Title: Operational structures as a foundation for probabilistic theories

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Abstract: Work on formulating general probabilistic theories in an operational context has tended to concentrate on the probabilistic aspects (convex cones and so on) while remaining relatively naive about how the operational structure is built up (combining operations to form composite systems, and so on). In particular, an unsophisticated notion of a background time is usually taken for granted. It pays to be more careful about these matters for two reasons. First, by getting the foundations of the operational structure correct it can be easier to prove theorems. And second, if we want to construct new theories (such as a theory of Quantum Gravity) we need to start out with a sufficiently general operational structure before we introduce probabilities. I will present an operational structure which is sufficient to provide a foundation for the probabilistic concepts necessary to formulate quantum theory. According to Bob Coecke, this operational structure corresponds to a symmetric monoidal category. I will then discuss a more general operational framework (which I call Object Oriented Operationalism) which provides a foundation for a more general probabilistic framework which may be sufficient to formulate a theory of Quantum Gravity. This more general operational structure does not admit an obvious category theoretic formulation.

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Operational Structures as a Foundation for probability theories

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Perimeter Institute

2nd June 2009 Categories, Quanta, Concepts

Problem: to find a theory of Quantum Gravity

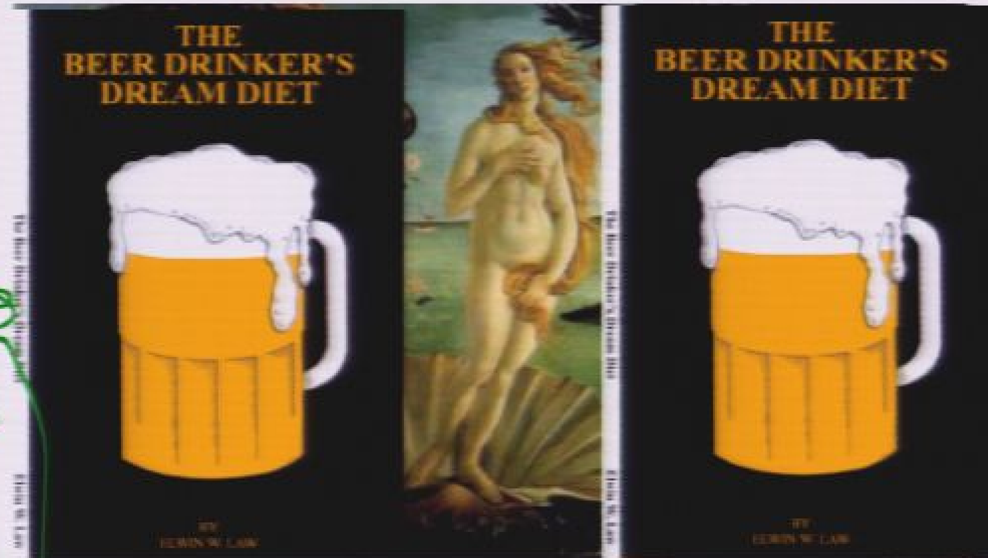
GR
beautiful
pristine
theory

QT
has problems.

Usually people try to solve QT's problems in an attempt to reconcile two theories.

Here, instead, we try to corrupt GR to enable a marriage of equals.

inject
meas
problem

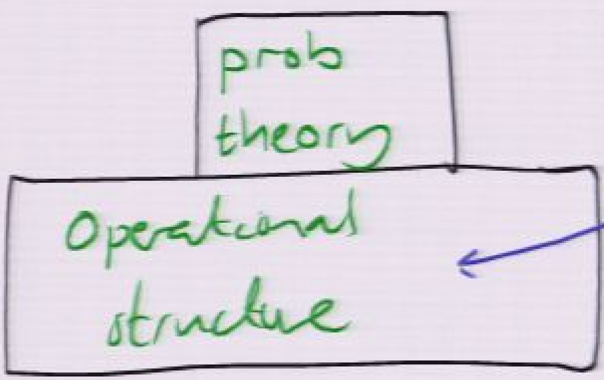


"The Copenhagen Interpretation of GR".

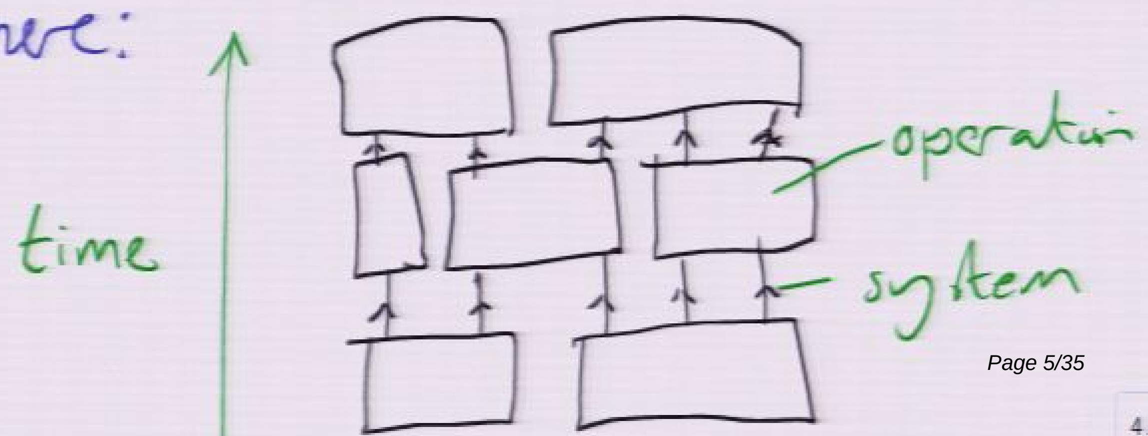
Actually will work toward an operational framework rich enough to accommodate both QT and Prob GR and work towards a theory of QG.

