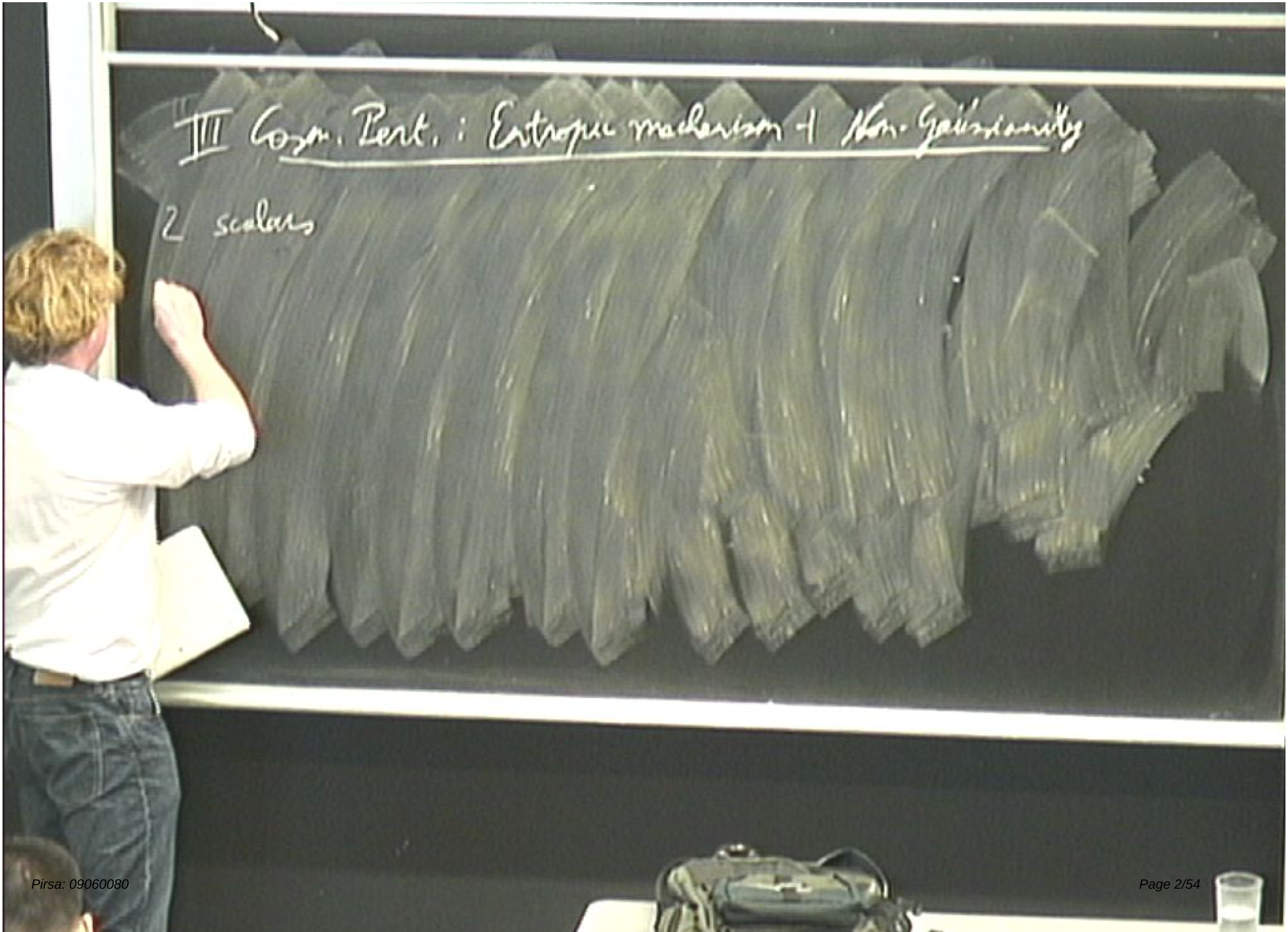
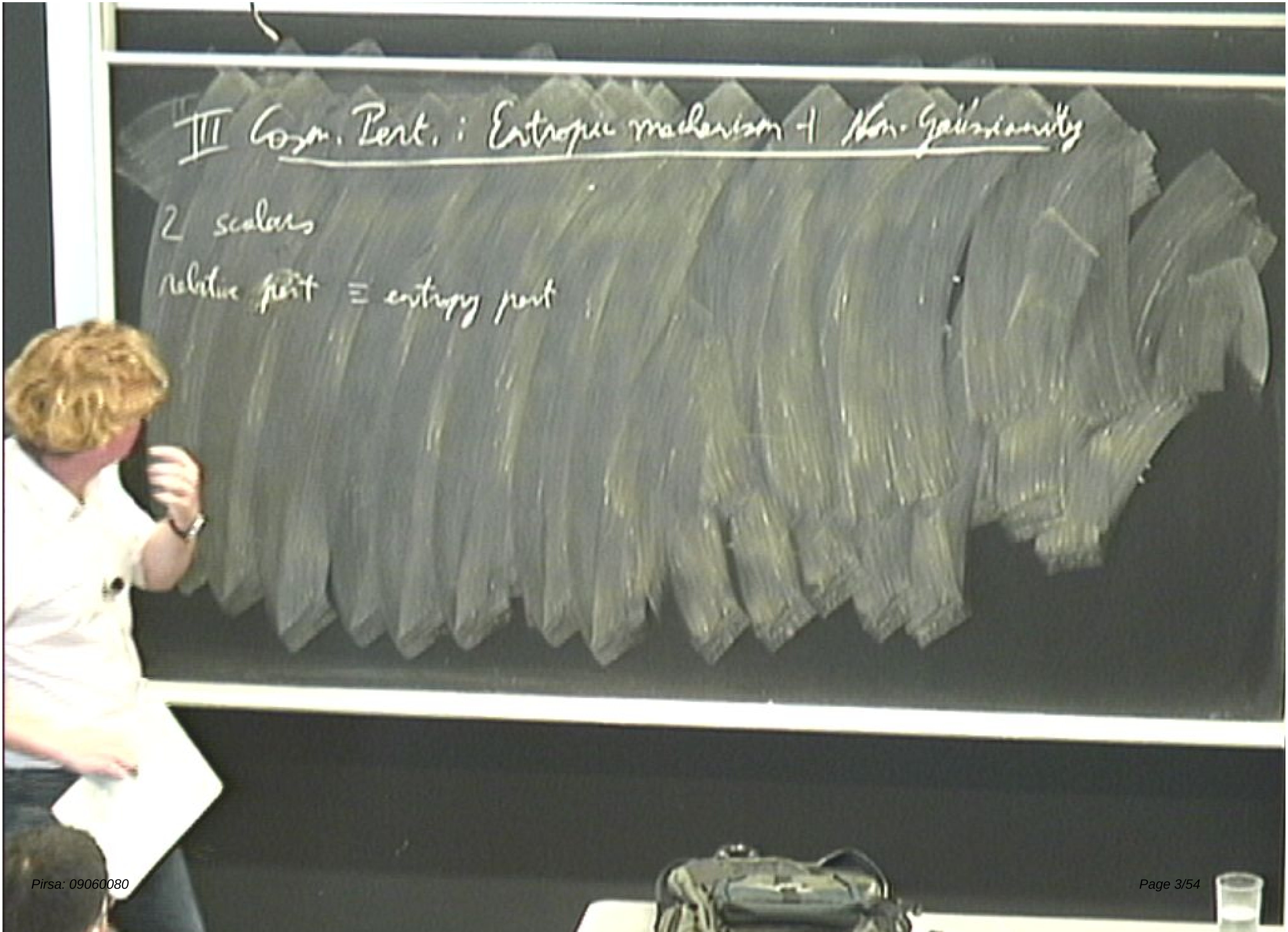
Title: Ekpyrotic/Cyclic Cosmology - Lecture 3

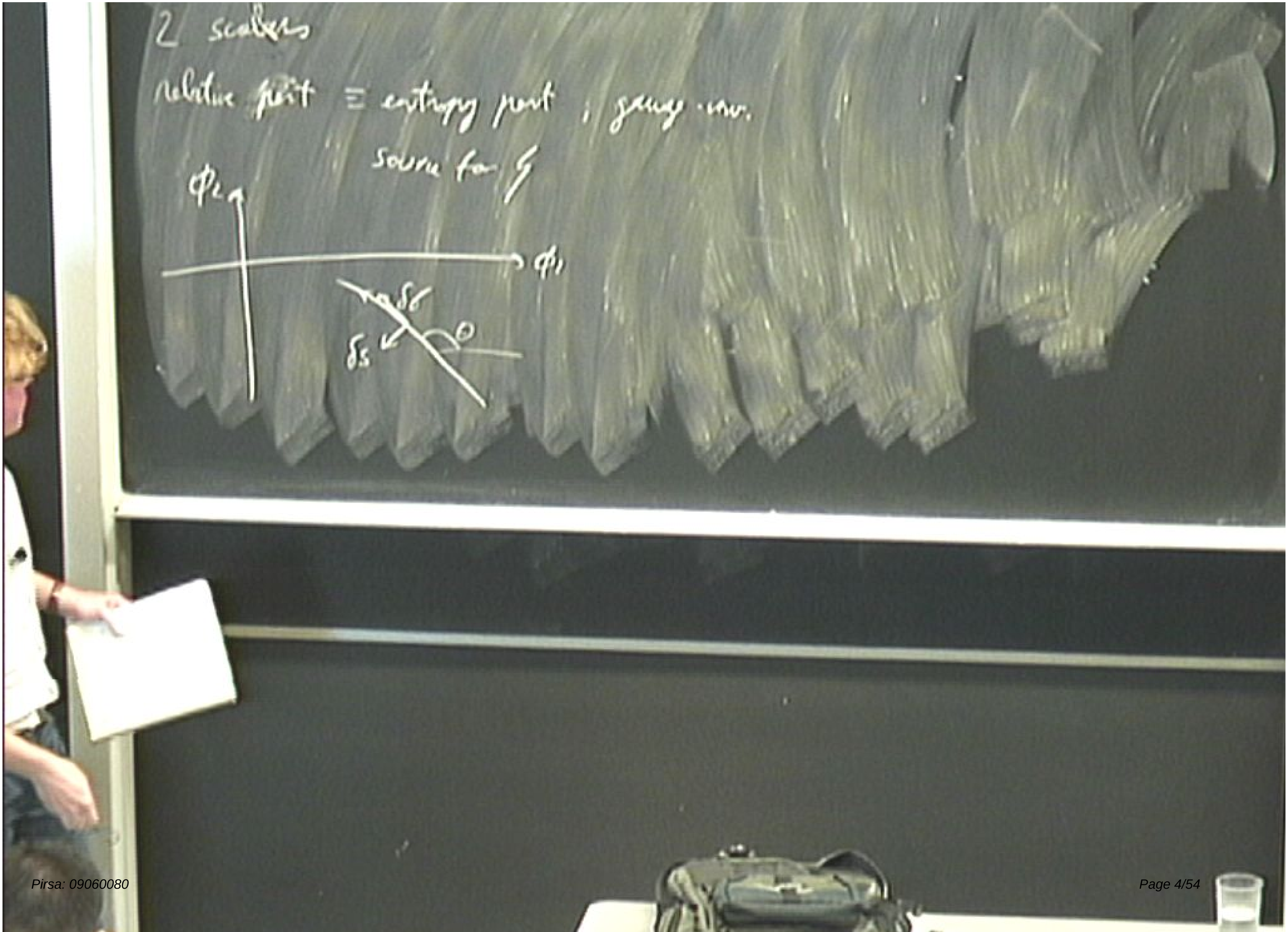
Date: Jun 29, 2009 10:00 AM

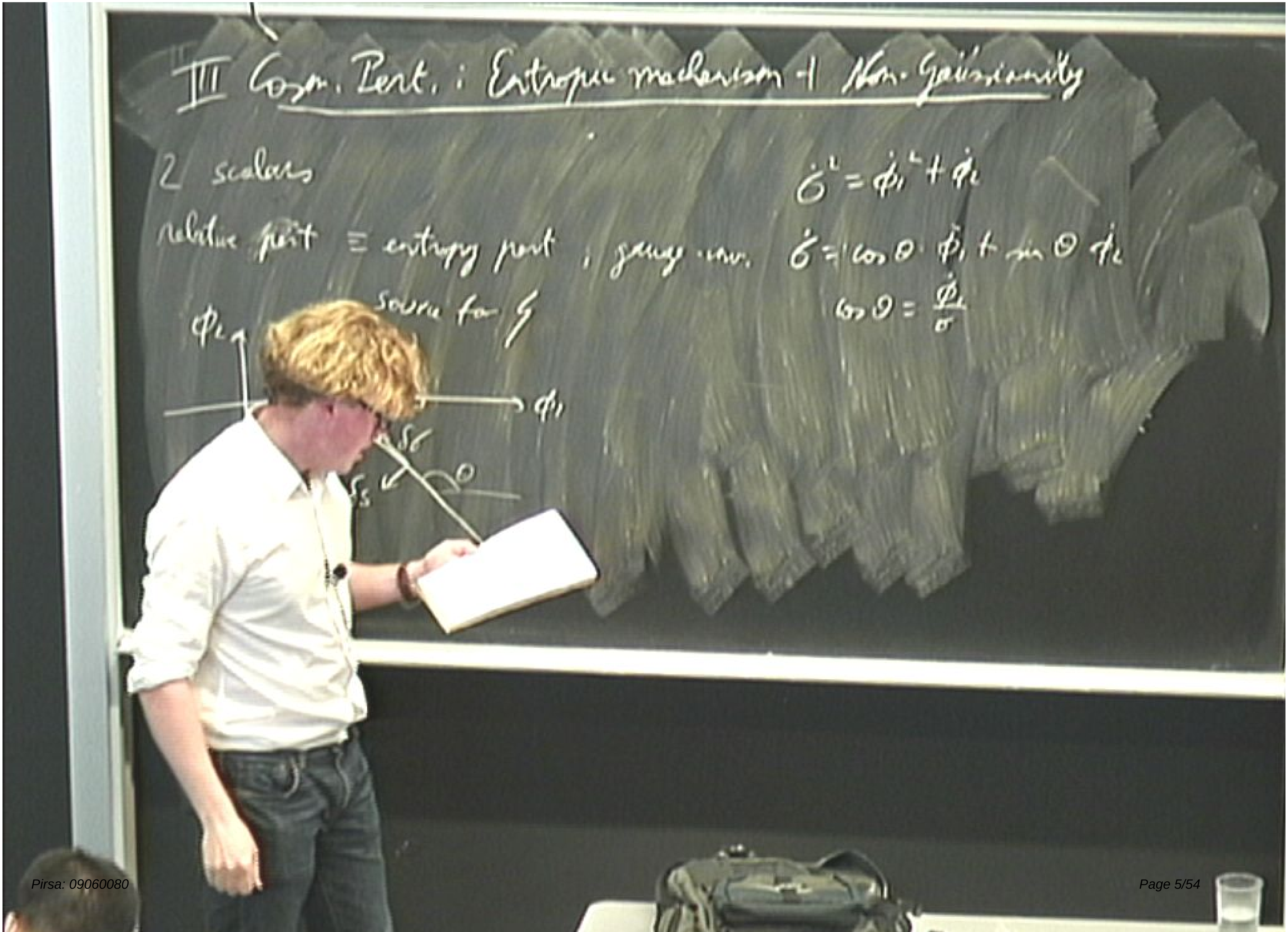
URL: http://pirsa.org/09060080

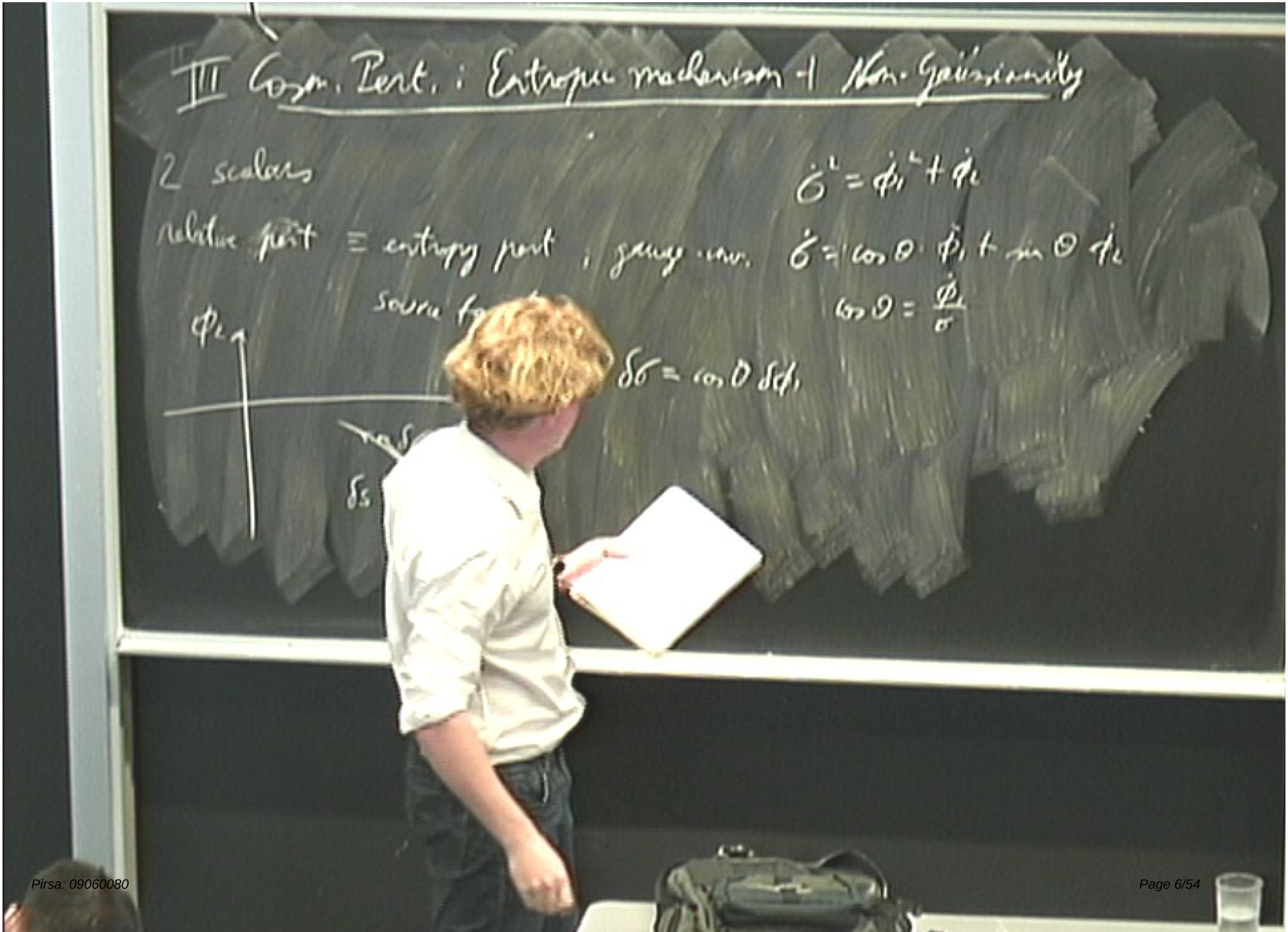
Abstract:

