Title: Emergent Electroweak Gravity

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Abstract: The relic neutrino background contains a gapless, spin-2 sound mode, as well as a spin-1 mode if there is a neutrino-antineutrino asymmetry. The self-coupling of the spin-2 mode is given by Z boson exchange in the Standard Model and is parametrically similar to Newton's constant given the expected density of relic neutrinos. I will describe this emergent gravity theory and also describe how emergent theories avoid the Weinberg-Witten theorem, when the constituent degrees of freedom live in a flat Lorentz invariant space.

Emergent Electroweak Gravity

Bob McElrath
CERN

Perimeter Institute, June 26, 2009

Outline

- Introduction
- Massive cosmological relics are quantum liquids (as opposed to a classical gas)
- Interactions of quantum liquids: fermionic cosmological relics have propagating zero-sound
- Neutrino pairing: Vector and Tensor Landau Zero Sound
- Sundry considerations: Renormalization, Cutoff, Wenberg-Witten Theorem
- Conclusions

Introduction: Where are we?

(right here, right now)

The universe is not empty.

Even "vacuum" contains long wavelength neutrinos and photons.

To leading order we're justified in ignoring them because

$$T_\nu, T_\gamma, N_\nu^{1/3}, N_\gamma^{1/3} << T_0, p_{F0} << M_W$$

.

Our field theories and experiments have accurately told us what lives at *high* energies ($W^\pm$, $Z^\pm$ and possibly H^0).

If I look at the scales that are known, the ratios of those scales seem to contain the Planck scale (M_Z^2/T_ν or M_Z^2/p_F).

Brief Review on the Cosmic Neutrino Background

Cosmic neutrinos decouple from the Big Bang plasma at a temperature around 2 MeV. At that time they have a thermal Fermi-Dirac distribution.

As the universe expands, their density and temperature red-shift, leading to

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 1.95\text{K}; \qquad n_{\nu_i} = n_{\overline{\nu}_i} = \frac{3}{22} n_\gamma = \frac{56}{\text{cm}^3}$$

where T_γ and n_γ are the measured temperature and number density of CMB photons. Thus at least two species must be non-relativistic today. If neutrinos cluster gravitationally, the density is enhanced [Singh, Ma; Ringwald, Wong].

Due to large mixing, the flavor composition is equilibrated. All three mass eigenstates have equal densities. [Lunardini, Smirnov]

The asymmetry $\eta_\nu = (n_\nu - n_{\overline{\nu}})/n_\gamma$ is related to the baryon asymmetry $\eta_b = (n_b - n_{\overline{b}})/n_\gamma \simeq 10^{-10}$, so that any asymmetry can be neglected and we will assume $n_\nu = n_{\overline{\nu}}$.

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Introduction: My Question

We never consider these ratios of scales. Why?

To my mind, this looks like a finite temperature/finite density theory with a small parameter ($p_F \simeq 10^{-3}$ eV) and a large parameter ($G_F^{-1/2} \simeq 10^{11}$ eV), and at each ratio of these scales, new dynamics arises.

Therefore I am led to the following question:

What is the Standard Model dynamics which arises proportional to: p_F, $p_F G_F$, $p_F^2 G_F^2$, $p_F^3 G_F$?

We *must* answer this question, because the answer may give corrections to gravitational dynamics, dark matter dynamics, the cosmological constant, and other anomalies such as Pioneer. Either these dynamics exists, or we should prove that it does not, before assuming that G_N, Λ (etc) are unrelated to the weak scale.

Introduction: Scales of the neutrino background

What scales do I know about? (note $p_F^3 = 3\pi^2 n$; $E_F = \sqrt{m^2 + p_F^2}$)

$p_F(\nu)$	2.34×10^{-4} eV	per flavor/anti
$\sqrt{\Delta m_{12}}$	8.94×10^{-3} eV	
$\sqrt{\Delta m_{23}}$	5.29×10^{-2} eV	
T_ν	1.68×10^{-4} eV	
$G_F^{-1/2}$	2.92×10^{12} eV	

What scales do I want to explain? (using p_F as representative of the low scale)

Λ	2.3×10^{-3} eV	$\mathcal{O}(p_F)$
$p_F(\chi)$	8.80×10^{-6} eV $\left(\frac{100\text{GeV}}{M_\chi}\right)$	$\mathcal{O}(p_F)$
$M_{\text{Pl}}^{-1} = \sqrt{G_N}$	$10^{-28} eV^{-1}$	$\mathcal{O}(p_F G_F)$
α_Λ	1.51×10^{-33} eV	$\mathcal{O}(p_F^3 G_F)$
α_{MOND}	2.63×10^{-34} eV	$\mathcal{O}(p_F^3 G_F)$
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Is this all a big coincidence?

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Introduction: The Hierarchy Problem

Nearly every talk in particle physics (including this one) begins with the following magical incantation:

If we compute the one-loop corrections to the Higgs mass... we find...

$$\delta m_H^2 \propto M_{\mathrm{Pl}}^2$$

The argument goes: Any new matter or cutoff that couples to the Higgs introduces a correction proportional to that scale. Since we know gravity exists, therefore we know there is a scale above the Higgs mass that introduces this correction.

This argument has one flaw... Is M_{Pl} a scale? Is it a cutoff to field theory? Could M_{Pl} be a ratio of scales instead (e.g. $(G_F T)^{-1}$)

We have no idea!

This has been entertained e.g. by Sundrum (2003) "soft gravitons" and Sakharov (1967) "induced gravity" among many others.

Introduction: Scales

The "scale" present in the problem is actually a coupling constant, G_N.

The scales at which gravity is tested include ~ 1 eV (CMB freeze-out) and ~ 1 MeV from Big Bang Nucleosynthesis (if we can ignore the ^{7}Li problem, and *no scale above that.*

It would be completely compatible with direct tests of gravity if gravity "turned off" at a scale $\gtrsim 1$ MeV

It is known that gravitational theories can emerge from field theory. e.g. "Analog Models", "Dumb Holes", [Sakharov, Liberati, Visser, etc] as well as a 3-dimensional gravity in ^{3}He [Volovik].

So let's make a radical assumption.

Introduction: I don't believe in Gravity

The existing models for Gravity (String Theory, Loop Quantum Gravity) insist that somehow gravity is the most fundamental theory. Particle physics "accidentally" falls out of these theories.

This seems backwards to me. The Standard Model (along with Quantum Field Theory) is the most precise, most predictive theory we have ever had.

Since Theoretical Physics is a game of deciding which important theoretical or experimental result I'm going to ignore so that I can write my next paper, let's ignore gravity.

Let's pretend that I just discovered gravity. But I have at my disposal all of particle physics and the Standard Model (including possibly the Higgs).

Assumptions: *flat* Lorentz invariant space + Standard Model.

What could there be in the vacuum that could cause these funny forces I just discovered? (photons, neutrinos)

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Introduction: Scattering Scales

This interactions of cosmic neutrinos are a theory of contact interactions in a quantum liquid at finite density and zero temperature. The fundamental parameters are the Fermi momentum p_F and G_F.

Let us examine the effective range expansion of neutrino self-scattering to get an idea of the scales:

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 l_0 + \dots$$

where $a = \sqrt{\sigma_{\nu\nu}/4\pi} \simeq T_\nu G_F$ is the s-wave scattering length and $l_0 = \sqrt{G_F}$ is the range of the potential. Thus we have the approximation regime $a \ll l_0$.

This is the *opposite* approximation regime to atomic and nuclear finite density systems, BEC's, and BCS superconductivity, so one must be careful when applying results from those fields, and we want to take $a \to 0$.

Therefore, the leading dynamics occurs due to this p-wave term.

Introduction: Scattering Scales Again

Note that the self-interactions of a weakly-interacting fluid can be expanded as

$$\mathcal{M} = \mathrm{Re}\mathcal{M} + i\mathrm{Im}\mathcal{M} = \alpha G_F + i\beta G_F^2$$

That is, the imaginary part of the matrix element is related, by the Optical Theorem, to the total scattering cross section. This is $\mathcal{O}(G_F^2)$. The real part however is only $\mathcal{O}(G_F)$.

Or, to repeat the last slide, $l_0 \gg a$. The range ($l_0 = \sqrt{G_F}$) is much larger than the scattering length ($a = T_\nu G_F$).

Therefore, *the dynamics of the real part of the matrix element are much, much more important than the scattering cross section for weakly interacting fluids.*

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What are neutrinos doing *today*

The dynamics of the neutrino background is given just by its kinetic term and self-interaction

$$\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi + \frac{g^2}{M_Z^2}\overline{\psi}\gamma^{\mu}\psi\overline{\psi}\gamma_{\mu}\psi$$

let us ignore the interactions for a few slides and concentrate on the kinetic term. It does 2 things:

- Gives rise to the 2 point function, transporting neutrinos in space
- Causes the expansion of the neutrino's wave packet

The latter effect is normally forgotten in QFT under the assumption that we have asymptotic localized particles. Is this a good assumption for a cosmological relic?

Wave packet expansion I/IV

Wave packets expand because different wave numbers move at different velocities in the presence of a mass or interaction. The wave number at $p = p_0 + \Delta p$ moves with velocity $v = (p_0 + \Delta p)/E$ while the wave number on the other side moves with velocity $v = (p_0 - \Delta p)/E$, and these wave numbers separate in space.

Thus the uncertainty of a wave packet evolves as

$$\Delta x(t)^2 = \Delta x_0^2 + \Delta v^2 t^2$$

In the relativistic case we must use

$$\Delta v = \frac{\Delta p}{E}(1 - v^2).$$

Assuming the uncertainty is given by the de Broglie wavelength

$$\Delta x_0 = \lambda/p = \lambda/\sqrt{3mkT}$$

allows us to derive the condition for a quantum liquid with $t = 0$ (or equivalently $\Delta p = 0$) from $\Delta x > n^{-1/3}$:

