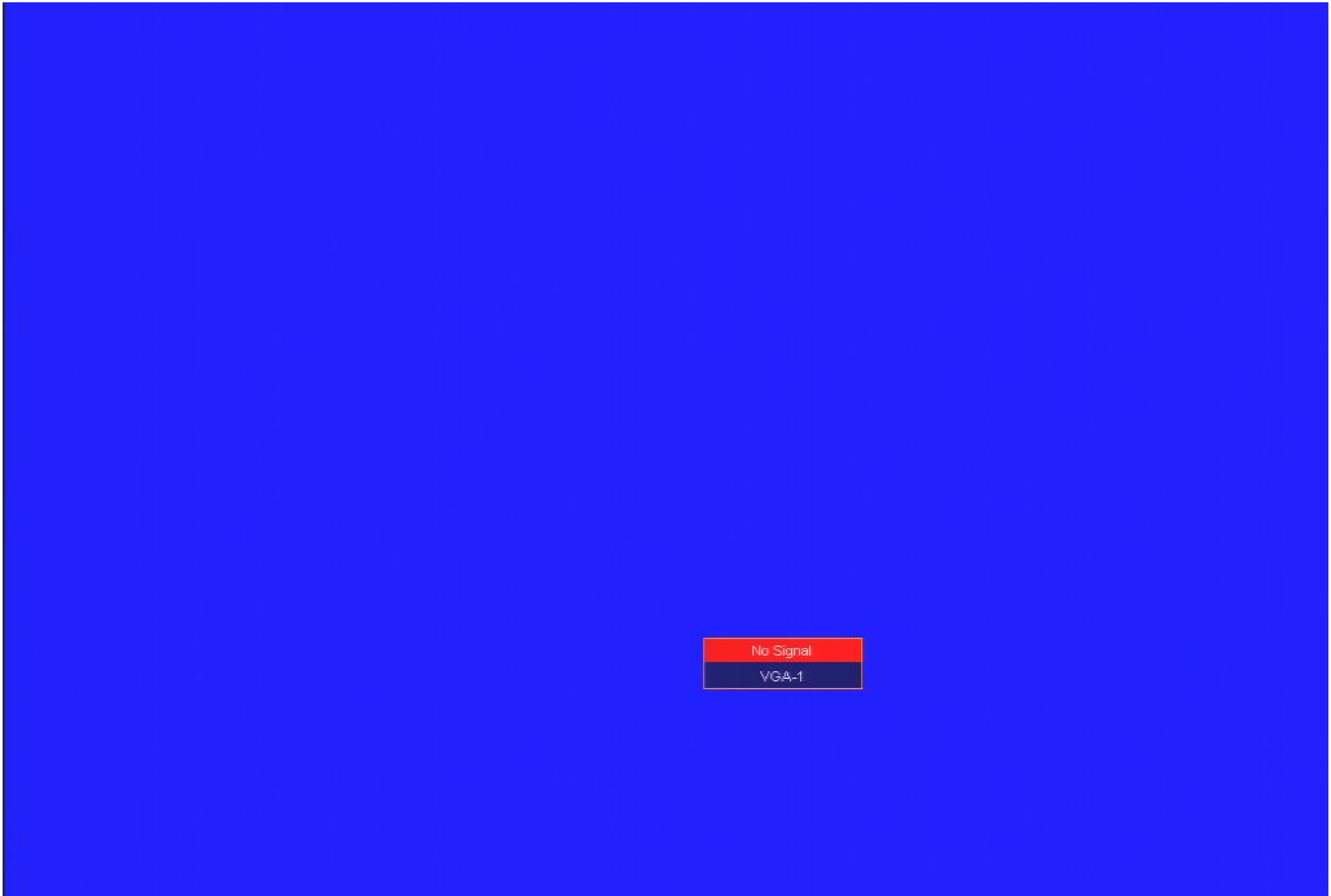
Title: Ekpyrotic/Cyclic Cosmology - Lecture 2

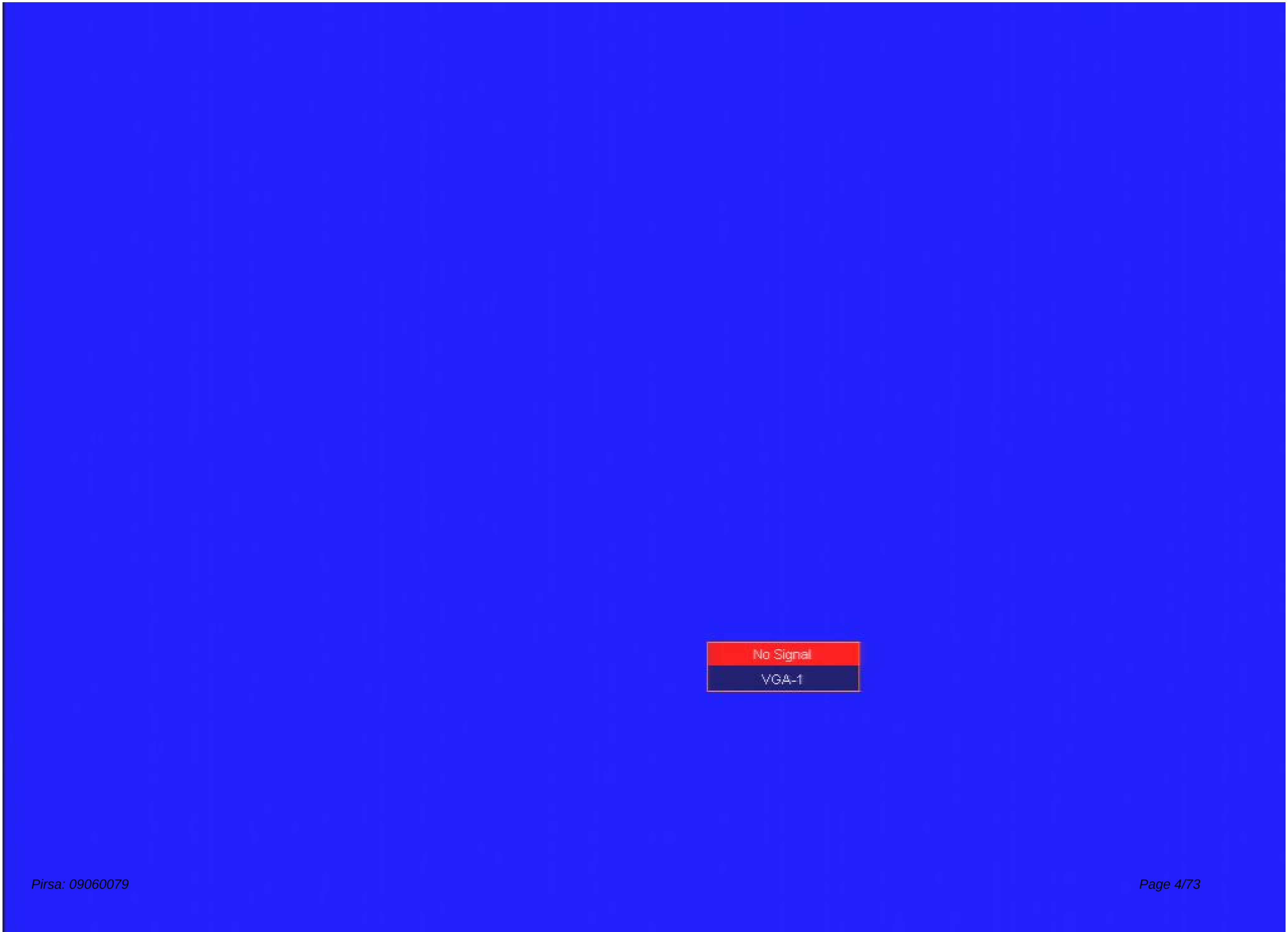
Date: Jun 29, 2009 09:00 AM

URL: http://pirsa.org/09060079

Abstract: TBA

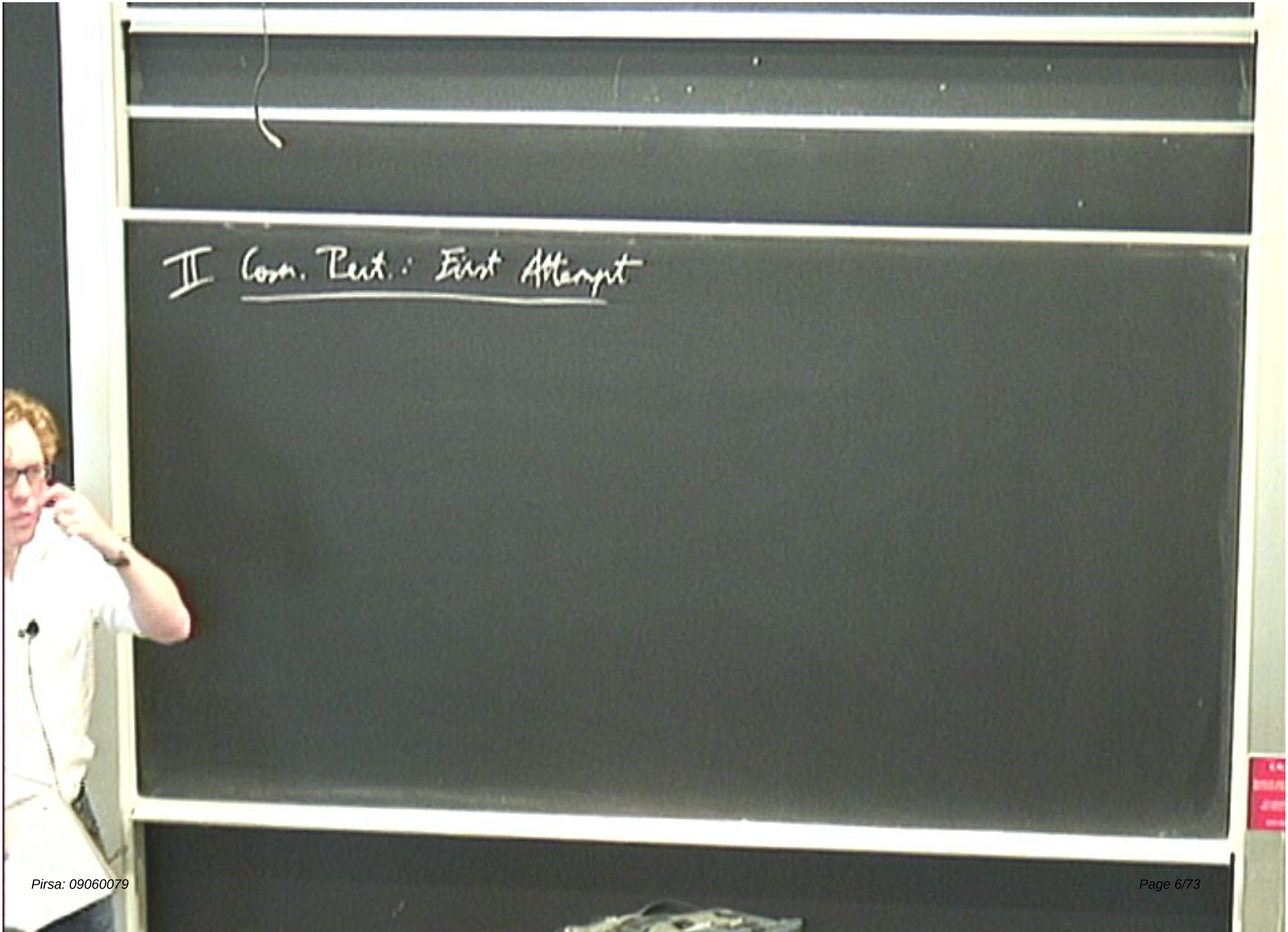

No Signal
VGA-1

No Signal
VGA-1


No Signal
VGA-1

No Signal

VGA-1


II Cosm. Pert.: First Attempt

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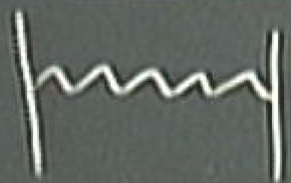
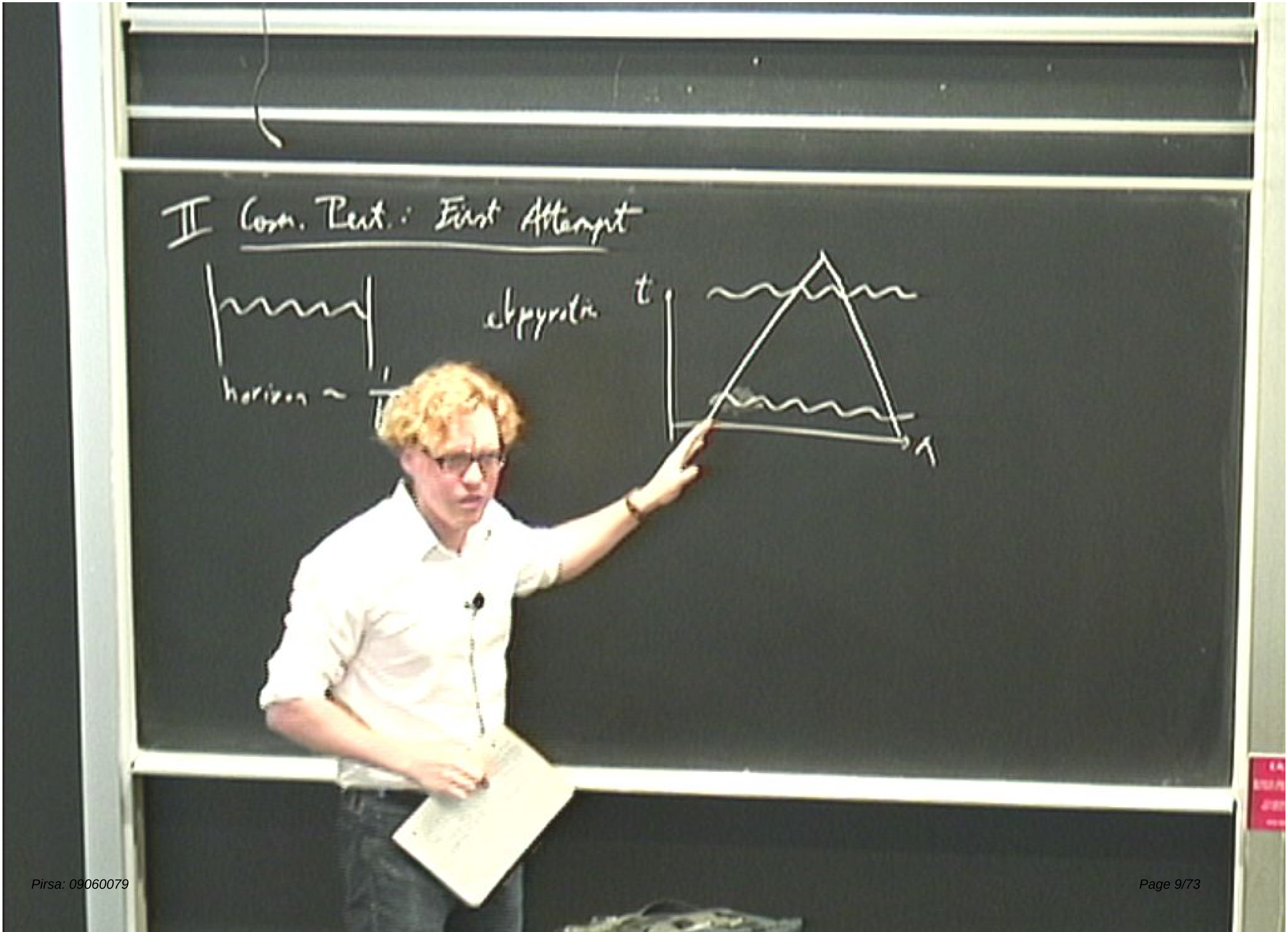
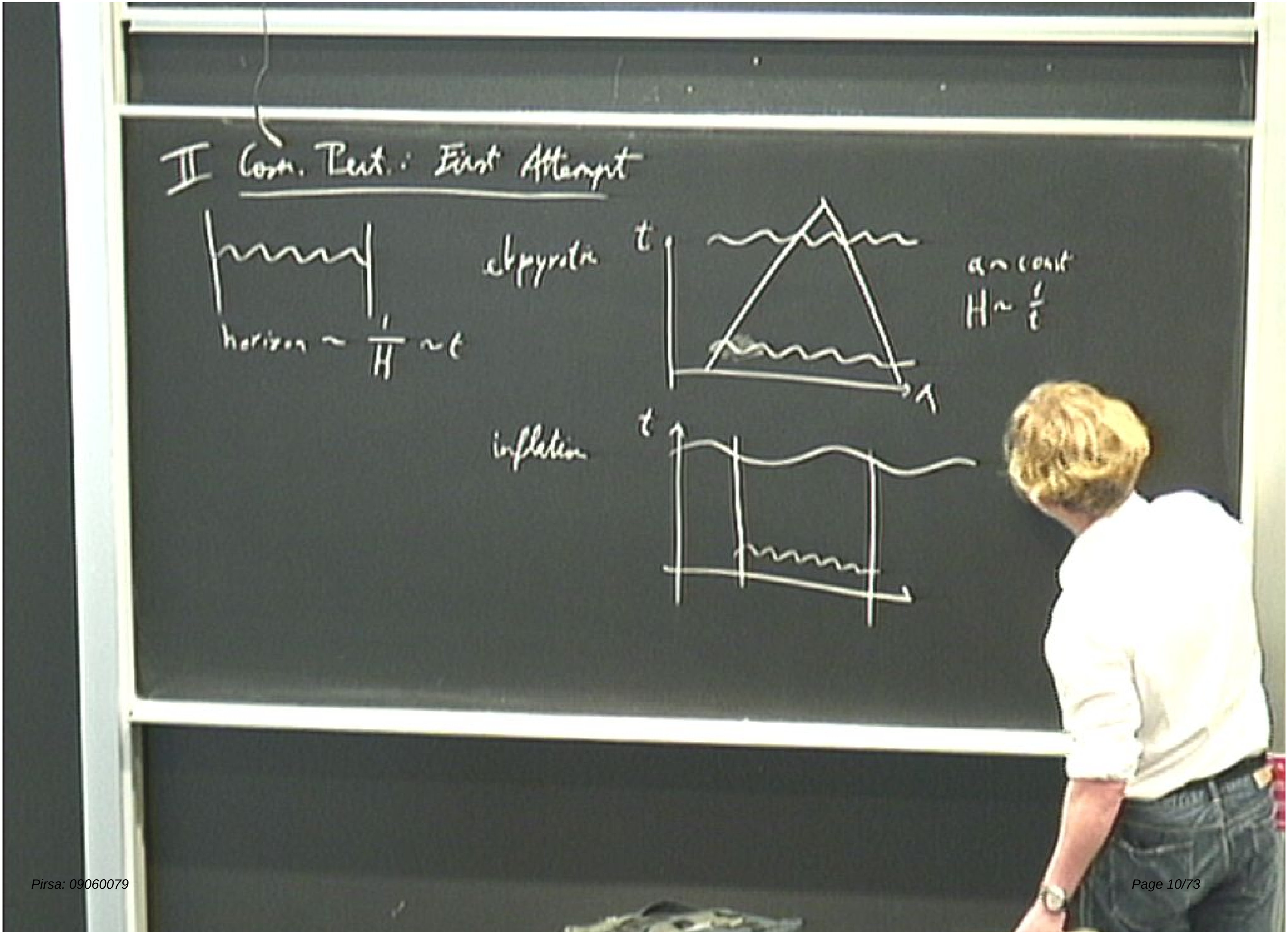


