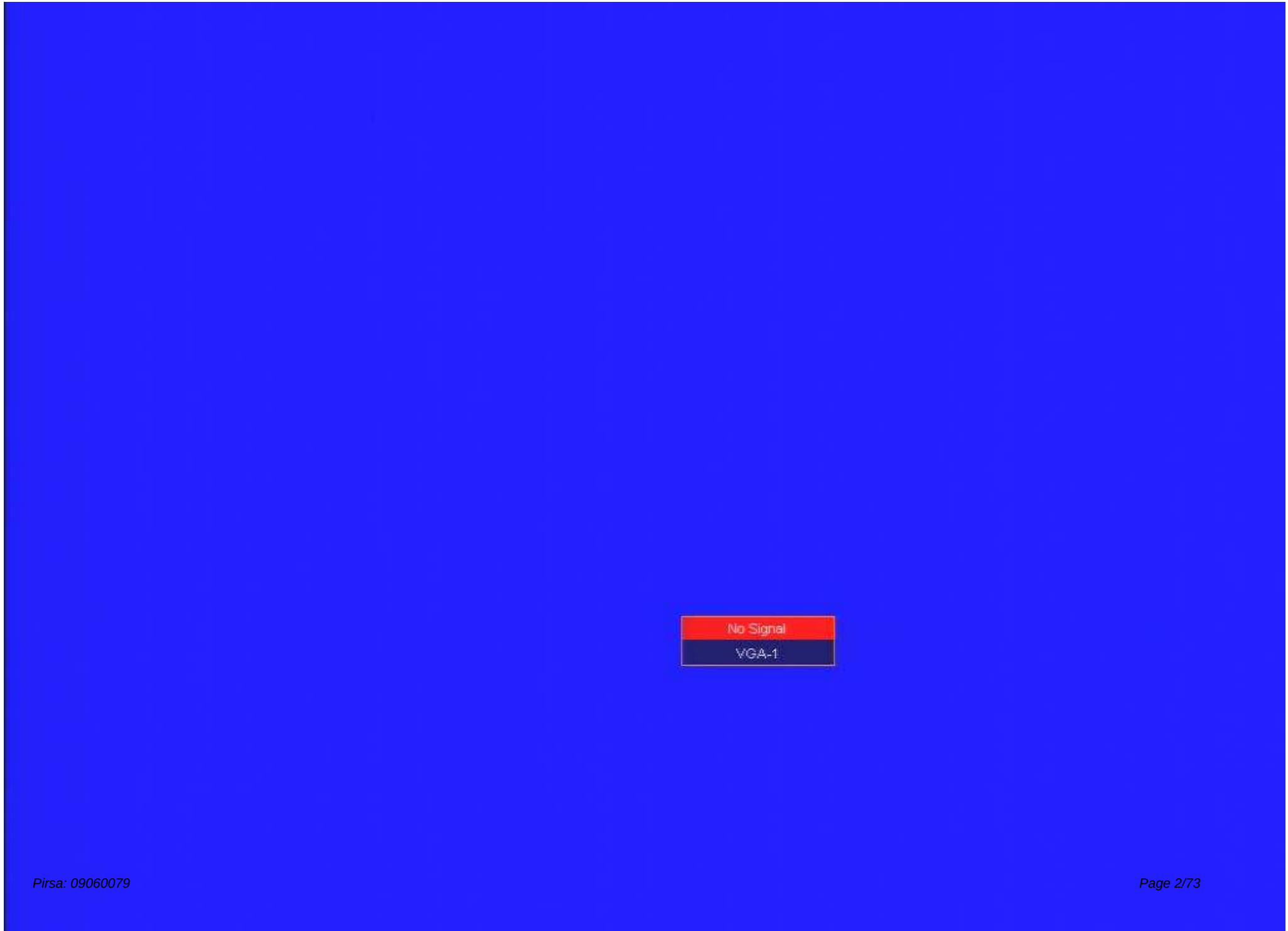
Title: Ekpyrotic/Cyclic Cosmology - Lecture 2

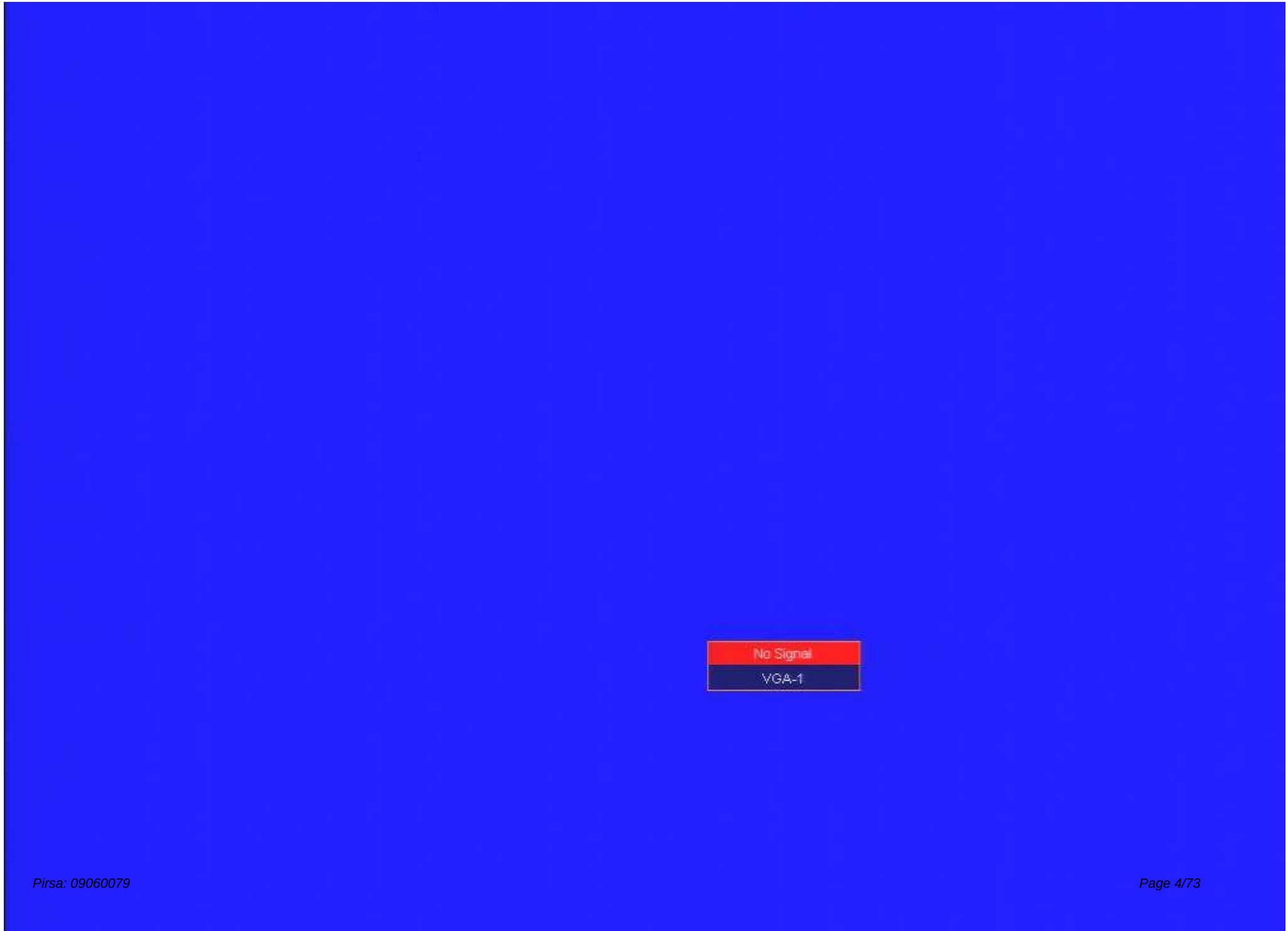
Date: Jun 29, 2009 09:00 AM

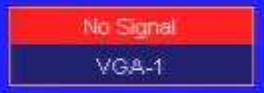
URL: http://pirsa.org/09060079

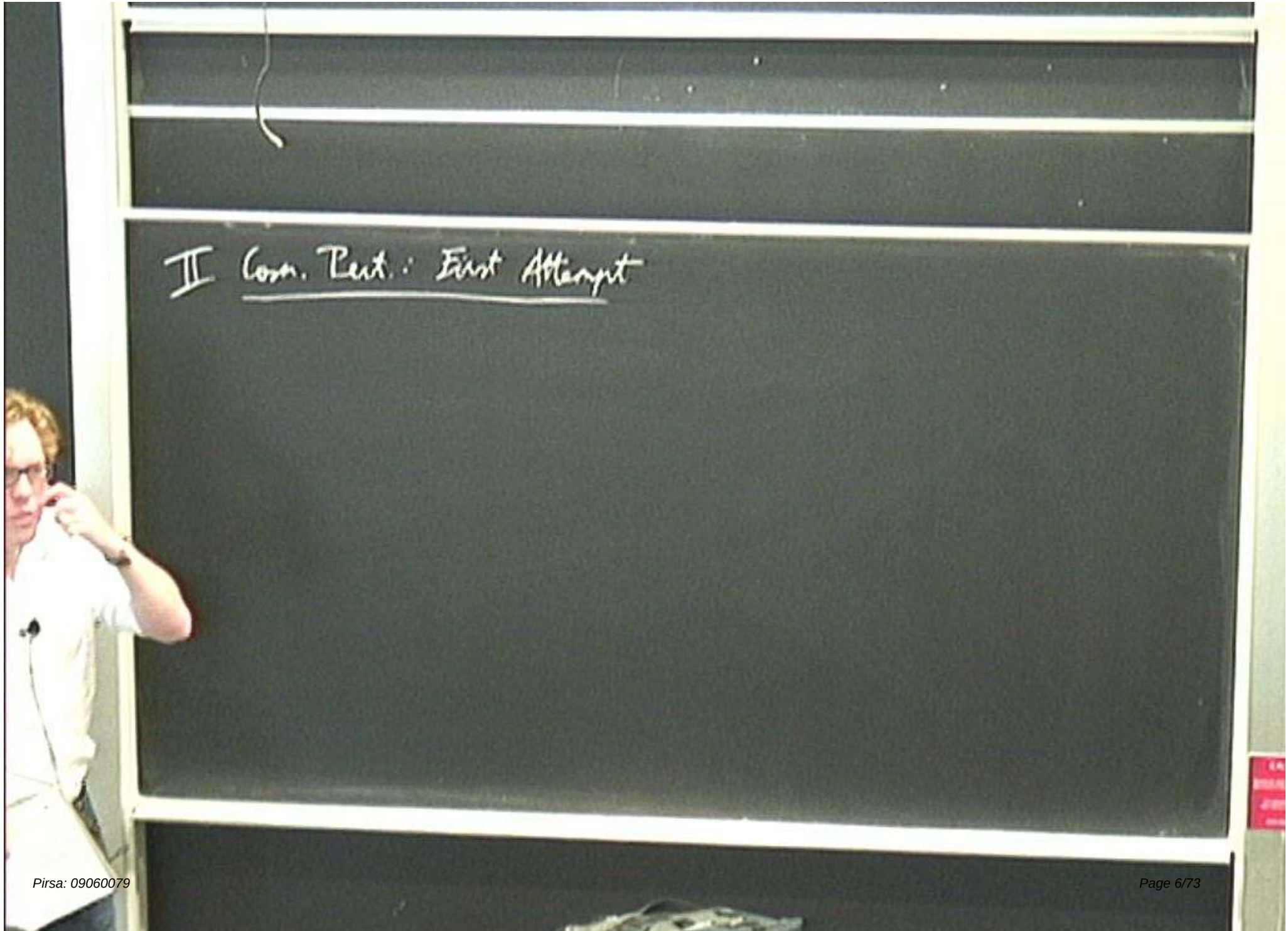
Abstract: TBA


No Signal
VGA-1

No Signal
VGA-1

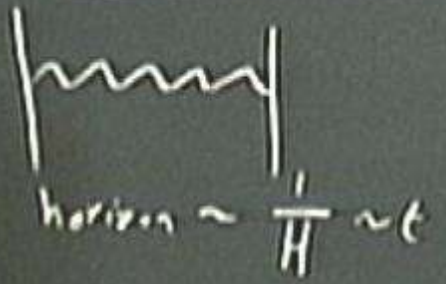

No Signal
VGA-1


No Signal
VGA-1

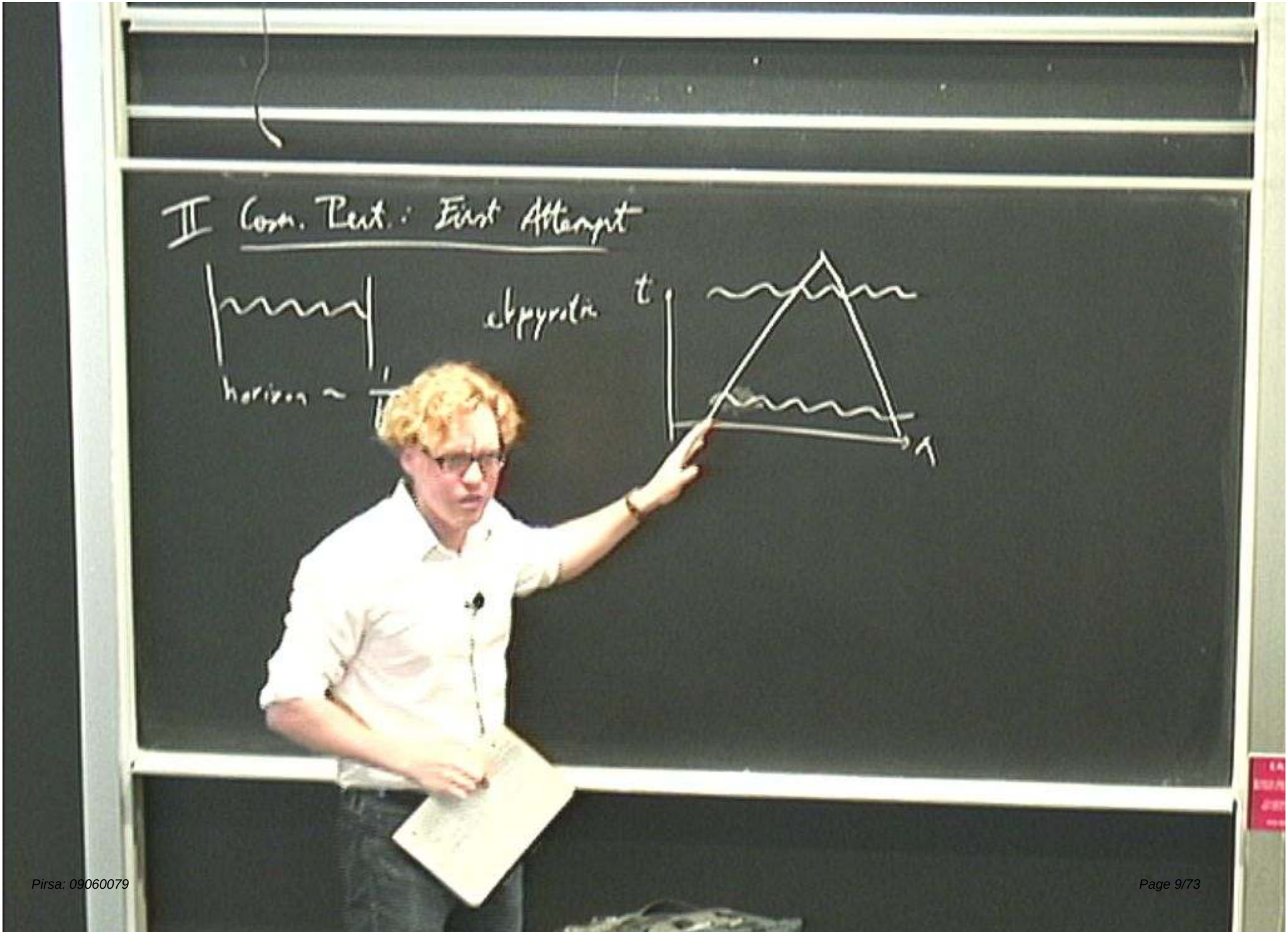

II Com. Pert.: First Attempt

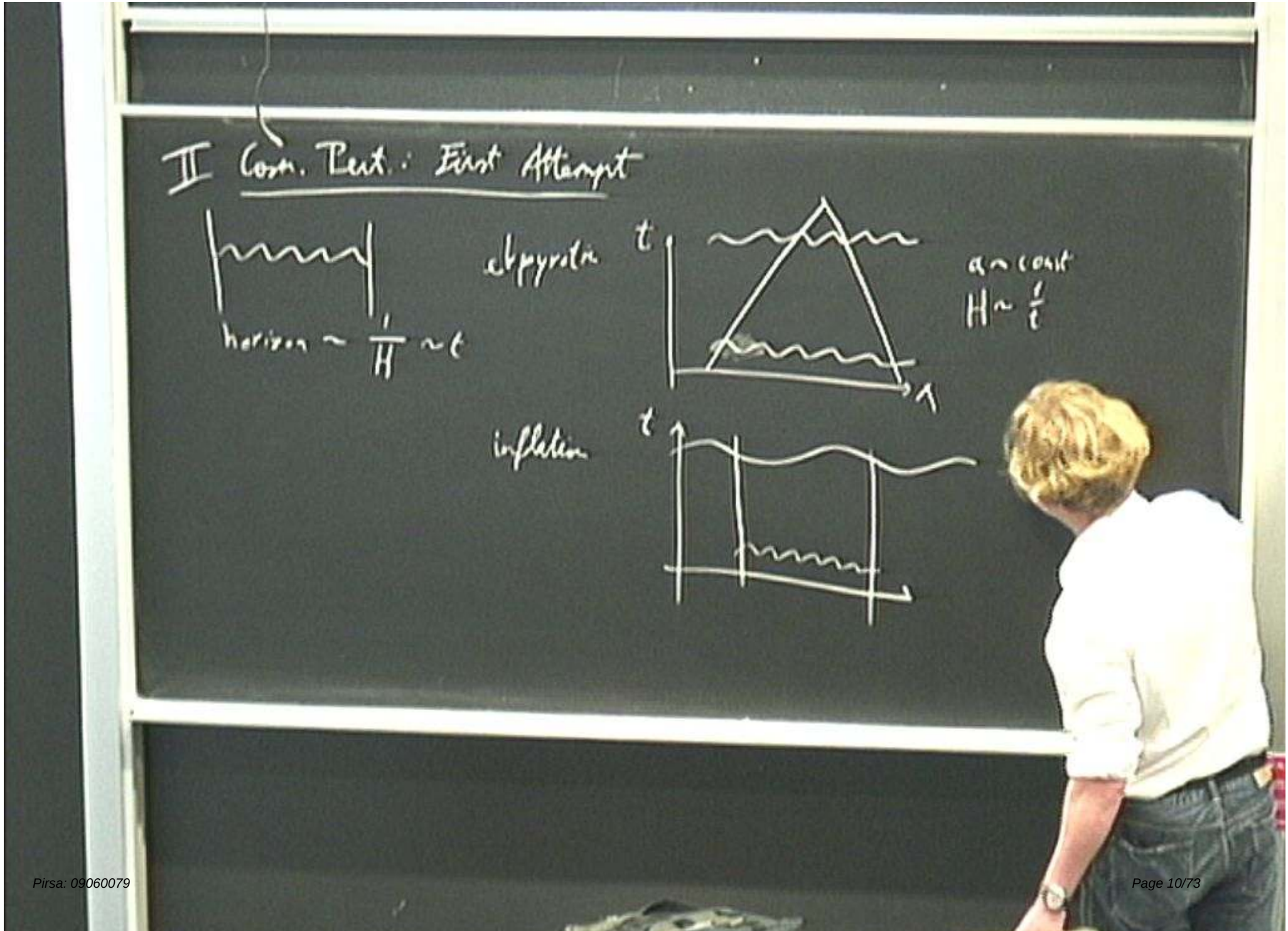
II Com. Test: First Attempt

II Cosm. Test.: First Attempt

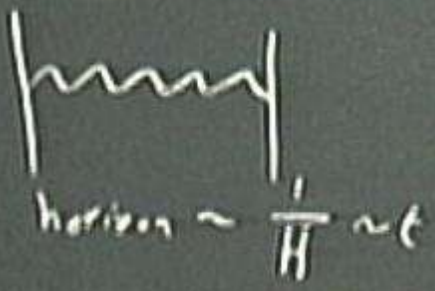


ekpyrotic

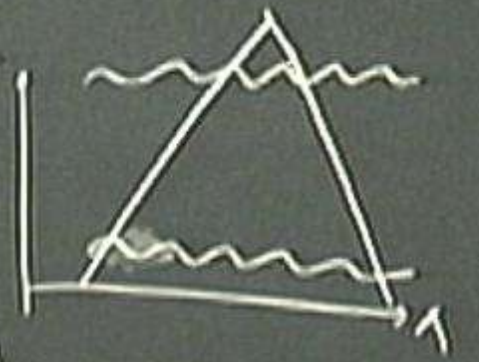

II Cosm. Test.: First Attempt
horizon ~
t


II Cosm. Test.: First Attempt
horizon ~ 1/H ~ t
adiabatic
t
a ~ const
H ~ 1/t
inflation
t