In the recent literature, general probabilistic theories are usually formulated in an operational context

– so operational structures form a foundation for general probabilistic theories.



researches into gen prob structures usually take naive approach here:

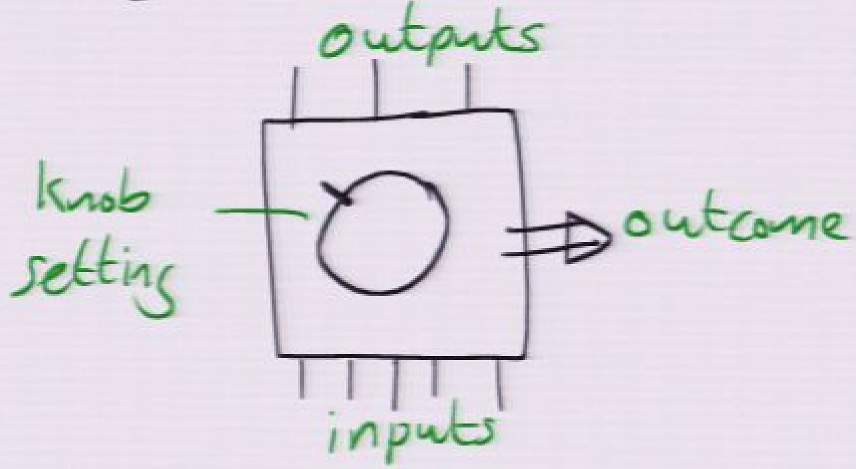


We can try to be more serious. Here's one way to proceed:

Basic building block is the Operation. An operation is a use of an apparatus having:

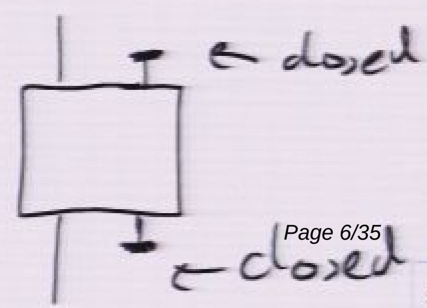
Knob settings: can adjust

Outcomes: can read off



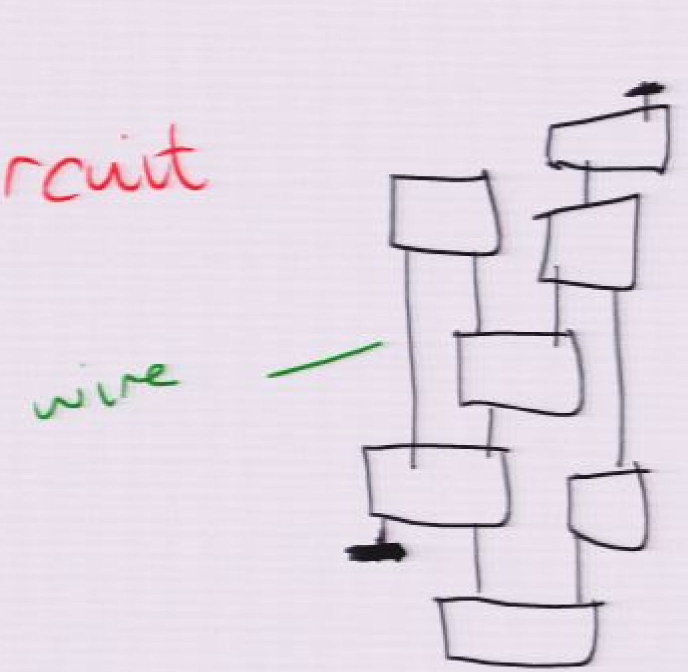
Inputs and Outputs come in different types a, b, c, ...

and can be open or closed.



Wires show how inputs connected to outputs of same type (Ikea idea)

A circuit



no open inputs or outputs left over

no closed loops

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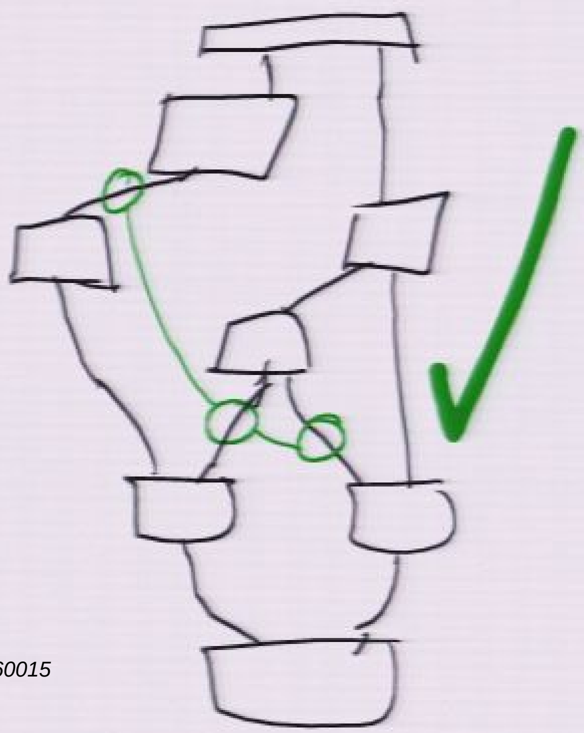
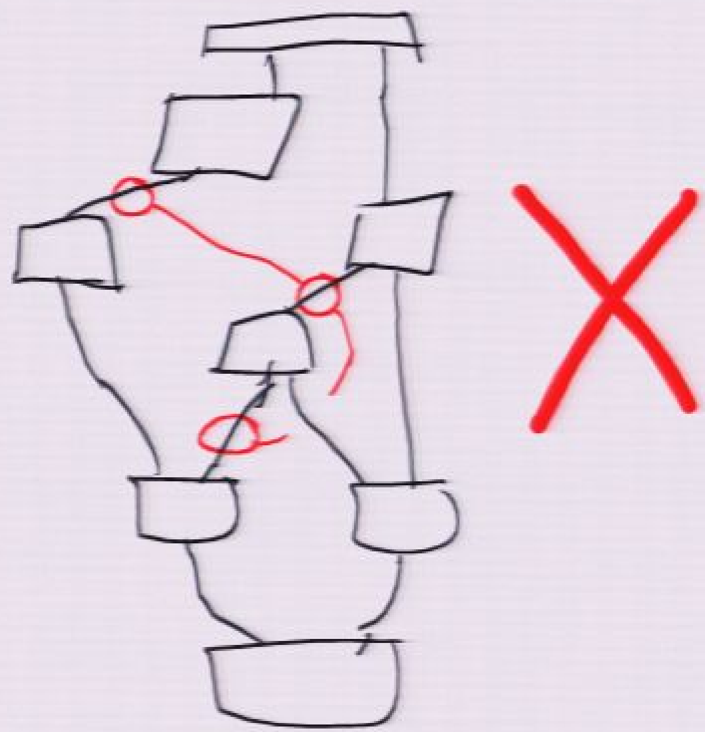
Time

A synchronous set of wires in a circuit is a set for which there exists no path from one wire to another in the set by tracing forward.