II Cosm. Pert.: First Attempt

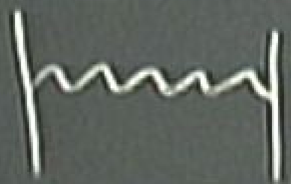
ekpyrotic

horizon $\sim \frac{1}{H} \sim t$


II Cosm. Pert. : First Attempt
ekpyrotic
t
horizon ~

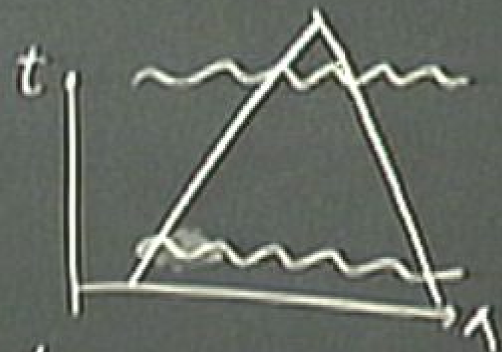

II Cosm. Pert.: First Attempt
horizon ~ 1/H ~ t
ekpyrotic
t
a ~ const
H ~ 1/t
inflation
t

II Cosm. Pert.: First Attempt



horizon $\sim \frac{1}{H} \sim t$

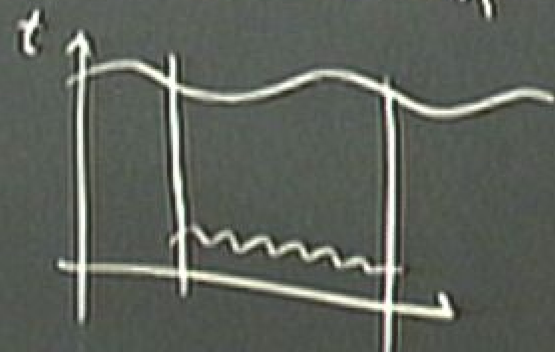
ekpyrotic



$a \sim \text{const}$

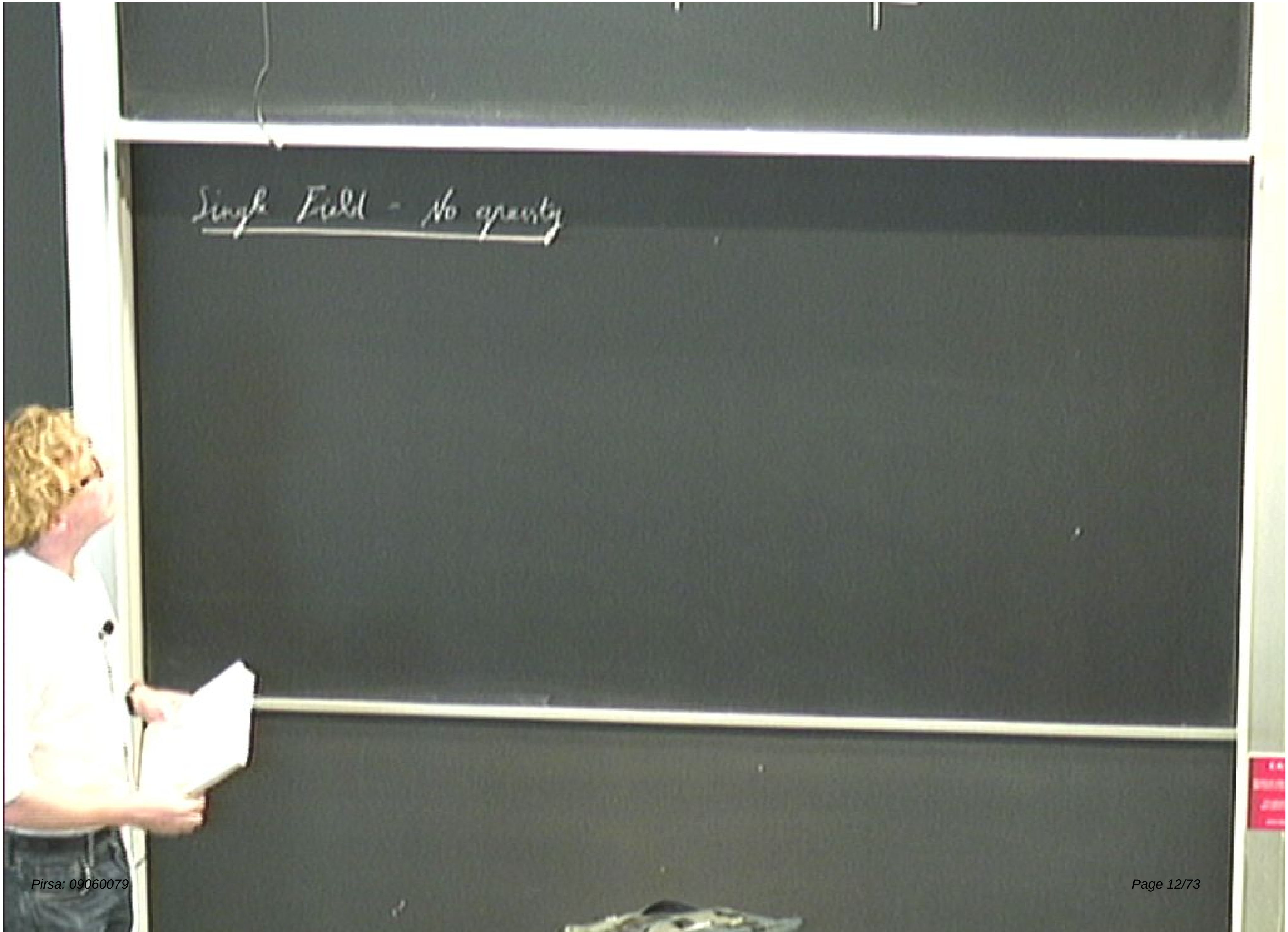
$H \sim \frac{1}{t}$

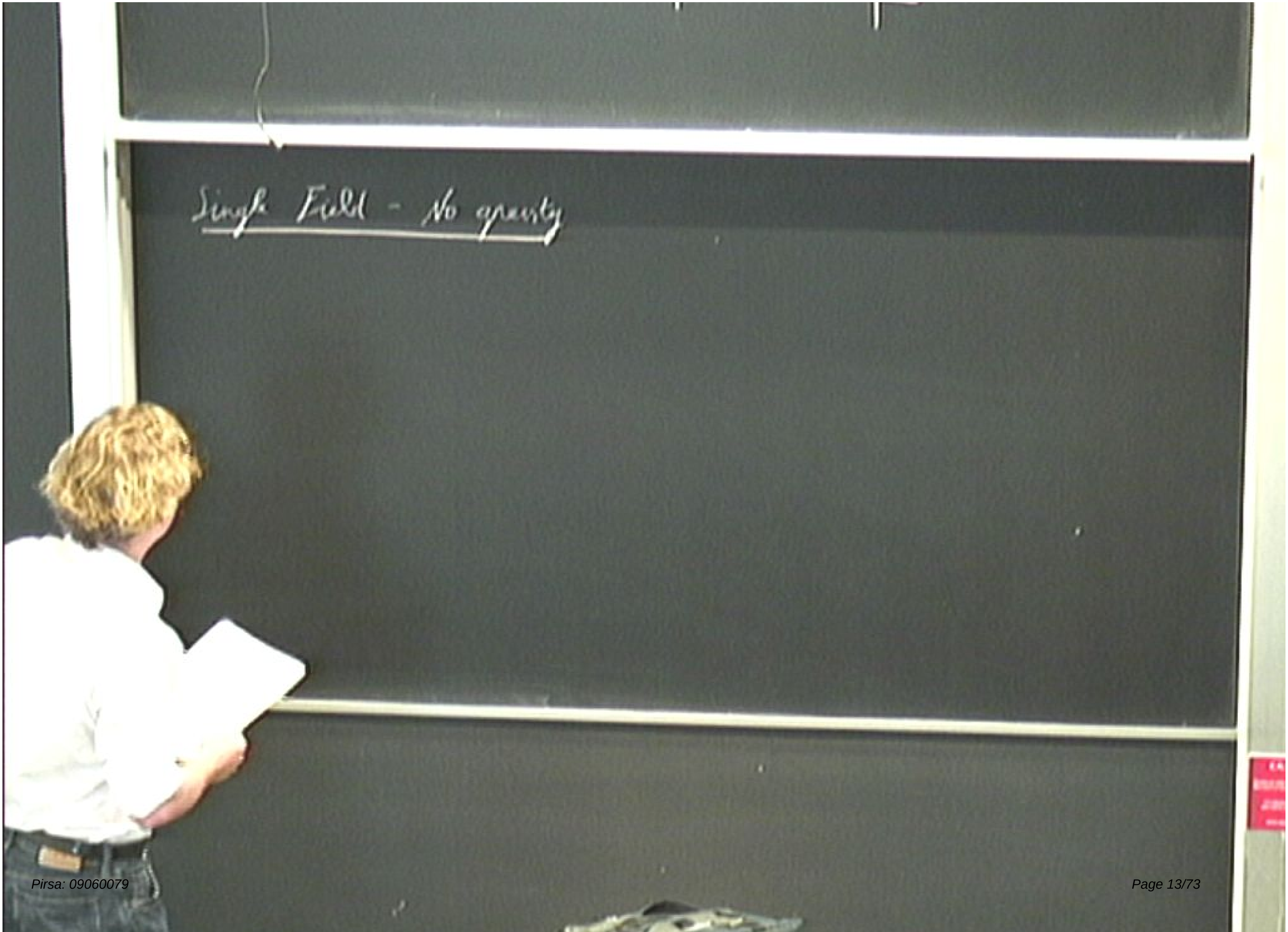
inflation

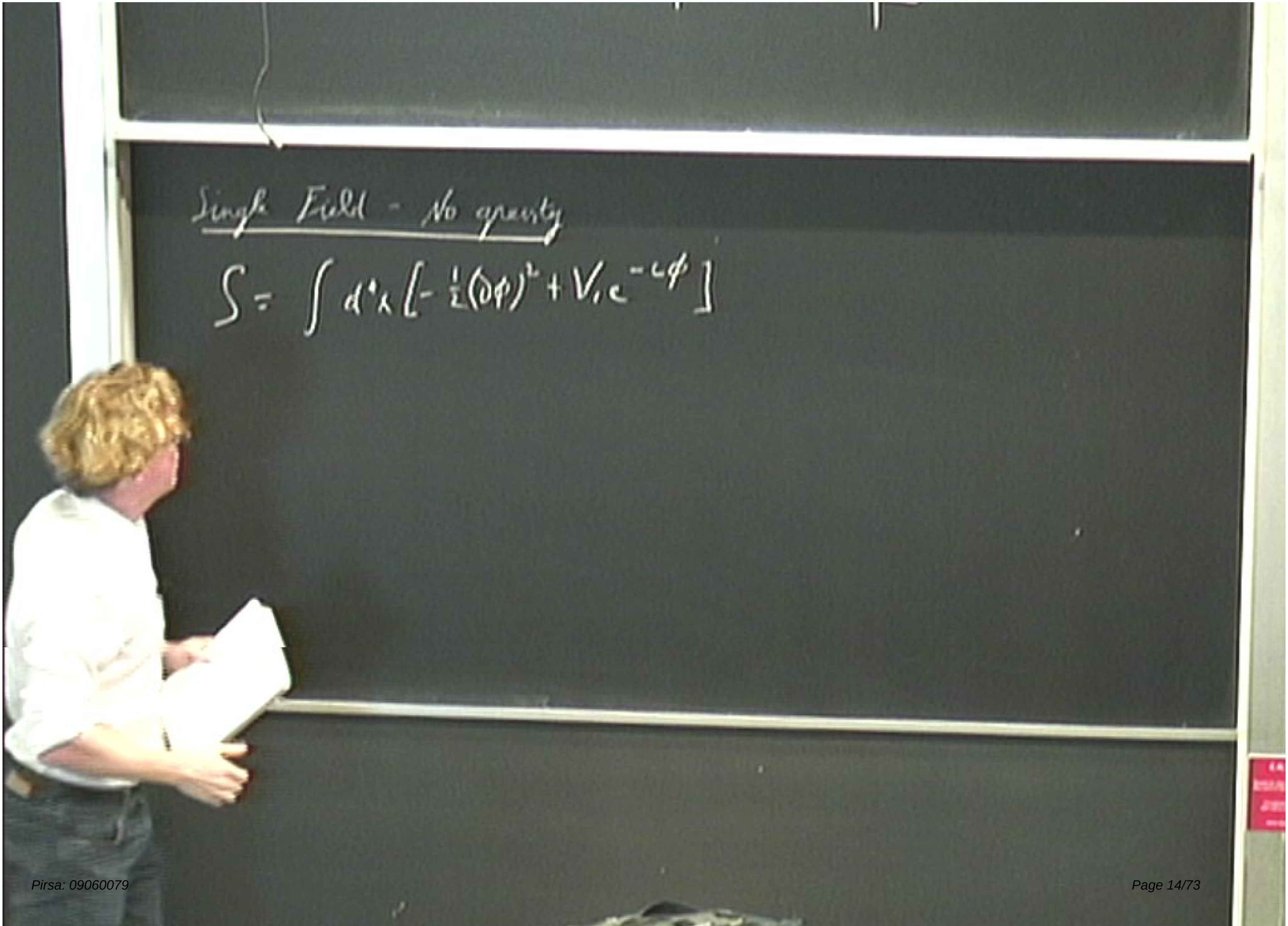


$H \sim \text{const}$

$a \sim e^{Ht}$


Single Field - No gravity


Single Field - No gravity


Single Field - No gravity
$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 + V_1 e^{-c\phi} \right]$

Single Field - No gravity

$$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 + V_0 e^{-c\phi} \right]$$

$$\phi = \bar{\phi}(t) + \delta\phi(t, x)$$

Single Field - No gravity

$$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 + V_1 e^{-c\phi} \right]$$

$$\phi = \bar{\phi}(t) + \delta\phi(t, x)$$

$$\delta\ddot{\phi} - \nabla^2\delta\phi + V_{,\phi\phi}\,\delta\phi = 0$$

Single Field - No gravity

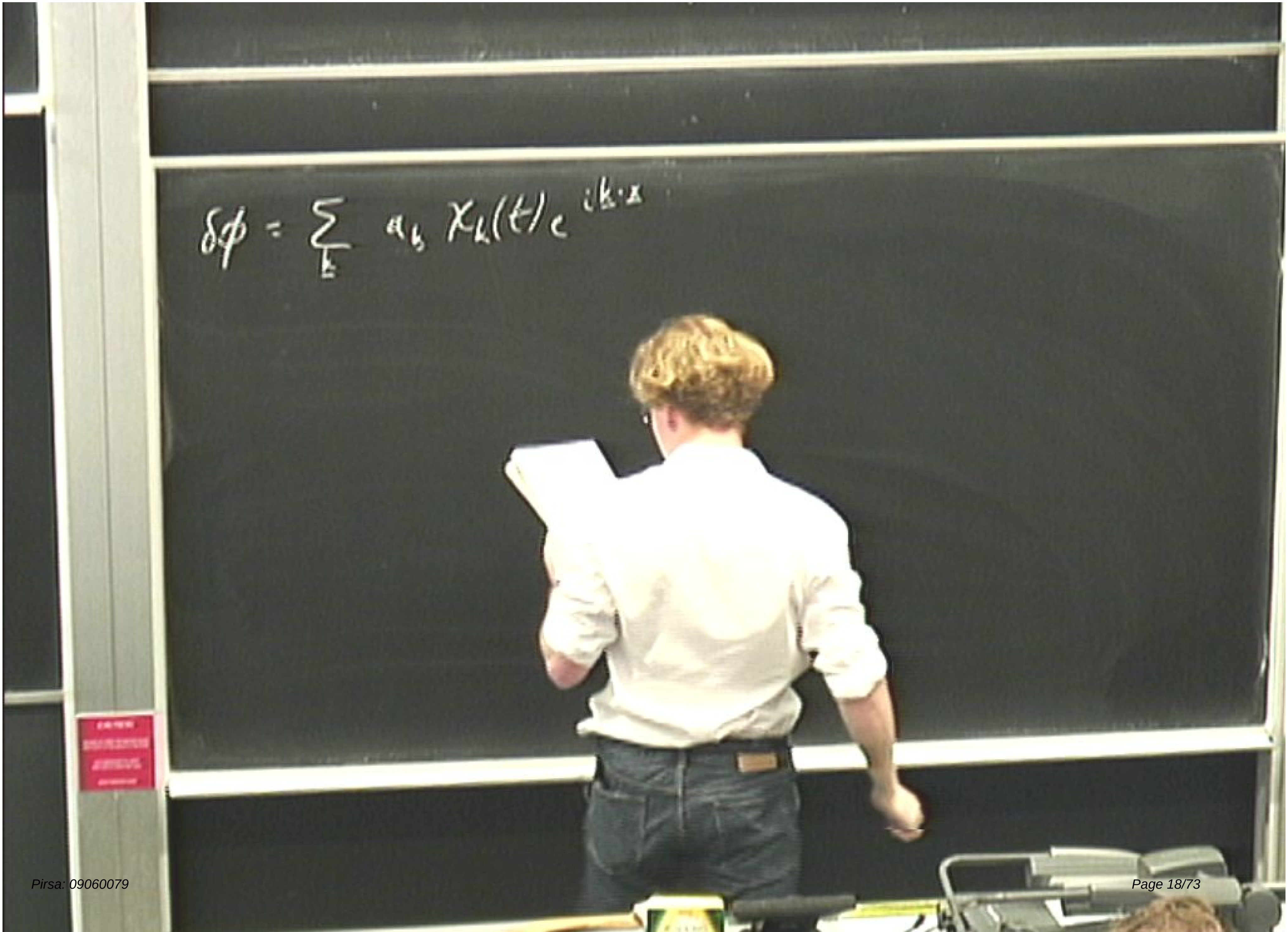
$$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 + V_1 e^{-c\phi} \right]$$

$$\phi = \bar{\phi}(t) + \delta\phi(t, x)$$

$$V = -\frac{1}{c^2 t^2}$$

$$\delta\ddot{\phi} - \nabla^2\delta\phi + V_{,\phi\phi}\,\delta\phi = 0$$

$$V_{,\phi\phi} = c^2 V = -\frac{2}{t^2}$$


$\delta\phi = \sum_k a_k \chi_k(t) e^{ik\cdot x}$

$$\delta\phi = \sum_{\mathbf{k}} \underset{\zeta}{a_{\mathbf{k}}}\, \chi_k(t)\, e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.$$

$$\delta\phi = \sum_{k} a_k \chi_k(t) e^{ik\cdot x} + h.c.$$

$\hookrightarrow$ annihilation operator $\quad a_k|0\rangle = 0$

$$\delta\phi = \sum_{\underline{k}} a_{\underline{k}}\, \chi_{\underline{k}}(t)\, e^{i\underline{k}\cdot\underline{x}} + h.c.$$

↳ positive frequency mode fcts

↳ annihilation operator $\quad a_{\underline{k}}|0\rangle = 0$

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↳ positive frequency mode fcts