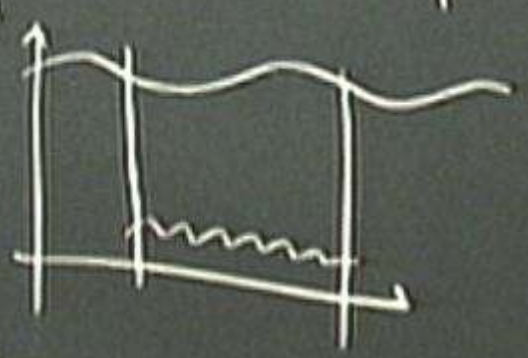
II Cosm. Test.: First Attempt

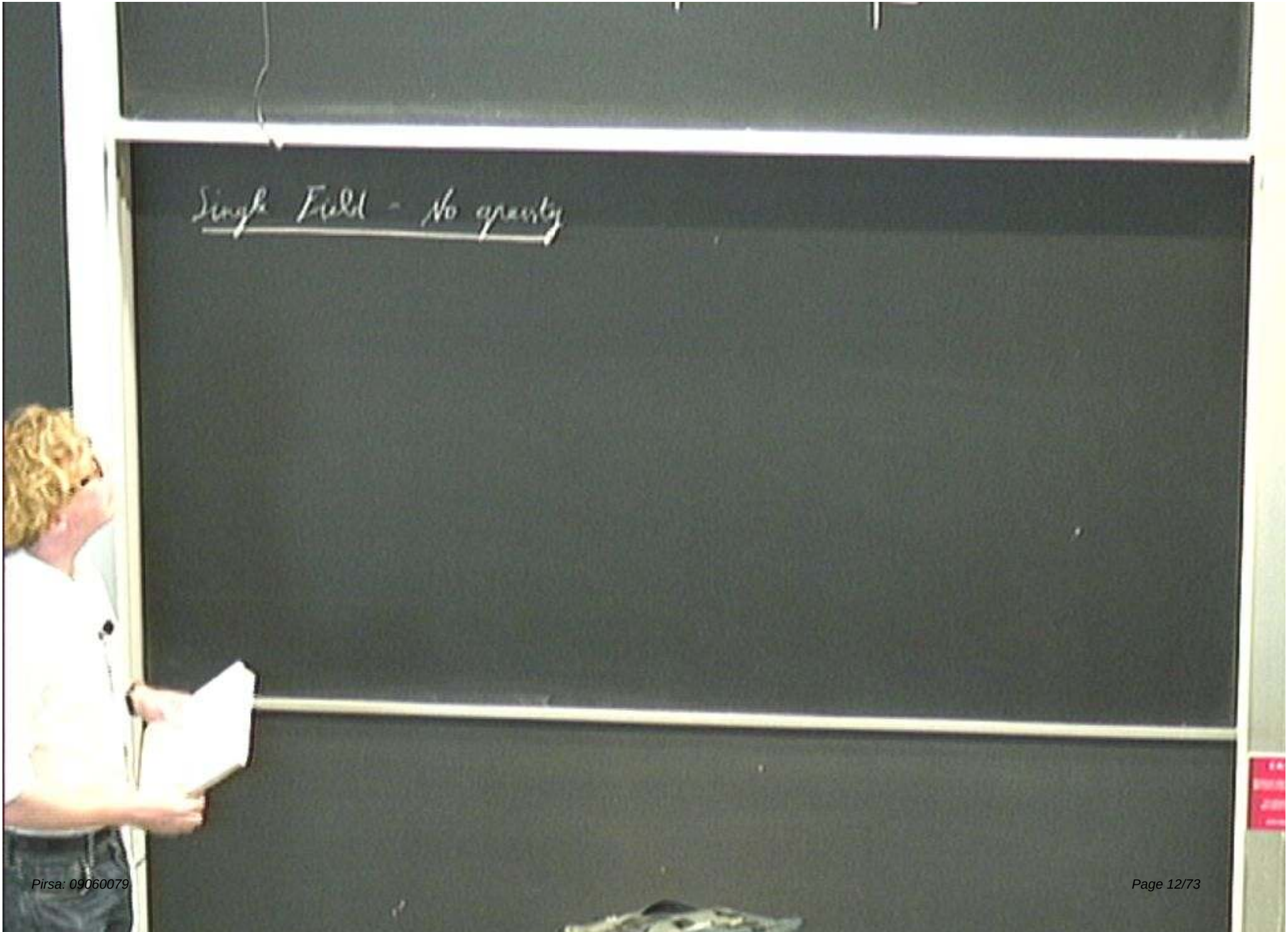


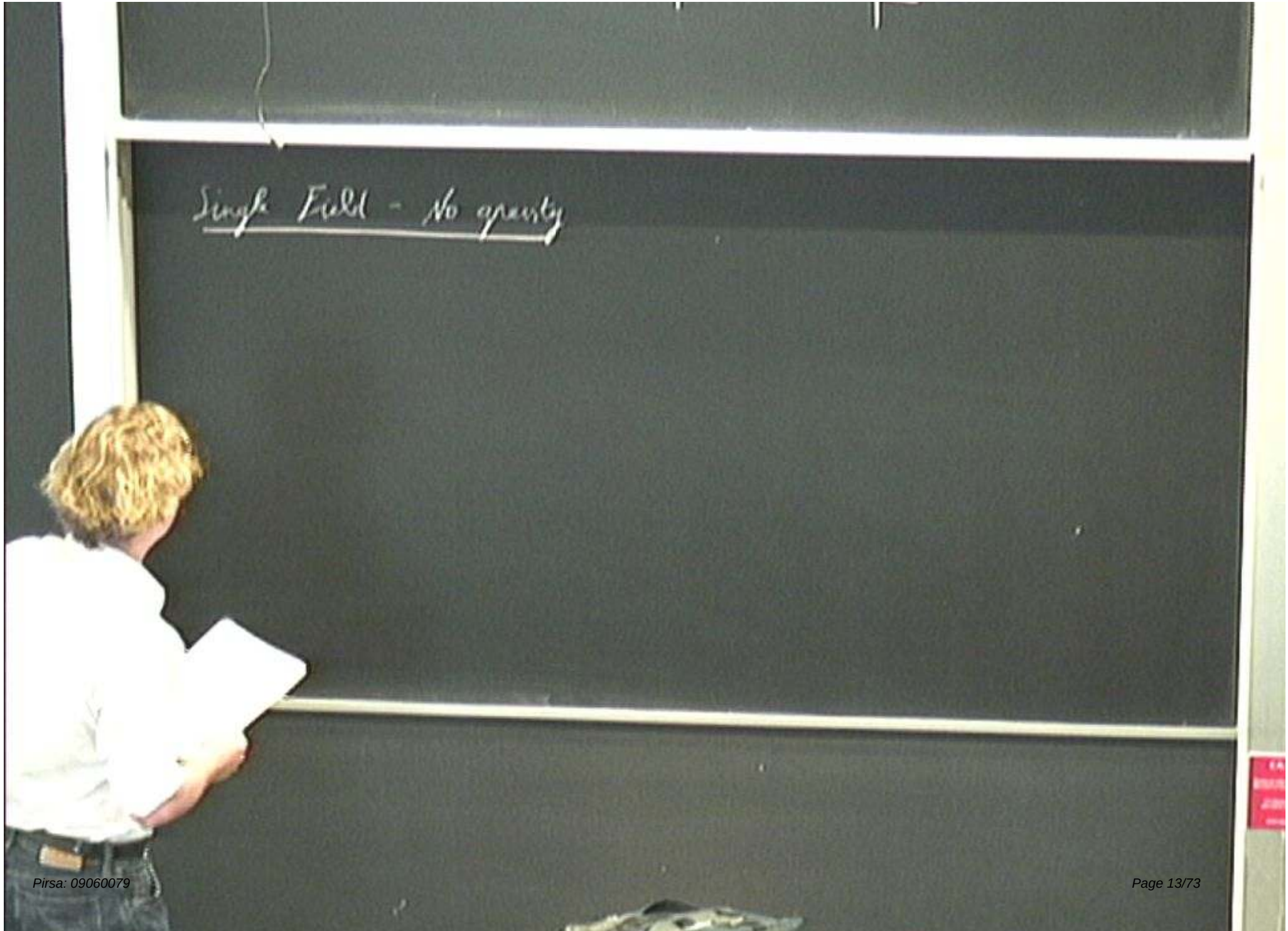
adiabatic



inflation




Single Field - No gravity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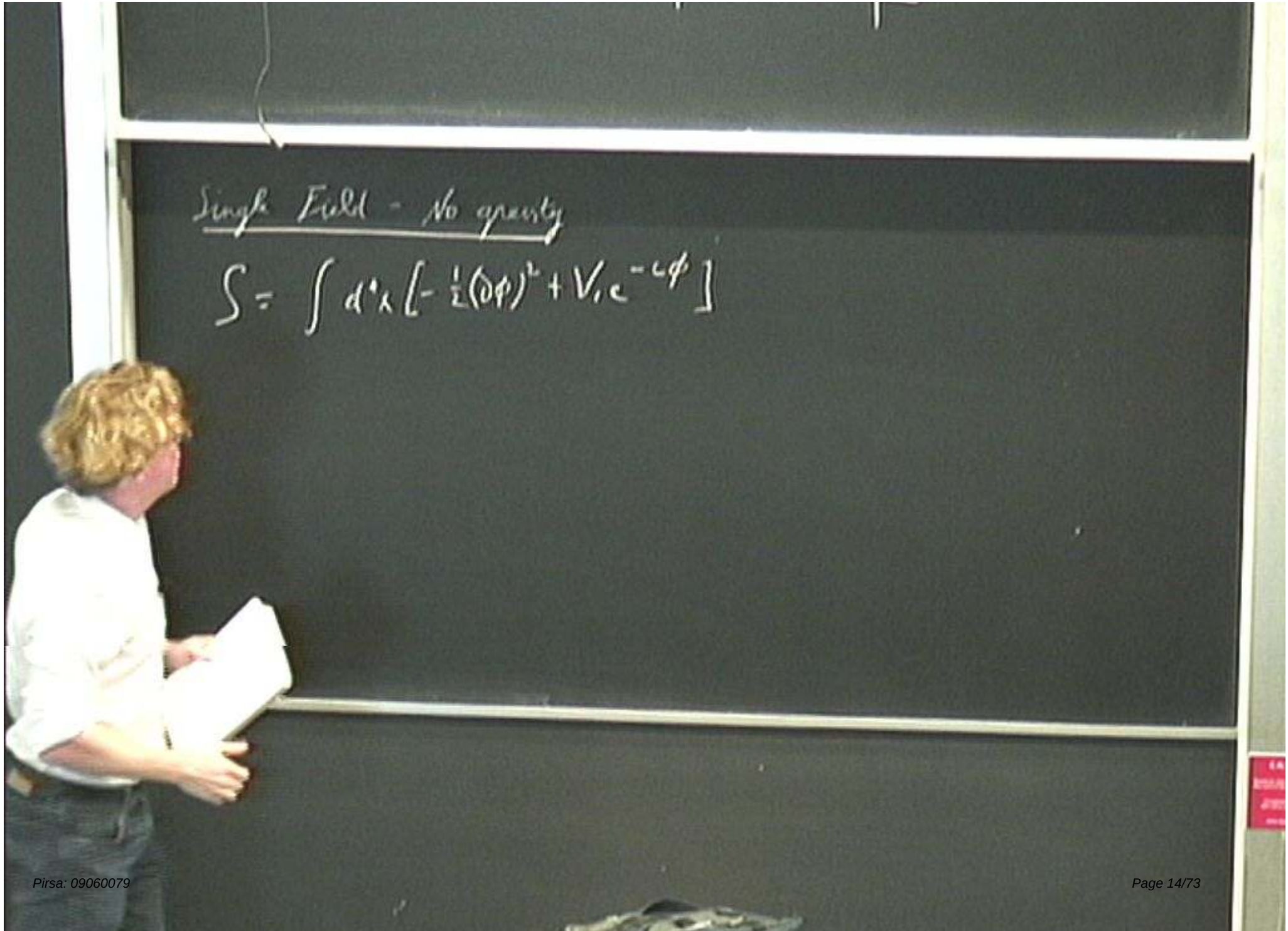
Single Field - No gravity
$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 + V_{inf}(\phi) \right]$

Single Field - No gravity

$$S = \int d^4x \left[-\tfrac{1}{2}(\partial\phi)^2 + V_1 e^{-c\phi} \right]$$

$$\phi = \bar{\phi}(t) + \delta\phi(t, \vec{x})$$

Single Field - No gravity

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$$\ddot{\delta\phi} - \nabla^2\delta\phi + V_{1\phi\phi}\,\delta\phi = 0$$

Single Field - No gravity

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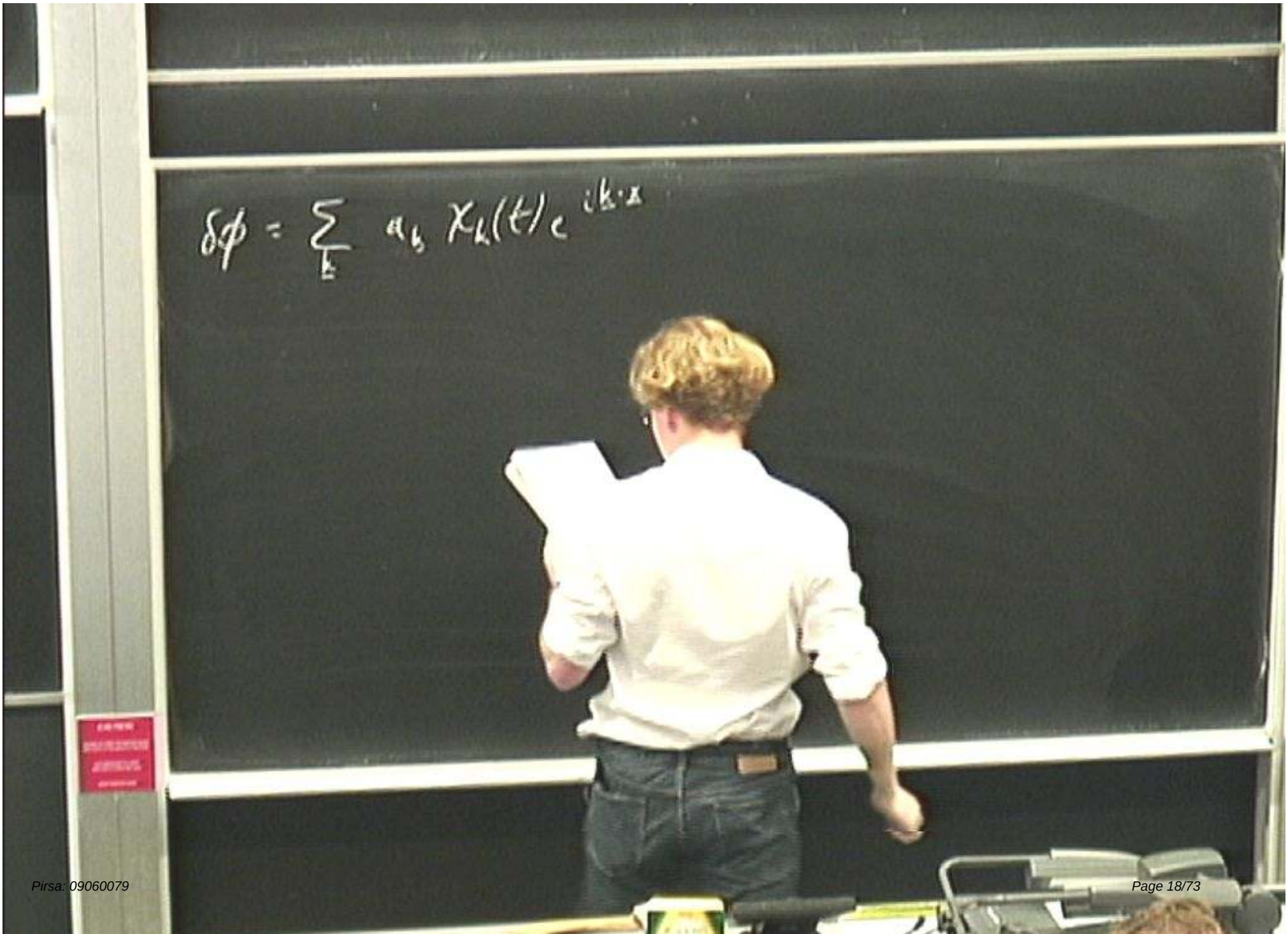
$$\phi = \bar{\phi}(t) + \delta\phi(t, \vec{x})$$

$$\ddot{\delta\phi} - \nabla^2\delta\phi + V_{,\phi\phi}\,\delta\phi = 0$$

$$\hookrightarrow V_{,\phi\phi} = c^2 V = -\frac{2}{t^2}$$

$$V = -\frac{1}{\epsilon t^2}$$


$\delta\phi = \sum_k a_k \chi_k(t) e^{i k \cdot x}$

$$\delta\phi = \sum_{\vec{k}} a_{\vec{k}} \chi_k(t) e^{i\vec{k}\cdot\vec{x}} + h.c.$$

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↳ annihilation operator $\quad a_{\vec{k}}|0\rangle = 0$

$$\delta\phi = \sum_{\vec{k}} a_{\vec{k}}\, \chi_k(t)\, e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}$$

↳ positive frequency mode function

↳ annihilation operator $\quad a_{\vec{k}}|0\rangle = 0$

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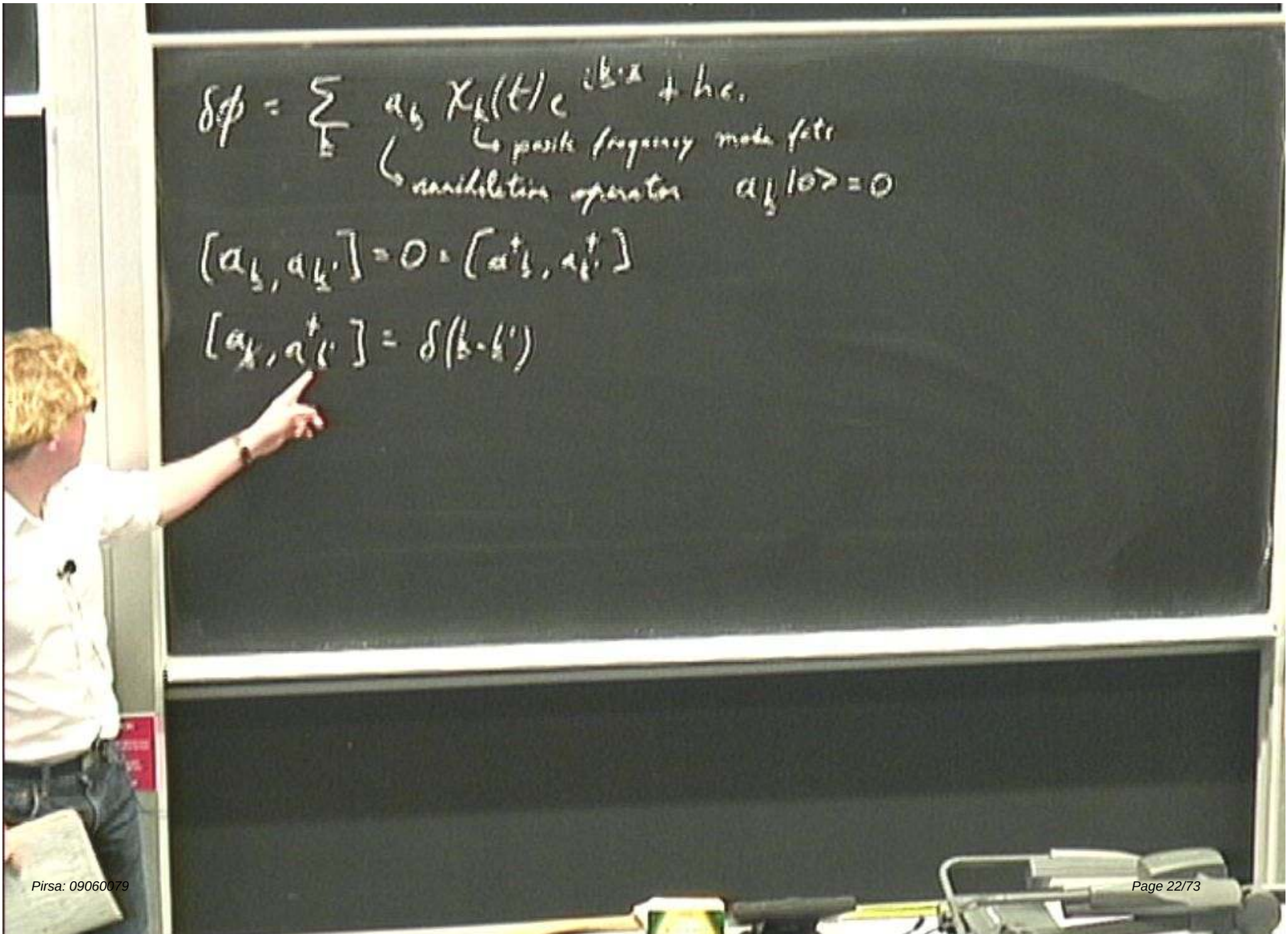
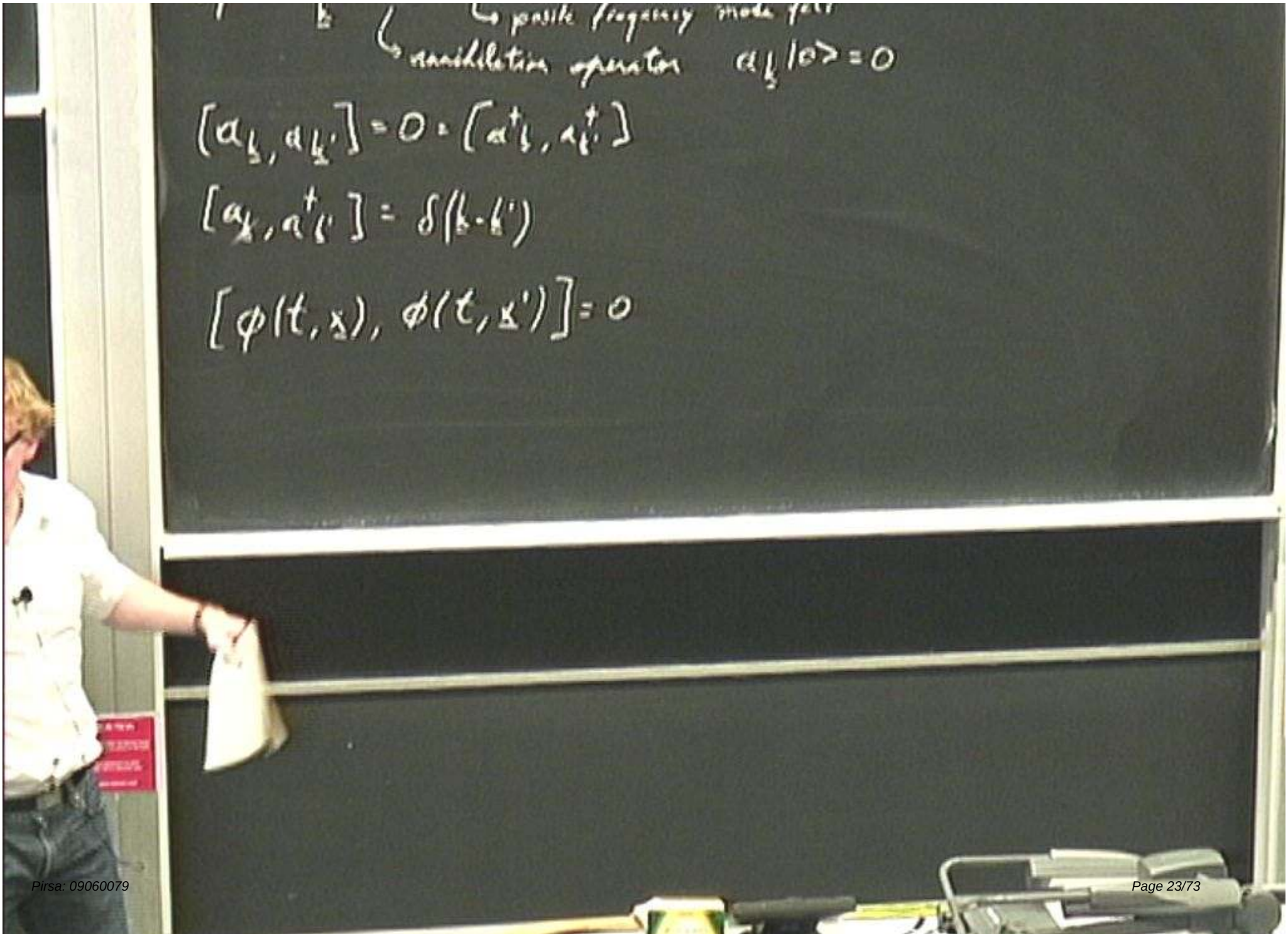
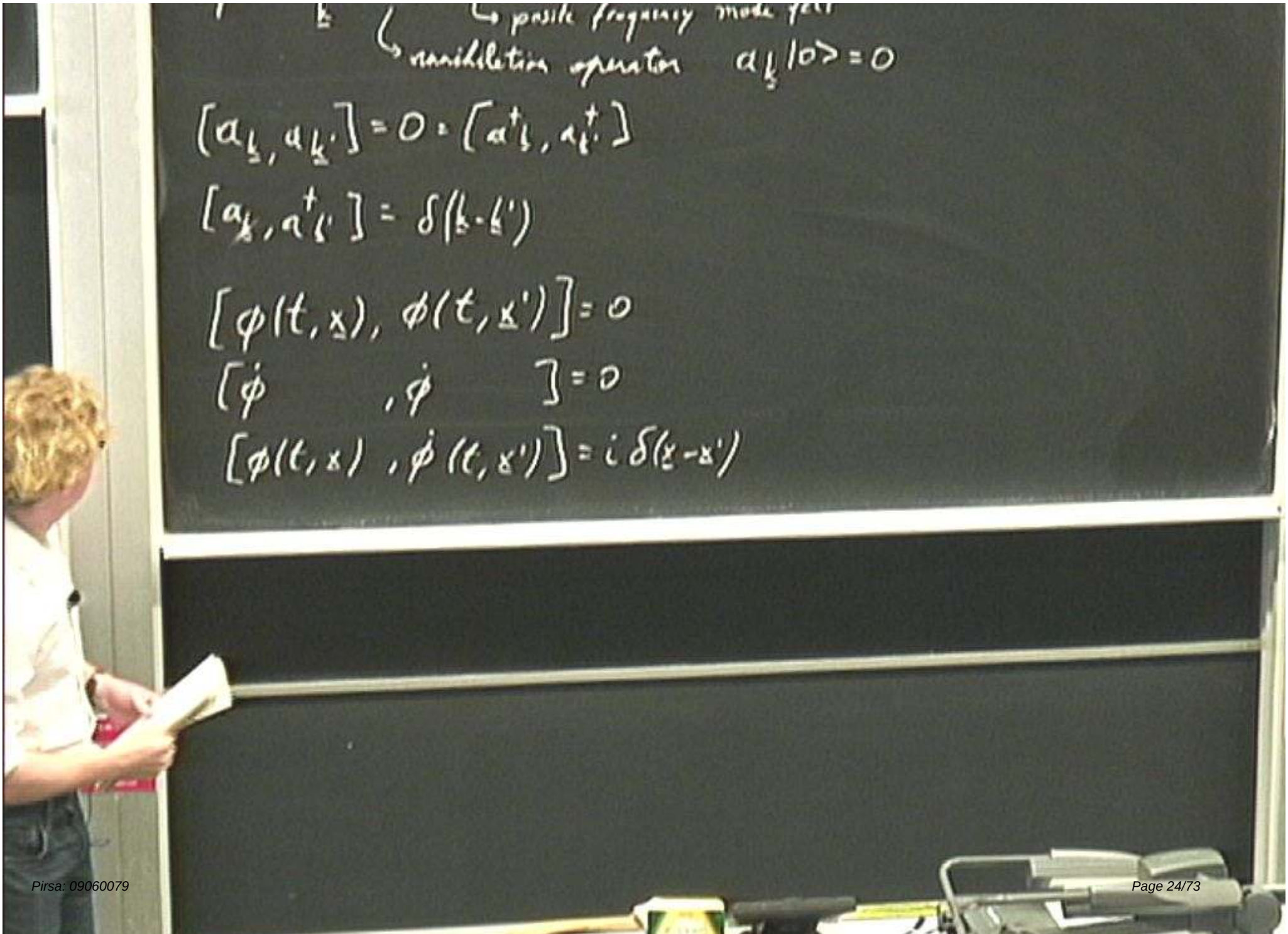

$\delta\phi = \sum_k a_k \chi_k(t) e^{ik\cdot x} + h.c.$
↳ positive frequency mode function
↳ annihilation operator $a_k|0\rangle = 0$
$[a_k, a_{k'}] = 0 = [a^\dagger_k, a^\dagger_{k'}]$
$[a_k, a^\dagger_{k'}] = \delta(k-k')$


