Title: Categorical Structures in AQFT

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Abstract: TBA

Categorical Structures in AQFT

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AQFT – connections between QM
& relativity.

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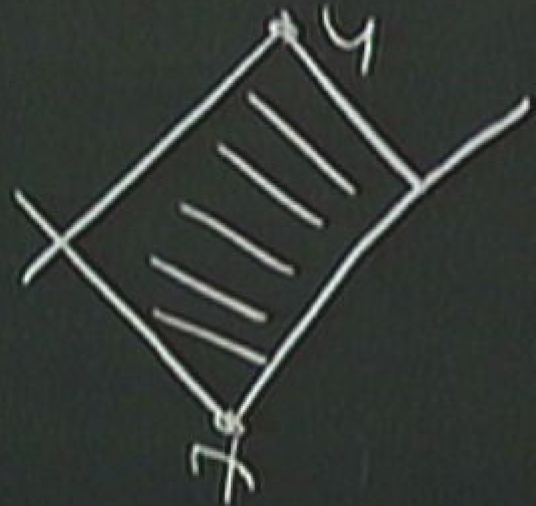
"causal ordering"

Given $x, y \in M$

$[x, y] = \{ z \mid x \leq z \leq y \}$



intervals
double cones
diamonds

The double cones are a poset $(K, \subseteq)$

subset inclusion.

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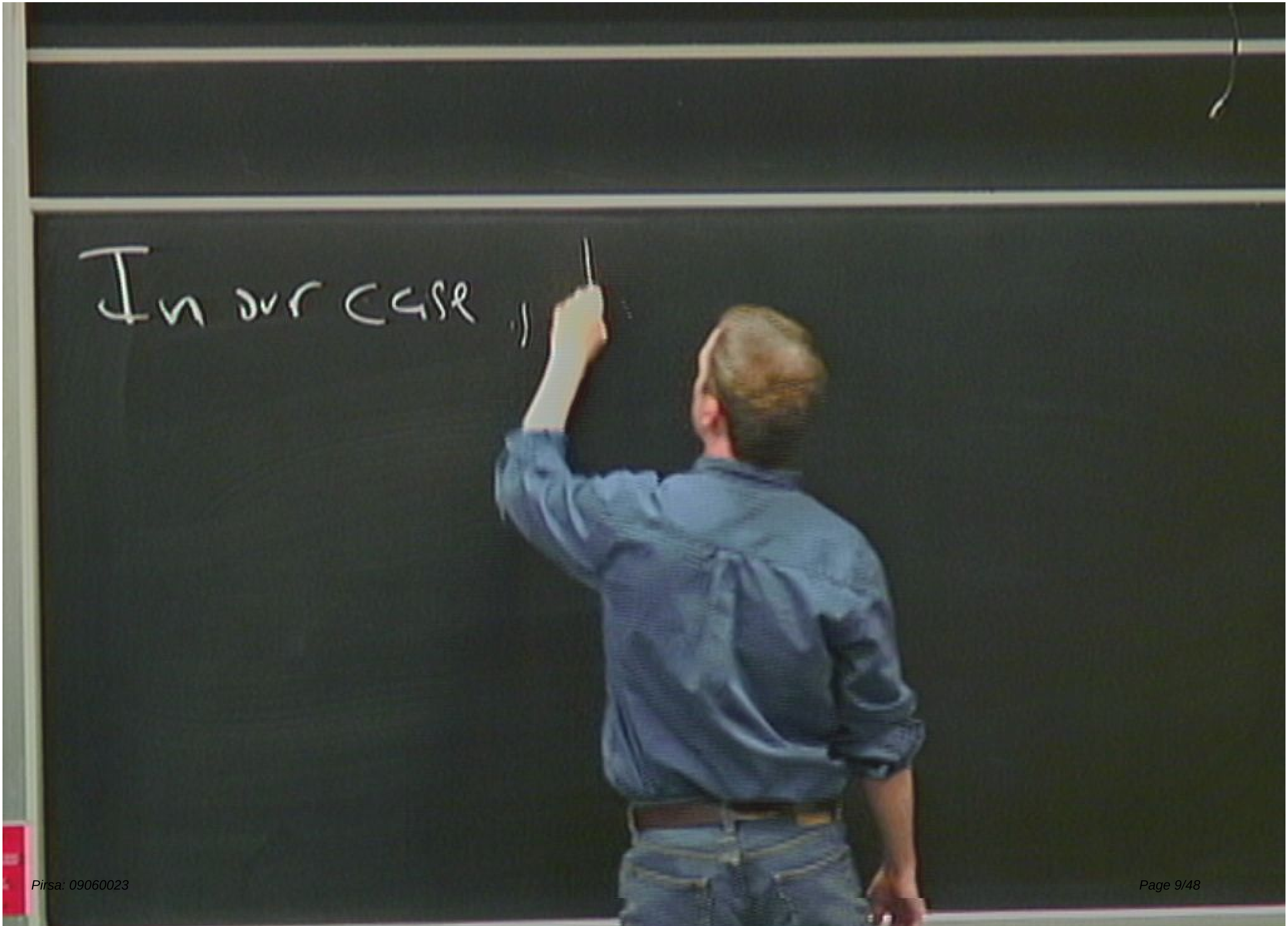
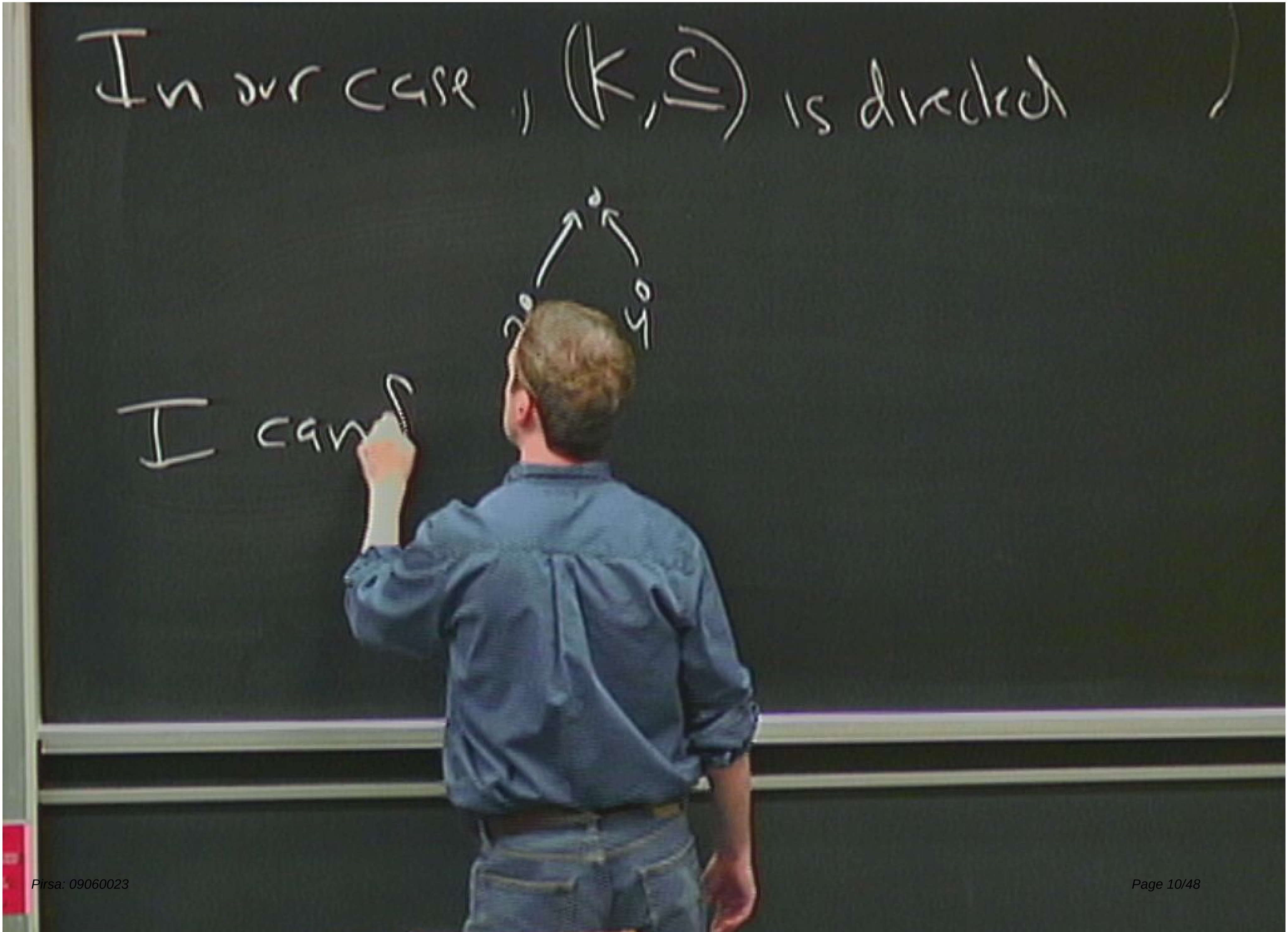
subset inclusion.