↳ annihilation operator $a_k |0\rangle = 0$

$$[a_k, a_{k'}] = 0 = [a^\dagger_k, a^\dagger_{k'}]$$

$$[a_k, a^\dagger_{k'}] = \delta(k - k')$$

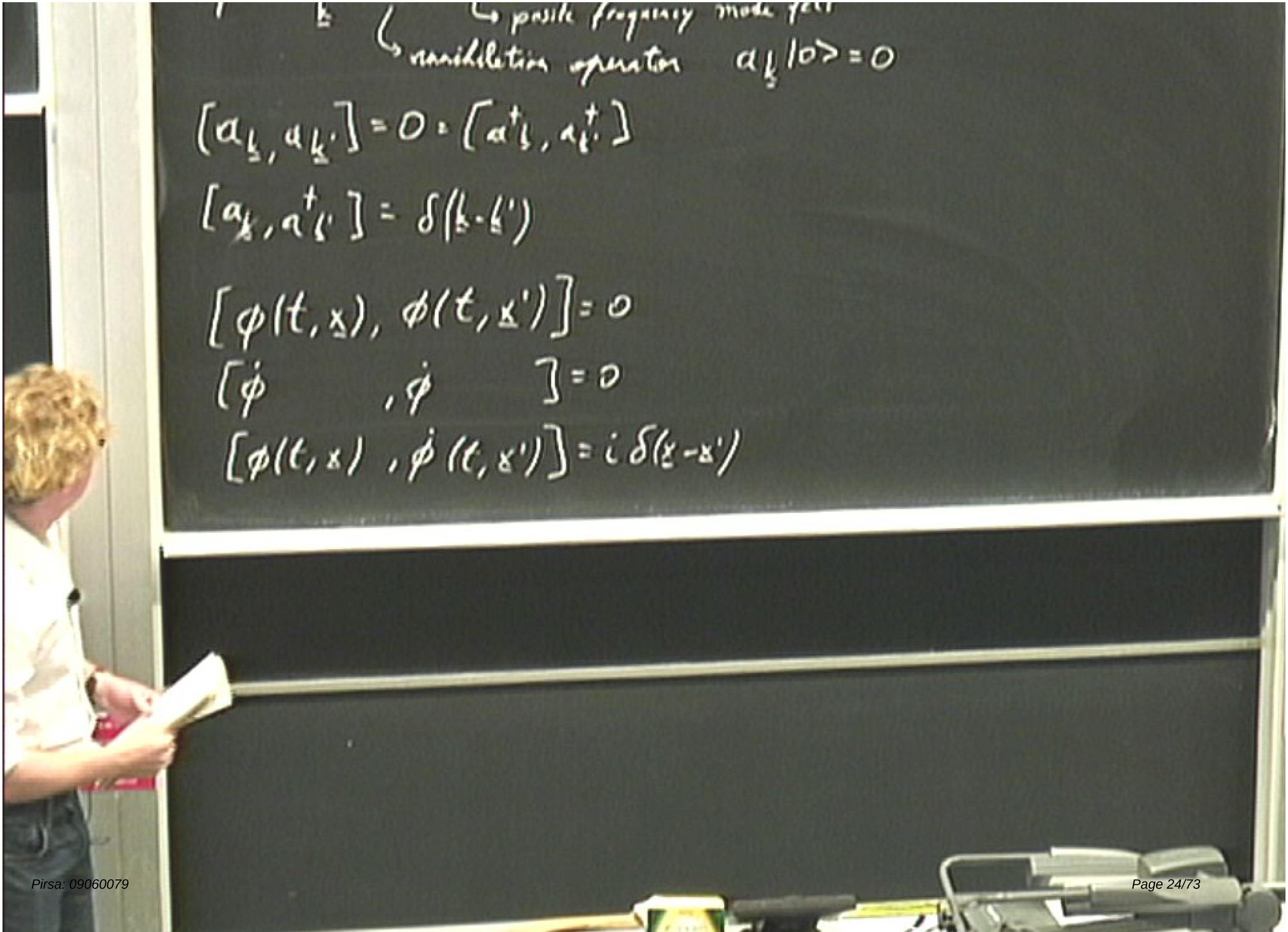
↳ positive frequency mode fn

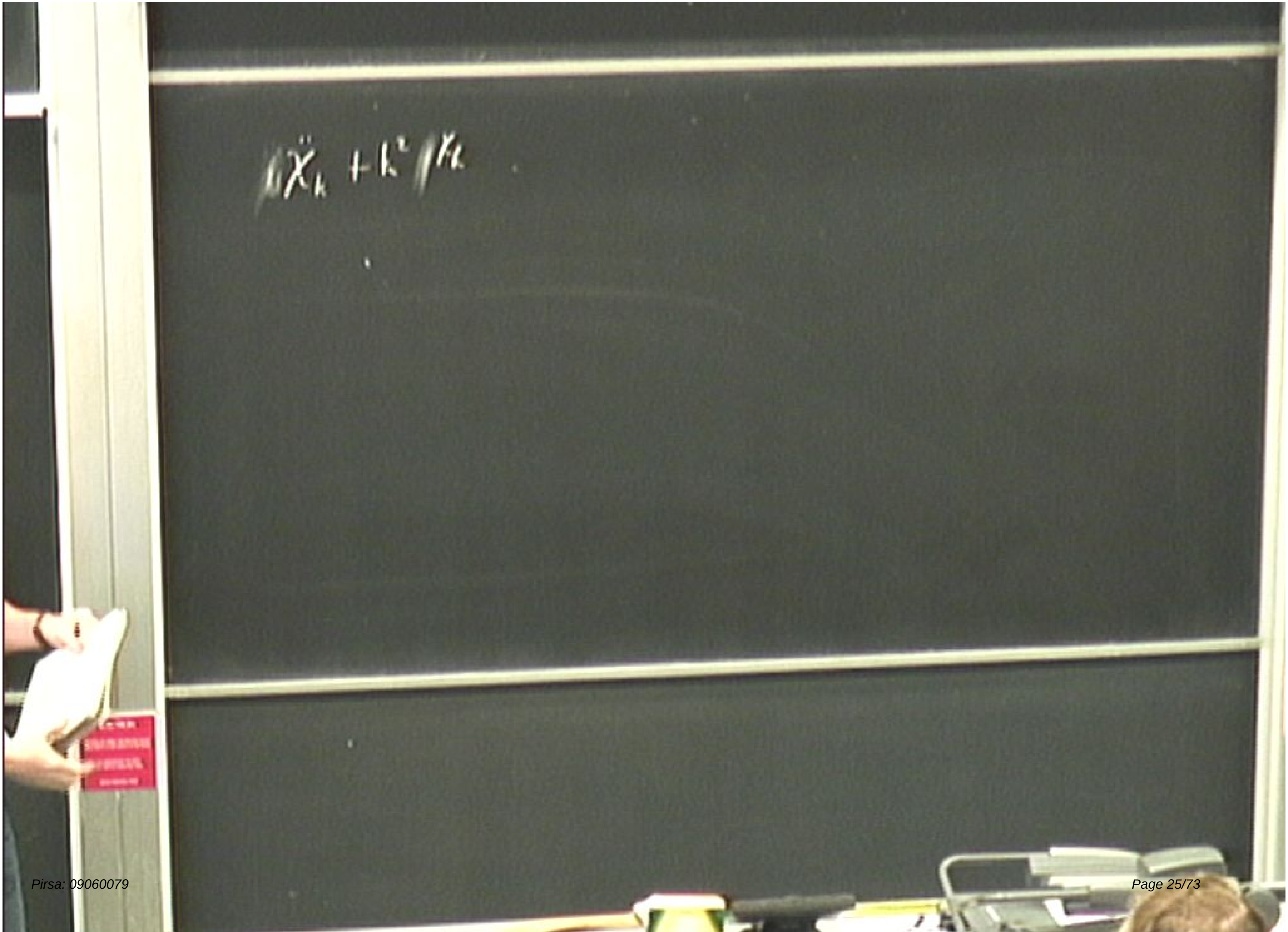
↳ annihilation operator $a_{\underline{k}}|0\rangle = 0$

$$[a_{\underline{k}}, a_{\underline{k}'}] = 0 = [a^\dagger_{\underline{k}}, a^\dagger_{\underline{k}'}]$$

$$[a_{\underline{k}}, a^\dagger_{\underline{k}'}] = \delta(\underline{k}-\underline{k}')$$

$$[\phi(t,\underline{x}), \phi(t,\underline{x}')] = 0$$


↳ positive frequency mode
↳ annihilation operator $a_k|0\rangle = 0$
$[a_k, a_{k'}] = 0 = [a^\dagger_k, a^\dagger_{k'}]$
$[a_k, a^\dagger_{k'}] = \delta(k-k')$
$[\phi(t,x), \phi(t,x')] = 0$
$[\dot\phi, \dot\phi] = 0$
$[\phi(t,x), \dot\phi(t,x')] = i\delta(x-x')$


$\ddot{X}_k + k^2 X_k$

$$\ddot{x}_k + k^2 x_k - \frac{2}{t^2} x_k = 0$$

$$\ddot{\chi}_k + k^2 \chi_k - \frac{2}{t^2}\chi_k = 0$$

2 solutions $e^{-ikt}\left(1 - \frac{i}{kt}\right)$, $e^{ikt}\left(1 + \frac{i}{kt}\right)$

$t \to -\infty$ want $\frac{1}{\sqrt{2k}} e^{-ikt}$

$$\ddot{\chi}_k + k^2 \chi_k - \frac{2}{t^2}\chi_k = 0$$

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sol. $\chi_k = \frac{1}{\sqrt{2k}}e^{-ikt}\left(1-\frac{i}{kt}\right)$

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sol. $X_k = \frac{1}{\sqrt{2k}} e^{-ikt}\left(1-\frac{i}{kt}\right)$ Bunch-Davies vacuum

$|t| \to 0$

$$\ddot{\chi}_k + k^2 \chi_k - \frac{2}{t^2}\chi_k = 0$$

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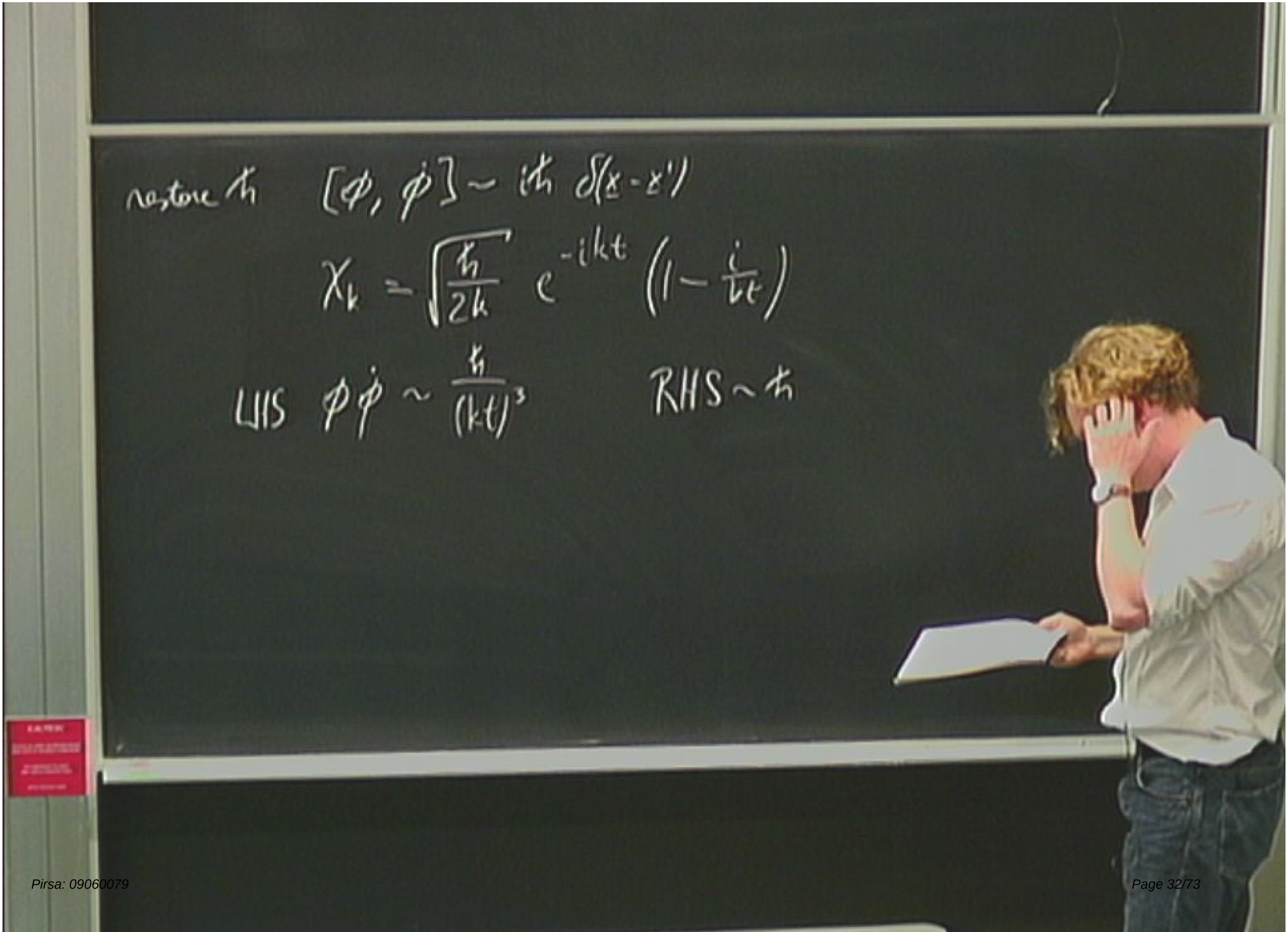
$t \to -\infty$ want $\frac{1}{\sqrt{2k}} e^{-ikt}$

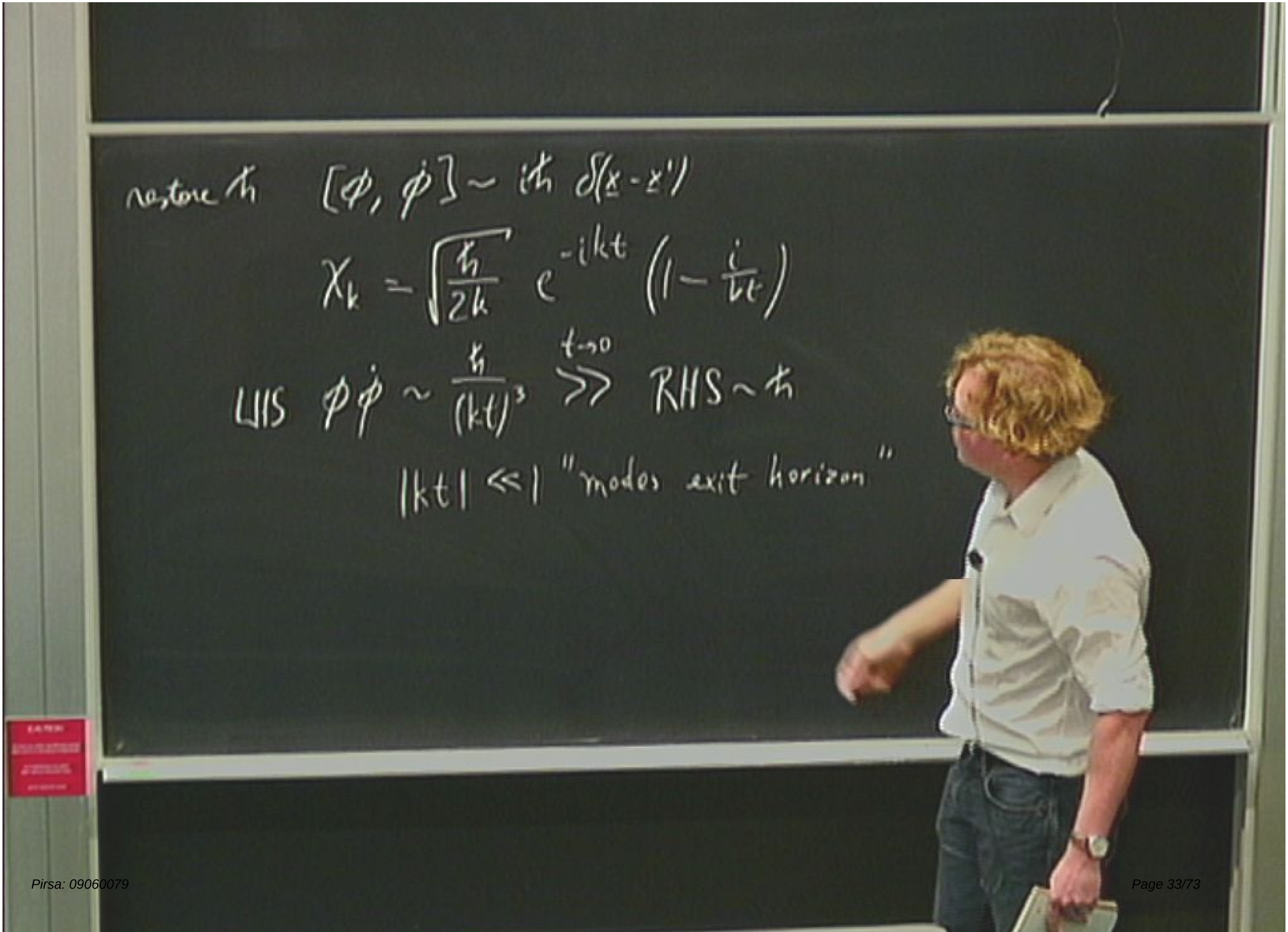
sol. $\chi_k = \frac{1}{\sqrt{2k}} e^{-ikt}\left(1-\frac{i}{kt}\right)$ Bunch-Davies vacuum

$|t| \to 0$ $|\chi_k| = \frac{-1}{\sqrt{2}\, k^{3/2}\, t}$

restore $\hbar$ $[\phi, \dot{\phi}] \sim i\hbar\, \delta(\underline{x} - \underline{x}')$

χ_k


restore $\hbar$ $[\phi, \dot{\phi}] \sim i\hbar\, \delta(\underline{x} - \underline{x}')$
$\chi_k = \sqrt{\frac{\hbar}{2k}}\, e^{-ikt} \left(1 - \frac{i}{kt}\right)$
LHS $\phi \dot{\phi} \sim \frac{\hbar}{(kt)^3}$
RHS $\sim \hbar$


restore $\hbar$ $[\phi, \dot{\phi}] \sim i\hbar\, \delta(\underline{x}-\underline{x}')$
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LHS $\phi\dot{\phi} \sim \frac{\hbar}{(kt)^3} \overset{t\to 0}{\gg}$ RHS $\sim \hbar$
$|kt| \ll 1$ "modes exit horizon"

