Title: Gravitational Wave Astronomy - Lecture 2

Date: Jun 24, 2009 11:30 AM

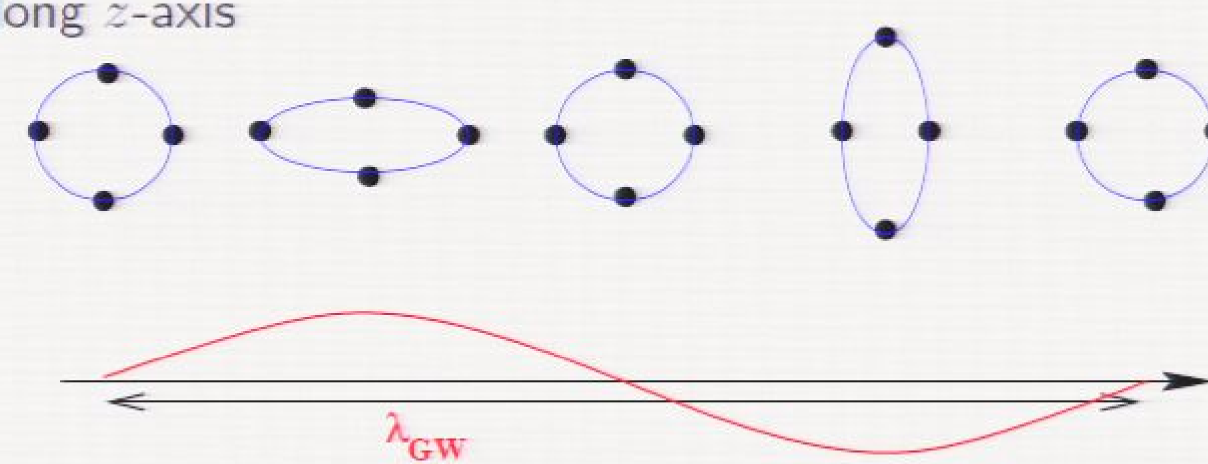
URL: http://pirsa.org/09060065

Abstract:

Interaction between GW and ring of free-falling particles

GW propagating along z-axis

- Case: $h_+ \neq 0$, $h_\times = 0$



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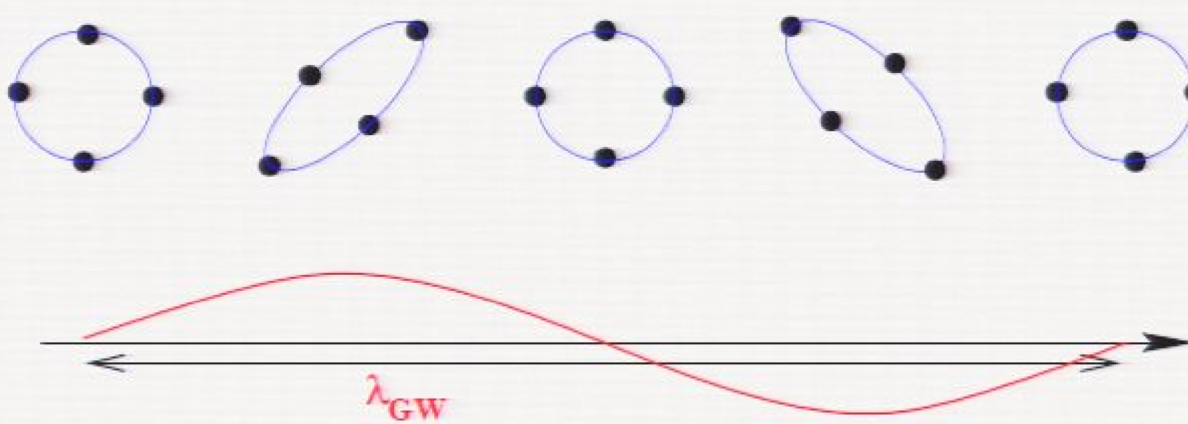
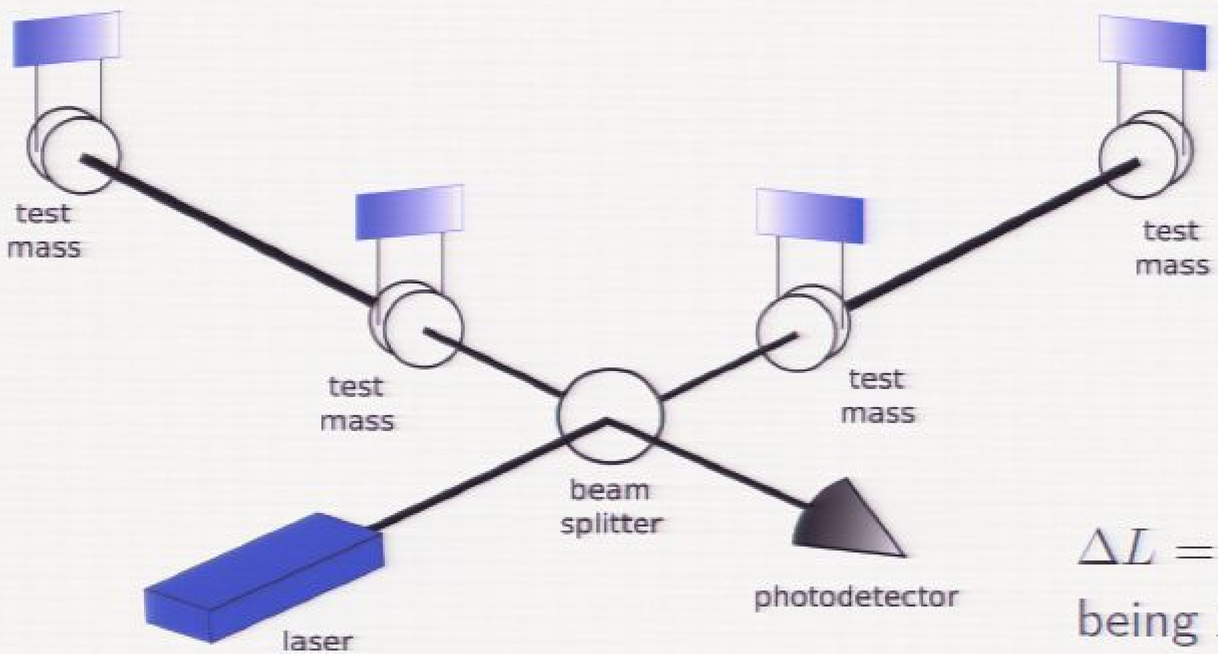


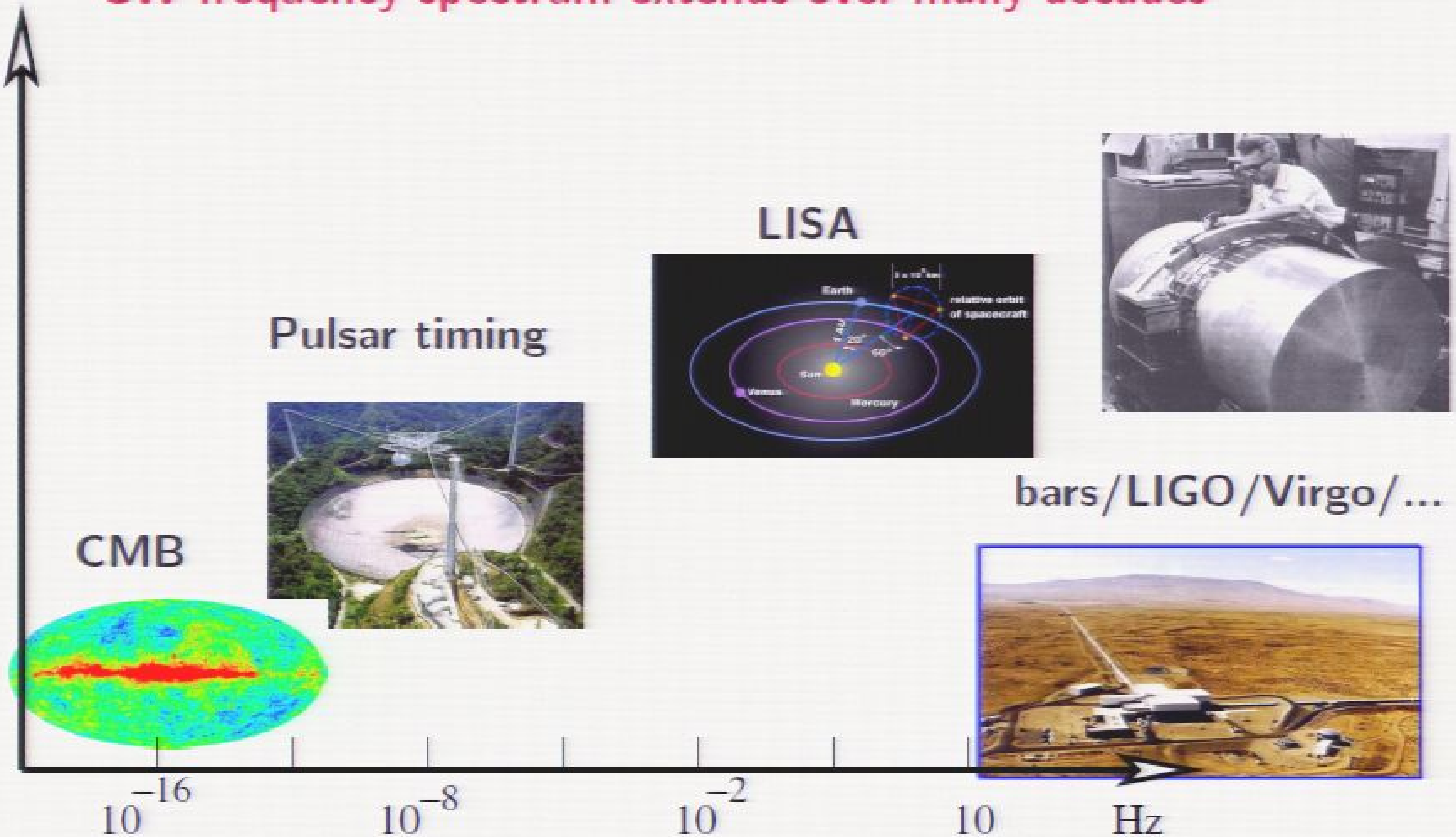
How to measure gravitational waves

Use light beams to measure the stretching and squeezing induced by GWs



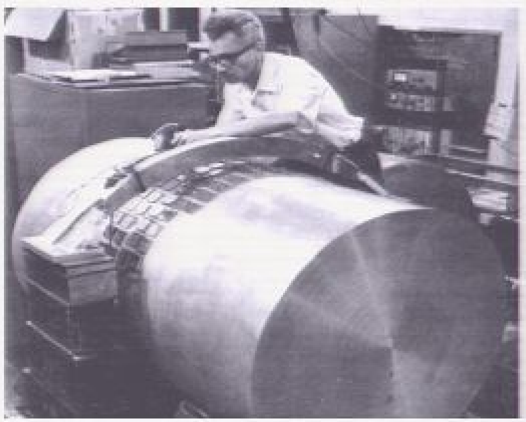
$\Delta L = L\, h \sim 10^{-16}$ cm
being $L = 4$km and $h \sim 10^{-21}$
$\Delta\phi \sim 10^{-8}$ rad

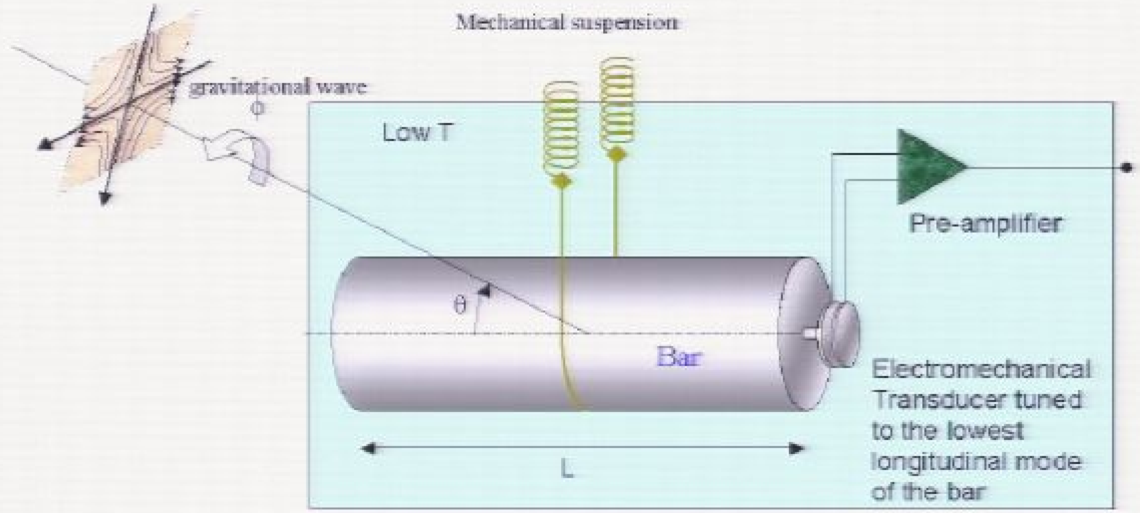
GW frequency spectrum extends over many decades



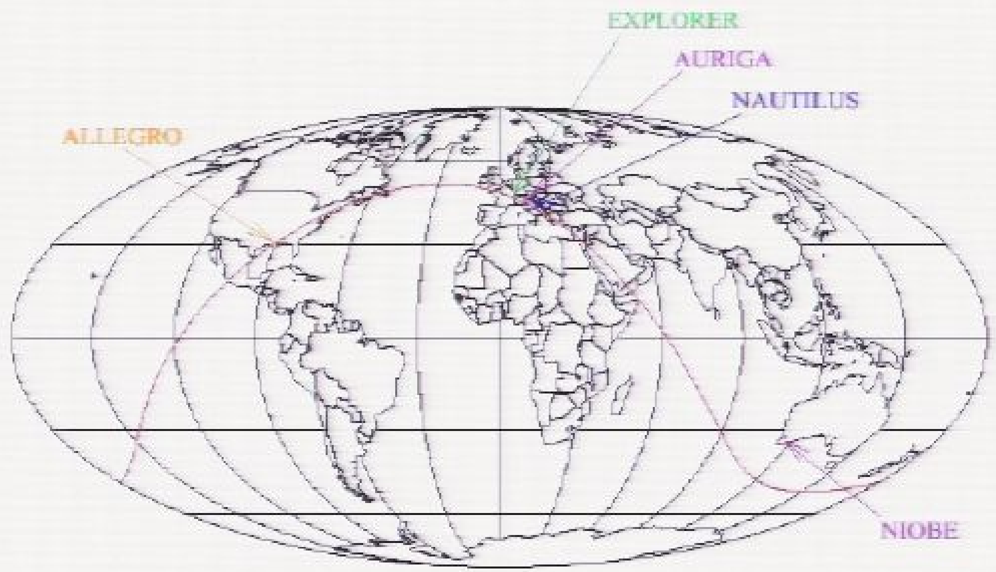
Direct observation with resonant-mass detectors

- Pioneering work by *Joe Weber* at Maryland U





Resonant-mass detectors in the world



Resonant bar or sphere detectors (GW frequency $\sim$ 1 kHz)

Nautilus (Rome) Explorer (CERN) Schenberg (Brasil) MiniGRAIL (Belgium)
Allegro (Louisiana) Niobe (Perth) Auriga (Padova)

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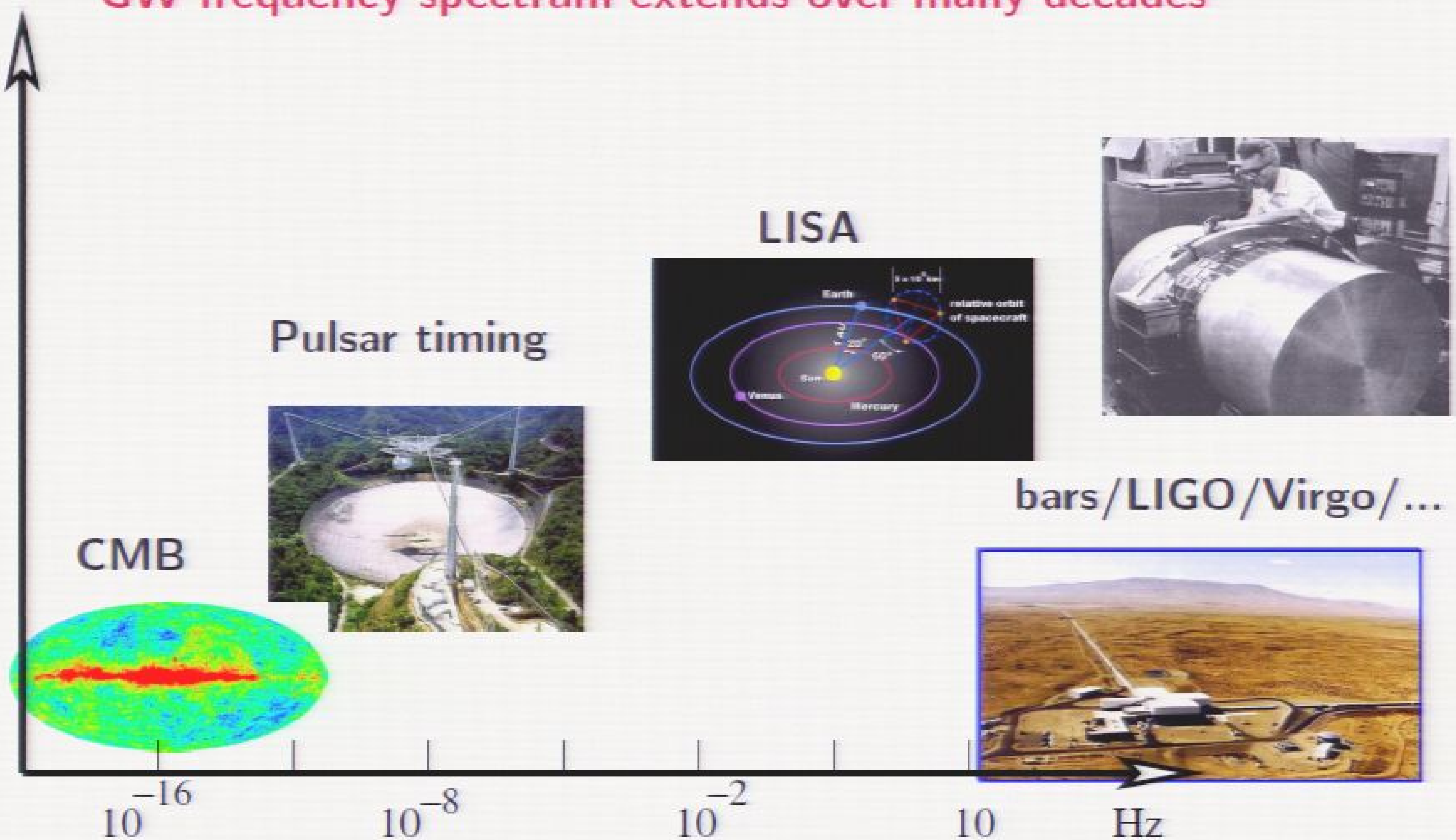

