Title: Measuring Problem in Eternal Inflation

Date: Jun 27, 2009 02:00 PM

URL: http://pirsa.org/09060078

Abstract:

The measure problem in eternal inflation

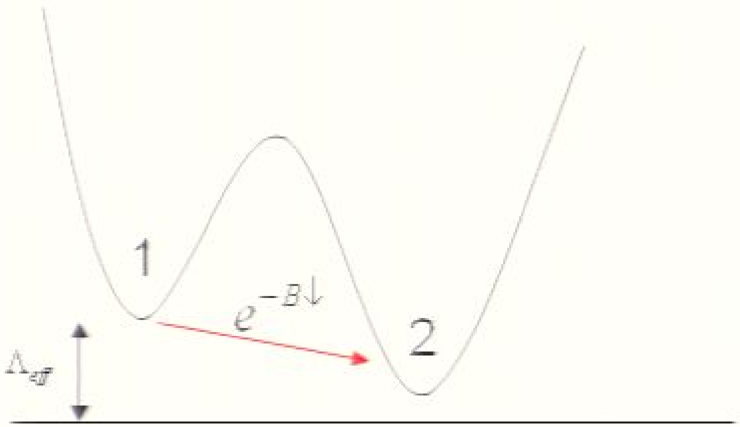
Jaume Garriga,
(U. Barcelona)

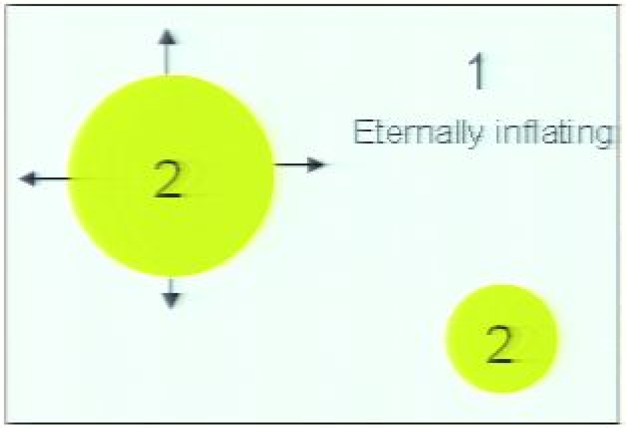
Eternal inflation is quite generic:

1- Models with metastable dS vacua

2- Models with a regime dominated by quantum diffusion

Metastable dS vacua





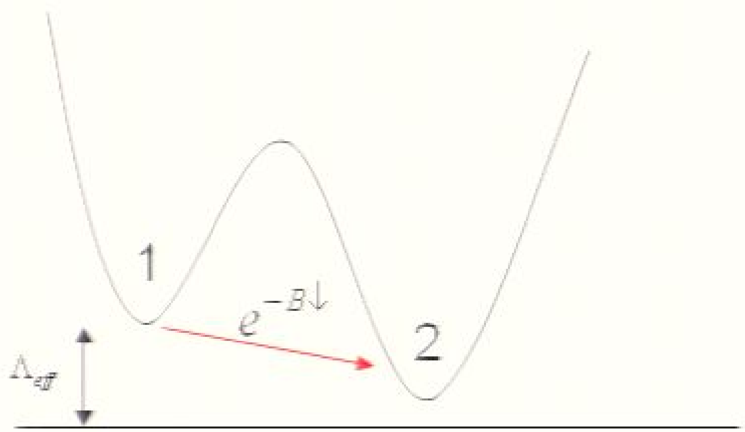
$\lambda \equiv$ decay rate per Hubble volume $\sim \; e^{-B} \ll 1$

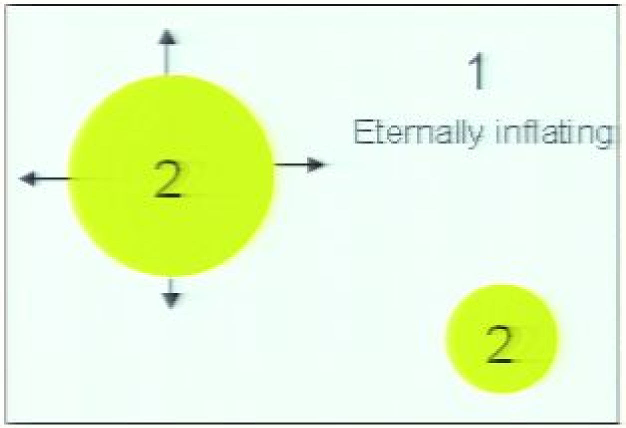
$$\frac{dV_1}{dt} = 3HV_1 - \lambda H V_1$$

$$V_1 = C\, e^{(3-\lambda)Ht}$$

Average volume grows unbounded for $\lambda < 3$.

Metastable dS vacua





$\lambda \equiv$ decay rate per Hubble volume $\sim e^{-B} \ll 1$

$$\frac{dV_1}{dt} = 3HV_1 - \lambda H V_1$$

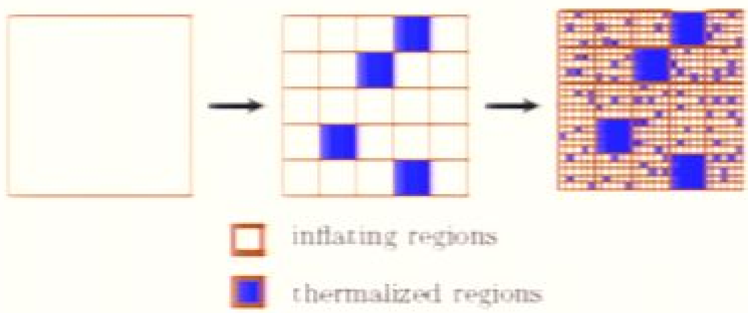
$$V_1 = C\, e^{(3-\lambda)Ht}$$

Average volume grows unbounded for $\lambda < 3$.

$\lambda < 3 \Longleftrightarrow$ **Eternal inflation**

Finite probability that transition is never complete

The eternally inflating fractal (Winitzki 02,05)



$p \equiv e^{-\lambda}$ survival prob.

$b \equiv e^{3}$ branching factor

$N \equiv Ht$ number of steps

Sierpinsky carpet

$V_1 = H^{-3}(bp)^N$ grows for $p > 1/b$

$X \equiv$ Probability that a Hubble sized region contains eternal points

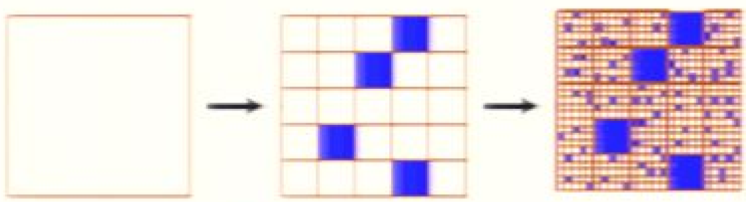
$$X = 1 - (1 - Xp)^b$$

For p $< 1/b$ (ie. $\lambda < 3$) only trivial solution $X = 0$,
but for p $> 1/b$, nontrivial solution $0 < X \leq 1$.

$E =$ Set of eternal points (non-empty with finite probability for $\lambda < 3$)

$$\dim E = 3 - \lambda$$

The eternally inflating fractal (Winitzki 02,05)



$p \equiv e^{-\lambda}$ survival prob.

$b \equiv e^{3}$ branching factor

$N \equiv Ht$ number of steps

Sierpinsky carpet

$V_1 = H^{-3}(bp)^N$ grows for $p > 1/b$

$X \equiv$ Probability that a Hubble sized region contains eternal points

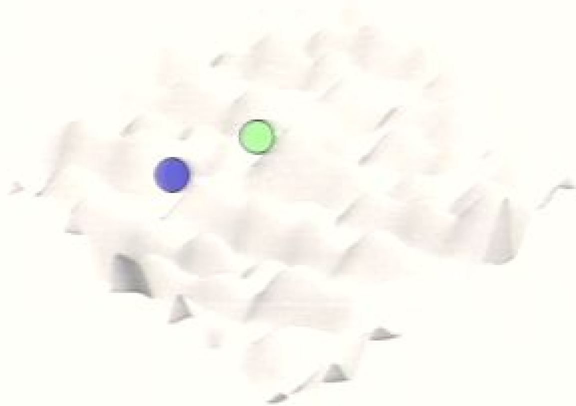
$$X = 1-(1-Xp)^b$$

For p $< 1/b$ (ie. $\lambda < 3$) only trivial solution $X = 0$, but for p $> 1/b$, nontrivial solution $0 < X \leq 1$.