$$T < \frac{n^{2/3}\lambda^2}{3mk}$$

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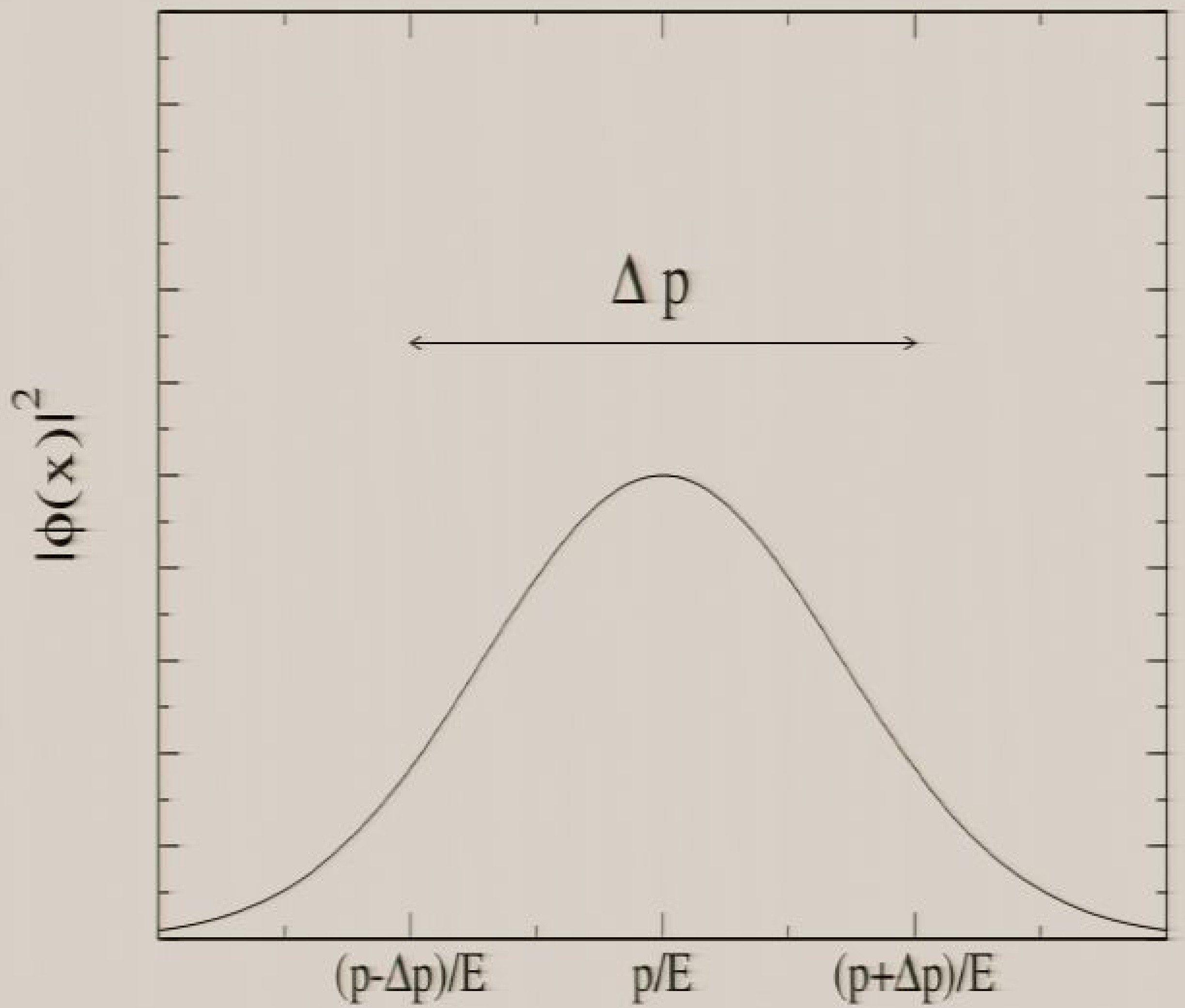

Wave packet expansion III/IV

The "quantum liquid" condition is:

$$\Delta x \gg n^{-1/3}.$$

The opposite limit is the "classical gas" limit, and is the limit used by scattering theory (particles are localized):

$$\Delta x \ll n^{-1/3} \sim b.$$

where b is the impact parameter in scattering theory. The temperature condition is valid only if scattering occurs sufficiently often that the time dependence of the wave packet can be neglected:

$$\tau \ll \frac{\Delta x}{\Delta v} = E\frac{\Delta x}{\Delta p}$$

where $\tau = (\sigma n v)^{-1}$ is the mean time between collisions. This holds for atomic and nuclear matter at the densities usually considered.

Notice that the other assumption $\Delta p = E\Delta v = 0$ implies $\Delta x = \infty$ and vacuum calculations are not appropriate. they must be done at finite density. (i.e. we're in a momentum eigenstate but there is *no empty space*)

Wave packet expansion conclusion

Putting everything together using $t = \tau$:

$$\frac{1}{p^2} + \frac{(1-v^2)^2}{\sigma^2 n^2} > \frac{1}{\lambda^2 n^{2/3}}. \tag{1}$$

If we can neglect the first term, which is valid for decoupled relics, we obtain the quantum liquid criterion for weakly coupled relics:

$$\sigma < \frac{\lambda(1-v^2)}{n^{2/3}}. \tag{2}$$

This is very (very very) well satisfied for both relic neutrinos and dark matter ($\sigma \simeq 10^{-56}\text{eV}^{-2}$, $n^{-2/3} \simeq 10^{-8}\text{eV}^{-2}$). This means:

1) We have to worry about the dynamics of a quantum liquid for any massive cosmological relic (dark matter, at least 2 flavors of neutrinos)

2) We need to worry about quantum liquid dynamics of massless relics (lightest neutrino, axions, photons) too, because $T \sim n^{-1/3}$ and the low-momentum components of the distribution function are a quantum liquid.

Where do we go from here?

How do we deal with this kind of quantum liquid, and what are its dynamics?

The wave packet Δx calculation is telling us that relics are *plane waves*. Therefore they are entirely described by their thermal distribution $n(p)$.

When $\Delta x \gg n^{-1/3}$, collective dynamics begin to be important. It doesn't make sense to compute Δx larger than this. The dynamical impact of Δx is the suppression of collective effects, and if $\Delta x ggn^{-1/3}$, one cannot observe this.

Fuller and Kishimoto recently calculated Δx for relic neutrinos and gave an answer of Gpc [Phys. Rev. Lett. 102, 201303 (2009)]. (a.k.a. "Ginormous Neutrinos")

What's different about a quantum liquid?

We have non-zero density everywhere. Particles are not isolated or localized.

$\Rightarrow$ Contact operators have expectation values in "vacuum".

This means that those contact operators can define propagating composite degrees of freedom.

For a Fermi liquid with repulsive interactions, this is *zero-sound*.

Just as with a BEC (cooper pair), it is the attractive interactions that define the propagating modes.

This is also index of refraction (forward-scattering) physics, which is important when there is no scattering! (look through a plate of glass)

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The tree diagrams are all finite. One is required to compute at one loop to see the infrared divergences corresponding to collective effects.

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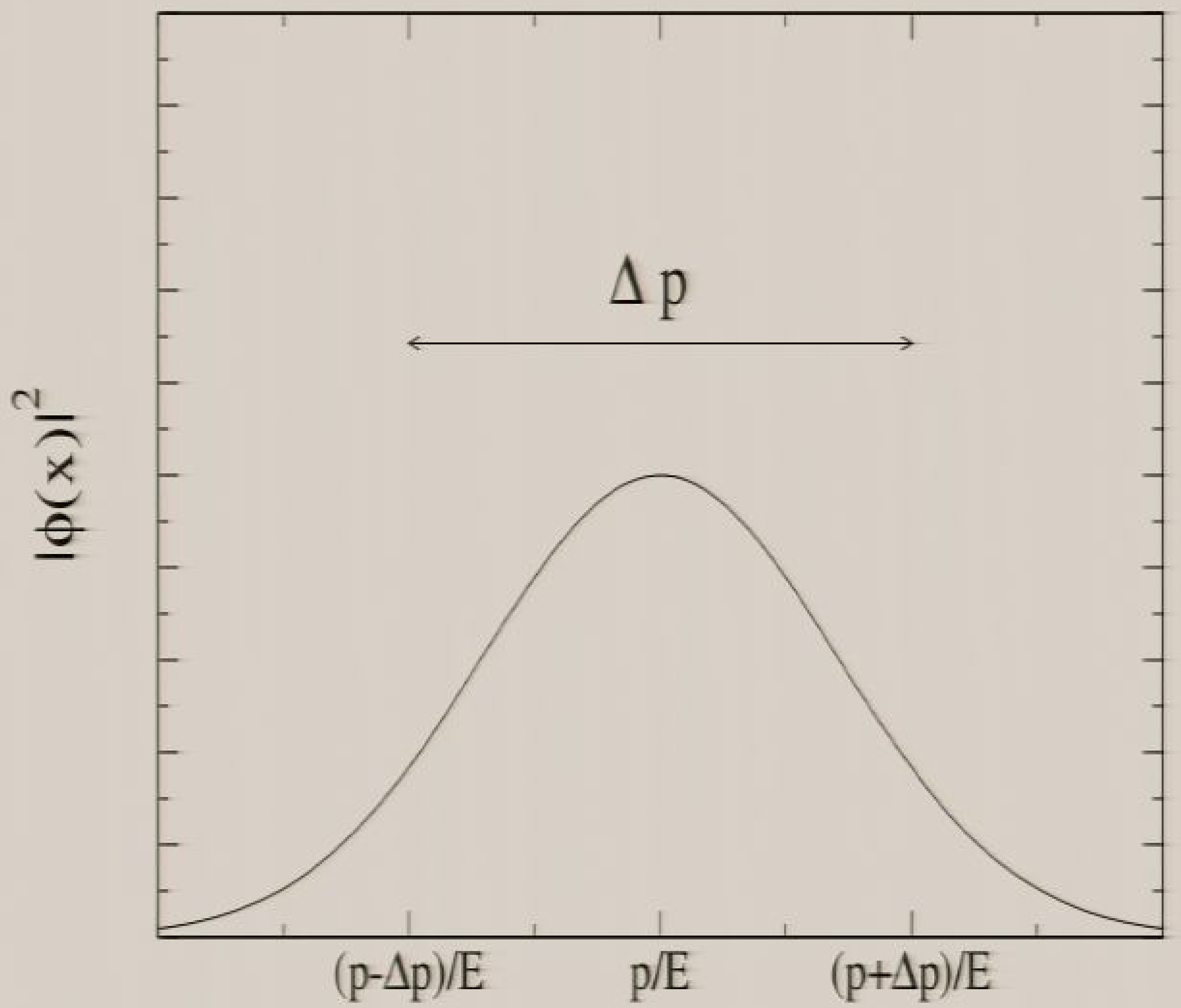

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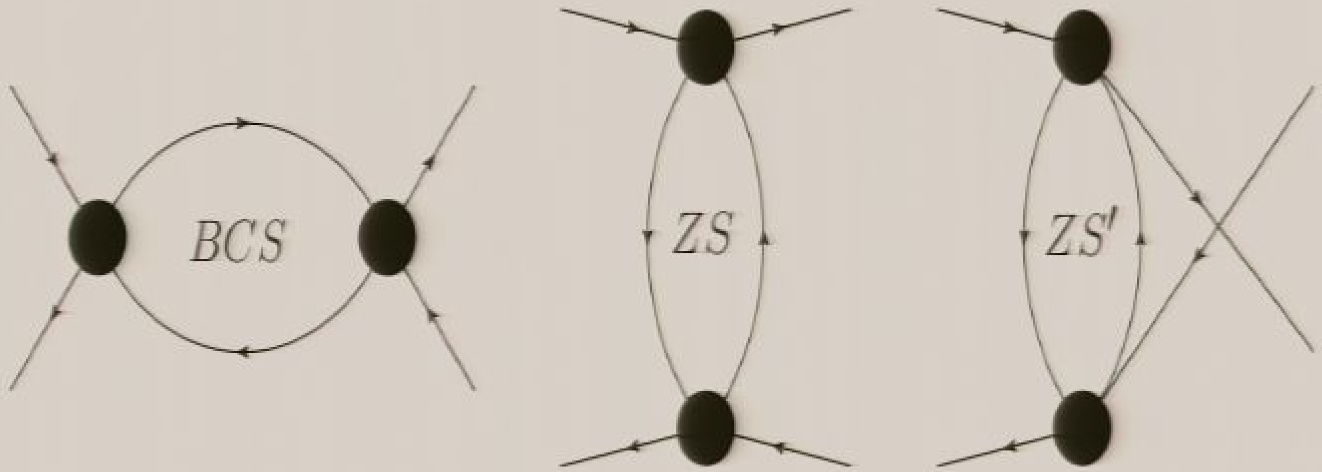

This set of diagrams has two singular limits: the BCS and Zero-Sound (forward scattering) limits.

