Title: Observational Probes of Early Universe Cosmology - Lecture 1

Date: Jun 25, 2009 10:00 AM

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Abstract:

This lecture:
Dynamics of the Universe and distance measurements

1. The current cosmological model: overview
2. From Einstein's field equations to the dynamics of the universe
3. Observations of H_0
4. The age of the Universe
5. Dark matter and baryon content
6. Dark energy and supernovae

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Evolution of density with a

- Rearrange the Friedman equations:

$$3\,(\dot{a}^2 + k)/a^2 = 8\,\pi\,G\,\rho(t)$$
$$(2a\,\ddot{a} + \dot{a}^2 + k)/a^2 = -8\,\pi\,G\,p(t)$$

- to give (k=0)
- *If* $p = w\,\rho$ then
- *If* also w=constant

$$d/dt(\rho a^3) = -p\,d/dt(a^3)$$

$$a^3\,d\rho = -(w+1)\,\rho\,3\,a^2\,da$$

$$\rho \propto a^{-3(w+1)}$$

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What is w?

- Defined by the equation $p = w \rho$
- Often called the "equation of state"
- Special values:
  - w=0 means p=0 e.g. matter
  - w=1/3 e.g. radiation
  - w=-1 looks just like a cosmological constant

Dark energy or cosmological constant?

$$G_k^i + \Lambda\, g_k^i = R_k^i - 1/2\, \delta_k^i\, R + \Lambda\, g_k^i = 8\pi\, G\, T_k^i$$

$$T_k^i = \mathrm{diag}[\rho(t), -p(t), -p(t), -p(t)]$$

- If w=-1 i.e. $p=-\rho$ then $T_k^i = g_k^i$
  - Indistinguishable from a cosmological constant
- Dark energy could have any w
  - Could even vary with time

Evolution of the density

$$\rho \propto a^{-3(w+1)}$$

- Matter dominated (w=0): $\rho \propto a^{-3}$
- Radiation dominated (w=1/3): $\rho \propto a^{-4}$
- Cosmological constant (w=-1): ρ =constant
- Dark energy with w<-1 e.g. w=-2: $\rho \propto a^{3}$
  - Energy density *increases* as is stretched out!
  - Eventually would dominate over even the energies holding atoms together! ("Big Rip")

What is dominant when?

Matter dominated (w=0): $\rho \propto a^{-3}$

Radiation dominated (w=1/3): $\rho \propto a^{-4}$

Dark energy (w~-1): ρ ~constant

- Radiation density decreases the fastest with time
  - Must increase fastest on going back in time
  - Radiation must dominate early in the Universe
- Dark energy with w~-1 dominates last

Radiation domination → Matter domination → Dark energy domination

Putting it all together

- $\rho_{tot} = \rho_r + \rho_m + \rho_{DE}$
- $\rho_r = \rho_{r\,0}\, a^{-4}$ since $\rho \propto a^{-4}$ and a today is 1
  - $\rho_{r0} \equiv$ radiation density today
  - $\Omega_{r0} \equiv \rho_{r0} / \rho_{crit}$ by definition
  - Usually 0 is dropped from Ω_{r0} for simplicity
- So $\rho_r = \rho_{crit}\, \Omega_r\, a^{-4}$
  - sim ρ_m, ρ_{DE}

$$\rho_{tot} = \rho_{crit} \left[\Omega_r\, a^{-2} + \Omega_m\, a^{-3} + \Omega_{DE}\, a^{-3(1+w)} \right]$$

Assuming w for the dark energy is constan

Most important equation in this lecture

$$(\dot{a}/a)^2 = 8\pi G \rho(t) /3$$

Friedman eqn1 for k=0

$$\rho(t) = \rho_{crit} [\Omega_r a^{-2} + \Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)}]$$

- Putting these together and using $H_0 \equiv (\dot{a}/a)_0$:

$$(\dot{a}/a)^2 = H_0^2 (\Omega_r a^{-2} + \Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)})$$

or if w varies with time then $\Omega_{DE}\, e^{-3\int_1^a (1+w(b))d\ln}$

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How does scale factor relate to time?

$$3\,(\dot{a}^2 + k)/a^2 = 8\,\pi\,G\,\rho(t)$$
$$(2a\,\ddot{a} + \dot{a}^2 + k)/a^2 = -8\,\pi\,G\,p(t)$$

Friedman eqns

$$\rho \propto a^{-3(w+1)}$$

Which we just derived

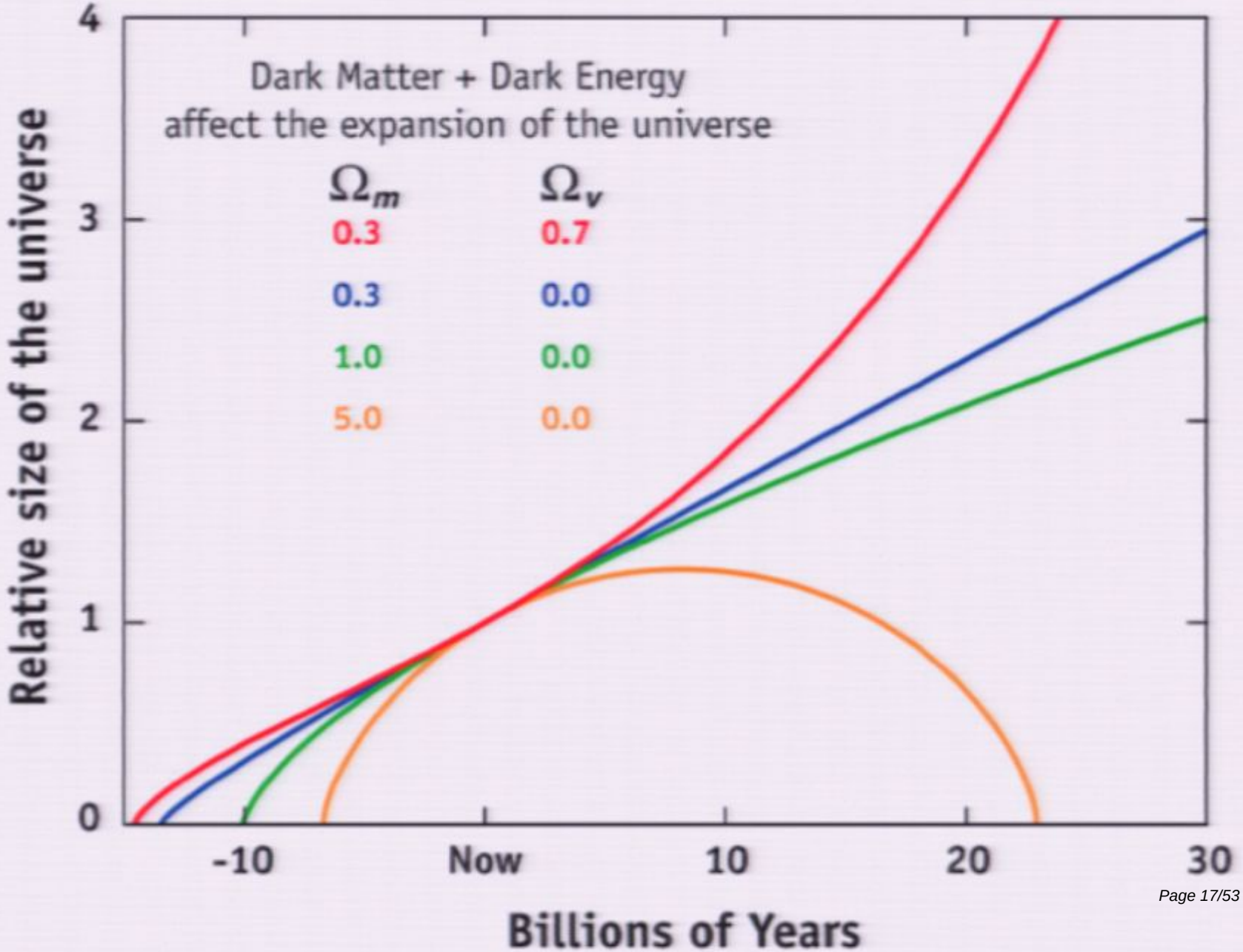
- Then if k=0 (flat Universe) $a^{(3w+1)/2}\,da = dt$
- If w>-1 and w=constant then: $a \propto t^{2/(3(w+1))}$
- If w=-1: $a \propto e^{\lambda t}$

Dynamics of the Universe

$$a \propto t^{2/(3(w+1))}$$

- Matter dominated (w=0): $a \propto t^{2/3}$
  - Decelerating
- Radiation dominated (w=1/3): $a \propto t^{1/2}$
  - Decelerating
- Cosmological constant (w=-1): $a \propto e^{\lambda t}$ (special)
  - Accelerating
- Where is the transition?
  - w>-1/3 decelerating
  - w<-1/3 accelerating

Expansion of the Universe



Redshift and co-moving distance

- Definition of redshift gives 1+z = 1/a
- Co-moving distance remains fixed as Universe expands by opposition to the physical distance
- Co-moving distance between different emitters and us
  - Turns out to be very useful
  - Rearrange to get

$$D = \int_{t(a)}^{t_0} c\frac{dt}{a}$$

$$D = -c\int_{a}^{1} da\frac{dt}{ada}$$

- Use $(\dot{a}/a)^2 = H_0^2(\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)})$

$$D = \int_0^1 \frac{c\frac{da}{a^2}}{H_0(\Omega_m a^{-3}+\Omega_{DE}a^{-3(1+w)})^{1/2}}$$

Luminosity & angular diameter distances

- Using distances is tricky. Imagine we are looking at a QSO at z=3 (a=1/4), what is the right distance to use to relate the flux we see and the luminosity? Or looking at an object of known size, which distance to use?
- Luminosity distance ≡ D_L
  - Defined such that $S = L / (4 \pi D_L^2)$
    - S = bolometric flux density
    - L = bolometric luminosity
  - $D_L = D (1+z)$
- Angular diameter distance ≡ D_A
  - Defined such that $\theta = r / D_A$
    - r=physical size of object
    - θ= angular size on sky
  - $D_A = D/(1+z)$
- Both D_A and D_L will be very important to measure e.g. but requires good templates, "standard candles" (SNe) or "rulers" (BAO, CMB)

