Title: Complementarity as a resource

Date: Jun 01, 2009 11:30 AM

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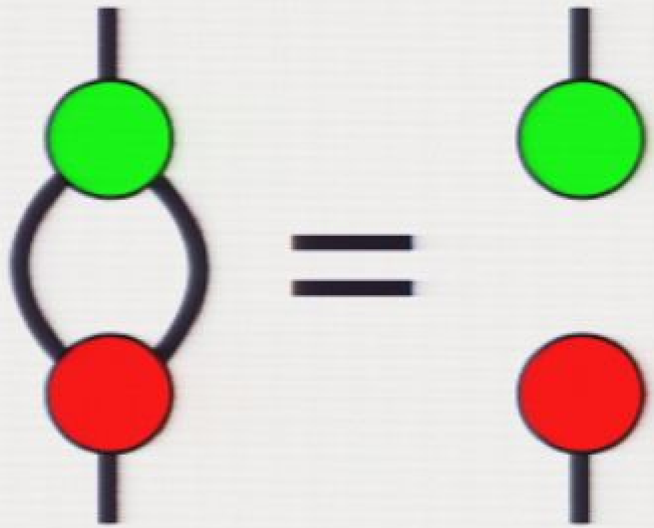
Abstract: TBA

COMPLEMENTARITY AS A RESOURCE

Bob Coecke & Ross Duncan



Ref: *Interacting quantum observables, secs 6,7,9.* **on arXiv soon!**

Cats tutorial: *Categories for the practicing physicist.* **arXiv:0905.3010.**

Survey: *Quantum picturalism.* **Contemporary Physics. Ask me for a copy.**

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To joint parts and processes we refer by $- \otimes -$,
and to consecutive processes by $- \circ -$.

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Our language := system, process, $\otimes$, $\circ$

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Graphical presentation. An equational statement is provable for SMCs *if and only if* it is provable in the corresponding graphical calculus. [Joyal & Street '91]

— *data of a compositional theory* —

Systems:

$$A \qquad B \qquad C$$

Processes:

$$A \xrightarrow{f} A \qquad A \xrightarrow{g} B \qquad B \xrightarrow{h} C$$

Compound systems:

$$A \otimes B \qquad \mathrm{I} \qquad A \otimes C \xrightarrow{f \otimes g} B \otimes D$$

Temporal composition:

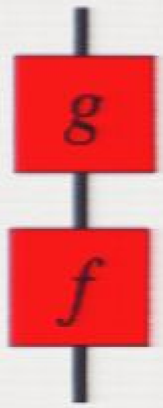
$$A \xrightarrow{h \circ g} C := A \xrightarrow{g} B \xrightarrow{h} C \qquad A \xrightarrow{1_A} A$$

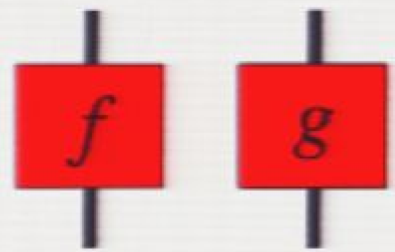
— *graphical formalism* —

$f \equiv$ 

$1_A \equiv$ 

— *graphical formalism* —

$g \circ f \equiv$ 

$f \otimes g \equiv$ 

— *states, effects and quantities* —

$\psi : \mathrm{I} \to A$ $\pi : A \to \mathrm{I}$ $\pi \circ \psi : \mathrm{I} \to \mathrm{I}$

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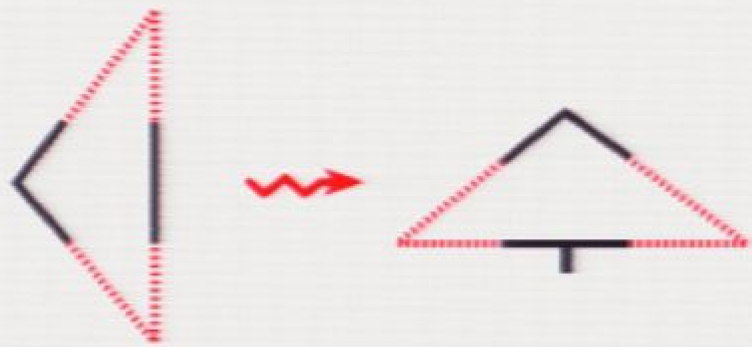

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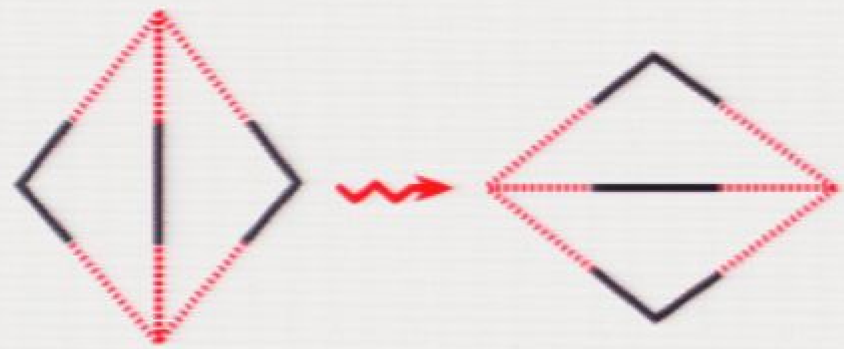
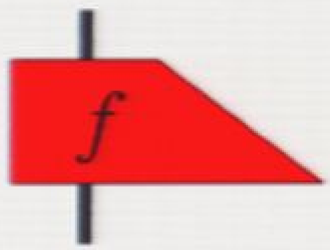
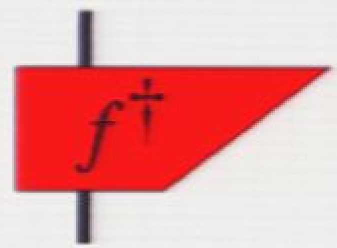


— *dagger symmetric monoidal category* —

$f : A \to B$ $\qquad$ $f^\dagger : B \to A$





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- fin. dim. Hilbert spaces
- linear maps
- tensor product
- adjoint

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- fin. sets
- relations
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$\mathcal{WP}(\mathbf{FHilb}) :=$ Pure states, operations, effects

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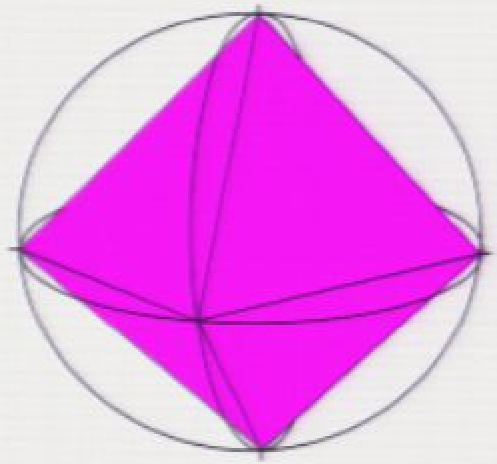
Stab := sub-†-SMC of **FHilb** generated by

- nth powers of qubits $\mathcal{Q}$
- unitaries on $\mathcal{Q}$ $\bigcap$ symmetries Bloch-octahedron
- $$\mathcal{Q} \to \mathcal{Q} \otimes \mathcal{Q} :: \begin{cases} |0\rangle \mapsto |00\rangle \\ |1\rangle \mapsto |11\rangle \end{cases} \quad + \quad \textit{its unit}$$



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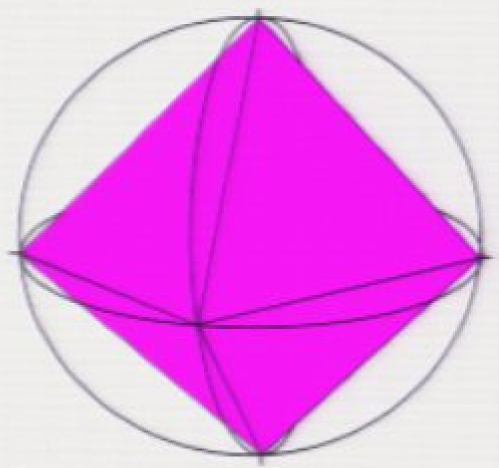

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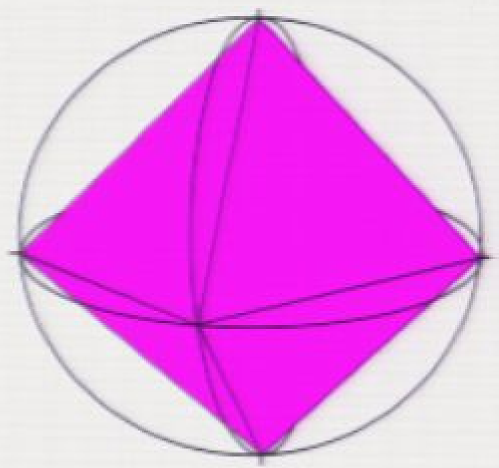

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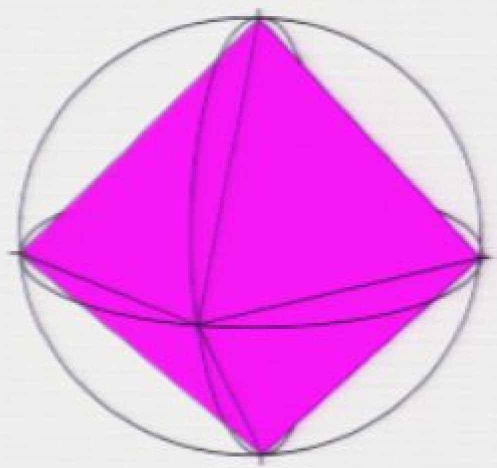

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Spek := sub-†-SMC of **FRel** generated by

- nth powers of quads $\mathbb{IV}$
- all permutations on $\mathbb{IV}$
- $$\mathbb{IV} \to \mathbb{IV} \times \mathbb{IV} :: \begin{cases} 1 \mapsto \{(1,1),(2,2)\} \\ 2 \mapsto \{(1,2),(2,1)\} \\ 3 \mapsto \{(3,3),(4,4)\} \\ 4 \mapsto \{(3,4),(4,3)\} \end{cases} \quad + \quad \textit{its unit}$$

Is there a notion of:

- **Classicality** (= conceptual)
- **Observable** (= opertional)
- **Basis** (= technical)

Which applies to all of the above categories?

Only **FHilb** and **FRel** have 'direct sum bases':

- Abramsky & Coecke. in LiCS'04 arXiv:quant-ph/0402130

OBSERVABLES

Coecke, Pavlovic, ... Vicary, ... Paquette, Perdrix

quant-ph/0608035, 0810.0812, 0902.0500, 0904.1997

observable := copying + deleting ability

$$= \delta : A \longrightarrow A \otimes A$$

$$= \epsilon : A \longrightarrow \mathrm{I}$$

A (non-deg) *observable* is:

$$A \xrightarrow{\delta} A \otimes A =$$

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such that:

1. ϵ is a *unit* for δ ;
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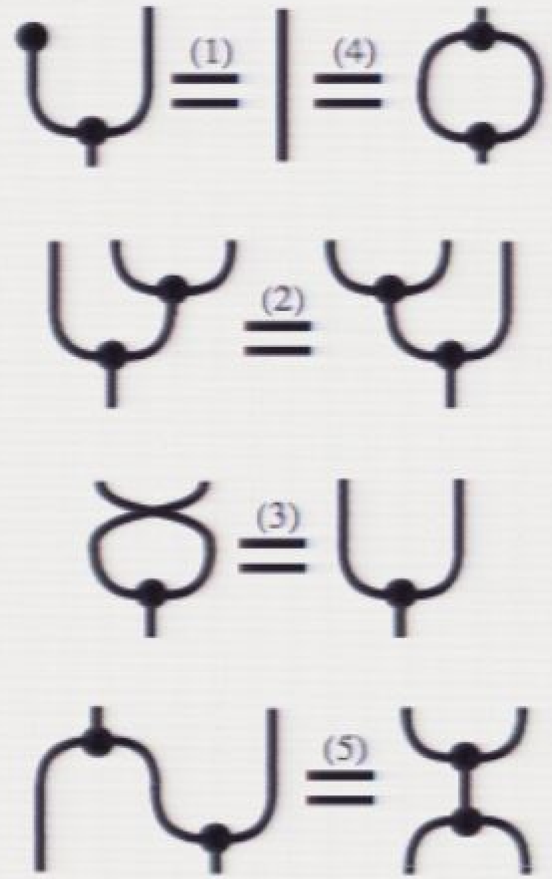

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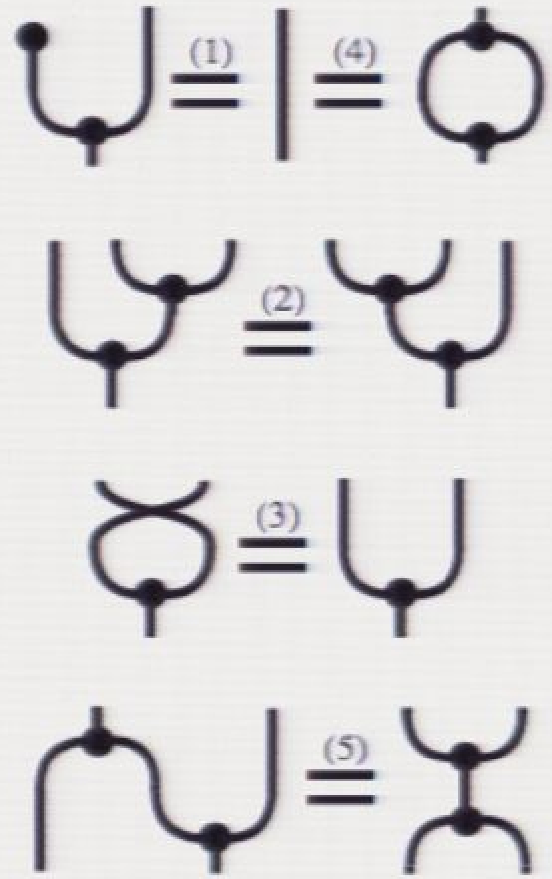

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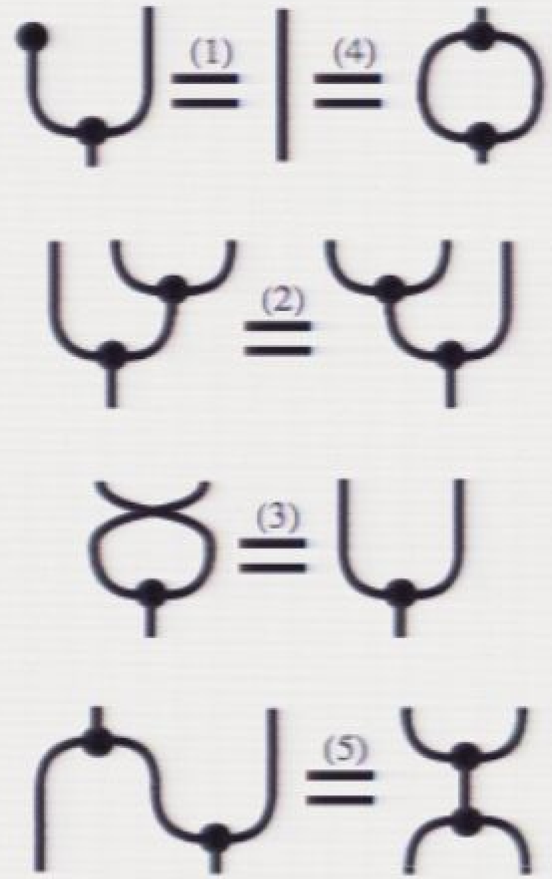

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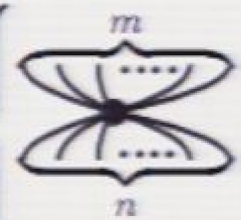
Encoding basis as linear map:

$$\sum_i c_i |i\rangle\langle i| \leftrightarrow \sum_i |ii\rangle\langle i|$$

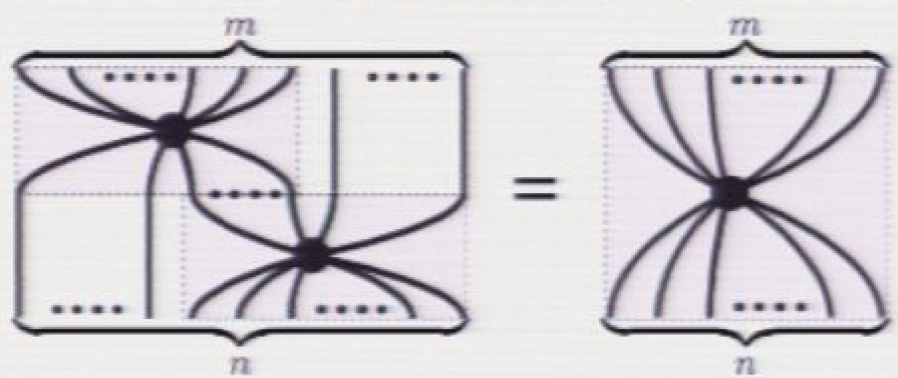
Examples:

- **FHilb**: indeed.
- **Stab**: indeed.
- $\mathcal{WP}$(**FHilb**): indeed.
- $\mathcal{CP}$(**FHilb**): indeed.
- **Spek**: indeed.
- $\vdots$
- **FRel**: lots more observables now.