All three of these diagrams have infrared singularities due to a background density.

*

*The Landau Theory of Fermi Liquids omits the ZS' diagram, and therefore does not properly have the correct interference and Pauli blocking due to it.

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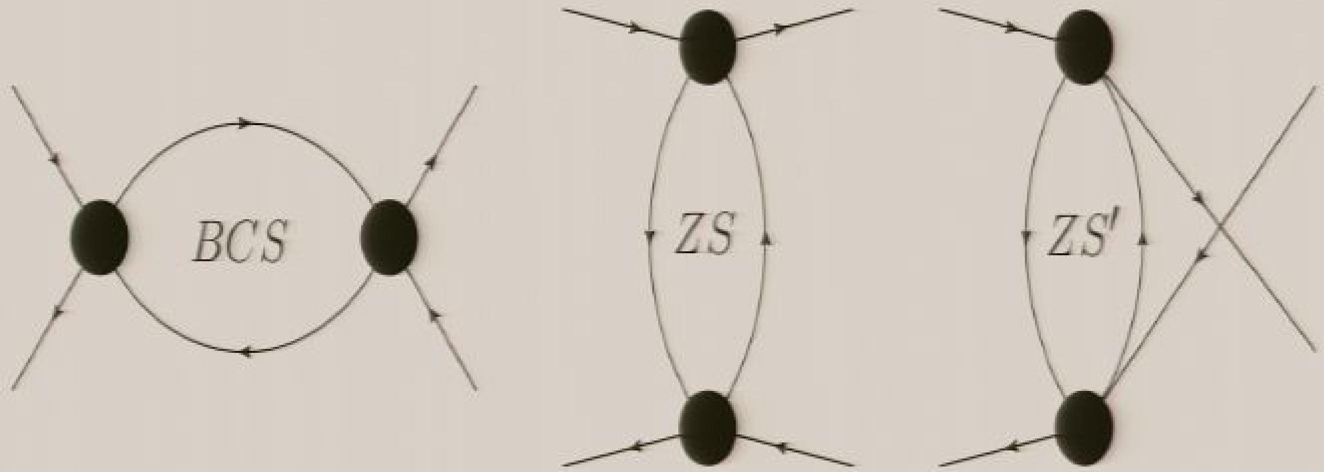

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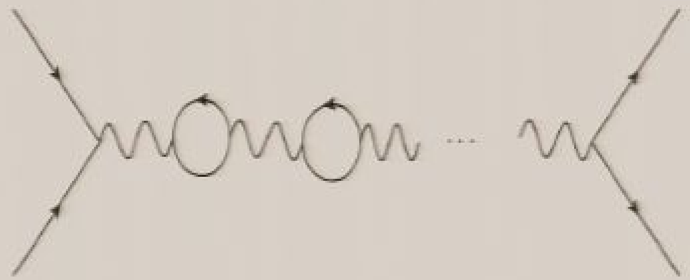
Vector Zero Sound



We can resum the divergence of the BCS diagram with Dyson's equation

$$\begin{aligned} D_{\mu\nu}(q) &= D^0_{\mu\nu}(q) + D^0_{\mu\alpha}(q)\Pi^{\alpha\beta}(q)D_{\beta\nu}(q) \\ &\Rightarrow [\delta_{\mu\nu} - D^0_{\mu\alpha}(q)\Pi^{\alpha\nu}(q)]D_{\nu\beta}(q) = D^0_{\mu\beta}(q) \end{aligned}$$

In terms of the vector boson self-energy,

$$\Pi_{\alpha\beta}(q) = ig^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma_\alpha G(k)\gamma_\beta G^0(k+q)]$$

and fermion Green's function $G(k)$. The poles occur when

$$\det[\delta_{\mu\nu} - D^0_{\mu\alpha}(q)\Pi^\alpha_\nu(q)] = 0$$

Vector Zero Sound Spectrum

We can factorize this dielectric function

$$\epsilon_{\mu\nu}(q) = \delta_{\mu\nu} - D^0_{\mu\alpha}(q)\Pi^\alpha_\nu(q)$$

$$\epsilon(q) = \det \epsilon_{\mu\nu}(q) = \epsilon_L(q)\epsilon_T^2(q) = 0$$

reflecting the three degrees of freedom of a massive vector boson (two transverse and one longitudinal).

$$\begin{aligned}
\epsilon_L(q) &= 1 - \frac{g^2 p_F E_F}{\pi^2}\frac{1 - C_0^2}{\vec{q}^2 - q_0^2 + M_Z^2}\Phi\left(\frac{C_0}{v_F}\right) \\
\epsilon_T(q) &= 1 + \frac{g^2 p_F^3}{2\pi^2 E_F}\frac{1}{\vec{q}^2 - q_0^2 + M_Z^2}\left[1 + \left(1 - \frac{C_0^2}{v_F^2}\right)\Phi\left(\frac{C_0}{v_F}\right)\right] \\
\Phi(y) &= -1 + \frac{1}{2}y \ln\left|\frac{y+1}{y-1}\right|
\end{aligned}$$

Solving $\epsilon_T(q) = 0$, one finds real solutions for q_0 only if

$$q_0^2 > M_Z^2 + |\vec{q}|^2 + \Omega^2; \qquad \Omega = \frac{g^2}{3\pi^2}\frac{k_F^3}{E_F}$$

Vector Zero Sound Properties

Solving $\epsilon_L(q) = 0$ and expanding around $C_0 = 1$ we can find an expression for the velocity of zero sound:

$$C_0^2 = 1 - \frac{\pi^2 M_Z^2}{g^2 k_F E_F \Phi(E_F/k_F)} = 1 + \frac{\pi^2 M_Z^2}{g^2 k_F E_F}\left(\frac{1}{1 - \frac{1}{2}\ln 2 + \ln(E_F/k_F - 1)}\right)$$

Thus a relativistic mode appears in the limit $E_F \rightarrow k_F$ or equivalently, $m \rightarrow 0$.

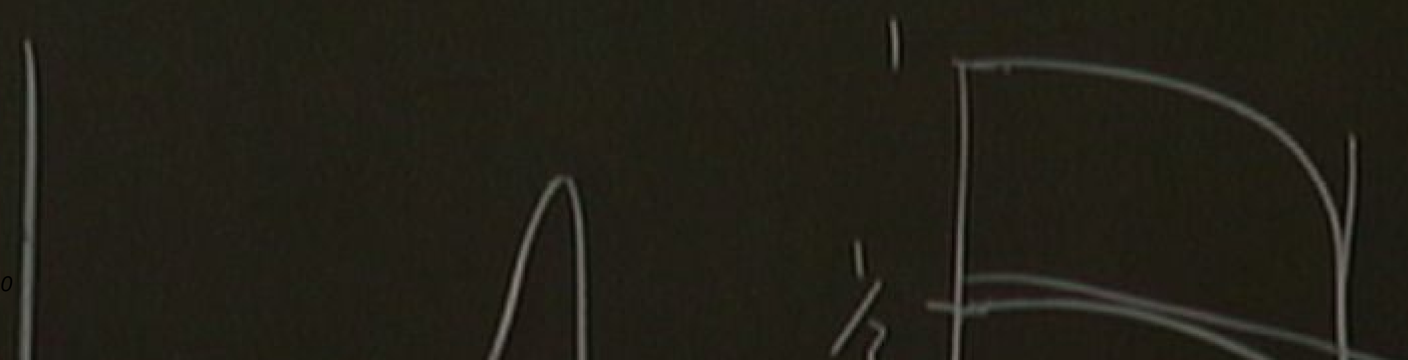
For finite m,

$$C_0 = \sqrt{\frac{\Omega^2}{M_Z^2 + \Omega^2}}; \qquad \Omega = \frac{g^2}{3\pi^2}\frac{k_F^3}{E_F}$$

and approaches c as $k_F \rightarrow \infty$. (high-density limit)

$\Rightarrow$ The Zero-Sound of a massless Fermi liquid is *relativistic*!

$$V_F = \frac{p_F}{\sqrt{m^2 + p_F^2}} \qquad E_F = \sqrt{m^2 + p_F^2}$$



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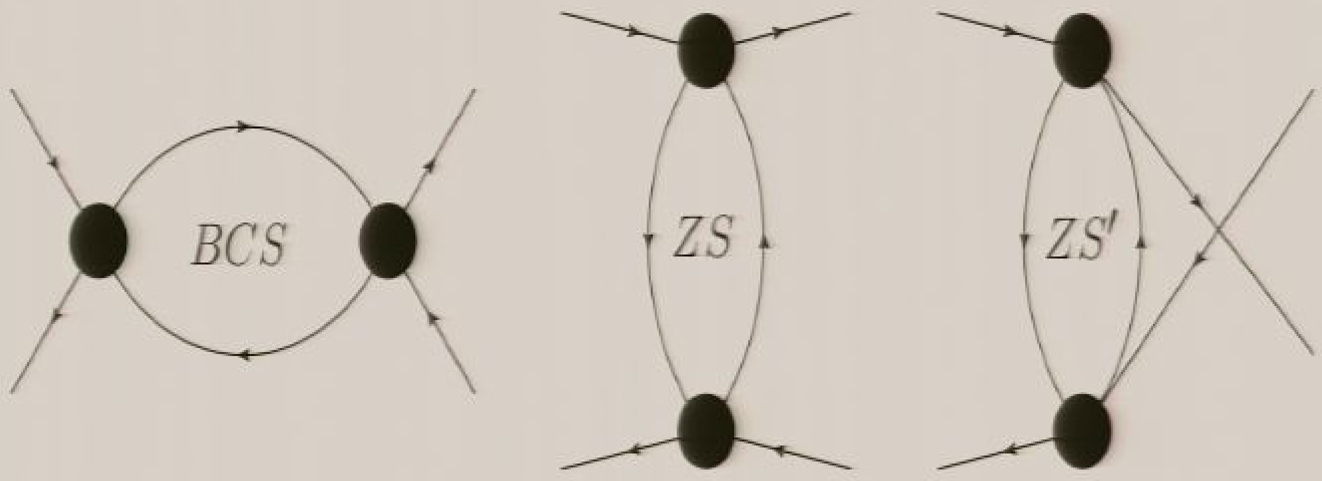
Fermi Liquid Self-Interactions