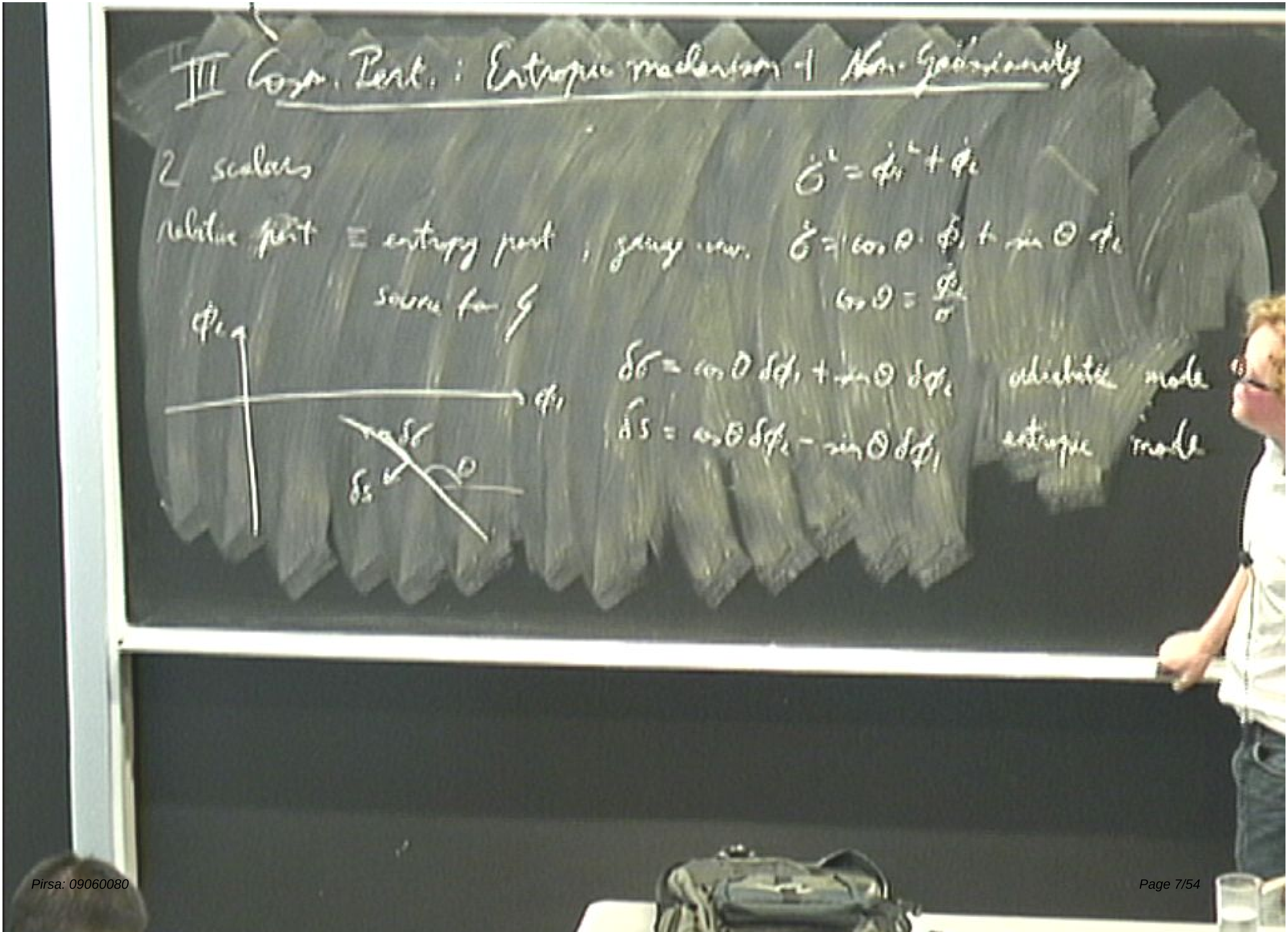

III Cosm. Pert.: Entropic mechanism + Non-Gaussianity
2 scalars

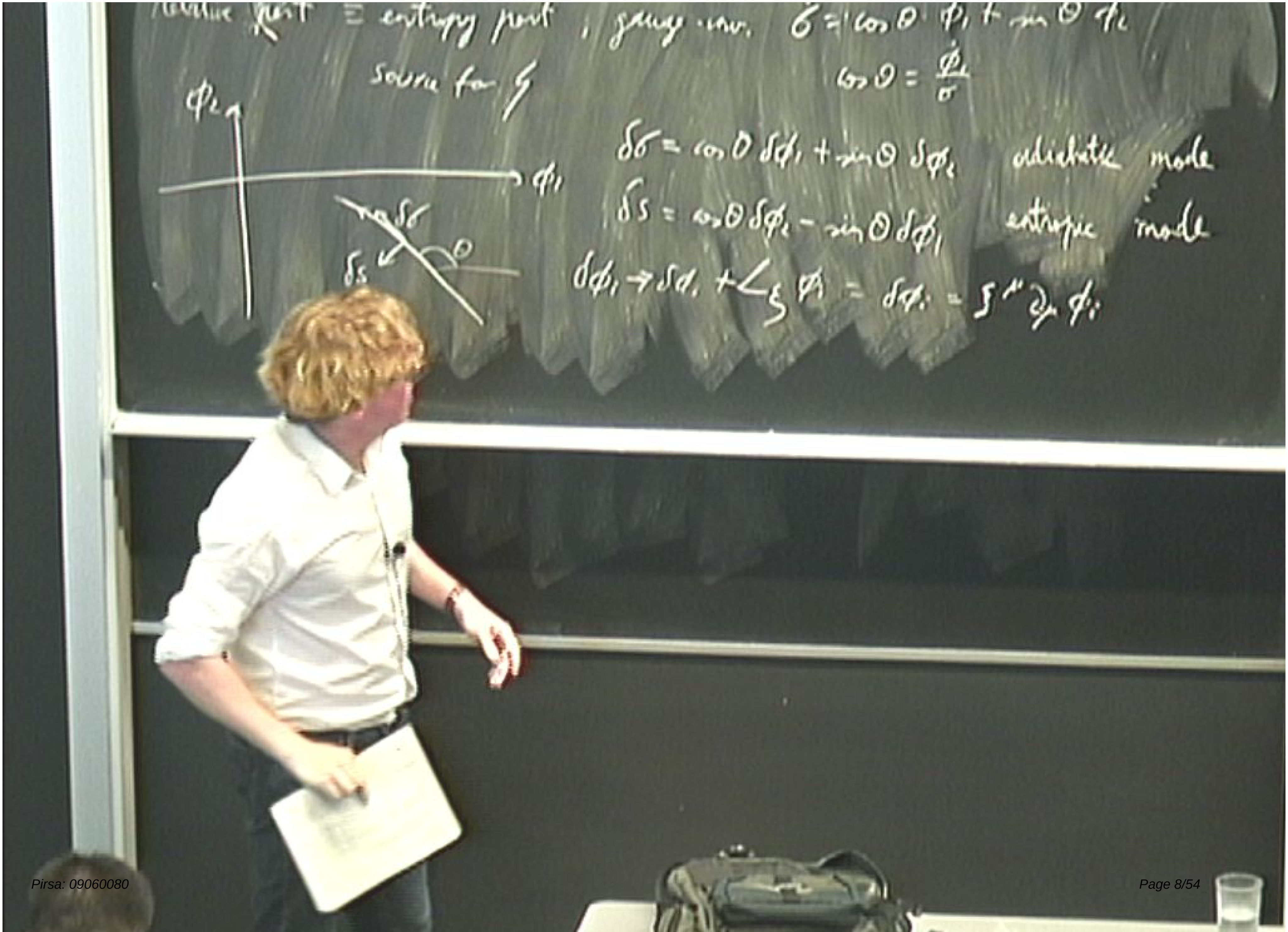

III Cosm. Pert. : Entropic mechanism + Non-Gaussianity
2 scalars
relative pert ≡ entropy pert

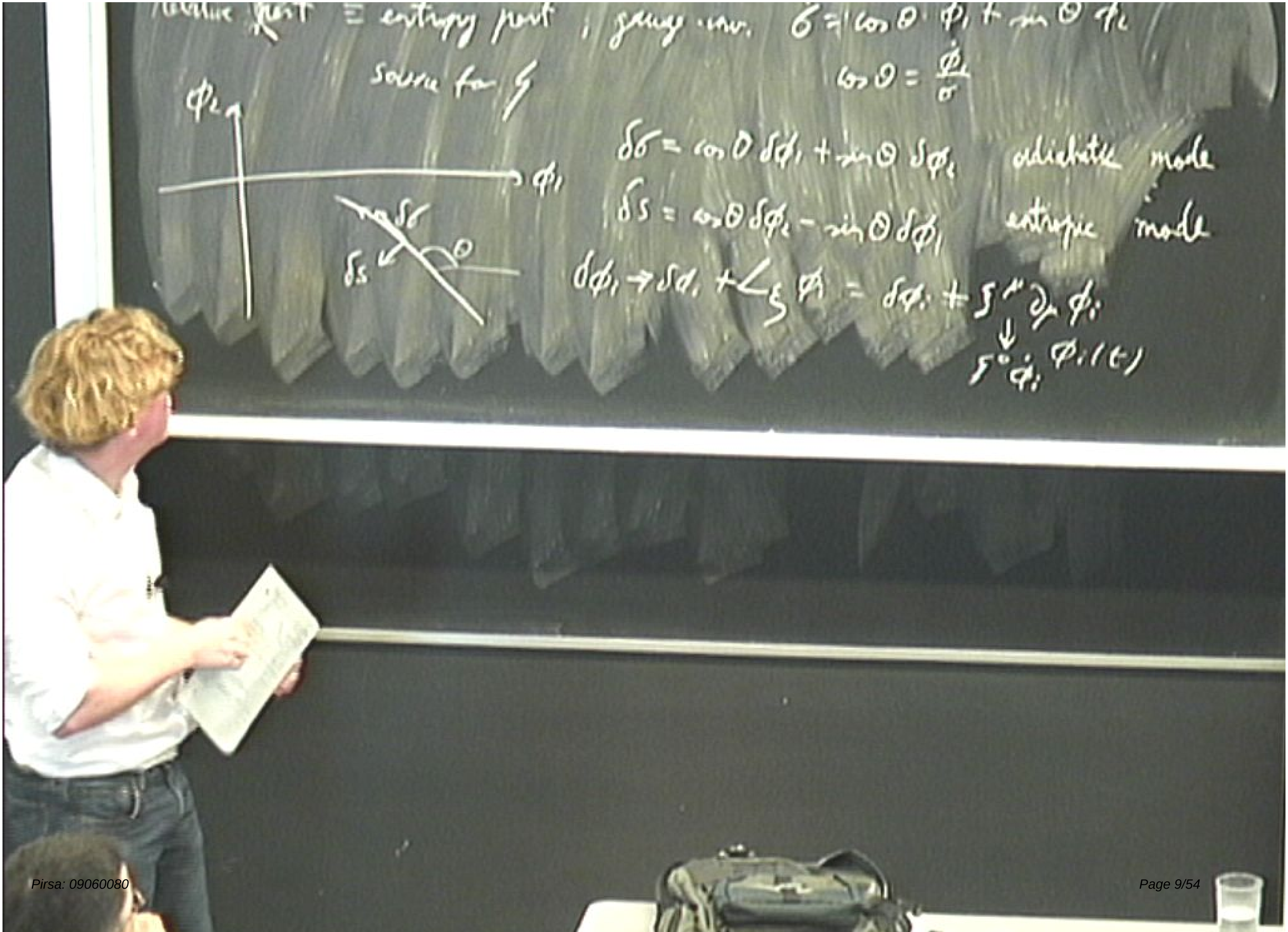

2 scalars
relative pert ≡ entropy pert ; gauge inv.
source for ζ
φ2
φ1
δσ
δs
θ

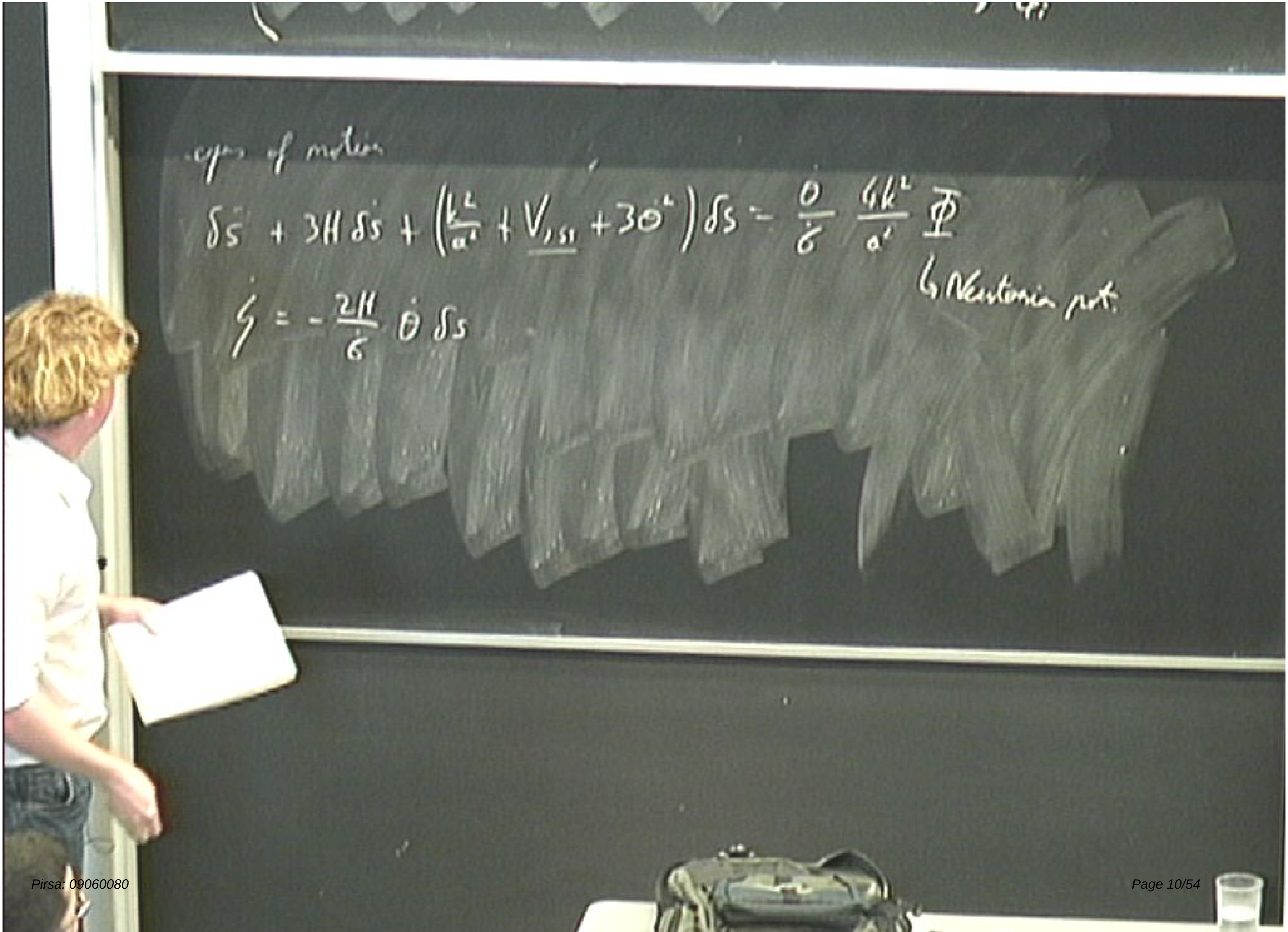

III Cosm. Pert.: Entropic mechanism + Non-Gaussianity
2 scalars
relative pert ≡ entropy pert ; gauge inv.
source for ζ
$\dot\sigma^2 = \dot\phi_1^2 + \dot\phi_2^2$
$\dot\sigma = \cos\theta\, \dot\phi_1 + \sin\theta\, \dot\phi_2$
$\cos\theta = \frac{\dot\phi_1}{\dot\sigma}$
ϕ_2
ϕ_1
$\delta\sigma$
δs
θ


III Cosm. Pert. : Entropic mechanism + Non-Gaussianity
2 scalars
$\dot\sigma^2 = \dot\phi_1^2 + \dot\phi_2^2$
relative pert ≡ entropy pert , gauge inv.
$\dot\sigma = \cos\theta \cdot \dot\phi_1 + \sin\theta\, \dot\phi_2$
$\cos\theta = \frac{\dot\phi_1}{\dot\sigma}$
ϕ_2
$\delta\sigma = \cos\theta\, \delta\phi_1$
δs


III Cosm. Pert.: Entropic mechanism + Non-Gaussianity
2 scalars
relative pert ≡ entropy pert, gauge inv.
source for ζ
σ̇² = φ̇₁² + φ̇₂²
σ̇ = cos θ · φ̇₁ + sin θ φ̇₂
cos θ = φ̇₁/σ̇
δσ = cos θ δφ₁ + sin θ δφ₂ adiabatic mode
δs = cos θ δφ₂ − sin θ δφ₁ entropic mode
φ₂
φ₁
δσ
δs
θ


≡ entropy pert
gauge inv.
σ = cos θ φ1 + sin θ φ2
source for ζ
δσ = cos θ δφ1 + sin θ δφ2
adiabatic mode
δs = cos θ δφ2 − sin θ δφ1
entropic mode


relative pert ≡ entropy pert ; gauge inv. $\sigma = \cos\theta\ \phi_1 + \sin\theta\ \phi_2$
source for ζ
$\cos\theta = \frac{\dot\phi_1}{\dot\sigma}$
ϕ_2
ϕ_1
$\delta\sigma$
δs
θ
$\delta\sigma = \cos\theta\, \delta\phi_1 + \sin\theta\, \delta\phi_2$ adiabatic mode
$\delta s = \cos\theta\, \delta\phi_2 - \sin\theta\, \delta\phi_1$ entropic mode
$\delta\phi_i \to \delta\phi_i + \mathcal{L}_\xi \phi_i = \delta\phi_i + \xi^\mu \partial_\mu \phi_i$
$\xi^0 \dot\phi_i$
$\phi_i(t)$


eqns of motion
$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{,ss} + 3\dot\theta^2\right)\delta s = \frac{\dot\theta}{\dot\sigma}\frac{4k^2}{a^2}\Phi$
↳ Newtonian pot.
$\dot\zeta = -\frac{2H}{\dot\sigma}\dot\theta\,\delta s$

eqns of motion

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + \underline{V_{,ss}} + 3\dot{\theta}^2\right)\delta s = \frac{\dot{\theta}}{\dot{\sigma}}\frac{4k^2}{a^2}\Phi$$

↳ Newtonian pot.

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\dot{\theta}\,\delta s$$

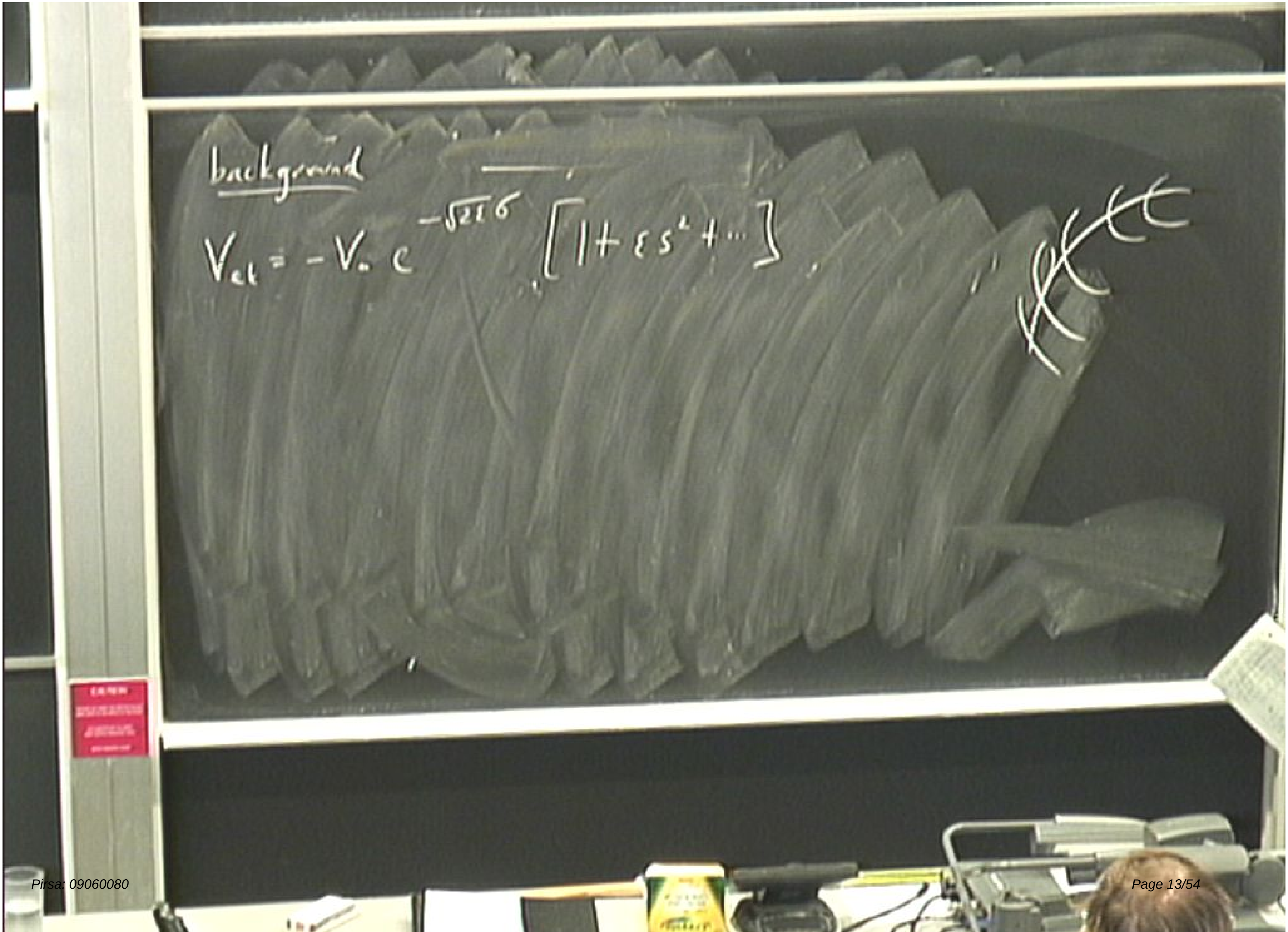
$\zeta(\delta\varphi_i, \delta\rho_i$, metric pert.$)$

eqns of motion

$$\begin{cases} \ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{,ss} + 3\dot{\theta}^2\right)\delta s = \frac{\dot{\theta}}{\dot{\sigma}}\frac{4k^2}{a^2}\Phi \\ \dot{\zeta} = -\frac{2H}{\dot{\sigma}}\dot{\theta}\,\delta s \quad \text{(large-scales)} \end{cases}$$

↳ Newtonian pot.