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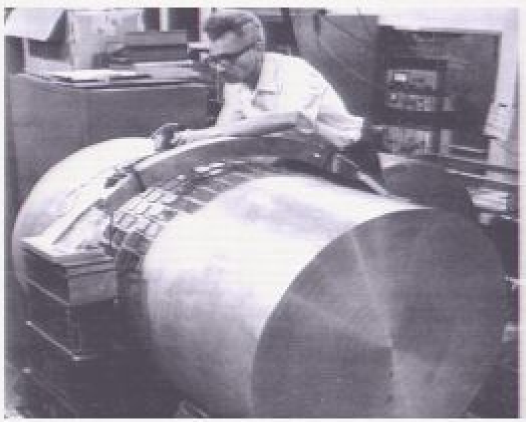

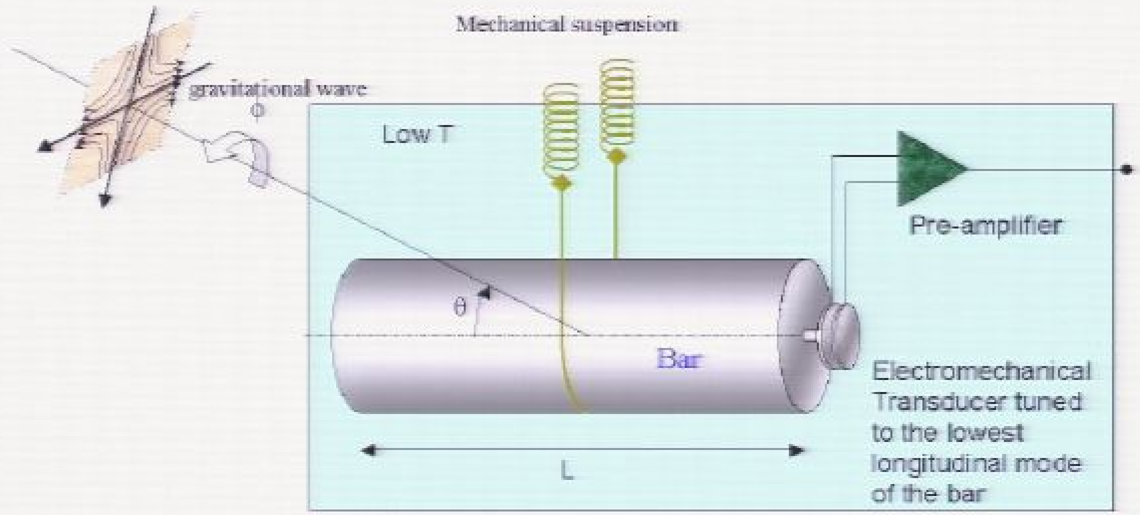

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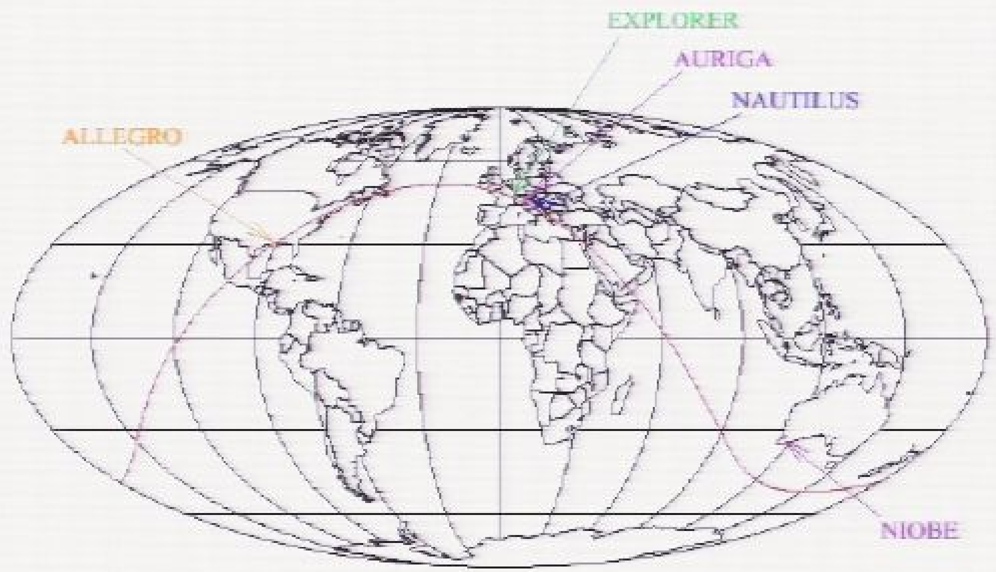

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International network of GW interferometers (frequency band $\sim 10\text{–}10^3$ Hz)

LIGO at Livingston (Lousiana) $\Rightarrow$

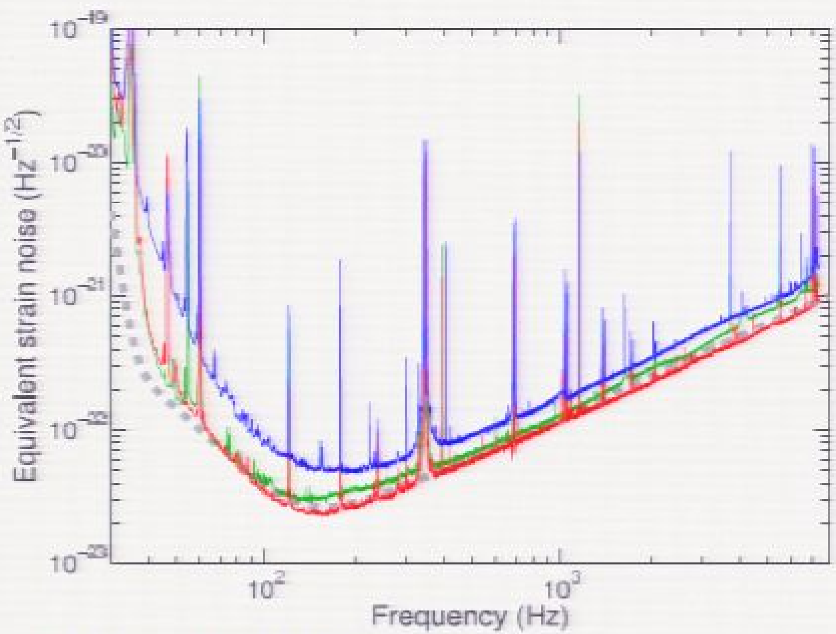
$\Leftarrow$ LIGO at Hanford (Washington State)

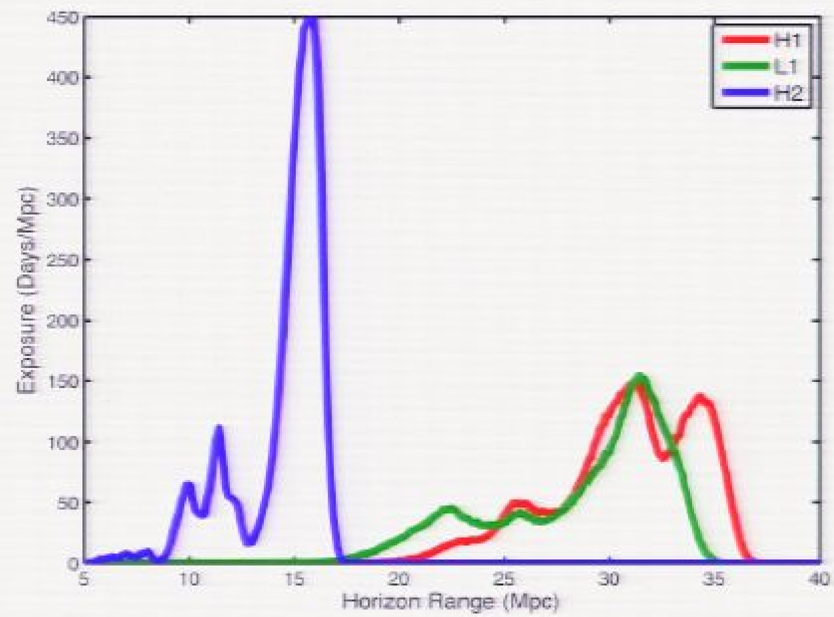
Virgo (France-Italy) $\Rightarrow$

$\Leftarrow$ GEO 600 (UK-Germany)

TAMA 300 (Japan)

Current status of LIGO detectors





Astrophysical reach:

NS-NS ($1.4M_\odot + 1.4M_\odot$): 31 Mpc, 250 L_{10} **Our own Milky Way galaxy has $\sim 1.7\,L_{10}$**

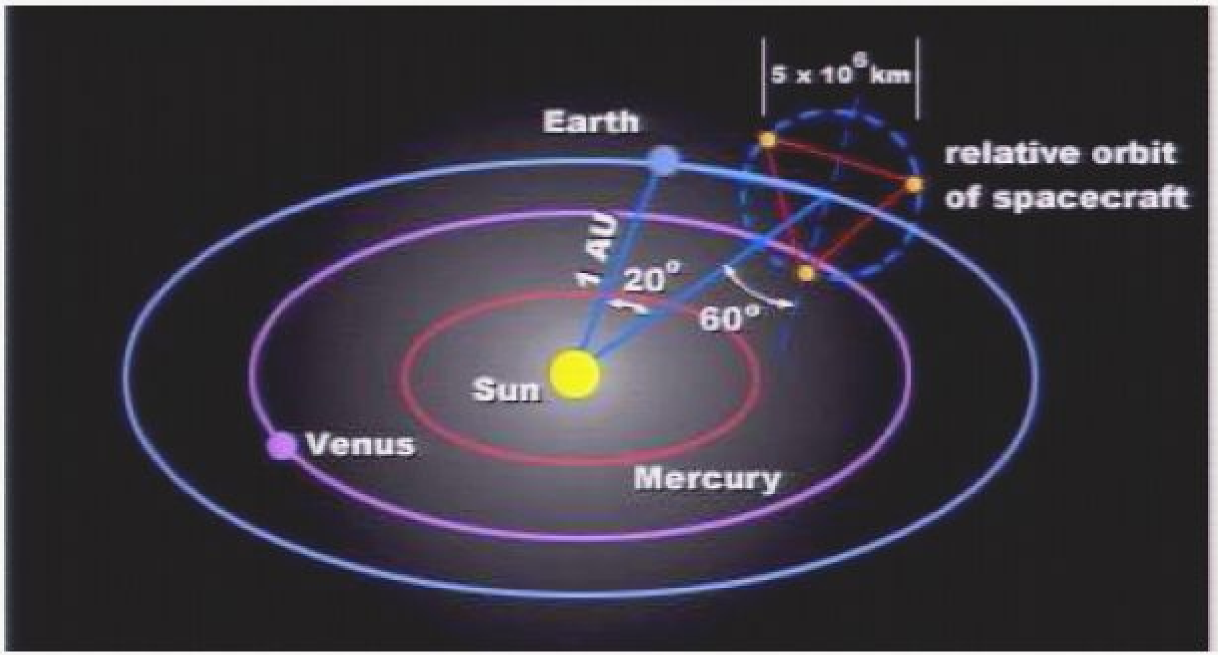
BH-BH ($10M_\odot + 10M_\odot$): 125 Mpc, 20,000 L_{10}

Data of S5 run at design sensitivity are currently under investigation

LISA: Laser Interferometer Space Antenna (frequency band: $10^{-4} - 0.1$ Hz)

LISA science goals complementary to ground-based interferometer ones

ESA/NASA mission in 2020?



GWs on the Earth: comparison with other kind of radiation

Supernova at 20 kpc:

- **From GWs:** $\sim 400 \frac{\rm erg}{\rm cm^2\, sec} \left(\frac{f_{\rm GW}}{\rm 1kHz}\right)^2 \left(\frac{h}{10^{-21}}\right)^2$ **during few msecs**
- **From neutrino:** $\sim 10^5 \frac{\rm erg}{\rm cm^2\, sec}$ **during 10 secs**
- **From optical radiation:** $\sim 10^{-4} \frac{\rm erg}{\rm cm^2\, sec}$ **during one week**

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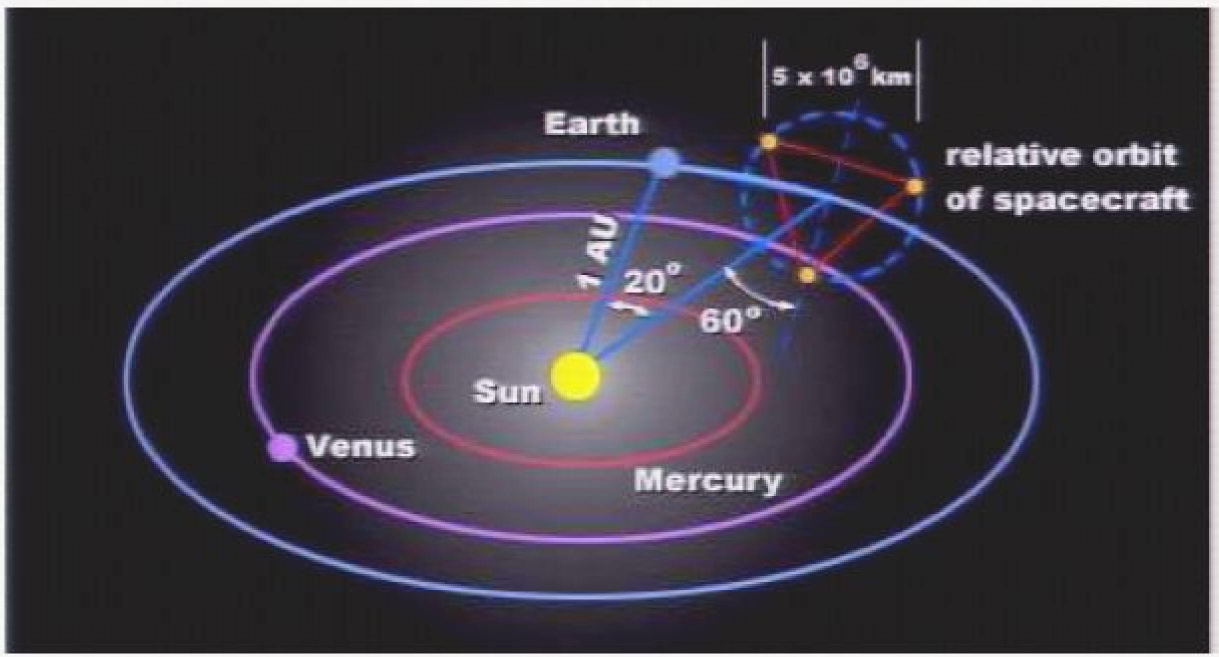