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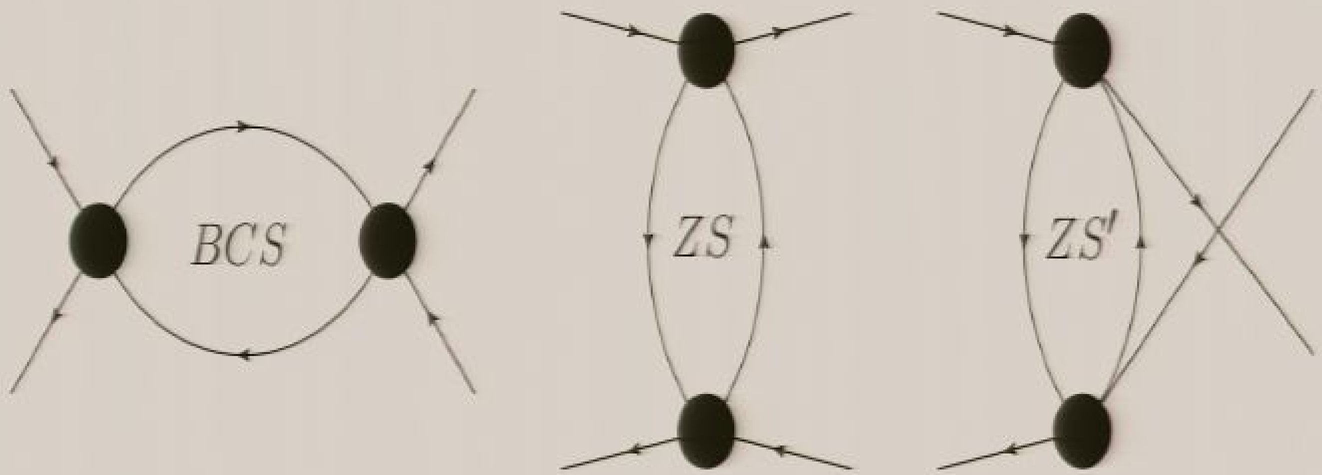

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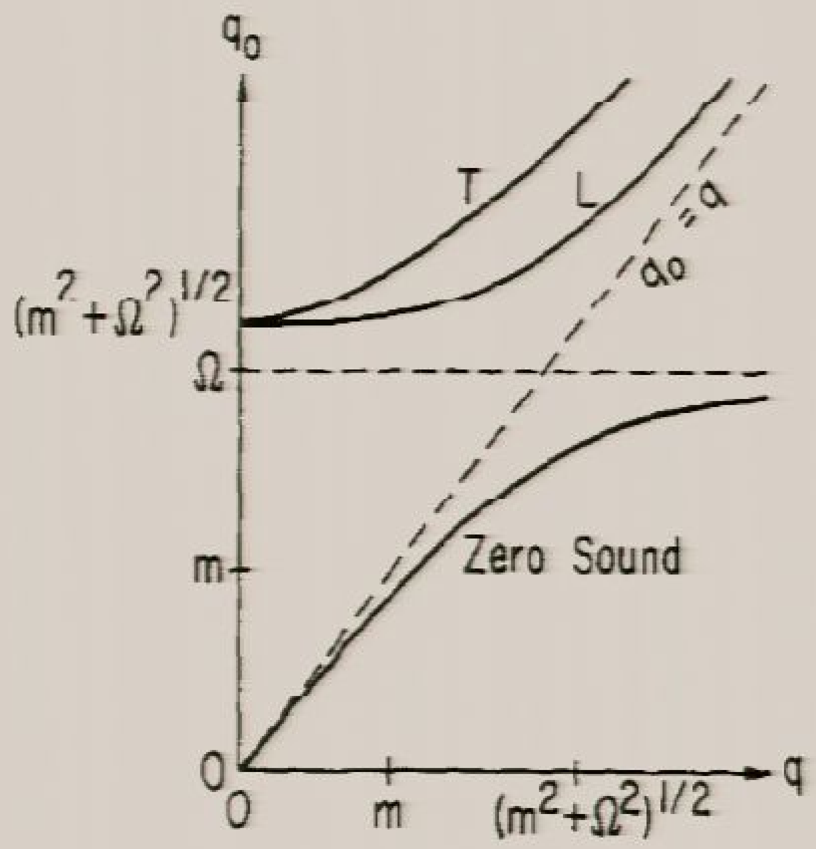
Massive Fermion Vector Zero Sound



Here, $m = M_Z$, $\Omega = \frac{g^2}{3\pi^2}\frac{k_F^3}{E_F}$. Note $E_F > k_F$ assumed.

Vector Zero Sound Operators

The effective one loop operator for this interaction is

$$\frac{g^4 \tilde{p}^2}{16\pi^2 M_Z^4}(\chi^\dagger \chi^\dagger)(\chi\chi)$$

Let us define the operator

$$A^\mu(x,y) = \frac{i}{2k_F}\left(\chi_x \tilde{\partial}_x^\mu \chi_y - \chi_x \tilde{\partial}_y^\mu \chi_y\right)$$

The momentum $\tilde{p}^\mu$ contains an index of refraction.

Because this mode is a goldstone boson, the theory has a cutoff at $2k_F$, so the effective action is an expansion in $\tilde{p}^\mu / k_F$.

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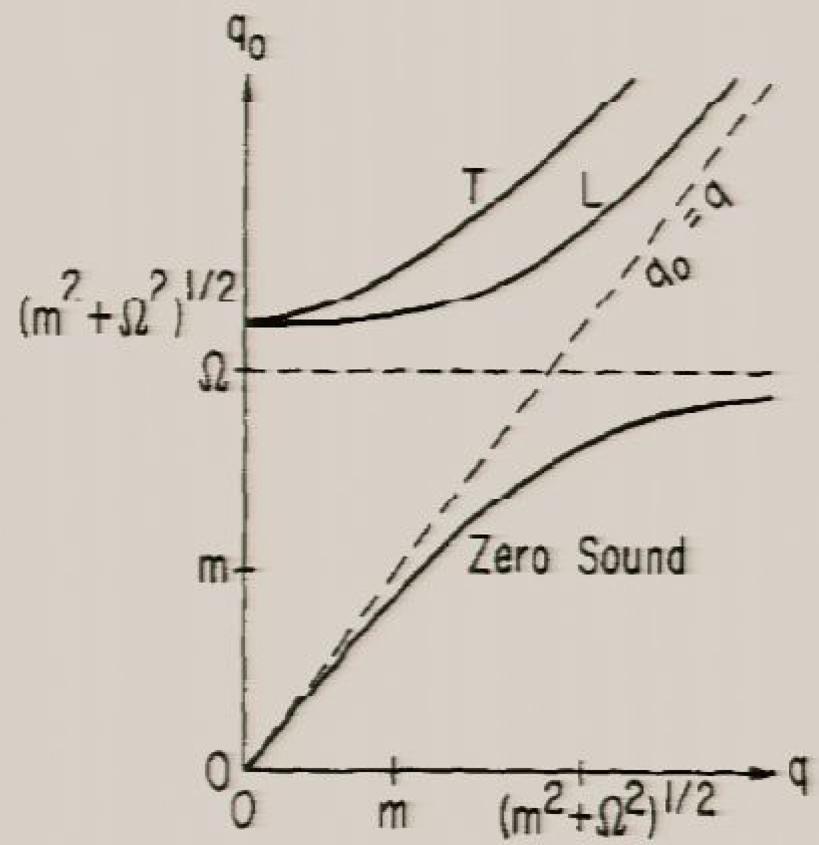

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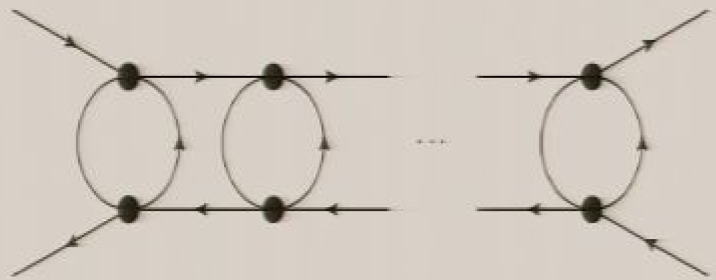
Tensor Zero Sound



We only resummed the bubble insertions of the gauge boson propagator to find vector Zero Sound. However this is not the only class of diagrams. Consider the resummation of the ZS and ZS' graphs.

This gives rise to a contribution that is simply a Fierz transformation of the previous operator

$$-\frac{g_Z^4 k_F^2}{4\pi^2 M_Z^4}\int_{xy}\left[(1-\eta_\nu)E_\mu^{a\dagger}E_a^\mu + \eta_\nu A_\mu^\dagger A^\mu\right].$$

where

$$\eta_\nu = \frac{n_\nu - n_{\overline{\nu}}}{n_\nu + n_{\overline{\nu}}}$$

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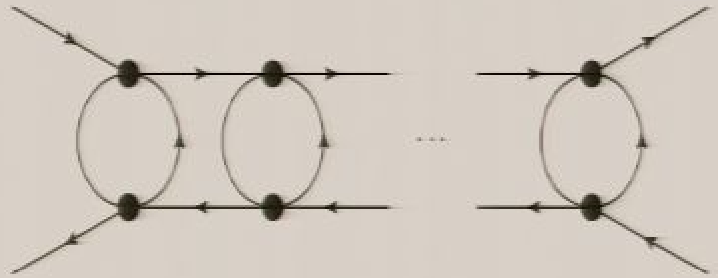

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Identify the neutrino condensate

Let us examine the possible quasi-particles containing one derivative:

$$A_\mu(x,y) = \frac{i}{2k_F}\left(\tilde{\partial}_\mu\chi(x)\epsilon\chi(y) - \chi(x)\epsilon\tilde{\partial}_\mu\chi(y)\right)$$

$$E^a_\mu(x,y) = \frac{i}{2k_F}\left(\tilde{\partial}_\mu\chi^\dagger(x)\overline{\sigma}^a\chi(y) - \chi^\dagger(x)\overline{\sigma}^a\tilde{\partial}_\mu\chi(y)\right)$$

These arise from integrating out the Z and including the 1-loop corrections from the previous slide(s). The 4-point interactions are

$$A^\dagger_\mu A^\mu; \qquad E^{a\dagger}_\mu E^\mu_a$$

these are the *same* interaction (related to each other by a Fierz transformation). The derivative is

$$\tilde{\partial}_\mu = (\partial_t, v\vec{\partial})$$

reflecting the fact that the dispersion relation for these states is $E = vp$ with $v < c$ (there is an index of refraction). The interaction terms are therefore

$$-\frac{g^4k_F^2}{4\pi^2M_Z^4}A^\dagger_\mu A^\mu; \qquad -\frac{g^4k_F^2}{4\pi^2M_Z^4}E^{a\dagger}_\mu E^\mu_a$$

these are clearly tachyonic mass terms.

Some Calculational details *

One can regard this problem as zero-temperature and finite density.

Temperature effects only affect cross sections and are down by $T^2 p_F^3 G_F^2$ which is much smaller than leading $p_F^2 G_F^2$ we're interested in.

The poles that occur due to finite density occur *regardless of the form of the distribution function*. The system is definitely out of equilibrium anyway.

