Title: Gravitational Wave Astronomy - Lecture 1

Date: Jun 24, 2009 09:00 AM

URL: http://pirsa.org/09060063

Abstract:

Gravitational Waves

Alessandra Buonanno
Department of Physics, University of Maryland

Summer School: Exploring the Cosmological Frontiers, Perimeter Institute, Waterloo

Lectures' content

- Lecture 1:
  - Linearization of Einstein equations
  - GW propagation: plane wave solution in transverse-traceless gauge
  - Key ideas underlying gravitational-wave detectors
- Lecture 2:
  - GWs carry off energy, angular momentum and linear momentum
  - Wave generation in Einstein theory (quadrupole formula)

Lectures' content

- Hulse-Taylor pulsar binary; double-pulsar binary

- Binary black hole coalescence: dynamics and GW emission

- Lecture 3:

  - Binary black hole coalescence: analytical and numerical modeling, tests of strong gravity

  - Brief review of other sources:

    Pulsars

    Supernovae

    GWs from the early Universe

References

Landau & Lifshitz: *Field Theory*, **Chap. 11, 13**

B. Schutz: *A first course in general relativity*, **Chap. 8, 9**

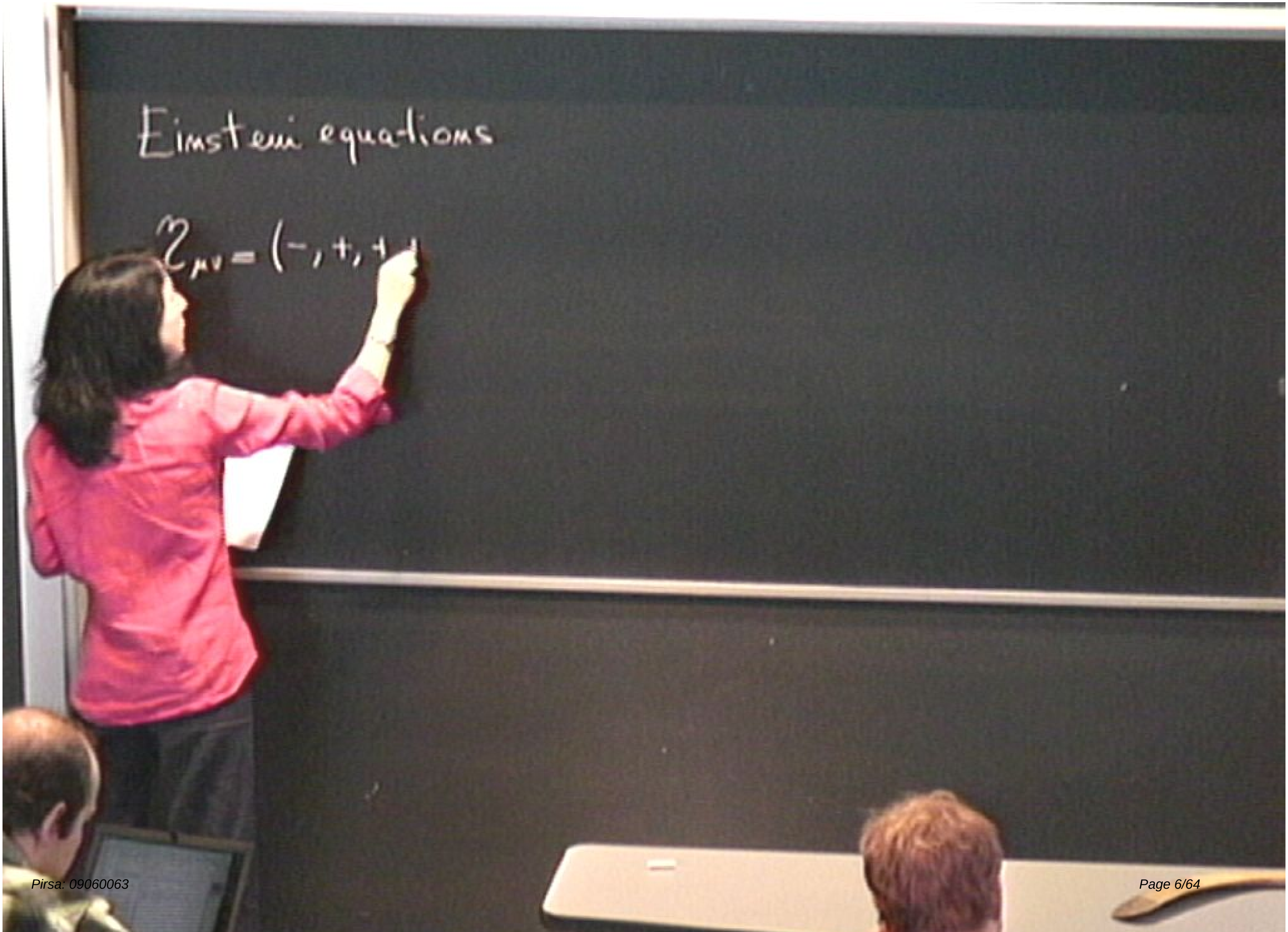
S. Weinberg: *Gravitation and Cosmology*, **Chap. 7, 10**

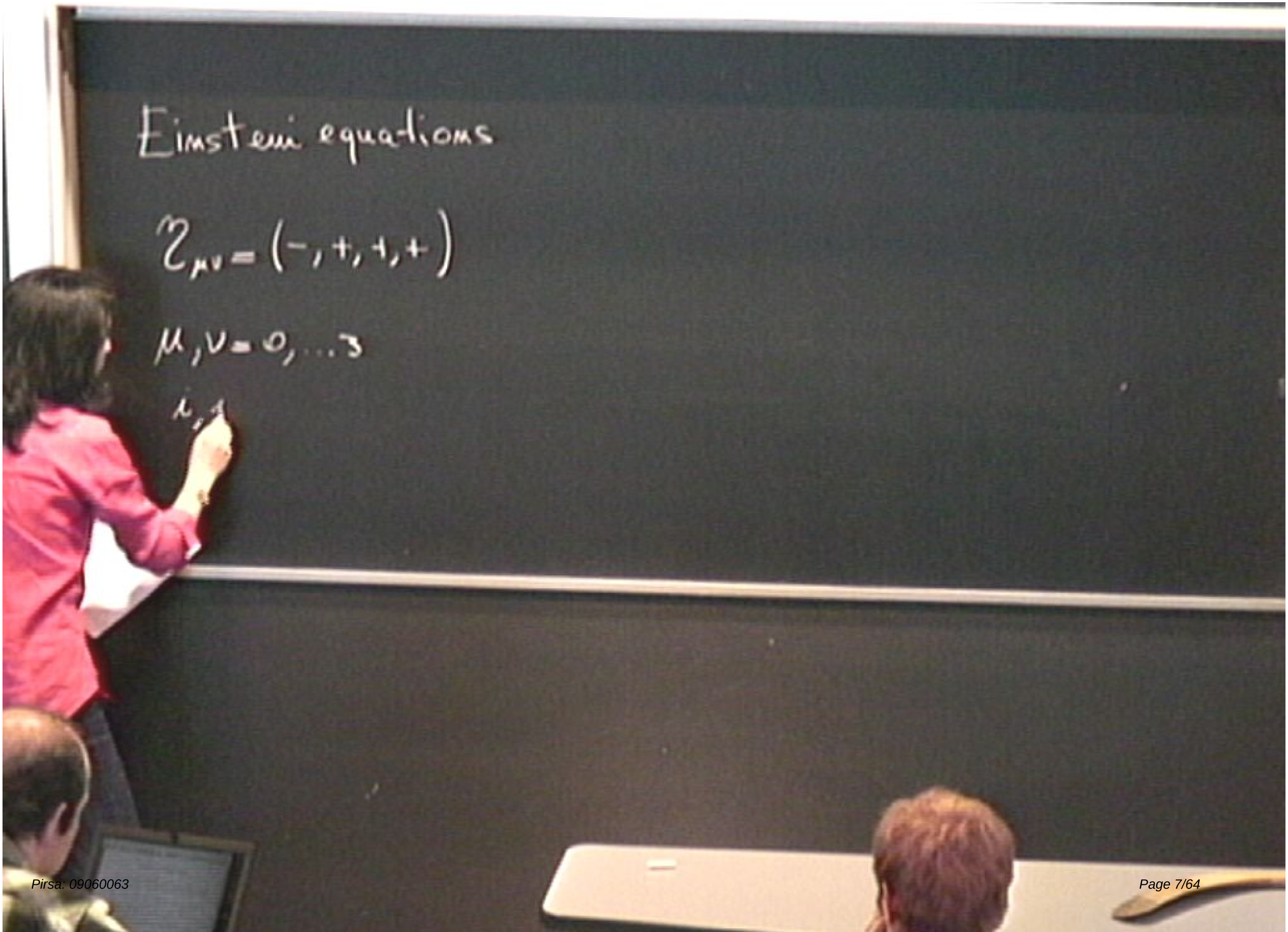
C. Misner, K.S. Thorne & A. Wheeler: *Gravitation*, **Chap. 8**

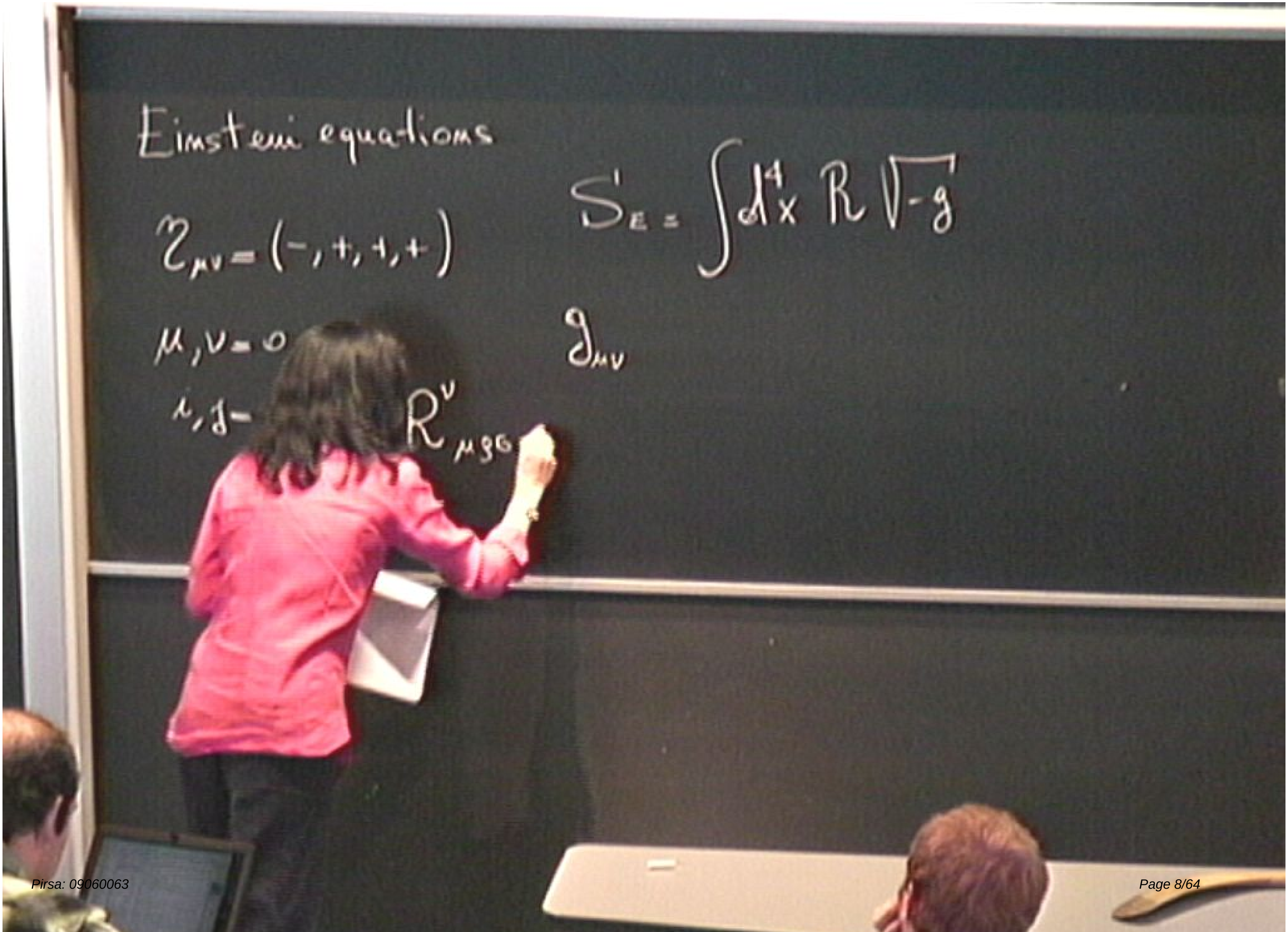
S. Carroll, *Spacetime and Geometry: An Introduction to GR*, **Chap. 7**

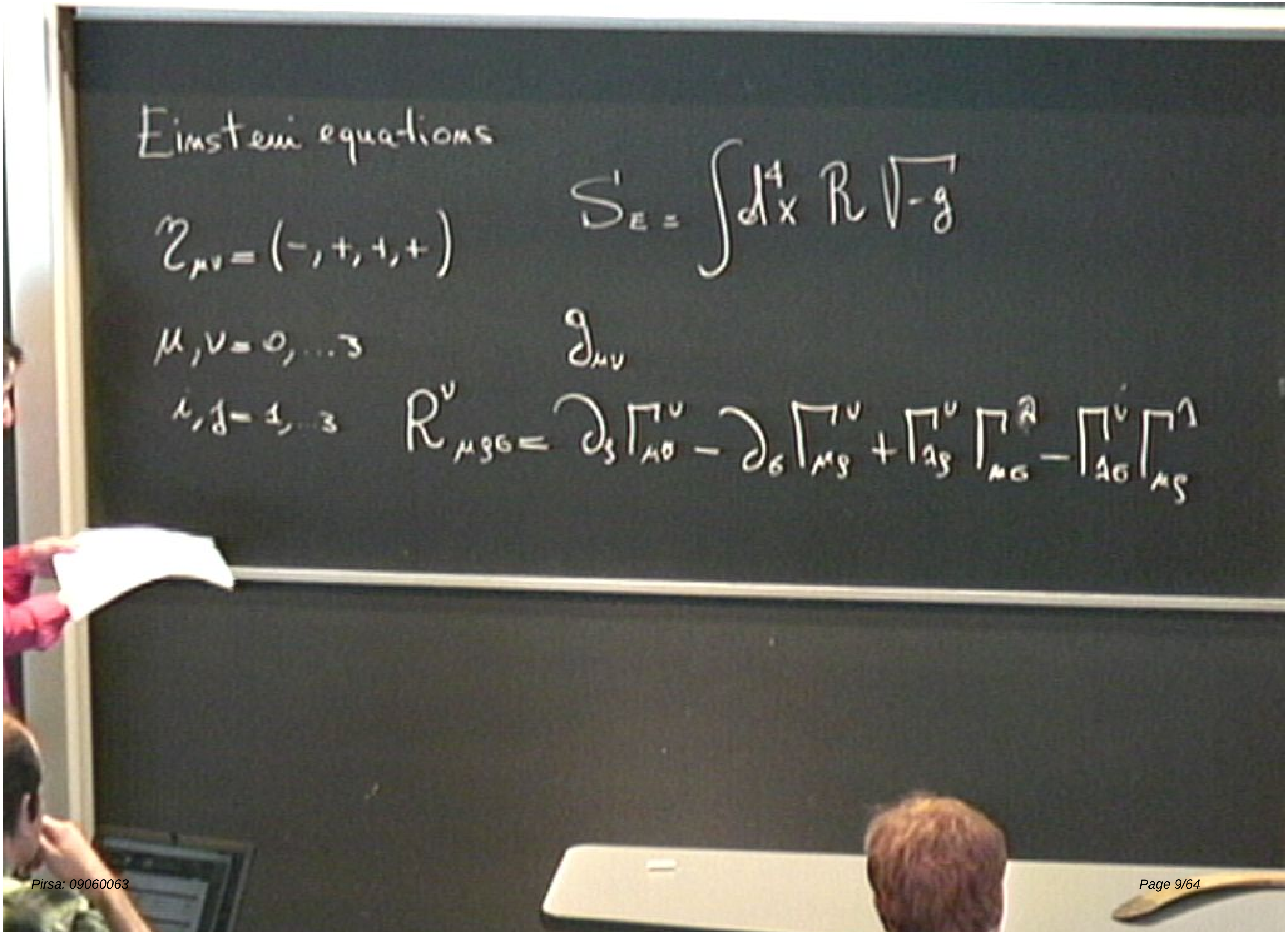
Course by K.S. Thorne available on the web: Lectures 4, 5 & 6

M. Maggiore: *Gravitational waves: Theory and Experiments* **(2007)**


Einstein equations
$\eta_{\mu\nu} = (-, +, +$


Einstein equations
$\eta_{\mu\nu} = (-,+,+,+)$
$\mu,\nu = 0,\dots 3$


Einstein equations
$\eta_{\mu\nu} = (-,+,+,+)$
$\mu,\nu = 0$
$i,j =$
$R^{\nu}{}_{\mu\rho\sigma}$
$S_E = \int d^4x \, R \sqrt{-g}$
$g_{\mu\nu}$


Einstein equations
$\eta_{\mu\nu} = (-,+,+,+)$
$\mu,\nu = 0,\dots 3$
$i,j = 1,\dots 3$
$S_E = \int d^4x\, R \sqrt{-g}$
$g_{\mu\nu}$
$R^{\nu}{}_{\mu\rho\sigma} = \partial_\rho \Gamma^{\nu}_{\mu\sigma} - \partial_\sigma \Gamma^{\nu}_{\mu\rho} + \Gamma^{\nu}_{\lambda\rho} \Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\nu}_{\lambda\sigma} \Gamma^{\lambda}_{\mu\rho}$