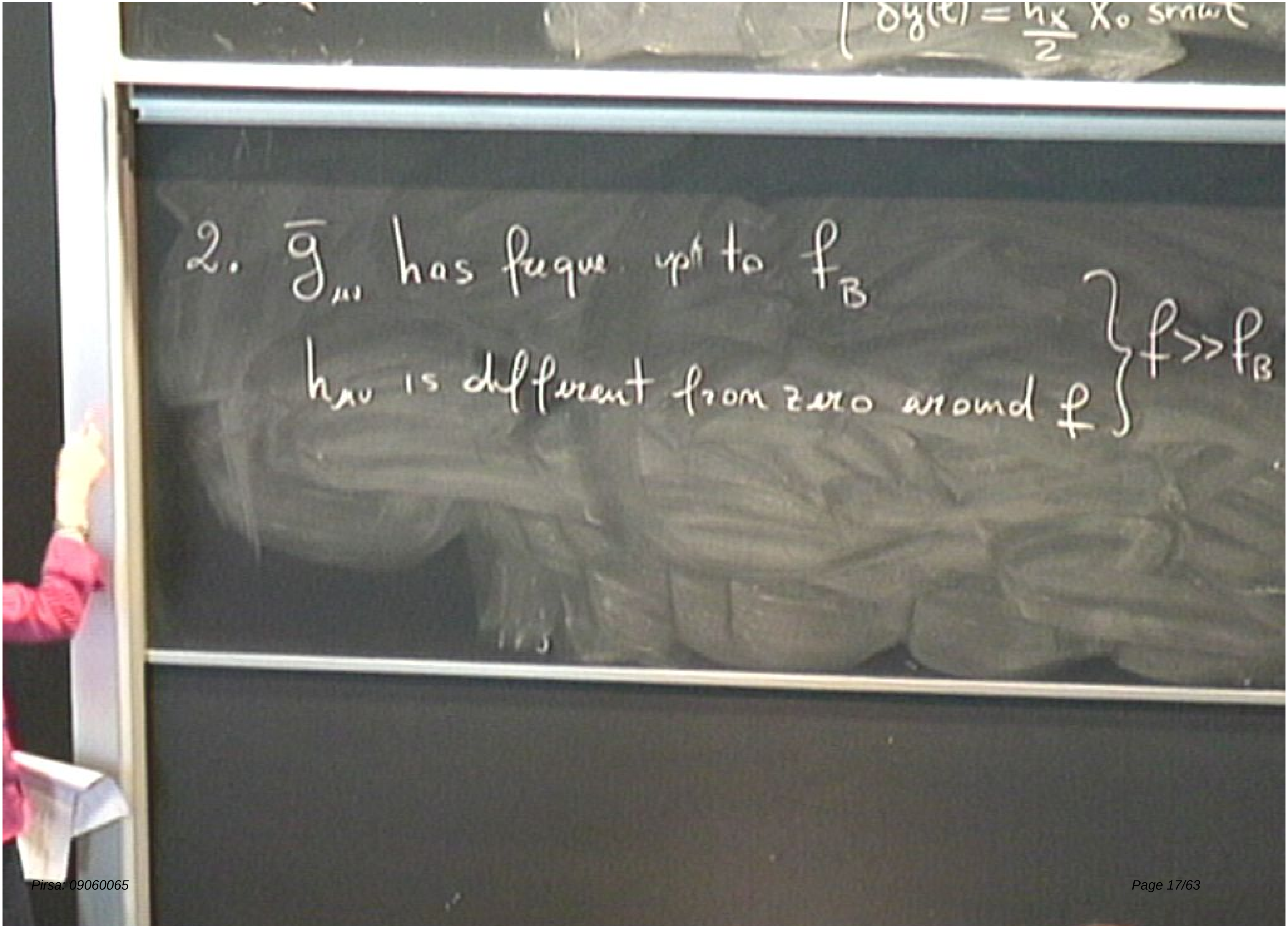
$\delta y(t) = \frac{h_\times}{2} x_0 \sin\omega t$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \|h_{\mu\nu}\| \ll 1$$

1. $\bar{g}_{\mu\nu}$ has typical scale L_B

$_{\mu\nu}$ " " wavelength λ_{GW}

$\Big\}\ \lambda_{GW} \ll L_B$


δy(t) = h_x/2 X_0 sin ωt
2. ḡ_μν has freque up to f_B
h_μν is different from zero around f
f >> f_B

$\delta y(t) = \frac{h_x}{2} x_0 \sin\omega t$

2. $\bar{g}_{\mu\nu}$ has freque upt to f_B

$h_{\mu\nu}$ is different from zero around f $\Big\}$ $f >> f_B$

$h, \lambda_{GW}/L_B \left(f_B/f \right)$

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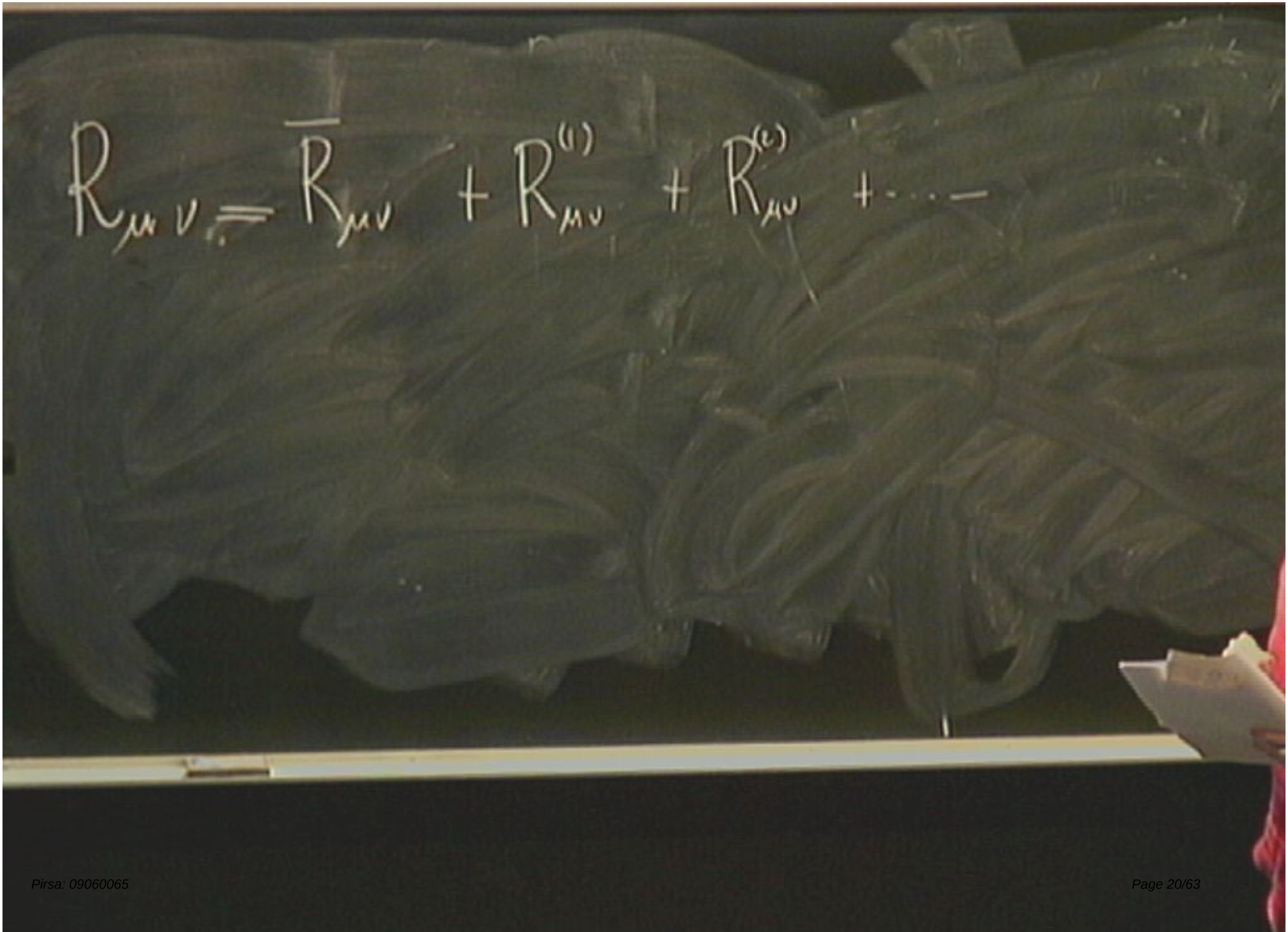
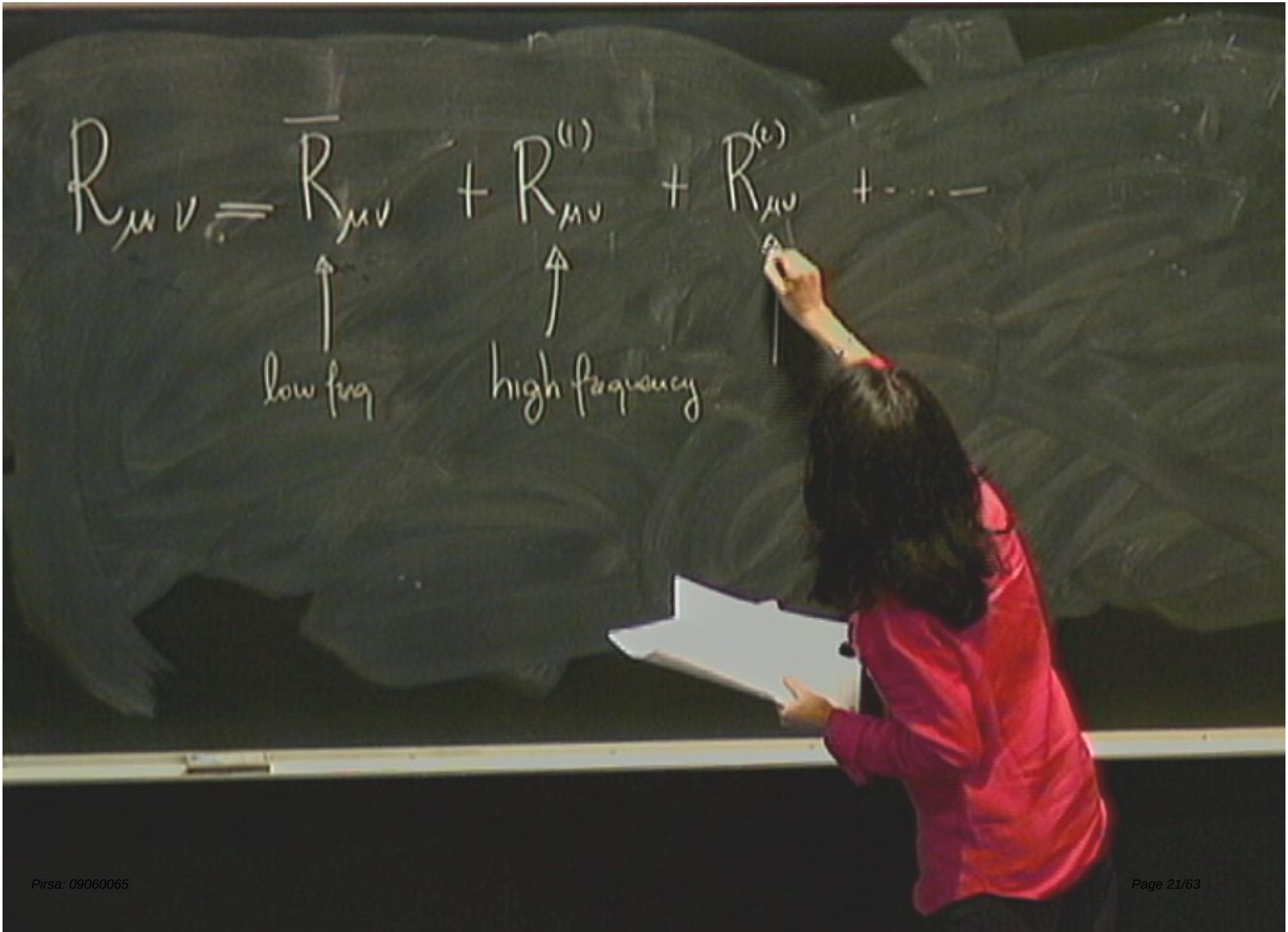
$h, \lambda_{GW}/L_B(f_B/f)$