$E =$ Set of eternal points (non-empty with finite probability for $\lambda < 3$)

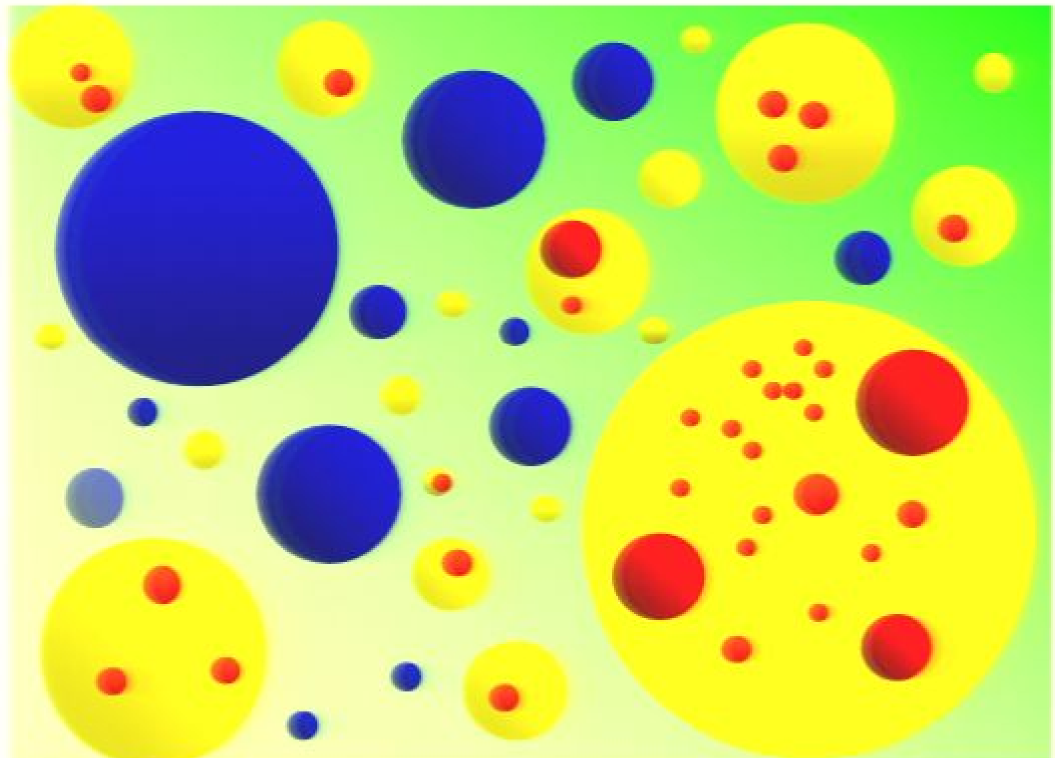
$$\dim E = 3 - \lambda$$

Eternally inflating multiverse



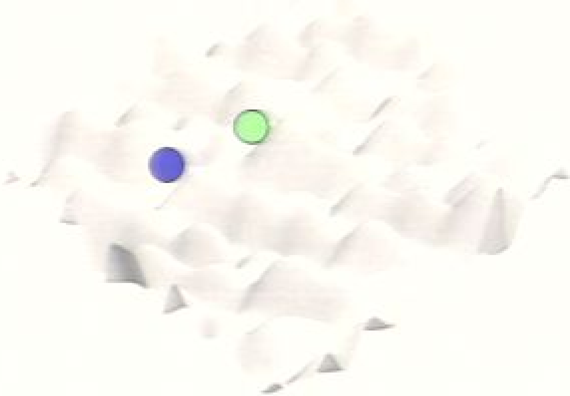
Field space
(Landscape of vacua)

$N_{vac} \sim 10^{500}$



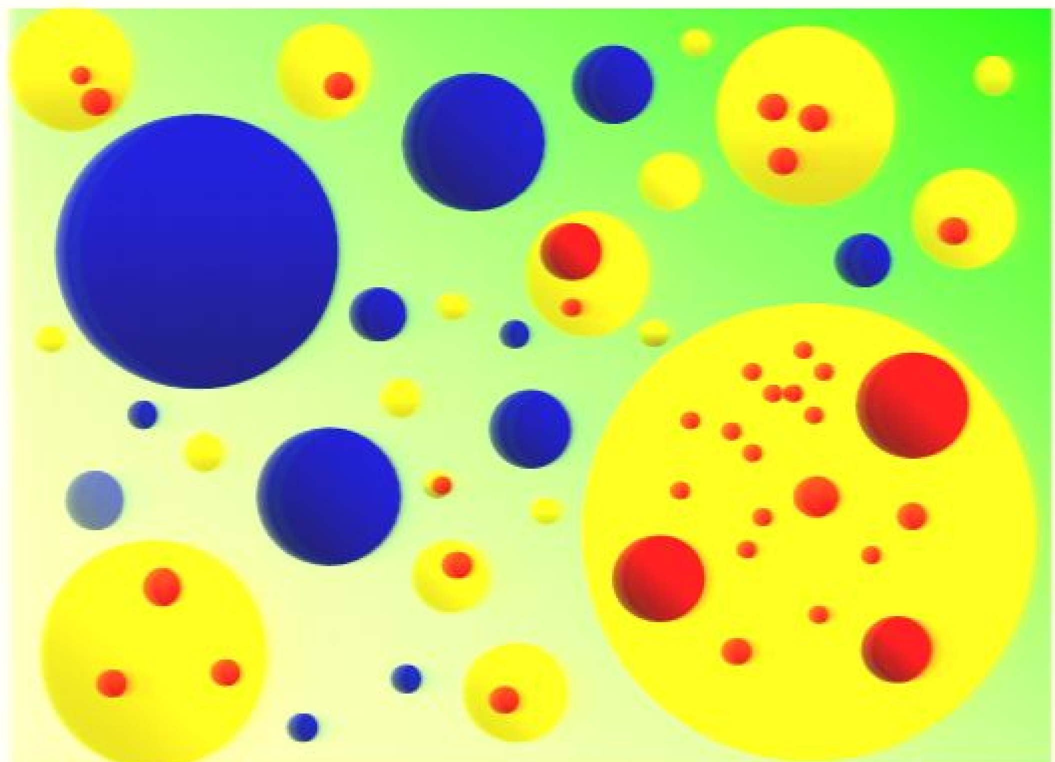
Physical space

Eternally inflating multiverse



Field space
(Landscape of vacua)

$$N_{vac} \sim 10^{500}$$

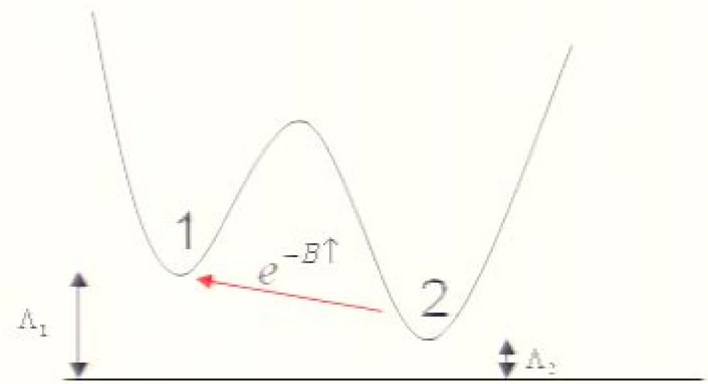


Physical space

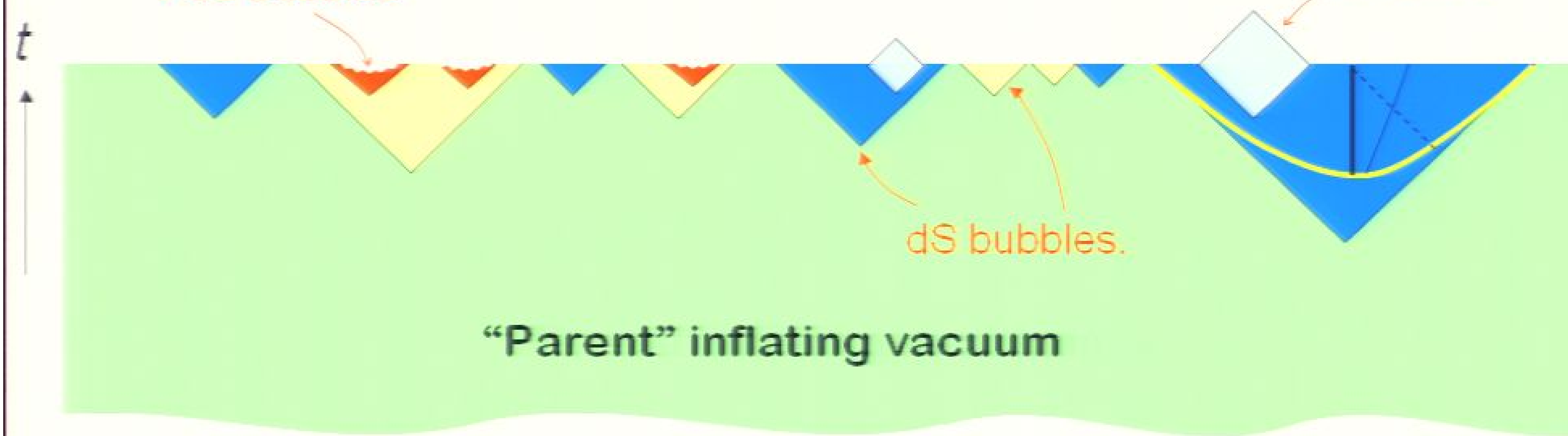
Tunneling uphill is also possible

$$\Gamma^{\uparrow} / \Gamma^{\downarrow} \sim e^{-S(2)+S(1)}$$

Entropy difference



Spacetime structure



- Bubbles nucleate and expand at nearly the speed of light.
- dS (**Inflating**)
  AdS
  Minkowski } **(Terminal bubbles)**

Attractor behaviour of volume distribution:

Fraction of volume $V_i(t)$ in inflating vacuum of type

$$dt = H^{\alpha}\, d\tau$$

$\alpha = 0$ **Proper time gauge**

$\alpha = 1$ **Scale factor gauge** $t = \log a$

$$\boxed{\frac{dV_i}{dt} = 3H_i^{1-\alpha} V_i + M_{ij} V_j}$$

rate equation

$$M_{ij} = \underbrace{\lambda_{ij}}_{\text{Gained from other vacua}} - \underbrace{\delta_{ij} \sum_r \lambda_{ri}}_{\text{Lost to other vacua}}$$

Gained from other vacua

Lost to other vacua

$$\lambda_{ij} = \frac{4\pi}{3} H_j^{-3-\alpha}\, \Gamma_{ij}$$

From bubbles of type "i" in vacuum "j".