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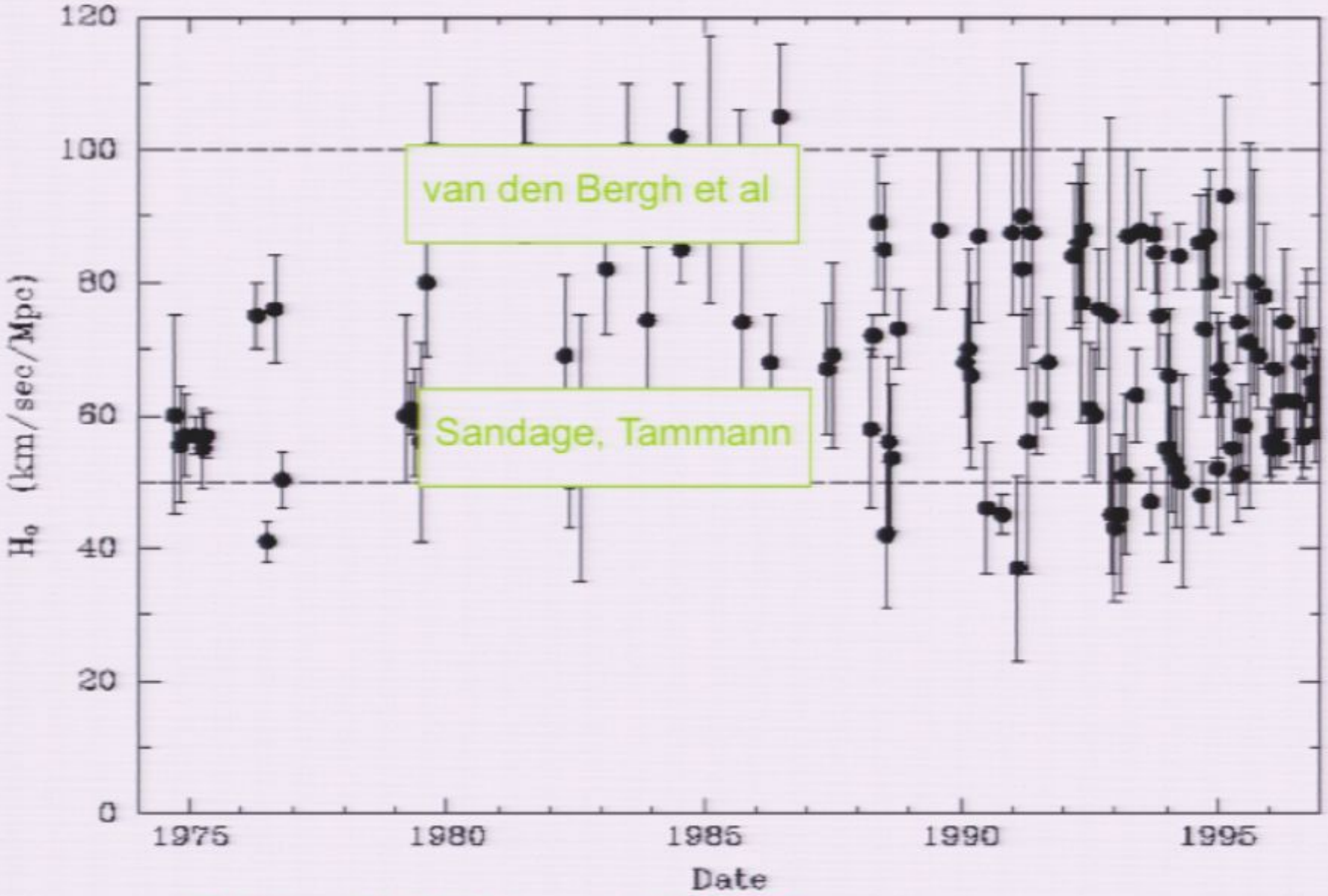
Measurements of the Hubble constant



Rob Kennicutt's Evolution of the Hubble constant 1975-1997. This is an updated (10 Mar 97) version of a graph which appeared in Kennicutt R. C., Freedman W. L. & Mould J. R. (1995) ``Measuring the Hubble Constant with the Hubble Space Telescope'' A.J. 110, 1476

Methods and distance ladder

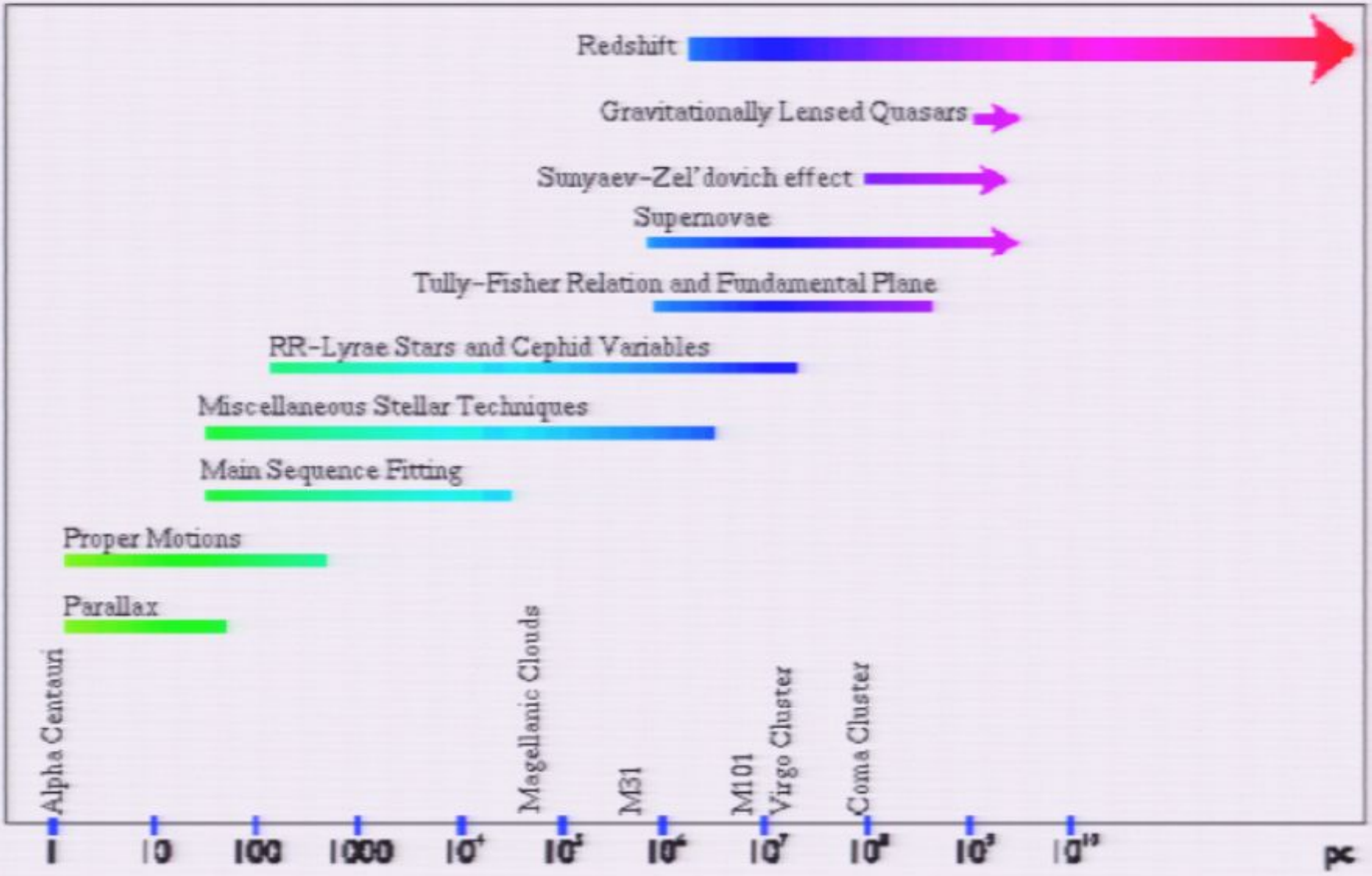
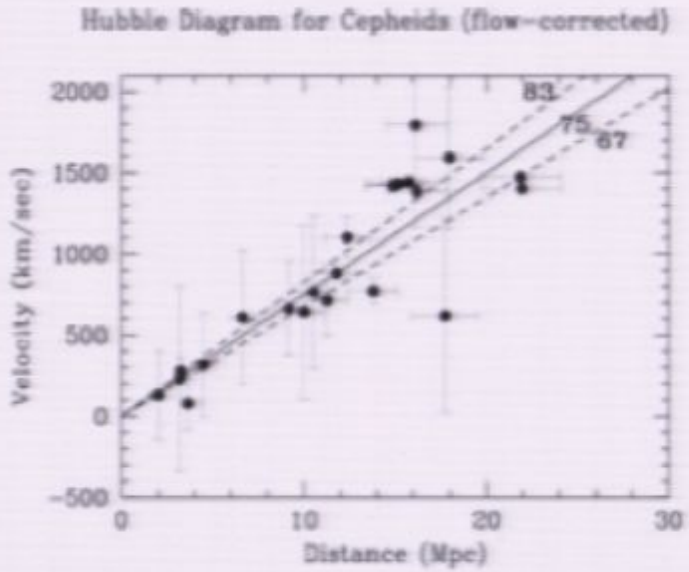


Figure 3.2: *The different distance estimators. This seemingly simple plot shows a grand overview of our efforts to measure distances in the Universe. Adapted from [Rowan-Robinson, 1985] and [Roth and Primack, 1996].*

www.astro.gla.ac.uk/ users/kenton/C185/

Hubble Space Telescope Key Project: Hubble constant

- Comprehensive study of 5 distance indicators
- Freedman et al



Method	H_0	Error (%)	References
36 Type Ia supernovae $4{,}000 < cz < 30{,}000$ km/sec	71	$\pm 2_r$ $\pm 6_s$	Hamuy et al. (1996), Riess et al. (1998), Jha et al. (1999), Gibson et al. (2000)
21 Tully–Fisher clusters $1{,}000 < cz < 9{,}000$ km/sec	71	$\pm 3_r$ $\pm 7_s$	Giovanelli et al. (1997), Aaronson et al. (1982, 19 Sakai et al. (2000)
11 FP clusters $1{,}000 < cz < 11{,}000$ km/sec	82	$\pm 6_r$ $\pm 9_s$	Jorgensen et al. (1996), Kelson et al. (2000)
SBF for 6 clusters $3{,}800 < cz < 5{,}800$ km/sec	70	$\pm 5_r$ $\pm 6_s$	Lauer et al. (1998), Ferrarese et al. (2000a)
4 Type II supernovae $1{,}900 < cz < 14{,}200$ km/sec	72	$\pm 9_r \pm 7_s$	Schmidt et al. (1994)

Combined values of H_0:

$H_0 = 72 \pm 2$ [random] km/sec/Mpc [Bayesian]

$H_0 = 72 \pm 3$ [random] km/sec/Mpc [frequentist]

$H_0 = 72 \pm 3$ [random] km/sec/Mpc [Monte Carlo]

