Title: Cosmological gravity and the AdS/CFT correspondence

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Abstract: Recently there has been renewed interest in cosmological gravity, namely 3d gravity coupled to a Chern-Simons term with parameter $\mu$, following claims that at $\mu =1$ the theory becomes chiral and stable. In this talk we will investigate cosmological gravity by setting up a concrete holographic dictionary with a dual 2d field theory and we will demonstrate that the theory is in fact not chiral in the limit $\mu = 1$. Instead our holographic dictionary implies that at $\mu =1$ the dual field theory becomes a logarithmic CFT (LCFT). Our results resolve confusions in the earlier literature and the resulting new class of holographic dualities involving LCFTs may have interesting applications to strongly correlated condensed matter systems.

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# Introduction

- Three dimensional gravity offers an interesting arena to investigate quantization of gravitational theories.
- However Einstein gravity in three dimensions has no propagating degrees of freedom so it is not a good toy model for higher dimensional gravitational theories.
- Adding higher derivative terms leads to propagating degrees of freedom but the theory then generically contains ghost-like excitations.
- The aim of this work is to discuss a particular theory, the topologically massive gravity, which has been conjectured to be free of such problems for a particular value of its parameter and analyze what we can learn using the AdS/CFT duality.

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# Topological Massive Gravity [Deser, Jackiw, Templeton]

- Topologically massive gravity is obtained by adding to the Einstein gravity the gravitational CS term,

$$S = \int d^3x \left( \sqrt{-g}(R - 2\Lambda) + \frac{1}{2\mu}(\Gamma d\Gamma + \Gamma^3) \right)$$

where $\Gamma$ is the 1-form Christoffel symbol.

- This theory admits asymptotically *AdS* solutions, for example the BTZ black hole, and has perturbative massive modes.
- When $\mu \neq 1$ however, the massive modes have negative energy and the theory is unstable.

# The "chiral point", $\mu = 1$

- When $\mu = 1$ the negative energy modes disappear, the left moving gravitational modes become pure gauge and the theory seems to only contain a purely right moving sector.
- This led to the conjecture that the theory is stable and consistent when $\mu = 1$ and dual to a $2d$ chiral CFT [Li, Song, Strominger (2008)].
- This created a lot of controversy as other authors found non-chiral modes and instabilities at the chiral point [Carlip et al], [Grumiller, Johansson], [Giribet et al] ...

# Fall-off conditions

- Part of the issue is the question:
  What are the correct fall-off conditions for the fields at infinity?
- The unstable modes have fall-off conditions different from those that the metric satisfies in 3*d* Einstein gravity (the Brown-Henneaux boundary conditions).
- So one is led to ask: are modified fall-off conditions allowed?

# Fall-off conditions [Regge, Teitelboim (1974)] ...

The traditional point of view is as follows:

1. Select physically "reasonable" fall-off conditions such that relevant solutions, for example black holes, satisfy them.
2. Check that conserved charges are finite with this choice.

One may consider different fall-off conditions as defining different theories.

# AdS/CFT: a new perspective

- The AdS/CFT correspondence provides a new perspective which leads to a comprehensive answer to such questions.
- The aim of this talk will be to explain the new insights and methodology originating from AdS/CFT and their applications to topologically massive gravity.

# Motivations?

- Renewed interest in three dimensional gravity theories followed Witten's proposal of a holographic duality between pure gravity and extremal CFTs.
- Here we find a correspondence between higher derivative theories and logarithmic CFTs, which immediately invites CMT applications.

# References

- This talk is based on

  KS, Marika Taylor, Balt C. van Rees
  Topologically Massive Gravity and the AdS/CFT Correspondence
  arXiv:0906.4926

# Outline

1. **AdS/CFT**
2. Topologically massive gravity
3. Holography for TMG and LCFT

# AdS/CFT: basics

- Recall that the $AdS_{d+1}$ metric is

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\eta_{ij}dx^i dx^j,$$

  with there being a conformal boundary at $r \to 0$.
- Thus we will need to give boundary conditions for all bulk fields at conformal infinity.

