

Title: Observational Probes of Early Universe Cosmology - Lecture 1

Date: Jun 25, 2009 10:00 AM

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Abstract:

This lecture:
Dynamics of the Universe
and distance measurements

1. The current cosmological model: overview
2. From Einstein's field equations to the dynamics of the universe
3. Observations of H_0
4. The age of the Universe
5. Dark matter and baryon content
6. Dark energy and supernovae

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Table

Table Format

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Header Row Header Column

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Evolution of density with a

- Rearrange the Friedman equations:

$$3 (\dot{a}^2 + k) / a^2 = 8 \pi G \rho(t)$$

$$(2a \ddot{a} + \dot{a}^2 + k) / a^2 = -8 \pi G p(t)$$

- to give ($k=0$)
- *If* $p = w \rho$ then
- *If* also $w=\text{constant}$

$$d/dt(\rho a^3) = -p d/dt(a^3)$$

$$a^3 d\rho = -(w+1) \rho 3 a^2 da$$

$$\rho \propto a^{-3(w+1)}$$

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What is w ?

- Defined by the equation $p = w \rho$
- Often called the “equation of state”
- Special values:
 - $w=0$ means $p=0$ e.g. matter
 - $w=1/3$ e.g. radiation
 - $w=-1$ looks just like a cosmological constant

Dark energy or cosmological constant'

$$G_k^i + \Lambda g_k^i = R_k^i - 1/2 \delta_k^i R + \Lambda g_k^i = 8\pi G T_k^i$$

$$T_k^i = \text{diag}[\rho(t), -p(t), -p(t), -p(t)]$$

- If $w = -1$ i.e. $p = -\rho$ then $T_k^i = g_k^i$
 - Indistinguishable from a cosmological constant
- Dark energy could have any w
 - Could even vary with time

Evolution of the density

$$\rho \propto a^{-3(w+1)}$$

- Matter dominated ($w=0$): $\rho \propto a^{-3}$
- Radiation dominated ($w=1/3$): $\rho \propto a^{-4}$
- Cosmological constant ($w=-1$): $\rho = \text{constant}$
- Dark energy with $w < -1$ e.g. $w=-2$: $\rho \propto a^3$
 - Energy density *increases* as is stretched out!
 - Eventually would dominate over even the energies holding atoms together! (“Big Rip”)

What is dominant when?

Matter dominated ($w=0$): $\rho \propto a^{-3}$

Radiation dominated ($w=1/3$): $\rho \propto a^{-4}$

Dark energy ($w \sim -1$): $\rho \sim \text{constant}$

- Radiation density decreases the fastest with time
 - Must increase fastest on going back in time
 - Radiation must dominate early in the Universe
- Dark energy with $w \sim -1$ dominates last



Putting it all together

- $\rho_{\text{tot}} = \rho_r + \rho_m + \rho_{\text{DE}}$
- $\rho_r = \rho_{r0} a^{-4}$ since $\rho \propto a^{-4}$ and a today is 1
 - $\rho_{r0} \equiv$ radiation density today
 - $\Omega_{r0} \equiv \rho_{r0} / \rho_{\text{crit}}$ by definition
 - Usually 0 is dropped from Ω_{r0} for simplicity
- So $\rho_r = \rho_{\text{crit}} \Omega_r a^{-4}$
 - sim ρ_m, ρ_{DE}

$$\rho_{\text{tot}} = \rho_{\text{crit}} [\Omega_r a^{-2} + \Omega_m a^{-3} + \Omega_{\text{DE}} a^{-3(1+w)}]$$

Most important equation in this lecture

$$\left(\frac{\dot{a}}{a}\right)^2 = 8 \pi G \rho(t) / 3$$

Friedman eqn1 for $k=0$

$$\rho(t) = \rho_{\text{crit}} \left[\Omega_r a^{-2} + \Omega_m a^{-3} + \Omega_{\text{DE}} a^{-3(1+w)} \right]$$

- Putting these together and using $H_0 \equiv (\dot{a}/a)_0$:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_r a^{-2} + \Omega_m a^{-3} + \Omega_{\text{DE}} a^{-3(1+w)} \right)$$

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How does scale factor relate to time?

$$\begin{aligned} 3 (\dot{a}^2 + k) / a^2 &= 8 \pi G \rho(t) \\ (2a \ddot{a} + \dot{a}^2 + k) / a^2 &= -8 \pi G p(t) \end{aligned}$$

Friedman eqns

$$\rho \propto a^{-3(w+1)}$$

Which we just derived

- Then if $k=0$ (flat Universe)

$$a^{(3w+1)/2} da = dt$$

- If $w > -1$ and $w = \text{constant}$ then:

$$a \propto t^{2/(3(w+1))}$$

- If $w = -1$:

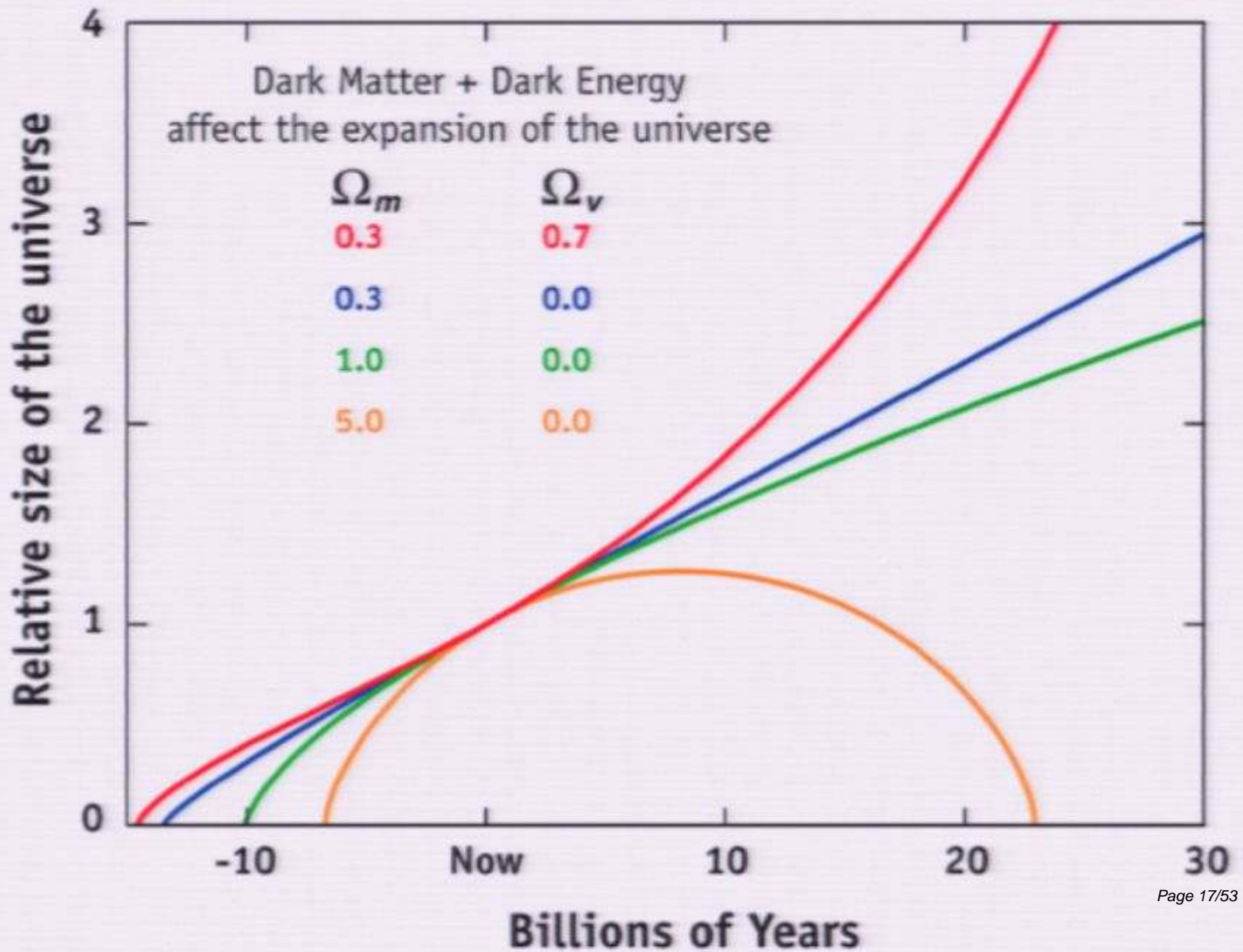
$$a \propto e^{\lambda t}$$

Dynamics of the Universe

$$a \propto t^{2/(3(w+1))}$$

- Matter dominated ($w=0$): $a \propto t^{2/3}$
 - Decelerating
- Radiation dominated ($w=1/3$): $a \propto t^{1/2}$
 - Decelerating
- Cosmological constant ($w=-1$): $a \propto e^{\lambda t}$ (special)
 - Accelerating
- Where is the transition?
 - $w > -1/3$ decelerating
 - $w < -1/3$ accelerating

EXPANSION OF THE UNIVERSE



Redshift and co-moving distance

- Definition of redshift gives $1+z = 1/a$
- Co-moving distance remains fixed as Universe expands by opposition to the physical distance
- Co-moving distance between different emitters and us
 - Turns out to be very useful
 - Rearrange to get

$$D = \int_{t(a)}^{t_0} c \frac{dt}{a}$$

$$D = -c \int_a^1 da \frac{dt}{a da}$$

- Use $(\dot{a}/a)^2 = H_0^2 (\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)})$

$$D = \int_0^1 \frac{c \frac{da}{a^2}}{H_0 (\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)})^{1/2}}$$