Einstein equations

$$\eta_{\mu\nu} = (-,+,+,+)$$

$$S_E = \int d^4x \, R \sqrt{-g}$$

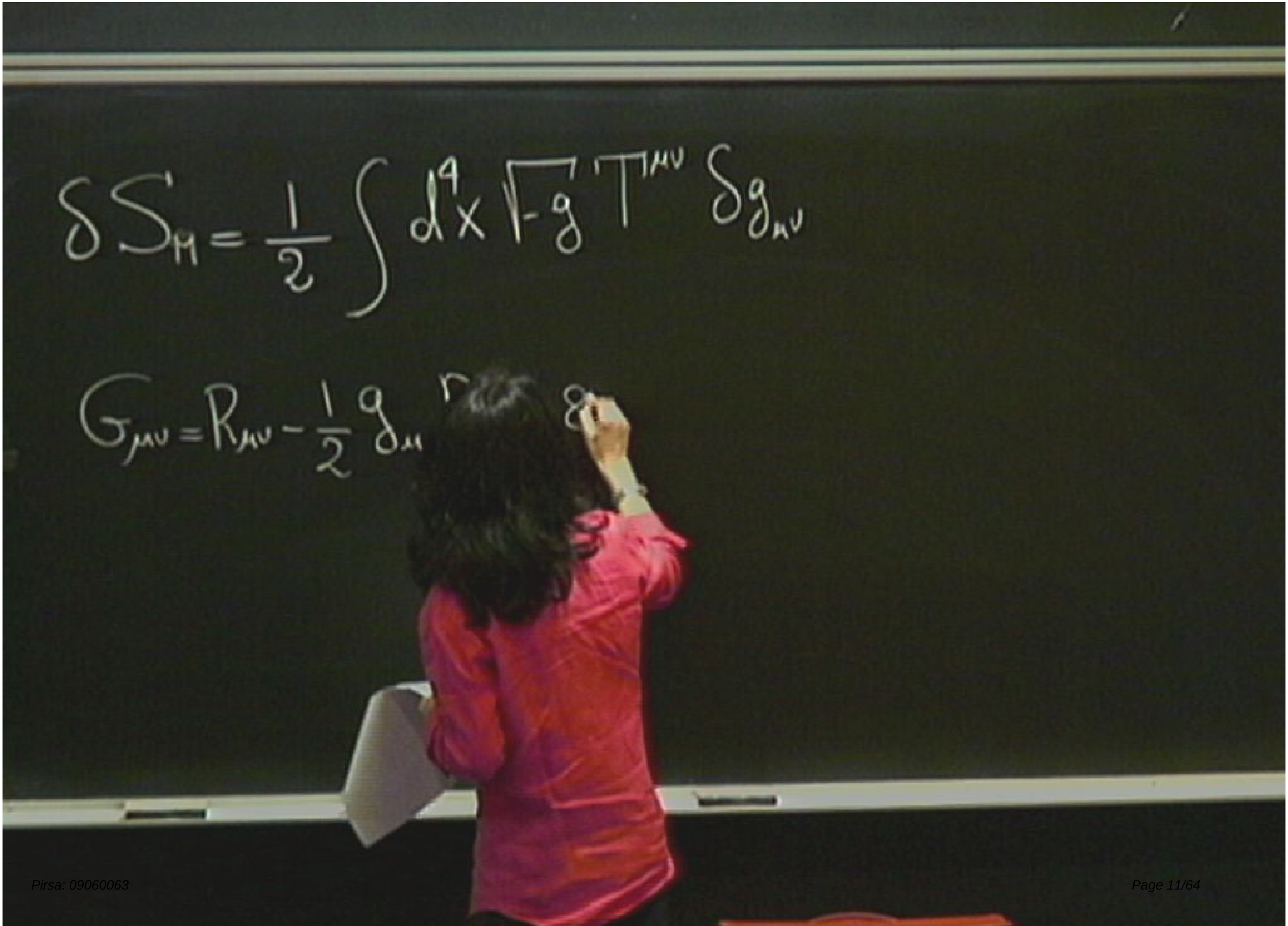
$\mu, \nu = 0, \ldots 3$

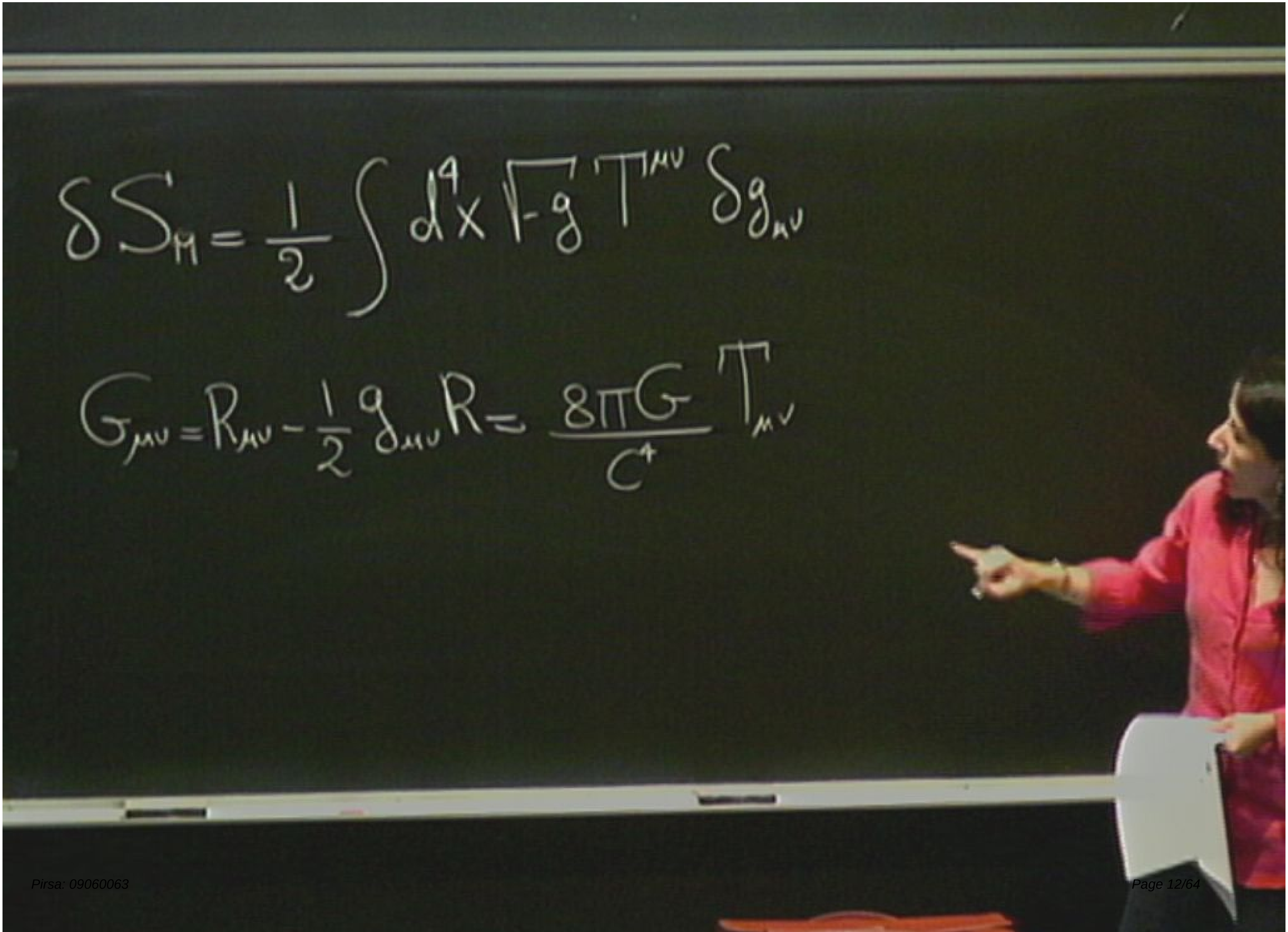
$g_{\mu\nu}$

$i, j = 1, \ldots 3$

$$R^{\nu}_{\ \mu\rho\sigma} = \partial_\rho \Gamma^{\nu}_{\mu\sigma} - \partial_\sigma \Gamma^{\nu}_{\mu\rho} + \Gamma^{\nu}_{\lambda\rho} \Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\nu}_{\lambda\sigma} \Gamma^{\lambda}_{\mu\rho}$$

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left(\partial_\rho g_{\lambda\nu} + \partial_\nu g_{\lambda\rho} - \partial_\lambda g_{\nu\rho} \right)$$


$\delta S_M = \frac{1}{2}\int d^4x \sqrt{-g}\, T^{\mu\nu}\, \delta g_{\mu\nu}$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu}$ … 8



$\delta S_M = \frac{1}{2} \int d^4x \sqrt{-g}\, T^{\mu\nu} \delta g_{\mu\nu}$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$

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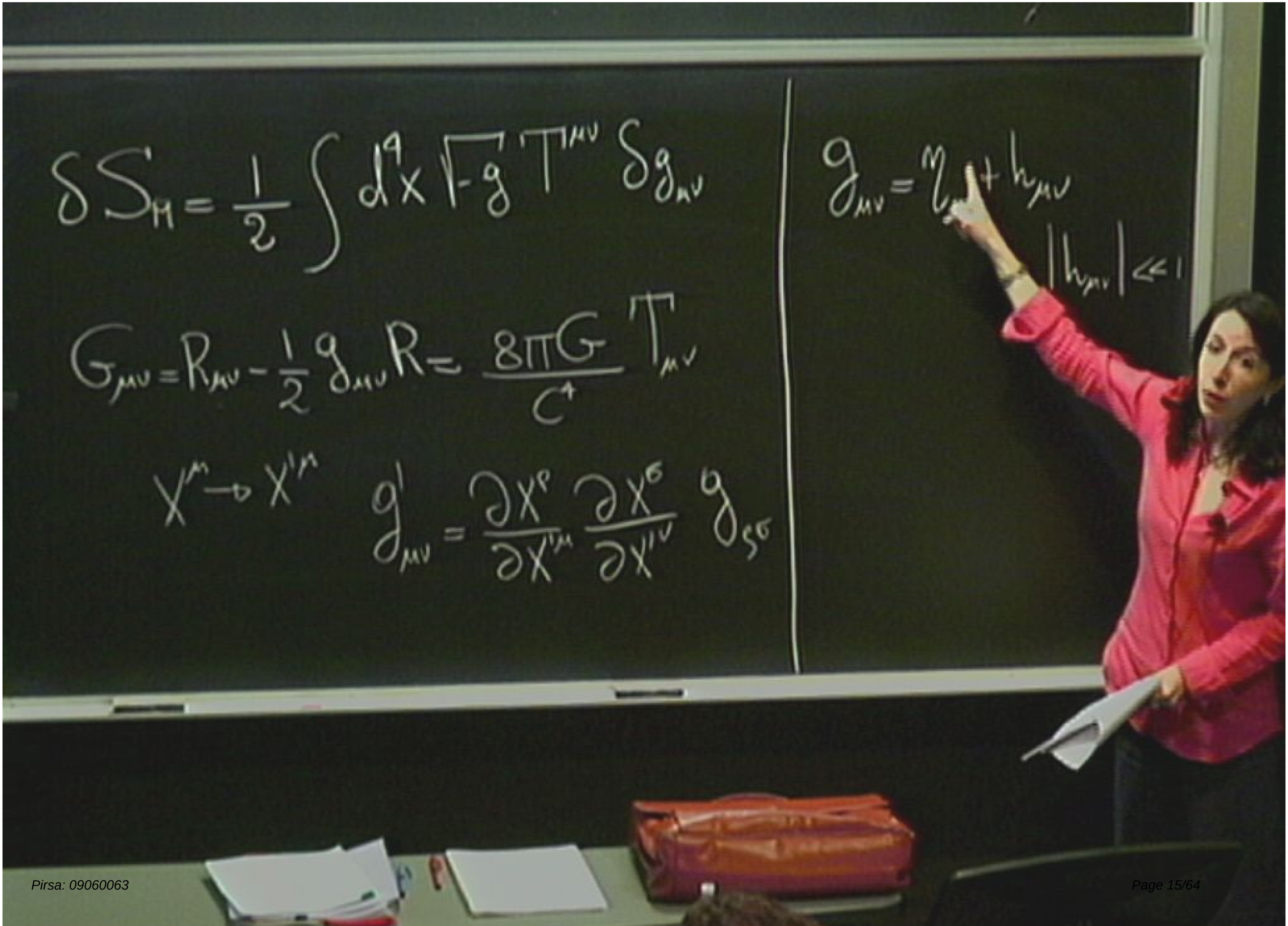
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$x^\mu \to x'^\mu \qquad g'_{\mu\nu} = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}\, g_{\rho\sigma}$$



$\delta S_M = \frac{1}{2} \int d^4x \sqrt{-g}\, T^{\mu\nu} \delta g_{\mu\nu}$
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$x^\mu \to x'^\mu$
$g'_{\mu\nu} = \frac{\partial x^\rho}{\partial x'^\mu} \frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}$
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$|h_{\mu\nu}| \ll 1$

$$\delta S_M = \frac{1}{2}\int d^4x \sqrt{-g}\, T^{\mu\nu}\, \delta g_{\mu\nu}$$

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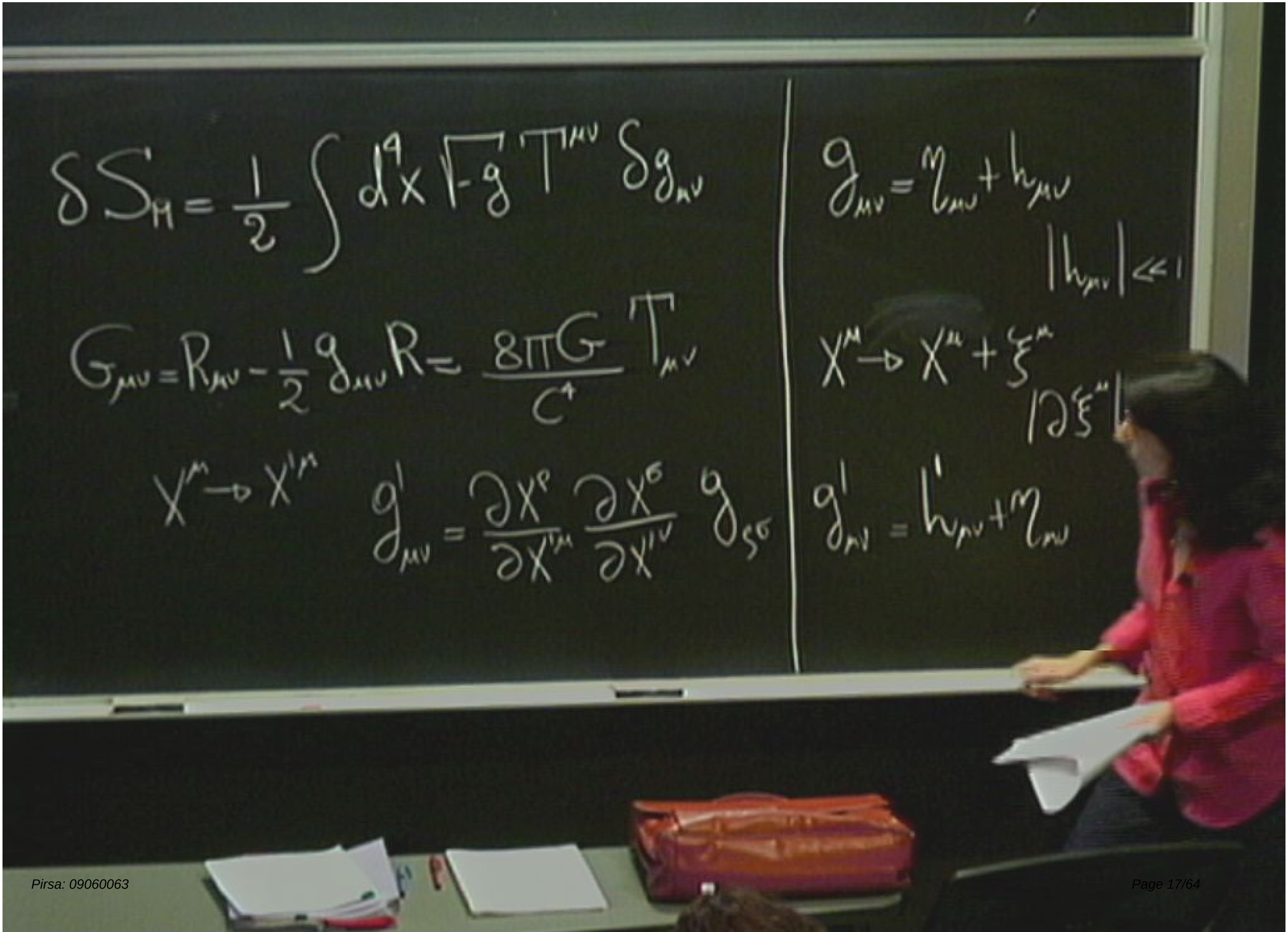
$$|h_{\mu\nu}| \ll 1$$

$$x^\mu \to x^\mu + \xi^\mu$$

$$|\partial \xi^\mu| \ll 1$$


$\delta S_M = \frac{1}{2}\int d^4x \sqrt{-g}\, T^{\mu\nu}\, \delta g_{\mu\nu}$
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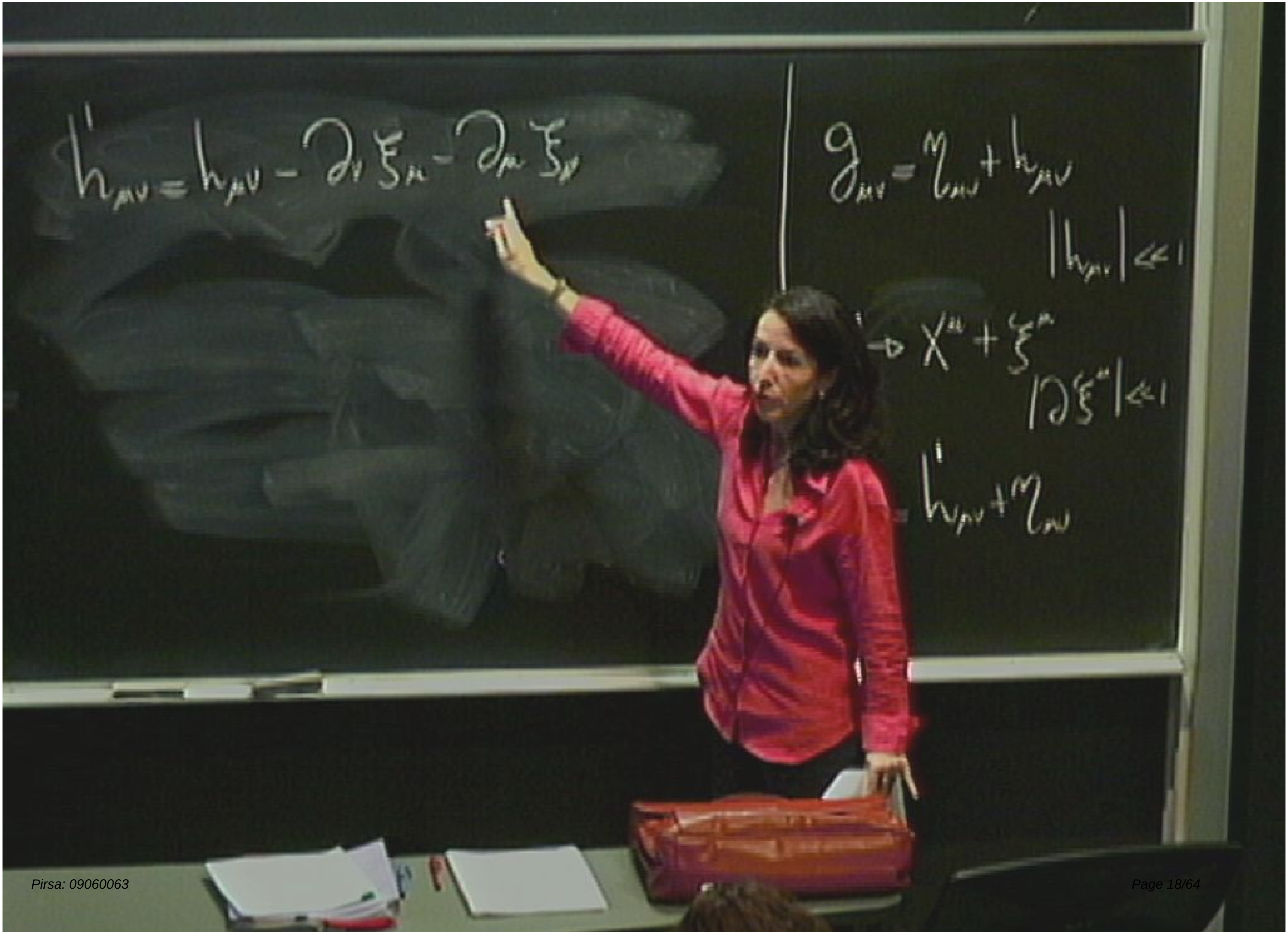
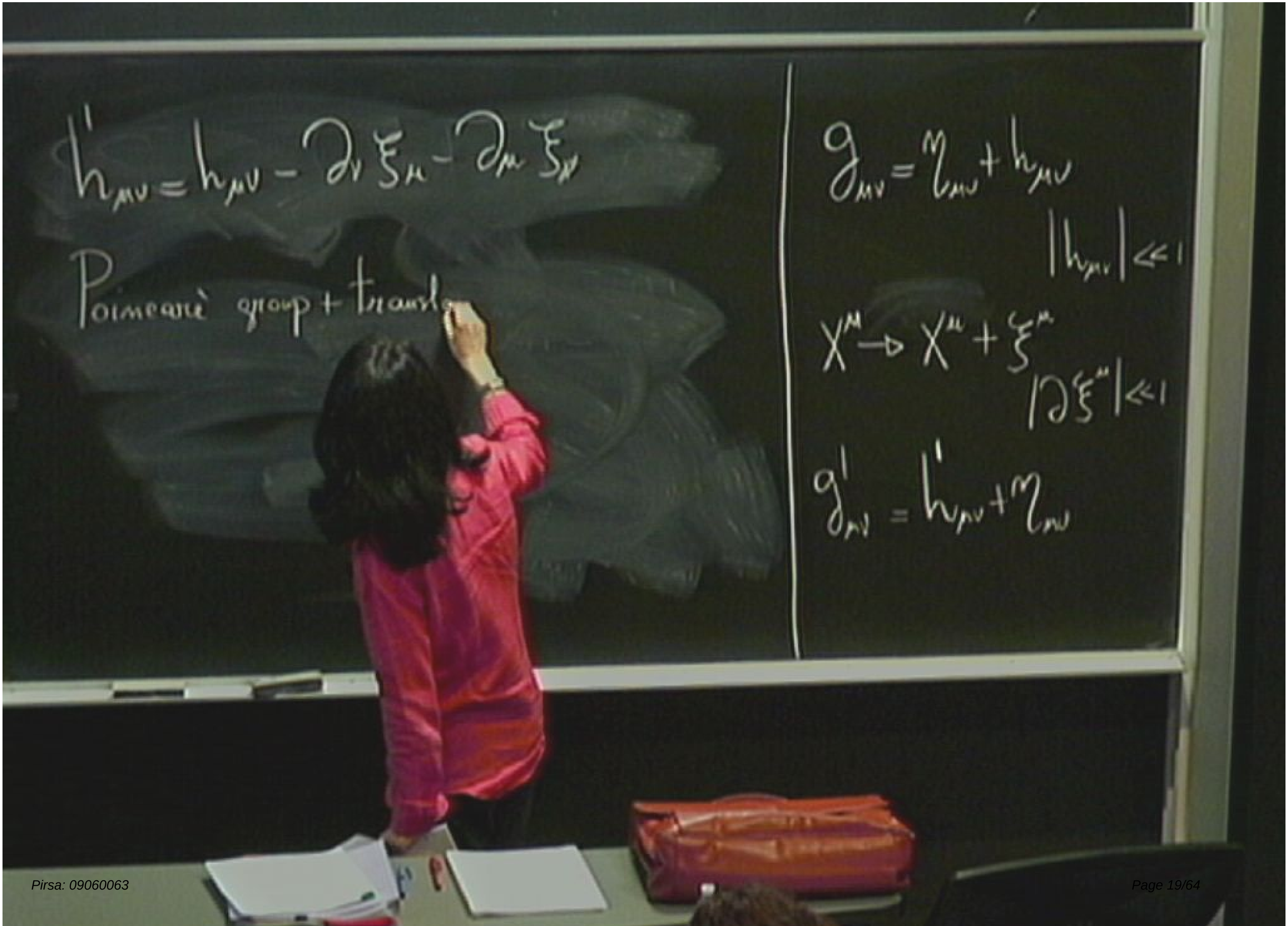
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$|h_{\mu\nu}| \ll 1$
$x^\mu \to x^\mu + \xi^\mu$
$|\partial \xi^\mu|$
$g'_{\mu\nu} = h'_{\mu\nu} + \eta_{\mu\nu}$


