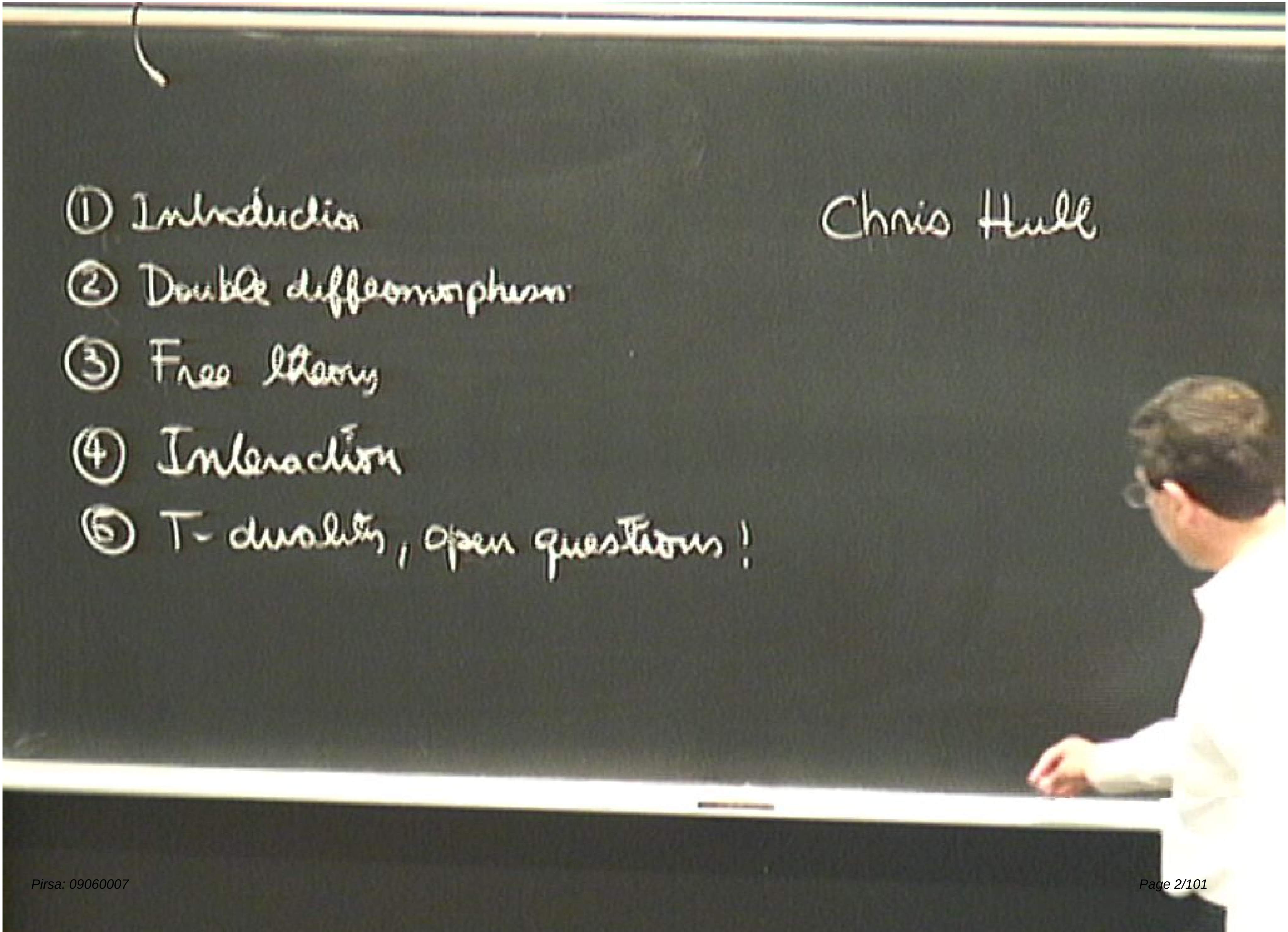
Title: Double Field Theory

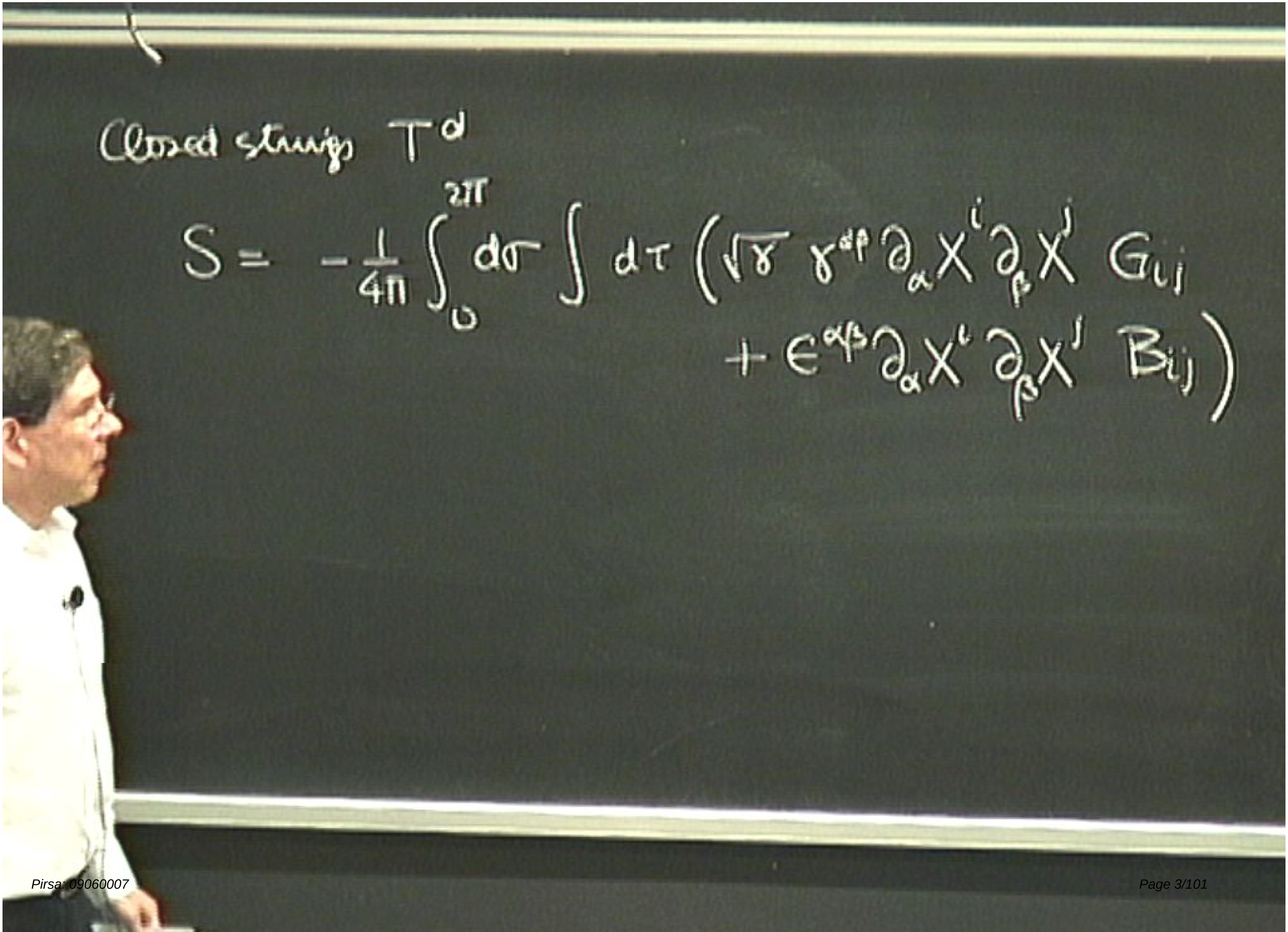
Date: Jun 09, 2009 11:00 AM

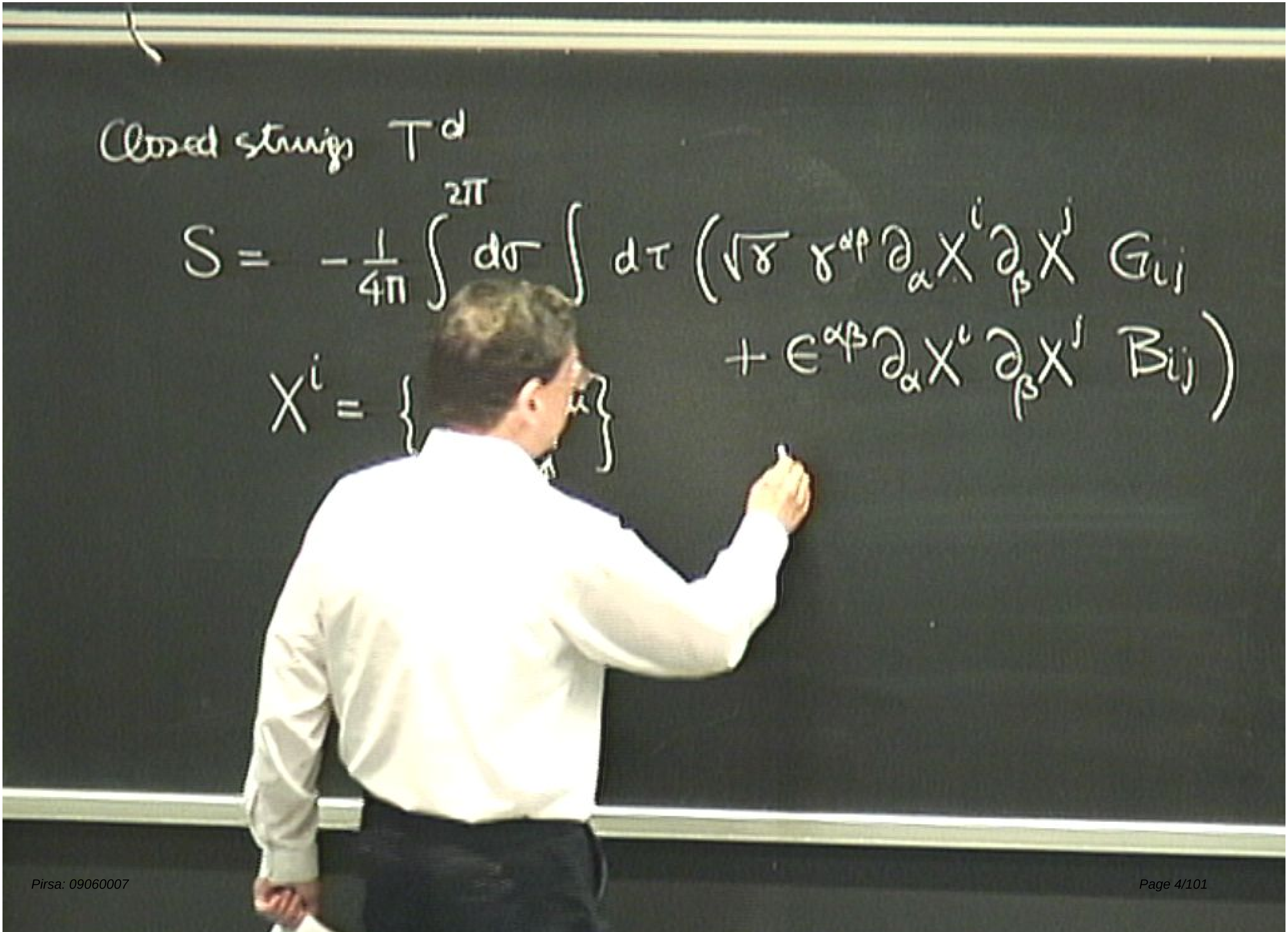
URL: http://pirsa.org/09060007

Abstract: The massless fields of closed string theory on toroidal backgrounds naturally depend on coordinates dual to momentum and coordinates dual to winding. Their dynamical theory, which contains gravitation, must include diffeomorphism and dual diffeomorphism invariance. We begin a serious attempt to construct this generalized form of field theory.