We'll say ~~iff~~ $U, V \in K$

$U \perp V$ if U, V are spacelike separated.


In our case,


In our case, (K, ⊆) is directed
I can

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$$\begin{tikzcd} & \bullet & \\ x \arrow[ur] & & y \arrow[ul] \end{tikzcd}$$

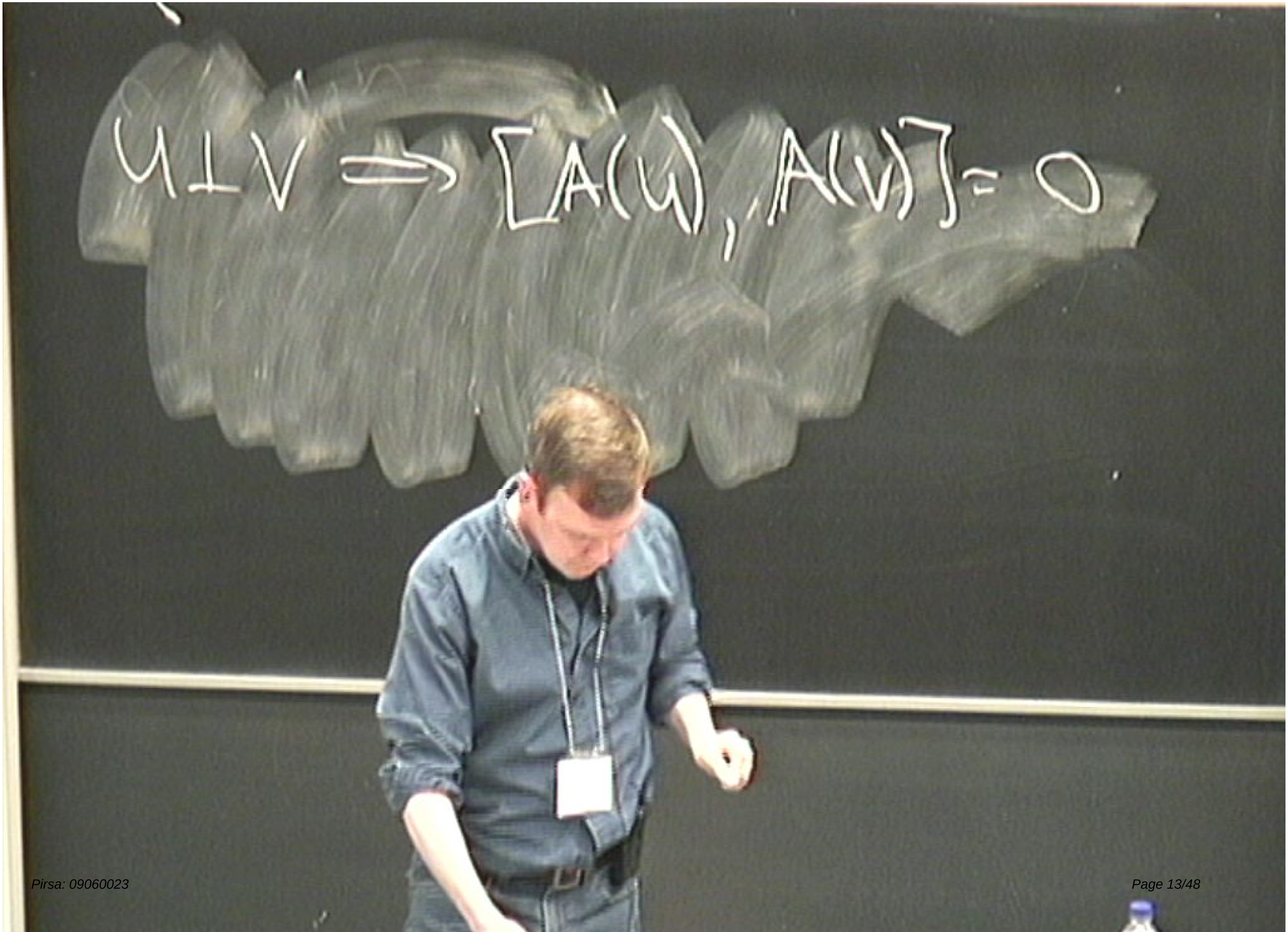
I can form the colimit

$$\hat{A} := \bigcup \{ A(u) \mid u \in K \}$$

In our case, $(K, \subseteq)$ is directed,

I can form the colimit

$$\underbrace{\hat{A}}_{\text{quasilocal algebra}} := \bigcup \{ \underbrace{A(u)}_{\text{local algebra(s)}} \mid u \in K \}$$


$u \perp v \Rightarrow [A(u), A(v)] = 0$

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What is the gauge group?