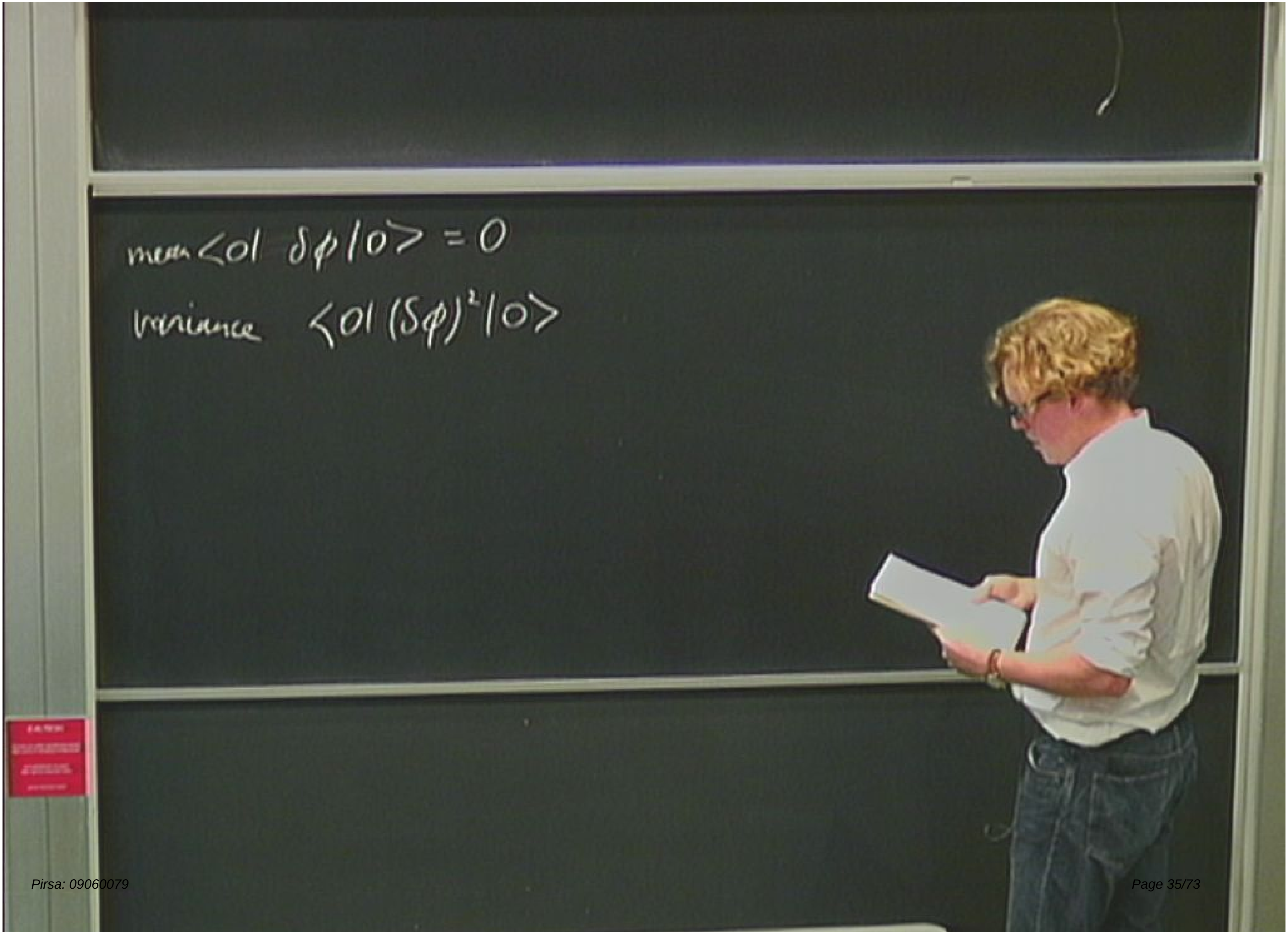
restore $\hbar$ $\quad [\phi, \dot{\phi}] \sim i\hbar\, \delta(\underline{x}-\underline{x}')$

$$\chi_k = \sqrt{\frac{\hbar}{2k}}\; e^{-ikt}\left(1-\frac{i}{kt}\right)$$

LHS $\phi\dot{\phi} \sim \frac{\hbar}{(kt)^3} \overset{t\to 0}{\gg}$ RHS $\sim \hbar$

$|kt| \ll 1$ "modes exit horizon"

port $\to$ classical


mean $\langle 0 | \delta\phi | 0 \rangle = 0$
variance $\langle 0 | (\delta\phi)^2 | 0 \rangle$

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variance $\langle 0 | (\delta\phi)^2 | 0 \rangle \equiv \int \frac{dk}{k} \Delta_\phi^2(k)$

$\hookrightarrow$ variance

$$\Delta_\phi^2(k) = \Delta_\phi^2(k_0) \left(\frac{k}{k_0}\right)^{n_s - 1}$$

mean $\langle 0|\,\delta\phi\,|0\rangle = 0$

variance $\langle 0|(\delta\phi)^2|0\rangle \equiv \int \frac{dk}{k}\,\Delta_\phi^2(k)$

variance

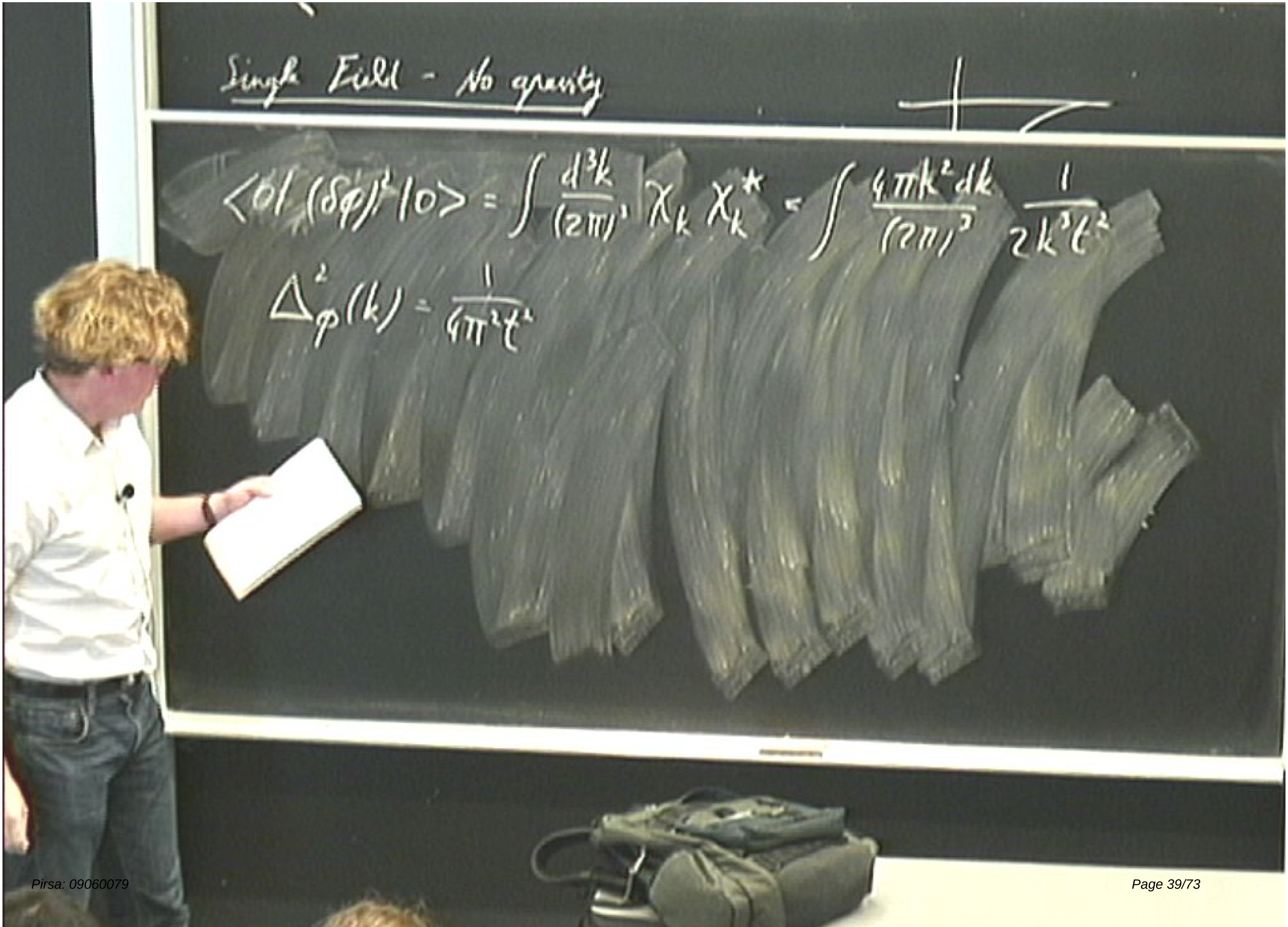
$$\Delta_\phi^2(k) = \Delta_\phi^2(k_0)\left(\frac{k}{k_0}\right)^{n_s-1} \quad \text{(scalar)}$$

spectral index

reference scale

Single Field - No gravity

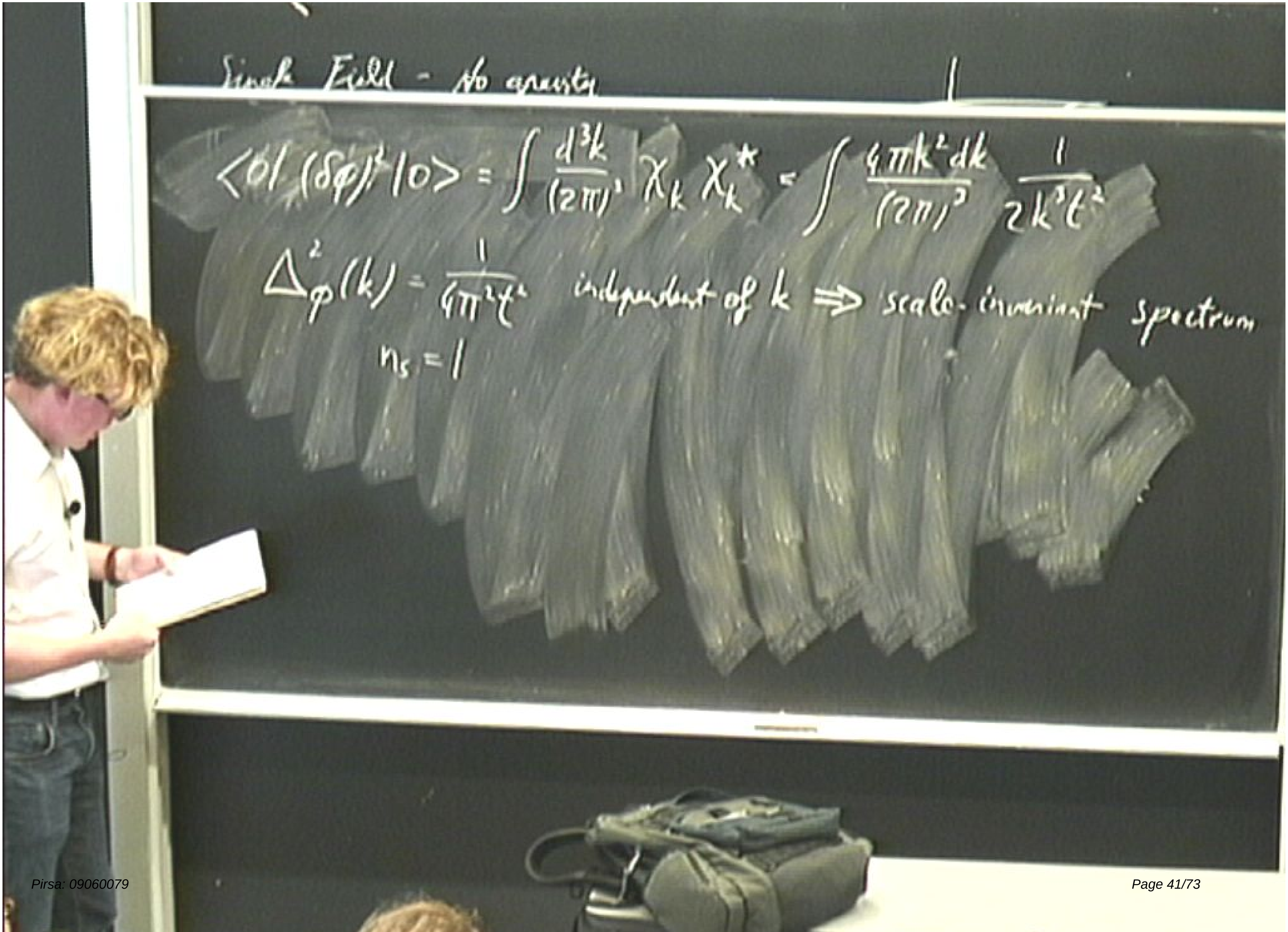
$$\langle 0|\,(\delta\varphi)^2\,|0\rangle = \int \frac{d^3k}{(2\pi)^3}\,\chi_k\,\chi_k^*$$


Single Field - No gravity
$\langle 0 | (\delta\varphi)^2 | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} \chi_k \chi_k^* = \int \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2k^3 t^2}$
$\Delta^2_\varphi(k) = \frac{1}{4\pi^2 t^2}$

Single Field - No gravity

$$\langle 0|\,(\delta\varphi)^2\,|0\rangle = \int \frac{d^3k}{(2\pi)^3}\,\chi_k\,\chi_k^* = \int \frac{4\pi k^2\,dk}{(2\pi)^3}\,\frac{1}{2k^3 t^2}$$

$\Delta^2_\varphi(k) = \dfrac{1}{4\pi^2 t^2}$ independent of $k \Rightarrow$ scale-invariant spectrum


Single Field - No gravity
$\langle 0| (\delta\phi)^2 |0\rangle = \int \frac{d^3k}{(2\pi)^3} \chi_k \chi_k^* = \int \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2k^3 t^2}$
$\Delta^2_\phi(k) = \frac{1}{4\pi^2 t^2}$ independent of $k \Rightarrow$ scale-invariant spectrum
$n_s = 1$

Single Field - No gravity

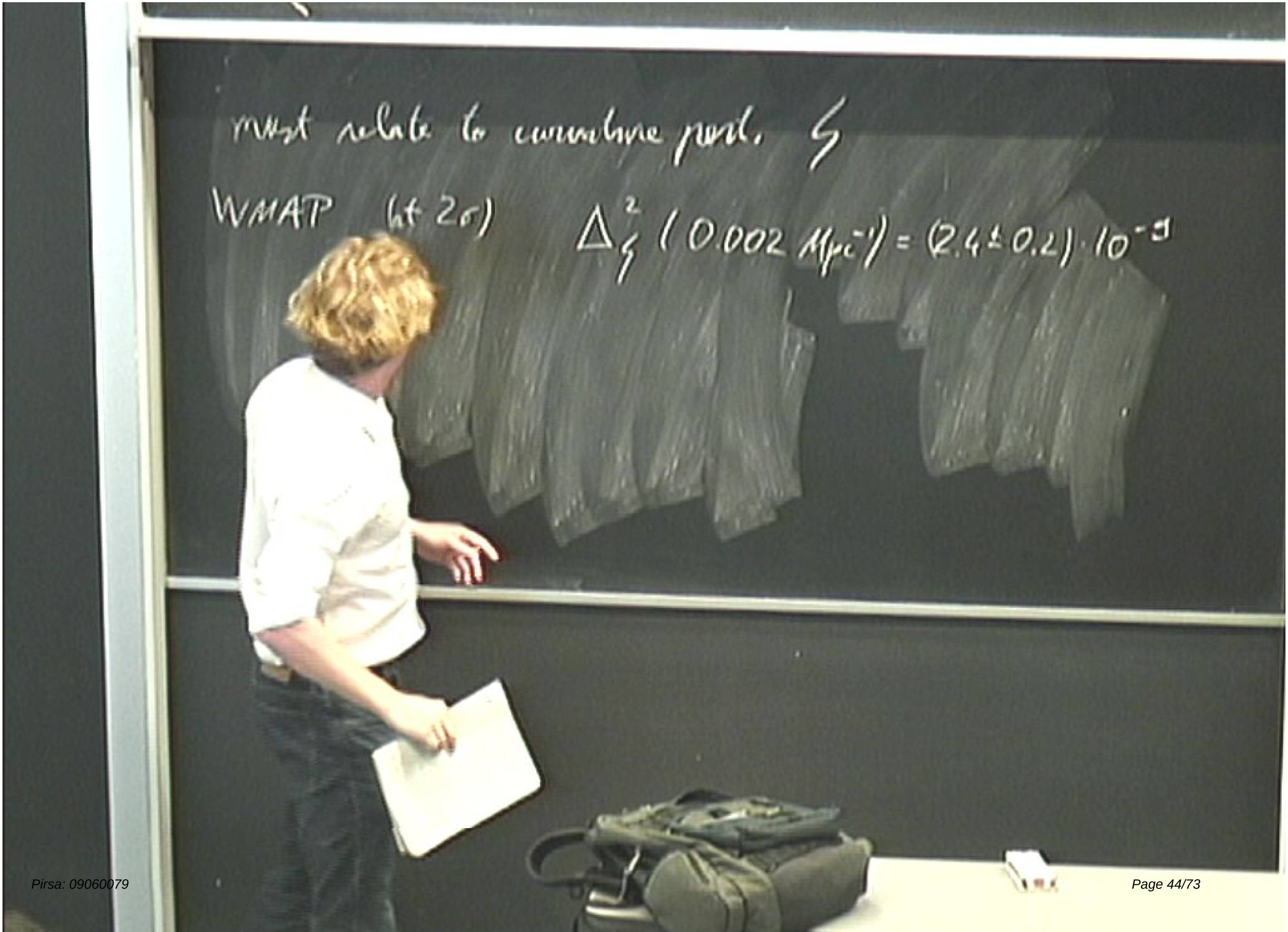
$$\langle 0| (\delta\phi)^2 |0\rangle = \int \frac{d^3k}{(2\pi)^3} \chi_k \chi_k^* = \int \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2k^3 t^2}$$

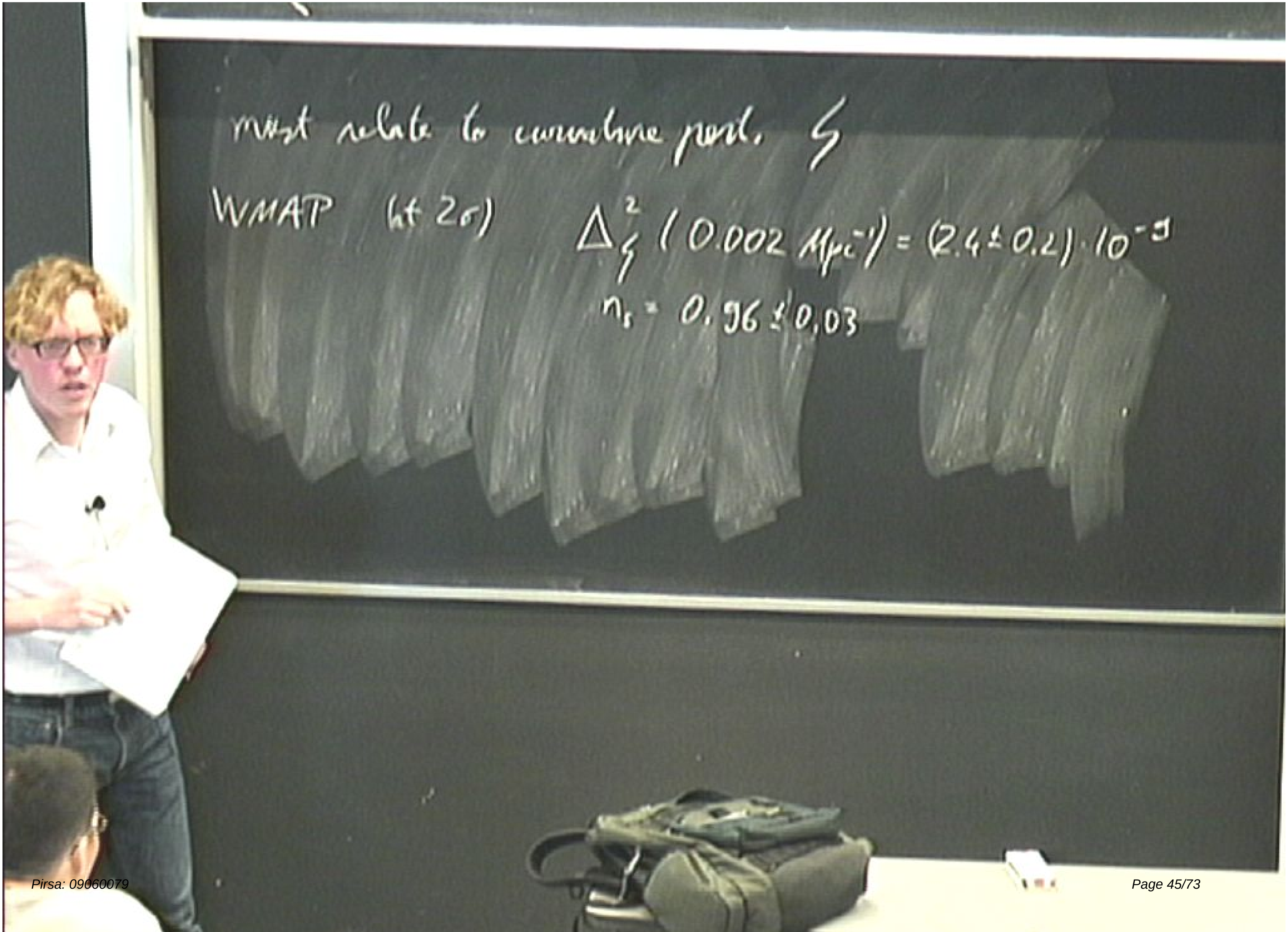
$\Delta_\phi^2(k) = \frac{1}{4\pi^2 t^2}$ independent of $k \Rightarrow$ scale-invariant spectrum