① Introduction
② Double diffeomorphism
③ Free theory
④ Interaction
⑤ T-duality, open questions!
Chris Hull


Closed strings T^d
$S = -\frac{1}{4\pi}\int_0^{2\pi} d\sigma \int d\tau \left(\sqrt{\gamma}\,\gamma^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j G_{ij} + \epsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j B_{ij}\right)$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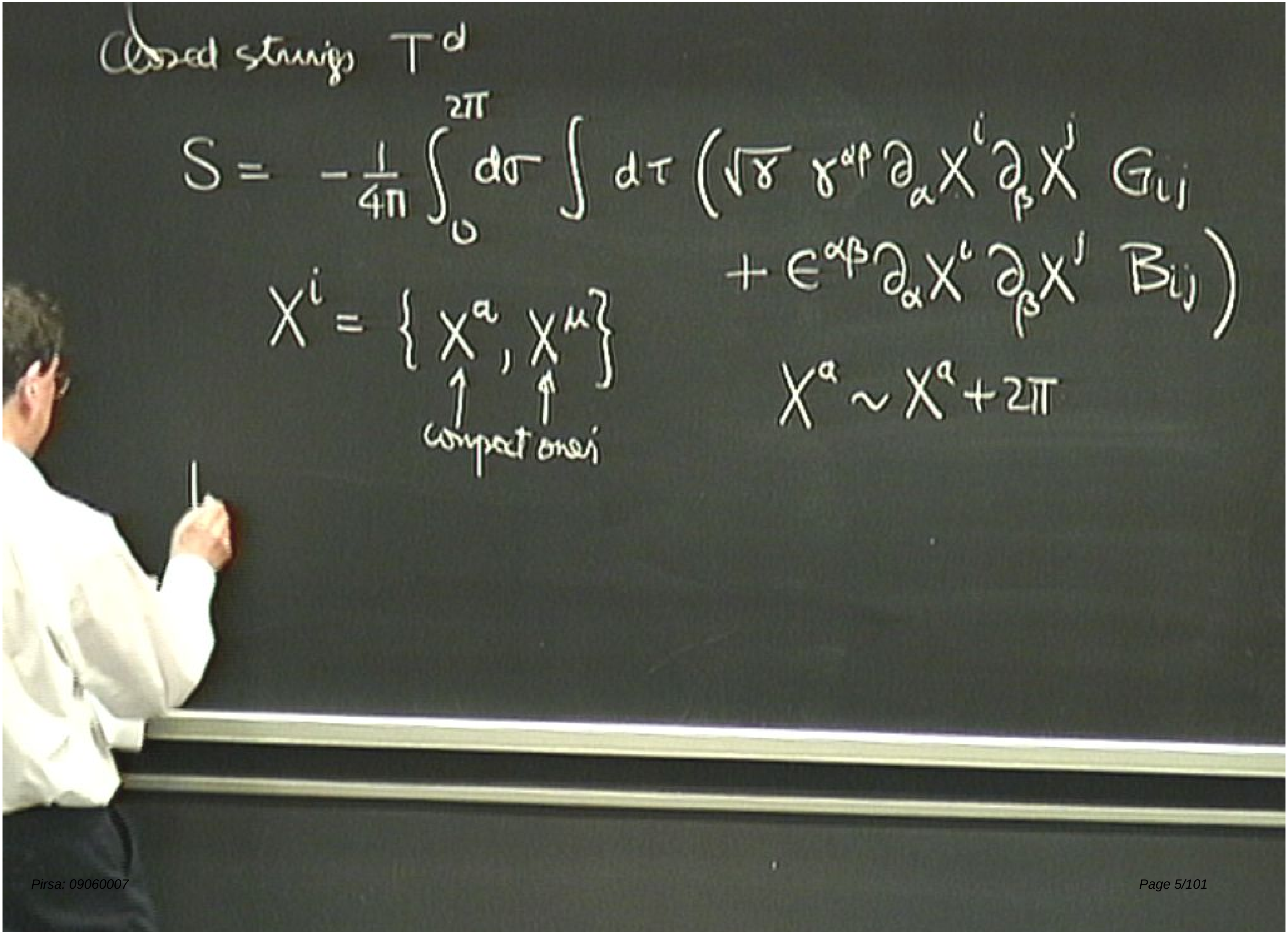
$X^i = \{$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$X^i = \{X^a, X^\mu\}$
compact ones
$X^a \sim X^a + 2\pi$

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$$\left(G_{ab} = \frac{R_a^2}{\alpha'} \delta_{ab} \right)$$

$$X^i(\tau,\sigma) = x^i + w^i\sigma + \tau G^{ij}(p_j \cdots\cdots) \cdots$$

$$2\pi P_i(\tau,\sigma) = p_i + \cdots$$

$$\tilde{X}_i(\tau,\sigma) = \tilde{x}_i + p_i\sigma + \tau[\cdots]\cdots$$

$$X^i(\tau,\sigma) = x^i + w^i\sigma + \tau G^{(i)}(p_1 \cdots\cdots) \cdots$$

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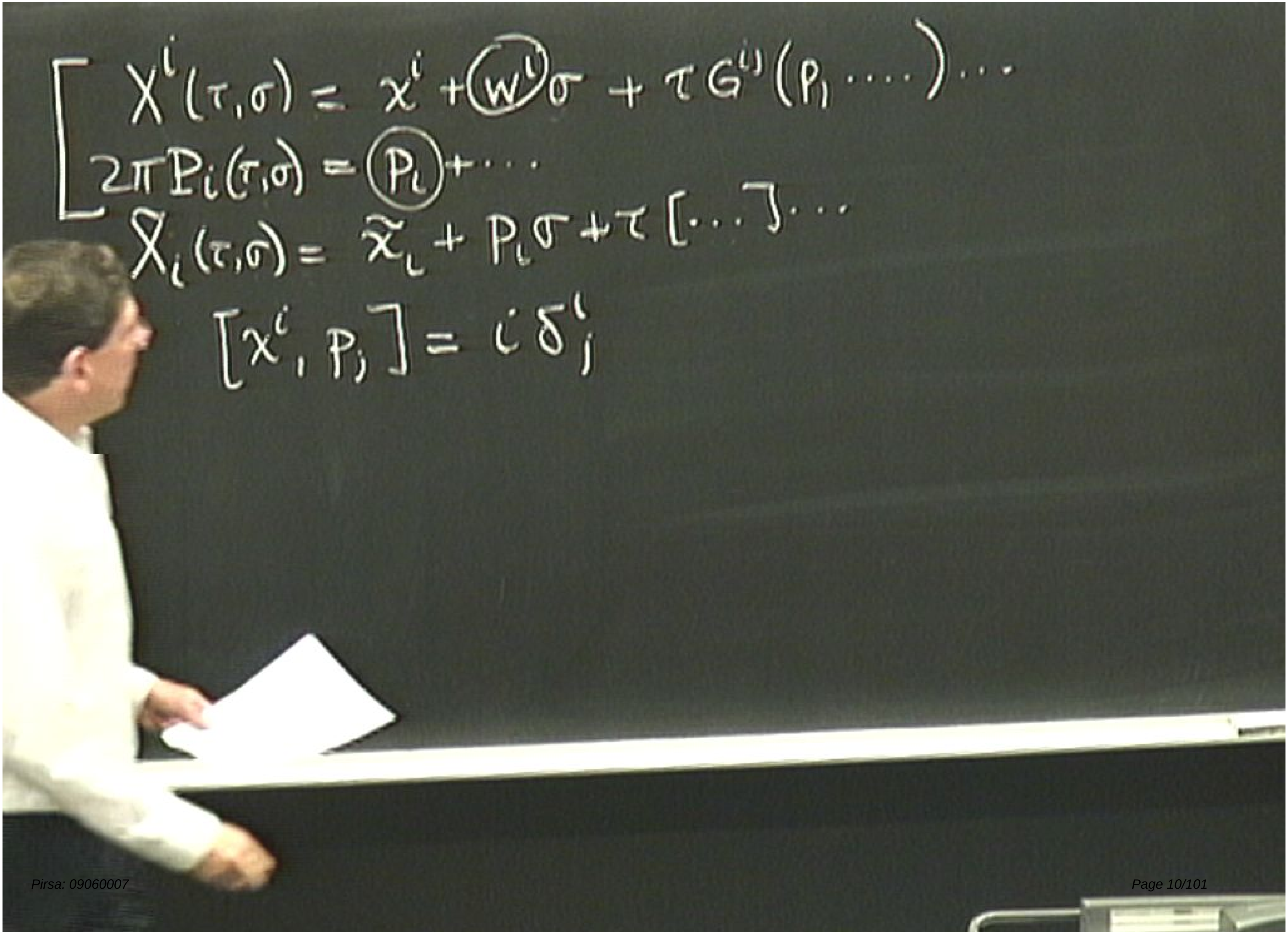
$X^i(\tau,\sigma) = x^i + w^i\sigma + \tau G^{ij}(p_j \cdots) \cdots$
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$[x^i, p_j] = i\delta^i_j$

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$$p_i = \frac{1}{i}\frac{\partial}{\partial x^i}$$

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$$X(\cdot) \sim X(\tau - \sigma)$$

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$$\left[\begin{array}{l} X^i(\tau,\sigma) = x^i + w^i\sigma + \tau G^{(i)}(p_i \cdots\cdots)\cdots \\ 2\pi P_i(\tau,\sigma) = p_i + \cdots \end{array}\right.$$

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p_i eigenvalues $n_i \in \mathbb{Z}$

w^i " $m^i \in \mathbb{Z}$

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$$|\psi\rangle = \sum$$

$$|\psi\rangle = \sum_{\{m^i, n_i\}} \xi_{i_1 \cdots i_p\, j_1 \cdots j_p}(m^i, n_i)\, \alpha_{-1}^{i_1} \cdots \alpha_{-1}^{i_p}\, \bar{\alpha}_{-1}^{j_1} \cdots \bar{\alpha}_{-1}^{j_p} |m, n\rangle$$

$$|\psi\rangle = \sum_{\{m^i, n\}} \xi_{i_1 \cdots i_p\, j_1 \cdots j_p}(m^i, n_1)\, \alpha_{-1}^{i_1} \cdots \alpha_{-1}^{i_p}\, \bar{\alpha}_{-1}^{j_1} \cdots \bar{\alpha}_{-1}^{j_p} |m, n\rangle$$

$$|\psi\rangle = \sum_{\{m^i, n_i\}} \xi_{i_1 \cdots i_p, j_1 \cdots j_p}(m^i, n_i)\, \alpha_{-1}^{i_1} \cdots \alpha_{-1}^{i_p}\, \tilde{\alpha}_{-1}^{j_1} \cdots \tilde{\alpha}_{-1}^{j_p} |m, n\rangle$$

$$|\psi\rangle = \sum_{\{m^i,\, n\}} \underline{\underline{\xi_{i_1 \cdots i_p\, j_1 \cdots j_p}(m^i, n)}}\; \alpha_{-1}^{i_1} \cdots \alpha_{-1}^{i_p}\, \bar{\alpha}_{-1}^{j_1} \cdots \bar{\alpha}_{-1}^{j_p}\, |m, n\rangle$$

$$\xi_{i_1}$$

$$|\psi\rangle = \sum_{\{m^i, n_i\}} \xi_{i_1 \cdots i_p, j_1 \cdots j_p}(m^i, n_i)\, \alpha_{-1}^{i_1} \cdots \alpha_{-1}^{i_p}\, \bar{\alpha}_{-1}^{j_1} \cdots \bar{\alpha}_{-1}^{j_p} |m, n\rangle$$

$$\xi_{i_1 \cdots i_p, j_1 \cdots j_p}(\tilde{x}, x) = \sum_{\{m^i, n_i\}} \xi_{i_1 \cdots j_1 \cdots}(m^i, n_i)\, e^{i m^i \tilde{x}_i}\, e^{i n_i x^i}$$

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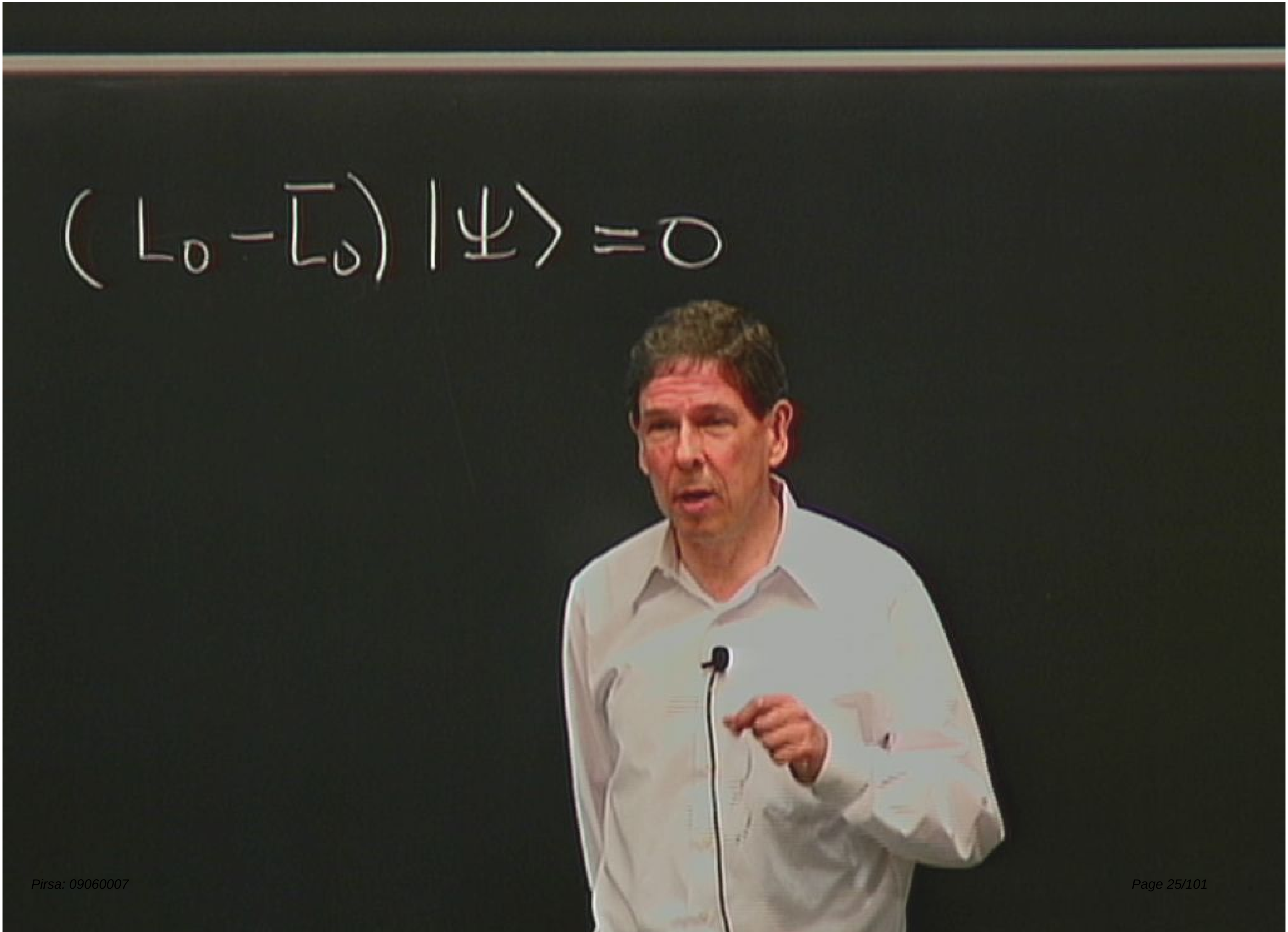
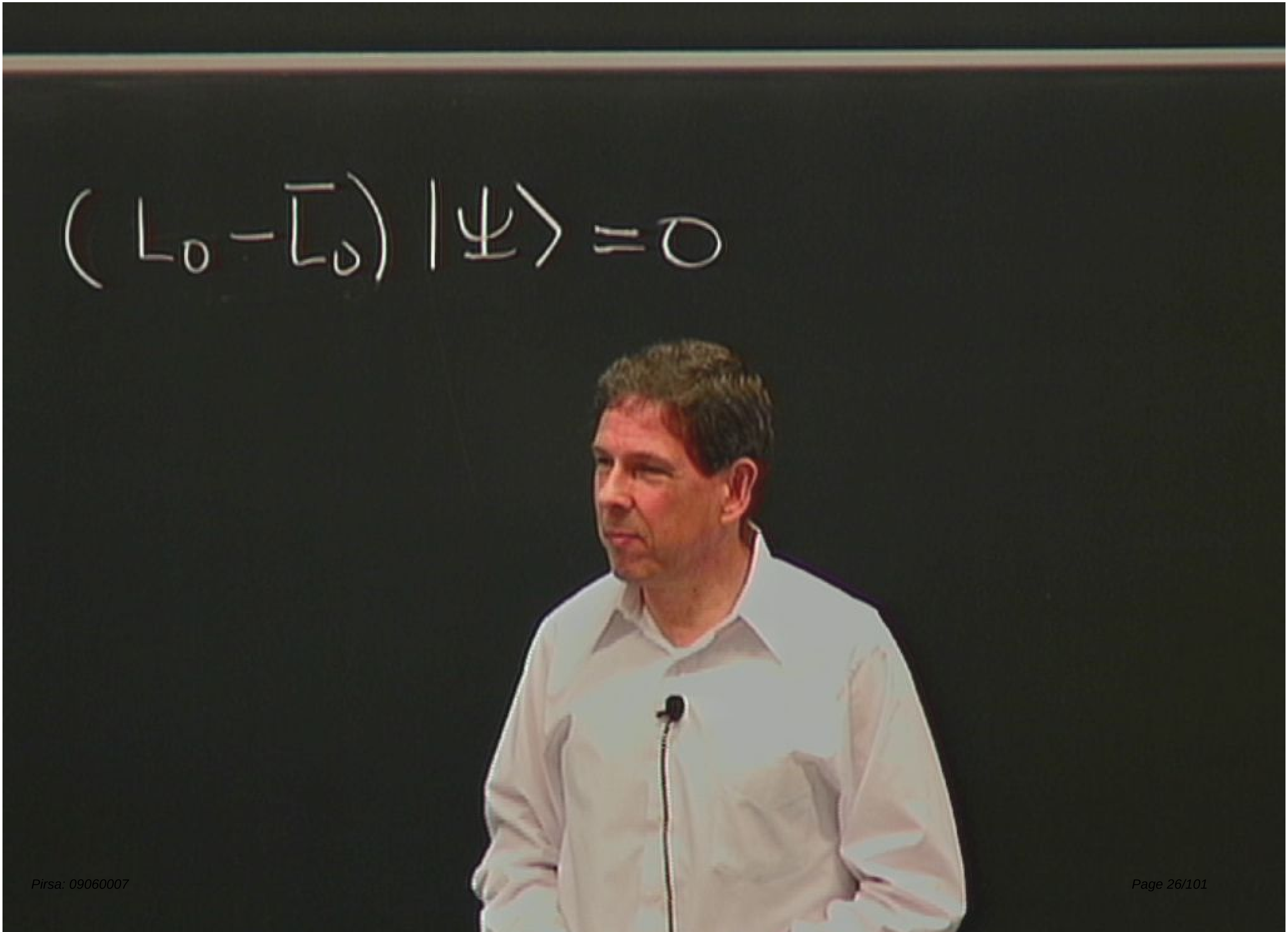