$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu \xi_\mu - \partial_\mu \xi_\nu$
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$|h_{\mu\nu}| \ll 1$
$\to x^\mu + \xi^\mu$
$|\partial \xi^\mu| \ll 1$
$h'_{\mu\nu} + \eta_{\mu\nu}$


$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu \xi_\mu - \partial_\mu \xi_\nu$
Poincaré group + transl
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$|h_{\mu\nu}| \ll 1$
$x^\mu \to x^\mu + \xi^\mu$
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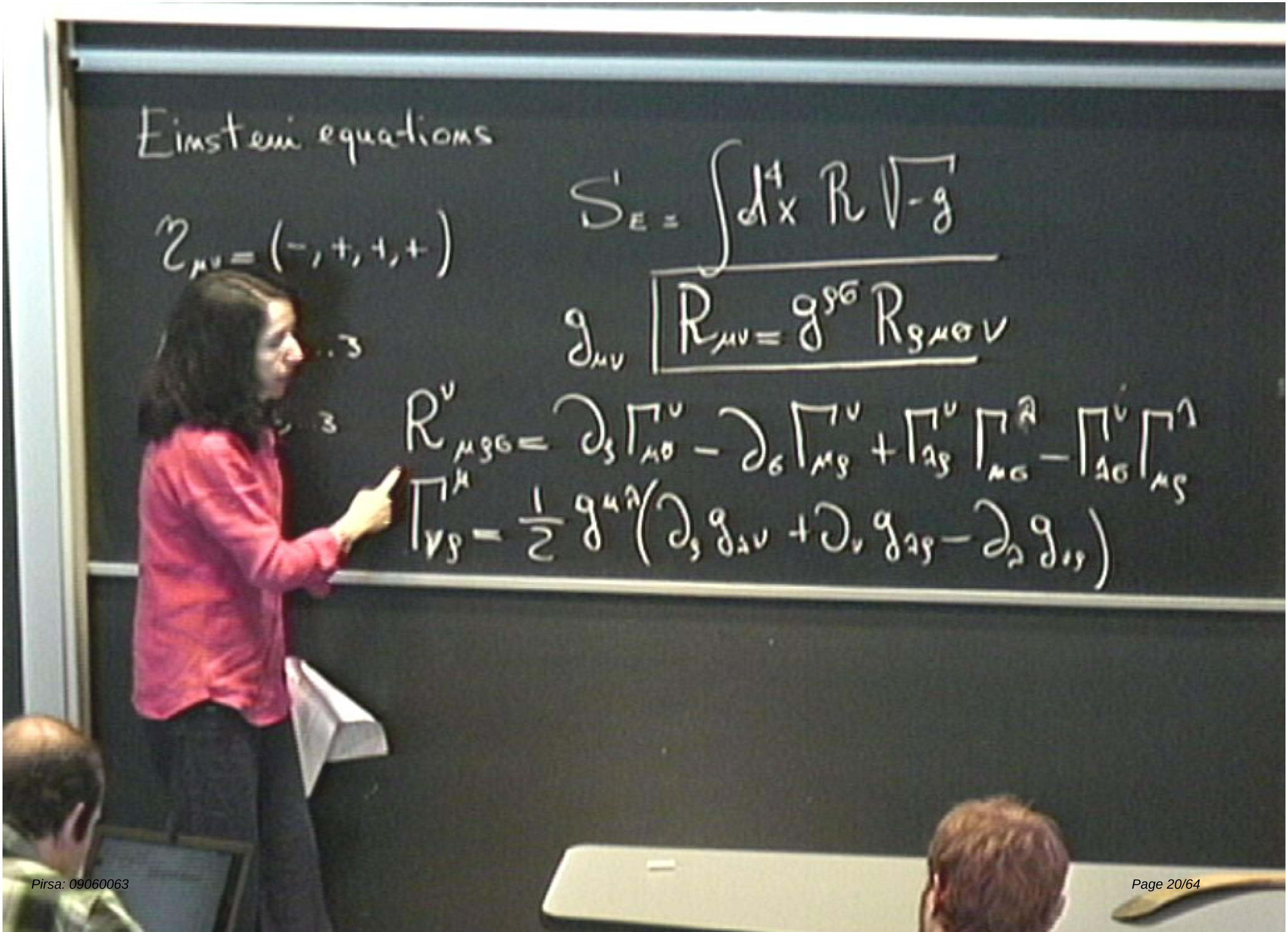
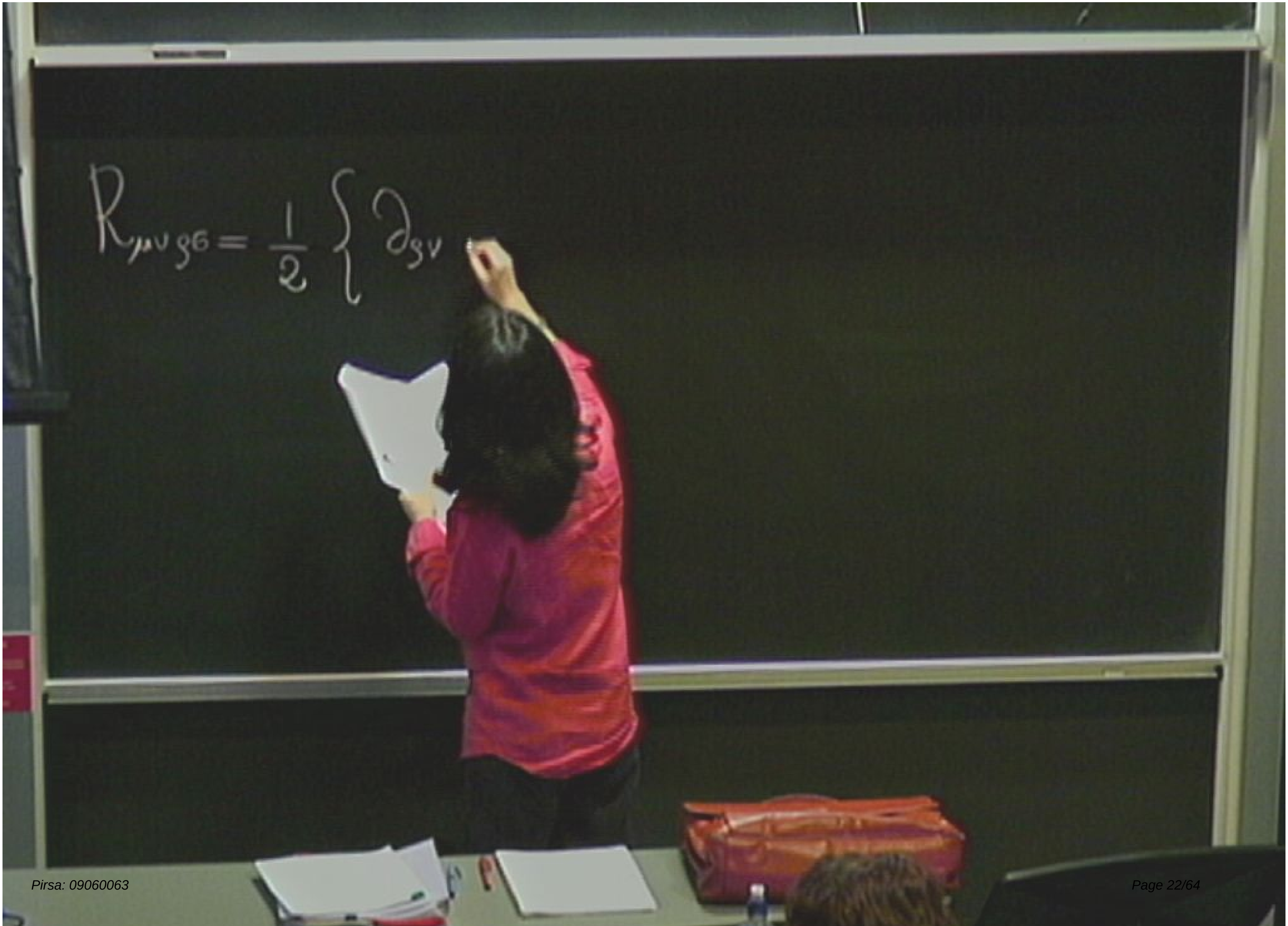
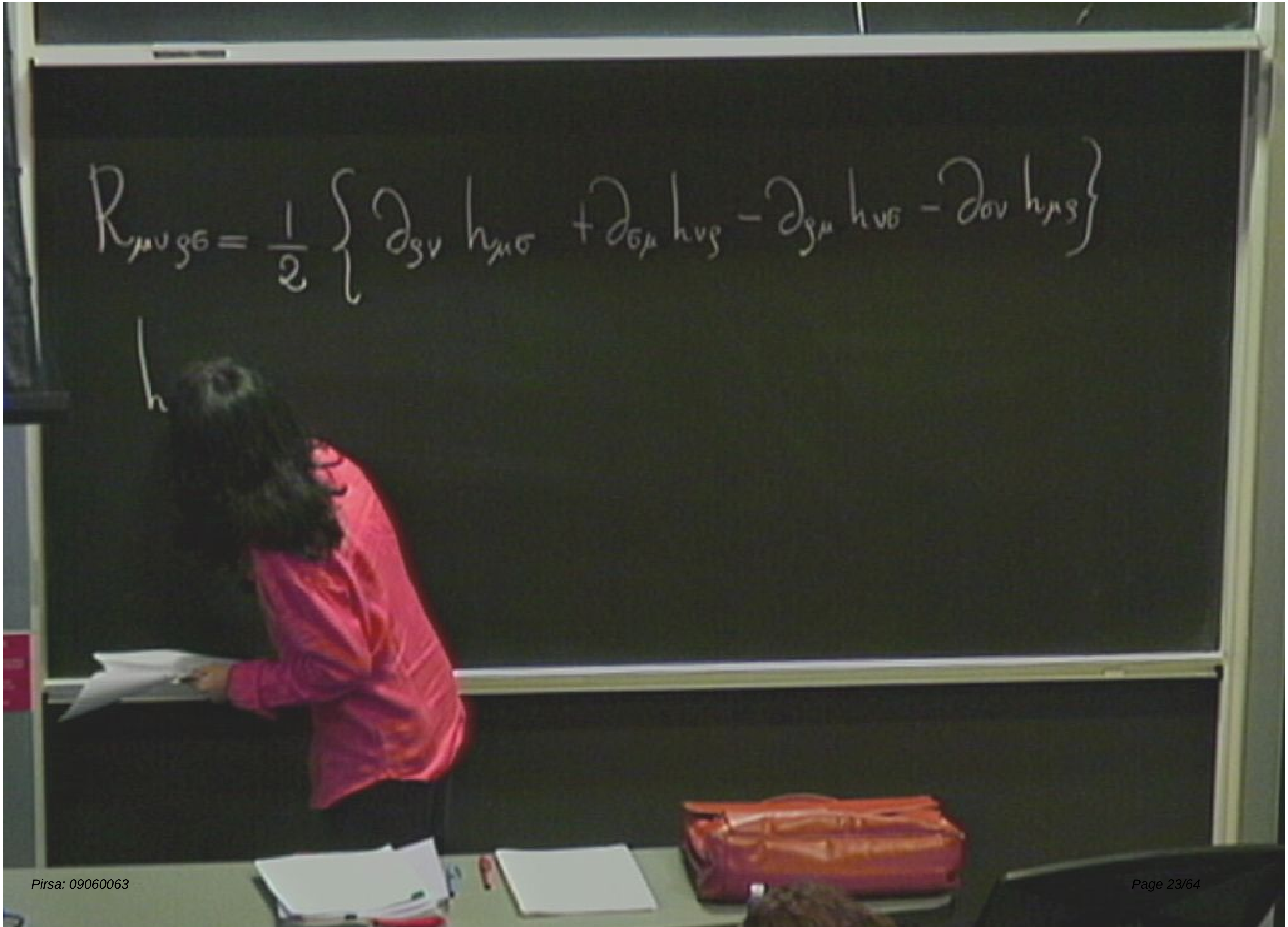
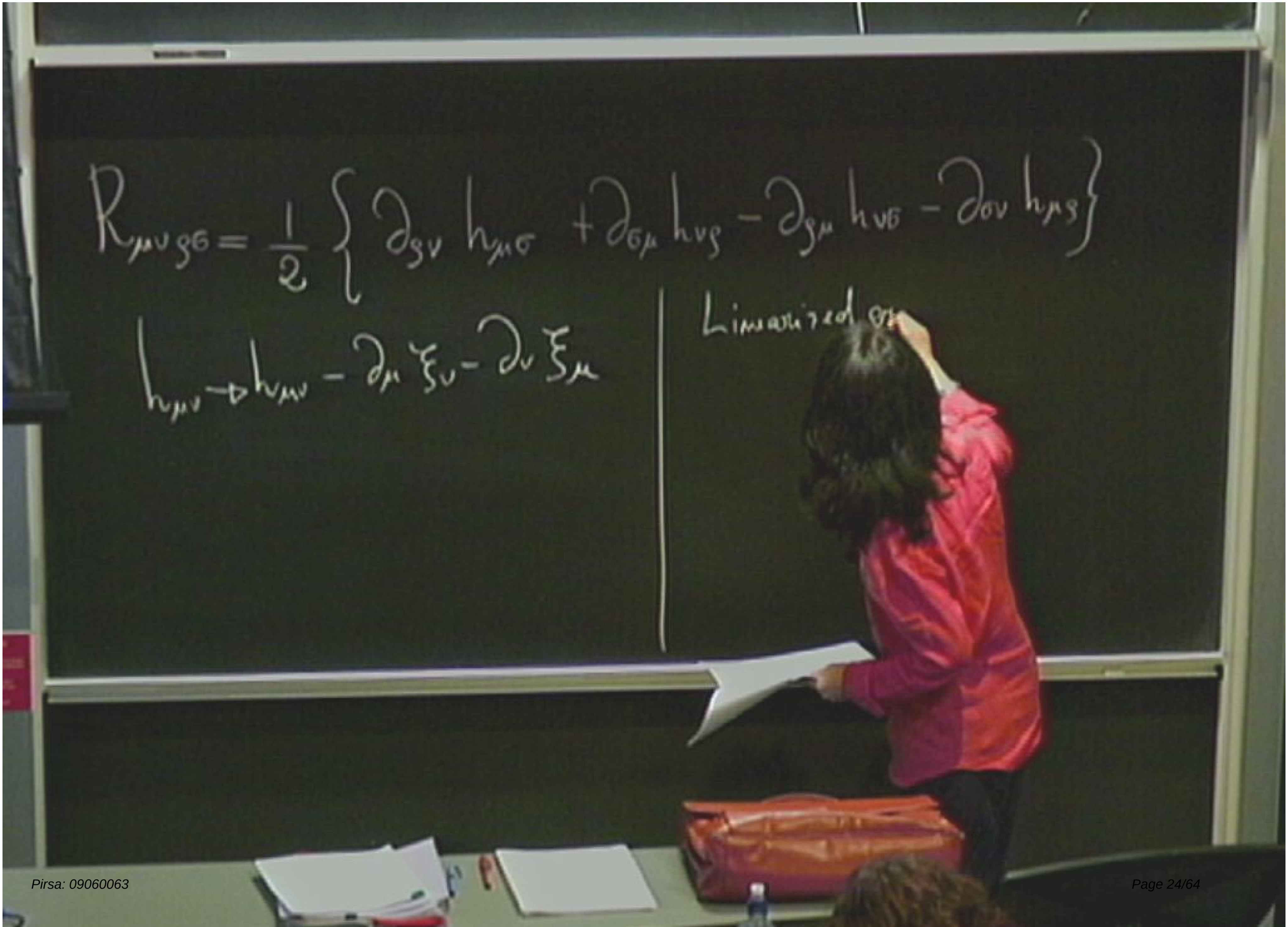