Luminosity & angular diameter distances

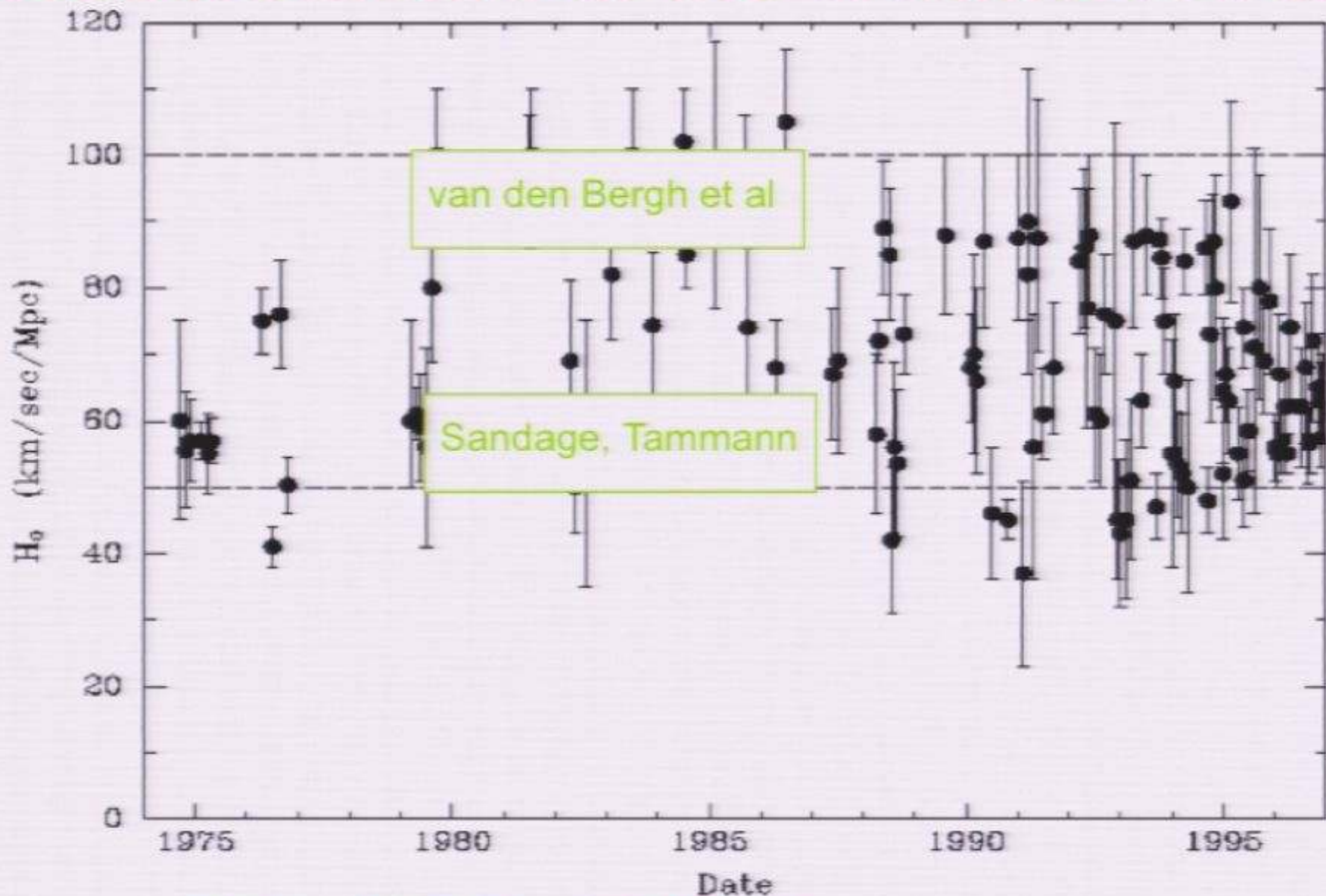
- Using distances is tricky. Imagine we are looking at a QSO at $z=3$ ($a=1/4$), what is the right distance to use to relate the flux we see and the luminosity? Or looking at an object of known size, which distance to use?
- Luminosity distance $\equiv D_L$
 - Defined such that $S = L / (4 \pi D_L^2)$
 - S = bolometric flux density
 - L = bolometric luminosity
 - $D_L = D (1+z)$
- Angular diameter distance $\equiv D_A$
 - Defined such that $\theta = r / D_A$
 - r = physical size of object
 - θ = angular size on sky
 - $D_A = D / (1+z)$
- Both D_A and D_L will be very important to measure e.g. but requires good templates, “standard candles” (SNe) or “rulers” (BAO, CMB)

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Measurements of the Hubble constant



Methods and distance ladder

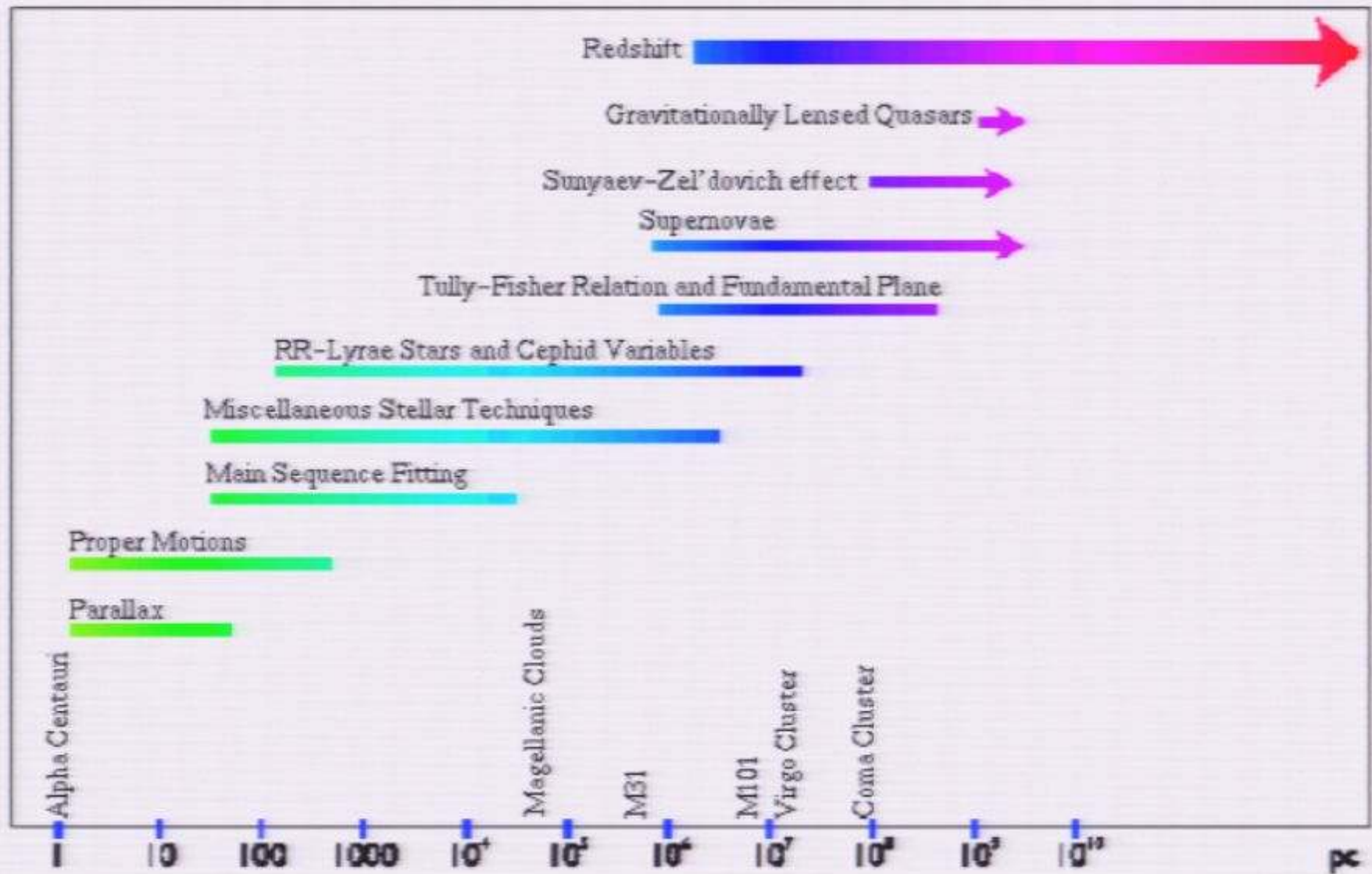
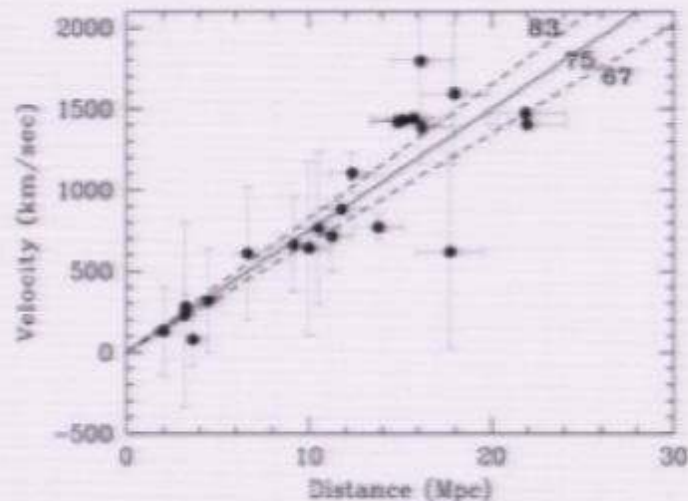


Figure 3.2: The different distance estimators. This seemingly simple plot shows a grand overview of our efforts to measure distances in the Universe. Adapted from [Rowan-Robinson, 1985] and [Roth and Primack, 1996].

Hubble Space Telescope Key Project: Hubble constant

- Comprehensive study of 5 distance indicators
- Freedman et al

Hubble Diagram for Cepheids (flow-corrected)



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Method	H_0	Error (%)	References
36 Type Ia supernovae 4,000 < cz < 30,000 km/sec	71	$\pm 2_r \pm 6_s$	Hamuy et al. (1996), Riess et al. (1998), Jha et al. (1999), Gibson et al. (2000)
21 Tully-Fisher clusters 1,000 < cz < 9,000 km/sec	71	$\pm 3_r \pm 7_s$	Giovanelli et al. (1997), Aaronson et al. (1982, 1990), Sakai et al. (2000)
11 FP clusters 1,000 < cz < 11,000 km/sec	82	$\pm 6_r \pm 9_s$	Jorgensen et al. (1996), Kelson et al. (2000)
SBF for 6 clusters 3,800 < cz < 5,800 km/sec	70	$\pm 5_r \pm 6_s$	Lauer et al. (1998), Ferrarese et al. (2000a)
4 Type II supernovae 1,900 < cz < 14,200 km/sec	72	$\pm 9_r \pm 7_s$	Schmidt et al. (1994)

