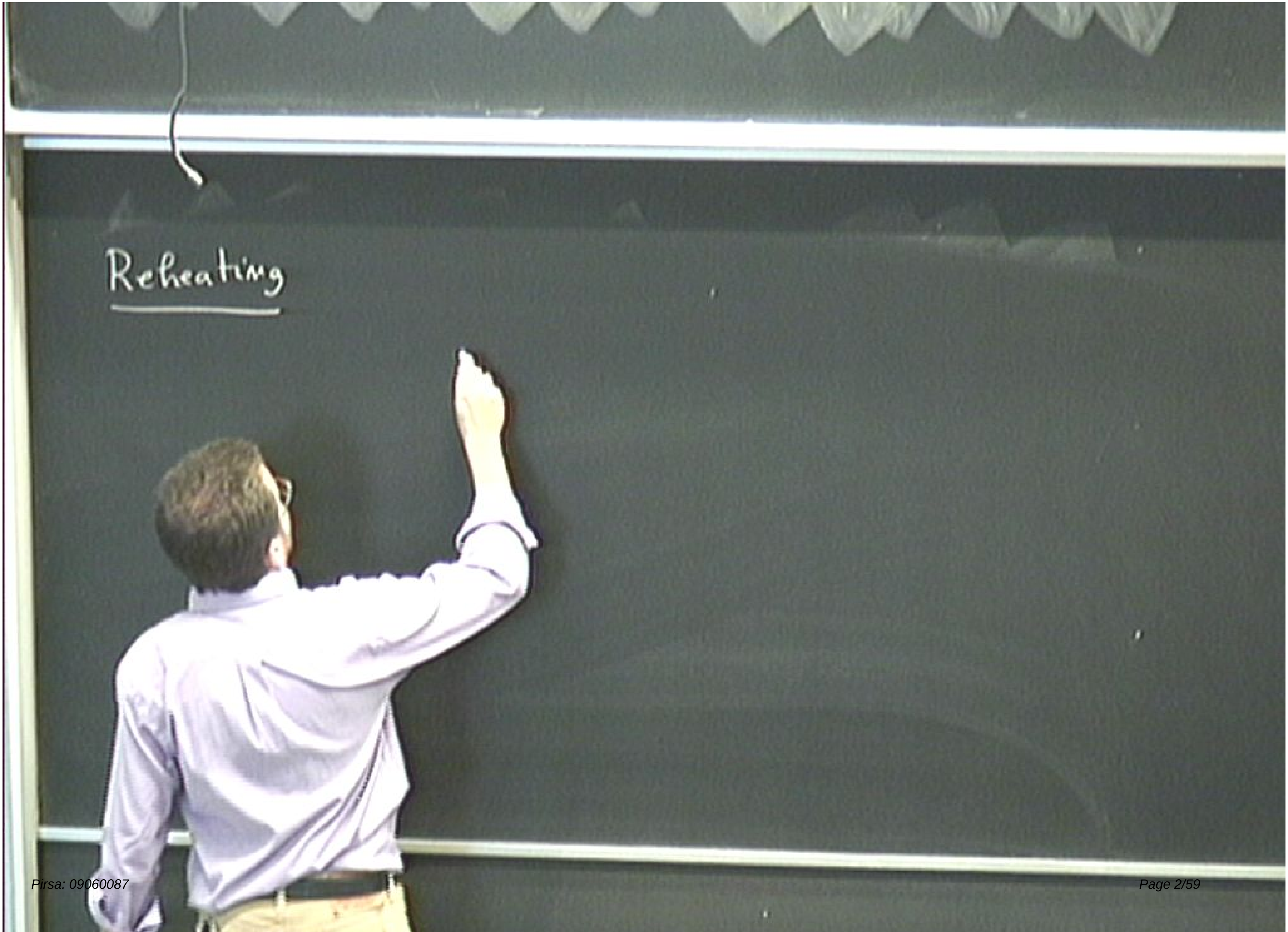
Title: Inflationary Cosmology - Lecture 3

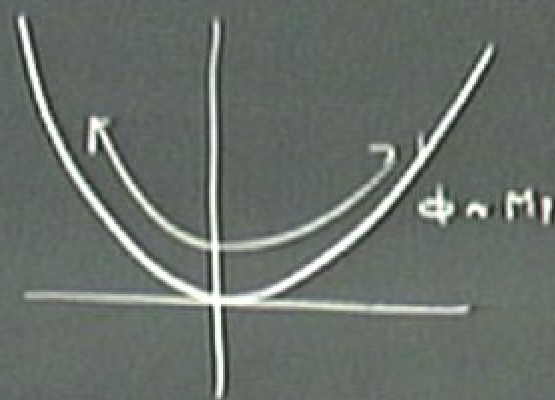
Date: Jun 30, 2009 11:30 AM

URL: http://pirsa.org/09060087

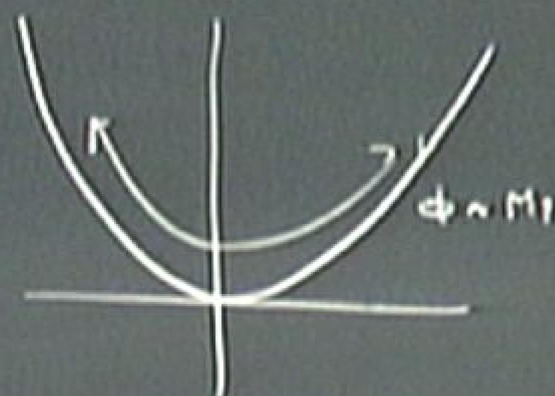
Abstract:


Reheating

## Reheating

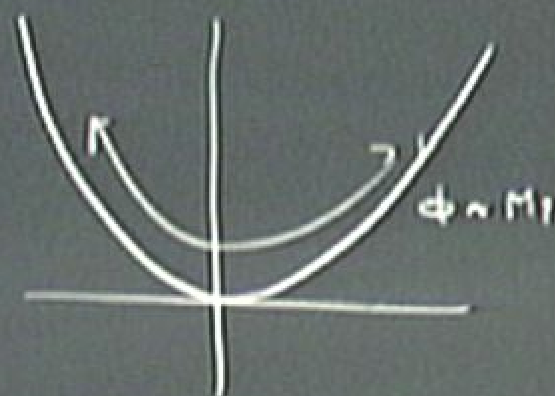


## Reheating



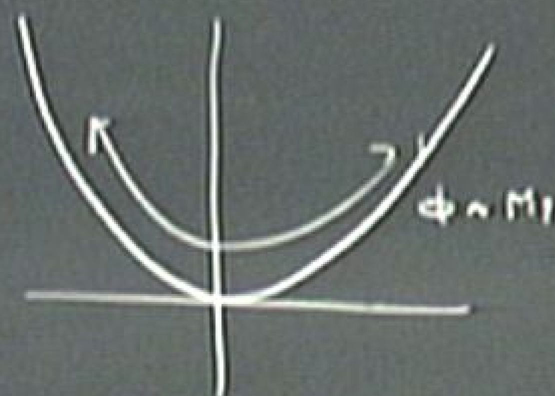

$$n_\phi = \frac{\epsilon_\phi}{m} = \frac{1}{2m}\left(\dot{\phi}^2 + m^2\phi^2\right) = \frac{1}{2} m \Phi^2$$

## Reheating




$$n_\phi = \frac{\epsilon_\phi}{m} = \frac{1}{2m}\left(\dot{\phi}^2 + m^2\phi^2\right) \simeq \frac{1}{2} m \Phi^2 \qquad 10^{92}\ \mathrm{cm}^{-3}$$

## Reheating

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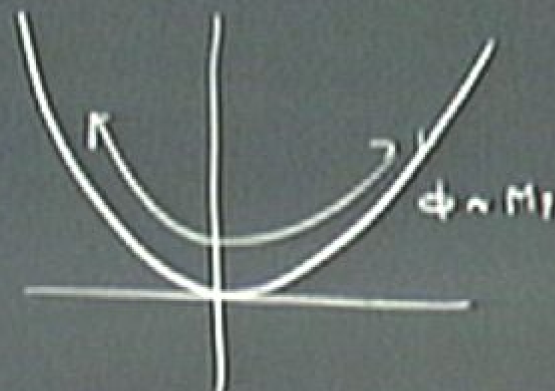
$\Gamma$

## Reheating

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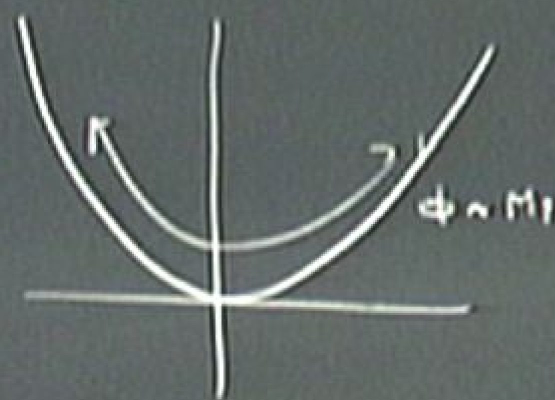
$$\Gamma \simeq H$$

## Reheating

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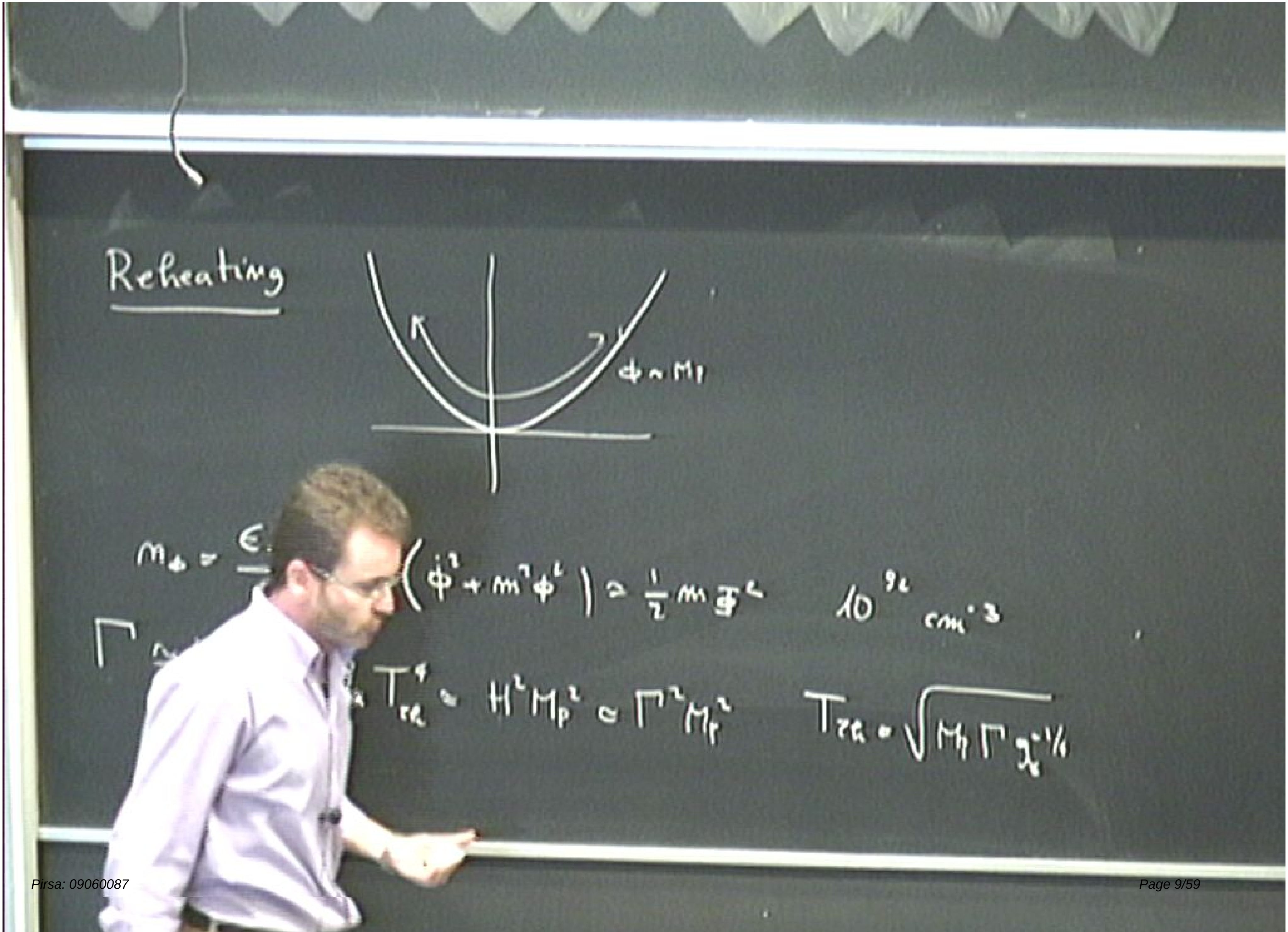
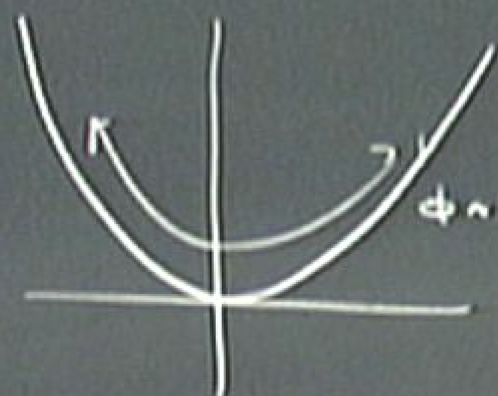
$$\Gamma \simeq H \qquad g_* T_{RH}^4 \simeq H^2 M_P^2 \simeq \Gamma^2 M_P^2$$


Reheating
φ ~ M_P
m_φ = ε
(φ̇² + m²φ²) ≈ ½ m Φ²
10^92 cm^-3
Γ ~
T_rh^4 ≈ H² M_p² ≈ Γ² M_p²
T_rh ≈ √(M_p Γ g_*^-1/2)