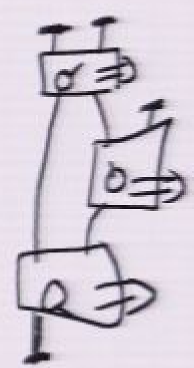
Important definition – now introduce probability

A closable set of operations, $\mathcal{O}$, is one such that

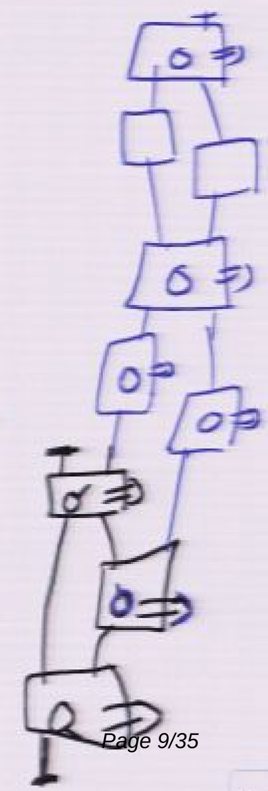
i) The probability for every set of outcomes at the operations is well defined for all circuits that can be built from $\mathcal{O}$ for all choices of knob settings

ii) This probability is unchanged when closed outputs are opened and extra operations wired on, so long as no closed inputs are opened.

well defined prob

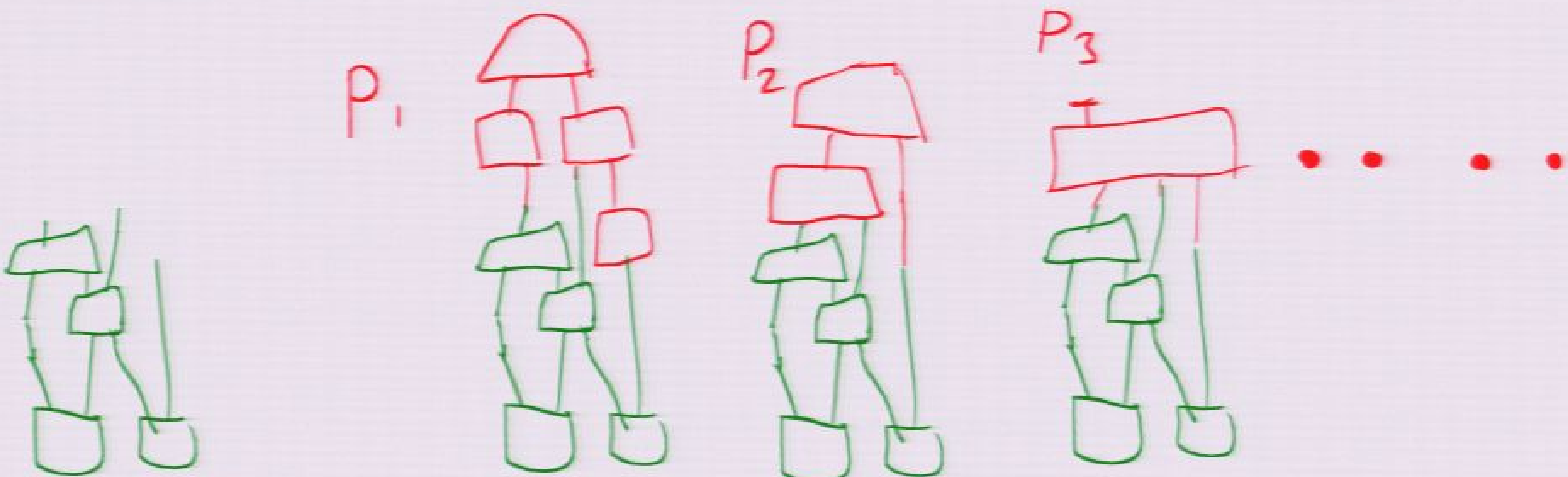


same prob



The State

The state associated with a preparation is that thing represented by any mathematical object that can be used to calculate the probability for any circuit with that preparation



preparation

have

probability theory

Operational structure

Can use this to prove theorems.

Eg. Two notions of purity of states equivalent

extremal states $\iff$ uncorelatable states

In naive op. structure have to introduce additional assump.

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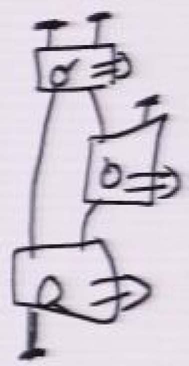

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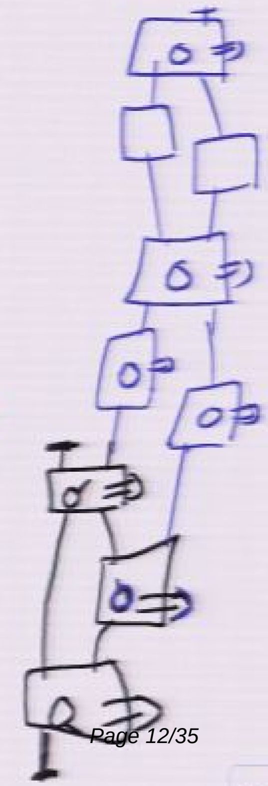

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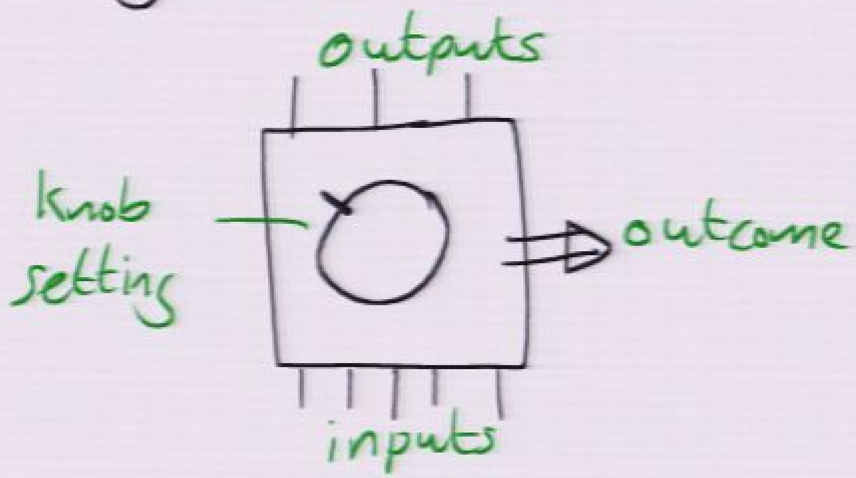

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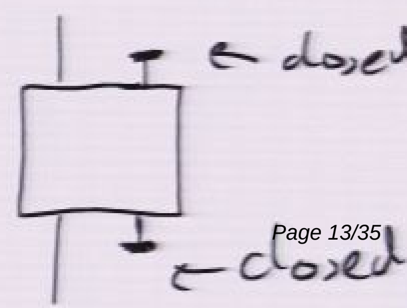