$(L_0 - \bar{L}_0)|\Psi\rangle = 0$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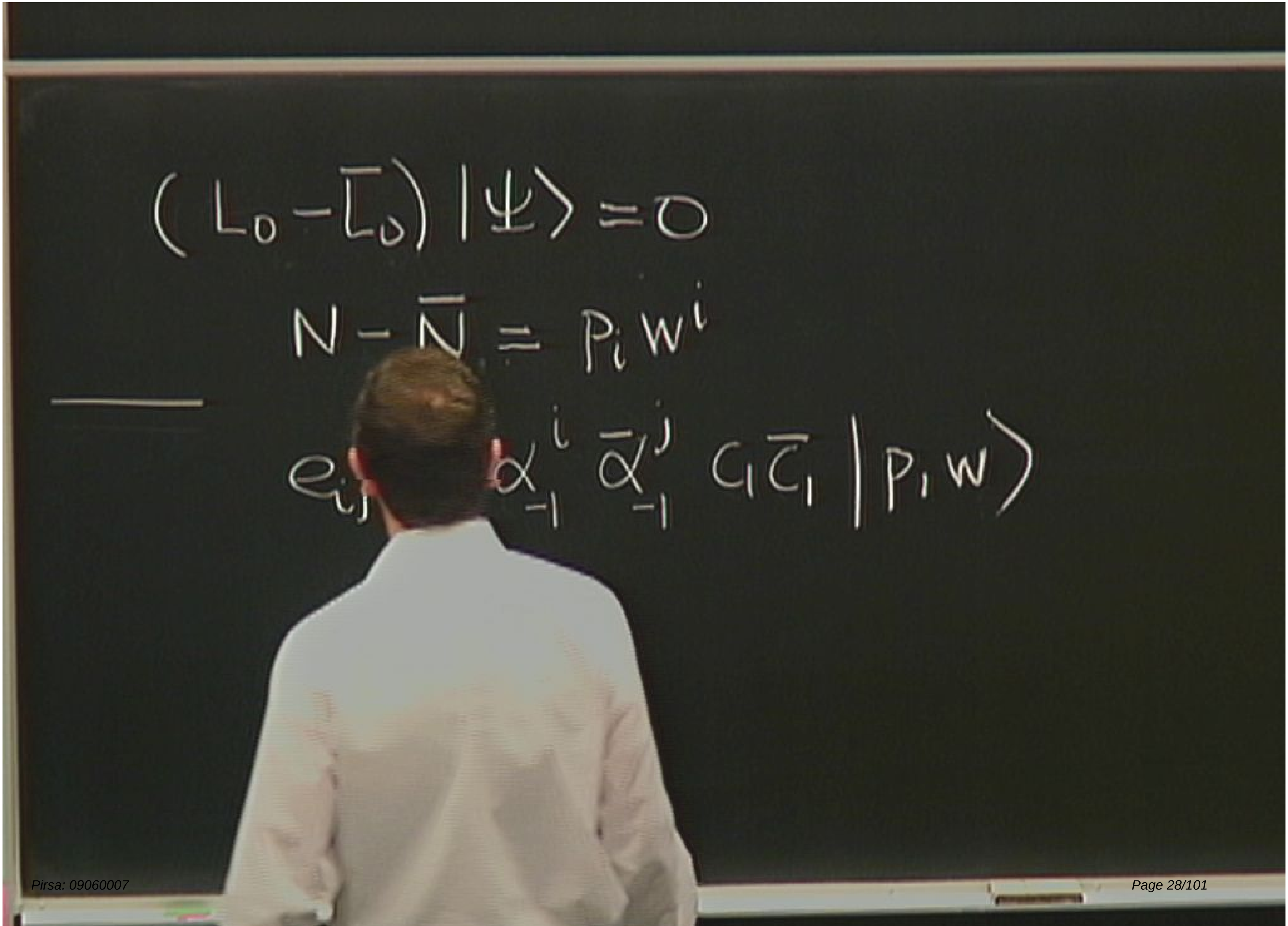
$$N - \bar{N} = p_i w^i$$


$(L_0 - \bar{L}_0)|\Psi\rangle = 0$
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$e_{ij}\ \alpha_{-1}^i \bar{\alpha}_{-1}^j c_1 \bar{c}_1 |p, w\rangle$

$$(L_0 - \bar{L}_0)|\Psi\rangle = 0$$

$$N - \bar{N} = p_i w^i$$

$$e_{ij}(p,w)\, \alpha^i_{-1}\, \bar{\alpha}^j_{-1}\, c_1 \bar{c}_1 \,|p,w\rangle$$

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$$d(p,w)\left(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1}\right)|$$

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$e_{ij}(x,\tilde{x})$; $d(x,\tilde{x})$ killed by $\Delta = \partial_i \tilde{\partial}^i$

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$$N - \bar{N} = p_i w^i$$

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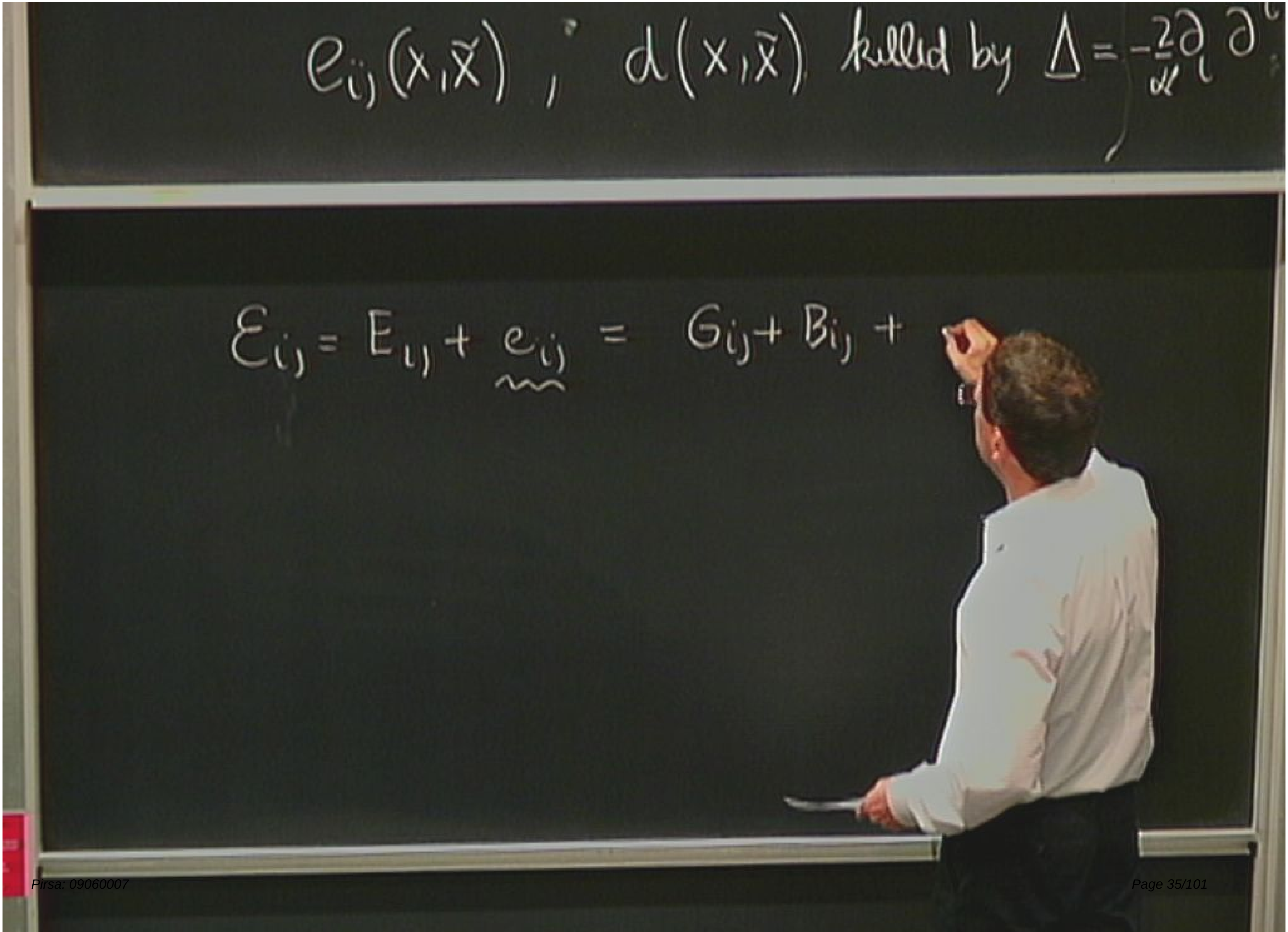
$e_{ij}(x,\tilde{x})$; $d(x,\tilde{x})$ killed by $\Delta = -\frac{2}{\alpha'} \partial_i \tilde{\partial}^i$


$e_{ij}(x,\vec{x})$; $d(x,\vec{x})$ killed by $\Delta = -\frac{2}{\alpha}\partial_i\partial^i$
$\mathcal{E}_{ij} = E_{ij} + e_{ij} = G_{ij} + B_{ij} +$

$e_{ij}(x,\vec{x})$; $d(x,\vec{x})$ killed by $\Delta = -\frac{2}{\alpha}\partial_i \partial^i$

$$\mathcal{E}_{ij} = E_{ij} + \underset{\sim\sim}{e_{ij}} = G_{ij} + \underline{\underline{B_{ij}}} + h_{ij} + \underline{\underline{b_{ij}}}$$

$e_{ij}(x, \vec{x})$, $d(x, \vec{x})$ killed by $\Delta = -\frac{2}{\alpha}\partial_i\partial$

$$\mathcal{E}_{ij} = E_{ij} + e_{ij} = G_{ij} + \underline{\underline{B_{ij}}} + h_{ij} + \underline{\underline{b_{ij}}}$$