Combined values of H_0 :

$$H_0 = 72 \pm 2 \text{ [random] km/sec/Mpc [Bayesian]}$$

$$H_0 = 72 \pm 3 \text{ [random] km/sec/Mpc [frequentist]}$$

$$H_0 = 72 \pm 3 \text{ [random] km/sec/Mpc [Monte Carlo]}$$

72 ± 8

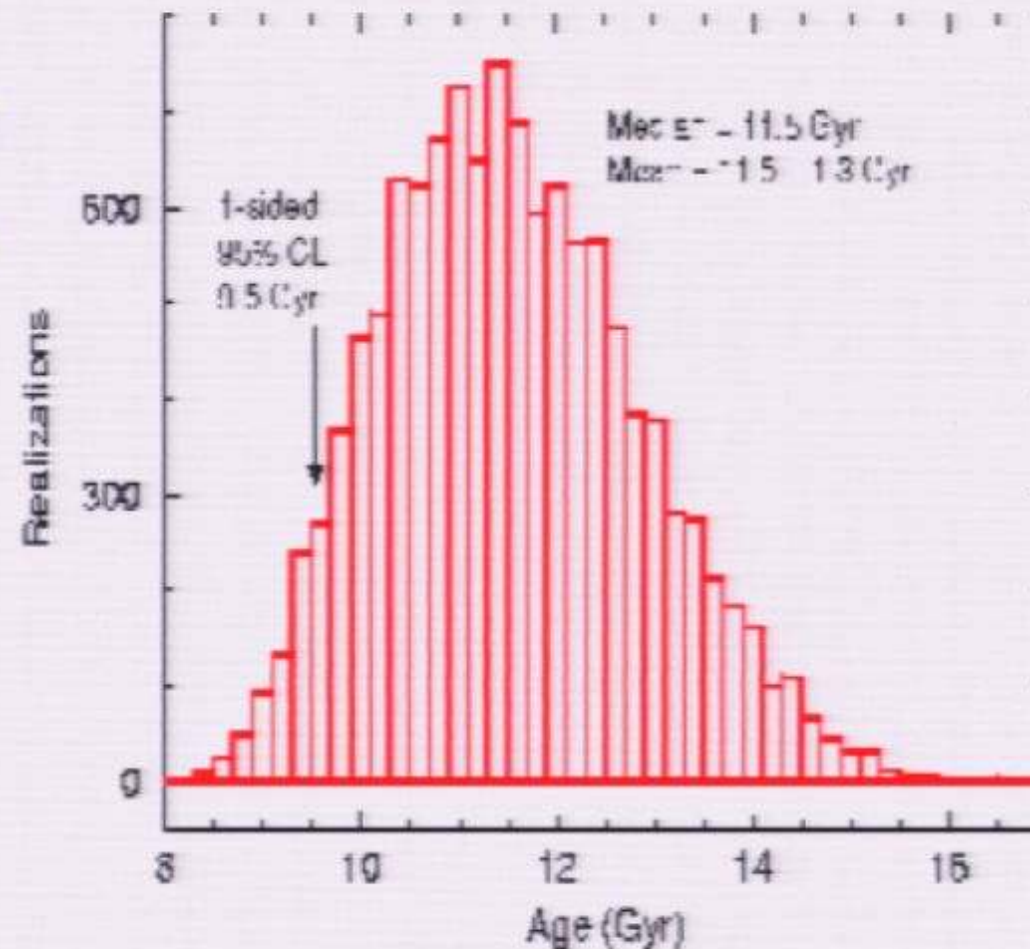
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Measurements of the age of universe

- Krauss + Chaboyer
 - stars age = 12.4 Gyr
 - estimate ~2 Gyrs min for formation
 - $t_0 > 10.2$ Gyr 95 per cent 1-tailed
- CMB + flatness \rightarrow
 $t_0 \sim 13.4$ Gyr



The age of the Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)} \right)$$

- Integrate to get: $t = \int_0^1 \frac{da/a}{H_0 (\Omega_m a^{-3} + \Omega_{DE} a^{-3(1+w)})^{1/2}}$

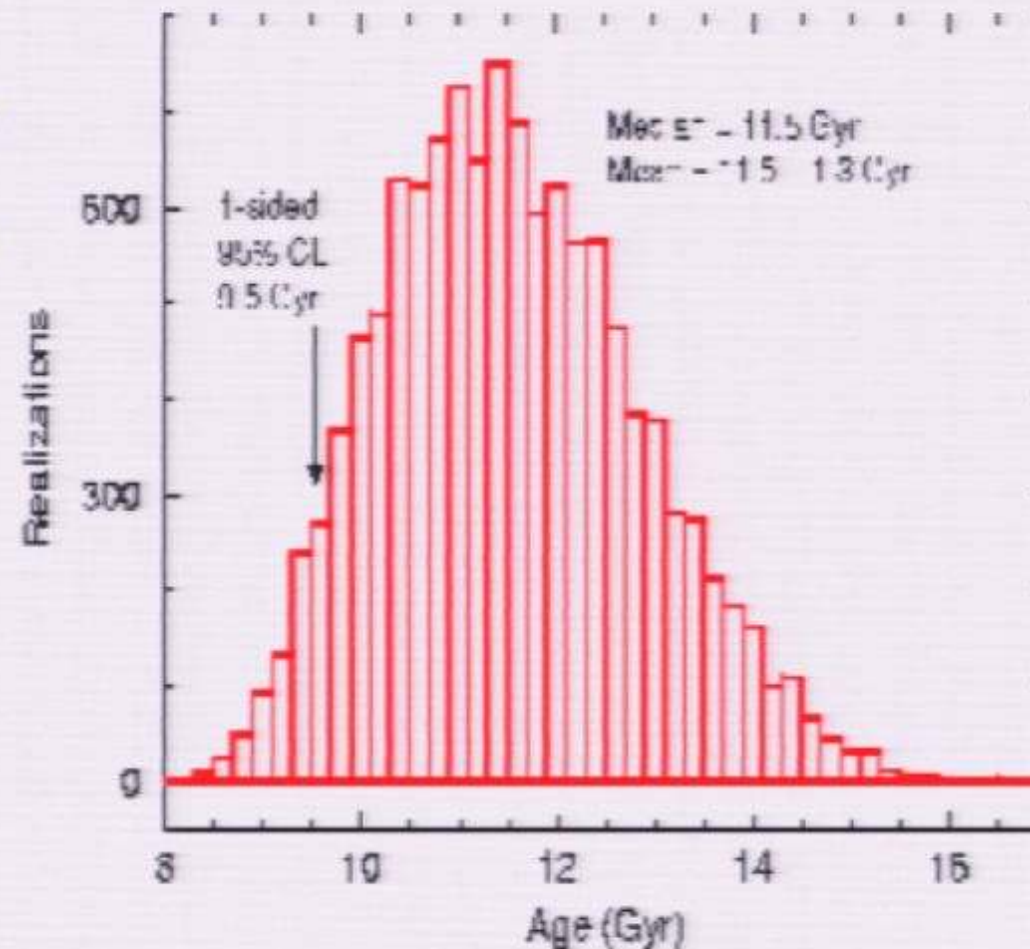
- If $w = -1$ (so $\Omega_{DE} = \Omega_\Lambda$): $t_0 = \frac{2}{3H_0} \frac{\tanh^{-1}(\sqrt{\Omega_\Lambda})}{\sqrt{\Omega_\Lambda}}$

- If $\Omega_{DE} = 0$ (so $\Omega_m = 1$ given $k=0$ assumption)

$$t = \int_0^1 a^{1/2} \frac{da}{H_0} = \frac{2}{3H_0}$$

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Dark Energy

- General properties
- Cosmological constant or quintessence?
- Parameterizing quintessence
 - Incorporating quintessence into predictions
 - Parameterizing $w(z)$
- Current measurements
- Prospects using supernovae

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Dark Energy

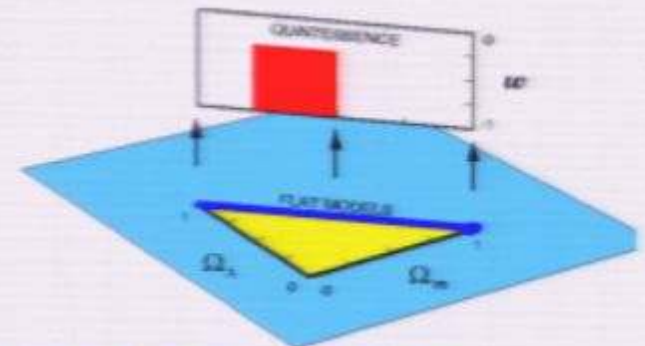
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Dark energy: general properties

- The dominant constituent of the universe
- Causes accelerated expansion
- Modifies growth rate of fluctuations (next lecture)

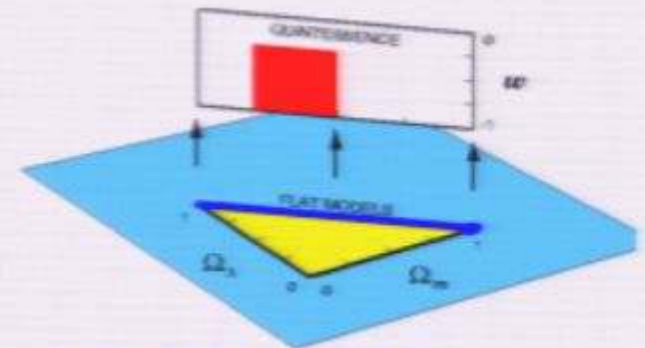
Cosmological constant or Quintessence

- Cosmological constant:
 - zero-point quantum fluctuations? factor of 10^{110}
 - coincidence of $\Omega_m \sim \Omega_\Lambda$ / why accelerates now
- Quintessence
 - time-dependent
 - spatially inhomogeneous
 - e.g. scalar field rolling down a potential
 - negative pressure – $w = \text{pressure}/\text{density}$