$-q \equiv$ M_{ij}

-
- γ_0
- γ_0

e.g. in scale factor gauge, the solution is of the form

$$V_i(a) = V_i^{(0)} a^{\gamma_0} + \sum_n V_i^{(n)} a^{\gamma_n} \to \boxed{V_i^{(0)}} a^{\gamma_0} \qquad (a \to \infty)$$

$$\gamma_0 = 3 - q$$

$-q \equiv$ Dominant eigenvalue of transition matrix M_{ij}

- $q \le \min_j \sum_i \lambda_{ij} \qquad 0 < q \ll 1$
- If γ_0 has positive real part, volume grows unbounded (that's eternal inflation).
- If γ_0 is real, positive, and non-degenerate, ⇒ late time volume distribution approaches unique attractor solution

$$\frac{V_i(a)}{V_{total}(a)} \propto V_i^{(0)} \qquad (a \to \infty)$$

Independent of initial conditions

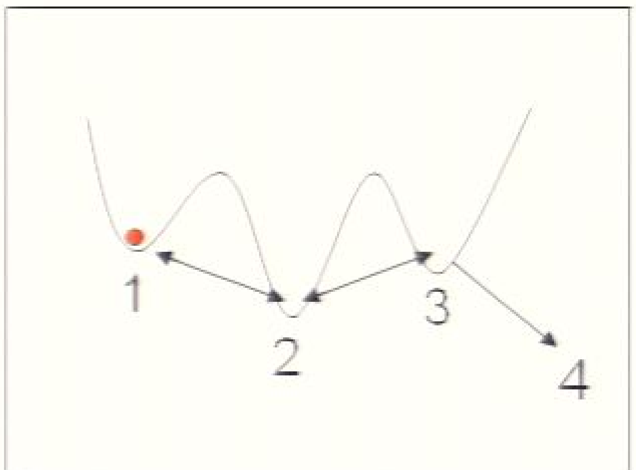
To each irreducible ``landscape'' there corresponds a unique attractor volume distribution.

$$V_i(t) \rightarrow \; V_i^{(0)} \; e^{(3-q)t} \qquad 0<q\ll 1$$

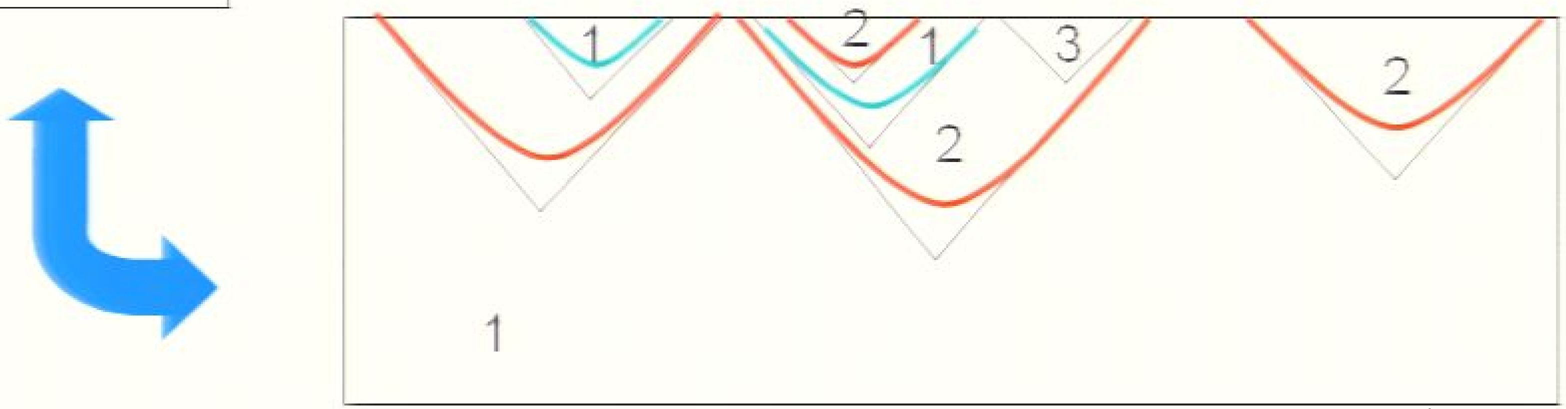
In this sense, initial conditions do not play a role.

J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)

THEORY

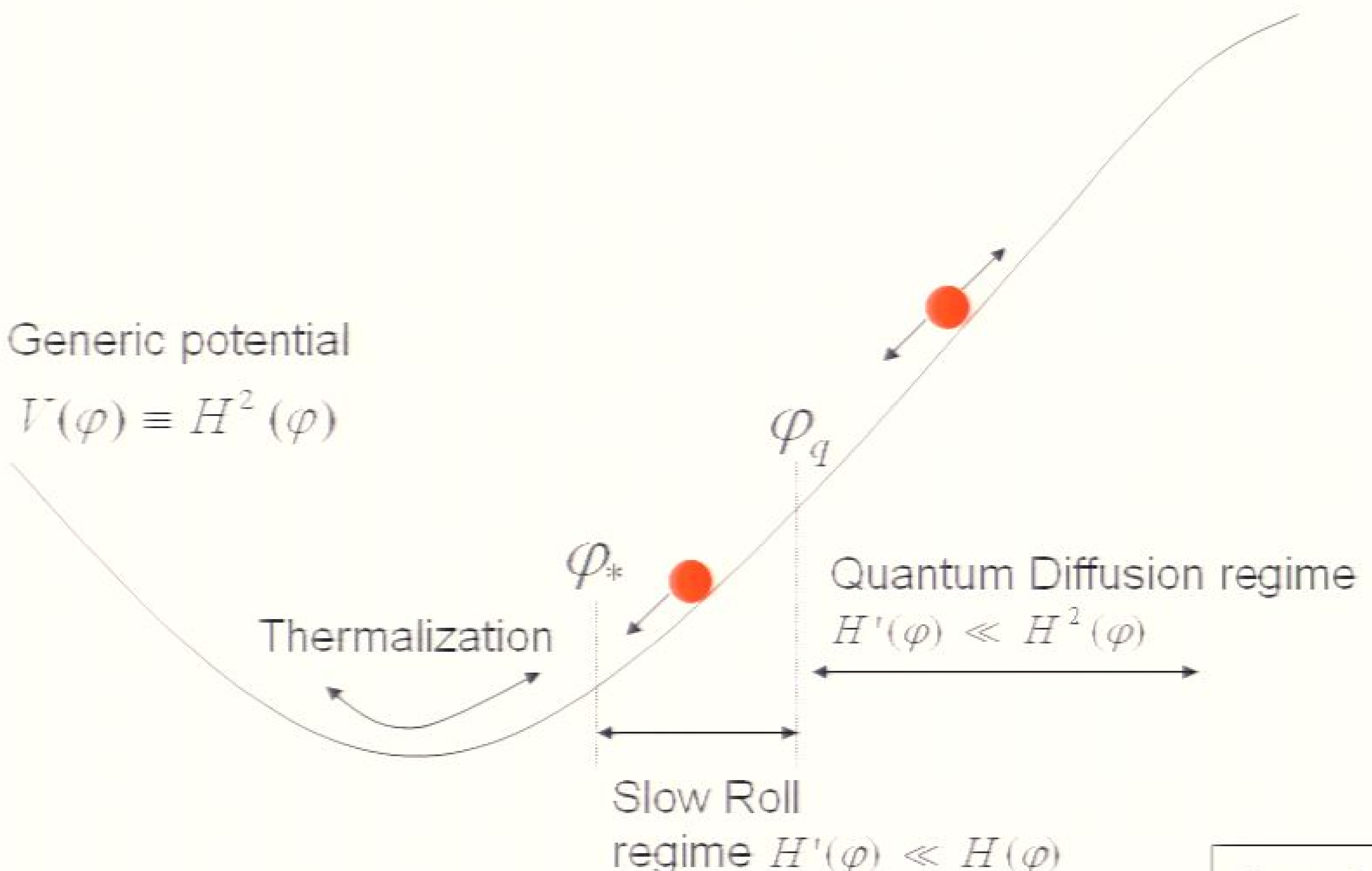


(Self-similar fractal)



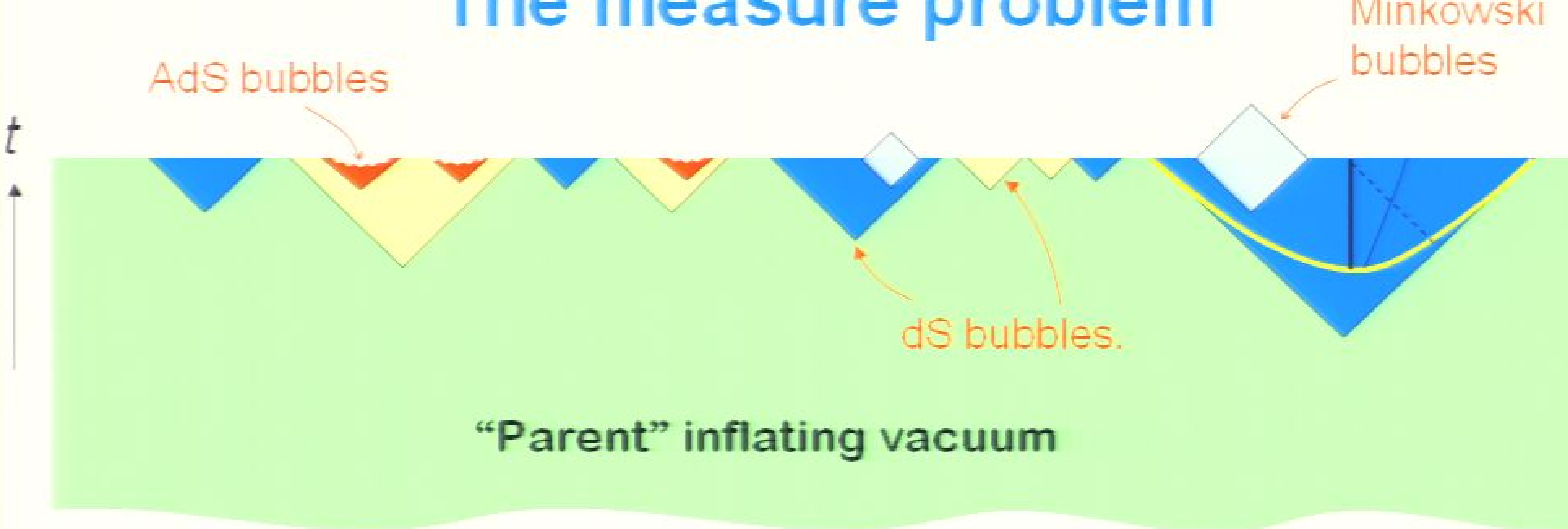
MULTIVERSE

Models with quantum diffusion



$$\delta\varphi / \delta t|_{QD} \sim H^2$$
$$\delta\varphi / \delta t|_{SR} \sim H'$$

The measure problem



- Everything that can happen will happen an infinite number of times. We have to learn how to compare these infinities. (Otherwise we cannot distinguish probable events from highly improbable & cannot make any predictions.)