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What is the gauge group?

Pick a representation of $\hat{\mathfrak{A}}$. $\pi_0 : \hat{\mathfrak{A}} \to \mathfrak{B}(H_0)$

"vacuum representation"

$\pi : \hat{A} \to \mathcal{B}(H)$ is a superselection sector or a DHR representation if

$\forall U \in K$, $\exists$ a unitar $H \to H_0$ induces

$$\pi\big|_{A(U^\perp)} \cong \pi_0\big|_{A(U^\perp)}$$

$$A(U^\perp) = C^* \text{ alg generated by } A(U')\big|_{U' \perp U}$$

Structure?

Let $\rho : \hat{A} \to \hat{A}$. ρ is <u>localized</u>

at $U \subset K$ if $\rho|_{A(U^\perp)} = \mathrm{id}$

Transportable if $\forall\, U' \in K$ $\exists \rho'$ localized at U' and a unitary $U \in \hat{A}$ s.t.

$$U\rho = \rho' U$$

Transportable if $\forall\, \mathcal{U}' \in K$ $\exists \rho'$ localized at $\mathcal{U}'$ and a unitary $U \in \hat{A}$ s.t.

$$U\rho = \rho' U$$

Let $\Delta(\mathcal{U})$ = localized, transportable endos at $\mathcal{U}$.

$$\Delta = \bigcup \Delta(\mathcal{U})$$

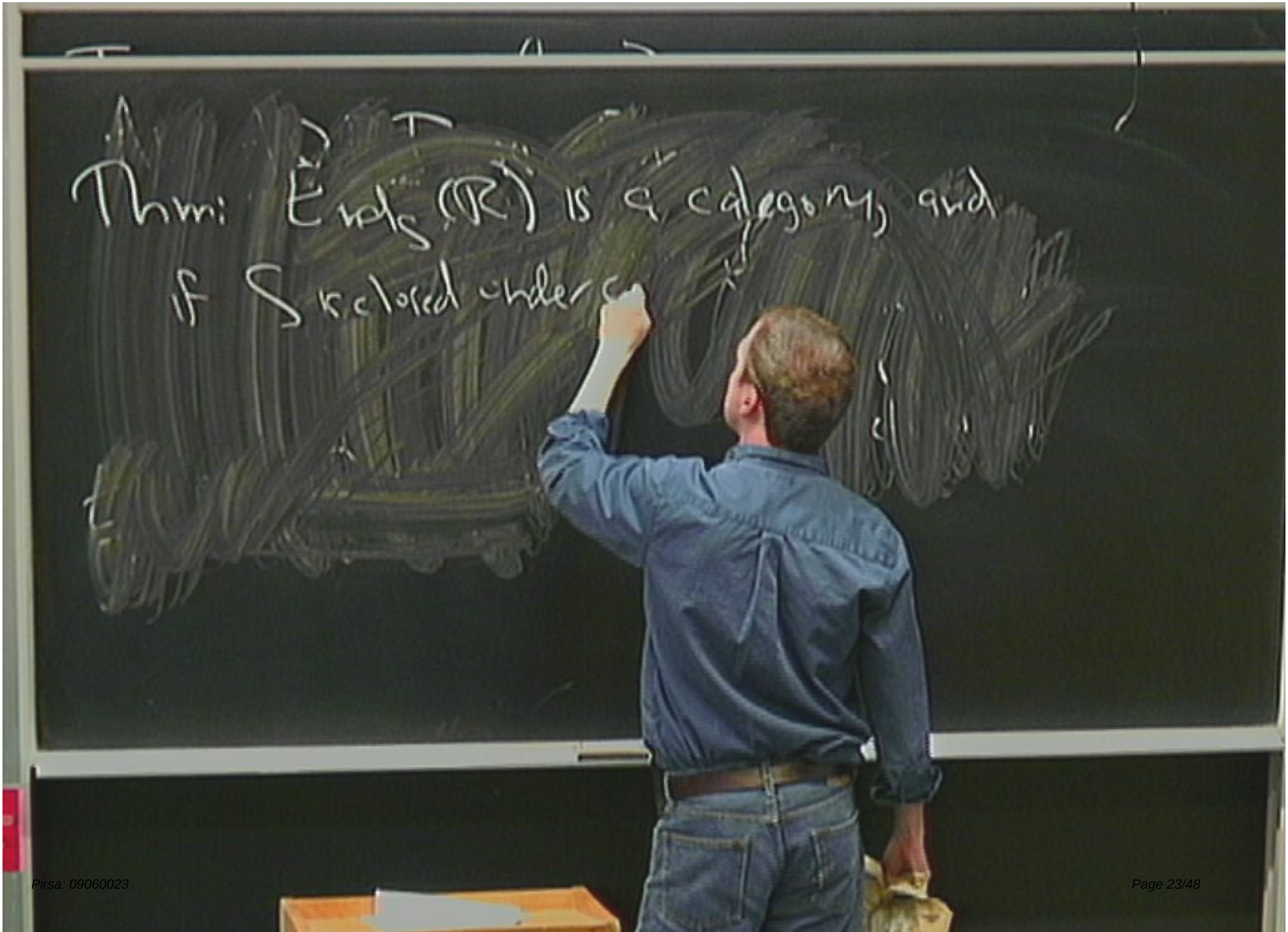
Let R be a ring.

Let $S \subseteq End(R)$

Define a category $End_S(R)$ as follows

- objects - elements of S
- arrows $\quad \rho, \rho' \in S \quad \rho \xrightarrow{r} \rho' \quad r \in R$

$$r\rho = \rho' r$$



Thm: $\mathrm{End}_S(\mathbb{R})$ is a category, and
if S is closed under

Thm: $\mathrm{Ends}_S(\mathbb{R})$ is a category, and if S is closed under composition and $\mathrm{id} \in S$, it's a monoidal category. $\otimes$ is composition of elements of S.

