Title: Quantum State and Process Measurement and Characterization

Date: Jun 03, 2009 04:00 PM

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Abstract: This talk will present an overview of work done in the past decade on quantum state and process tomography, describing the basic notions at an introductory level, and arguing for a pragmatic approach for data reconstruction. The latest results include recent numerical comparison of different reconstruction techniques, aimed at answering the question: "is 'the best' the enemy of 'good enough'?"

Quantum State and Process Measurement and Characterization

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3 June 2009

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title page from last time I talked on this topic:

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Universität Innsbruck
5 May 2004

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ONTARIO PHOTONICS CONSORTIUM

State of a Single Qubit

- Photon polarization based qubits

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

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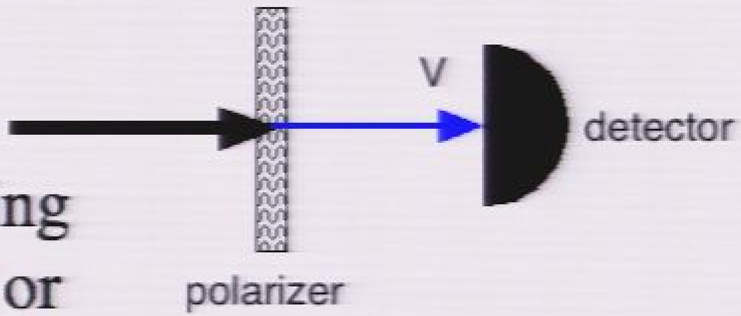

- Measure Single Copy by projecting on to $|V\rangle\langle V|$: get answer "click" or "no click"
  - *One bit of information about α and β: you know one of them is non-zero*

State of a Single Qubit

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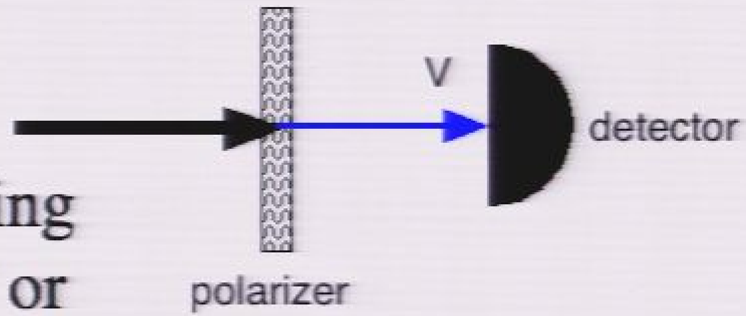
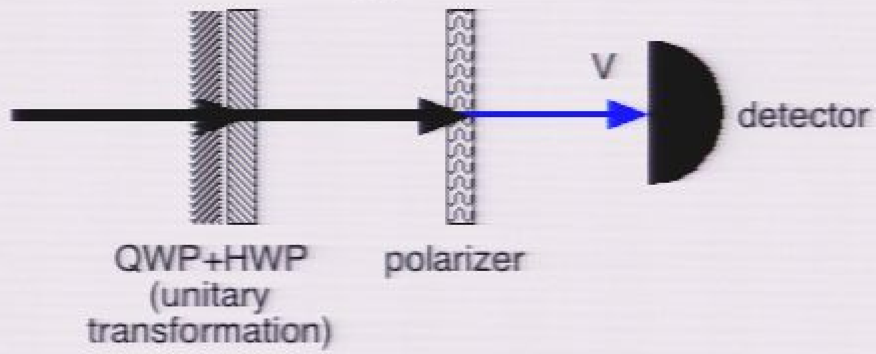


- Measure Single Copy by projecting on to $|V\rangle\langle V|$: get answer "click" or "no click"
  - *One bit of information about α and β: you know one of them is non-zero*
- Measure multiple (assumed identical) copies: frequency of "clicks" gives estimate of $|\beta|^2$

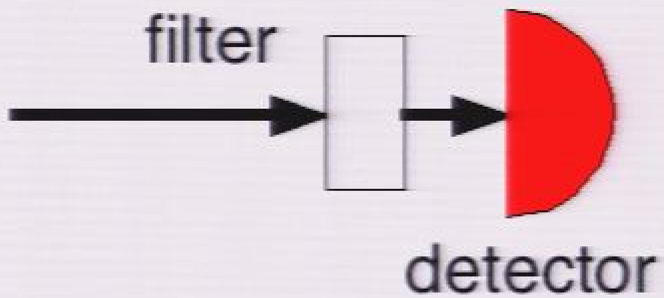
- Find relative phase of α and β by performing a unitary operation before beam splitter:



e.g.: $|\psi\rangle \rightarrow |\psi'\rangle = \frac{1}{\sqrt{2}}(\alpha+\beta)|H\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)|V\rangle$

- Frequency of "clicks" now gives an estimate of $\frac{1}{2}|\alpha-\beta|^2$
- Systematic way of getting all the data needed….

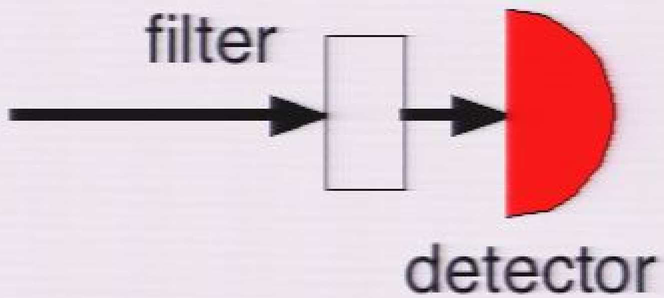
Stokes Parameters



Measure intensity with four different filters:

G. G. Stokes, *Trans Cambridge Philos Soc* **9** 399 (1852)

Stokes Parameters



Measure intensity with four different filters:

(i) 50% intensity $n_0 = \frac{N}{2}\ \mathrm{Tr}\{\rho\} = \frac{N}{2}\ \mathrm{Tr}\{\Pi_0 \rho\}$

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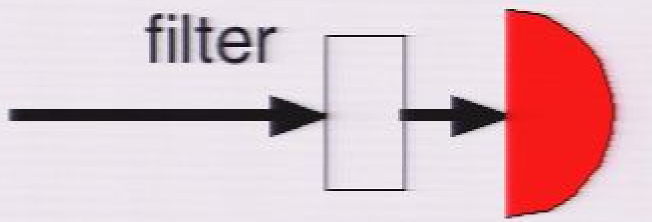
Stokes Parameters



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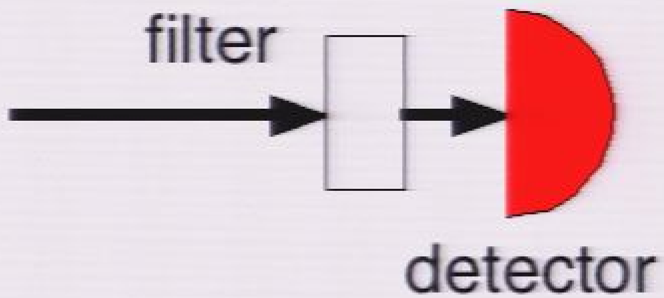

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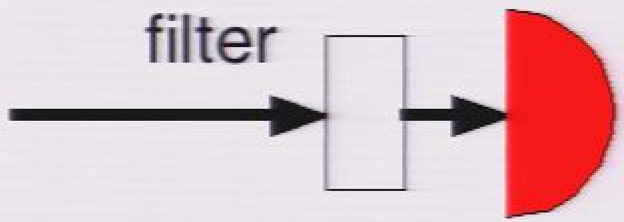
Stokes Parameters



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(iii) 45° polarizer $n_2 = N\ \mathrm{Tr}\{|D\rangle\langle D|\rho\} \equiv N\ \mathrm{Tr}\{\Pi_2 \rho\}$

(iv) RCP $n_3 = N\ \mathrm{Tr}\{|R\rangle\langle R|\rho\} = N\ \mathrm{Tr}\{\Pi_3 \rho\}$

G. G. Stokes, *Trans Cambridge Philos Soc* **9** 399 (1852)

Stokes Parameters

$$S_0 = 2n_0 = N\left(\langle R|\rho|R\rangle + \langle L|\rho|L\rangle\right)$$

$$S_1 = 2(n_1 - n_0) = N\left(\langle R|\rho|L\rangle + \langle L|\rho|R\rangle\right)$$

$$S_2 = 2(n_2 - n_0) = iN\left(\langle R|\rho|L\rangle - \langle L|\rho|R\rangle\right)$$

$$S_3 = 2(n_3 - n_0) = N\left(\langle R|\rho|R\rangle - \langle L|\rho|L\rangle\right)$$

- These 4 parameters completely specify polarization of beam
- Beam is an ensemble of photons…

$$\rho = \frac{1}{2}\sum_{i=0}^{4}\frac{S_i}{S_0}\sigma_i$$

Pauli matrices

2 Qubit Quantum States

- Pure states
  - *Ideal case*

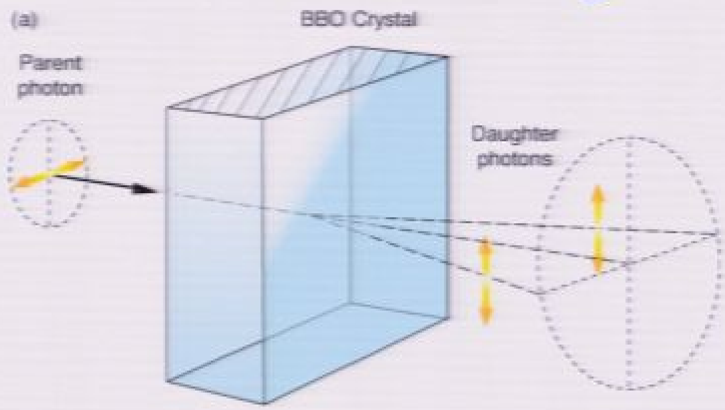
$$|\psi\rangle = a|HH\rangle + b|HV\rangle + c|VH\rangle + d|VV\rangle$$

- Mixed states
  - *Quantum state is random: need <u>averages</u> and <u>correlations</u> of coefficients*

$$\text{Density Matrix } \rho = \begin{pmatrix} \overline{|a|^2} & \overline{a^*b} & \overline{a^*c} & \overline{a^*d} \\ \overline{b^*a} & \overline{|b|^2} & \overline{b^*c} & \overline{b^*d} \\ \overline{c^*a} & \overline{c^*b} & \overline{|c|^2} & \overline{c^*d} \\ \overline{d^*a} & \overline{d^*b} & \overline{d^*c} & \overline{|d|^2} \end{pmatrix}$$

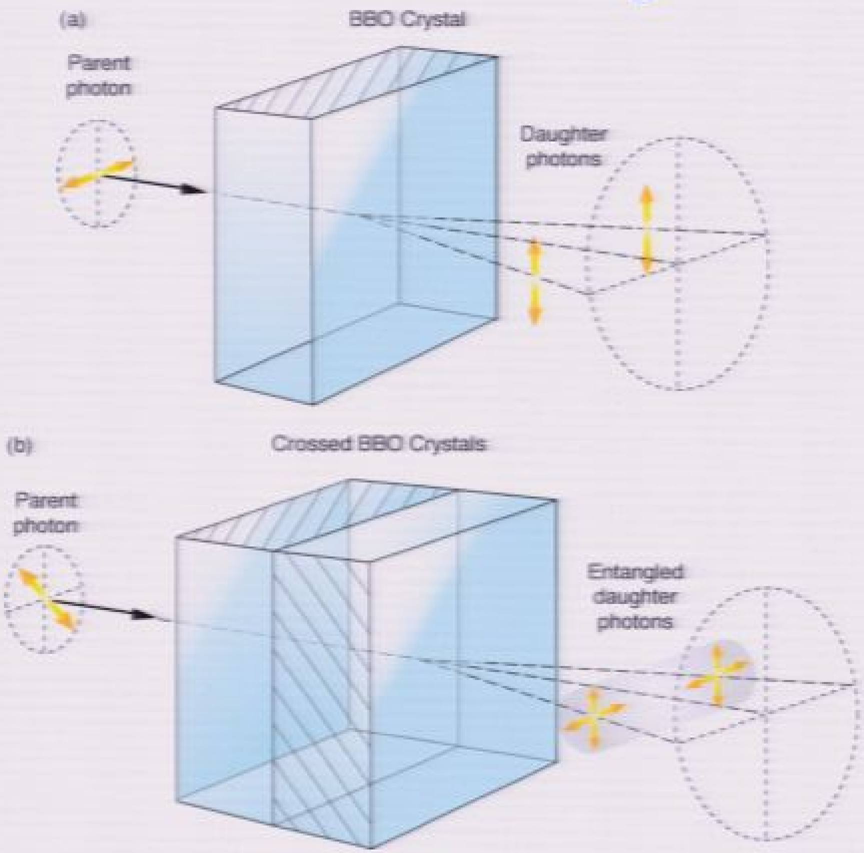
State Creation by OPDC

State Creation by OPDC



$$|H\rangle \rightarrow |VV\rangle$$

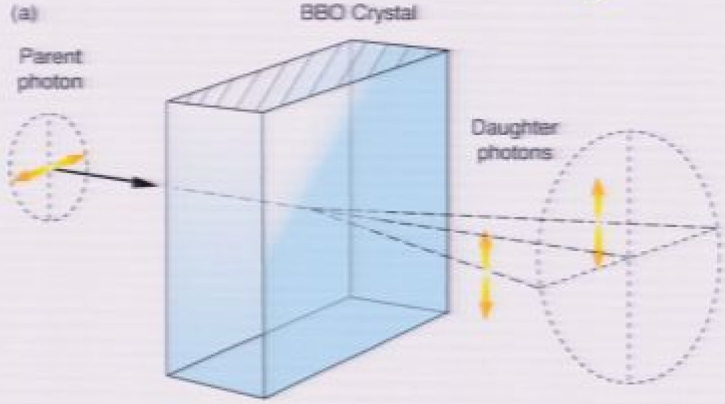
State Creation by OPDC



$$|H\rangle \rightarrow |VV\rangle$$

$$\cos\theta|H\rangle + e^{i\phi}\sin\theta|V\rangle \rightarrow$$
$$\cos\theta|VV\rangle + e^{i\phi}\sin\theta|HH\rangle$$