↳ positive frequency mode part
↳ annihilation operator $a_k|0\rangle = 0$
$[a_k, a_{k'}] = 0 = [a^\dagger_k, a^\dagger_{k'}]$
$[a_k, a^\dagger_{k'}] = \delta(k-k')$
$[\varphi(t,x), \phi(t,x')] = 0$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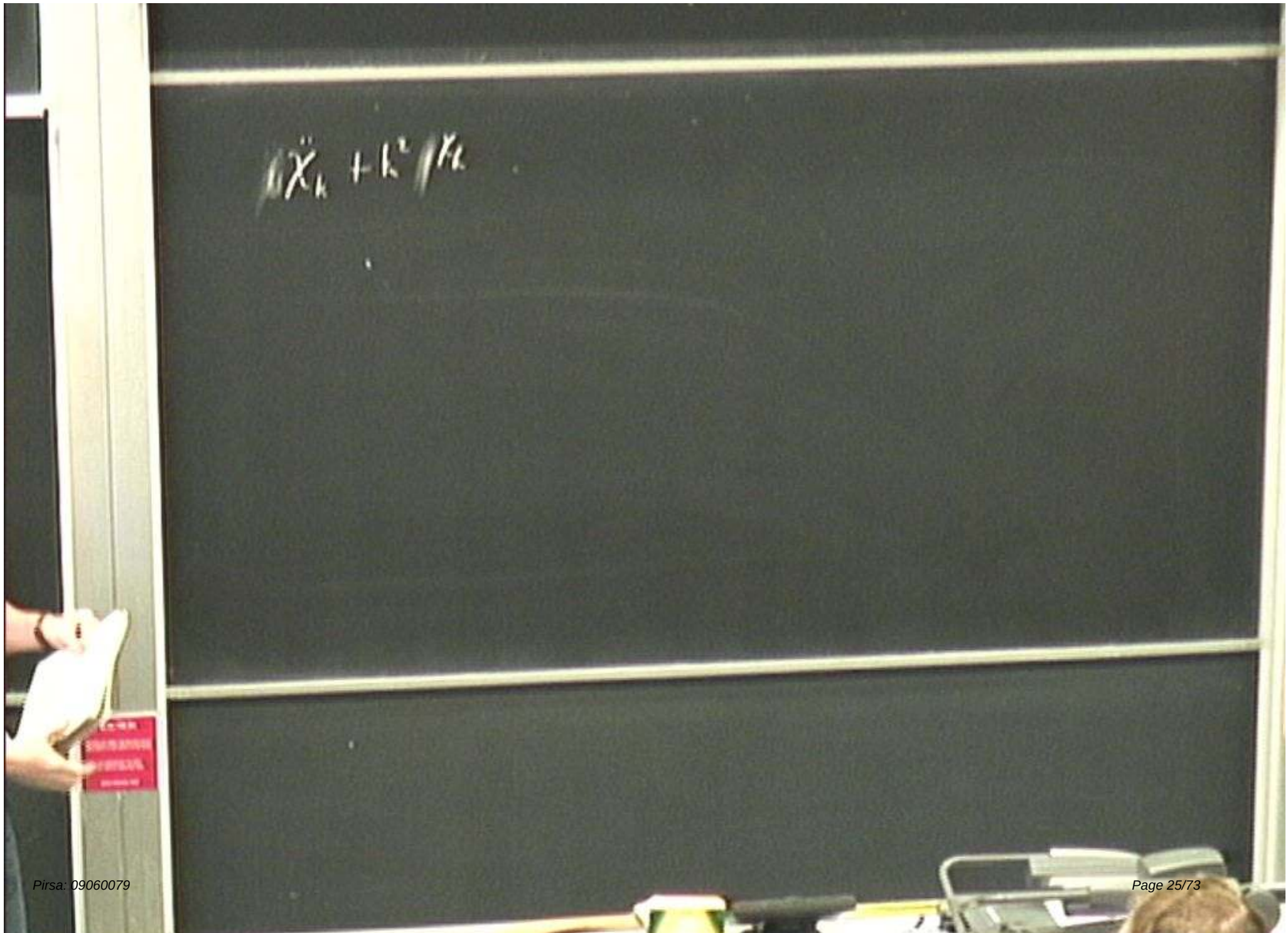
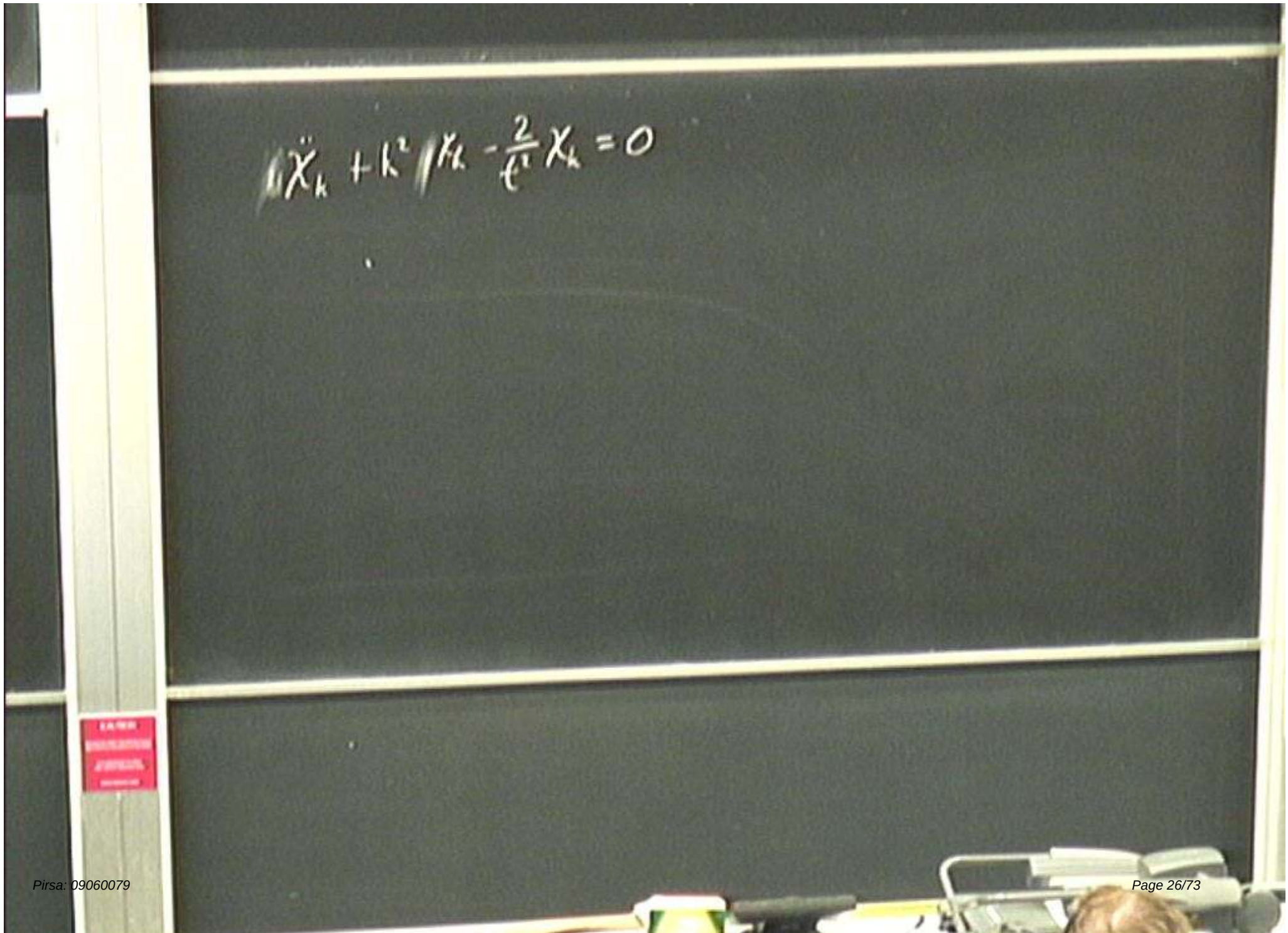
$[\varphi(t,x), \phi(t,x')] = 0$
$[\dot\phi \quad , \dot\phi \quad] = 0$
$[\phi(t,x) \; , \dot\phi(t,x')] = i\,\delta(x-x')$


$\ddot{X}_k + k^2 X_k$


$\ddot{X}_k + k^2 X_k - \frac{2}{t^2} X_k = 0$

$$\ddot{X}_k + k^2 X_k - \frac{2}{t^2} X_k = 0$$

2 solutions: $e^{-ikt}\left(1 - \frac{i}{kt}\right)$, $e^{ikt}\left(1 + \frac{i}{kt}\right)$

$t \to -\infty$ want $\frac{1}{\sqrt{2k}} e^{-ikt}$

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sol. $X_k = \frac{1}{\sqrt{2k}} e^{-ikt}\left(1 - \frac{i}{kt}\right)$

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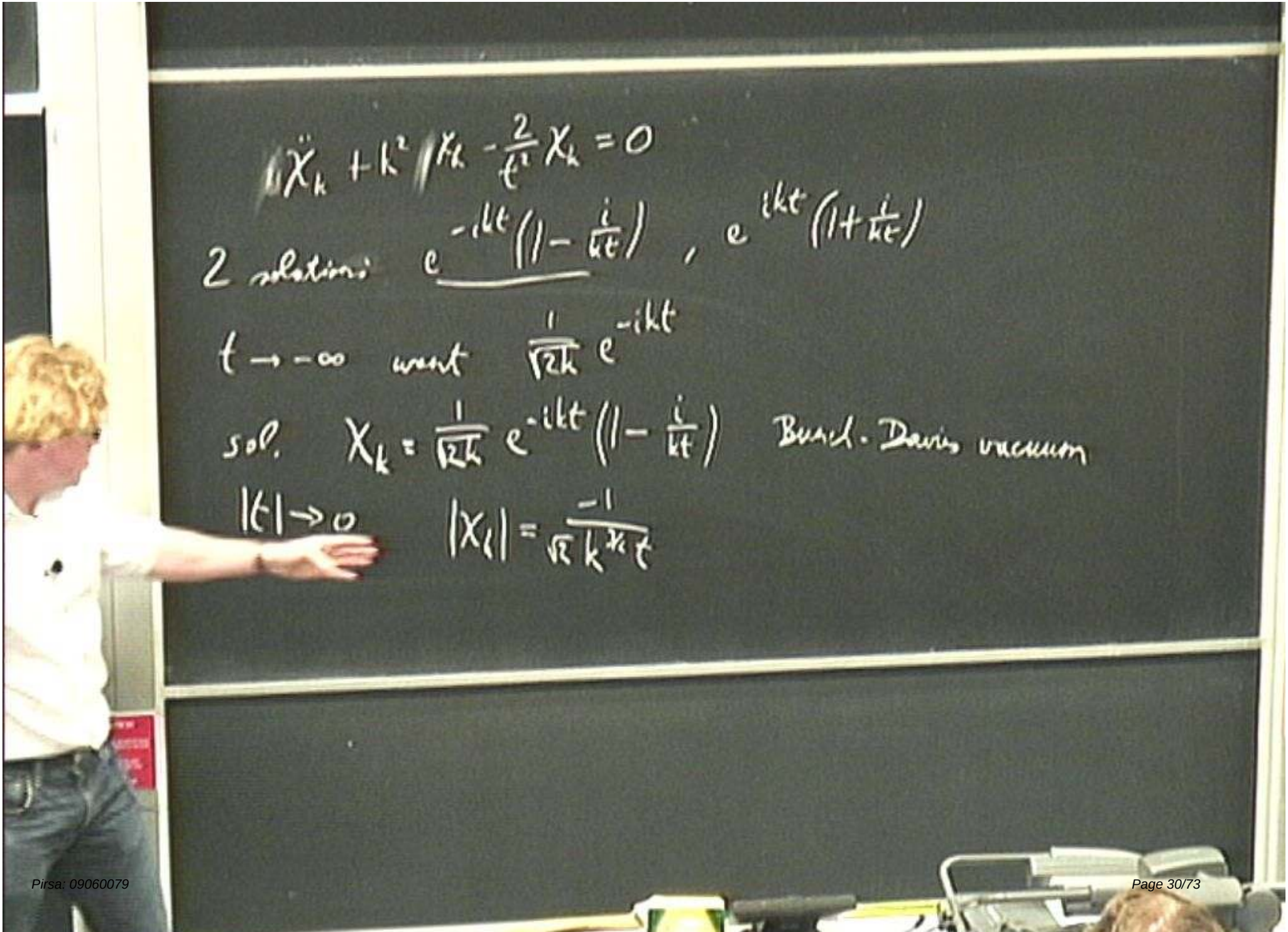
$|t| \to 0$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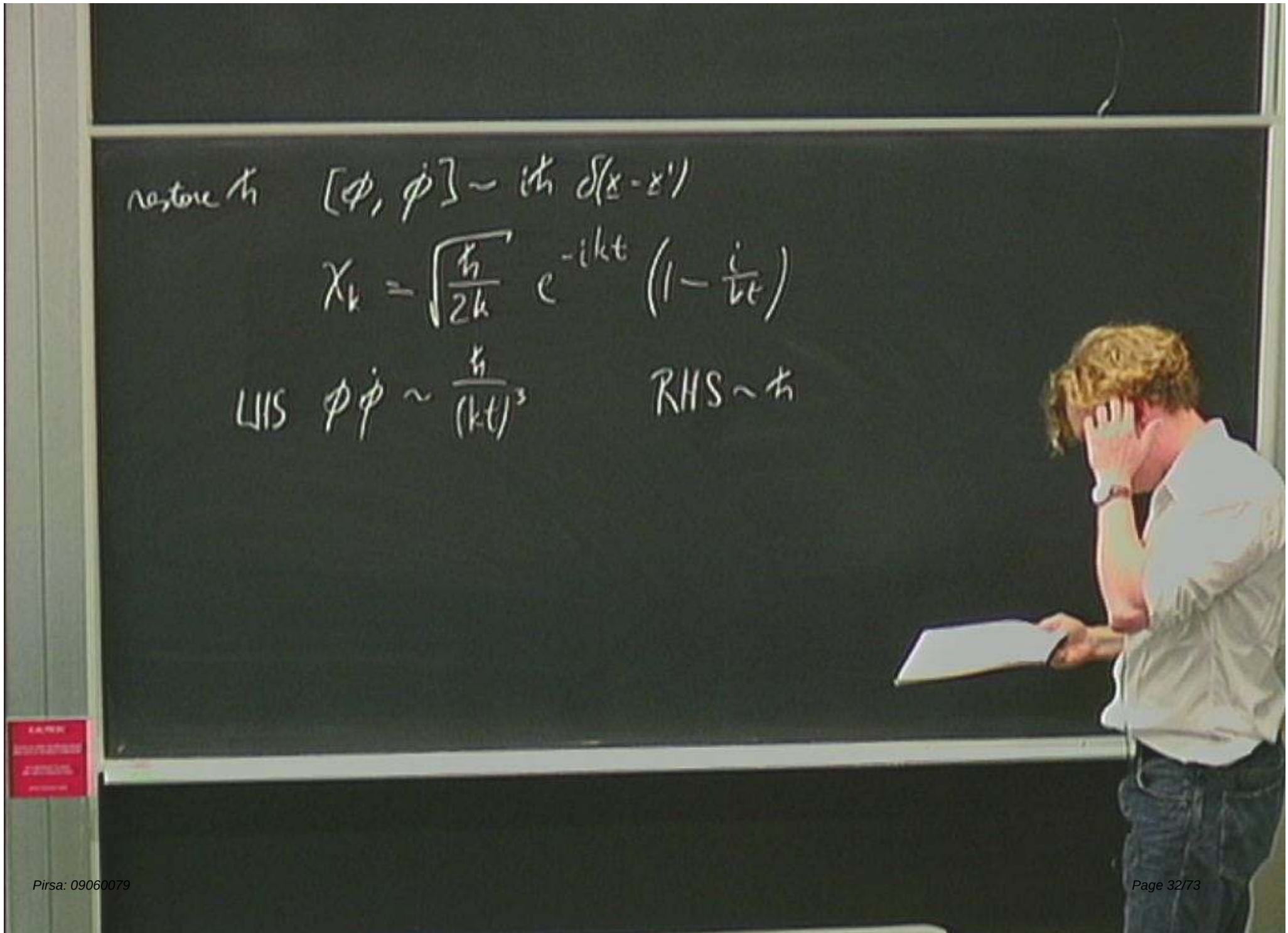
$|t| \to 0$ $|X_k| = \frac{-1}{\sqrt{2}\, k^{3/2} t}$

restore th $[\phi, \dot{\phi}] \sim i\hbar\, \delta(x - x')$

χ_k


restore ħ [φ, φ̇] ~ iħ δ(x - x')
$\chi_k = \sqrt{\frac{\hbar}{2k}}\, e^{-ikt}\left(1 - \frac{i}{kt}\right)$
LHS $\varphi\dot{\varphi} \sim \frac{\hbar}{(kt)^3}$
RHS ~ ħ

restore $\hbar$ $[\phi, \dot{\phi}] \sim i\hbar\, \delta(\vec{x} - \vec{x}')$

$$\chi_k = \sqrt{\frac{\hbar}{2k}}\, e^{-ikt}\left(1 - \frac{i}{kt}\right)$$

LHS $\phi\dot{\phi} \sim \frac{\hbar}{(kt)^3} \overset{t \to 0}{\gg}$ RHS $\sim \hbar$

$|kt| \ll 1$ "modes exit horizon"

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LHS $\phi\dot{\phi} \sim \frac{\hbar}{(kt)^3} \overset{t\to 0}{\gg}$ RHS $\sim \hbar$

$|kt| \ll 1$ "modes exit horizon"

$\hbar \to 0$ → classical

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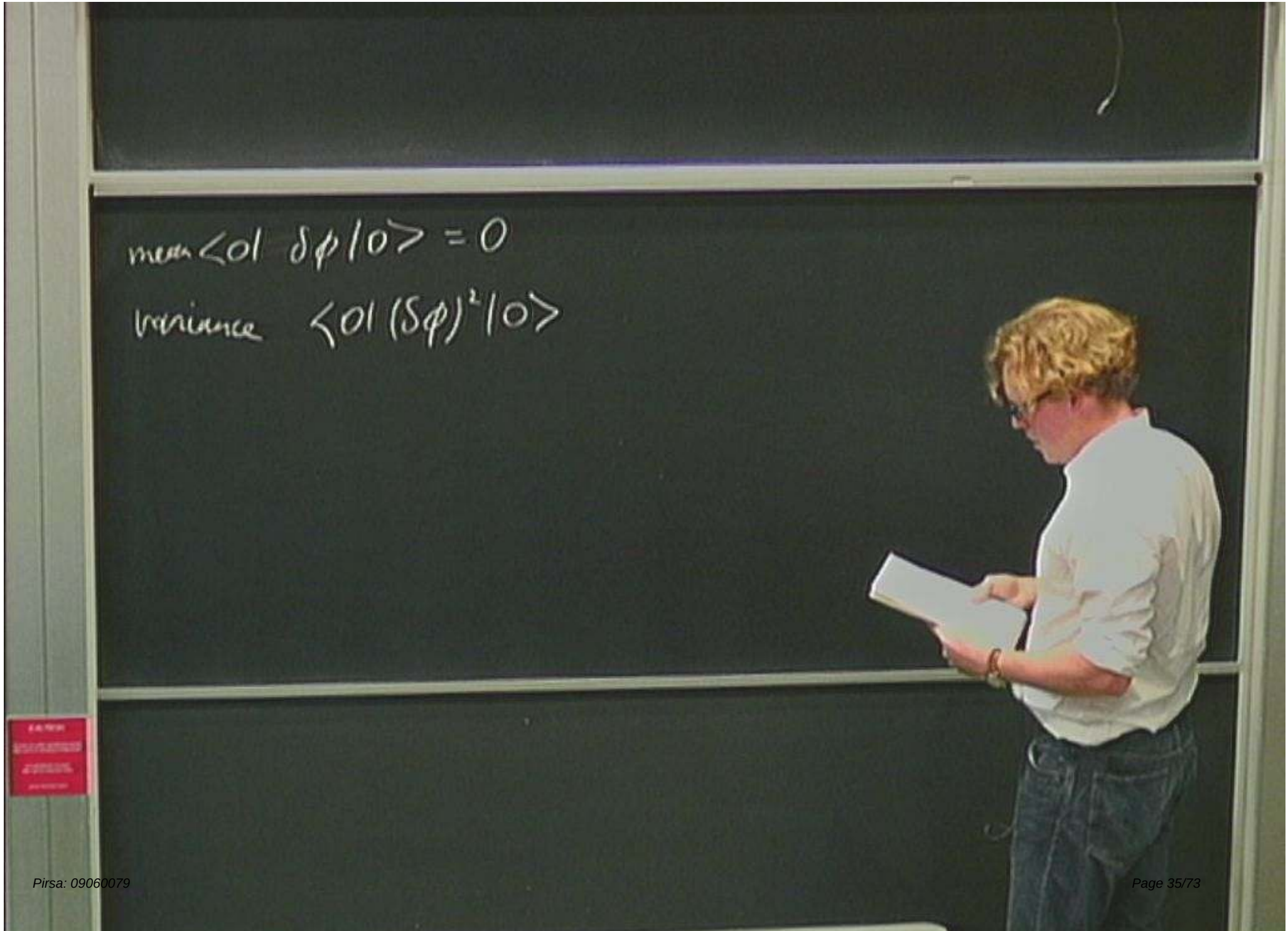
mean $\langle 0|\delta\phi|0\rangle = 0$
variance $\langle 0|(\delta\phi)^2|0\rangle$

mean $\langle 0| \delta\phi |0\rangle = 0$

variance $\langle 0|(\delta\phi)^2|0\rangle \equiv \int \frac{dk}{k} \Delta_\phi^2(k)$

(↳ variance)

$$\Delta_\phi^2(k) = \Delta_\phi^2(k_0) \left(\frac{k}{k_0}\right)^{n_s - 1}$$

mean $\langle 0| \delta\phi |0\rangle = 0$

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↳ variance

$\Delta^2_\phi(k) = \Delta^2_\phi(k_0) \left(\frac{k}{k_0}\right)^{n_s - 1}$ (scalar)

↳ spectral index

↳ reference scale

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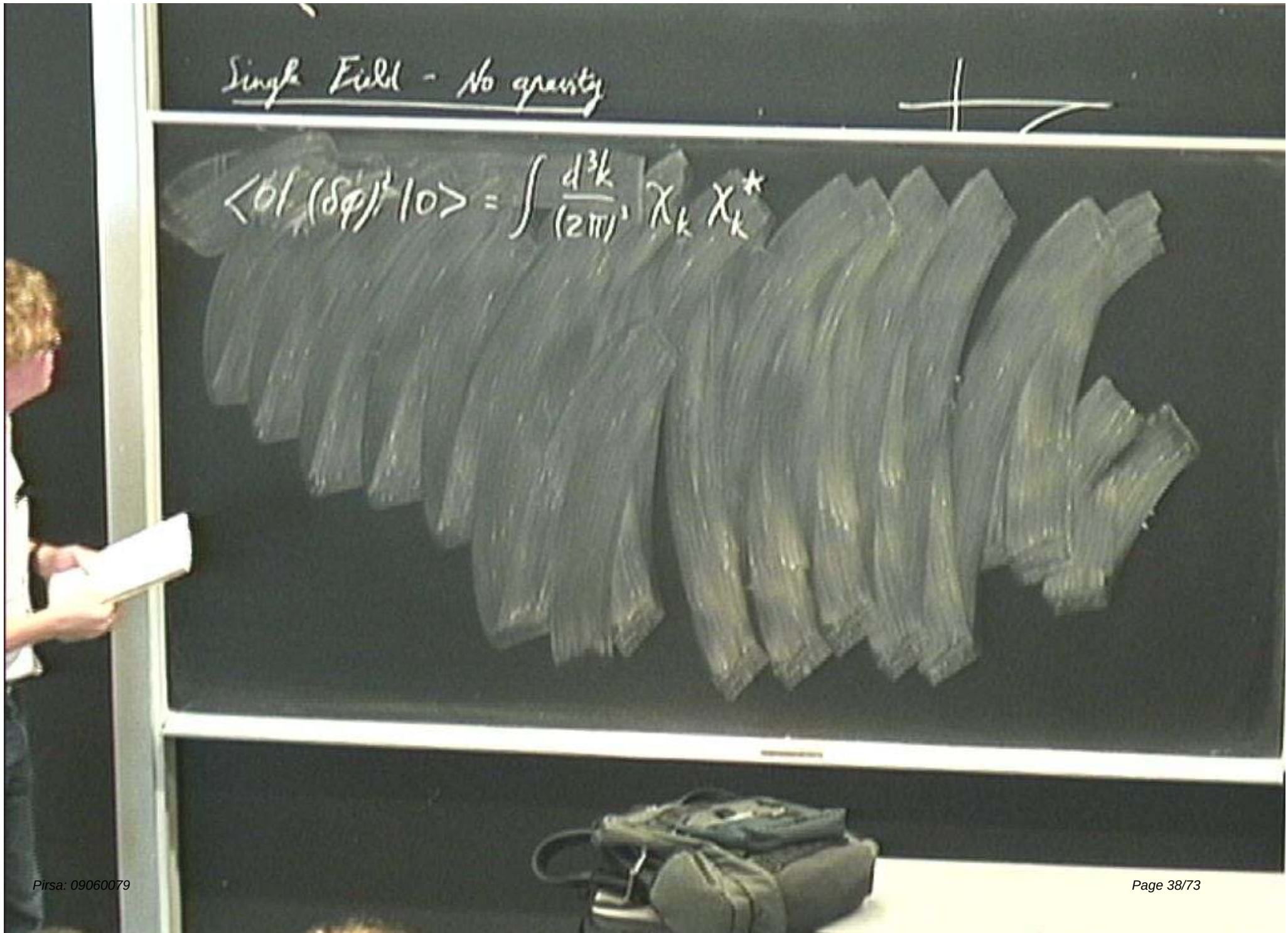
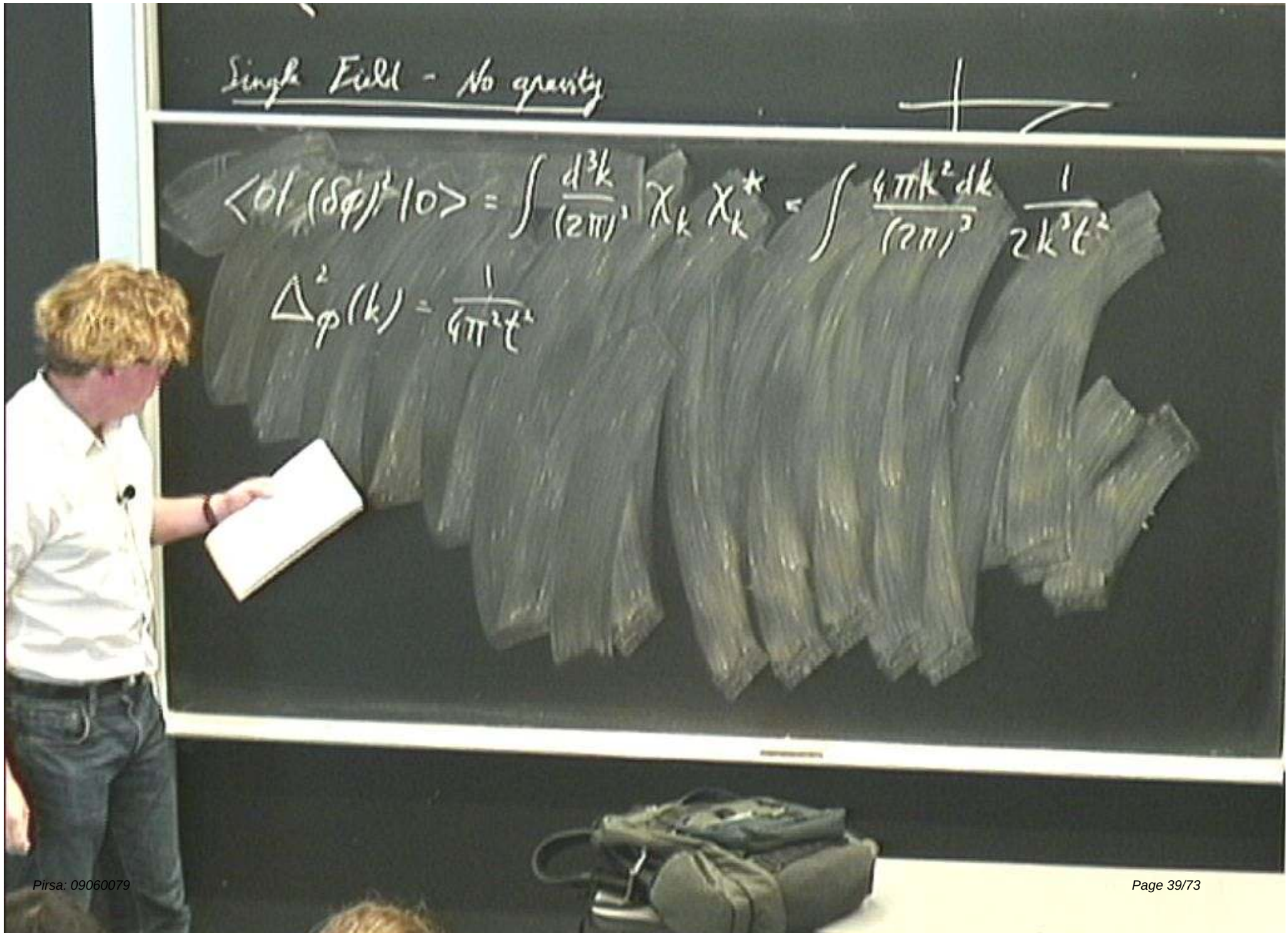