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# AdS/CFT: basics

In the AdS/CFT correspondence:

- The fields $\phi^I_{(0)}$ parameterizing these boundary conditions at conformal infinity are identified with sources that couple to operators $O^I$ of the dual CFT.
- The on-shell action, $S_{onshell}[\phi_{(0)}]$, is the generating functional of CFT correlation functions:

$$\langle O \rangle = \frac{\delta S_{onshell}[\phi_{(0)}]}{\delta \phi_{(0)}}, \quad \langle O(x) O(y) \rangle = \frac{\delta^2 S_{onshell}[\phi_{(0)}]}{\delta \phi_{(0)}(x) \delta \phi_{(0)}(y)},$$

# AdS/CFT: expectations and requirements

These identifications imply new intuitions about the boundary conditions:

- In QFT the sources coupling to operators are unconstrained, since one functionally differentiates w.r.t. them.
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# Asymptotically AdS spacetimes

Consider the case where the bulk field is the metric.

- In the physics literature, prior to the AdS/CFT correspondence, there were a number of works on Asymptotically AdS spacetimes [Ashtekar, Magnon (1984)] [Henneaux, Teitelboim (1985)] ...
  In all these works the metric approaches that of AdS as conformal infinity is approached.
- For AdS/CFT however one needs more general boundary conditions. The boundary conditions must be parameterized by an unconstrained metric, since the metric acts as a source for the energy momentum tensor $T_{ij}$ of the dual CFT.

# Asymptotically locally AdS spacetimes

- Fortunately this more general set-up has been developed in the mathematics literature [Fefferman-Graham (1985)]. The corresponding spacetimes are called Asymptotically locally AdS spacetimes (AlAdS).
- An AlAdS spacetime admits the following metric in a finite neighborhood of the conformal boundary, located at $r = 0$:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x, r) dx^i dx^j$$

where

$$\lim_{r \to 0} g_{ij}(x, r) = g_{(0)ij}(x)$$

is an arbitrary non-degenerate metric.

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# Asymptotically locally AdS spacetimes

- One should emphasize that the only requirement put on $g_{ij}(x, r)$ a priori is that it should have a non-degenerate limit as $r \to 0$.
- The precise form of the expansion $g_{ij}(x, r)$ is determined by solving the bulk field equations asymptotically.
- This reduces to solving algebraic equations, so the **most general** asymptotic solution can be readily found for any given bulk theory that admits AlAdS solutions.

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# Fefferman-Graham expansion

For Einstein gravity in $(d+1)$ dimensions,

$$g_{ij}(x,r) = g_{(0)ij}(x) + r^2 g_{(2)ij} + \cdots + r^d (g_{(d)ij} + \log r^2 h_{(d)ij}) + \cdots$$

- The blue coefficients are locally determined in terms of $g_{(0)}$.
- $g_{(d)ij}$ is only partially determined by asymptotics. This coefficient is related via AdS/CFT to the 1-point function of $T_{ij}$ and thus to bulk conserved charges.
- $h_{(d)}$ is non-zero when $d$ is even and is related to the Weyl anomaly of the boundary theory, [de Haro, Solodukhin, Skenderis (2000)]