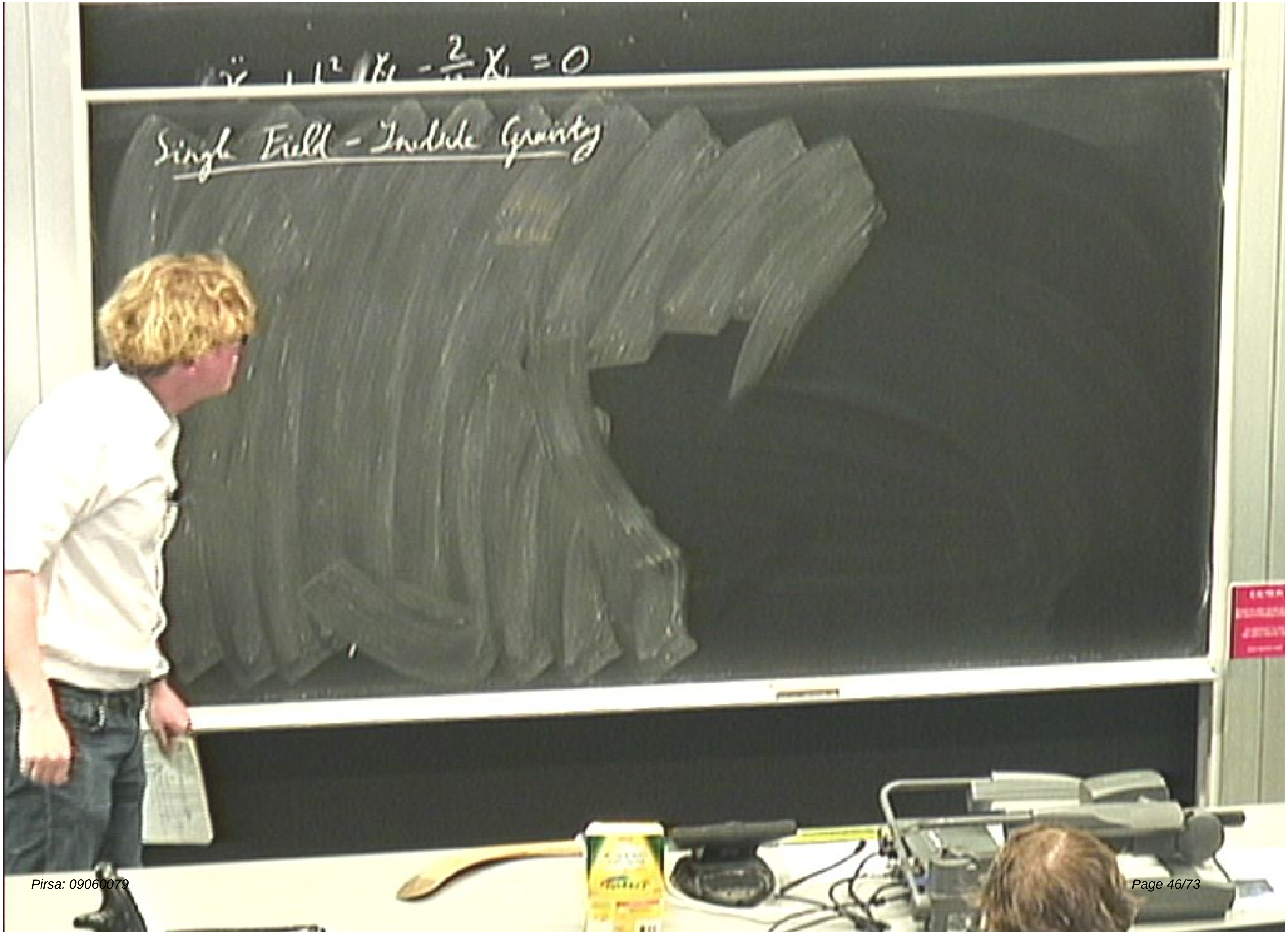
$n_s = 1$

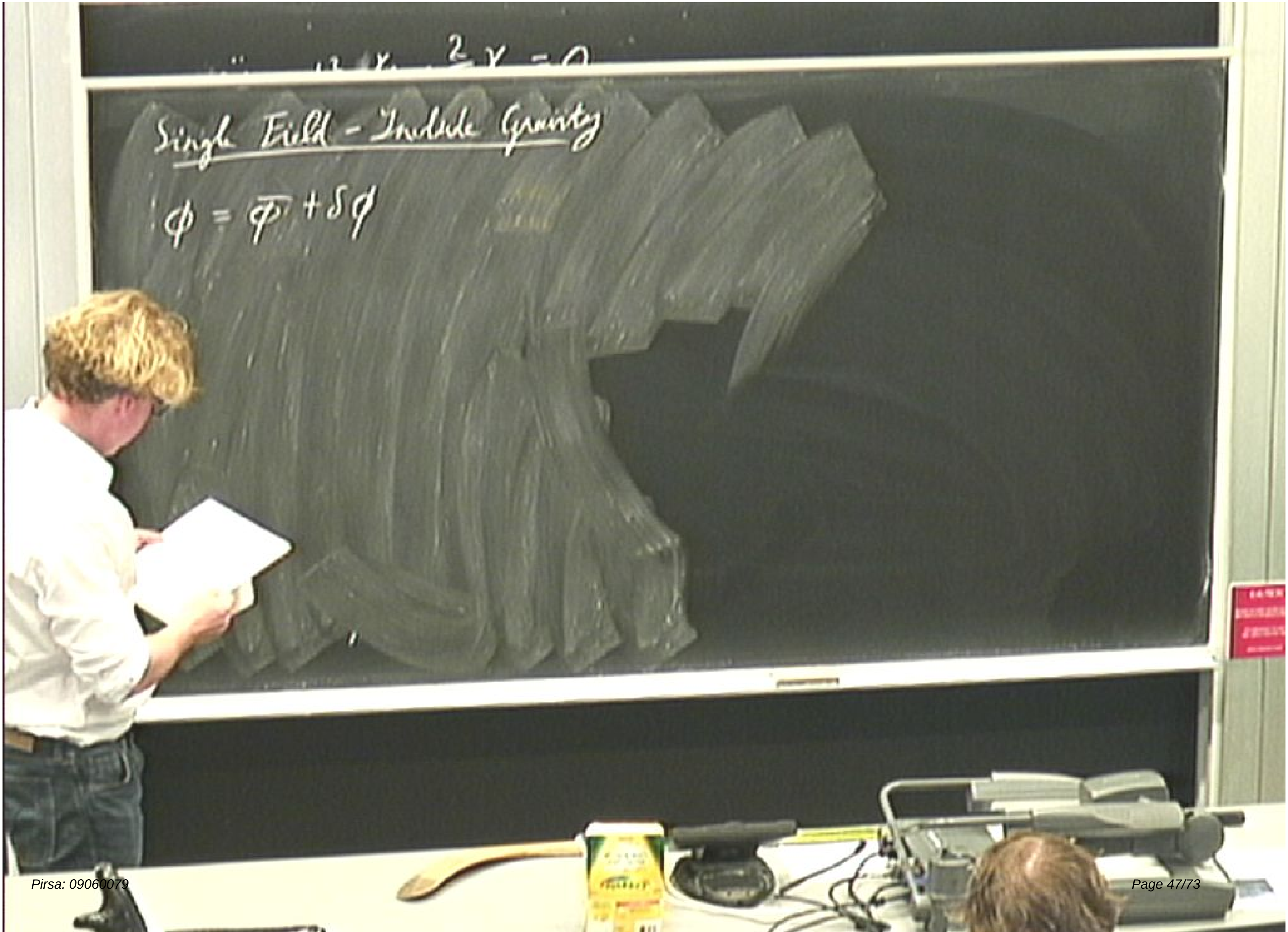
power spectrum $P(k) \equiv |\chi_k|^2 = \frac{2\pi^2}{k^3} \Delta_\phi^2(k)$


must relate to curvature pert.


must relate to curvature pert. ζ
WMAP (at 2σ)
Δ²_ζ (0.002 Mpc⁻¹) = (2.4 ± 0.2) · 10⁻⁹


must relate to curvature pert. ζ
WMAP (at 2σ)
Δ_ζ^2 (0.002 Mpc^-1) = (2.4 ± 0.2) · 10^-9
n_s = 0.96 ± 0.03


Single Field - Induced Gravity


Single Field - Include Gravity
$\phi = \bar{\phi} + \delta\phi$

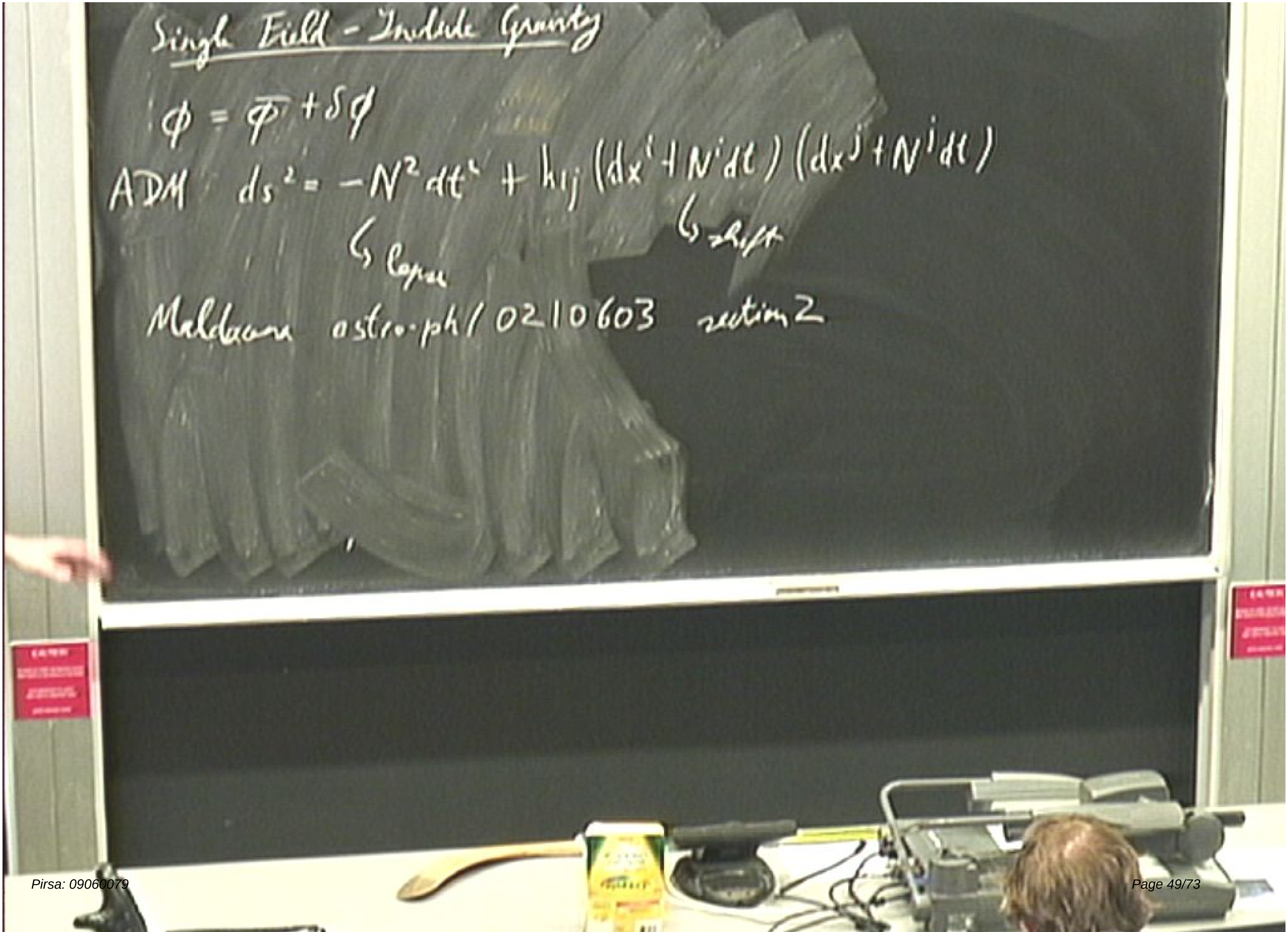
Single Field - Include Gravity

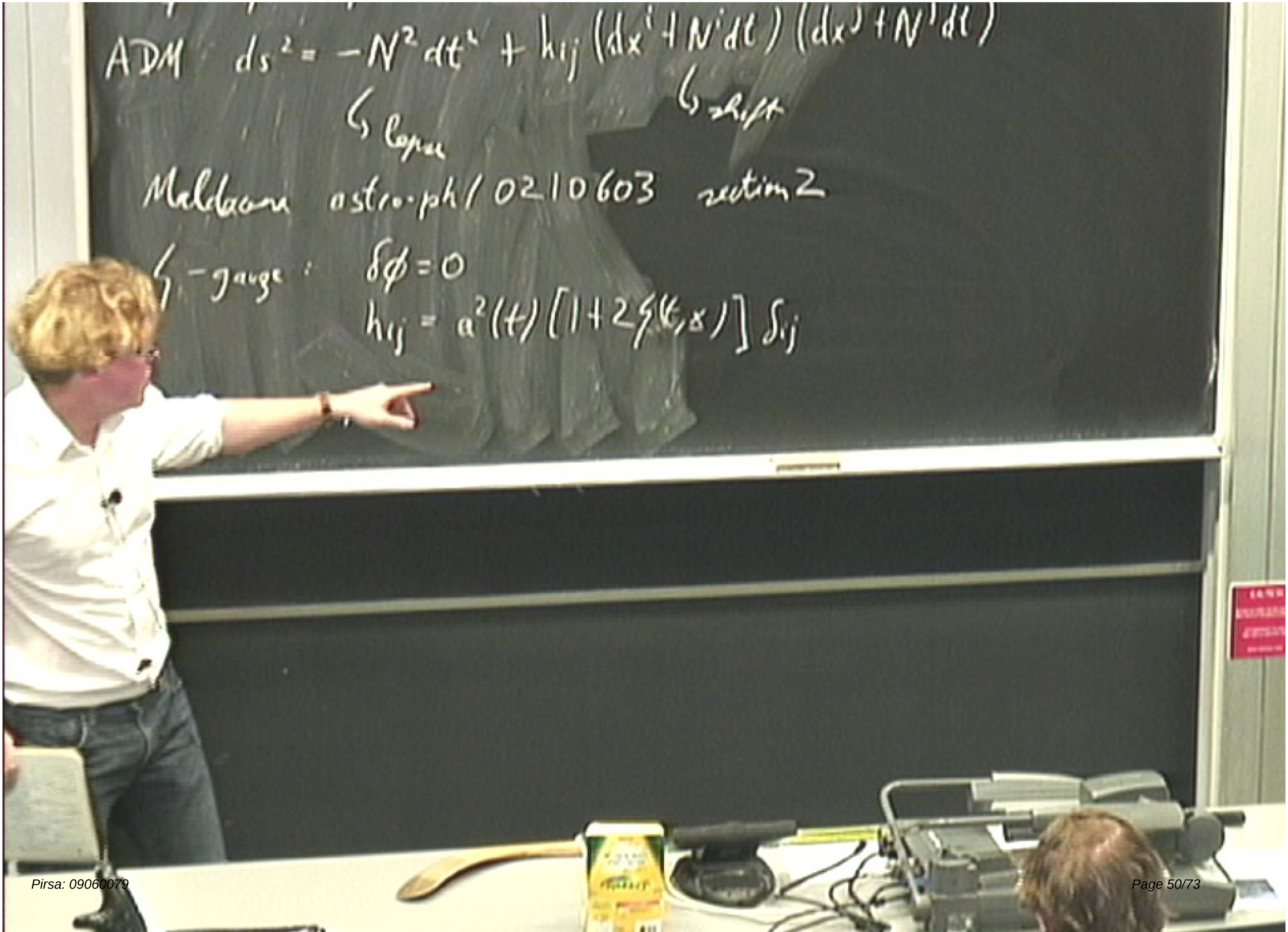
$$\phi = \bar{\phi} + \delta\phi$$

ADM $\quad ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

lapse

shift


Single Field - Include Gravity
$\phi = \bar{\phi} + \delta\phi$
ADM $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
↳ lapse
↳ shift
Maldacena astro-ph/0210603 section 2


ADM $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
↳ lapse
↳ shift
Maldacena astro-ph/0210603 section 2
ζ-gauge: $\delta\phi = 0$
$h_{ij} = a^2(t)[1 + 2\zeta(t,x)]\delta_{ij}$