An *observable* is:

$\Bigg\{$  $\Bigg|\ n, m \in \mathbb{N} \Bigg\}$

invariant under flipping and swapping, and such that:



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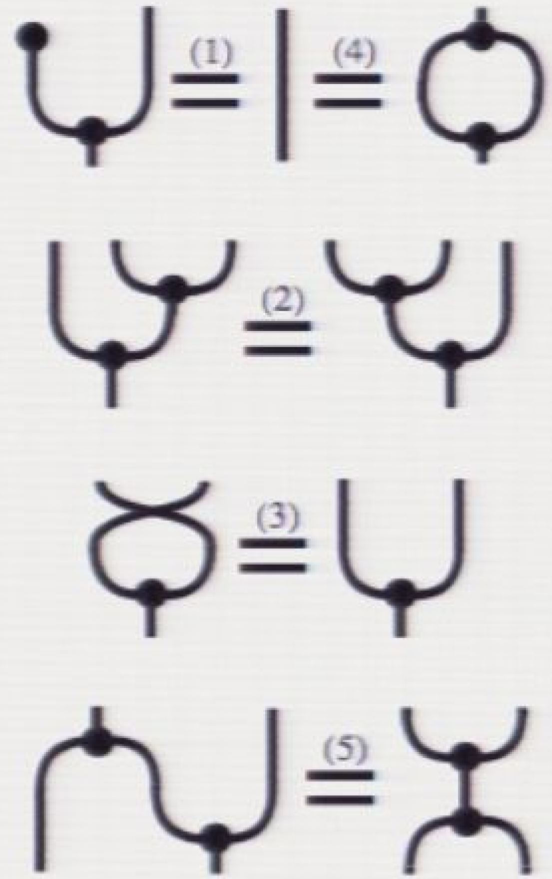

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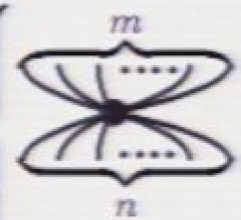
$$\left\{ \right.$$  $$\left| \; n, m \in \mathbb{N} \right\}$$

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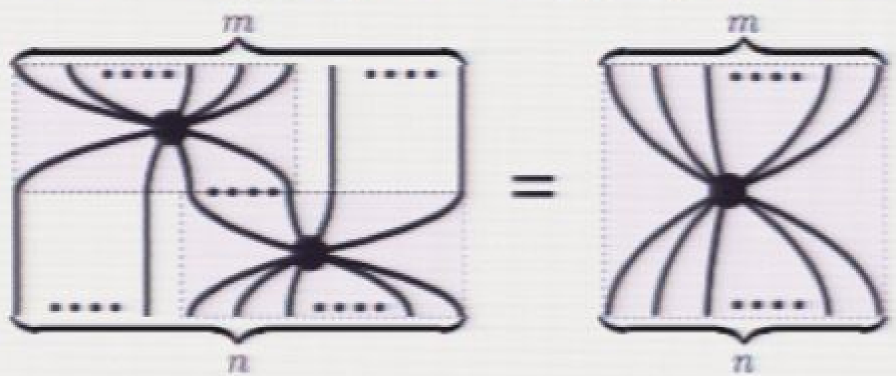

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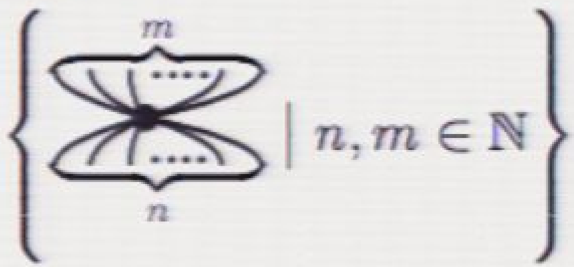

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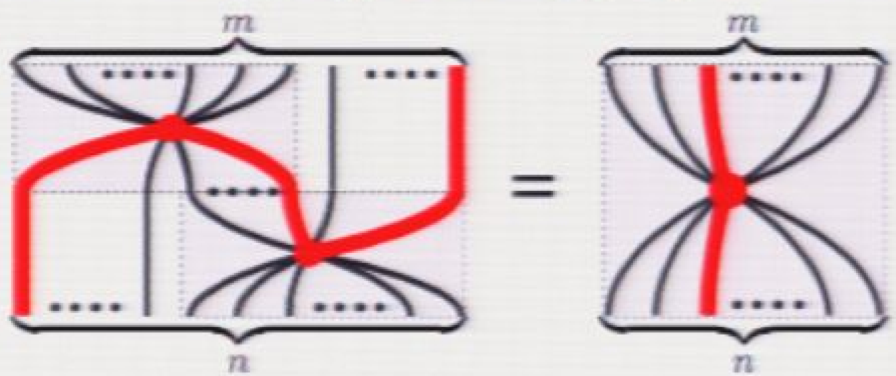

spiders  and  are Bell state/effect

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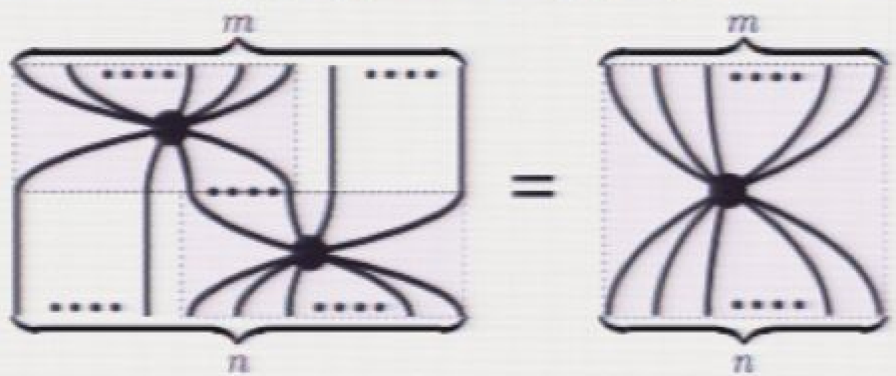

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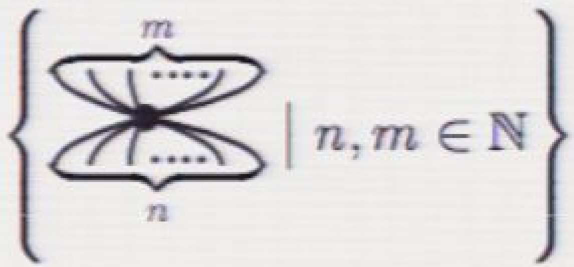

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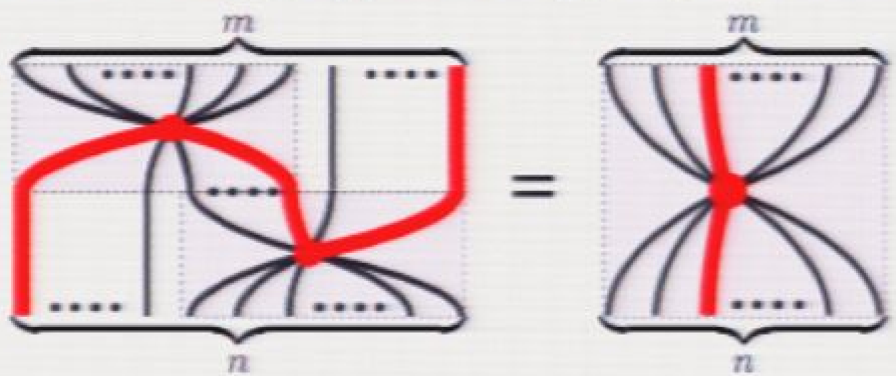

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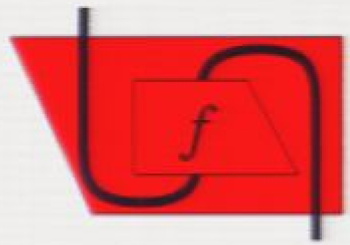

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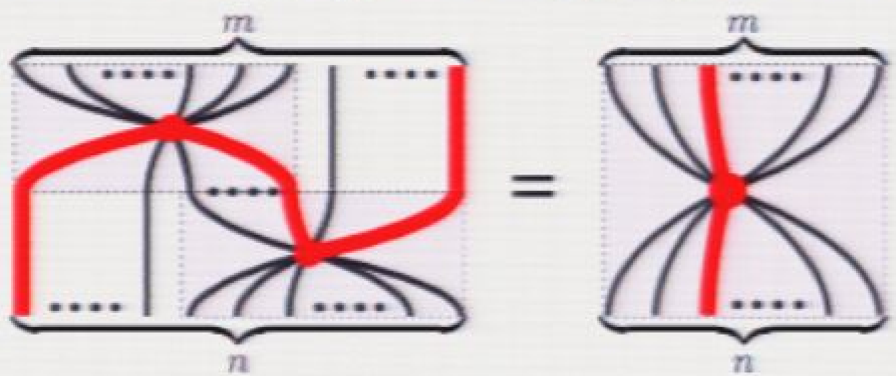
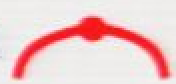


spiders  and  are Bell state/effect

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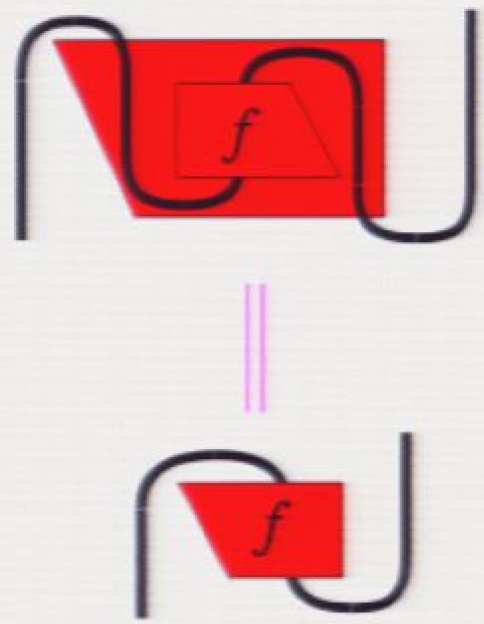

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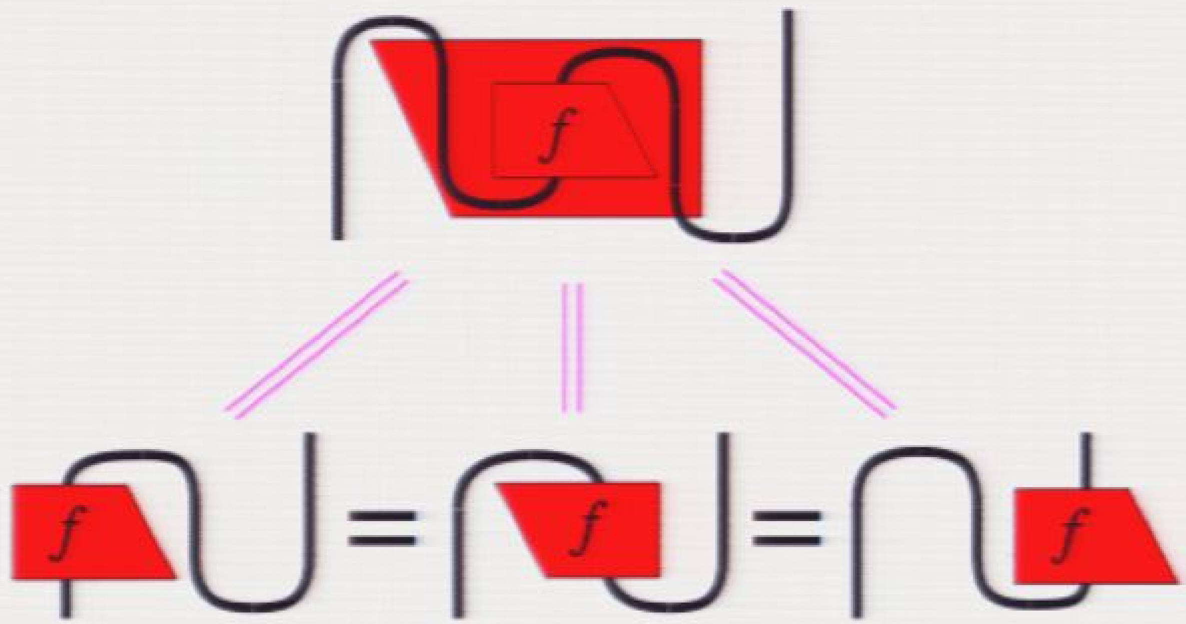

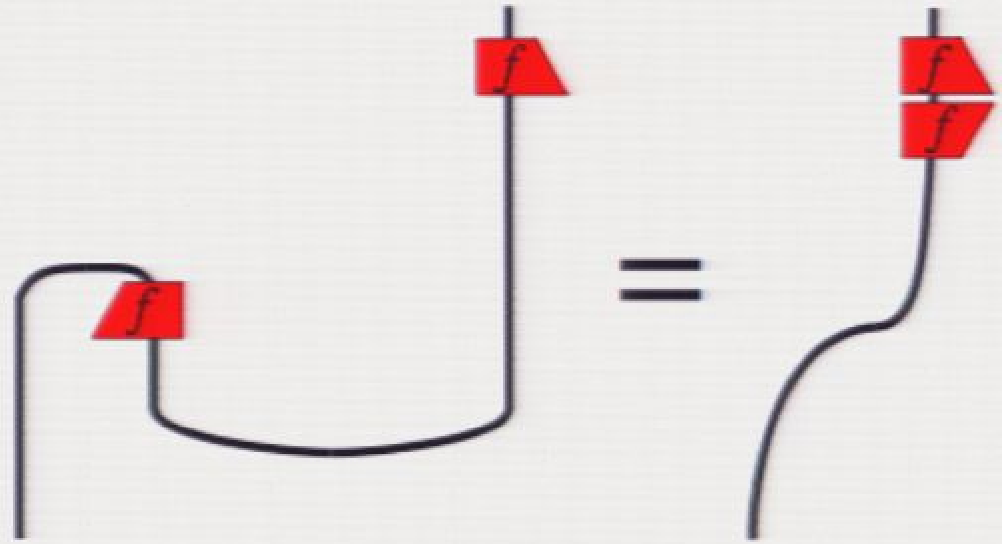

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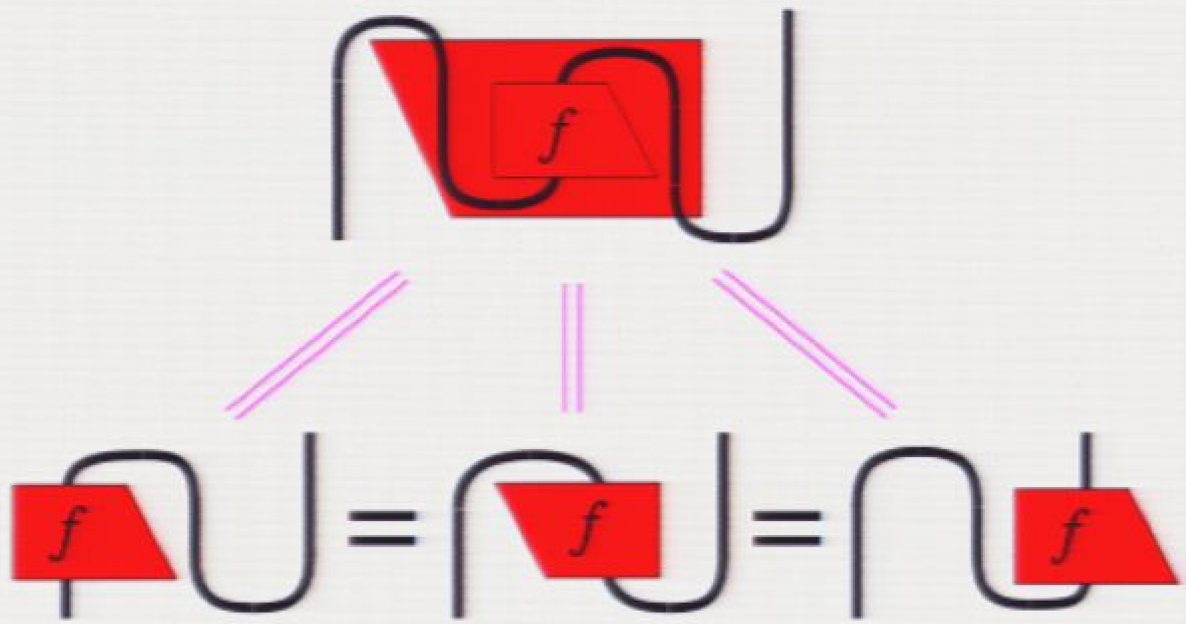