State Creation by OPDC



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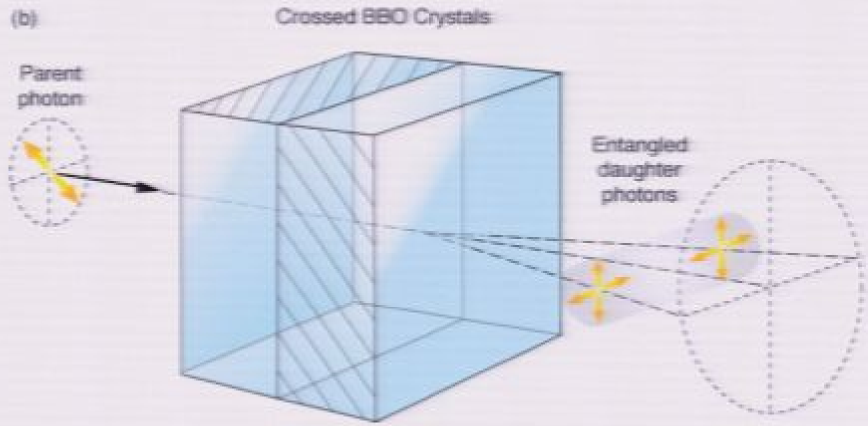


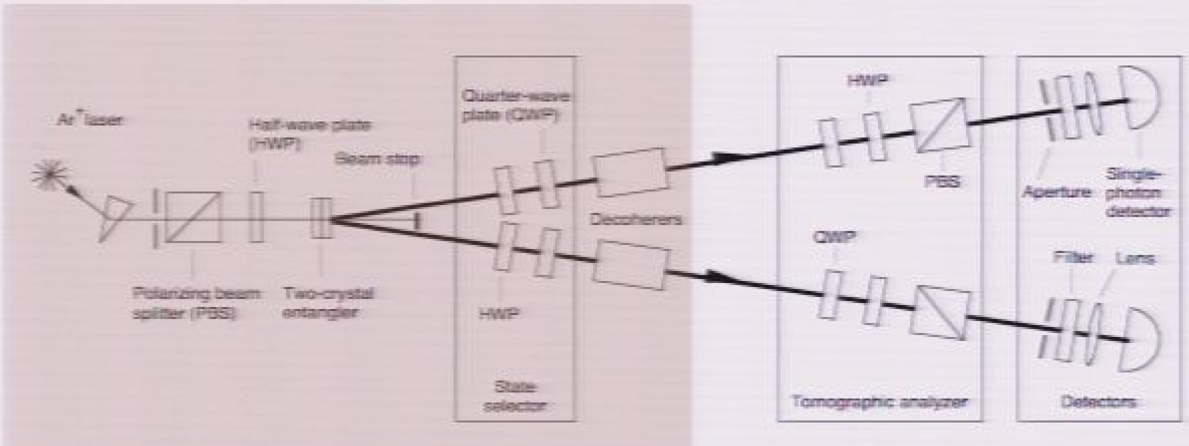
$$\cos\theta|H\rangle + e^{i\phi}\sin\theta|V\rangle \rightarrow$$
$$\cos\theta|VV\rangle + e^{i\phi}\sin\theta|HH\rangle$$

Schmidt decomposition : $|\psi\rangle = a|\phi\varphi\rangle + b|\phi_{\perp}\varphi_{\perp}\rangle$

Arbitrary state: change basis…

2 Qubit Quantum State Tomography

source



measurement

Coincidence Rate measurements for two photons

$$n_{a,b} = N \ Tr\{(\Pi_a \otimes \Pi_b)\rho\}$$

Linear combination of $n_{a,b}$ yields the *two-photon Stokes parameters*:

$$S_{a,b} = N \; Tr\{(\sigma_a \otimes \sigma_b)\rho\}$$

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From the two-photon Stokes parameters, we can get an estimate of the *density matrix*:

$$\rho = \sum_{a,b=0}^{3} \frac{S_{a,b}}{S_{0,0}} (\sigma_a \otimes \sigma_b)$$

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- **Doesn't give the right answer….**

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Maximum Likelihood Tomography*

- Maximum Likelihood fit to "physical" density matrix
  - *Density matrix must be Hermitian, normalized, **non-negative***
  - *Numerically Minimize the function:*

$$\chi^2(t_1, t_2, \ldots t_{16}) = \sum_{a,b=0}^{3} \left(\mathrm{Tr}\{\rho(t_1, t_2, \ldots t_{16})(\Pi_a \otimes \Pi_b)\} - n_{a,b} \right)^2 / n_{a,b}$$

* D. F. V. James, et al., *Phys Rev A* **64**, 052312 (2001).

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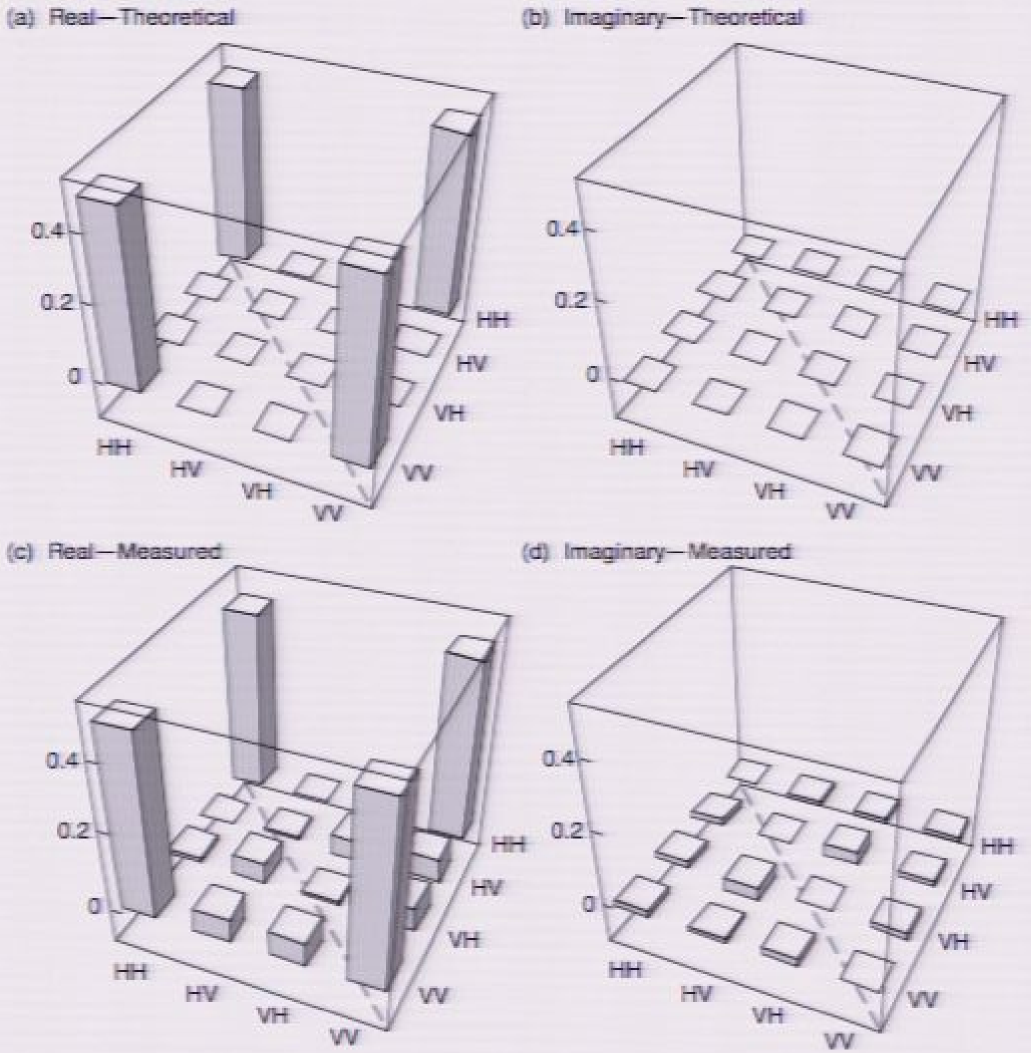
  - *where:* $\rho = \mathrm{TT}^\dagger / Tr\{\mathrm{TT}^\dagger\}$ *and*

$$T(t_1,t_2,\ldots t_{16}) = \begin{pmatrix} t_1 & 0 & 0 & 0 \\ t_5 + it_6 & t_2 & 0 & 0 \\ t_{11} + it_{12} & t_7 + it_8 & t_3 & 0 \\ t_{15} + it_{16} & t_{13} + it_{14} & t_9 + it_{10} & t_4 \end{pmatrix}$$

* D. F. V. James, et al., *Phys Rev A* **64**, 052312 (2001).

Example: Measured Density Matrix



Quantum State Tomography I

- Sublevels of Hydrogen (partial) (*Ashburn et al, 1990*)
- Optical mode (*Raymer et al., 1993*)
- Molecular vibrations (*Walmsley et al, 1995*)
- Motion of trapped ion (*Wineland et al., 1996*)
- Motion of trapped atom (*Mlynek et al., 1997*)
- Liquid state NMR (*Chaung et al, 1998*)
- Entangled Photons (*Kwiat et al, 1999*)*
- Entangled ions (*Blatt et al., 2002; 8 ions: 2005*)
- Superconducting qubits (*Martinis et al., 2006)*

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*A. G. White, D. F. V. James, P. H. Eberhard and P. G. Kwiat, *Phys Rev Lett* **83**, 3103 (1999).

Characterizing the State

Purity

$$\textit{Entropy} \quad S = -Tr\{\rho \ln \rho\} = -\sum_i \lambda_i \ln \lambda_i$$

$$\textit{"Linear Entropy"} \quad \tfrac{4}{3}\left(1 - Tr\{\rho^2\}\right)$$

Fidelity: how close are two states?