## Reheating

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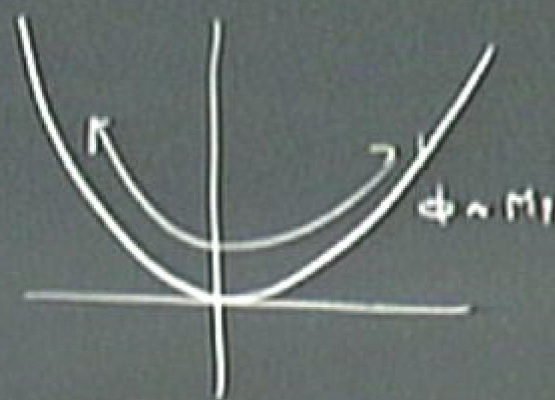
## (P)Reheating

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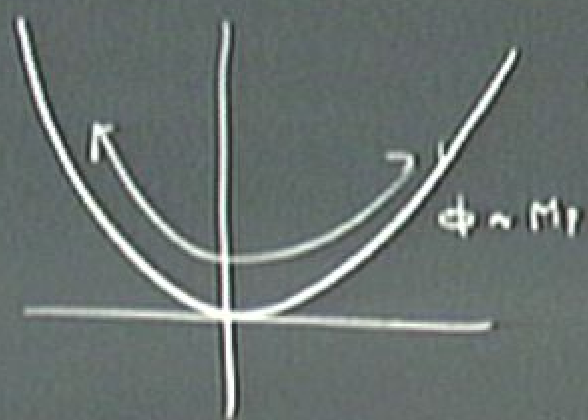


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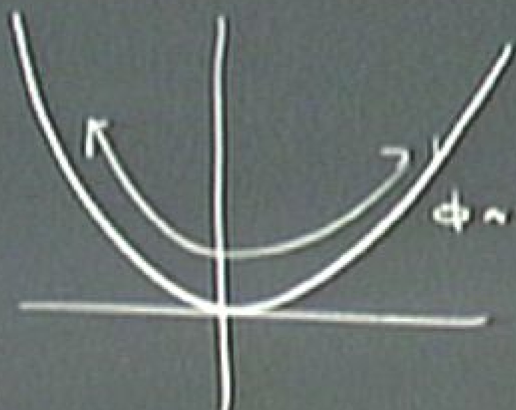


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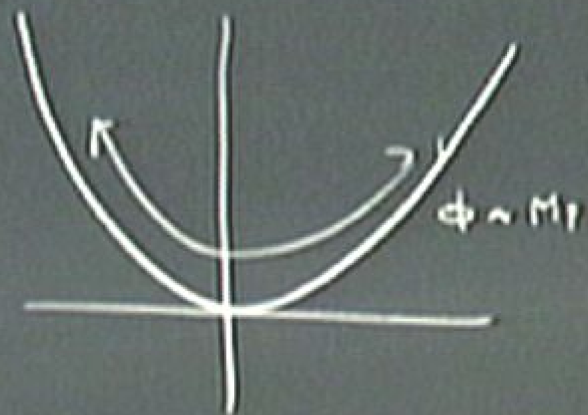
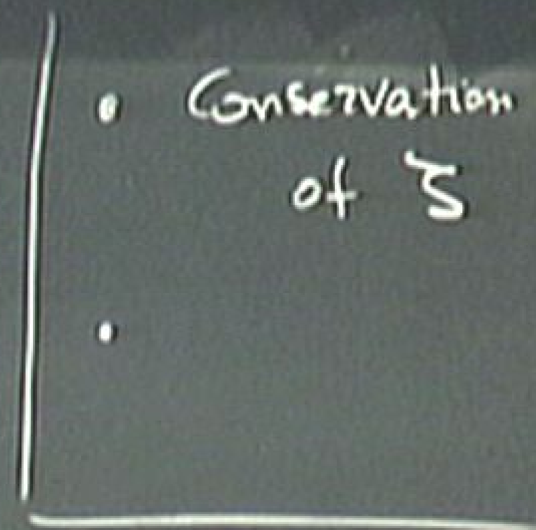



• Conservation of $\zeta$

•



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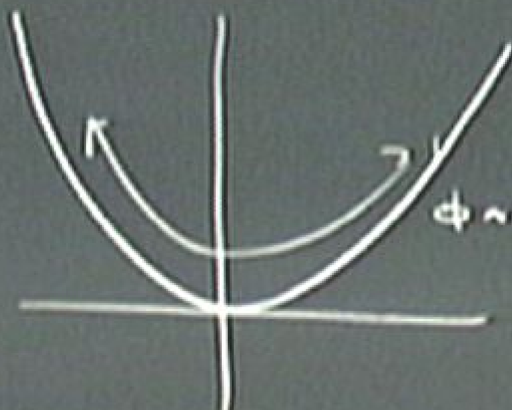
(P)Reheating




• Conservation of $\zeta$

$10^{15}\,GeV \longleftrightarrow 1\,MeV$

$$n_\phi = \frac{\epsilon_\phi}{m} = \frac{1}{2m}\left(\dot\phi^2 + m^2\phi^2\right) \simeq \frac{1}{2} m \Phi^2 \qquad 10^{92}\ cm^{-3}$$

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- Classification of infl. models:

$$\frac{1}{\sqrt{\epsilon}} \frac{H}{M_P} \simeq 10^{-5}$$

(1) High energy models

$$V \propto \phi^n \qquad \epsilon, \eta = \left(\frac{M_P}{\phi}\right)^2$$

$$\frac{H}{M_P} \quad \text{GW signal}$$

$$\Delta \phi$$

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① High energy models

$$V \propto \phi^n \qquad \epsilon, \eta = \left(\frac{M_P}{\phi}\right)^2 \sim \frac{1}{N}$$

$$\Delta\phi \gg M_P$$

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$$V \sim 10^{16}\,\text{GeV} \sim M_{GUT}$$

RHCB

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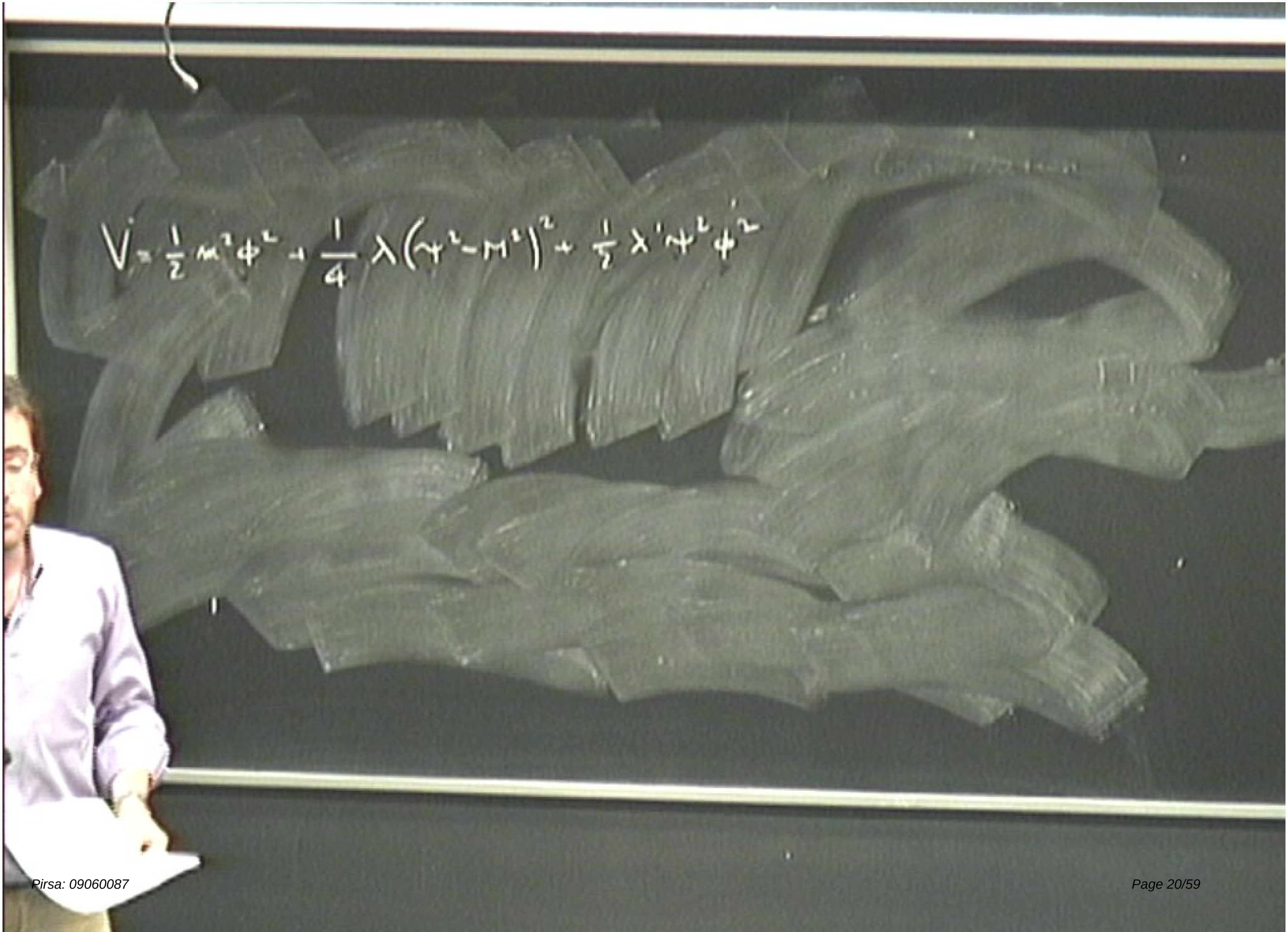
$$V \sim 10^{?}\,\text{GeV} \sim M_{GUT}$$

(2) Hybrid models


$V = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda (\psi^2 - M^2)^2 + \frac{1}{2} \lambda' \psi^2 \phi^2$

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$\epsilon \lll 1$

• Classification of infl models:

$$\frac{1}{\sqrt{\epsilon}}\,\frac{H}{M_P} \simeq 10^{-5}$$

$$\frac{H}{M_P} \quad \text{GW signal}$$

$$\Delta\phi \simeq \frac{\dot\phi}{H} \simeq \frac{V'}{H^2} \simeq \sqrt{\epsilon}\, M_P$$

① High energy models

$$V \propto \phi^n \qquad \epsilon, \eta \simeq \left(\frac{M_P}{\phi}\right)^2 \sim \frac{1}{N}$$

$$\Delta\phi \gg M_P$$

$$V \sim 10^{16}\,\text{GeV} \sim M_{GUT}$$

② Hybrid models

- Classification of infl. models:

$$\frac{1}{\sqrt{\epsilon}} \frac{H}{M_P} \simeq 10^{-5}$$

$$\frac{H}{M_P}$$ GW signal

$$\Delta\phi \simeq \frac{\dot{\phi}}{H} \simeq \frac{V'}{H^2} \simeq \sqrt{\epsilon}\, M_P$$

$\boxed{\text{Lyth bound}}$

① High energy models

$$V \propto \phi^n \qquad \epsilon, \eta = \left(\frac{M_P}{\phi}\right)^2 \sim \frac{1}{N}$$

$$\Delta\phi \gg M_P$$

$$V \sim 10^{4}\ \text{GeV} \sim M_{GUT}$$

② Hybrid models

· Classification of infl. models:

$$\frac{1}{\sqrt{\epsilon}} \cdot \frac{H}{M_P} \simeq 10^{-5}$$

$$\frac{H}{M_P} \quad \text{GW signal}$$

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② Hybrid models

$\hbar \neq 0 = 1$

- Harmonic oscillator
- Free field in dS

$\hbar \neq 0 = 1$

- Harmonic oscillator
- Free field in dS
- Scalar + tensor spectrum

$\hbar \neq 0 = 1$

- Harmonic oscillator
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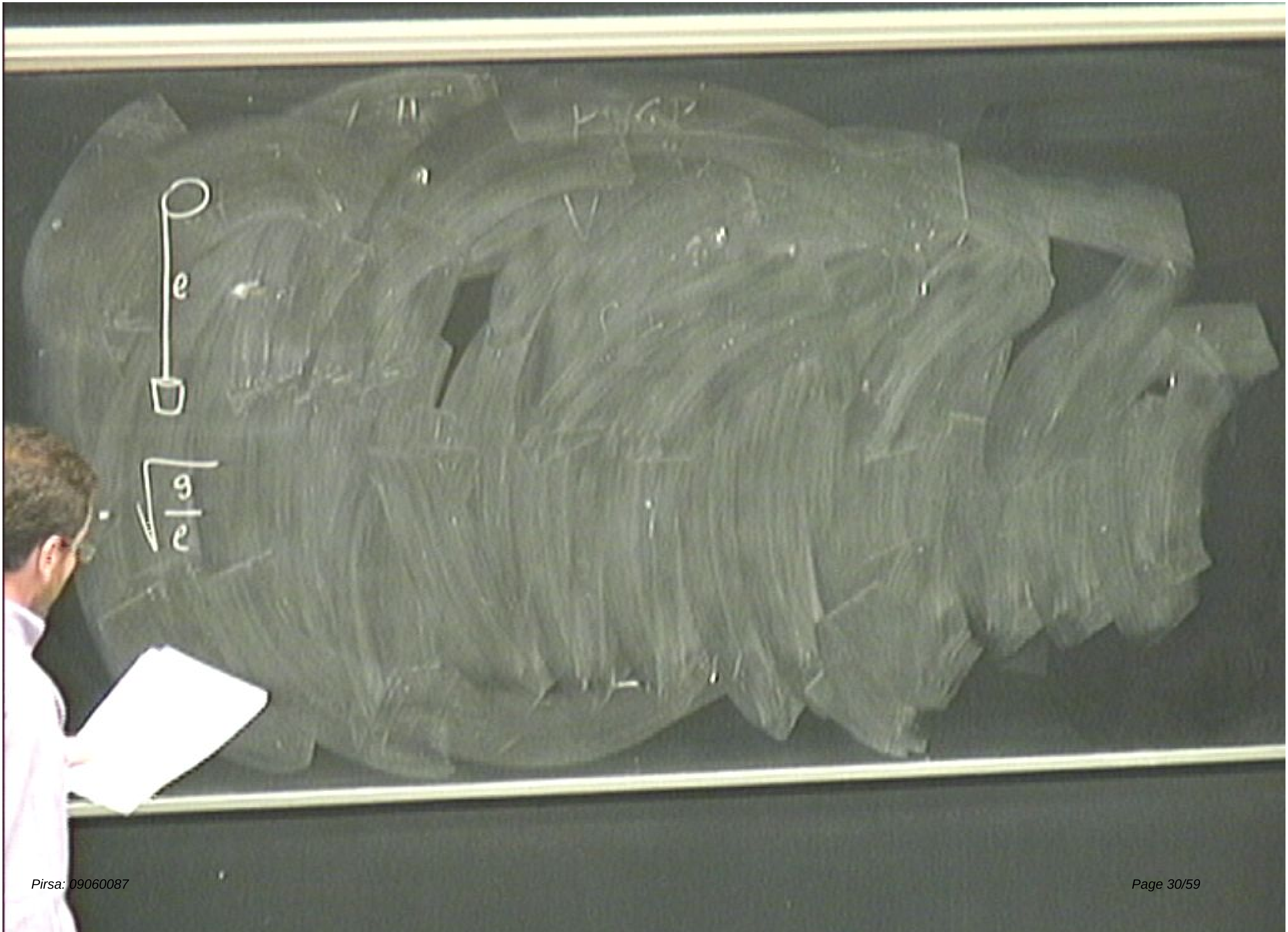
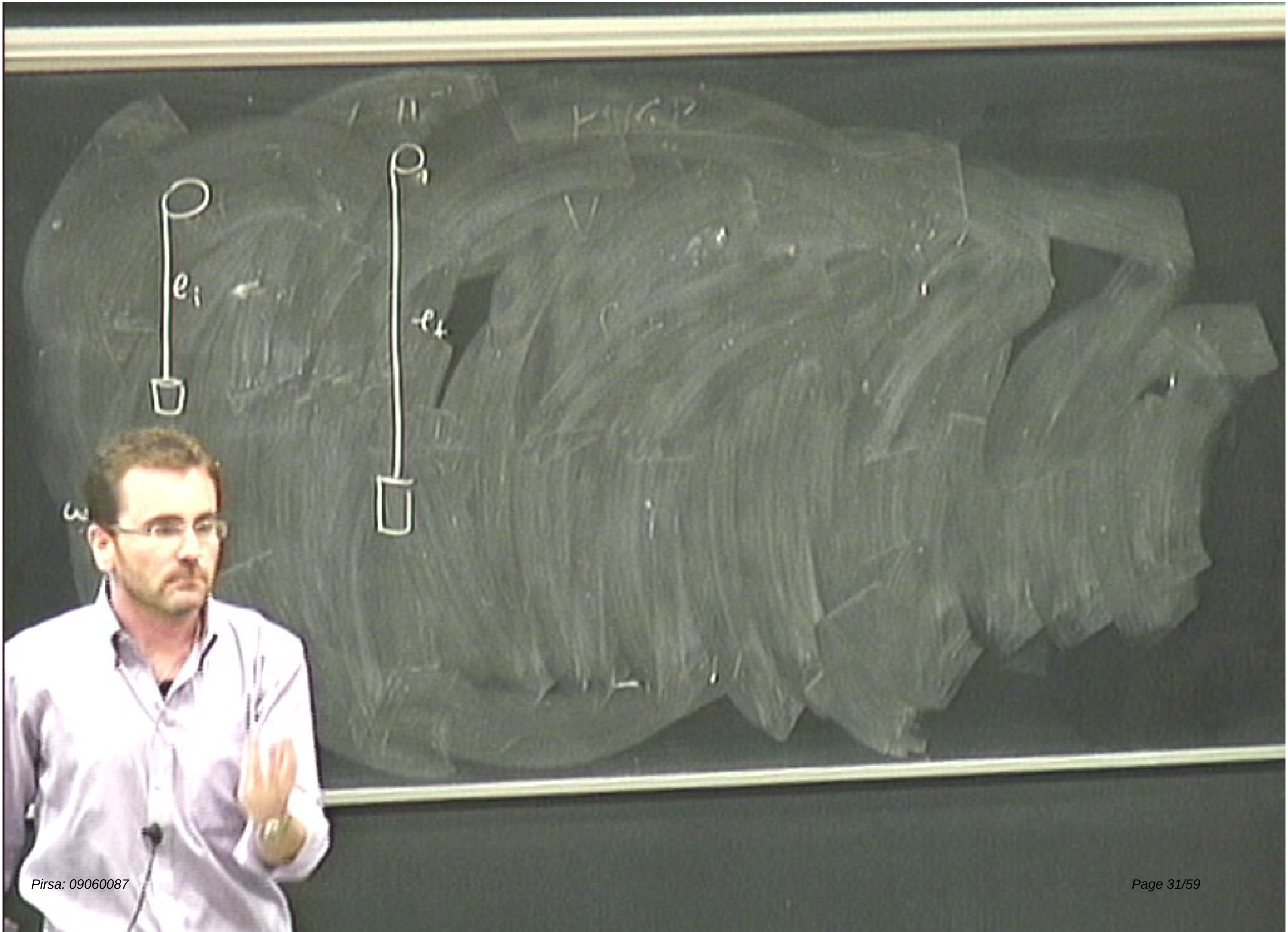
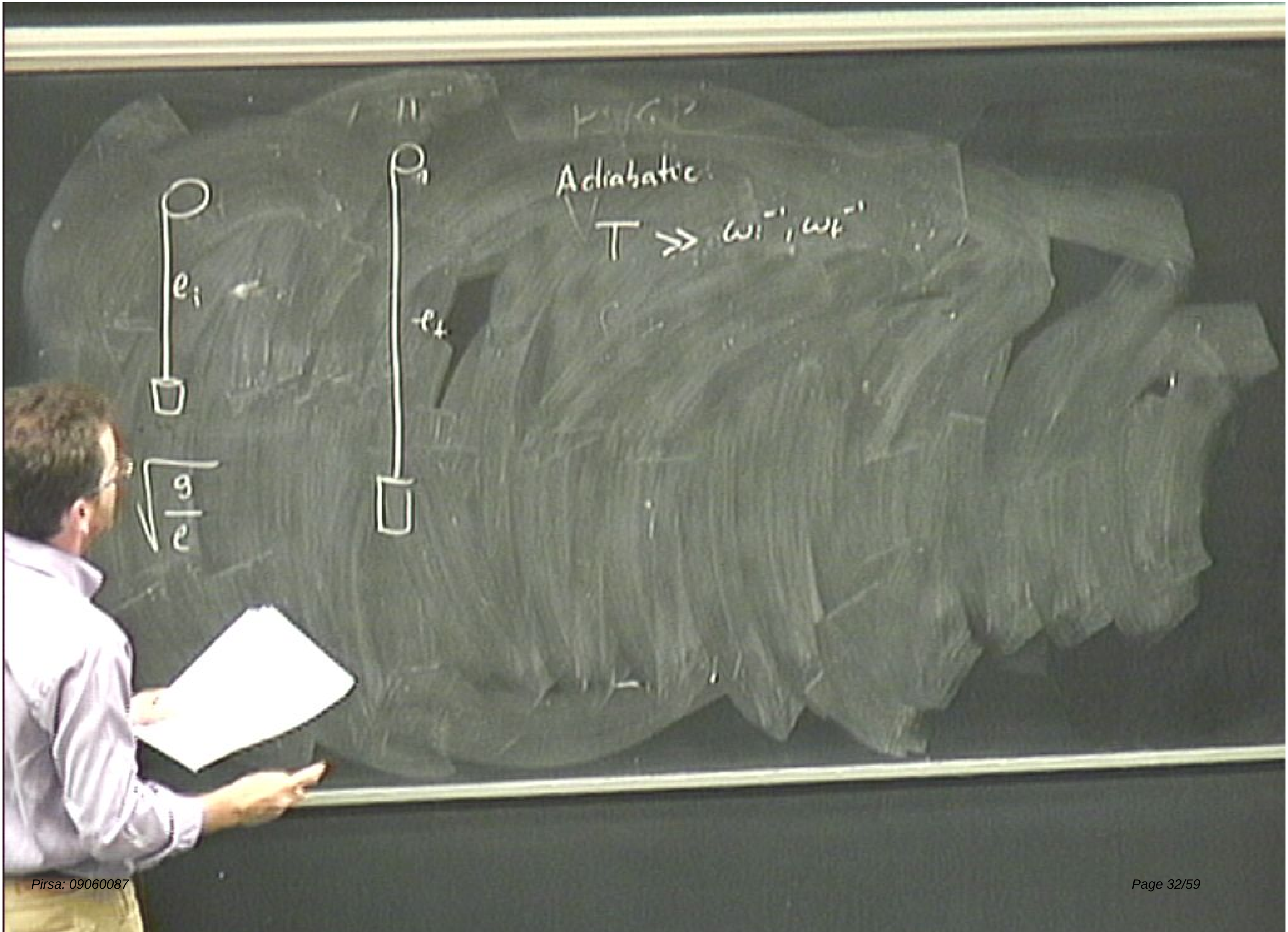
- Scalar + tensor spectrum

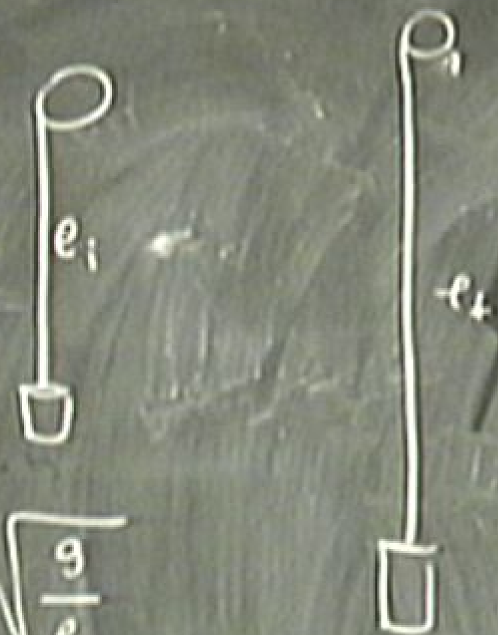
$\hbar \neq 0 = 1$ . Free field theory

- Harmonic oscillator
- Free field in dS
- Scalar + tensor spectrum


ℓ
$\sqrt{\frac{g}{\ell}}$




Adiabatic:
$T \gg \omega_i^{-1}, \omega_f^{-1}$
$\ell_i$
$\ell_f$
$\sqrt{\frac{g}{\ell}}$