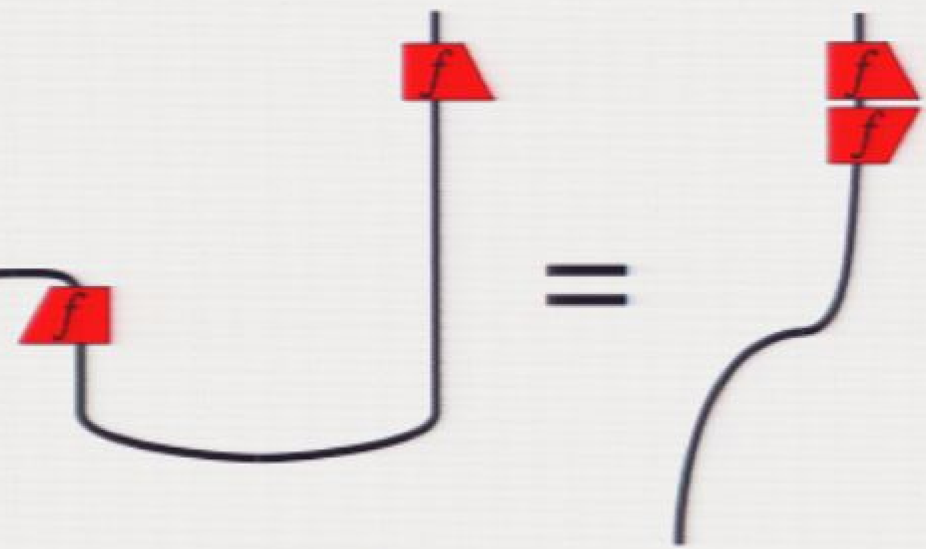

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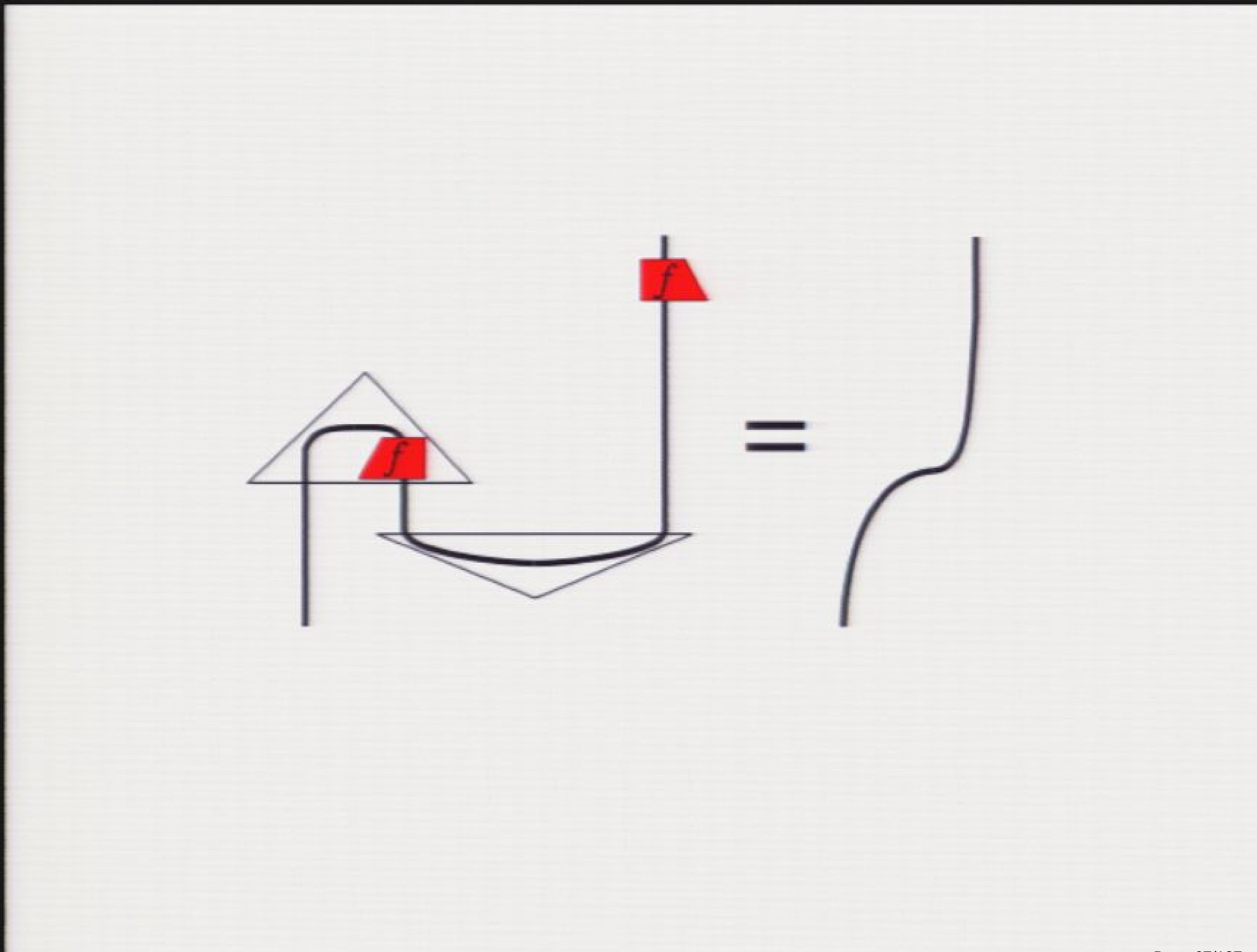

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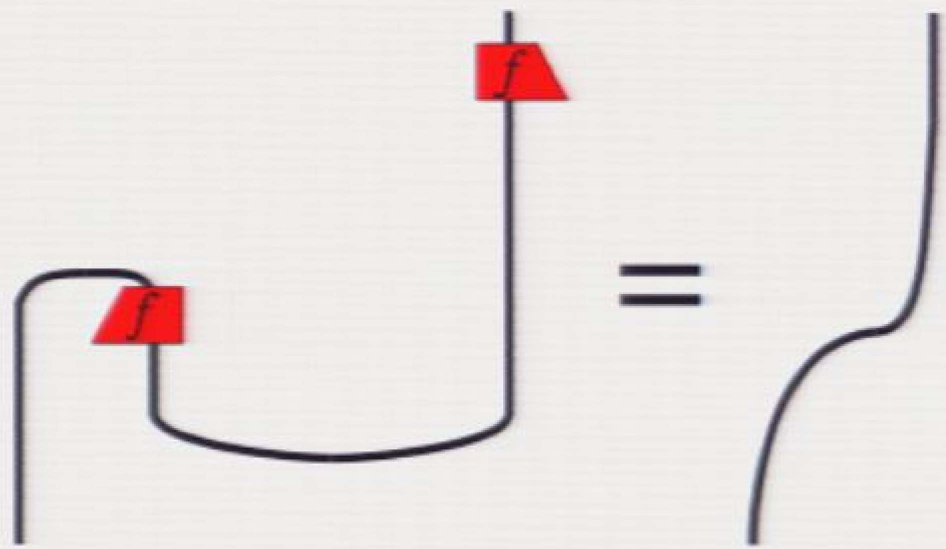
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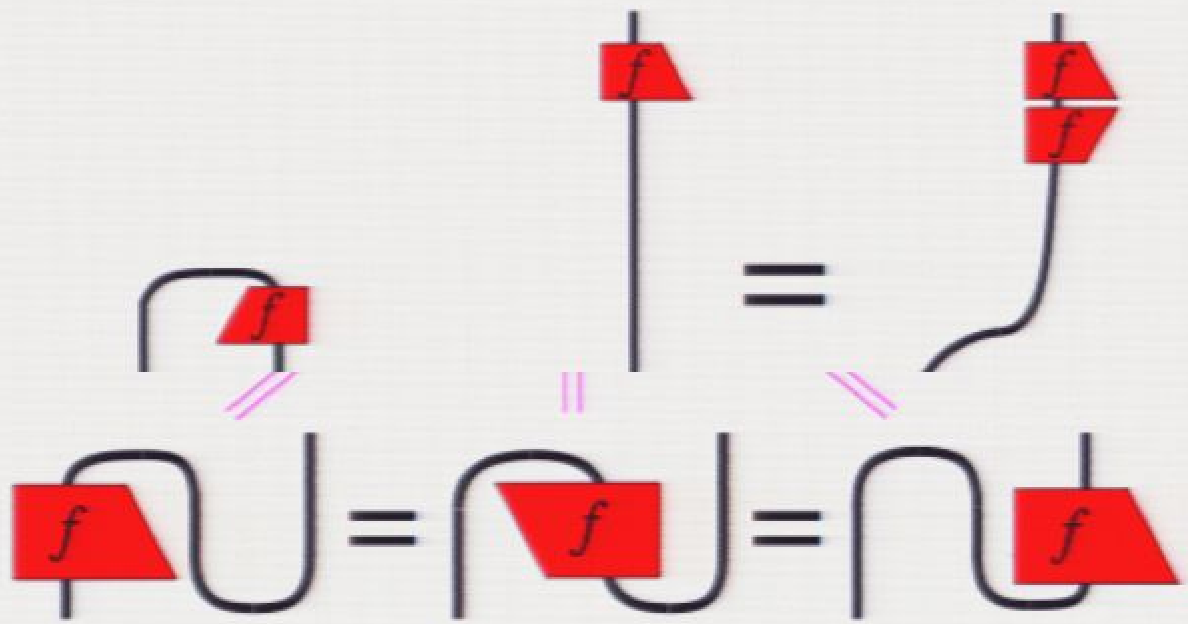
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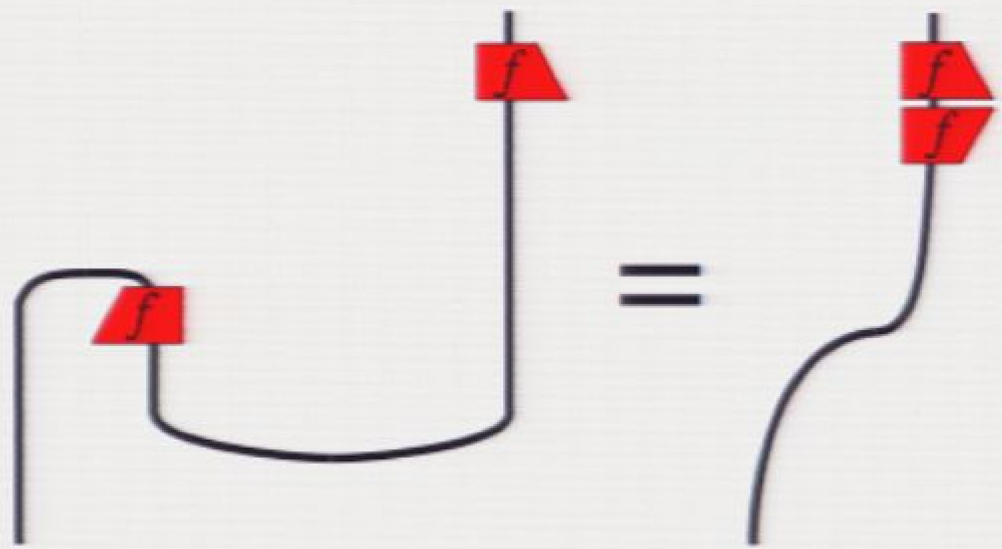

f
f
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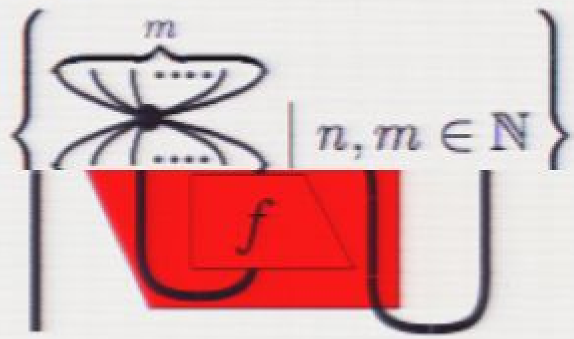
=

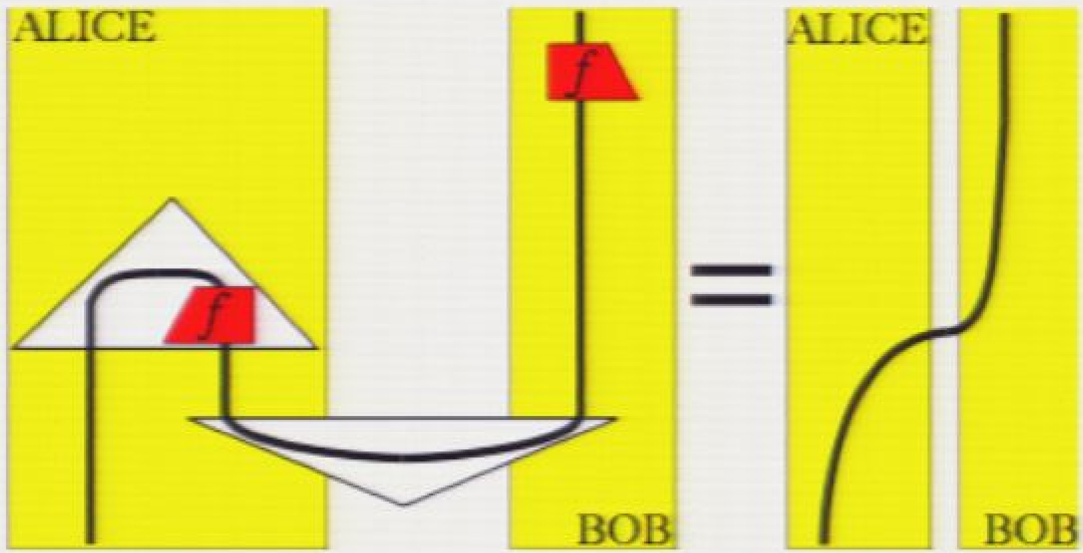
An *observable* is:



COMPLEMENTARY OBSERVABLES

Coecke-Duncan ICALP'08

Def. Two observables are complementary (=unbiased) if the classical states for one are unbiased for the other.



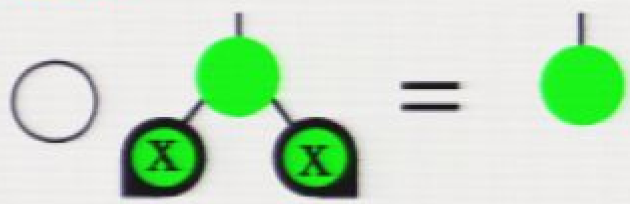
⇒ **quantum teleportation**

Abramsky & Coecke in LiCS'04; arXiv:quant-ph/0402130

Def. Two observables are complementary (=unbiased) if the classical states for one are unbiased for the other.