$$h_{(d)} = \frac{\delta}{\delta g_{(0)}} \int (conformal\ anomaly)$$

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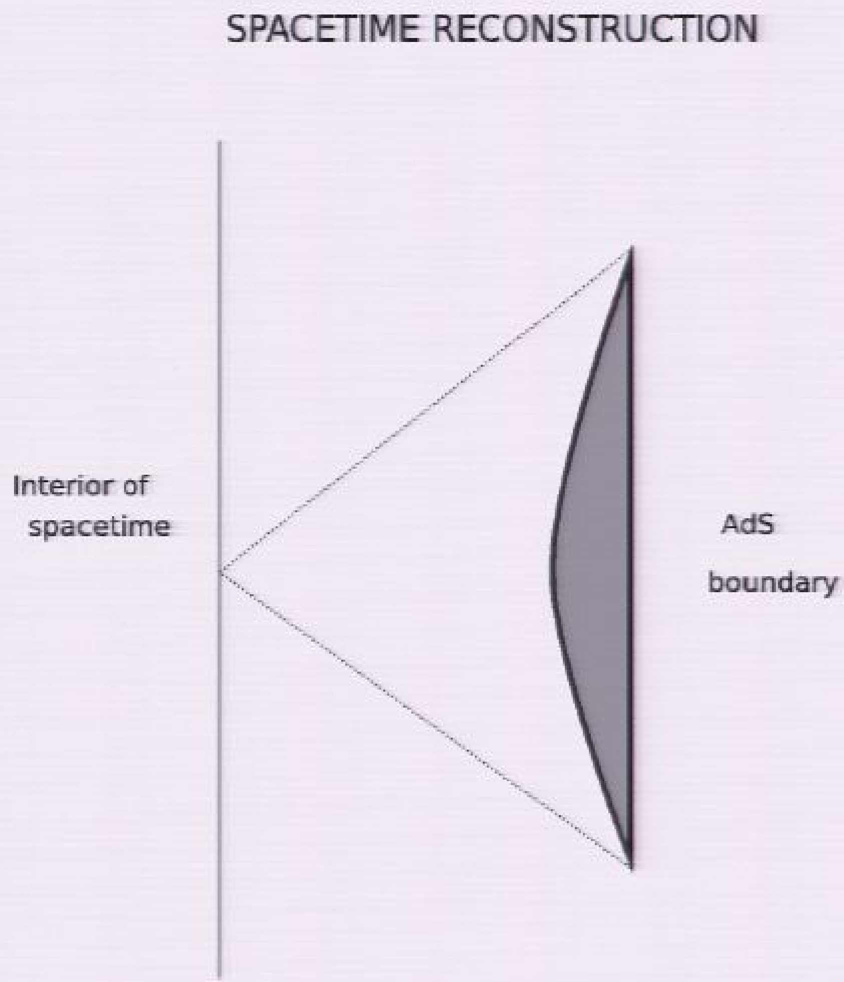
# Holographic reconstruction of spacetime

The global reconstruction (including horizons) is an important open question.



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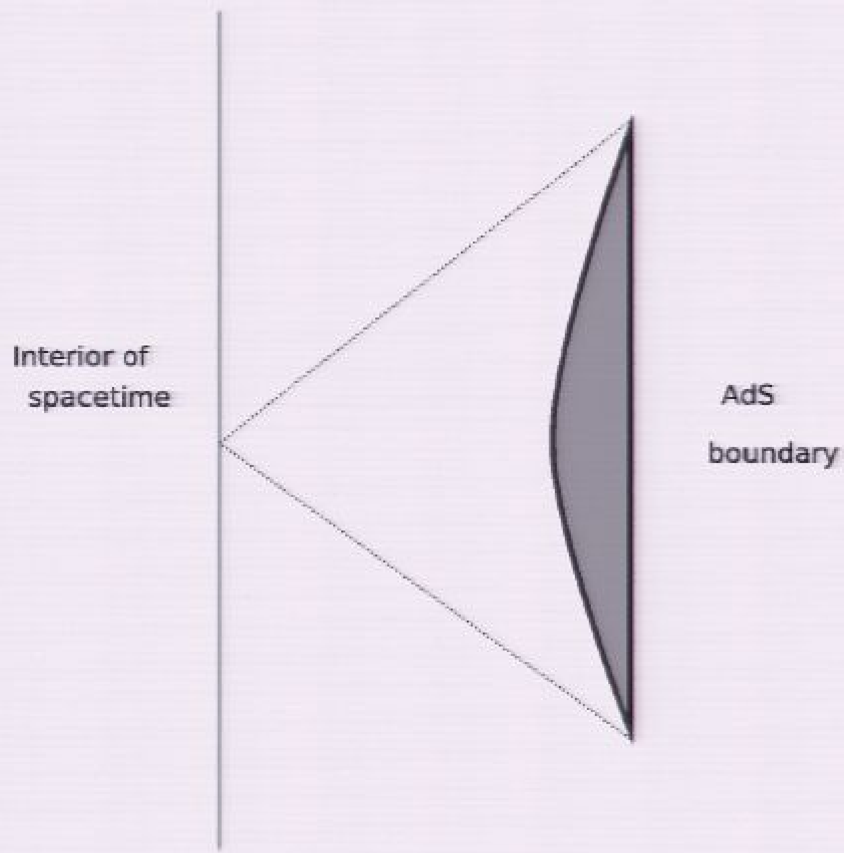