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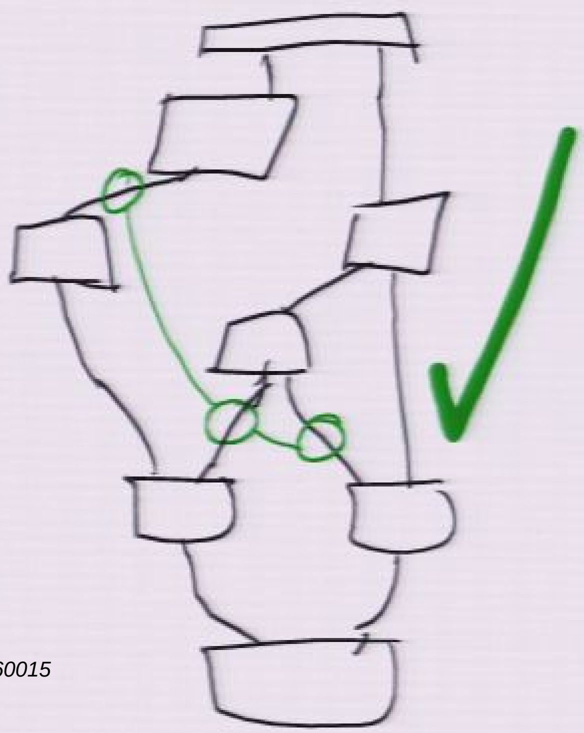

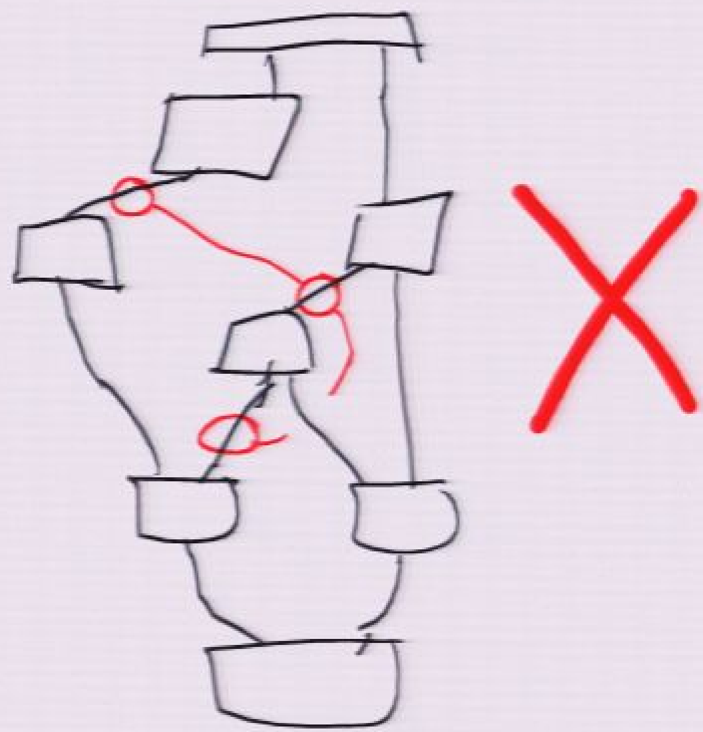

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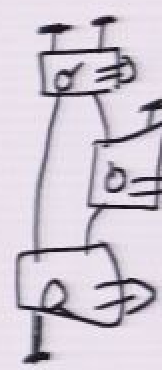

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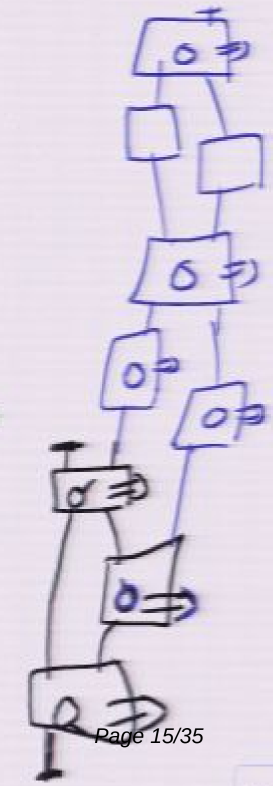

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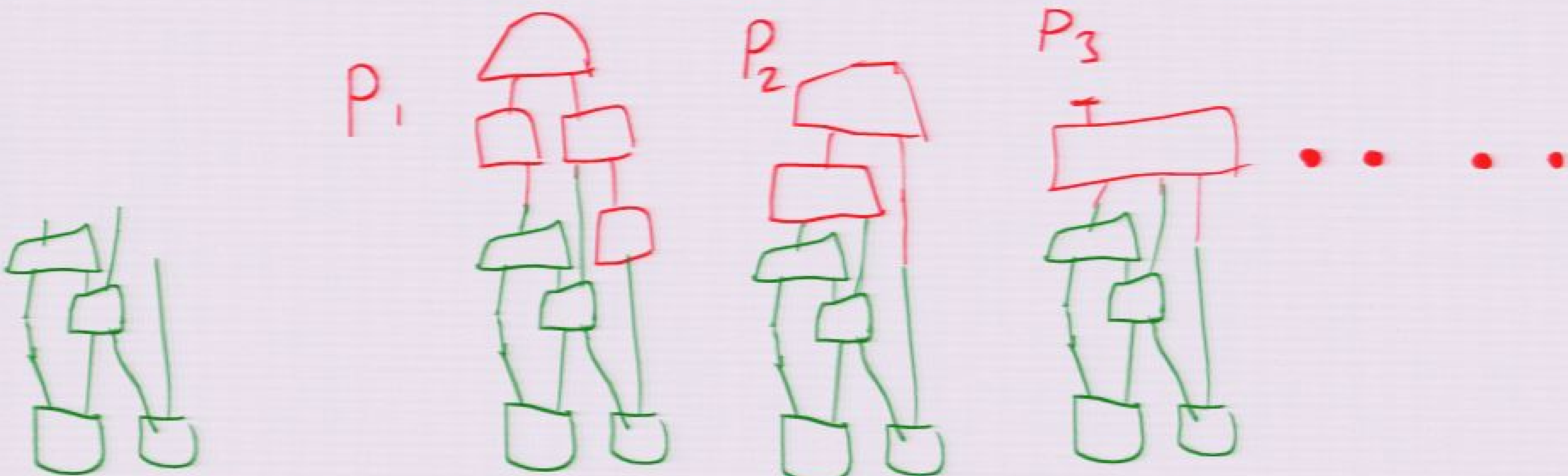

preparation

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Use linear compression

state $\Longleftrightarrow$ $\begin{pmatrix} \vdots \\ p_\alpha \\ \vdots \end{pmatrix} \longrightarrow \underline{p} = \begin{pmatrix} \vdots \\ p_k \\ \vdots \end{pmatrix}$ $k \in \Omega$

$$\boxed{|\Omega| \equiv K}$$

such that $p_\alpha = \underline{r}_\alpha \cdot \underline{p}$

$$\left[\text{c.f.} \qquad p_\alpha = tr(\hat{A}_\alpha \hat{\rho}) \right]$$

10 / 27

have

probability theory

Operational structure

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Eg. Two notions of purity of states equivalent

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Can use this operational structure to set up postulates for QT:

P1 Information Systems having, or constrained to have, the same information carrying capacity have the same properties.