Single Field - No gravity
$\langle 0| (\delta\varphi)^2 |0\rangle = \int \frac{d^3k}{(2\pi)^3} \chi_k \chi_k^*$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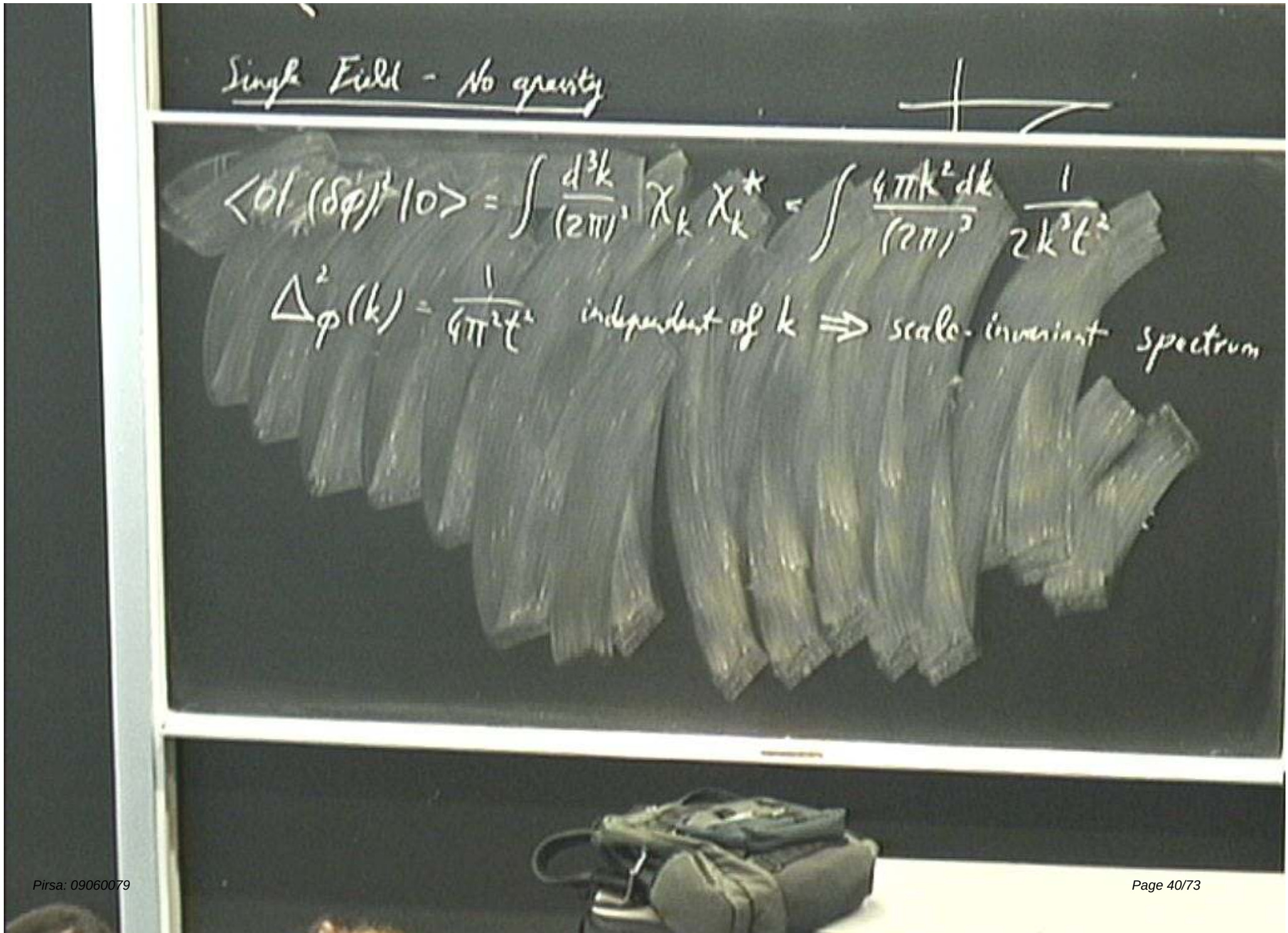
$\langle 0| (\delta\varphi)^2 |0\rangle = \int \frac{d^3k}{(2\pi)^3} \chi_k \chi_k^* = \int \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2k^3\epsilon^2}$
$\Delta^2_\varphi(k) = \frac{1}{4\pi^2\epsilon^2}$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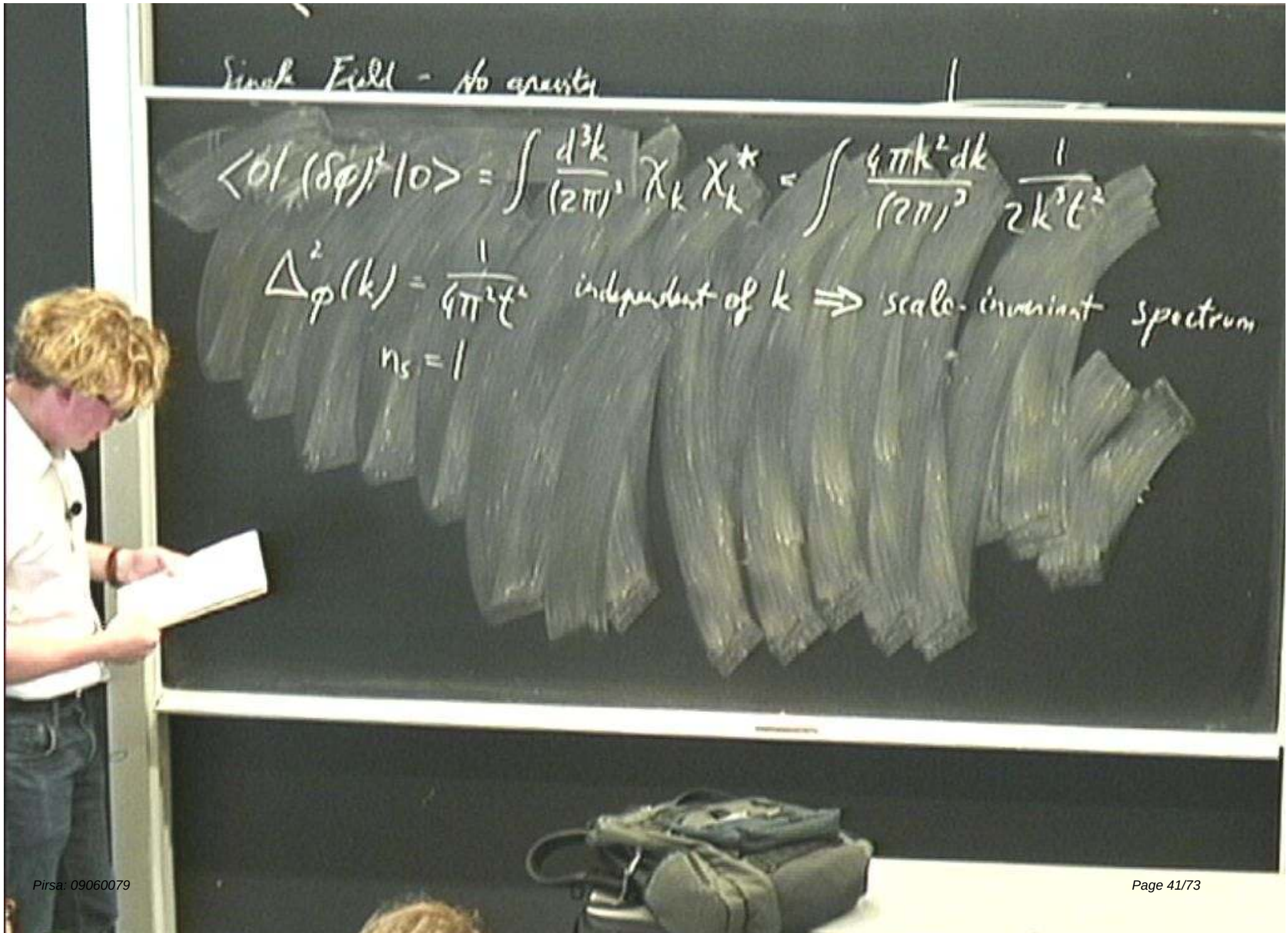
$\Delta^2_\varphi(k) = \frac{1}{4\pi^2 t^2}$ independent of $k \Rightarrow$ scale-invariant spectrum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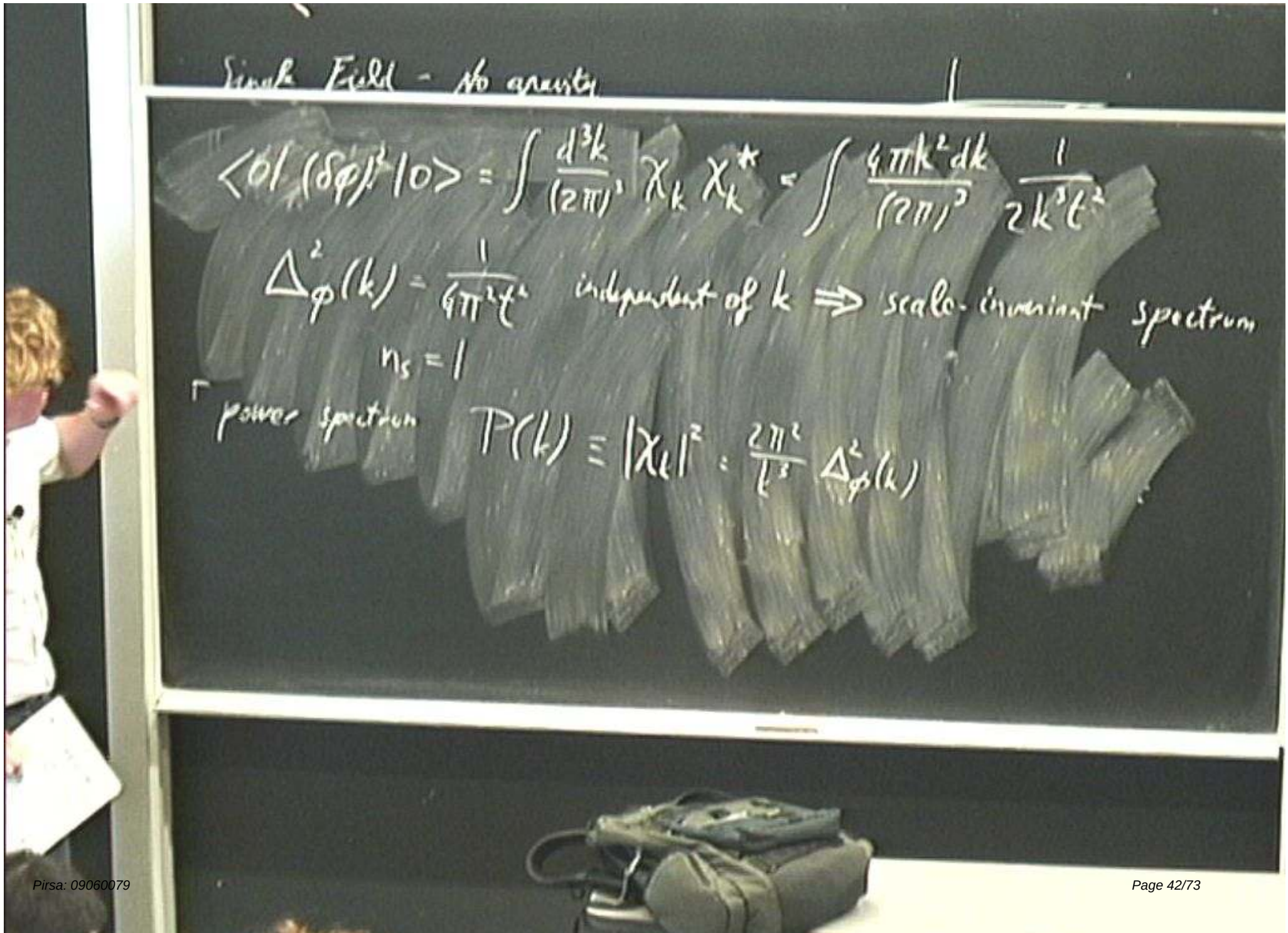
$n_s = 1$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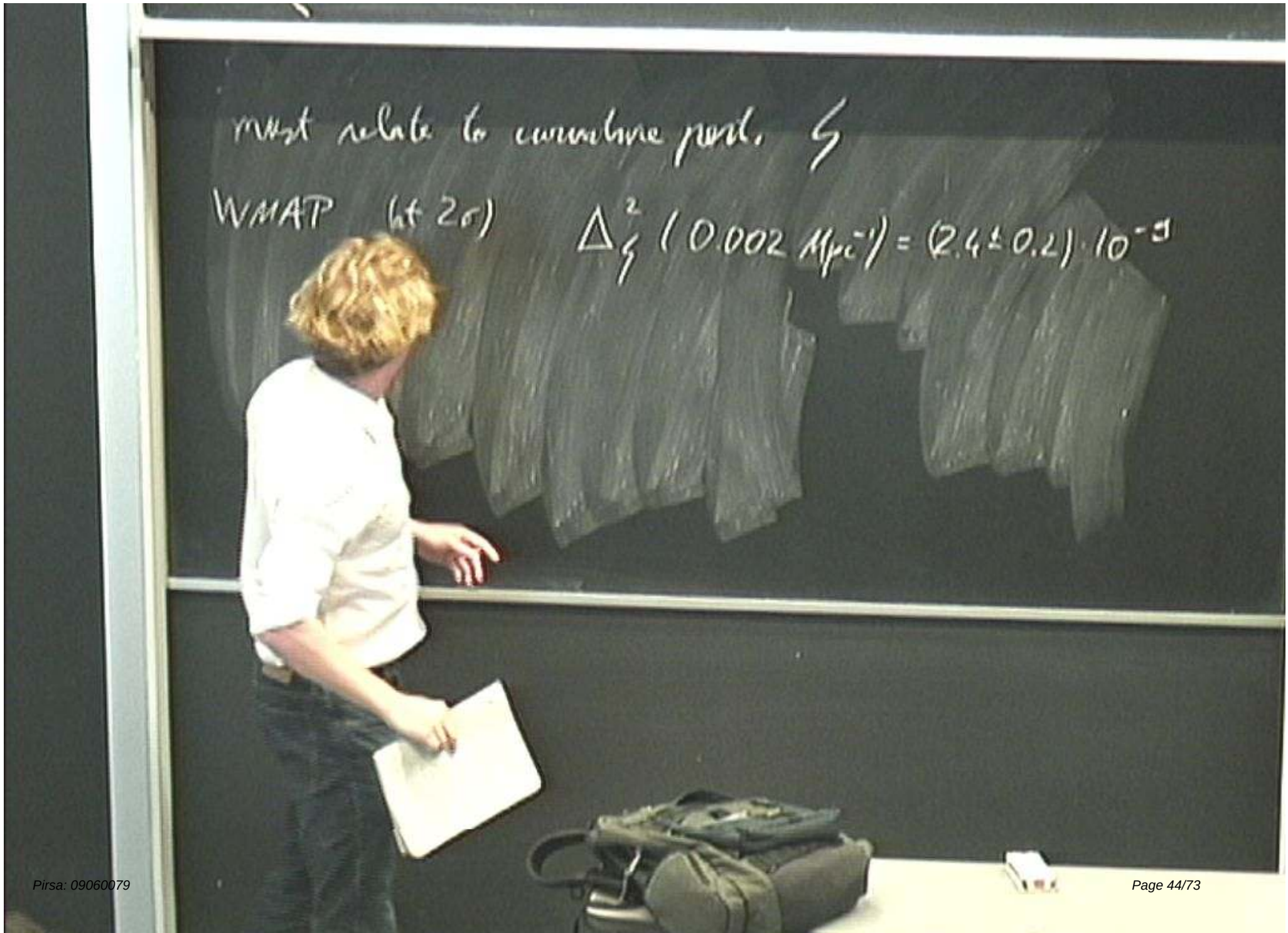
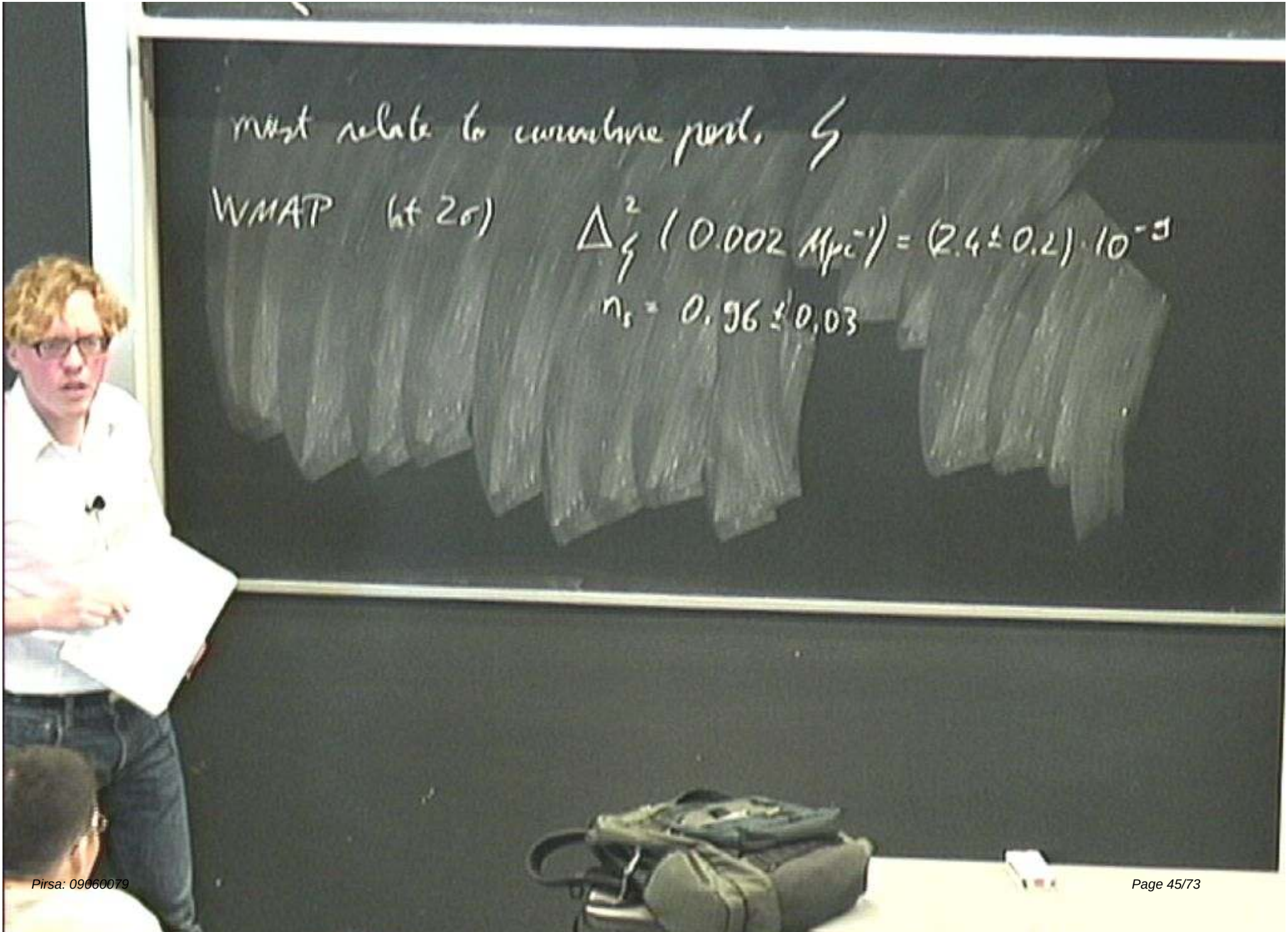
$\Delta^2_\varphi(k) = \frac{1}{4\pi^2 t^2}$ independent of $k \Rightarrow$ scale-invariant spectrum
$n_s = 1$
power spectrum $P(k) \equiv |\chi_t|^2 = \frac{2\pi^2}{t^3} \Delta^2_\varphi(k)$

