

Title: Inflationary Cosmology - Lecture 4

Date: Jul 01, 2009 10:00 AM

URL: <http://pirsa.org/09070002>

Abstract:

Massless scalar in an expanding FRW

ck^2

Massless scalar in an expanding FRW

$$ds^2 = a^2(\eta) [-d\eta^2 + d\vec{x}^2]$$

$$S = \int d^4x a^4(\eta) \left[\frac{1}{2} (\partial_\eta \phi)^2 - \frac{1}{2} (\nabla \phi)^2 \right]$$

Massless scalar in an expanding FRW

$$ds^2 = a^2(\eta) [-d\eta^2 + d\vec{x}^2]$$

$$S = \int d^4x a^4(\eta) \left[\frac{1}{2} (\partial_\eta \phi)^2 - \frac{1}{2} (\nabla \phi)^2 \right]$$

$$= a^4 \phi$$

$$S = \int d^4x \left[\frac{1}{2} (\partial_\eta \psi)^2 - \frac{a^4}{2} (\partial_i \psi)^2 \right]$$

Massless scalar in an expanding FRW

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$$S = \int d^4x a^4(\eta) \left[\frac{1}{2} (\partial_\eta \phi)^2 - \frac{1}{2} (\nabla \phi)^2 \right]$$

$$y \equiv a \phi$$

$$S = \int d^4x \left[\frac{1}{2} (\partial_\eta y - \frac{\dot{a}}{a} y)^2 - \frac{1}{2} (\partial_i y)^2 \right]$$

Massless scalar in an expanding FRW

$$ds^2 = a^2(\eta) [-d\eta^2 + d\vec{x}^2]$$

$$S = \int d^4x a^4(\eta) \left[\frac{1}{2} (\partial_\eta \phi)^2 - \frac{1}{2} (\nabla \phi)^2 \right]$$

$$y \equiv a\phi$$

$$S = \int d^4x \left[\frac{1}{2} \left(\partial_\eta y - \frac{\dot{a}}{a} y \right)^2 - \frac{1}{2} (\partial_i y)^2 \right]$$

$$\mathcal{H} = \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2} p(k) p^+(k) + \frac{1}{2} k^2 y(k) y^+(k) + \frac{1}{2} \frac{a'}{a} [y(k) p^+(k) - p(k) y^+(k)] \right]$$

$$\mathcal{H} = \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2} p(k) p^\dagger(k) + \frac{1}{2} k^2 y(k) y^\dagger(k) \right. \\ \left. + \frac{1}{2} \frac{a'}{a} [y(k) p^\dagger(k) - p(k) y^\dagger(k)] \right]$$

$$\mathcal{H} = \int \frac{d^3k}{(2\pi)^3} \left[\frac{k}{2} [\hat{a}(k) \hat{a}^\dagger(k) + \hat{a}^\dagger(k) \hat{a}(k)] \right. \\ \left. + i \frac{a'}{a} (\hat{a}^\dagger(k) \hat{a}^\dagger(-k) - \hat{a}(k) \hat{a}(-k)) \right]$$

$$\begin{aligned}
 \mathcal{H} &= \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \left[\hat{a}(k) \hat{a}^\dagger(k) + \hat{a}^\dagger(k) \hat{a}(k) \right] \\
 &\quad + i \frac{a'}{a} \left(\hat{a}^\dagger(k) \hat{a}^\dagger(-k) - \hat{a}(k) \hat{a}(-k) \right) \\
 \begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix} &= \begin{pmatrix} -ik \\ ik \end{pmatrix} \begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{H} &= \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \left[\hat{a}(k) \hat{a}^\dagger(k) + \hat{a}^\dagger(k) \hat{a}(k) \right] \\
 &+ i \frac{a}{2} \left(\hat{a}^\dagger(k) \hat{a}^\dagger(-k) - \hat{a}(k) \hat{a}(-k) \right) \\
 &\begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix}^{-1} \gamma \begin{pmatrix} -ik & p/a \\ p/a & ik \end{pmatrix} \begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{H} = & \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \left[\hat{a}(k) \hat{a}^\dagger(k) + \hat{a}^\dagger(k) \hat{a}(k) \right] \\
 & + i \frac{a'}{a} \left(\hat{a}^\dagger(k) \hat{a}^\dagger(-k) - \hat{a}(k) \hat{a}(-k) \right) \quad k \gtrsim \frac{a'}{a} - aH \\
 \begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix} = & \begin{pmatrix} -ik & p/a_- \\ p/a_- & ik \end{pmatrix} \begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix}
 \end{aligned}$$