Galileo: “ Infinite numerable sets cannot be said to be bigger or smaller than one another.”

Example

- $\mathbb{S}=\{1^2,2^2,3^2,\ldots\} \subset \mathbb{N}=\{1,2,3,\ldots\}$

  Suggests that $\mathbb{S}$ smaller than $\mathbb{N}$

- But for each $n \in \mathbb{N}$ there is one and only one $n^2 \in \mathbb{S}$

  Suggests that $\mathbb{S}$ NOT smaller than $\mathbb{N}$

Sequences

The elements of $\mathbb{N}$ can be arranged into the natural sequence

$$1,2,3,2^2,5,6,7,8,3^2,10,11,12,13,14,15,4^2,17,\ldots$$

Frequency of occurrences

Cutting off the sequence at finite length L,
the relative frequency of perfect squares is

$$P(n \in \mathbb{S}) \sim L^{-1/2} \to 0 \qquad (L \to \infty)$$

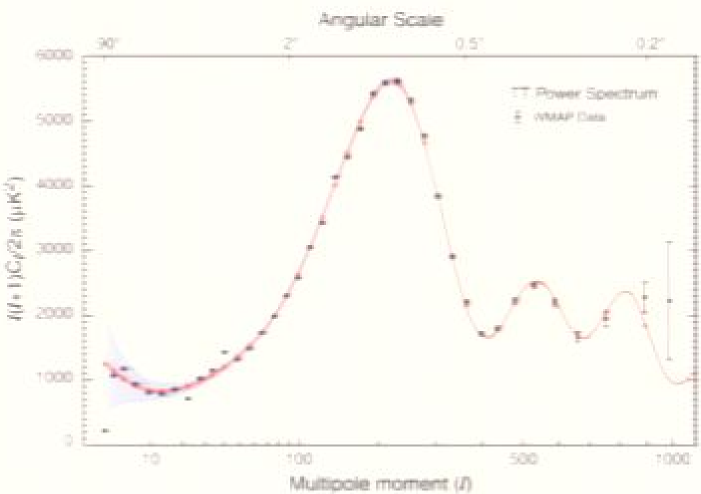
Perfect squares are very rare in the natural sequence.

More generally: We can use infinite sequences define probabilities, as the frequencies of occurrence.

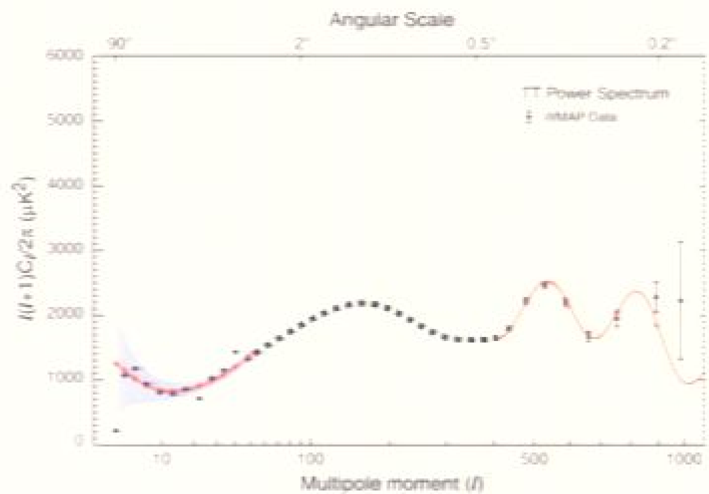
Giordano Bruno: "The universe is infinite, homogeneous, and full of planets like the Earth, orbiting stars like the Sun" (c.1600)

In an infinite and homogeneous universe:

We see this CMB



Some people, very far away, will see this "unlikely" CMB



We say this one is "more likely", because we think of a specific "sequence", where this occurs more often.

Some proposals for the probability measure

Measure's galore:

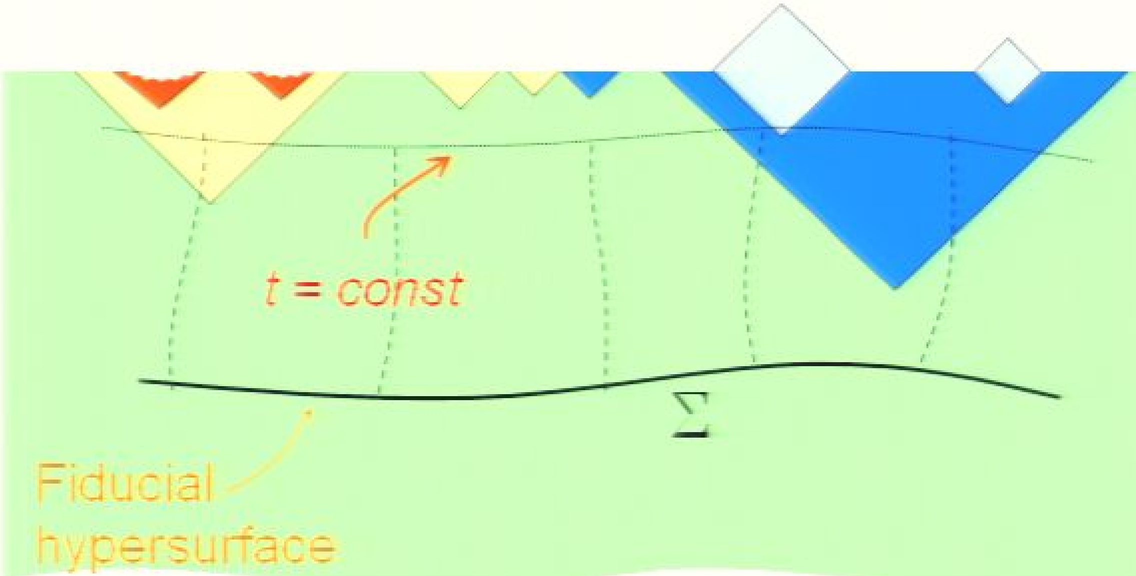
- Proper time cutoff — *Garcia-Bellido, Linde & Linde (1994)* *Vilenkin (1995)*
- Scale factor cutoff — *Garcia-Bellido, Linde & Linde (1994)* *De Simone, Guth, Salem & Vilenkin 2008)*
- Pocket-based — *J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)* *Easther, Lim & Martin (2005)*
- Causal-patch — *Bousso (2006); Susskind (2007)*
- Reheating volume — *Winitzki (2008)*

Empirical approach:

Investigate different measure proposals and discard those which suffer from internal inconsistencies or strongly disagree with observations.

Global time cutoff:

Count only observations that were made before some time *t*.



Garcia-Bellido, Linde & Linde (1994); Vilenkin (1995)

Possible choices of t :

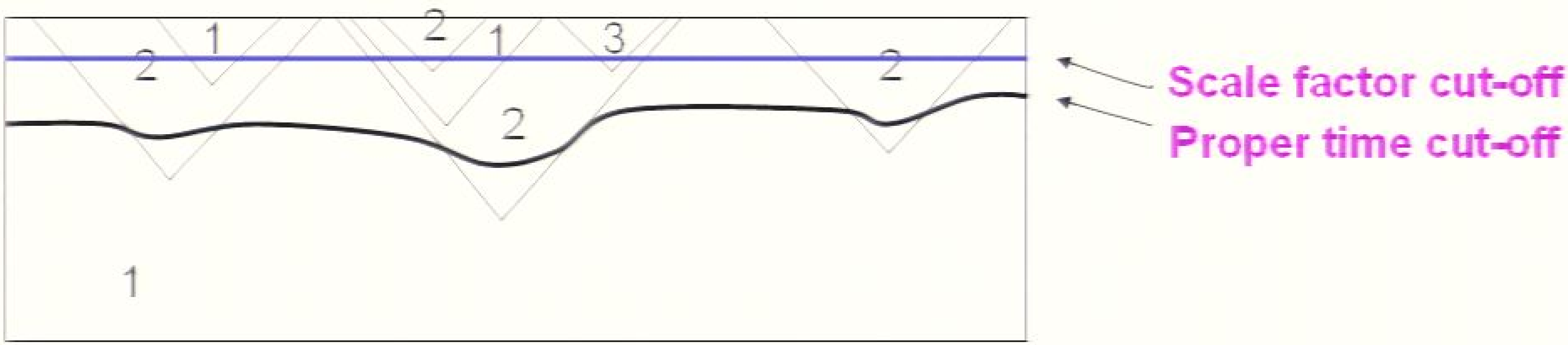
(i) proper time $t = \tau$ along geodesics orthogonal to Σ ;

(ii) scale-factor time, $t = \ln a$.