must relate to curvature part. S


must relate to curvature part. ζ
WMAP (at 2σ)
$\Delta^2_\zeta(0.002\ \mathrm{Mpc}^{-1}) = (2.4 \pm 0.2) \cdot 10^{-9}$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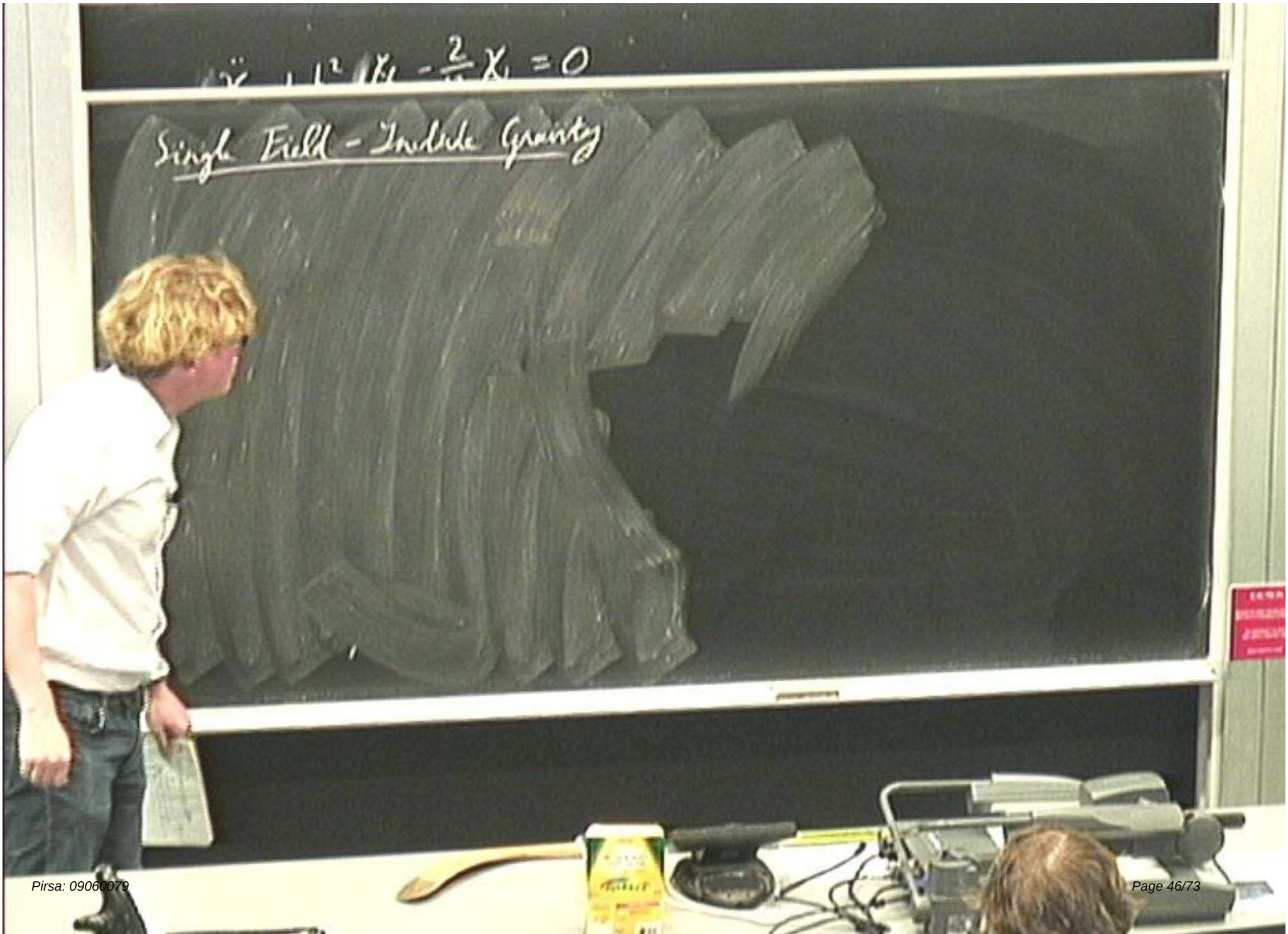
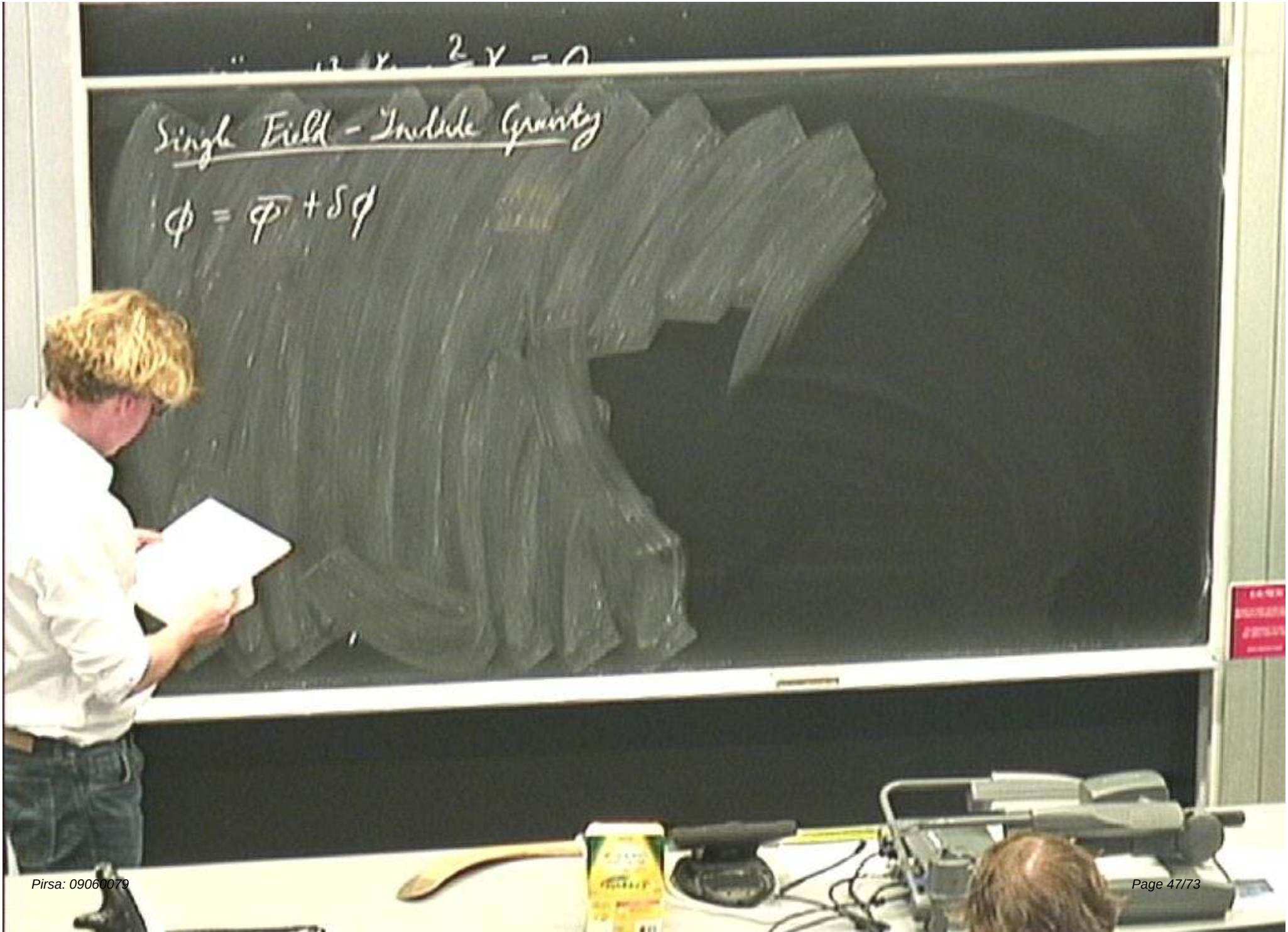
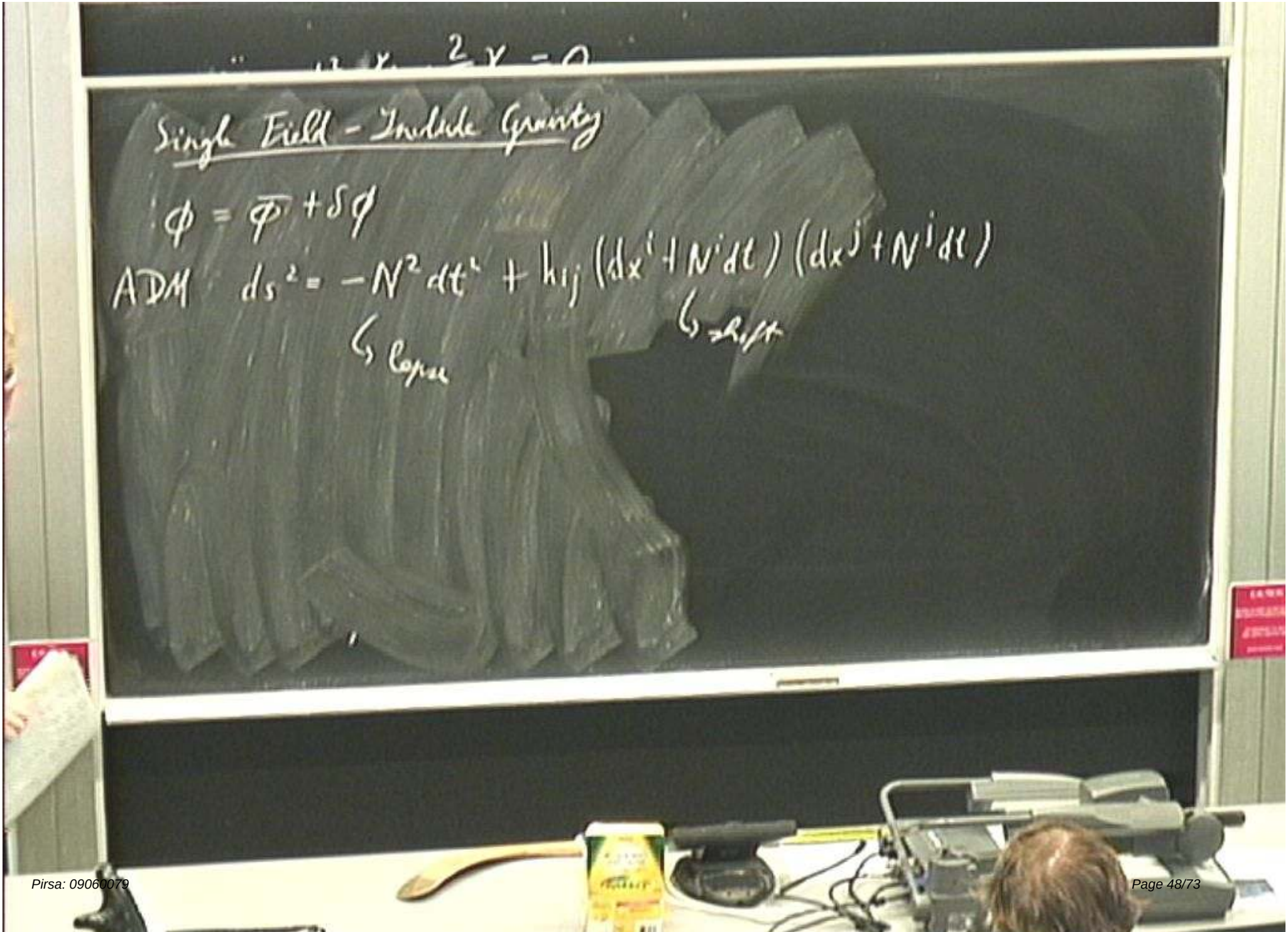
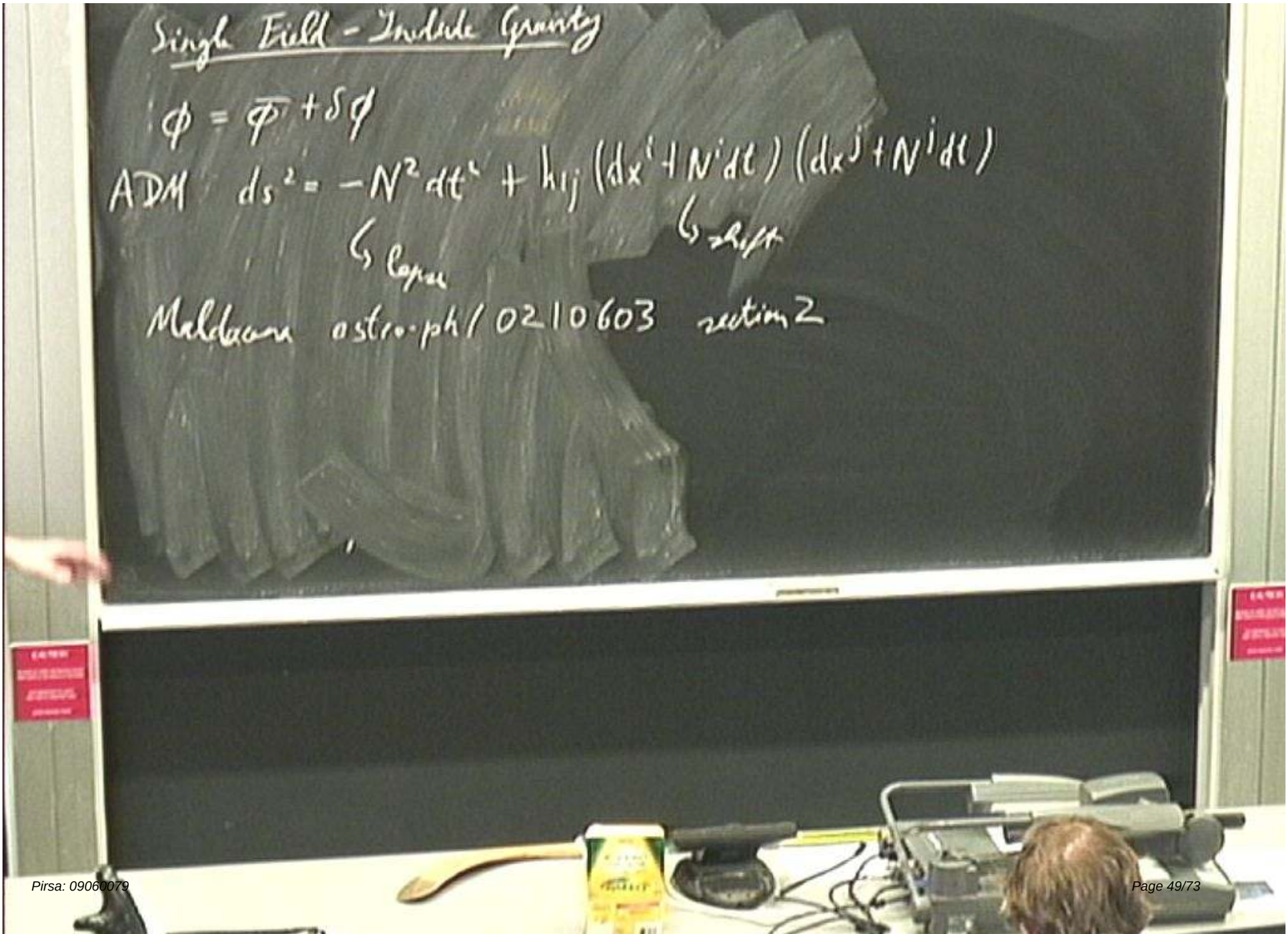
$n_s = 0.96 \pm 0.03$


Single Field - Inside Gravity


Single Field - Inside Gravity
$\phi = \bar{\varphi} + \delta\varphi$


Single Field - Include Gravity
$\phi = \bar{\phi} + \delta\phi$
ADM $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
↳ Lapse
↳ Shift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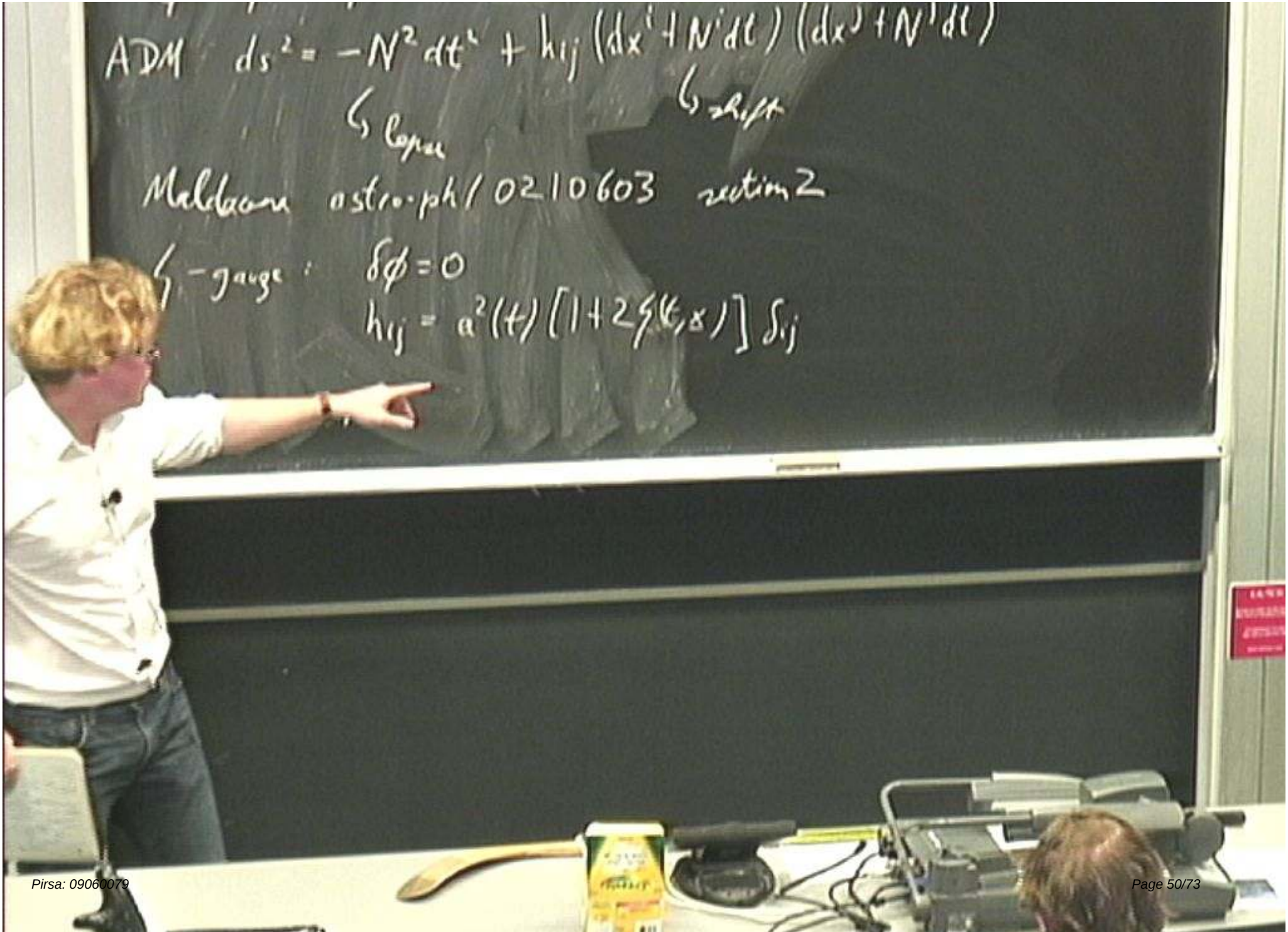
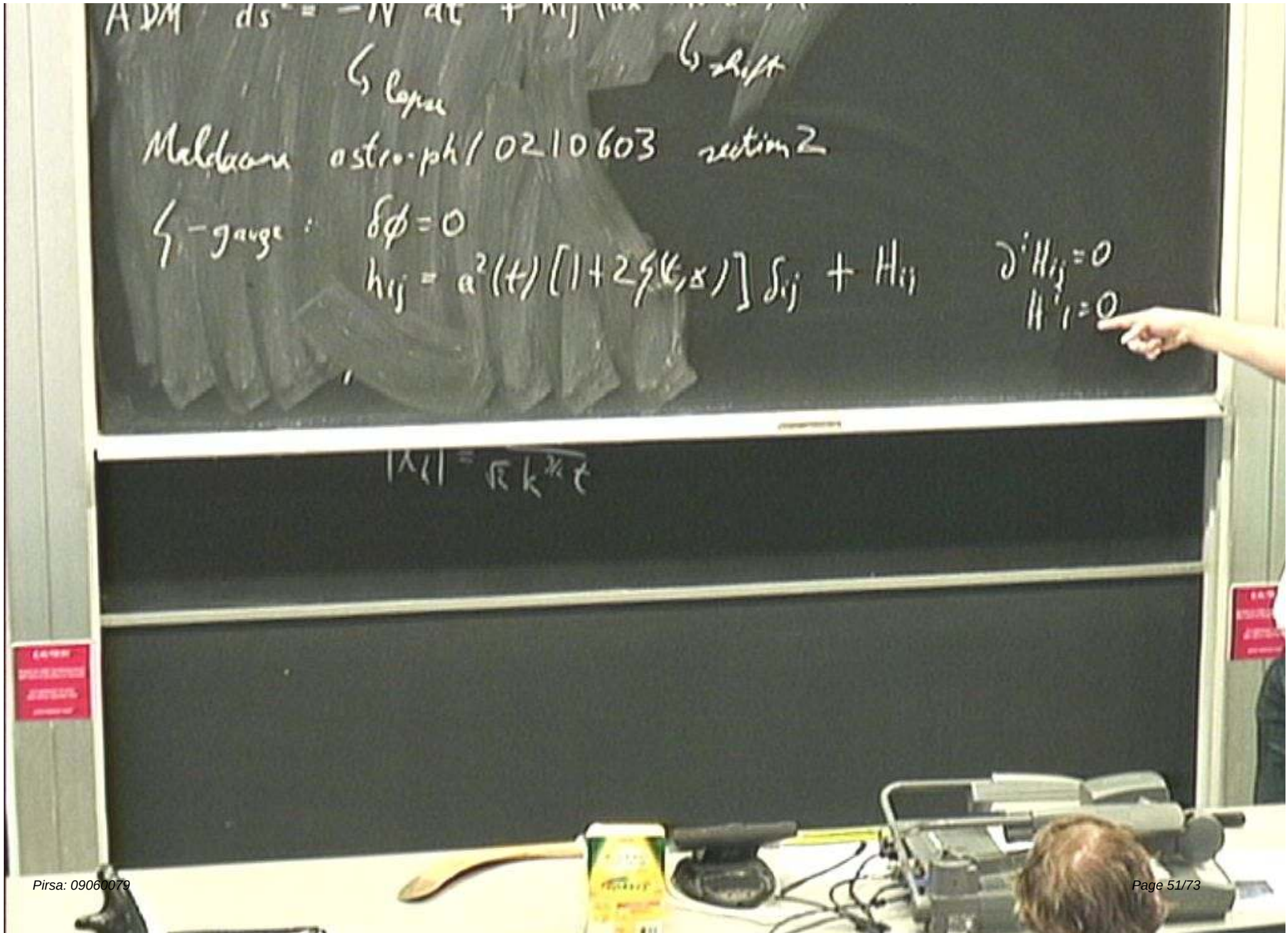
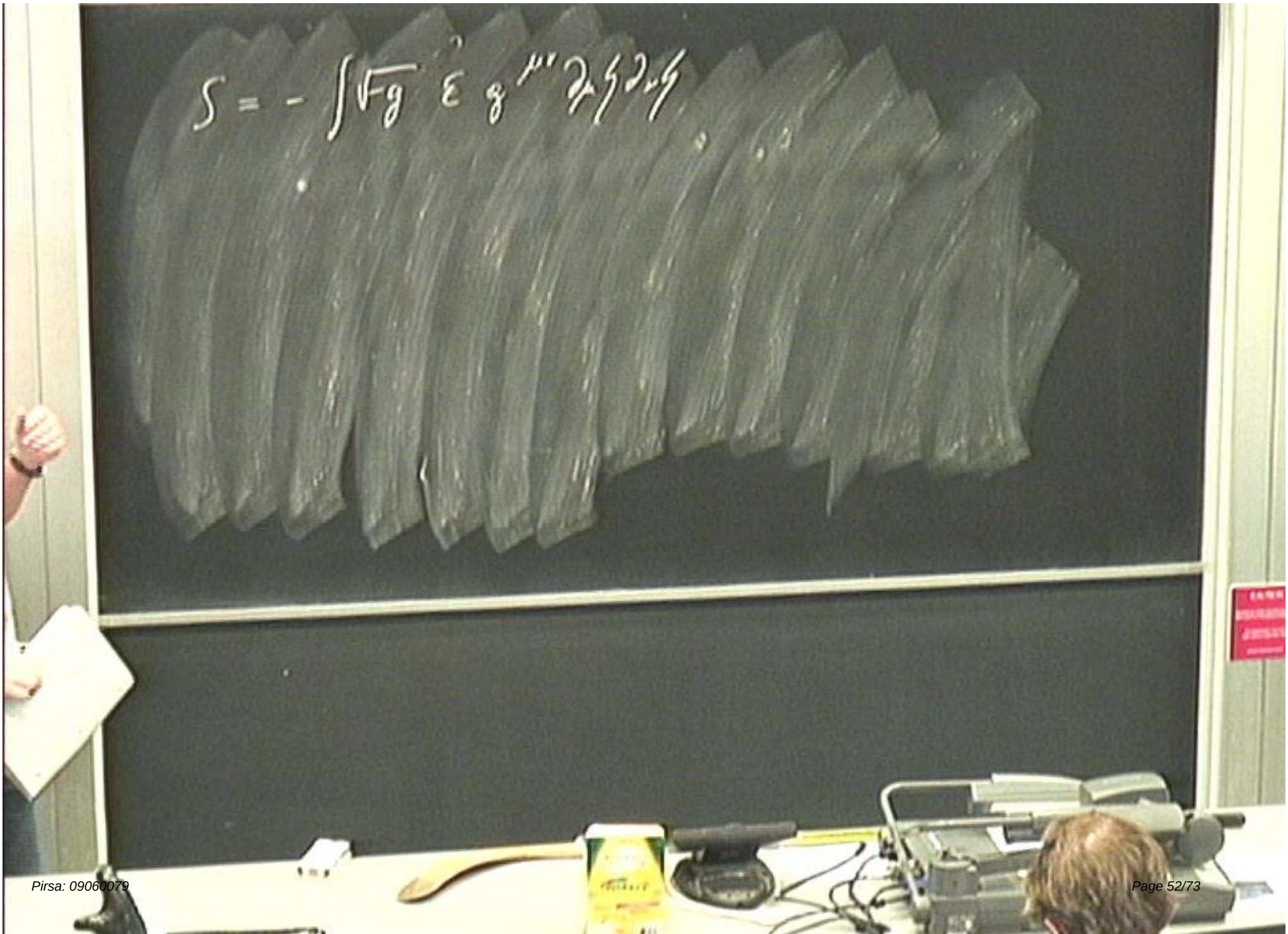
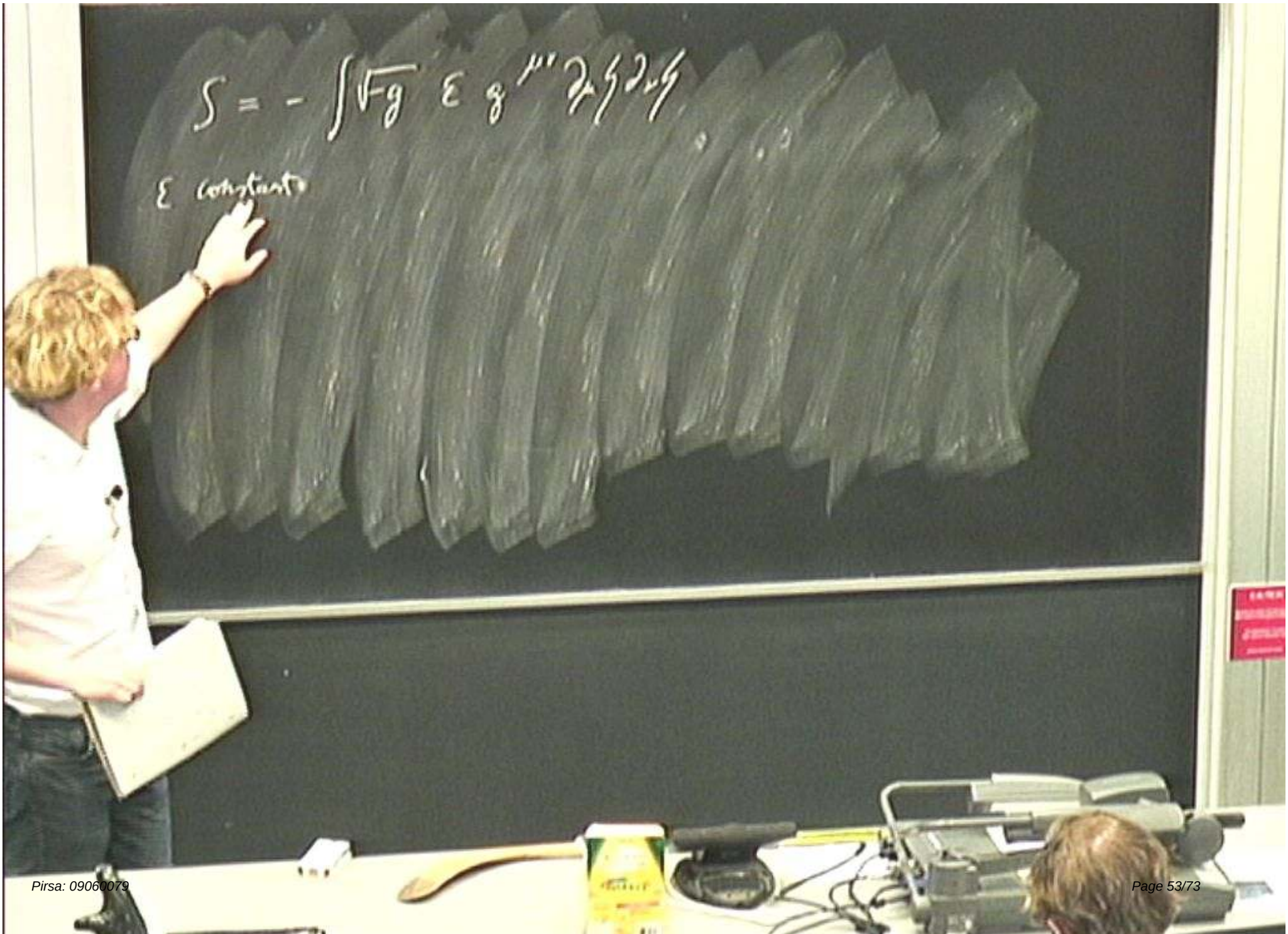
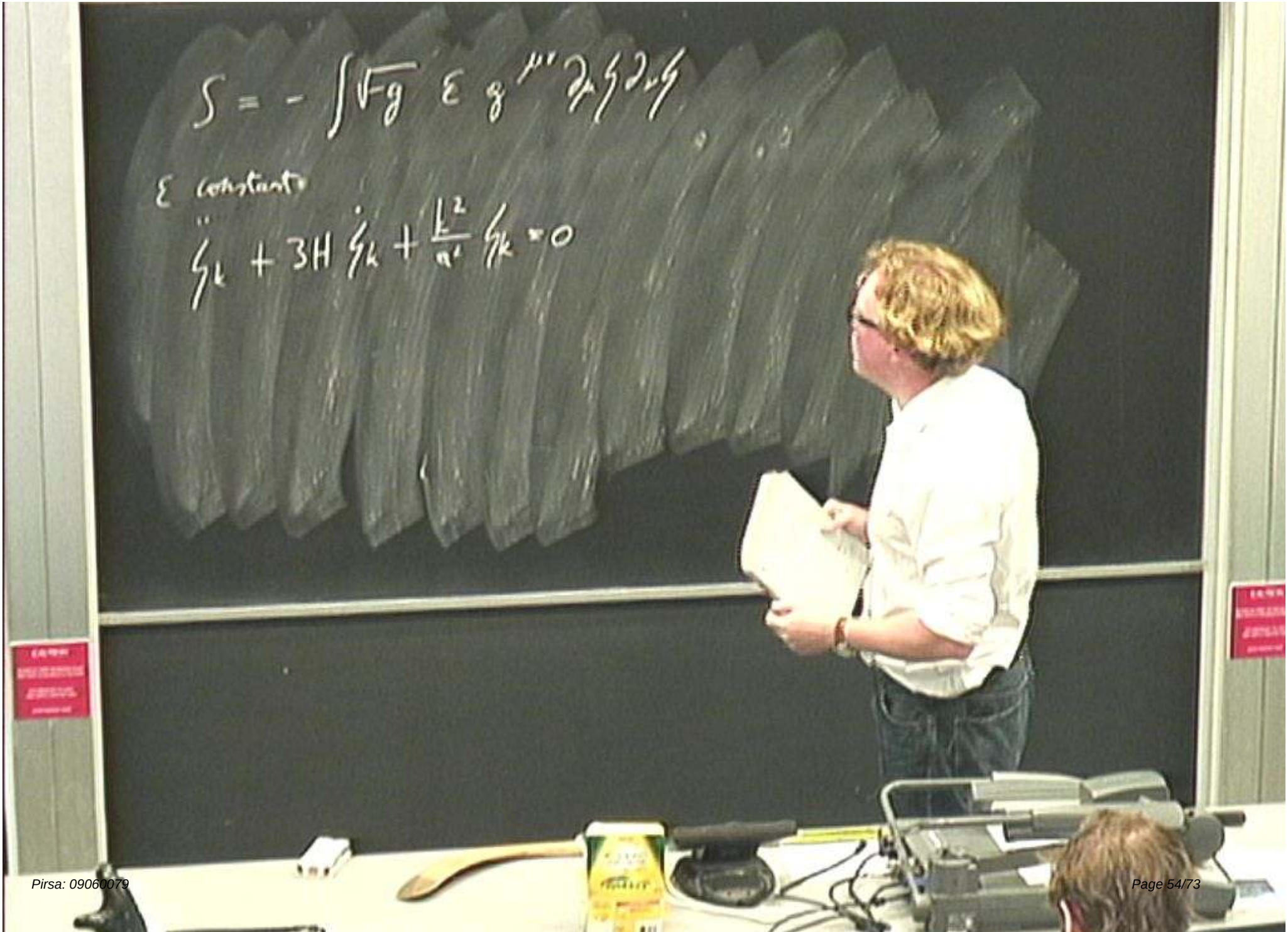
Maldacena astro-ph/0210603 section 2