ADM $ds^2 = -N^2 dt^2 + h_{ij}(dx^i$

↳ lapse ↳ shift

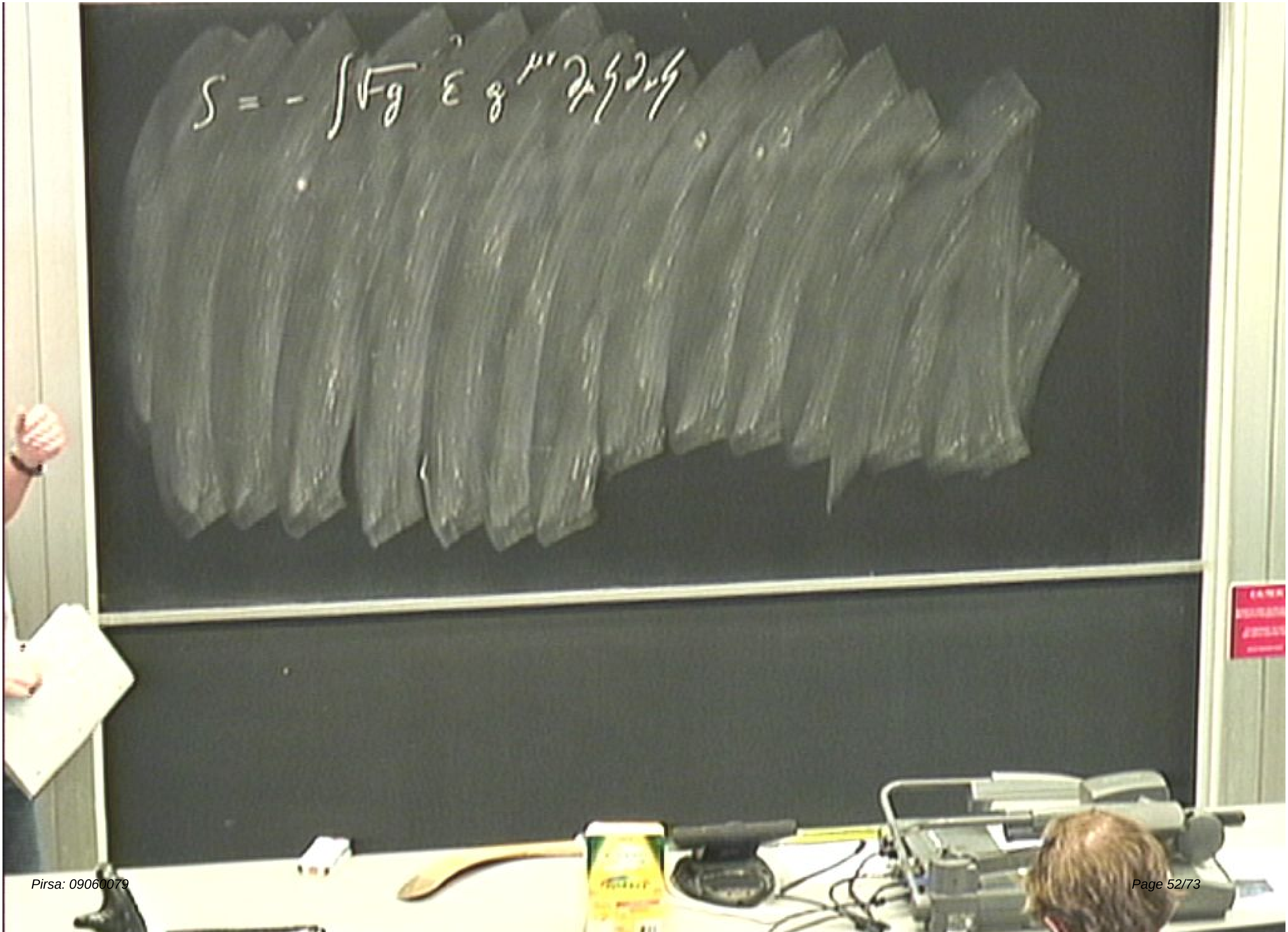
Maldacena astro-ph/0210603 section 2

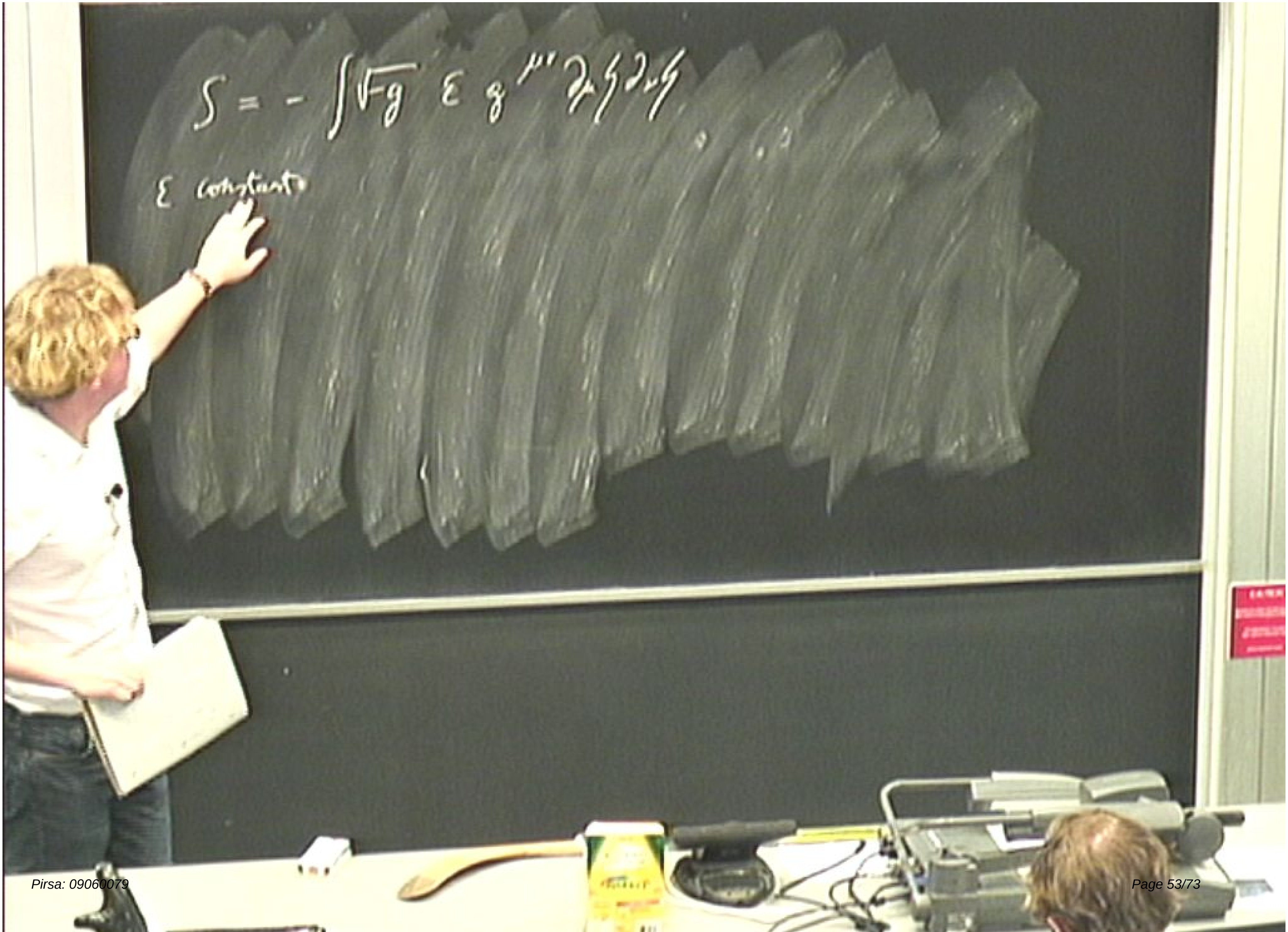
ζ-gauge: $\delta\phi = 0$

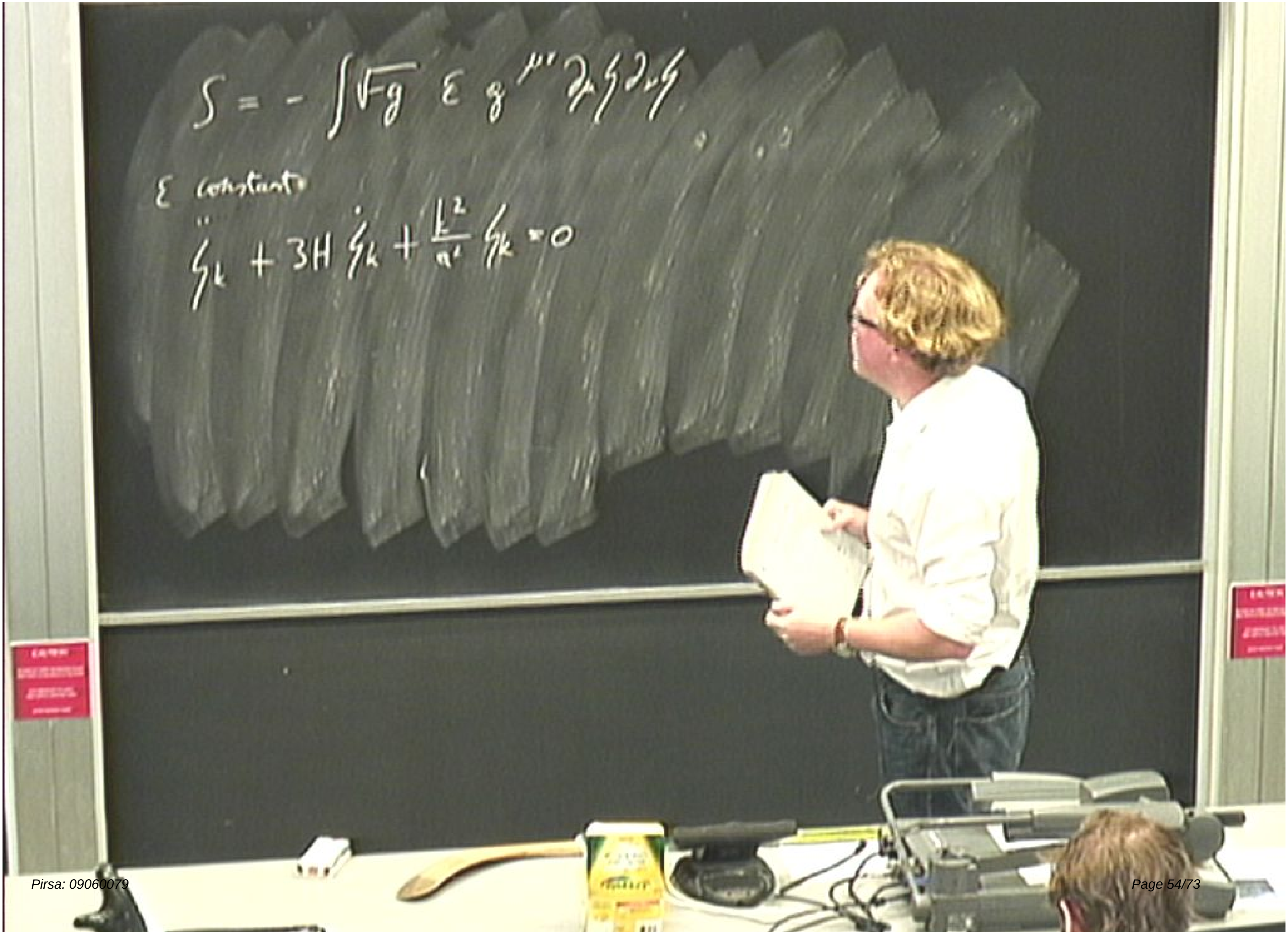
$h_{ij} = a^2(t)\left[1 + 2\zeta(t,x)\right]\delta_{ij} + H_{ij}$ $\quad \partial^i H_{ij} = 0$

$H^i{}_i = 0$

$|\lambda_\zeta| = \sqrt{2}\, k^{3/2} t$


$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$


$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
ε constant


$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
ε constant
$\ddot{\zeta}_k + 3H \dot{\zeta}_k + \frac{k^2}{a^2} \zeta_k = 0$

$$S = -\int \sqrt{-g}\ \epsilon\ g^{\mu\nu}\partial_\mu \zeta \partial_\nu \zeta$$

$$\epsilon = \frac{3}{2}(1+w) = \frac{c^2}{2}$$

$$V \propto -e^{-c\phi}$$

ϵ constant

$$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$$

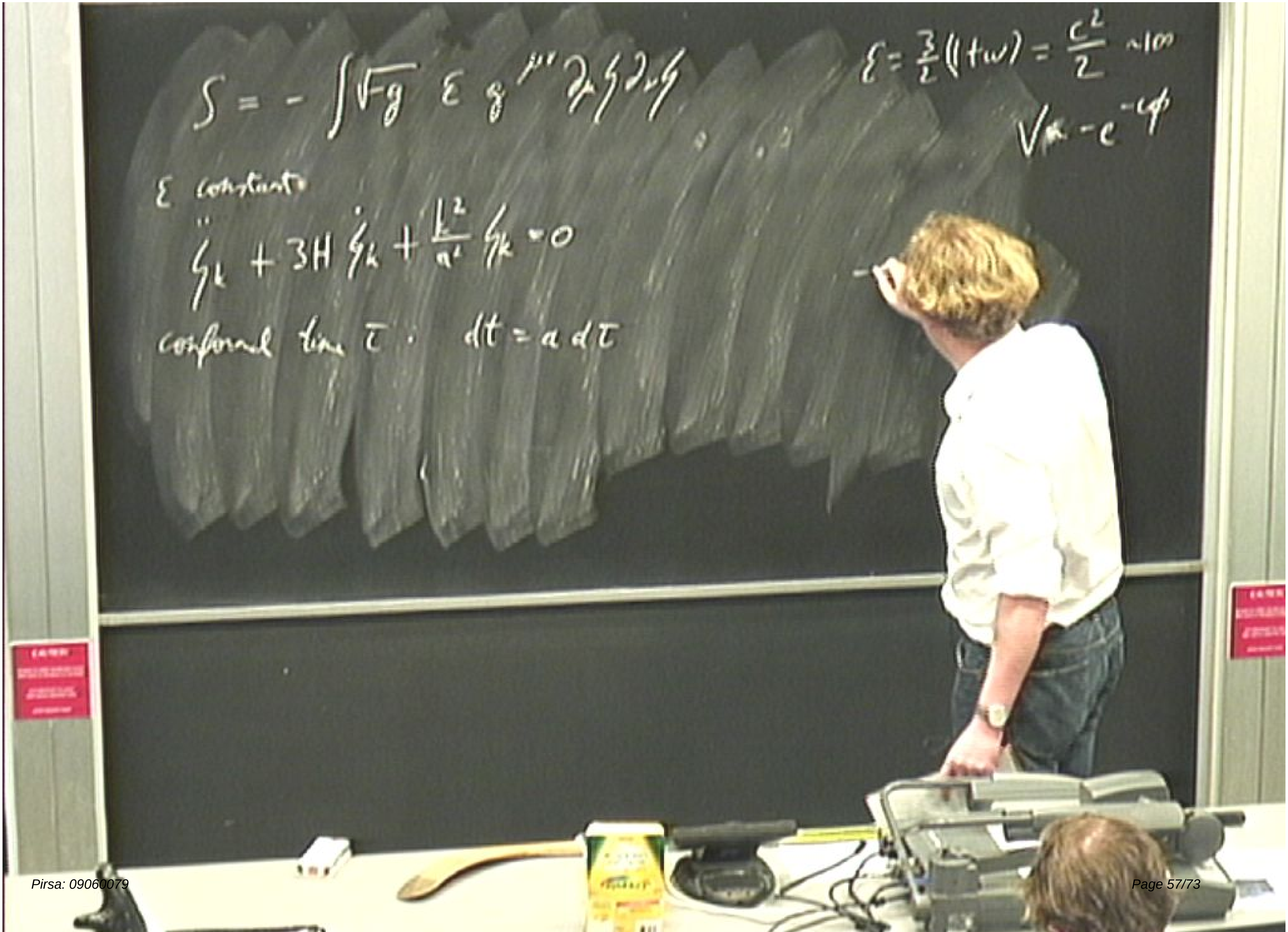
$$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$$

$$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2} \sim 100$$

$$V \sim -e^{-c\phi}$$

ε constant

$$\ddot{\zeta}_k + 3H \dot{\zeta}_k + \frac{k^2}{a^2} \zeta_k = 0$$


$S = -\int \sqrt{-g}\, \varepsilon\, g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta$
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$V \sim -e^{-c\phi}$
ε constant
$\ddot{\zeta}_k + 3H \dot{\zeta}_k + \frac{k^2}{a^2} \zeta_k = 0$
conformal time τ : $dt = a\, d\tau$

$$S = -\int \sqrt{-g}\,\varepsilon\, g^{\mu\nu}\partial_\mu \zeta \partial_\nu \zeta$$

$$\varepsilon = \frac{3}{2}(1+w) = \frac{c^2}{2} \sim 100$$

$$V \sim -e^{-c\phi}$$

ε constant

$$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$$

FRW $-dt^2 + a^2 ds^2$

$= a^2(-d\tau^2 + ds^2)$

conformal time τ: $dt = a\, d\tau$

$$S = -\int \sqrt{-g}\; \epsilon\; g^{\mu\nu}\partial_\mu \zeta \,\partial_\nu \zeta$$

$$\epsilon = \frac{3}{2}(1+w) = \frac{c^2}{2} \sim 100$$

$$V \propto -e^{-c\phi}$$

ϵ constant

$$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$$

FRW $-dt^2 + a^2 dx^2$

$$= a^2\left(-d\tau^2 + dx^2\right)$$

conformal time τ: $dt = a\, d\tau$

$\frac{d}{d\tau} \equiv {}'$

$$\zeta_k'' + 2\frac{a'}{a}\zeta_k' + k^2 \zeta_k = 0$$

$$S = -\int \sqrt{-g}\ \epsilon\ g^{\mu\nu}\partial_\mu\zeta\,\partial_\nu\zeta$$

$$\epsilon = \frac{3}{2}(1+w) = \frac{c^2}{2} \sim 100$$

$$V \sim -e^{-c\phi}$$

ϵ constant

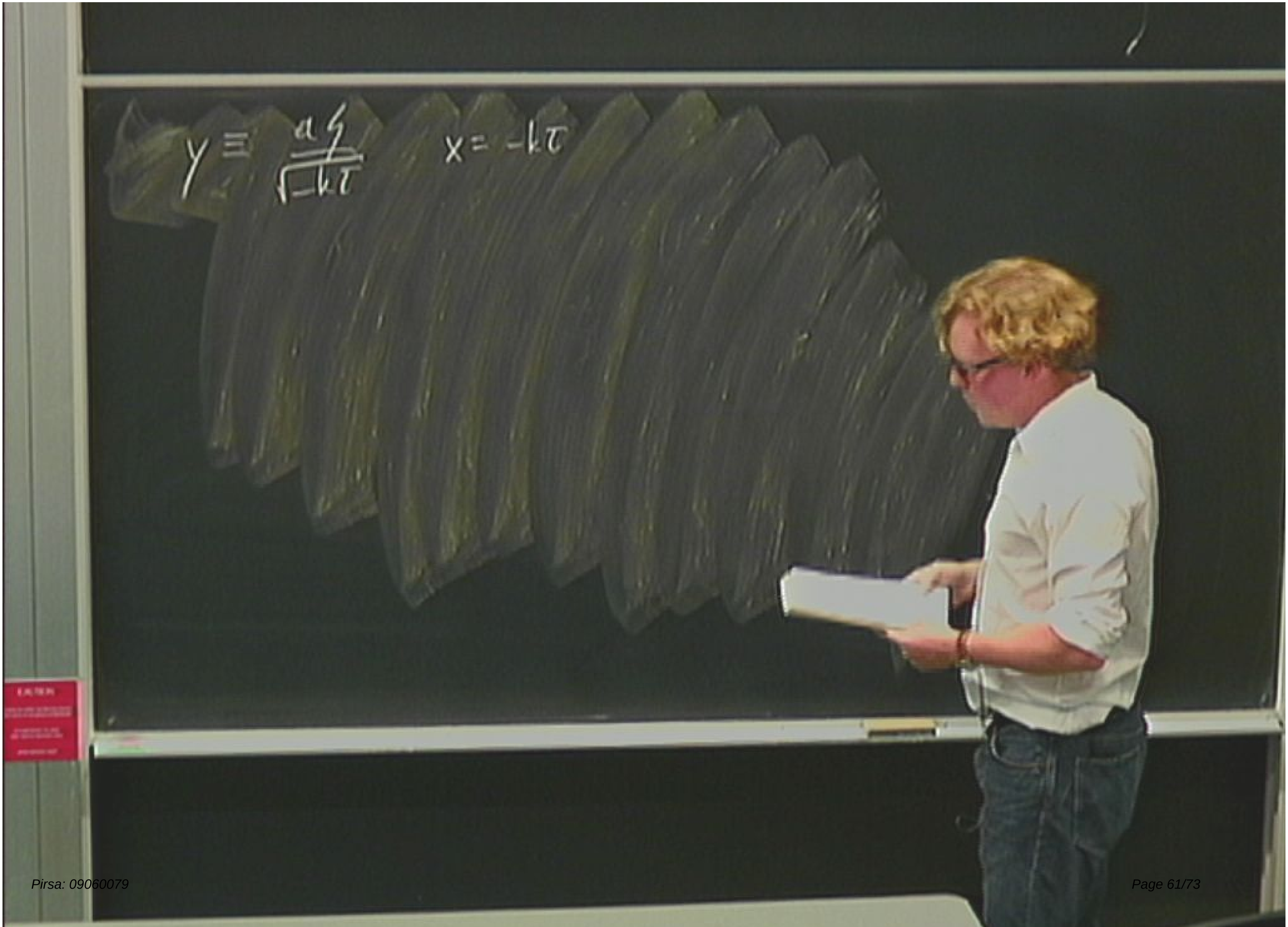
$$\ddot{\zeta}_k + 3H\dot{\zeta}_k + \frac{k^2}{a^2}\zeta_k = 0$$