Cosmological constant or Quintessence

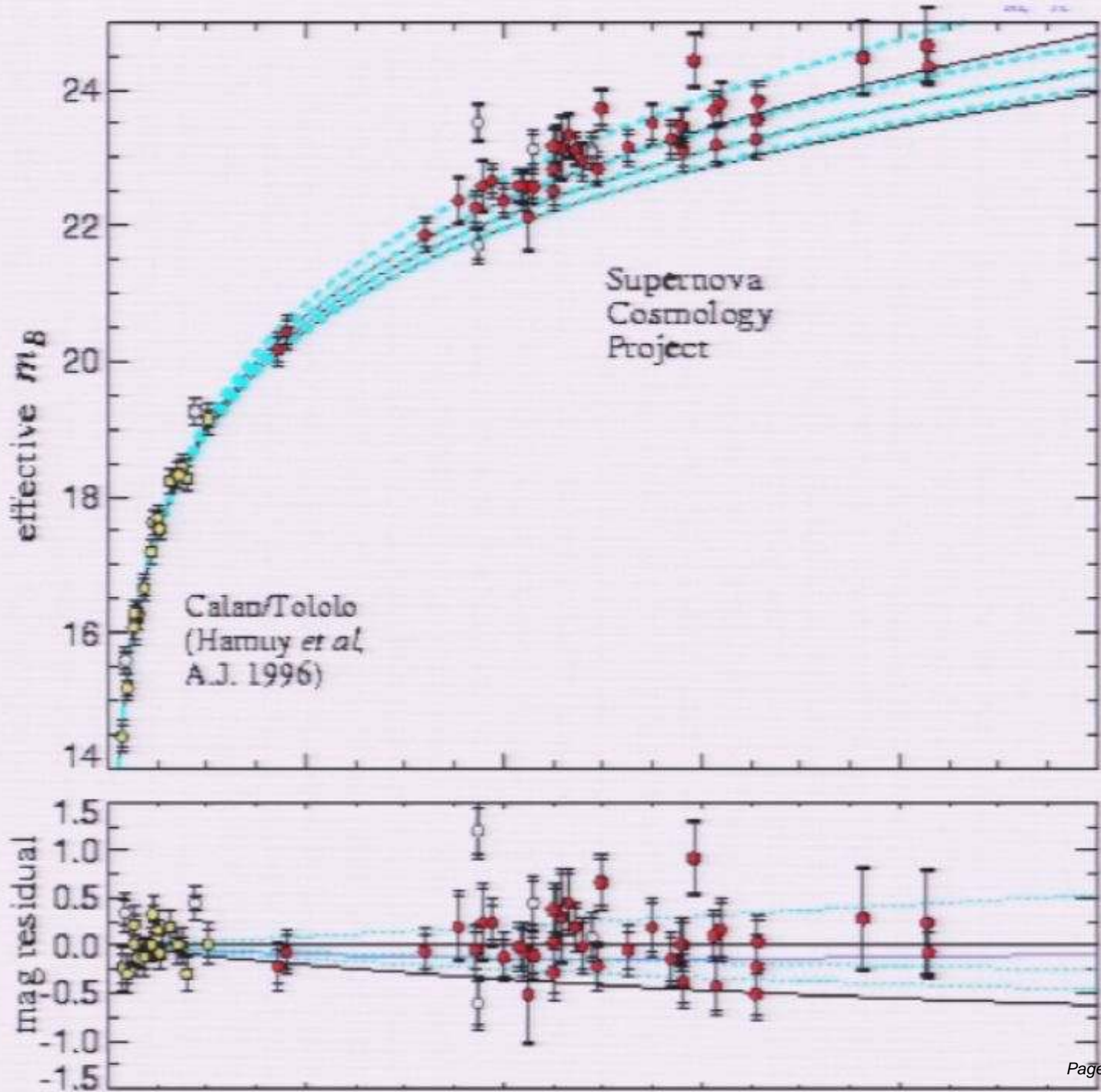
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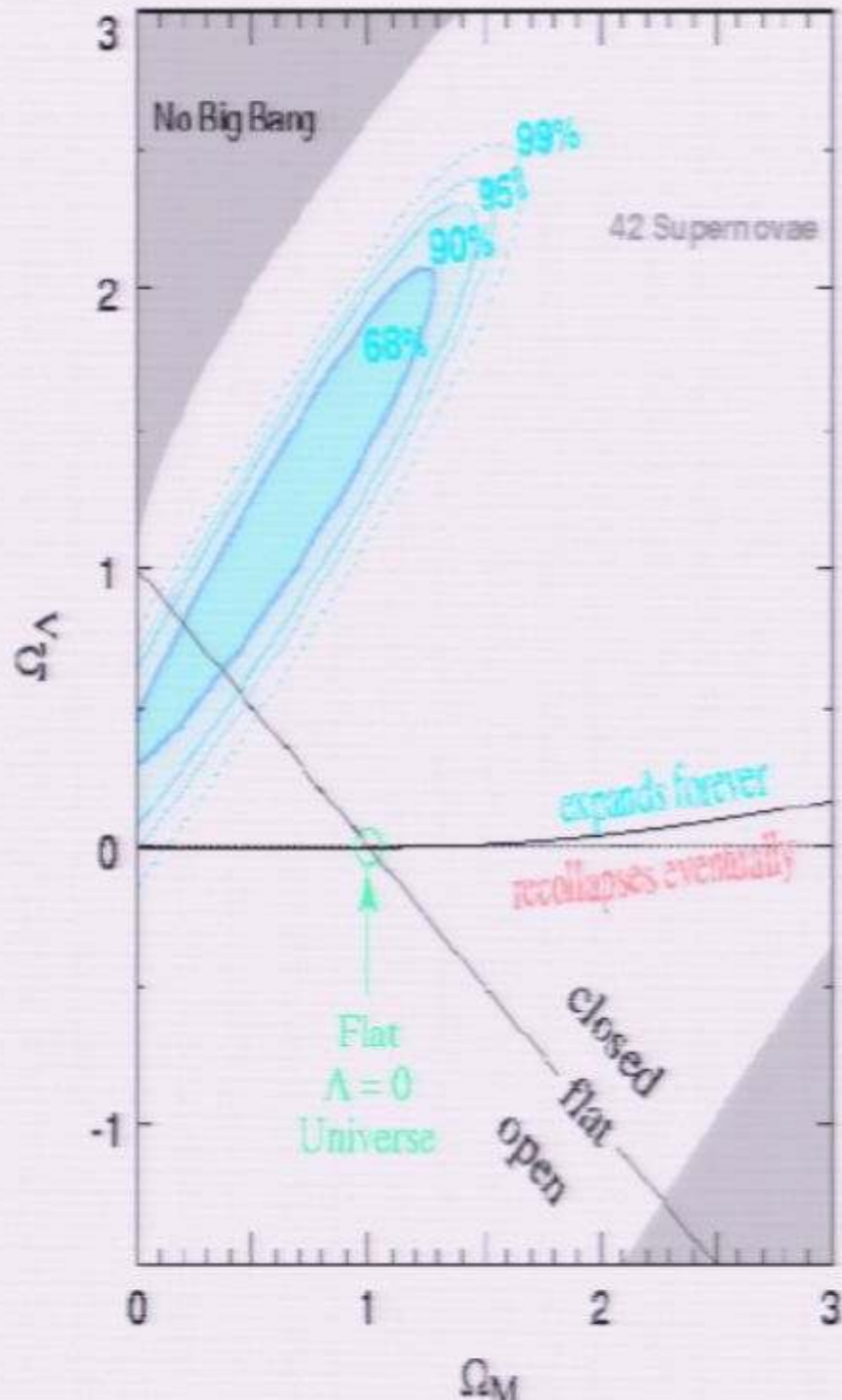


Supernovae Ia

- Supposed to be good standard candles that we can self-calibrate
- Measure expansion rate as a function n of z
- Systematics? e.g.
 - evolution
 - grey dust
- Tests e.g. $z=1.7$ supernova