Thm (DR) $\mathrm{End}_\Delta(\hat{A})$ is symmetric, monoidal

C^*-category

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- dagger category
- enriched over Banach spaces

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C^* category

+ dagger category ✓
- enriched over Banach spaces
+ coherence conditions

Thm (DR) $\mathrm{End}_{\Delta}(\hat{A})$ is symmetric monoidal

C^* category

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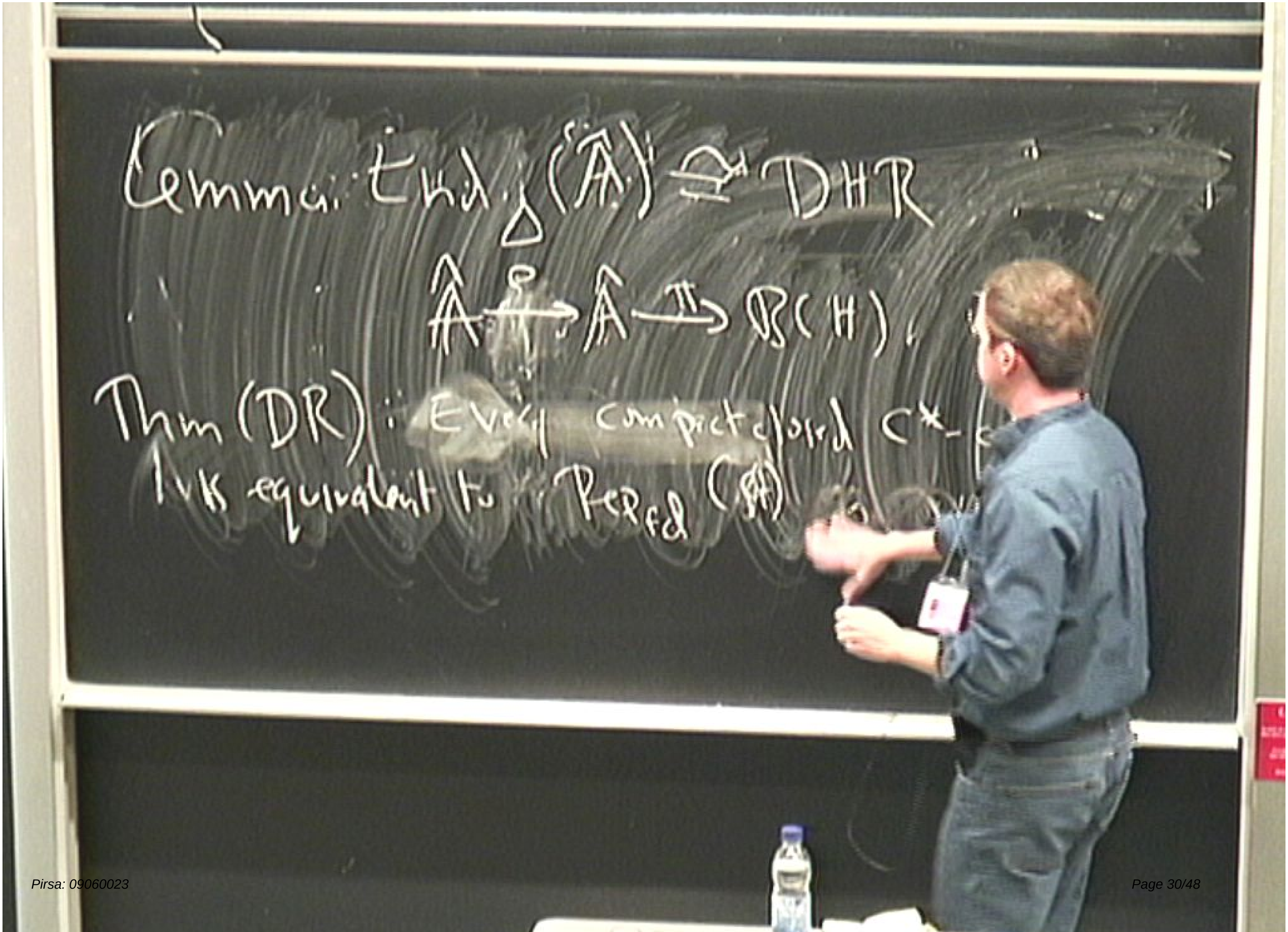
Lemma: Th. $\Delta(\hat{A}) \cong$ DHR

$$\hat{A} \xrightarrow{\rho} \hat{A} \xrightarrow{\pi} \mathcal{B}(H)$$


Lemma: End_Δ(Â) ≅ DHR
Â --ρ--> Â --π--> B(H)
Thm (DR): Every compact closed C*-
is equivalent to Rep_fd (G)

Lemma: $\mathrm{End}_{\Delta}(\hat{\hat{A}}) \cong \mathrm{DHR}$

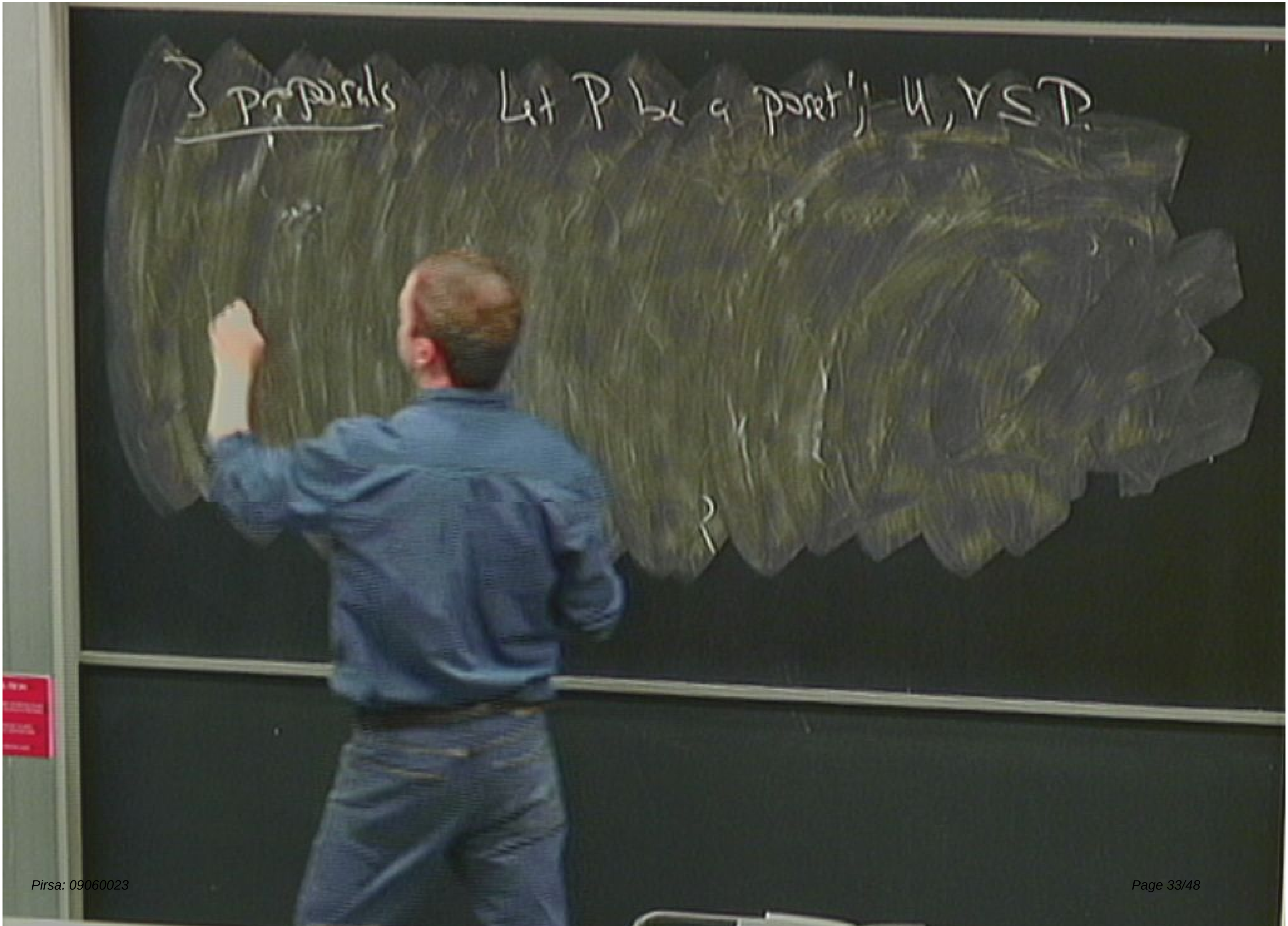
$$\hat{\hat{A}} \xrightarrow{\rho} \hat{A} \xrightarrow{\pi} \mathcal{B}(H)$$