Pure states: $F(\psi_1, \psi_2) = |\langle \psi_1 | \psi_2 \rangle|^2$

Mixed states: $Tr\{\rho_1 \rho_2\}$ doesn't work: $Tr\{\rho^2\} \neq 1$

$$F(\rho_1, \rho_2) = \left[Tr\left\{\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}\right\}\right]^2$$

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Measures of Entanglement

- Pure states

$$|\psi\rangle = \alpha|HH\rangle + \beta|HV\rangle + \gamma|VH\rangle + \delta|VV\rangle$$

- How much entanglement is in this state?
  - *Entropy of reduced density matrix of one photon*

$$E = -Tr\{\rho_A \ln(\rho_A)\} \quad \text{where} \quad \rho_A = Tr_B\{|\psi\rangle\langle\psi|\}$$

  - *Concurrence:*

$$C = 2|\alpha\delta - \beta\gamma|$$

  - *Concurrence is equivalent to Entanglement :*

$$E = h\left[\left(1-\sqrt{1-C^2}\right)/2\right], \text{ where } h[x] = -x\ln(x) - (1-x)\ln(1-x)$$

  - *C=0 implies separable state*
  - *C=1 implies maximally entangled state (e.g. Bell states)*

Entanglement in Mixed States

- Mixed states can be de-composed into incoherent sums of pure states:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

- "Average" Concurrence: dependent on decomposition

$$\overline{C} = \sum_i p_i C(\psi_i)$$

- "Minimized Average Concurrence":

$$\overline{C}_{min} = \min_{\{\psi_i\}} \sum_i p_i C(\psi_i)$$

  - *Independent of decomposition*
  - *C=0 implies separable state*
  - *C=1 implies maximally entangled state (e.g. Bell states)*
  - *Analytic expression (Wootters, '98) makes things very convenient!*

Two Qubit Mixed State Concurrence

$$R = \rho\Sigma\rho^T\Sigma$$

Transpose (in computational basis)

"spin flip matrix"

$$\Sigma = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues of R (in decreasing order)

$$C = \mathrm{Max}\left[\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0\right]$$

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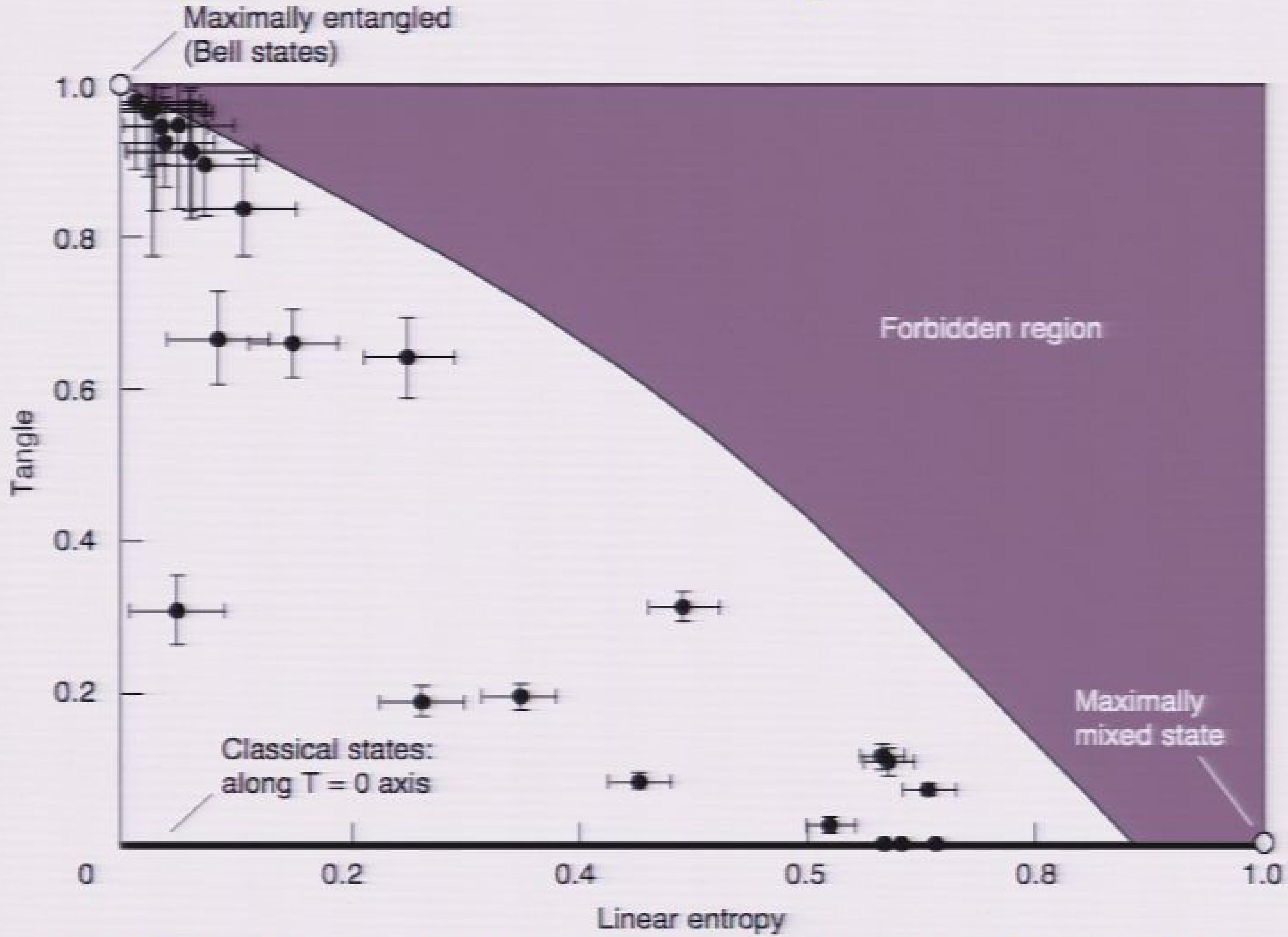
"Map" of Hilbert Space*



* D.F.V. James and P.G .Kwiat, *Los Alamos Science*, 2002

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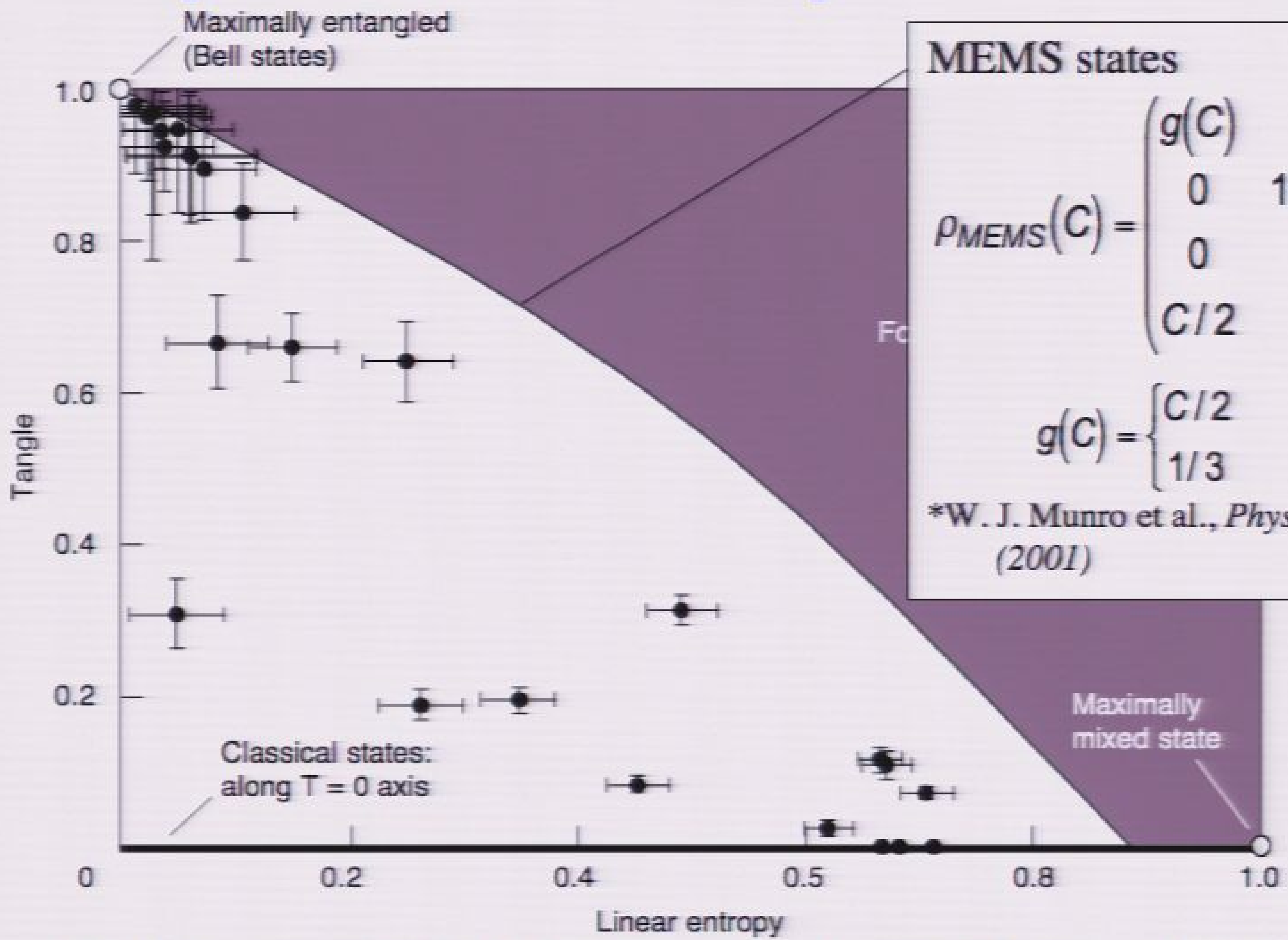

MEMS states

$$\rho_{MEMS}(C) = \begin{pmatrix} g(C) & 0 & 0 & C/2 \\ 0 & 1-2g(C) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C/2 & 0 & 0 & g(C) \end{pmatrix}$$

$$g(C) = \begin{cases} C/2 & \text{if } C \geq 2/3 \\ 1/3 & \text{if } C < 2/3 \end{cases}$$

*W. J. Munro et al., *Phys Rev A* ***64***, *030302-1 (2001)*

* D.F.V. James and P.G .Kwiat, *Los Alamos Science*, 2002

Process Tomography

- **Trace Preserving Completely Positive Maps**: Every thing that could possibly happen to a quantum state

$$\rho' = \sum_i E_i \rho E_i^\dagger; \qquad \sum_i E_i^\dagger E_i = I$$

"operator-sum formalism"
"Kraus operators"

set of basis matrices, e.g.:

$$\Gamma_\nu = \frac{1}{2}\sigma_{m(\nu)} \otimes \sigma_{n(\nu)}$$

$$\Gamma_1 = \frac{1}{2}\sigma_0 \otimes \sigma_1, \quad \Gamma_2 = \frac{1}{2}\sigma_0 \otimes \sigma_2, \quad \Gamma_4 = \frac{1}{2}\sigma_1 \otimes \sigma_0, etc, etc.$$

Trace orthogonality: $Tr\{\Gamma_\mu \Gamma_\nu\} = \delta_{\nu,\mu}$

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Decompose the Kraus operators: $E_i = \sum_\nu \varepsilon_{i,\nu}\Gamma_\nu$

then-

$$\rho' = \sum_{\mu,\nu} \chi_{\mu\nu}\Gamma_\mu \rho \Gamma_\nu$$

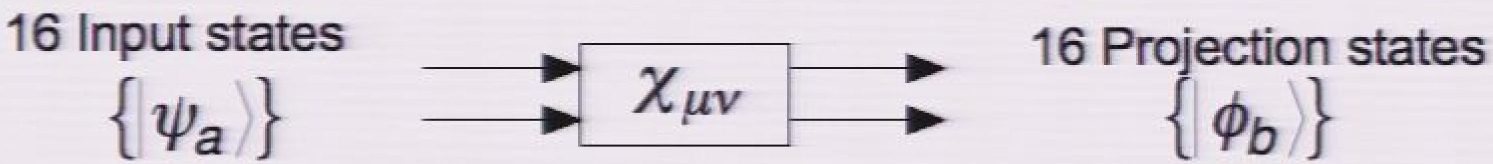
where-

$$\sum_i \varepsilon_{i,\nu}\varepsilon^*{}_{i,\mu} = \chi_{\nu\mu}$$

χ is a Hermitian, positive 16x16 matrix("error correlation matrix"), with the constraints-

$$\sum_{\mu,\nu} \chi_{\mu\nu} Tr\{\Gamma_\mu \Gamma_\lambda \Gamma_\nu\} = Tr\{\Gamma_\lambda\}$$

Process Tomography*



- Estimate probability from counts 16x16 = 256 data:

$$p_{ab} = \sum_{\mu\nu} \chi_{\mu\nu} \langle \phi_b | \Gamma_\mu | \psi_a \rangle \langle \psi_a | \Gamma_\nu | \psi_b \rangle$$

- Recover $\chi_{\mu\nu}$ by linear inversion
  - *problematic in constraining positivity*
  - *close analogy with state tomography…*

*I. L. Chuang and M. A. Nielsen, *J. Mod Op.* **44**, 2455 (1997)

