-1}\delta M)$ gives the metric on the space of all M's.

Generalised geometry is $E_7/SU(8)$.

$X^{\alpha\beta} F_\alpha F_\beta$

$F_\alpha(g_{ij}, A_{ijk})$


a)
π


a) π̂ not π
Only find CS term in d=10

a) $\hat{\pi}$ not π

Only find CS term in $d=10$

b) Fermions Lazy

c)

a) $\hat{\pi}$ not π

Only find CS term in $d=10$

b) Fermions Lazy

c) $\lambda=8$ → Too many fields.

a) $\hat{\pi}$ not π

Only find CS term in $d=10$

b) Fermions Lazy

c) $\lambda = 8 \rightarrow$ Too many fields.

d) Other extra fields

a) $\hat{\pi}$ not π

Only find CS term in $d=10$

b) Fermions Lazy

c) $\lambda=8 \to$ Too many fields.

d) Other extra fields

e) Why not covariant.

a) $\hat{\pi}$ not π

Only find CS term in $d=10$

b) Fermions Lazy

c) $\lambda=8$ → Too many fields.

d) Other extra fields

e) Why not covariant.

f) Singularities, billiards →