72 ±8

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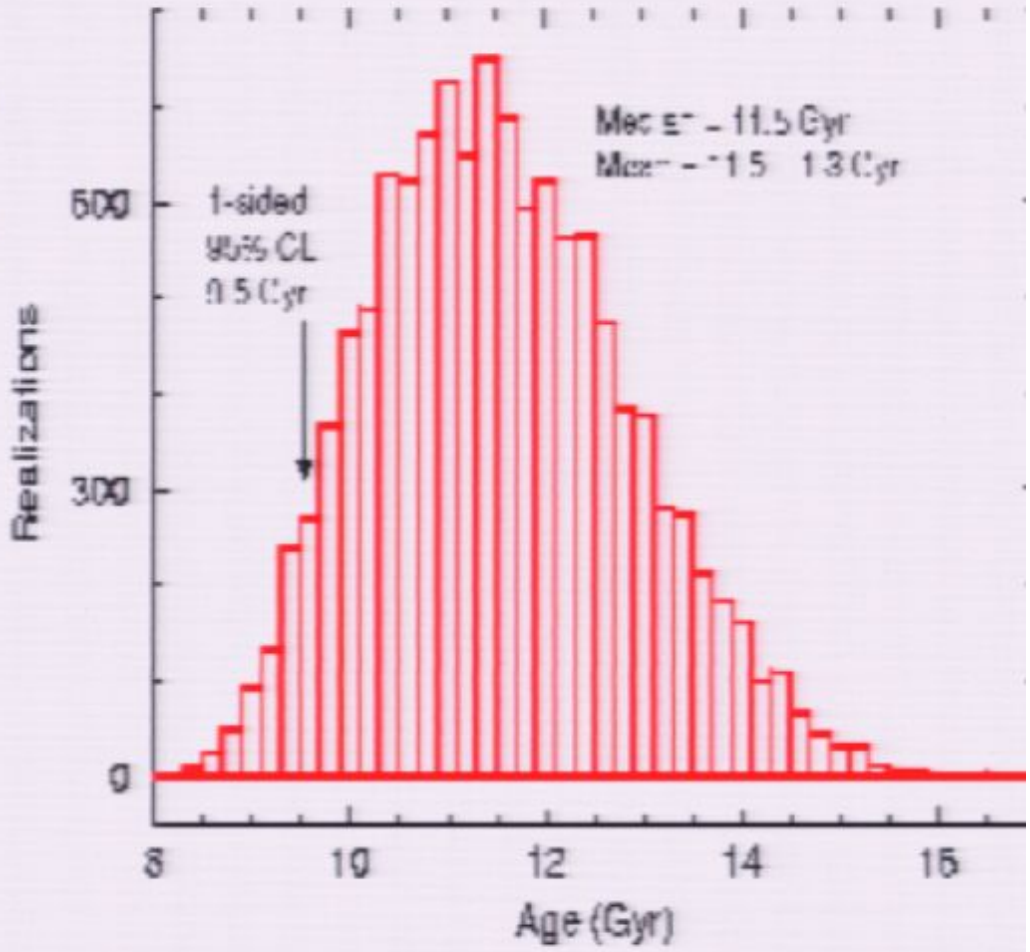
Measurements of the age of universe

- Krauss + Chaboyer
  - stars age = 12.4 Gyr
  - estimate ~2 Gyrs min for formation
  - t_0>10.2 Gyr 95 per cent 1-tailed
- CMB + flatness -> t_0~13.4 Gyr



The age of the Universe

$$(\mathring{a}/a)^2 = H_0^2 \left(\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)}\right)$$

- Integrate to get: $t = \int_0^1 \frac{da/a}{H_0(\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)})^{1/2}}$

- If w=-1 (so $\Omega_{DE}=\Omega_\Lambda$): $t_0 = \frac{2}{3H_0} \frac{\tanh^{-1}(\sqrt{\Omega_\Lambda})}{\sqrt{\Omega_\Lambda}}$

- If $\Omega_{DE}=0$ (so $\Omega_m=1$ given k=0 assumption)

$$t = \int_0^1 a^{1/2} \frac{da}{H_0} = \frac{2}{3H_0}$$

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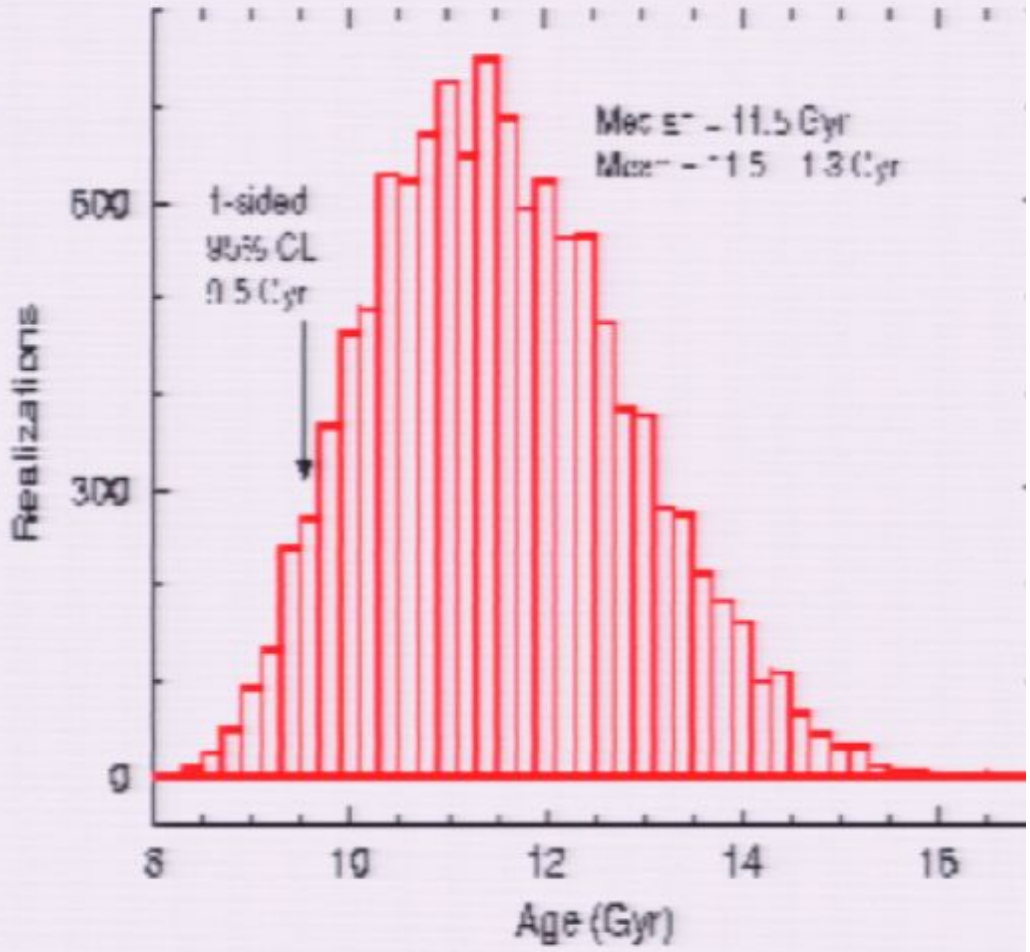

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Dark Energy

- General properties
- Cosmological constant or quintessence?
- Parameterizing quintessence
  - Incorporating quintessence into predictions
  - Parameterizing w(z)
- Current measurements
- Prospects using supernovae

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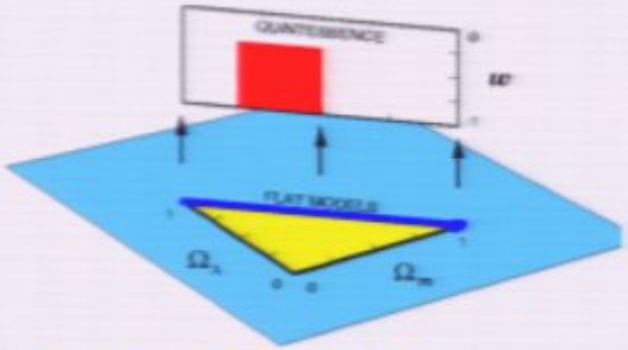
Dark energy: general properties

- The dominant constituent of the universe
- Causes accelerated expansion
- Modifies growth rate of fluctuations (next lecture)

Cosmological constant or Quintessence

- Cosmological constant:
  - zero-point quantum fluctuations? factor of 10^{110}
  - coincidence of $\Omega_m \sim \Omega_\Lambda$ / why accelerates now
- Quintessence
  - time-dependent
  - spatially inhomogeneous
  - e.g. scalar field rolling down a potential
  - negative pressure - w=pressure/density