Maximum Likelihood Process Tomography

- Numerically optimize

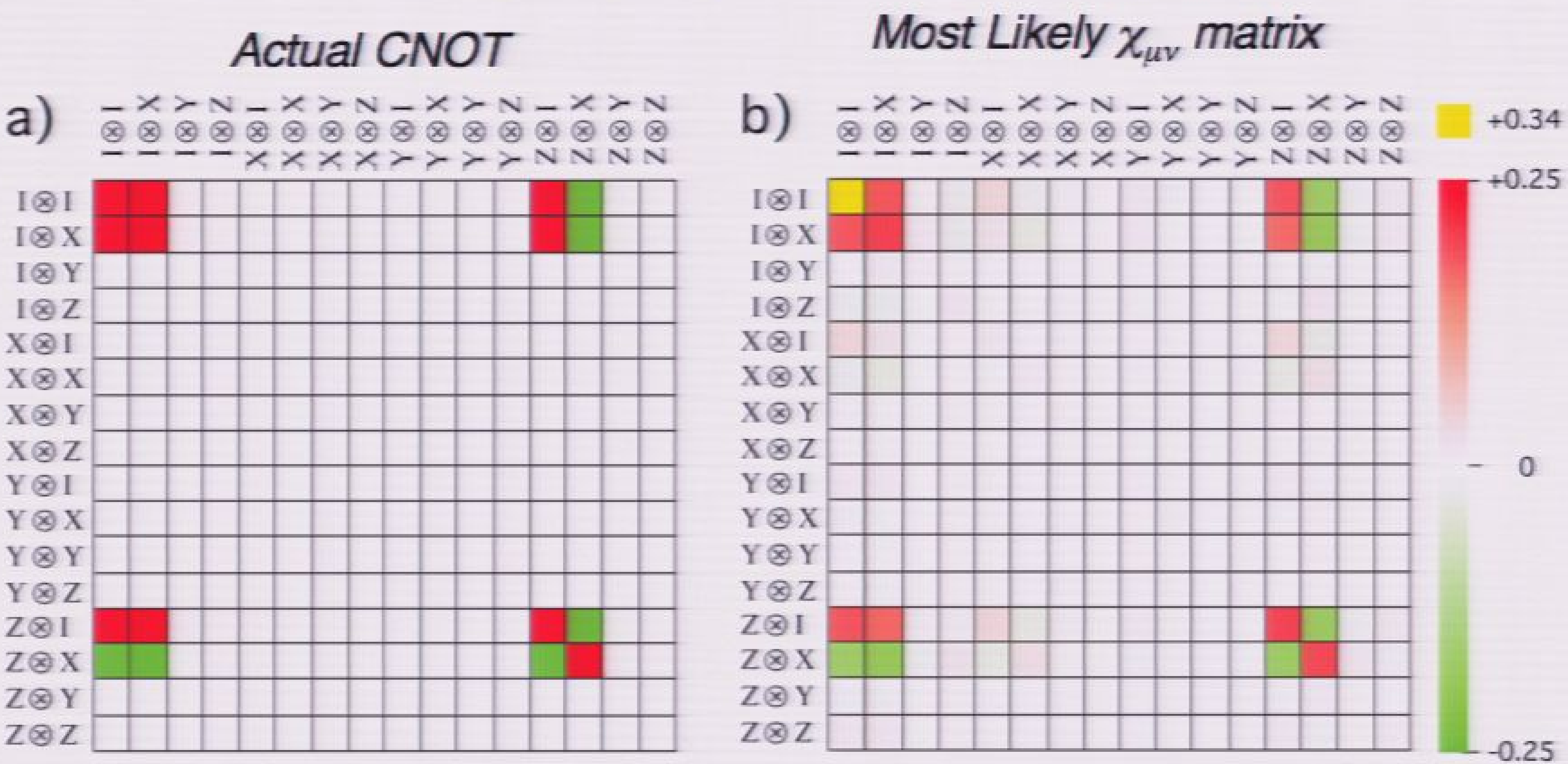
$$\chi^2(t_1,\dots t_{256}) = \sum_{a,b=1}^{16} \left(\sum_{\mu\nu} \chi_{\mu\nu}(t_1,\dots t_{256}) \langle \phi_b | \Gamma_\mu | \psi_a \rangle \langle \psi_a | \Gamma_\nu | \psi_b \rangle - p_{ab} \right)^2 \Big/ p_{ab}$$

where: $\chi_{\mu\nu} = T^\dagger T$ *(256 free parameters)*

- Constraints on $\chi_{\mu\nu}$:
  - *positive*
  - *Hermitian*
  - *additional constraint for physically allowed process:*

$$\sum_{\mu,\nu} \chi_{\mu\nu} Tr\{\Gamma_\mu \Gamma_\lambda \Gamma_\nu\} = Tr\{\Gamma_\lambda\}$$

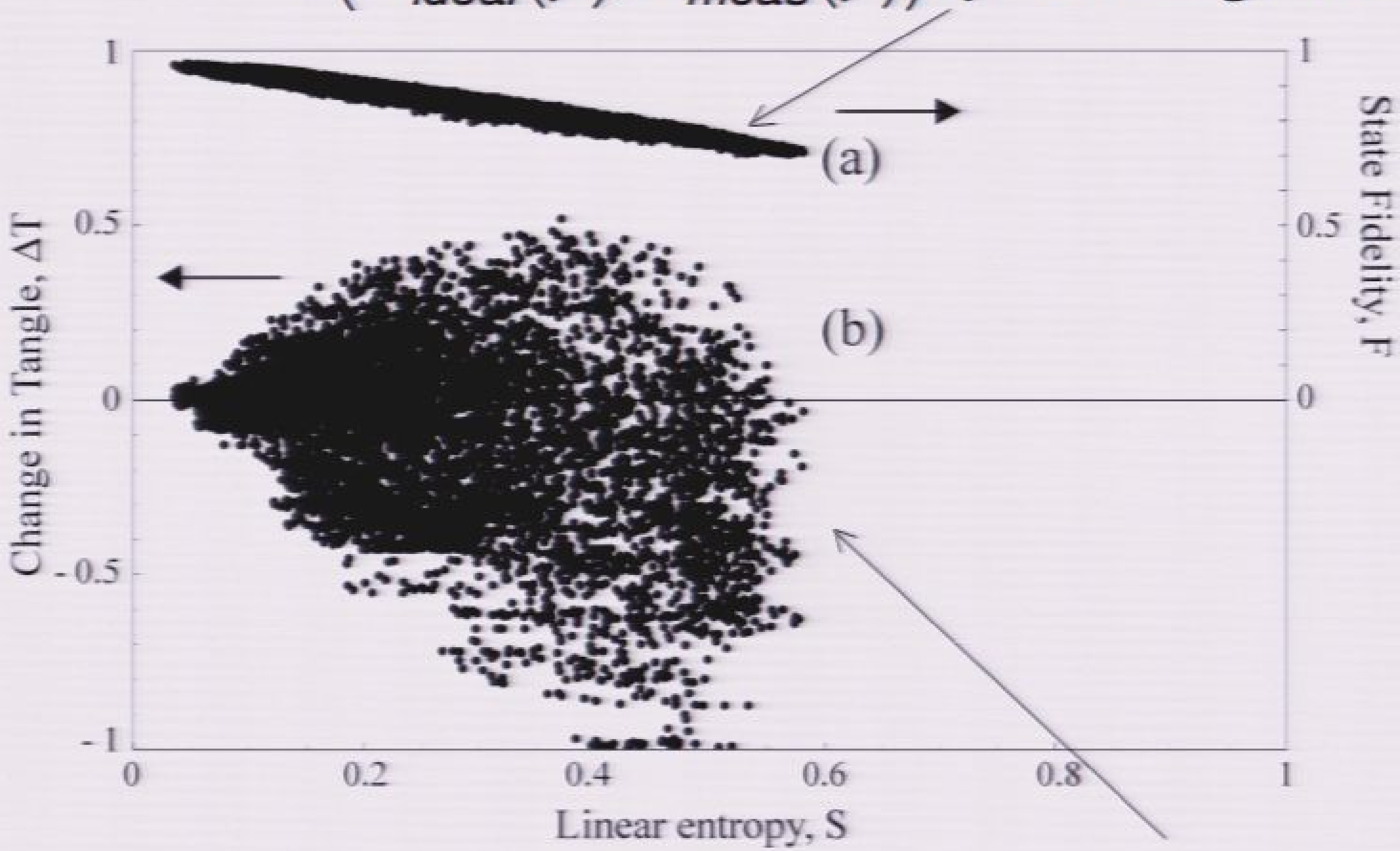
Process Tomography of UQ Optical CNOT*



*J. L. O'Brien, G. J. Pryde, A. Gilchrist, D. F. V. James, N. K. Langford, T. C. Ralph and A. G. White, "Quantum process tomography of a controlled-NOT gate," *Phys Rev Lett*, **93**, 080502 (2004); quant-ph/0402166.

Characterizing Processes

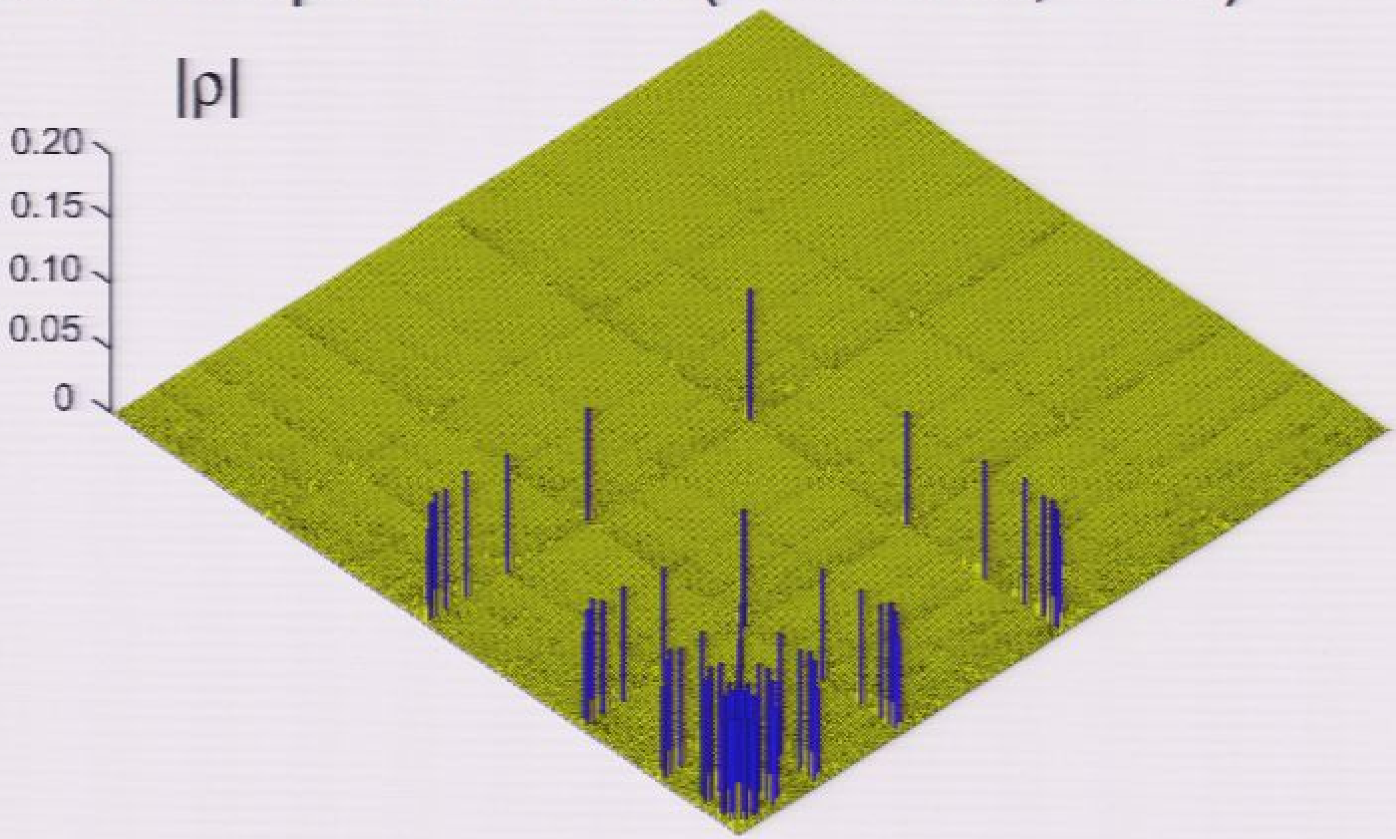
$F(\mathcal{E}_{ideal}(\rho), \mathcal{E}_{meas}(\rho))$ plotted against $S(\rho)$



$T(\mathcal{E}_{meas}(\rho)) - T(\rho)$ plotted against $S(\mathcal{E}_{meas}(\rho)) - S(\rho)$

Scalability?