Eimstein equations
$\eta_{\mu\nu} = (-,+,+,+)$
$S_E = \int d^4x \, R \sqrt{-g}$
$g_{\mu\nu}$
$R_{\mu\nu} = g^{\rho\sigma} R_{\rho\mu\sigma\nu}$
$R^{\nu}_{\ \mu\rho\sigma} = \partial_\rho \Gamma^{\nu}_{\mu\sigma} - \partial_\sigma \Gamma^{\nu}_{\mu\rho} + \Gamma^{\nu}_{\lambda\rho} \Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\nu}_{\lambda\sigma} \Gamma^{\lambda}_{\mu\rho}$
$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} (\partial_\rho g_{\lambda\nu} + \partial_\nu g_{\lambda\rho} - \partial_\lambda g_{\nu\rho})$

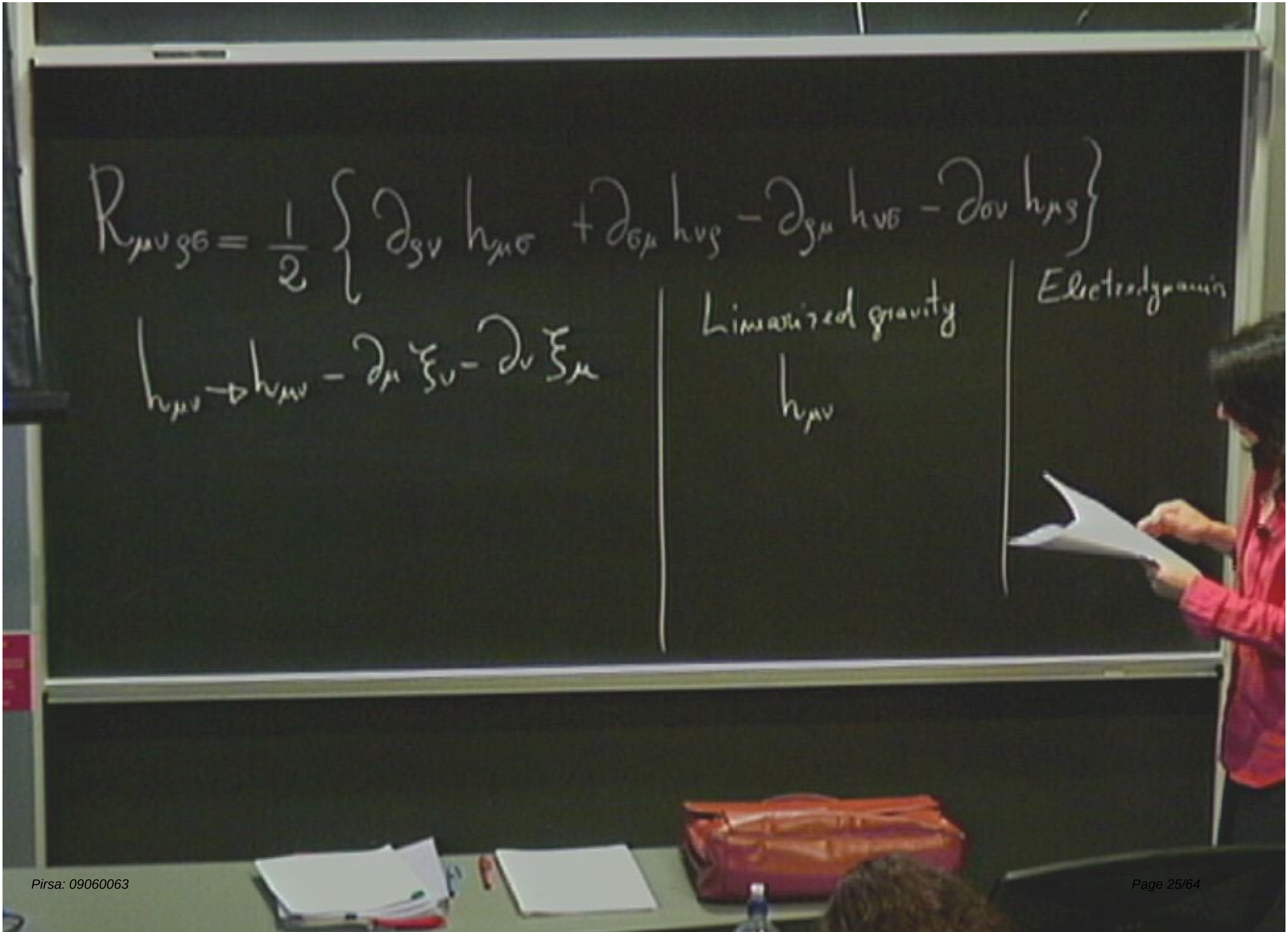

$R_{\mu\nu} g\sigma =$


$R_{\mu\nu\rho\sigma} = \frac{1}{2} \{ \partial_{\rho\nu}$



$R_{\mu\nu\rho\sigma} = \frac{1}{2}\{\partial_{\rho\nu} h_{\mu\sigma} + \partial_{\sigma\mu} h_{\nu\rho} - \partial_{\rho\mu} h_{\nu\sigma} - \partial_{\sigma\nu} h_{\mu\rho}\}$
h


$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left\{ \partial_{\rho\nu} h_{\mu\sigma} + \partial_{\sigma\mu} h_{\nu\rho} - \partial_{\rho\mu} h_{\nu\sigma} - \partial_{\sigma\nu} h_{\mu\rho} \right\}$
$h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$
Linearized e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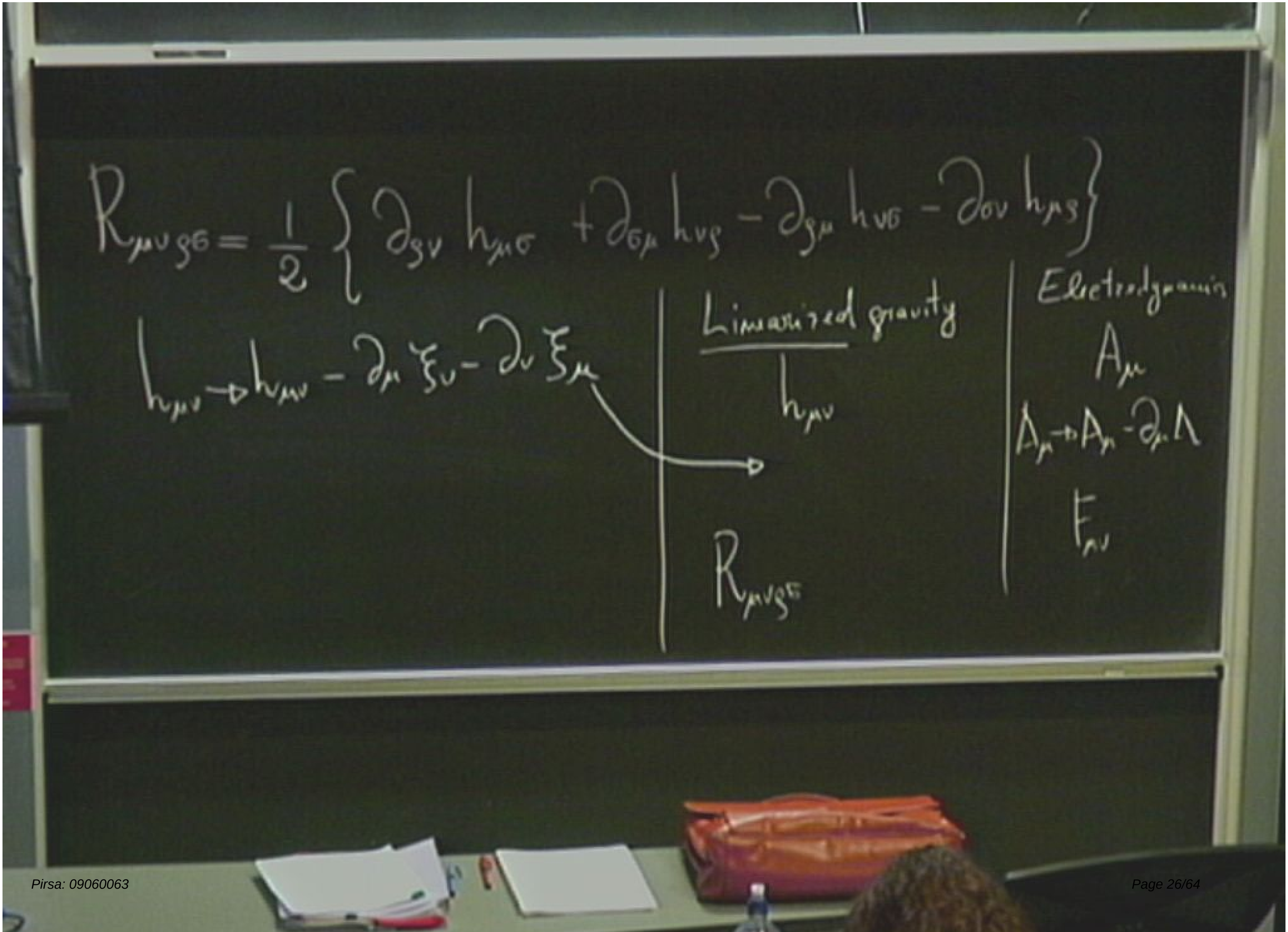
Linearized gravity
$h_{\mu\nu}$
Electrodynamics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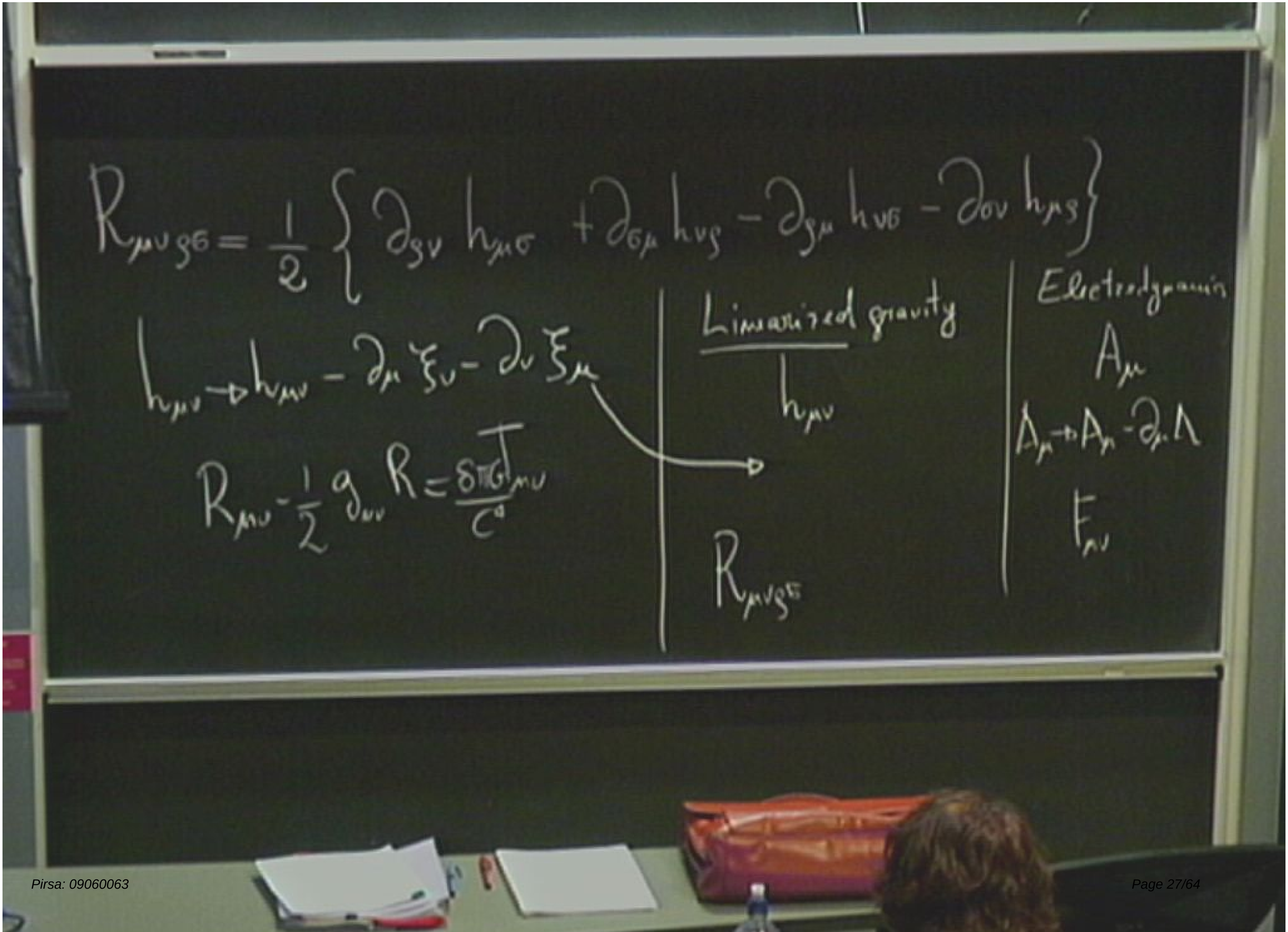
Linearized gravity
$h_{\mu\nu}$
$R_{\mu\nu\rho\sigma}$
Electrodynamics
A_μ
$A_\mu \to A_\mu - \partial_\mu \Lambda$
$F_{\mu\nu}$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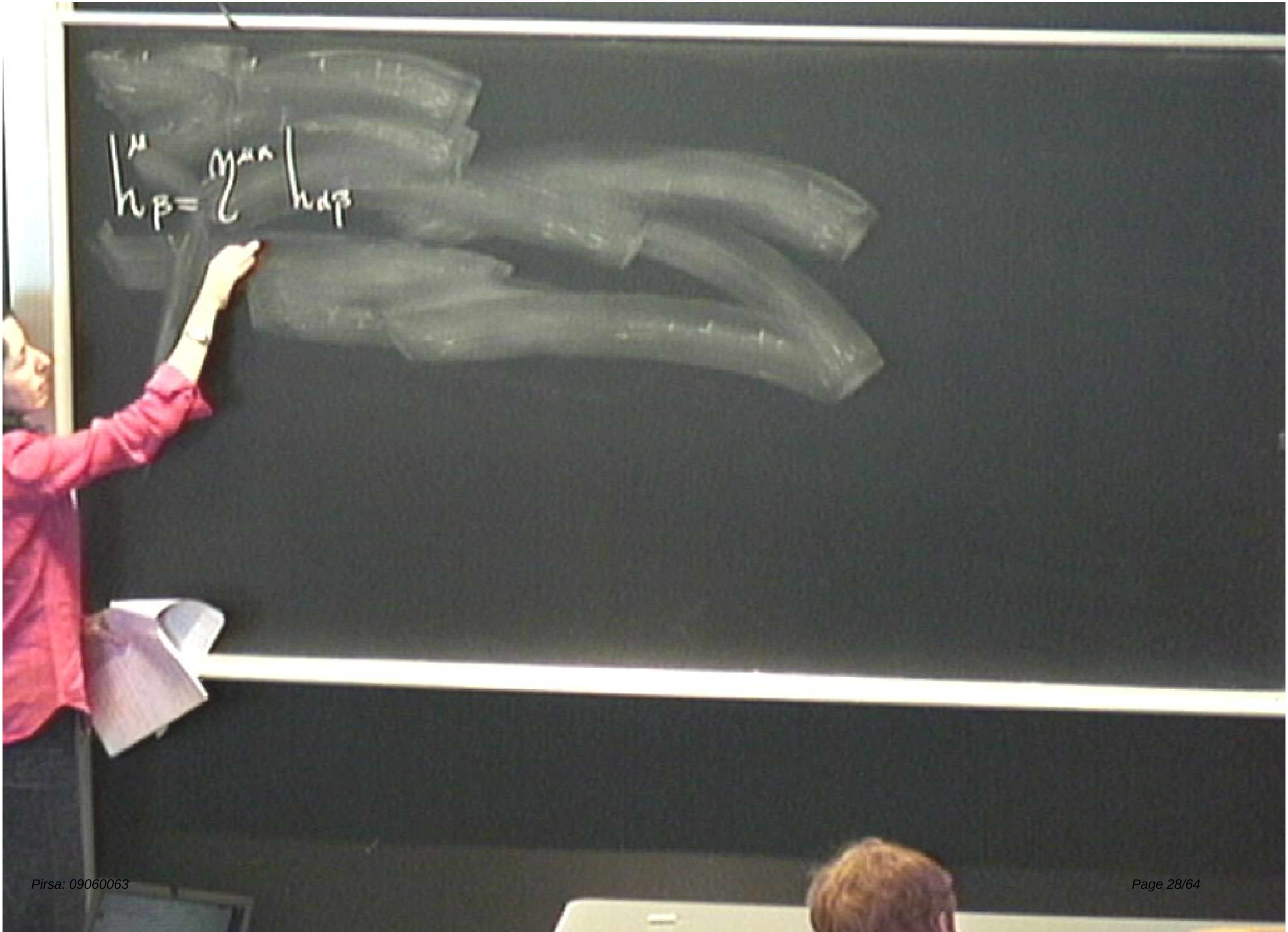

$h^{\mu}{}_{\beta} = \eta^{\mu\alpha} h_{\alpha\beta}$

$$h^{\mu}{}_{\beta} = \eta^{\mu\alpha} h_{\alpha\beta}$$

$$h = h^{\kappa}{}_{\kappa}$$

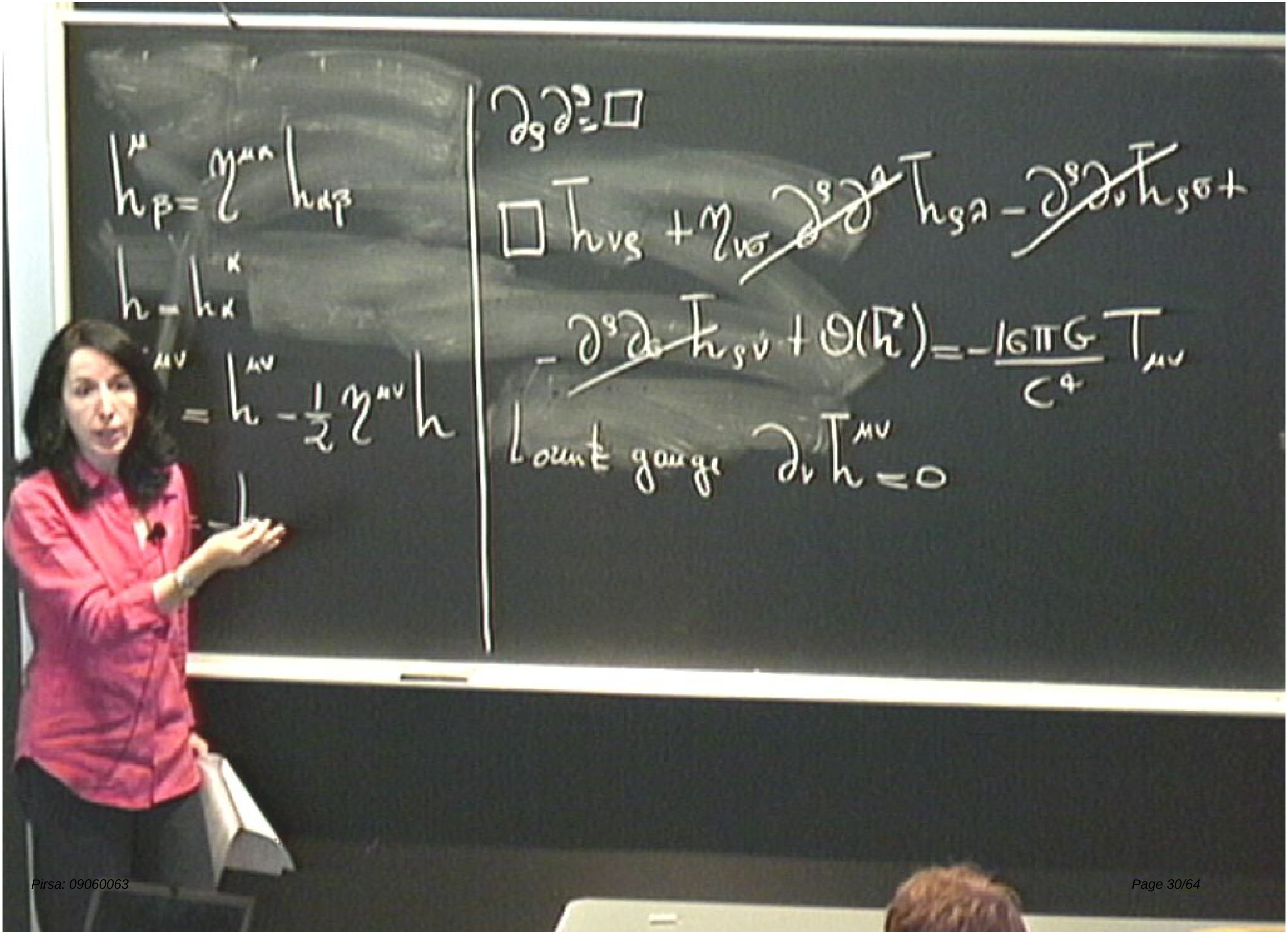
$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$$

$$\bar{h} = -h$$

$$\partial_\rho \partial^\rho = \Box$$

$$\Box \bar{h}_{\nu\rho} + \eta_{\nu\sigma} \partial^\rho \partial^\lambda \bar{h}_{\rho\lambda} - \partial^\rho \partial_\nu \bar{h}_{\rho\sigma} +$$

$$- \partial^\rho \partial_\sigma \bar{h}_{\rho\nu} + O(\bar{h}^2) = -\frac{16\pi G}{c^4} T_{\mu\nu}$$