# Fefferman-Graham expansion for $d = 2$

For Einstein gravity in $d = 2$

$$g_{ij}(x, r) = g_{(0)ij}(x) + r^2 g_{(2)ij} + \cdots$$

- $h_{(2)}$ vanishes because the integral of the conformal anomaly is a topological quantity (the Euler number).
- The Brown-Henneaux boundary conditions are as above with $g_{(0)ij}(x) = \delta_{ij}$.
- The precise form of this expansion is special to Einstein gravity; coupling to matter changes the coefficients.
- For example, coupling Einstein gravity to a free massless scalar induces a logarithmic term in the expansion, i.e. $h_{(2)} \neq 0$ in this theory.

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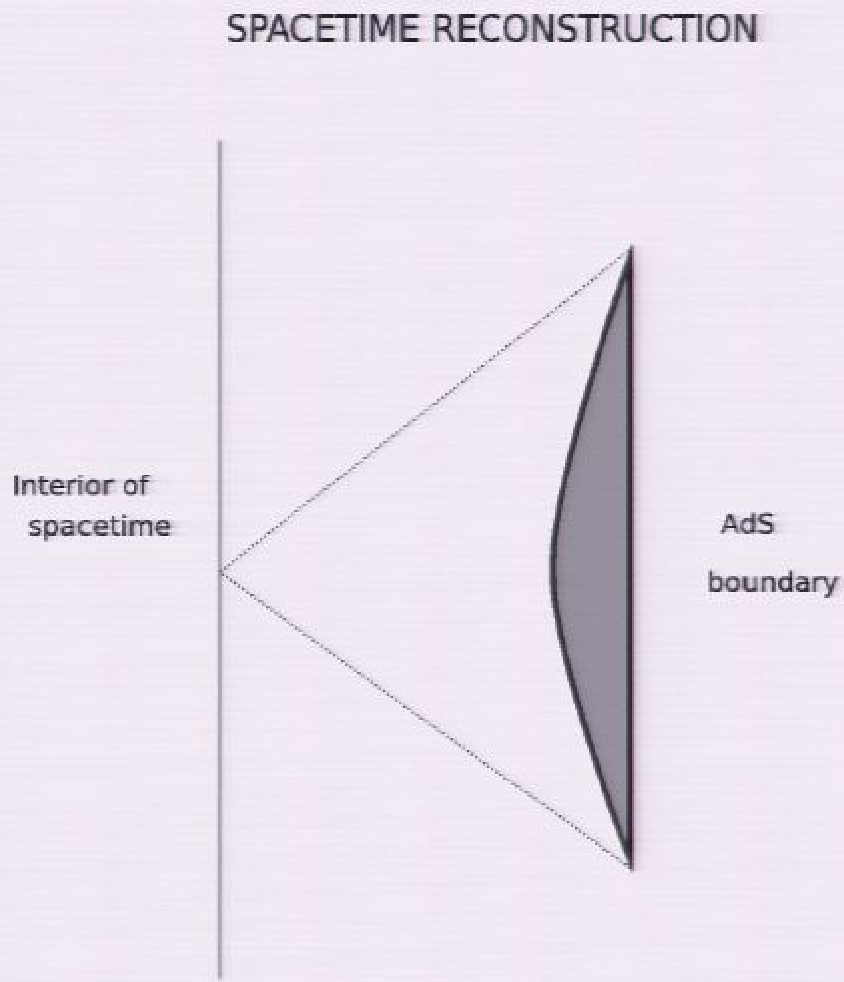