Def. Classicality:

Def. Unbiasedness:



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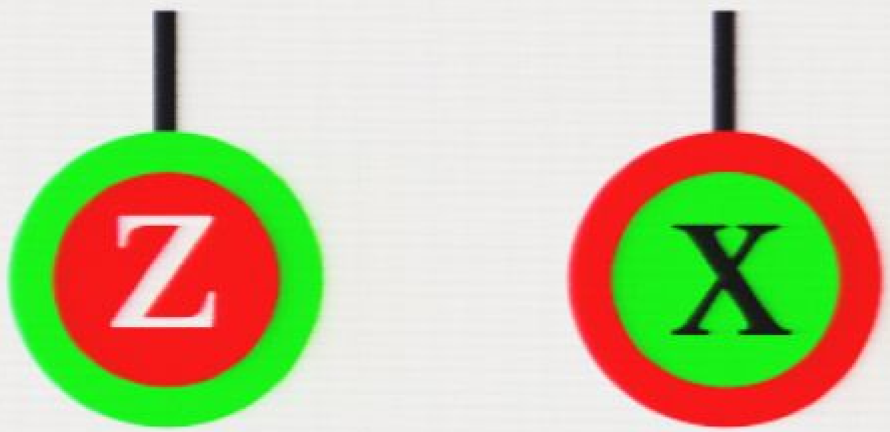

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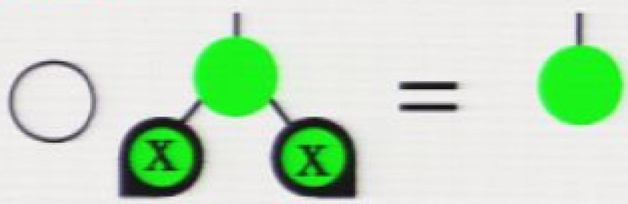

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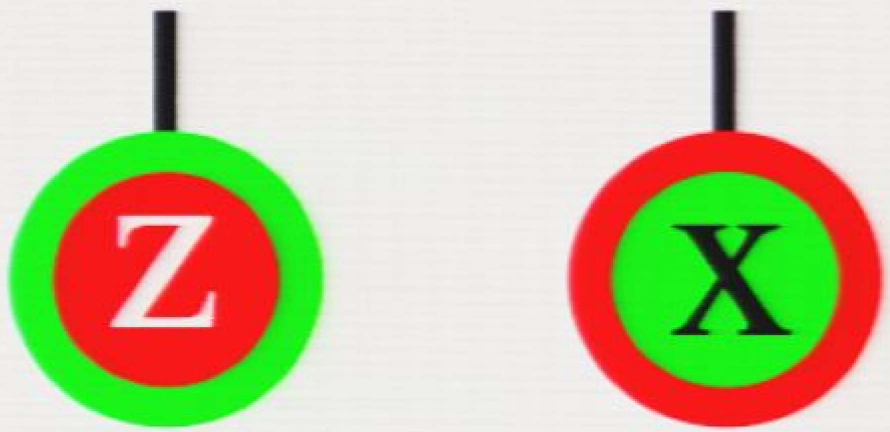
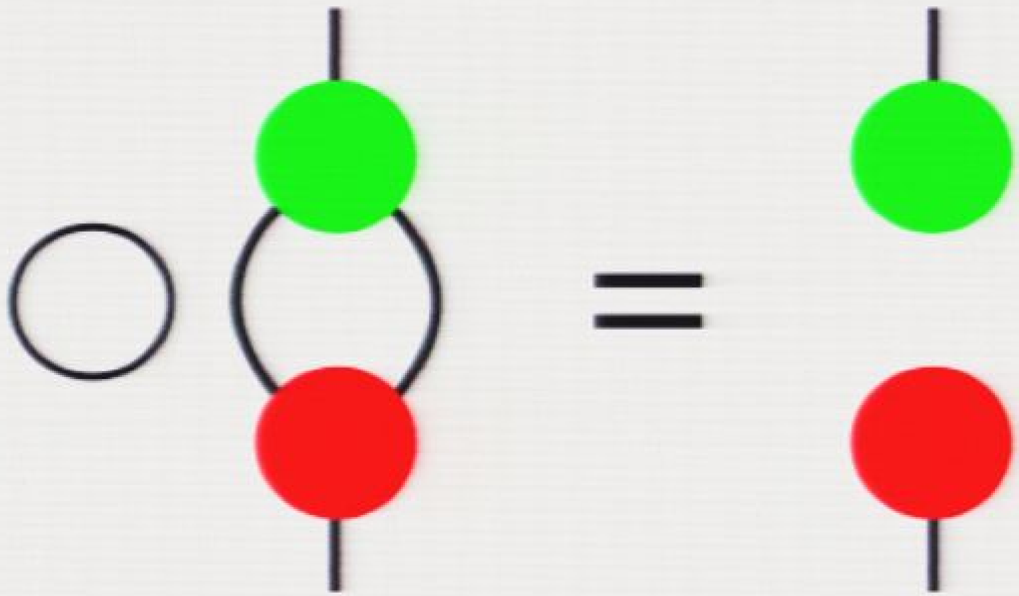
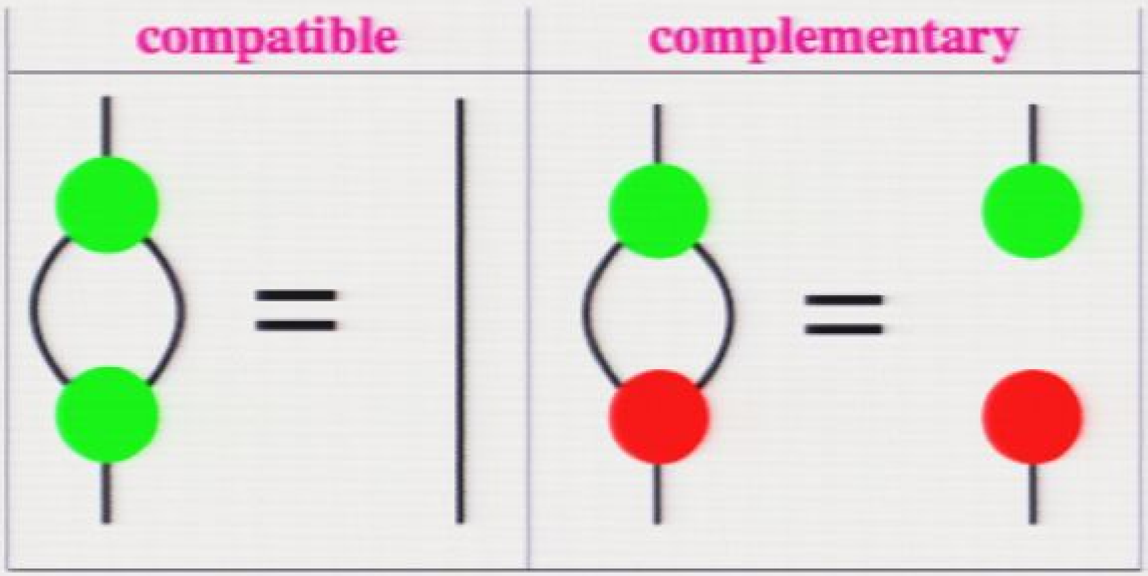


Thm. Observables are complementary if and only if:




compatible
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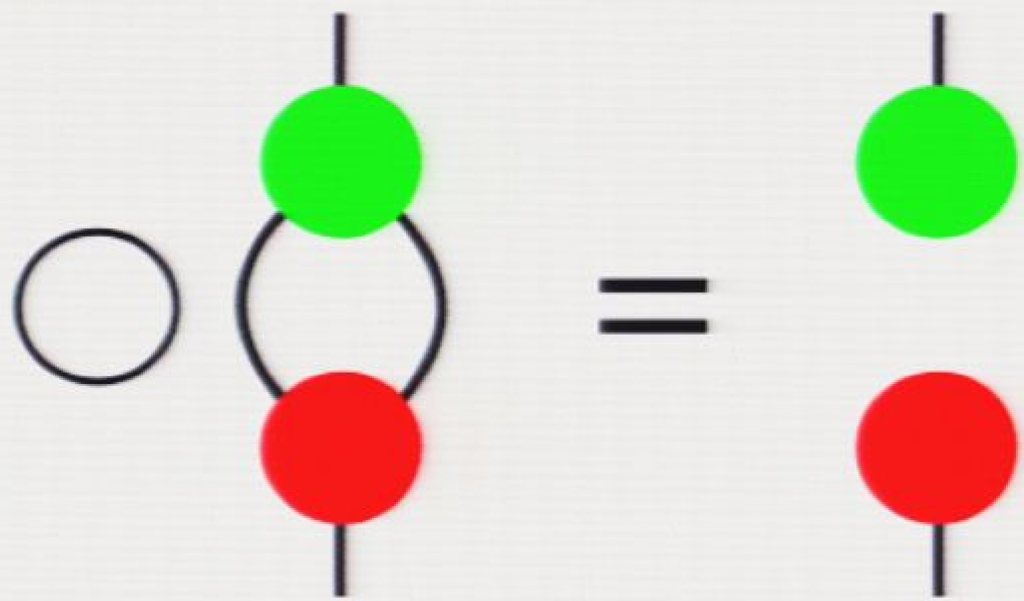

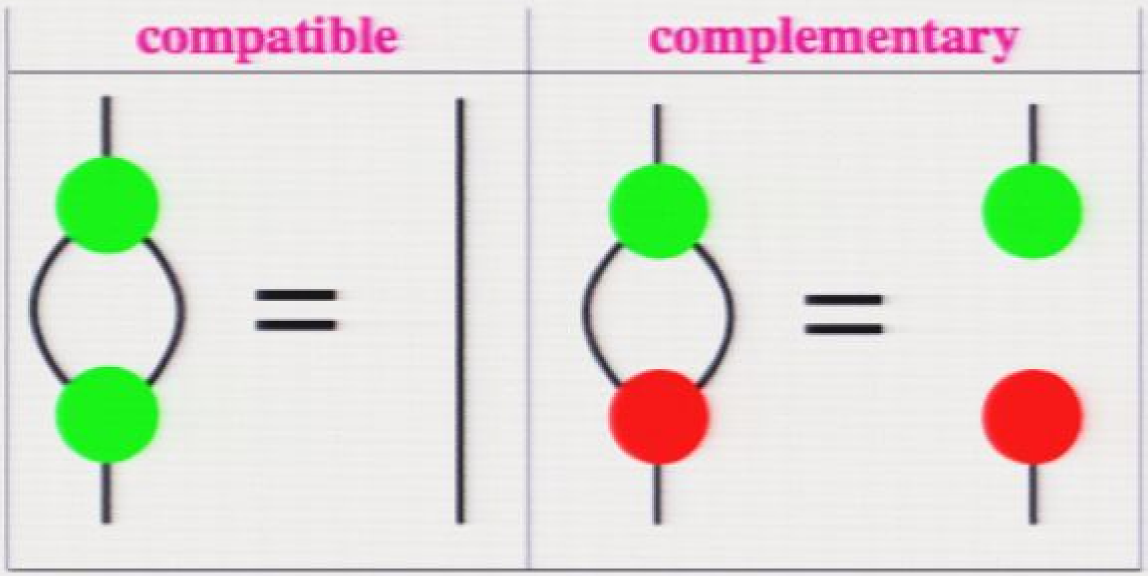
compatible
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=
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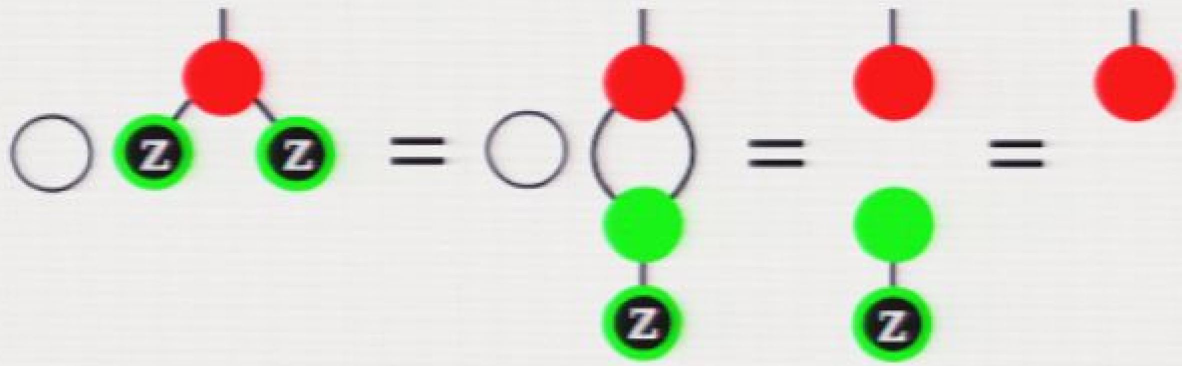
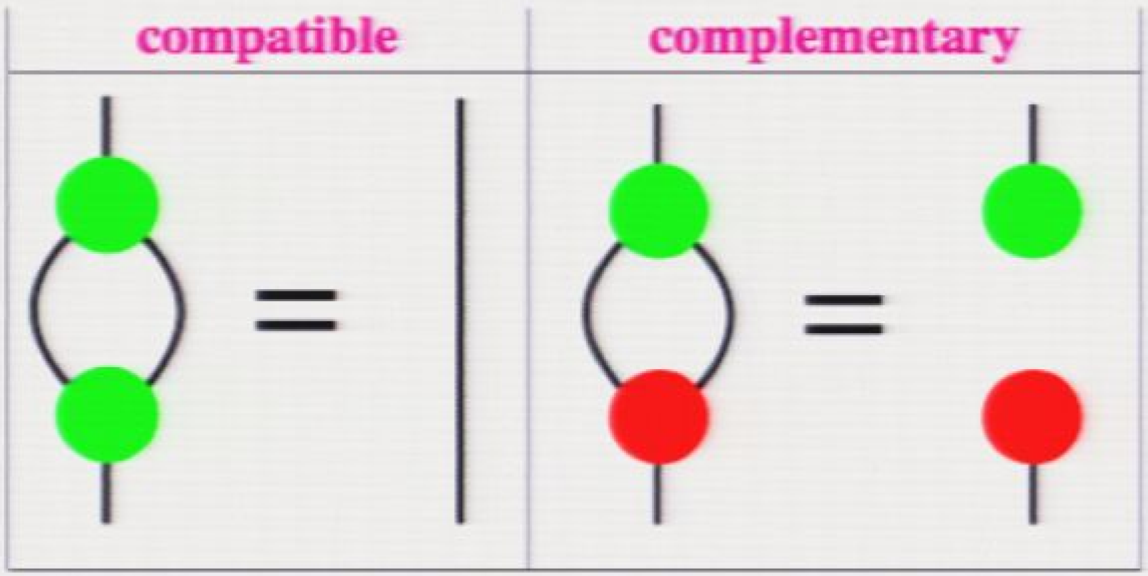
Proof. Hopf law $\Longrightarrow$ [class $\Rightarrow$ unbiased]

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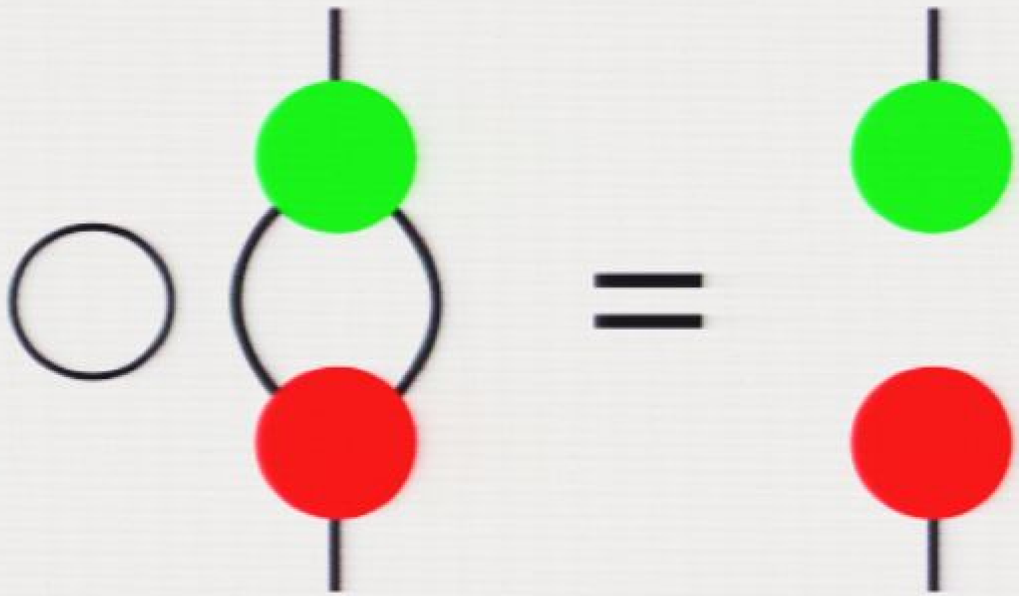