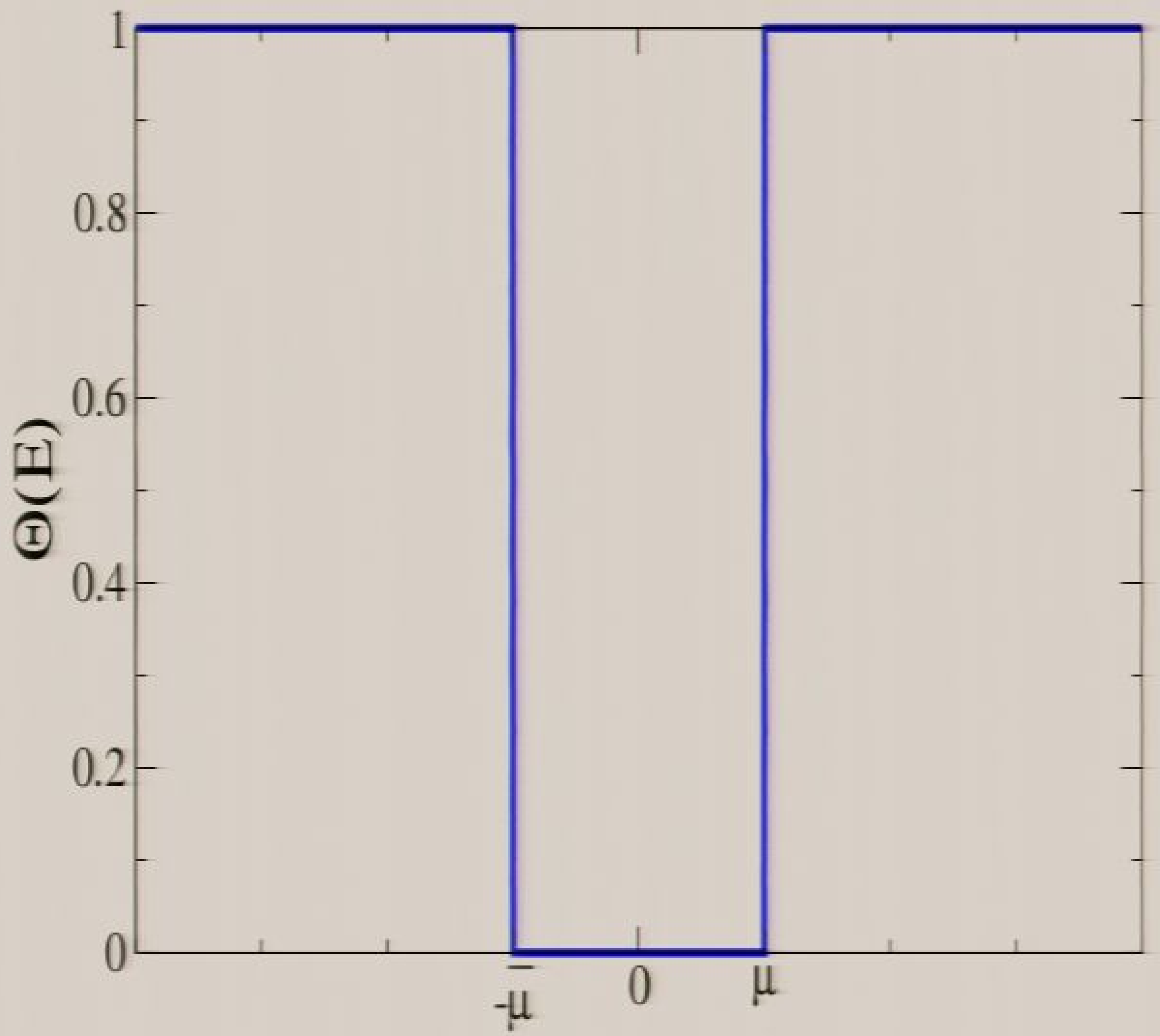
Then one can write the fermion propagator as:

$$S_F(p) = \Theta(\mu - E)\frac{i}{\not{p} - m + i\epsilon} + \Theta(\bar{\mu} + E)\frac{i}{\not{p} - m - i\epsilon}$$
$$= \frac{i}{\not{p} - m + i\epsilon} - \left(\frac{i}{\not{p} - m + i\epsilon} - \frac{i}{\not{p} - m - i\epsilon}\right)(\Theta(E - \mu) - \Theta(\bar{\mu} + E))$$

We're going to Pauli-block some of the momentum modes from the loop integral.

* Bob McElrath, to appear

The zero-temperature distribution function



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\begin{aligned}
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&= \frac{i}{\not p - m + i\epsilon} - \left(\frac{i}{\not p - m + i\epsilon} - \frac{i}{\not p - m - i\epsilon}\right)\left(\Theta(E - \mu) - \Theta(\bar{\mu} + E)\right)
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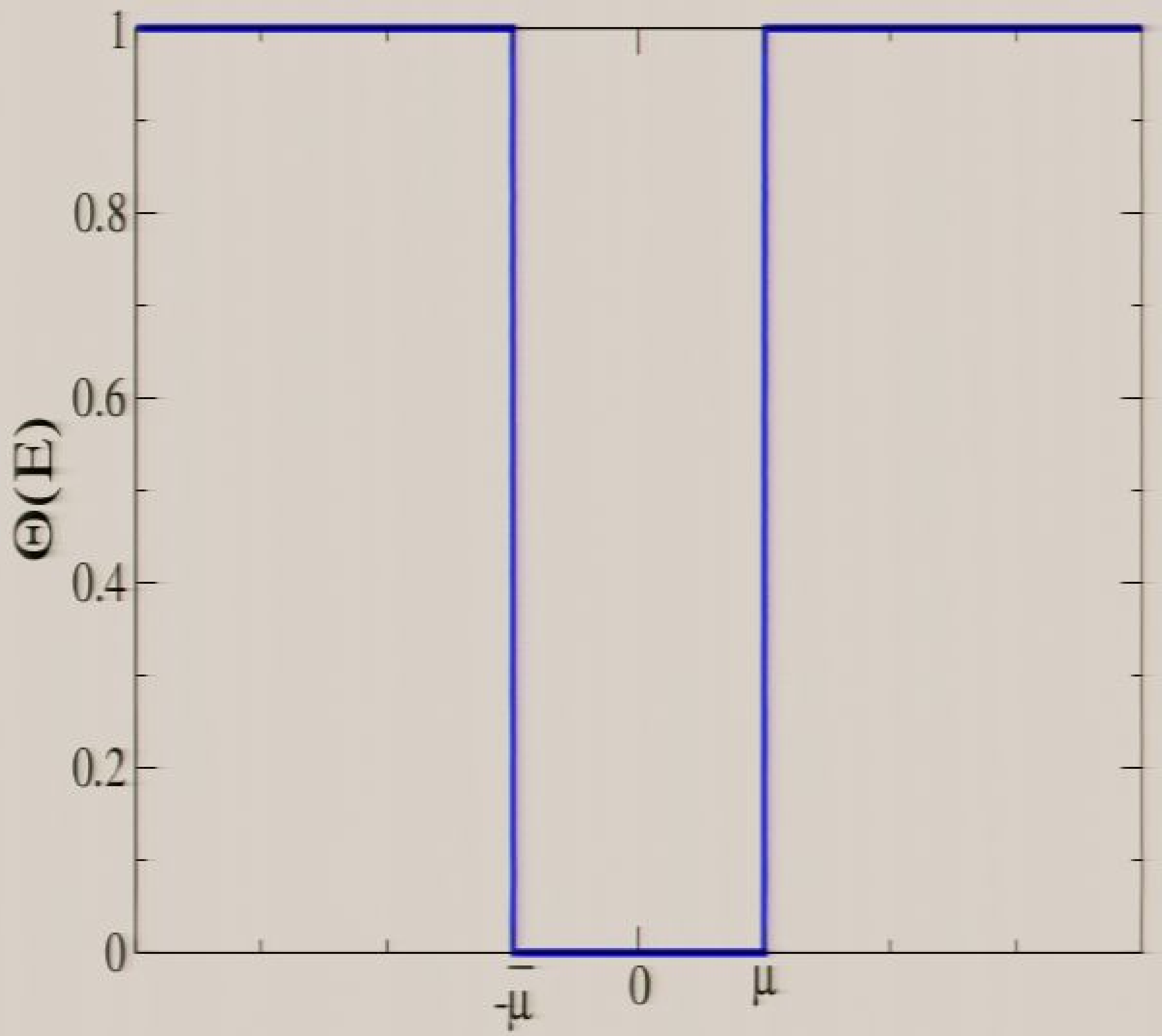

More Calculational Details

As long as the momentum modes that get Pauli blocked have $p < M_Z^2/T$, then we don't care *which* momentum modes are blocked, and it's equivalent to consider a degenerate distribution $\Theta(\mu - E)$.

The number of modes that are blocked is defined by the density parameter, $p_F = (3\pi n)^{1/3}$ or $E_F = \mu(T=0) = \sqrt{m^2 + p_F^2}$.

This is *almost* equivalent to putting in a chemical potential. A chemical potential μ is a Lagrange multiplier which forces conservation of $N = n_f - n_{\bar{f}}$: $\mu^\alpha \bar{\psi}\gamma_\alpha\psi$. In the rest frame, $\mu^\alpha = (\mu, \vec{0})$.

This is only appropriate in equilibrium where particle-antiparticle pairs are quickly annihilated.

For relic neutrinos and dark matter, we need to *separately conserve* n_ν and $n_{\bar{\nu}}$, necessitating two "chemical potentials" μ and $\bar{\mu}$ (but remember $(n_\nu - n_{\bar{\nu}})/(n_\nu + n_{\bar{\nu}}) \sim 10^{-10}$). What is conserved is $E_\nu N_\nu + E_{\bar{\nu}} N_{\bar{\nu}}$, which is the same as conserving $T^{\mu\nu}$.

Yet More Calculational Details: Renormalization

One might consider doing a Taylor expansion around $q = 0$ on the gauge boson propagator which would generate $(E_\mu^a)^2$. Since this is an irrelevant operator, it has a polynomial running anyway, and we can absorb Lorentz-invariant functions like q^2 into the definition of G_F or g_Z^2/M_Z^2.

If we choose to renormalize at the scale $q^2 = p_F^2$, we can choose that at that scale, the *only* operator that appears is

$$\frac{g_Z^2}{M_Z^2}\chi^\dagger\bar{\sigma}^a\chi\chi^\dagger\bar{\sigma}_a\chi$$

Then at one-loop we generate

$$-\frac{g_Z^4 k_F^2}{4\pi^2 M_Z^4}\int_{xy}\left[(1-\eta_\nu)E_\mu^{a\dagger}E_a^\mu + \eta_\nu A_\mu^\dagger A^\mu\right].$$

which are clearly proportional to the renormalization scale p_F (and would disappear if we renormalize around $q = 0$!)

We are not at zero temperature or density, and if we renormalize around $q = 0$ we miss important physics...

Introduce the condensate into the action (A)

There are two nice ways to introduce the composite into the action.

(A) The Hubbard Stratonivich transformation (a.k.a. Saddle Point integration):

Multiply the action by

$$const = \int \mathcal{D}[X^a_\mu] \exp\left\{\lambda \frac{i}{\hbar} \int d^4x d^4y \, X^a_\mu(x,y) X^{\dagger b}_\nu(x,y) \eta_{ab} \eta^{\mu\nu}\right\}$$

where

$$X^a_\mu = E^a_\mu - \frac{i}{2p_F}\left(\chi^\dagger \bar{\sigma}^a \tilde{\partial}_\mu \chi - \tilde{\partial}_\mu \chi^\dagger \bar{\sigma}^a \chi\right)$$

This removes the induced quartic term in favor of E^a_μ, leaving the only a kinetic and tree level interaction term for ψ.