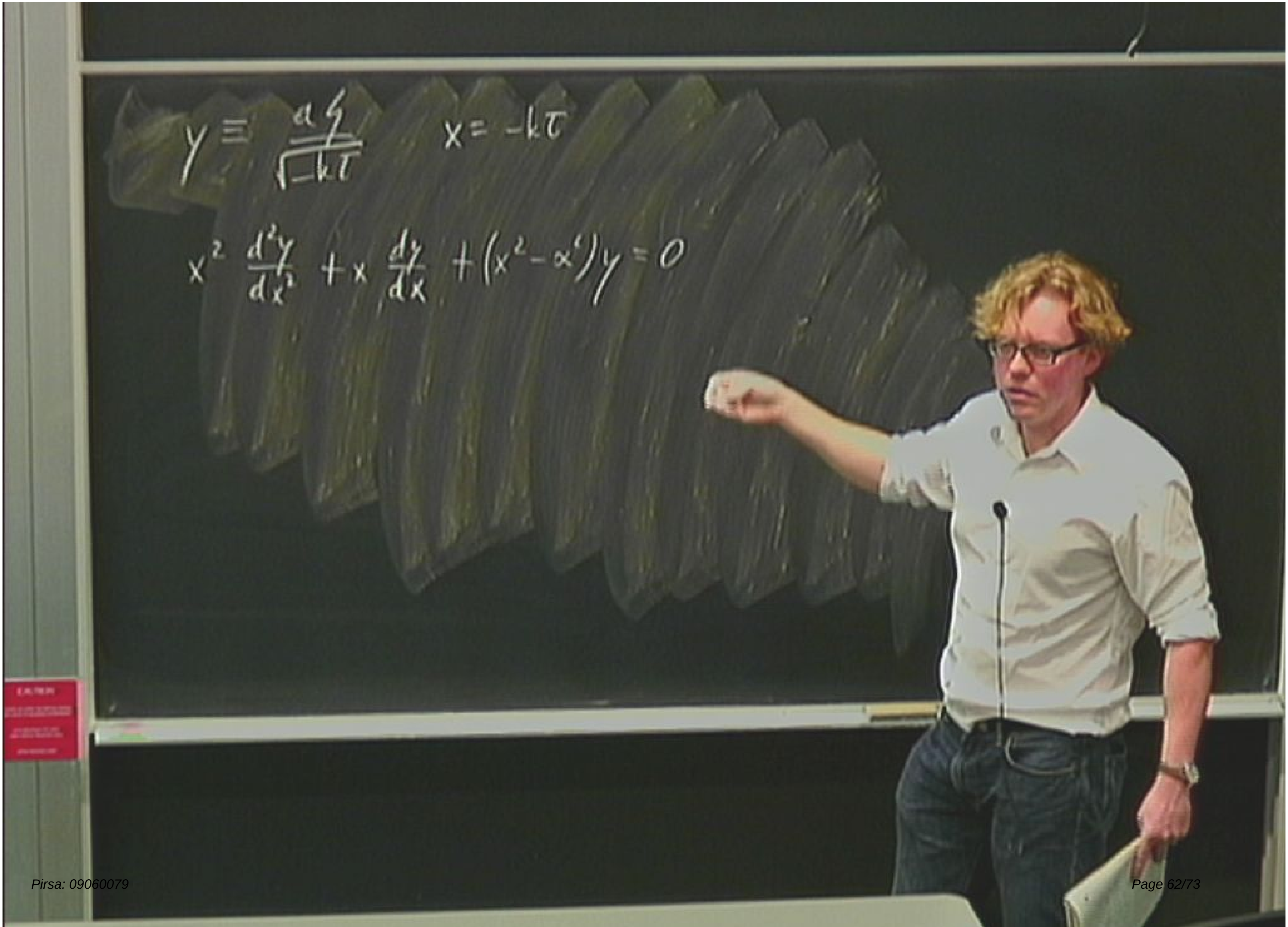
FRW $-dt^2 + a^2 dx^2$

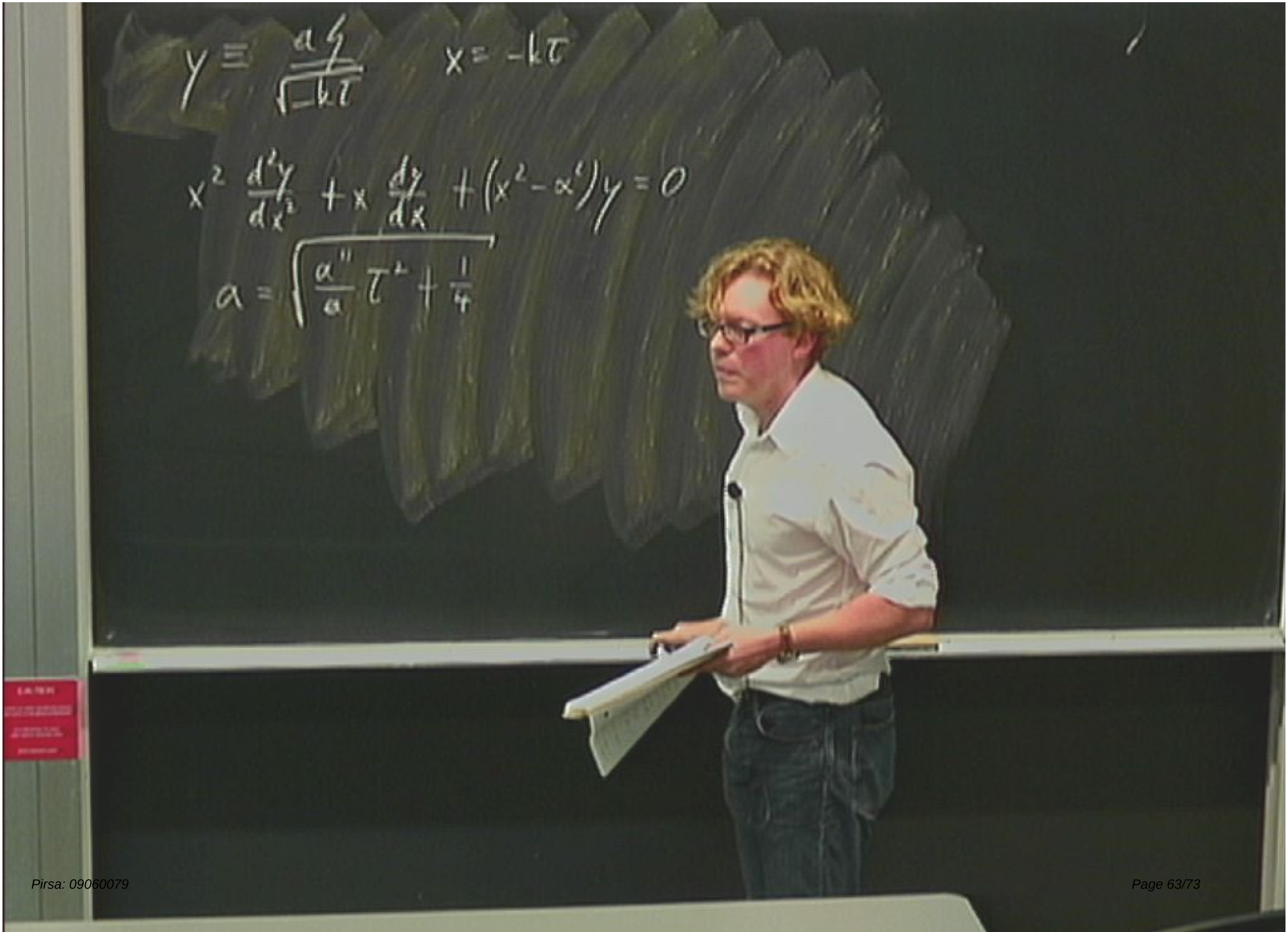
$$= a^2\left(-d\tau^2 + dx^2\right)$$

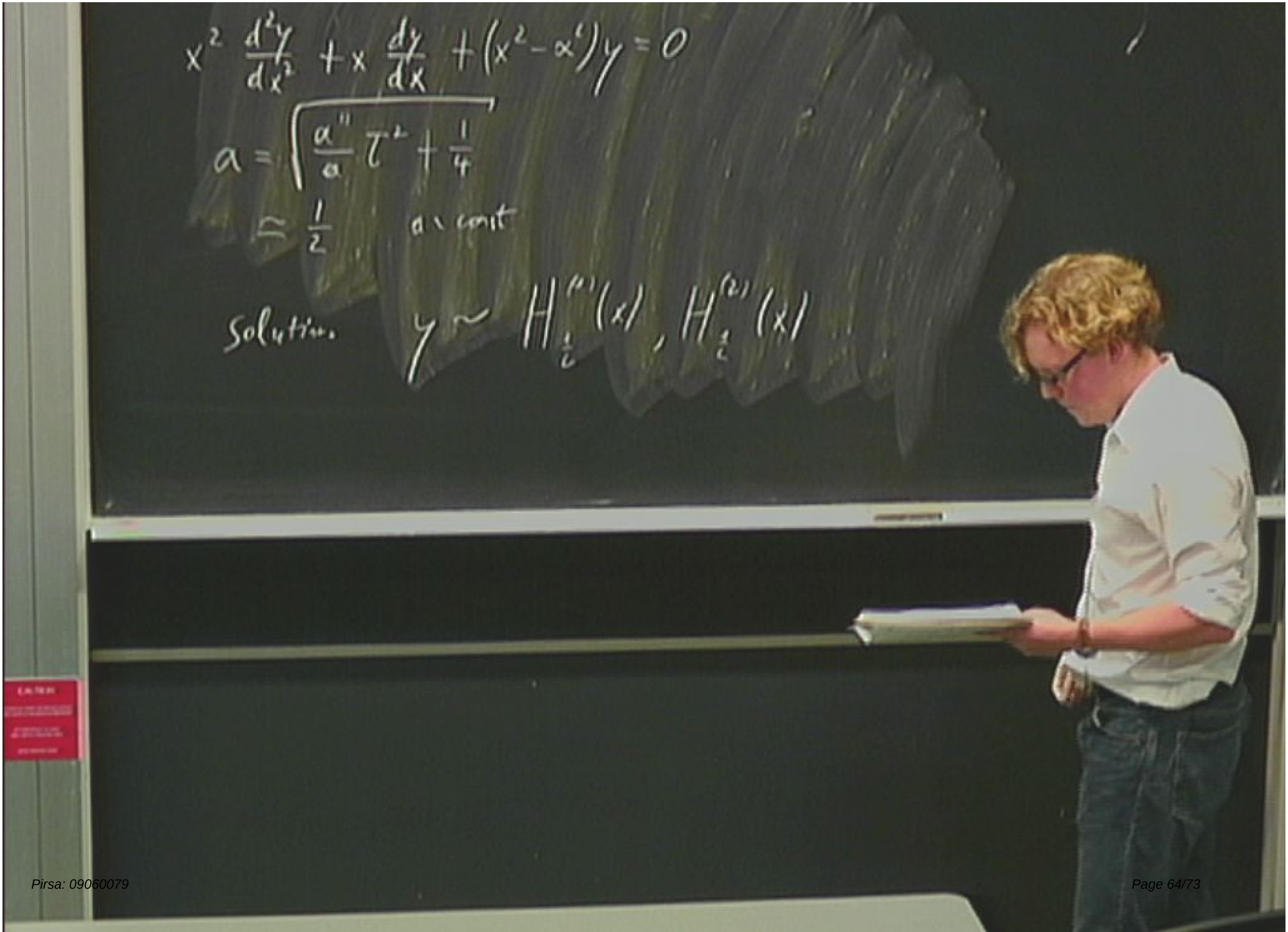
conformal time τ: $dt = a\,d\tau$

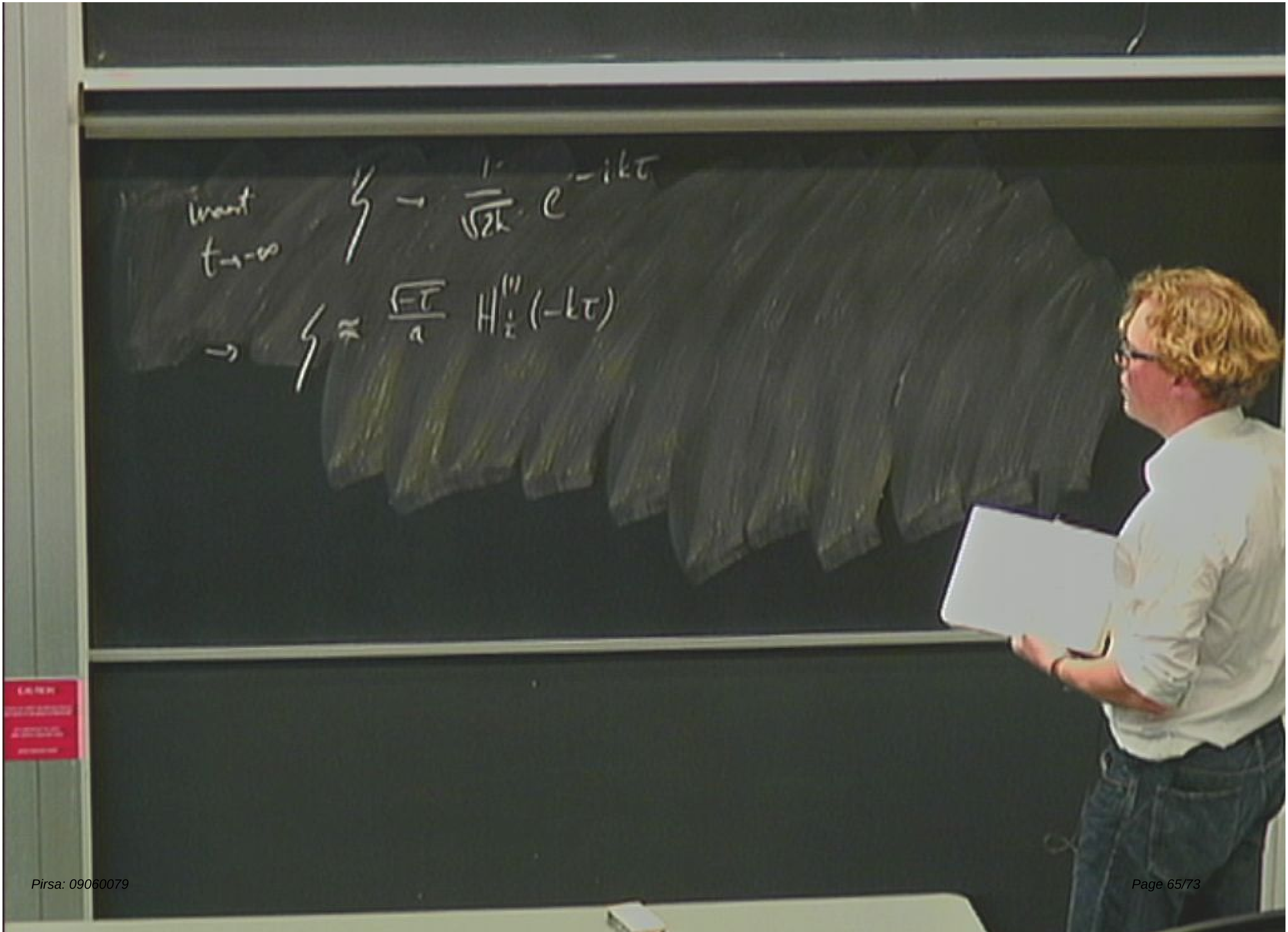
$\frac{d}{d\tau} \equiv {}'$ $\qquad \zeta_k'' + 2\frac{a'}{a}\zeta_k' + k^2\zeta_k = 0$

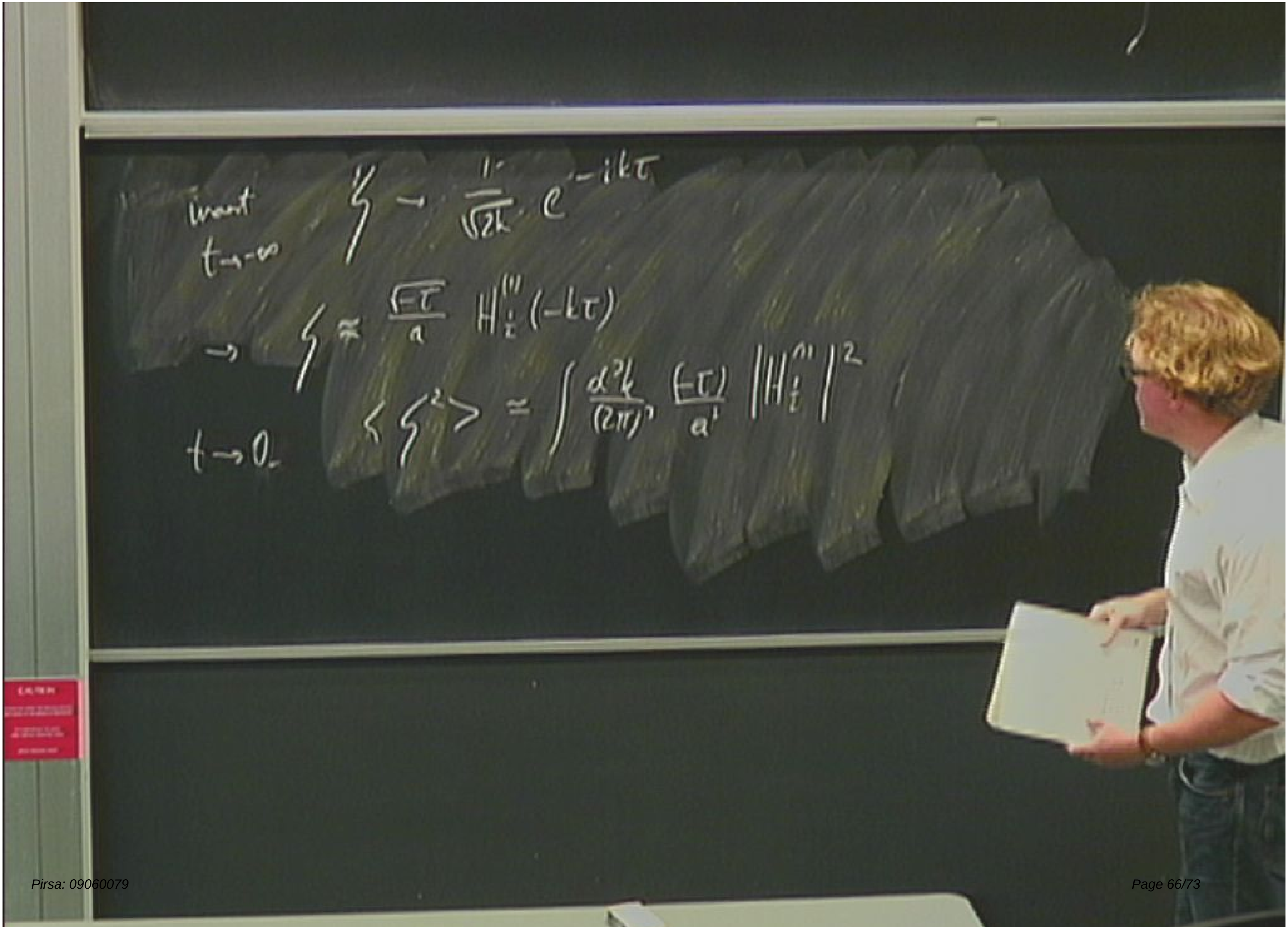

$y \equiv \frac{a\zeta}{\sqrt{-k\tau}}$
$x = -k\tau$


$y \equiv \frac{a\zeta}{\sqrt{-k\tau}}$
$x = -k\tau$
$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0$