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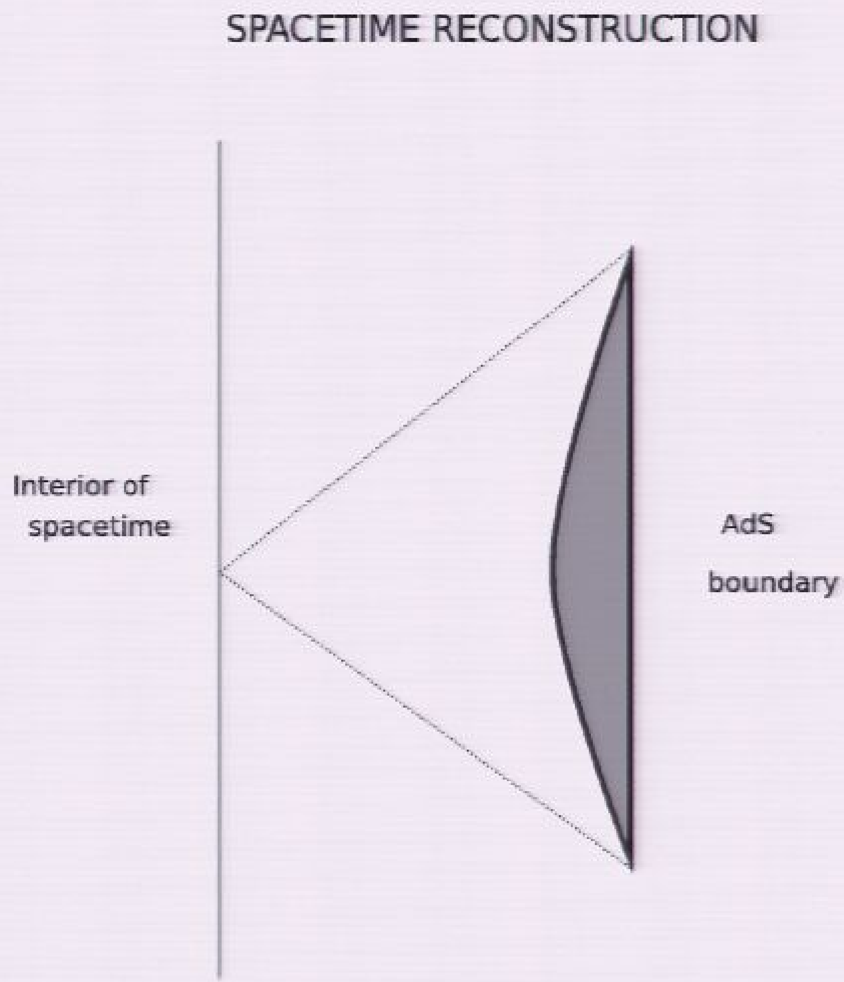

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# Conserved charges for AlAdS spacetimes

The AdS/CFT duality implies a new approach to conserved charges:

- In QFT the energy is computed using the energy momentum tensor,

$$E = \langle H \rangle = \int_C d^{d-1}x \langle T_{00} \rangle$$

  Generically, this expression needs renormalization due to UV infinities.

- In the AdS/CFT correspondence,

$$\langle T_{ij} \rangle = \frac{\delta S_{onshell}[g_{(0)}]}{\delta g_{(0)}^{ij}}$$

  This expression is infinite, due to infinite volume of spacetime (IR divergences) and needs renormalization.

# Holographic charges

- One can holographically renormalize the theory by adding local boundary covariant counterterms [Henningson, Skenderis (1998)].
- One then obtains a finite 1-point function for $T_{ij}$ for general AlAdS spacetimes [de Haro, Solodukhin, Skenderis (2000)]:

$$\langle T_{ij} \rangle \sim g_{(d)ij} + X_{ij}[g_{(0)}]$$

  $X_{ij}[g_{(0)}]$ known local function of $g_{(0)}$.
- One can prove rigorously from first principles (e.g. using Noether's method or Wald's covariant phase space methods) that the holographic charges are the correct gravitational conserved charges [Papadimitriou, Skenderis (2005)].

# Outline

1. AdS/CFT
2. **Topologically massive gravity**
3. Holography for TMG and LCFT

# Topological Massive Gravity [Deser, Jackiw, Templeton]

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$$S = \int d^3x \left( \sqrt{-g}(R - 2\Lambda) + \frac{1}{2\mu}(\Gamma d\Gamma + \Gamma^3) \right)$$

where $\Gamma$ is the 1-form Christoffel symbol.

- The equations of motion are:

$$R_{\mu\nu} + 2g_{\mu\nu} + \frac{1}{\mu}\epsilon_\mu^{\ \rho\sigma}\nabla_\rho R_{\sigma\nu} = 0,$$

and these admit asymptotically *AdS* solutions, for example the BTZ black hole, as well as perturbative massive modes expanding around *AdS*.