$h^{\mu}_{\beta} = \eta^{\mu\alpha} h_{\alpha\beta}$
$h = h^{\alpha}_{\alpha}$
$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$
$\partial_\rho \partial^\rho = \Box$
$\Box \bar{h}_{\nu\rho} + \cancel{\eta_{\nu\sigma} \partial^\rho \partial^\lambda} \bar{h}_{\rho\lambda} - \cancel{\partial^\rho \partial_\nu} \bar{h}_{\rho\sigma} +$
$- \cancel{\partial^\rho \partial_\sigma} \bar{h}_{\rho\nu} + O(\bar{h}^2) = -\frac{16\pi G}{c^4} T_{\mu\nu}$
Lorentz gauge $\partial_\nu \bar{h}^{\mu\nu} = 0$

$h^{\mu}{}_{\beta} = \eta^{\mu\alpha} h_{\alpha\beta}$

$h = h^{\kappa}{}_{\kappa}$

$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$

$\bar{h} = -h$

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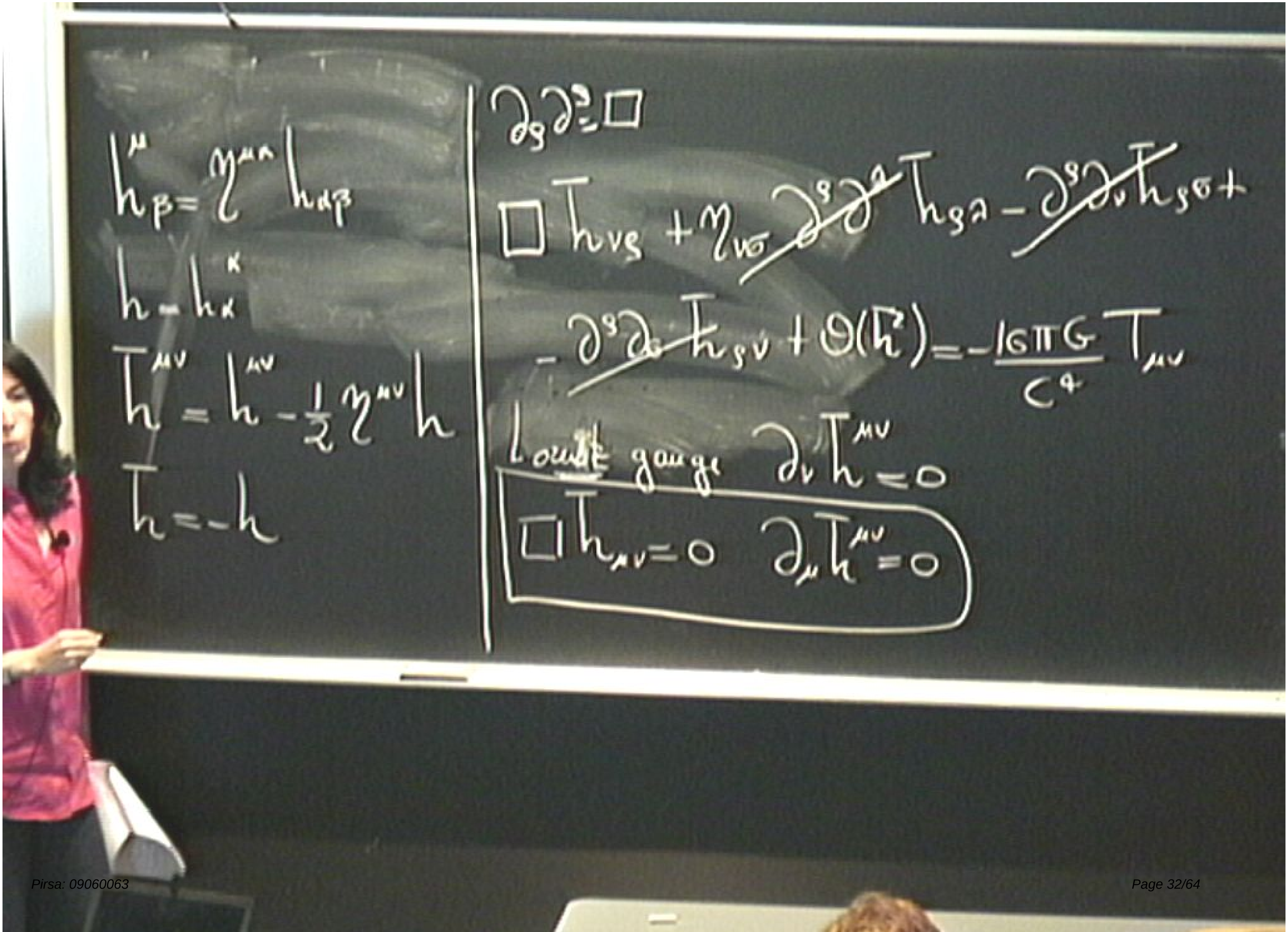
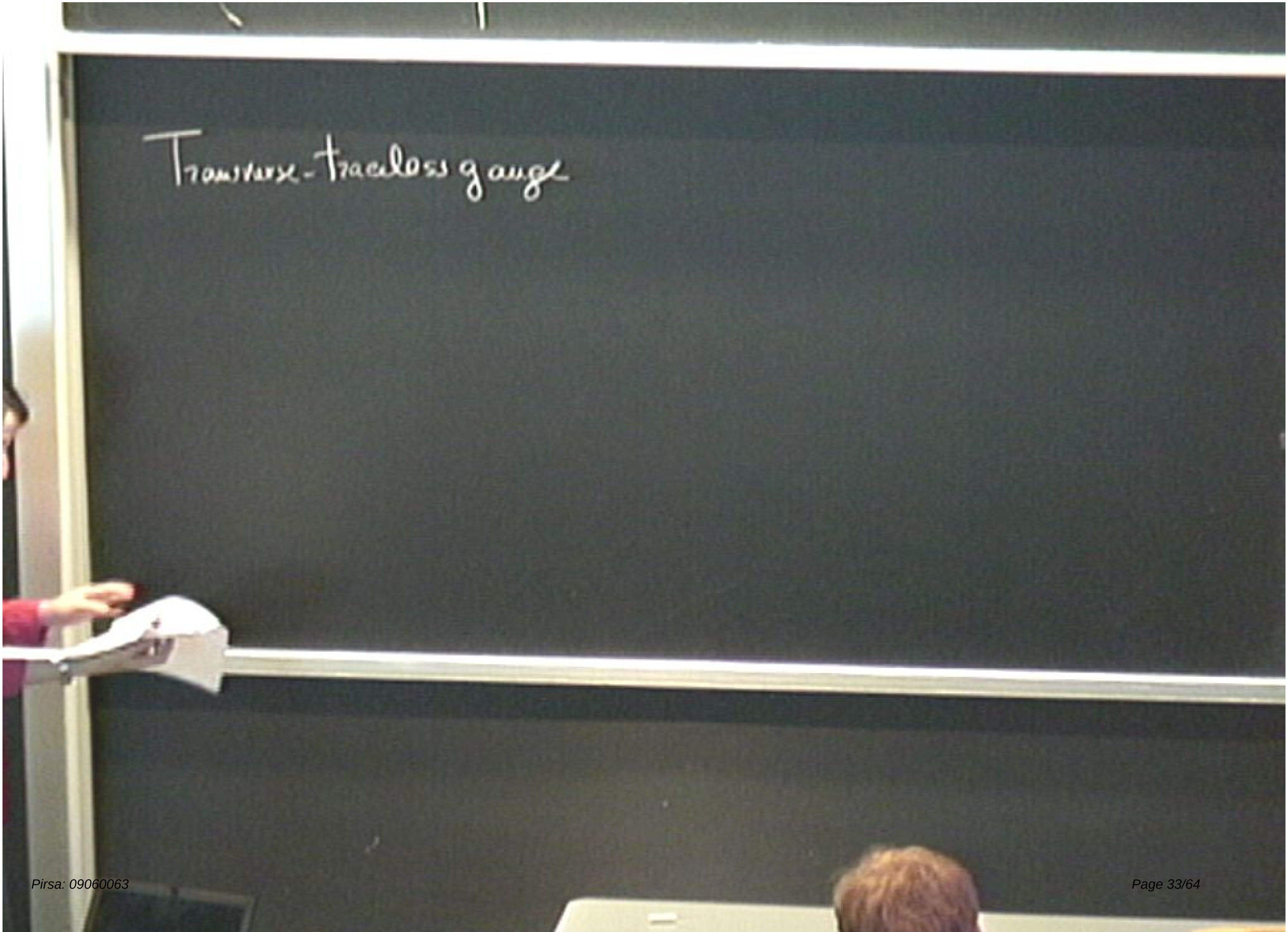
$\Box \bar{h}_{\nu\rho} + \eta_{\nu\sigma} \cancel{\partial^\rho \partial^\lambda} \bar{h}_{\rho\lambda} - \cancel{\partial^\rho \partial_\nu} \bar{h}_{\rho\sigma} +$

$- \cancel{\partial^\rho \partial_\sigma} \bar{h}_{\rho\nu} + O(\bar{h}^2) = -\frac{16\pi G}{c^4} T_{\mu\nu}$

Lorentz gauge $\partial_\nu \bar{h}^{\mu\nu} = 0$


$h^{\mu}{}_{\beta} = \eta^{\mu\alpha} h_{\alpha\beta}$
$h = h^{\kappa}{}_{\kappa}$
$\bar{h}^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$
$\bar{h} = -h$
$\partial_\rho \partial^\rho = \Box$
$\Box \bar{h}_{\nu\rho} + \eta_{\nu\sigma} \cancel{\partial^\rho \partial^\lambda} \bar{h}_{\rho\lambda} - \cancel{\partial^\rho \partial_\nu} \bar{h}_{\rho\sigma} +$
$- \cancel{\partial^\rho \partial_\sigma} \bar{h}_{\rho\nu} + O(h^2) = -\frac{16\pi G}{c^4} T_{\mu\nu}$
Lorentz gauge $\partial_\nu \bar{h}^{\mu\nu} = 0$
$\boxed{\Box \bar{h}_{\mu\nu} = 0 \quad \partial_\mu \bar{h}^{\mu\nu} = 0}$


Transverse-traceless gauge

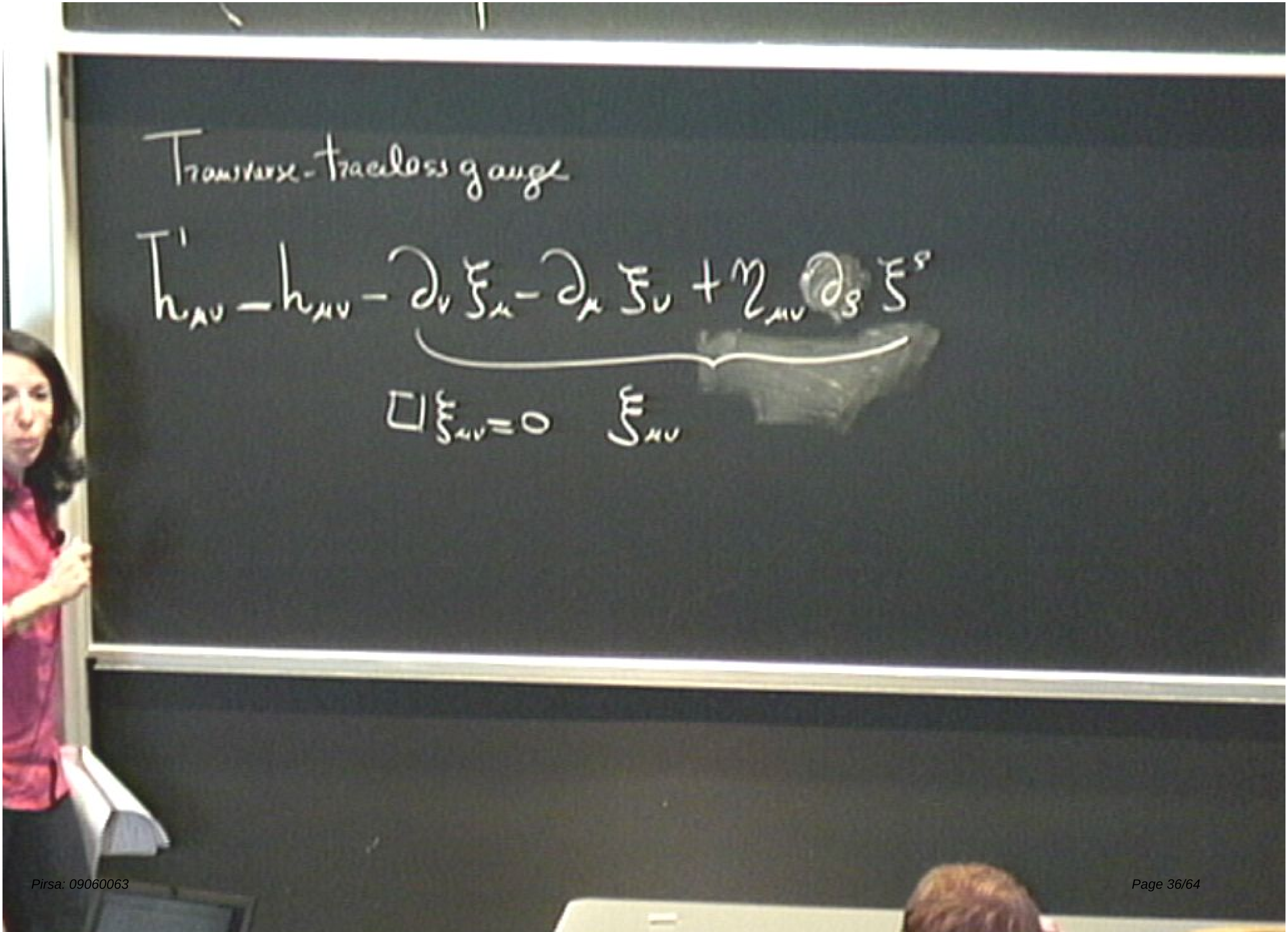
Transverse-traceless gauge

$$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho$$

Transverse-traceless gauge

$$h'_{\mu\nu} = h_{\mu\nu} - \underbrace{\partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho}$$

$\Box \xi_{\mu\nu} = 0 \quad \xi_{\mu\nu}$


Transverse-traceless gauge
$h'_{\mu\nu} = h_{\mu\nu} - \underbrace{\partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho}$
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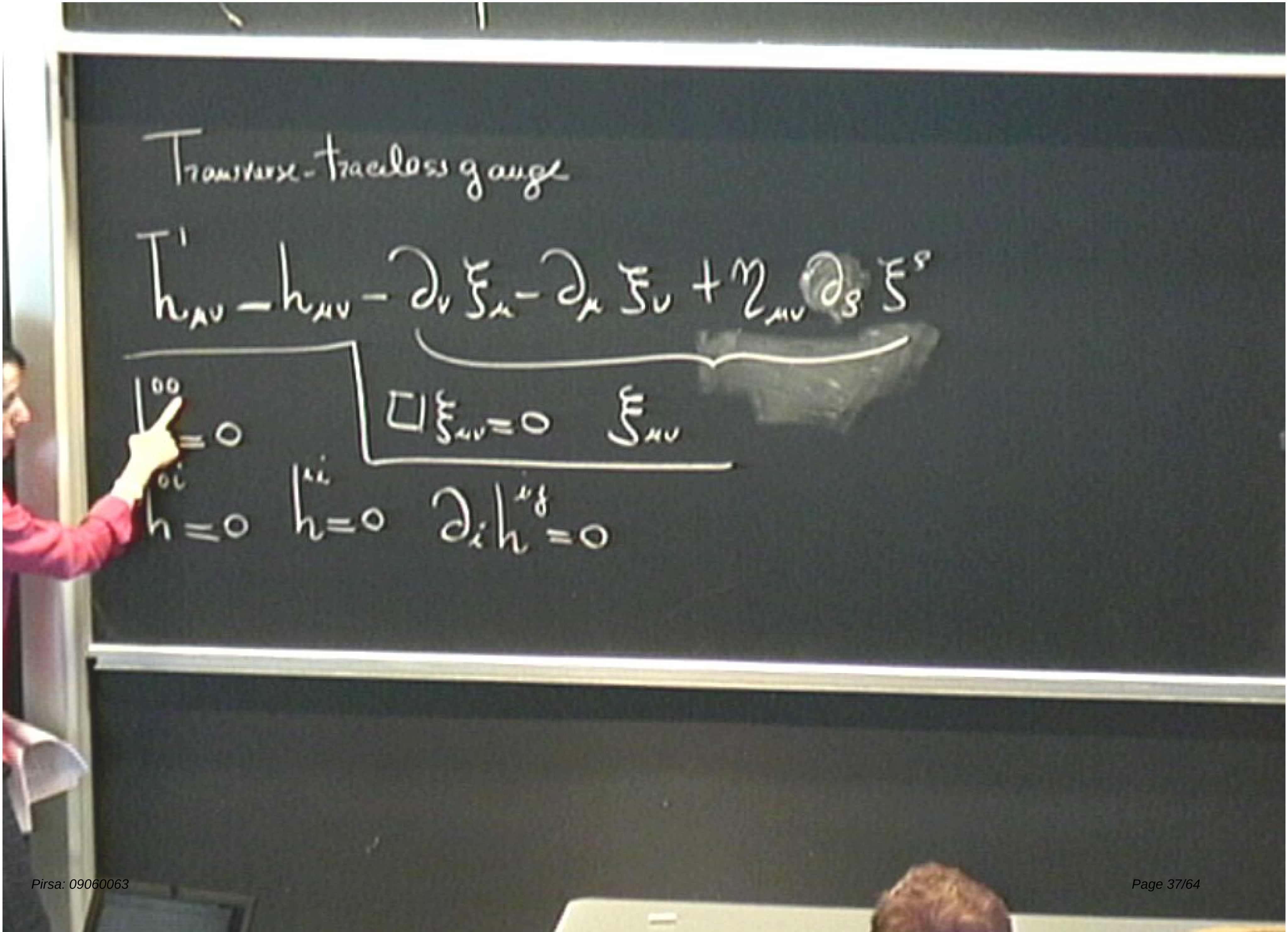