Note that this transformation is only valid if the effective coupling $\lambda < 0$ (attractive). Were $\lambda > 0$ this would not be a gaussian integral, and this transformation would be invalid.

Introduce the condensate into the action (B)

(B) By Legendre Transformation

Following the "Nonequilibrium Quantum Field Theory" (a.k.a. 2PI) developed by Cornwall, Jackiw, and Tomboulis, one can insert a pair current N_μ^a. First let us note that

$$\frac{\delta \Gamma}{\delta E_\mu^a} = E_\mu^{a\dagger} + \epsilon_\mu^a = 0 \qquad (3)$$

where $\epsilon_\mu^a = \delta_\mu^a \delta^4(x-y)$, which comes from the fermion's kinetic term:

$$\int_x \chi^\dagger \partial_\mu \bar{\sigma}_\mu \chi = \int_{xy} E_\mu^a \delta_a^\mu \delta^4(x-y)$$

Therefore we will need to shift E_μ^a, so that the equations of motion for E_μ^a are quadratic. Thus we have a generating functional:

$$W[\eta, N_\mu^a] = -i\hbar \ln \int \mathcal{D}[\chi] \exp\left\{\frac{i}{\hbar}\left[S[\chi] + \int_x \chi\eta + \frac{1}{2}\int_{xy} N_\mu^a (E_a^\mu + \epsilon_a^\mu)\right]\right\}. \qquad (4)$$

Neutrino condensate symmetry breaking

The original action has a particle number global symmetry:

$$\chi \to e^{-i\theta}\chi$$

This is broken by the condensation of the field A^μ which is a $\chi\chi$ condensate. A_μ also breaks translation invariance. $\Rightarrow$ the long wavelength fluctuations around the order parameter A^μ are a weakly[2] coupled goldstone vector boson.

E^a_μ is a $\chi^\dagger\chi$ condensate so does not break the $U(1)$ number conservation symmetry. However notice that it transforms as a bi-vector under $SO(3,1)_{spin} \times SO(3,1)_{space}$, and that

$$\int_x i\chi^\dagger \not{\partial}\chi = \int_x \frac{i}{2}(\partial_\mu\chi^\dagger\bar{\sigma}^\mu\chi - \chi^\dagger\bar{\sigma}^\mu\partial_\mu\chi) = \int_{xy} E^a_\mu \delta^\mu_a \delta^4(x-y)$$

in other words, the fermion kinetic term is linear in the field E^a_μ (a tadpole) and indicates that if E^a_μ is a propagating field, we need to shift E^a_μ to remove the fermion's kinetic term.

Neutrino condensate symmetry breaking

The effective potential has an enhanced symmetry: $SO(3,1)_{\text{space}}$ (greek indices) $\times SO(3,1)_{\text{spin}}$ (roman indices):

$$\int_x \chi^\dagger \overline{\sigma}^a \chi \chi^\dagger \overline{\sigma}_a \chi - \int_{xy} E_\mu^{a\dagger} E_a^\mu - \int_{xy} A^{\mu\dagger} A_\mu$$

The only possible symmetry breaking terms $(\chi^\dagger \overline{\sigma}^\mu \partial_\mu \chi)^n \mathcal{O}$ are removed by the leading order equations of motion (and generate higher-order interactions for the zero-sound).

Now let us add the appropriate Lagrange multipliers to fix the charge under the two global symmetries (Lorentz and $U(1)$ particle number):

$$\omega^{\alpha\beta} T_{\alpha\beta} + \mu^\alpha \chi^\dagger \overline{\sigma}_\alpha \chi = \omega_a^\alpha \chi^\dagger \overline{\sigma}^a \partial_\alpha \chi + \mu^\alpha \chi^\dagger \overline{\sigma}_\alpha \chi$$

Thus E_μ^a and A^α are the goldstone boson, long wavelength fluctuations around the symmetry breaking Lagrange multipliers $\omega^{\mu\nu}$ (which fixes the stress-energy of the system) and chemical potential μ^α (which fixes the number density asymmetry of the system)

Lorentz Symmetry Breaking

The expectation value for E^a_μ has a simpler interpretation in terms of the stress tensor for a massless fermion:

$$\langle \tau^{\mu\nu}\rangle = \frac{1}{2}\langle E^a_\lambda\rangle \left[\delta^\nu_a \eta^{\lambda\mu} + \delta^\mu_a \eta^{\lambda\nu} + 2\delta^\lambda_a \eta^{\mu\nu}\right]$$

The Lorentz symmetry is actually two symmetries, spacetime and spin:

$$\tilde{L}_{\mu\nu} = i(x_\mu \tilde{\partial}_\nu - x_\nu \tilde{\partial}_\mu); \qquad S_{ab} = \frac{i}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a)$$

the neutrino transforms as a scalar $(0,0)$ under the first group and a spinor $(\frac{1}{2},0)$ under the second group.

Note that $\tilde{L}_{\mu\nu}$ is not the original Lorentz symmetry, but the *approximate* symmetry which emerges once indices of refraction are taken into account:

$$n = 1 - \eta_\nu k_F^2 G_F + \det(\omega_{\mu\nu}) G_F^2$$

The Weinberg Witten Theorem I

Weinberg and Witten (1980) told us that for any massless spin 2 object with a conserved Lorentz covariant stress tensor, its self-scattering matrix elements are zero.

This is generally used to "rule-out" a composite graviton, and indeed it does rule out a meson-like composite graviton.

However the theory of neutrino zero-sound is *NOT* Lorentz covariant. The fundamental theory is, but p_F breaks it! This results in the following Lorentz-breaking objects*

	value today	flat space (WW) limit
$\langle E^a_\mu \rangle$	$\mathcal{O}(10^{-3})$ eV	0
$n = v/c$	$1 - G_F^2 p_F^4 \simeq 1$	1
p_F	$\mathcal{O}(10^{-3})$ eV	0
G_N	$\mathcal{O}(p_F^2 G_F^2)$	0
M_{Pl}	$\mathcal{O}(1/p_F G_F)$	∞

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The effective potential has an enhanced symmetry: $SO(3,1)_{\text{space}}$ (greek indices) $\times SO(3,1)_{\text{spin}}$ (roman indices):

$$\int_x \chi^\dagger \bar{\sigma}^a \chi \chi^\dagger \bar{\sigma}_a \chi - \int_{xy} E^{a\dagger}_\mu E^\mu_a - \int_{xy} A^{\mu\dagger} A_\mu$$

The only possible symmetry breaking terms $(\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi)^n \mathcal{O}$ are removed by the leading order equations of motion (and generate higher-order interactions for the zero-sound).

Now let us add the appropriate Lagrange multipliers to fix the charge under the two global symmetries (Lorentz and $U(1)$ particle number):

$$\omega^{\alpha\beta} T_{\alpha\beta} + \mu^\alpha \chi^\dagger \bar{\sigma}_\alpha \chi = \omega^\alpha_a \chi^\dagger \bar{\sigma}^a \partial_\alpha \chi + \mu^a \chi^\dagger \bar{\sigma}_a \chi$$

Thus E^a_μ and A^α are the goldstone boson, long wavelength fluctuations around the symmetry breaking Lagrange multipliers $\omega^{\mu\nu}$ (which fixes the stress-energy of the system) and chemical potential μ^α (which fixes the number density asymmetry of the system)

Neutrino condensate symmetry breaking

The original action has a particle number global symmetry:

$$\chi \to e^{-i\theta}\chi$$

This is broken by the condensation of the field A^{μ} which is a $\chi\chi$ condensate. A_{μ} also breaks translation invariance. $\Rightarrow$ the long wavelength fluctuations around the order parameter A^{μ} are a weakly^2 coupled goldstone vector boson.

E^a_{μ} is a $\chi^{\dagger}\chi$ condensate so does not break the $U(1)$ number conservation symmetry. However notice that it transforms as a bi-vector under $SO(3,1)_{spin} \times SO(3,1)_{space}$, and that

$$\int_x i\chi^{\dagger}\partial\!\!\!/\chi = \int_x \frac{i}{2}(\partial_{\mu}\chi^{\dagger}\overline{\sigma}^{\mu}\chi - \chi^{\dagger}\overline{\sigma}^{\mu}\partial_{\mu}\chi) = \int_{xy} E^a_{\mu}\delta^{\mu}_a\delta^4(x-y)$$

in other words, the fermion kinetic term is linear in the field E^a_{μ} (a tadpole) and indicates that if E^a_{μ} is a propagating field, we need to shift E^a_{μ} to remove the fermion's kinetic term.

Introduce the condensate into the action (B)

(B) By Legendre Transformation

Following the "Nonequilibrium Quantum Field Theory" (a.k.a. 2PI) developed by Cornwall, Jackiw, and Tomboulis, one can insert a pair current N_μ^a. First let us note that

$$\frac{\delta\Gamma}{\delta E_\mu^a} = E_\mu^{a\dagger} + \epsilon_\mu^a = 0 \tag{3}$$

where $\epsilon_\mu^a = \delta_\mu^a \delta^4(x-y)$, which comes from the fermion's kinetic term:

$$\int_x \chi^\dagger \partial_\mu \bar{\sigma}_\mu \chi = \int_{xy} E_\mu^a \delta_a^\mu \delta^4(x-y)$$

Therefore we will need to shift E_μ^a, so that the equations of motion for E_μ^a are quadratic. Thus we have a generating functional:

$$W[\eta, N_\mu^a] = -i\hbar \ln \int \mathcal{D}[\chi] \exp\left\{\frac{i}{\hbar}\left[S[\chi] + \int_x \chi\eta + \frac{1}{2}\int_{xy} N_\mu^a (E_a^\mu + \epsilon_a^\mu)\right]\right\}. \tag{4}$$

Yet More Calculational Details: Renormalization

One might consider doing a Taylor expansion around $q = 0$ on the gauge boson propagator which would generate $(E_\mu^a)^2$. Since this is an irrelevant operator, it has a polynomial running anyway, and we can absorb Lorentz-invariant functions like q^2 into the definition of G_F or g_Z^2/M_Z^2.

If we choose to renormalize at the scale $q^2 = p_F^2$, we can choose that at that scale, the *only* operator that appears is

$$\frac{g_Z^2}{M_Z^2}\chi^\dagger\bar{\sigma}^a\chi\chi^\dagger\bar{\sigma}_a\chi$$

Then at one-loop we generate

$$-\frac{g_Z^4 k_F^2}{4\pi^2 M_Z^4}\int_{xy}\left[(1-\eta_\nu)E_\mu^{a\dagger}E_a^\mu + \eta_\nu A_\mu^\dagger A^\mu\right].$$

which are clearly proportional to the renormalization scale p_F (and would disappear if we renormalize around $q = 0$!)

We are not at zero temperature or density, and if we renormalize around $q = 0$ we miss important physics...

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The Weinberg Witten Theorem I

Weinberg and Witten (1980) told us that for any massless spin 2 object with a conserved Lorentz covariant stress tensor, its self-scattering matrix elements are zero.

This is generally used to "rule-out" a composite graviton, and indeed it does rule out a meson-like composite graviton.