$t \to \infty$ ➡ steady-state distribution.

The distribution does not depend on the choice of Σ
-- but depends on what we use as *t*.

Different choices of time variable $dt = H^{\alpha}\, d\tau$
give rise to diferent
results for the dominant eigenvalue $\Gamma_i^{\tau(0)}(\alpha)$



The regularized probability distributions will be different

Proper time cutoff leads to "youngness paradox"

Linde & Mezhlumian (1996), Guth (2001), Tegmark (2004), Bousso, Freivogel & Yang (2007)

Volume in regions of any kind grows as

$$V_j \propto e^{\kappa\tau}, \quad \kappa \sim H_{\max} \sim M_{Pl}.$$

Driven by fastest-expanding vacuum

Observers who take less time to evolve are rewarded by a huge volume factor.

Observers who evolve faster than us by $\Delta\tau = 1\,\mathrm{Gyr}$ and measure $T_{CMB} = 2.9\mathrm{K}$ are more numerous by

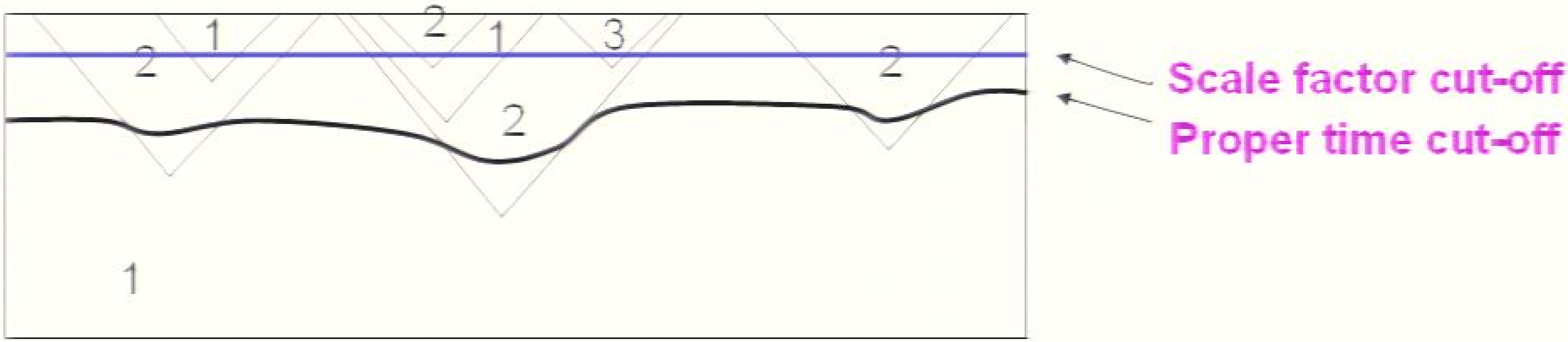
$$\exp(\kappa\,\Delta\tau) = \exp(10^{60})$$

Proper-time cutoff is ruled out.

Different choices of time variable give rise to diferent results for the dominant eigenvalue

$$dt = H^{\alpha}\, d\tau$$

$$\Gamma_i^{(0)}(\alpha)$$



The regularized probability distributions will be different

Proper time cutoff leads to "youngness paradox"

Linde & Mezhlumian (1996), Guth (2001), Tegmark (2004), Bousso, Freivogel & Yang (2007)

Volume in regions of any kind grows as

$$V_j \propto e^{\kappa\tau}, \quad \kappa \sim H_{\max} \sim M_{Pl}.$$

Driven by fastest-expanding vacuum

Observers who take less time to evolve are rewarded by a huge volume factor.

Observers who evolve faster than us by $\Delta\tau = 1\,\mathrm{Gyr}$ and measure $T_{CMB} = 2.9\mathrm{K}$ are more numerous by

$$\exp(\kappa\,\Delta\tau) = \exp(10^{60})$$

Proper-time cutoff is ruled out.

Scale-factor cutoff – a mild youngness bias

Growth of volume: $V_j \propto e^{(3-q)t} = a^{(3-q)}$

$$q \propto \lambda_{\min} \ll 1$$

– decay rate of the slowest-decaying vacuum

The probability of living at T = 2.9K is enhanced only by $(T/T_0)^3 \approx 1.2$.

Not ruled out and may have interesting observational consequences.

Proper time cutoff leads to “youngness paradox”

Linde & Mezhlumian (1996), Guth (2001), Tegmark (2004), Bousso, Freivogel & Yang (2007)

Volume in regions of any kind grows as

$$V_j \propto e^{\kappa\tau}, \quad \kappa \sim H_{\max} \sim M_{Pl}.$$

Driven by fastest-expanding vacuum

Observers who take less time to evolve are rewarded by a huge volume factor.

Observers who evolve faster than us by $\Delta\tau = 1\,\mathrm{Gyr}$ and measure $T_{CMB} = 2.9\mathrm{K}$ are more numerous by

$$\exp(\kappa\,\Delta\tau) = \exp(10^{60})$$

Proper-time cutoff is ruled out.

Scale-factor cutoff – a mild youngness bias

Growth of volume: $V_j \propto e^{(3-q)t} = a^{(3-q)}$

$$q \propto \lambda_{\min} \ll 1$$ – decay rate of the slowest-decaying vacuum

The probability of living at T = 2.9K
is enhanced only by $(T/T_0)^3 \approx 1.2$.

Not ruled out and may have interesting observational consequences.

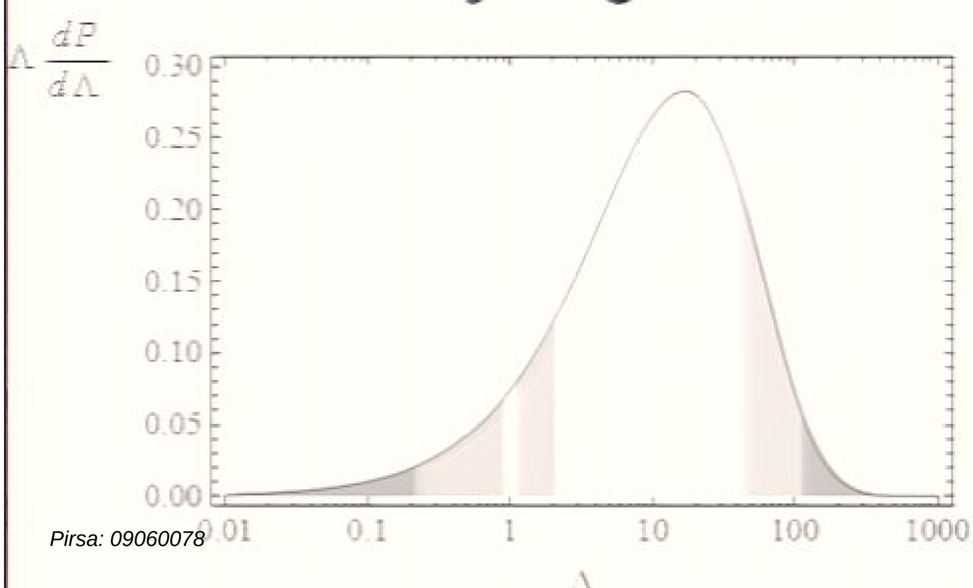
Probability distribution for Λ in scale factor measure

De Simone, Guth, Salem & Vilenkin (2008)

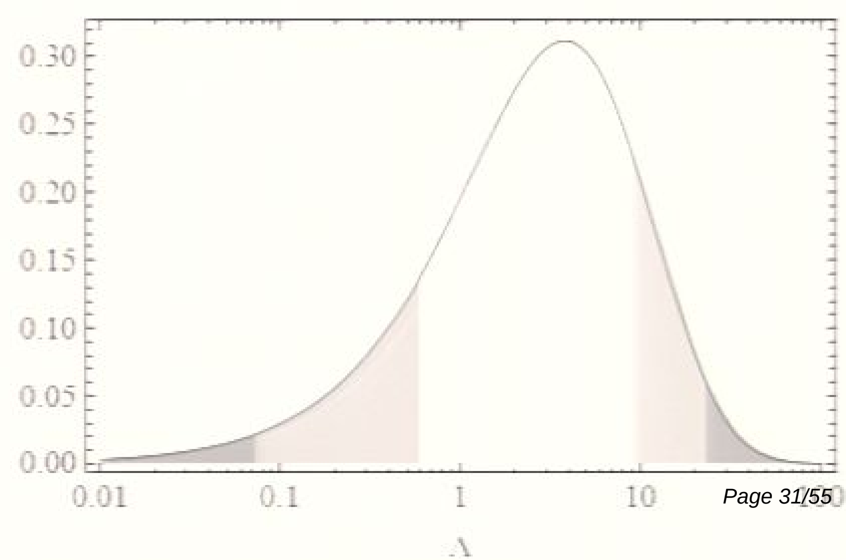
Assumptions:

- Number of observers is proportional to the fraction of matter clustered in large galaxies ($M \geq 10^{12} M_{\odot}$).
- Observers do their measurements at a fixed time = 5 Gyr after galactic halo collapse.

Without youngness bias



Including youngness bias



Pocket-based measure

J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)
Easther, Lim & Martin (2005)

$$P_j \propto p_j w_j$$

p_j – bubble abundance.

Select $\mathcal{N}$ geodesics at random out of the congruence. Count only bubbles crossed by at least one geodesic. Take limit $\mathcal{N} \to \infty$.

w_j – weight factor.

Sample equal comoving volumes in all bubbles.
(All bubble spacetimes are identical at early times.)