$\rightarrow > 600$

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$$\mathcal{H} = \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \left[\hat{a}(k) \hat{a}^\dagger(k) + \hat{a}^\dagger(k) \hat{a}(k) \right]$$

$$+ i \frac{a'}{a} \left(\hat{a}^\dagger(k) \hat{a}^\dagger(-k) - \hat{a}(k) \hat{a}(-k) \right)$$

$$k \gtrsim \frac{a'}{a} - aH$$

$$\begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix} = \begin{pmatrix} -ik & p/a' \\ p/a' & ik \end{pmatrix} \begin{pmatrix} \hat{a}(k) \\ \hat{a}^\dagger(-k) \end{pmatrix}$$

$v_1 > 600$

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$$\begin{pmatrix} \hat{Q} \\ \hat{Q}^+ \end{pmatrix} \xrightarrow{\hat{Q}^+} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \hat{Q} \\ \hat{Q}^+ \end{pmatrix}$$

$$y(F) \propto Q$$

$$p(F) \propto Q^{-1}$$



$$\begin{pmatrix} \hat{Q}_+ \\ \hat{Q}_- \end{pmatrix} = \begin{pmatrix} \hat{Q}_+ \\ \hat{Q}_- \end{pmatrix} \begin{pmatrix} 0 & - \\ - & 0 \end{pmatrix} \begin{pmatrix} \hat{Q}_+ \\ \hat{Q}_- \end{pmatrix}$$

$$\begin{aligned} Y(E) &\propto Q \\ P(E) &\propto Q^{-1} \end{aligned} \Rightarrow \phi = \text{const}$$

$$a^e = \frac{-1}{H^2 \eta^2}$$

$$\frac{Hk}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta}$$

$$a^{\pm} = \frac{-1}{i\eta^2} \quad \eta \in (-\infty, 0)$$

$$\frac{H_k}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta} = \phi_k^{cc}(\eta)$$

$$(1, \vec{x}) = \int \frac{d^3p_k}{(2\pi)^3} \phi_k(\eta) e^{i\vec{k}\cdot\vec{x}}$$

$$\phi_k(\eta) = \phi_k$$

$$a^{\pm} = \frac{-1}{i\sqrt{2\pi}} \quad \eta \in (-\infty, 0)$$

$$\frac{H(\eta)}{\sqrt{2\pi}} (1 - i\eta) e^{i\eta} = \phi_k^{(+) }(\eta)$$

$$\phi(\mathbf{r}, \mathbf{r}') = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$\phi_k(\eta) = \phi_k^{(+)}(\eta) \hat{a}_k^{\dagger} + \phi_k^{(-)}(\eta) \hat{a}_k$$

$$a^{\pm} = \frac{-1}{H^2 \eta^2} \quad \eta \in (-\infty, 0)$$

$$\frac{H}{\sqrt{2k^3}} e^{ik\eta + i\frac{k}{a} \Delta\eta}$$

$$\frac{H}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta} = \phi_k^{cc}(\eta)$$

$$\phi(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\phi_k(\eta) = \phi_k^{\circ}(\eta) \hat{a}_k^{\dagger} + \phi_k^{**}(\eta) \hat{a}_k$$

$$a^{\pm} = \frac{-1}{H^2 \eta^2} \quad \eta \in (-\infty, 0)$$

$$\frac{H}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta} = \phi_k^{cc}(\eta)$$

$$\Phi(\mathbf{x}, \bar{x}) = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\phi_k(\eta) = \phi_k^{\circ}(\eta) \hat{a}_k^{\dagger} + \phi_k^{**}(\eta) \hat{a}_k$$