• record: 8 qubit W-state (Blatt et al., 2006)



• Why not more? N qubit state tomography requires $\mathbf{4^N}$**-1** measurements (& numerical optimization in a 4^N-1 dimensional space)

Characterizing Processes

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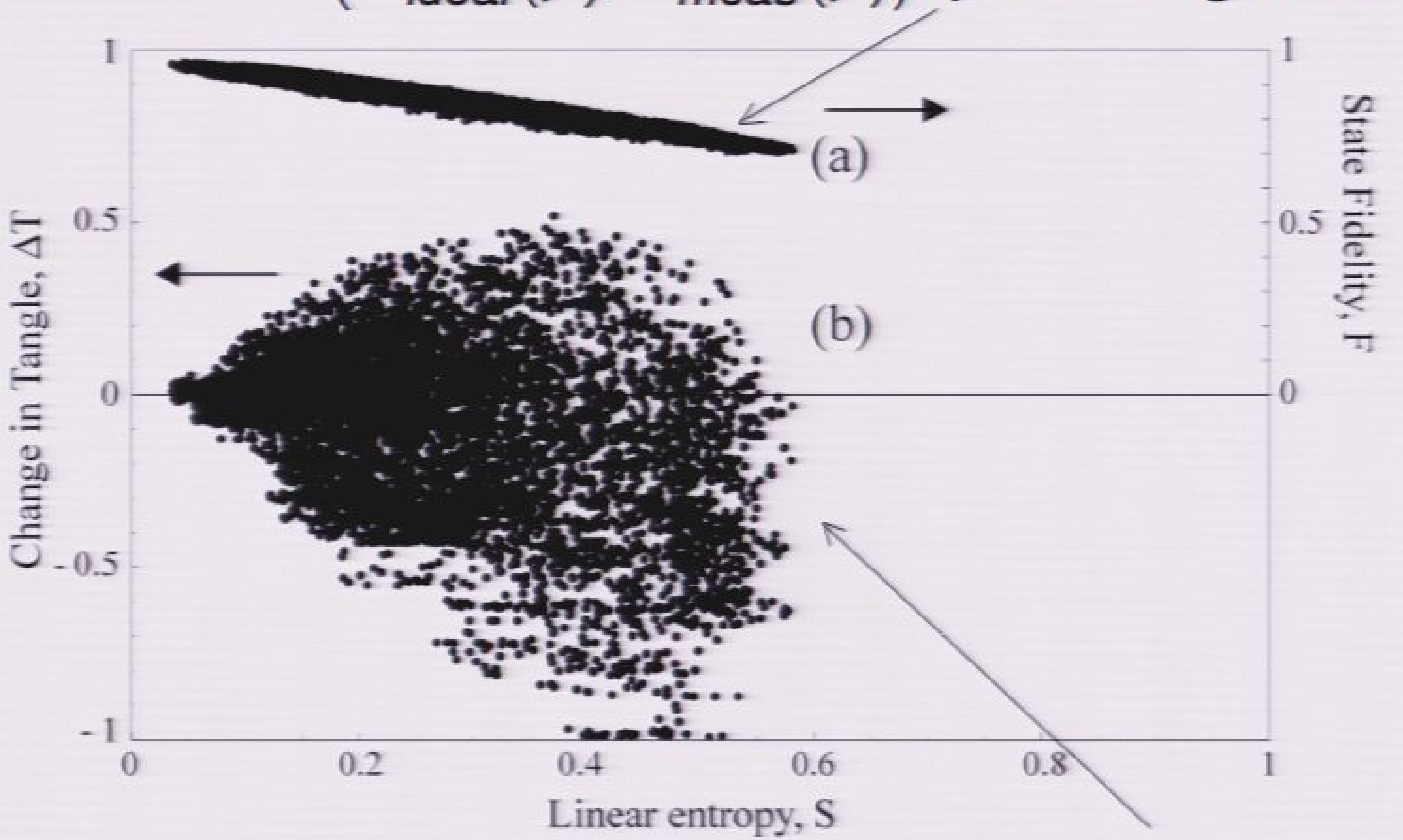

$T(\mathcal{E}_{meas}(\rho)) - T(\rho)$ plotted against $S(\mathcal{E}_{meas}(\rho)) - S(\rho)$

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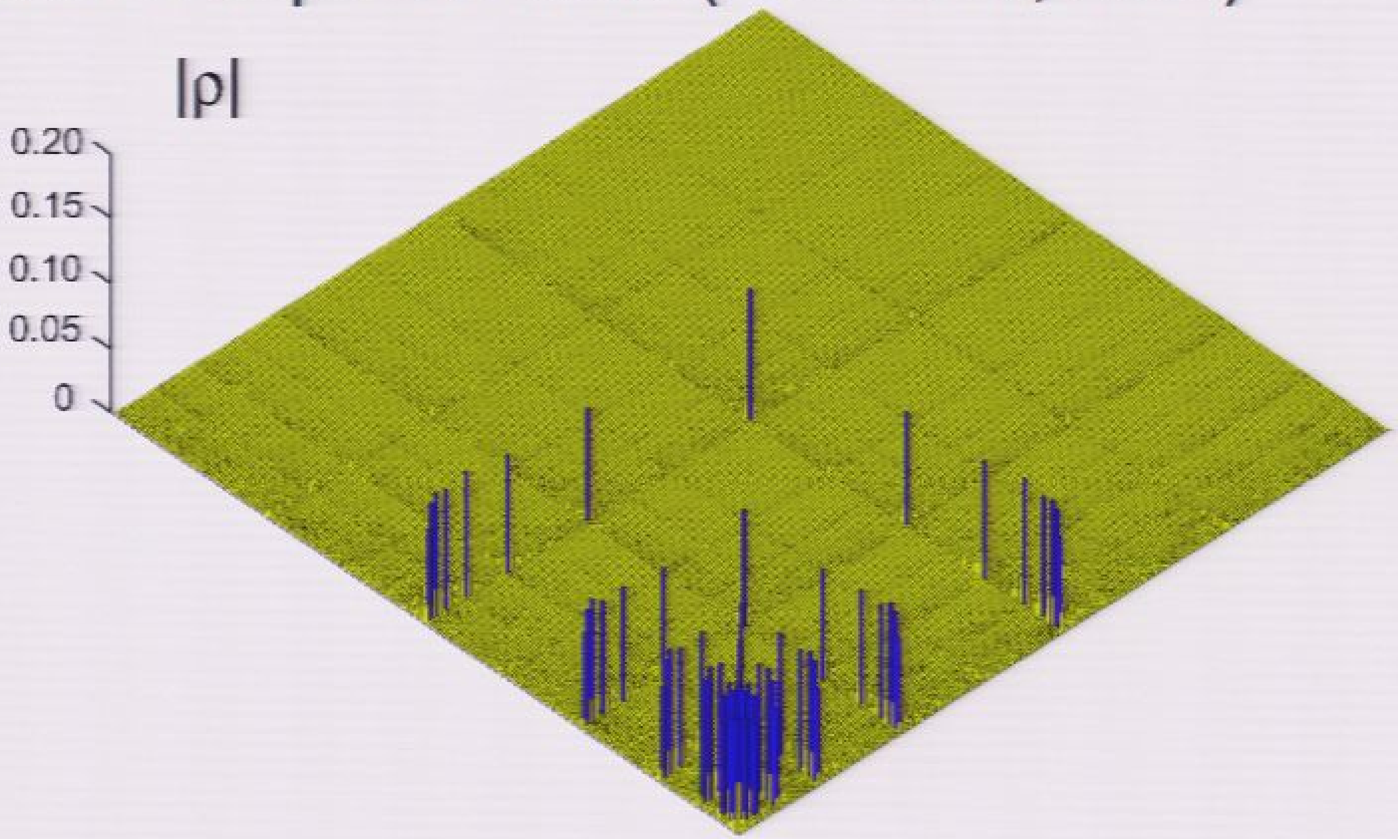


- Why not more? N qubit state tomography requires **4^N-1** measurements (& numerical optimization in a 4^N-1 dimensional space)

Fixes?

- **Measurements**: you can get a good guess at the density matrix with fewer measurements (it still requires exponential searching) (Aaronson, 2006)
- **Direct Characterization**: In some cases you can get the information you need more directly, without the tedious mucking around with the density matrix (e.g. entanglement witnesses; noise characterization)
- **Push the envelope**: How far can we go using smart computer science before we hit the wall?

Pushing the Envelope*

Max Kaznady

- **Step 1:** find a smart computer scientist:

- data storage and handling

- optimization algorithms (conjugate gradient technique implemented via matrix calculus)

  - density matrix via the Cholesky decompostion $\hat{\rho} = \hat{T}\hat{T}^{\dagger} / Tr\{\hat{T}\hat{T}^{\dagger}\}$
  - search space of $\hat{T}$ matrices to find a state $\hat{\rho}$ which 'best' fits observations using conjugate gradient technique.
  - use matrix calculus to find an *analytic* form for $\partial F(\hat{T}) / \partial\hat{T}$.
  - code implemented in matlab, see:
    http://www.physics.utoronto.ca/~dfvj/NST/index.html

* M. Kaznady and D. F. V. James, *Phys Rev A* **79**, 022109 (2009); arXiv:0809.2376

Is “the best” the enemy of “good enough”

Are we being too pedantic in looking for the optimal density matrix to fit a given data set, when a simpler numerical technique produces a good estimation (i.e. within the error bars)?

Alternatives:

- *‘quick and dirty’*: zero out the negative eigenvalues rather than perform an exhaustive optimization.

- *‘forced purity’*: we are trying to make **pure states**, so why not use that fact?

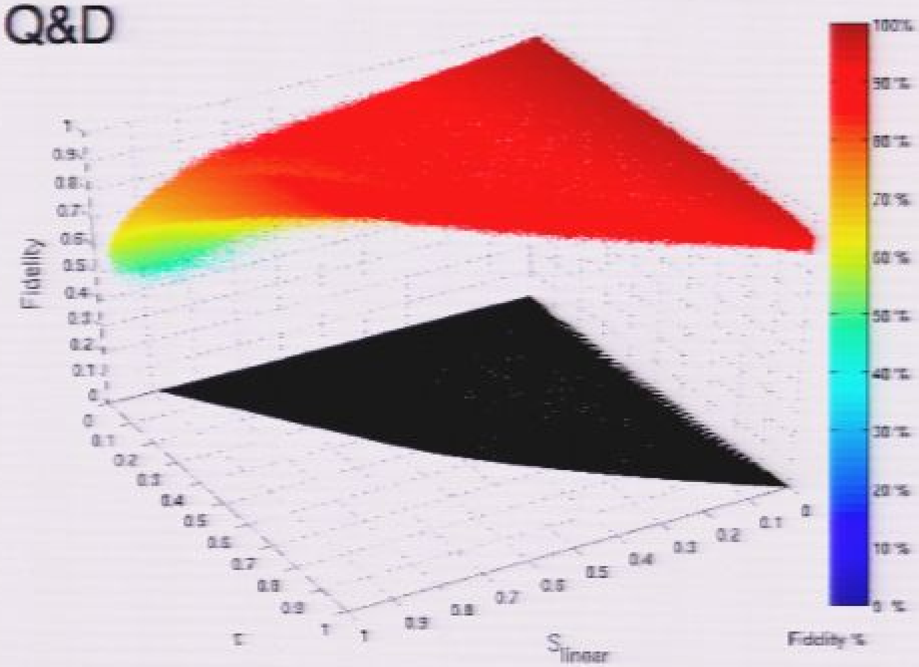
Both give positive matrices quickly: but are they the actual states in question?

Numerical experiments

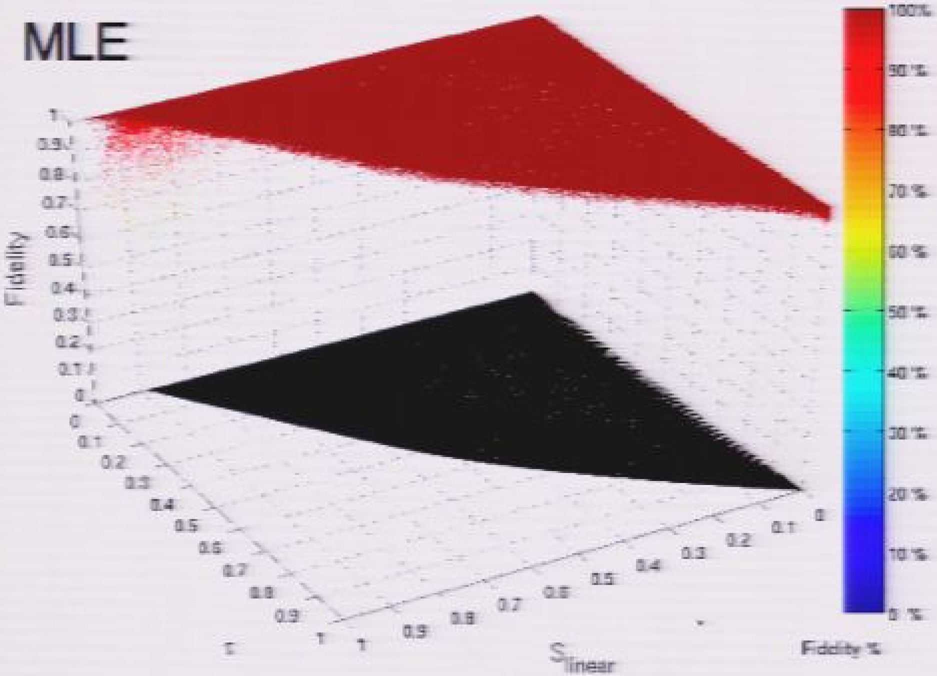
- choose a state
- simulate measurement data with a Poisson RNG
- estimate state using code
-compare estimated and actual state