2. $\int \sqrt{-g}\, R \qquad g_{ij} = \eta_{ij} + h_{ij}$

$$S_0^{(2)} = \int dx \Big[\tfrac{1}{4} h^{ij} \partial^2 h_{ij} - \tfrac{1}{4} h \partial^2 h + \tfrac{1}{2} (\partial^i h_{ij})^2 + \tfrac{1}{2} h \partial_i \partial_j h^{ij} \Big]$$

$$= \int dx \, R(\partial)$$

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$$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$$

$$\tilde{\delta} h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \qquad h_{ij}, \epsilon_i, \tilde{\epsilon} \quad (x, \tilde{x})$$

$$S = \int dx\, d\tilde{x} \left[\, R(\partial) + R(\tilde{\partial}) \right]$$

$$S = \int dx\, d\tilde{x} \left[R(\partial) + R(\tilde{\partial}) \right]$$

$$\tilde{\delta} S = \int dx\, d\tilde{x} \left[\tilde{\delta} R(\partial) \right]$$

$$= \int dx\, d\tilde{x} \Big[(\tilde{\partial}_j h^{ij}) \partial^k \left(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i \right)$$

$$+ (\partial_i \partial_j h^{ij} - \partial^2 h)\, \tilde{\partial} \cdot \tilde{\epsilon}$$

$$+ (\partial^i h_{ij} - \partial_j h)(\partial \cdot \tilde{\partial}) \tilde{\epsilon}^j \Big]$$

$$S = \int dx\, d\tilde{x} \left[R(\partial) + R(\tilde{\partial}) \right]$$

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$$+ \left(\partial^j h_{ij} - \partial_j h \right) (\partial \cdot \tilde{\partial}) \tilde{\epsilon}^i \Big]$$

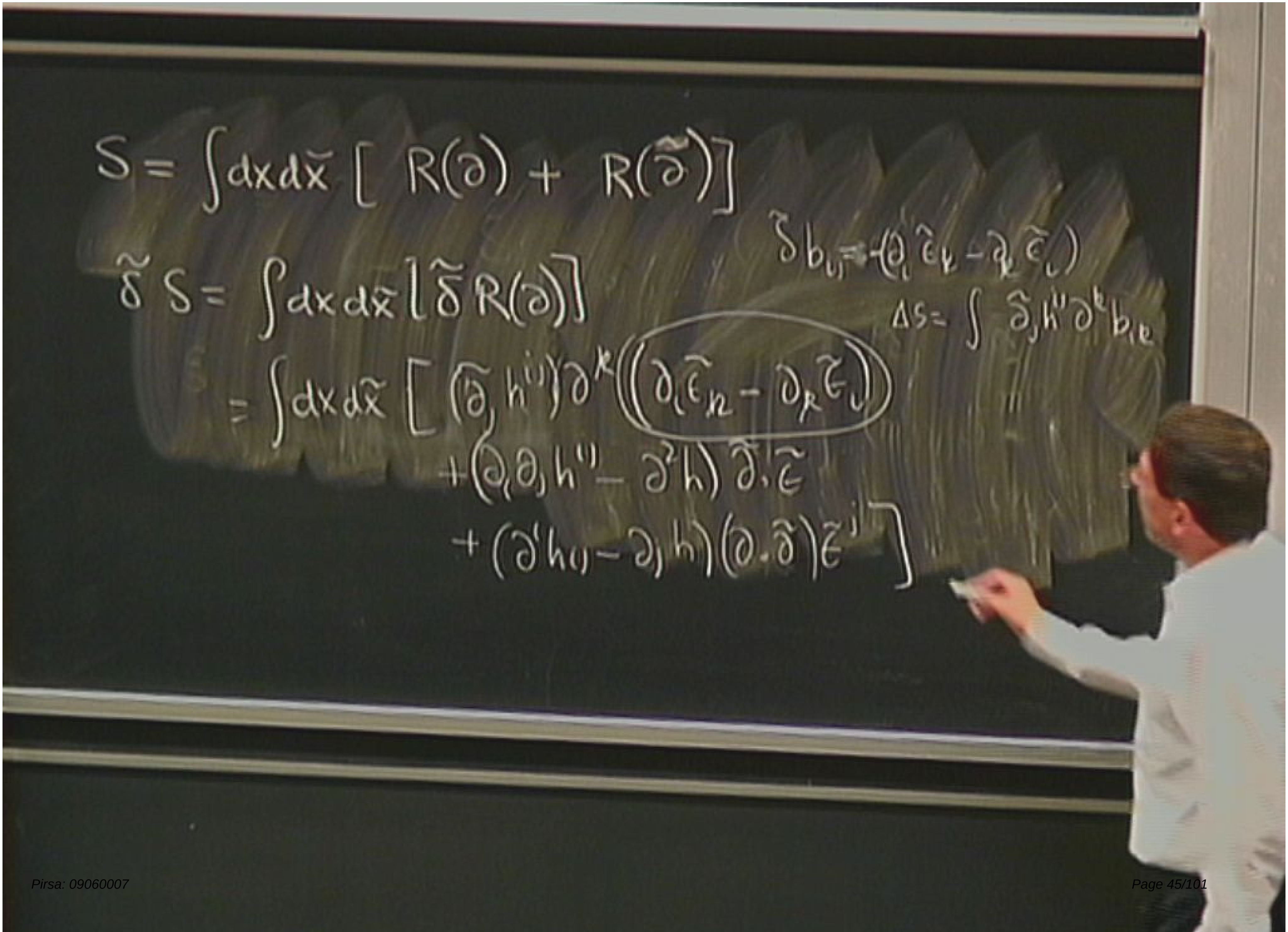
$$S = \int dx\, d\tilde{x}\, \left[R(\partial) + R(\tilde{\partial}) \right]$$

$$\delta b_{ik} = -(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$$

$$\tilde{\delta} S = \int dx\, d\tilde{x}\, \left[\tilde{\delta} R(\partial) \right]$$

$$\Delta S = \int \tilde{\partial}_j h^{ij} \partial^k b_{ik}$$

$$= \int dx\, d\tilde{x}\, \Big[(\tilde{\partial}_j h^{ij}) \partial^k \big(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i \big) + (\partial_i \partial_j h^{ij} - \partial^2 h)\, \tilde{\partial} \cdot \tilde{\epsilon} + (\partial^i h_{ij} - \partial_j h)(\partial \cdot \tilde{\partial}) \tilde{\epsilon}^j \Big]$$


$S = \int dx\, d\tilde{x}\, [\, R(\partial) + R(\tilde{\partial}) \,]$
$\delta b_{ik} = -(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$
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$\Delta S = \int \tilde{\partial}_j h^{ij} \partial^k b_{ik}$
$= \int dx\, d\tilde{x}\, \Big[(\tilde{\partial}_j h^{ij}) \partial^k (\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$
$+ (\partial_i \partial_j h^{ij} - \partial^2 h)\, \tilde{\partial} \cdot \tilde{\epsilon}$
$+ (\partial^i h_{ij} - \partial_j h)(\partial \cdot \tilde{\partial}) \tilde{\epsilon}^j \Big]$

$$S = \int dx\,d\tilde{x}\,\left[\, R(\partial) + R(\tilde{\partial})\right]$$

$$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$$

$$\tilde{\delta} S = \int dx\,d\tilde{x}\,[\tilde{\delta} R(\partial)]$$

$$\Delta S = \int \tilde{\partial}_j h^{ij} \partial^k b_{ik}$$

$$= \int dx\,d\tilde{x}\,\Big[(\tilde{\partial}_j h^{ij})\partial^k (\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$$

$$\delta d = -\tilde{\partial}\cdot\tilde{\epsilon}$$

$$+ (\partial_i \partial_j h^{ij} - \partial^2 h)(\tilde{\partial}\cdot\tilde{\epsilon})$$

$$+ (\partial^i h_{ij} - \partial_j h)(\partial\cdot\tilde{\partial})\tilde{\epsilon}^j \Big]$$

$$S = \int dx\, d\tilde{x} \, [\, R(\partial) + R(\tilde{\partial})\,]$$

$$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$$

$$\tilde{\delta} S = \int dx\, d\tilde{x} \, [\tilde{\delta} R(\partial)]$$

$$\Delta S = \int \tilde{\partial}_j h^{ij} \partial^k b_{ik}$$

$$= \int dx\, d\tilde{x} \, \Big[(\tilde{\partial}_j h^{ij}) \partial^k \big((\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i) \big)$$

$$\delta d = -\tilde{\partial} \cdot \tilde{\epsilon}$$

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$$+ (\partial^i h_{ij} - \partial_j h)(\underline{\partial \cdot \tilde{\partial}})\tilde{\epsilon}^j \Big]$$

$$S = \int dx\, d\tilde{x}\, [\, R(\partial) + R(\tilde{\partial})]$$

$$\tilde{\delta} S = \int dx\, d\tilde{x}\, [\tilde{\delta} R(\partial)]$$

$$= \int dx\, d\tilde{x}\, \Big[(\tilde{\partial}_j h^{ij})\, \partial^k \big(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i \big) + \big(\partial_i \partial_j h^{ij} - \partial^2 h\big) \big(\partial \cdot \tilde{\epsilon}\big) + \big(\partial^i h_{ij} - \partial_j h\big) \underline{\underline{(\partial \cdot \tilde{\partial})}}\, \tilde{\epsilon}^j \Big]$$

$$\tilde{\delta} b_{ij} = -(\partial_i \tilde{\epsilon}_k - \partial_k \tilde{\epsilon}_i)$$

$$\Delta S = \int \tilde{\partial}_j h^{ij} \partial^k b_{ik}$$

$$\delta d = -\tilde{\partial} \cdot \tilde{\epsilon}$$

(3)

$$S = \int dx\, d\tilde{x}\, \Big[\, R(\partial) + R(\tilde{\partial}) + \tfrac{1}{2} b_{ij}\, \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 \Big)$$

H^2

(3)

$$S = \int dx\, d\tilde{x}\, \Big[\, R(\partial) + R(\tilde{\partial})$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4\, d\, \partial^i \tilde{\partial}^j b_{ij}$$

③

$$S = \int dx\, d\tilde{x}\, \Big[\, R(\partial, d) + R(\tilde{\partial}, d)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})\Big)$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4\, d\, \partial^i \tilde{\partial}^j b_{ij}$$

$$S = \int dx\, d\tilde{x} \, [\, R(\partial, h) + R(\tilde{\partial}, h)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4\, d\, \partial^i \tilde{\partial}^j b_{ij} \,]$$

$$S = \int dx\, d\tilde{x}\, \Big[R(\partial, \mathcal{H}) + R(\tilde{\partial}, \mathcal{H})$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \Big]$$

δh_{ij}

$\delta b_{ij} =$

δd

$$S = \int dx\, d\tilde{x} \, \Big[R(\partial, h) + R(\tilde{\partial}, h)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j h_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \Big]$$

δh_{ij}

$\delta b_{ij} = -(\partial_i \tilde{\varepsilon}_j - \partial_j \tilde{\varepsilon}_i)$

δd

$$S = \int dx\, d\tilde{x}\, \Big[R(\partial, h) + R(\tilde{\partial}, h)$$

$$+ \frac{1}{2} b_{ij} \partial^2 b_{ij} + \frac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \Big]$$

$$\delta h_{ij} =$$

$$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$$

$$\delta d =$$

$$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon$$

$$S = \int dx\, d\tilde{x}\, \Big[R(\partial, h) + R(\tilde{\partial}, h)$$

$$+ \tfrac{1}{2} b_{ij}\partial^2 b_{ij} + \tfrac{1}{2}(\partial^j h_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik}\, \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4\, d\, \partial^i \tilde{\partial}^j b_{ij} \Big]$$

$\delta h_{ij} =$

$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$

$\delta d =$

$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$

$\delta b_{ij} =$

$\delta d = -\tfrac{1}{2} \partial \cdot \epsilon$

$$S = \int dx\, d\tilde{x} \,\Big[\, R(\partial, h) + R(\tilde{\partial}, h)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

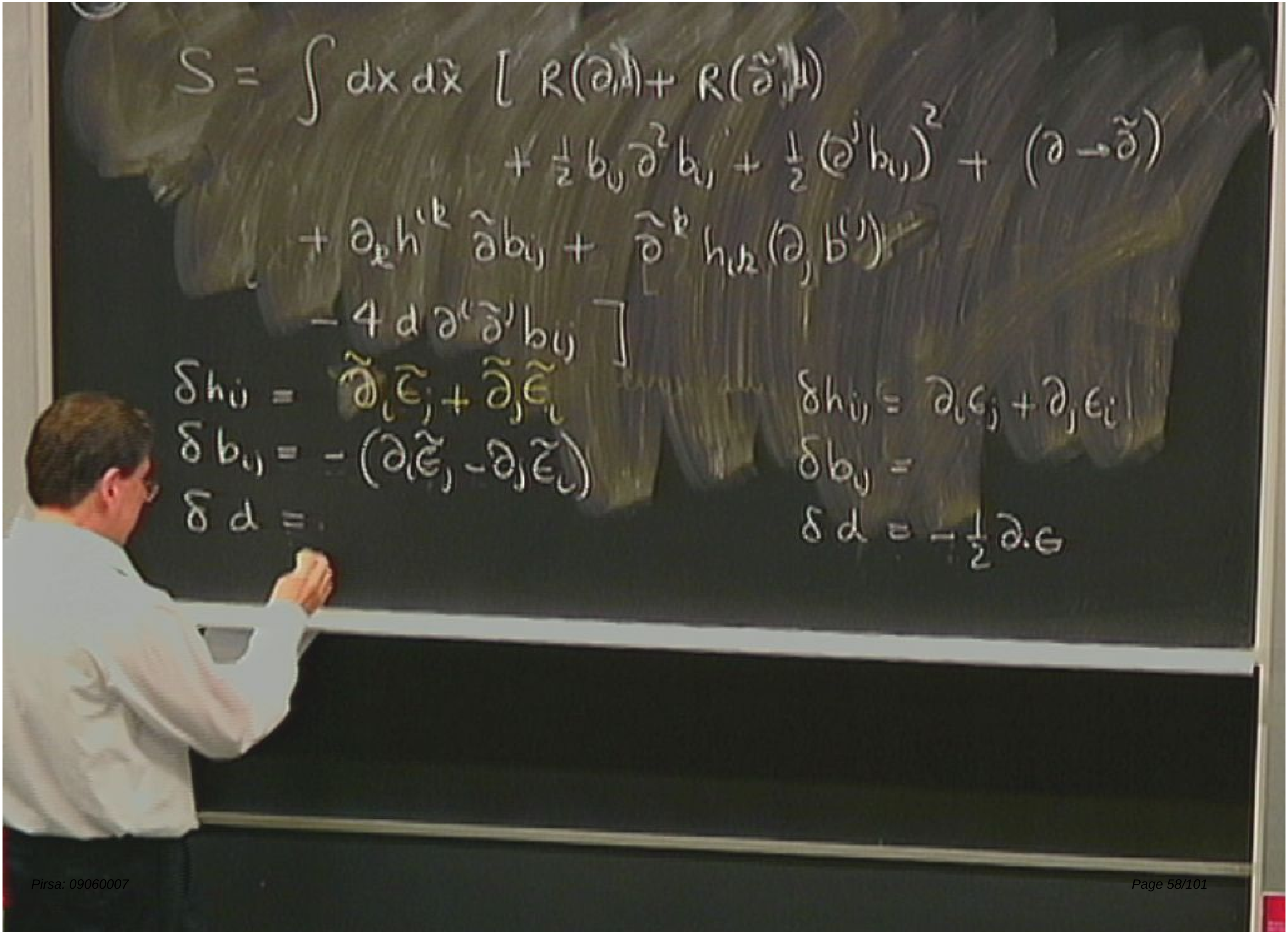
$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \,\Big]$$

$$\delta h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \qquad \delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$$

$$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i) \qquad \delta b_{ij} =$$

$$\delta d = \qquad \delta d = -\tfrac{1}{2} \partial \cdot \epsilon$$


$S = \int dx\, d\tilde{x}\, [\, R(\partial) + R(\tilde{\partial})$
$+ \frac{1}{2} b_{ij} \partial^2 b_{ij} + \frac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$
$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$
$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \,]$
$\delta h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i$
$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$
$\delta d =$
$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$
$\delta b_{ij} =$
$\delta d = -\frac{1}{2} \partial \cdot \epsilon$

$$S = \int dx\, d\tilde{x}\ \Big[\ R(\partial,) + R(\tilde{\partial},)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