$$\omega = \sqrt{\frac{g}{\ell}}$$

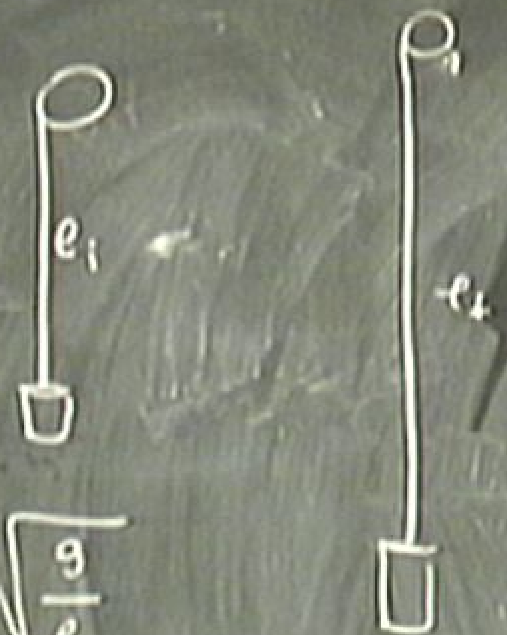
- Adiabatic

  $T \gg \omega_i^{-1}, \omega_f^{-1}$

  Vacuum stays there

- Sudden limit

  $T \ll \omega_i^{-1}, \omega_f^{-1}$

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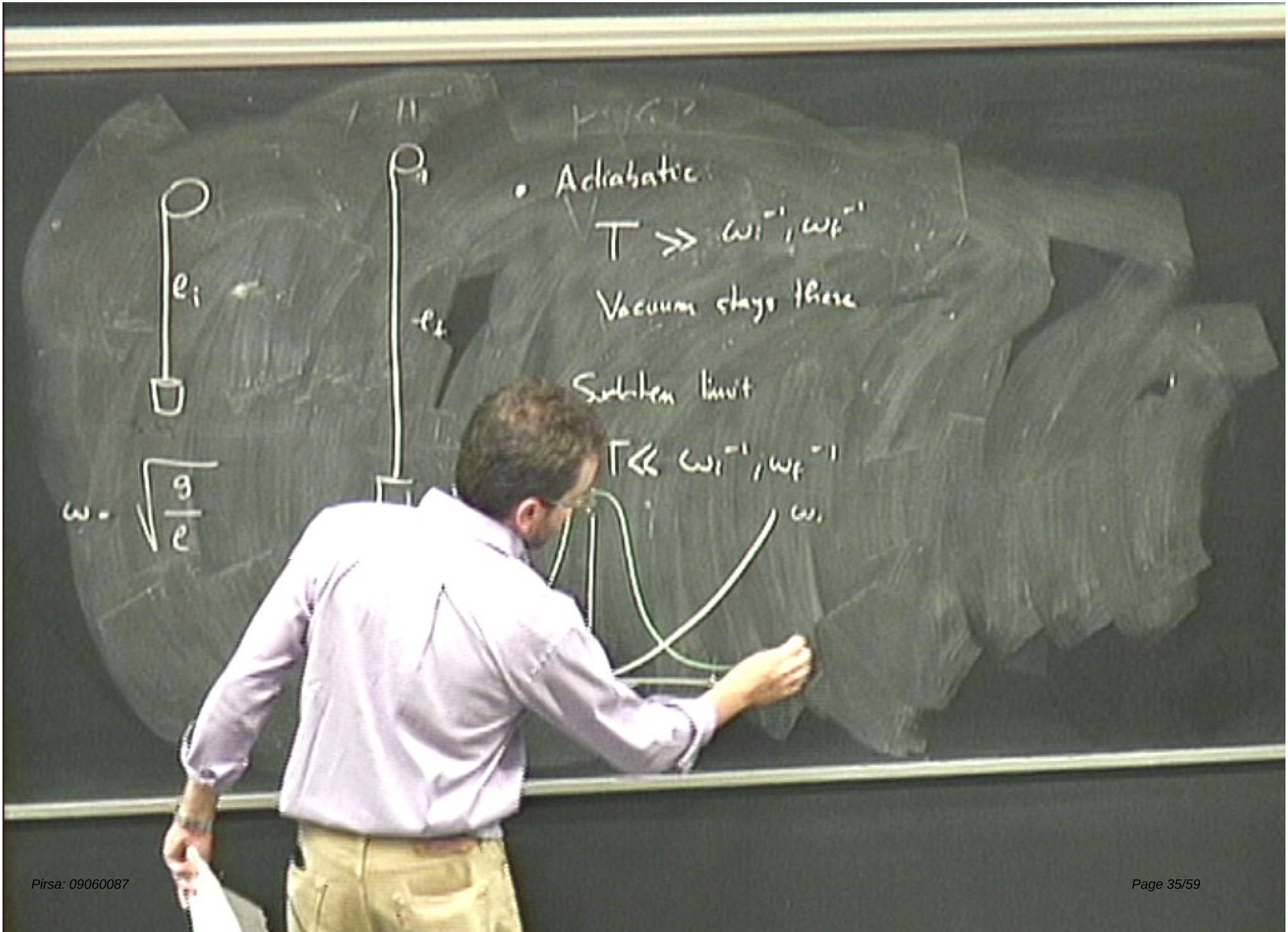
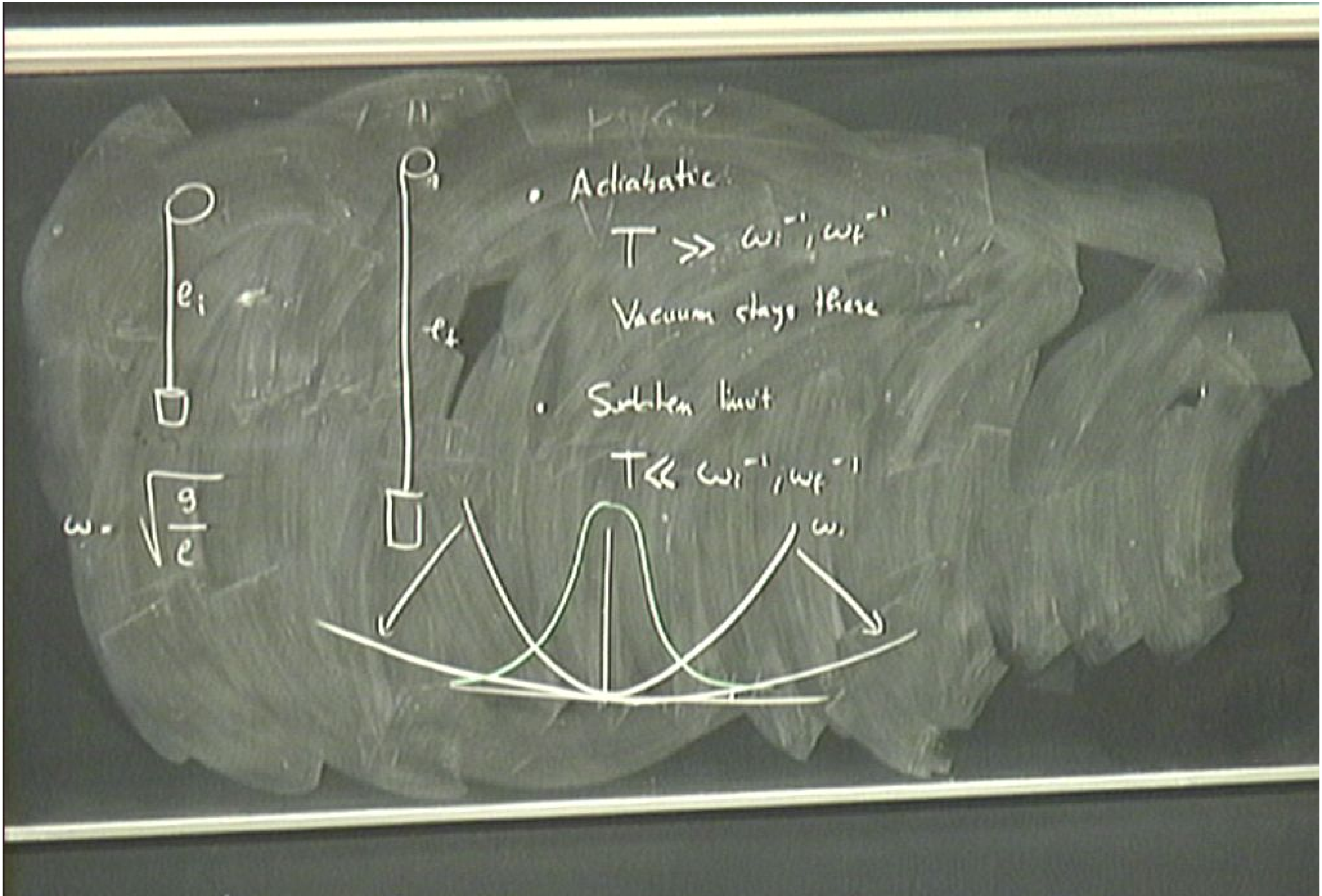
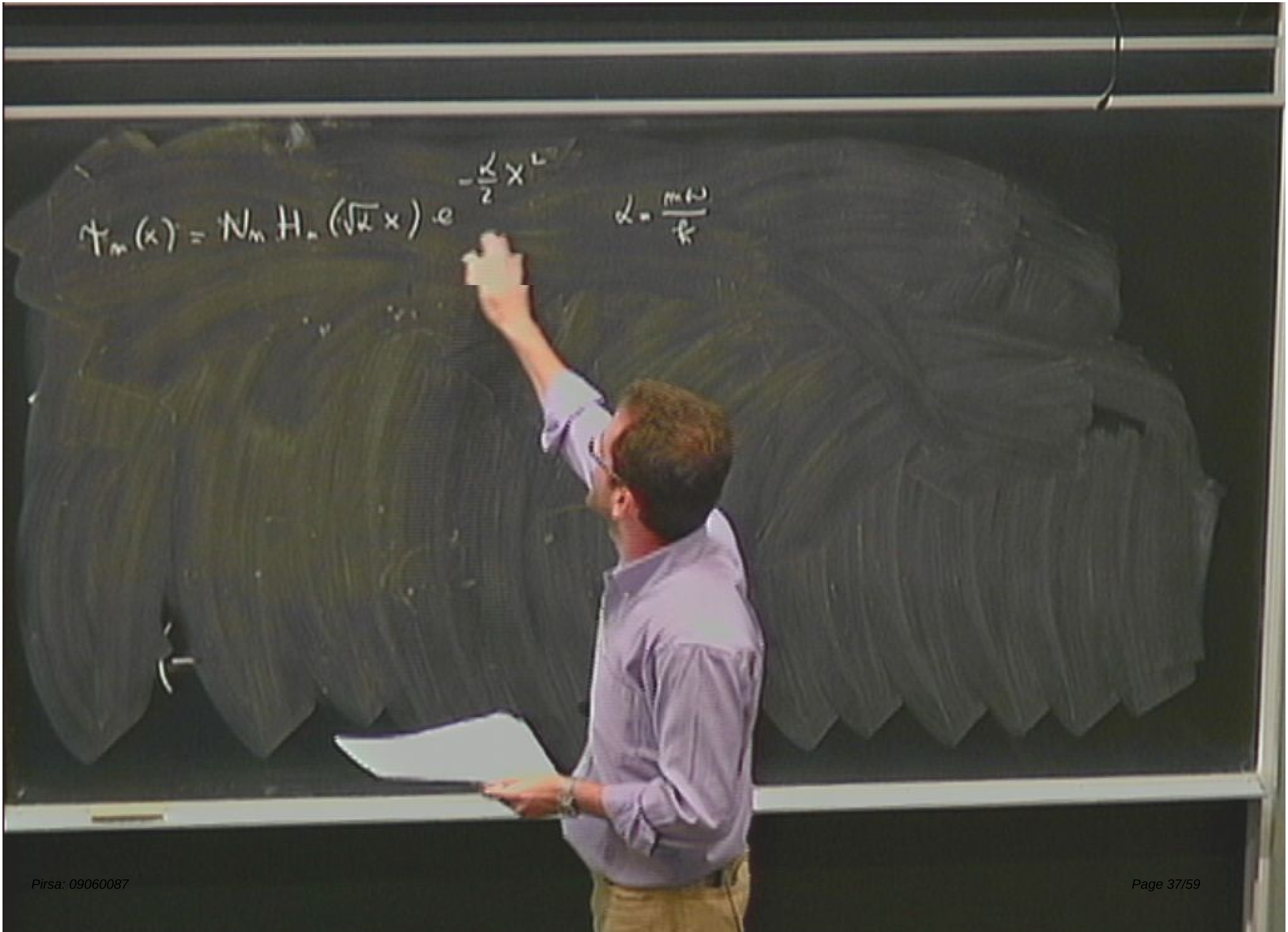
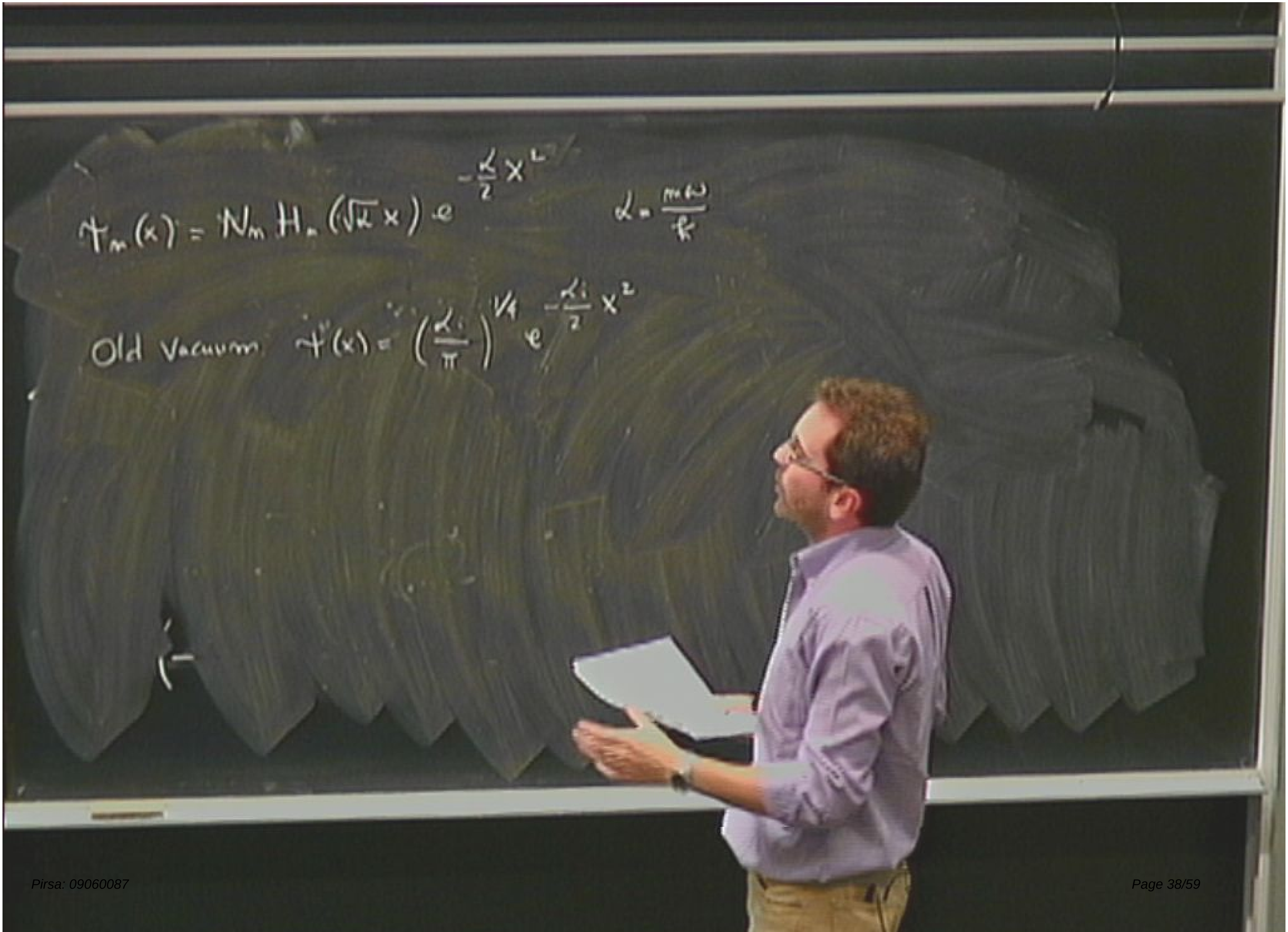


Adiabatic
T >> ωi⁻¹, ωf⁻¹
Vacuum stays there
Sudden limit
T << ωi⁻¹, ωf⁻¹
ω = √(g/ℓ)
ℓi
ℓf
ωi