$\zeta(\delta\varphi_1, \delta\varphi_2, \text{metric pert.})$


background
$V_{eff} = -V_0 e^{-\sqrt{2\lambda}\sigma} [1 + \epsilon s^2 + \cdots]$

background

$$V_{ek} = -V_0\, e^{-\sqrt{2\epsilon}\,\sigma}\left[1+\epsilon s^2+\dots\right]$$

ekpyrotic pot. for σ

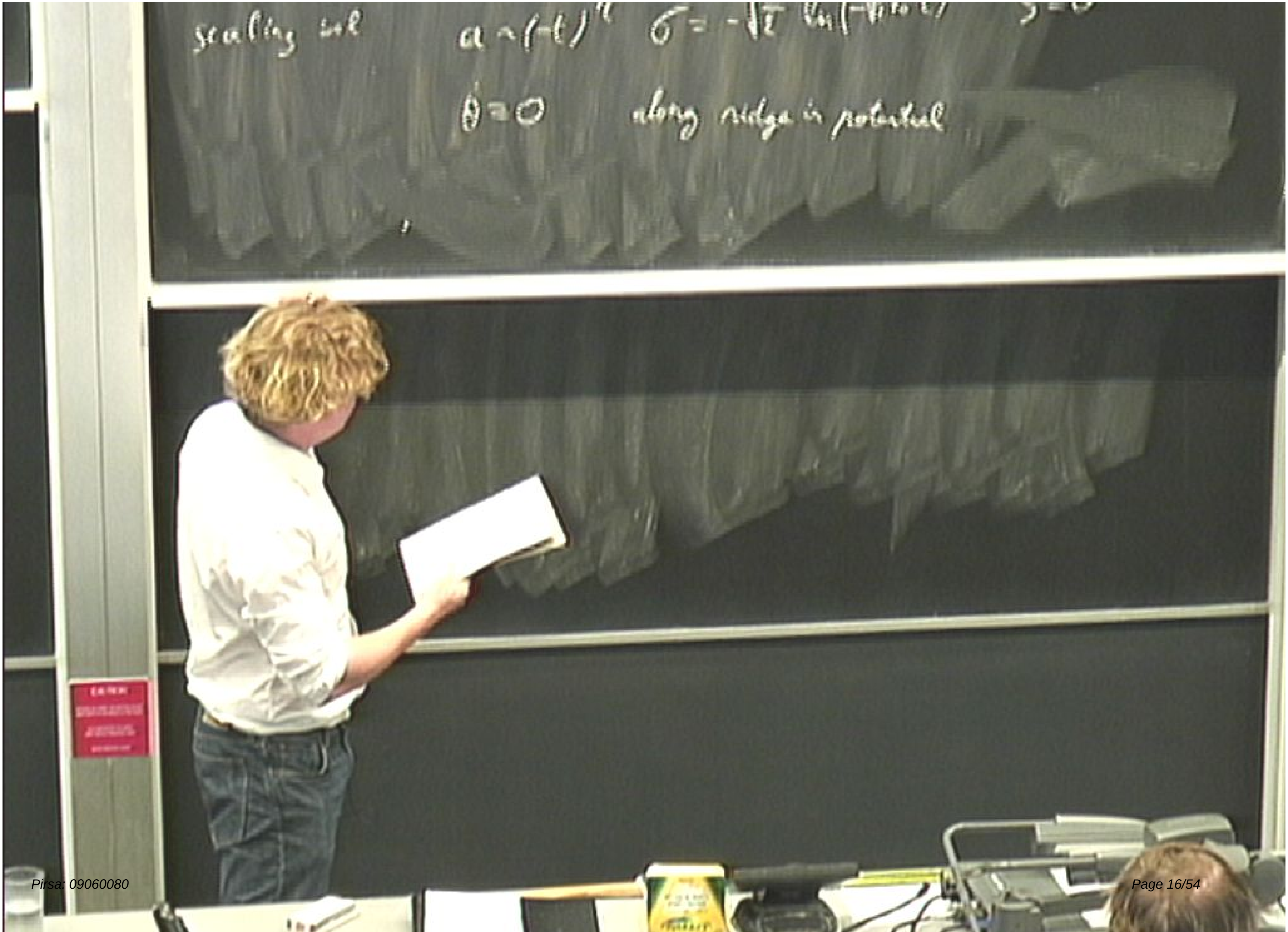
tachyonic pot. for s

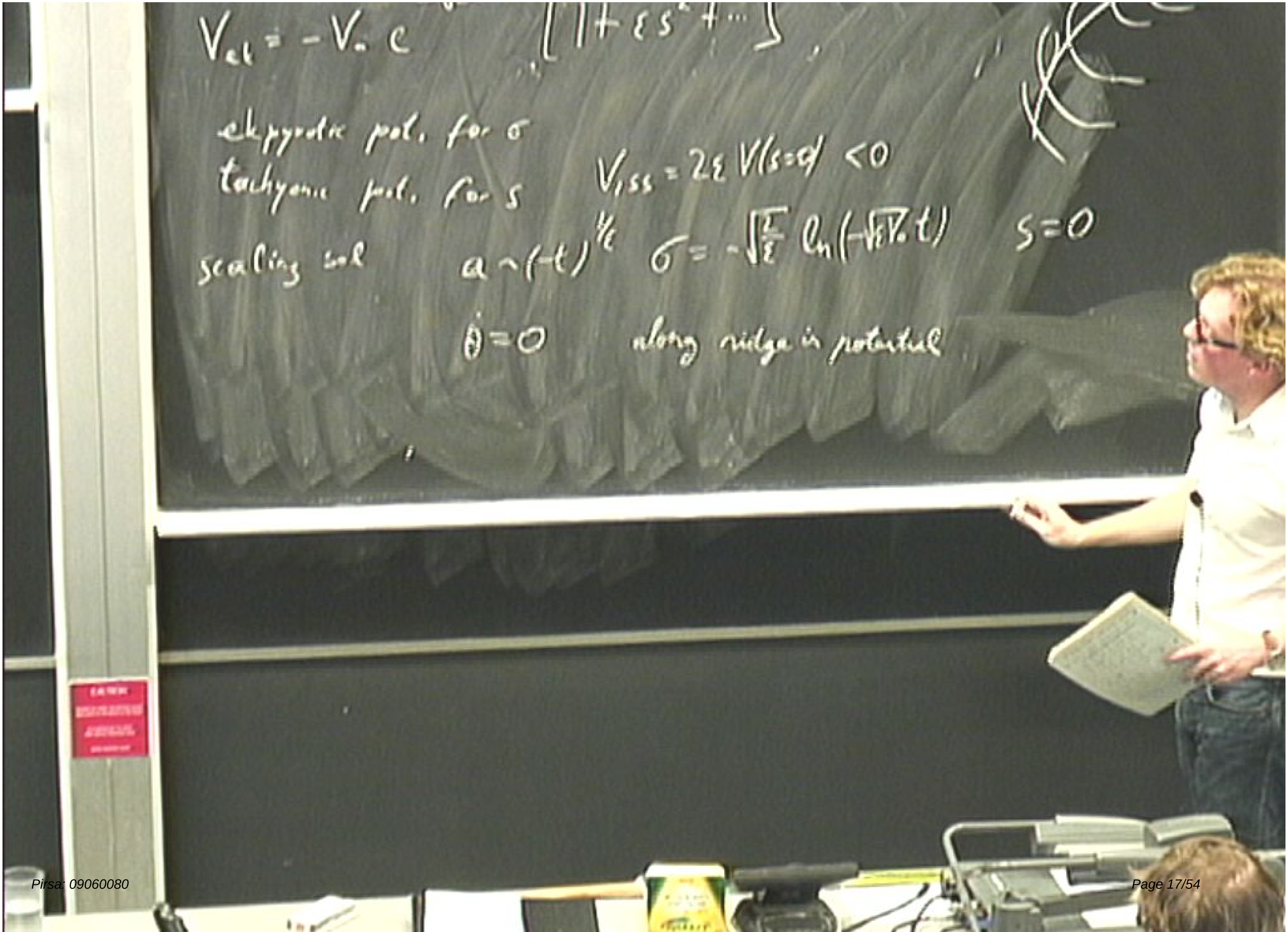
$$V_{ek} = -V_0\, e^{\cdots} \left[1 + \varepsilon s^2 + \cdots \right]$$

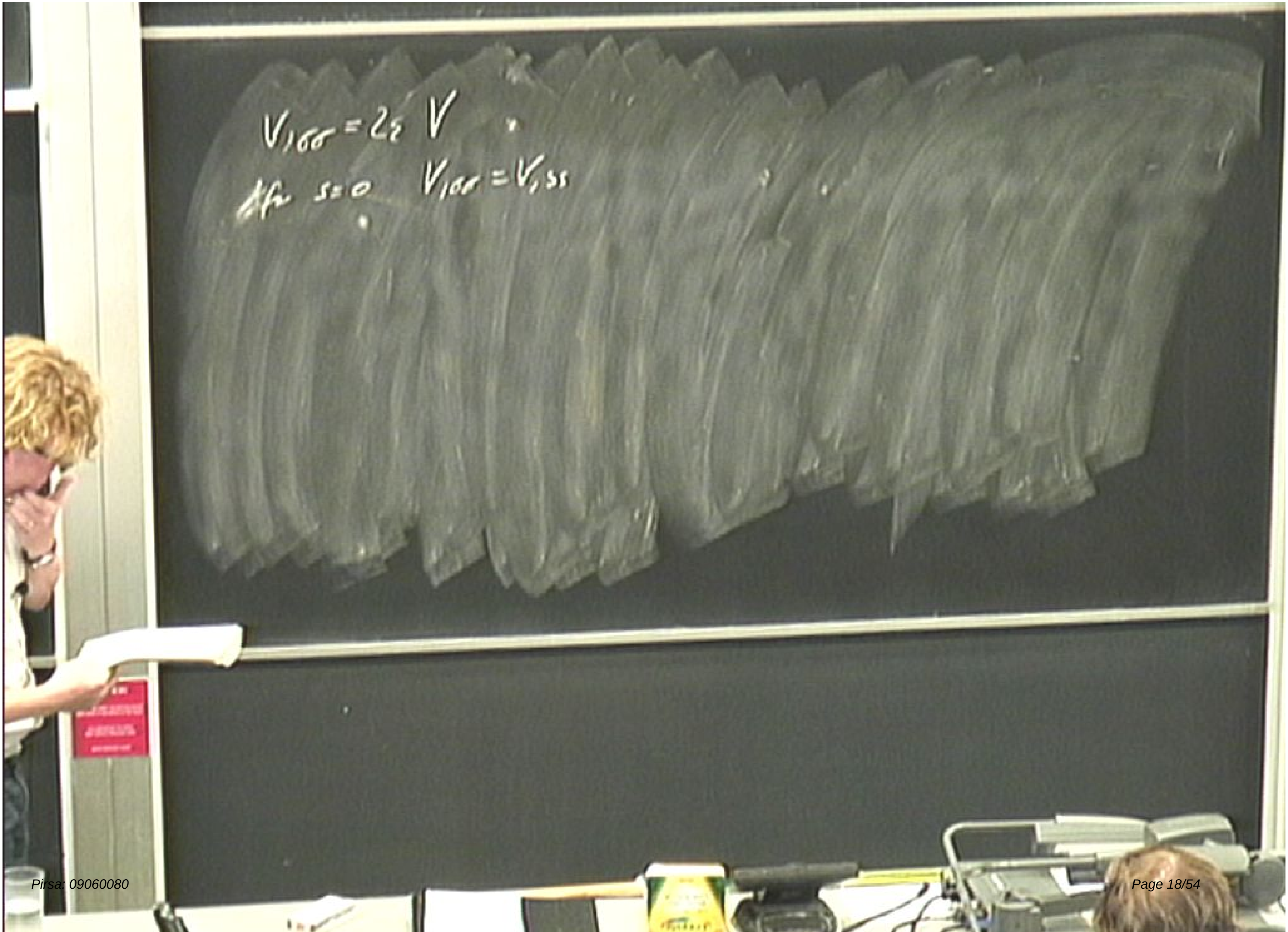
ekpyrotic pot. for σ

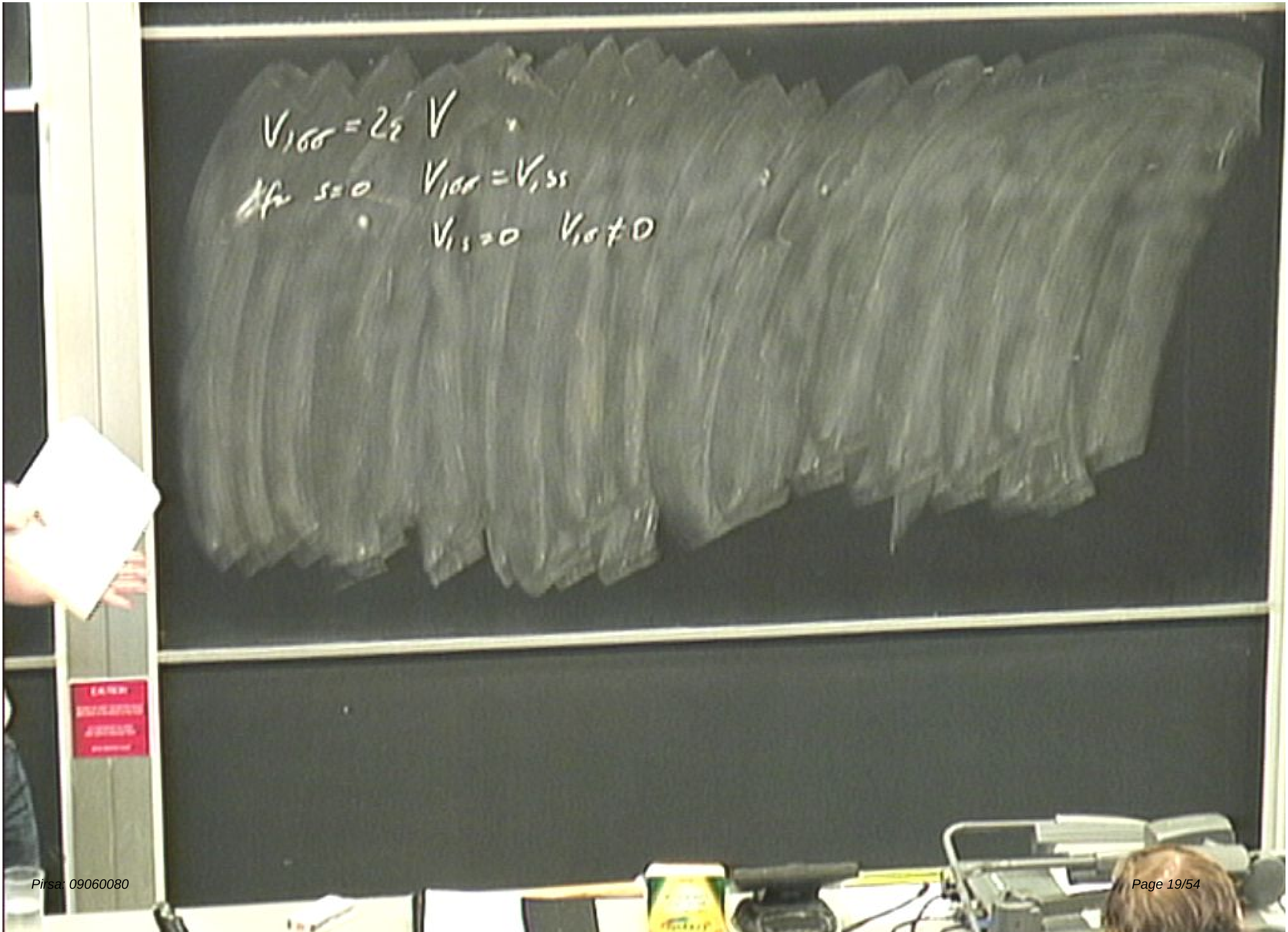
tachyonic pot. for s

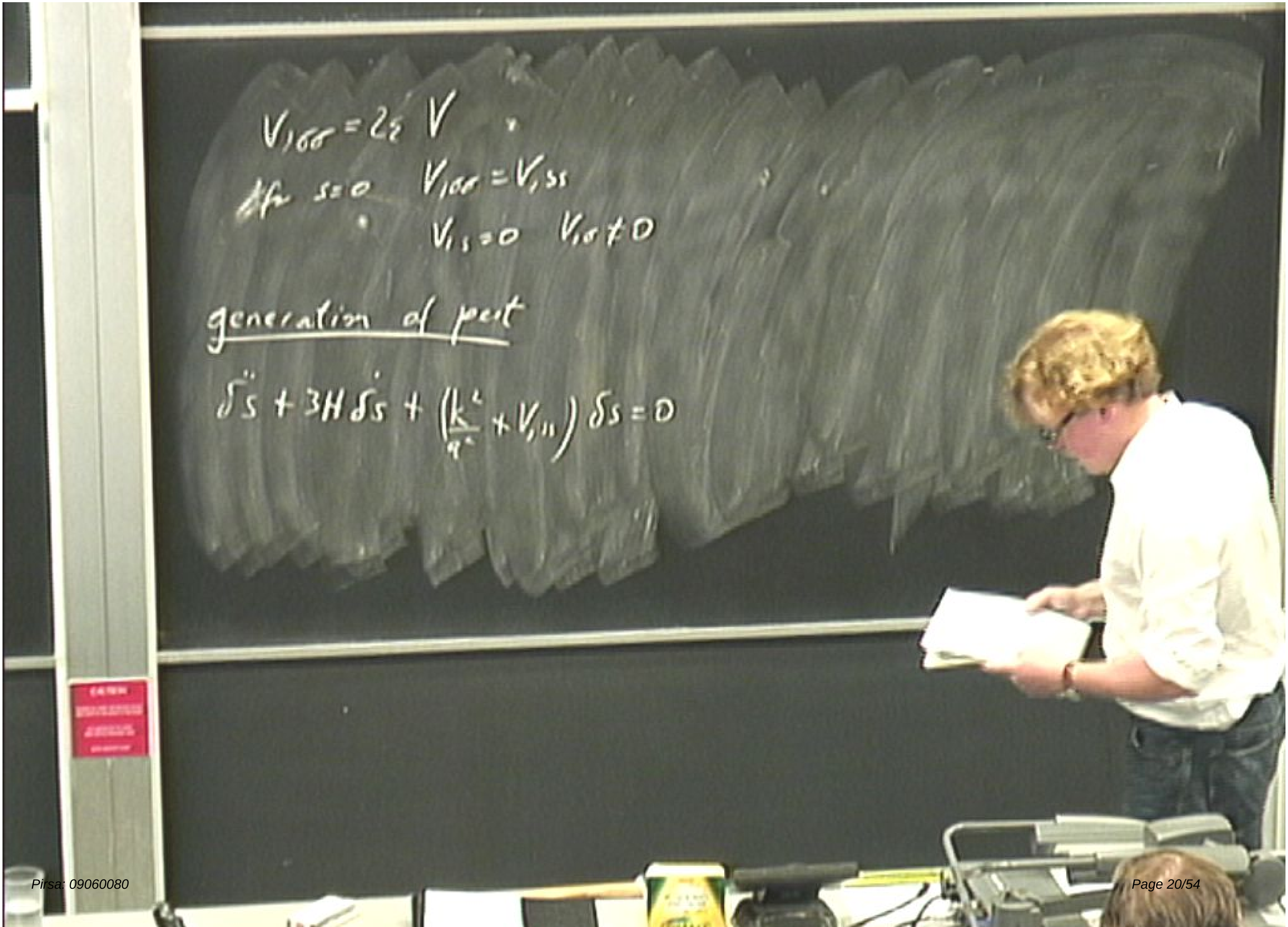
scaling sol $\quad a \sim (-t)^{1/\varepsilon} \quad \sigma = -\sqrt{\frac{2}{\varepsilon}}\, \ln\left(-\sqrt{\varepsilon V_0}\, t\right) \quad s = 0$


scaling sol
$\dot\theta = 0$ along ridge in potential


V_eff = -V_0 e [1 + ε s^2 + ...]
ekpyrotic pot. for σ
tachyonic pot. for s
V,ss = 2ε V(s=0) < 0
scaling sol
a ~ (-t)^{1/ε}
σ = -√(2/ε) ln(-√(εV_0) t)
s = 0
θ̇ = 0
along ridge in potential


$V_{,\sigma\sigma} = 2\xi\, V$
for $s=0$ $V_{,\sigma\sigma} = V_{,ss}$


V1σσ = 2ξ V
s=0 V1σσ = V1ss
V1s = 0 V1σ ≠ 0


V,σσ = 2ε V
for s=0 V,σσ = V,ss
V,s = 0 V,σ ≠ 0
generation of pert
δs̈ + 3H δṡ + (k²/a² + V,ss) δs = 0

$V_{,\sigma\sigma} = 2\xi V$

if $s=0$ $\quad V_{,\sigma\sigma} = V_{,ss}$

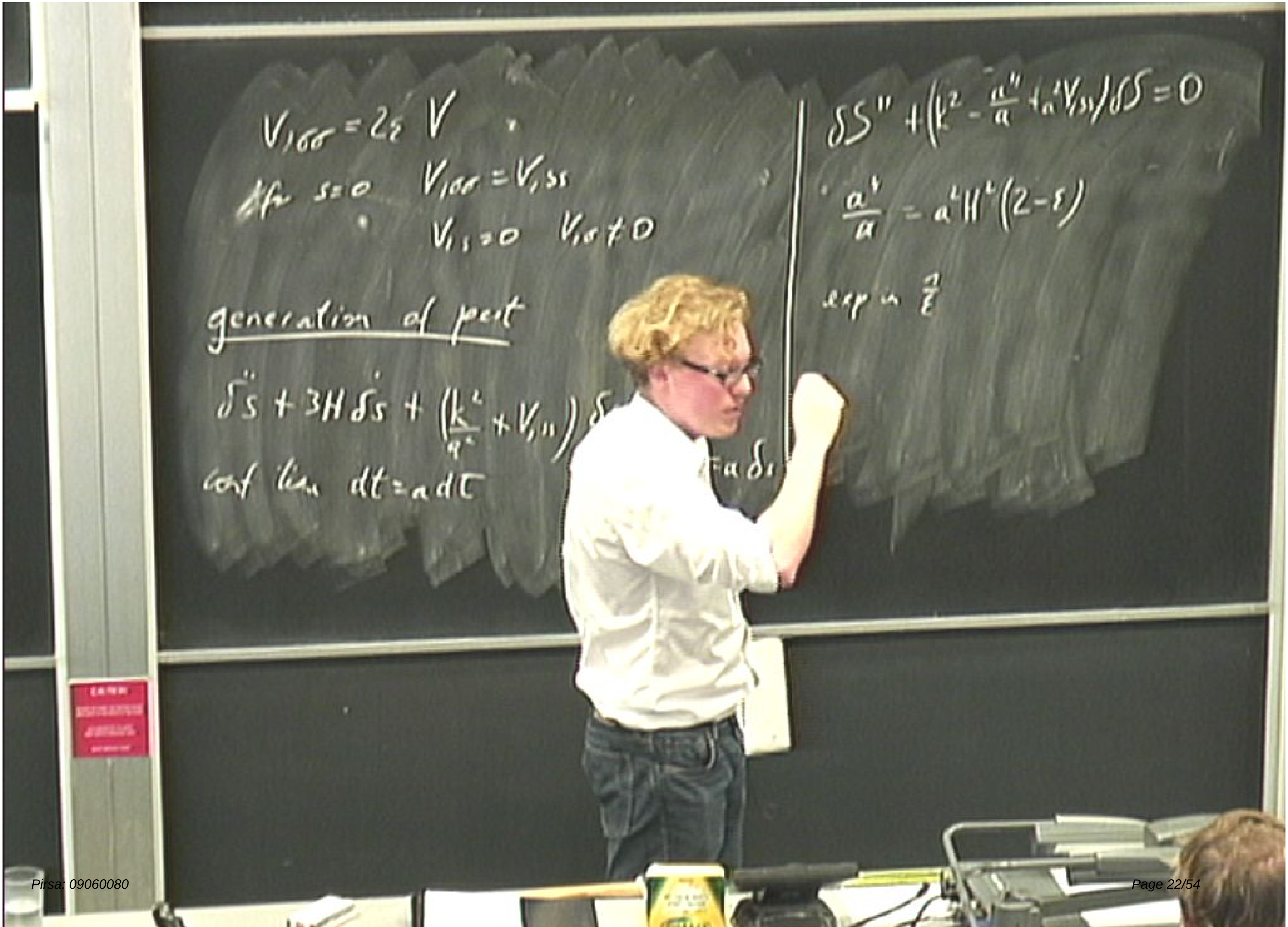
$V_{,s} = 0 \quad V_{,\sigma} \neq 0$

<u>generation of pert</u>

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{,ss}\right)\delta s = 0$$

conf time $dt = a\,d\tau \qquad$ def $\quad \delta S = a\,\delta s$

$$\delta S'' + \left(k^2 - \frac{a''}{a} + a^2 V_{,ss}\right)\delta S = 0$$


V,σσ = 2ε V
s=0 V,σσ = V,ss
V,s = 0 V,σ ≠ 0
generation of pert
conf time dt = a dτ
δS'' + (k² - a''/a + a²V,ss) δS = 0
a''/a = a²H²(2-ε)