- When $\mu \neq 1$ however, the massive modes have negative energy and the theory is unstable.

# Is TMG chiral at $\mu = 1$?

In exploring the TMG theory (motivated by Witten's earlier holography conjecture), Strominger et al have claimed that the dual 2d CFT is chiral at $\mu = 1$, i.e.

1. There are no left moving modes satisfying Brown-Henneaux boundary conditions.
2. The left moving central charge $c_L$ is zero at $\mu = 1$.
3. The theory is stable at $\mu = 1$.

A holographic correspondence between TMG and a chiral CFT was then proposed.

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# Non-chiral mode in TMG

However, a non-chiral mode of TMG was found in [Grumiller-Johansson] which has the asymptotic form

$$g_{ij}(x,r) = \delta_{ij} + r^2(g_{(2)ij} + \log r^2 h_{(2)ij}) + \cdots$$

and differs from the Brown-Henneaux boundary condition because of the $h_{(2)}$ logarithmic term.

- There followed a discussion of whether such boundary conditions are consistent.
- Subsequently it was proven by [Henneaux,Martinez,Troncoso, 0901] that conserved charges are finite with such boundary conditions.

# Non-chiral mode in TMG

From our holographic perspective:

1. A subleading log is not surprising, as the subleading coefficient routinely changes when the bulk action is modified from pure Einstein gravity.
2. The form of the asymptotic expansion should not be fixed by hand but rather derived by solving the bulk equations asymptotically.

We will return to the most general asymptotic solution of TMG shortly.

# Holographic methodology

The new holographic methodology replacing previous approaches is:

1. Derive the most general solution of the bulk equations with general Dirichlet boundary conditions for all fields.
2. General results guarantee that the conserved charges are well-defined and can be obtained from the holographic 1-point functions.

The framework also immediately allows one to go further and compute two and higher point functions.

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# Application to TMG

Let us now consider holography for the topologically massive gravity, at first focussing on the "chiral point". There is an important new element compared to earlier holographic discussions:

- The field equations are third order in derivatives, so there are two independent boundary data: the metric and the extrinsic curvature.
- The boundary metric $g_{(0)}$ is the source for the energy momentum tensor $T_{ij}$.
- The boundary field $b_{(0)ij}$ parameterizing the extrinsic curvature is a source for a new operator $t_{ij}$.

# TMG at the chiral point

We need one further ingredient. It turns out that $t_{ij}$ is obtained as a limit of an irrelevant operator (as $\mu \to 1$).

- In CFT, when one couples an irrelevant operator, this generates severe UV divergences and the theory is no longer conformal in the UV.
- In gravity, a source for an irrelevant operator introduces severe IR divergences and the background is not asymptotically AdS.
- In both cases, one bypasses the problems by treating the source perturbatively.

We will work to first order in $b_{(0)}$, which suffices for the computation of 2-point functions.

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# Results: asymptotic solution

The most general asymptotic solution is

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x, r) dx^i dx^j$$

with

$$g_{ij}(x, r) = b_{(0)ij} \log r^2 + g_{(0)ij} + r^2 (g_{(2)ij} + b_{(2)ij} \log r^2) + \cdots$$

- Only $b_{(0)\bar{z}\bar{z}}$ can be non-zero and is the source for the new operator $t_{zz}$.
- The coefficients $g_{(2)}$ and $b_{(2)}$ are constrained partially by the asymptotic analysis.

# Interpretation of asymptotic solution

- The unconstrained terms $g_{(0)ij}$ and $b_{(0)\bar{z}\bar{z}}$ relate to the boundary values of the metric and extrinsic curvature respectively.
- They act as sources for boundary operators, the stress energy tensor and the new operator $t_{zz}$ respectively.
- The fact that $g_{(2)}$ and $b_{(2)}$ are constrained partially by the asymptotic analysis relates to operator Ward identities.