$$\frac{H}{\sqrt{2k^3}} k\eta e^{ik\eta + i\left(\frac{k}{a}\right) \Delta\eta} \quad k_{\mu}$$

$$a^{\pm} = \frac{-1}{H^2 \eta^2} \quad \eta \in (-\infty, 0)$$

$$\frac{H}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta} = \phi_k^{cc}(\eta)$$

$$\phi(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\phi_k(\eta) = \phi_k^o(\eta) \hat{a}_k^{\dagger} + \phi_k^{*o}(\eta) \hat{a}_k$$

$$\frac{H}{\sqrt{2k^3}} k\eta e^{ik\eta + i\left(\frac{k}{a}\right) \Delta\eta}$$

$$a^{\pm} = \frac{-1}{H^2 \eta^2} \quad \eta \in (-\infty, 0)$$

$$\frac{Hk}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta} = \phi_k^{(+)}$$

$$\phi(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\phi_k(\eta) = \phi_k^{(+)}(\eta) \hat{a}_k^{\dagger} + \phi_k^{(-)*}(\eta) \hat{a}_{-k}$$

$$\frac{Hk}{\sqrt{2k^3}} a_{\mathbf{k}}^{\pm} e^{i(k\eta + \frac{k}{\alpha} \Delta\eta)}$$

$$a^{\pm} = \frac{-1}{H^2 \eta^2} \quad \eta \in (-\infty, 0)$$

$$\frac{Hk}{\sqrt{2k^3}} (1 - ik\eta) e^{ik\eta} = \phi_k^{cc}(\eta)$$

$$\Phi(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\phi_k(\eta) = \phi_k^o(\eta) \hat{a}_k^{\dagger} + \phi_k^{cc*}(\eta) \hat{a}_k$$

$$\frac{Hk}{\sqrt{2k^3}} a_{\mathbf{k}}^{\dagger} e^{i(k\eta + \frac{k}{\alpha} \Delta\eta)}$$

Normalization

$$\langle \phi_E(\eta) \phi_E(\eta) \rangle =$$

$$\langle \phi_{\vec{k}}(\eta) \phi_{\vec{k}'}(\eta) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') |\phi_{\vec{k}}|^2$$

$$= (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3} (1 + k^2 \eta^2)$$

$$\langle \phi_{\vec{k}}(\eta) \phi_{\vec{k}'}(\eta) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') |\phi_{cc}|^2$$

$$= (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3} \underbrace{(1 + k^2 \eta^2)}_{\text{Scale-invariant}} \approx (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3}$$

Scale-invariant

$$\langle \phi_{\vec{k}}(\eta) \phi_{\vec{k}'}(\eta) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') |\phi_{cc}|^2$$

$$= (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3} \left(1 + \frac{k^2 \eta^2}{4}\right) \approx (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3}$$

Scale-invariant

$$\langle \phi(x, \eta) \phi(x', \eta) \rangle = -\frac{H^2}{(2\pi)^2} \log \frac{|x - x'|}{L}$$

$$\langle \phi_{\vec{k}}(\eta) \phi_{\vec{k}'}(\eta) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') |\phi_{\vec{k}}|^2$$

$$= (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3} \left(1 + \frac{k^2 \eta^2}{3}\right) \approx (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3}$$

$$\langle \phi(x, \eta) \phi(x', \eta) \rangle = -\frac{H^2}{(2\pi)^2} \log \frac{|x - x'|}{L}$$

Scale-invariant

Scalar spectrum in spatially flat gauge

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Scalar spectrum in spatially flat gauge