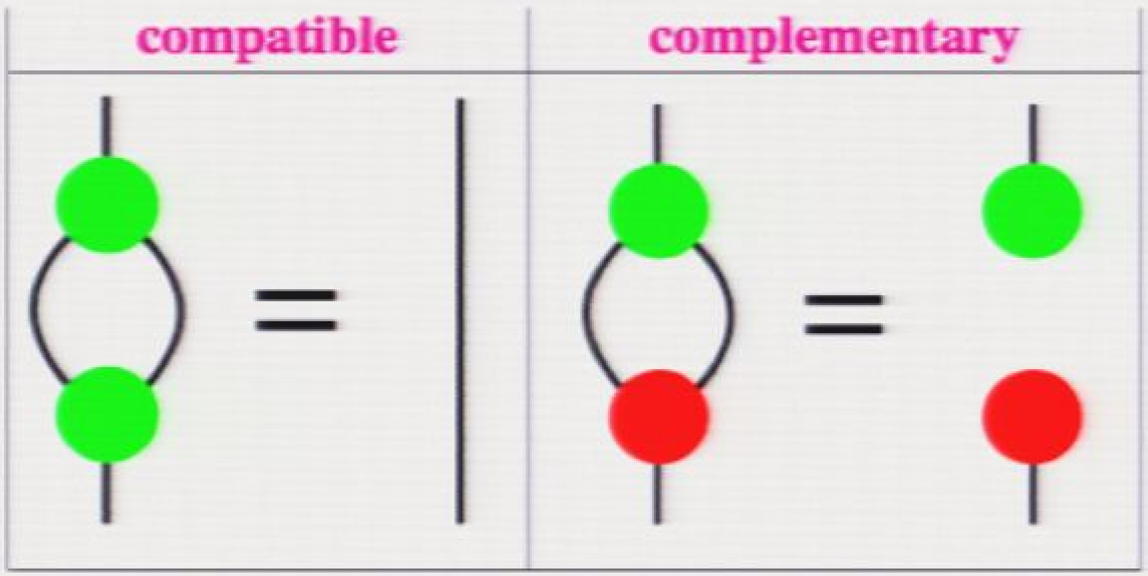
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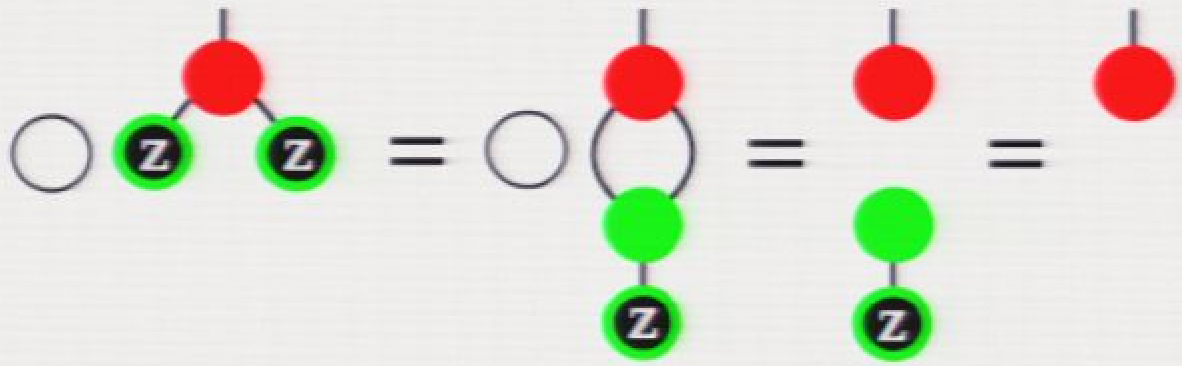

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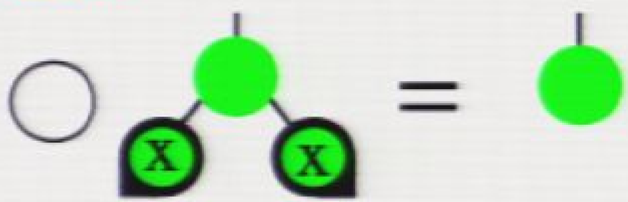

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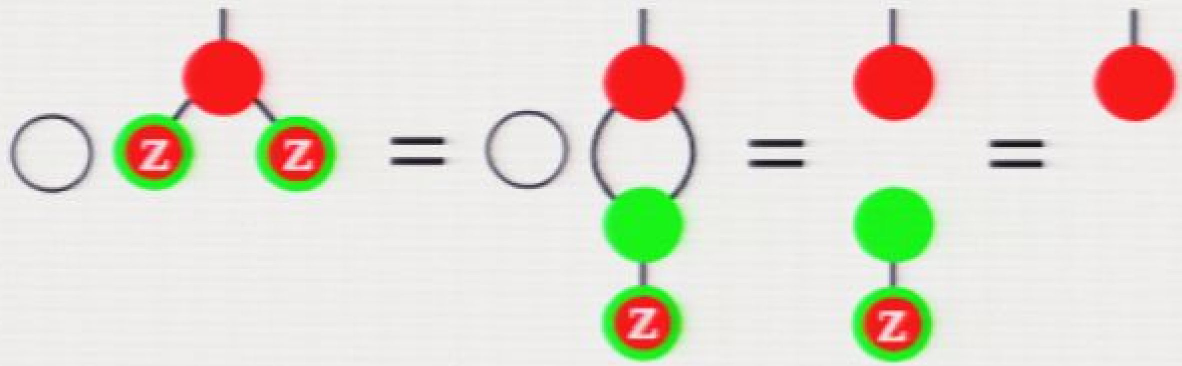

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Observable with class states $\mathcal{B}$ has *vector basis* iff

$$\forall z \in \mathcal{B} : f \circ z = g \circ z \;\Rightarrow\; f = g\,.$$

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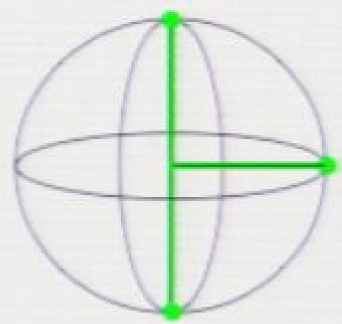

E.g. $\mathcal{WP}$(**FHilb**), **Stab**, **Spek**, **FRel**, ...

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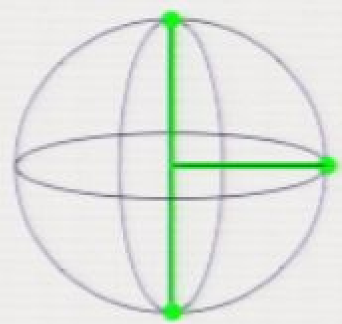

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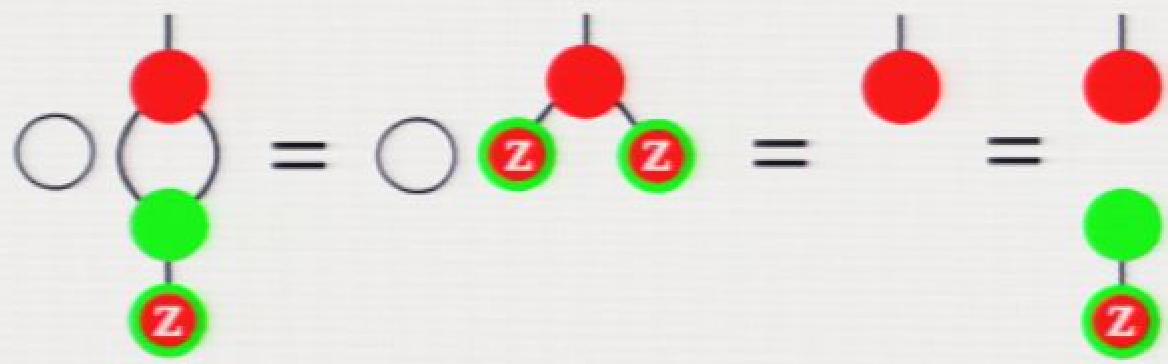

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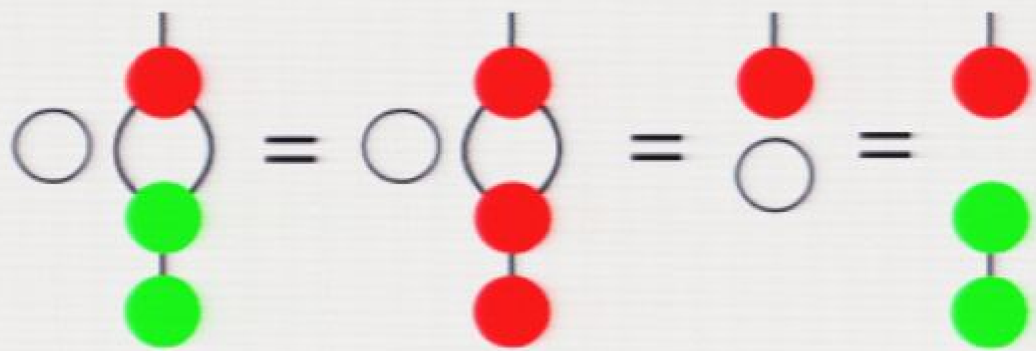

EXAMPLES

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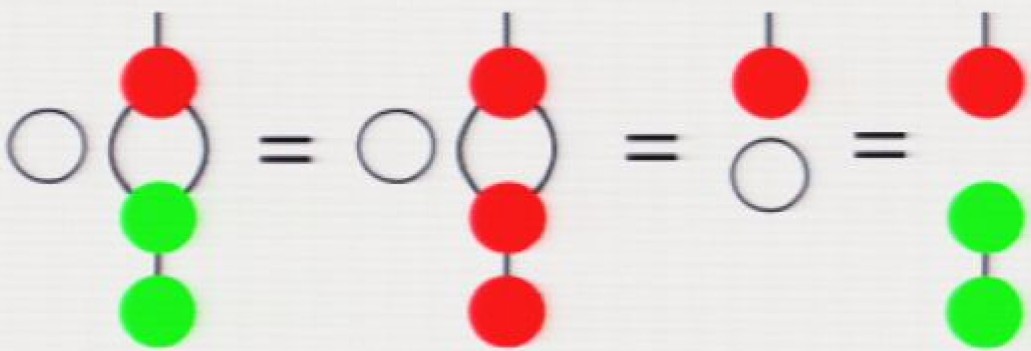

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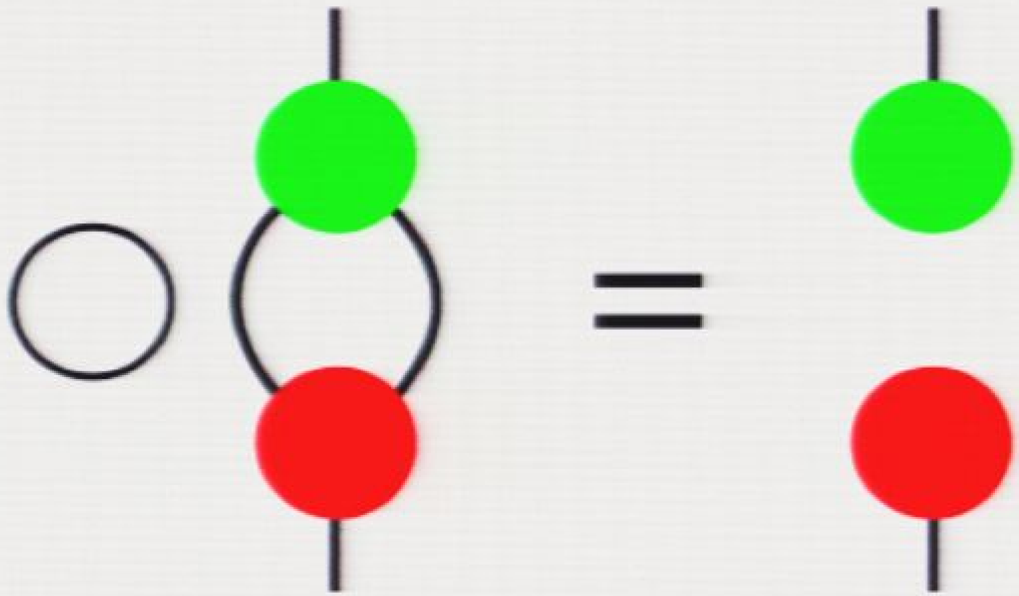

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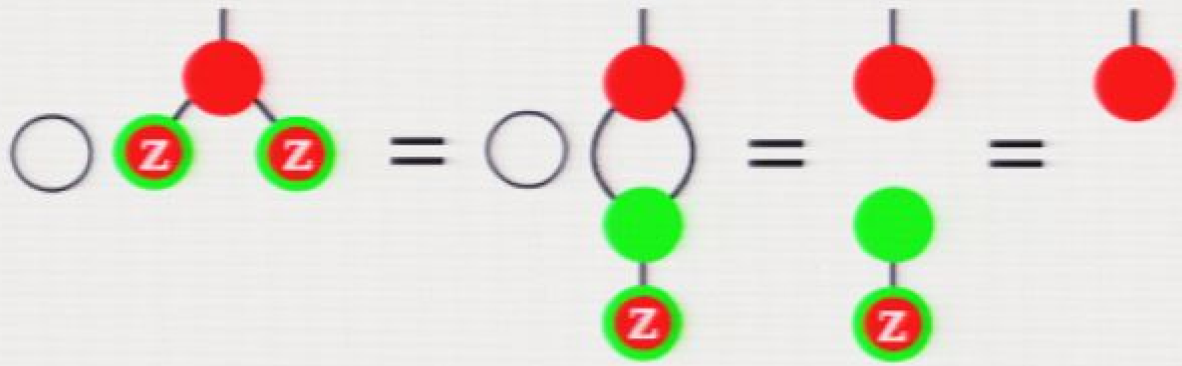


EXAMPLES

Z- and X-spin in FHilb:

$\delta_Z : |i\rangle \mapsto |ii\rangle$

$\epsilon_Z : \sqrt{2}|+\rangle \mapsto 1$

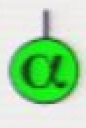
$\delta_X : |\pm\rangle \mapsto |\pm\pm\rangle$

$\epsilon_X : \sqrt{2}|0\rangle \mapsto 1$

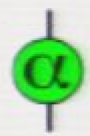
where

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

For qubits in **FHilb** with green $\equiv \{|0\rangle, |1\rangle\} \equiv Z$:

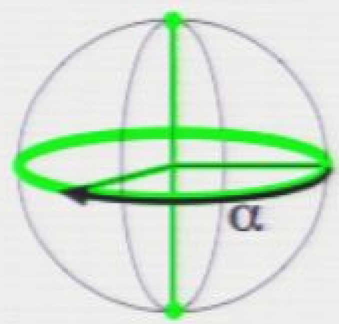


$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_Z$

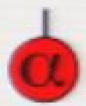


$= Z_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_Z$

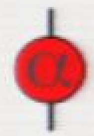
These are relative phases for Z, hence in X-Y:



For qubits in **FHilb** with red $\equiv \{|+\rangle, |-\rangle\} \equiv X$:

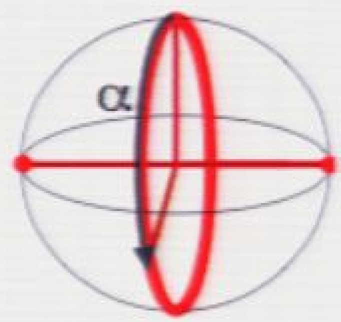


$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_X$



$= X_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_X$

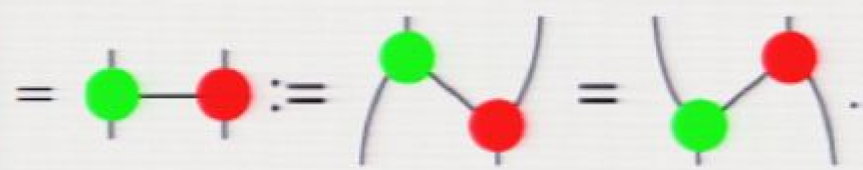
These are relative phases for X, hence in Z-Y:



Thm. Every linear map in $\mathbf{FHilb}_2$ can be expressed in the language of a pair of complementary observables and the corresponding phases, that is, it can be written down using only red and green decorated spiders.

$$\Lambda^Z(\gamma) \circ \Lambda^X(\beta) \circ \Lambda^Z(\alpha) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



'Z- and X-spin' in **FRel**:

- Copying-Deleting canonically (from biproducts):

$$\delta_Z :: \begin{cases} |0\rangle \mapsto |00\rangle \\ |1\rangle \mapsto |11\rangle \end{cases} \qquad \epsilon_Z :: \begin{cases} |0\rangle \mapsto 1 \\ |1\rangle \mapsto 1 \end{cases}$$

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- Copying-Deleting magically:

$$\delta_X :: \begin{cases} |0\rangle \mapsto |00\rangle + |11\rangle \\ |1\rangle \mapsto |01\rangle + |10\rangle \end{cases} \qquad \epsilon_X :: \begin{cases} |0\rangle \mapsto 1 \\ |1\rangle \mapsto 0 \end{cases}$$

Coecke & Edwards. Toy quantum categories. arXiv:0808.1037

'Z-, X- and Y-spin' in **Spek**:

$$\mathbb{IV} \to \mathbb{IV} \times \mathbb{IV} :: \begin{cases} 1 \mapsto \{(1,1),(2,2)\} \\ 2 \mapsto \{(1,2),(2,1)\} \\ 3 \mapsto \{(3,3),(4,4)\} \\ 4 \mapsto \{(3,4),(4,3)\} \end{cases} \quad + \quad \textit{its unit}$$

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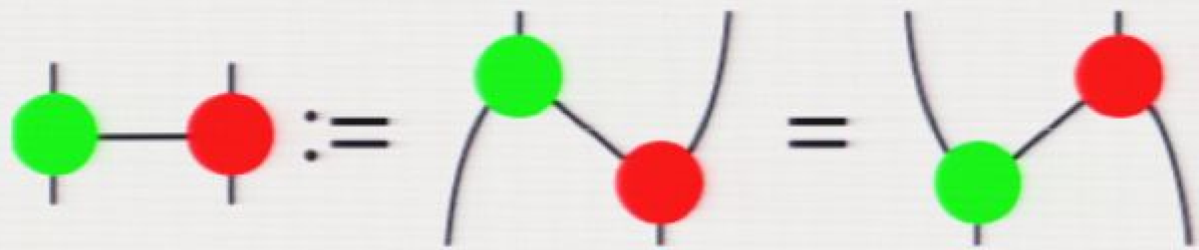
CALCULATIONS

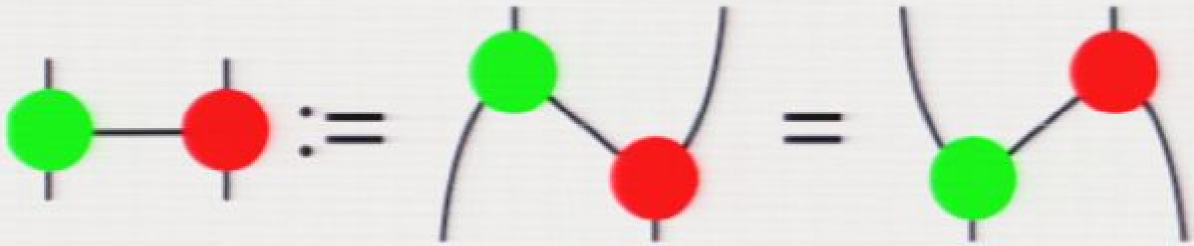
— *one* CX *gate* —



i.e.

$$(\delta_Z^\dagger \otimes 1) \circ (1 \otimes \delta_X) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

— *one* CX *gate* —



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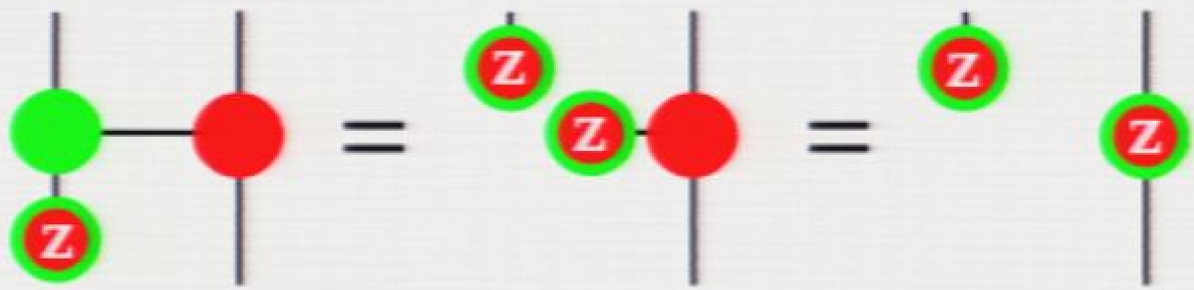

— *two* CX *gates* —

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = ?$$

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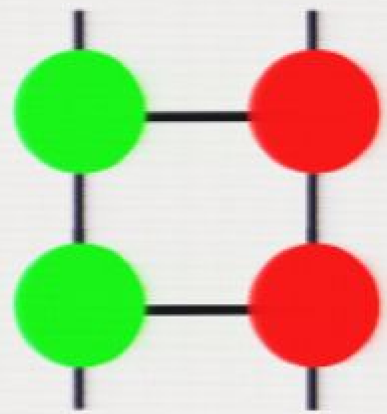

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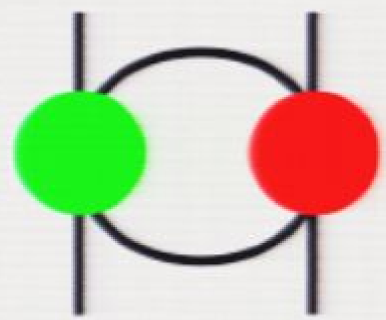

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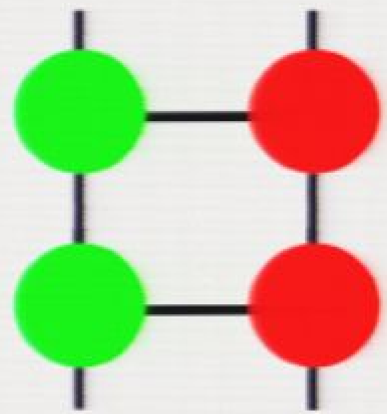

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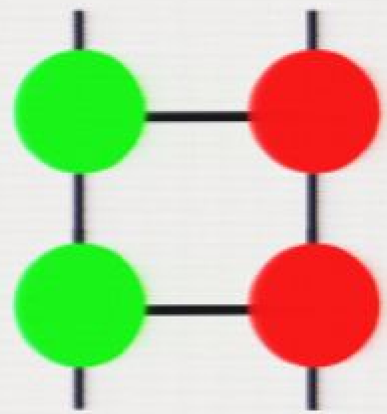

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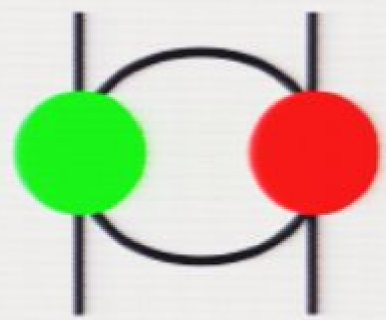

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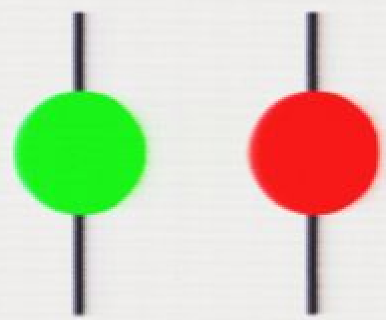
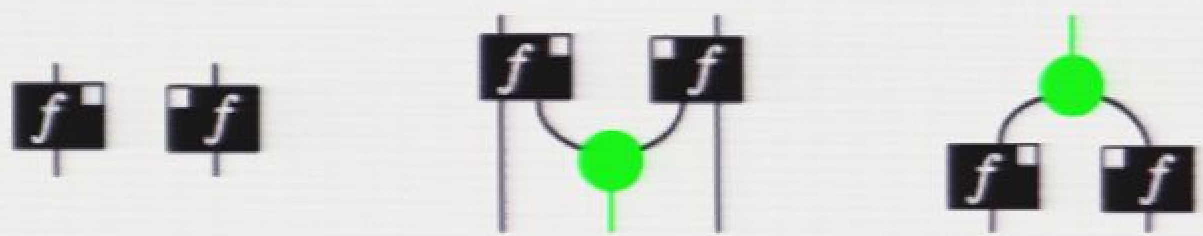


— *two* CX *gates* —

— *teleportation with classical communication* —

Classical $\equiv$ one wire

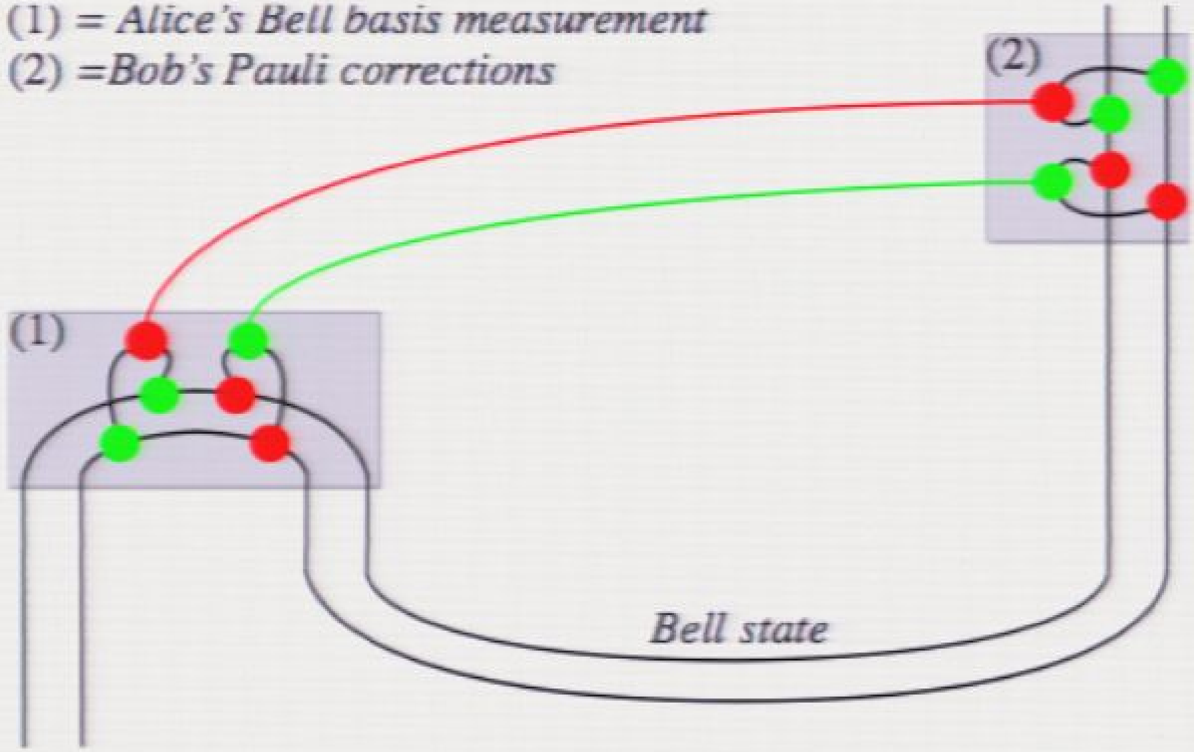
Quantum $\equiv$ two wires



Coecke, Paquette & Pavlovic. Class and quant structuralism. arXiv:0904.1997

— teleportation with classical communication —

(1) = *Alice's Bell basis measurement*
(2) =*Bob's Pauli corrections*



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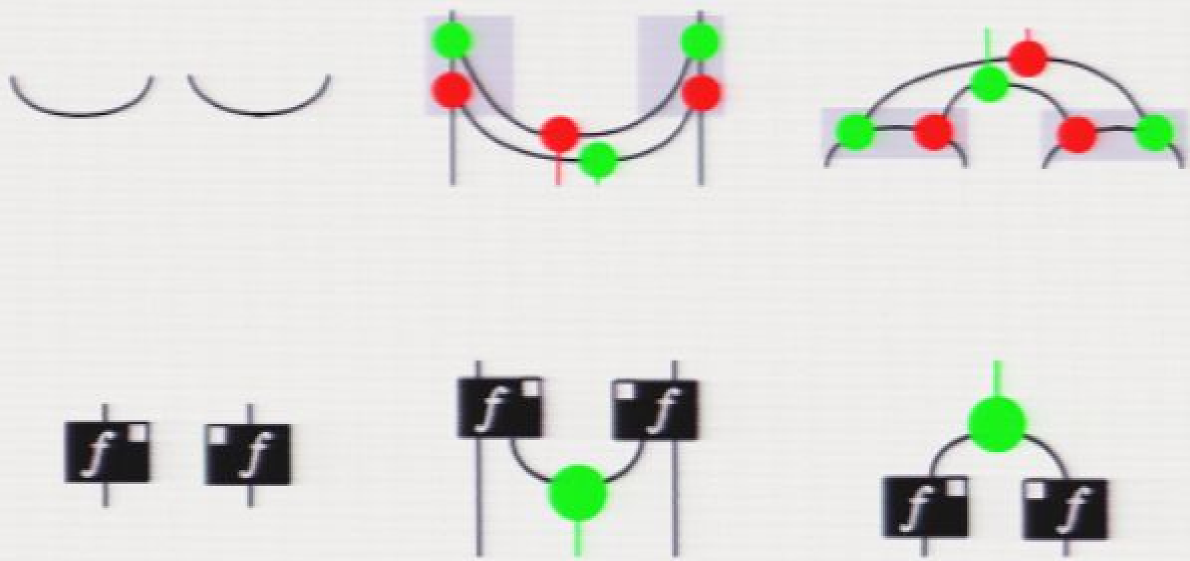

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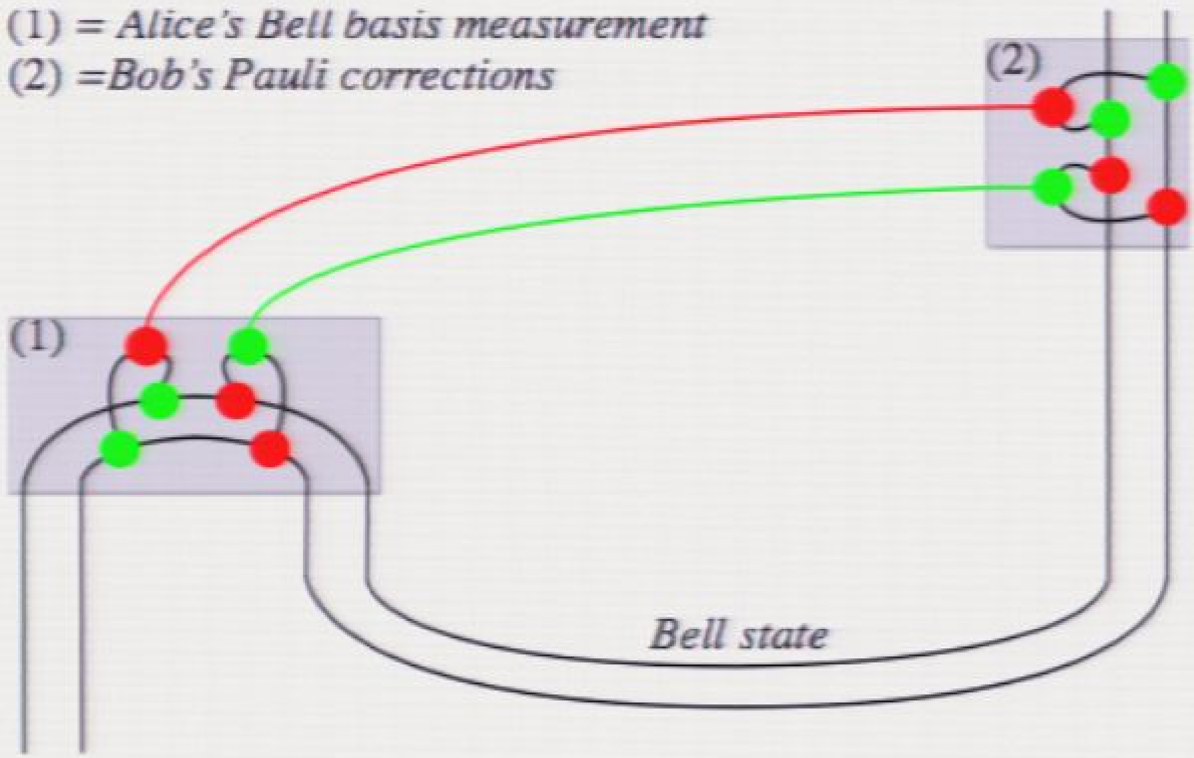