$V_{,\sigma\sigma} = 2\varepsilon V$

for $s=0$ $\quad V_{,\sigma\sigma} = V_{,ss}$

$V_{,s} = 0 \quad V_{,\sigma} \neq 0$

<u>generation of pert</u>

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{,ss}\right)\delta s = 0$$

conf time $dt = a\,d\tau$ $\qquad$ def $\delta S = a\,\delta s$

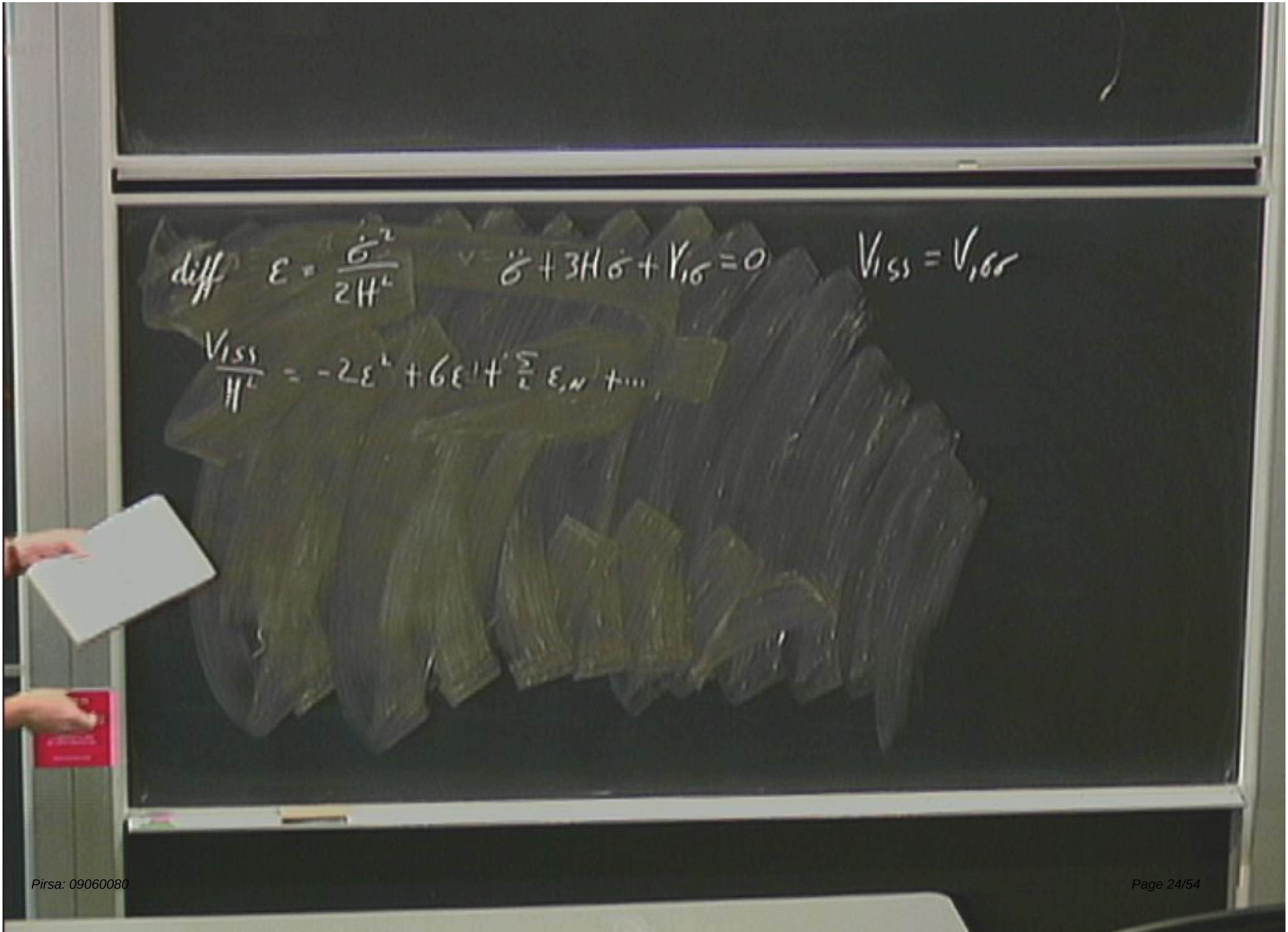
$$\delta S'' + \left(k^2 - \frac{a''}{a} + a^2 V_{,ss}\right)\delta S = 0$$

$$\frac{a''}{a} = a^2 H^2 (2-\varepsilon)$$

exp in ε

$dN = d\ln a$

$\frac{d\varepsilon}{dN}$


diff $\varepsilon = \frac{\dot{\sigma}^2}{2H^2}$
$\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0$
$V_{,ss} = V_{,\sigma\sigma}$
$\frac{V_{,ss}}{H^2} = -2\varepsilon^2 + 6\varepsilon' + \sum \varepsilon_{,N} + \dots$

diff $\quad \epsilon = \frac{\dot{\sigma}^2}{2H^2} \qquad \ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0 \qquad V_{,ss} = V_{,\sigma\sigma}$

$$\frac{V_{,ss}}{H^2} = -2\epsilon^2 + 6\epsilon + \frac{5}{2}\epsilon_{,N} + \dots$$

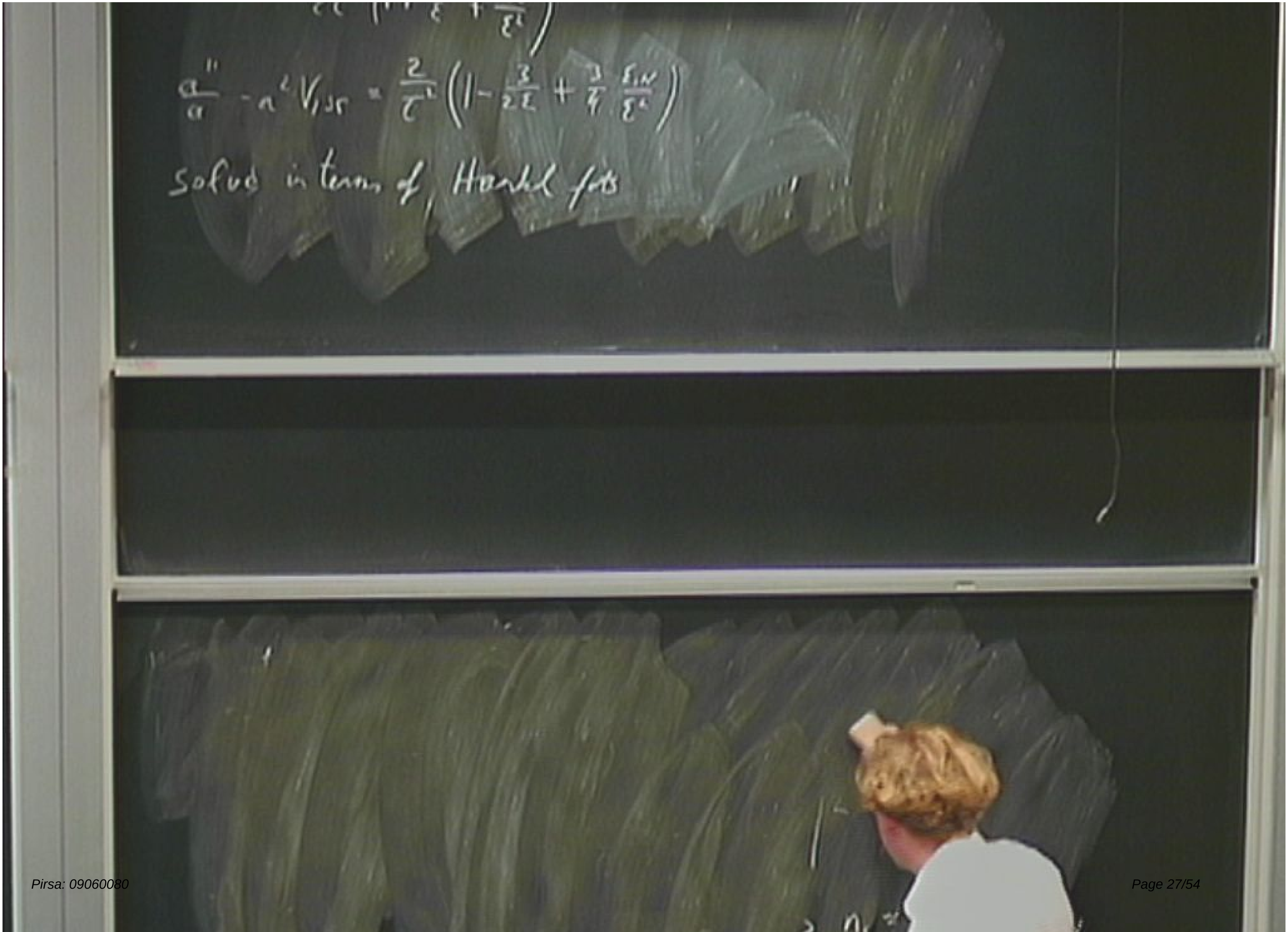
$$aH = \frac{1}{\epsilon\tau}\left(1 + \frac{1}{\epsilon} + \frac{\epsilon_{,N}}{\epsilon^2}\right)$$

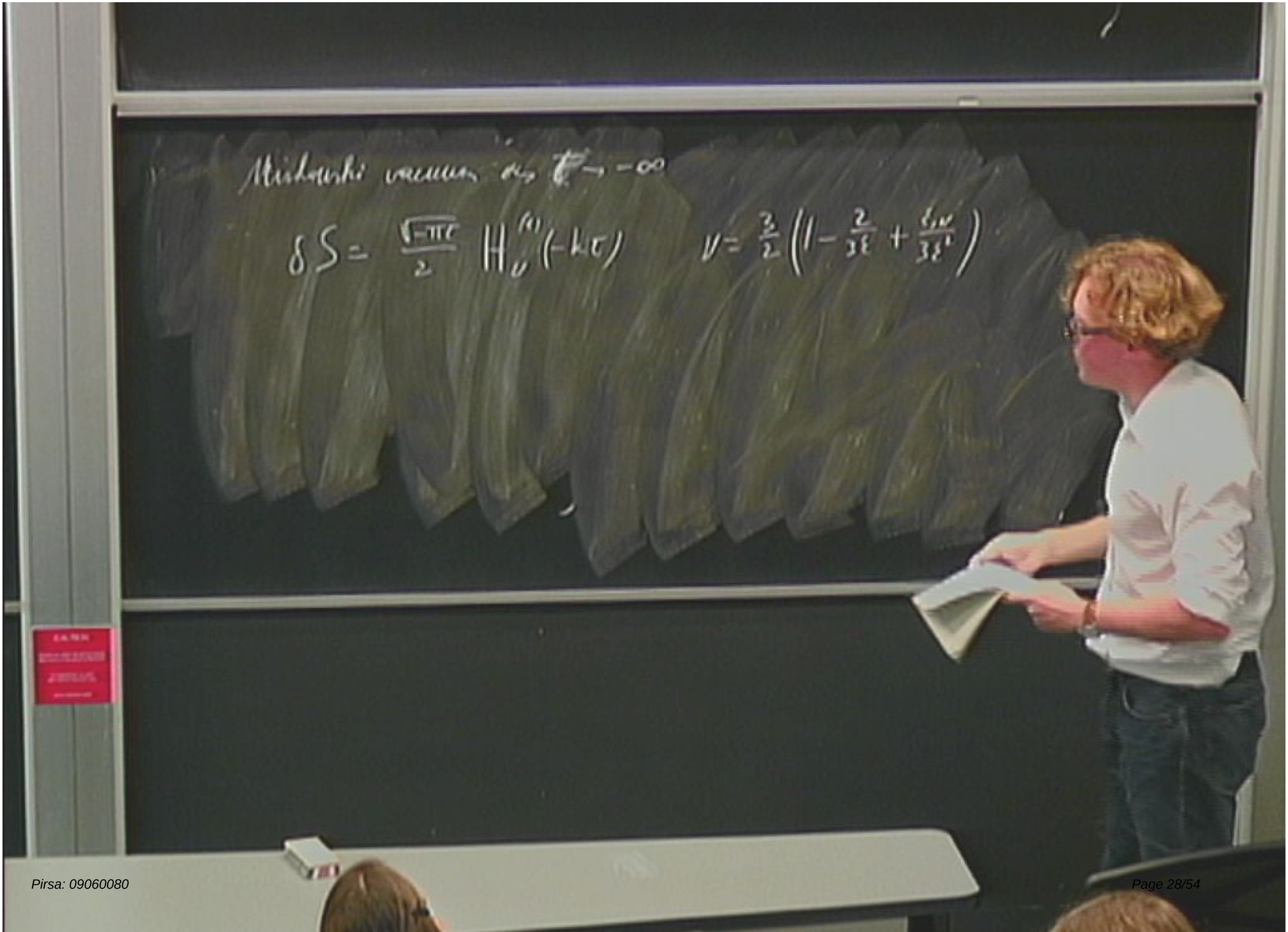
diff $\varepsilon = \frac{\dot{\sigma}^2}{2H^2}$ $\qquad \ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0$ $\qquad V_{,ss} = V_{,\sigma\sigma}$

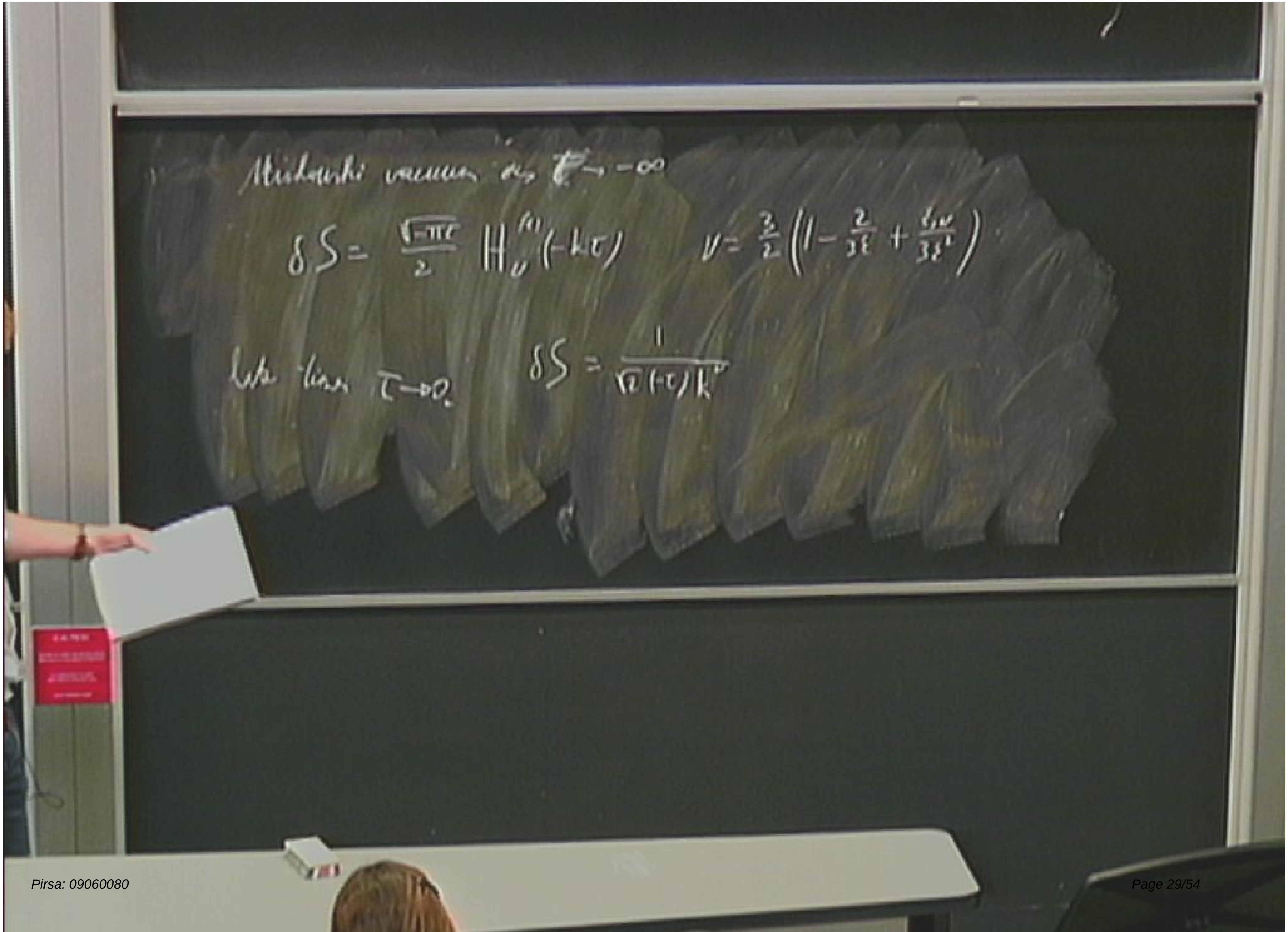
$$\frac{V_{,ss}}{H^2} = -2\varepsilon^2 + 6\varepsilon' + \frac{5}{2}\varepsilon_{,N} + \dots$$

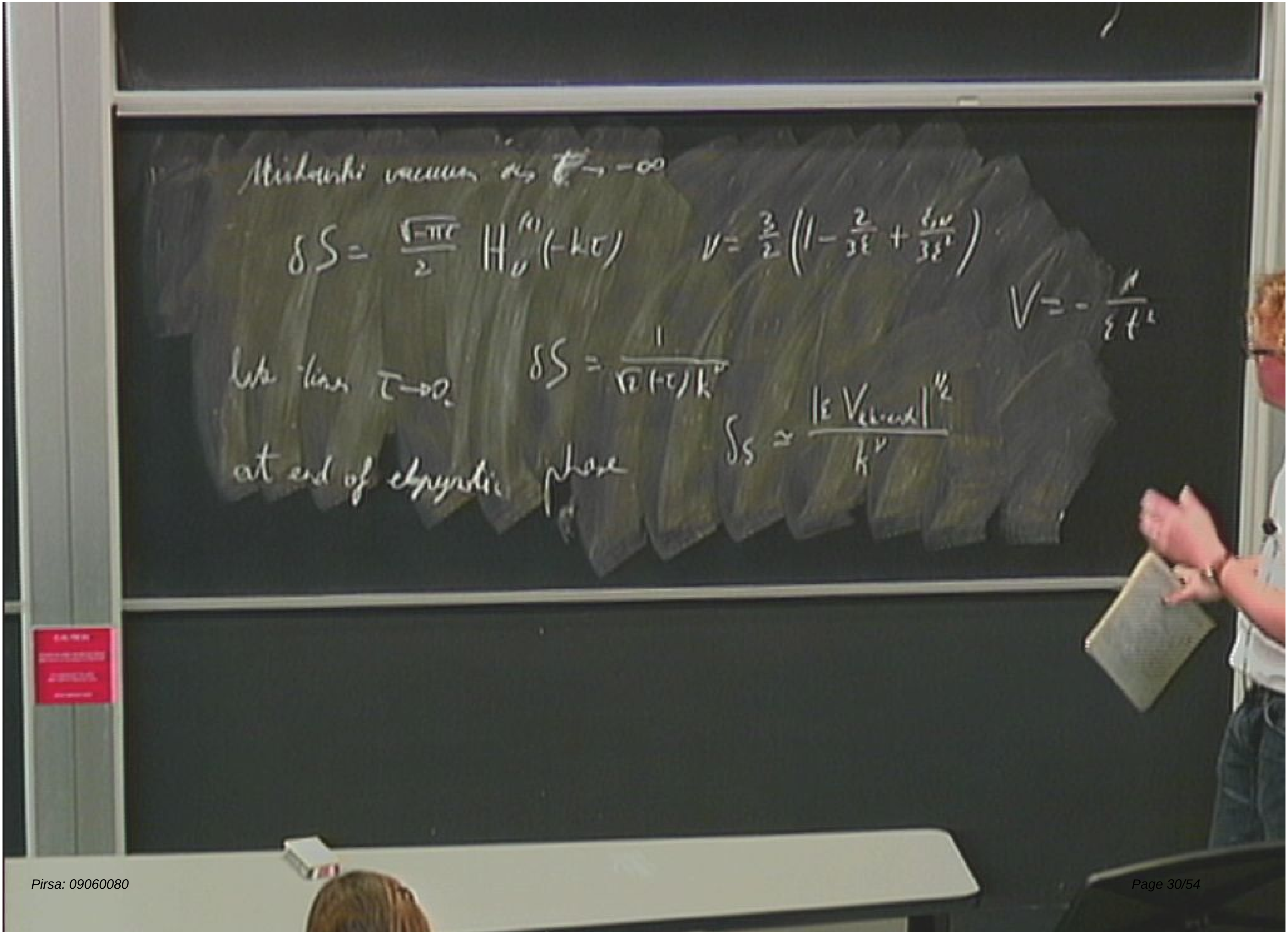
$$aH = \frac{1}{\varepsilon\tau}\left(1 + \frac{1}{\varepsilon} + \frac{\varepsilon_{,N}}{\varepsilon^2}\right)$$