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Scalar spectrum in spatially flat gauge

$$\phi(t, \vec{x}) = \phi_0(t)$$



Scalar spectrum in spatially flat gauge

$$\phi(t, \vec{x}) = \phi_c(t) + \varphi(t, \vec{x})$$

Scalar spectrum in spatially flat gauge

$$\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$$

$$g_{ij} = a^2(t) \delta_{ij}$$

Scalar spectrum in spatially flat gauge

$$\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$$

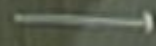


$$g_{ij} = a^2(t) \delta_{ij}$$



Scalar spectrum in spatially flat gauge

$$\phi(t, \vec{x}) = \phi_0(t) + \psi(t, \vec{x})$$



$$g_{ij} = a^2(t) \delta_{ij}$$

Action for ψ is
a massless scalar in d/S
up to slow-roll corrections

$$\delta t = - \frac{\varphi(t, \vec{x})}{\dot{\varphi}}$$

$$a^i(t + \delta t) \delta_{ij} = a^i(t) \left(1 - \frac{2H\varphi}{\dot{\varphi}} \right) \delta_{ij}$$

$$\delta t = - \frac{\varphi(t; \vec{x})}{\dot{\varphi}}$$

$$a^i(t + \delta t) \delta_{ij} = a^i(t) \left(1 - \frac{2H\varphi}{\dot{\varphi}} \right) \delta_{ij}$$

(3) R

$$1 + 2\mathcal{S}(\vec{x}, t)$$

$$\delta t = - \frac{\varphi(t, \bar{x})}{\dot{\phi}}$$

$$a^i(t + \delta t) \delta_{ij} = a^i(t) \left(1 - \frac{2H\varphi}{\dot{\phi}} \right) \delta_{ij}$$

$$(3) R = - \frac{4}{a^2} \nabla^2 \zeta$$

$$1 + 2\zeta(\bar{x}, t)$$

$$\delta t = - \frac{\varphi(t, \vec{x})}{\dot{\phi}}$$

$$a^i(t + \delta t) \delta_{ij} = a^i(t) \left(1 - \frac{2H\varphi}{\dot{\phi}} \right) \delta_{ij}$$

$$(3) R = - \frac{4}{a^2} \nabla^2 \zeta$$

$$1 + 2\zeta(\vec{x}, t)$$

Conservation of ζ
on super H scales

$$\langle \zeta_{\mathbf{E}}(\eta) \zeta_{\mathbf{E}'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{E} + \mathbf{E}') \frac{H^4}{2k^3} \frac{H^2}{\phi^2}$$

after
Hubble
crossing



$$\langle \zeta_{\mathbf{E}}(\eta) \zeta_{\mathbf{E}'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{E} + \mathbf{E}') \frac{H^4}{2k^3} \frac{H^2}{\phi^2}$$

after
Hubble
crossing

$$\langle \zeta \zeta \rangle \sim \frac{H^4}{\phi^2}$$

$$\zeta \sim \frac{H^2}{\phi}$$

$$\langle \zeta_E(\eta) \zeta_{E'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{H^2}{2k^3} \frac{H^2}{\phi^2}$$

after
Hubble
crossing

$$\langle \zeta \zeta \rangle \sim \frac{H^4}{\phi^2}$$

$$\zeta \sim \frac{H^2}{\phi} \sim \frac{H}{\phi/H}$$



$$\langle \zeta_{\mathbf{k}}(\eta) \zeta_{\mathbf{k}'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{H^2}{2k^2} \cdot \frac{H^2}{\phi^2}$$

after
Hubble
crossing

$$\langle \zeta \zeta \rangle \sim \frac{H^4}{\phi^2} \approx (10^{-5})^2$$

$$\zeta \sim \frac{H^2}{\phi} \sim \frac{H}{\phi/H}$$

$$\langle \zeta_{\mathbf{k}}(\eta) \zeta_{\mathbf{k}'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{H^2}{2k^2} \cdot \frac{H^2}{\phi^2}$$

after
Hubble
crossing

$$\langle \zeta \zeta \rangle \sim \frac{H^2}{\phi^2} \approx (10^{-5})^2$$

$$\sim \frac{H^2}{\phi} \sim \frac{H}{\phi/H}$$

$$V = \frac{1}{2} m^2 \phi^2$$

$$\left(\frac{H}{\sqrt{4\pi}} \frac{1}{\sqrt{\epsilon}} \right)^2$$

$$\langle \zeta_{\vec{k}}(\eta) \zeta_{\vec{k}'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^2} \frac{H^2}{\phi^2}$$