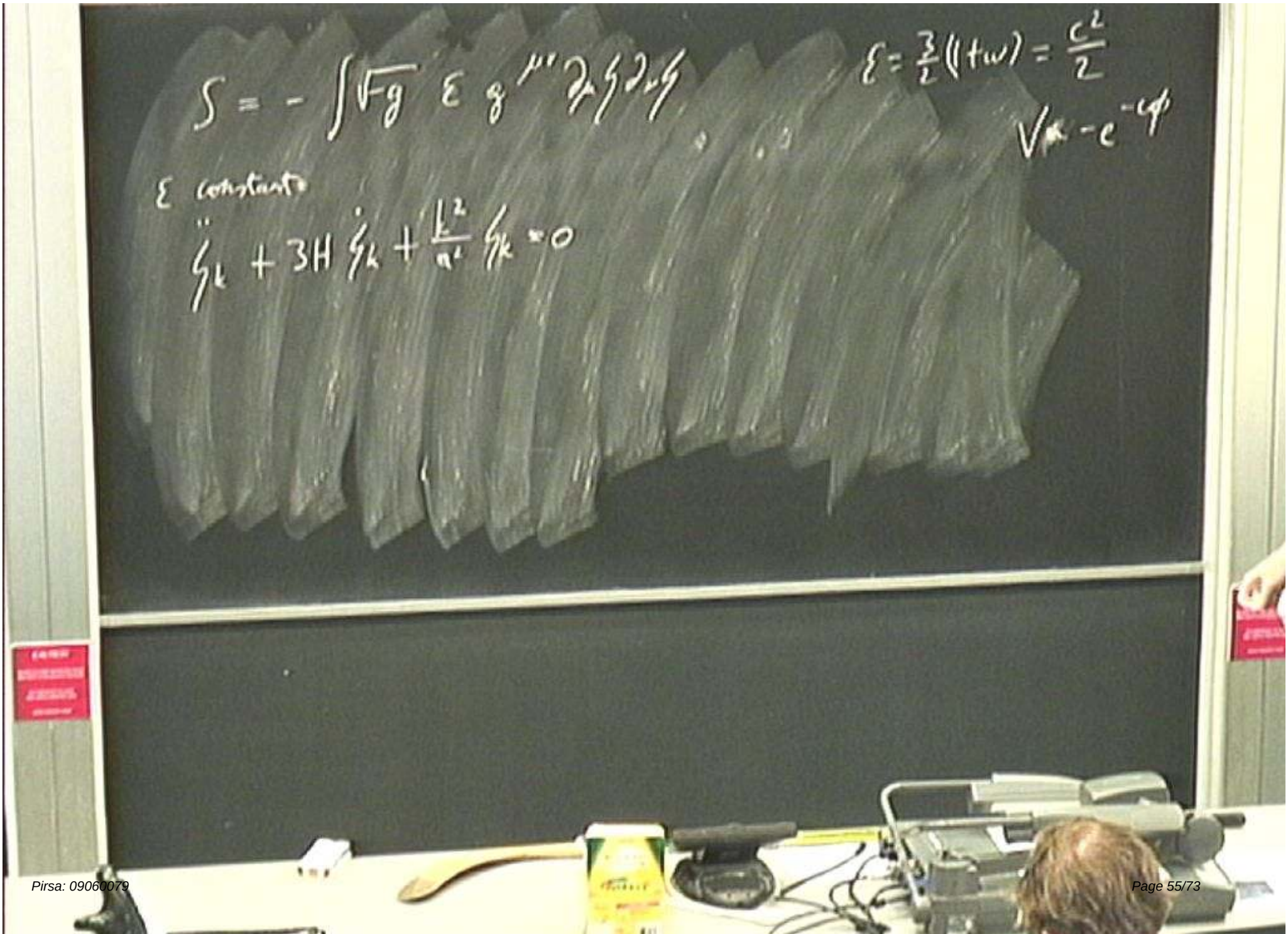

ADM $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
↳ Lapse
↳ Shift
Maldacena astro-ph/0210603 section 2
↳ -gauge: $\delta\phi = 0$
$h_{ij} = a^2(t)[1 + 2\zeta(t,x)]\delta_{ij}$

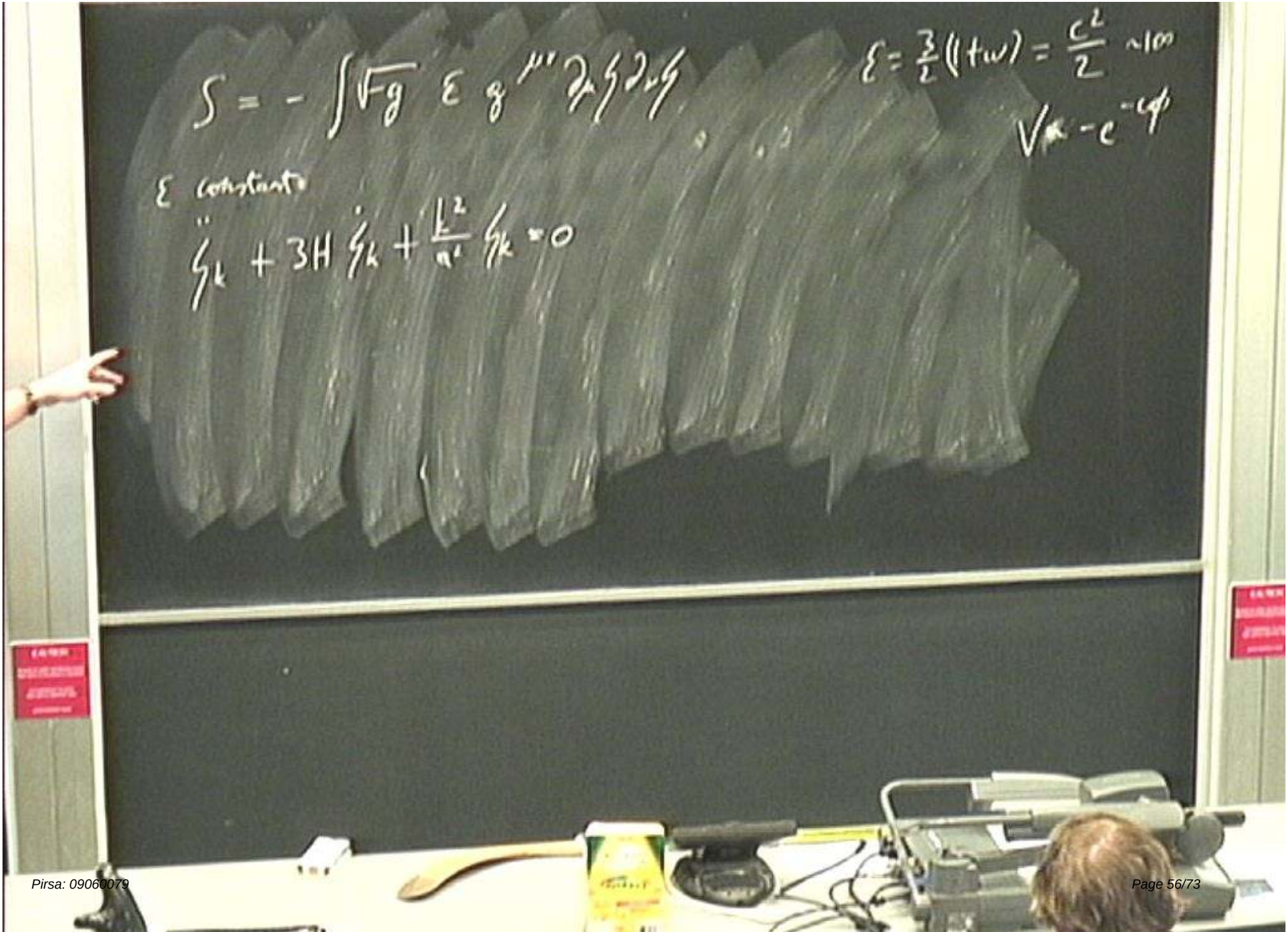

ADM
Maldacena astro-ph/0210603 section 2
ζ - gauge : δφ = 0
$h_{ij} = a^2(t)\left[1 + 2\zeta(t,x)\right]\delta_{ij} + H_{ij}$
$\partial^i H_{ij} = 0$
$H^i{}_i = 0$
$|\lambda| = \sqrt{\epsilon}\, k^{3/2}\tau$

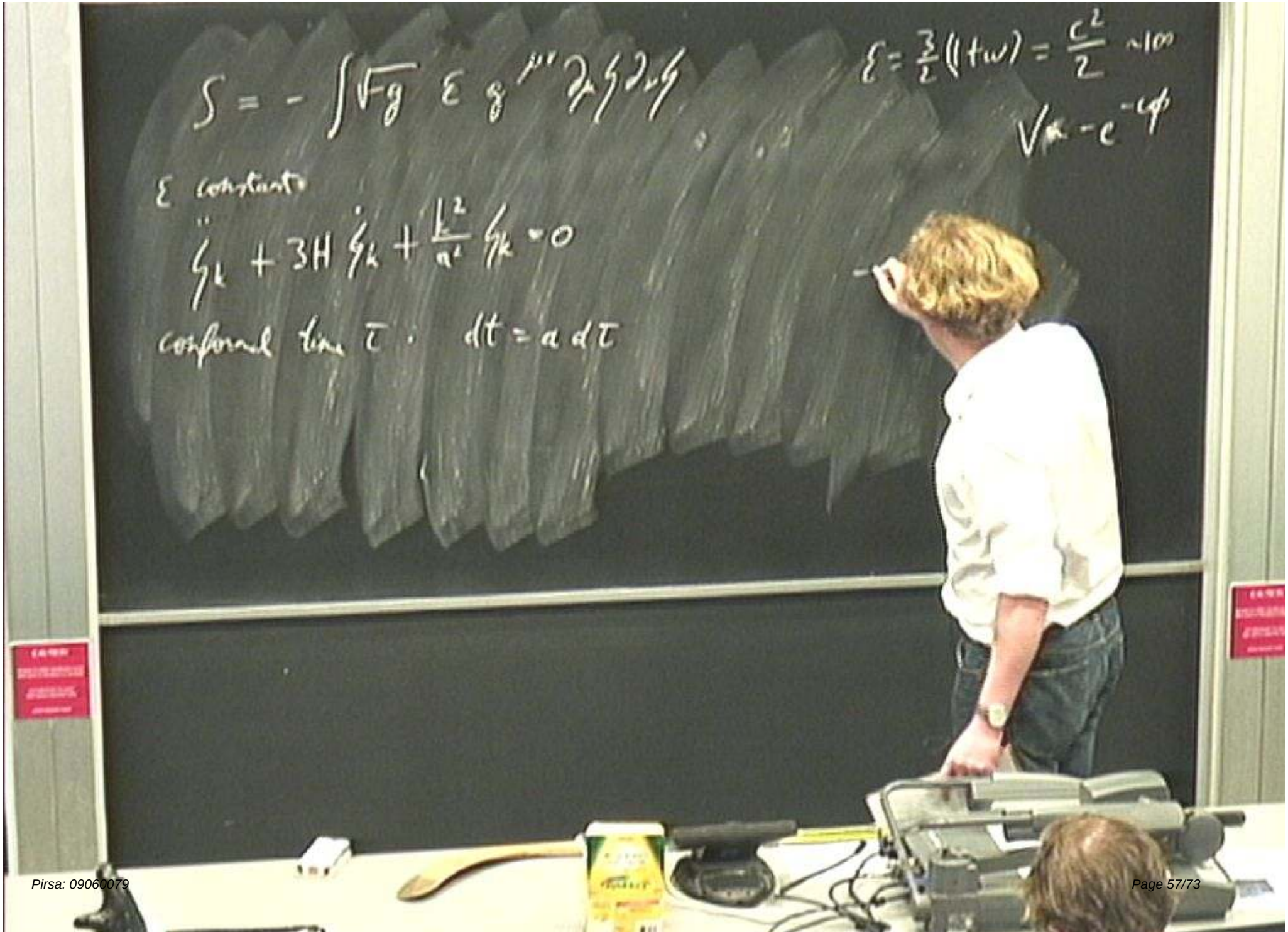

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$

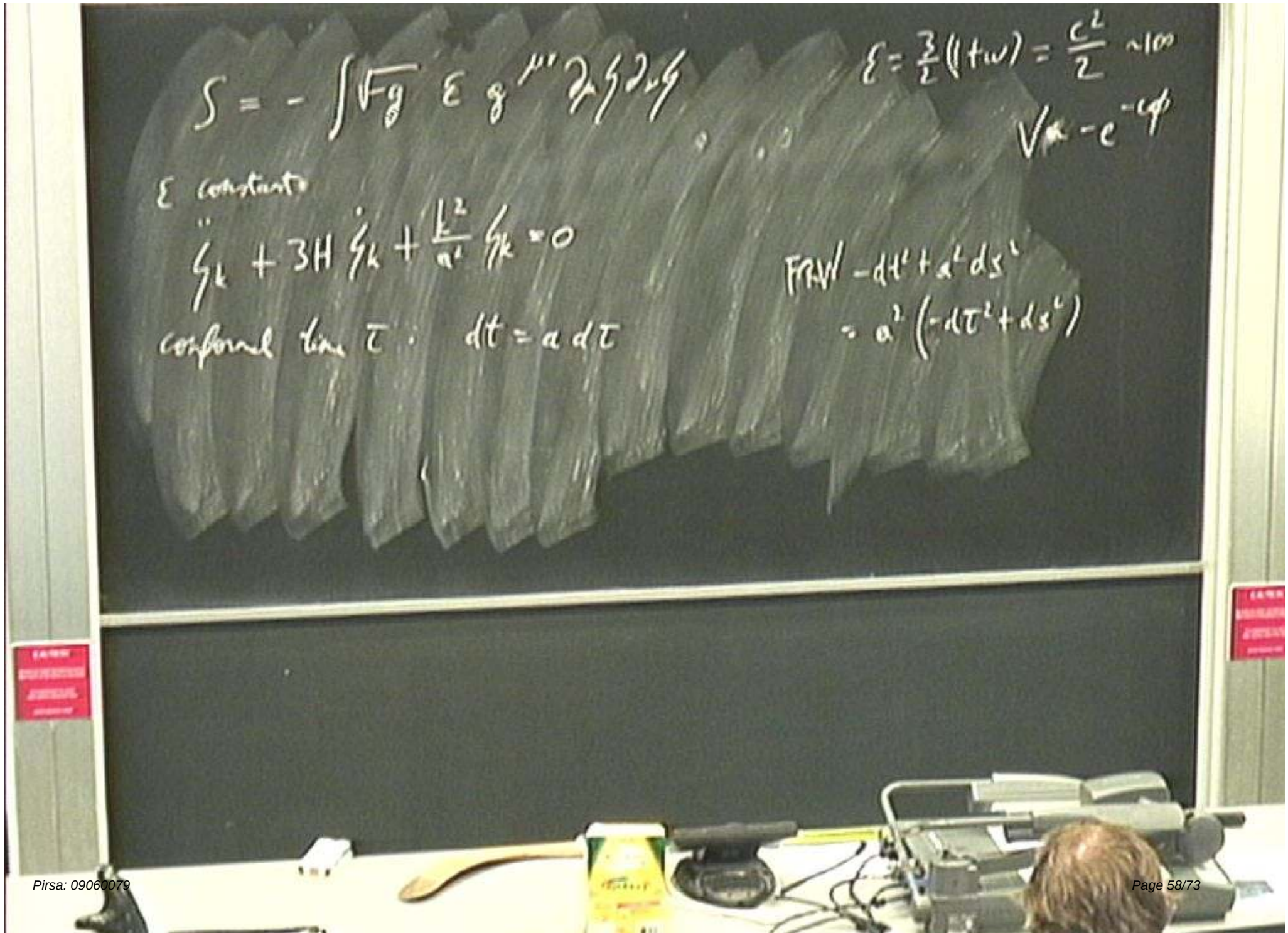

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu_1 \nu_1} \ldots$
ε cosmological

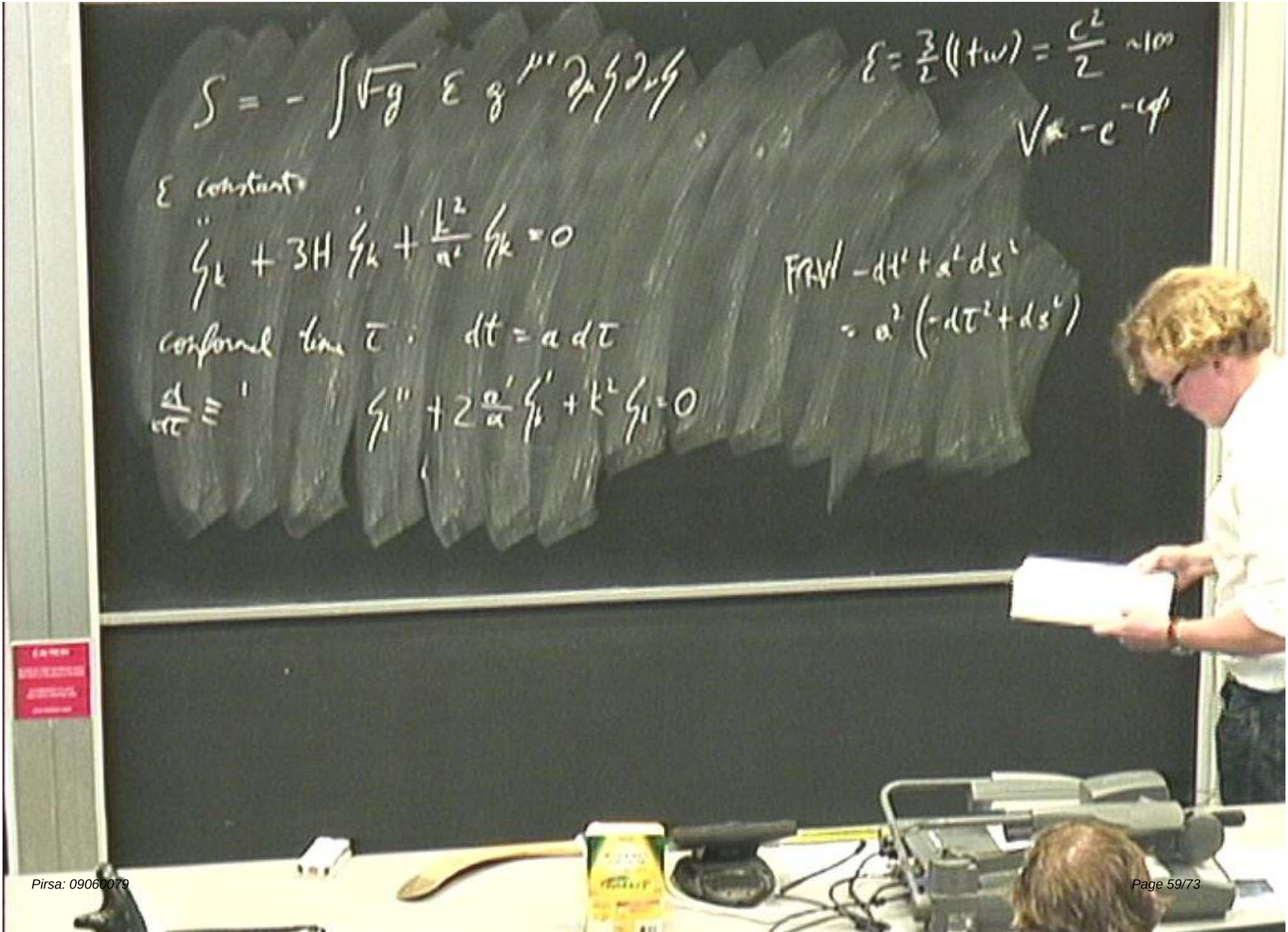

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu}\, \partial_\mu \zeta\, \partial_\nu \zeta$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$

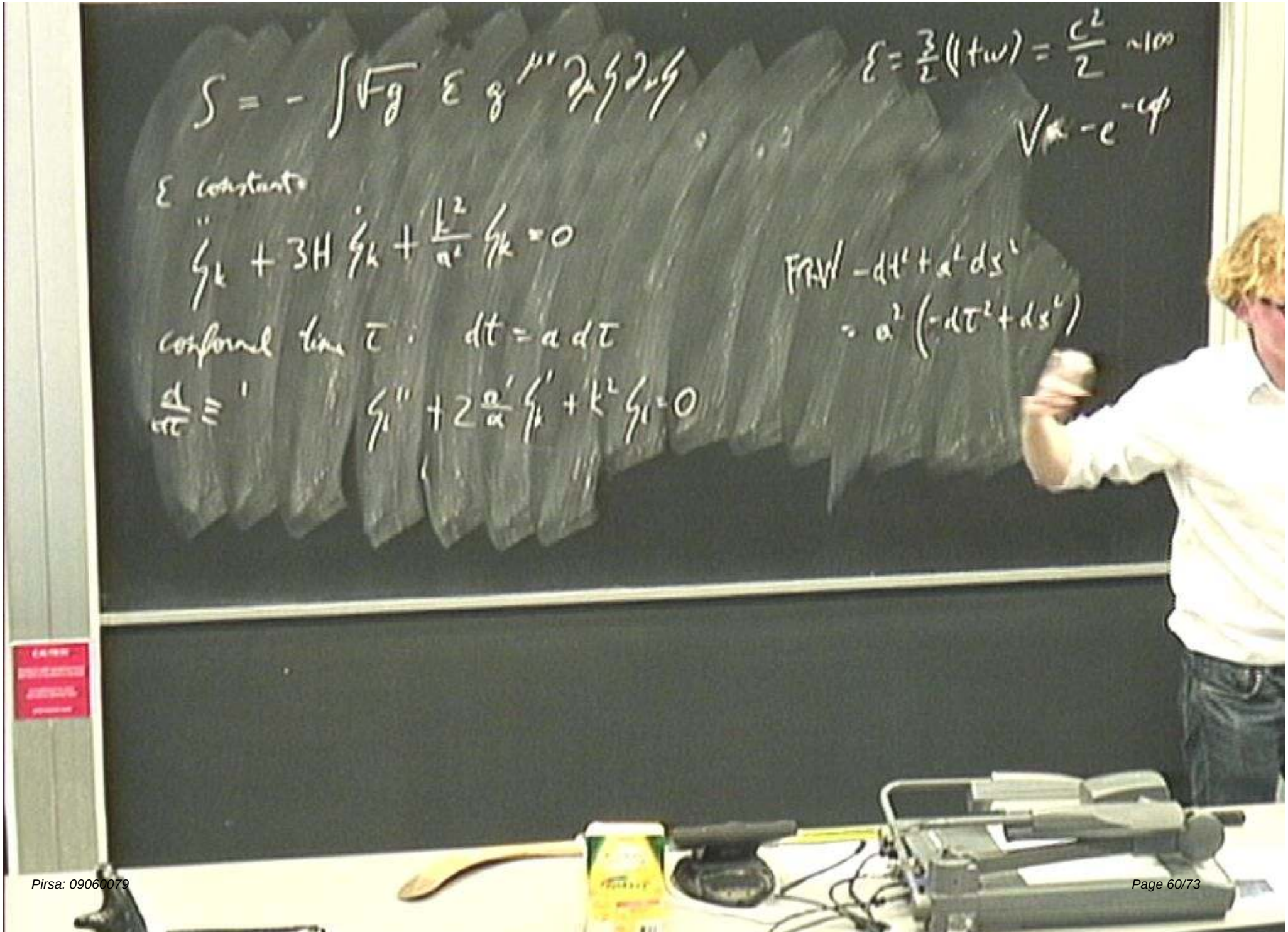

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2}$
$V \propto e^{-c\varphi}$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$