P2 Composites $K_{AB} = K_A K_B$ $N_{AB} = N_A N_B$

P3 Continuity There exists a continuous reversible transformation between any two pure states

P4 Simplicity For each N, K takes the smallest value consistent with the other postulates

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The operational structure described here

is according to Bob Coecke a

symmetric monoidal category

so maybe category theory is a way of formalising

operational structures.

However, it does have a background time.

In Quantum Gravity we expect indefinite

causal structure. So no background time

The structure discussed above will not do.

Need a more general operational structure for QG.

It is not clear whether category theory will help.

The problem, at least for symmetric monoidal categories, is that

tensor $\otimes$ $\equiv$ space like (more or less)

composition $\circ$ $\equiv$ time like

This enforces definite causal structure.

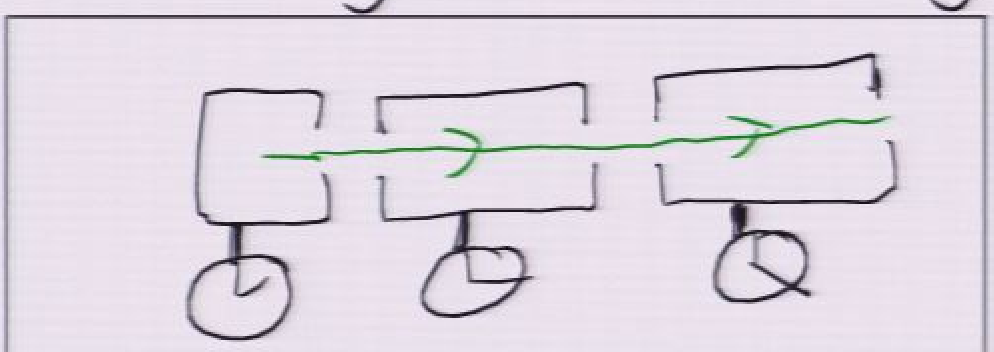
I have some preliminary results to present on building an operational structure that may accommodate QG

– I don't know whether category theory will help here

Object Oriented Operationalism (Beta version)

Want an abstract characterisation of operational structures rich enough to describe all salient aspects of an experiment.

structure above only captures connections between apparatuses due to a system passing in between (operationally this corresponds to the alignment of apertures along with some timing considerations)



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Two concepts concerning relationships between apparatuses

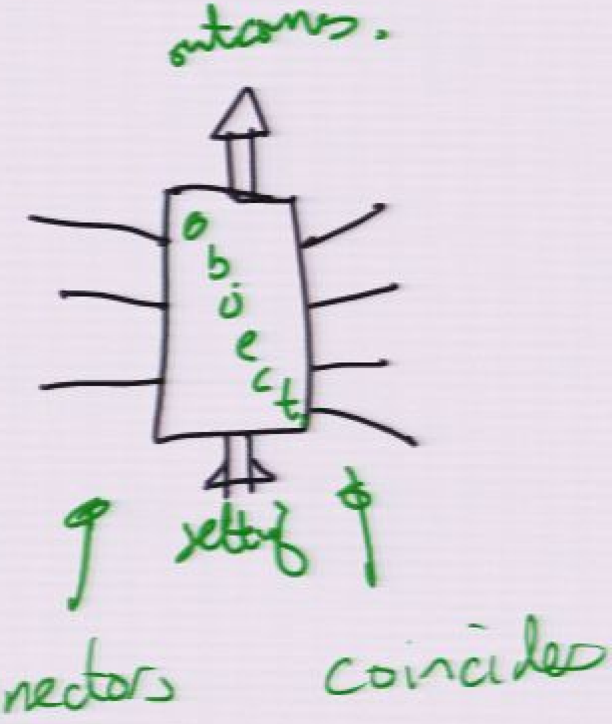
connections are given – the theory makes predictions given a set of connections. } eg alignment of apparatuses

coincidences are things a theory can make predictions about (though can condition on them also). } eg coincidences between marks on rulers

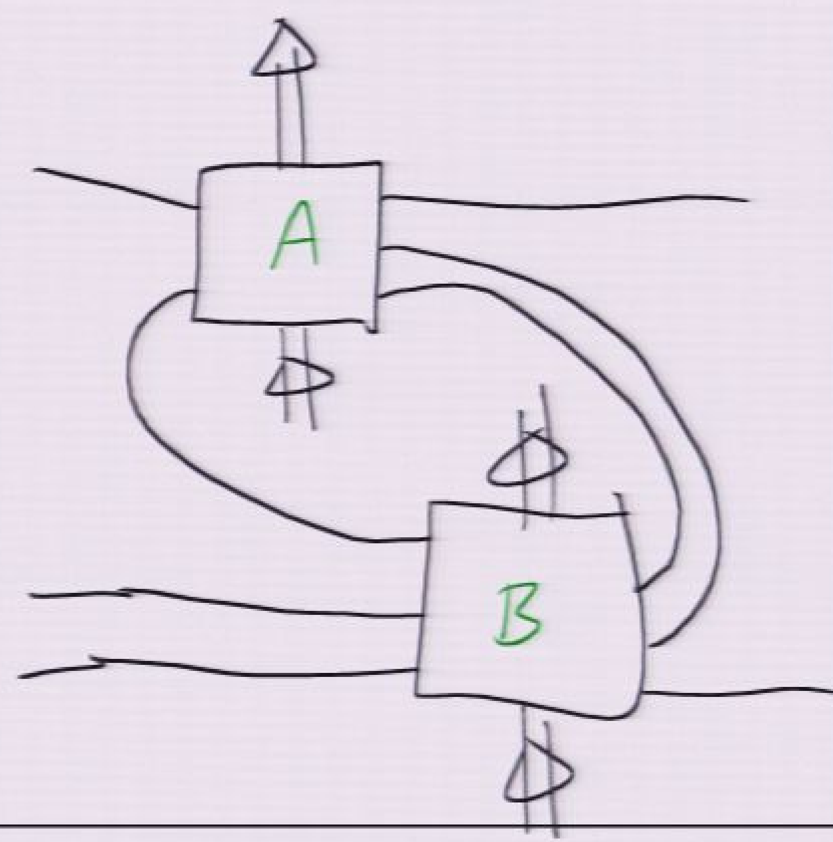
Theory predicts things like prob(coincidences | connections)

Also things like prob(coincidences_1 | coincidences_2, connections)