$$m_B \propto \ln D_L$$





Dark energy density

Ω_{Λ}

1.5

1.0

0.5

0.0

0.0

0.2

0.4

0.6

0.8

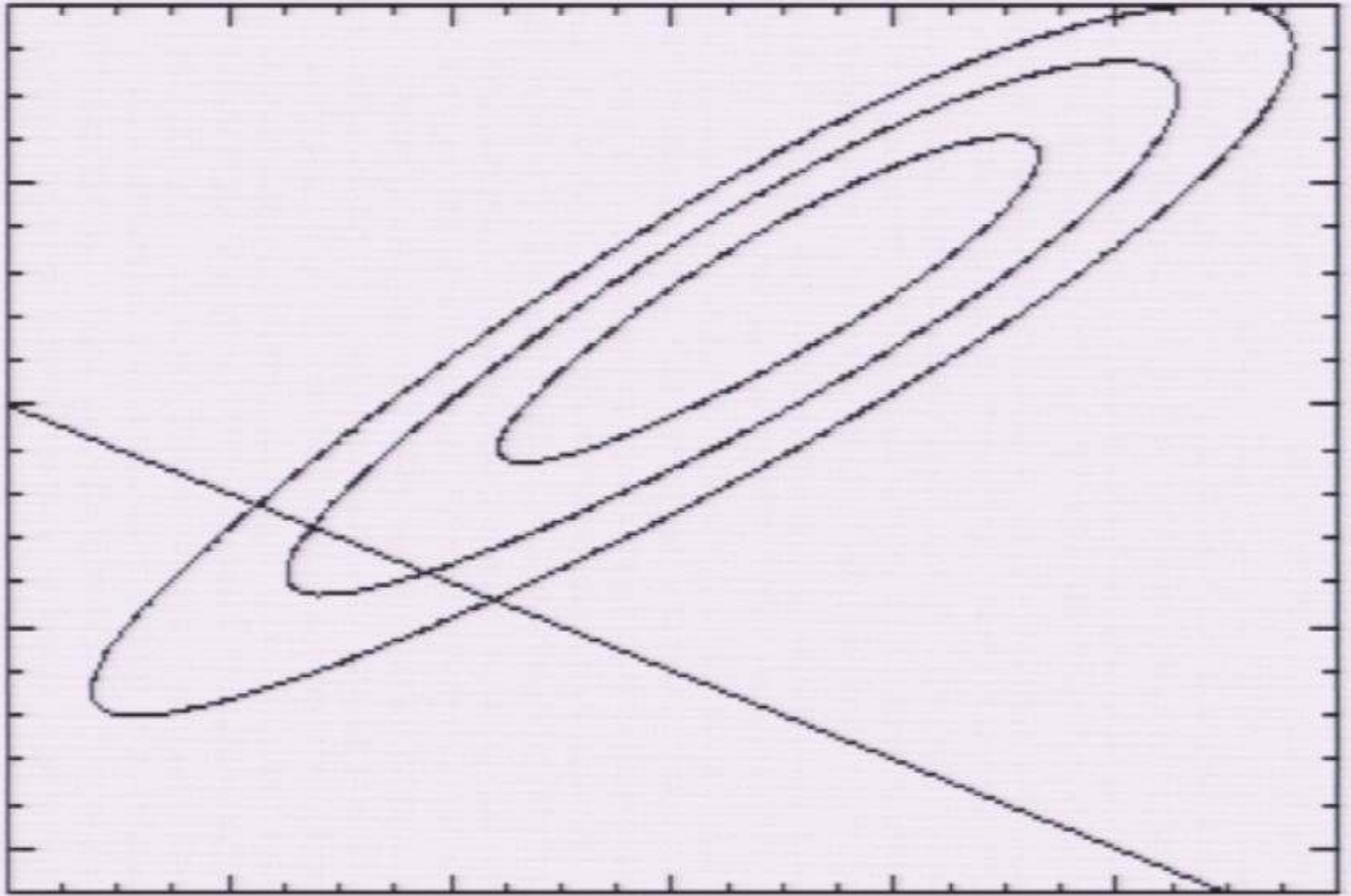
1.0

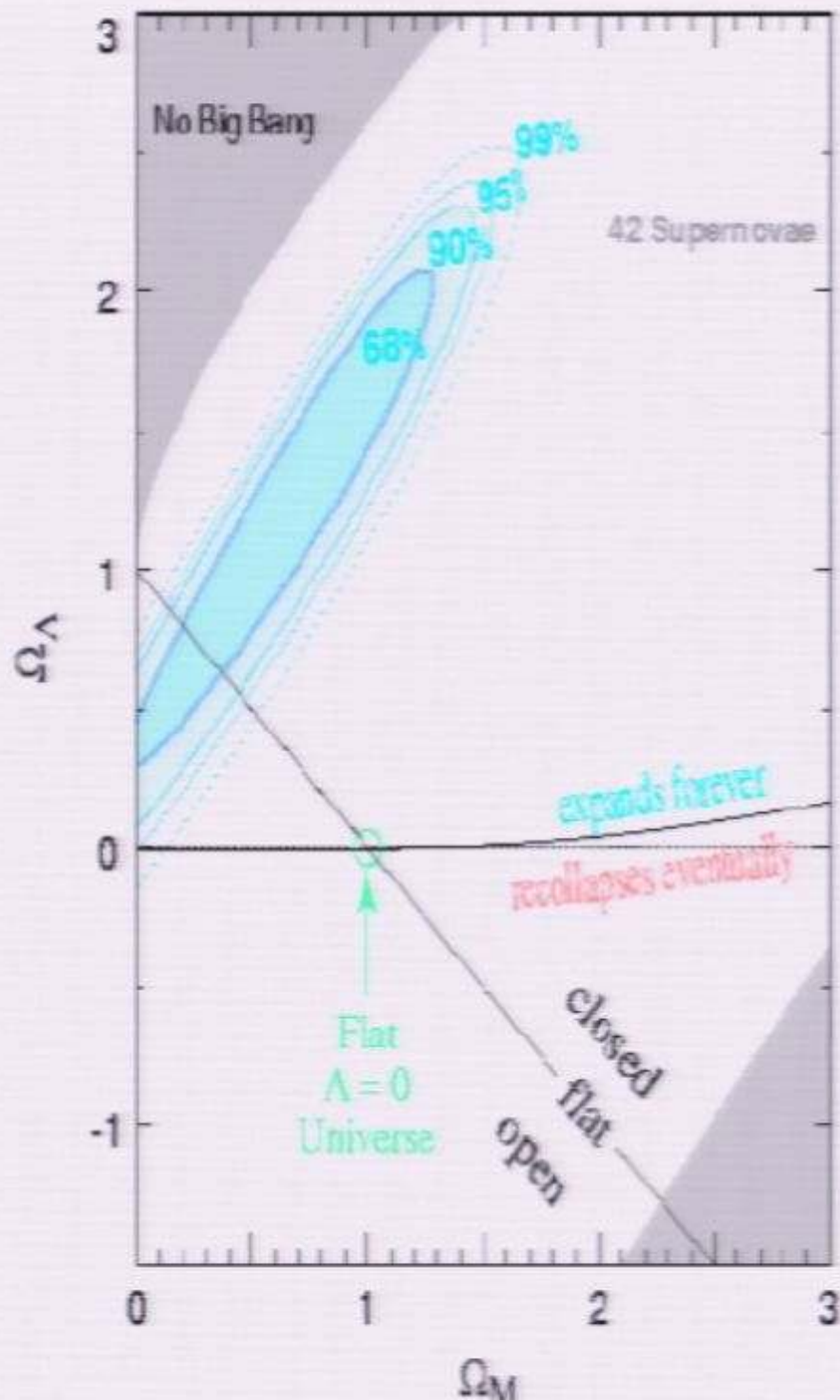
1.2

Ω_m

Dark matter density

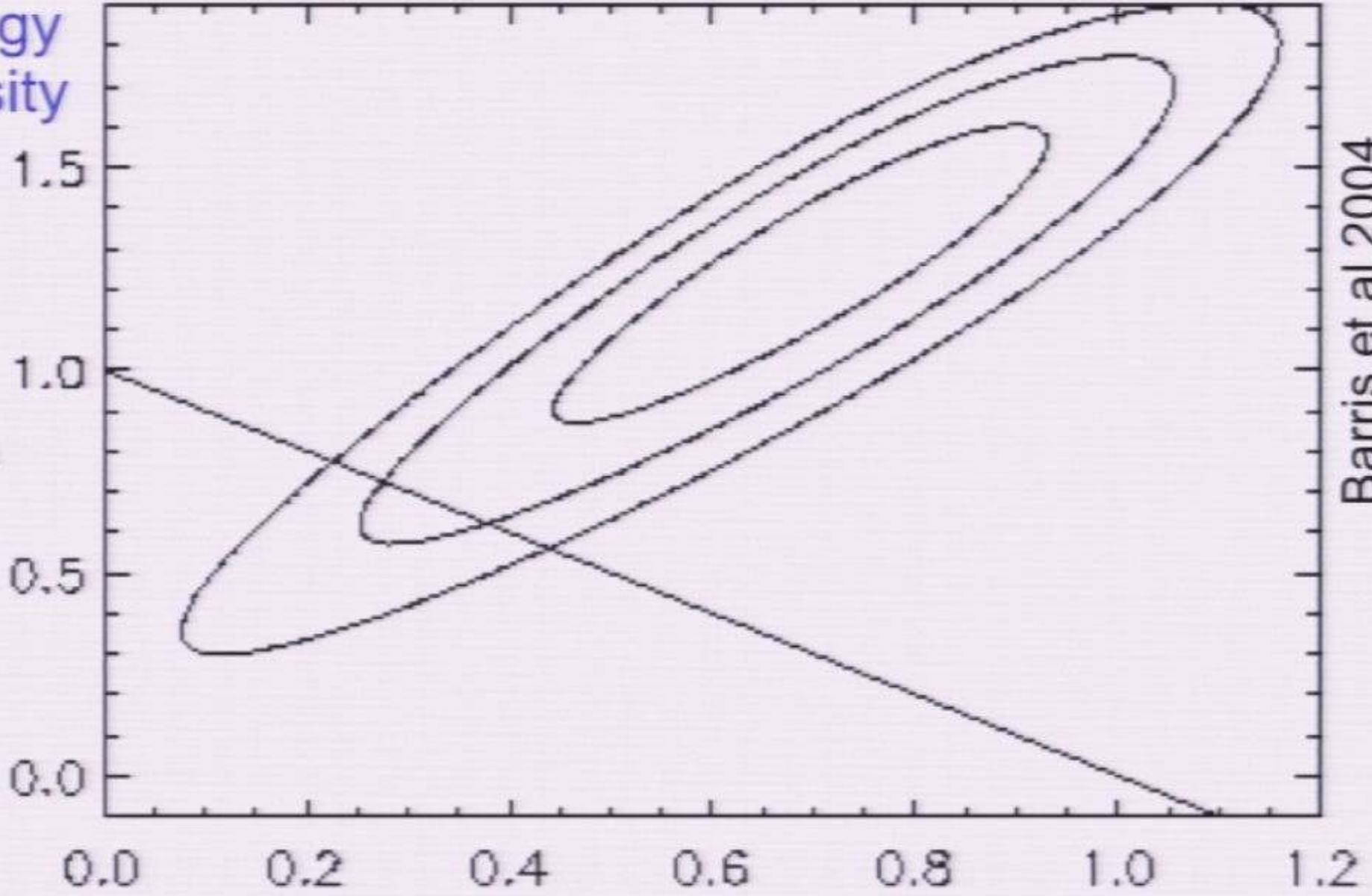
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Dark energy density