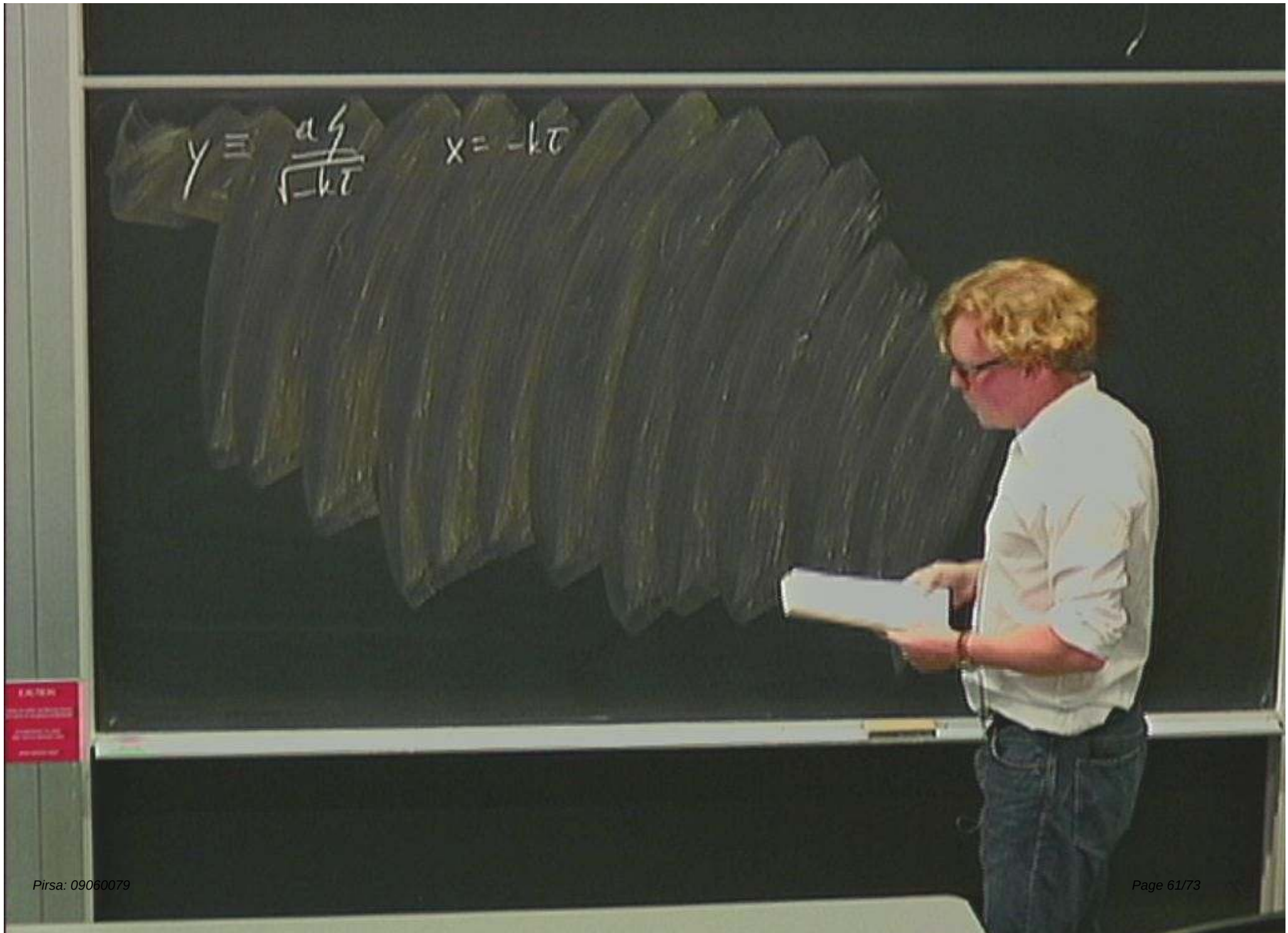

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2} \sim 100$
$V \propto e^{-c\varphi}$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$

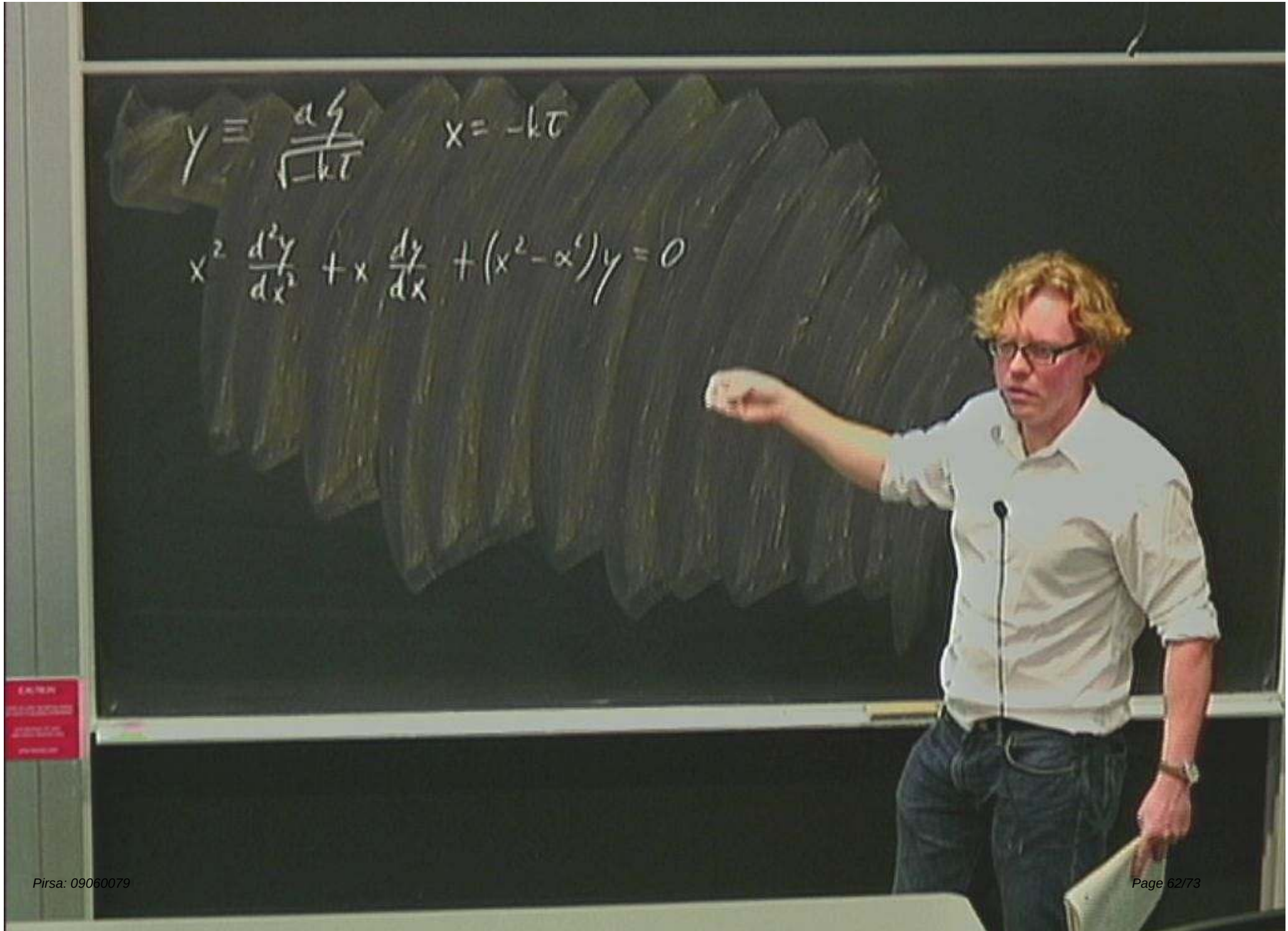

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu}\, \partial_\mu \zeta\, \partial_\nu \zeta$
$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2}$ ~100
$V_{\text{eff}} - e^{-4\phi}$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$
conformal time τ : $dt = a\, d\tau$

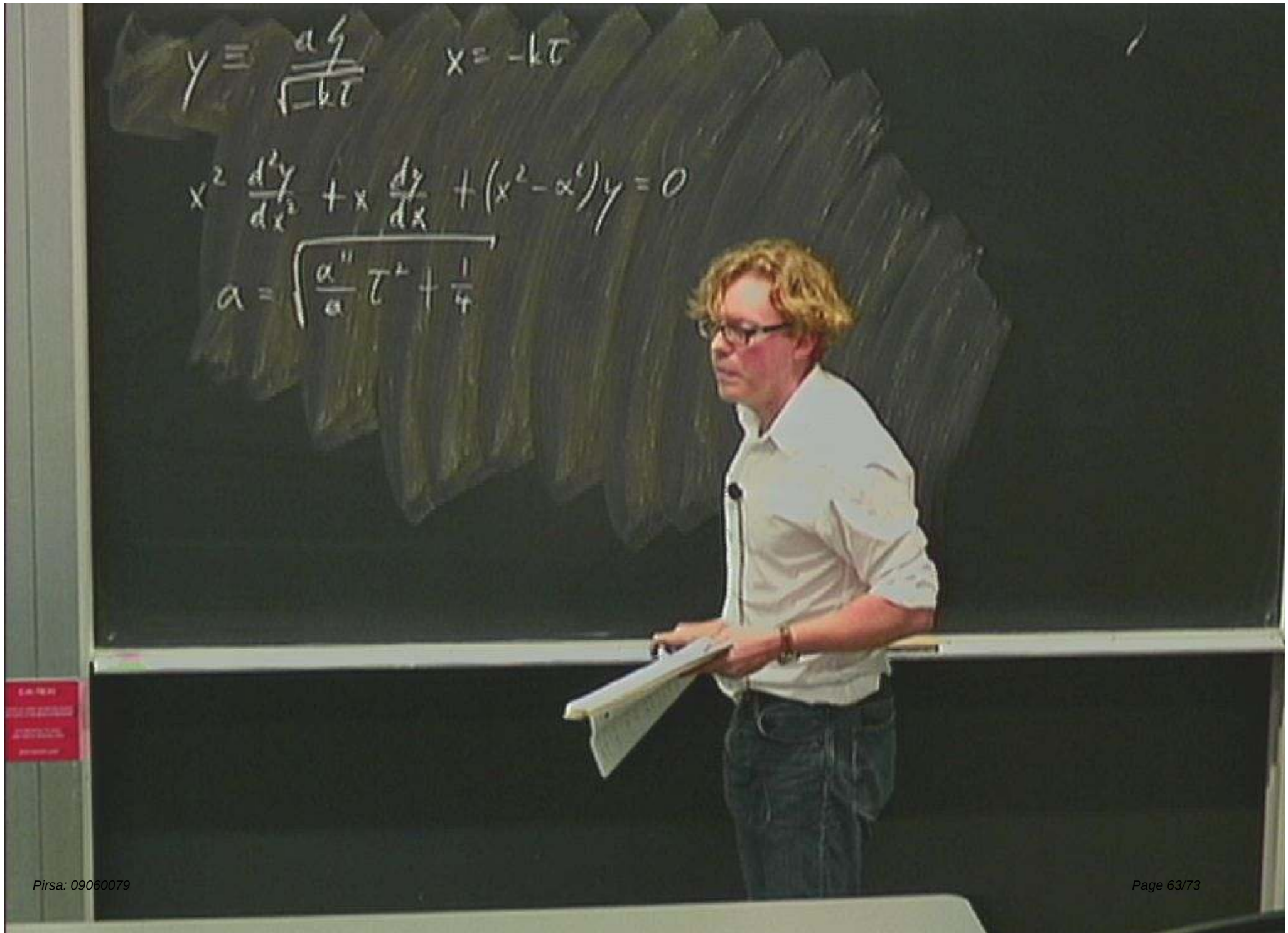

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2} \sim 100$
$V \propto e^{-c\varphi}$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$
conformal time τ : $dt = a\, d\tau$
FRW $-dt^2 + a^2 ds^2$
$= a^2(-d\tau^2 + ds^2)$

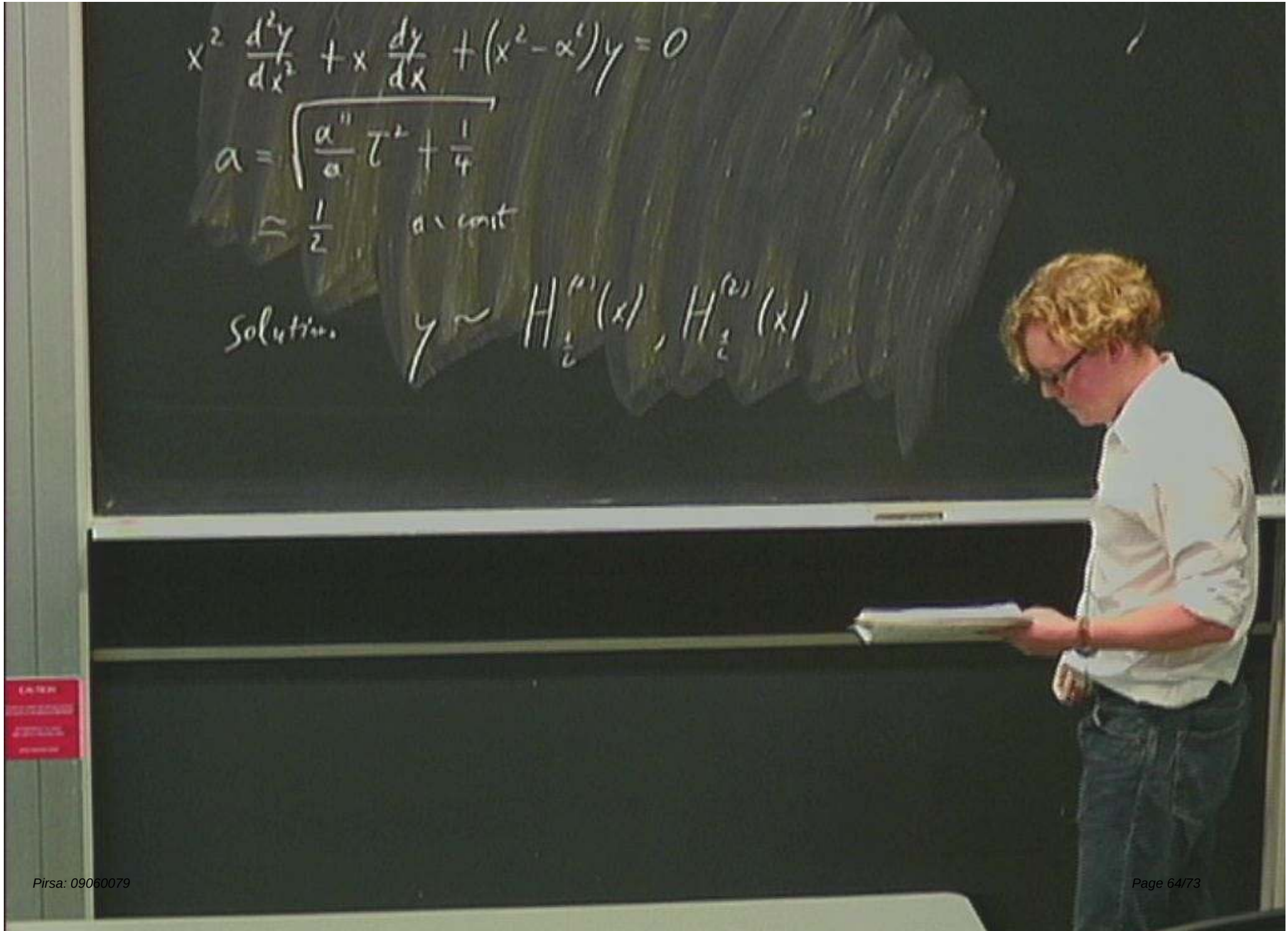

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2}$ ~100
$V \sim c^{-4}\varphi$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$
conformal time τ : $dt = a\, d\tau$
$\frac{d}{d\tau} \equiv '$
$\zeta_k'' + 2\frac{a'}{a}\zeta_k' + k^2\zeta_k = 0$
FRW $-dt^2 + a^2 ds^2$
$= a^2(-d\tau^2 + ds^2)$

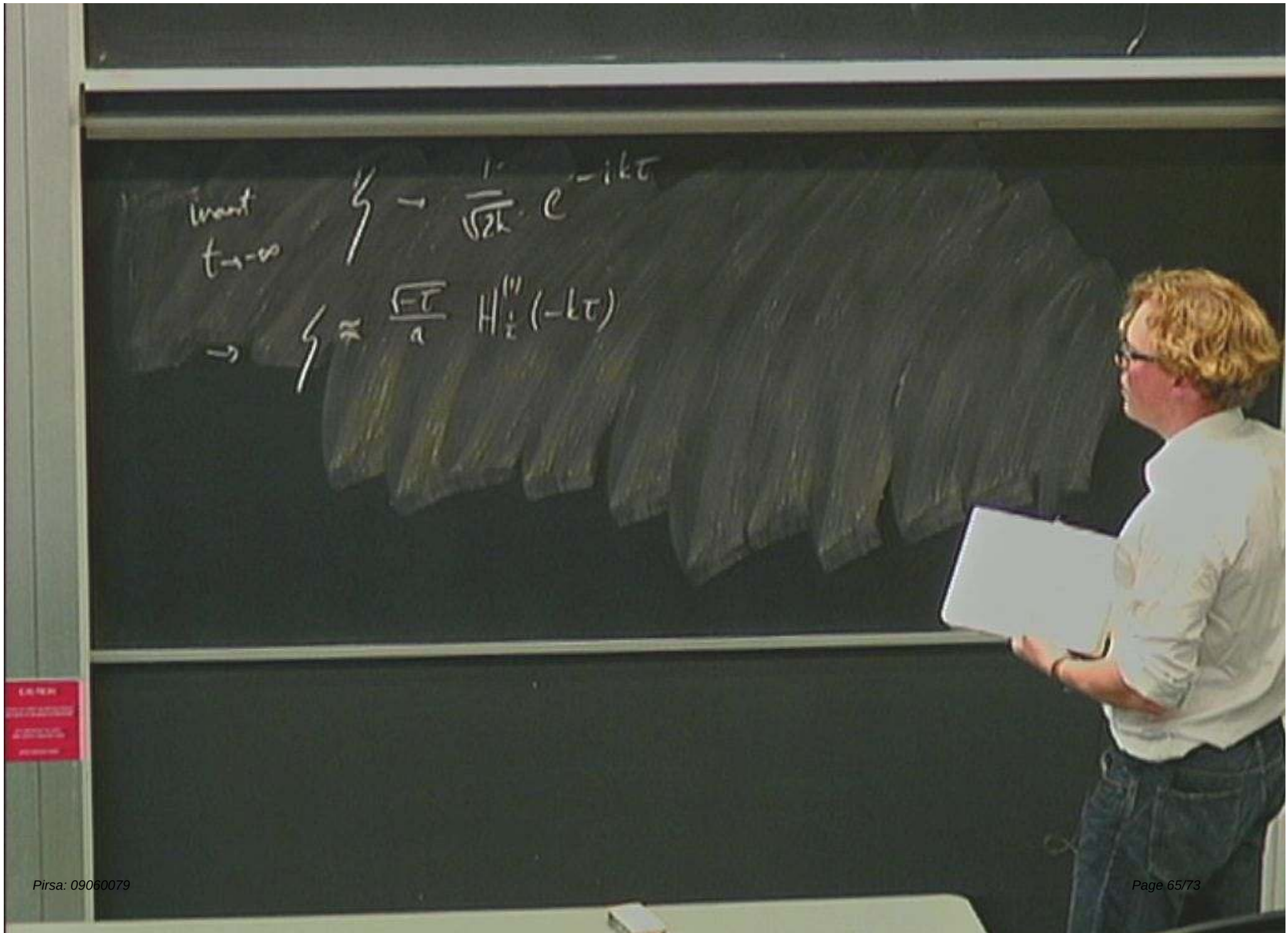

$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2}$ ~100
$V \sim e^{-\varphi}$
ε constant
$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$
conformal time τ : $dt = a\, d\tau$
$\frac{d}{d\tau} \equiv '$
$\zeta_k'' + 2\frac{a'}{a}\zeta_k' + k^2\zeta_k = 0$
FRW $-dt^2 + a^2 dx^2$
$= a^2(-d\tau^2 + dx^2)$

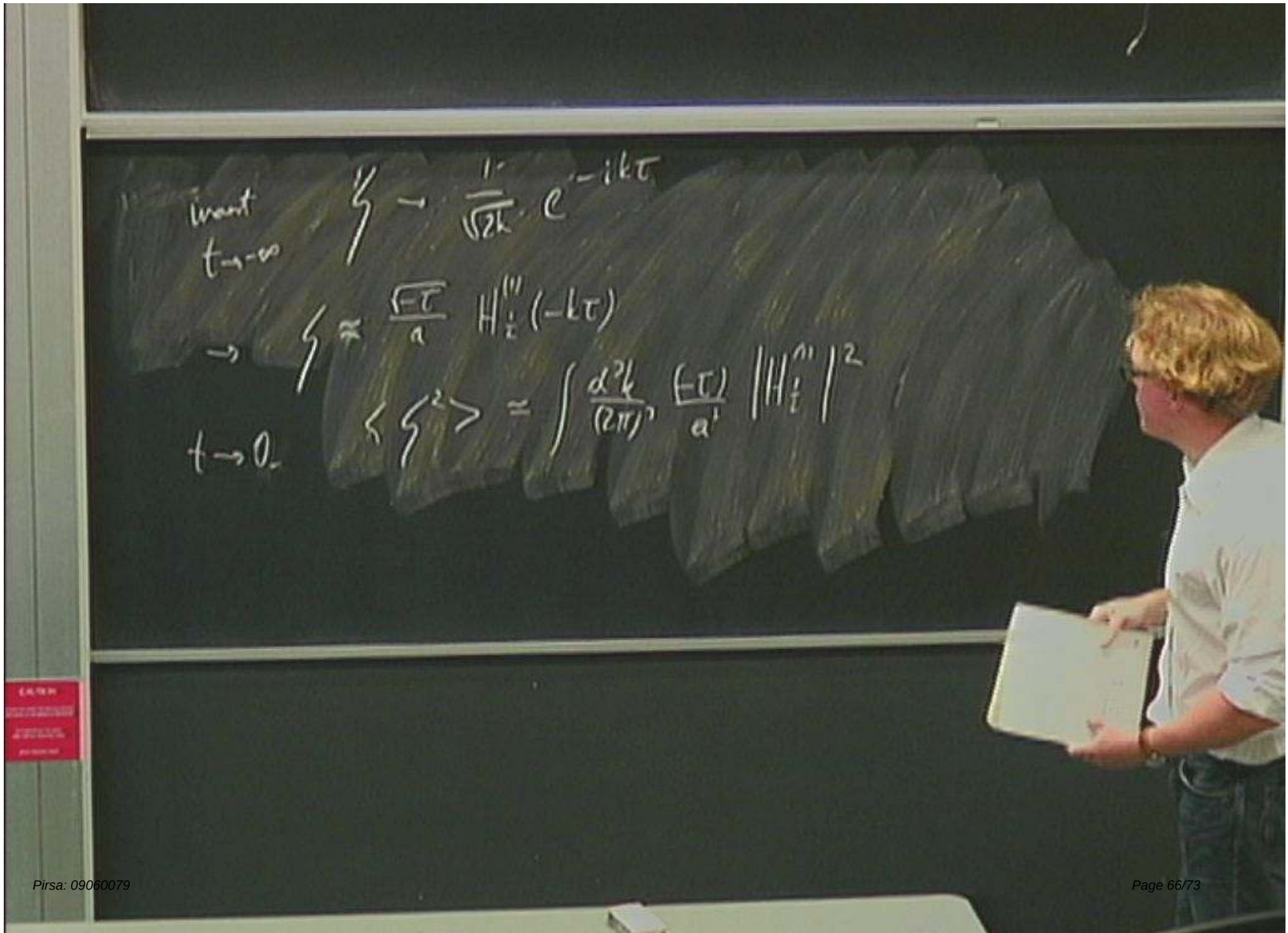

$y \equiv \frac{a\zeta}{\sqrt{-k\tau}}$
$x = -k\tau$

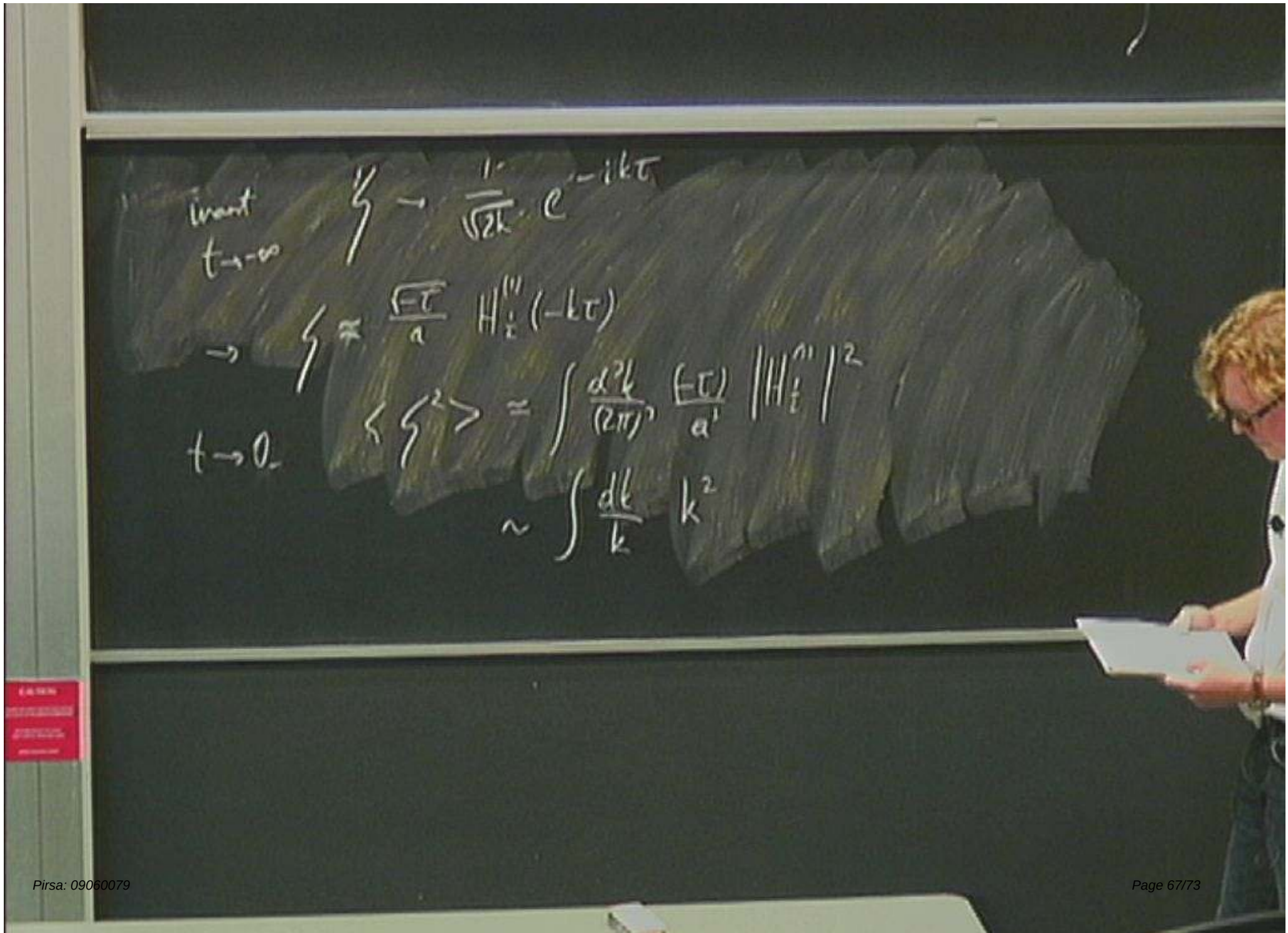

$y \equiv \frac{a\zeta}{\sqrt{-k\tau}}$
$x = -k\tau$
$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2)y = 0$