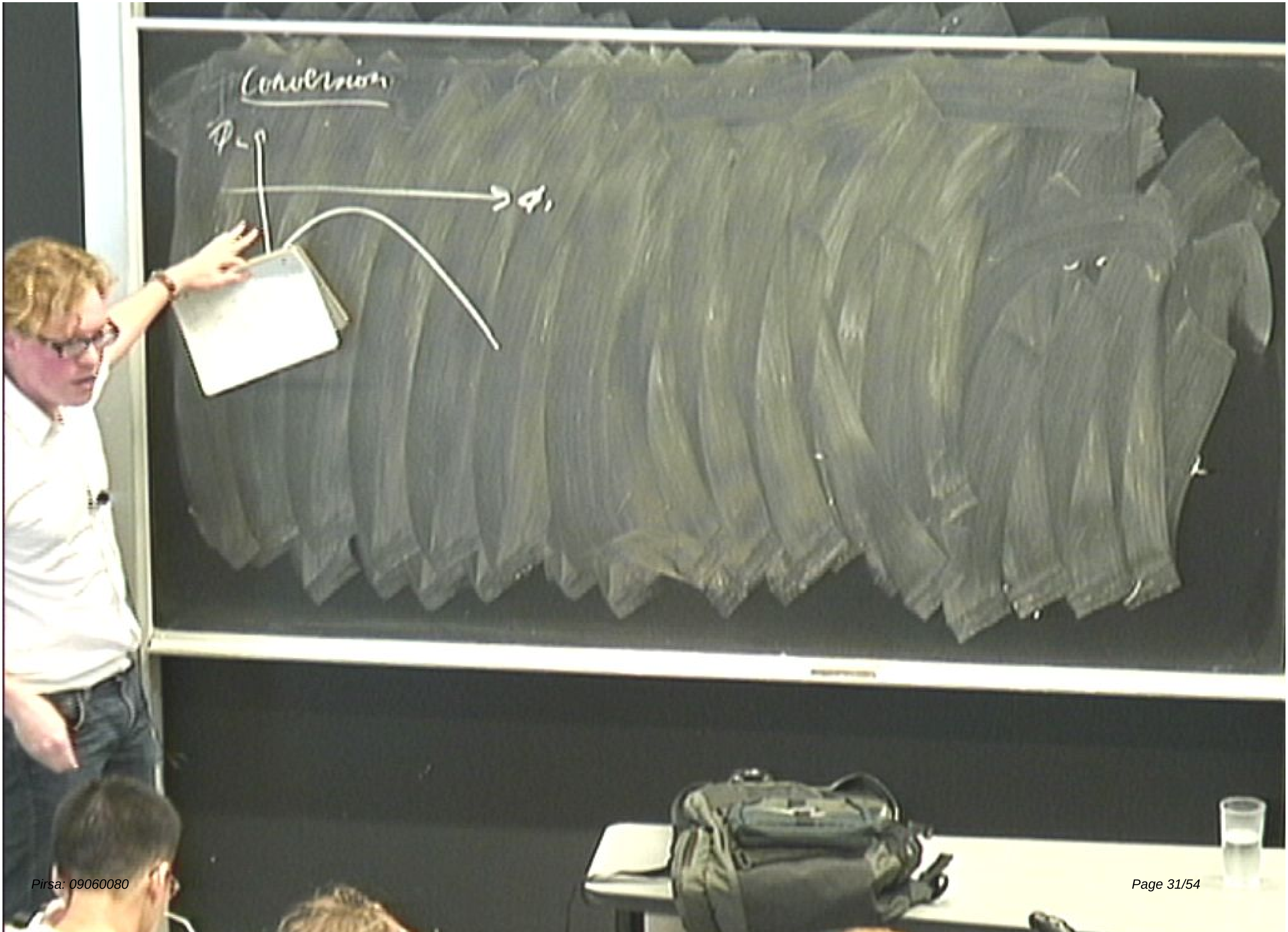
$$\frac{a''}{a} - a^2 V_{,ss} = \frac{2}{\tau^2}\left(1 - \frac{3}{2\varepsilon} + \frac{3}{4}\frac{\varepsilon_{,N}}{\varepsilon^2}\right)$$

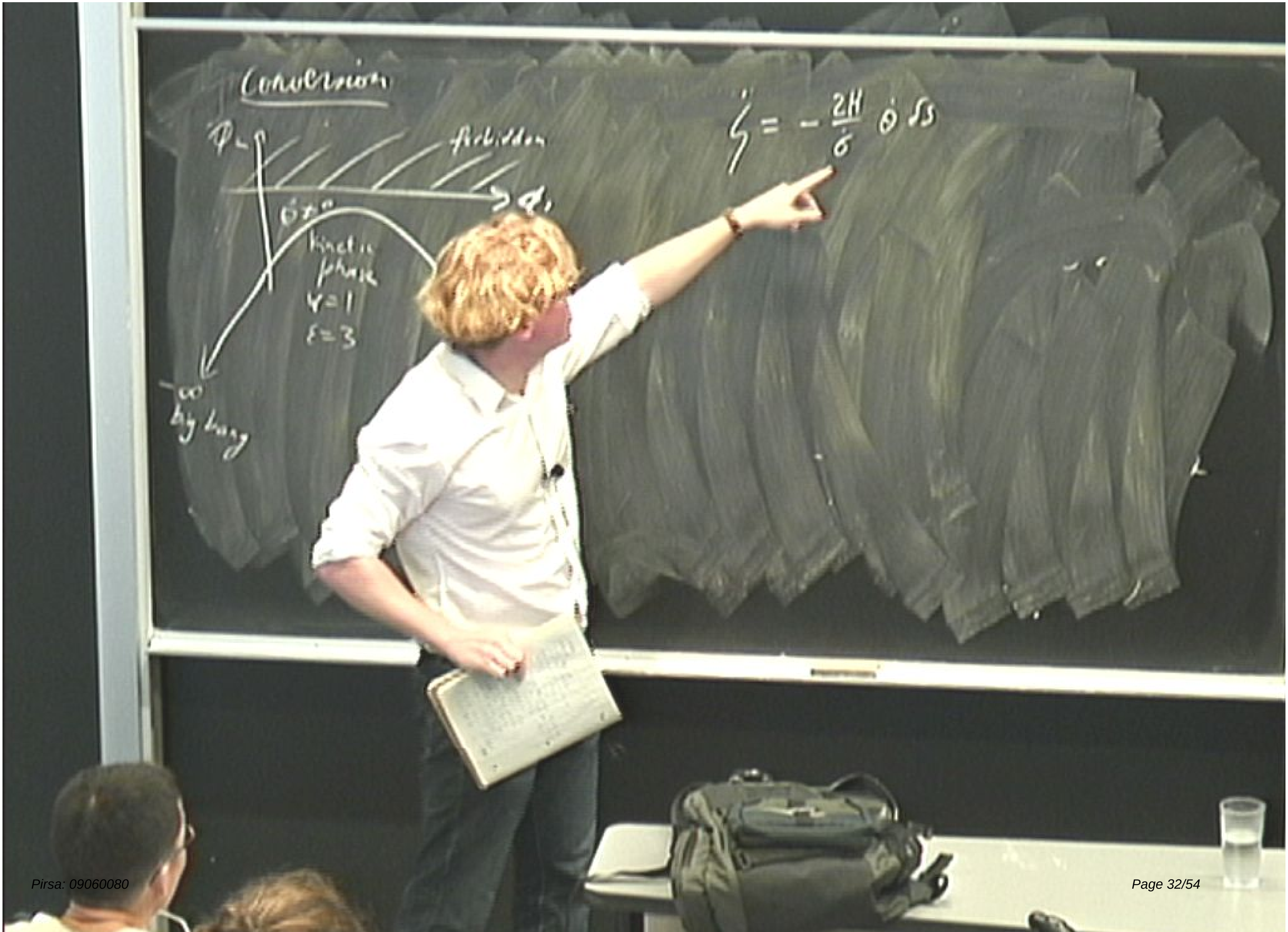

$\frac{a''}{a} - a^2 V_{,\phi\phi} = \frac{2}{\tau^2}\left(1 - \frac{3}{2}\epsilon + \frac{3}{4}\frac{\epsilon_1 N}{\epsilon^2}\right)$
Solved in terms of Hankel fcts

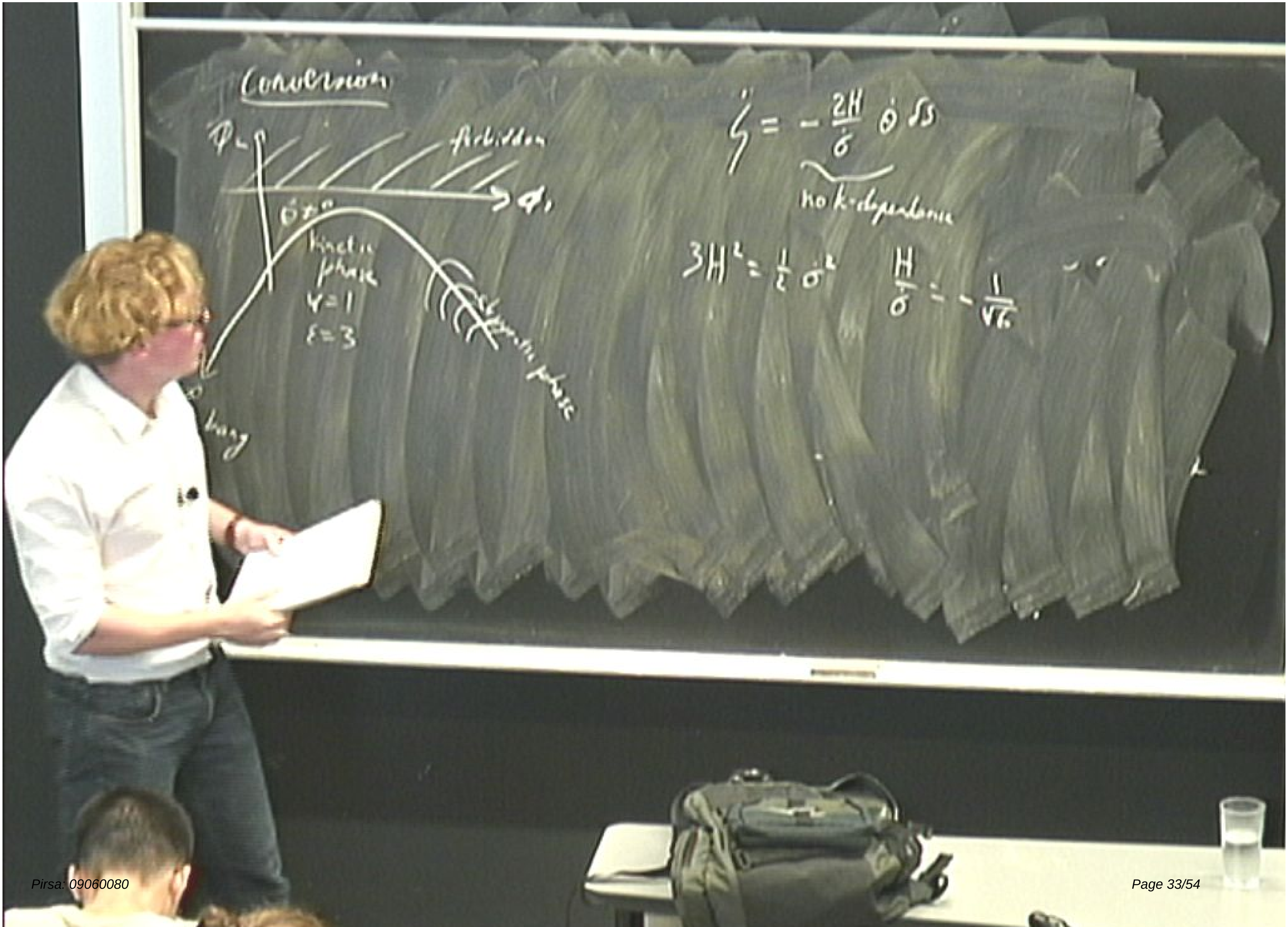

Minkowski vacuum as τ → −∞
δS = √(−πτ)/2 H_ν^(1)(−kτ)

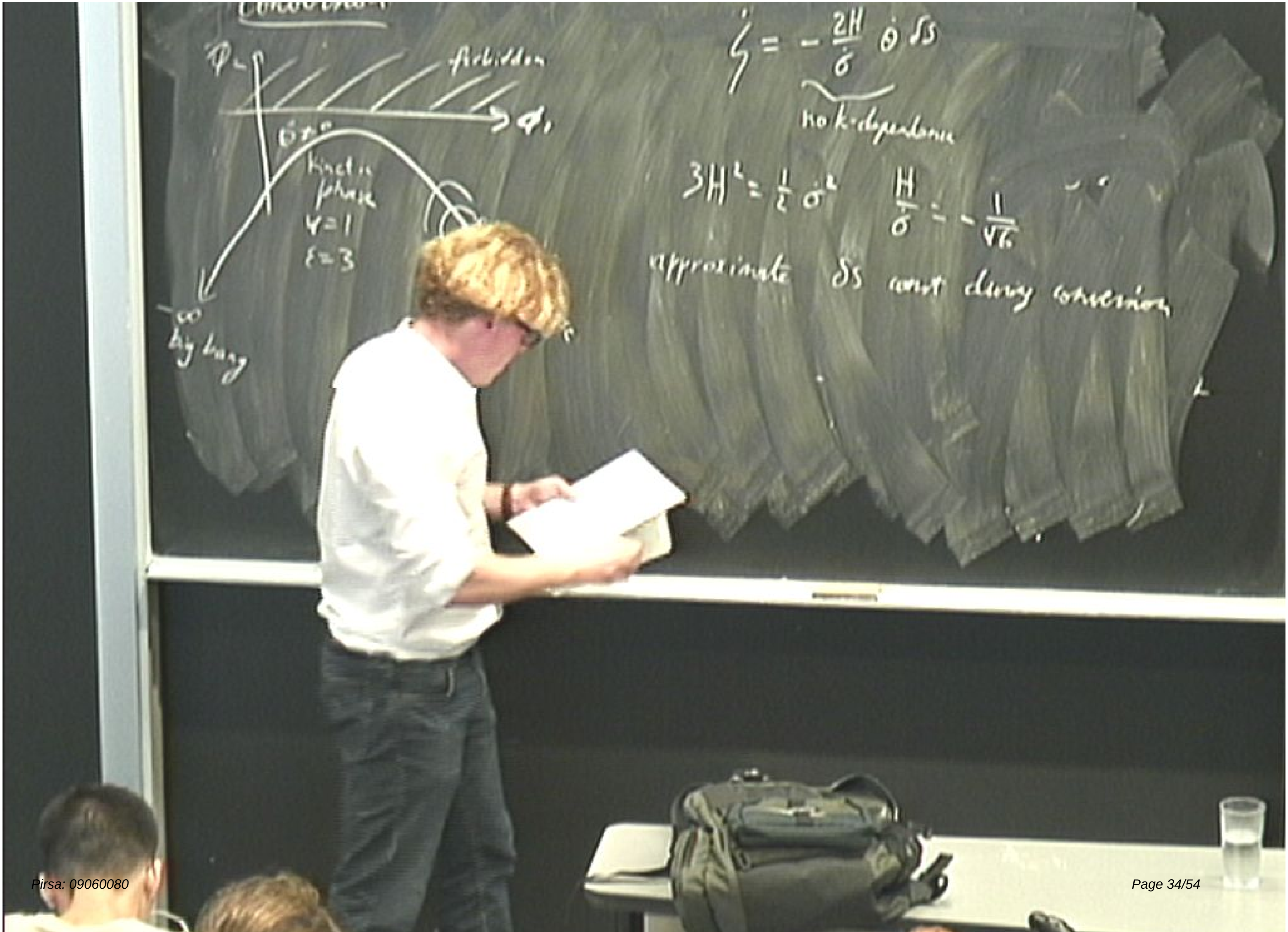

Minkowski vacuum as τ → −∞
δS = √(−πτ)/2 H_ν^(1)(−kτ)
late time τ → 0
δS = 1/(√2 (−τ) k

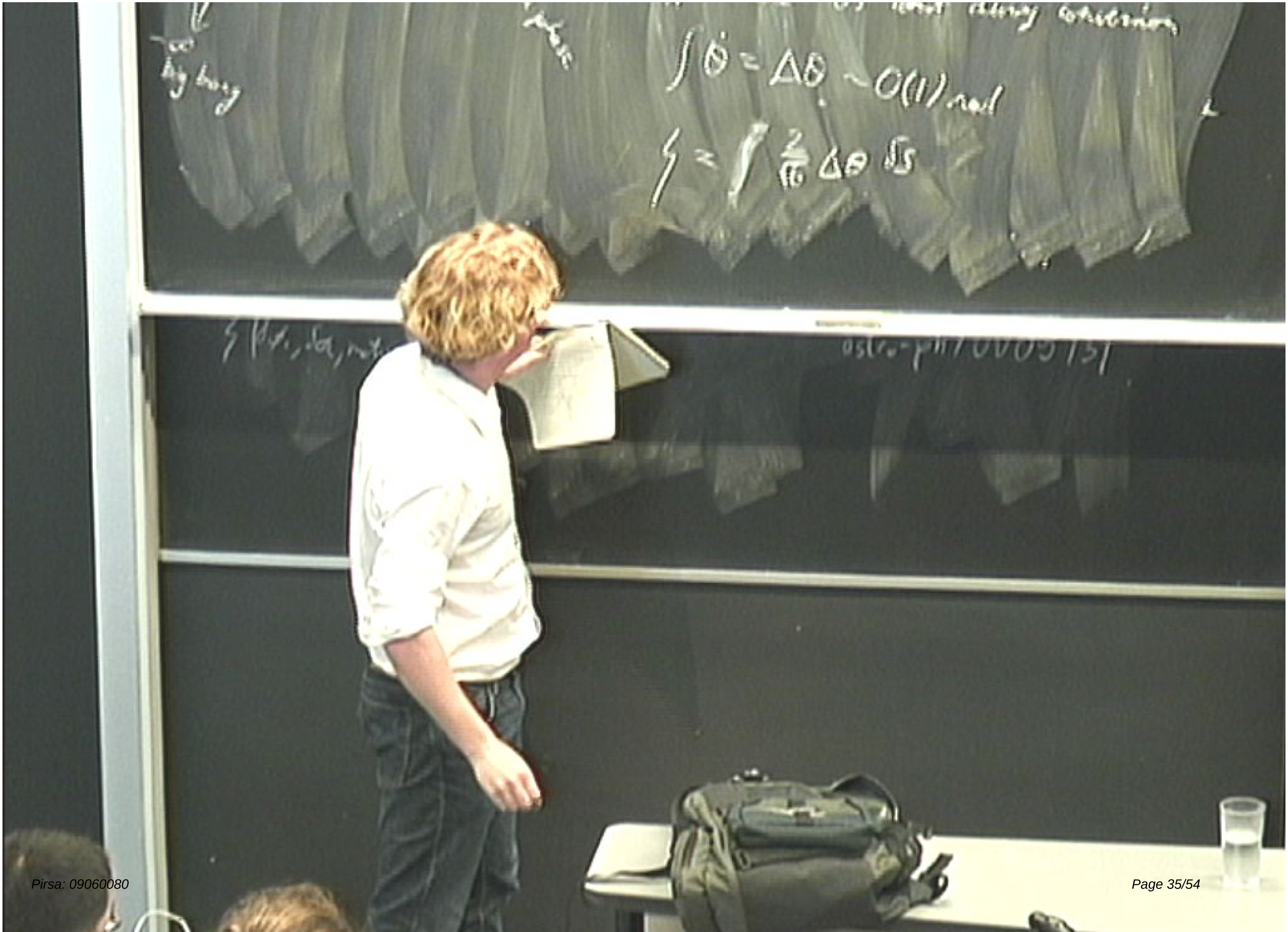

Minkowski vacuum as τ → −∞
δS = √(−πτ)/2 H_ν^(1)(−kτ)
ν = 3/2 (1 − 2/3ε + ...)
late time τ → 0₋
δS = 1/(√2 (−τ) k^ν)
δS ≃ |ε V_ekpyrotic|^(1/2) / k^ν
at end of ekpyrotic phase