Large inflation inside bubbles is rewarded:

$$w_j \propto Z_j^3$$

expansion factor during inflation

(This leads to some problems…)

"Q catastrophe"

Feldstein, Hall & Watari (2005)
J.G. & Vilenkin (2006)
Graesser & Salem (2007)

The expansion factor Z_j depend exponentially on the shape of inflaton potential $V(\varphi)$.

$P_j \propto Z_j^3$ ⟹ Parameters sensitive to $V(\varphi)$ have exponential probability distributions.

In particular the amplitude of density perturbations Q.

“Q catastrophe”

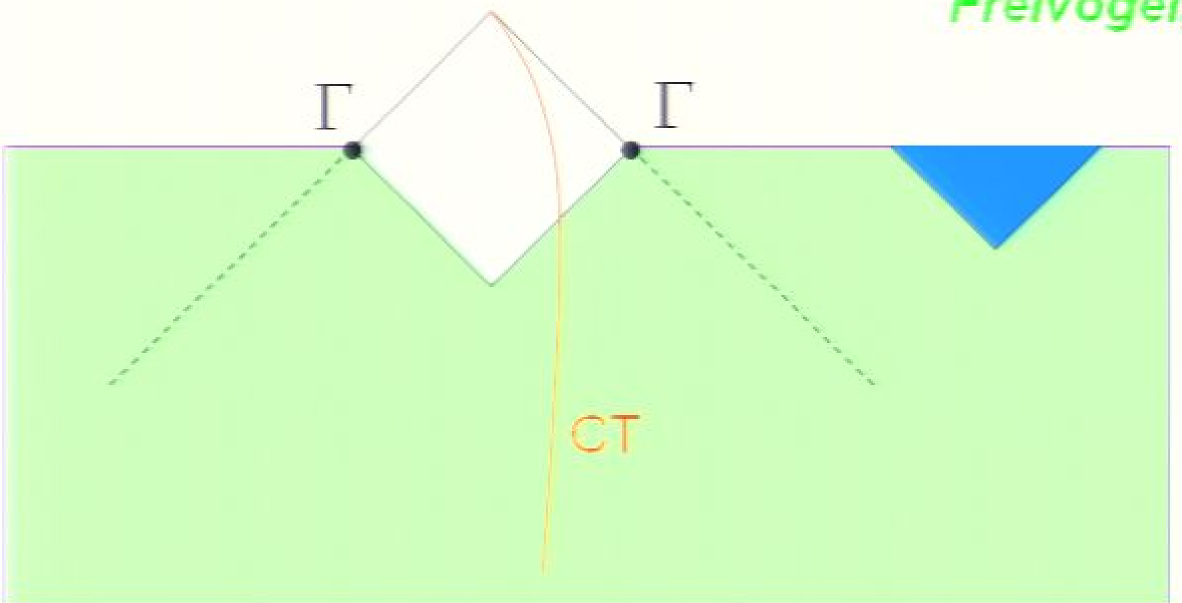
Feldstein, Hall & Watari (2005)
J.G. & Vilenkin (2006)
Graesser & Salem (2007)

The expansion factor Z_j depend exponentially on the shape of inflaton potential $V(\varphi)$.

$P_j \propto Z_j^3$ ⟹ Parameters sensitive to $V(\varphi)$ have exponential probability distributions.

In particular the amplitude of density perturbations Q.

Census-taker (causal patch) measure



Freivogel, Sekino, Susskind & Yeh (2006)
Susskind (2008)
Related idea: ***Bousso (2007).***

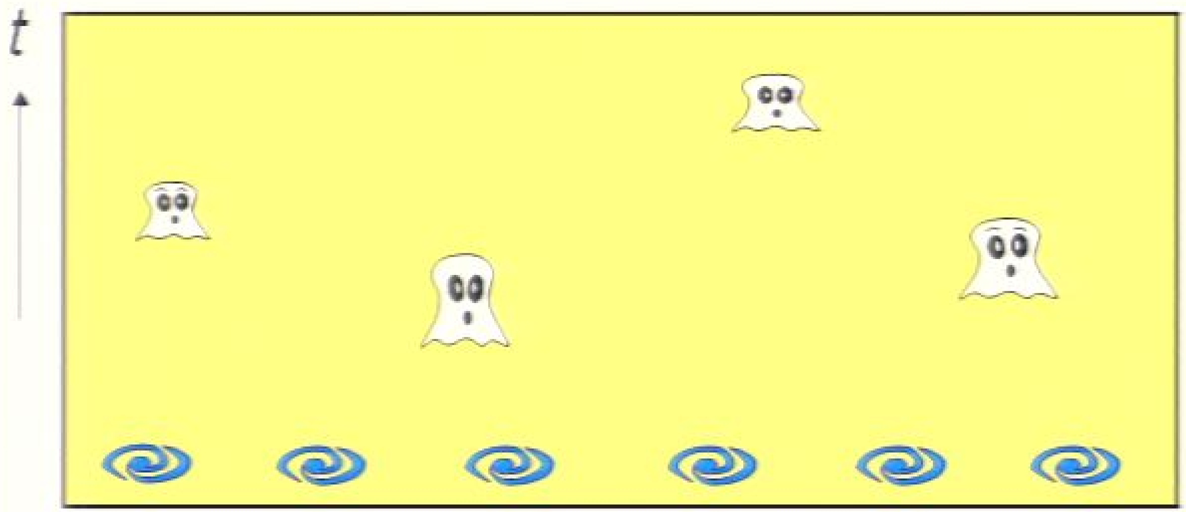
Include only spacetime region accessible to a single observer (CT).

CT will see an infinite number of other bubbles (that collide with his bubble), but their abundances depend on the nucleation rates in the parent vacuum.

→ *The measure depends on the initial state.*

Holography: The 4D theory in the region accessible to CT is equivalent (*dual*) to a Euclidean 2D field theory on Γ.

Freak observers ("Boltzmann brains")



Page (2006),
Bousso & Freivogel (2006)

Observers popping out as quantum fluctuations outnumber regular observers.

$$\Gamma_{BB} \sim \exp(-M / T_{dS})$$

In causal patch and in scale factor measure:

Bousso, Freivogel & Yang (2008)

BBs are not a problem, provided that

- $\Gamma_{BB} / \Gamma_{vac} << 1$

for all vacua that can support BBs.

Is this satisfied in the landscape?

	Youngness paradox	Q catastrophe	Dependence on initial state	Boltzmann Brain Parado
Proper time cutoff				
Scale factor cutoff				
Pocket-based measure				
Causal patch measure				
Insert your own				

A measure from fundamental theory

(JG, A. Vilenkin, 08,09)

- The dynamics of the multiverse may be encoded in its future boundary (suitably defined).

  Inspired by holographic ideas: *Quantum dynamics of a spacetime region is describable by a boundary theory.*

- The measure can be obtained by imposing a UV cutoff in the boundary theory.
  Related to scale-factor cutoff.

Holography

Maldacena (1998)
Strominger (2001)
Freivogel, Sekino,
Susskind & Yeh (2006)

AdS_{d+1}/CFT_d correspondence

Maldacena (1998)

Euclidean AdS:

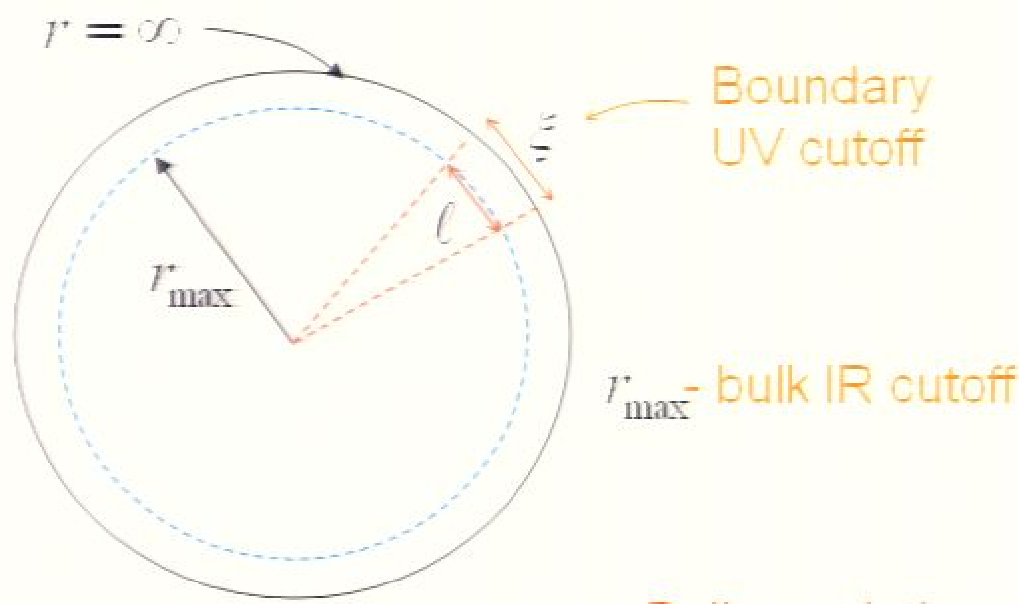
$$ds^2 = dr^2 + \sinh^2 r \, d\Omega_D^2$$

Regulate boundary theory:

Integrate out short-wavelength modes of wavelength $\lambda_B < \xi$.