- **Fidelities for 2-qubit states:**

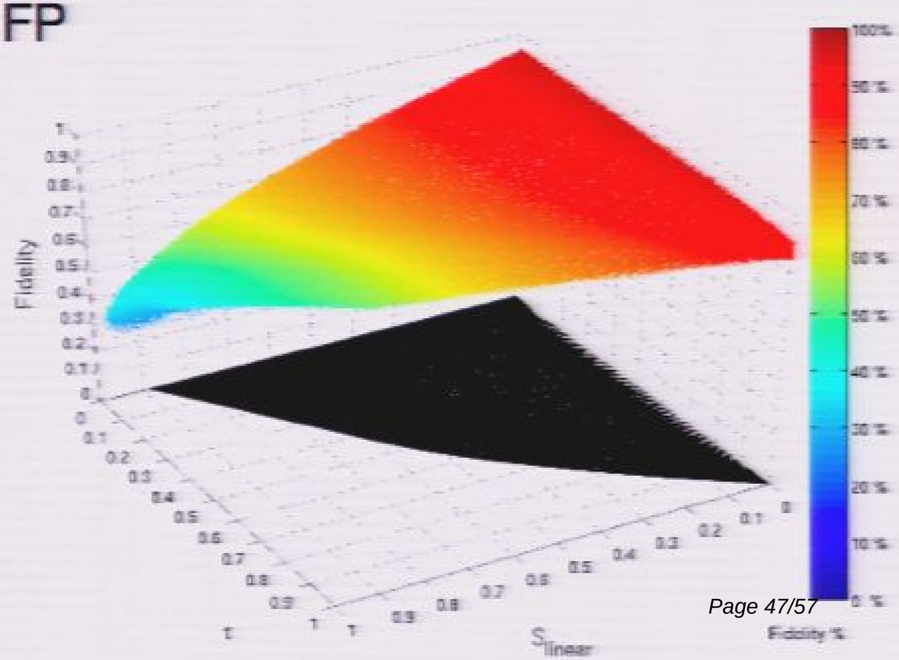
Q&D



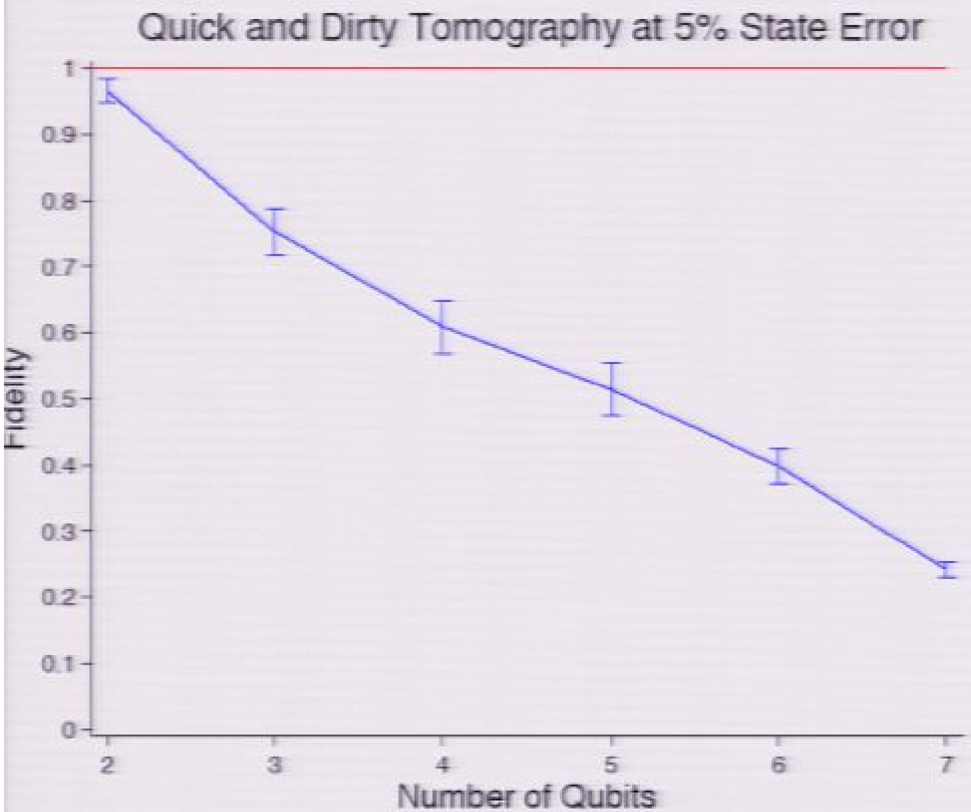
MLE



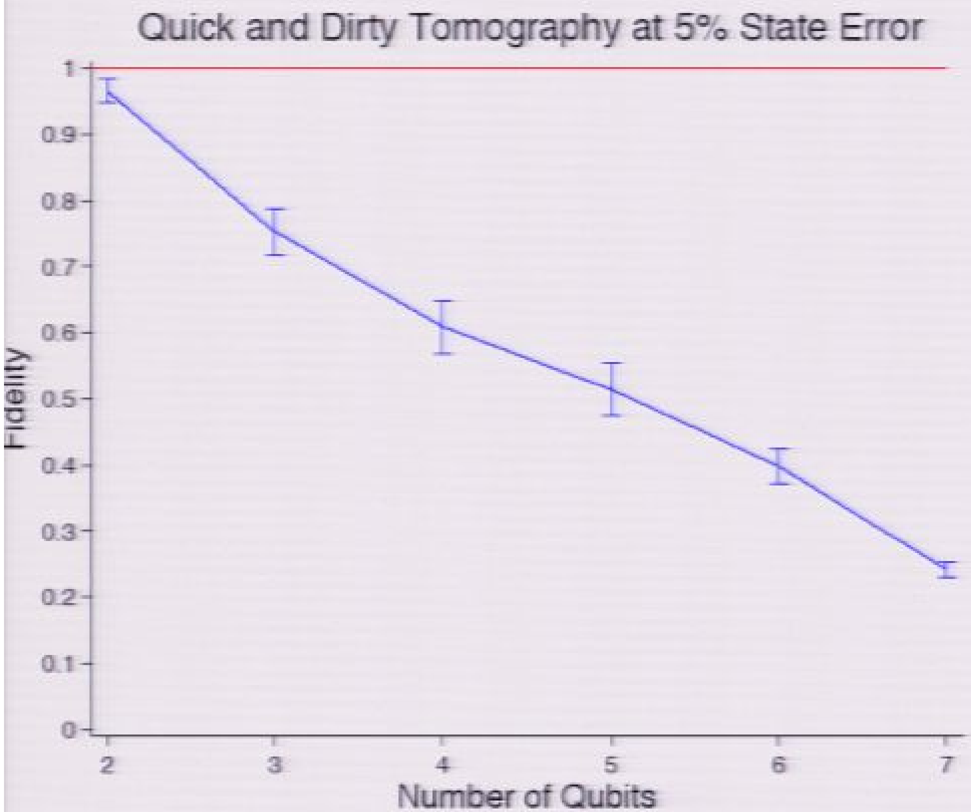
FP

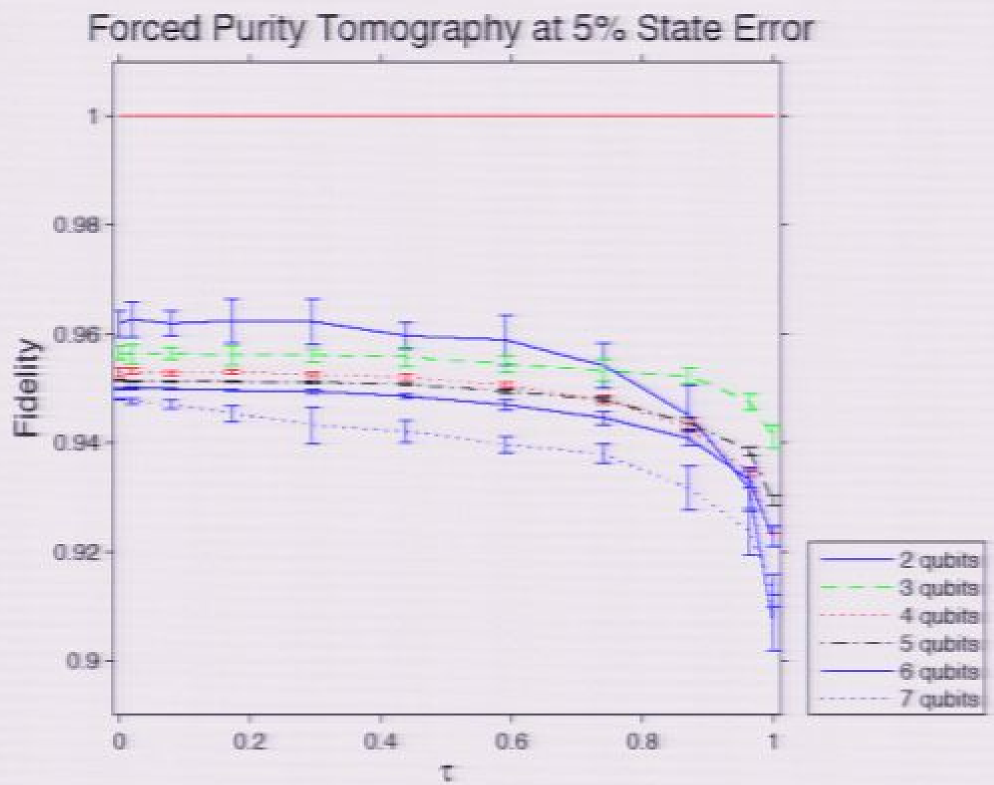


- Larger numbers of qubits...



- Larger numbers of qubits...



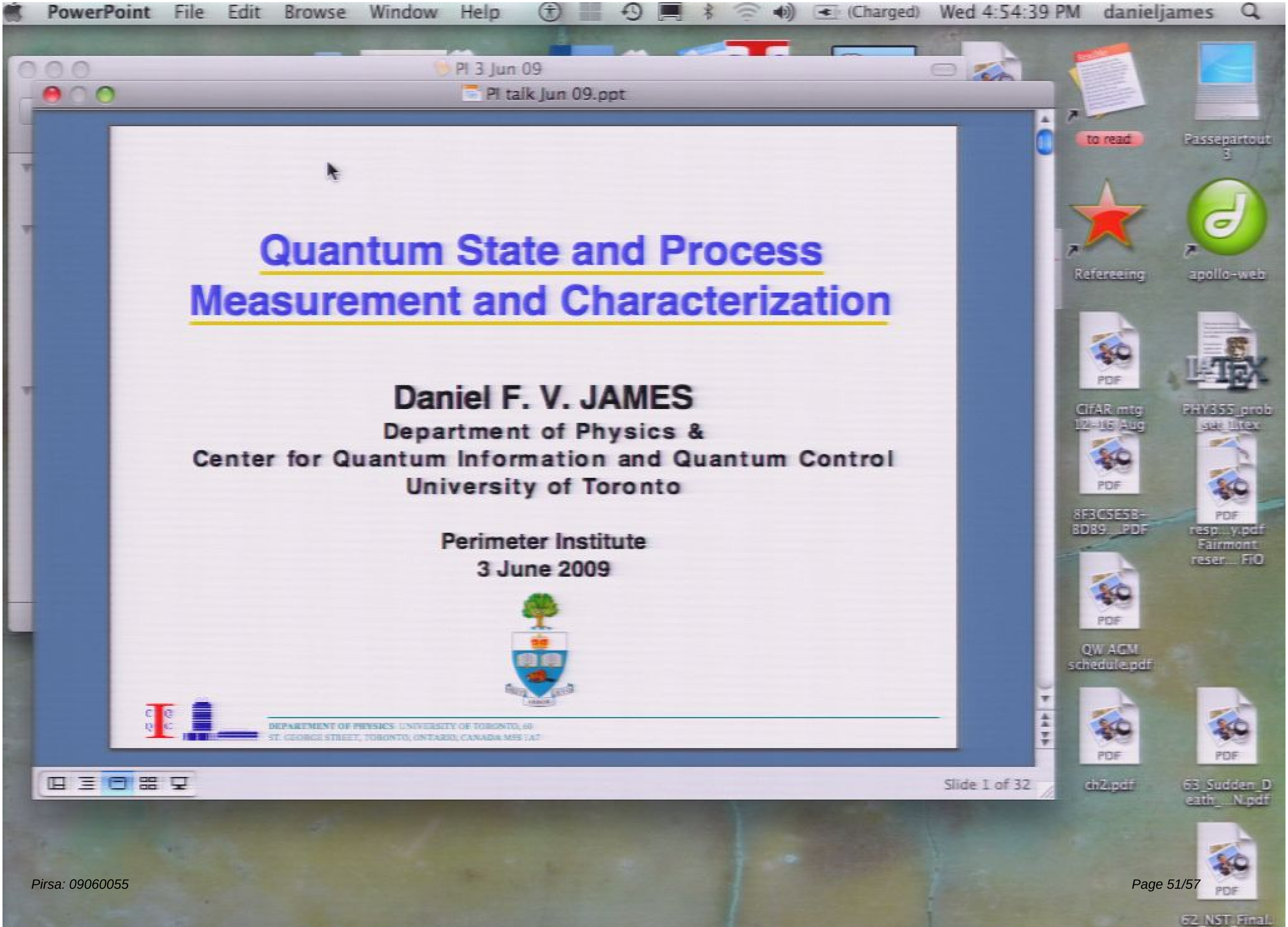


Conclusions

• Maybe these techniques can give reasonably good characterization of a dozen or so qubits….