$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \ldots$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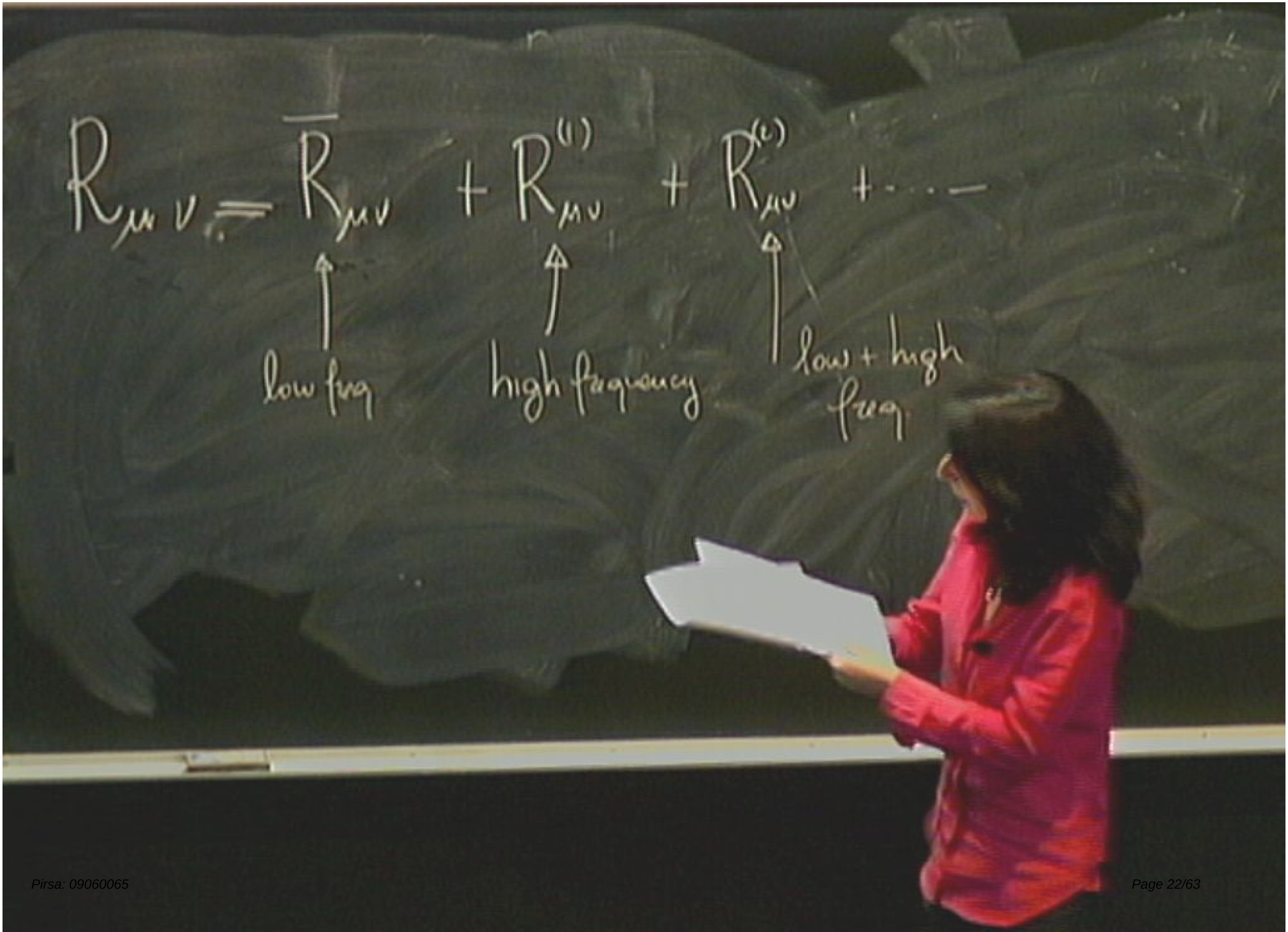
low freq
high frequency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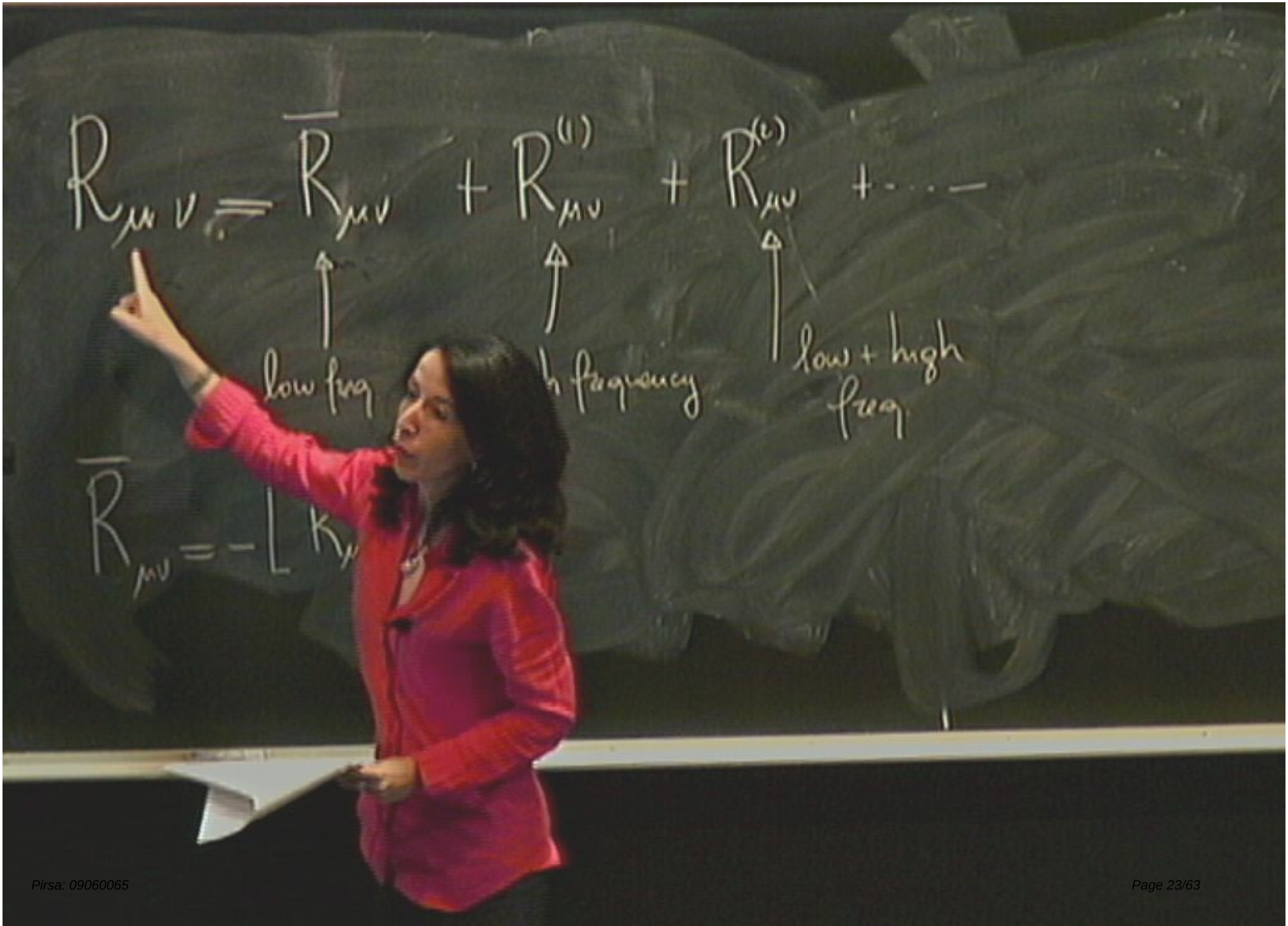
low freq
high frequency
low + high freq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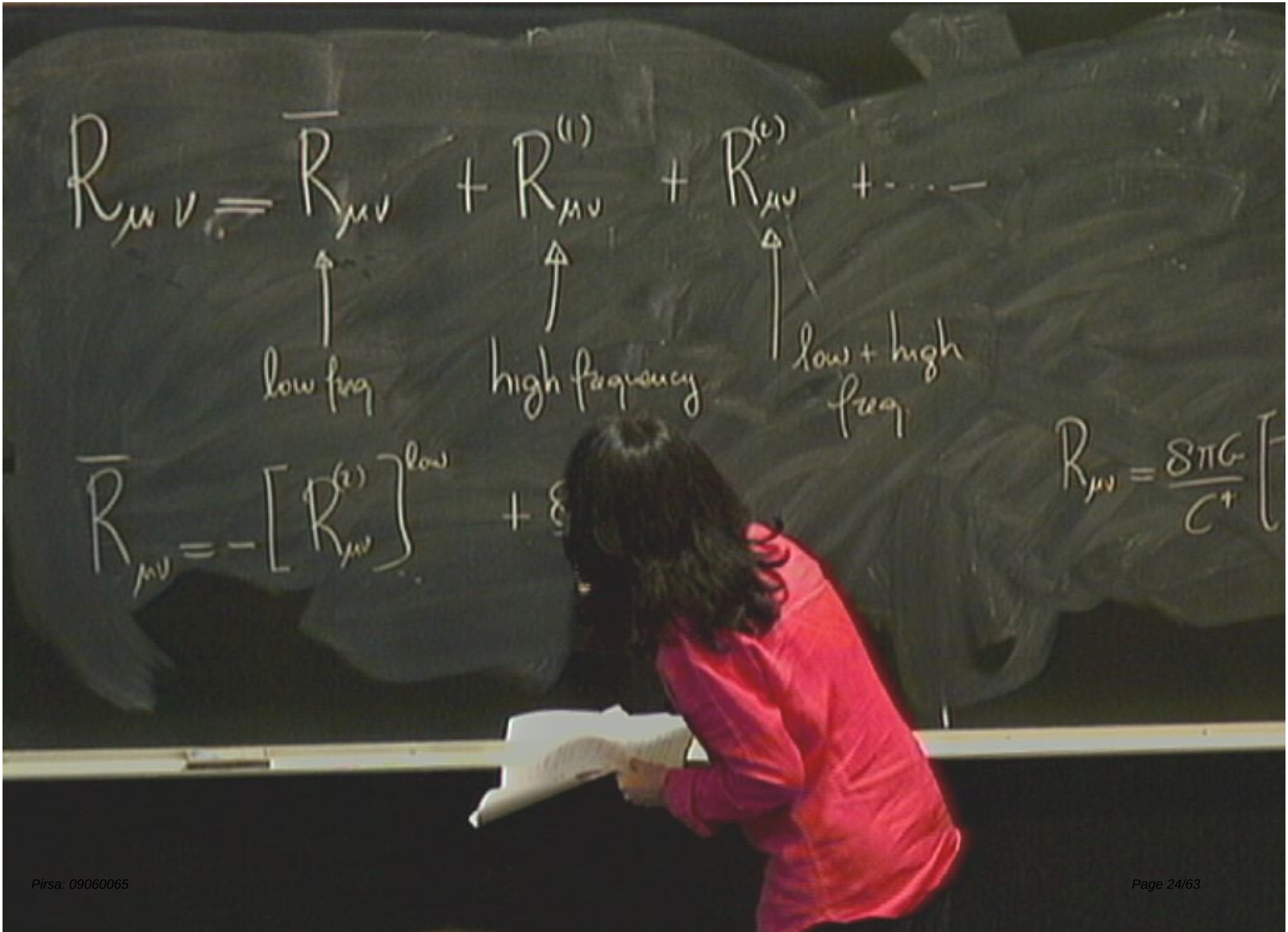
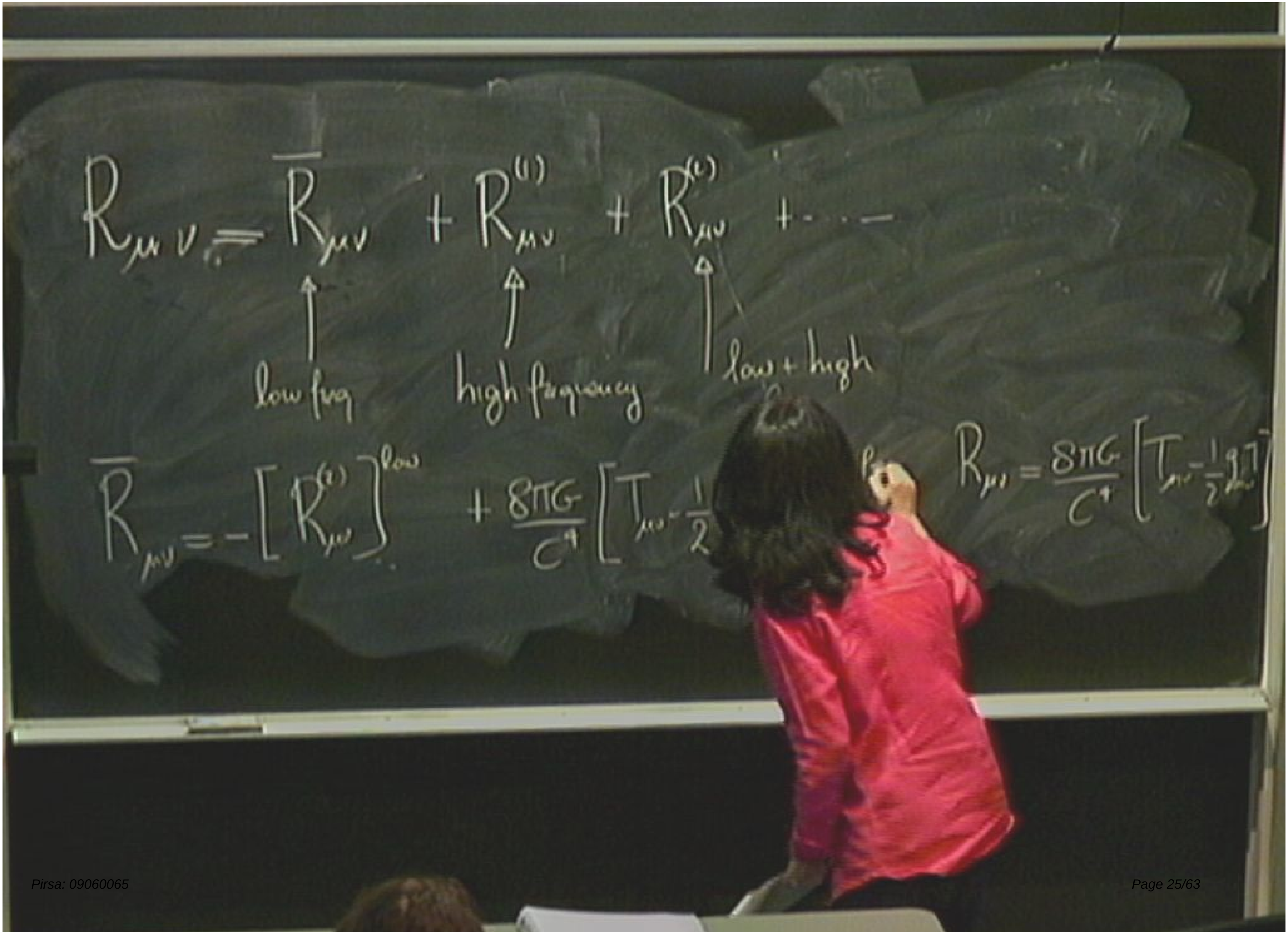
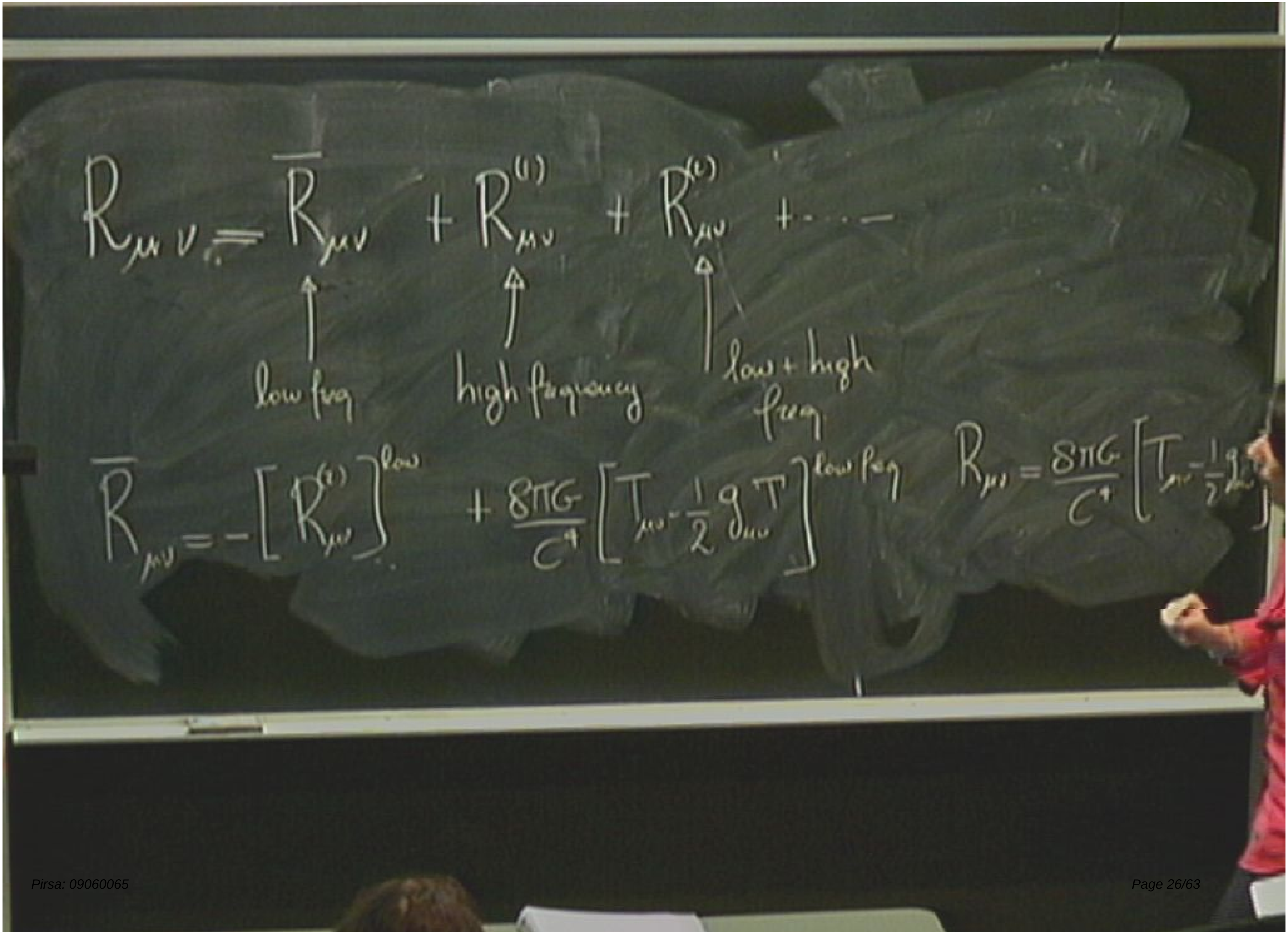
low freq
low + high freq


$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \ldots$
low freq
high frequency
low + high freq
$\bar{R}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{low} + 8$
$R_{\mu\nu} = \frac{8\pi G}{c^4}\left[\right.$



$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \dots$
low freq
high frequency
low + high
$\bar{R}_{\mu\nu} = -[R^{(2)}_{\mu\nu}]^{low} + \frac{8\pi G}{c^4}[T_{\mu\nu} - \frac{1}{2}$
$R_{\mu\nu} = \frac{8\pi G}{c^4}[T_{\mu\nu} - \frac{1}{2} g T]$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low freq
high frequency
low + high freq
$\bar{R}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{low} + \frac{8\pi G}{c^4}\left[T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right]^{low\ freq}$
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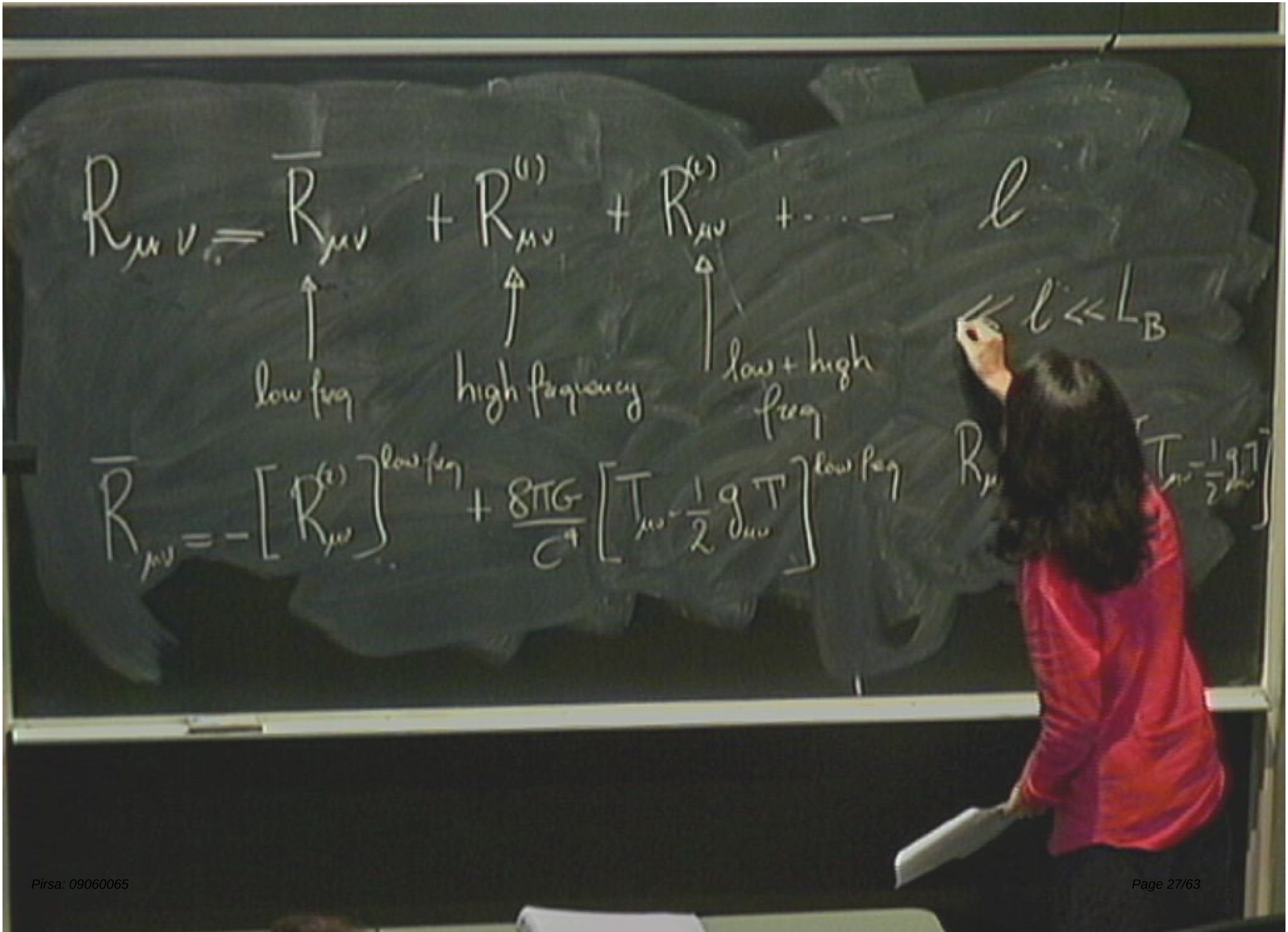
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$\ll \ell \ll L_B$
low freq
high frequency
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$$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \dots \qquad \ell$$