Ω_{Λ}

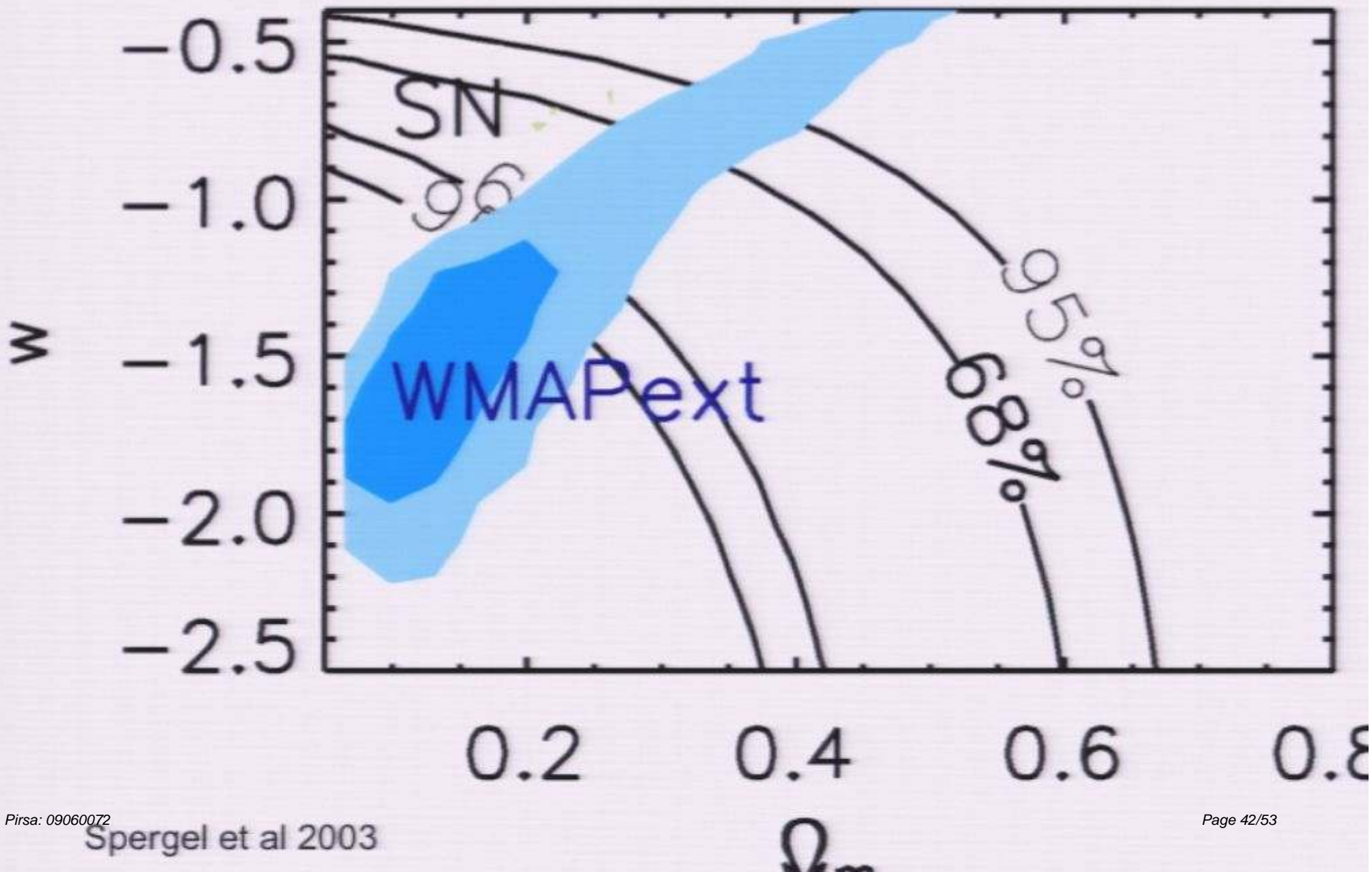


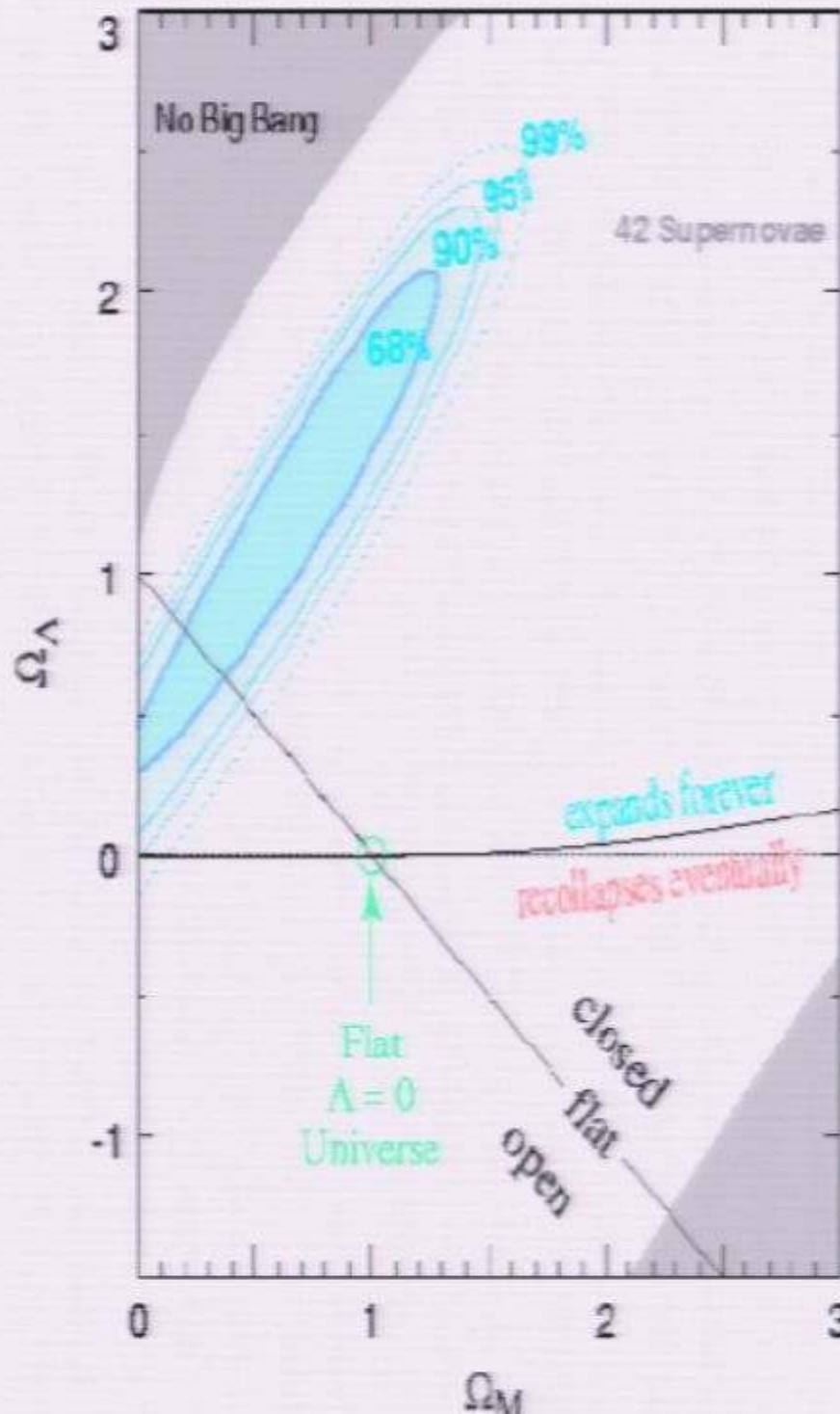
Barris et al 2004

Ω_m

Dark matter density

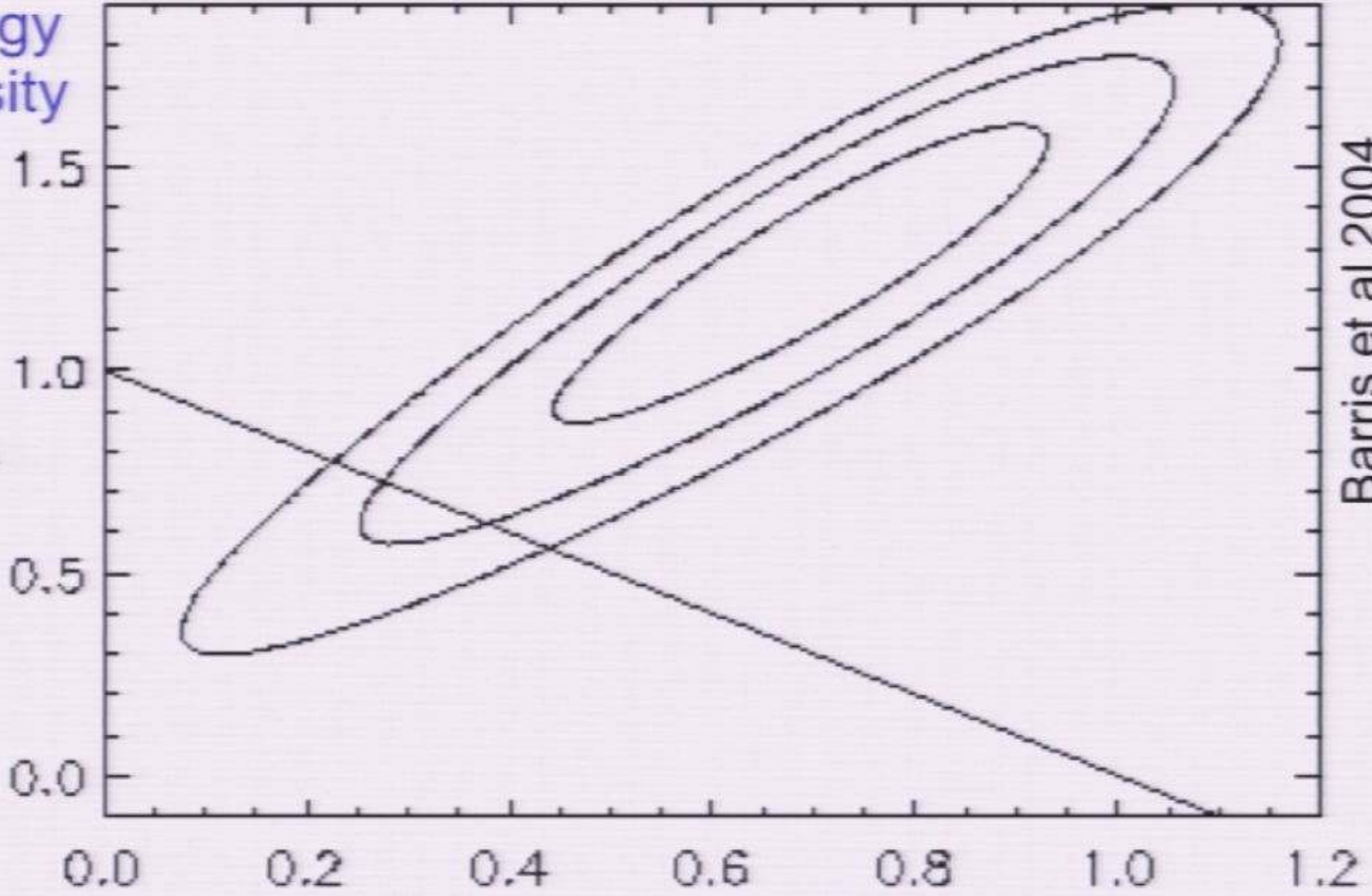
Or assume flat universe:





Dark energy density

Ω_{Λ}

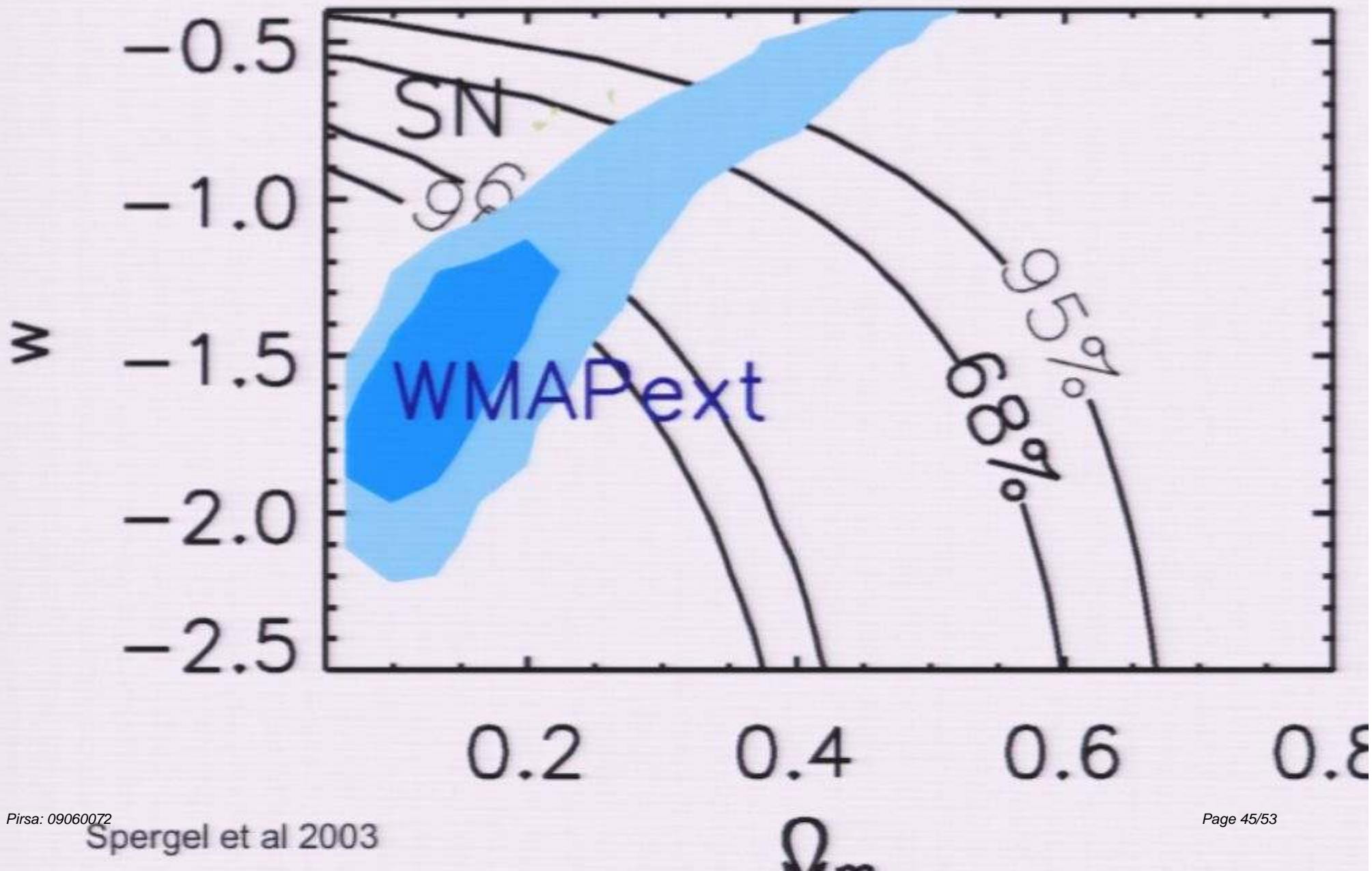


Barris et al 2004

Ω_m

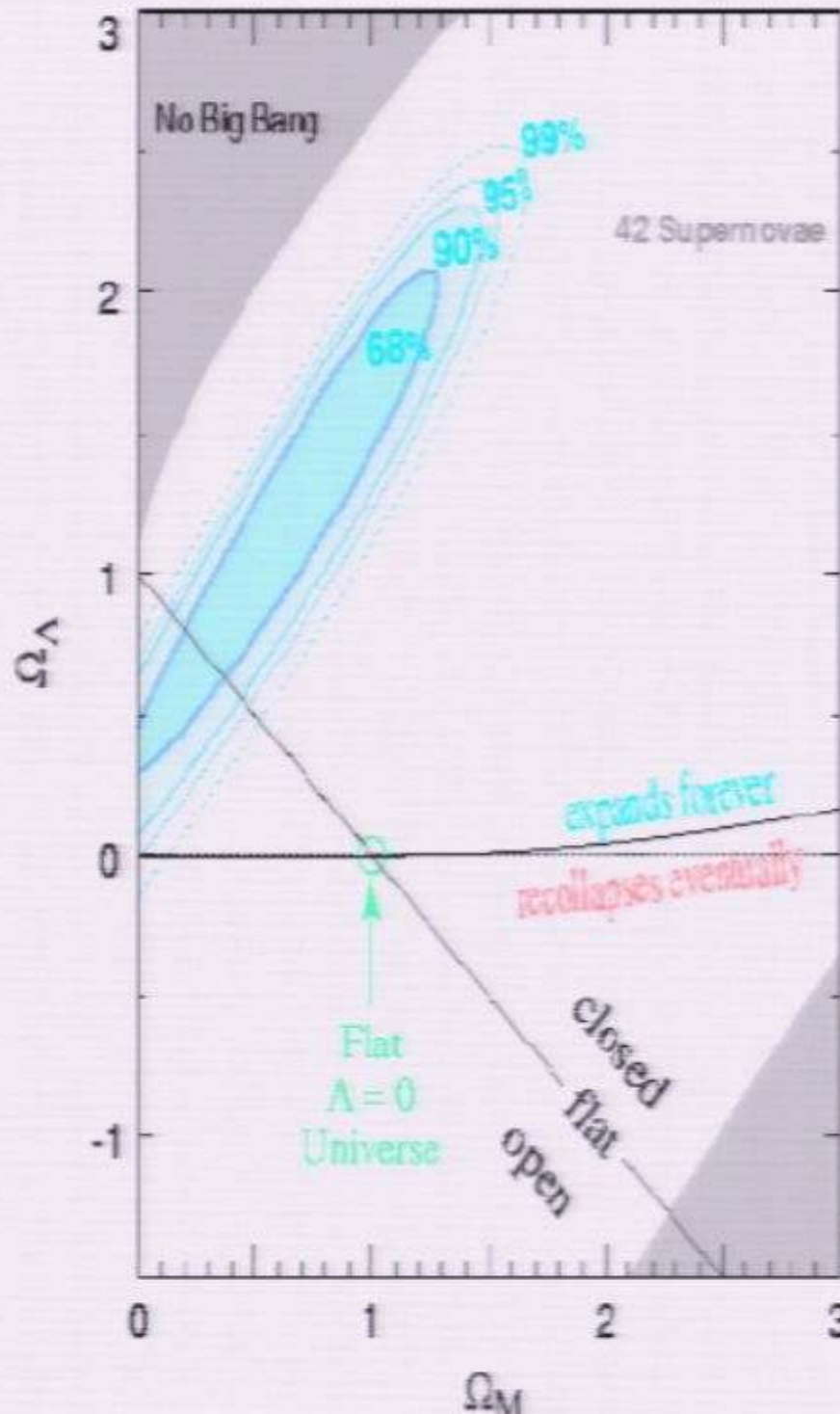
Dark matter density

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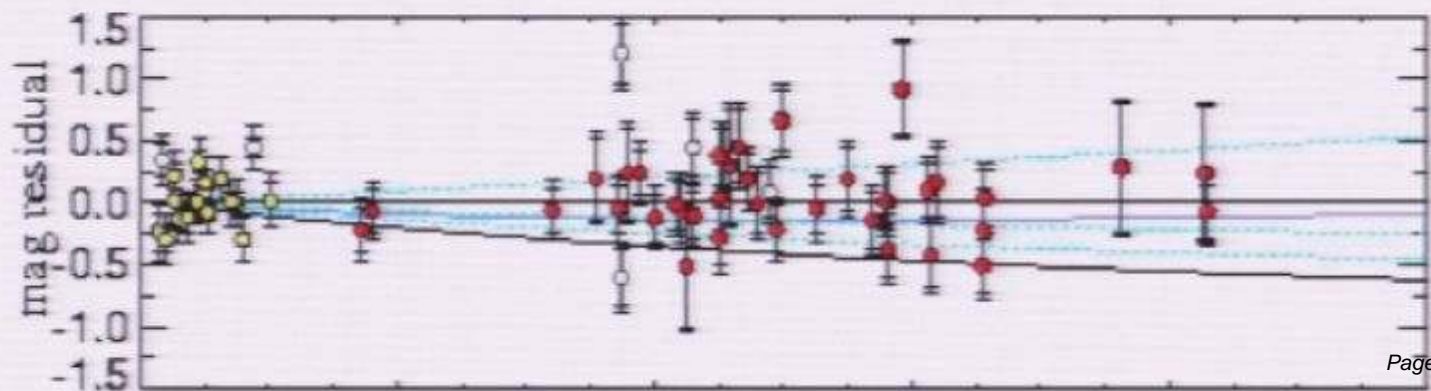
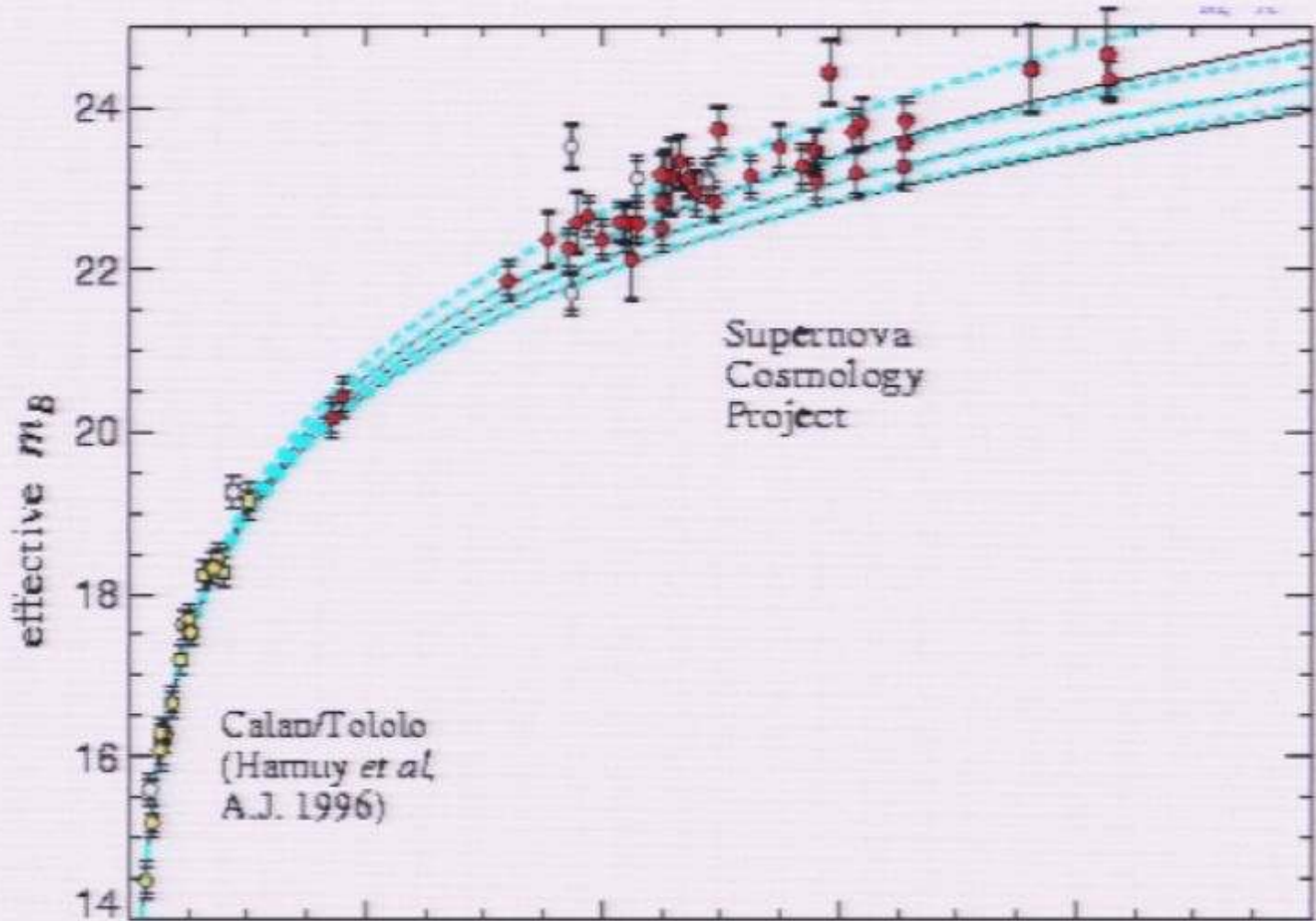