Transverse-traceless gauge
$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho$
$h^{00} = 0$
$\Box \xi_{\mu\nu} = 0$ $\xi_{\mu\nu}$
$h^{0i} = 0$ $h^{ii} = 0$ $\partial_i h^{ij} = 0$

Transverse-traceless gauge

$$h'_{\mu\nu} = h_{\mu\nu} - \underbrace{\partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho}$$

$$h^{00} = 0 \qquad \boxed{\Box \xi_{\mu\nu} = 0 \quad \xi_{\mu\nu}}$$

$$h^{0i} = 0 \quad h^{ii} = 0 \quad \partial_i h^{ij} = 0 ; \; h^{ij} \neq 0$$

Transverse-traceless gauge

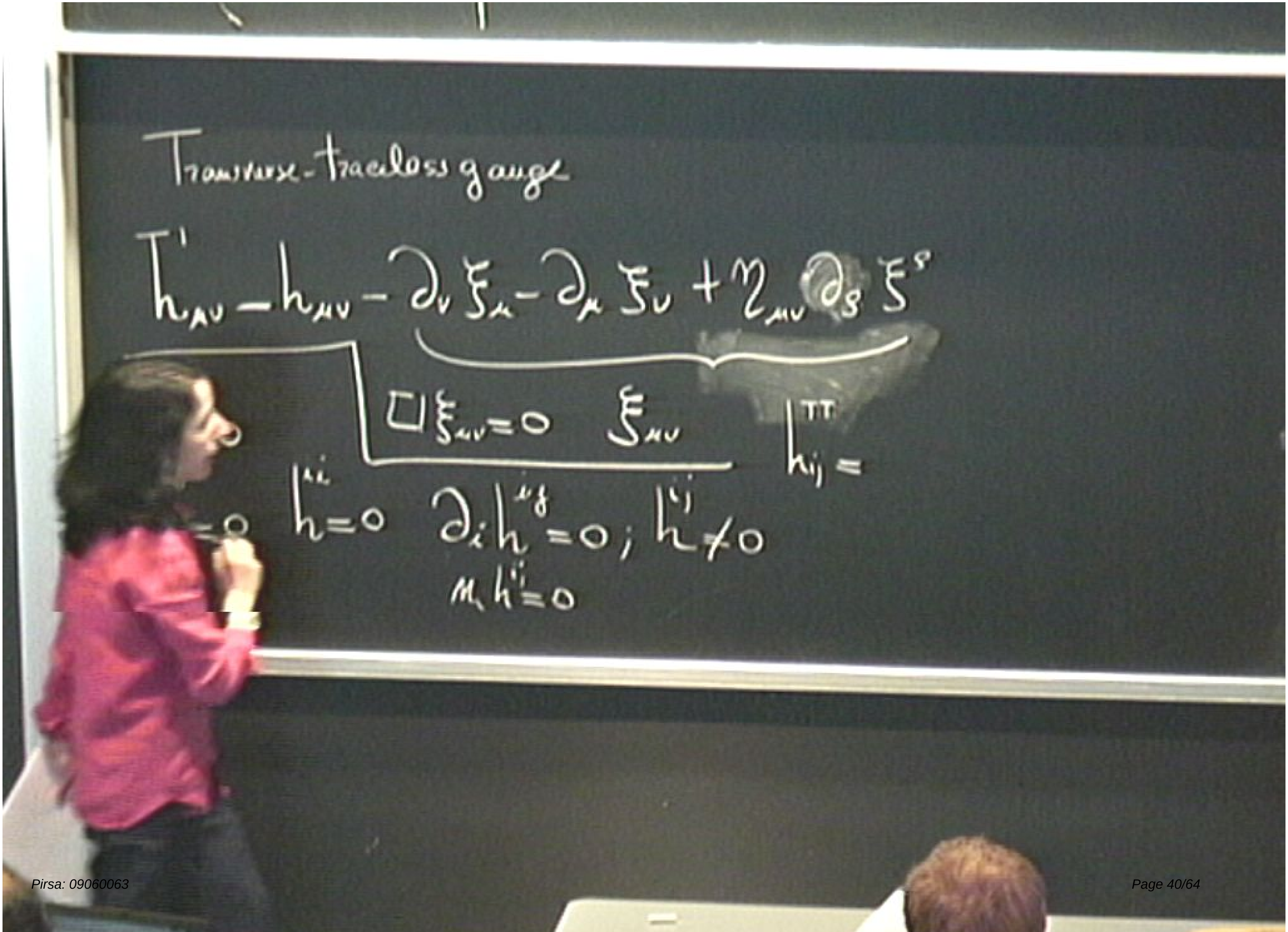
$$h'_{\mu\nu} = h_{\mu\nu} - \underbrace{\partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu}\partial_\delta \xi^\delta}$$

$h'^{00} = 0$

$\boxed{\Box \xi_{\mu\nu} = 0 \quad \xi_{\mu\nu}}$

$h'^{0i} = 0 \quad h'^{i}{}_{i} = 0 \quad \partial_i h'^{ij} = 0 \,; \; h'^{ij} \neq 0$

$n_i h'^{ij} = 0$


Transverse-traceless gauge
h'_{μν} = h_{μν} − ∂_ν ξ_μ − ∂_μ ξ_ν + η_{μν} ∂_ρ ξ^ρ
□ξ_{μν} = 0   ξ_{μν}
h^{TT}_{ij} =
=0   h'^{μi}   h'=0   ∂_i h'^{ij} = 0 ; h'^{ij} ≠ 0
n_i h'^{ij} = 0

Transverse-traceless gauge

$$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho$$

$h^{00} = 0$

$\Box \xi_{\mu\nu} = 0 \quad \xi_{\mu\nu}$

$h^{0i} = 0 \quad h^{i}{}_{i} = 0 \quad \partial_i h^{ij} = 0 ;$

$n_i h^{ij} = 0$

$$= \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos \omega t$$

Transverse-traceless gauge

$$h'_{\mu\nu} = h_{\mu\nu} - \underbrace{\partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho}$$

$h^{00} = 0$ $\quad \Box \xi_{\mu\nu} = 0 \quad \xi_{\mu\nu}$

$h^{0i} = 0 \quad h^{i}_{i} = 0 \quad \partial_i h^{ij} = 0 \, ; \; h^{ij} \neq 0$

$n_i h^{ij} = 0$

$$h^{TT}_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos\omega\left(t - \frac{z}{c}\right)$$

Transverse-traceless gauge $\quad (h_+ \pm i h_\times)$

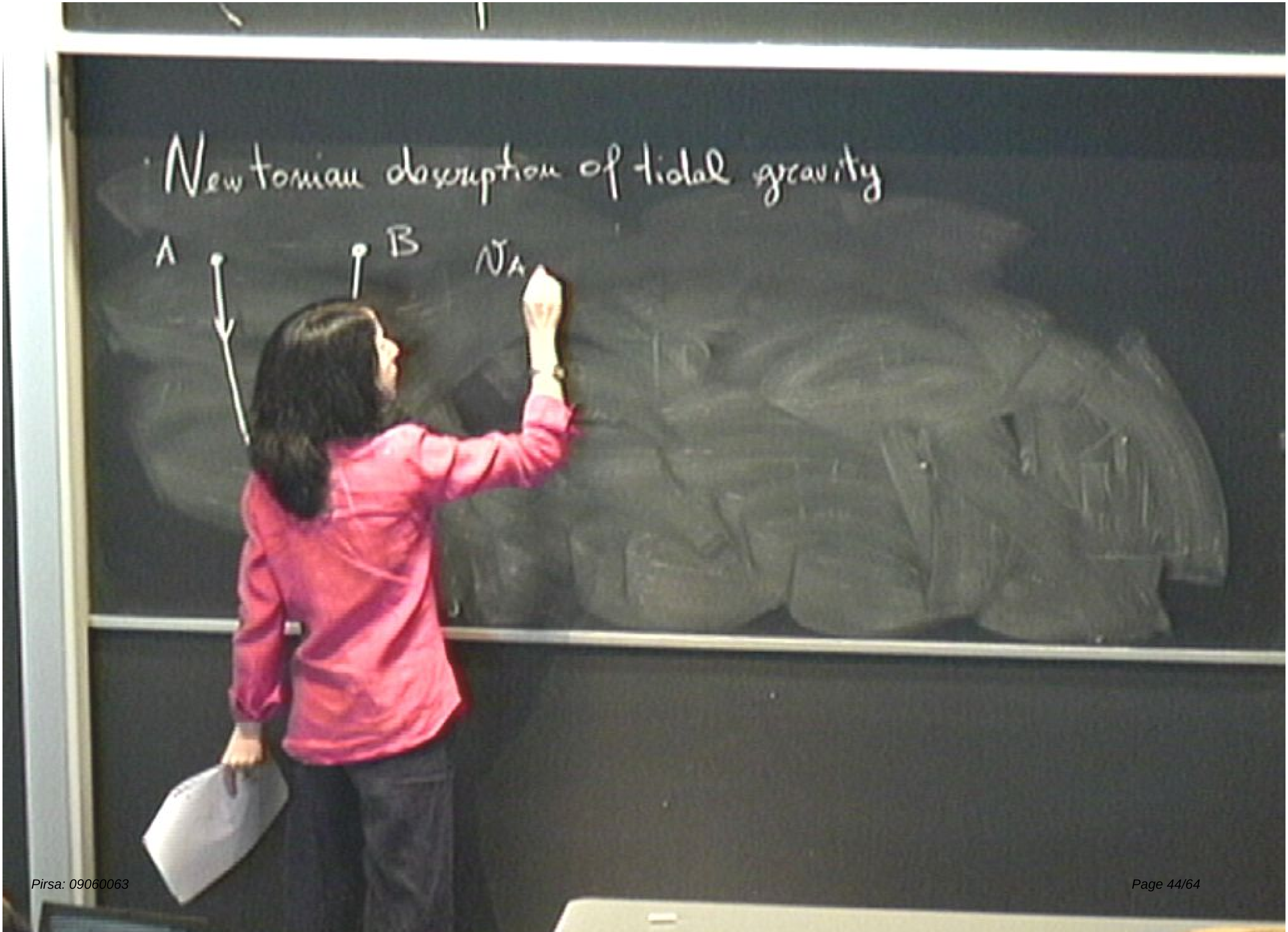
$$h'_{\mu\nu} = h_{\mu\nu} - \underbrace{\partial_\nu \xi_\mu - \partial_\mu \xi_\nu + \eta_{\mu\nu} \partial_\rho \xi^\rho}$$

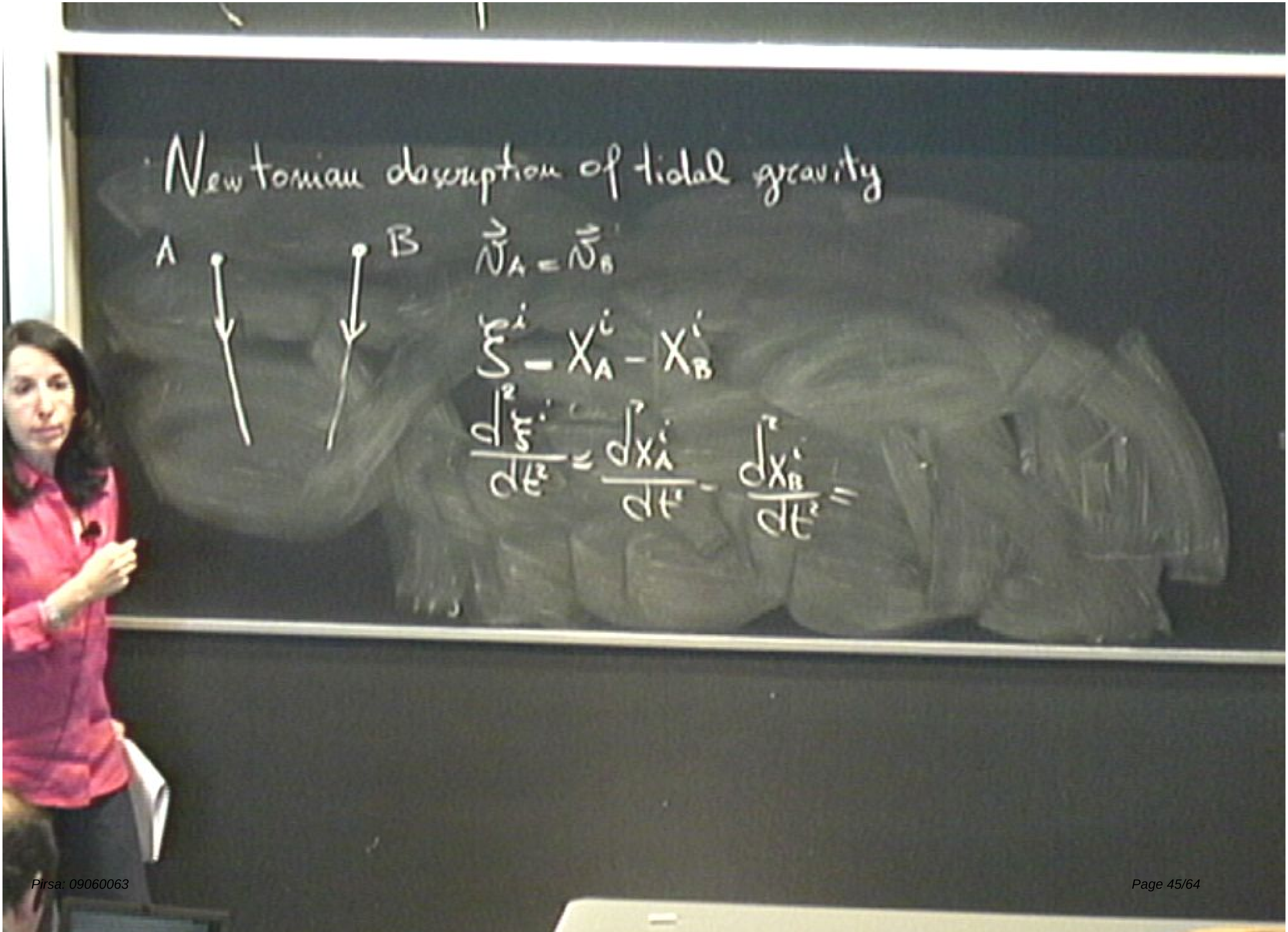
$h^{00} = 0$ $\quad \boxed{\Box \xi_{\mu\nu} = 0 \quad \xi_{\mu\nu}}$

$$h^{TT}_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos\omega(t - \tfrac{z}{c})$$

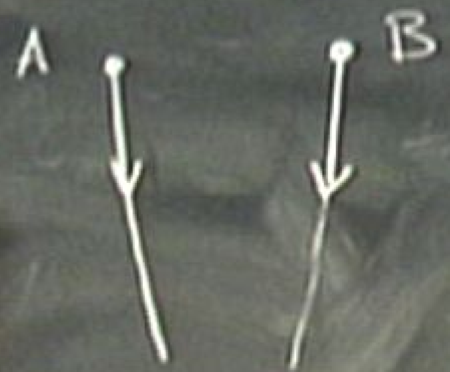
$h^{0i} = 0 \quad h^{i}{}_{i} = 0 \quad \partial_i h^{ij} = 0 \,; \; h^{ij} \neq 0$

$n_i h^{ij} = 0$


Newtonian description of tidal gravity
A
B
vA


Newtonian description of tidal gravity
A
B
$\vec{v}_A = \vec{v}_B$
$\xi^i = X_A^i - X_B^i$
$\frac{d^2\xi^i}{dt^2} = \frac{d^2X_A^i}{dt^2} - \frac{d^2X_B^i}{dt^2} =$