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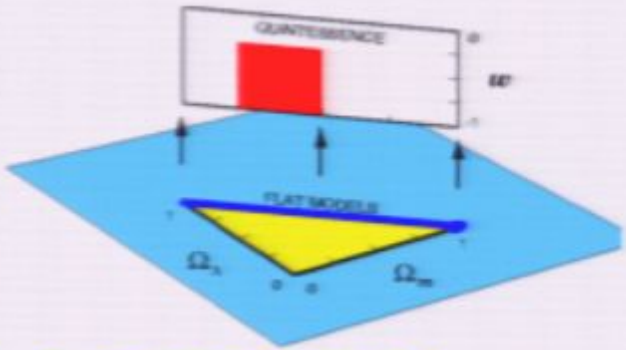
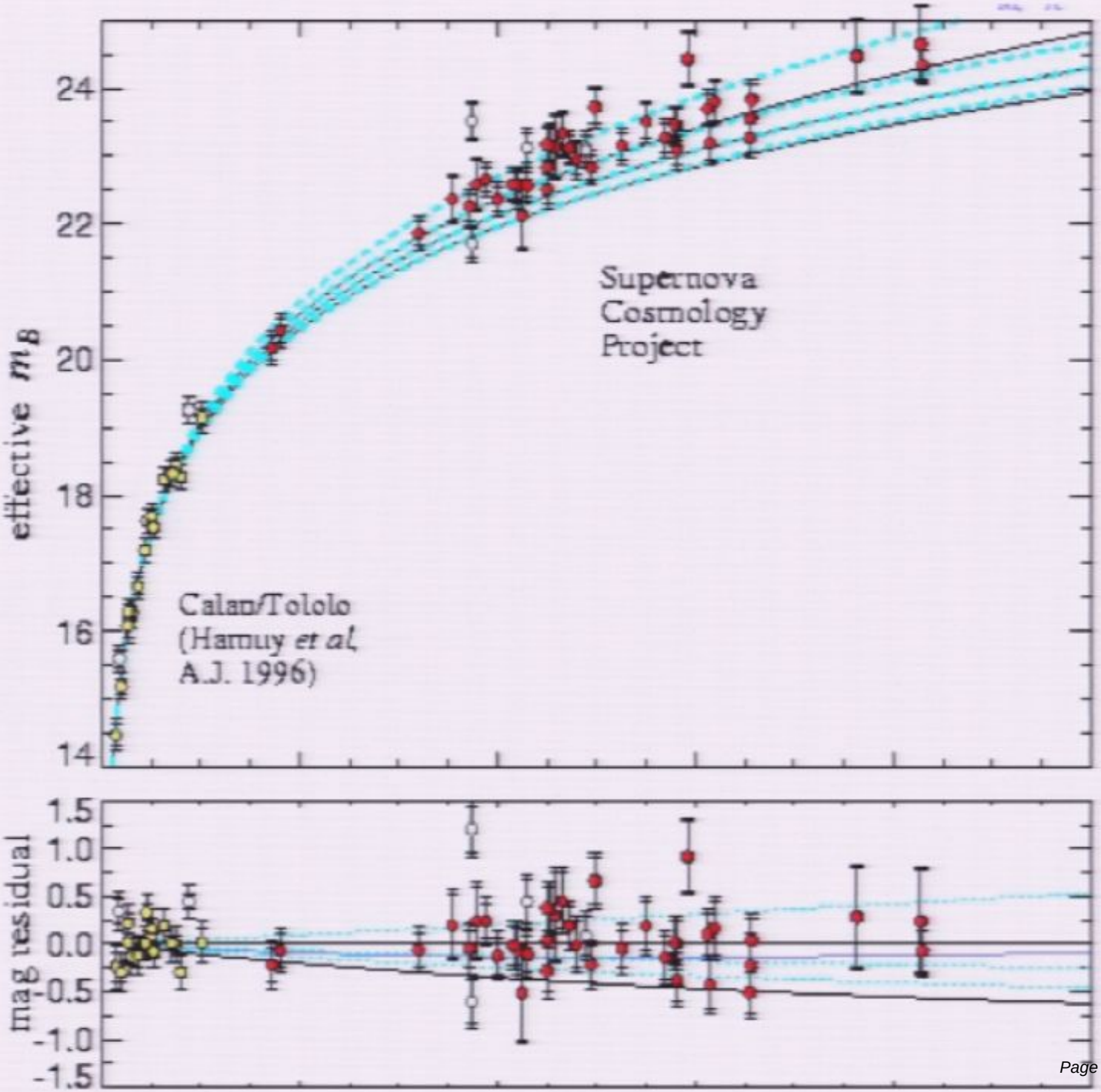
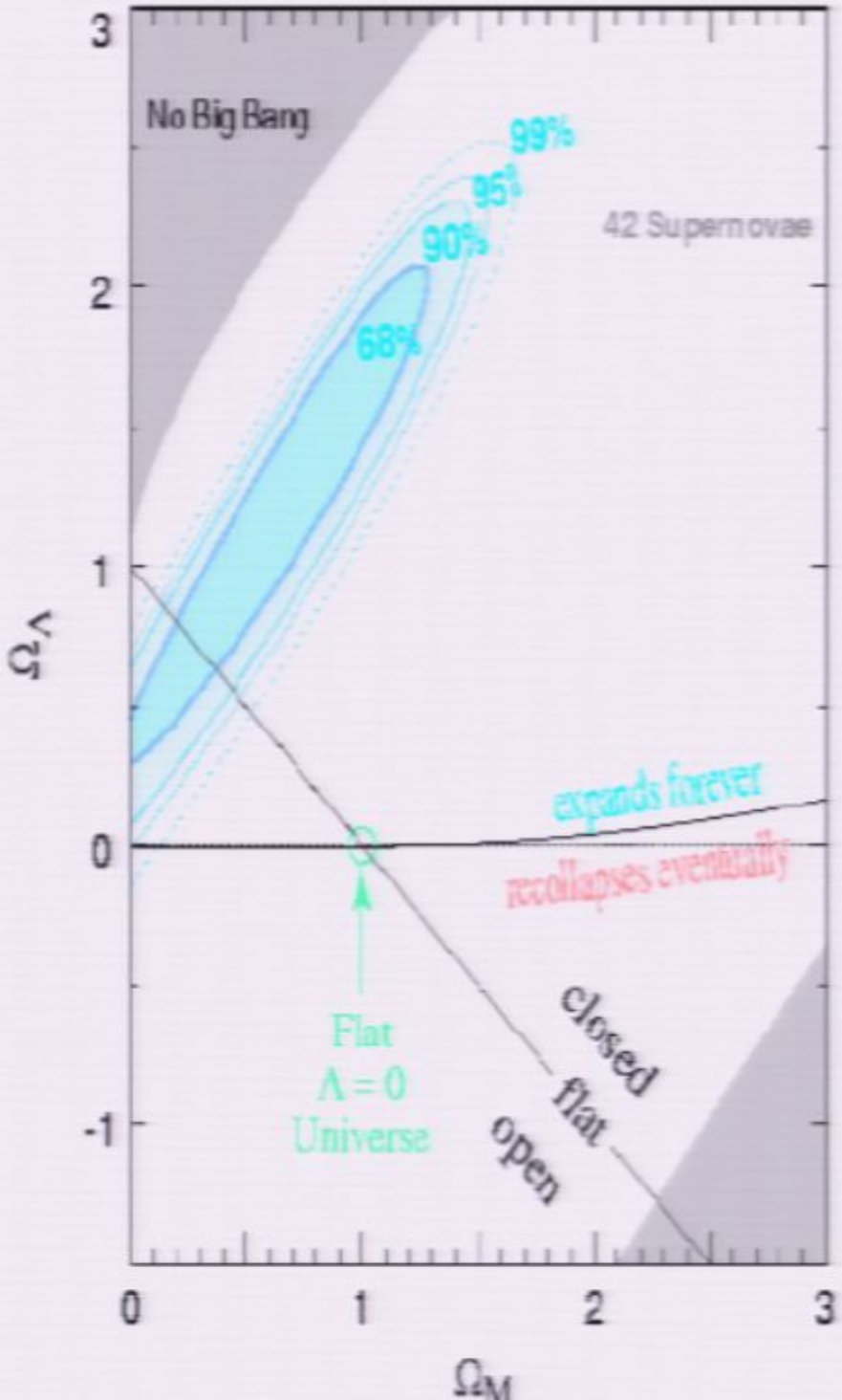


Supernovae Ia

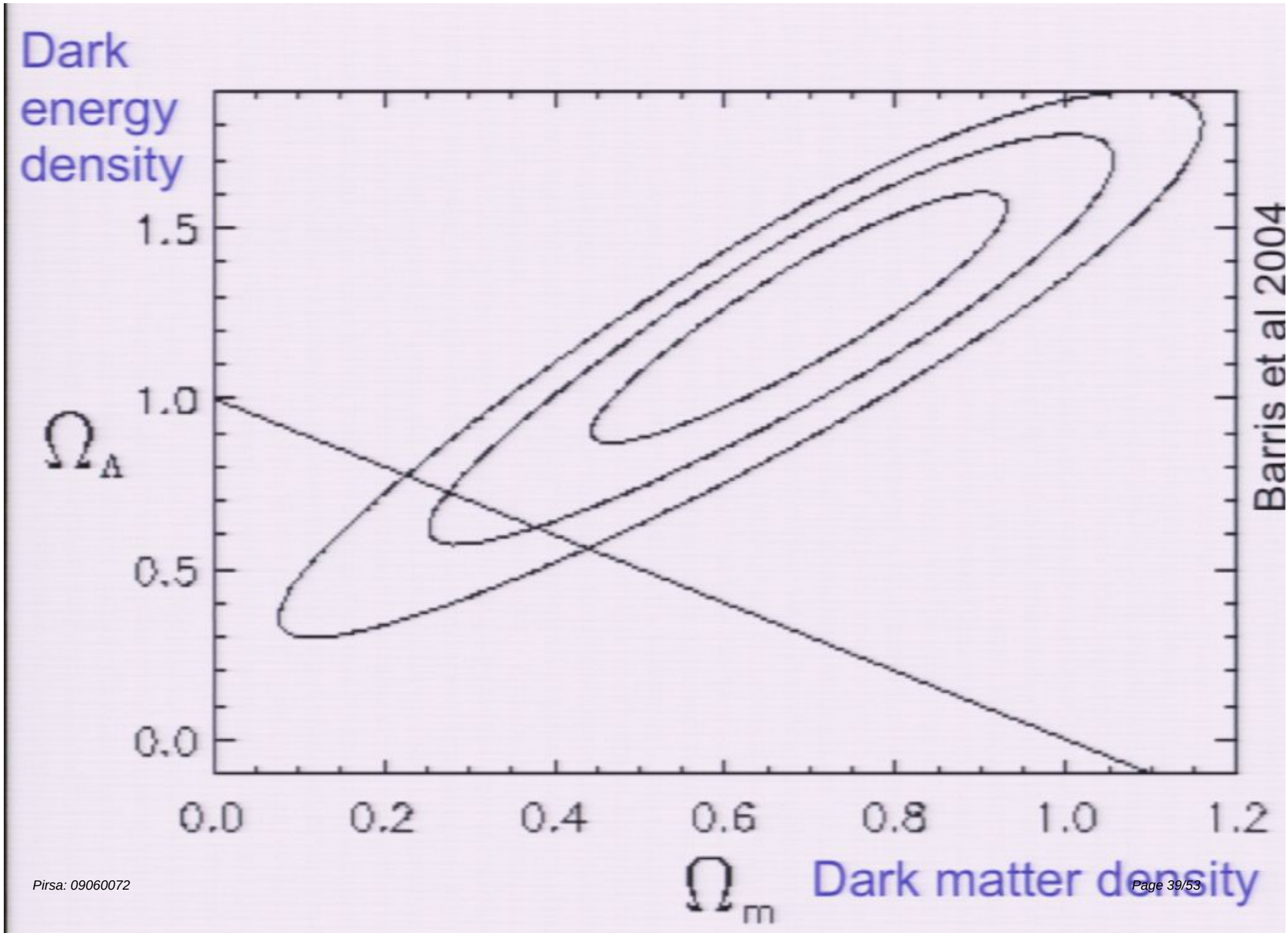
- Supposed to be good standard candles that we can self-calibrate
- Measure expansion rate as a function n of z
- Systematics? e.g.
  - evolution
  - grey dust
- Tests e.g. z=1.7 supernova