Object

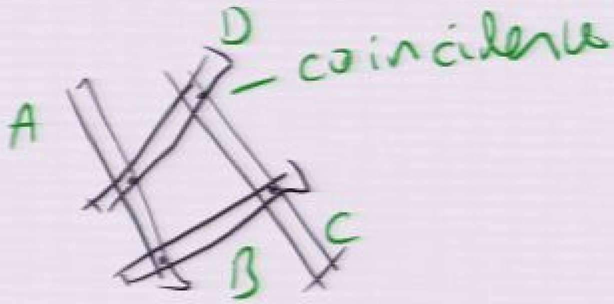


Composite Object



note: no time direction is implied by connections, coincidences in general

Eg rulers floating in space



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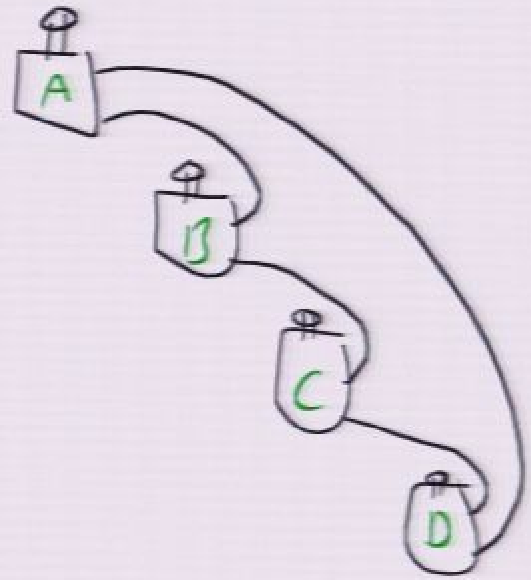
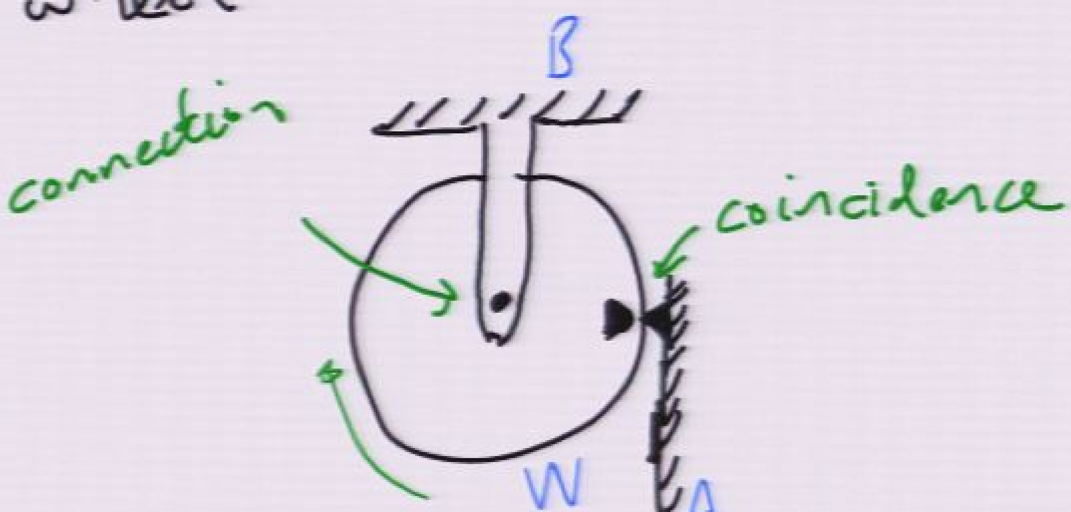


Eg wheel



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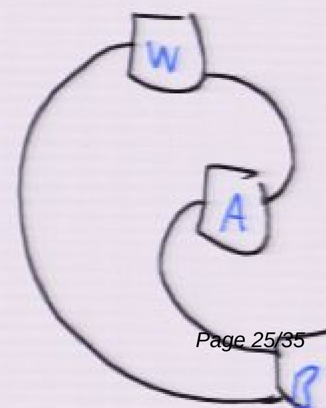

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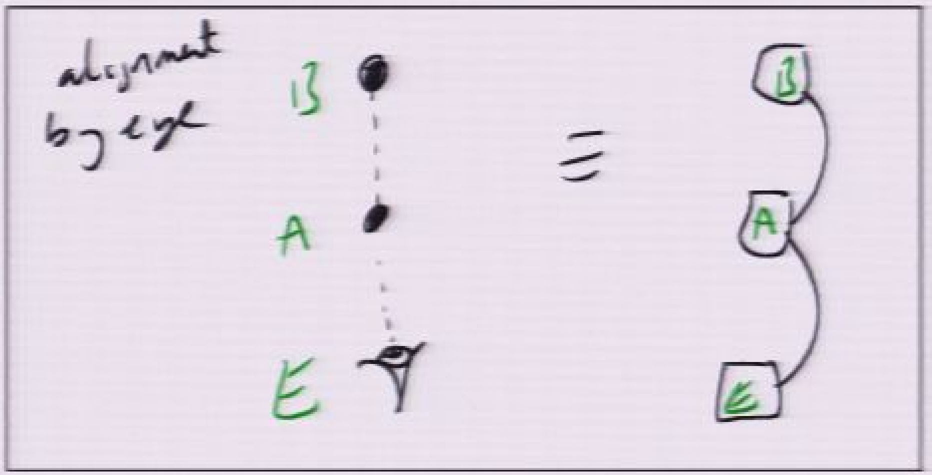
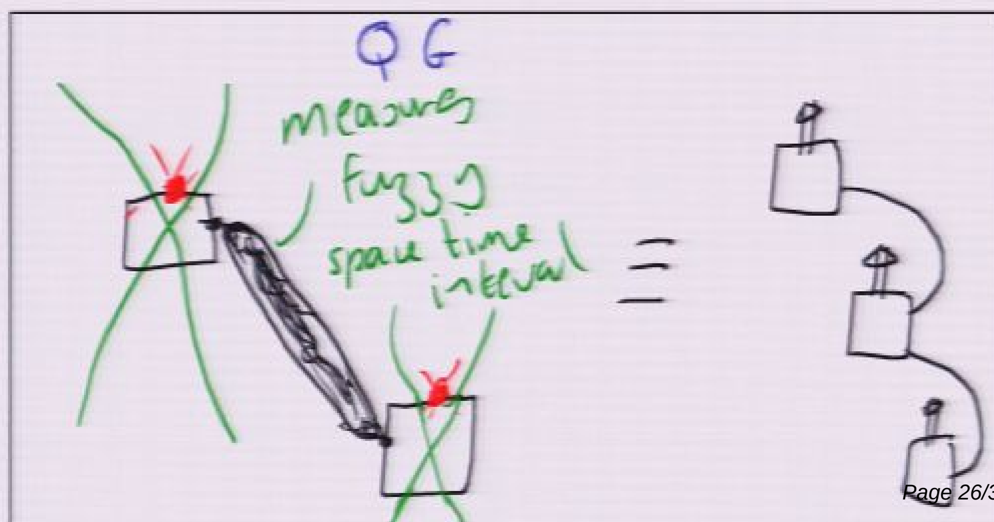
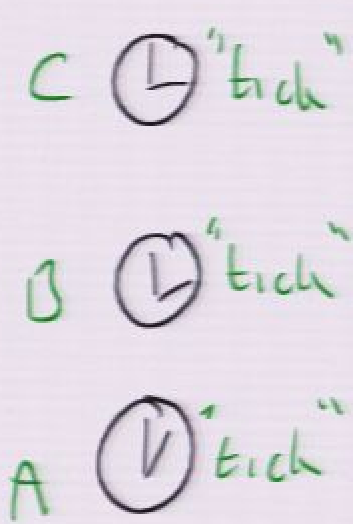
More general notions of connection / coincidence