$\bar{R}_{\mu\nu}$ ↑ low freq

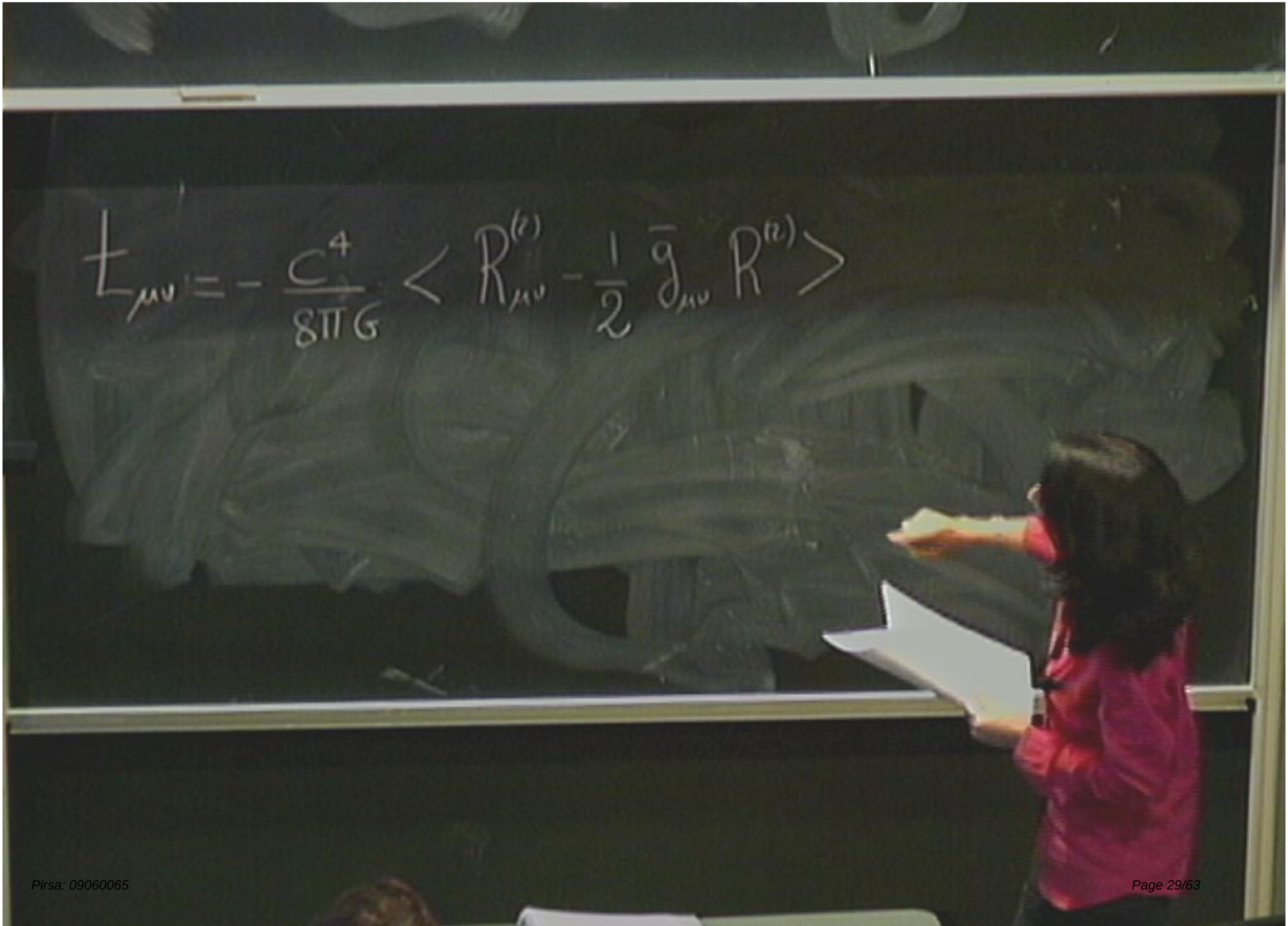
$R^{(1)}_{\mu\nu}$ ↑ high frequency

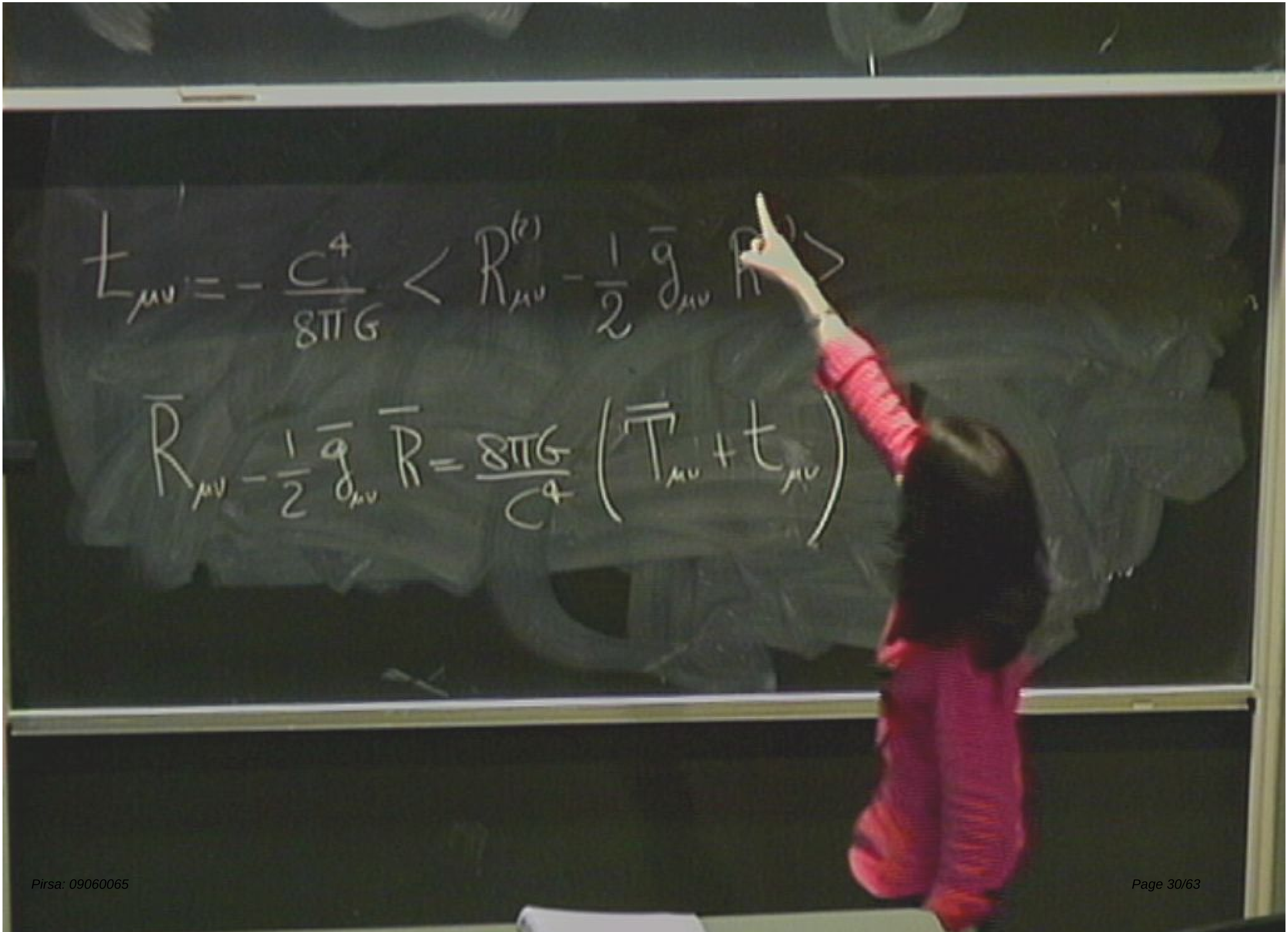
$R^{(2)}_{\mu\nu}$ ↑ low + high freq

$$\lambda_{GW} \ll \ell \ll L_B$$

$$\bar{R}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\text{low freq}} + \frac{8\pi G}{c^4}\left[T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right]^{\text{low freq}}$$

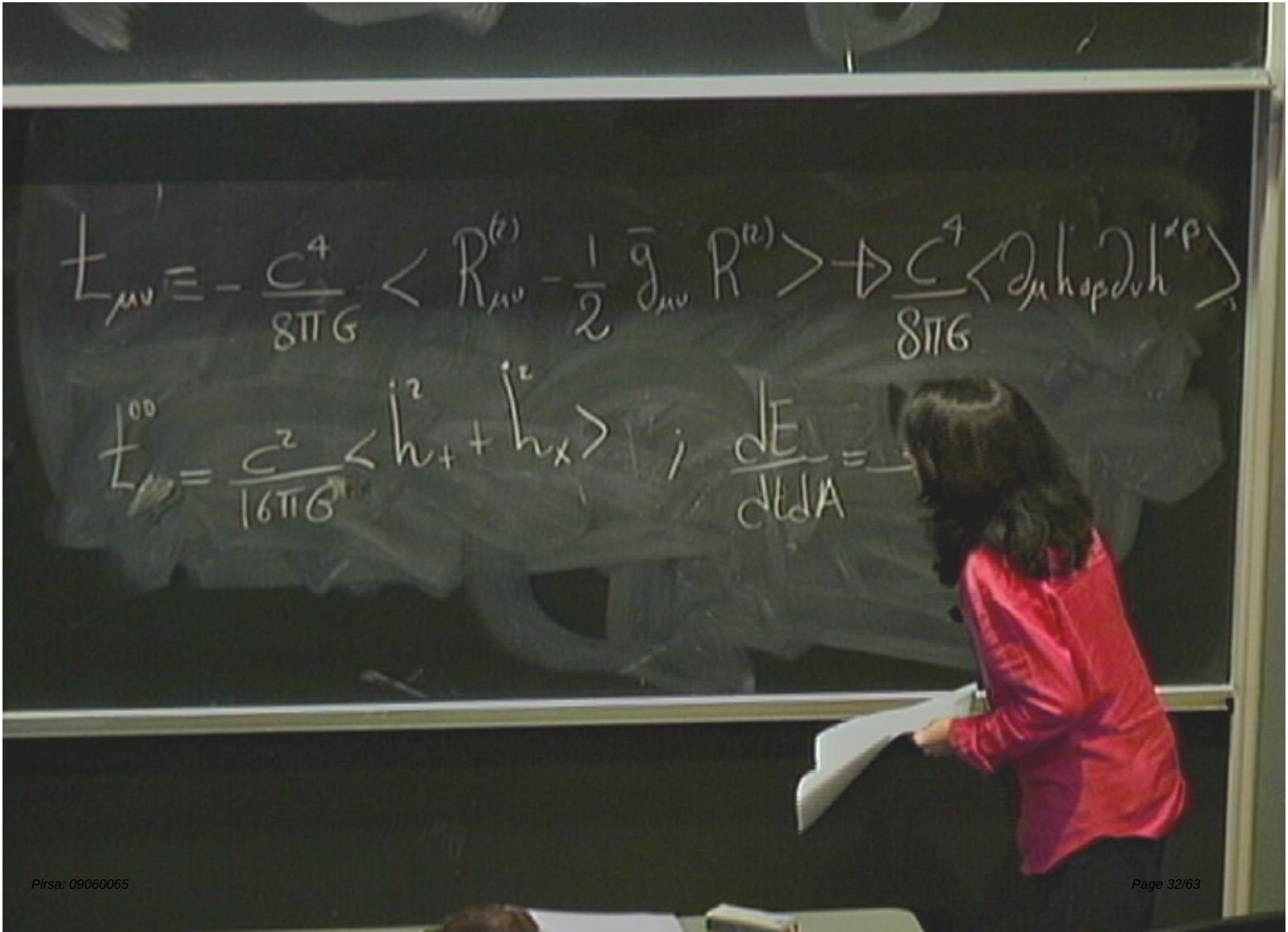
$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left[T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right]$$


$t_{\mu\nu} = - \frac{c^4}{8\pi G} < R^{(2)}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} R^{(2)} >$


$t_{\mu\nu} = -\frac{c^4}{8\pi G} \langle R^{(2)}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} R \rangle$
$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{R} = \frac{8\pi G}{c^4} (\bar{T}_{\mu\nu} + t_{\mu\nu})$

$$t_{\mu\nu} \equiv -\frac{c^4}{8\pi G}\left\langle R^{(2)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu} R^{(2)}\right\rangle + \frac{c^4}{8\pi G}\left\langle \partial_\mu h_{\alpha\beta}\,\partial_\nu h^{\alpha\beta}\right\rangle$$

$$t^{00} = \frac{c^2}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$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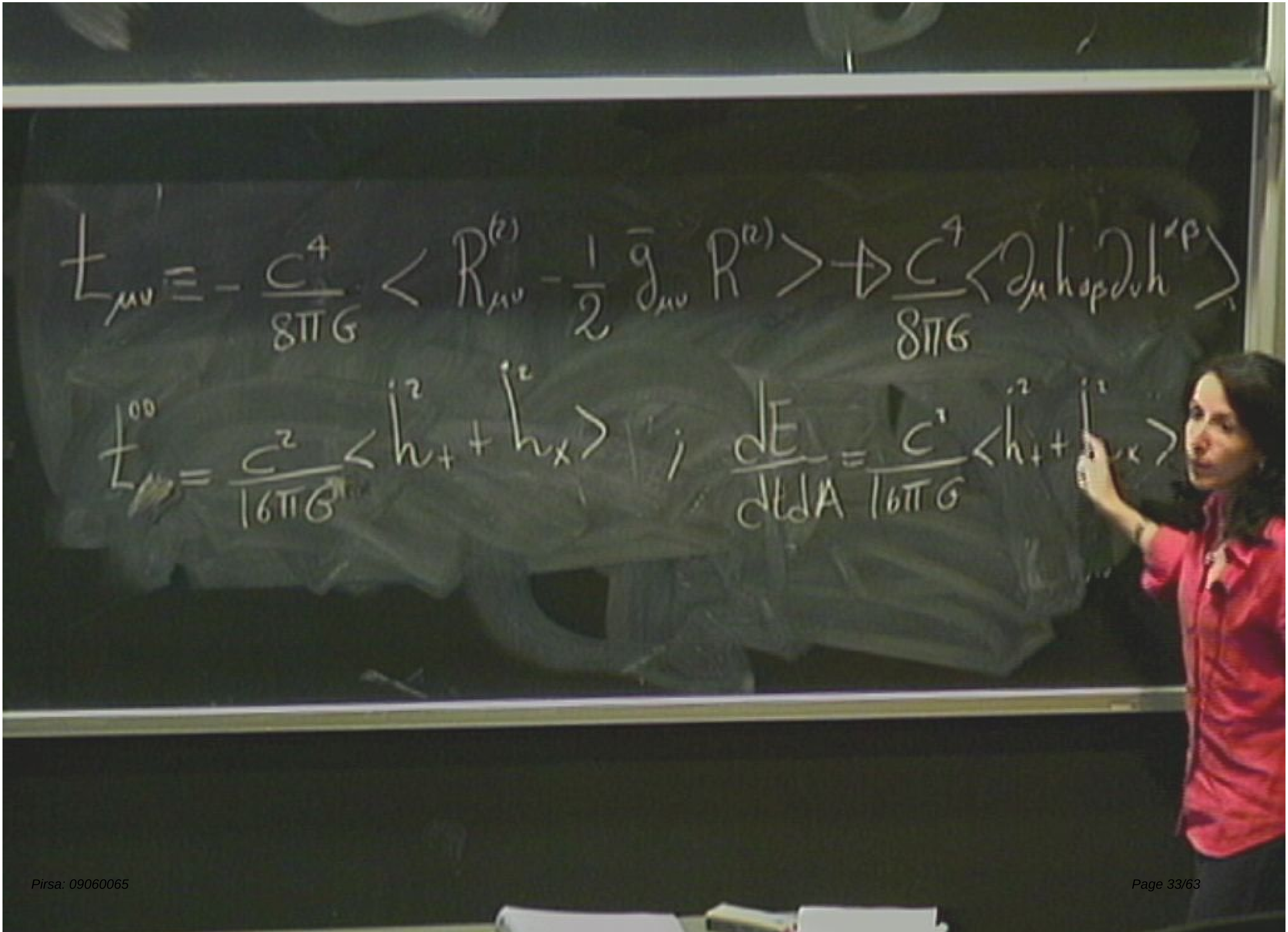
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Multipolar decomposition of waves in linear gravity