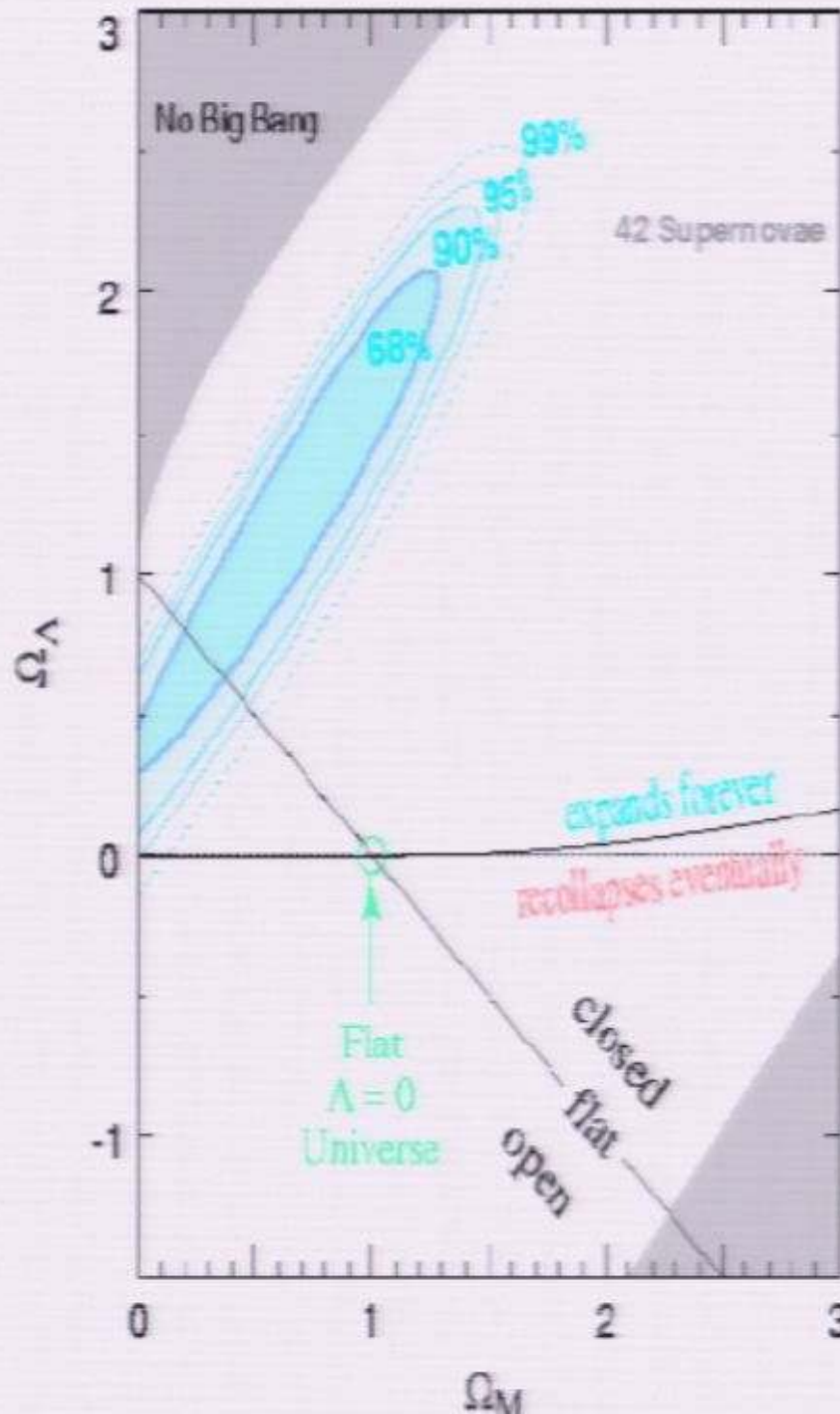
Future supernova data

- SNAP satellite proposed
 - Would measure ~2000 SNIa $0.1 < z < 1.7$
 - Funding situation currently uncertain
- SNFactory ongoing
 - To measure ~300 SNIa at $z \sim 0.05$

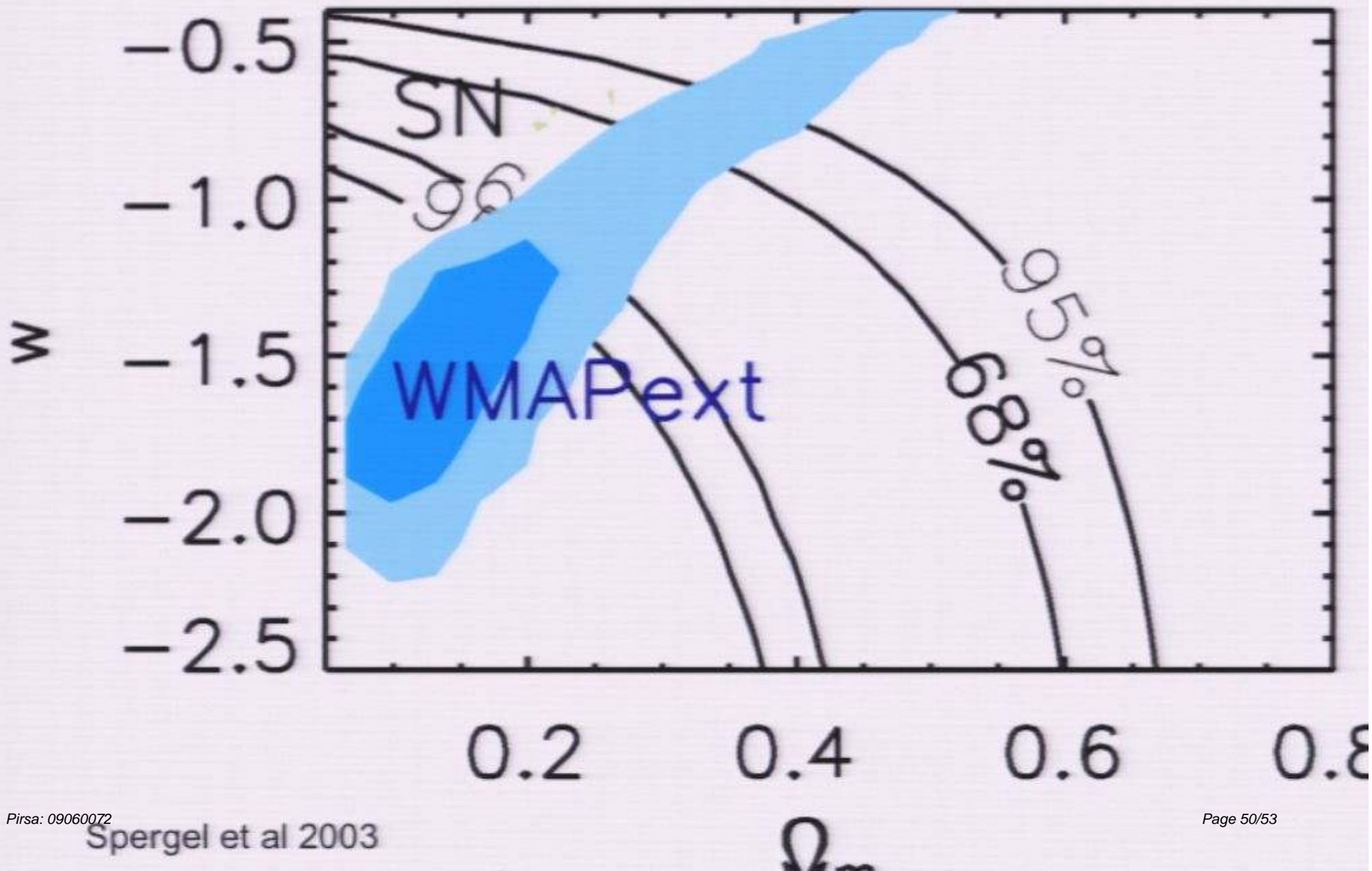


$$m_B \propto \ln D_L$$



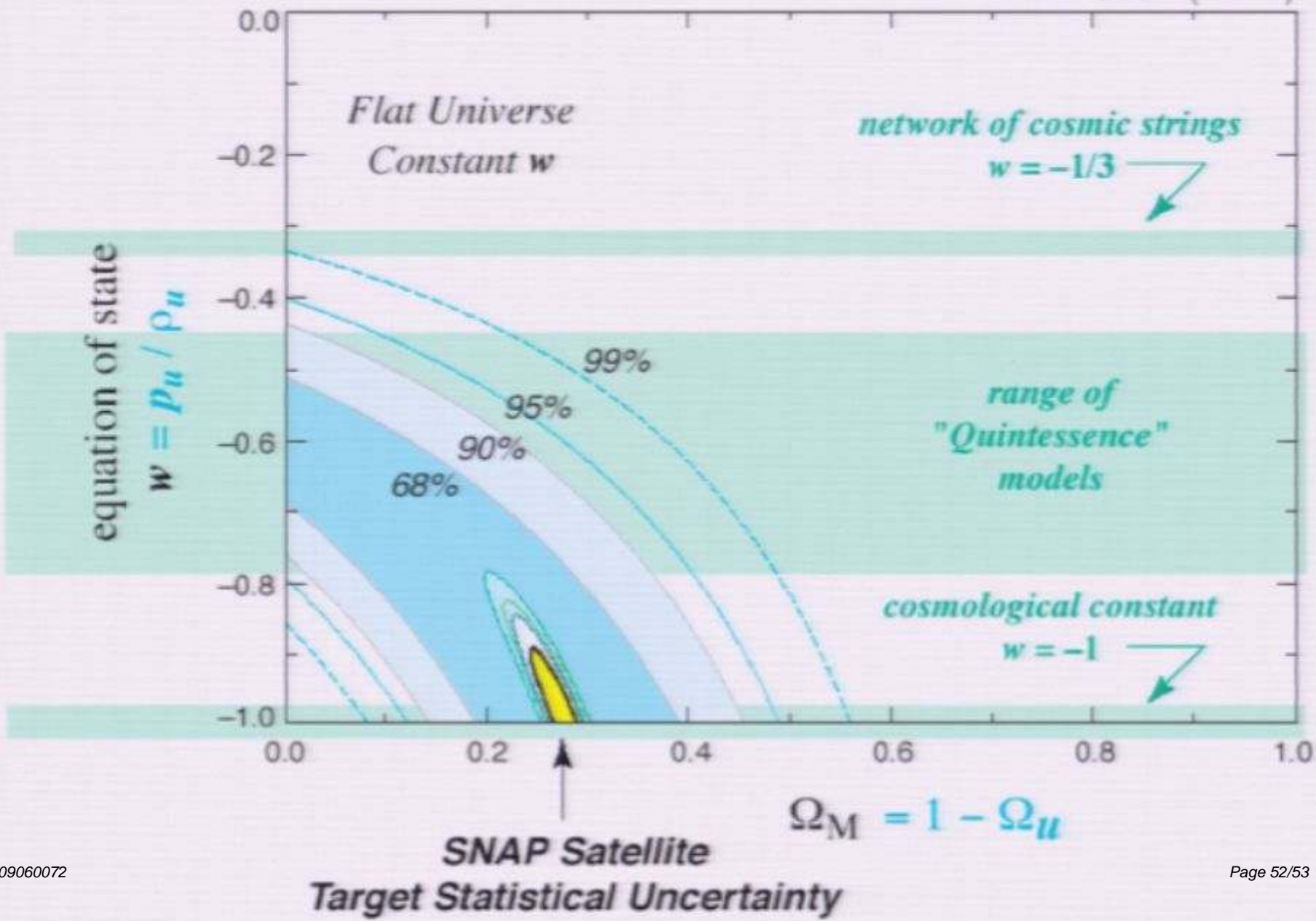


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To take away from this lecture:

- Equations for expansion of Universe as a function of time
- Equations for age of Universe
- Appreciation of observations of H_0 , t_0
- Evidence for dark energy from supernovae

Next lecture: Baryonic Acoustic Oscillations