• Beyond that, how an we know quantum computers is doing what it's meant to?

- *well characterized components.*
- *error correction: you can't know if it's bust or not, so you'd best fix it anyway.*
- *answers are easy to check.*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Quantum State and Process Measurement and Characterization
Daniel F. V. JAMES
Department of Physics &
Center for Quantum Information and Quantum Control
University of Toronto
Perimeter Institute
3 June 2009
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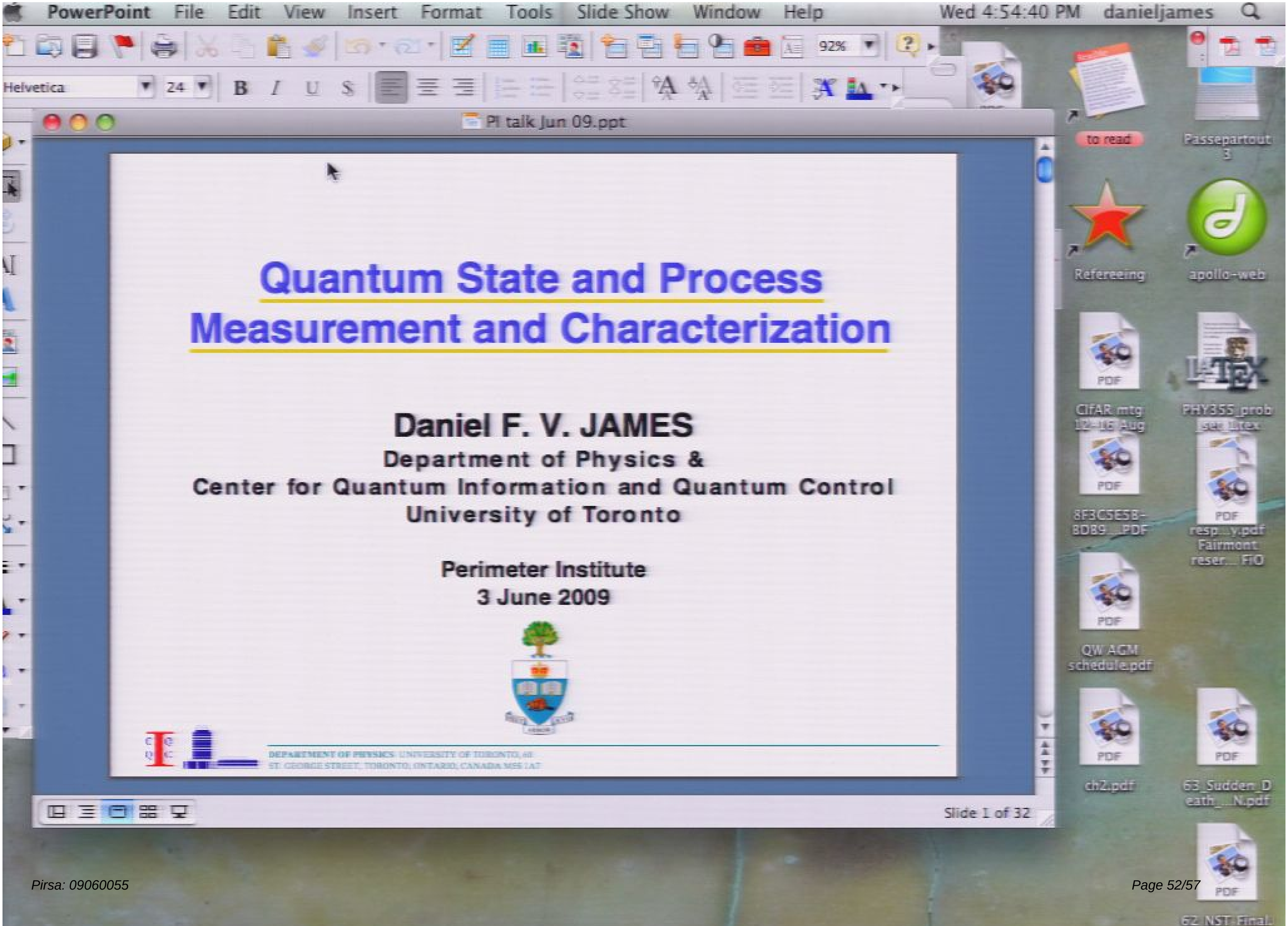
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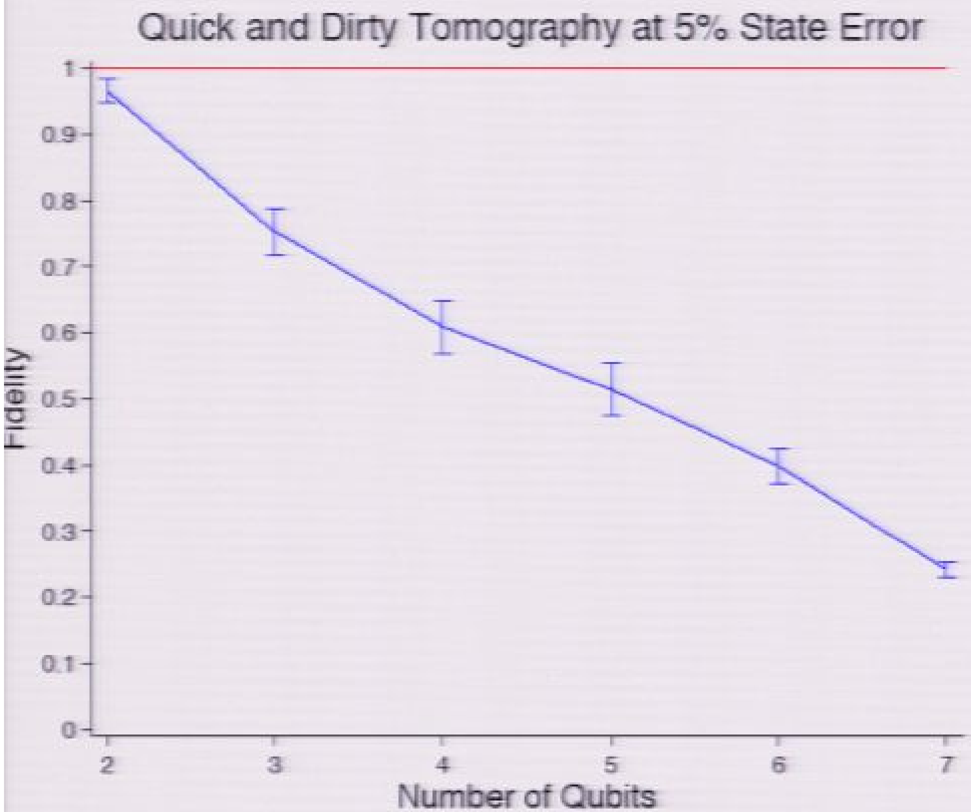
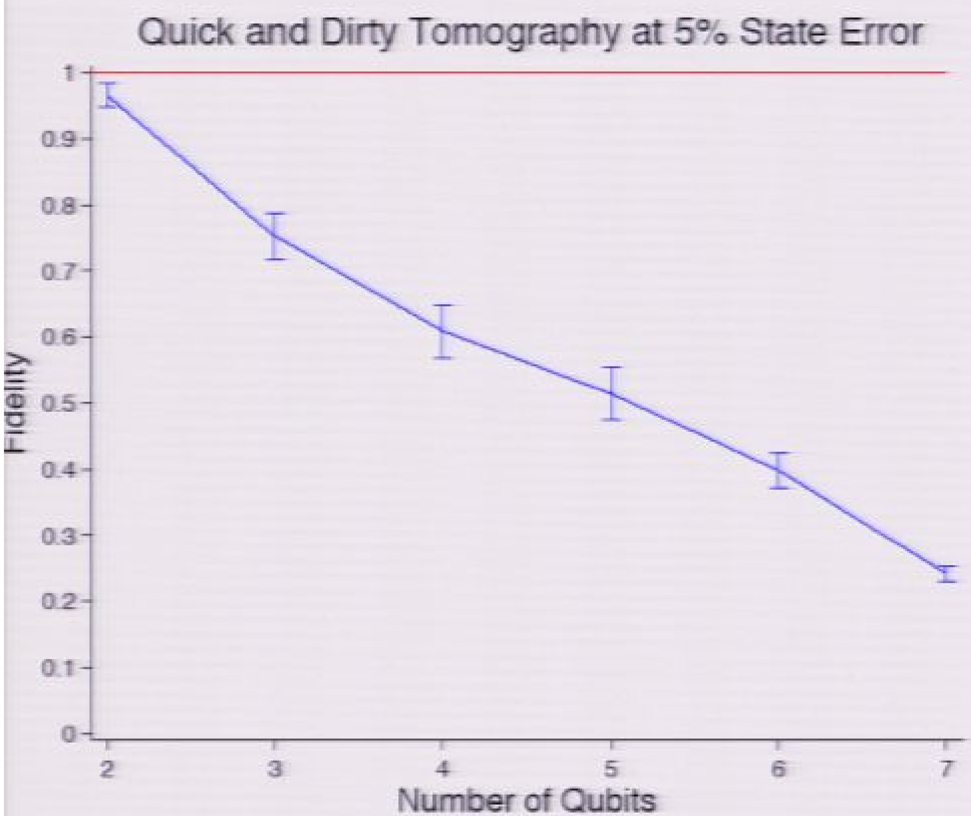
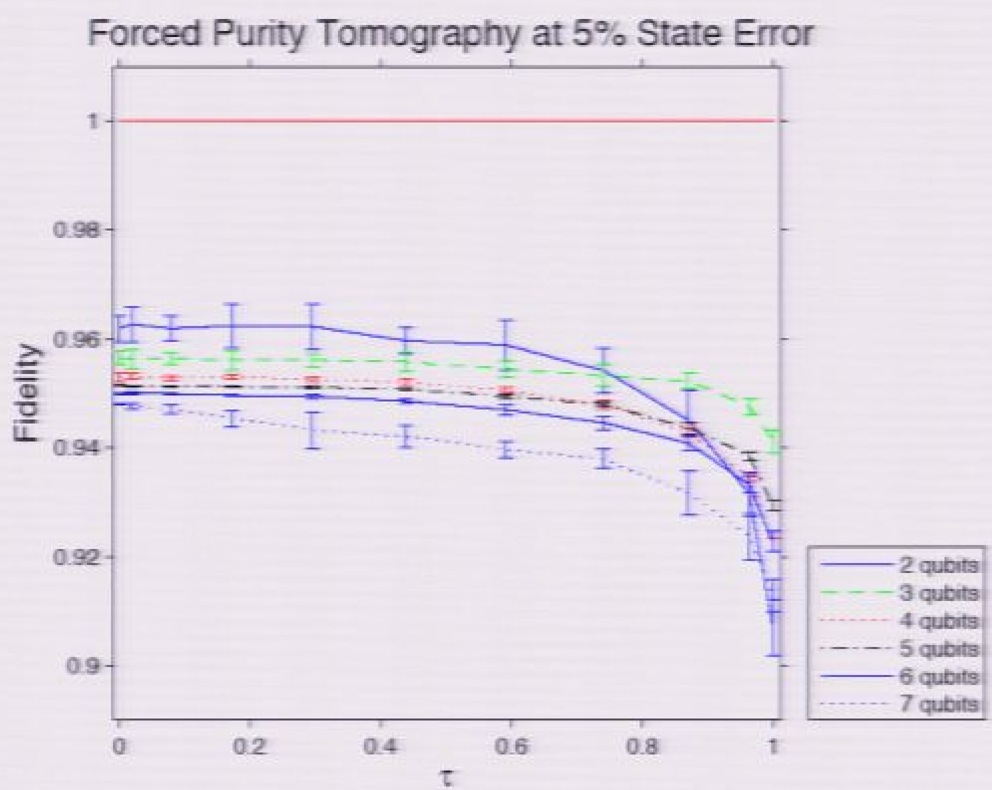
Slide 1 of 32

- Larger numbers of qubits...