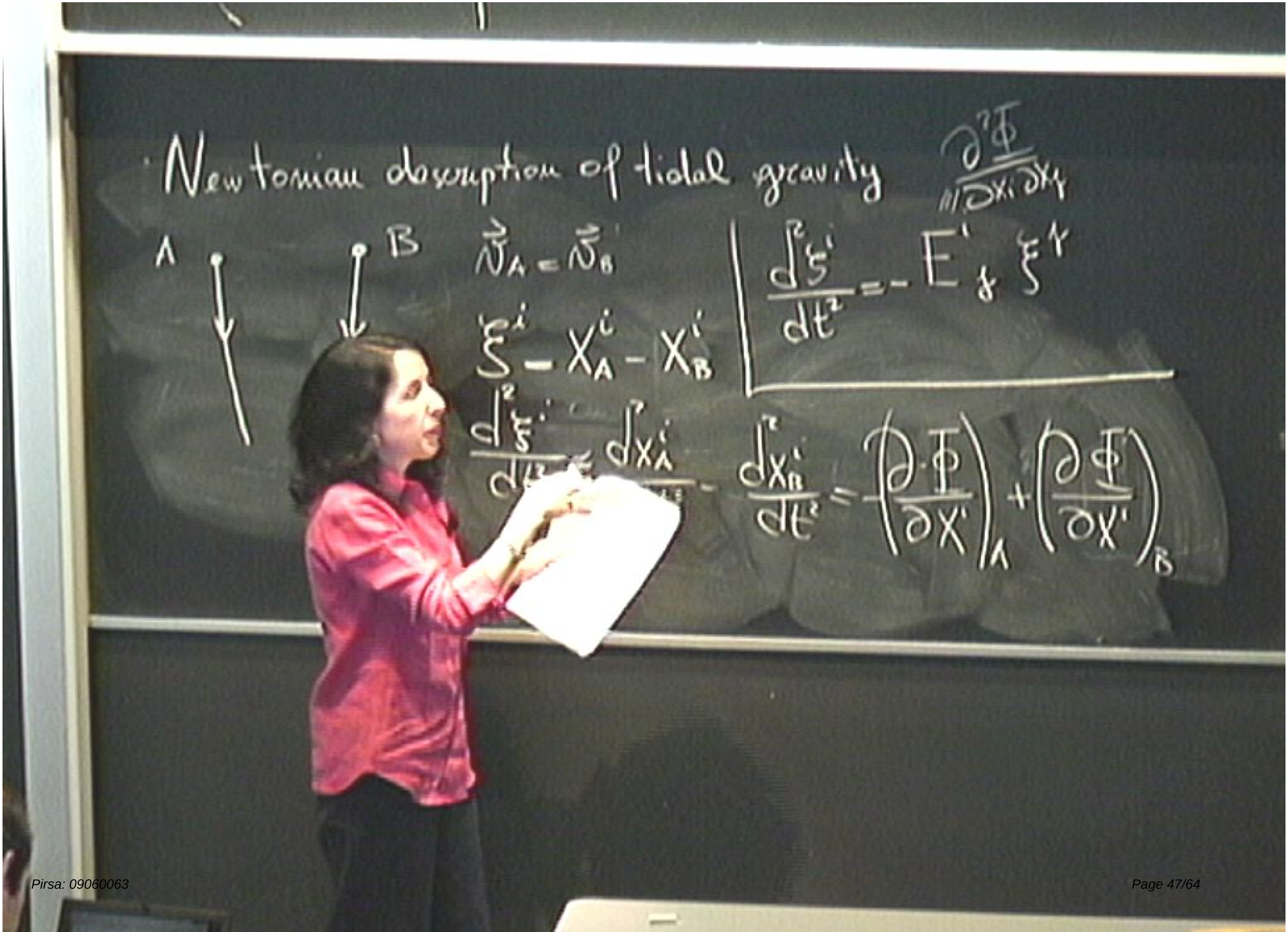
Newtonian description of tidal gravity

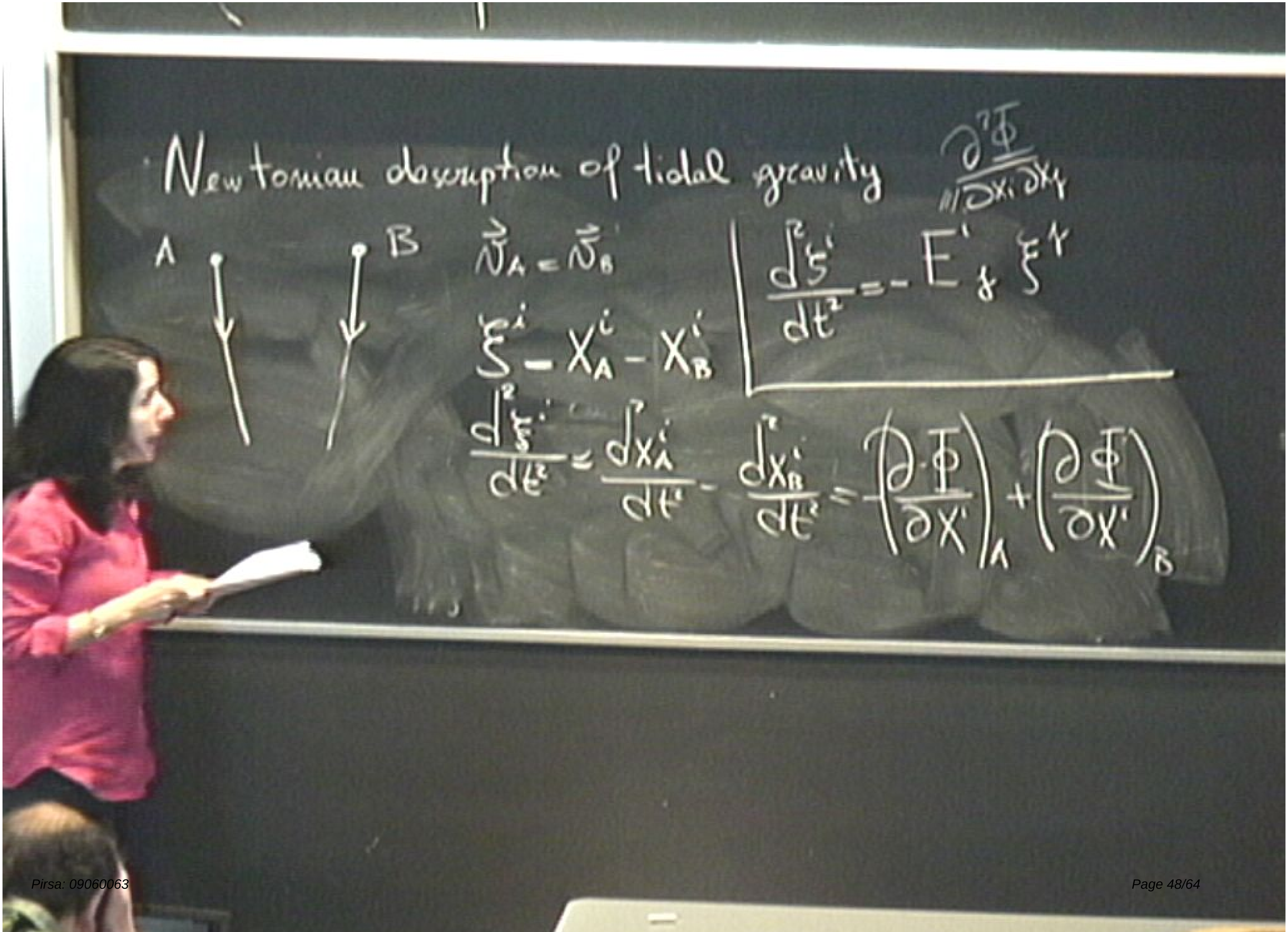


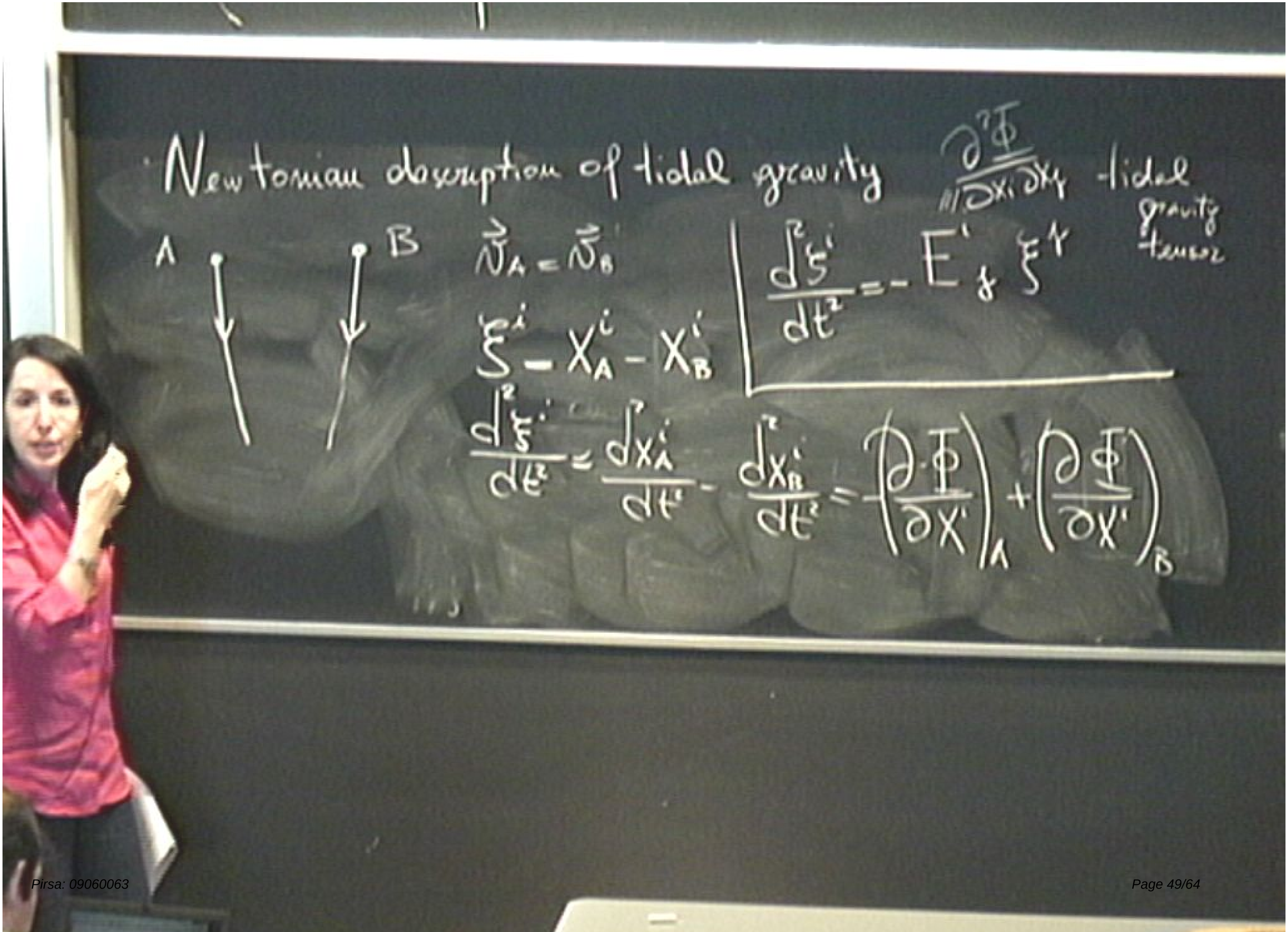
$$\vec{v}_A = \vec{v}_B$$

$$\xi^i = X^i_A - X^i_B$$

$$\frac{d^2\xi^i}{dt^2} = \frac{d^2X^i_A}{dt^2} - \frac{d^2X^i_B}{dt^2} = -\left(\frac{\partial\Phi}{\partial x^i}\right)_A + \left(\frac{\partial\Phi}{\partial x^i}\right)_B$$


Newtonian description of tidal gravity
$\frac{\partial^2 \Phi}{\partial x_i \partial x_j}$
A
B
$\vec{v}_A = \vec{v}_B$
$\xi^i = X^i_A - X^i_B$
$\frac{d^2\xi^i}{dt^2} = -E^i{}_j \xi^j$
$\frac{d^2\xi^i}{dt^2} = \frac{d^2 X^i_A}{dt^2} - \frac{d^2 X^i_B}{dt^2} = -\left(\frac{\partial \Phi}{\partial x^i}\right)_A + \left(\frac{\partial \Phi}{\partial x^i}\right)_B$


Newtonian description of tidal gravity
$\frac{\partial^2 \Phi}{\partial x_i \partial x_j}$
A
B
$\vec{v}_A = \vec{v}_B$
$\xi^i = X^i_A - X^i_B$
$\frac{d^2\xi^i}{dt^2} = -E^i_j \xi^j$
$\frac{d^2\xi^i}{dt^2} = \frac{d^2X^i_A}{dt^2} - \frac{d^2X^i_B}{dt^2} = -\left(\frac{\partial \Phi}{\partial x^i}\right)_A + \left(\frac{\partial \Phi}{\partial x^i}\right)_B$


Newtonian description of tidal gravity
$\frac{\partial^2\Phi}{\partial x_i \partial x_j}$ tidal gravity tensor
A
B
$\vec{v}_A = \vec{v}_B$
$\xi^i = X_A^i - X_B^i$
$\frac{d^2\xi^i}{dt^2} = -E^i{}_j \xi^j$
$\frac{d^2\xi^i}{dt^2} = \frac{d^2x_A^i}{dt^2} - \frac{d^2x_B^i}{dt^2} = -\left(\frac{\partial\Phi}{\partial x^i}\right)_A + \left(\frac{\partial\Phi}{\partial x^i}\right)_B$

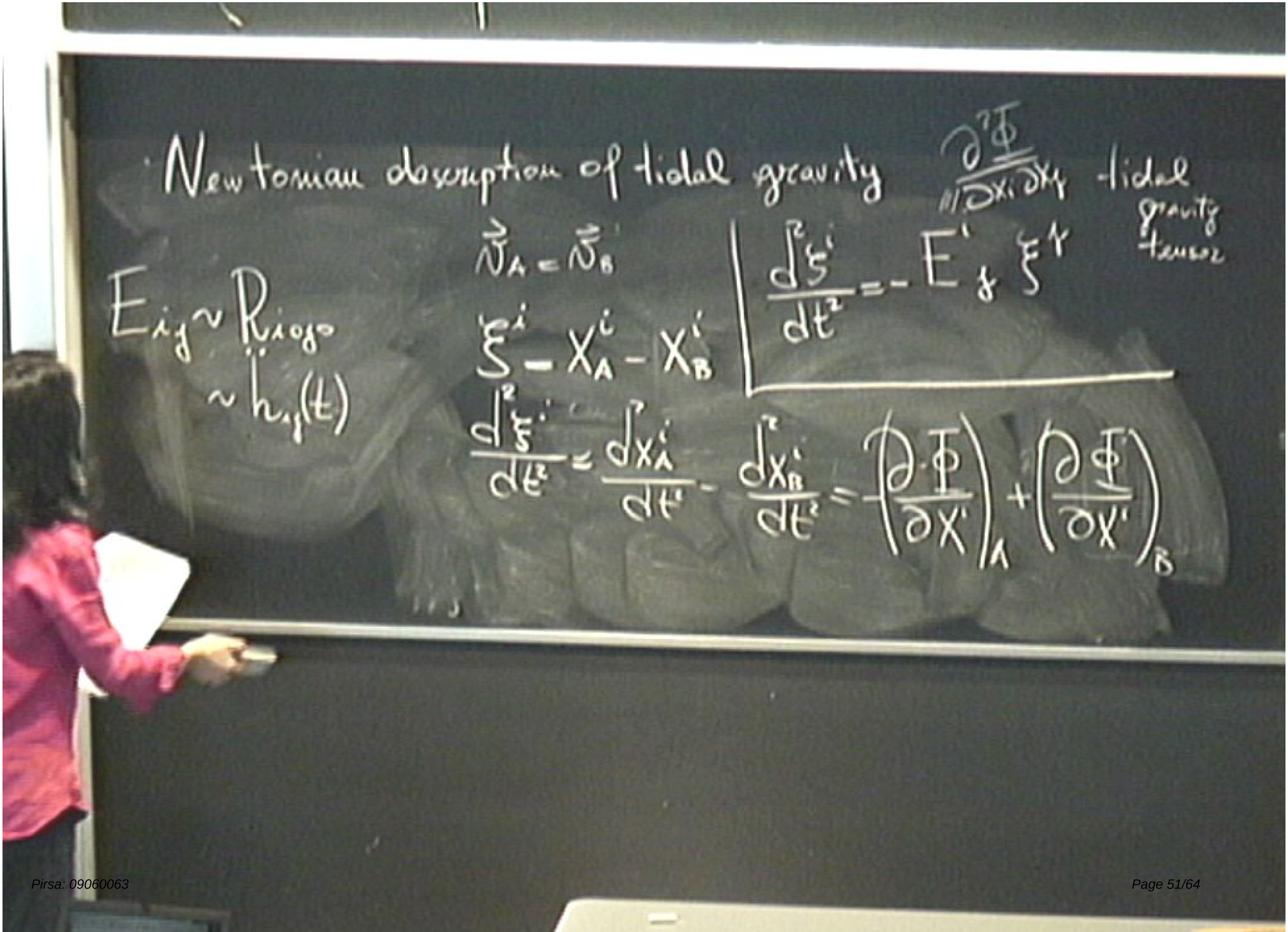
Newtonian description of tidal gravity $\qquad \dfrac{\partial^2 \Phi}{\partial x^i \partial x^k}$ tidal gravity tensor

$$\vec{v}_A = \vec{v}_B$$

$$\xi^i = X_A^i - X_B^i$$

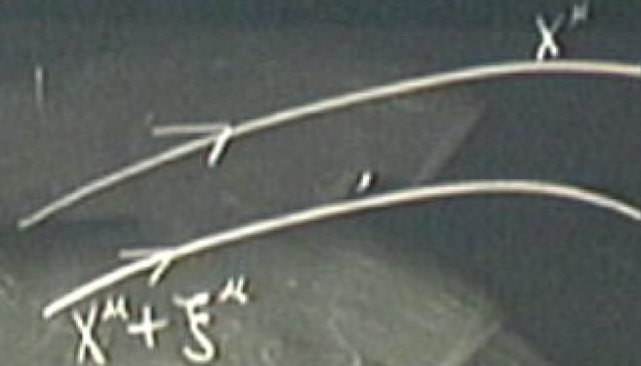
$$\frac{d^2\xi^i}{dt^2} = -E^i{}_k\, \xi^k$$

$$\frac{d^2\xi^i}{dt^2} = \frac{d^2 X_A^i}{dt^2} - \frac{d^2 X_B^i}{dt^2} = -\left(\frac{\partial \Phi}{\partial x^i}\right)_A + \left(\frac{\partial \Phi}{\partial x^i}\right)_B$$


Newtonian description of tidal gravity
$\frac{\partial^2 \Phi}{\partial x_i \partial x_j}$ tidal gravity tensor
$E_{ij} \sim R_{i0j0} \sim \ddot{h}_{ij}(t)$
$\vec{v}_A = \vec{v}_B$
$\xi^i = X_A^i - X_B^i$
$\frac{d^2\xi^i}{dt^2} = -E^i{}_j\, \xi^j$
$\frac{d^2\xi^i}{dt^2} = \frac{d^2X_A^i}{dt^2} - \frac{d^2X_B^i}{dt^2} = -\left(\frac{\partial \Phi}{\partial x^i}\right)_A + \left(\frac{\partial \Phi}{\partial x^i}\right)_B$

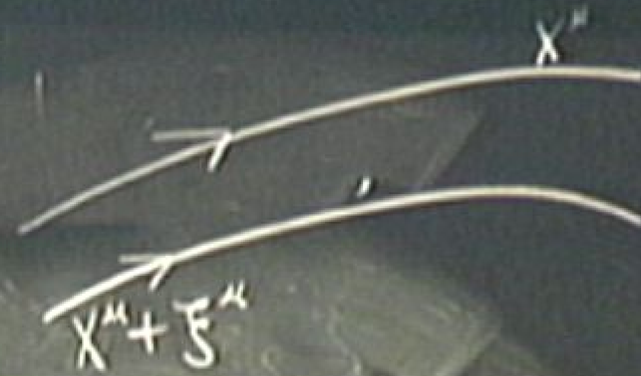
$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

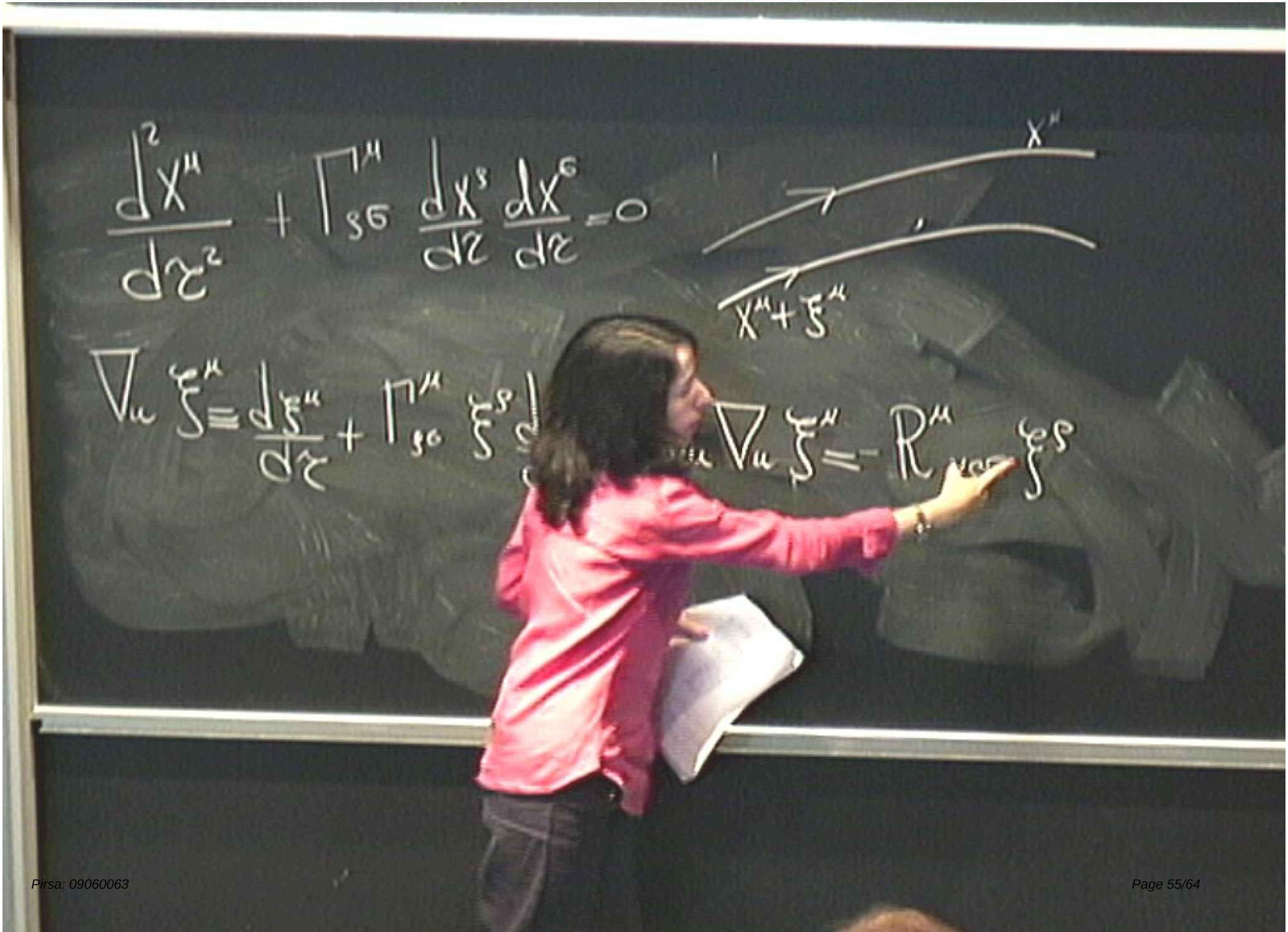


∇

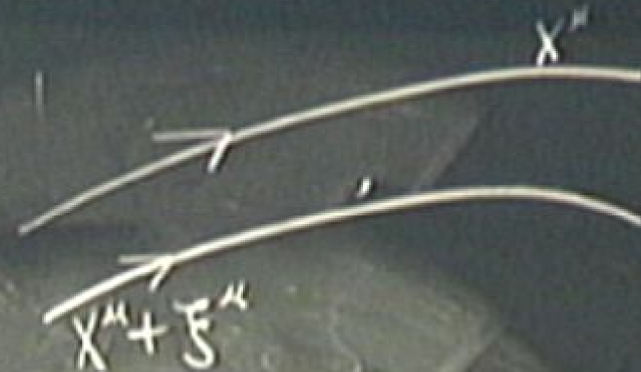
$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$



$$\nabla_u \xi^\mu \equiv \frac{d\xi^\mu}{d\tau} + \Gamma^\mu_{\rho\sigma}\,\xi^\rho\frac{dx^\sigma}{d\tau}$$