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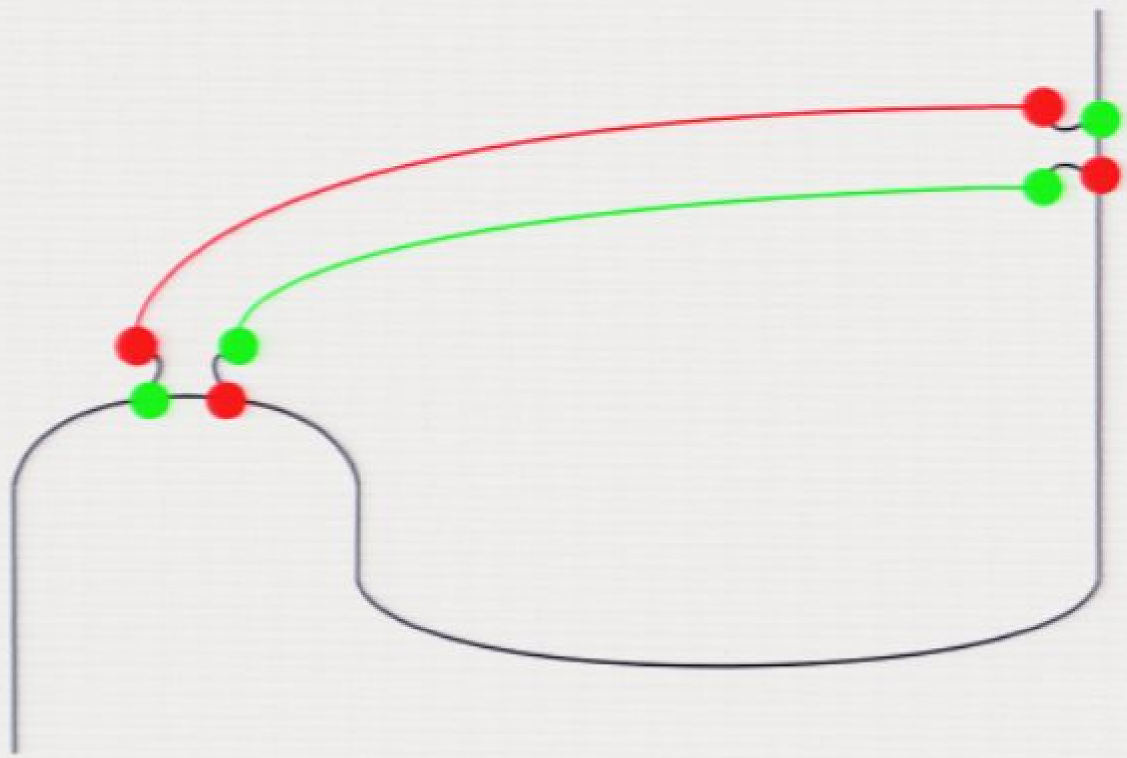

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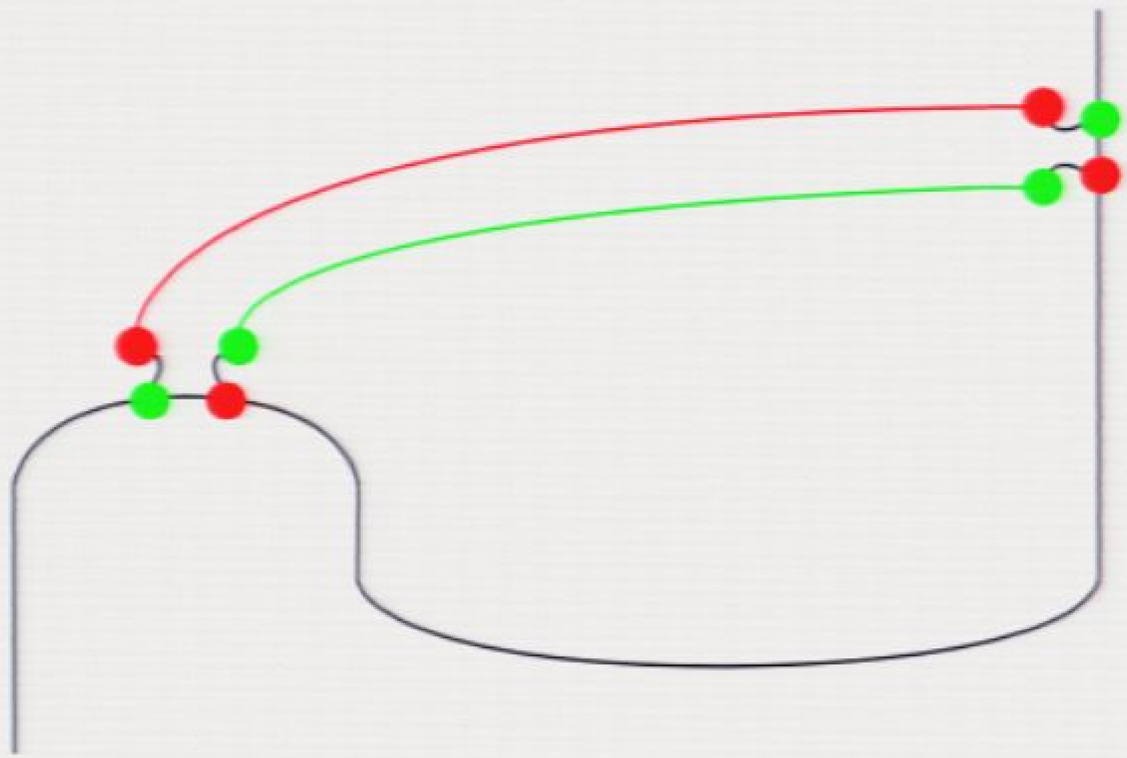

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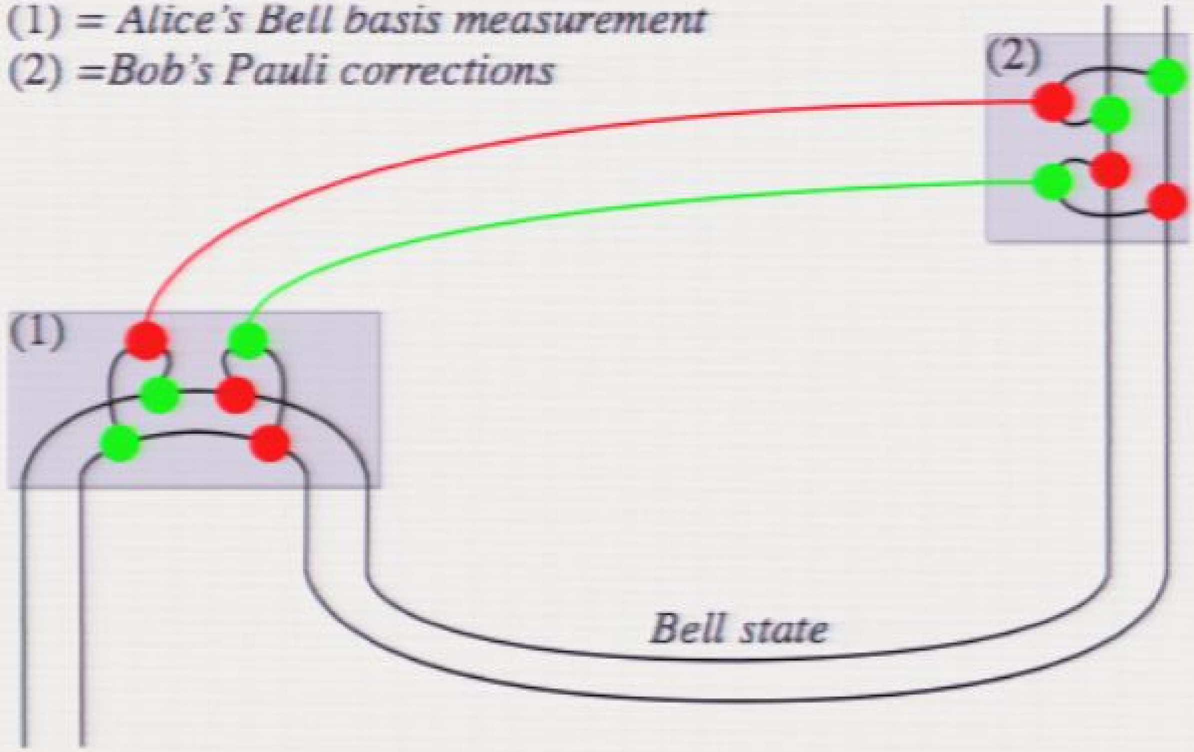

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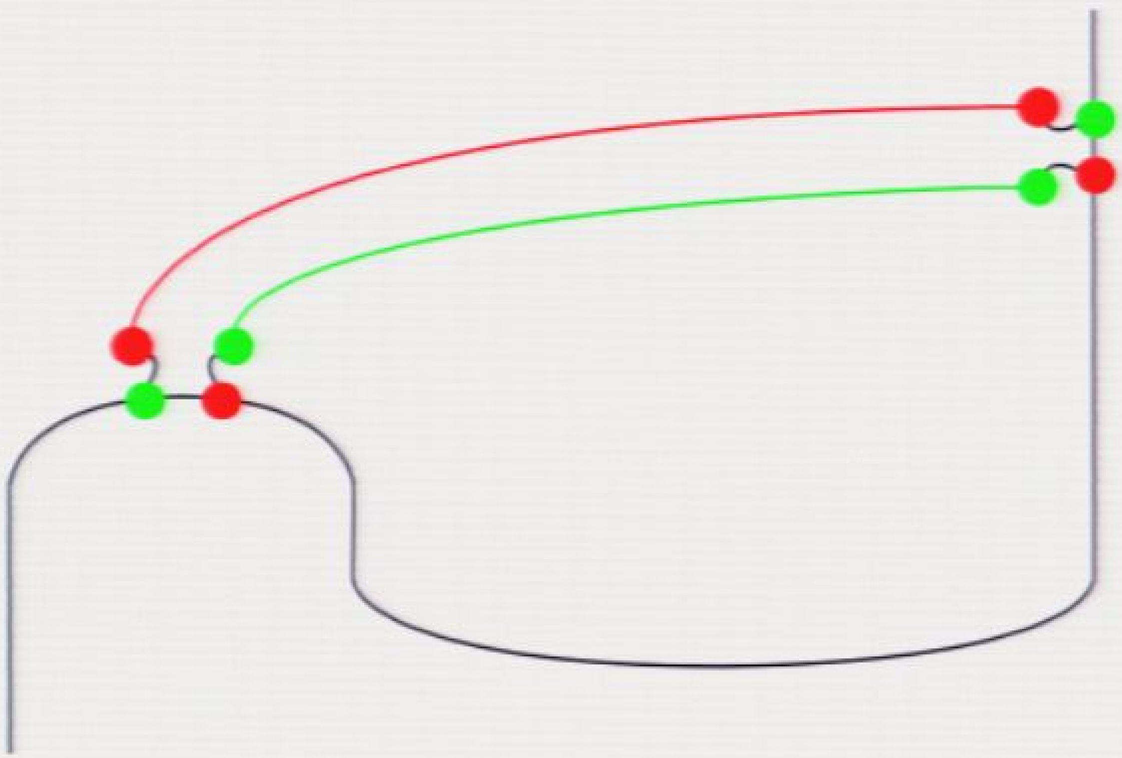