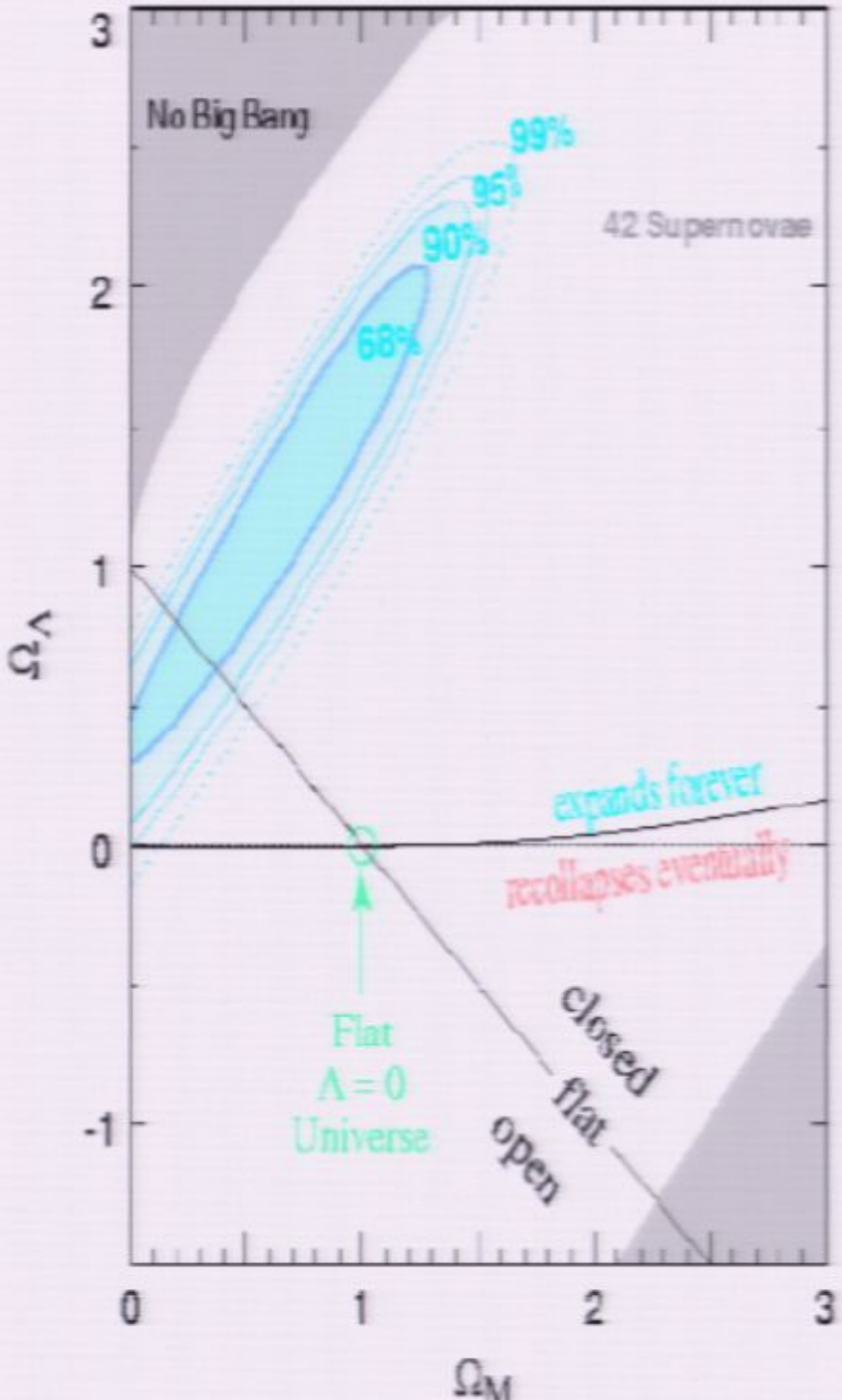
$m_B \propto \ln D_L$





Perlmutter et al


Dark energy density
1.5
1.0
0.5
0.0
Ω_Λ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Ω_m
Dark matter density
Barris et al 2004



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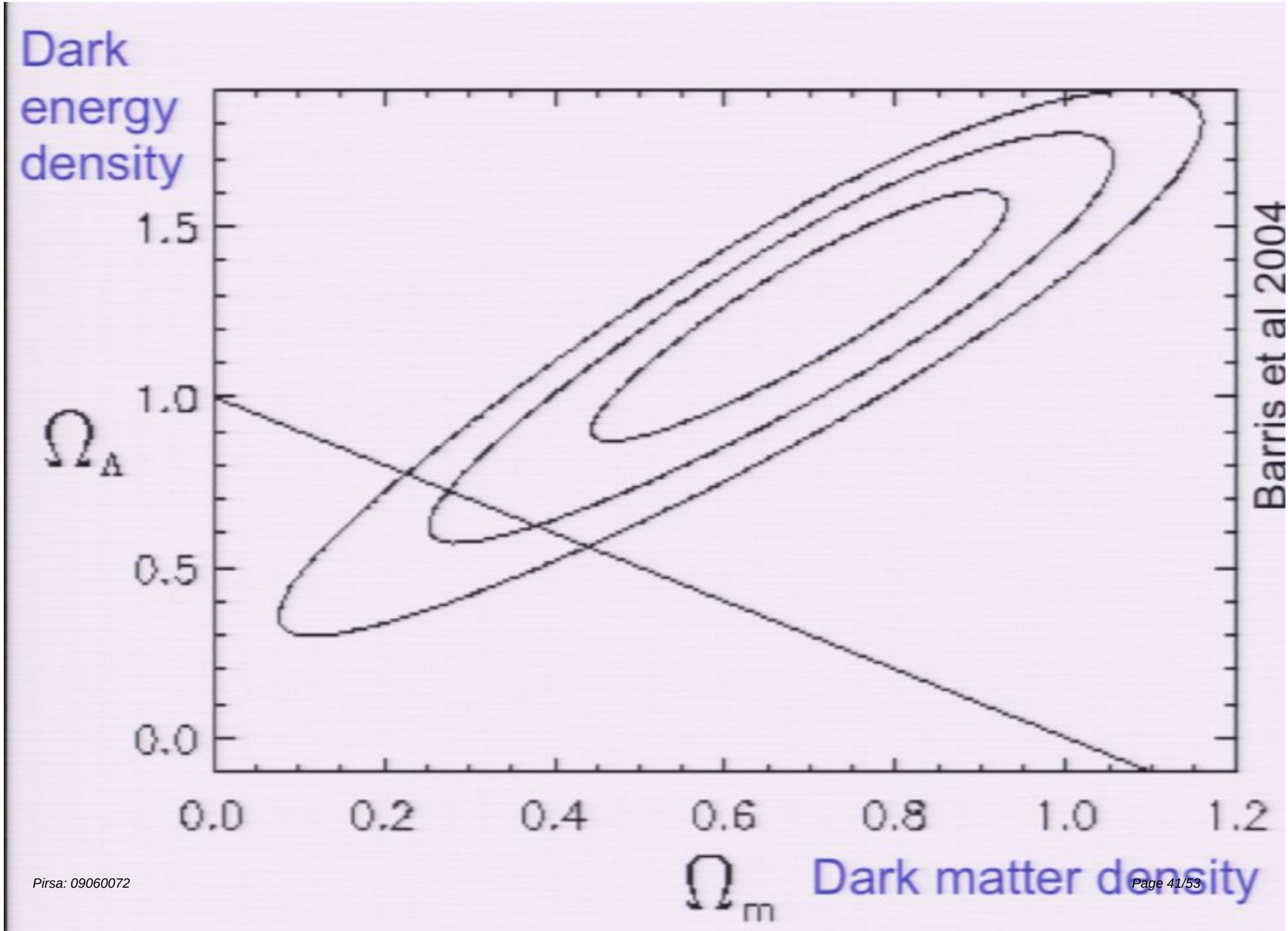
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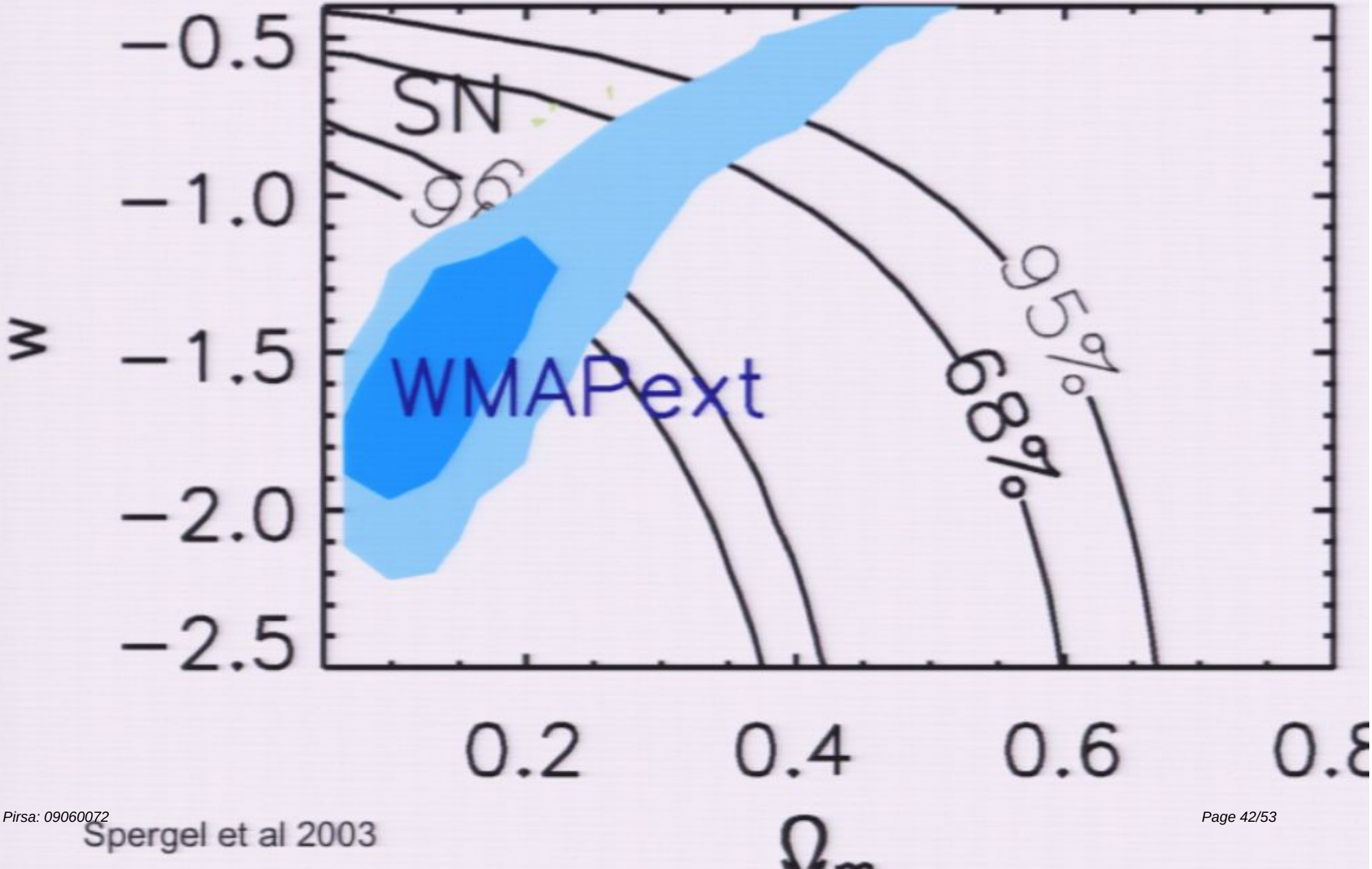
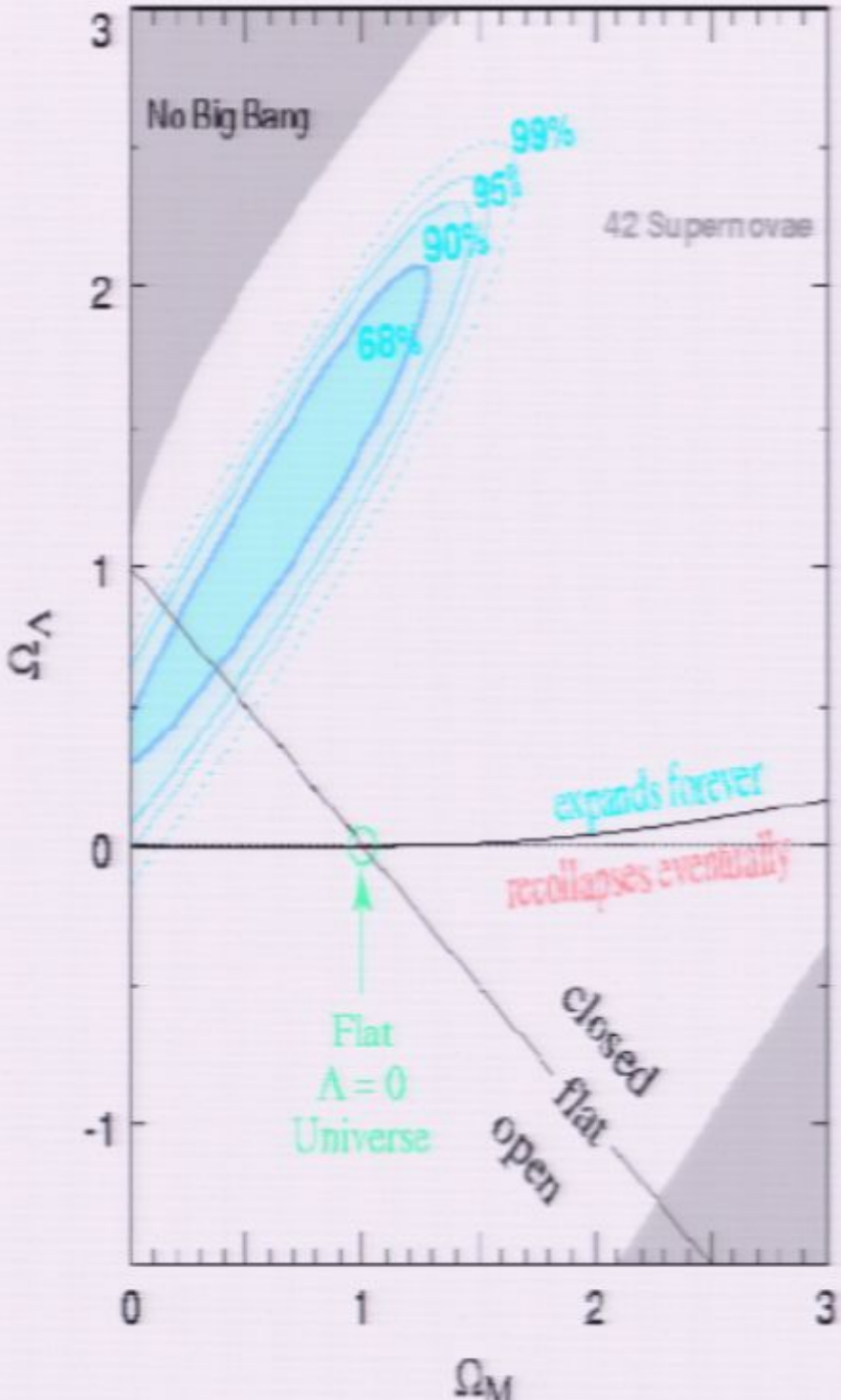
Or assume flat universe:



Spergel et al 2003



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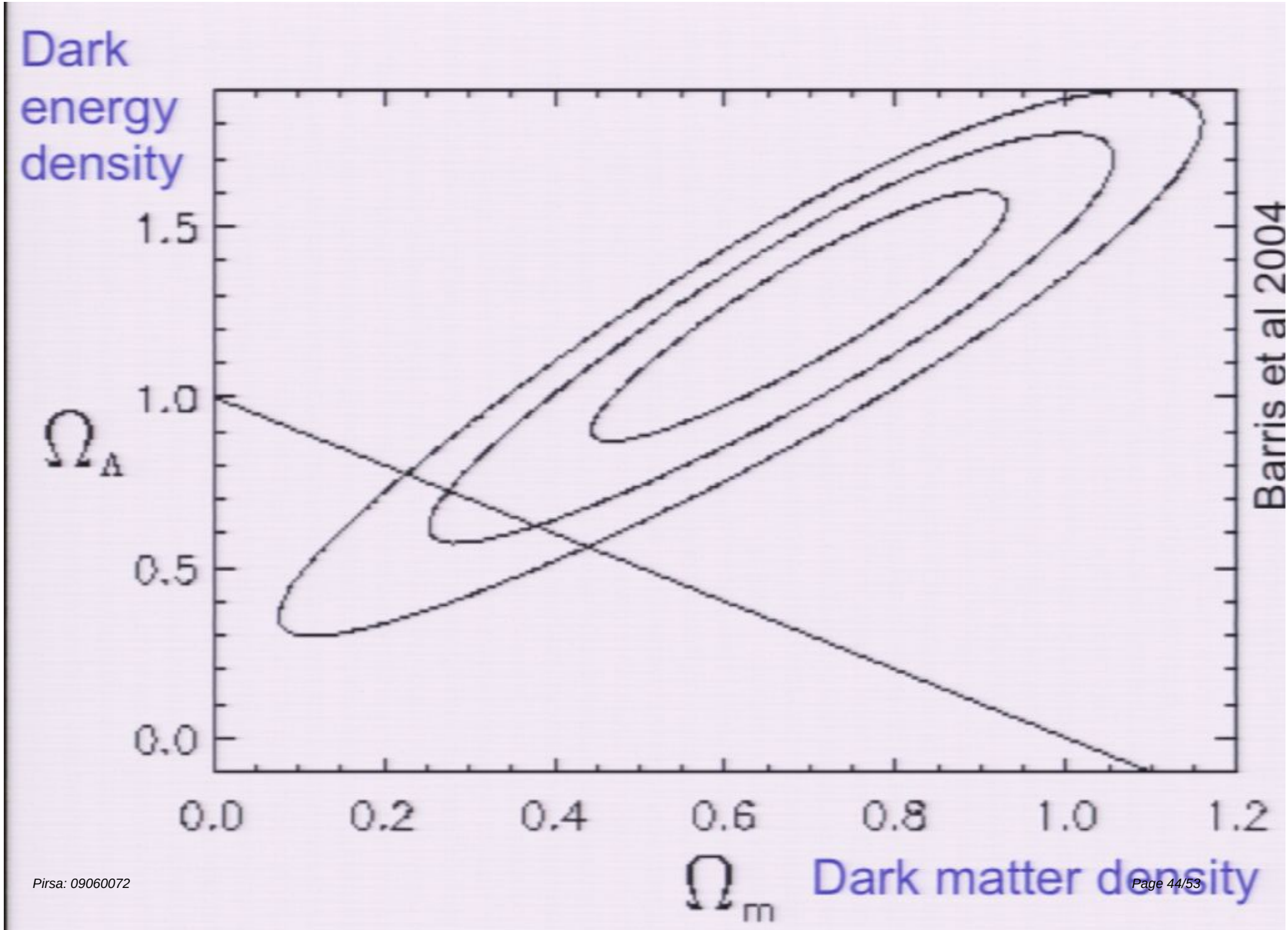
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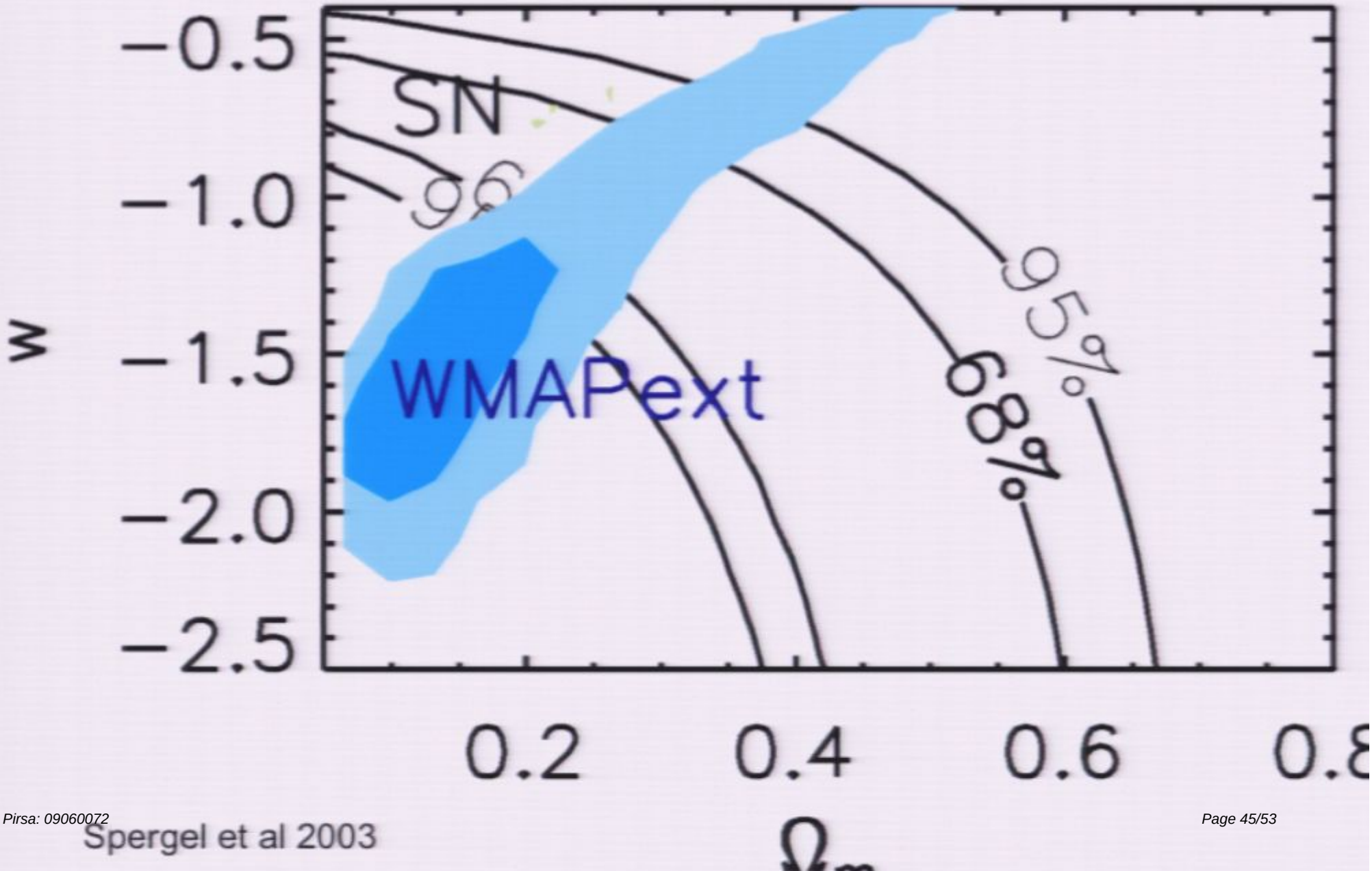
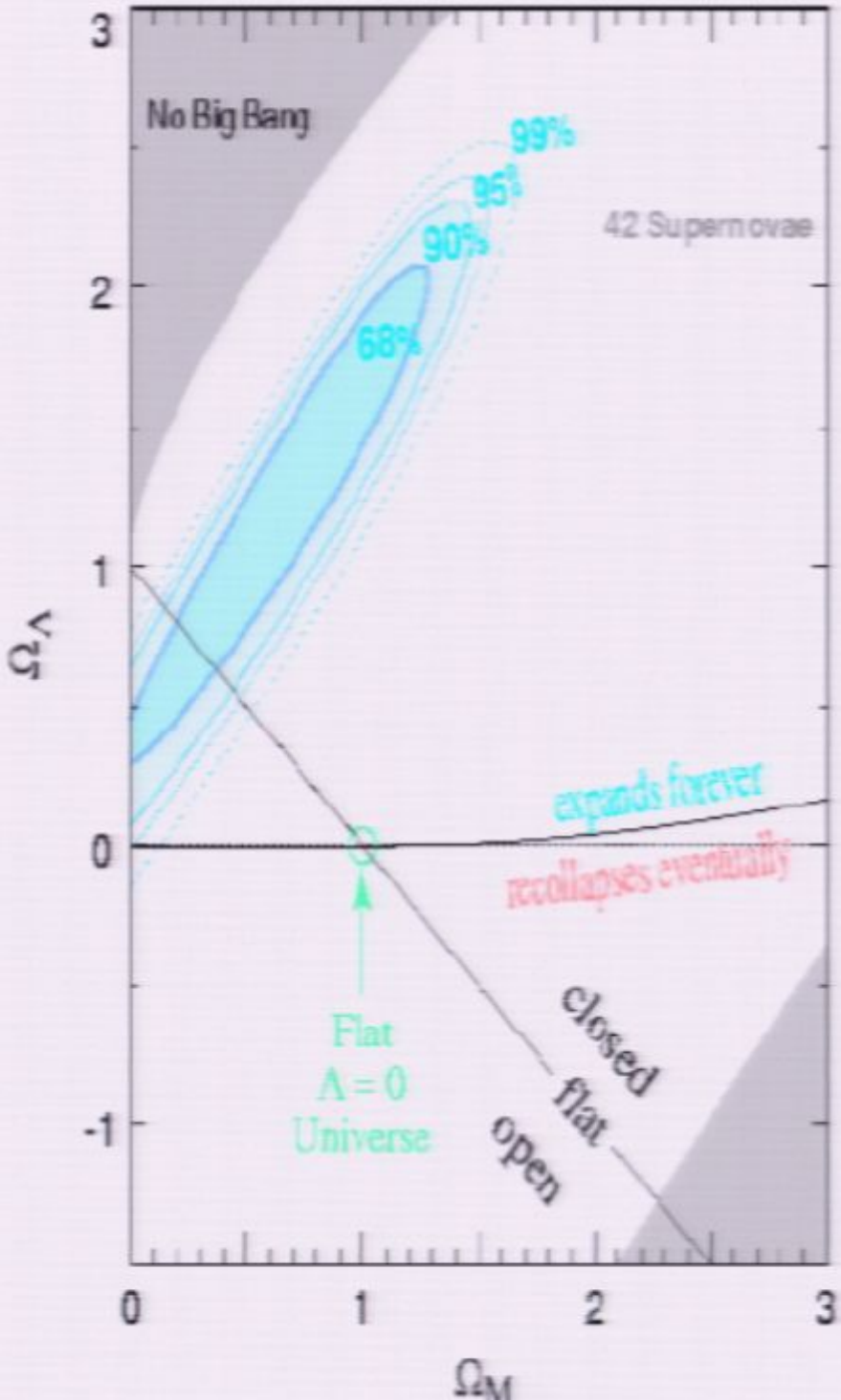