The corresponding bulk modes have minimum wavelength

$$\lambda_{\min}(r) = \xi \sinh r.$$



Require $\lambda_{\min} < \ell$

$$\Longrightarrow \quad \sinh r_{\max} = \ell / \xi$$

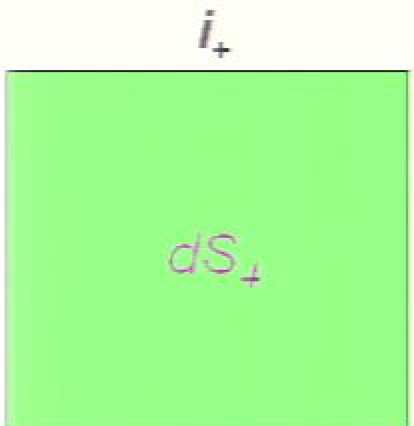
$$r_{\max} \to \infty \quad \Leftrightarrow \quad \xi \to 0.$$

Variation of r_{max} $\Leftrightarrow$ RG flow in the boundary theory.

dS/CFT correspondence

Strominger (2001)

Is the 4D theory describing an asymptotically de Sitter space equivalent to a 3D Euclidean CFT at future infinity i_+?



$$ds^2 = -dt^2 + H^{-2}\cosh^2(Ht)\, d\Omega_3^2$$

Future infinity is $S_3 : \;\; t \to \infty.$

Some potential problems:

1- Scaling dimensions are complex for bulk fields with

$$m^2 > \left(\frac{d}{2}\right)^2 H^2$$

(these fields oscillate in time)

2- In String Theory, dS space is metastable, so there is no such thing as asymptotically dS space.

dS/CFT and the wave function of the universe

$$\varphi(t \to \infty) = \bar{\varphi}(x^i)$$

$$\begin{cases} Z_{d+1}^{\text{Bulk}}[\bar{\varphi}] = \int D\varphi \, e^{i S_{\text{Bulk}}[\varphi]} \equiv \Psi[\bar{\varphi}] \\ e^{i W_{CFT}[\bar{\varphi}]} = \int D\psi \, e^{i S_{CFT}[\psi, \bar{\varphi}]} \end{cases}$$

$$\Psi[\bar{\varphi}] = e^{i W_{CFT}[\bar{\varphi}]}$$

Maldacena 02
Larssen, McNees 04
J.G., Vilenkin 08,09

$$ds^2 = a^2(\eta)[-d\eta^2 + d\mathbf{x}^2]. \qquad a(\eta) = -1/H\eta$$

Gaussian wave functional

$$\Psi[\bar{\phi}(\mathbf{x})] \propto e^{\frac{i}{2}\ a^{d-1}\int d^d\mathbf{k}\ \frac{v'_{\mathbf{k}}}{v_{\mathbf{k}}}|\phi_{\mathbf{k}}|^2}.$$

$$\phi(\mathbf{x}) = \int d^d\mathbf{k} \frac{e^{i\mathbf{kx}}}{(2\pi)^{d/2}}\phi_{\mathbf{k}}.$$

$$v^*_{\mathbf{k}} v'_{\mathbf{k}} - v_{\mathbf{k}} v'^*_{\mathbf{k}} = i a^{1-d}$$

Choice of vacuum (for m=0)

$$v_{\mathbf{k}}(\eta) = \frac{\pi^{1/2}}{2} a^{-d/2} H^{(1)}_{d/2}(k\eta)$$

Bunch-Davies

⇓ (d+1=5)

$$\Psi[\bar{\phi}(\mathbf{x})] \propto \exp\left[\frac{i}{2} \int d^4\mathbf{k} \left(\frac{-H^{-1}k^2a^2}{2} + \frac{1}{8}[i\pi + 2\gamma + \ln(k^2\eta^2)]k^4H^{-3} + O(a^{-2})\right)|\phi_{\mathbf{k}}|^2\right]$$

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8}k^4 H^{-3}\right)|\phi_{\mathbf{k}}|^2\right]$$

$$\langle \phi^*_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \frac{8H^3}{\pi}k^{-4}\delta(\mathbf{k}'-\mathbf{k})$$

Imaginary part of W_{CFT} determines the amplitude of cosmological perturbations.

But there is also the non-local part

$$W_{CFT} = \frac{H^{-3}}{16}\int d^4\mathbf{k}\ k^4 \ln(k^2/H^2)\ |\phi_{\mathbf{k}}|^2 + \text{analytic.}$$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\, k^4 \ln k^2$$

Expected form in a CFT

(As in Gubser, Klebanov, Polyakov 98)

Expected form in weakly coupled CFT

$$W[g_{ij}] = \int d^4x \sqrt{g} \left[c_1 R \ln(\Box/\mu^2) R + c_2 R_{ij} \ln(\Box/\mu^2) R^{ij} + c_3 R_{ijkl} \ln(\Box/\mu^2) R^{ijkl} \right] + \text{analytic}.$$

Coefficient of logarithmically divergent term is the trace anomaly

$$a_2 \sim \int d^4x \sqrt{g} \left[c_1 R^2 + c_2 R_{ij} R^{ij} + c_3 R_{ijkl} R^{ijkl} \right].$$

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8}k^4 H^{-3}\right)|\phi_{\mathbf{k}}|^2\right]$$

$$\langle \phi^*_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \frac{8H^3}{\pi}k^{-4}\delta(\mathbf{k}'-\mathbf{k})$$

Imaginary part of W_{CFT} determines the amplitude of cosmological perturbations.

But there is also the non-local part

$$W_{CFT} = \frac{H^{-3}}{16}\int d^4\mathbf{k}\; k^4 \ln(k^2/H^2)\;|\phi_{\mathbf{k}}|^2 + \text{analytic}.$$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\, k^4 \ln k^2$$

Expected form in a CFT

(As in Gubser, Klebanov, Polyakov 98)

Expected form in weakly coupled CFT

$$W[g_{ij}] = \int d^4x \sqrt{g} \left[c_1 R \ln(\Box/\mu^2) R + c_2 R_{ij} \ln(\Box/\mu^2) R^{ij} + c_3 R_{ijkl} \ln(\Box/\mu^2) R^{ijkl} \right] + \text{analytic.}$$

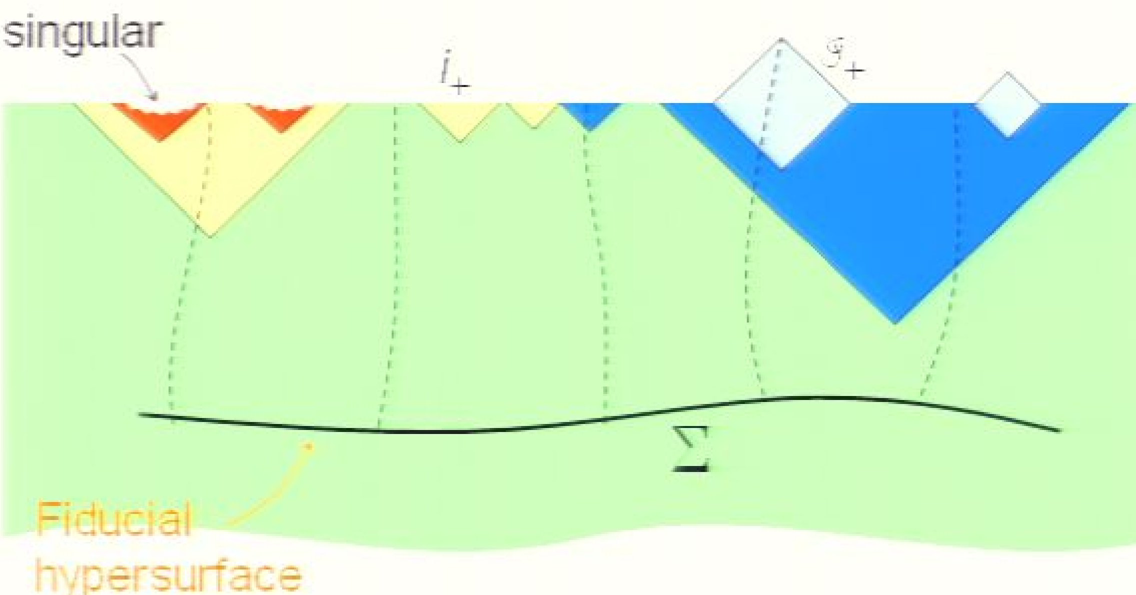
Coefficient of logarithmically divergent term is the trace anomaly

$$a_2 \sim \int d^4x \sqrt{g} \left[c_1 R^2 + c_2 R_{ij} R^{ij} + c_3 R_{ijkl} R^{ijkl} \right].$$

Proposal:

The boundary theory lives at the future boundary of the multiverse (suitably defined).