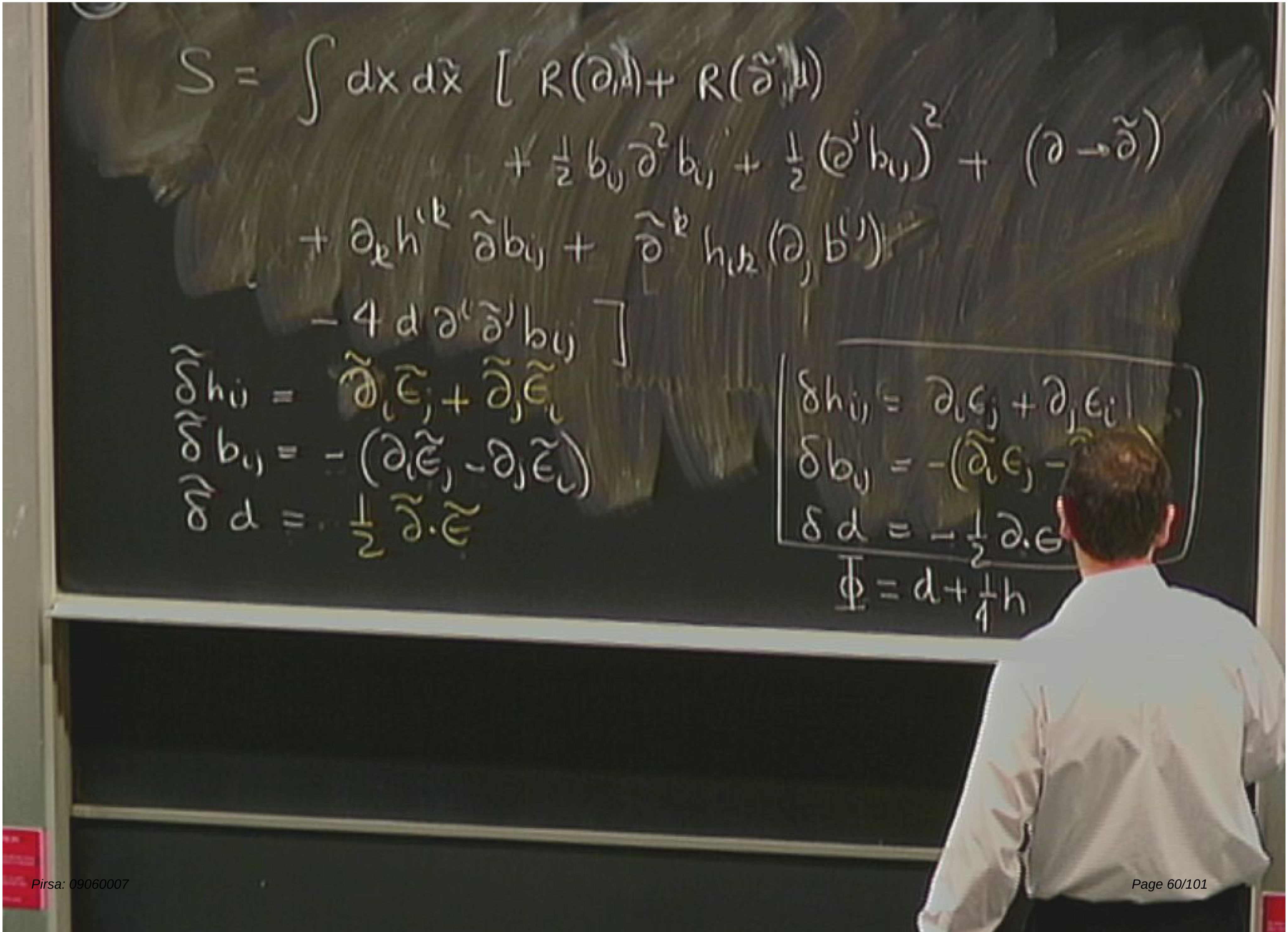
$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij}\ \Big]$$

$$\delta h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \qquad \delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$$

$$\delta b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i) \qquad \delta b_{ij} =$$

$$\delta d = \tfrac{1}{2} \tilde{\partial} \cdot \tilde{\epsilon} \qquad \delta d = -\tfrac{1}{2} \partial \cdot \epsilon$$


$S = \int dx\, d\tilde{x}\, [\, R(\partial, \ldots) + R(\tilde{\partial}, \ldots)$
$+ \frac{1}{2} b_{ij} \partial^2 b_{ij} + \frac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$
$+ \partial_k h^{ik} \tilde{\partial}^j b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$
$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \,]$
$\tilde{\delta} h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i$
$\tilde{\delta} b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$
$\tilde{\delta} d = \frac{1}{2} \tilde{\partial} \cdot \tilde{\epsilon}$
$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$
$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j -$
$\delta d = -\frac{1}{2} \partial \cdot \epsilon$
$\Phi = d + \frac{1}{4} h$

$$S = \int dx\, d\tilde{x} \, [\, R(\partial, d) + R(\tilde{\partial}, d)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j b_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d \partial^i \tilde{\partial}^j b_{ij} \,]$$

$$\tilde{\delta} h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i$$

$$\tilde{\delta} b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$$

$$\tilde{\delta} d = \tfrac{1}{2} \tilde{\partial} \cdot \tilde{\epsilon}$$

$$\boxed{\begin{aligned} \delta h_{ij} &= \partial_i \epsilon_j + \partial_j \epsilon_i \\ \delta b_{ij} &= -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) \\ \delta d &= -\tfrac{1}{2} \partial \cdot \epsilon \end{aligned}}$$

$$\Phi = d + \tfrac{1}{4} h \qquad \delta \Phi = 0$$

$$S = \int dx\, d\tilde{x} \Big[R(\partial, h) + R(\tilde{\partial}, h)$$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j h_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \Big]$$

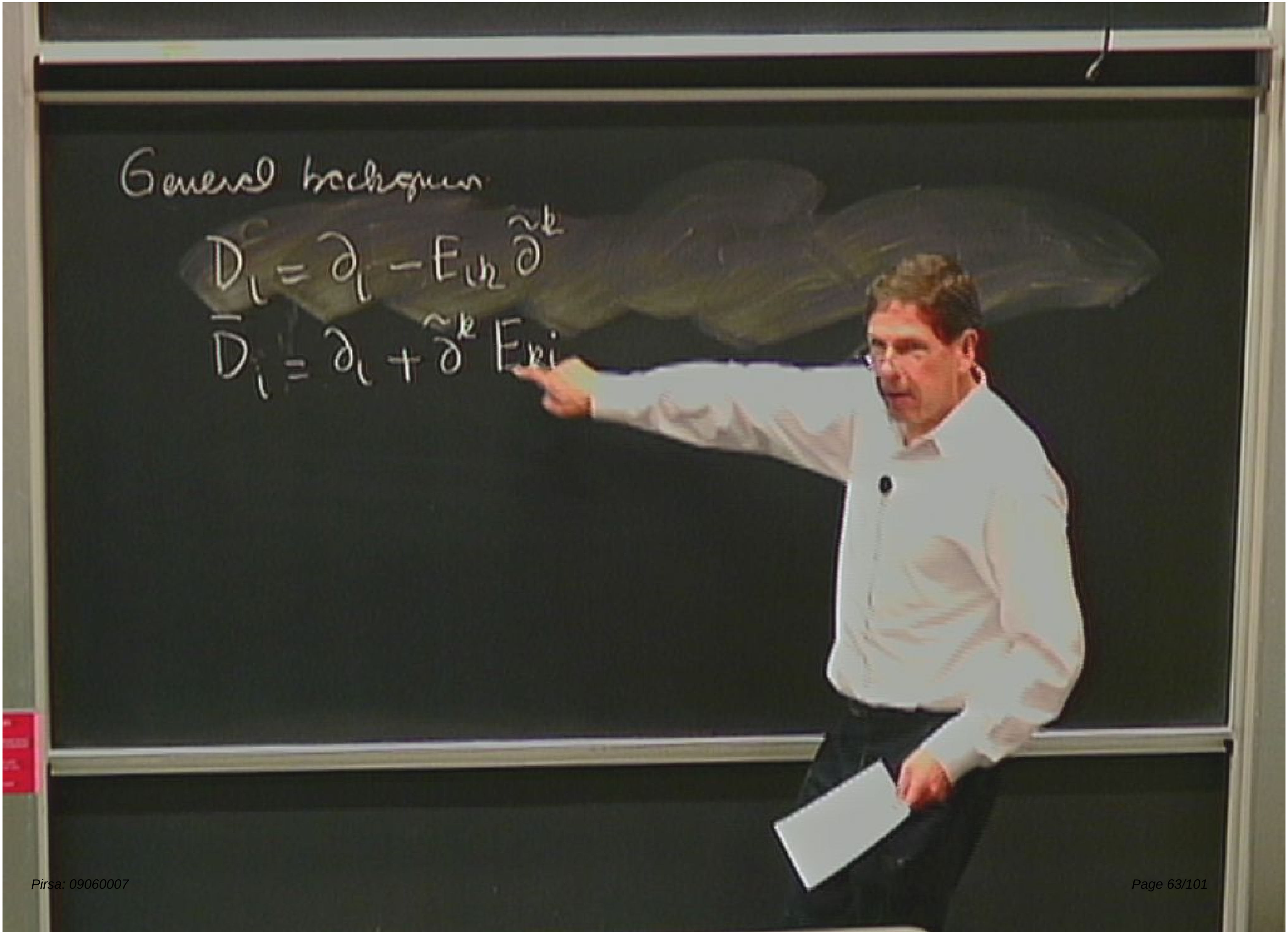
$$\tilde{\delta} h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i$$

$$\tilde{\delta} b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$$

$$\tilde{\delta} d = \tfrac{1}{2} \tilde{\partial} \cdot \tilde{\epsilon}$$

$$\boxed{\begin{aligned} \delta h_{ij} &= \partial_i \epsilon_j + \partial_j \epsilon_i \\ \delta b_{ij} &= -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) \\ \delta d &= -\tfrac{1}{2} \partial \cdot \epsilon \end{aligned}}$$

$$\tilde{\Phi} = d - \tfrac{1}{4} h, \quad \tilde{\delta} \tilde{\Phi} = 0 \; ; \quad \Phi = d + \tfrac{1}{4} h \quad \delta \Phi = 0$$


General technique
$D_i = \partial_i - E_{ik}\tilde{\partial}^k$
$\bar{D}_i = \partial_i + \tilde{\partial}^k E_{ki}$