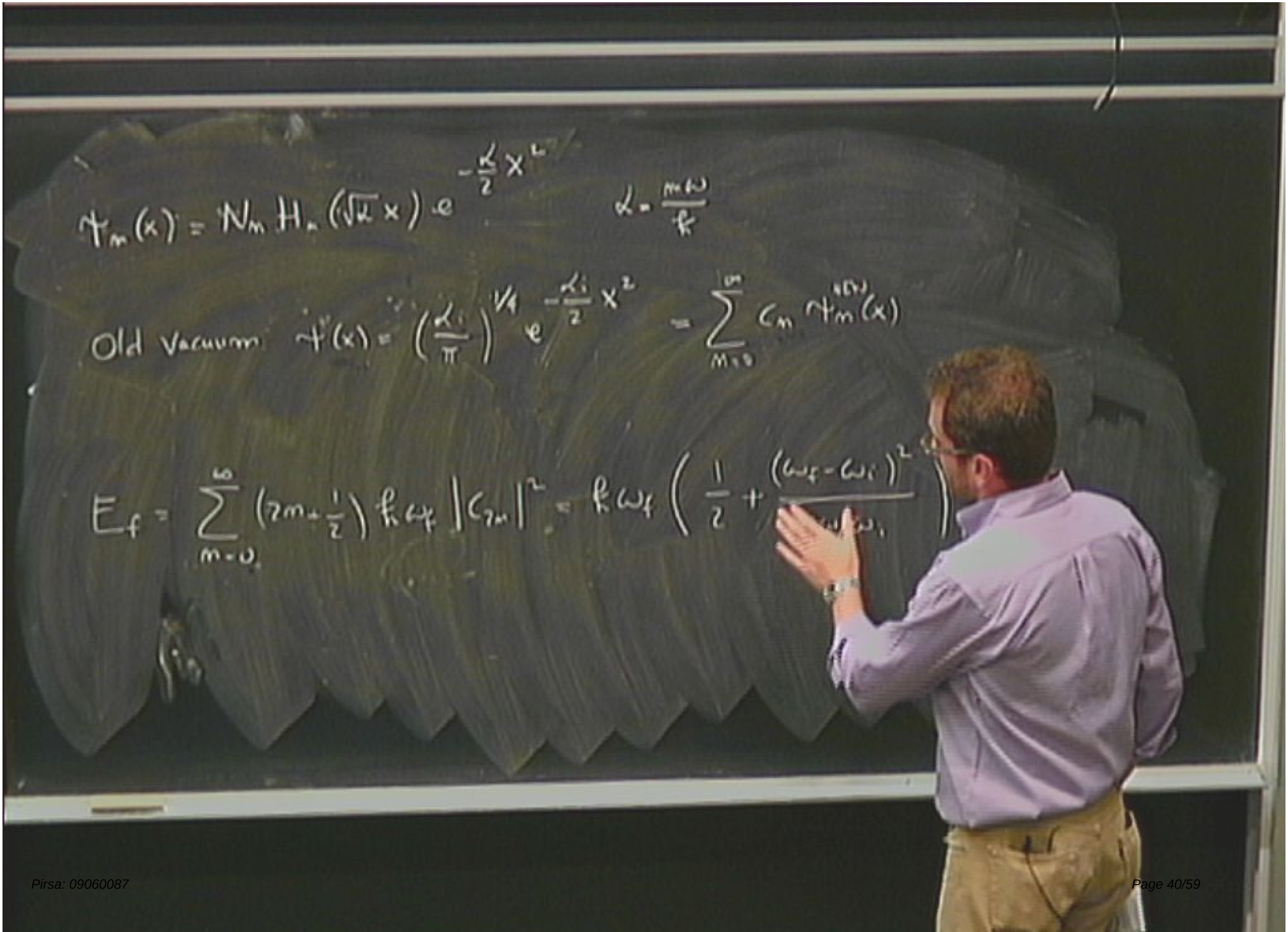

$\ell_i$
$\ell_f$
$\omega = \sqrt{\frac{g}{\ell}}$
• Adiabatic:
$T \gg \omega_i^{-1}, \omega_f^{-1}$
Vacuum stays there
• Sudden limit
$T \ll \omega_i^{-1}, \omega_f^{-1}$
$\omega_i$


$\psi_n(x) = N_n H_n(\sqrt{\alpha}\, x)\, e^{-\frac{\alpha}{2}x^2}$
$\alpha = \frac{m\omega}{\hbar}$


$\psi_n(x) = N_n H_n(\sqrt{\alpha}\, x)\, e^{-\frac{\alpha}{2}x^2}$
$\alpha = \frac{m\omega}{\hbar}$
Old Vacuum $\psi^{i}(x) = \left(\frac{\alpha_i}{\pi}\right)^{1/4} e^{-\frac{\alpha_i}{2}x^2}$

$$\psi_m(x) = N_m H_m(\sqrt{\alpha}\, x)\, e^{-\frac{\alpha}{2} x^2} \qquad \alpha = \frac{m\omega}{\hbar}$$

Old Vacuum: $\psi^{i}(x) = \left(\frac{\alpha_i}{\pi}\right)^{1/4} e^{-\frac{\alpha_i}{2} x^2} = \sum_{m=0}^{\infty} c_m \psi_m^{NEW}(x)$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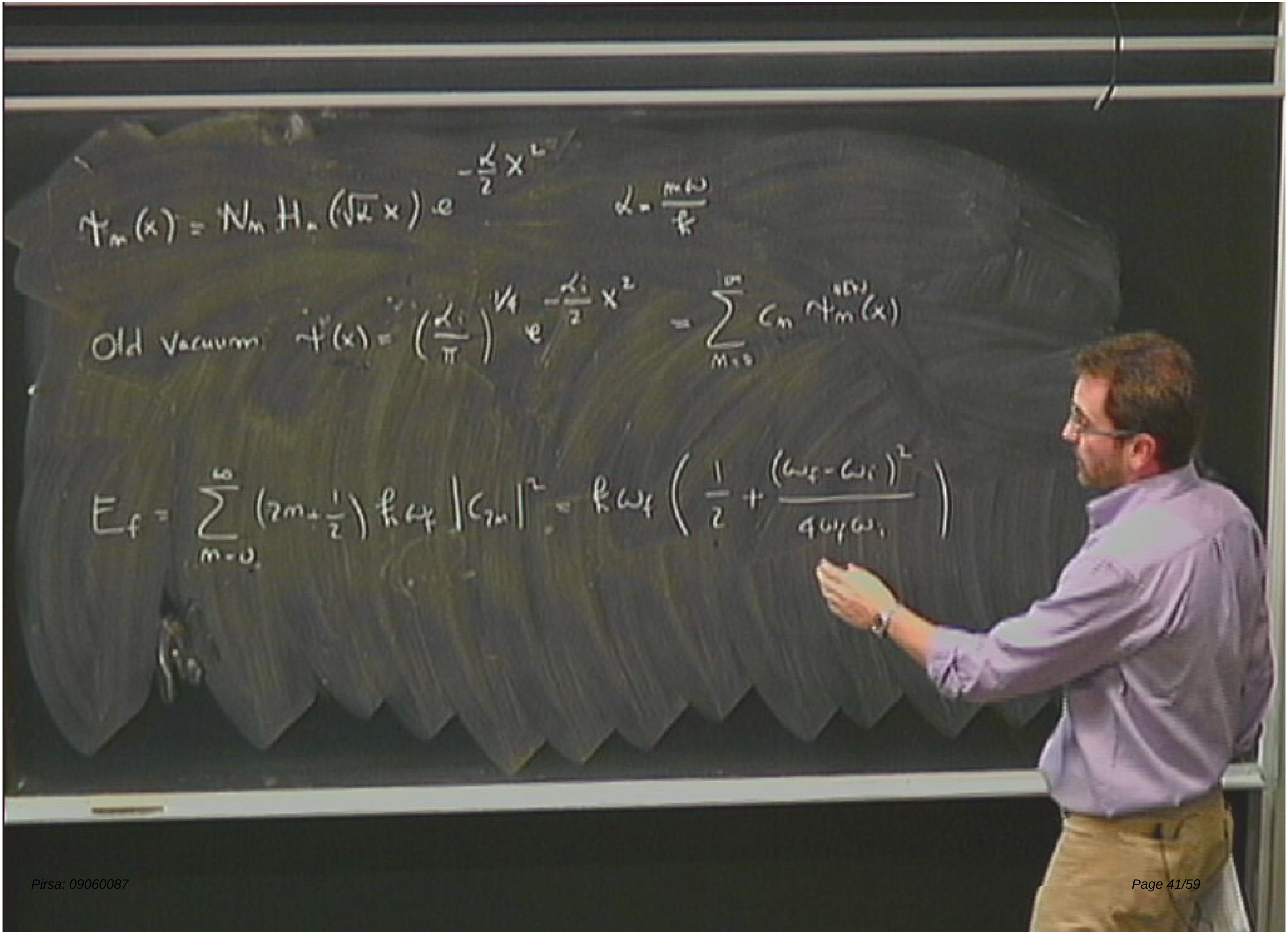
Old vacuum $\psi^i(x) = \left(\frac{\alpha_i}{\pi}\right)^{1/4} e^{-\frac{\alpha_i}{2}x^2} = \sum_{n=0}^{\infty} c_n \psi_n^{(f)}(x)$
$E_f = \sum_{n=0}^{\infty} (2n+\frac{1}{2})\hbar\omega_f |c_{2n}|^2 = \hbar\omega_f \left(\frac{1}{2} + \frac{(\omega_f-\omega_i)^2}{[illegible]}\right)$



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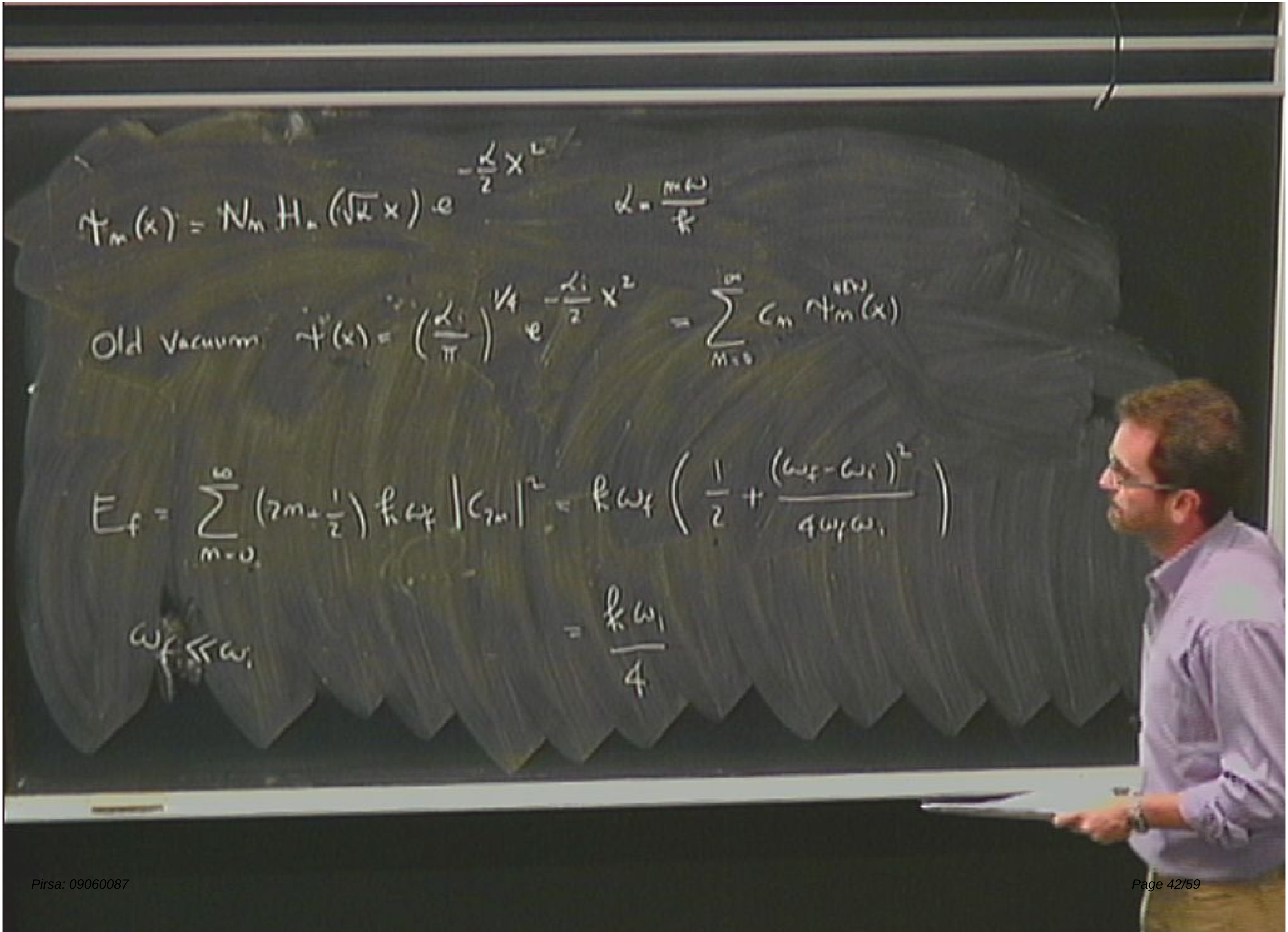
$E_f = \sum_{m=0}^{\infty} (2m + \frac{1}{2}) \hbar\omega_f |c_{2m}|^2 = \hbar\omega_f \left( \frac{1}{2} + \frac{(\omega_f - \omega_i)^2}{4\omega_f\omega_i} \right)$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$E_f = \sum_{n=0}^{\infty} (2n+\frac{1}{2})\hbar\omega_f |c_{2n}|^2 = \hbar\omega_f \left(\frac{1}{2} + \frac{(\omega_f-\omega_i)^2}{4\omega_f\omega_i}\right)$
$\omega_f \ll \omega_i$
$= \frac{\hbar\omega_i}{4}$

$$\psi_m(x) = N_m H_m(\sqrt{\alpha}\, x)\, e^{-\frac{\alpha}{2}x^2} \qquad \alpha = \frac{m\omega}{\hbar}$$