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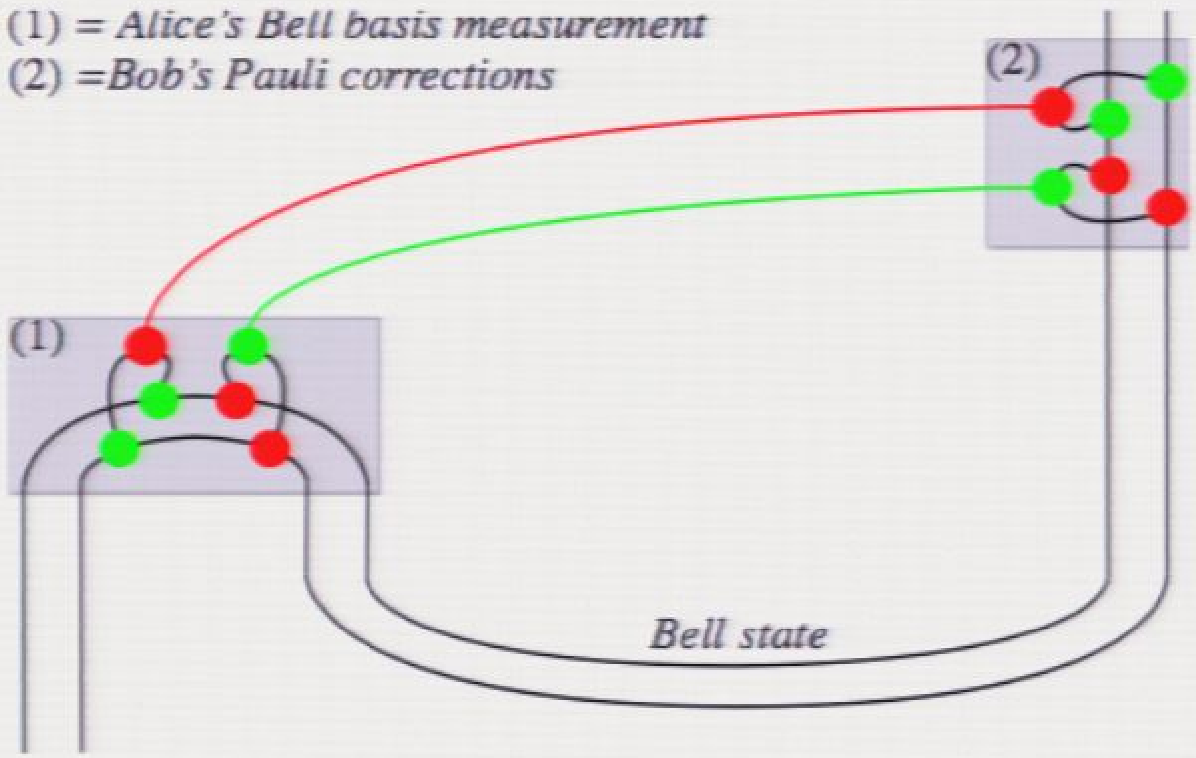
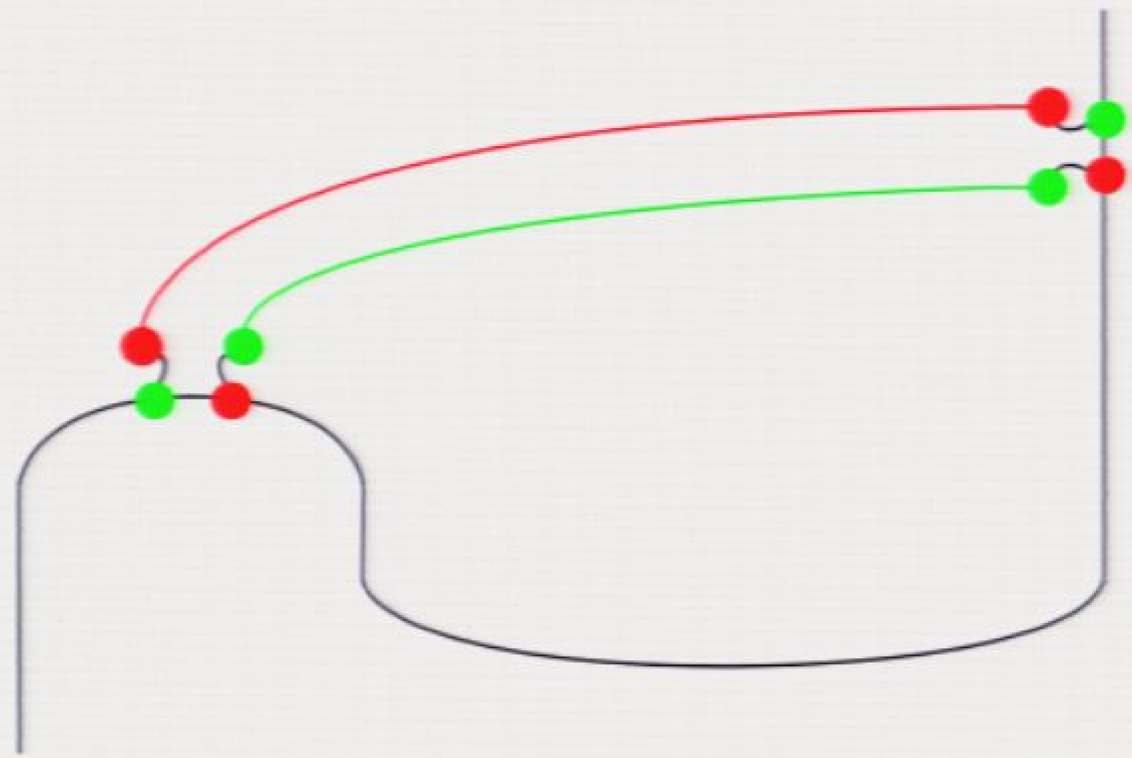
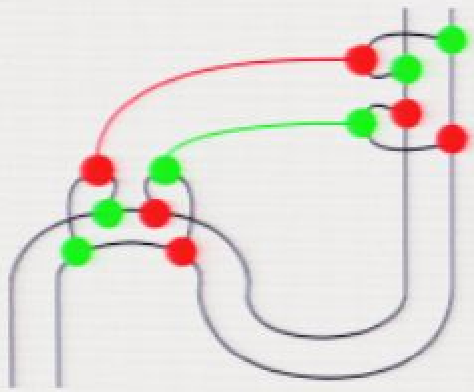
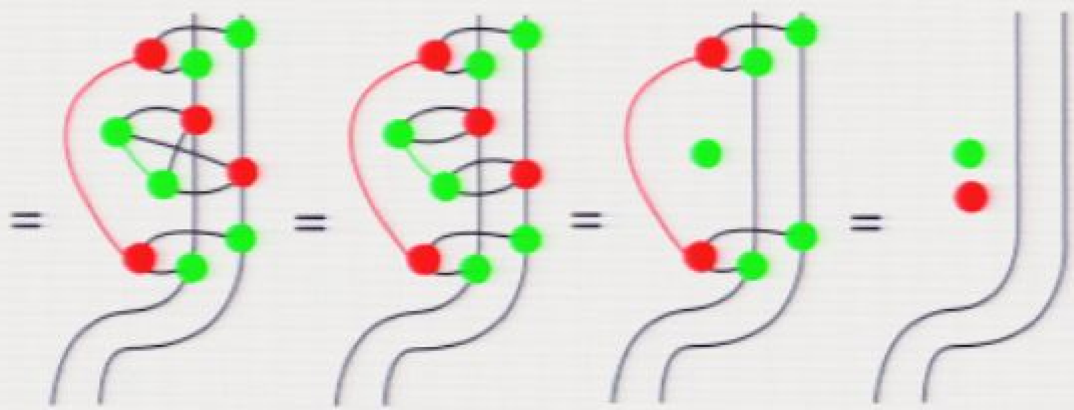


— teleportation with classical communication —



— teleportation with classical communication —





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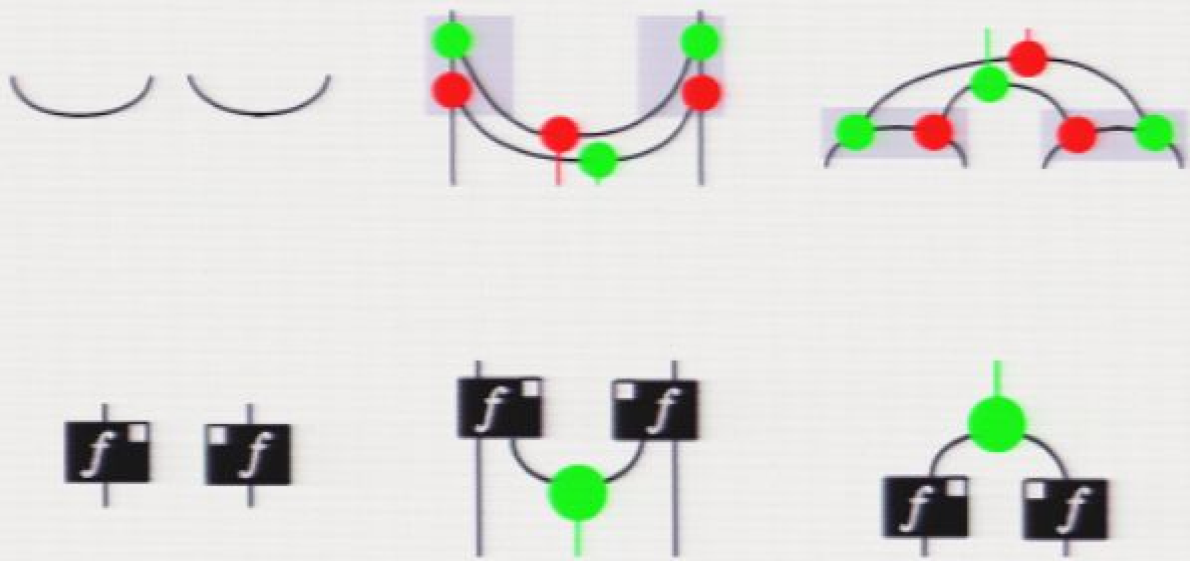
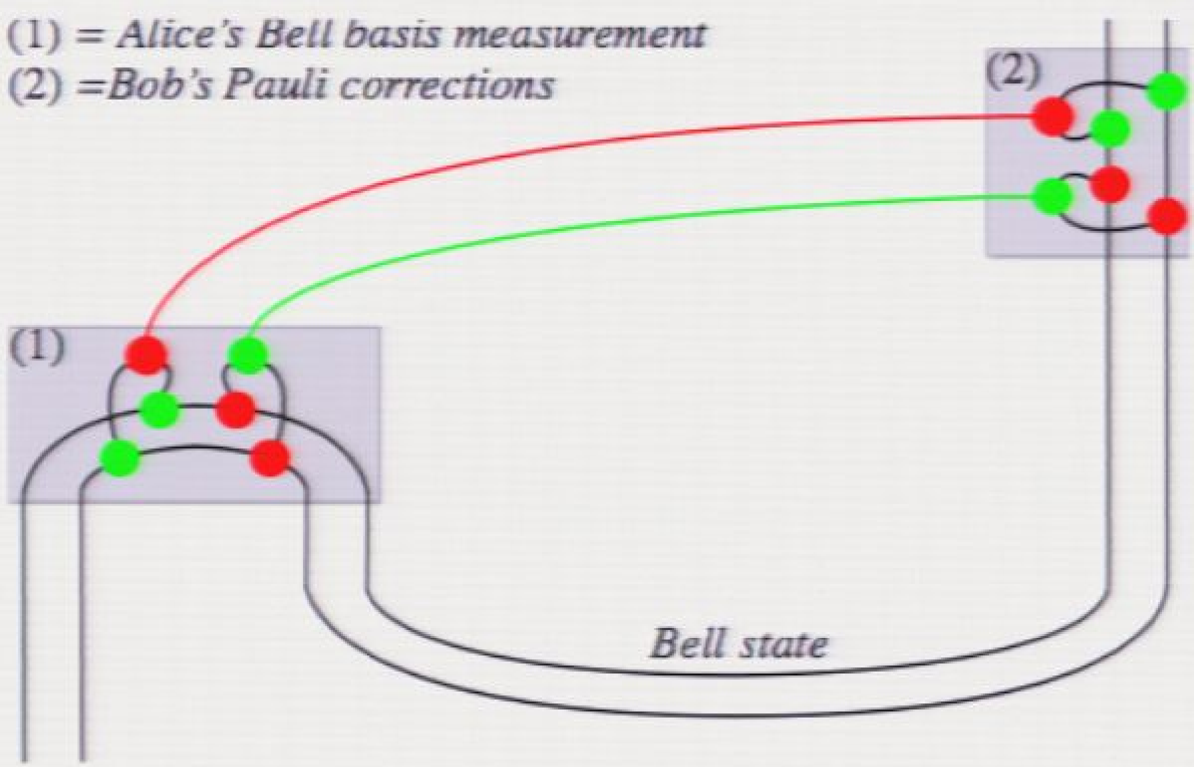
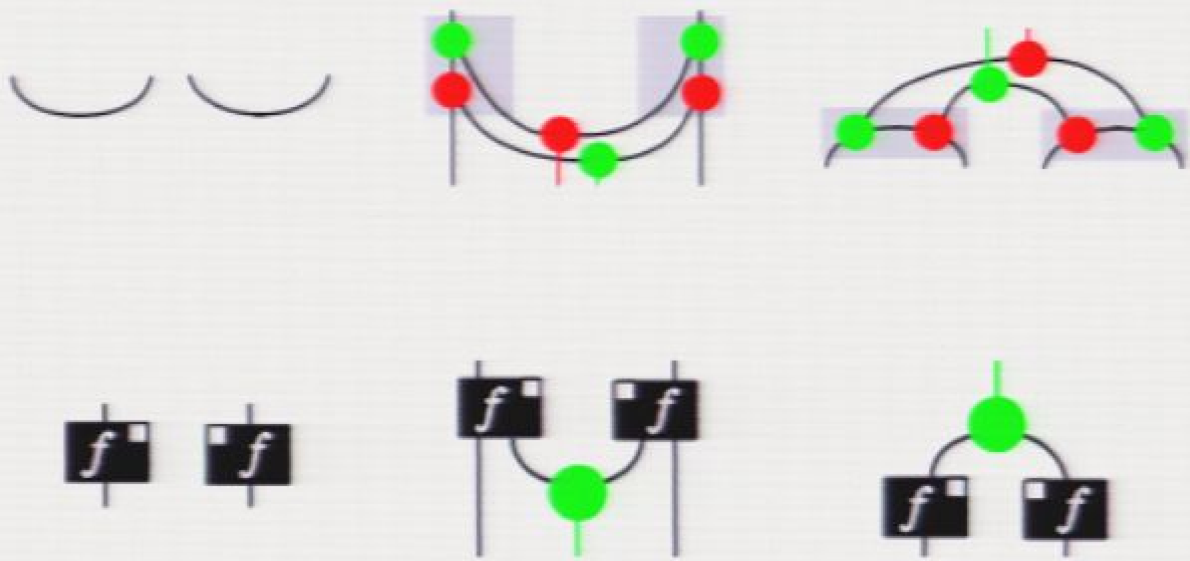


— teleportation with classical communication —

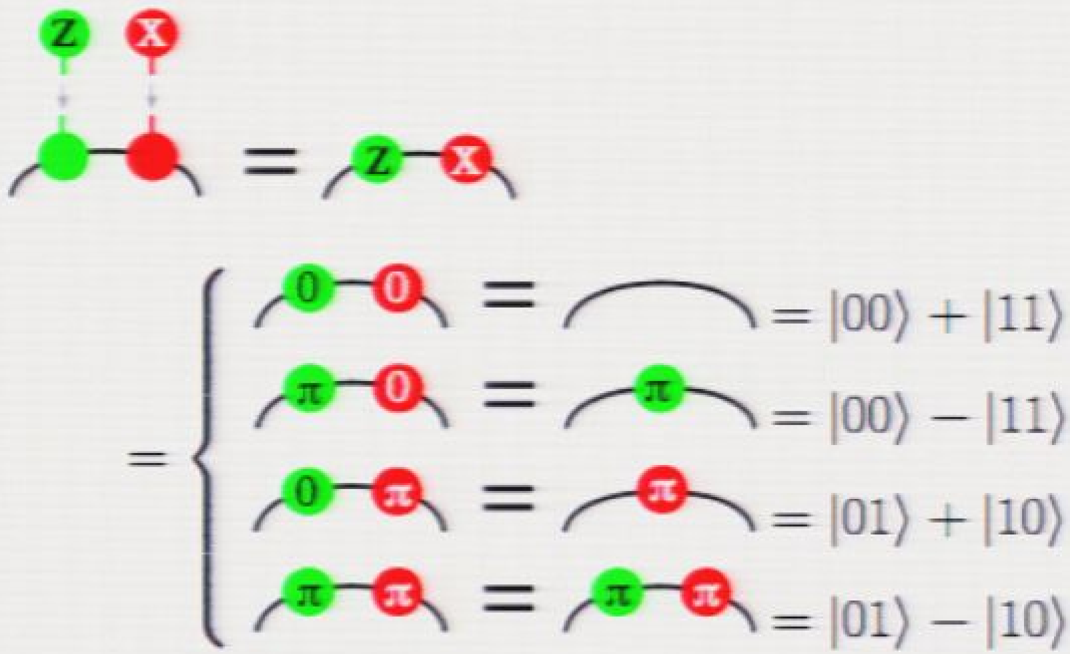


— teleportation with classical communication —



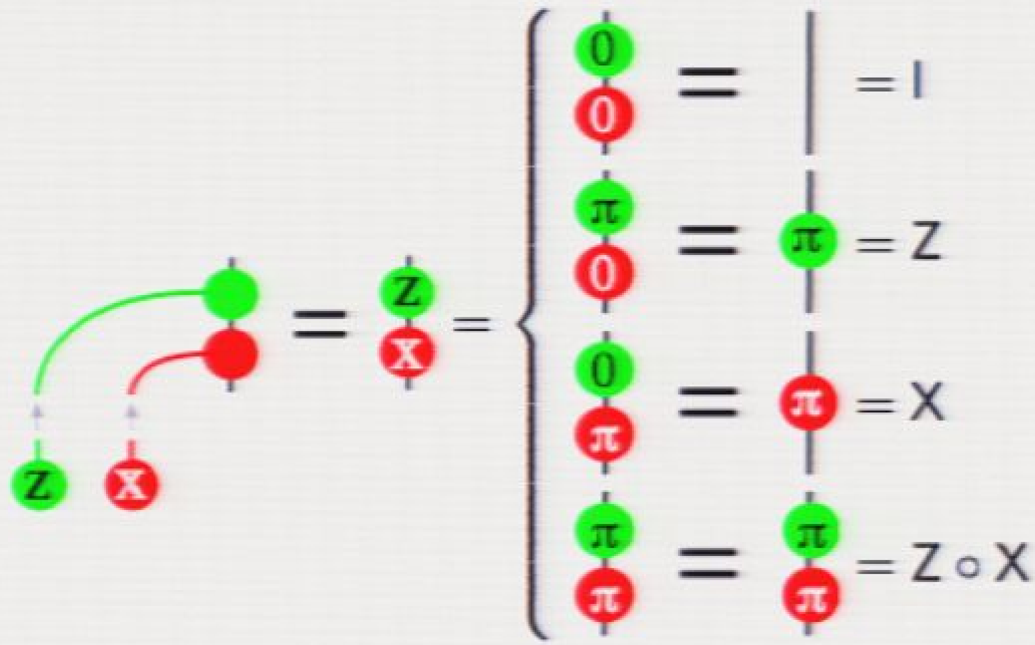
— *teleportation with classical communication* —

Postselecting Bell-basis measurements:



— *teleportation with classical communication* —

Selecting the Pauli corrections we obtain:



— *teleportation with classical communication* —