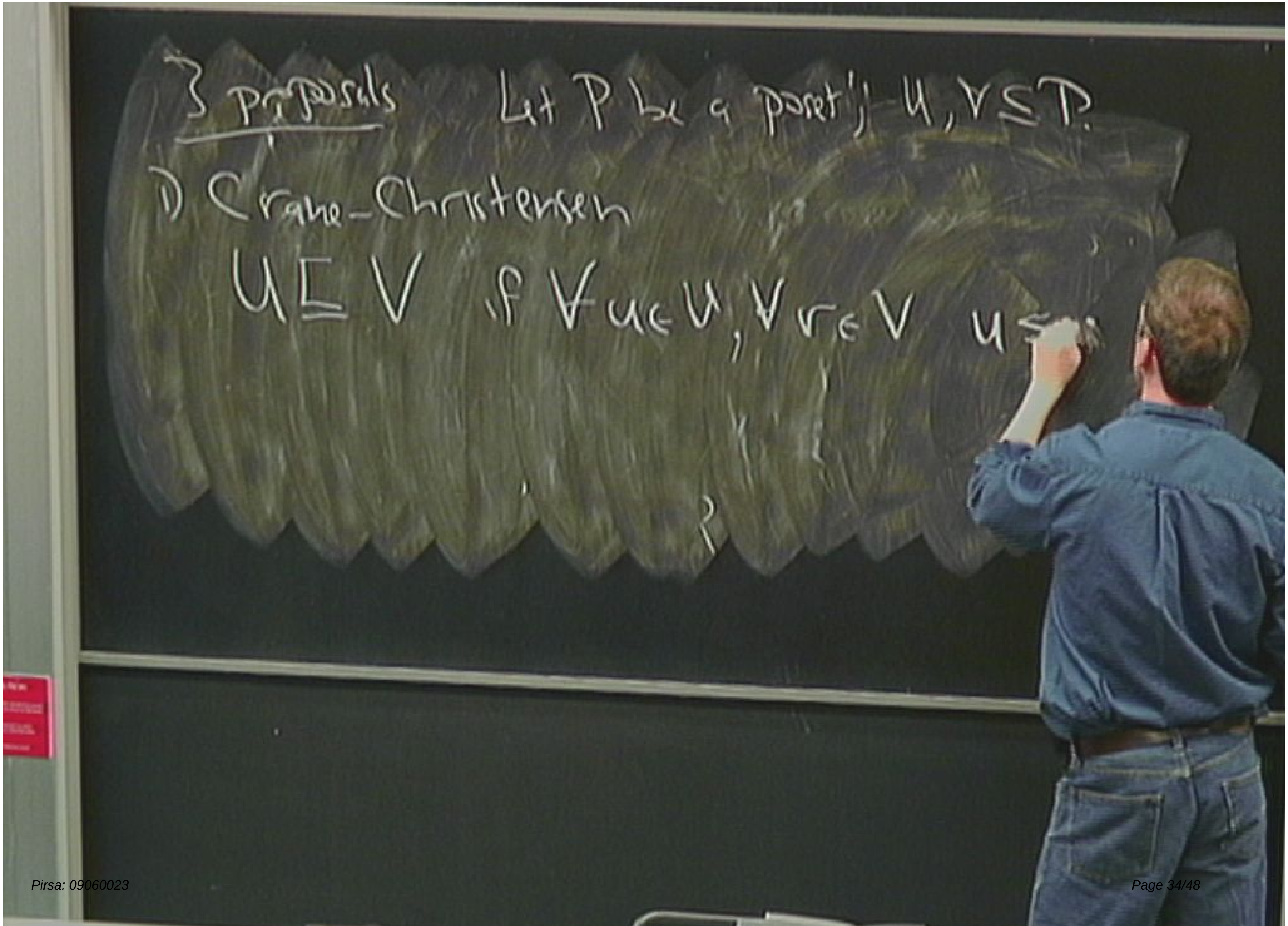
Thm (DR): Every compact closed C^*-category is equivalent to $\mathrm{Rep}_{fd}(G)$, where G is a compact group, unique up to iso.

Goals

1) $(K, \leq)$ Find a different ordering on K which represents causality. Given this, can then model causal evolution.

2) Bring Heidegger categories into the defn.


3 proposals
Let P be a poset; u, v ∈ P.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Let P be a poset; U, V ⊆ P.
1) Crane-Christensen
U ⊑ V if ∀u∈U, ∀v∈V u ≤

3 proposals   Let P be a poset; $U, V \subseteq P$.

1) Crane–Christensen

$U \sqsubseteq V$ if $\forall u \in U, \forall v \in V \;\; u \leq v$.

2)

3 proposals

Let P be a poset; $U, V \subseteq P$.