$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$
x^μ
$x^\mu + \xi^\mu$
$\nabla_u \xi^\mu = \frac{d\xi^\mu}{d\tau} + \Gamma^\mu_{\rho\sigma}\xi^\rho$
$\nabla_u \xi^\mu = - R^\mu \xi^\rho$

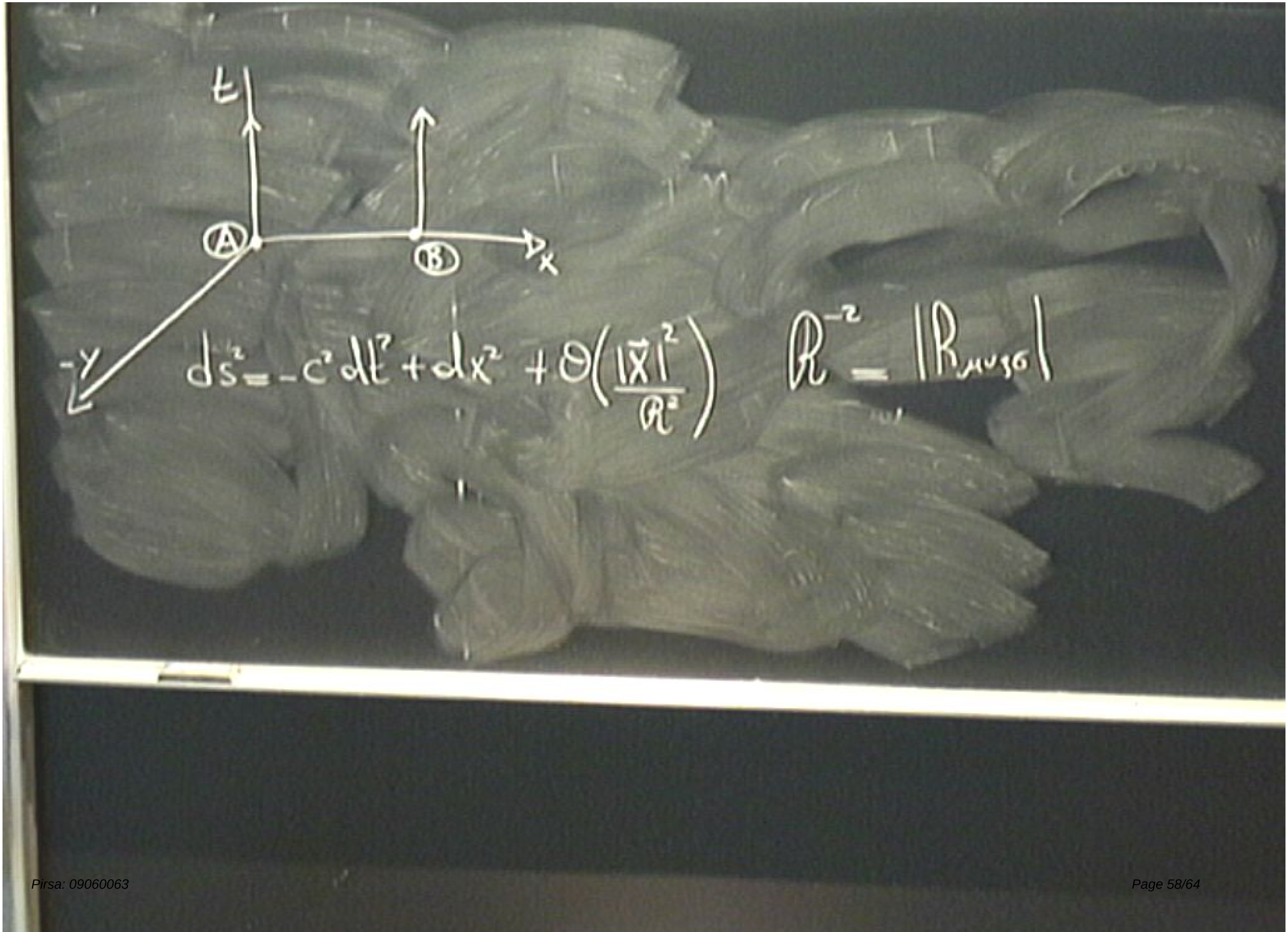
$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

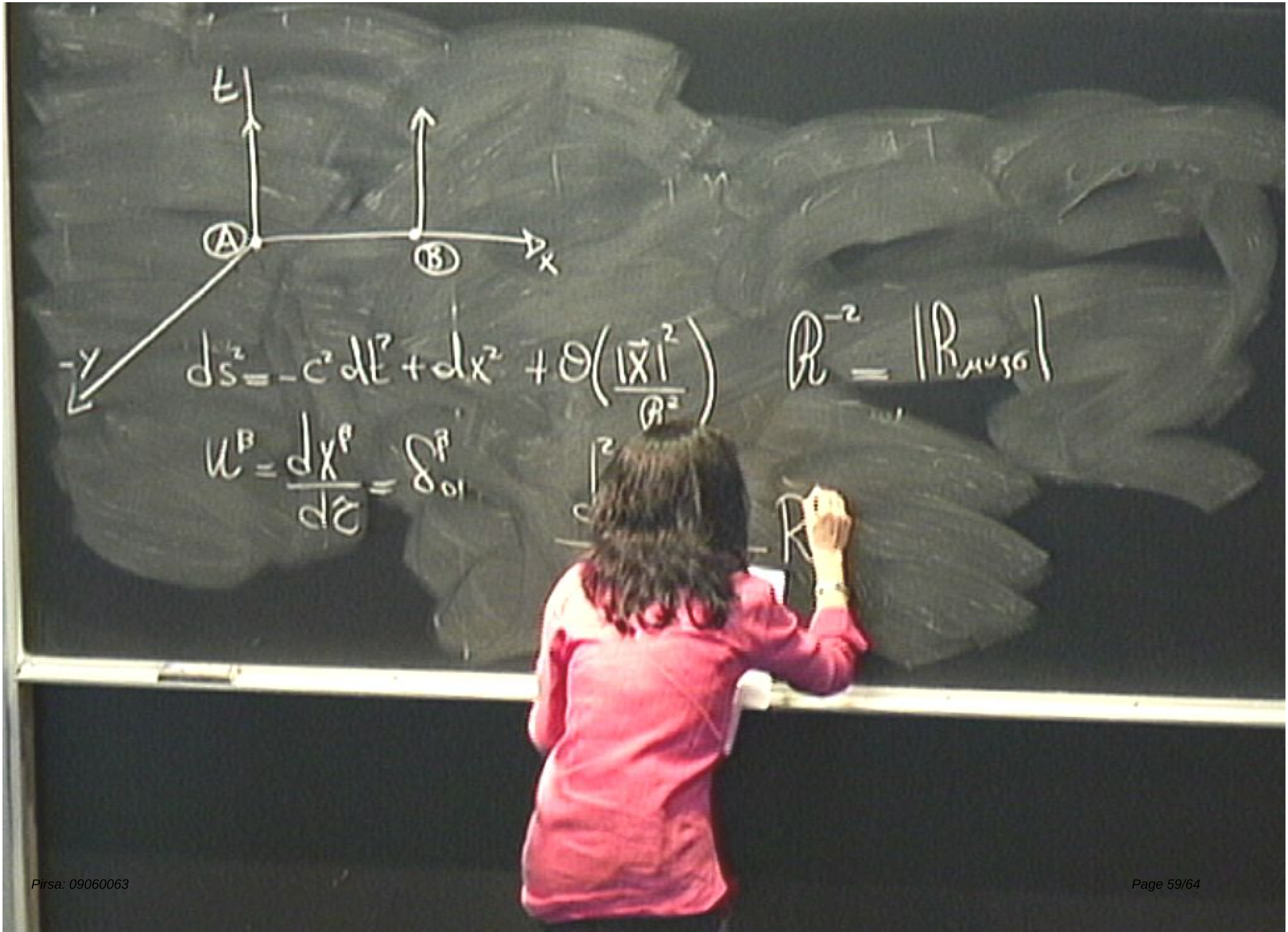


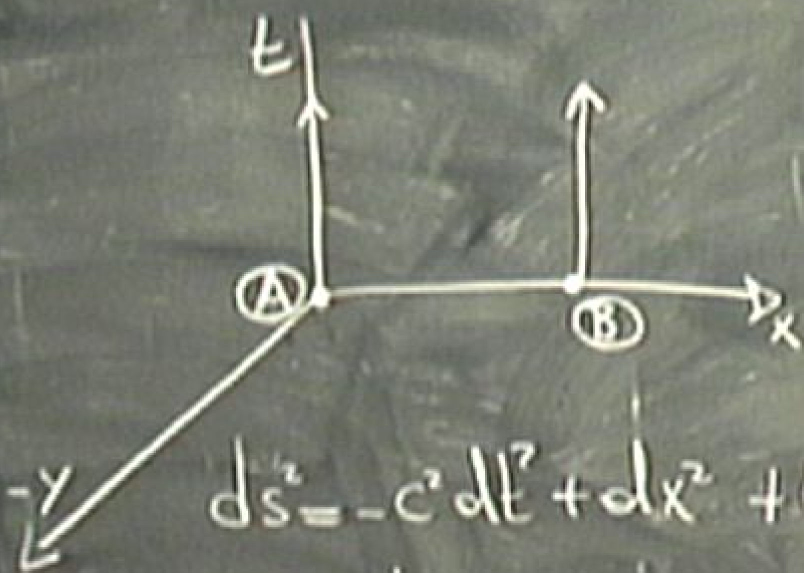
$$\nabla_u \xi^\mu = \frac{d\xi^\mu}{d\tau} + \Gamma^\mu_{\rho\sigma}\,\xi^\rho\frac{dx^\sigma}{d\tau}$$

$$\nabla_u \nabla_u \xi^\mu = -R^\mu{}_{\nu\rho\sigma}\,\xi^\rho\frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau}$$


t
A
B
x
-y


t
A
B
x
-y
$ds^2 = -c^2 dt^2 + dx^2 + O\left(\frac{|\vec{x}|^2}{R^2}\right)$
$R^{-2} \sim |R_{\mu\nu\rho\sigma}|$


t
A
B
x
-y
$ds^2 = -c^2 dt^2 + dx^2 + \mathcal{O}\left(\frac{|\vec{x}|^2}{R^2}\right)$
$R^{-2} = |R_{\mu\nu\rho\sigma}|$
$u^\beta = \frac{dx^\beta}{d\tau} = \delta^\beta_0$



$$ds^2 = -c^2 dt^2 + dx^2 + O\left(\frac{|\vec{x}|^2}{\mathcal{R}^2}\right) \qquad \mathcal{R}^{-2} = |R_{\mu\nu\rho\sigma}|$$

$$u^\beta = \frac{dx^\beta}{d\tau} = \delta^\beta_0$$

$$\boxed{\frac{d^2\xi^\beta}{dt^2} = -R^\alpha{}_{0\kappa 0}\,\xi^\kappa}$$

$$R^{TT}_{j0k0} = -\frac{1}{2}\ddot{h}^{TT}_{jk}$$

$$ds^2 = -c^2 dt^2 + dx^2 + O\left(\frac{|\vec{x}|^2}{\mathcal{R}^2}\right) \qquad \mathcal{R}^{-2} = |R_{\mu\nu\rho\sigma}|$$

$$u^\beta = \frac{dx^\beta}{d\tau} = \delta^\beta_0$$

$$\boxed{\frac{d^2\xi^j}{dt^2} = \frac{1}{2}\ddot{h}_{jk}\,\xi^k}$$

$$R^{TT}_{j0k0} = -\frac{1}{2}\ddot{h}^{TT}_{jk}$$

$$ds^2 = -c^2 dt^2 + dx^2 + \mathcal{O}\left(\frac{|\vec{x}|^2}{\mathcal{R}^2}\right) \qquad \mathcal{R}^{-2} = |R_{\mu\nu\rho\sigma}|$$

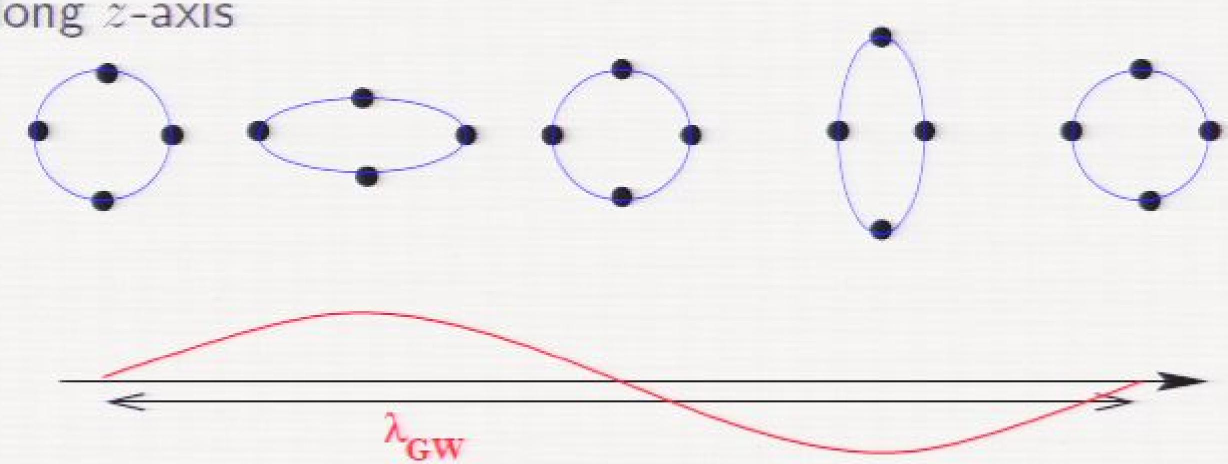
$$u^\beta = \frac{dx^\beta}{d\tau} = \delta^\beta_0$$

$$\boxed{\frac{d^2\xi^j}{dt^2} = \frac{1}{2}\ddot{h}_{jk}\,\xi^k}$$

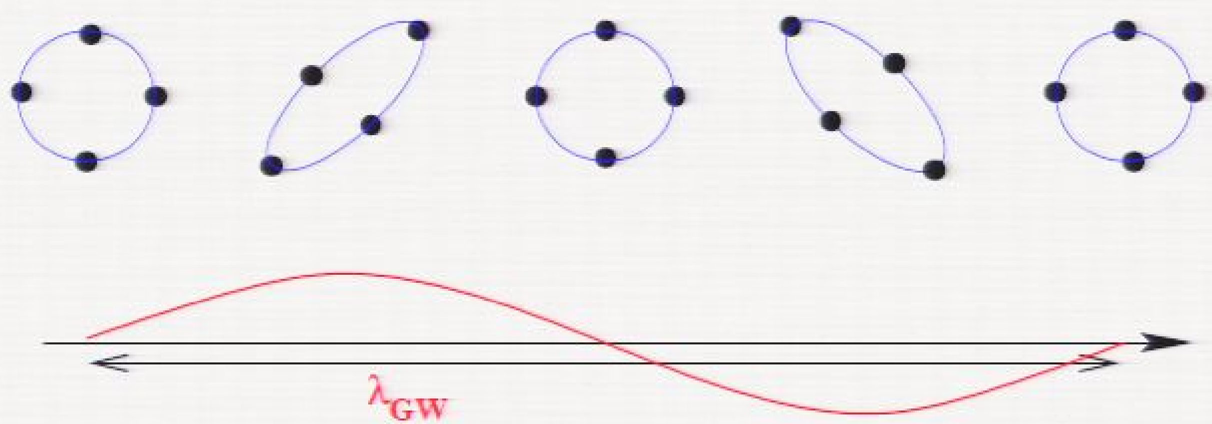
Interaction between GW and ring of free-falling particles

GW propagating along z-axis

- Case: $h_+ \neq 0$
  $h_\times = 0$

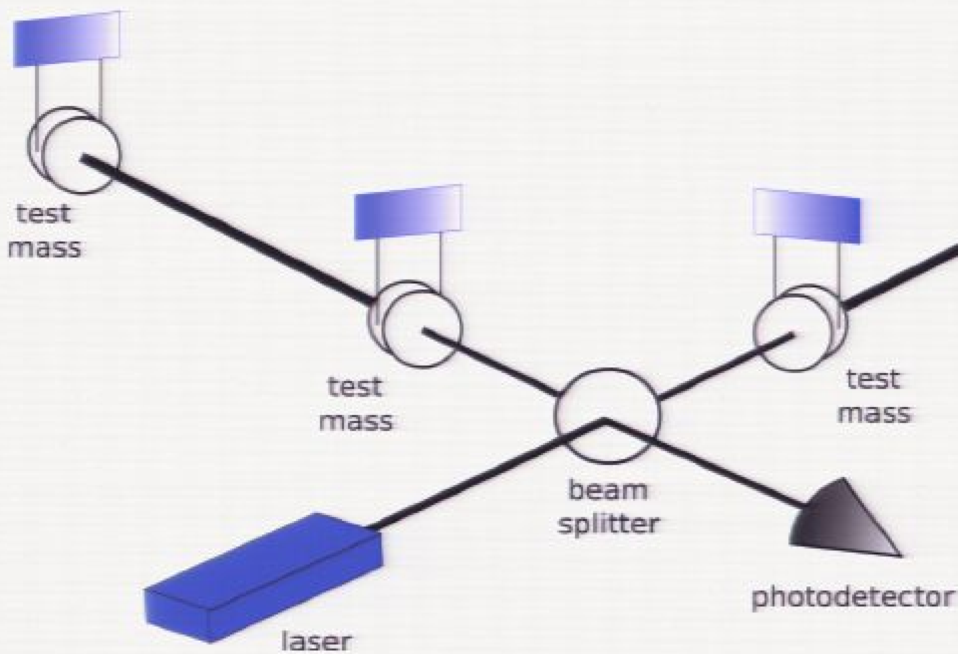


- Case: $h_\times \neq 0$
  $h_+ = 0$



How to measure gravitational waves

Use light beams to measure the stretching and squeezing induced by GWs



$\Delta L = L\,h \sim 10^{-16}$ cm

being $L = 4$km and $h \sim 10^{-21}$

$\Delta\phi \sim 10^{-8}$ rad