General technique

$$D_i = \partial_i - E_{ik}\tilde{\partial}^k$$

$$\bar{D}_i = \partial_i + \tilde{\partial}^k E_{ki}$$

$G^{ij}\,,\ G_{ij}$

$$D^i = G^{ij} D_j$$

$$\bar{D}^i = G^{ij} \bar{D}_j$$

General techniques

G^{ij} , G_{ij}

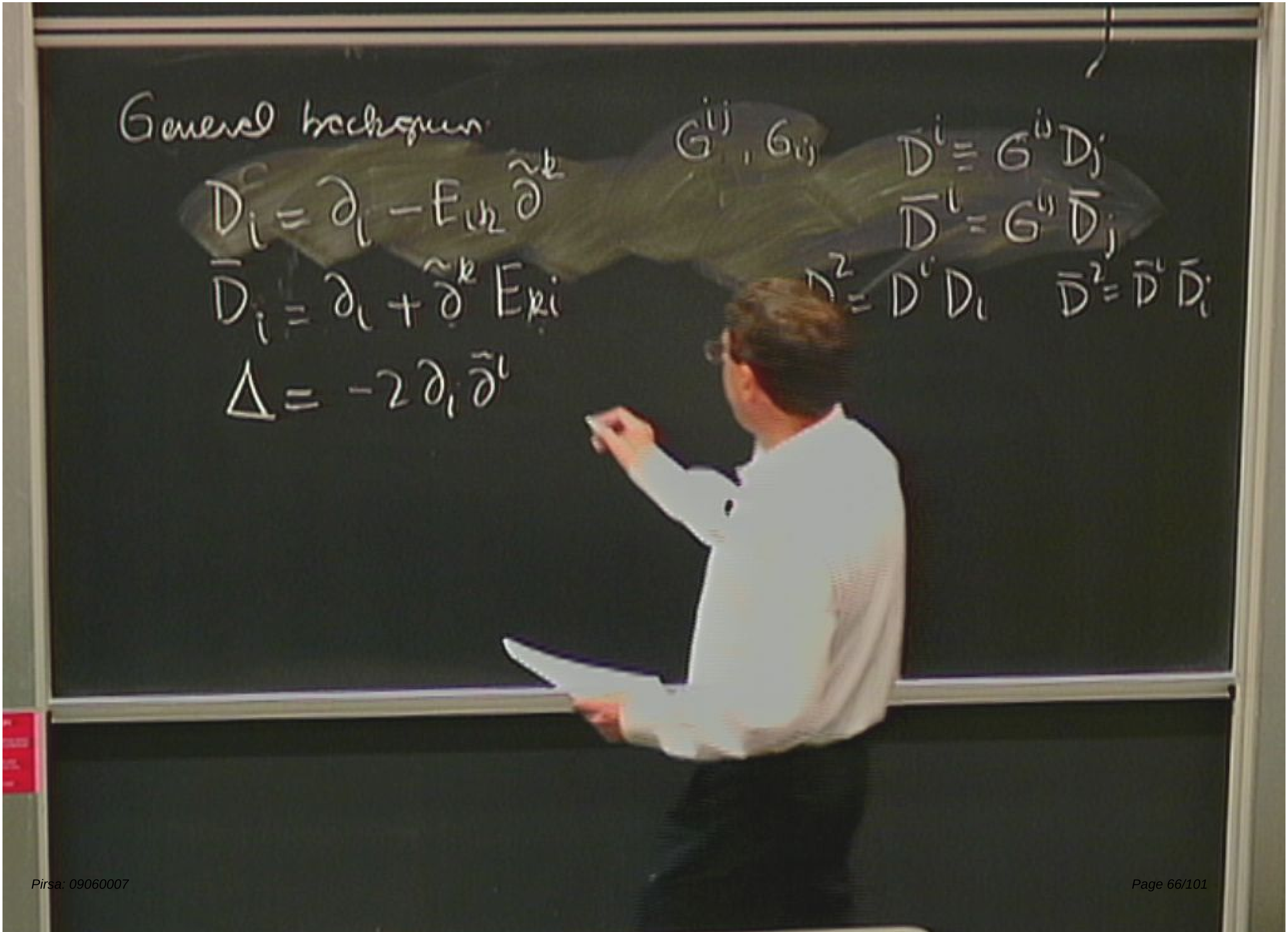
$$D_i = \partial_i - E_{ik}\tilde{\partial}^k$$

$$\bar{D}_i = \partial_i + \tilde{\partial}^k E_{ki}$$

$$D^i = G^{ij} D_j$$

$$\bar{D}^i = G^{ij} \bar{D}_j$$

$$D^2 = D^i D_i \qquad \bar{D}^2 = \bar{D}^i \bar{D}_i$$


General background
G^{ij}, G_{ij}
$D_i = \partial_i - E_{ik}\tilde{\partial}^k$
$\bar{D}_i = \partial_i + \tilde{\partial}^k E_{ki}$
$\Delta = -2\partial_i\tilde{\partial}^i$
$D^i = G^{ij}D_j$
$\bar{D}^i = G^{ij}\bar{D}_j$
$D^2 = D^i D_i$
$\bar{D}^2 = \bar{D}^i \bar{D}_i$

General background

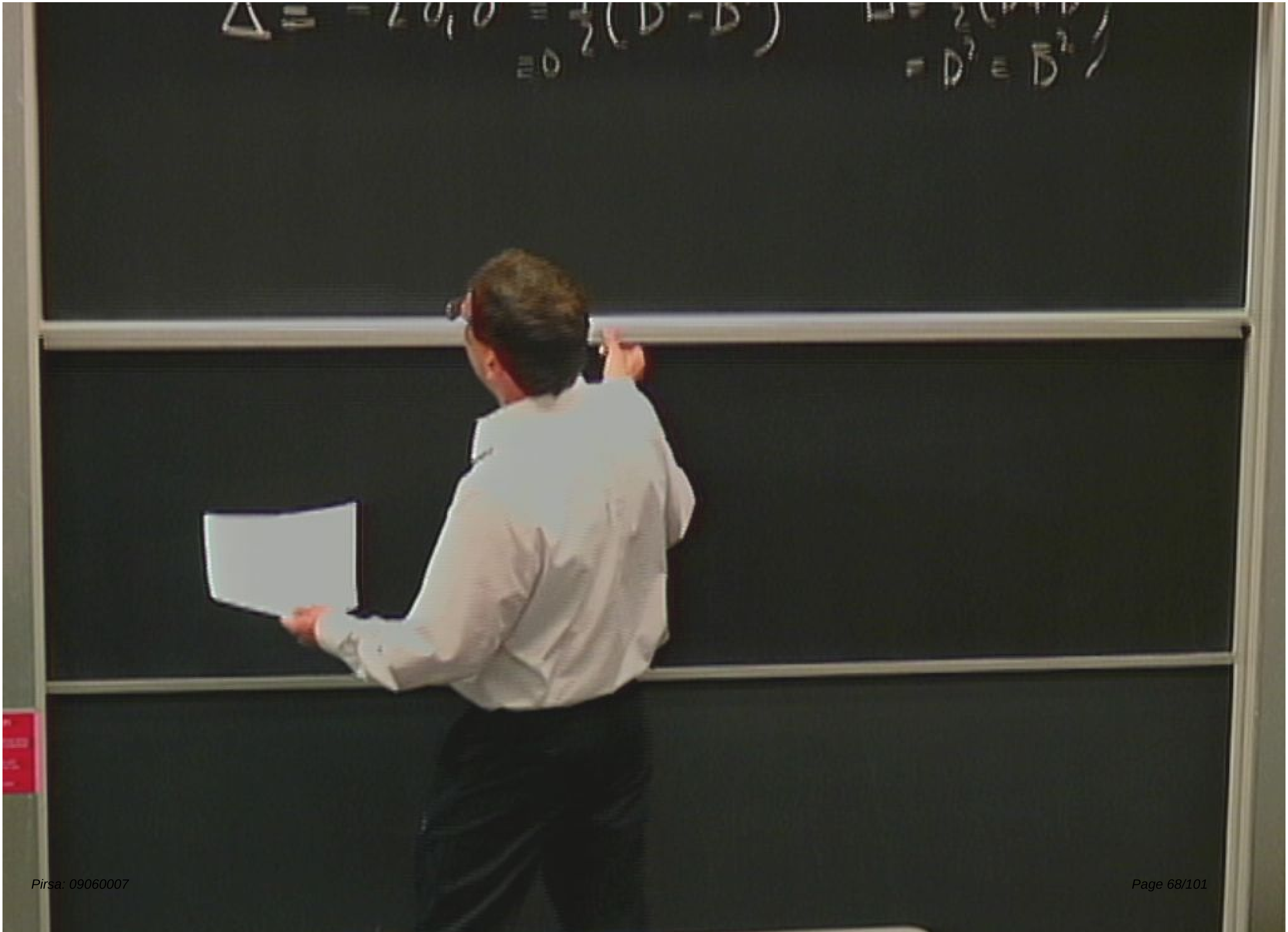
$$G^{ij}\,,\ G_{ij}$$

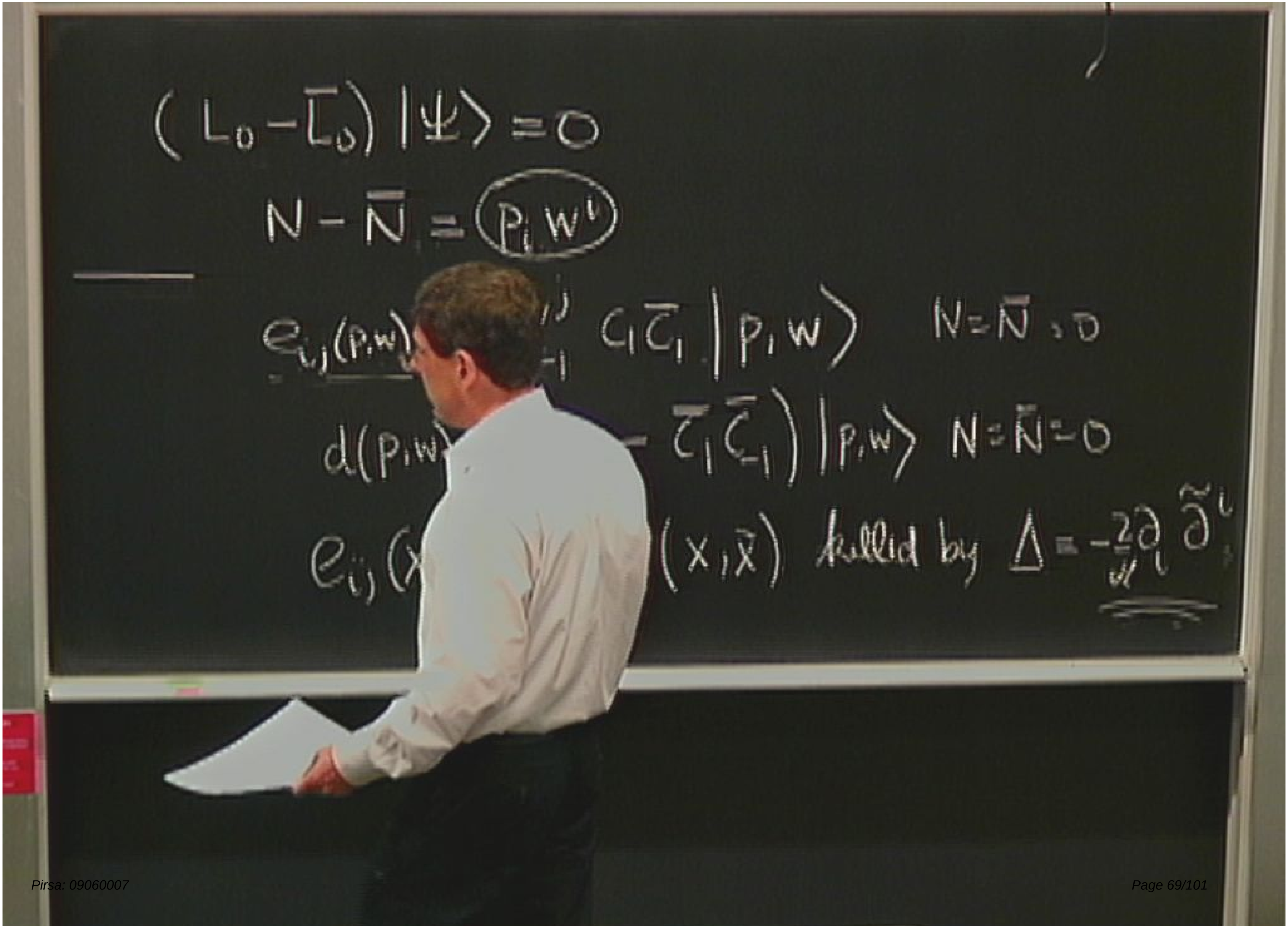
$$D_i = \partial_i - E_{ik}\tilde{\partial}^k \qquad D^i = G^{ij} D_j$$

$$\bar{D}_i = \partial_i + \tilde{\partial}^k E_{ki} \qquad \bar{D}^i = G^{ij} \bar{D}_j$$

$$D^2 = D^i D_i \qquad \bar{D}^2 = \bar{D}^i \bar{D}_i$$

$$\Delta = -2\,\partial_i \tilde{\partial}^i = \tfrac{1}{2}\left(D^2 - \bar{D}^2\right) \qquad \square = \tfrac{1}{2}\left(D^2 + \bar{D}^2\right)$$





$(L_0 - \bar{L}_0)\,|\Psi\rangle = 0$
$N - \bar{N} = p_i w^i$
$e_{ij}(p,w)$ $c_1 \bar{c}_1\,|p,w\rangle$ $N = \bar{N} = 0$
$d(p,w)$ $\bar{c}_1 \bar{c}_1)\,|p,w\rangle$ $N = \bar{N} = 0$
$e_{ij}(x$ $(x,\bar{x})$ killed by $\Delta = -\frac{2}{\alpha'}\partial_i \tilde{\partial}^i$