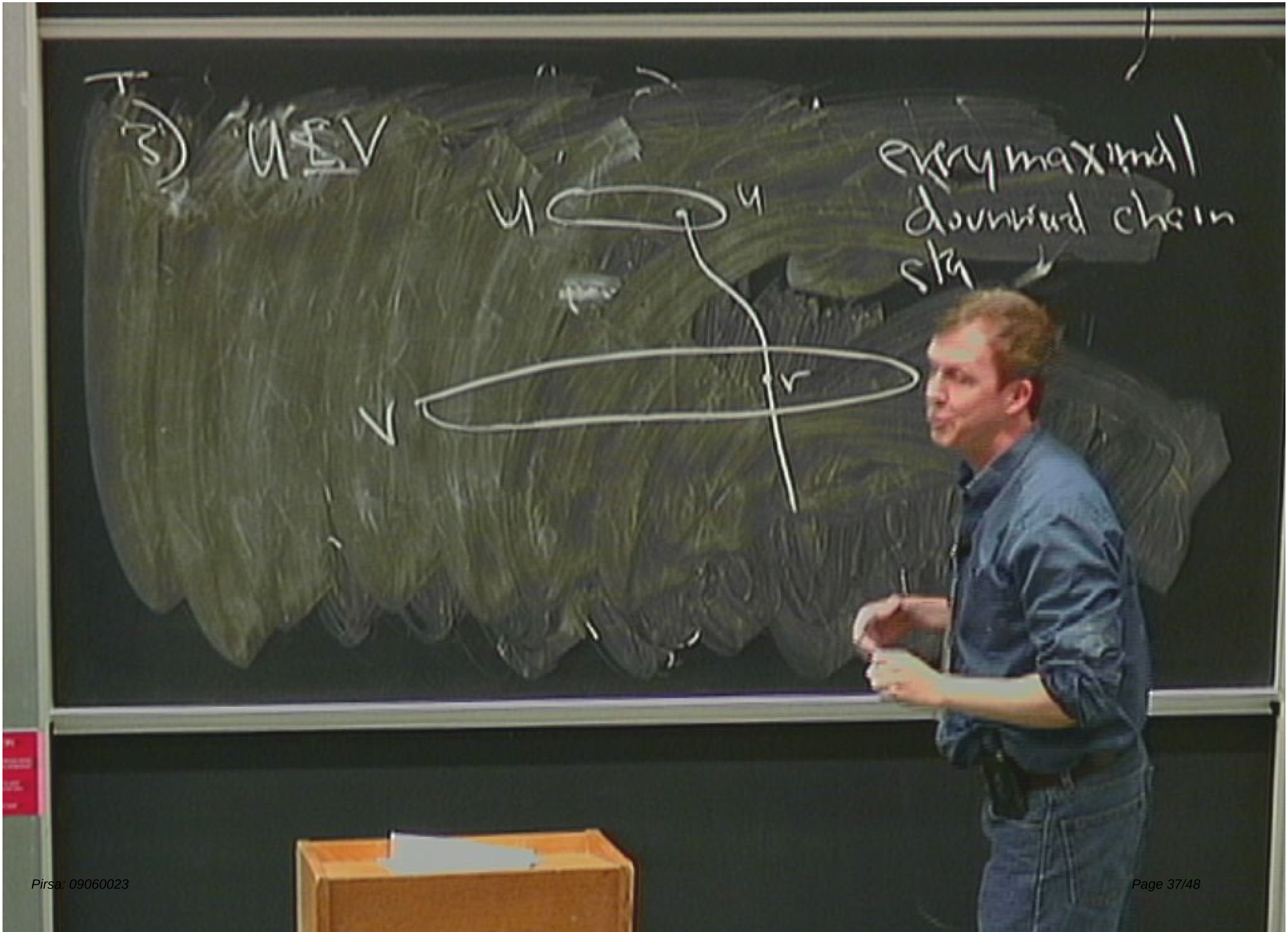
1) Crane-Christensen

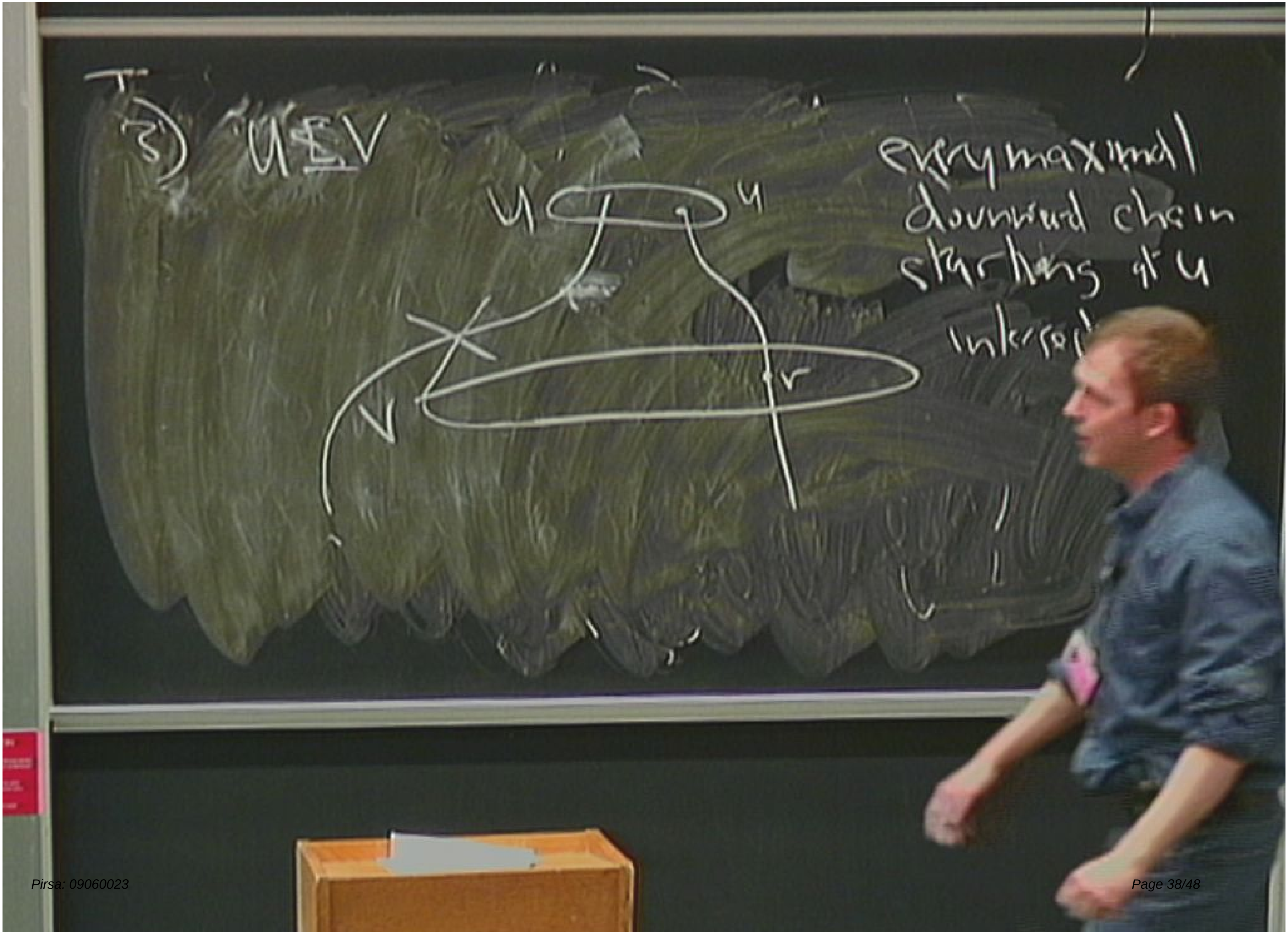
$U \sqsubseteq V$ if $\forall u \in U, \forall v \in V \; u \leq v$.