$y \equiv \frac{a\zeta}{\sqrt{-k\tau}}$ $x = -k\tau$
$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0$
$\alpha = \sqrt{\frac{a''}{a}\tau^2 + \frac{1}{4}}$


$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0$
$\alpha = \sqrt{\frac{a''}{a} \tau^2 + \frac{1}{4}}$
$\simeq \frac{1}{2}$
$a \sim$ const
Solution
$y \sim H^{(1)}_{\frac{1}{2}}(x), H^{(2)}_{\frac{1}{2}}(x)$


invert
t→-∞
$\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\to \zeta \approx \frac{\sqrt{-\tau}}{a} H^{(1)}_{\frac{3}{2}}(-k\tau)$

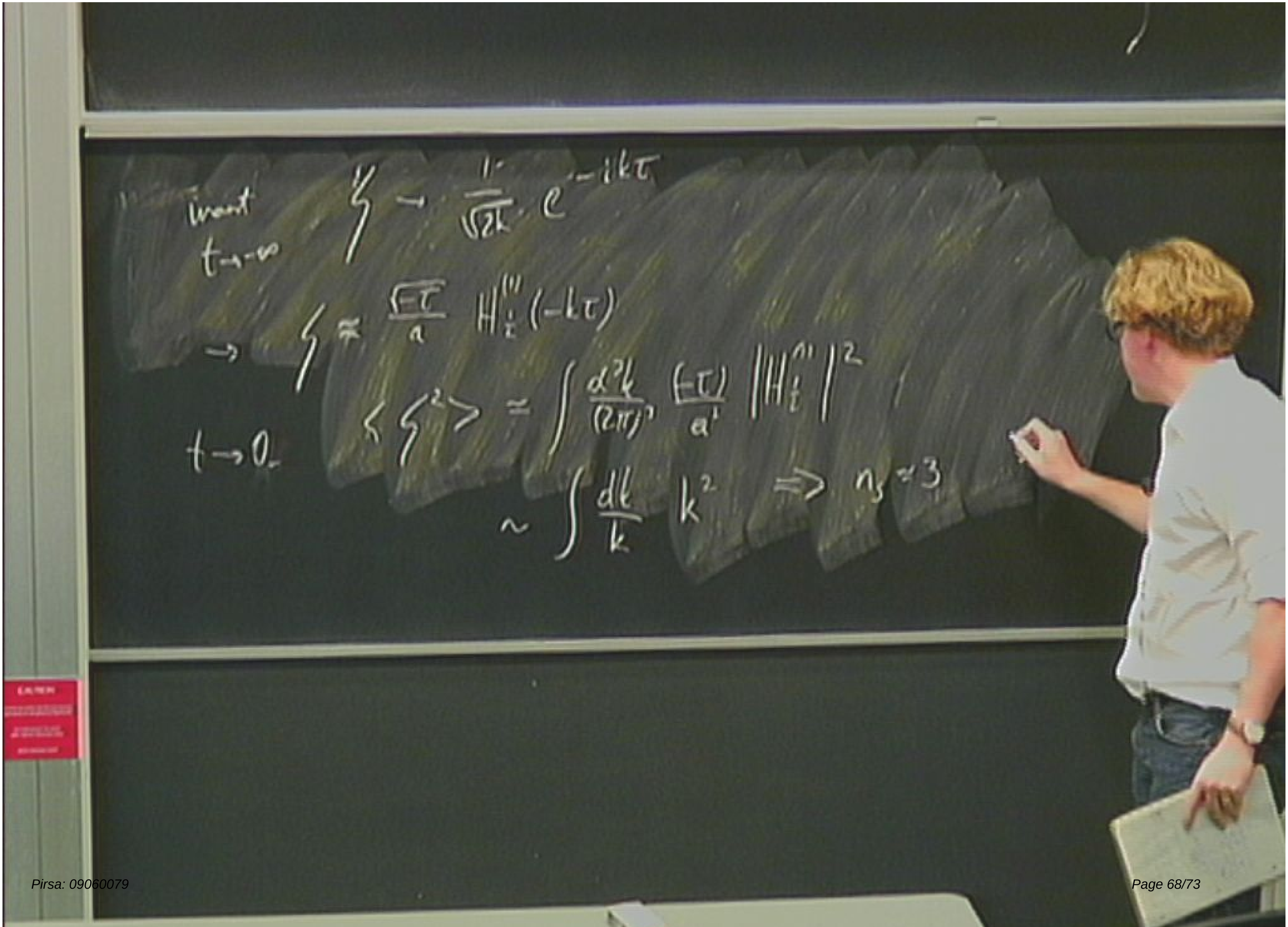

want
$t \to -\infty$ $\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\to$ $\zeta \approx \frac{\sqrt{-\tau}}{a} H^{(1)}_{\frac{3}{2}}(-k\tau)$
$\langle \zeta^2 \rangle \simeq \int \frac{d^3k}{(2\pi)^3} \frac{(-\tau)}{a^2} \left| H^{(1)}_{\frac{3}{2}} \right|^2$
$t \to 0_-$

want $t \to -\infty$ $\quad \zeta \to \frac{1}{\sqrt{2k}}\, e^{-ik\tau}$

$\to \quad \zeta \approx \frac{\sqrt{-\tau}}{a}\, H^{(1)}_{\frac{3}{2}}(-k\tau)$

$t \to 0_- \quad \langle \zeta^2 \rangle \approx \int \frac{d^3k}{(2\pi)^3}\, \frac{(-\tau)}{a^2}\, \left| H^{(1)}_{\frac{3}{2}} \right|^2$

$\sim \int \frac{dk}{k}\, k^2$


want
$t \to -\infty$
$\zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$\to \zeta \approx \frac{\sqrt{-\tau}}{a} H^{(1)}_{\frac{3}{2}}(-k\tau)$
$t \to 0_-$
$\langle \zeta^2 \rangle \simeq \int \frac{d^3k}{(2\pi)^3} \frac{(-\tau)}{a^2} |H^{(1)}_{\frac{3}{2}}|^2$
$\sim \int \frac{dk}{k} k^2 \Rightarrow n_s \simeq 3$

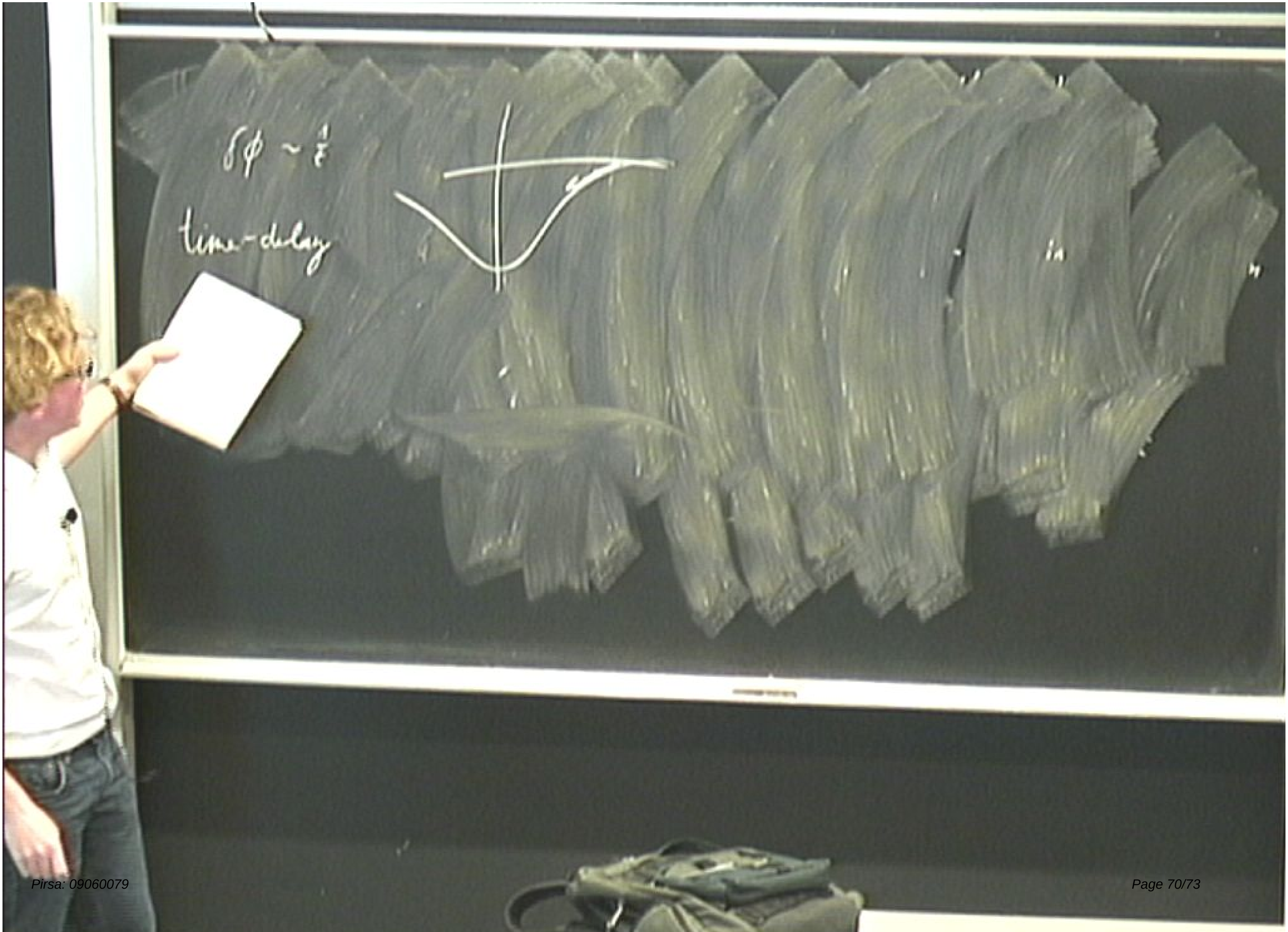
invert $t\to-\infty$ $\quad \zeta \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$

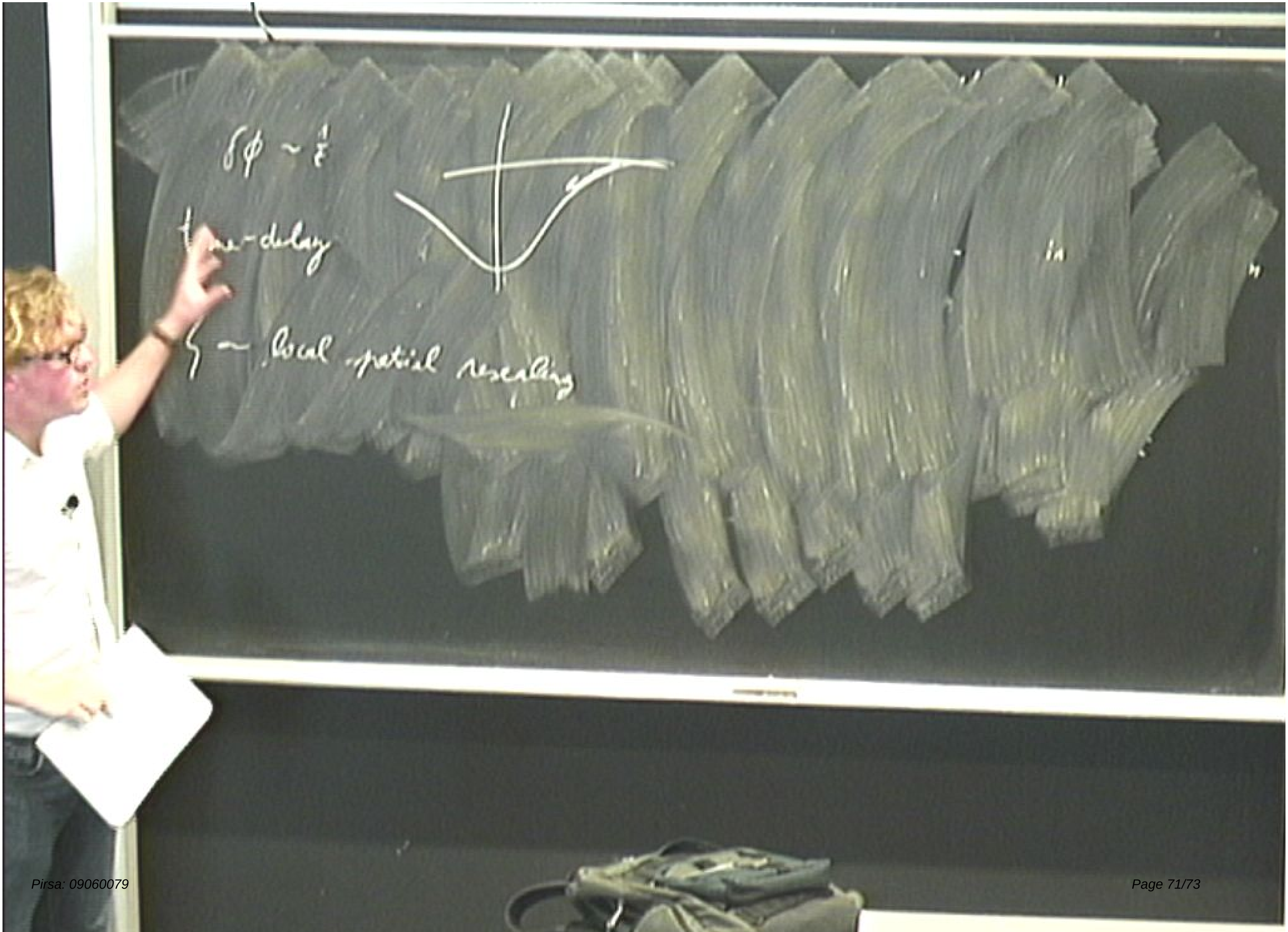
$\to \quad \zeta \approx \frac{\sqrt{-\tau}}{a} H^{(1)}_{\frac{3}{2}}(-k\tau)$

$t\to 0_- \quad \langle \zeta^2 \rangle \approx \int \frac{d^3k}{(2\pi)^3} \frac{(-\tau)}{a^2} \left| H^{(1)}_{\frac{3}{2}} \right|^2$

$\sim \int \frac{dk}{k} k^2 \Rightarrow n_s \approx 3$ blue

disagrees with obs!


$\delta\phi \sim \frac{1}{r}$
time-delay

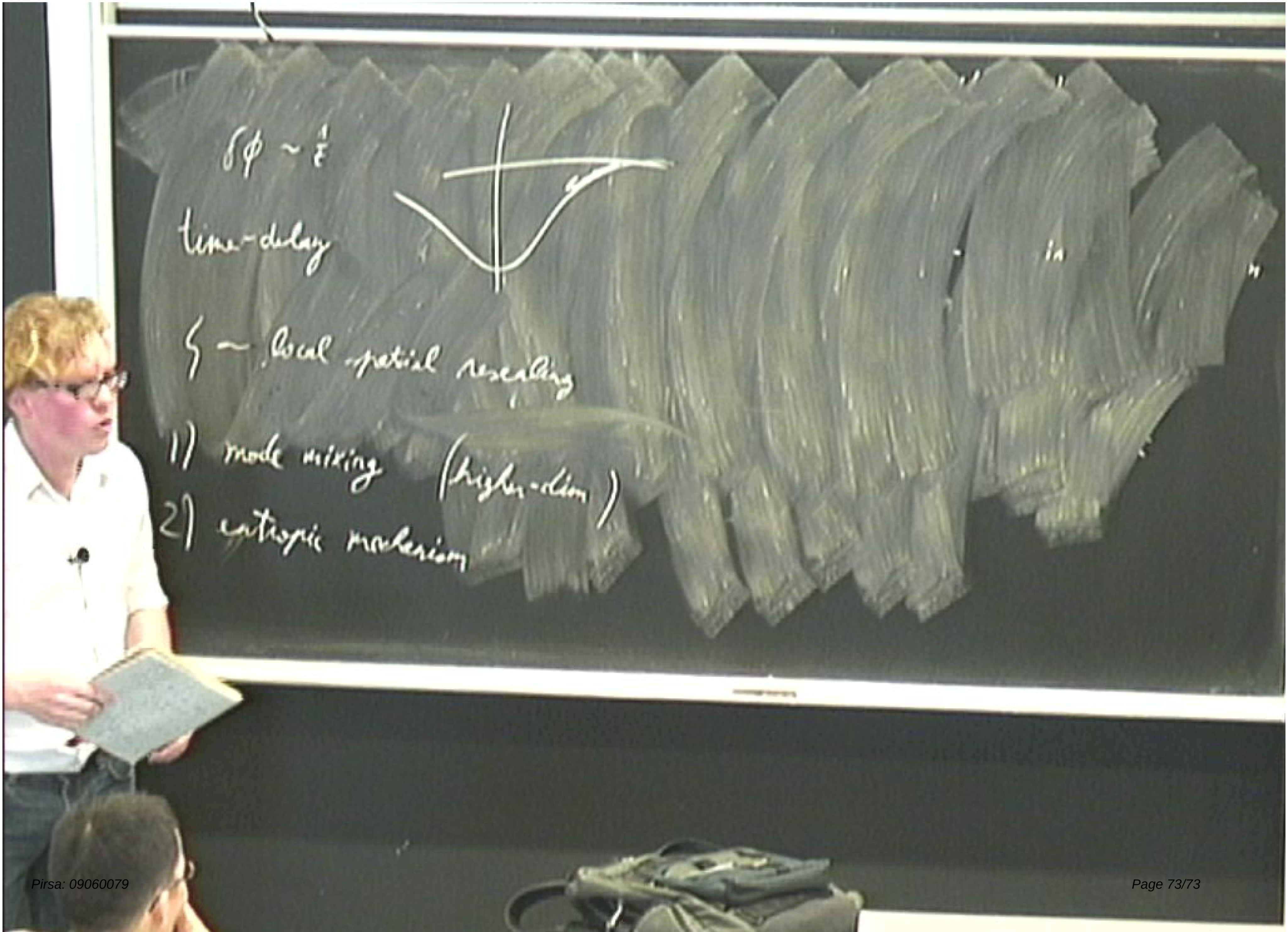

δφ ~ 1/t
time-delay
ζ ~ local spatial rescaling

$\delta\phi \sim \frac{1}{t}$

time-delay

$\zeta \sim$ local spatial rescaling

ii) mode mixing


time-delay
ξ ~ local spatial rescaling
1) mode mixing (higher-dim)
2) entropic mechanism