However the theory of neutrino zero-sound is *NOT* Lorentz covariant. The fundamental theory is, but p_F breaks it! This results in the following Lorentz-breaking objects*

	value today	flat space (WW) limit
$\langle E_\mu^a \rangle$	$\mathcal{O}(10^{-3})$ eV	0
$n = v/c$	$1 - G_F^2 p_F^4 \simeq 1$	1
p_F	$\mathcal{O}(10^{-3})$ eV	0
G_N	$\mathcal{O}(p_F^2 G_F^2)$	0
M_{Pl}	$\mathcal{O}(1/p_F G_F)$	∞

* Alejandro Jenkins and Bob McElrath, to appear

The Weinberg Witten Theorem II

Thus this theory evades the Weinberg-Witten Theorem (1980): the emergent graviton does not propagate in flat Minkowski space. It lives only in a curved space. As $p_F \to 0$, $\langle E^a_\mu \rangle \to 0$ and we return to Minkowski space, and in that limit, $G_N \to 0$ and the emergent graviton *disappears from the theory*. The smallness of Lorentz violation is directly related to the smallness of the coupling G_N. We can write

$$G_N \propto \frac{1-n}{k_F^2} = k_F^2 G_F^2$$

Thus there is a conserved stress tensor for the gravitational sector of this theory, but it does not live in the same space as the gravitational theory itself.

As stringers would prefer to word it: The WW theorem implies that *spacetime itself* must be emergent. In the present context, it is the space containing the *index of refraction* that is the emergent spacetime. This graviton lives *only* in that emergent space. The neutrino's stress tensor *does not*.

The Weinberg Witten Theorem III

The operator we generated was

$$\frac{\tilde{p}^2}{M_Z^4}\chi^\dagger\bar{\sigma}^a\chi\chi^\dagger\bar{\sigma}_a\chi.$$

In a Lorentz invariant space ($\tilde{p} = p$ and $n = 1$), this is simply a quadratic running for my irrelevant 4-fermion operator.

I could choose to absorb this correction by a choice of the finite part of my counter-term for the low-energy effective theory:

$$G_F(p^2) = G_F - p^2 G_F^2$$

This is a beautiful restatement of the Weinberg-Witten theorem:

In a Lorentz Invariant theory, an emergent spin-2 operator can be absorbed by a renormalization counterterm choice

Or,

gravity is *Lorentz breaking.*

The Cutoff

This theory has a cutoff defined by the density $2k_F$. A^μ and E^a_μ are rearrangements of *existing* modes in the background. Therefore they cannot carry energy density larger than $2k_F$. They are exactly stable below $2k_F$. As such, this is an implementation of Sundrum's "soft graviton", and the cosmological constant is $\Lambda \propto k_F^4$.

Above $2k_F$ these states acquire a width. This width is proportional to the mean free path and can be regarded as the decay of the spin-density perturbation back into free neutrinos. This width is extremely small. (very long lifetime)

If we ask when this width becomes large, this occurs when the CM energy puts the Z on pole. For a probe with energy E, this occurs when

$$E = M_Z^2/T_\nu \simeq M_{Pl}$$

Therefore, in the *lab* frame, this low-energy effective gravitational description of the relic neutrinos is valid throughout the range of energies we have explored (and even above k_F).

This Quasi-Particle is a Graviton

We already know what a $SO(3,1)$ bi-vector is: the vierbein (tetrad):

$$g_{\mu\nu}(x,y) = E^a_\mu(x,y)E^b_\nu(x,y)\eta_{ab}$$

This field has an internal global $SO(3,1)$ symmetry due to the spin Lorentz invariance.

This is different from the first-order (Palatini) formulation of gravity (which uses a *local* internal Lorentz symmetry).

Thus the fermion spin dependence is not a gauge symmetry, but is a physical observable in this theory. The spin distribution of the fermion gives rise to *Torsion*.

Such a theory was explored by Hebecker and Wetterich [2003; Wetterich 2003, 2004]. They conclude that the addition of torsion, due to a global, rather than local Lorentz symmetry is at present *unobservable*.

This theory differs from that of Hebecker and Wetterich due to the presence of the $SO(3,1) \times SO(3,1)$ symmetry breaking structure, and the associated metric $\eta_{\mu\nu}$. (e.g. they don't have $(E^a_\mu)^2$ or $(E^a_\mu)^4$)

Conclusions

If the universe contains a massless fermionic relic (such as a neutrino), then the long-wavelength fluctuations around its vacuum stress tensor is a goldstone graviton. If it has an asymmetry, then it is accompanied by a gravitationally-coupled goldstone vector boson.

these are acoustic quasi-particles ("zero sound" or "phonons") in the Cosmic Neutrino Background.

This theory is entirely natural. The highest scale in the theory is M_Z. The cutoff is k_F, generating a natural cosmological constant of the correct order.

This theory may also contain the keys to galactic rotation curves, neutrino mass, and cosmic expansion, at the next order in $\sqrt{p_F^2 G_F}$.

This theory is *supremely testable* and *falsifiable* (unlike other gravity theories). We can make W's, Z's, and neutrinos. It contains zero free parameters.