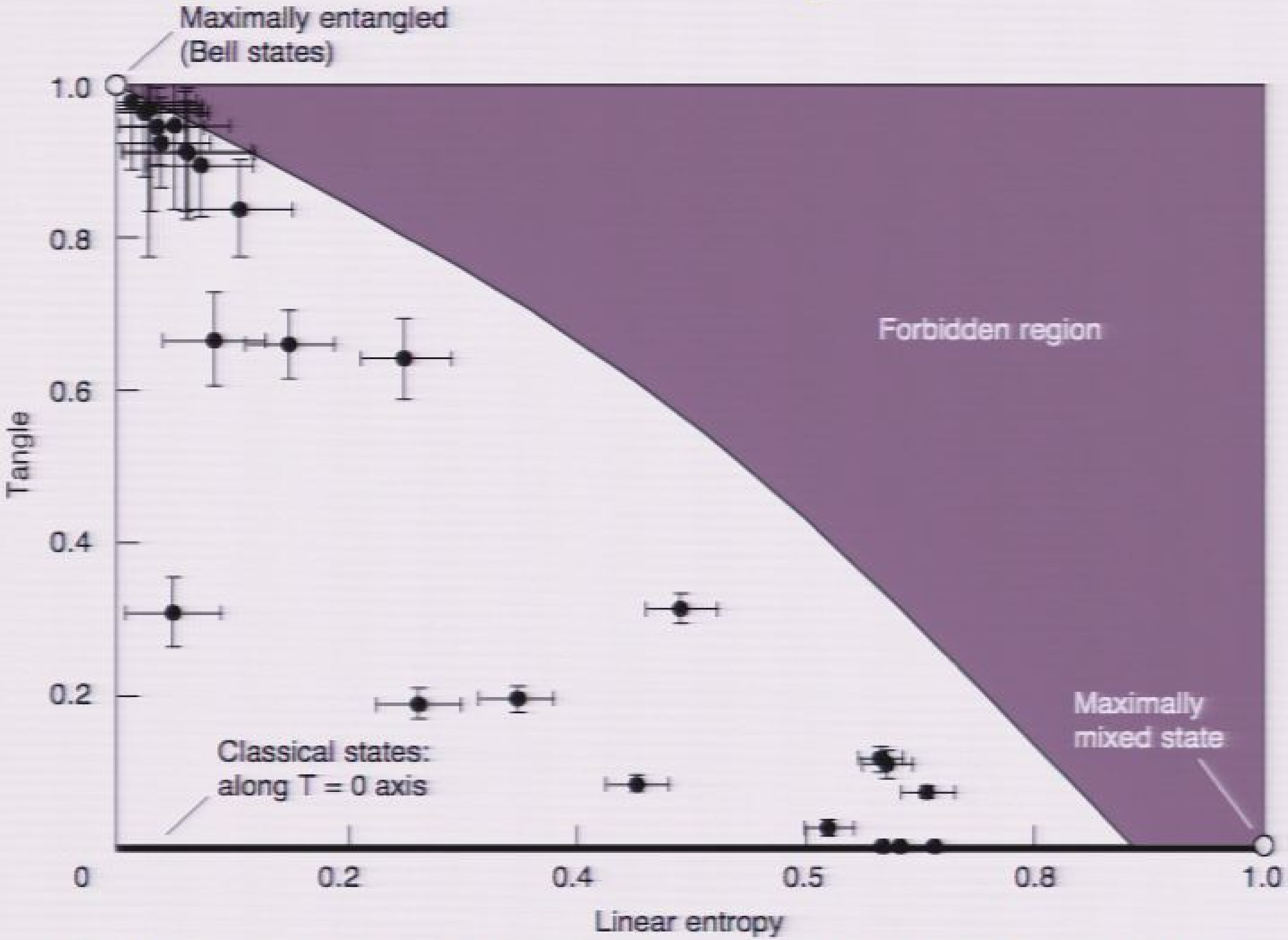


- Larger numbers of qubits...



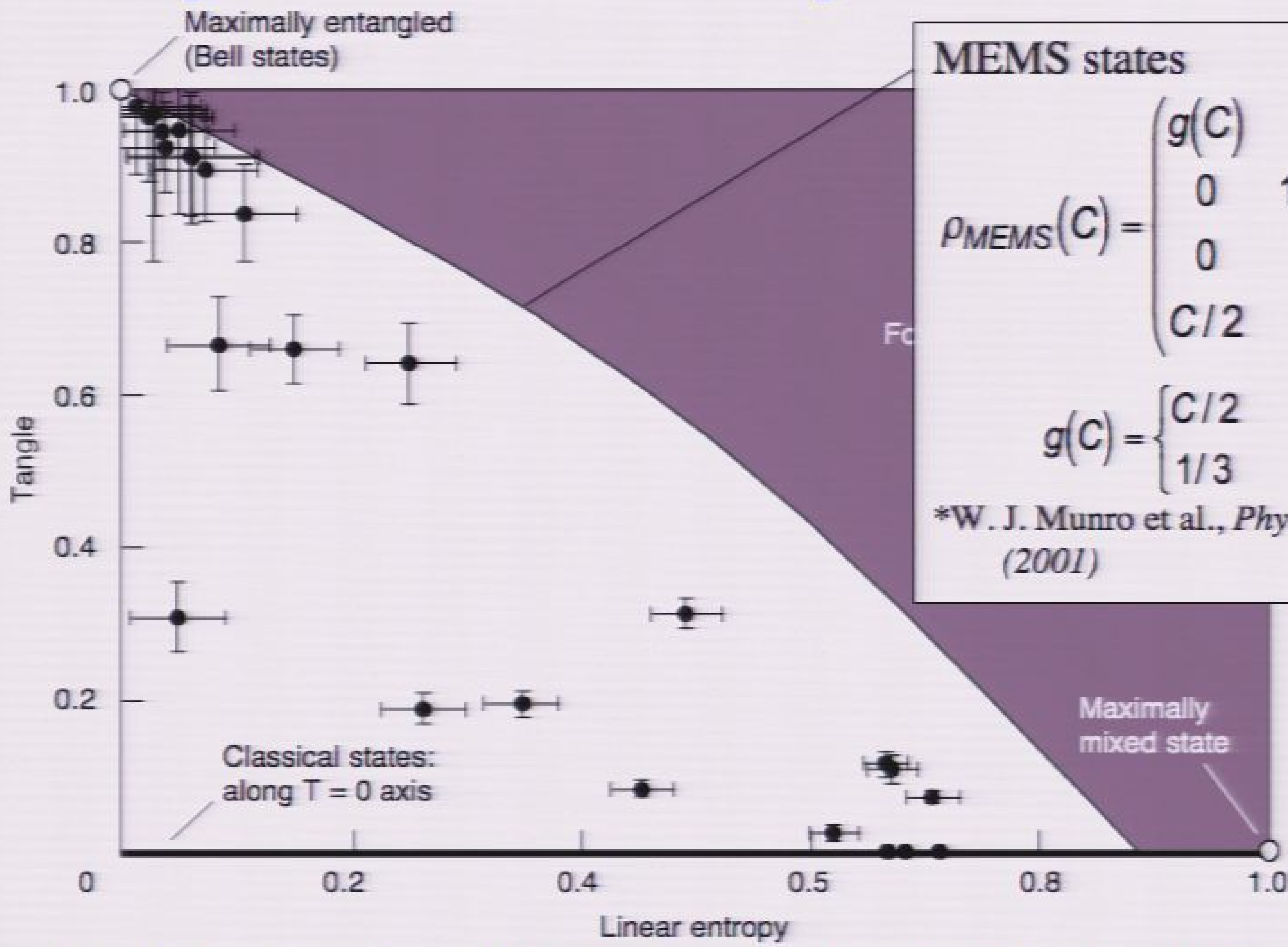


"Map" of Hilbert Space*



* D.F.V. James and P.G .Kwiat, *Los Alamos Science*, 2002

“Map” of Hilbert Space*



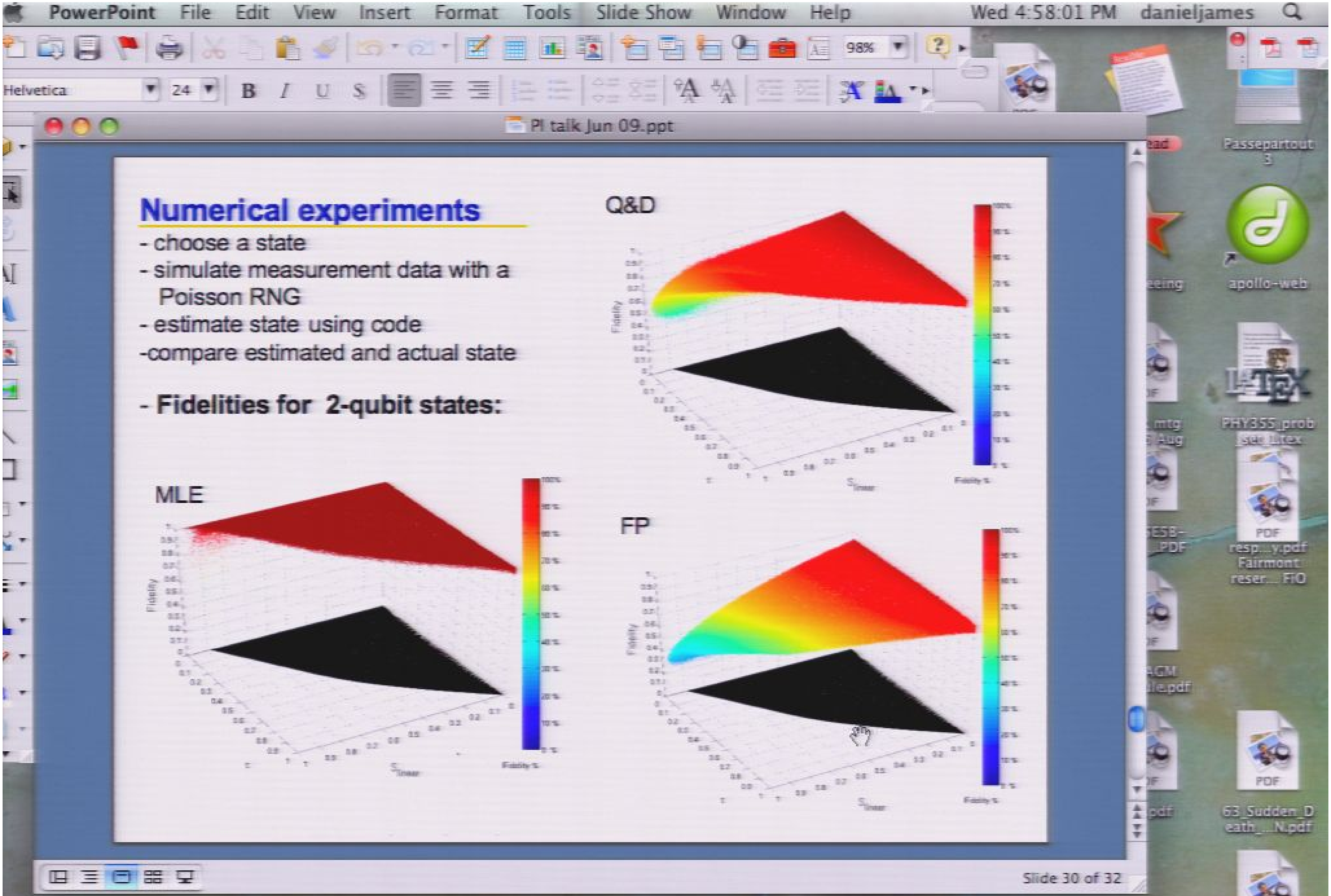
MEMS states

$$\rho_{MEMS}(C) = \begin{pmatrix} g(C) & 0 & 0 & C/2 \\ 0 & 1-2g(C) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C/2 & 0 & 0 & g(C) \end{pmatrix}$$

$$g(C) = \begin{cases} C/2 & \text{if } C \geq 2/3 \\ 1/3 & \text{if } C < 2/3 \end{cases}$$

*W. J. Munro et al., *Phys Rev A **64**, 030302-1 (2001)*

* D.F.V. James and P.G .Kwiat, *Los Alamos Science*, 2002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PI talk Jun 09.ppt
Numerical experiments
- choose a state
- simulate measurement data with a Poisson RNG
- estimate state using code
-compare estimated and actual state
- Fidelities for 2-qubit states:
Q&D
MLE
FP
Fidelity
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