after
Hubble
crossing

$$\langle \zeta \zeta \rangle \sim \frac{H^4}{\phi^2} \approx (10^{-5})^2$$

$$\zeta \sim \frac{H^2}{\phi} = \frac{H}{\phi/H}$$

$$V = \frac{1}{2} m^2 \phi^2$$

$$\epsilon \sim \frac{1}{2N}$$

$$\frac{H}{M_{Pl}} \sim 10^{-4}$$

$$\left(\frac{H}{M_{Pl}} \frac{1}{\sqrt{\epsilon}} \right)^2$$

$$H \sim 10^{13} \text{ GeV} \rightarrow m \sim 10^{13} \text{ GeV}$$

$$V \sim (10^{16} \text{ GeV})^4$$

$$a^{\pm} = \frac{-1}{H^{\pm} \eta^{\pm}} \quad \eta \in (-\infty, 0)$$

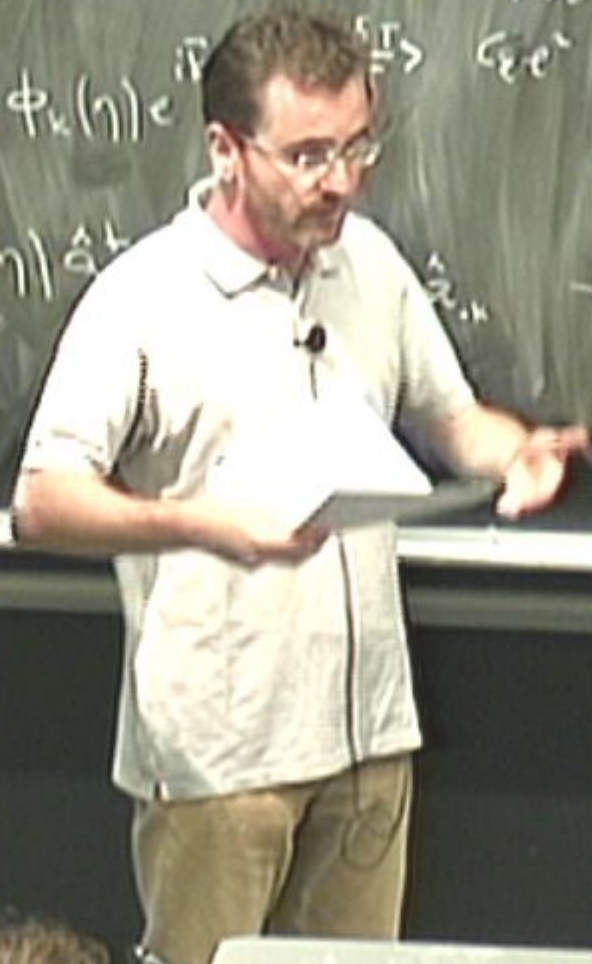
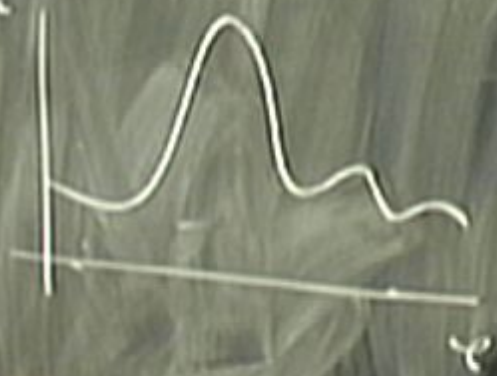
$$\frac{H^{\pm}}{\sqrt{2k^{\pm}}} (1 - ik^{\pm} \eta) e^{ik^{\pm} \eta} = \phi_k^{\pm}(\eta)$$

$$\Phi(t; \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \phi_k(\eta) e^{i\vec{k} \cdot \vec{x}}$$

$$\phi_+(\eta) = \phi_+^o(\eta) \hat{\Delta}^+$$

$$\frac{1}{\sqrt{2k^{\pm}}} a_{\vec{k}}^{\pm} e^{ik^{\pm} \eta + i\left(\frac{k}{a}\right) \delta \eta}$$