# Results: 1-point functions

- The holographic 1-point functions can be computed in complete generality:

$$\begin{aligned}\langle T_{ij}\rangle &= \frac{1}{4G_N}\Big(g_{(2)ij} + \frac{1}{2}R[g_{(0)}]g_{(0)ij} \\ &\quad -\frac{1}{2}\Big(\epsilon_i{}^k g_{(2)kj} + (i \leftrightarrow j)\Big) - 2b_{(2)ij} + \frac{1}{2}A_{ij}[g_{(0)ij}]\Big) \\ \langle t_{zz}\rangle &= \frac{1}{2G_N}(g_{(2)zz} + b_{(2)zz})\end{aligned}$$

and are expressed in terms of coefficients in the asymptotic expansions.

# Example: Conserved charges for BTZ black hole

- The energy momentum tensor $T_{ij}$ can be used to obtain the conserved charges.
- For example one can compute the conserved charges for the BTZ black hole. The stress energy tensor becomes chiral at $\mu = 1$,

$$T_{\bar{z}\bar{z}} = \frac{2}{G_N}(r_+ + r_-)^2, \quad T_{zz} = 0,$$

and the conserved charges are

$$\begin{aligned} M &= -\int d\phi T^t_t = \frac{\pi}{4G_N}(r_+ + r_-)^2 \\ J &= -\int d\phi T^t_\phi = M \end{aligned}$$

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&\quad - \frac{1}{2}\Big( \epsilon_i{}^k g_{(2)kj} + (i \leftrightarrow j) \Big) - 2b_{(2)ij} + \frac{1}{2} A_{ij}[g_{(0)ij}] \Big) \\
\langle t_{zz} \rangle &= \frac{1}{2G_N} ( g_{(2)zz} + b_{(2)zz} )
\end{aligned}
$$

and are expressed in terms of coefficients in the asymptotic expansions.

# Example: Conserved charges for BTZ black hole

- The energy momentum tensor $T_{ij}$ can be used to obtain the conserved charges.
- For example one can compute the conserved charges for the BTZ black hole. The stress energy tensor becomes chiral at $\mu = 1$,

$$T_{\bar{z}\bar{z}} = \frac{2}{G_N}(r_+ + r_-)^2, \quad T_{zz} = 0,$$

and the conserved charges are

$$\begin{aligned} M &= -\int d\phi T^t_t = \frac{\pi}{4G_N}(r_+ + r_-)^2 \\ J &= -\int d\phi T^t_\phi = M \end{aligned}$$

# Results: anomalies

- $T_{ij}$ satisfies the expected anomalous CFT Ward identities:

$$\begin{aligned}\langle T^i_i \rangle &= \frac{1}{4G_N}\left(\frac{1}{2}R[g_{(0)}] + \frac{1}{2}A^i_i[g_{(0)}]\right) \\ \nabla^j \langle T_{ij} \rangle &= \frac{1}{4G_N}\left(\frac{1}{4}\epsilon_{ij}\nabla^j R[g_{(0)}] + \frac{1}{2}\nabla^j A_{ij}[g_{(0)}]\right)\end{aligned}$$

where $A_{ij}[g_{(0)}]$ is a certain non-covariant functional of the metric and connection.

# Consistent v covariant stress energy tensor

- $T_{ij}$ follows from the variation of an effective action and its anomaly under a diffeomorphism $\zeta$:

$$H_\zeta = \int d^2x \sqrt{-g}\zeta^i \nabla^j T_{ij}$$

satisfies Wess-Zumino consistency conditions:

$$E_{\zeta_1} H_{\zeta_2} - E_{\zeta_2} H_{\zeta_1} == H_{[\zeta_2,\zeta_1]}.$$

- The consistent anomaly is not covariant. One can find an improvement term $Y_{ij}$ such that the improved stress energy tensor:

$$\hat{T}_{ij} = T_{ij} + Y_{ij}$$

transforms as a tensor, but then $\hat{T}_{ij}$ is not the variation of an effective action.

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# Two-point functions

- To compute two point functions in the dual theory, one looks for regular solutions of the linearized field equations about the AdS background.
- In the asymptotic expansion of these regular solutions, the normalizable modes will be non-locally related to the sources, allowing functional differentiation of the one point functions.
- N-point functions would be obtained by solving field equations at order $(N - 1)$.