- Multipole expansion in terms of mass moments (I_L) and mass-current moments (J_L) of the source

$$h \sim \overbrace{\frac{G}{c^2}\frac{I_0}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^3}\frac{\dot{I}_1}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^4}\frac{\ddot{I}_2}{r}}^{\text{mass quadrupole}} + \cdots$$

$$\cdots + \underbrace{\frac{G}{c^4}\frac{\dot{J}_1}{r}}_{\text{can't oscillate}} + \underbrace{\frac{G}{c^5}\frac{\ddot{J}_2}{r}}_{\text{current quadrupole}} + \cdots$$

- Typical strength: $h \sim \frac{G}{c^4}\frac{M\,L^2}{P^2}\frac{1}{r} \sim \frac{G(E_{\rm kin}/c^2)}{c^2\,r}$

If $E_{\rm kin}/c^2 \sim 1 M_\odot$**, depending on** $r \quad \Rightarrow \quad h \sim 10^{-23}\text{–}10^{-17}$

Quadrupole nature of GW emission [naive way]

EM theory: Luminosity $\propto \ddot{\mathbf{d}}^2$ $\mathbf{d} = e\,\mathbf{x} \Rightarrow$ electric dipole moment

- GW theory: electric dipole moment $\Rightarrow$ mass dipole moment

$$\mathbf{d} = \sum_i m_i\,\mathbf{x}_i \Rightarrow \dot{\mathbf{d}} = \sum_i m_i\,\dot{\mathbf{x}}_i = \mathbf{P}$$

Conservation of momentum $\Rightarrow$ no mass dipole radiation exists in GR

- GW theory: magnetic dipole moment $\Rightarrow$ current dipole moment

$$\mu = \sum_i m_i\,\mathbf{x}_i \times \dot{\mathbf{x}}_i = \mathbf{J}$$

Conservation of angular momentum $\Rightarrow$ no current dipole radiation exists in GR

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Comparison between GW and EM luminosity

$$\mathcal{L}_{\rm GW} = \frac{G}{5c^5}\,(\dddot{I}_2)^2 \qquad I_2 \sim \epsilon\, M\, R^2$$

$R \rightarrow$ typical source's dimension, $M \rightarrow$ source's mass, $\epsilon \rightarrow$ deviation from sphericity

$$\dddot{I}_2 \sim \omega^3\,\epsilon\,M\,R^2 \text{ with } \omega \sim 1/P \quad \Rightarrow \quad \mathcal{L}_{\rm GW} \sim \frac{G}{c^5}\,\epsilon^2\,\omega^6\,M^2\,R^4$$

$$\mathcal{L}_{\rm GW} \sim \frac{c^5}{G}\,\epsilon^2\,\left(\frac{G\,M\,\omega}{c^3}\right)^6\,\left(\frac{R\,c^2}{G\,M}\right)^4 \Rightarrow \quad \frac{c^5}{G} = 3.6 \times 10^{59} \text{ erg/sec (huge!)}$$

- For a steel rod of $M = 490$ tons, $R = 20$ m and $\omega \sim 28$ rad/sec: $GM\omega/c^3 \sim 10^{-32}$, $Rc^2/GM \sim 10^{25} \rightarrow \mathcal{L}_{\rm GW} \sim 10^{-27}$ erg/sec $\sim 10^{-60}\,\mathcal{L}^{\rm EM}_{\rm sun}$!

- If we introduce $R_S = 2GM/c^2$ and $\omega = (v/c)\,(c/R)$

$$\mathcal{L}_{\rm GW} = \frac{c^5}{G}\,\epsilon^2\,\left(\frac{v}{c}\right)^6\,\left(\frac{R_S}{R}\right) \quad \underbrace{\Rightarrow}_{v\sim c,\ R\sim R_S} \quad \mathcal{L}_{\rm GW} \sim \epsilon^2\,\frac{c^5}{G} \sim 10^{26}\,\mathcal{L}^{\rm EM}_{\rm sun}!$$

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If $E_{\text{kin}}/c^2 \sim 1M_\odot$**, depending on** r $\Rightarrow$ $h \sim 10^{-23}$–10^{-17}

GWs on the Earth: comparison with other kind of radiation

Supernova at 20 kpc:

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- **From neutrino:** $\sim 10^5 \frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{sec}}$ **during 10 secs**
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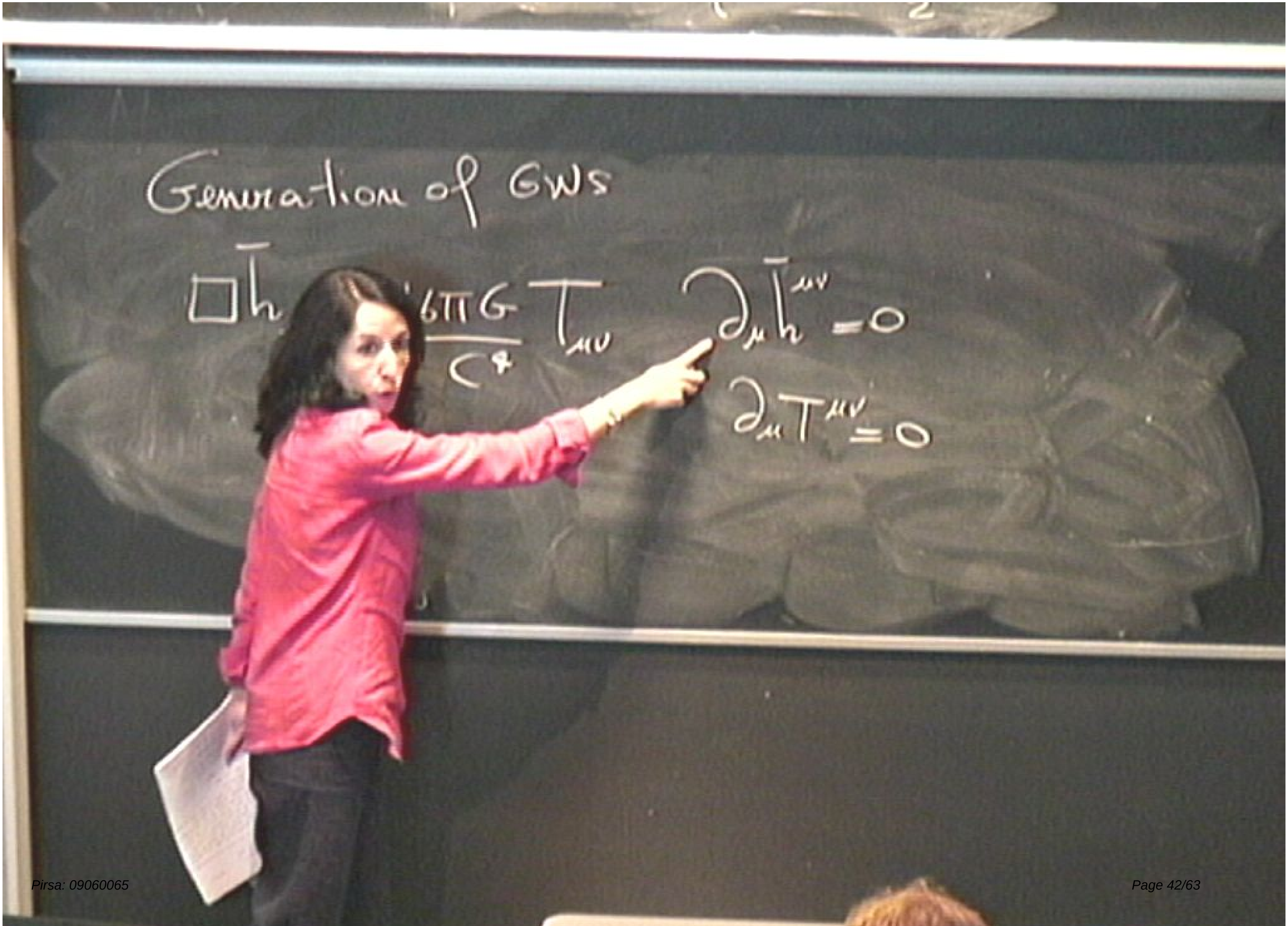
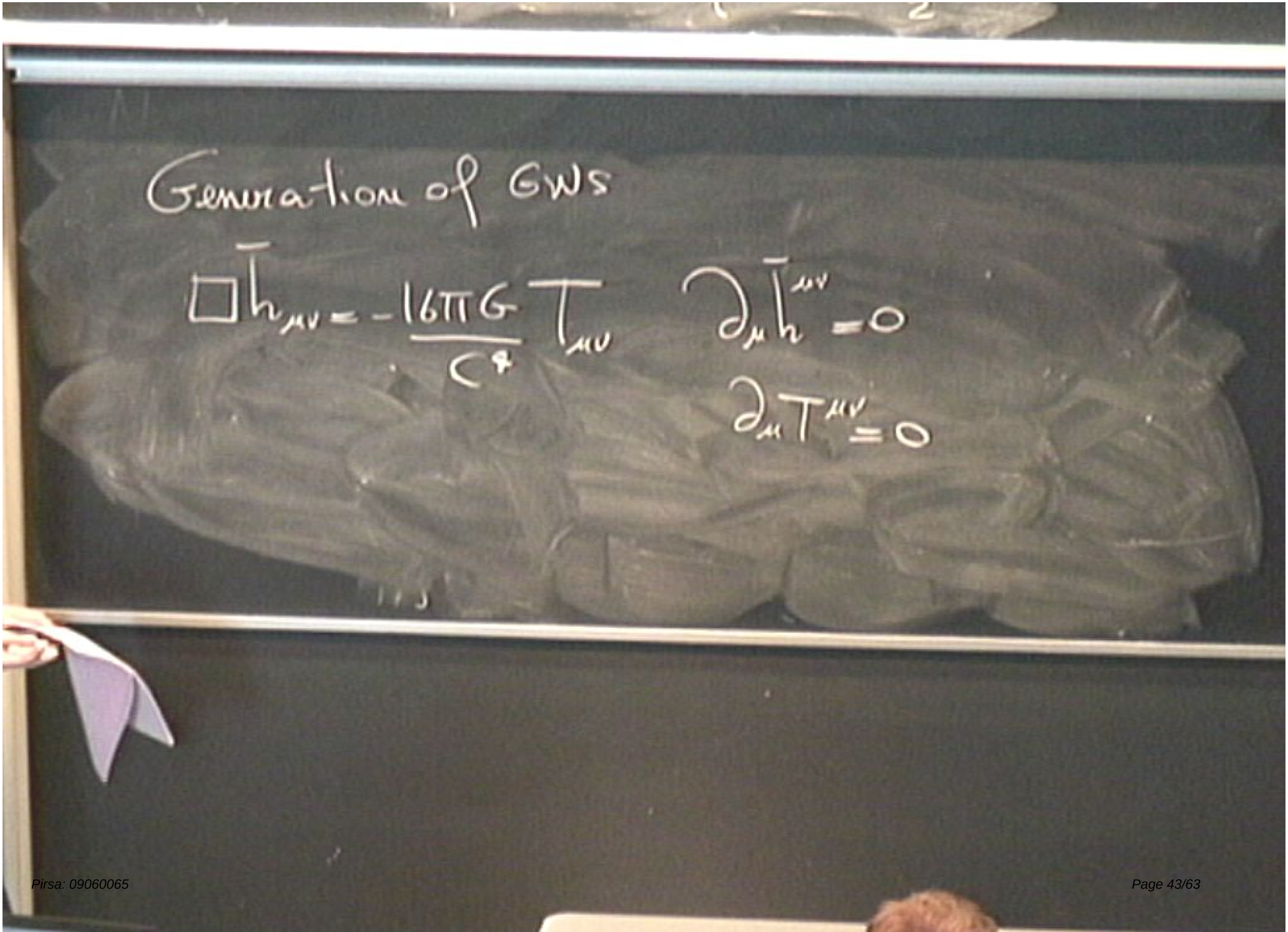
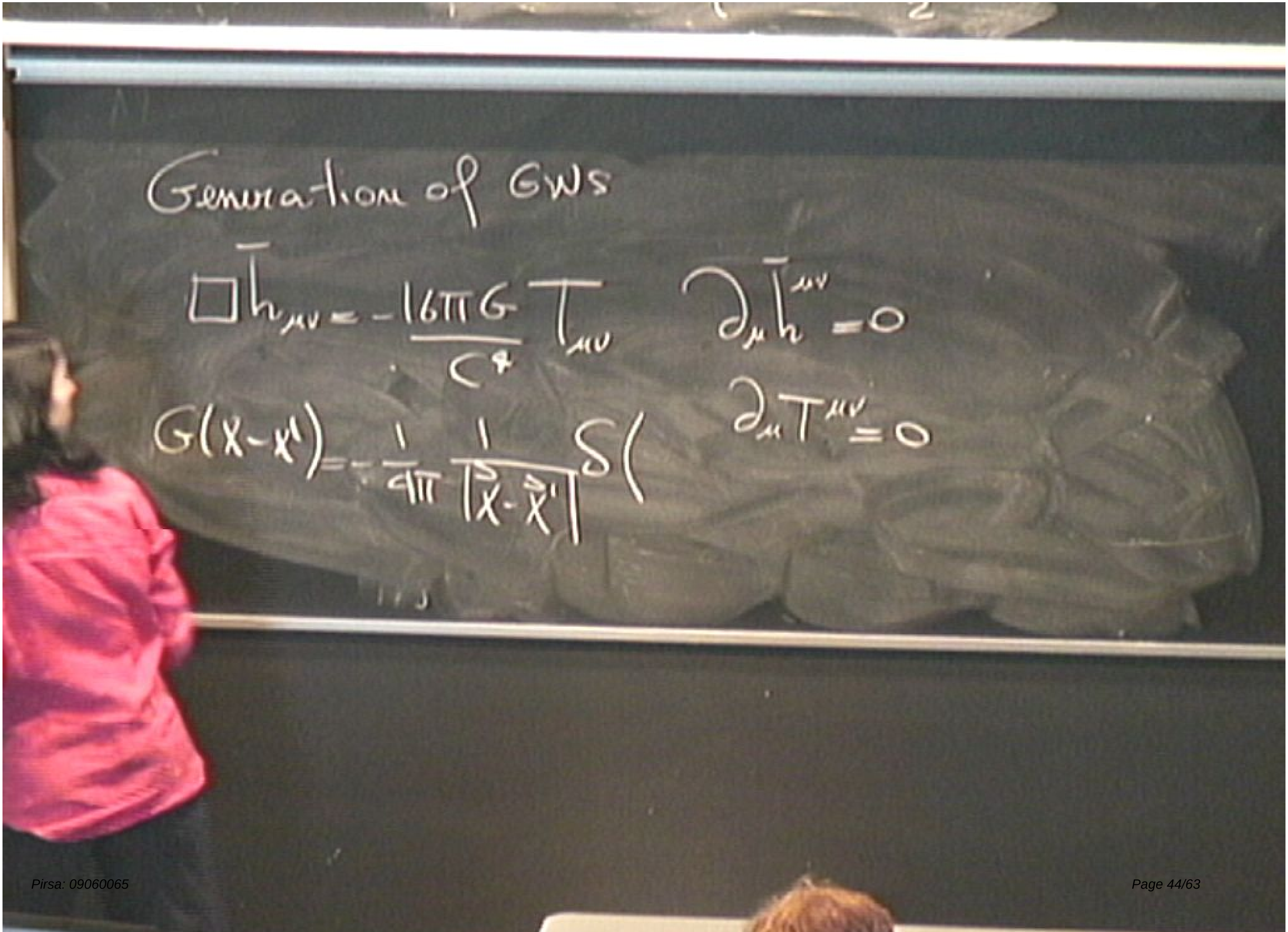
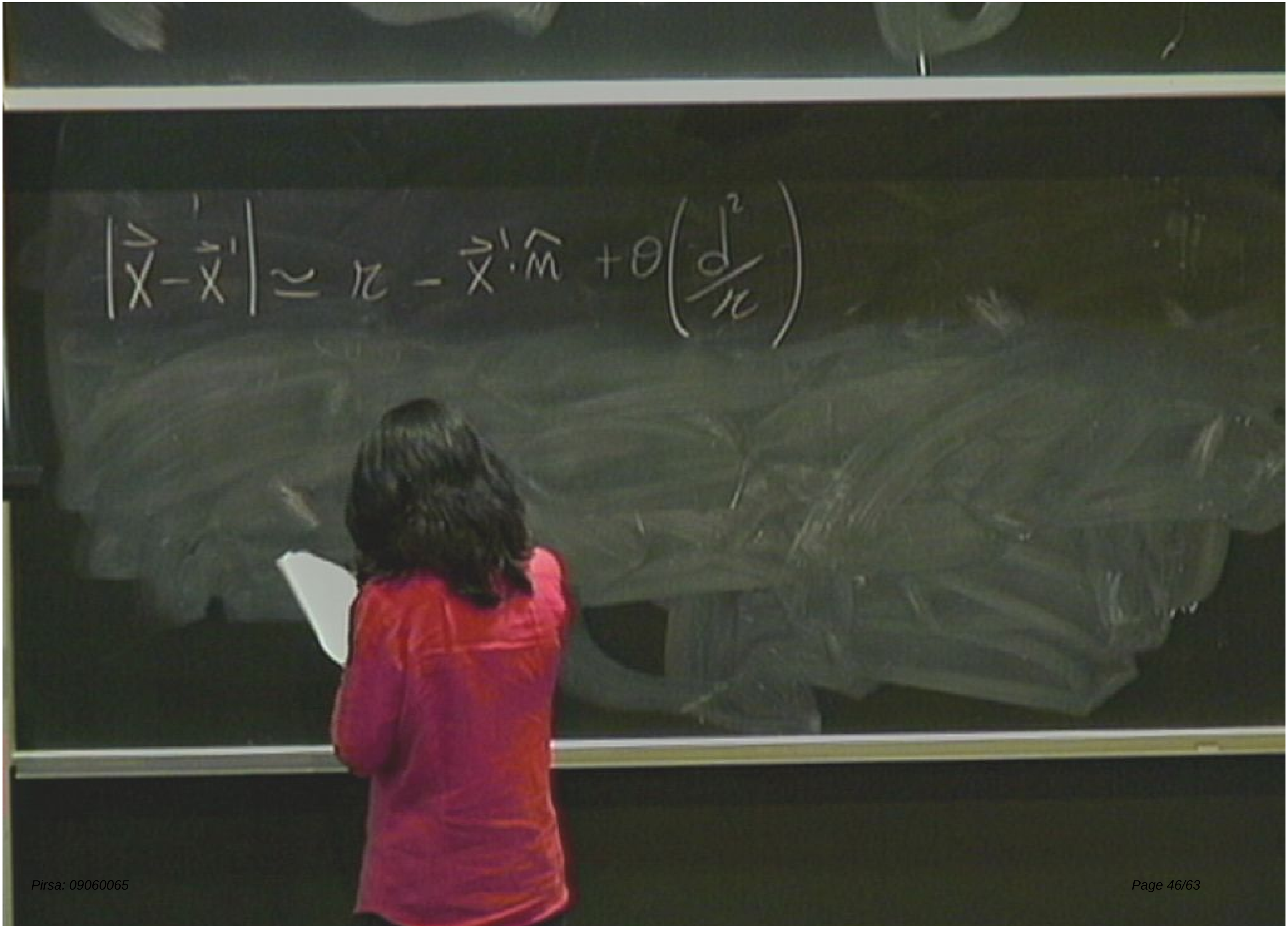