Normalization



$$\langle \zeta_{\vec{k}}(\eta) \zeta_{\vec{k}'}(\eta') \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^3} \frac{H^2}{\phi^2}$$

after
Hubble
crossing

$$\langle \zeta \zeta \rangle \sim \frac{H^4}{\phi^2} \approx (10^{-5})^2$$

$$\frac{H^2}{\phi} \rightarrow \frac{H}{\phi/H}$$

$$V = \frac{1}{2} m^2 \phi^2$$

$$\epsilon \sim \frac{1}{N}$$

$$\frac{H}{M_{\text{Pl}}} \sim 10^{-4}$$

$$\left(\frac{H}{M_{\text{Pl}}} \frac{1}{\sqrt{\epsilon}} \right)^2$$

$$H \sim 10^{13} \text{ GeV} \rightarrow m \sim 10^{13} \text{ GeV}$$

$$V \sim (10^{16} \text{ GeV})^4$$

$$1) \langle \sum_{\vec{k}} \langle \eta' \rangle \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{H^2}{2k^2} \cdot \frac{H^2}{\phi^2}$$

after
Hubble
crossing $\frac{k}{a} \sim H$

$$\gg \frac{H^4}{\phi^2} \approx (10^{-5})^2$$

$$\frac{H^2}{\phi} \sim \frac{H}{\phi/H}$$

$$V = \frac{1}{2} m^2 \phi^2$$

$$E \sim \frac{1}{2}$$

$$\frac{H}{M_{Pl}} \sim 10^{-4}$$

$$\left(\frac{H}{\frac{1}{\sqrt{2}} \frac{M_{Pl}}{\sqrt{\epsilon}}} \right)^2$$

$$H \sim 10^{13} \text{ GeV} \rightarrow m \sim 10^{13} \text{ GeV}$$

$$V \sim (10^{16} \text{ GeV})^4$$

Scale depen



Scale dependence

$$K^{-3+(n_s-1)}$$

$$\frac{d}{d \log k} \log \frac{H^4}{\phi} \quad \left| \begin{array}{l} \frac{k}{a} \sim H \\ d \log k = H dt \end{array} \right.$$

$$\frac{k}{a} \sim H$$

$$10^{-9}$$

$$m \sim 10^{13} \text{ GeV}$$

Scale dependence:

$$K^{-3+(m_s-1)}$$

$$m_s - 1 = \frac{d}{d \log k} \log \frac{H^4}{\phi} \quad \left| \begin{array}{l} \frac{k}{a} \sim H \\ d \log k = H dt \end{array} \right. = H^{-1} \frac{d}{dt}$$

$$\frac{k}{a} \sim H$$

$$10^{-6}$$

$$m \sim 10^{13} \text{ GeV}$$

Scale dependence:

$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$K^{-3+(n_s-1)}$$

$$n_s - 1 = \frac{d}{d \log k} \log \frac{H^4}{k^2} \quad \left| \begin{array}{l} \frac{k}{a} = H \\ d \log k = H dt \end{array} \right. \Rightarrow H^{-1} \frac{d}{dt} \log \frac{H^4}{V^{1/2}} = -6\epsilon$$

Scale dependence:

$$\epsilon = -\frac{\dot{H}}{H^2} \quad V'' \quad \eta$$

$$K^{-3+(m_s-1)}$$

$$m_s - 1 = \frac{d}{d \log k} \log \left(\frac{H^4}{\phi^2} \right) \Bigg|_{\frac{k}{a} \sim H} = H^{-1} \frac{d}{dt} \log \left(\frac{H^4}{V^{1/2}} \right) = -6\epsilon + 2\eta$$

$d \log k = H dt$

$m_s - 1 < 0$ red spectrum

$m_s - 1 > 0$ blue spectrum

$$K = -3 + (m_s - 2)$$

$$m_s - 1 = \frac{d}{d \log k} \log \frac{H^4}{\phi^2} \Big|_{\frac{d \log k}{dt} \sim H} = H^{-1} \frac{d}{dt} \log \frac{H^4}{V^{1/2}} = -6\epsilon + 2\eta$$

$m_s - 1 < 0$ red spectrum

$m_s - 1 > 0$ blue spectrum

$$m_s \approx 0.96$$



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