Spergel et al 2003

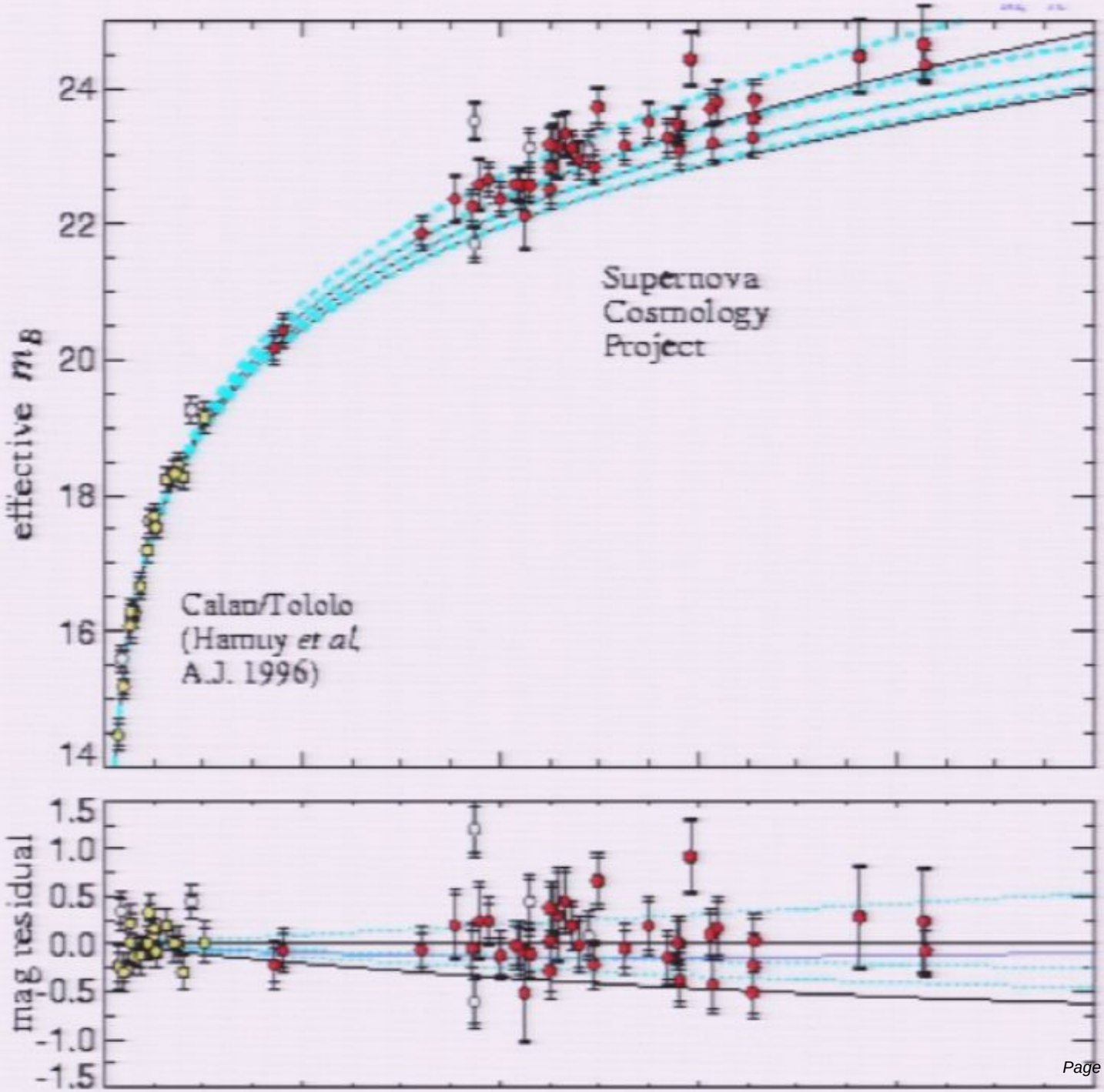
Future supernova data

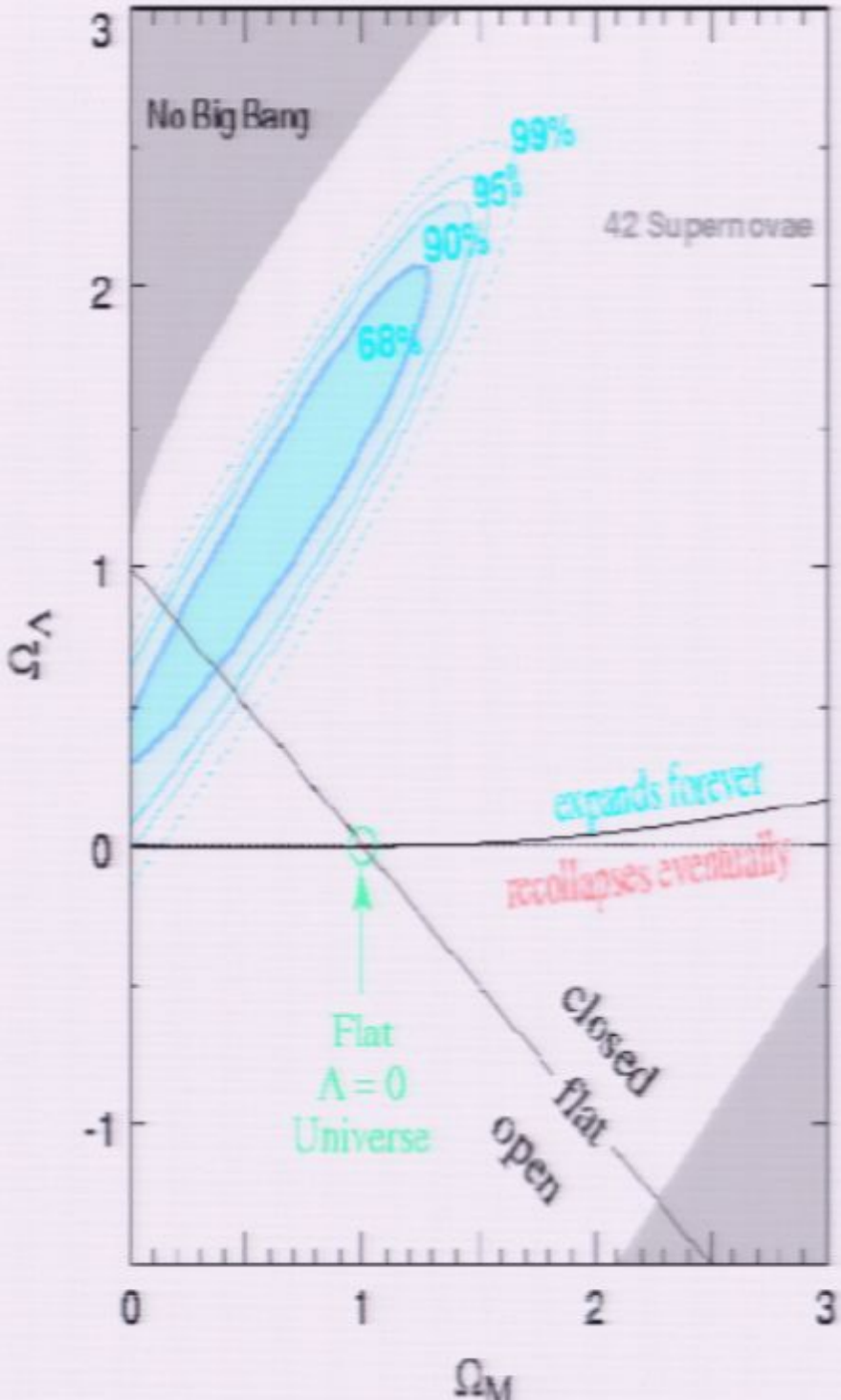
- SNAP satellite proposed
  - Would measure ~2000 SNIa $0.1<z<1.7$
  - Funding situation currently uncertain
- SNFactory ongoing
  - To measure ~300 SNIa at z~0.05



Perlmutter et al

$m_B \propto \ln D_L$





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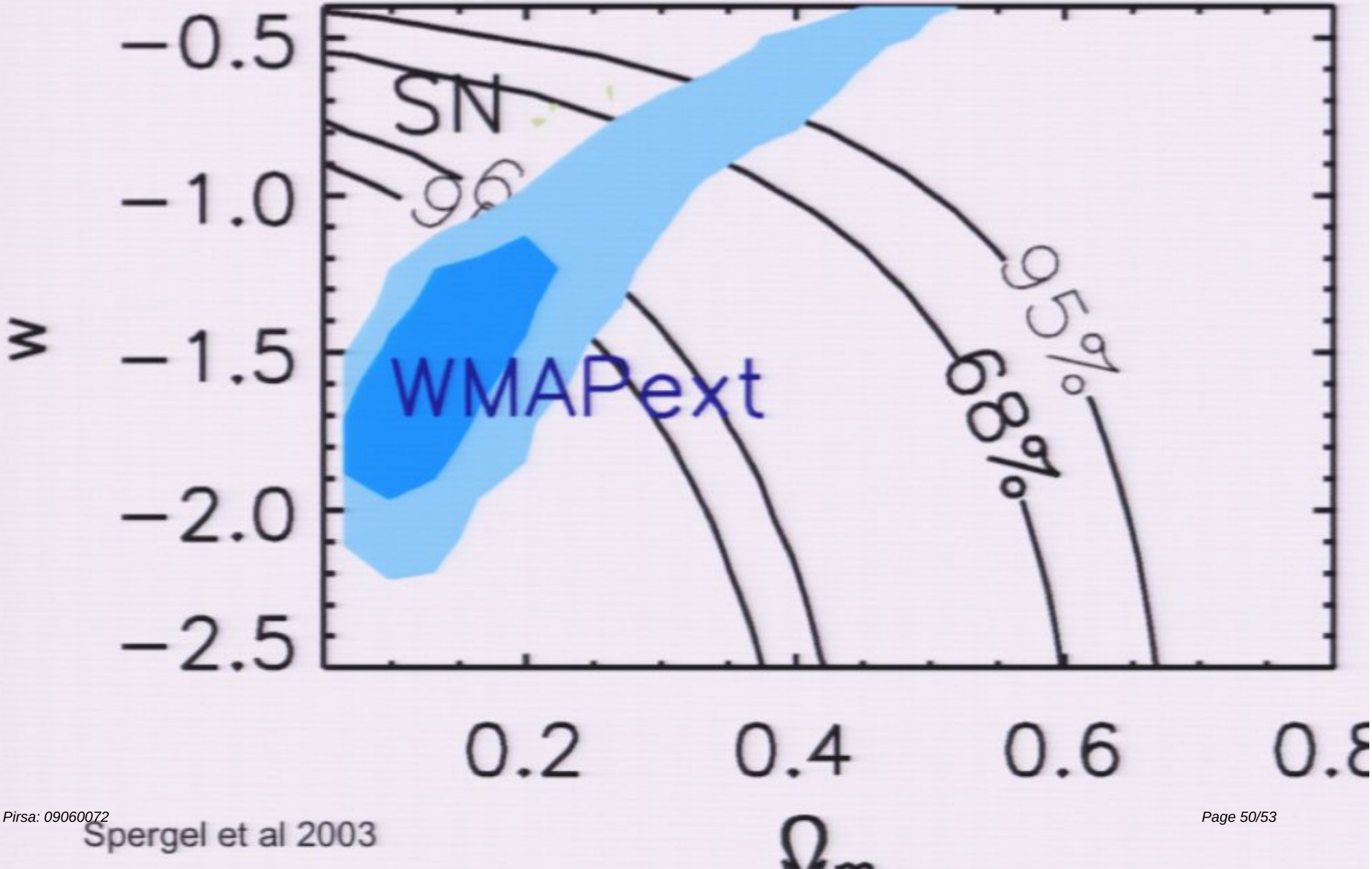

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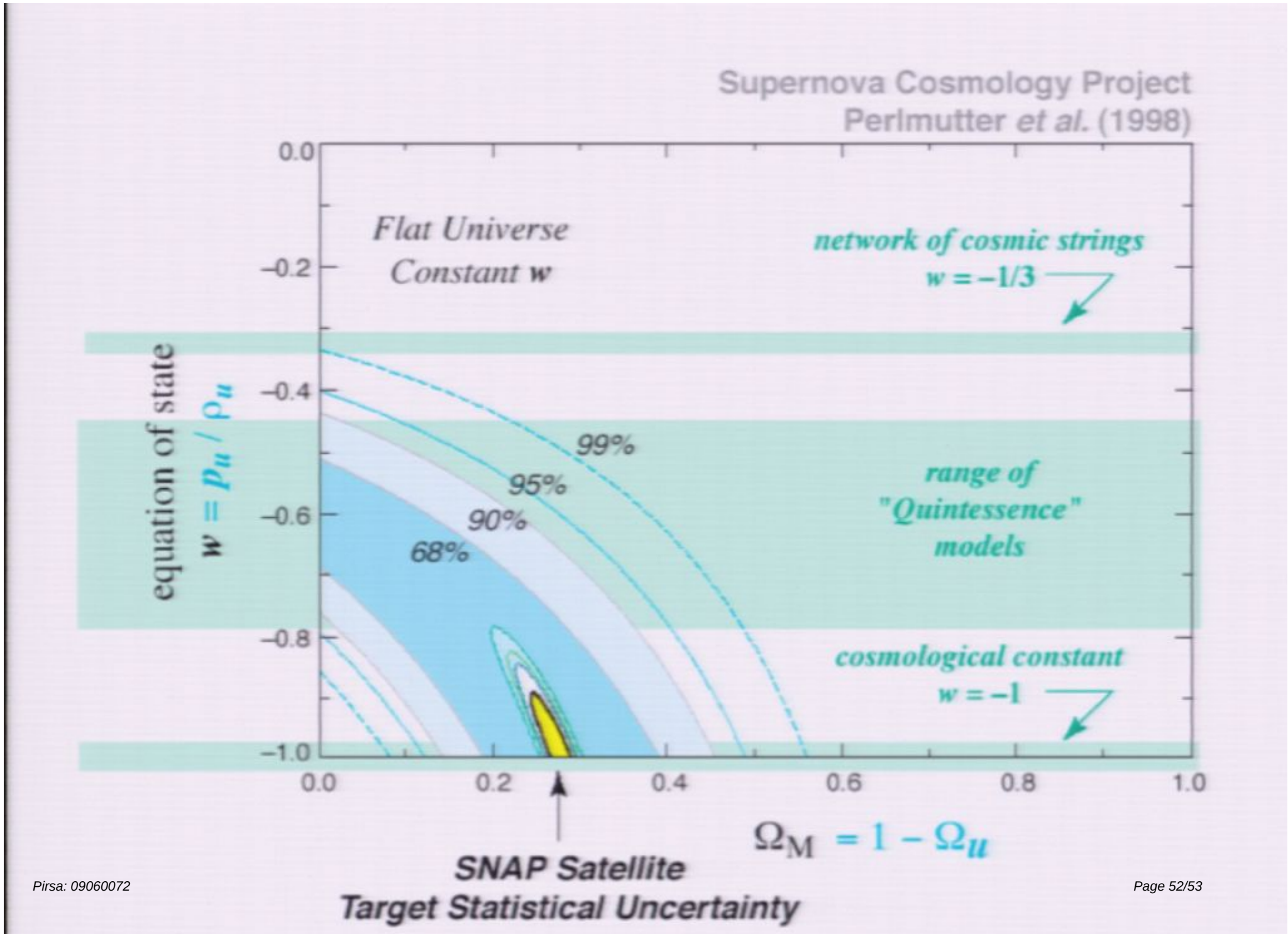
Supernova Cosmology Project
Perlmutter et al. (1998)
Flat Universe
Constant w
network of cosmic strings
w = –1/3
range of
"Quintessence"
models
cosmological constant
w = –1
99%
95%
90%
68%
equation of state
w = p_u / ρ_u
0.0
–0.2
–0.4
–0.6
–0.8
–1.0
0.0
0.2
0.4
0.6
0.8
1.0
Ω_M = 1 – Ω_u
SNAP Satellite
Target Statistical Uncertainty

To take away from this lecture:

- Equations for expansion of Universe as a function of time
- Equations for age of Universe
- Appreciation of observations of H_0, t_0
- Evidence for dark energy from supernovae

Next lecture: Baryonic Acoustic Oscillations