Old vacuum $\psi(x) = \left(\frac{\alpha_i}{\pi}\right)^{1/4} e^{-\frac{\alpha_i}{2}x^2} = \sum_{m=0}^{\infty} c_m \psi_m^{(f)}(x)$

$$E_f = \sum_{m=0}^{\infty} \left(2m+\frac{1}{2}\right)\hbar\omega_f\, |c_{2m}|^2 = \hbar\omega_f\left(\frac{1}{2} + \frac{(\omega_f-\omega_i)^2}{4\omega_f\omega_i}\right)$$

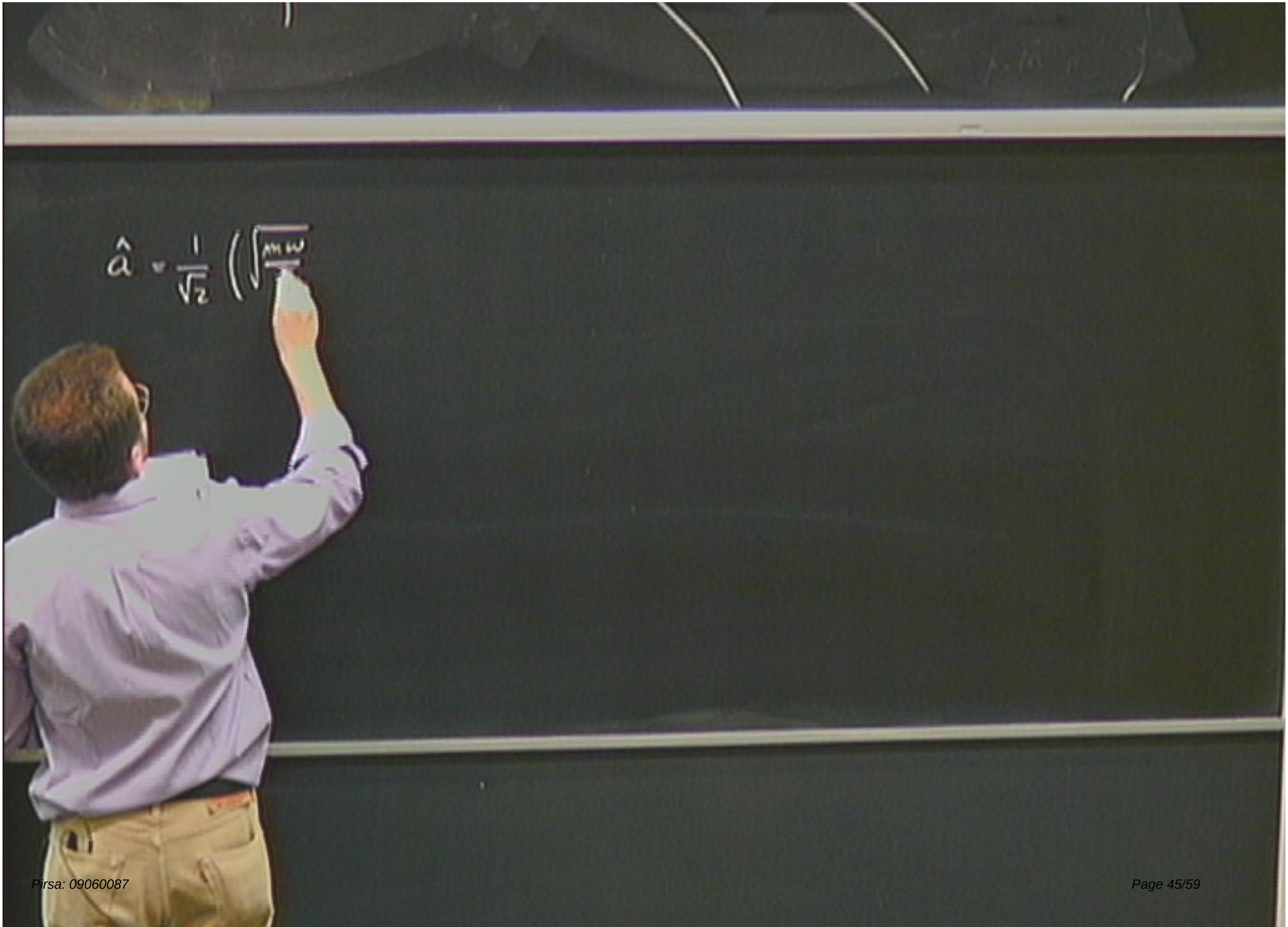
$\omega_f \ll \omega_i$
$$= \frac{\hbar\omega_i}{4} \gg \frac{\hbar\omega_f}{2}$$

$$\psi_m(x) = N_m H_m(\sqrt{\alpha}\, x)\, e^{-\frac{\alpha}{2}x^2} \qquad \alpha = \frac{m\omega}{\hbar}$$

Old vacuum: $\psi(x) = \left(\frac{\alpha_i}{\pi}\right)^{1/4} e^{-\frac{\alpha_i}{2}x^2} = \sum_{m=0}^{\infty} c_m \psi_m^{new}(x)$

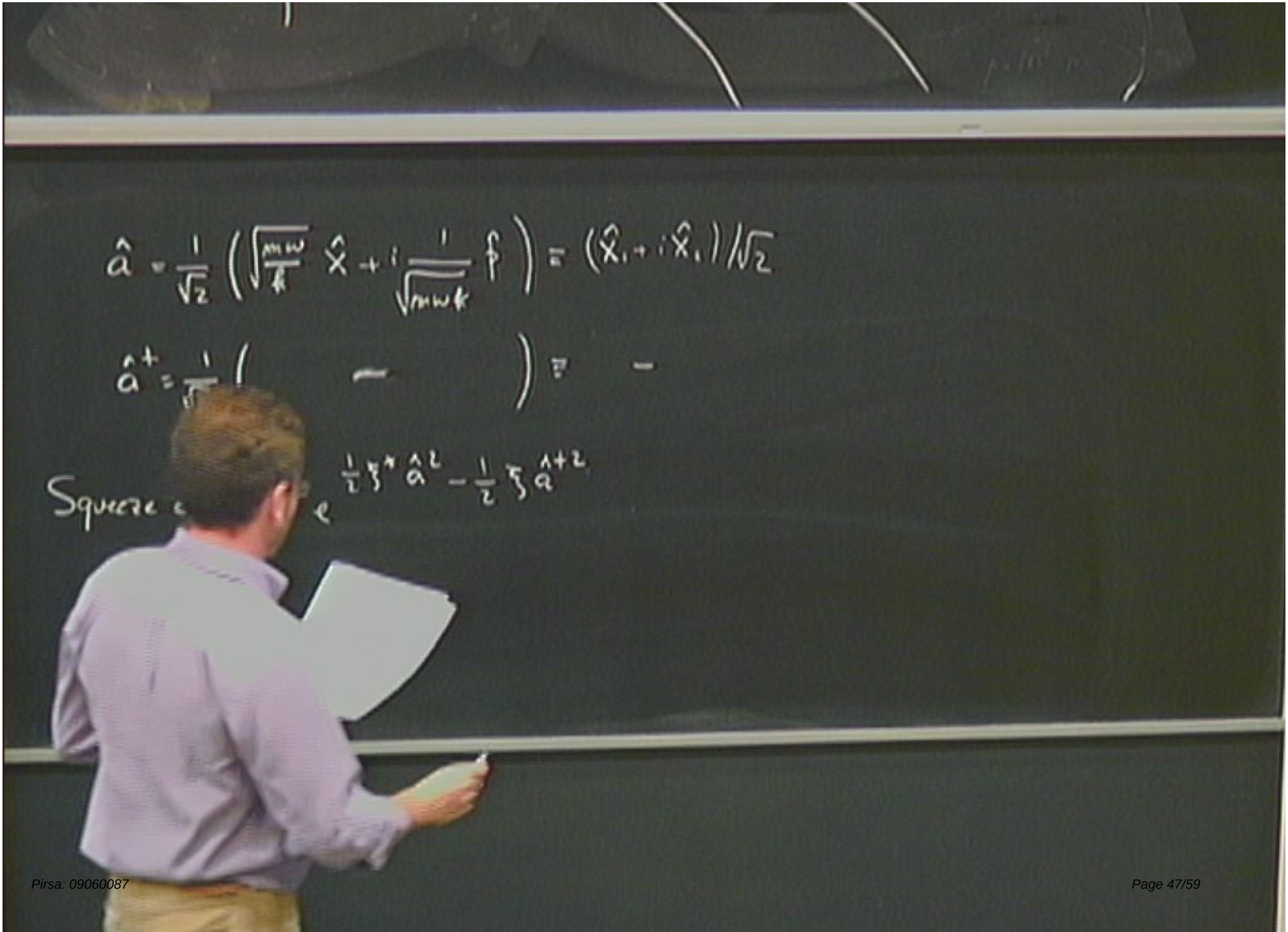
$$E_f = \sum_{m=0}^{\infty} \left(2m+\frac{1}{2}\right)\hbar\omega_f\, |c_{2m}|^2 = \hbar\omega_f\left(\frac{1}{2} + \frac{(\omega_f-\omega_i)^2}{4\omega_f\omega_i}\right)$$

$\omega_f \ll \omega_i$ $\qquad = \frac{\hbar\omega_i}{4} \gg \frac{\hbar\omega_f}{2}$

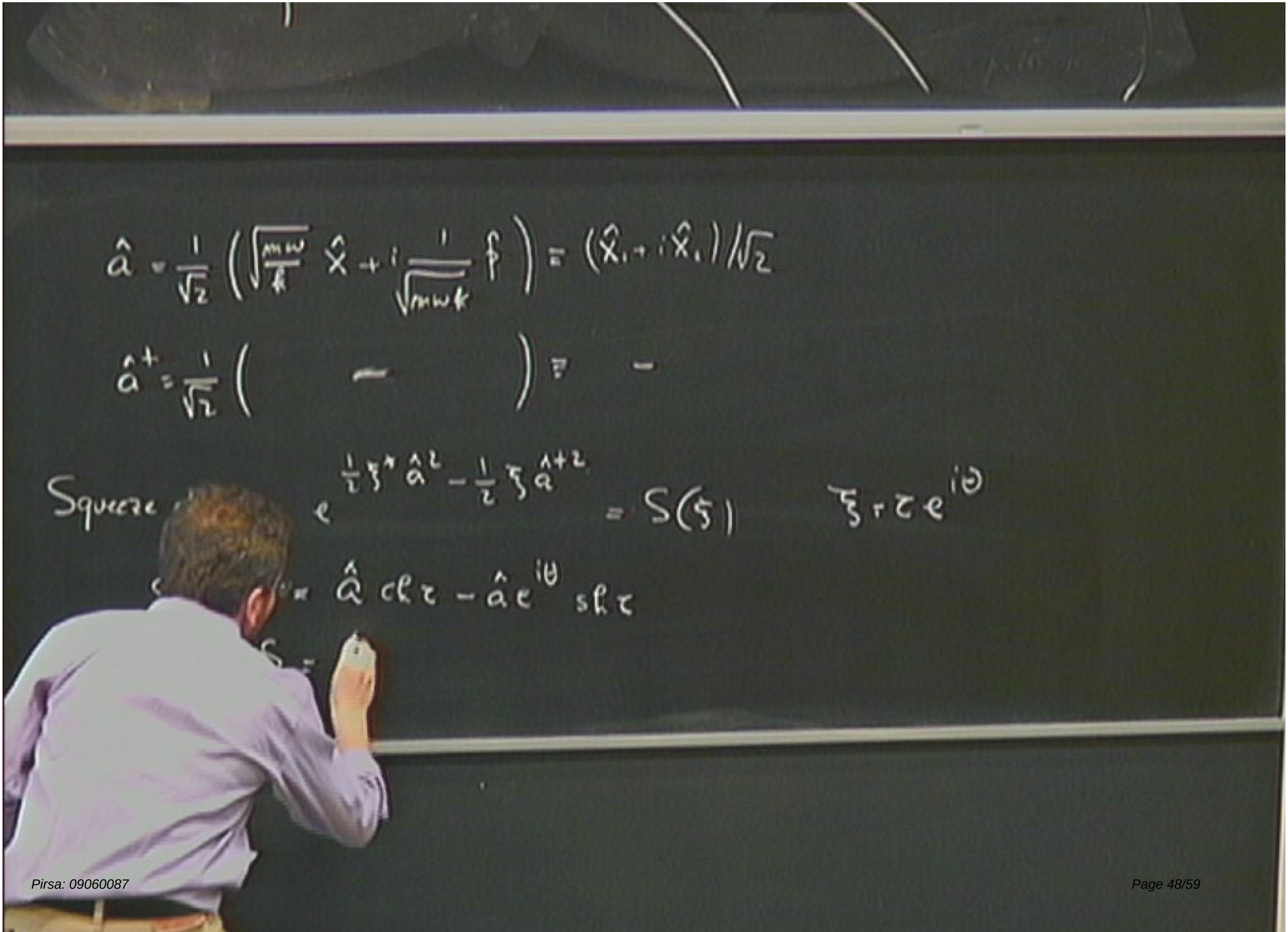

$\hat{a} = \frac{1}{\sqrt{2}} \Big( \sqrt{m\omega}$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) \equiv (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$$

$$\hat{a}^{+} = \frac{1}{\sqrt{2}}\left( \quad - \quad \right) \equiv \quad -$$



$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$
$\hat{a}^\dagger = \frac{1}{\sqrt{2}}\left( \quad - \quad \right) = \quad -$
Squeeze
$e^{\frac{1}{2}\xi^* \hat{a}^2 - \frac{1}{2}\xi \hat{a}^{\dagger 2}}$