Generation of GWs
$\partial_\mu \bar{h}^{\mu\nu} = 0$
$\partial_\mu T^{\mu\nu} = 0$


Generation of GWs
$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$
$\partial_\mu \bar{h}^{\mu\nu} = 0$
$\partial_\mu T^{\mu\nu} = 0$


Generation of GWs
$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$
$\partial_\mu \bar{h}^{\mu\nu} = 0$
$G(x-x') = -\frac{1}{4\pi} \frac{1}{|\vec{x}-\vec{x}'|} \delta($
$\partial_\mu T^{\mu\nu} = 0$

d → typical dimension of the source

$$\bar{h}_{\mu\nu} = \frac{4G}{c^4}\int d^3x' \frac{1}{|\vec{x}-\vec{x}'|}\, T_{\mu\nu}\left(t-\frac{|\vec{x}-\vec{x}'|}{c}, \vec{x}'\right)$$


$|\vec{x}-\vec{x}'| \simeq r - \vec{x}'\cdot\hat{n} + O\left(d^2/r\right)$

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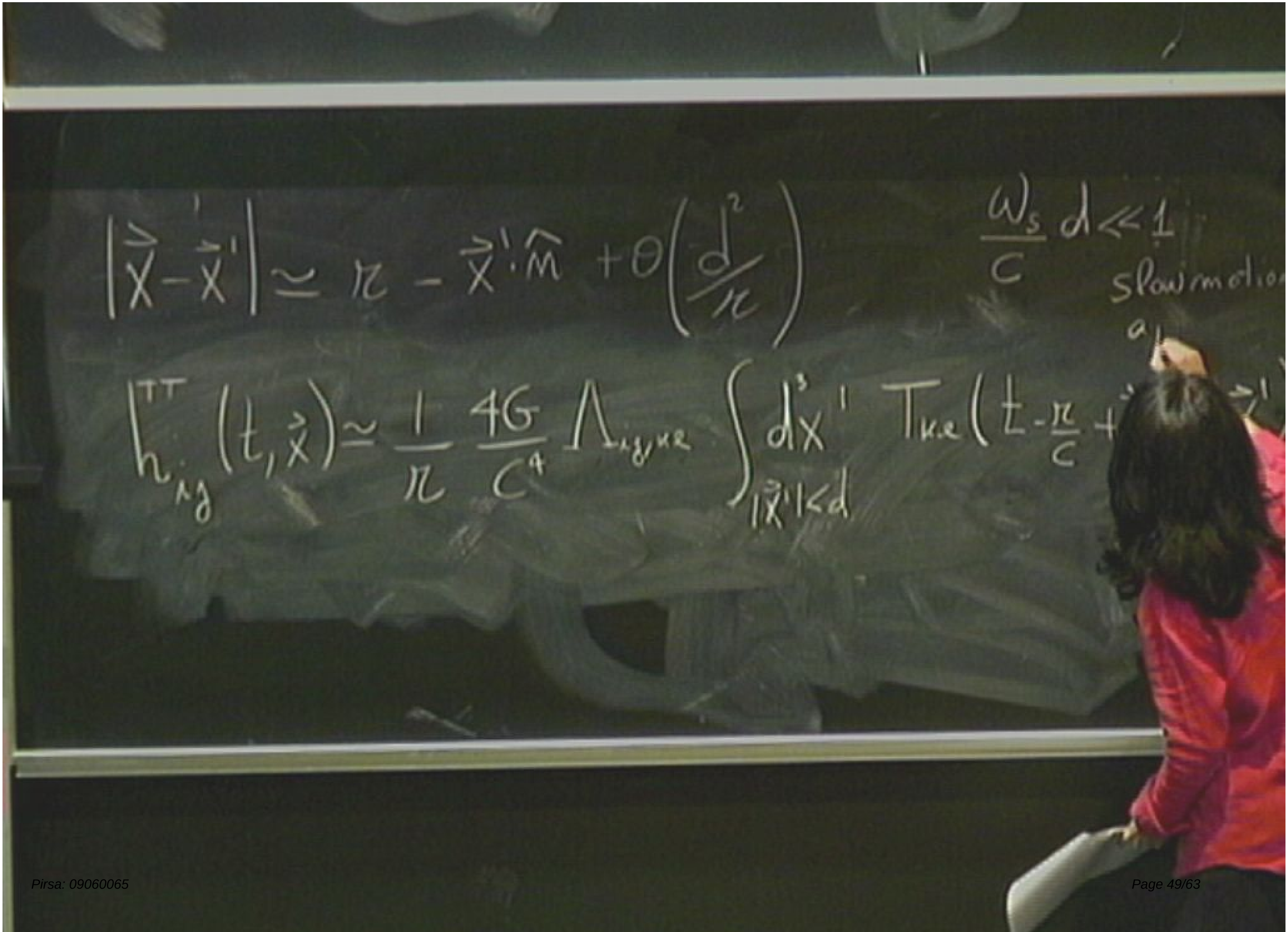
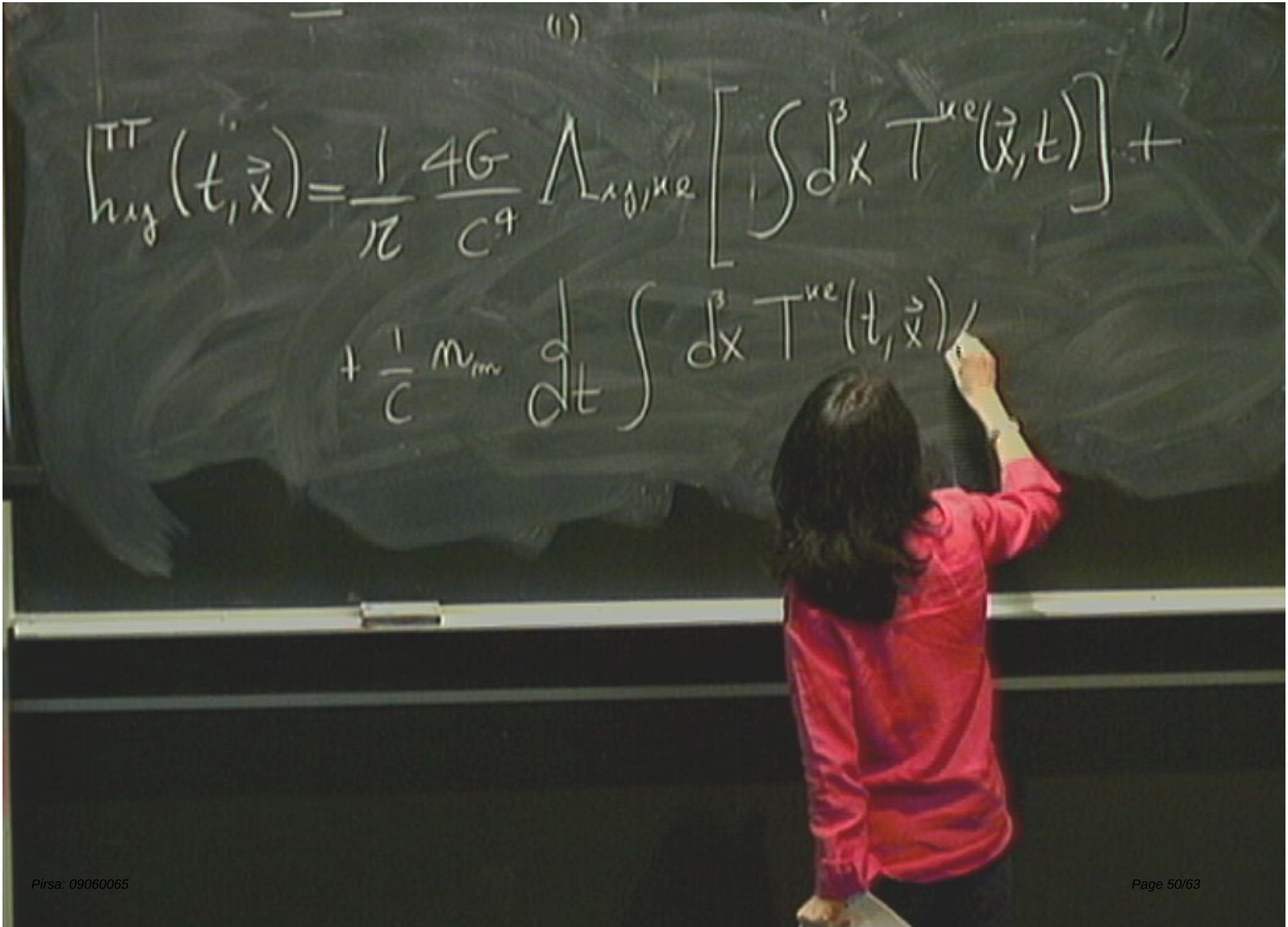
h^{TT}

$$|\vec{x}-\vec{x}'| \simeq r - \vec{x}'\cdot\hat{n} + O\left(\frac{d^2}{r}\right)$$

$$h^{TT}_{ij}(t,\vec{x}) \simeq \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}\int_{|\vec{x}'|<d} d^3x'\; T_{kl}\left(t-\frac{r}{c}+\frac{\vec{x}'\cdot\hat{n}}{c}, \vec{x}'\right)$$


$|\vec{x} - \vec{x}'| \simeq r - \vec{x}' \cdot \hat{n} + O\left(\frac{d^2}{r}\right)$
$\frac{\omega_s}{c} d \ll 1$
slow motion
$h^{TT}_{ij}(t, \vec{x}) \simeq \frac{1}{r} \frac{4G}{c^4} \Lambda_{ij,kl} \int_{|\vec{x}'|<d} d^3x' \, T_{kl}\left(t - \frac{r}{c} + \right.$


$h_{ij}^{TT}(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}\left[\int d^3x\, T^{kl}(\vec{x},t)\right] +$
$+ \frac{1}{c} n_m \frac{d}{dt}\int d^3x\, T^{kl}(t,\vec{x})$

$$h^{TT}_{ij}(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}\left[\int d^3x\, T^{kl}(\vec{x},t) + \right.$$

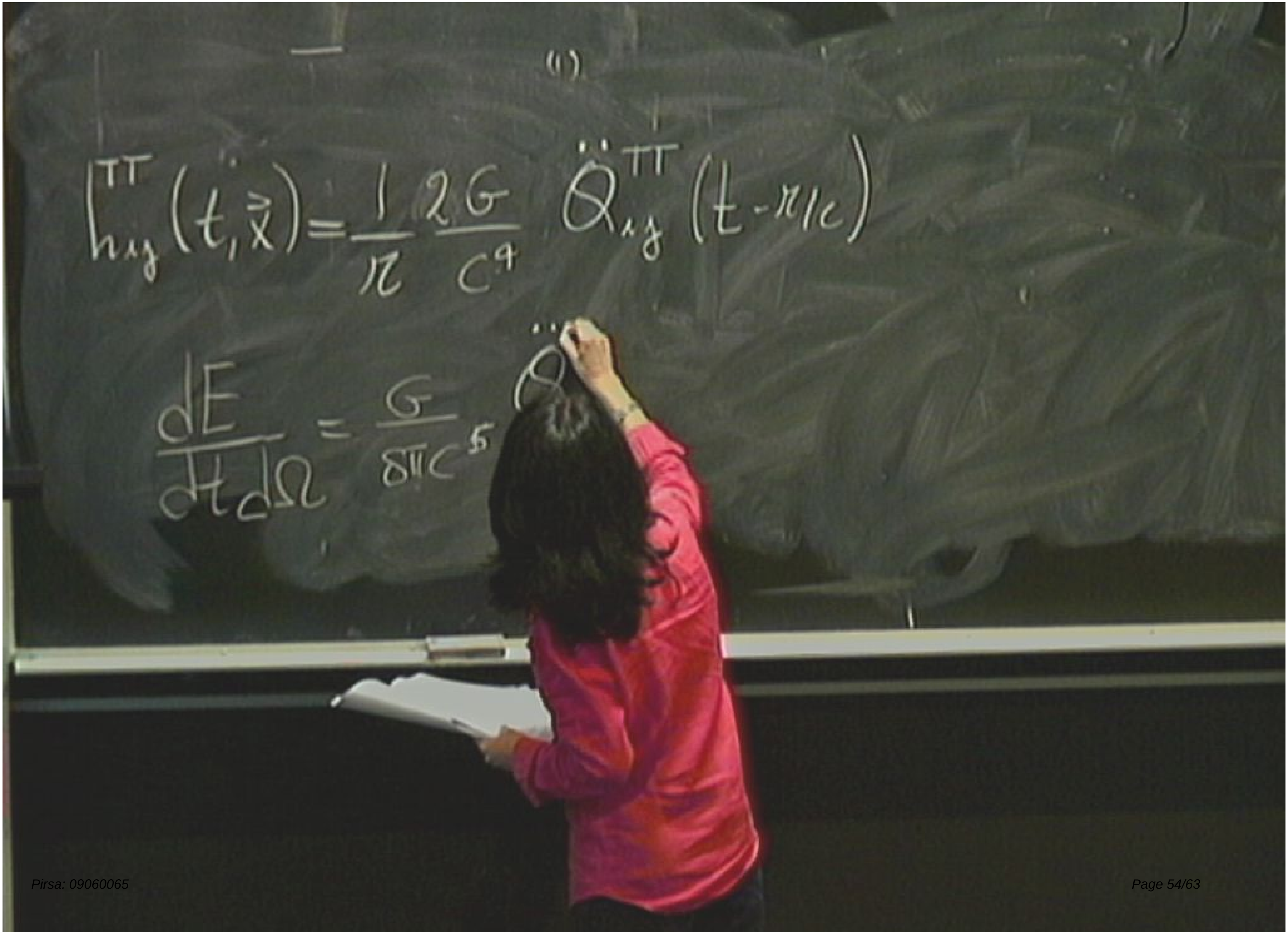
$$\left. + \frac{1}{c} n_m \frac{d}{dt}\int d^3x\, T^{kl}(t,\vec{x})x^m + \theta\left(\frac{1}{c^2}\right)\right]$$

$$h^{TT}_{ij}(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}\left[\int d^3x\, T^{kl}(\vec{x},t) + \frac{1}{c} n_m \frac{d}{dt}\int d^3x\, T^{kl}(t,\vec{x})\, x^m + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

$$\frac{v}{c} \ll 1 \qquad \lambda_{GW} \gg d$$

$$h^{TT}_{xy}(t,\vec{x}) = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}^{TT}_{xy}(t - r/c)$$

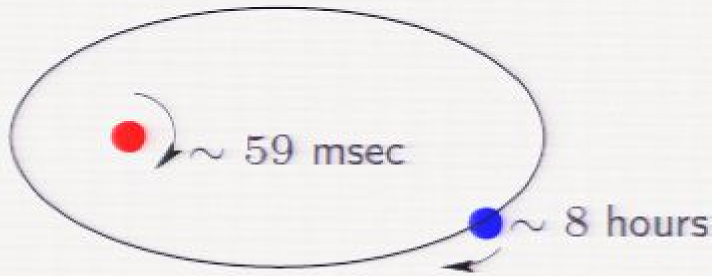
$$Q_{ij} = \int d^3x\, \rho \left(x_i x_j - \frac{1}{3}\delta_{ij} x^2 \right)$$


$h^{TT}_{ij}(t,\vec{x}) = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}^{TT}_{ij}(t - r/c)$
$\frac{dE}{dt\,d\Omega} = \frac{G}{8\pi c^5}$