Future infinity



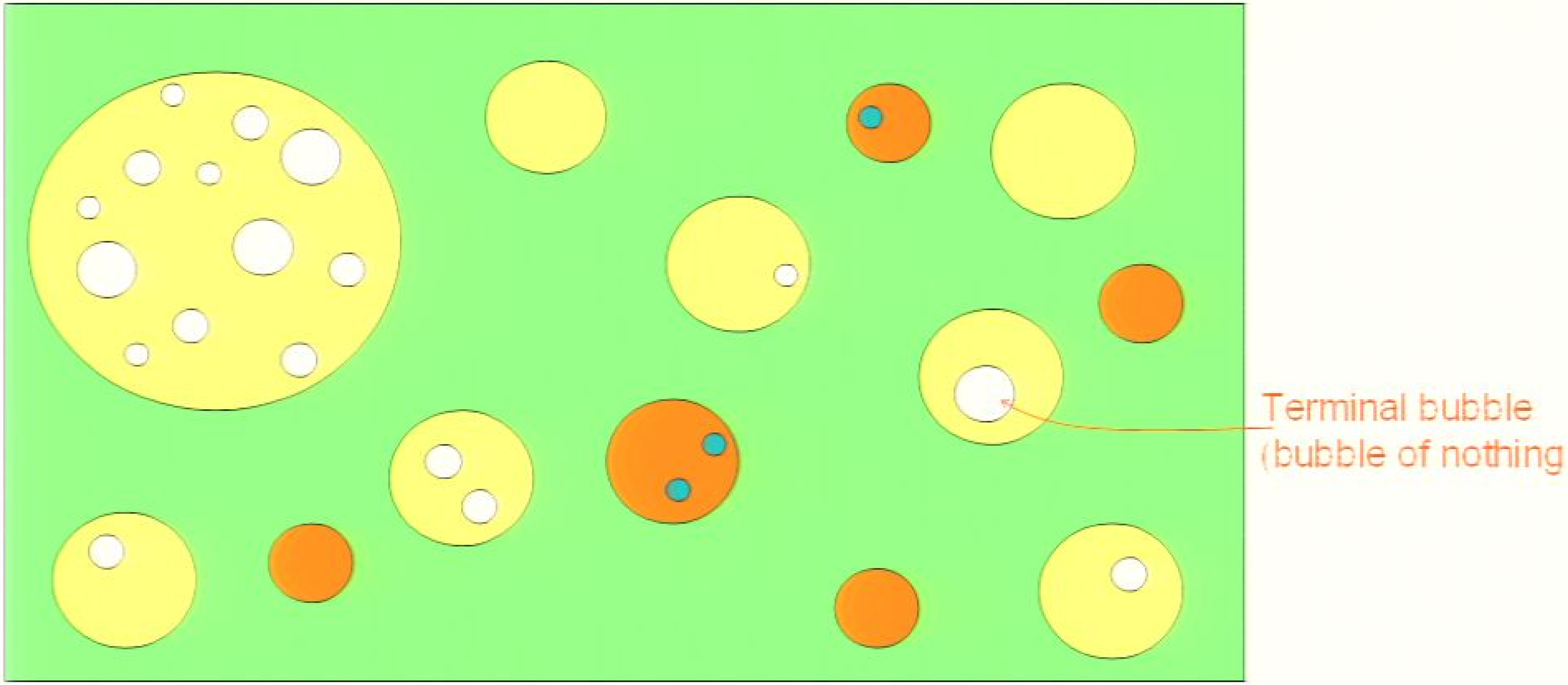
- Geodesic congruence projects bubbles onto Σ.
  $\Rightarrow$ Map of future infinity.
- Excise images of Minkowski bubbles. (They are described by the *2D* boundary degrees of freedom. *(FSSY)*
- AdS bubbles can be excised in a similar way (?).

(Horowitz, Hertog 05)

The future infinity F includes eternal points (and the boundaries of excised terminal bubbles).

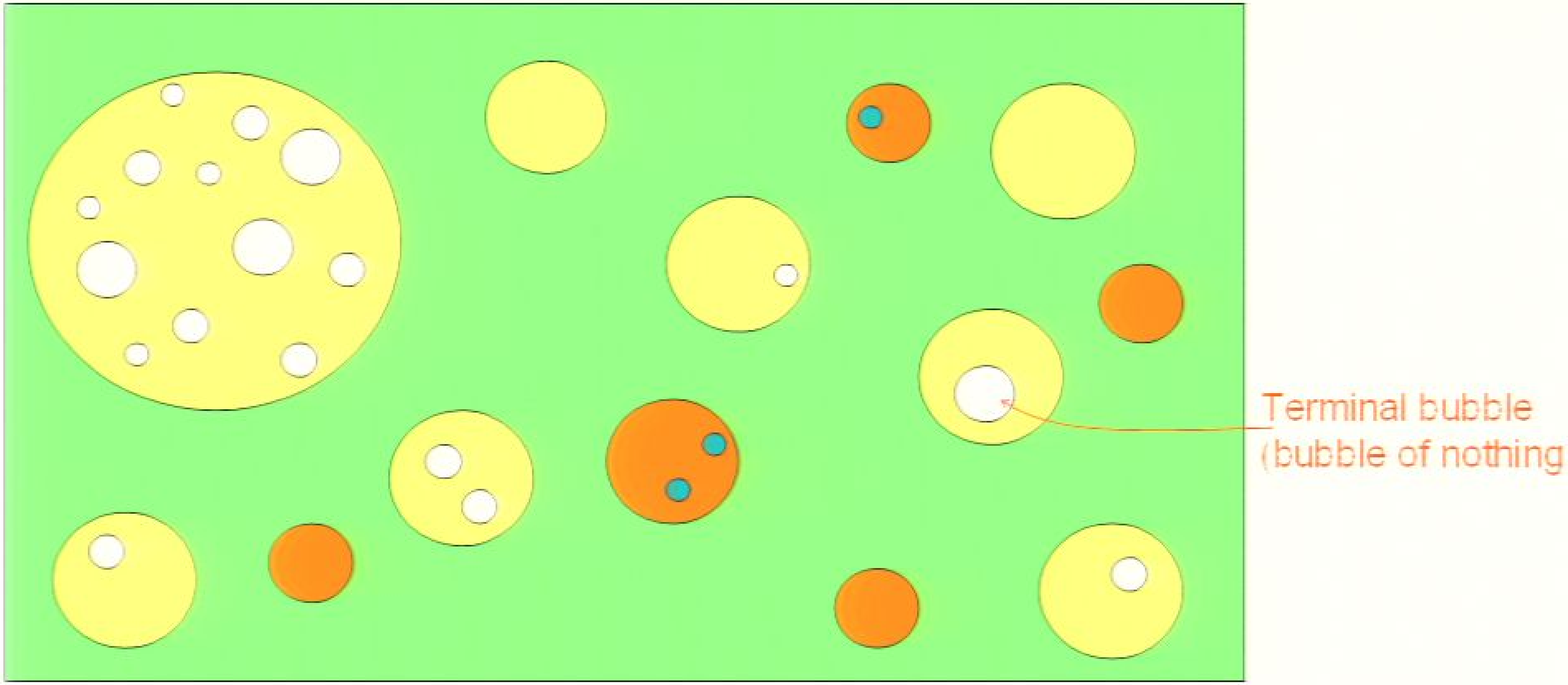
The metric $g_{ij}(x)$ on Σ defines a metric on F

Structure of the future boundary F



- Each bubble becomes a fractal "sponge" in the limit.
- Terminal bubbles correspond to holes (with 2D CFTs on their boundaries).
- Bubbles walls are sources in the boundary theory.

Structure of the future boundary F



- Each bubble becomes a fractal "sponge" in the limit.
- Terminal bubbles correspond to holes (with 2D CFTs on their boundaries).
- Bubbles walls are sources in the boundary theory.

Conformal invariance of the boundary theory

- $\langle TT \rangle$ correlator has the form expected in a CFT.
- Distribution of nested bubbles is invariant under the Euclidean conformal group in the UV
- Correlator of bubble fluctuations also has the form expected in CFT.

BUBBLE FLUCTUATIONS

Bulk calculation leads to:

$$W[\bar{\delta}] = \frac{TR_0}{H^2} \sum_{LM} \frac{\Delta(\Delta+2)}{16} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right) + 2\left(\psi(L) + \frac{1}{L}\right) + i\pi + 2\gamma \right] |\delta_{LM}|^2 + \ldots$$

Expected result in weakly coupled CFT

$$a_{3/2} = \int d\Sigma_2 \left[d_1 \left(K_{ab}K^{ab} - \frac{1}{2}K^2 \right) + d_2 \hat{R} \right] \propto \int d\Omega \; \delta\Delta(\Delta+2)\delta.$$

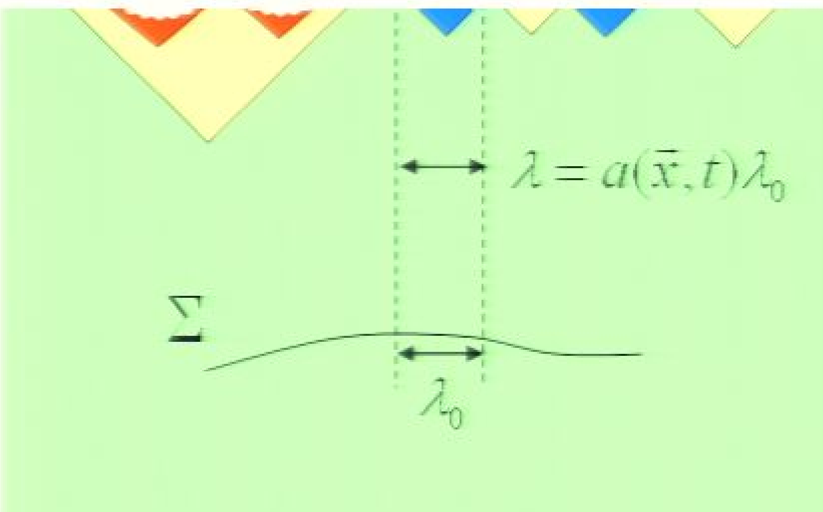
Boundary measure

Renormalization of boundary theory:

Integrate out short-wavelength modes $\lambda_0 < \xi$ ← Boundary UV cutoff

The corresponding 4D modes have minimum wavelength

$$\lambda_{\min}(\bar{x}, t) = a(\bar{x}, t)\,\xi.$$



Require $\lambda_{\min} < \ell$ ← Bulk resolution scale

⟹ $a_{\max} = \ell / \xi$ -- scale factor cutoff

$$\xi \to 0 \quad \Rightarrow \quad a_{\max} \to \infty .$$

UV cutoff on the boundary ⟺ (IR) scale factor cutoff in 4D.

RG flow on the boundary ⟺ scale-factor time evolution.

Correspondence is accurate on super-horizon scales.

Summary

- The dynamics of the multiverse may be encoded in its future boundary F, in terms of a theory which is conformal in the UV.
- The measure can be obtained by imposing a UV cutoff in the boundary theory.
- This measure is closely related to the scale factor cutoff.
- The measure may perhaps be formulated even if there is no dual theory, based on the property of conformal invariance of the wave function.
- We hope to further explore these ideas at the upcoming workshop on holographic cosmology.