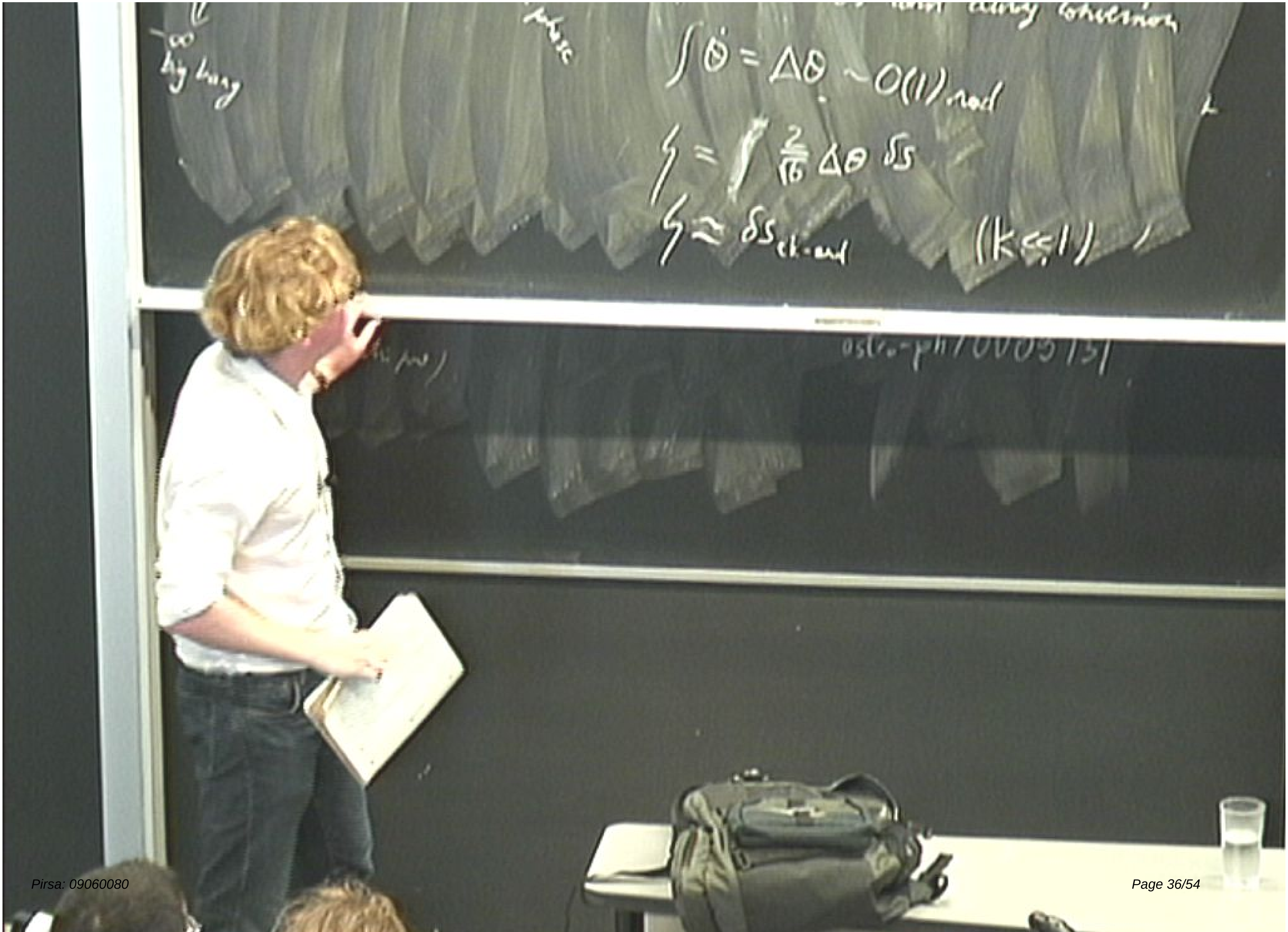


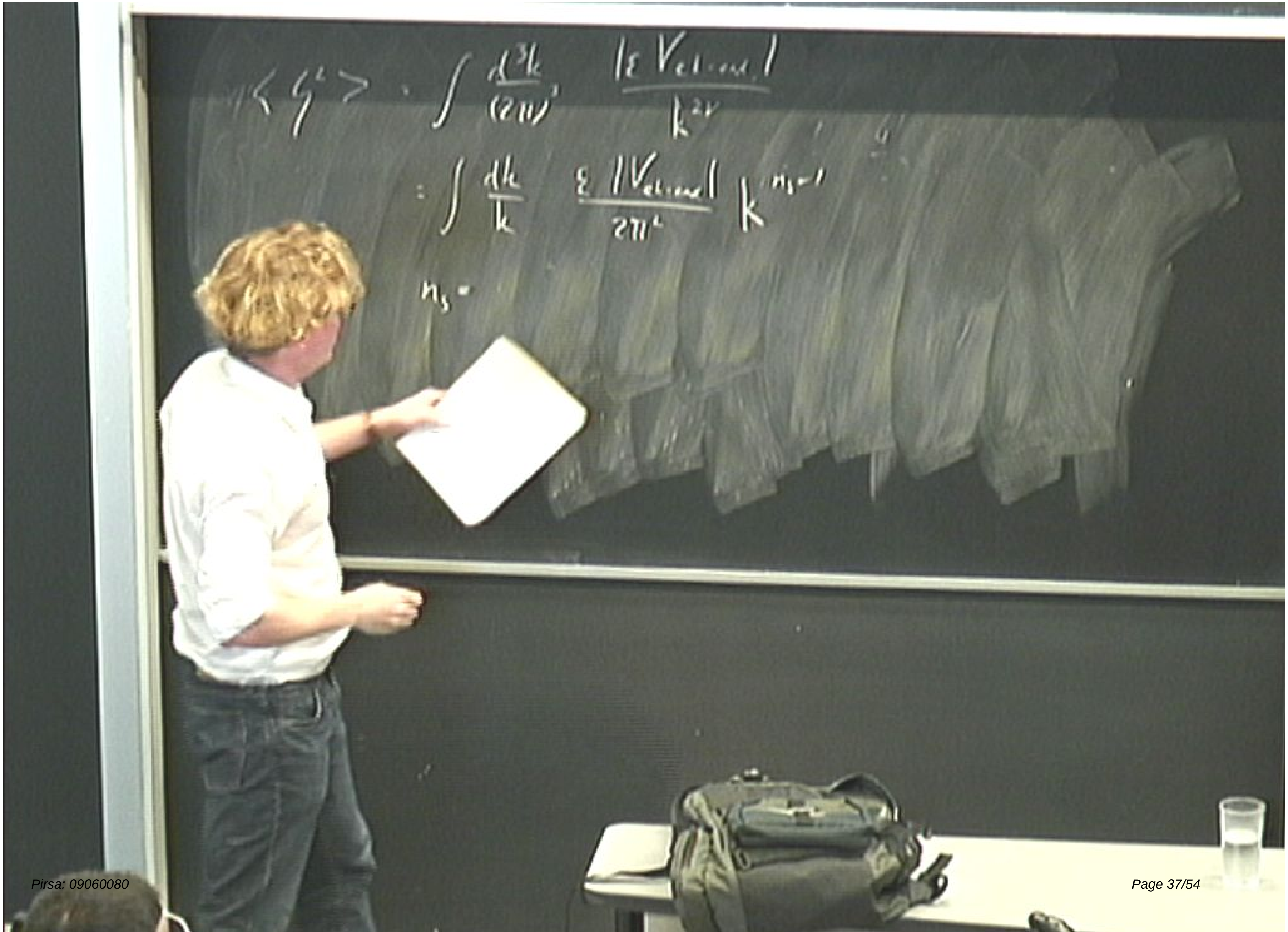

Conclusion
forbidden
kinetic phase
w=1
ε=3
-∞
big bang

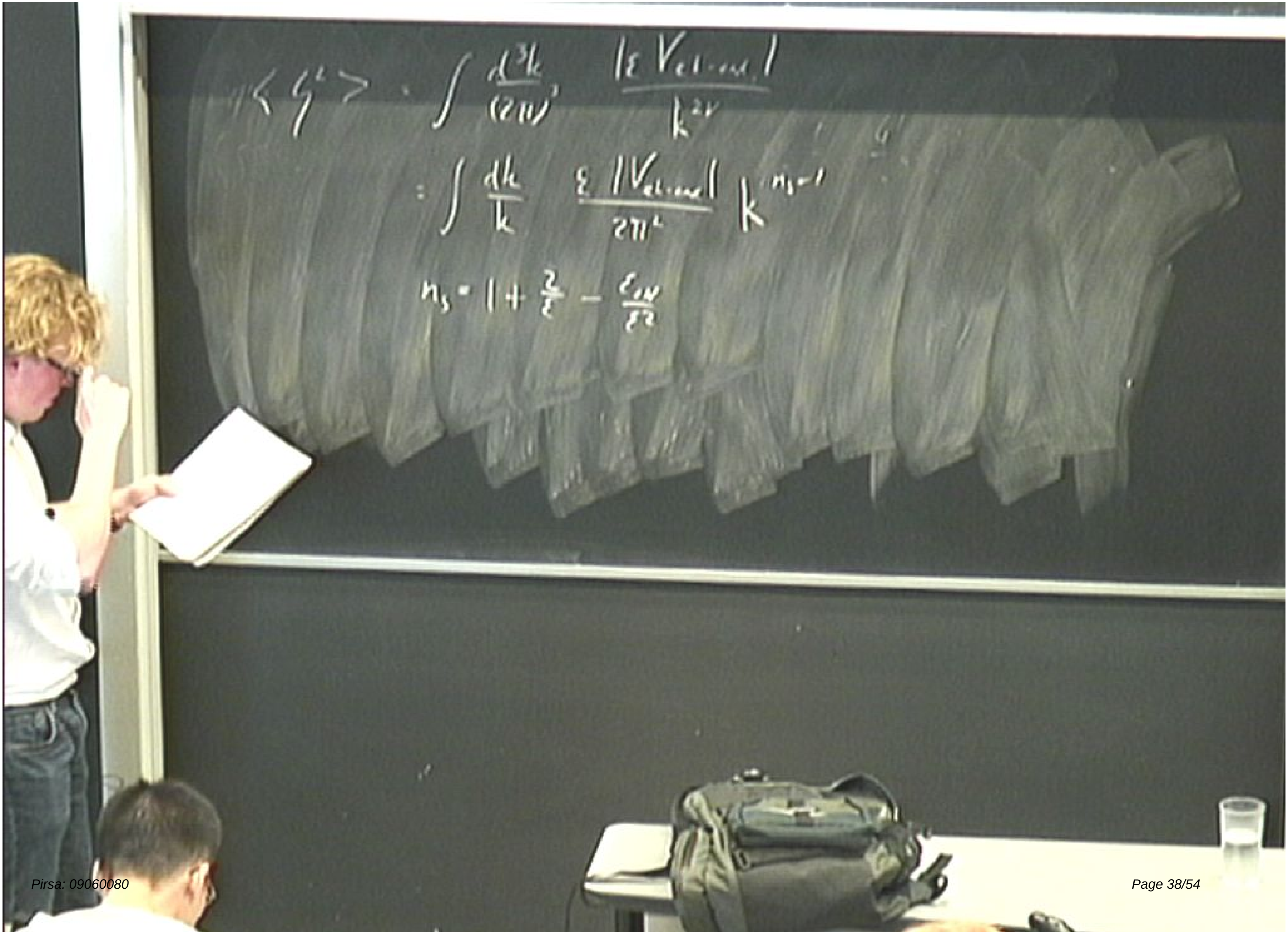

Conversion
forbidden
kinetic phase
w=1
ε=3
bang
ekpyrotic phase
no k-dependence
$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\dot{\theta}\,\delta s$
$3H^2 = \frac{1}{2}\dot{\sigma}^2$
$\frac{H}{\dot{\sigma}} = -\frac{1}{\sqrt{6}}$

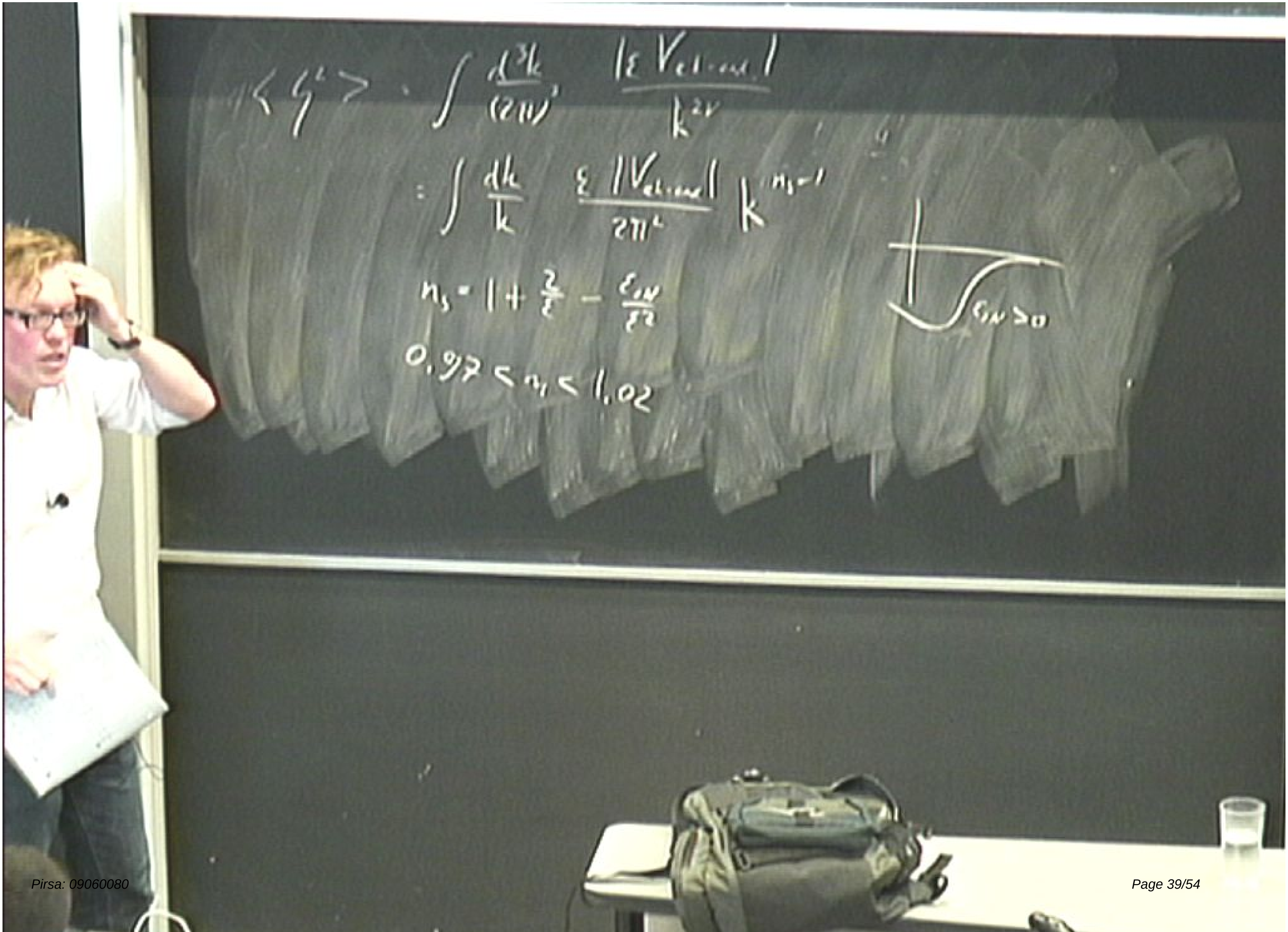

forbidden
kinetic phase
big bang
no k-dependence
approximate δs const during conversion

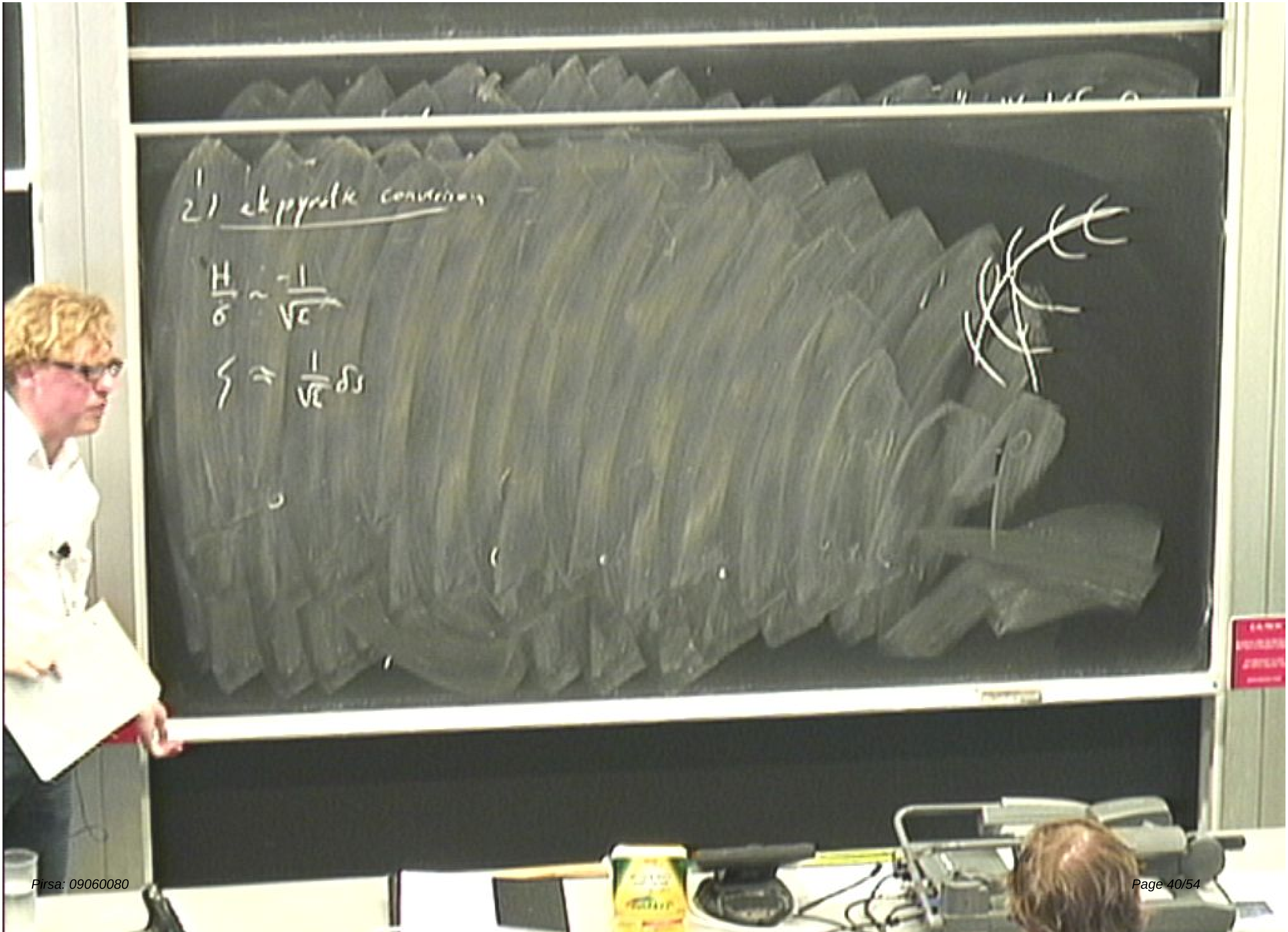



big bang
∫θ̇ = Δθ ~ O(1) rad
ζ = ∫ 2/√6 Δθ δs
ζ ≃ δs_ck-end (k≪1)


$\langle \zeta^2 \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{|\varepsilon V_{el}|}{k^{2\nu}}$
$= \int \frac{dk}{k} \frac{\varepsilon |V_{el}|}{2\pi^2} k^{n_s-1}$
$n_s =$


$n_s = 1 + \frac{2}{\epsilon} - \frac{\epsilon_{,N}}{\epsilon^2}$


$n_s = 1 + \frac{2}{\varepsilon} - \frac{\varepsilon_{,N}}{\varepsilon^2}$
$0.97 < n_s < 1.02$