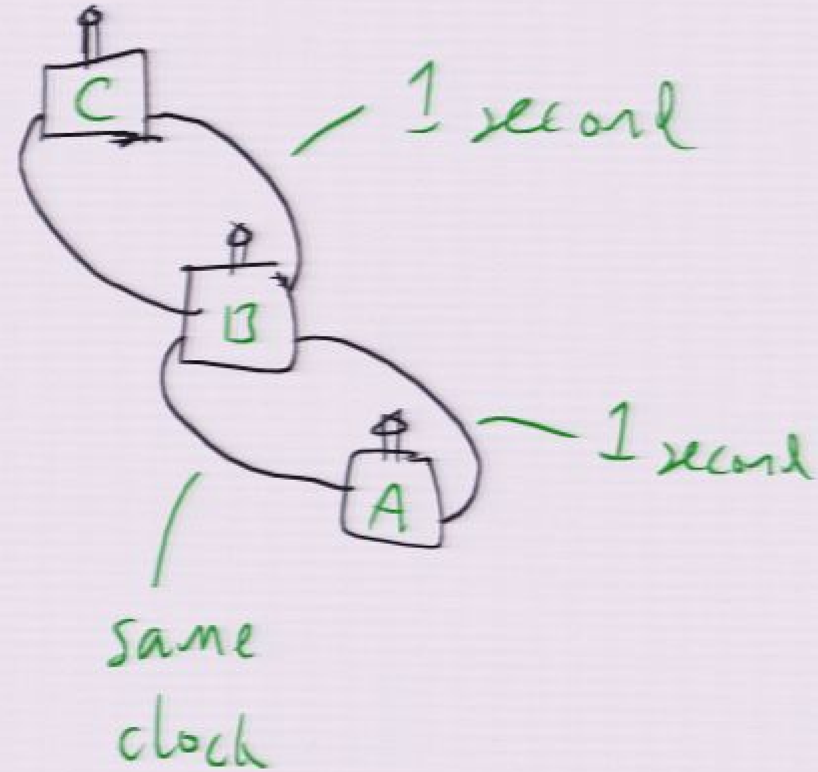




Eg ticking clock.



$\equiv$



(recall an object is one use of an apparatus).

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An operation is closed if it has no open connectors or coinciders.

An operation, A, can be closed if there exists another operation, B, which can form a closed composite operation with A.

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Introduce probability (Causaloid 2.0 Beta)

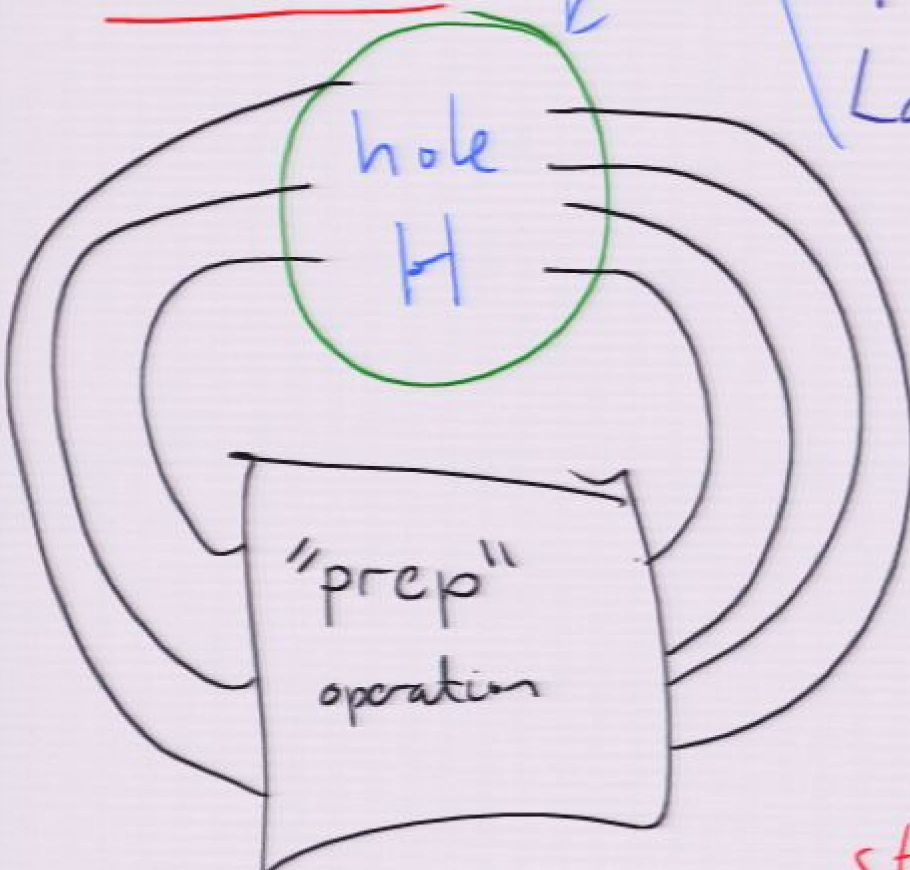
Set of operations is closable if

1) all operations can be closed

2) all closed operations have a well defined probability (i.e independent of other disconnected operations).

Consider only closable sets of operations.