$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$
$\hat{a}^{+} = \frac{1}{\sqrt{2}}\left( \quad - \quad \right) = \quad -$
Squeeze
$e^{\frac{1}{2}\xi^{*}\hat{a}^{2} - \frac{1}{2}\xi\hat{a}^{+2}} = S(\xi)$
$\xi = r e^{i\theta}$
$\hat{a}\, ch\, r - \hat{a} e^{i\theta}\, sh\, r$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$$

$$\hat{a}^{+} = \frac{1}{\sqrt{2}}\left( \qquad - \qquad \right) = \qquad -$$

Squeeze operator: $e^{\frac{1}{2}\xi^{*}\hat{a}^{2} - \frac{1}{2}\xi\hat{a}^{+2}} = S(\xi)$ $\quad \xi = \tau e^{i\theta}$

$$S^{+}\hat{a}\,S = \hat{a}\,\mathrm{ch}\,\tau - \hat{a}^{+}e^{i\theta}\,\mathrm{sh}\,\tau$$

$$S^{+}\hat{a}^{+}S = \hat{a}^{+}\mathrm{ch}\,\tau - \hat{a}\,e^{-i\theta}\,\mathrm{sh}\,\tau$$

Bogolubov transformation

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$$

$$|\xi\rangle = S(\xi)|0\rangle$$

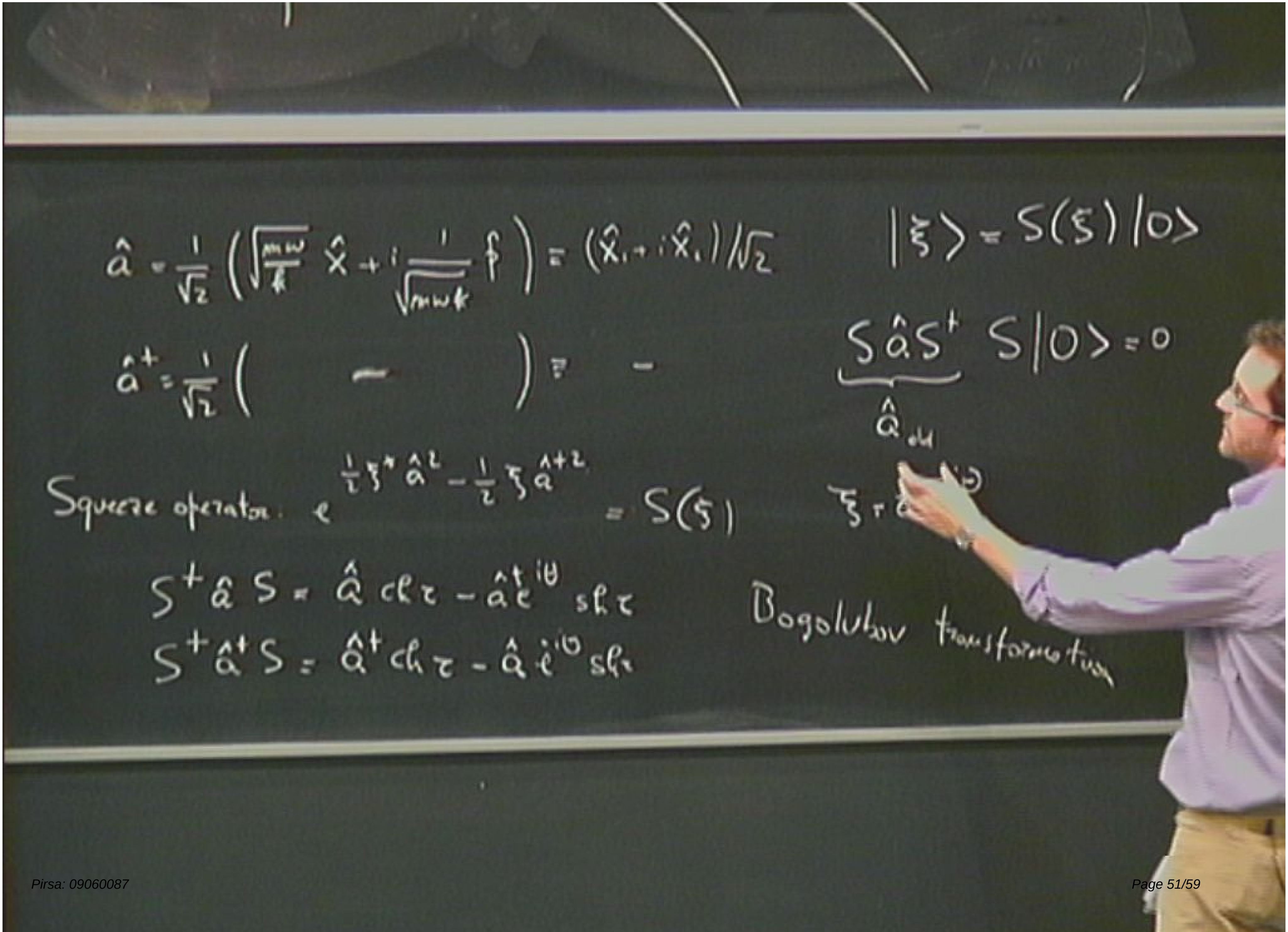
$$\hat{a}^+ = \frac{1}{\sqrt{2}}\left( \quad - \quad \right) = \quad -$$

Squeeze operator: $e^{\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{+2}} = S(\xi)$ $\quad \xi = \tau e^{i\theta}$

$$S^+\hat{a}S = \hat{a}\,\mathrm{ch}\,\tau - \hat{a}^+ e^{i\theta}\,\mathrm{sh}\,\tau$$

$$S^+\hat{a}^+S = \hat{a}^+\,\mathrm{ch}\,\tau - \hat{a}\,e^{-i\theta}\,\mathrm{sh}\,\tau$$

Bogolubov transformation


$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$
$|\xi\rangle = S(\xi)|0\rangle$
$\hat{a}^{+} = \frac{1}{\sqrt{2}}\left( \quad - \quad \right) = \quad -$
$\underbrace{S\hat{a}S^{+}}\, S|0\rangle = 0$
Squeeze operator: $e^{\frac{1}{2}\xi^{*}\hat{a}^{2} - \frac{1}{2}\xi\hat{a}^{+2}} = S(\xi)$
$\xi =$
$S^{+}\hat{a}S = \hat{a}\,\mathrm{ch}\,r - \hat{a}^{+}e^{i\theta}\,\mathrm{sh}\,r$
$S^{+}\hat{a}^{+}S = \hat{a}^{+}\,\mathrm{ch}\,r - \hat{a}\,e^{-i\theta}\,\mathrm{sh}\,r$
Bogolubov transformation

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$$

$$|\xi\rangle = S(\xi)\,|0\rangle$$

$$\hat{a}^{+} = \frac{1}{\sqrt{2}}\left( \quad - \quad \right) = \quad -$$

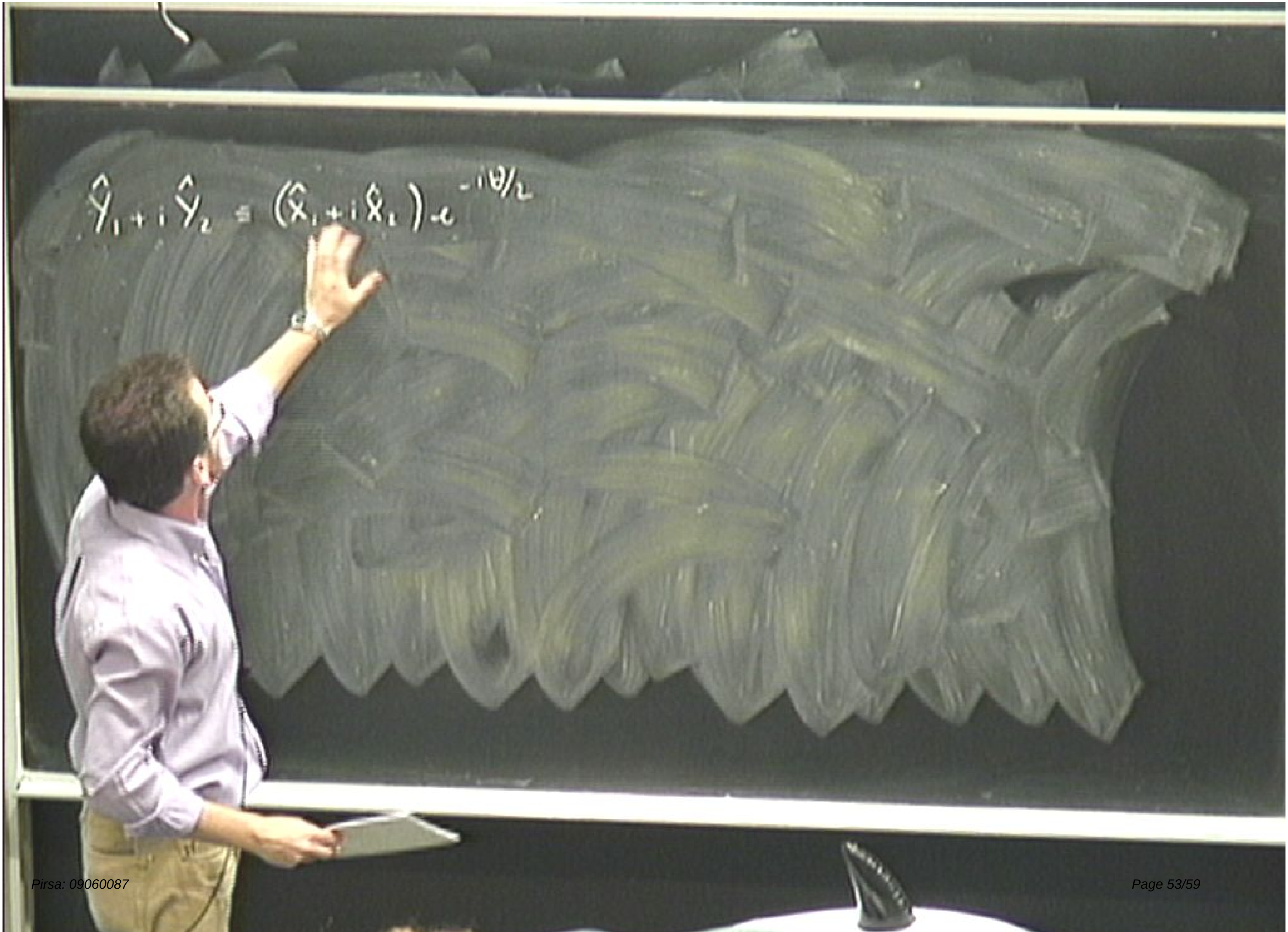
$$\underbrace{S\hat{a}S^{+}}_{\hat{a}_{old}}\,S|0\rangle = 0$$

Squeeze operator: $e^{\frac{1}{2}\xi^{*}\hat{a}^{2} - \frac{1}{2}\xi\hat{a}^{+2}} = S(\xi)$ $\qquad \xi = \tau e^{i\theta}$

$$S^{+}\hat{a}S = \hat{a}\,\mathrm{ch}\,\tau - \hat{a}^{+}e^{i\theta}\,\mathrm{sh}\,\tau$$

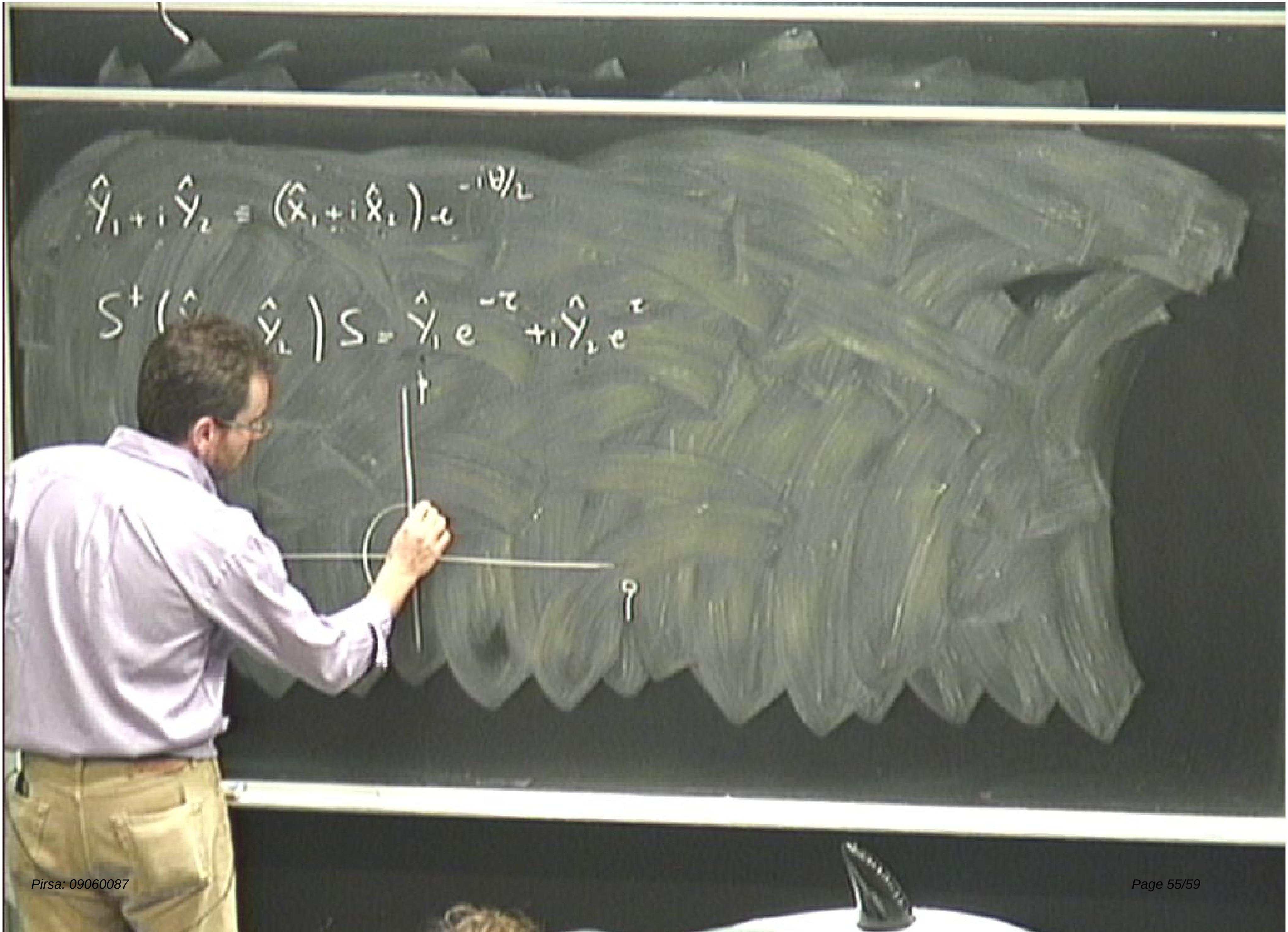
$$S^{+}\hat{a}^{+}S = \hat{a}^{+}\,\mathrm{ch}\,\tau - \hat{a}\,e^{-i\theta}\,\mathrm{sh}\,\tau$$