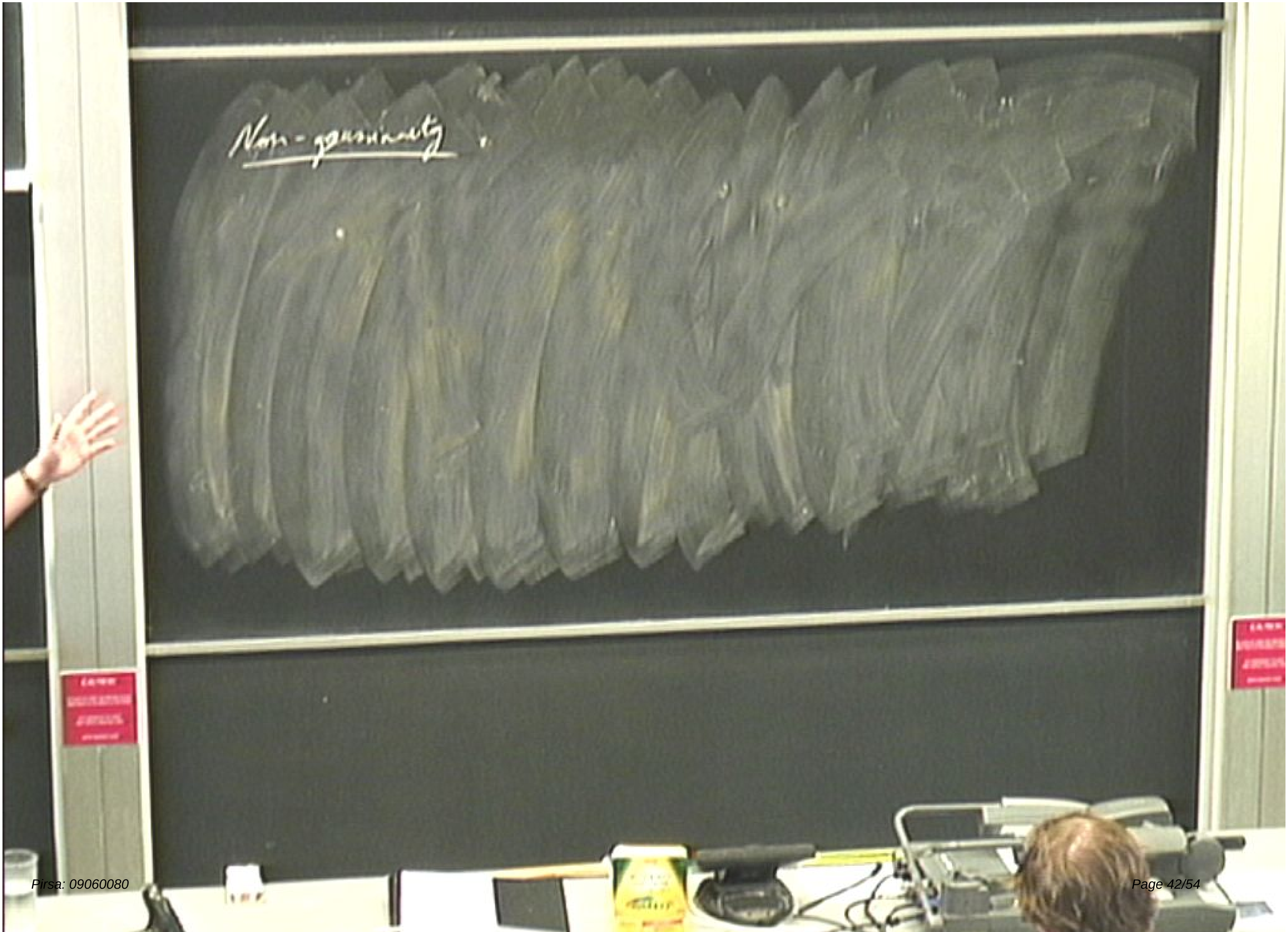

2) ekpyrotic conditions
H/σ ~ 1/√ε
ζ ≈ 1/√ε δs

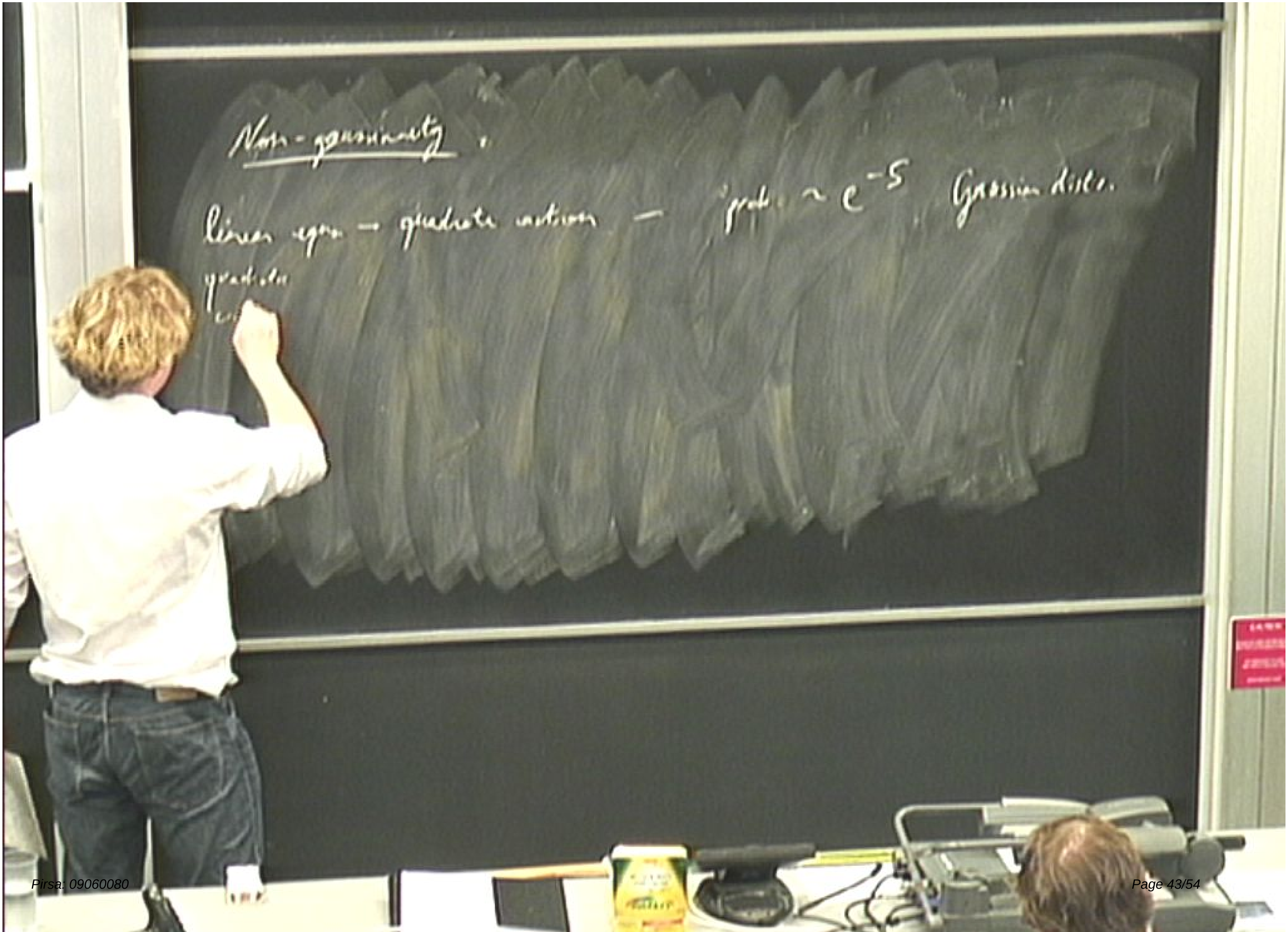
2) ekpyrotic conversion

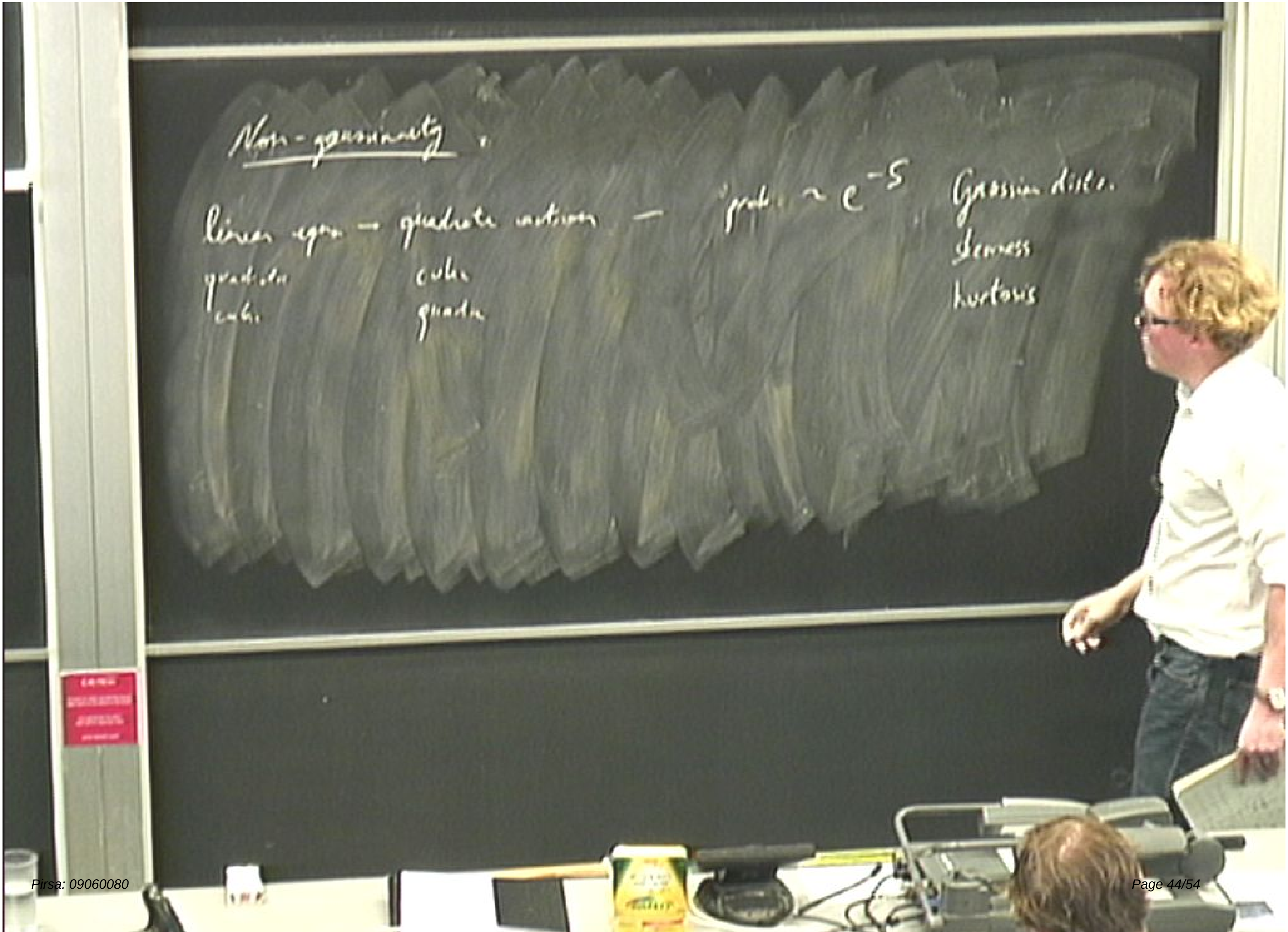
$$\frac{H}{\sigma} \sim \frac{-1}{\sqrt{\epsilon}}$$

$$\zeta \approx \frac{1}{\sqrt{\epsilon}} \delta s$$

GW – don't expect to see any


Non-gaussianity


Non-gaussianity :
linear eqn → quadratic action — prob. ~ e^-S Gaussian distr.
quadratic


Non-gaussianity :
linear eqn — quadratic action — prob ~ e^-S
Gaussian distn
skewness
kurtosis
quadratic
cubic
cubic
quartic

Non-gaussianity :

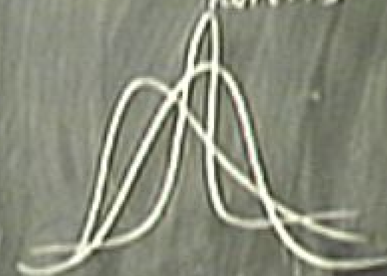
linear eqns → quadratic action — prob. $\sim e^{-S}$ Gaussian distn.