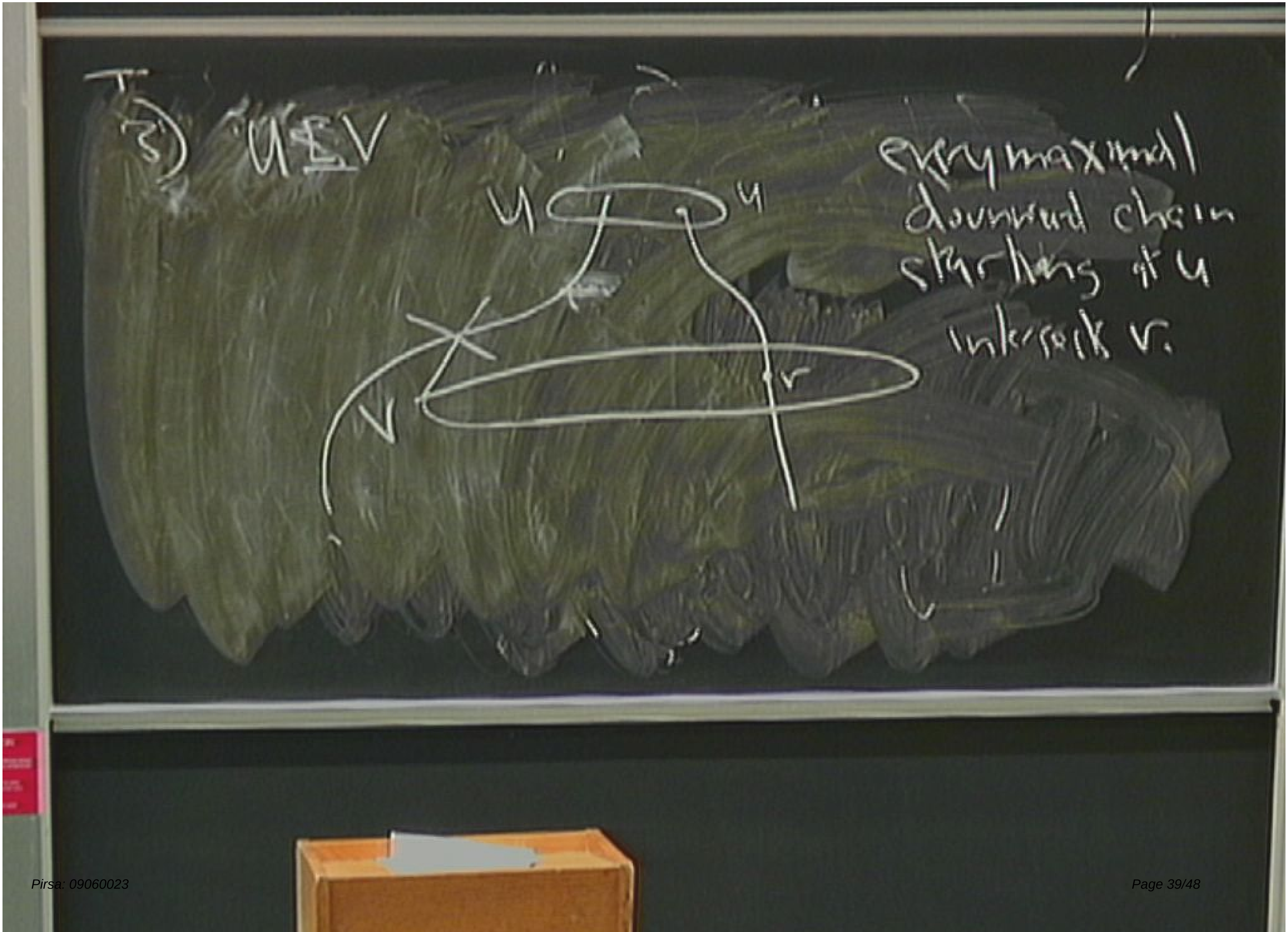
2) Egli-Milner ordering

$U \sqsubseteq V$ if $\forall u \in U, \exists v \in V \; u \leq v$ &

$\forall v \in V, \exists u \in U \; u \leq v$


3) $u \le v$
every maximal
downward chain
stg
u
u
v
v
r


3) u ≤ v
u
u
v
v
every maximal
downward chain
starting at u


3) U ≤ V
u
u
v
v
every maximal
downward chain
starting at u
intersects V.

Let $(K, \leq)$ be a causal set

Suppose a functor

$B : (K, \leq) \longrightarrow DAG$

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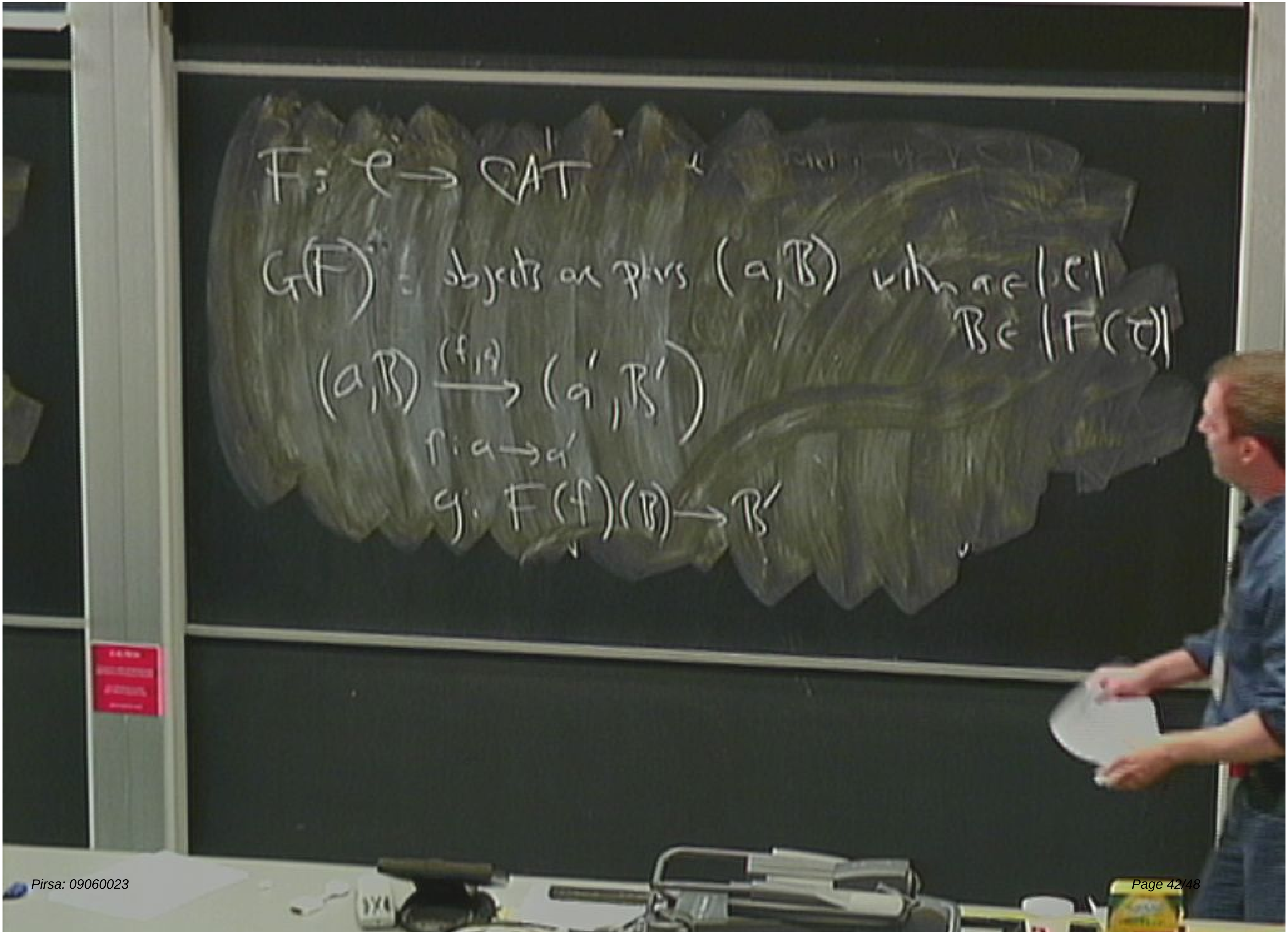
Suppose a functor