Indirect observation of gravitational wa

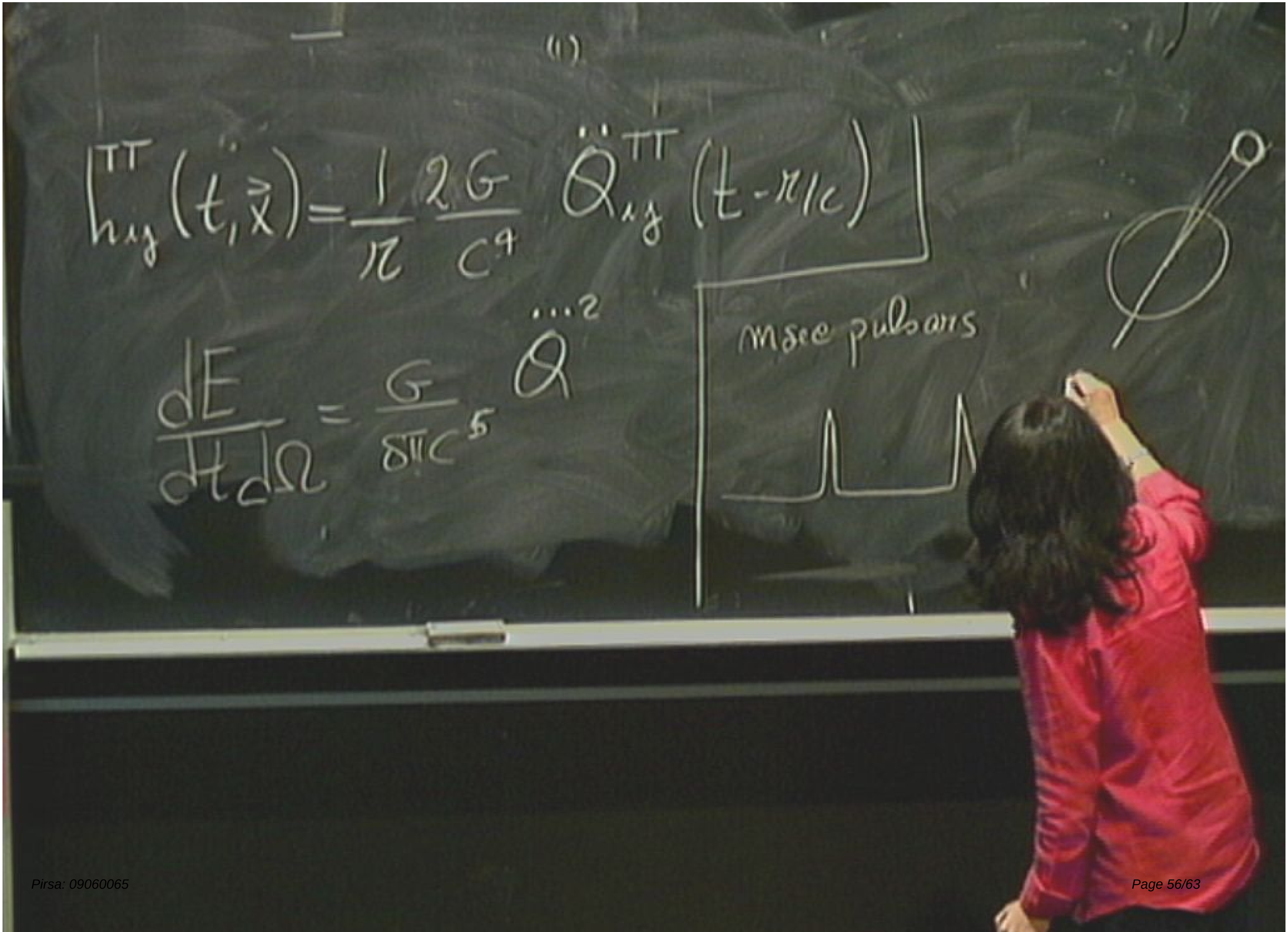
Neutron Binary System: PSR 1913 +16 - Timing Pulsars

Hulse & Taylor discovery (1974)



Separated by $\sim 10^6$ Km, $m_1 = 1.4 M_\odot$, $m_2 = 1.36 M_\odot$, eccentricity $= 0.617$

- Prediction from GR: rate of change of orbital period
- Emission of gravitational waves:
  - due to loss of orbital energy
  - orbital decay in agreement with GR at the level of 0.5%


$h^{TT}_{xy}(t,\vec{x}) = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}^{TT}_{xy}(t-r/c)$
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msec pulsars

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Hulse-Taylor binary: cumulative shift of periastron time

To show agreement with GR, they compared the *observed* orbital phase with a theoretical template phase

If f_b varies slowly with time, then to first order in a Taylor expansion

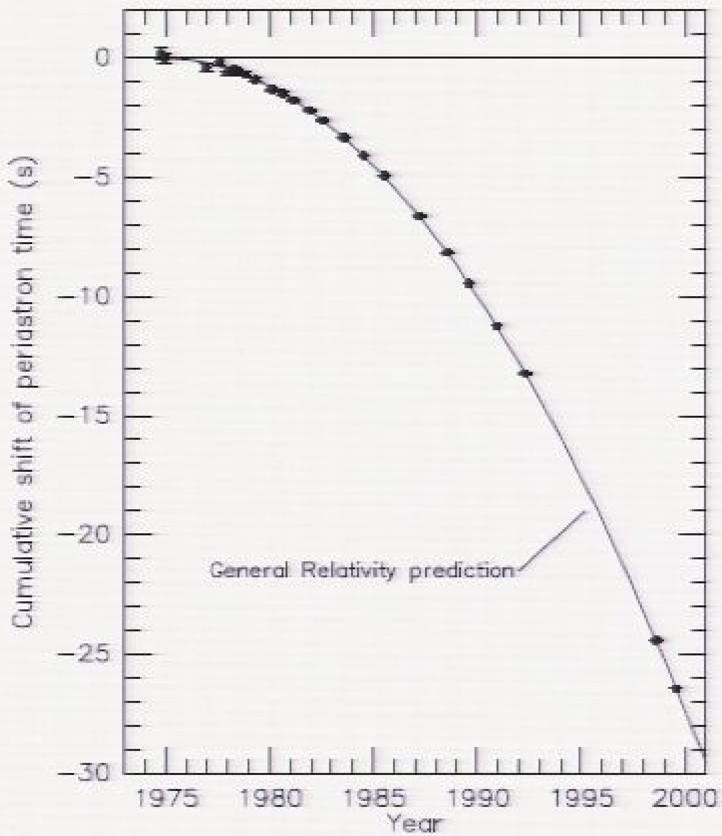
$$\Phi_b(t) = 2\pi\, f_b\, t + \pi \dot{f}_b\, t^2$$

Assuming that t_P is the periastron passage time defined as

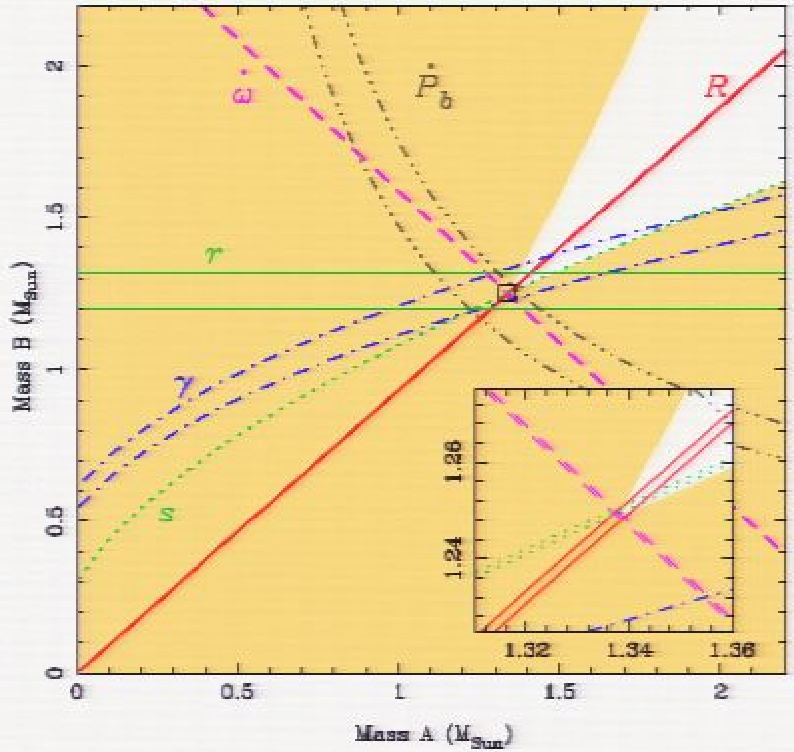
$\Phi(t_p) = 2\pi N$ N being an integer

$$2\pi N = 2\pi f_b\, t_p + \pi \dot{f}_b\, t_p^2 \quad \Rightarrow \quad t_p - N/f_b = -\frac{1}{2}\dot{f}_b/f_b\, t^2$$

Hulse-Taylor binary: cumulative shift of periastron time

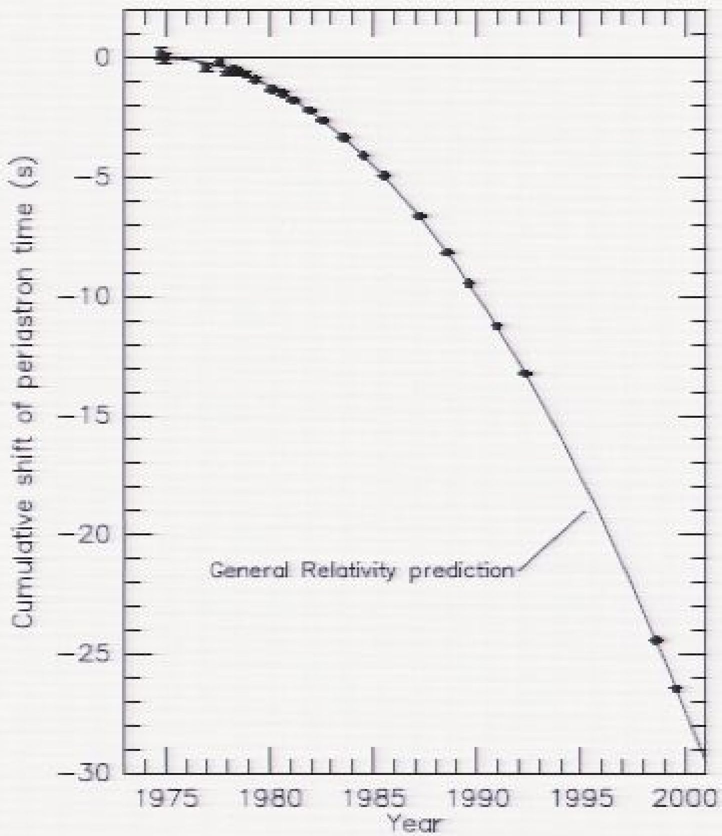


[from Taylor & Weisberg 2000]



[from Kramer et al. 2005]

Hulse-Taylor binary: cumulative shift of periastron time



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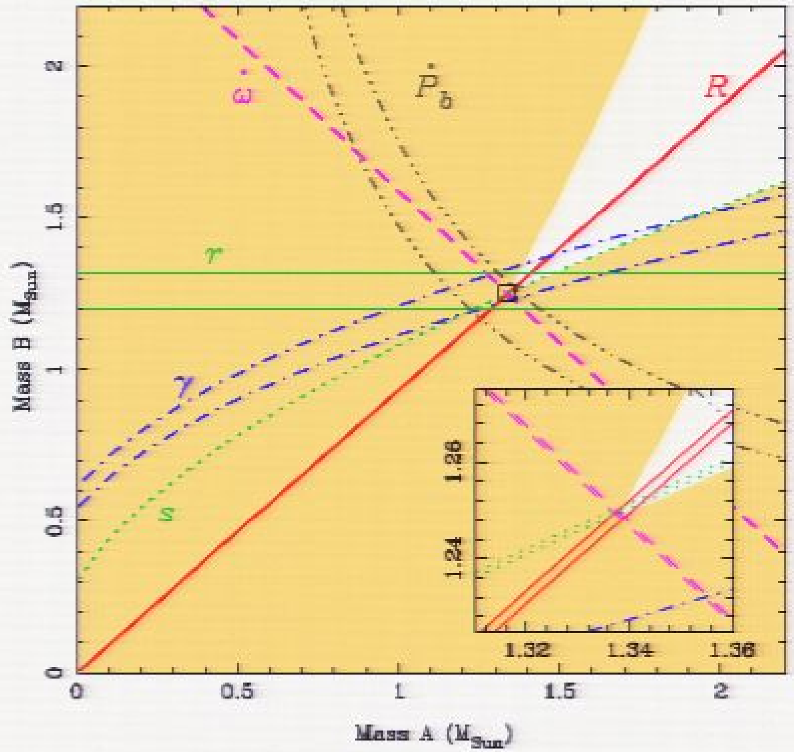

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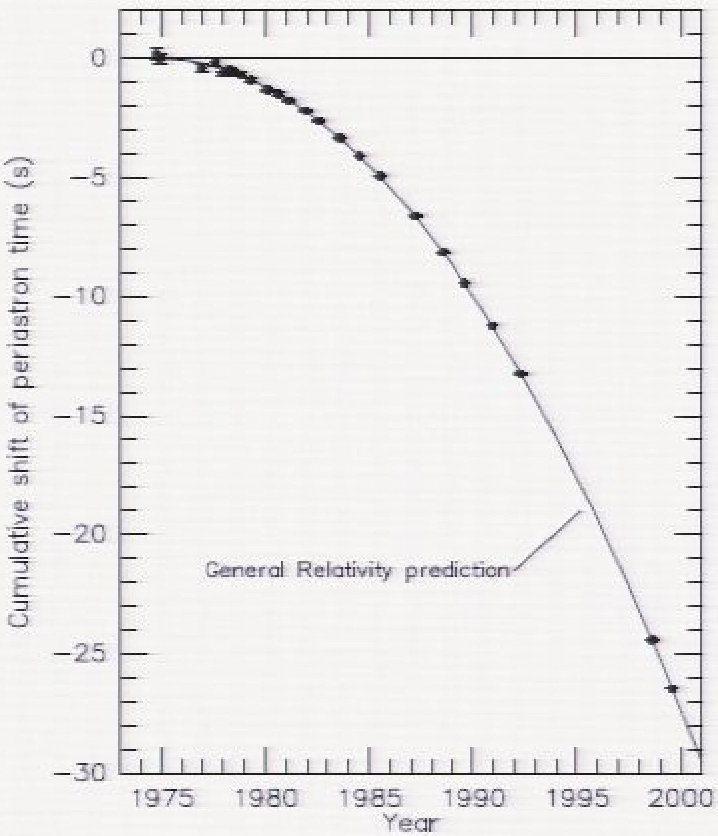
Hulse-Taylor binary: cumulative shift of periastron time



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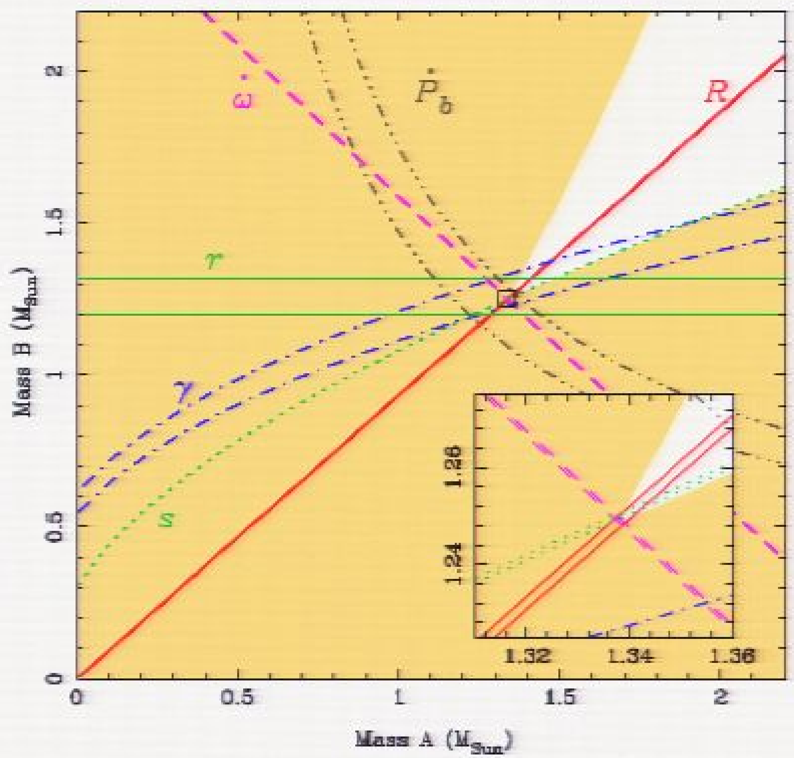

[from Kramer et al. 2005]

Known double pulsar binaries

PSR J0737-3039 [Burgay et al. 03; Lyne et al. 2004]

rot. period A 22.7 ms
rot. period B 2773.6 ms

orb. period 2 h and 45 min (will merge in 85 Myr!)
$e = 0.088$
distance $\sim 0.6 kpc$ (close!)

$\Delta\phi = 16.900(2) \mathrm{deg/yr}$ (large!) $\dot{P} = -1.20(8) 10^{-12}$