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From the general solution of the linearized equations of motion we extract the following non-zero 2-point functions:

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\begin{aligned}
\langle t_{zz}(z,\bar{z})t_{zz}(0)\rangle &= \frac{(3/G_N)\log|z|^2}{z^4}, \\
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Note in particular the logarithmic and non-diagonalizable structure in the left moving sector.

# 2-point functions in a logarithmic CFT

These are precisely the non-zero 2-point functions of a Logarithmic CFT with central charges

$$c_L = 0, \qquad c_R = \frac{3}{G_N}.$$

In the left moving sector the operators $(t_{zz}, T_{zz})$ form a logarithmic pair with non-diagonalizable two point functions, and "new anomaly" parameter $b = -3/G_N$ such that

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- We also analyzed the theory in the neighborhoord around $\mu = 1$.
- Letting $\mu = 2\lambda + 1$, near $\lambda = 0$, the general solution to the linearized equations of motion is expanded near the boundary as

$$h_{ij} = h_{(-2\lambda)ij}\rho^{-\lambda} + h_{(0)ij} + h_{(2)ij}\rho + h_{(2-2\lambda)ij}\rho^{1-\lambda} + h_{(2+2\lambda)ij}\rho^{\lambda+1} + \dots,$$

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$$\langle X_{ij} \rangle = \frac{\lambda(1+\lambda)}{2G_N(2\lambda+1)}(h_{(2+2\lambda)ij})_L,$$

and the two point functions are obtained using exact regular solutions to the linearized equations.

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- From these expressions we see that

$$(c_L, c_R) = \frac{3}{2G_N}\left(1-\frac{1}{\mu}, 1+\frac{1}{\mu}\right)$$

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# Avoiding the $c \to 0$ catastrophe

- These correlation functions smoothly reduce to those at $\mu \to 1$; the operator $t_{zz}$ is given by

$$t_{zz} = -\frac{1}{\lambda}(X_{zz} - T_{zz}).$$

  and we recover the value of $b$ given previously.
- In fact, our discussion mirrors the degeneration of a CFT to a logarithmic CFT as $c \to 0$ discussed by [Kogan, Nichols (2004)]; as here their logarithmic partner of the stress energy tensor originates from another primary whose dimension approaches $(2, 0)$ in the $c \to 0$ limit.
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- From the form of the 2-point functions one see that the CFT contains a state $|X\rangle$ of negative norm. Moreover one can show that $\langle X|H|X\rangle < 0$ in that state.
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# Summary

- Topologically massive gravity at the "chiral point" is dual to Logarithmic CFT and therefore it is not unitary.
- A more appropriate name for the $\mu = 1$ theory would perhaps be the "logarithmic point"!
- The expressions for the two point functions indicate problems with unitarity and positivity, as there are zero morm states at $\mu = 1$; negative norm states at $\mu \neq 1$ and negative conformal weights at $\mu < 1$.

# A chiral subsector?

- One could still try to restrict to the right-moving sector of the theory, as proposed by Strominger et al, which could potentially yield a consistent chiral theory.
- A necessary condition would be $\langle t\bar{T}\bar{T}\rangle = 0$, which indeed holds for certain LCFTs. It would be interesting to compute this 3-point function holographically for TMG at $\mu = 1$.
- However there is no known example of a LCFT admitting such a truncation to a chiral sector....

# Applications of AdS/LCFT duality

- Whilst it seems problematic to consider TMG as a fundamental theory, this and other AdS/LCFT dualities could nonetheless have interesting applications to **condensed matter systems**.
- LCFTs arise in describing critical systems with quenched disorder, as well as in describing transitions between quantum Hall plateaus.

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# Conclusions and outlook

- We have provided further evidence that TMG is dual to a LCFT at the "chiral point".
- It would be natural to use similar techniques to analyse the "new massive gravity" of Bergshoeff, Holm and Townsend.
- Both theories also admit "warped AdS" solutions, whose asymptotics indicate qualitatively different UV behavior of the dual field theory, and it would be interesting to extend the holographic setup to this class of solutions.

# Conclusions and outlook

- More generally, we see that an increasing number of classes of quantum field theories can be realized holographically.
- This is important for exploring whether gravity is always holographic, but also provides many interesting new holographic models which can be applied to CMT and elsewhere.