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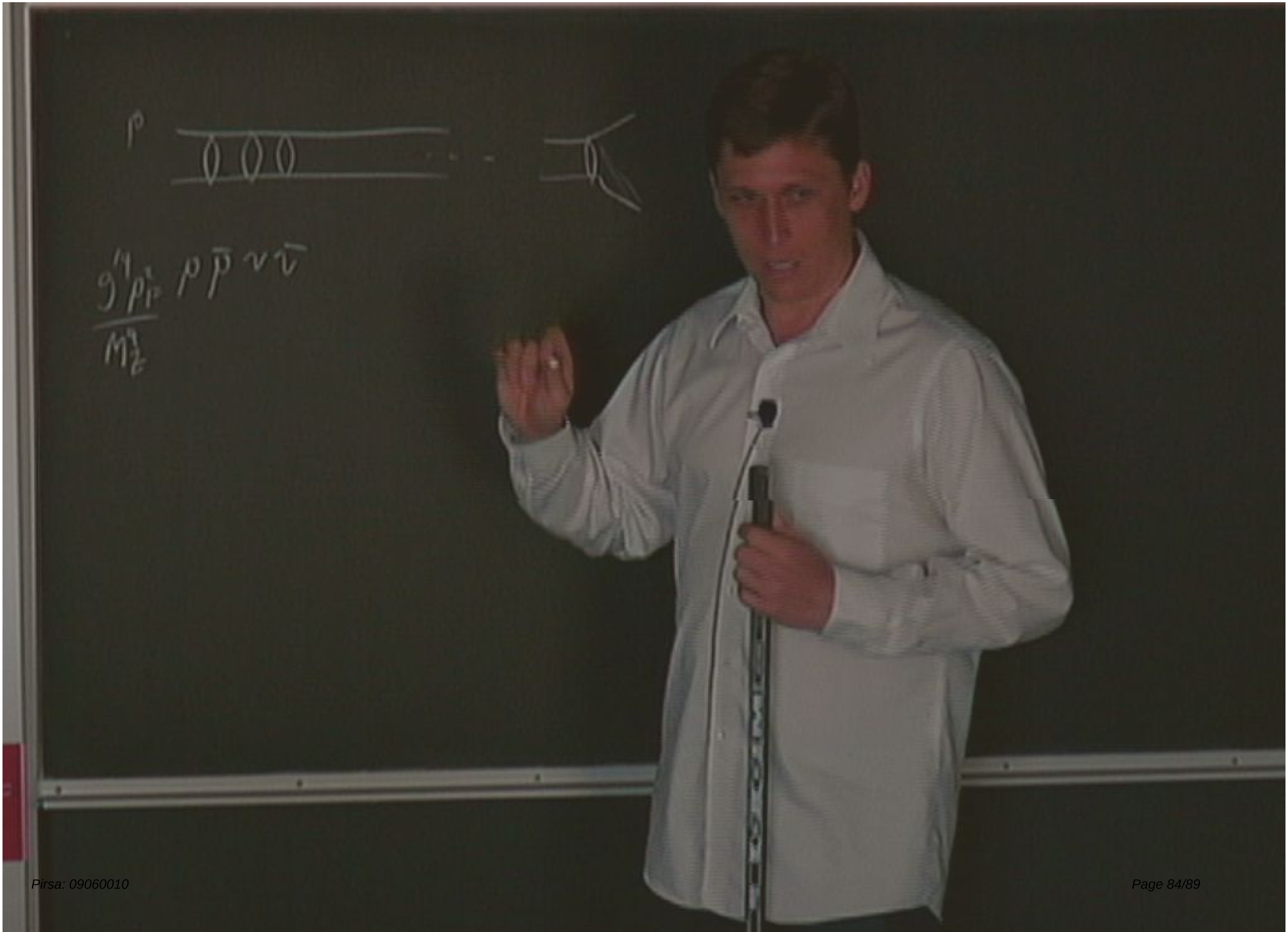
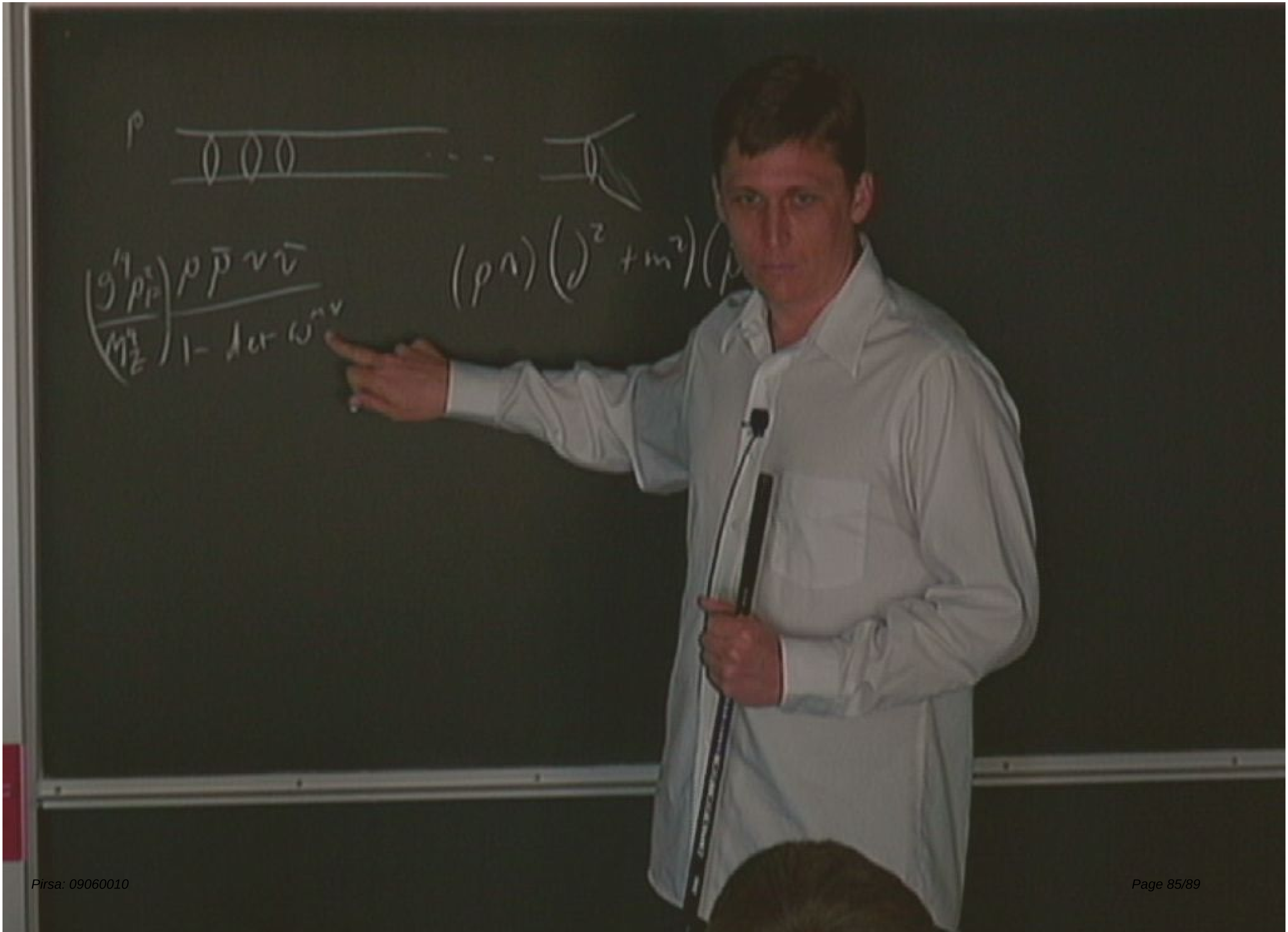
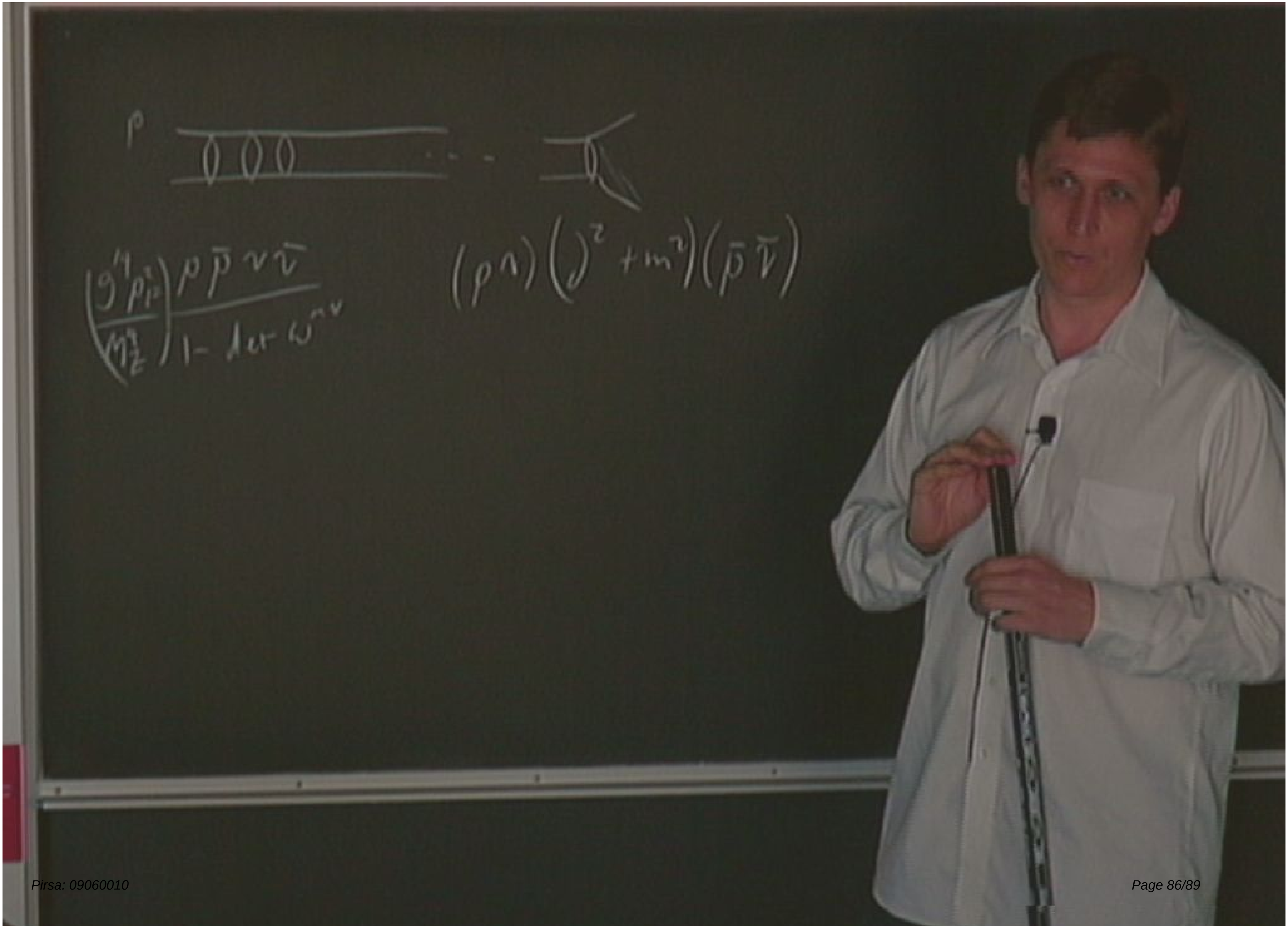
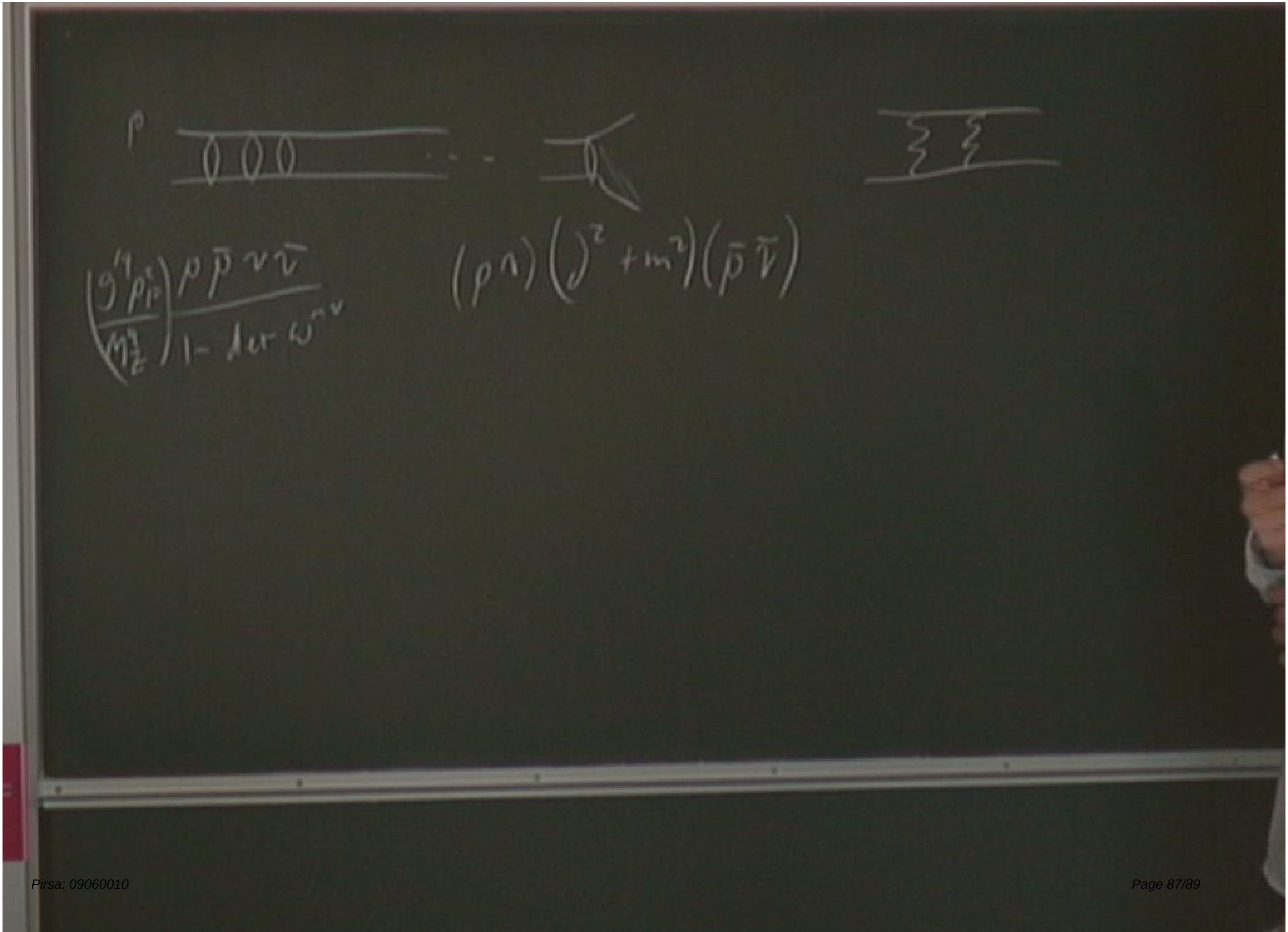
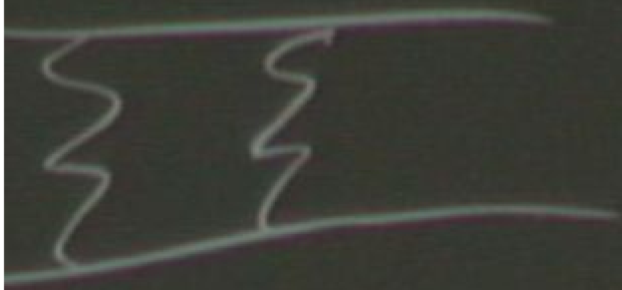







ρ
(g⁴ρ_p² / M_Z⁴) ρ p̄ ν ν̄ / 1 - det ω^μν
(ρ ν)(∂² + m²)(p̄ ν̄)



$$E \simeq \frac{M_Z^2}{\rho_F} \sim M_{pl}$$

Dismiss Bad Ideas

The low scale could be $T_\gamma, T_\nu, p_F(\gamma), p_F(\nu)$ or m_ν.

- Photons are boring: 4-γ vertex is dimension 8, and self-interaction cross section approximately $10^{-14} p_F^4 G_F^2$. (i.e. it may be interesting, but is very sub-leading)

- The combination $T_\nu^2 G_F^2$ is the self-interaction cross section of neutrinos. This would seem to be a hydrodynamic theory. However then one has to confront the flux. The inverse mean free path of a neutrino is

$$\lambda^{-1} = (\sigma n)^{-1} = T_\nu^2 G_F^2 p_F^3 \simeq \mathcal{O}(p_F^5 G_F^2)$$

  and much larger than the horizon size, and the interaction rate is too low to be interesting.

- If m_ν is a fundamental Lagrangian parameter it would only arise in combination with p_F or T_ν.

These come in at higher order in ratios of p_F and G_F than phenomena we can (and have) seen. effects that could be relevant for (leading order) gravity.