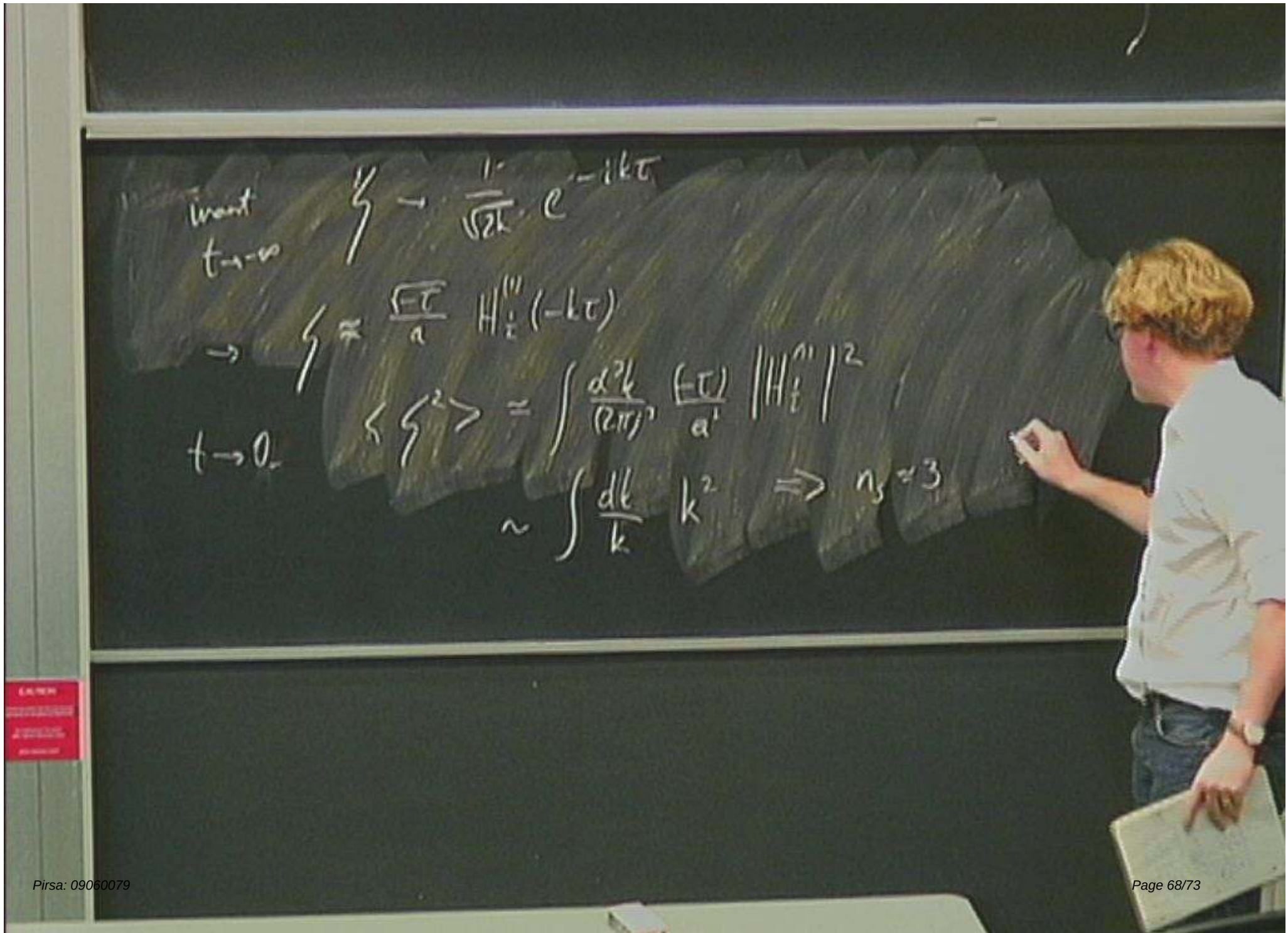

$y \equiv \frac{a\zeta}{\sqrt{-k\tau}}$
$x = -k\tau$
$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2)y = 0$
$\alpha = \sqrt{\frac{a''}{a}\tau^2 + \frac{1}{4}}$

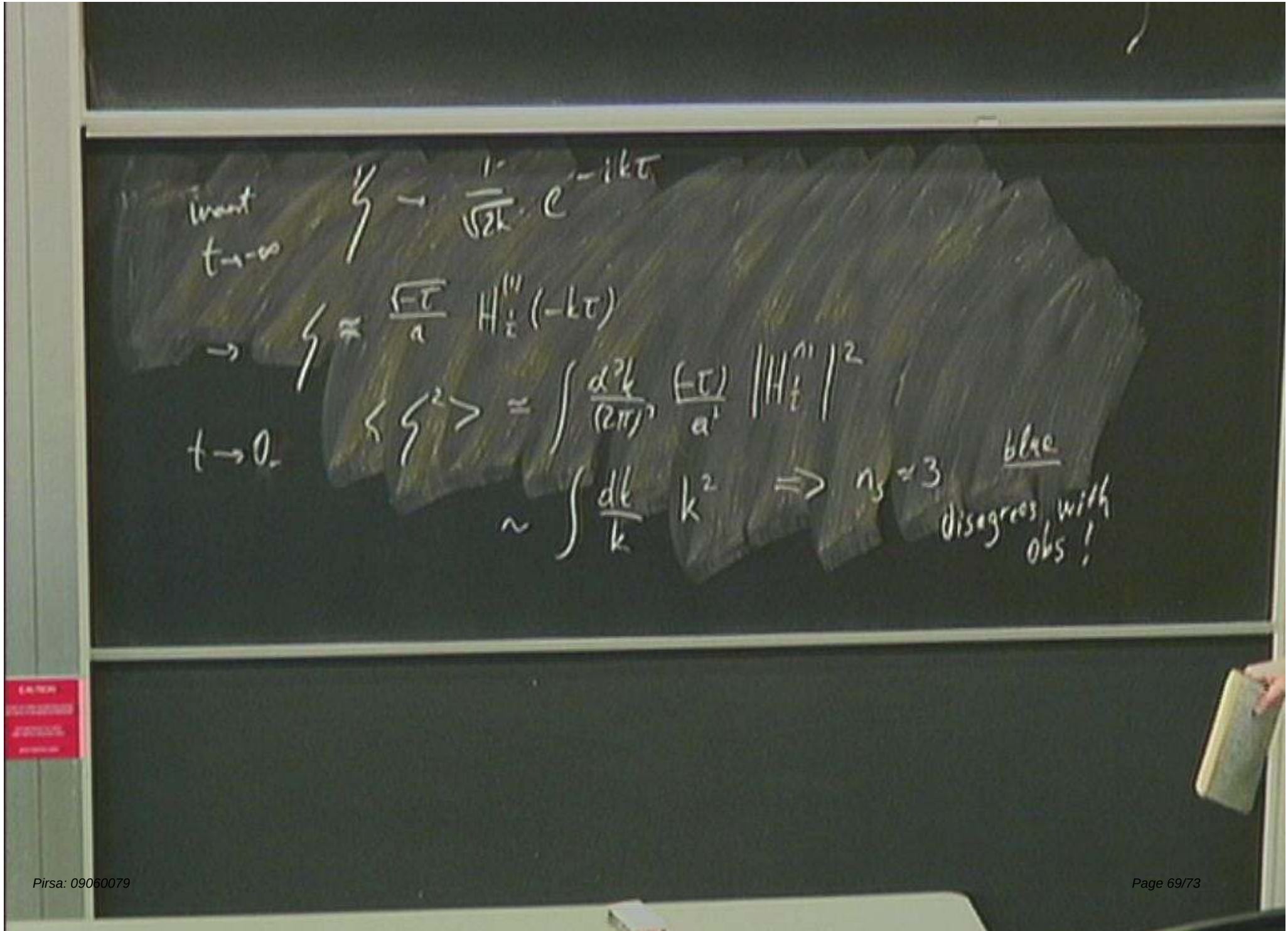

$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0$
$a = \sqrt{\frac{a''}{a} \tau^2 + \frac{1}{4}}$
$\simeq \frac{1}{2}$ a = const
Solution: $y \sim H_{\frac{1}{2}}^{(1)}(x), H_{\frac{1}{2}}^{(2)}(x)$

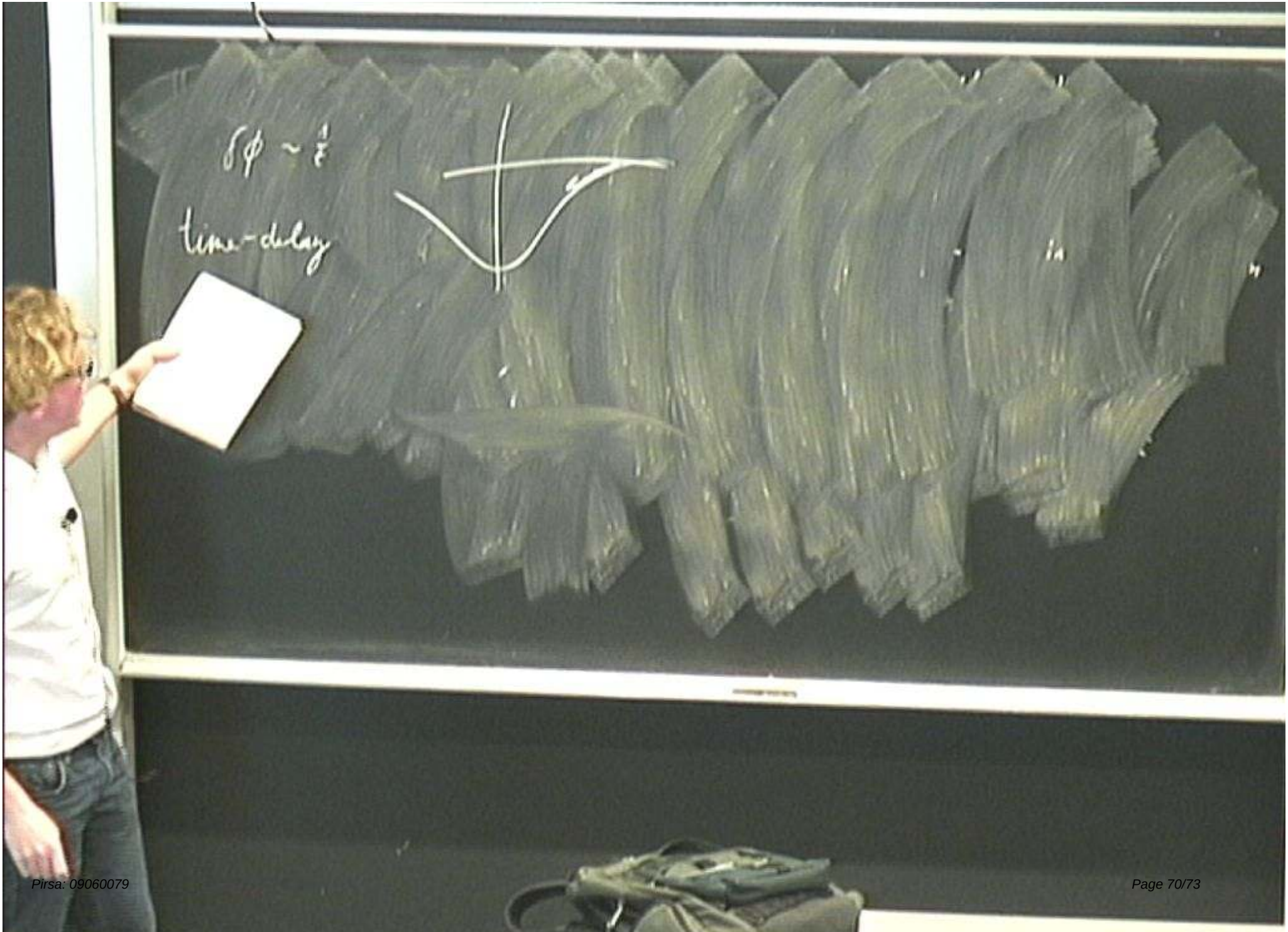

$\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\zeta \approx \frac{\sqrt{\tau}}{a} H^{(1)}_{\frac{1}{2}}(-k\tau)$

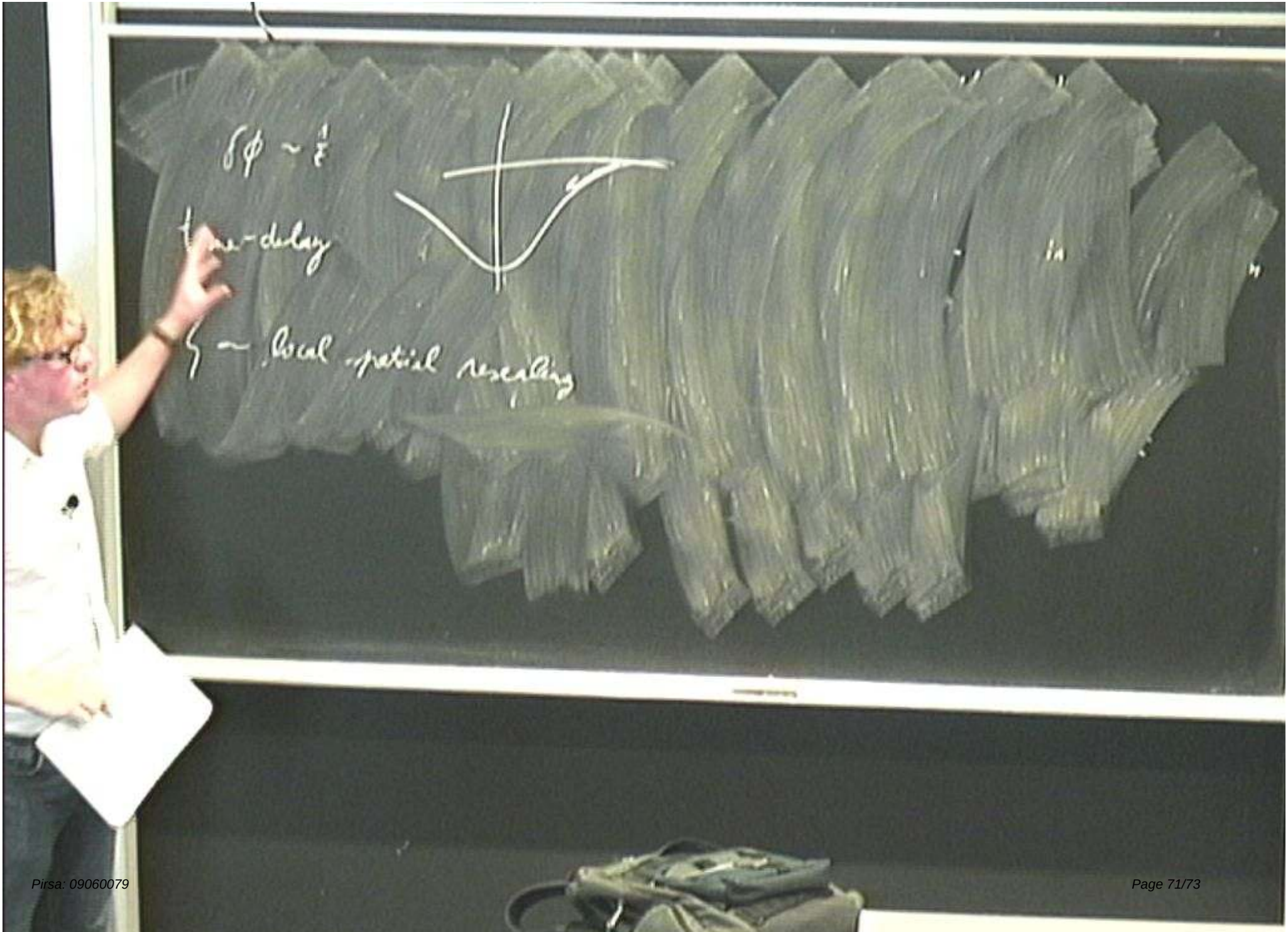

$\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\zeta \approx \frac{\sqrt{-\tau}}{a} H^{(1)}_{\nu}(-k\tau)$
$\langle \zeta^2 \rangle \simeq \int \frac{d^3k}{(2\pi)^3} \frac{(-\tau)}{a^2} |H^{(1)}_{\nu}|^2$

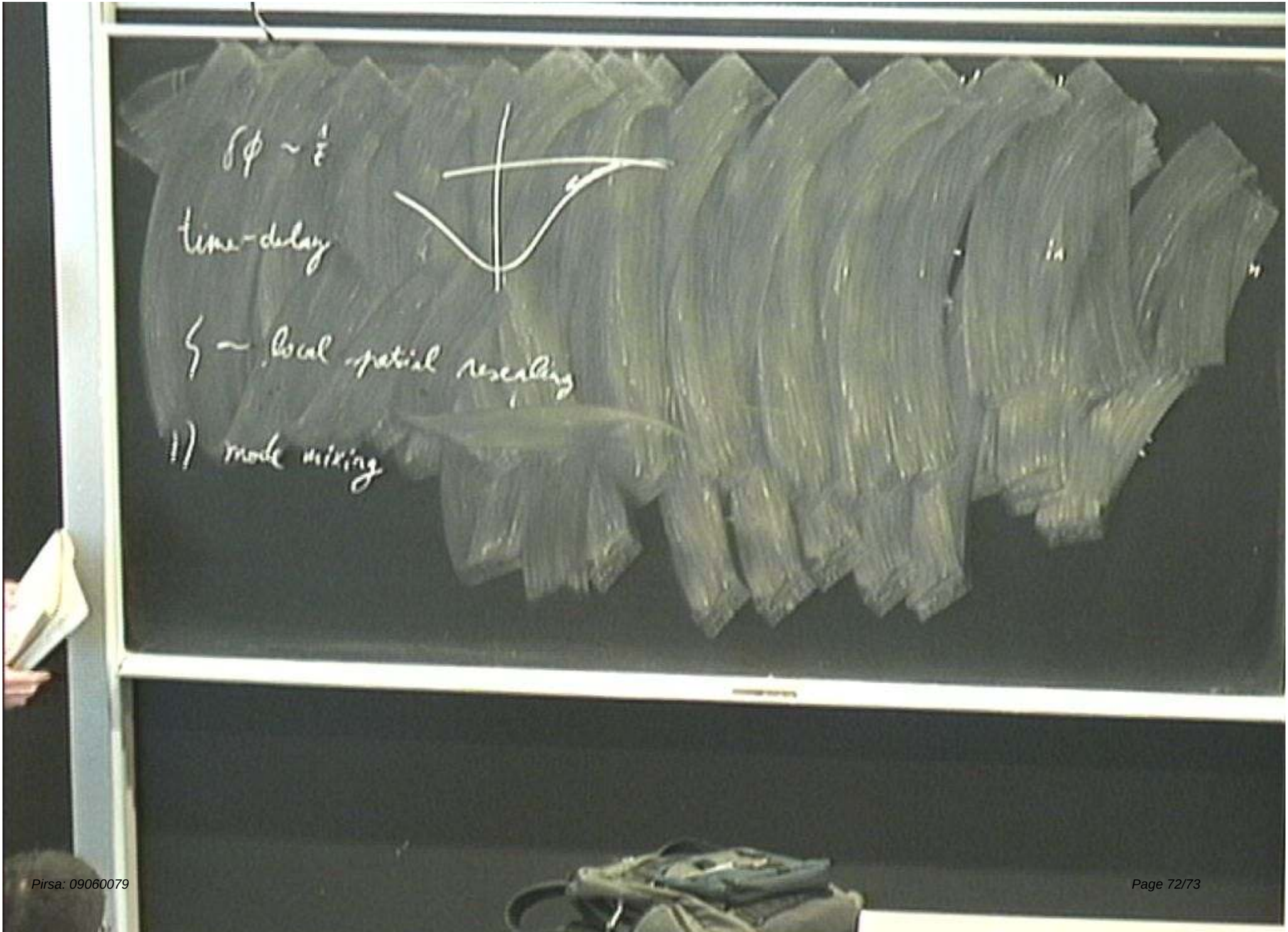

invert
$t \to -\infty$
$\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\to \zeta \propto \frac{H\tau}{a} H_\nu^{(1)}(-k\tau)$
$t \to 0$
$\langle \zeta^2 \rangle \simeq \int \frac{d^3k}{(2\pi)^3} \frac{H\tau}{a^2} \left| H_\nu^{(1)} \right|^2$
$\sim \int \frac{dk}{k} k^2$

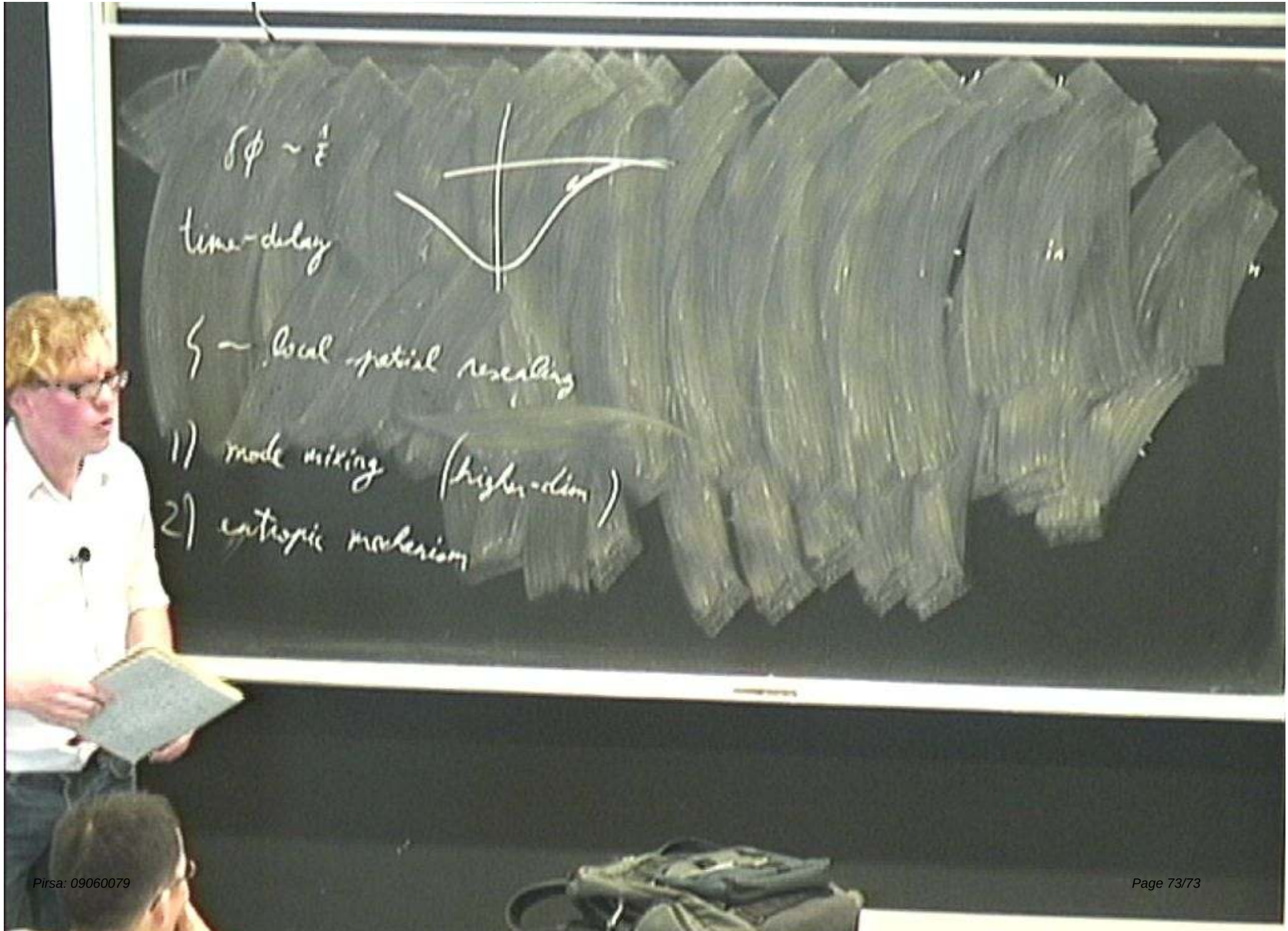

$t \to -\infty$
$\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\to \zeta \propto \frac{-\tau}{a} H_{\frac{1}{2}}^{(1)}(-k\tau)$
$t \to 0$
$\langle \zeta^2 \rangle \propto \int \frac{d^3k}{(2\pi)^3} \frac{(-\tau)}{a^2} |H_{\frac{1}{2}}^{(1)}|^2$
$\sim \int \frac{dk}{k} k^2 \Rightarrow n_s = 3$


instant
$t \to -\infty$
$\zeta \sim \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\to \zeta \propto \frac{\sqrt{-\tau}}{a} H^{(1)}_{\nu}(-k\tau)$
$t \to 0$
$\langle \zeta^2 \rangle \propto \int \frac{d^3k}{(2\pi)^3} \frac{(-\tau)}{a^2} |H^{(1)}_{\nu}|^2$
$\sim \int \frac{dk}{k} k^2 \Rightarrow n_s \simeq 3$
blue
disagrees with obs !


$\delta\phi \sim \frac{1}{t}$
time-delay


δφ ~ 1/ξ
time-delay
ξ ~ local spatial rescaling


$\delta\phi \sim \frac{1}{\xi}$
time-delay
$\zeta \sim$ local spatial rescaling
1) mode mixing


δφ ~ 1/ε
time-delay
ζ ~ local-spatial rescaling
1) mode mixing (higher-dim)
2) anisotropic modulations