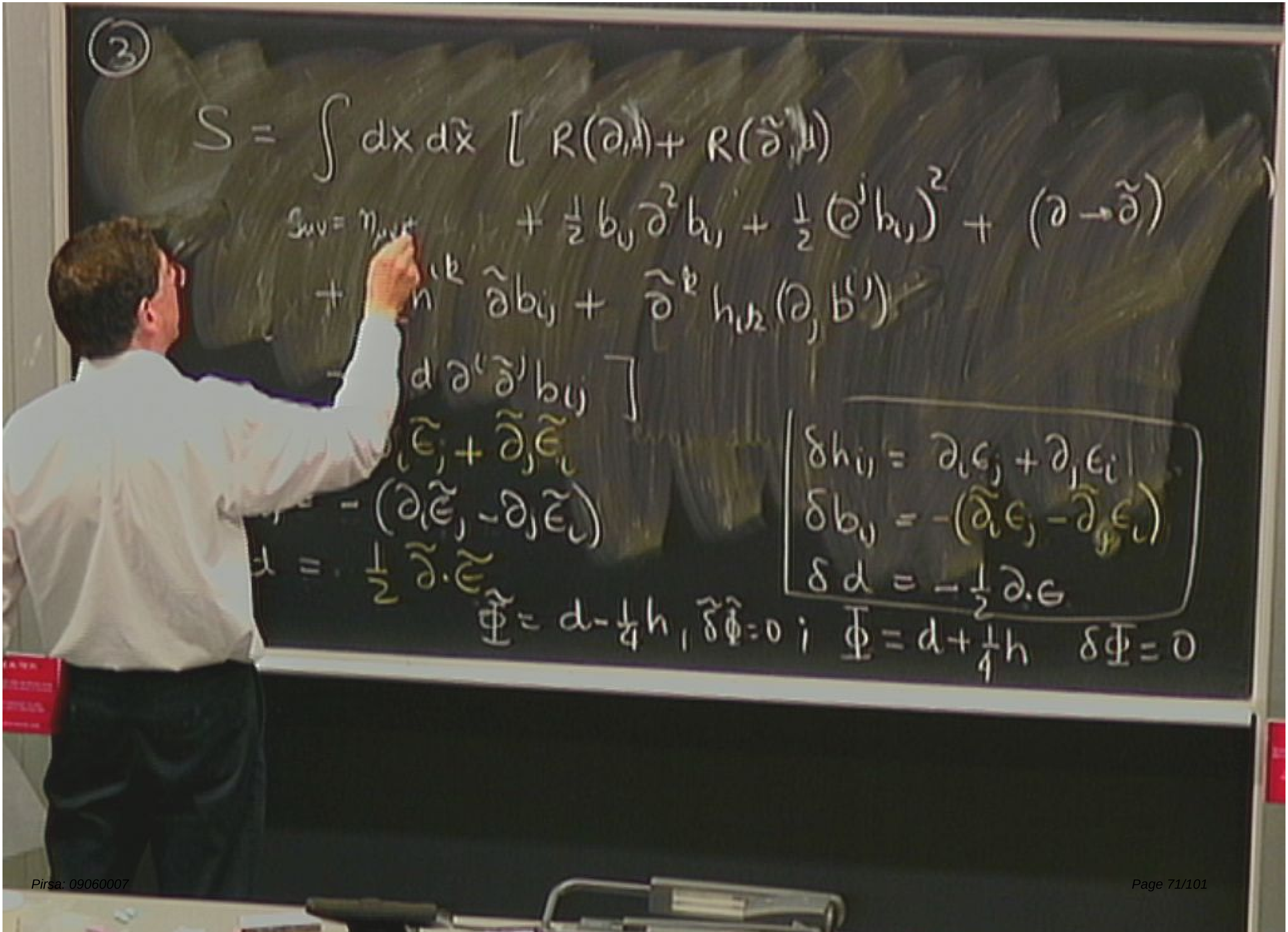
$$(L_0 - \bar{L}_0)\,|\Psi\rangle = 0$$

$$N - \bar{N} = p_i w^i$$

$$e_{ij}(p,w)\,\alpha^i_{-1}\,\bar{\alpha}^j_{-1}\,c_1\bar{c}_1\,|p,w\rangle \qquad N = \bar{N} = 0$$

$$d(p,w)\left(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1}\right)|p,w\rangle \qquad N = \bar{N} = 0$$

$$e_{ij}(x,\tilde{x}) \; ; \; d(x,\tilde{x}) \text{ killed by } \Delta = -\frac{2}{\alpha'}\partial_i \tilde{\partial}^i$$



3
$S = \int dx\, d\tilde{x}\, [\, R(\partial, h) + R(\tilde{\partial}, h)$
$g_{\mu\nu} = \eta_{\mu\nu} + \quad + \frac{1}{2} b_{ij} \partial^2 b_{ij} + \frac{1}{2} (\partial^j h_{ij})^2 + (\partial \to \tilde{\partial})$
$+ \quad h^{ik}\, \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$
$d\, \partial^i \tilde{\partial}^j b_{ij}\ ]$
$\partial_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i$
$- (\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$
$d = \frac{1}{2} \tilde{\partial} \cdot \tilde{\epsilon}$
$\delta h_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i$
$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i)$
$\delta d = -\frac{1}{2} \partial \cdot \epsilon$
$\tilde{\Phi} = d - \frac{1}{4} h,\ \tilde{\delta} \tilde{\Phi} = 0\ ;\ \Phi = d + \frac{1}{4} h \quad \delta \Phi = 0$

③

$$S = \int dx\, d\tilde{x}\, [\, R(\partial, \mathcal{H}) + R(\tilde{\partial}, \mathcal{H})$$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$+ \tfrac{1}{2} b_{ij} \partial^2 b_{ij} + \tfrac{1}{2} (\partial^j h_{ij})^2 + (\partial \to \tilde{\partial})$$

$$+ \partial_k h^{ik} \tilde{\partial} b_{ij} + \tilde{\partial}^k h_{ik} (\partial_j b^{ij})$$

$$- 4 d\, \partial^i \tilde{\partial}^j b_{ij} \,]$$

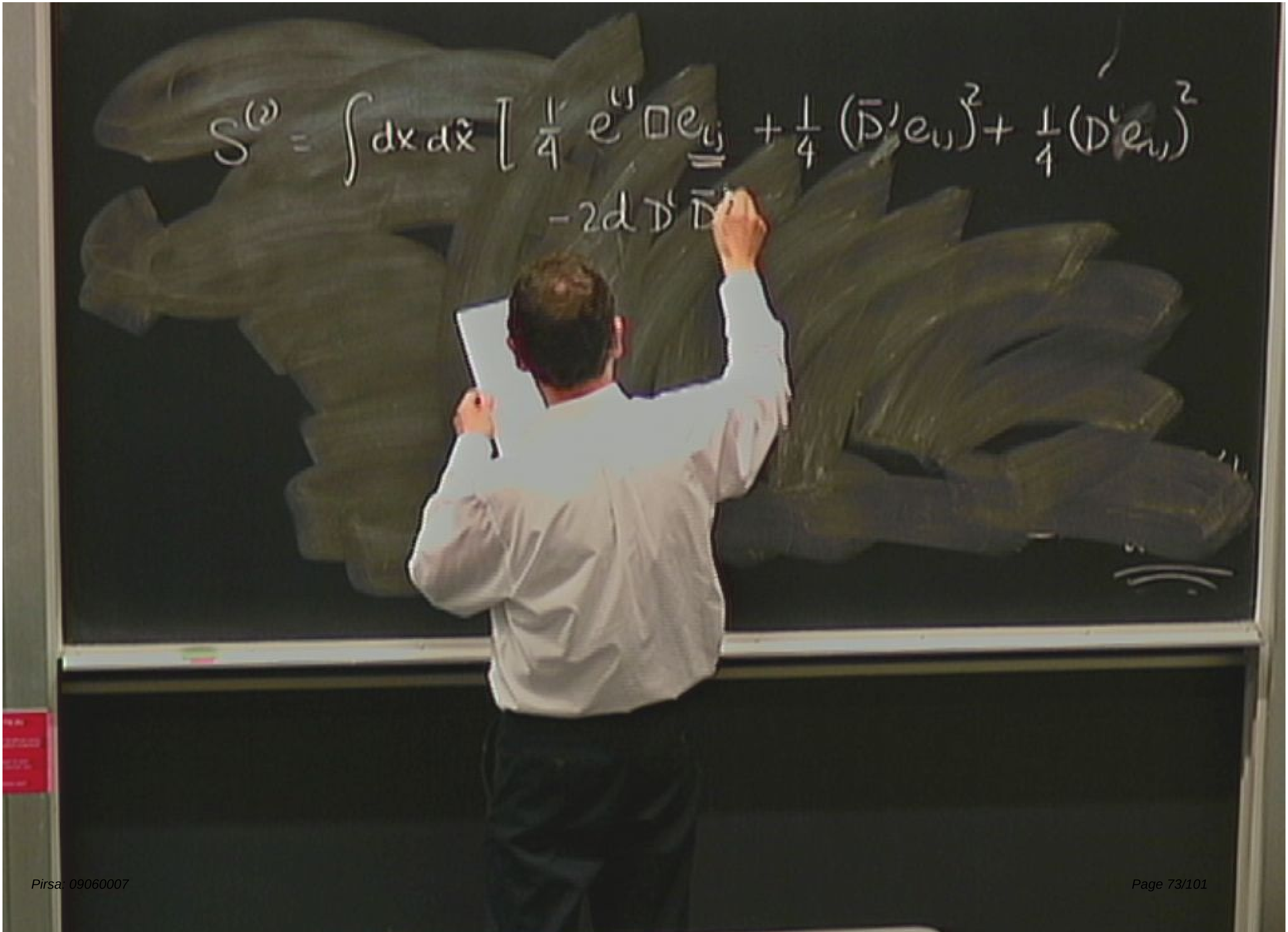
$$\tilde{\delta} h_{ij} = \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i$$

$$\tilde{\delta} b_{ij} = -(\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i)$$

$$\tilde{\delta} d = \tfrac{1}{2} \tilde{\partial} \cdot \tilde{\epsilon}$$

$$\boxed{\begin{aligned} \delta h_{ij} &= \partial_i \epsilon_j + \partial_j \epsilon_i \\ \delta b_{ij} &= -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) \\ \delta d &= -\tfrac{1}{2} \partial \cdot \epsilon \end{aligned}}$$

$$\tilde{\Phi} = d - \tfrac{1}{4} h, \quad \tilde{\delta} \tilde{\Phi} = 0 \; ; \quad \Phi = d + \tfrac{1}{4} h \quad \delta \Phi = 0$$


$S^{(2)} = \int dx\, d\tilde{x} \left[\frac{1}{4} e^{ij} \Box e_{ij} + \frac{1}{4} (\bar{D}^j e_{ij})^2 + \frac{1}{4} (D^i e_{ij})^2 \right.$
$-2d\, D^i \bar{D}^j$

$$S^{(2)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e^{ij} \Box e_{ij} + \frac{1}{4} (\bar{D}^j e_{ij})^2 + \frac{1}{4} (D^i e_{ij})^2 - 2 d\, D^i \bar{D}^j e_{ij} - 4\, d \Box d \Big]$$

$$S^{(2)} = \int dx\, d\tilde{x} \left[\frac{1}{4} e^{ij} \Box e_{ij} + \frac{1}{4} (\bar{D}^j e_{ij})^2 + \frac{1}{4} (D^i e_{ij})^2 - 2 d D^i \bar{D}^j e_{ij} - 4\, d \Box d \right]$$

$$\delta_\lambda e_{ij} = \bar{D}_j \lambda_i \qquad \delta_{\bar\lambda} e_{ij} = D_i \bar{\lambda}_j$$

$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda \qquad \delta_{\bar\lambda} d = -\frac{1}{4} \bar{D} \cdot \bar{\lambda}$$

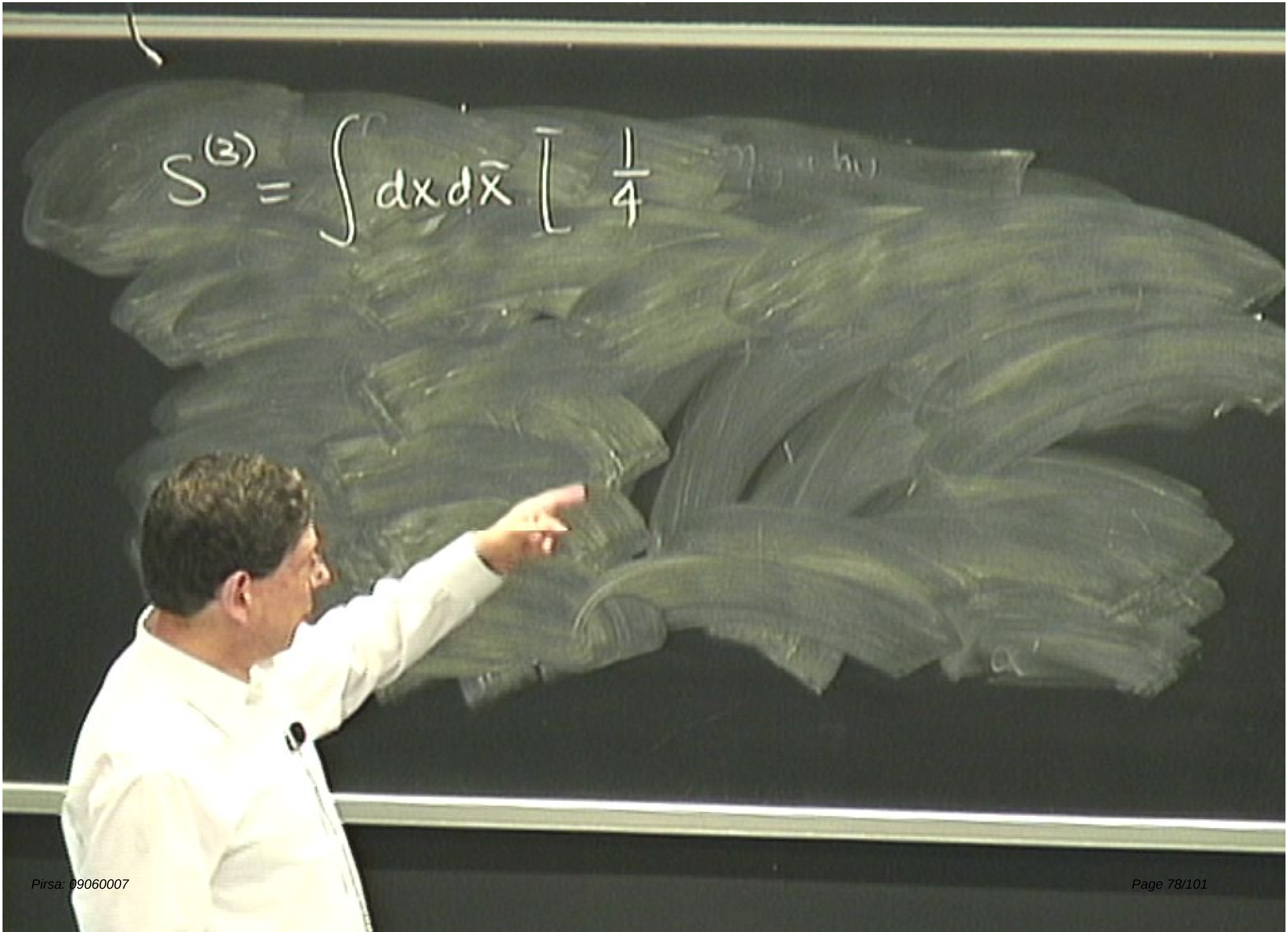
$$S^{(2)} = \int dx\, d\tilde{x} \left[\frac{1}{4} e^{ij} \Box e_{ij} + \frac{1}{4} (\bar{D}^j e_{ij})^2 + \frac{1}{4} (D^i e_{ij})^2 - 2 d D^i \bar{D}^j e_{ij} - 4 d \Box d \right]$$

$$\boxed{\begin{array}{ll} \delta_\lambda e_{ij} = \bar{D}_j \lambda_i & \delta_{\bar{\lambda}} e_{ij} = D_i \bar{\lambda}_j \\ \delta_\lambda d = -\frac{1}{4} D\cdot\lambda & \delta_{\bar{\lambda}} d = -\frac{1}{4} \bar{D}\cdot\bar{\lambda} \end{array}}$$

$$S^{(2)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e^{ij} \Box e_{ij} + \frac{1}{4} (\bar{D}^j e_{ij})^2 + \frac{1}{4} (D^i e_{ij})^2$$
$$- 2d\, D^i \bar{D}^j e_{ij} - 4\, d \Box d \Big]$$

$$\boxed{\begin{array}{ll} \delta_\lambda e_{ij} = \bar{D}_j \lambda_i & \delta_{\bar{\lambda}} e_{ij} = D_i \bar{\lambda}_j \\ \delta_\lambda d = -\frac{1}{4} D \cdot \lambda & \delta_{\bar{\lambda}} d = -\frac{1}{4} \bar{D} \cdot \bar{\lambda} \end{array}}$$