Bogolubov transformation


$\hat{q}_1 + i\hat{q}_2 = (\hat{x}_1 + i\hat{x}_2)e^{-i\theta/2}$

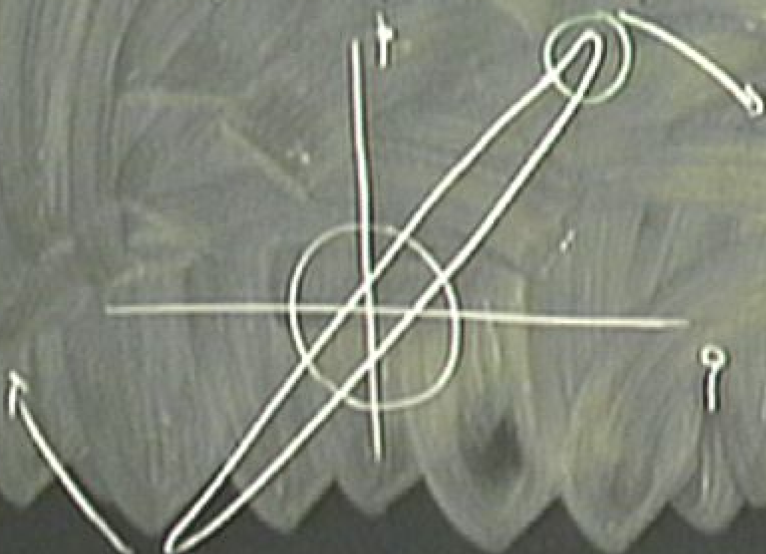
$$\hat{Y}_1 + i\hat{Y}_2 = (\hat{X}_1 + i\hat{X}_2)\, e^{-i\theta/2}$$

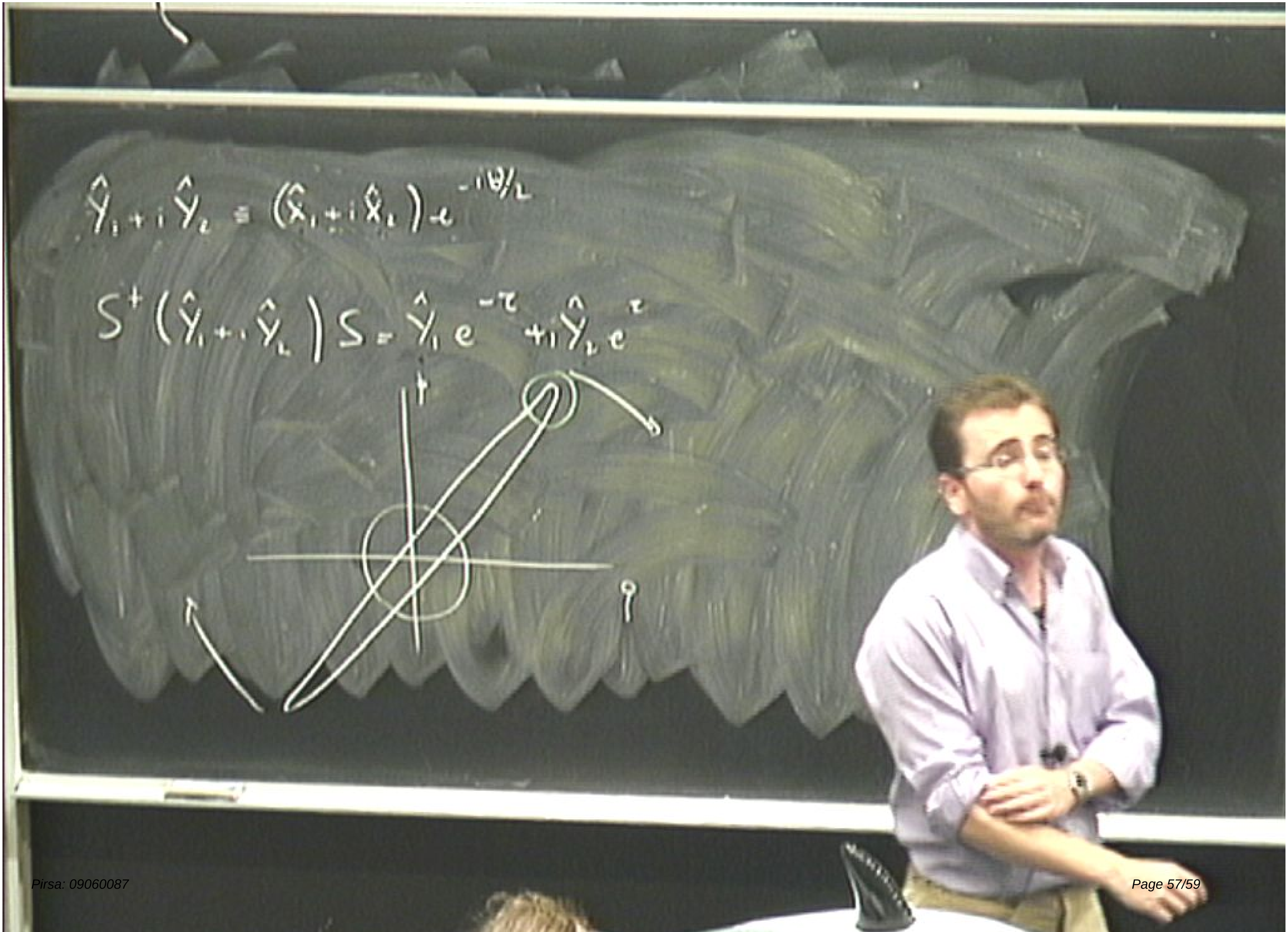
$$S^{\dagger}(\hat{Y}_1 + i\hat{Y}_2)\, S = \hat{Y}_1 e^{-r} + i\hat{Y}_2 e^{r}$$


Ŷ₁ + i Ŷ₂ = (X̂₁ + i X̂₂) e^{-iθ/2}
S⁺ ( Ŷ₂ ) S = Ŷ₁ e^{-r} + i Ŷ₂ e^{r}

$$\hat{y}_1 + i\hat{y}_2 = (\hat{x}_1 + i\hat{x}_2)\,e^{-i\theta/2}$$

$$S^\dagger(\hat{y}_1 + i\hat{y}_2)S = \hat{y}_1 e^{-r} + i\hat{y}_2 e^{r}$$

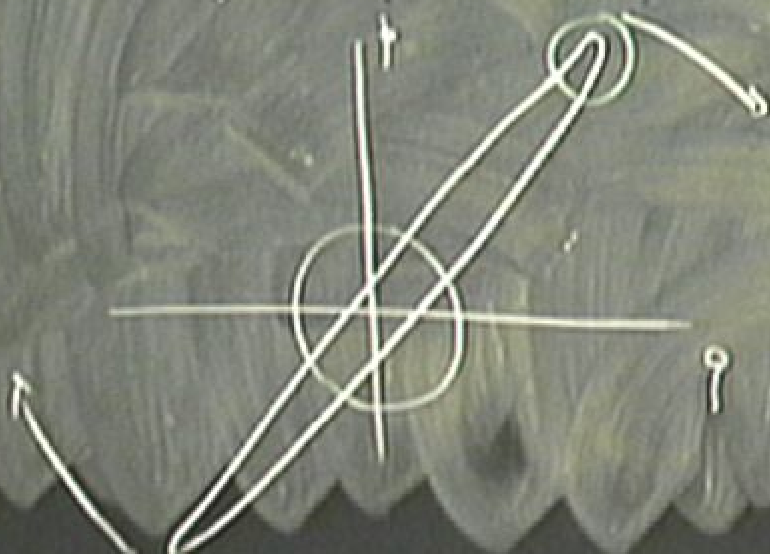


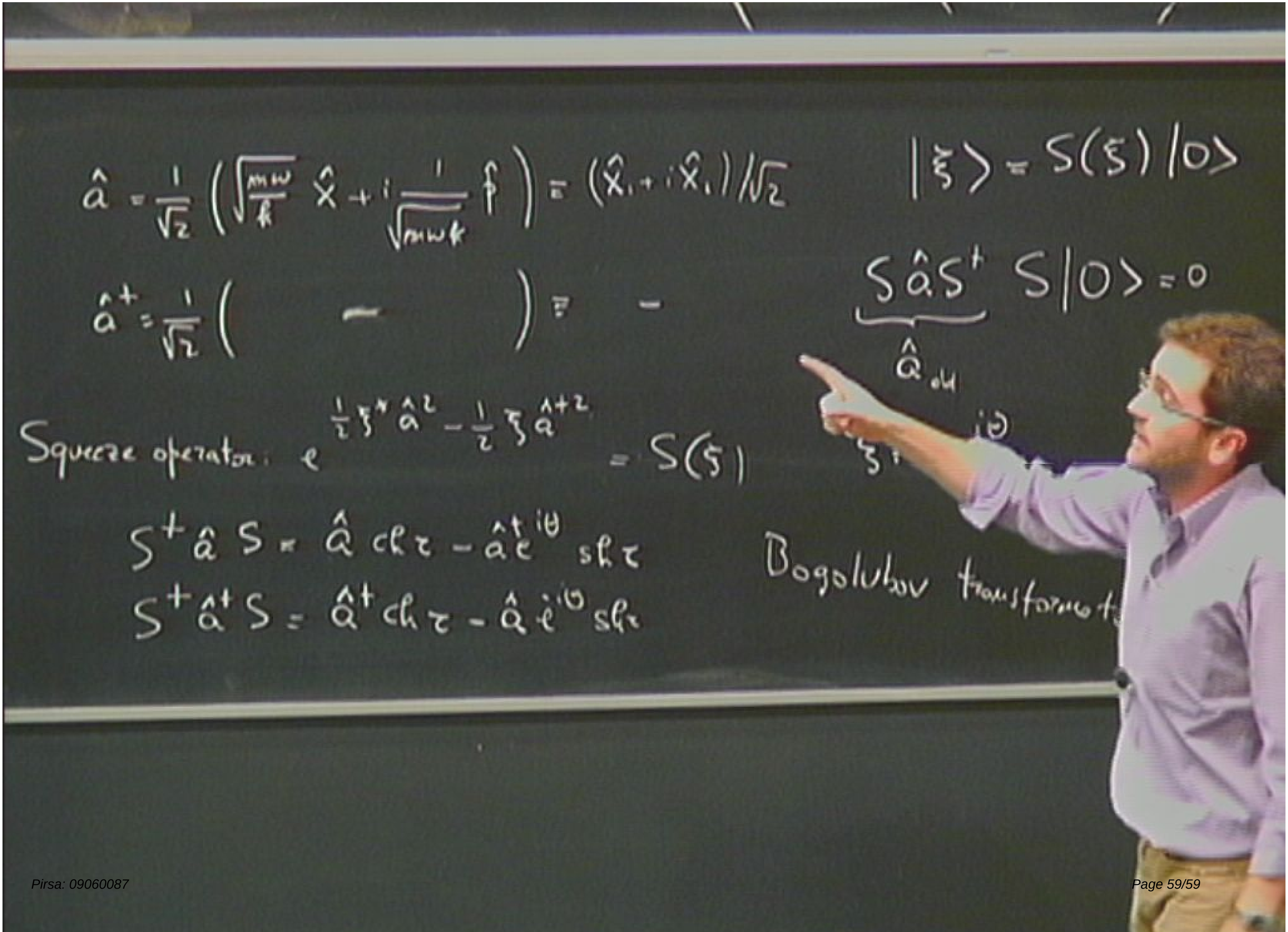


$\hat{Y}_1 + i\hat{Y}_2 = (\hat{X}_1 + i\hat{X}_2)e^{-i\theta/2}$
$S^{+}(\hat{Y}_1 + i\hat{Y}_2)S = \hat{Y}_1 e^{-\tau} + i\hat{Y}_2 e^{\tau}$

$$\hat{Y}_1 + i\hat{Y}_2 = (\hat{X}_1 + i\hat{X}_2)\, e^{-i\theta/2}$$

$$S^{\dagger}(\hat{Y}_1 + i\hat{Y}_2)\, S = \hat{Y}_1 e^{-r} + i\hat{Y}_2 e^{r}$$




$\hat{a} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,\hat{x} + i\frac{1}{\sqrt{m\omega\hbar}}\,\hat{p}\right) = (\hat{X}_1 + i\hat{X}_2)/\sqrt{2}$
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Squeeze operator: $e^{\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{+2}} = S(\xi)$
$S^+\hat{a}S = \hat{a}\,ch\,r - \hat{a}^+e^{i\theta}\,sh\,r$
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Bogolubov transformati