Holes



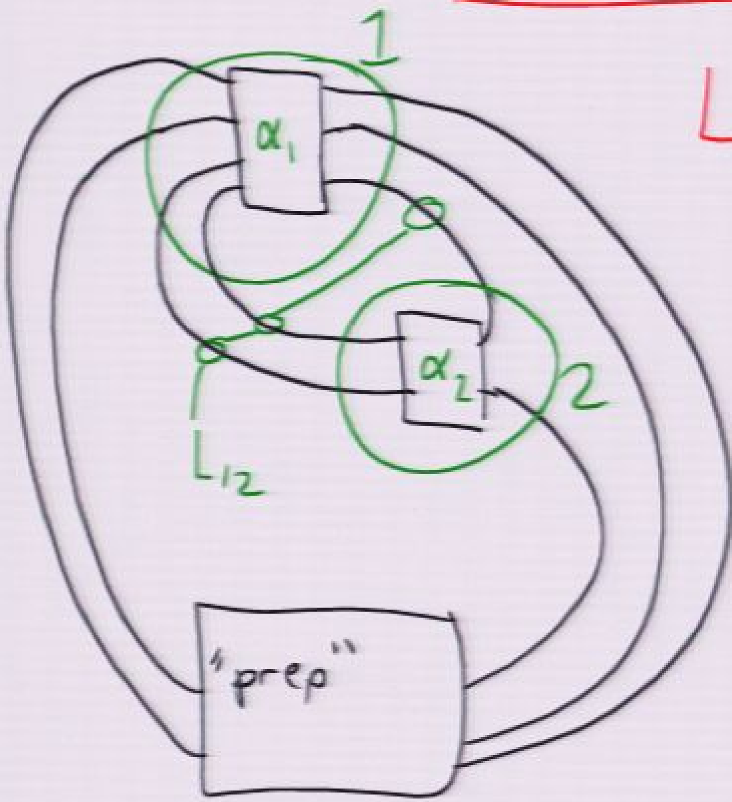
Can insert many operations in hole. Label them with α

$$\text{"prep"} \iff \begin{pmatrix} \vdots \\ p_\alpha \\ \vdots \end{pmatrix} \qquad \underline{p} = \begin{pmatrix} \vdots \\ p_k \\ \vdots \end{pmatrix} \quad k \in \Omega$$

$$\text{s.t. } p_\alpha = \underline{r}_\alpha \cdot \underline{p} \qquad \forall \text{ "prep"s}$$

Vector $\underline{r}_\alpha$ is associated with operation.

Causaloid product



L_{12} specifies connections and coincidences

$$\Gamma^{L_{12}}_{\alpha_1 \alpha_2} = \Gamma_{\alpha_1} \otimes^{L_{12}} \Gamma_{\alpha_2}$$

$$\Gamma^{L_{12}}_{\alpha_1 \alpha_2} \Big|_{l_1 l_2 \in \Omega_{12}} = \sum_{k_1 k_2 \in \Omega_1 \times \Omega_2} \Lambda^{k_1 k_2}_{l_1 l_2}(L_{12})\, \Gamma_{\alpha_1}\Big|_{k_1} \Gamma_{\alpha_2}\Big|_{k_2}$$

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Can use the vectors $\underline{r}$ to calculate

1) if a probability is well defined

2) if it is well defined then its value.

without reference to a background time

(details not presented here

but same as in Causaloid papers on arXiv)

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Conclusions

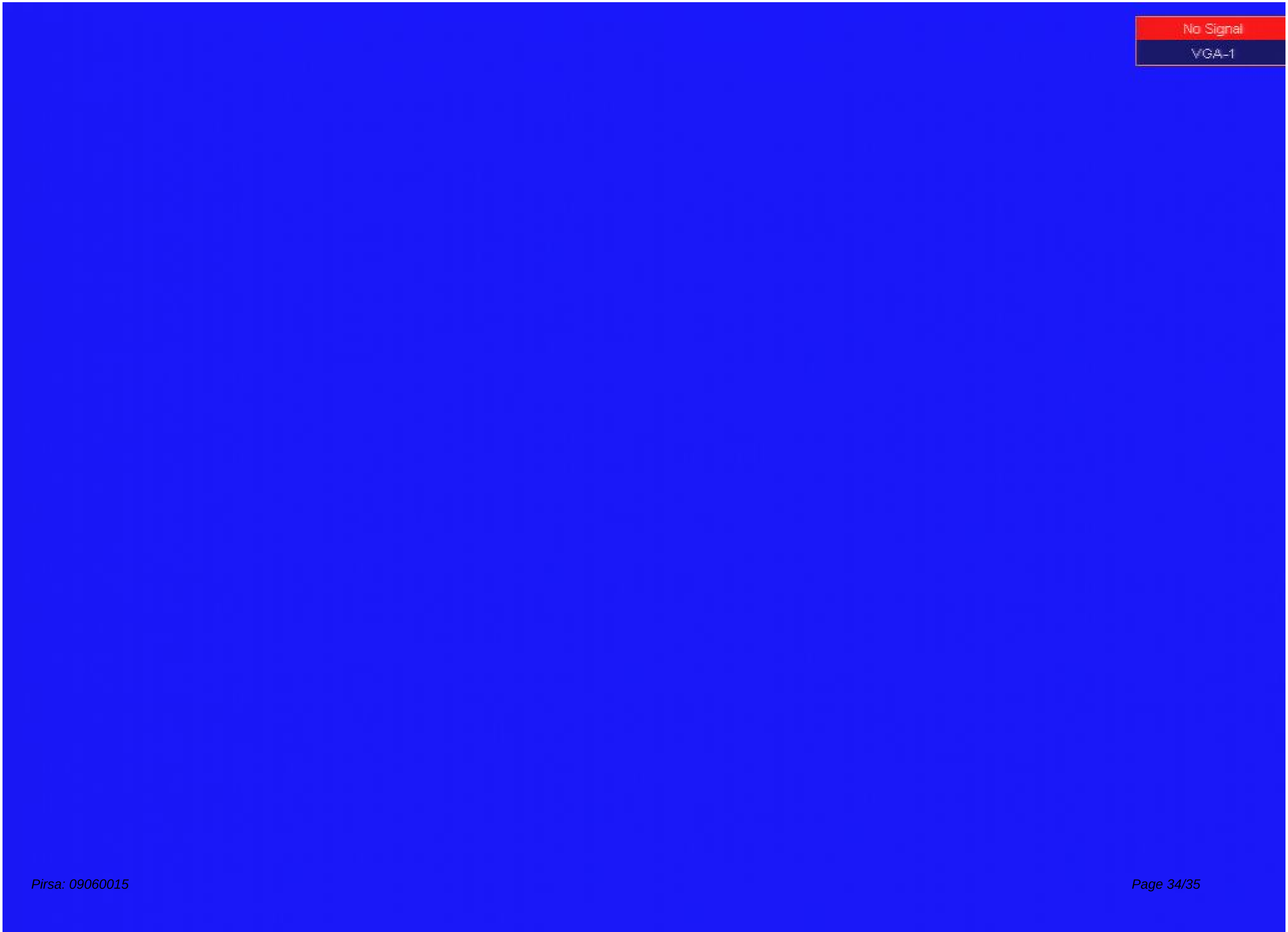
1) Operational structures are a foundation for general probabilistic theories

2) Can give an operational structure sufficient for quantum theory - this has a background time.

3) Object Oriented Operationalism beta provides a more general operational framework (GR? QG?) category?

4) Causaloid 2.0 beta adds probabilistic flesh to these structural bones.

5) Have technique for injecting measurement problem into General Relativity


No Signal
VGA-1

No Signal
VGA-1