$$B : (K, \leq) \longrightarrow DAG$$

Let $G(B)$ be the groth. category


$F: \mathcal{C} \to CAT$
$G(F)$: objects are pairs (a, β) with $a \in |\mathcal{C}|$
$\beta \in |F(a)|$
$(a,\beta) \xrightarrow{(f,g)} (a',\beta')$
$f: a \to a'$
$g: F(f)(\beta) \to \beta'$

$F: \mathcal{C} \to \mathcal{CAT}$

$G(F)$ = objects are pairs (a, B) with $a \in |\mathcal{C}|$, $B \in |F(a)|$

$$(a, B) \xrightarrow{(f,g)} (a', B')$$

$f : a \to a'$

$g : F(f)(B) \to B'$ in $F(a')$

$G(\mathbb{R})$

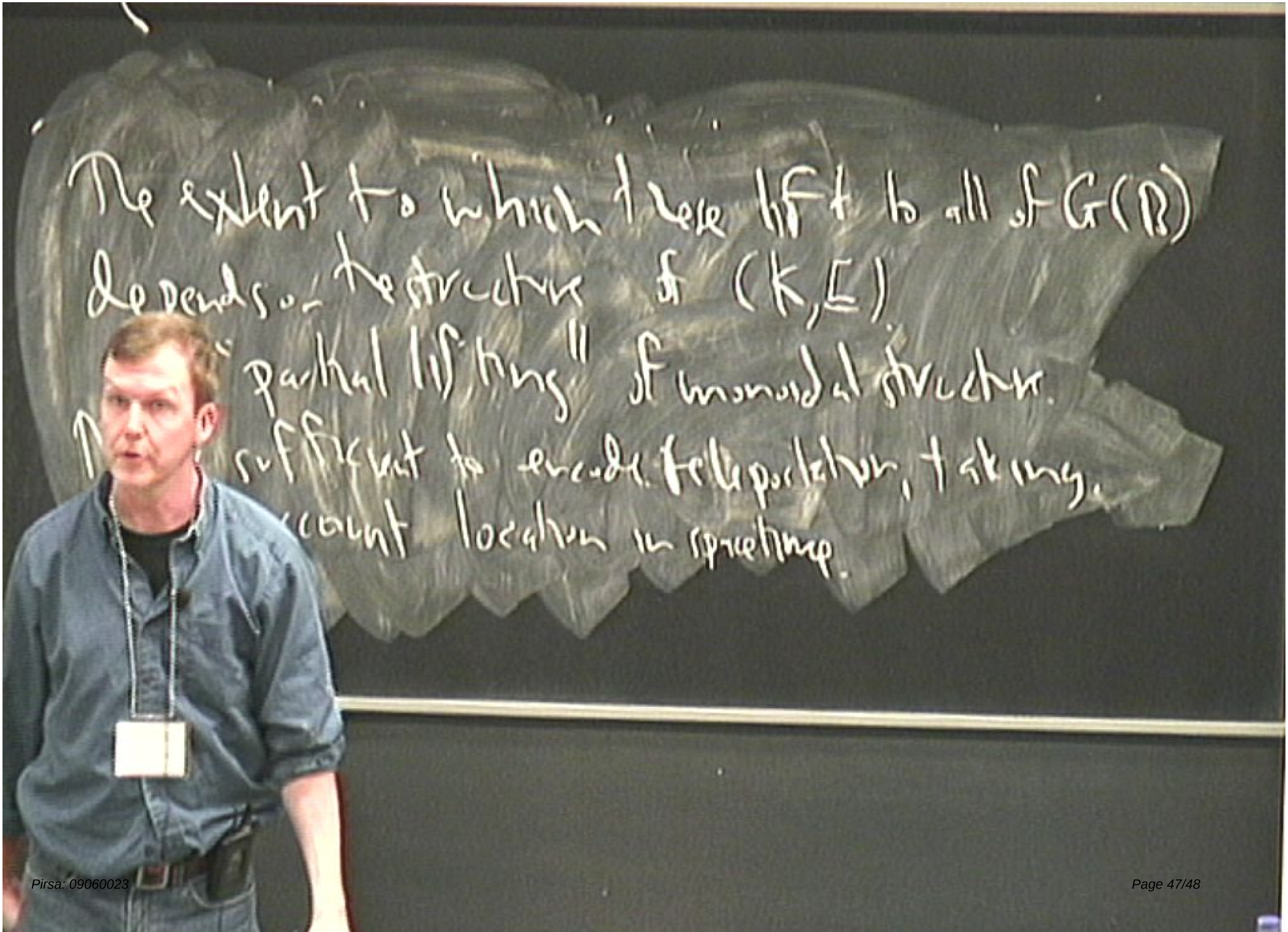
$G(\mathbb{B})$ is neither a dagger category,

nor monoidal.

But it is locally so

In each fibre, I have dagger and tensor operators.

The extent to which these lift to all of $G(R)$
depends on the structure of (K, L)
"partial liftings" of monoidal structure.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"partial lifting" of monoidal structure.

This is sufficient to encode teleportation, taking into account location in spacetime.