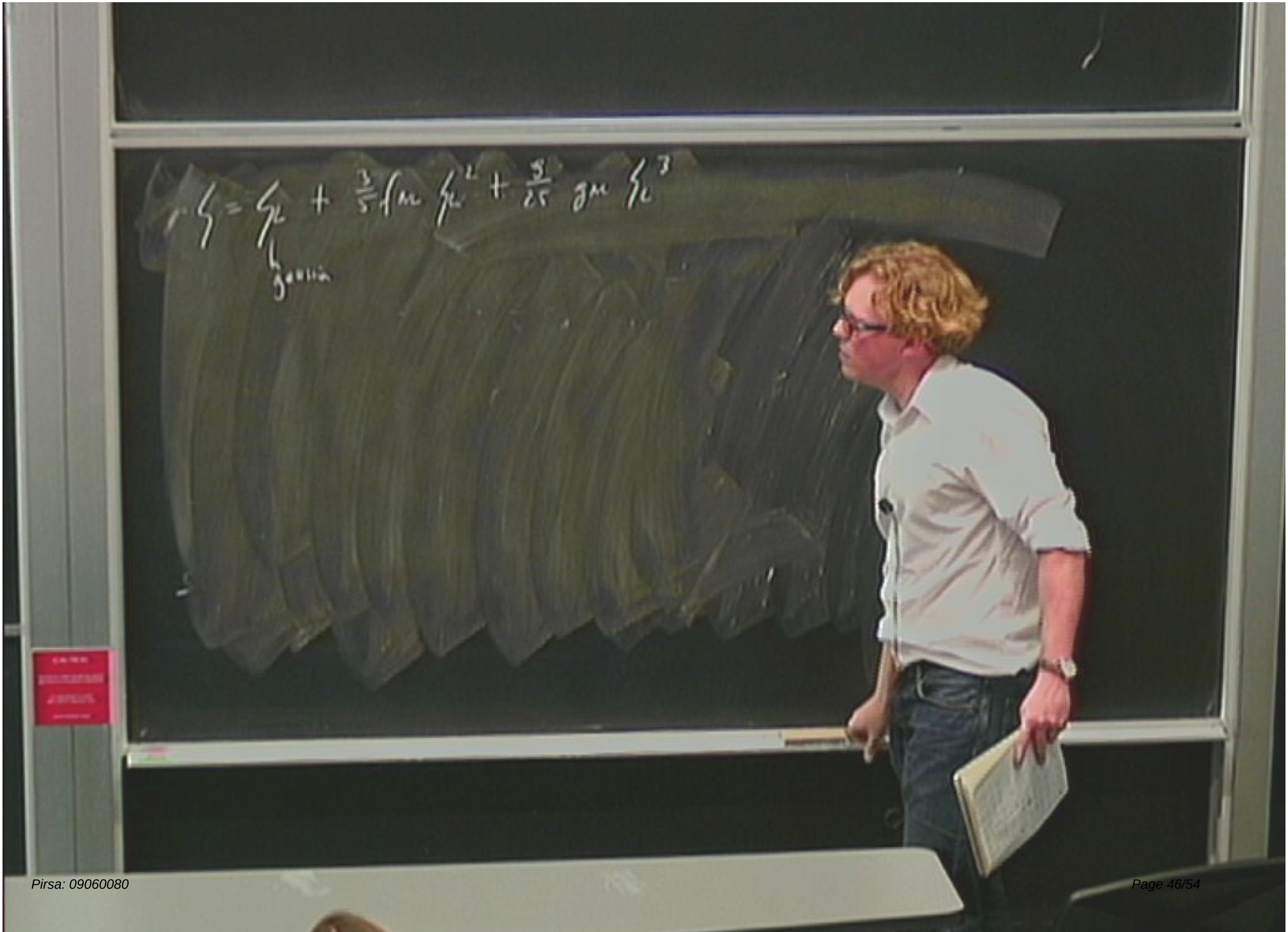
quadratic cubic

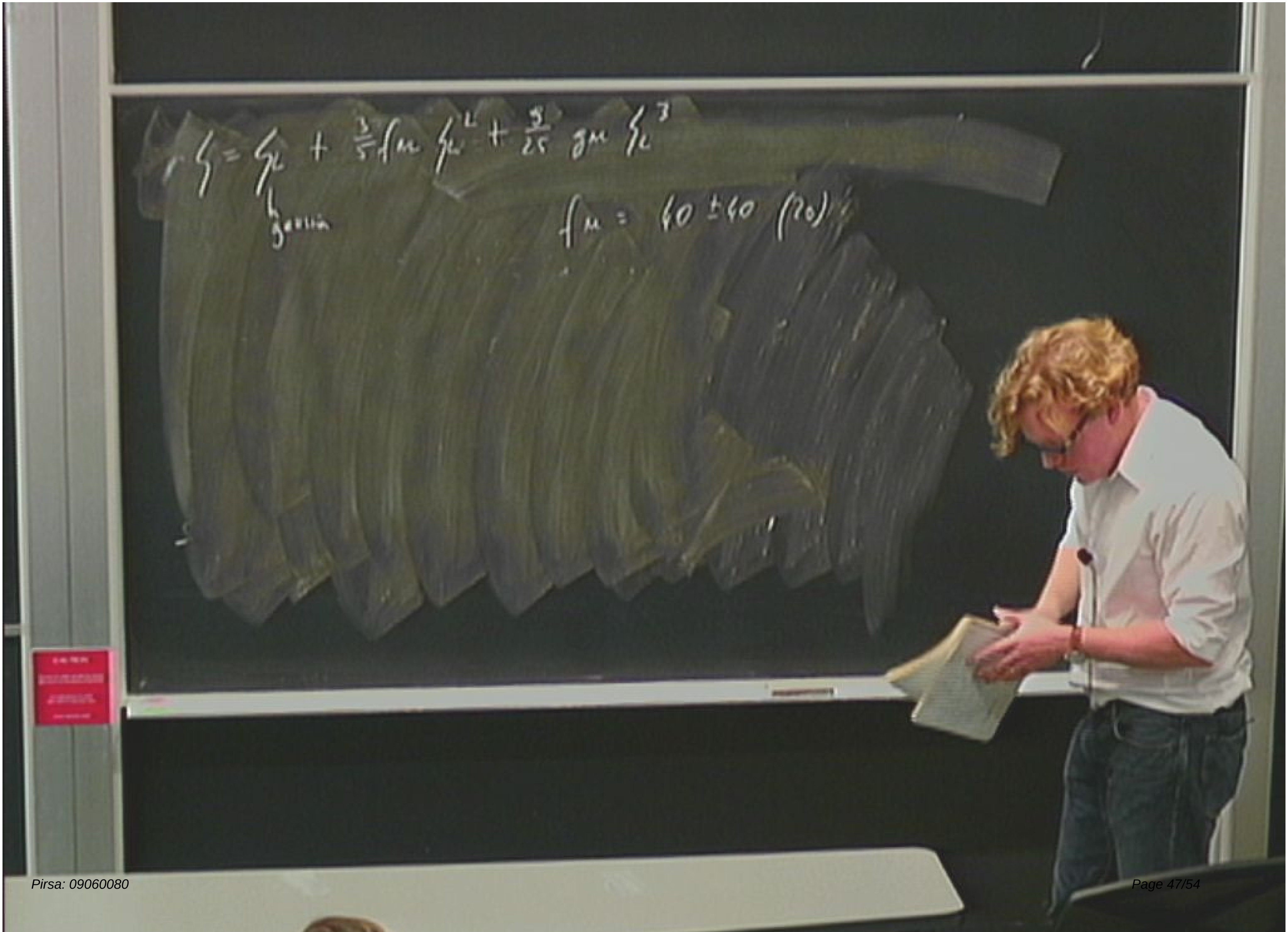
cubic quartic

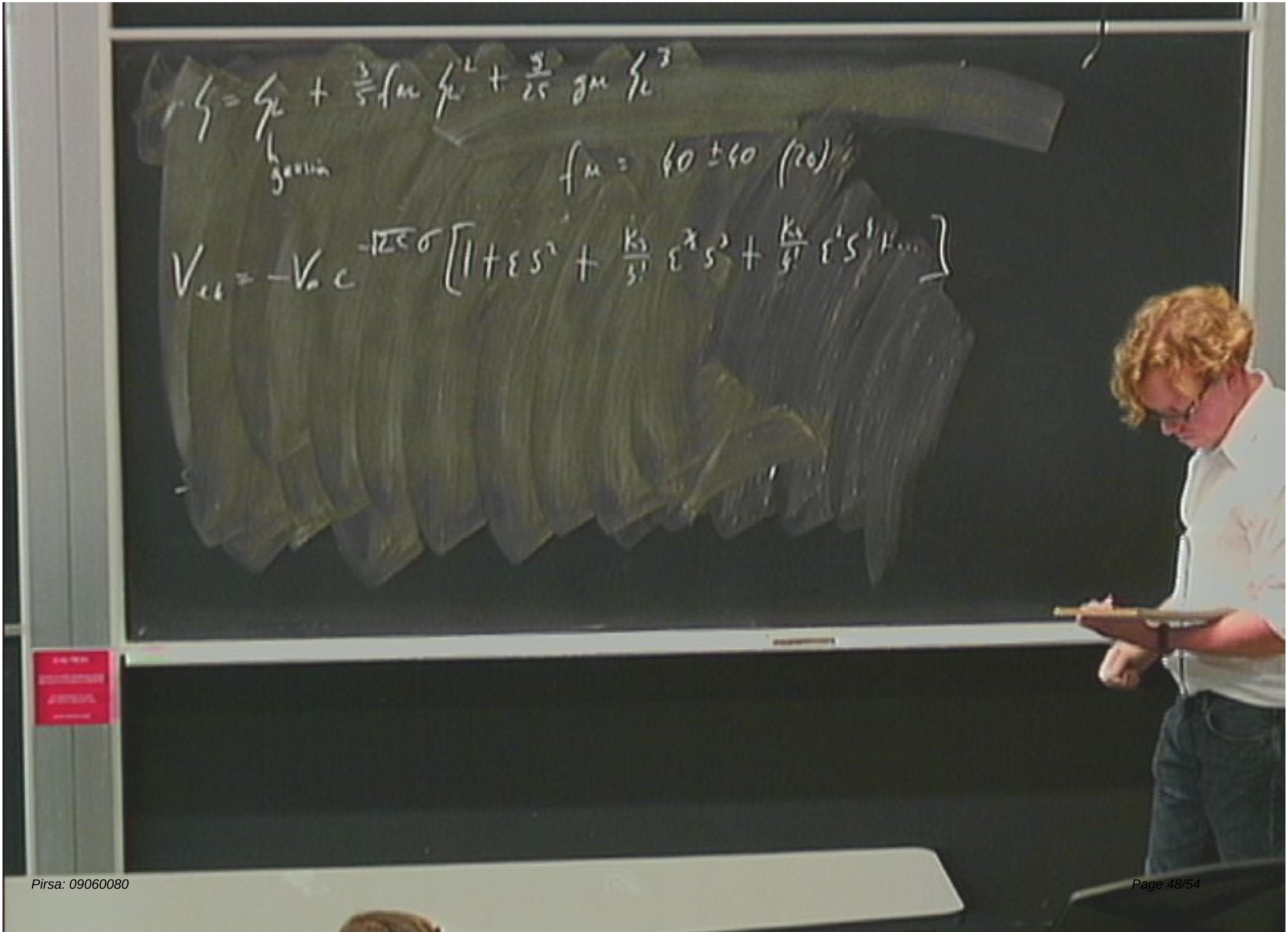
skewness

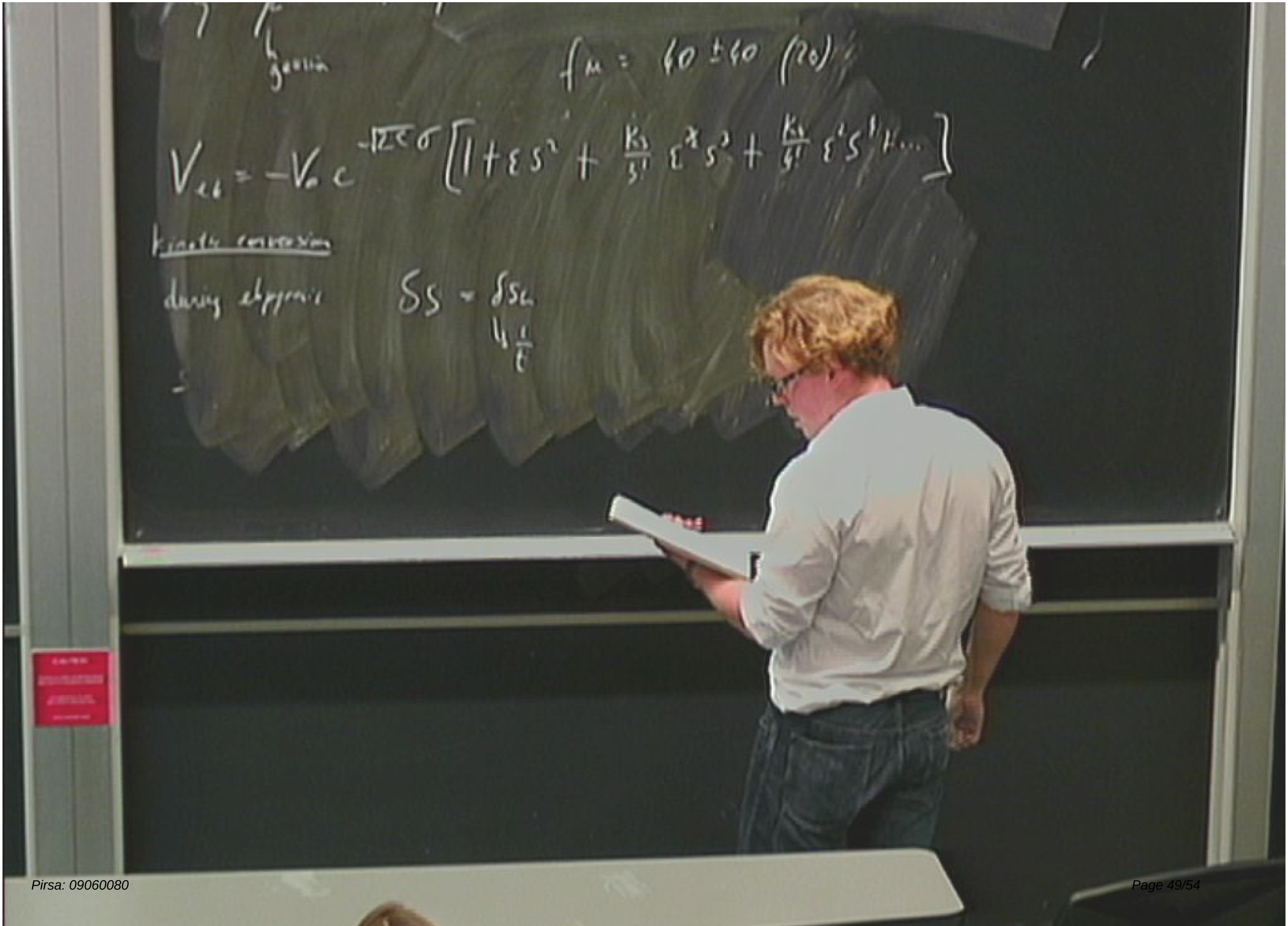
kurtosis

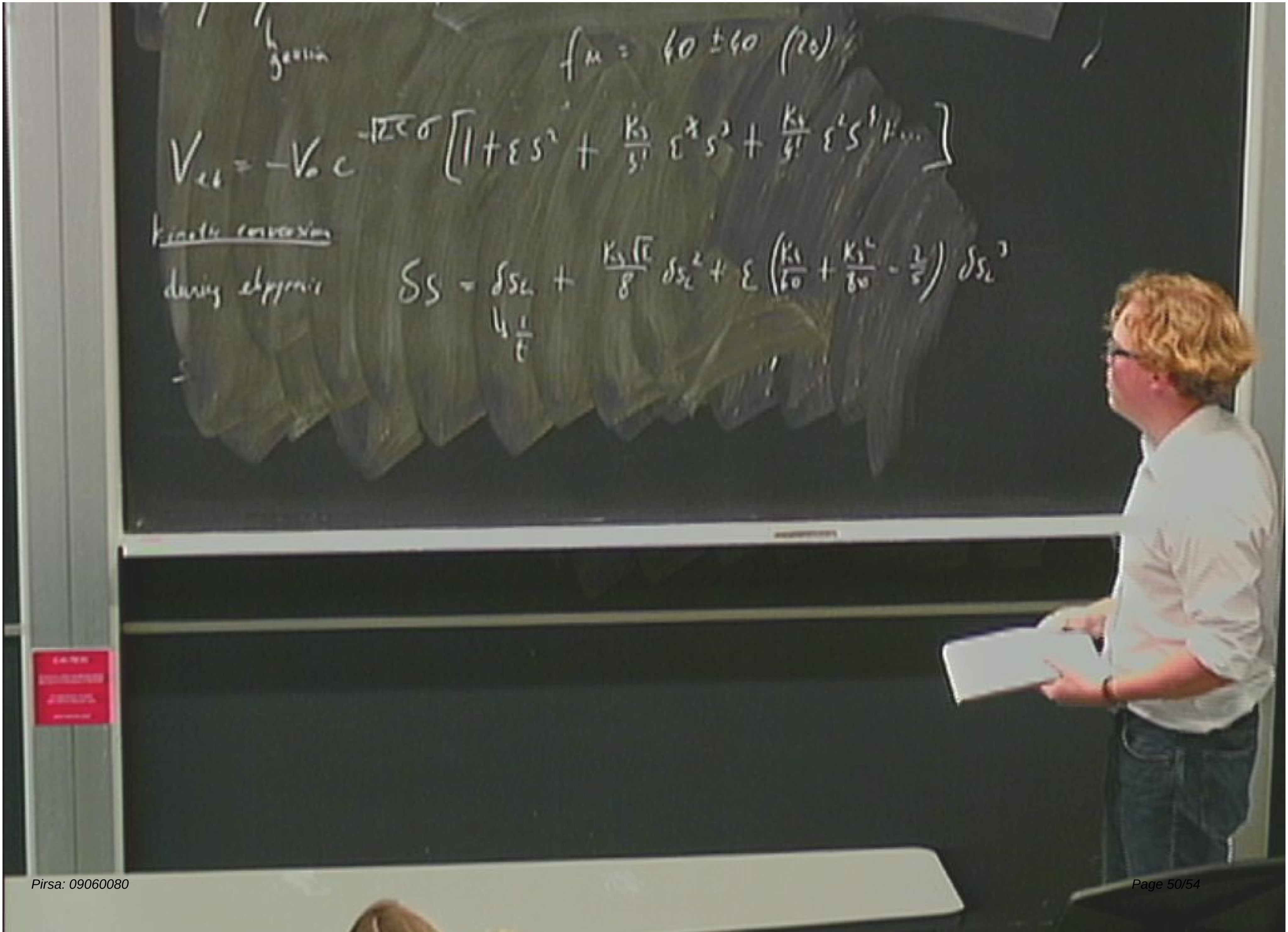


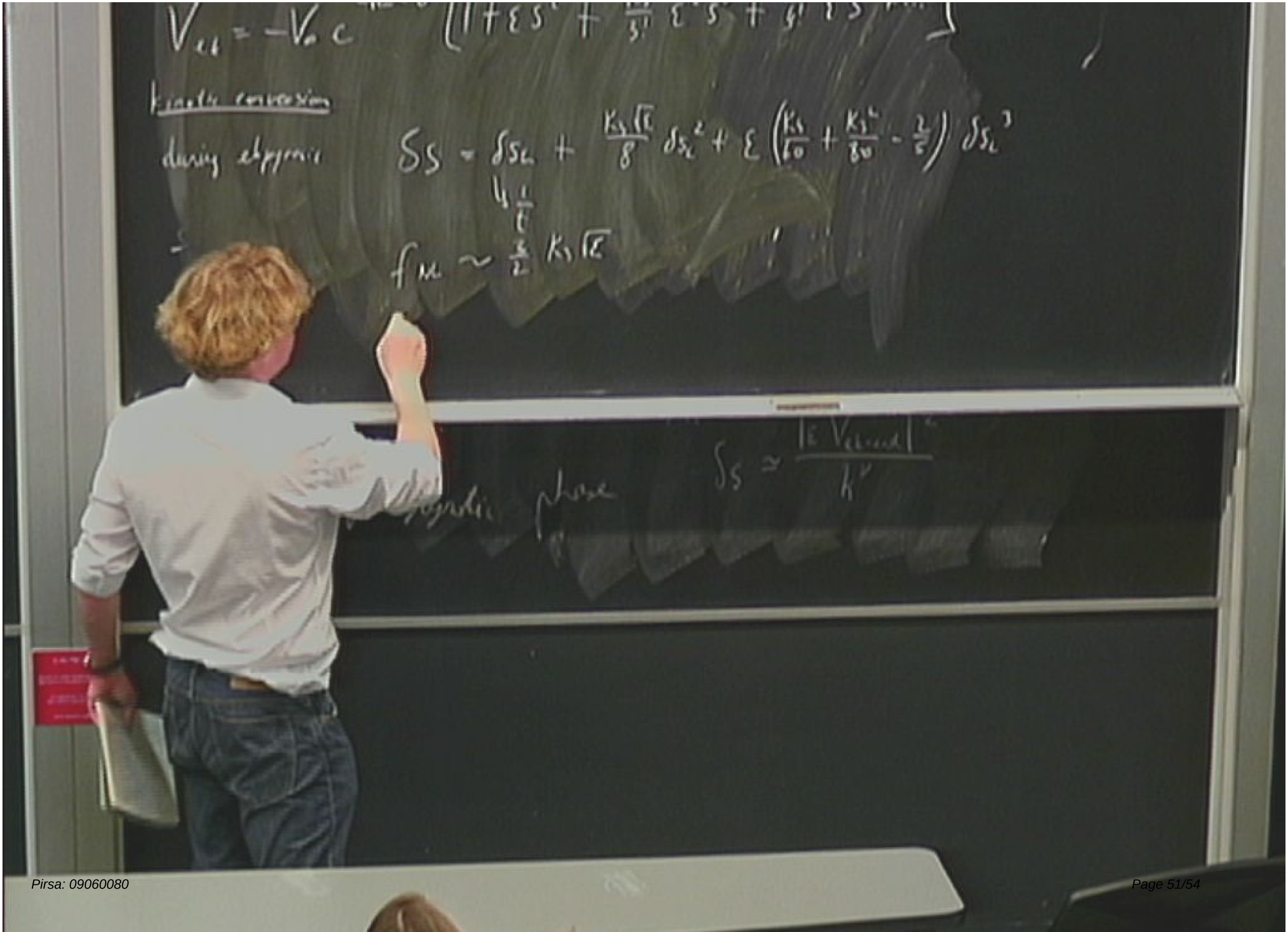

$\zeta = \zeta_g + \frac{3}{5} f_{NL} \zeta_g^2 + \frac{9}{25} g_{NL} \zeta_g^3$
gaussian

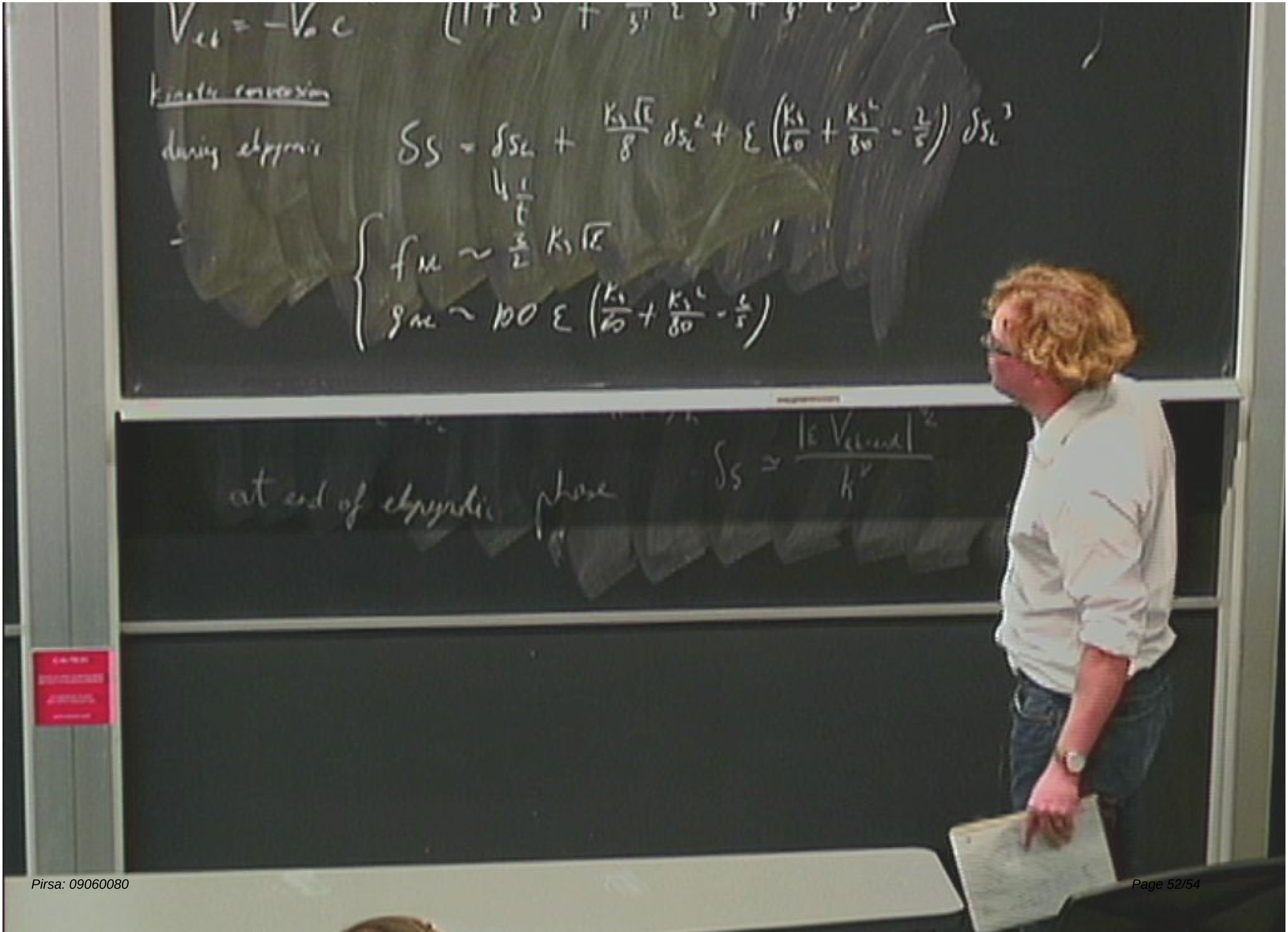

$\zeta = \zeta_g + \frac{3}{5} f_{NL} \zeta_g^2 + \frac{9}{25} g_{NL} \zeta_g^3$
gaussian
$f_{NL} = 40 \pm 40 \ (20)$

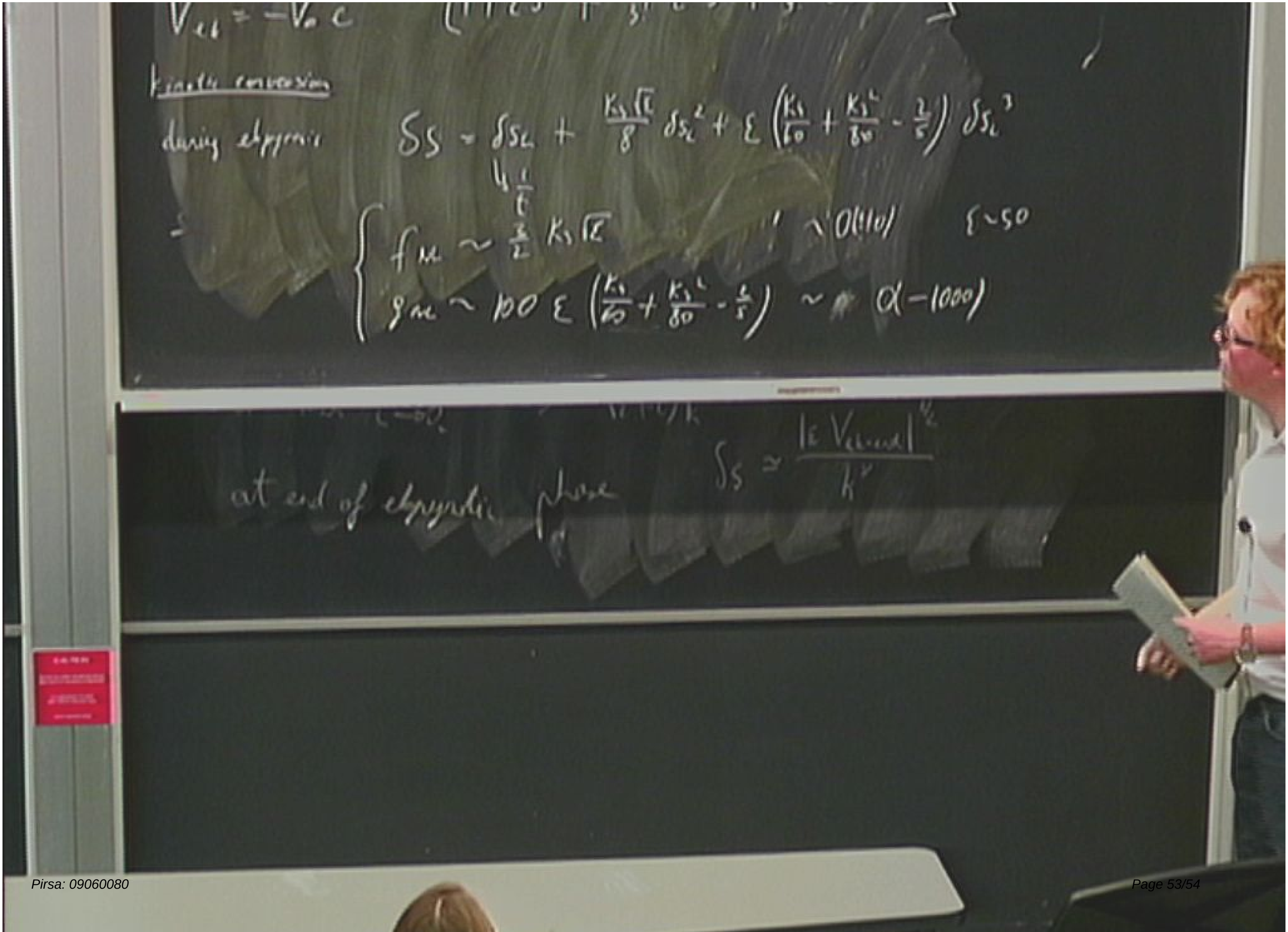

$\zeta = \zeta_L + \frac{3}{5} f_{NL} \zeta_L^2 + \frac{9}{25} g_{NL} \zeta_L^3$
gaussian
$f_{NL} = 60 \pm 40 \ (2\sigma)$
$V_{eff} = -V_0 e^{-\sqrt{2\varepsilon}\sigma}\left[1 + \varepsilon \sigma^2 + \frac{k_3}{3!}\varepsilon^{3/2}\sigma^3 + \frac{k_4}{4!}\varepsilon^2\sigma^4 + \dots\right]$

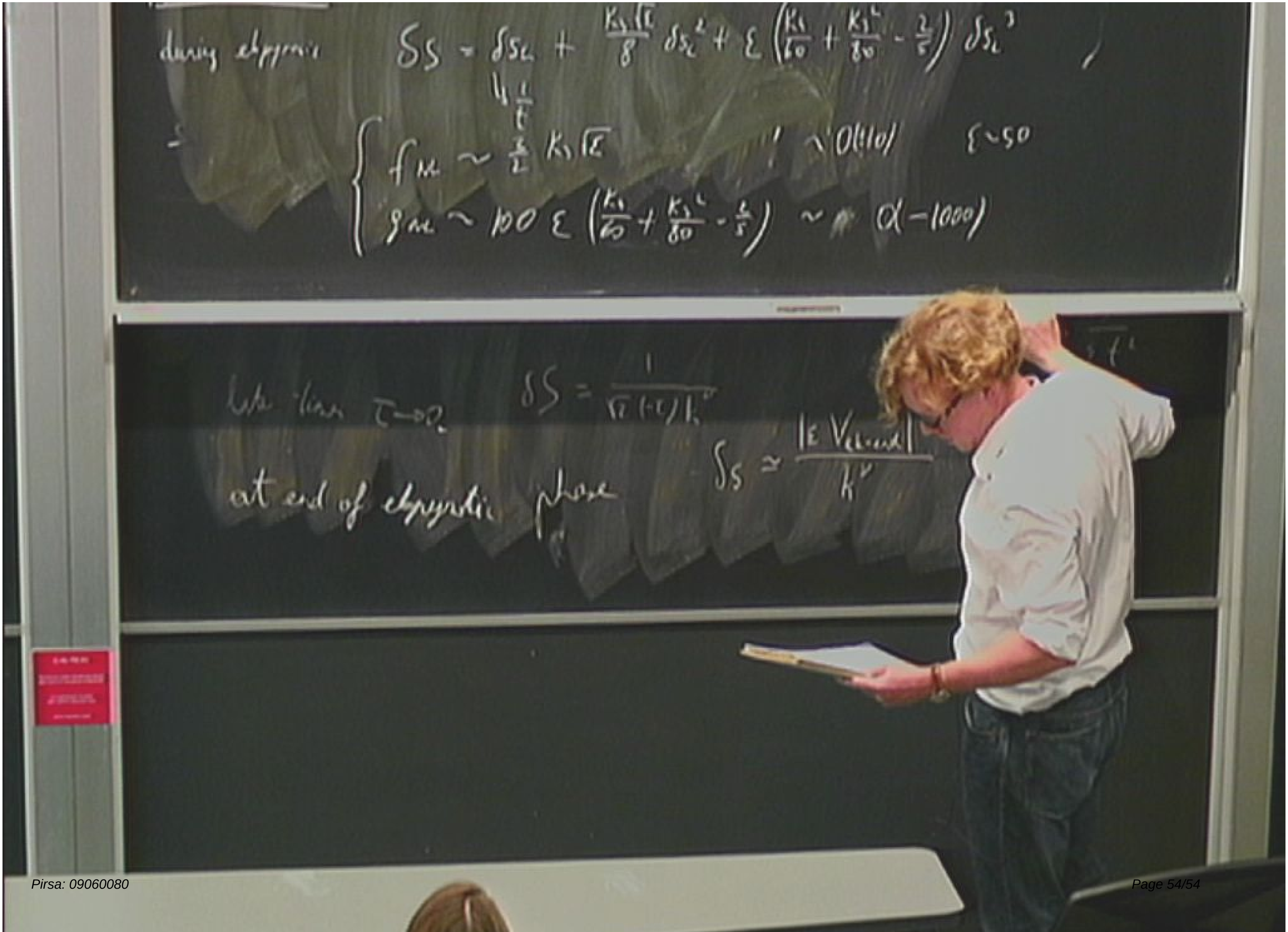

kinetic conversion


gaussian
$f_{NL} = 60 \pm 40\ (20)$
$V_{ek} = -V_0 e^{-\sqrt{2\epsilon}\sigma}\left[1 + \epsilon s^2 + \frac{K_3}{3!}\epsilon^{3/2} s^3 + \frac{K_4}{4!}\epsilon^2 s^4 + \dots\right]$
kinetic conversion
during ekpyrosis
$\delta s = \delta s_L + \frac{K_3\sqrt{\epsilon}}{8}\delta s_L^2 + \epsilon\left(\frac{K_4}{60} + \frac{K_3^2}{80} - \frac{2}{5}\right)\delta s_L^3$
$\propto \frac{1}{t}$


kinetic conversion
during ekpyrosis
phase


kinetic conversion
during ekpyrosis
at end of ekpyrotic phase


kinetic conversion
during ekpyrosis
at end of ekpyrotic phase


during ekpyrosis
f_NL ~ 3/2 κ_3 √ε
O(±10)
ε ~ 50
g_NL ~ 100 ε (κ_4/60 + κ_3²/80 − 2/5) ~ O(−1000)
at end of ekpyrotic phase