$\Delta e_{ij} = 0 \quad \Delta d = 0$


$S^{(3)} = \int dx\, d\tilde{x} \left[\frac{1}{4} \right.$

$$S^{(3)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e_{ij} \Big((D^i e_{kl})(\bar{D}^j e^{kl}) - (D^i e_{kl})(\bar{D}^l e^{kj}) - (\bar{D}^k e^{il})(\bar{D}^j e_{kl}) \Big)$$

$$S^{(3)} = \int dx\,d\tilde{x}\Big[\tfrac{1}{4}e_{ij}\Big((D^i e_{kl})(\bar{D}^j e^{kl}) - (D^i e_{kl})(\bar{D}^l e^{kj}) - (\bar{D}^k e^{il})(\bar{D}^j e_{kl})\Big)$$

$$+\tfrac{1}{2}d\big[(D^i e_{ij})^2 + \ldots$$

$$+\,4e_{ij}\,d\,D^i\bar{D}^j d + 4d^2\Box d\Big]$$

$$S^{(3)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e_{ij} \Big((D^i e_{kl})(\bar{D}^j e^{kl}) - (D^i e_{kl})(\bar{D}^l e^{kj}) - (\bar{D}^k e^{il})(\bar{D}^j e_{kl}) \Big)$$

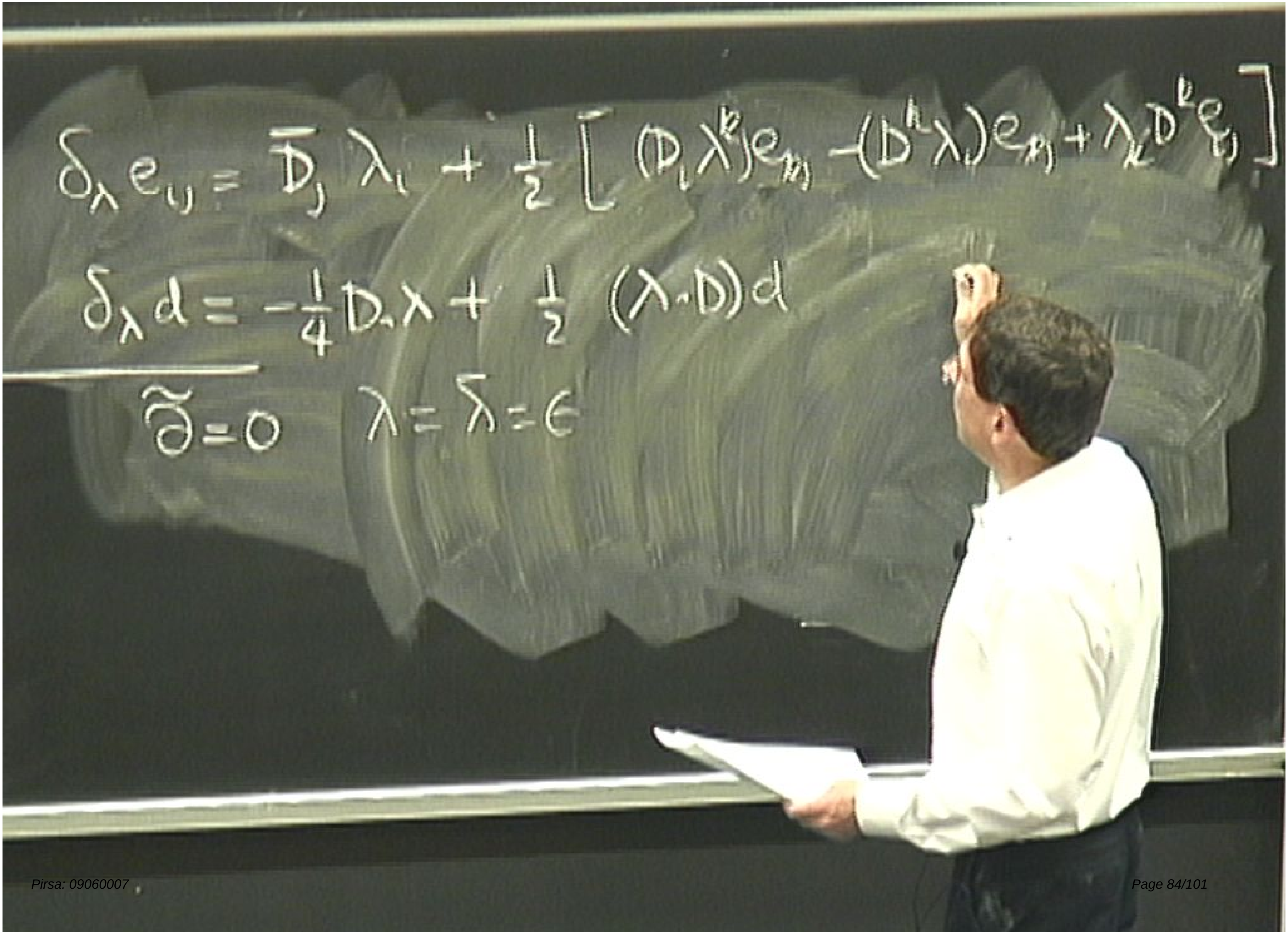
$$+ \frac{1}{2} d \big[(D^i e_{ij})^2 + \ldots$$

$$+ 4 e_{ij}\, d\, D^i \bar{D}^j d + 4 d^2 \Box d \Big]$$

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]$$

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]$$

$$\delta_\lambda d = -\frac{1}{4} D\cdot\lambda + \frac{1}{2} (\lambda\cdot D) d$$


$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2} \left[(D_i \lambda^k) e_{(kj)} - (D^k \lambda_i) e_{(kj)} + \lambda_k D^k e_{(ij)} \right]$
$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} (\lambda \cdot D) d$
$\tilde{\partial} = 0 \quad \lambda = \bar{\lambda} = \epsilon$

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]$$

$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} (\lambda \cdot D) d$$

$$\tilde{\partial} = 0 \qquad \lambda = \bar{\lambda} = \epsilon$$

$$\delta_\epsilon \Big(\underbrace{e_{ij} + \frac{1}{2} e_i{}^k e_{kj}}_{e^+_{(ij)}} \Big) = \partial_i \epsilon_j + \partial_j \epsilon_i + (\partial_i \epsilon^k) e^+_{kj} + (\partial_j \epsilon^k) e^+_{ik} + \epsilon^k \partial_k e^+_{ij}$$

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]$$

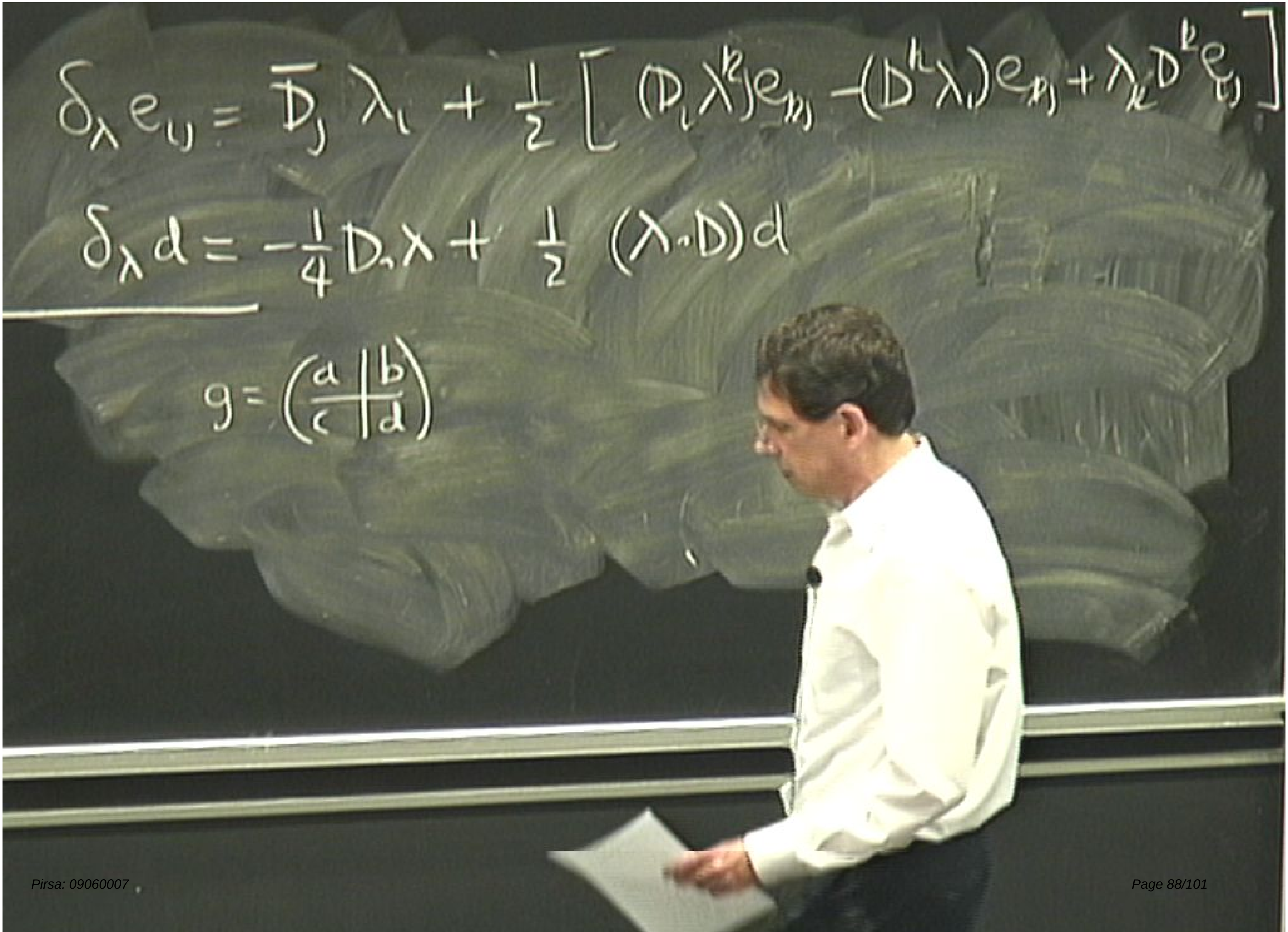
$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} (\lambda \cdot D) d$$

$$\tilde{\partial} = 0 \qquad \lambda = \bar{\lambda} = \epsilon$$

$$\delta_\epsilon \Big(\underbrace{e_{ij} + \frac{1}{2} e_i{}^k e_{kj}}_{e^+_{ij}} \Big) = \partial_i \epsilon_j + \partial_j \epsilon_i + (\partial_i \epsilon^k) e^+_{kj} + (\partial_j \epsilon^k) e^+_{ik} + \epsilon^k \partial_k e^+_{ij}$$

$\partial = 0 \qquad \tilde{\partial} \qquad \lambda = -\bar{\lambda} = \tilde{\varepsilon}$

$\delta_{\tilde{\varepsilon}}\left(e_{ij} - \frac{1}{2} e_{ik} e^{k}{}_{j} \right) =$ correct Lie derivative!



$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{(kj)} - (D^k \lambda_i) e_{(kj)} + \lambda_k D^k e_{(ij)} \right]$
$\delta_\lambda d = -\frac{1}{4} D\cdot\lambda + \frac{1}{2} (\lambda\cdot D) d$
$g = \left(\begin{array}{c|c} a & b \\ c & d \end{array}\right)$

$$\delta_\lambda e_{ij} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]$$

$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} (\lambda \cdot D) d$$

$$g = \left(\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right) \qquad E' = (aE + b)(cE + d)^{-1}$$

$$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix}$$

$$S(E, e_{ij}, d)$$

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]$$

$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} (\lambda \cdot D) d$$

$$g = \left(\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right) \qquad E' = (aE+b)(cE+d)^{-1}$$

$$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix} \qquad \begin{aligned} M &= d^t - E c^t \\ \bar{M} &= d^t + E^t c^t \end{aligned}$$

$$S(E; e_{ij}, d)$$

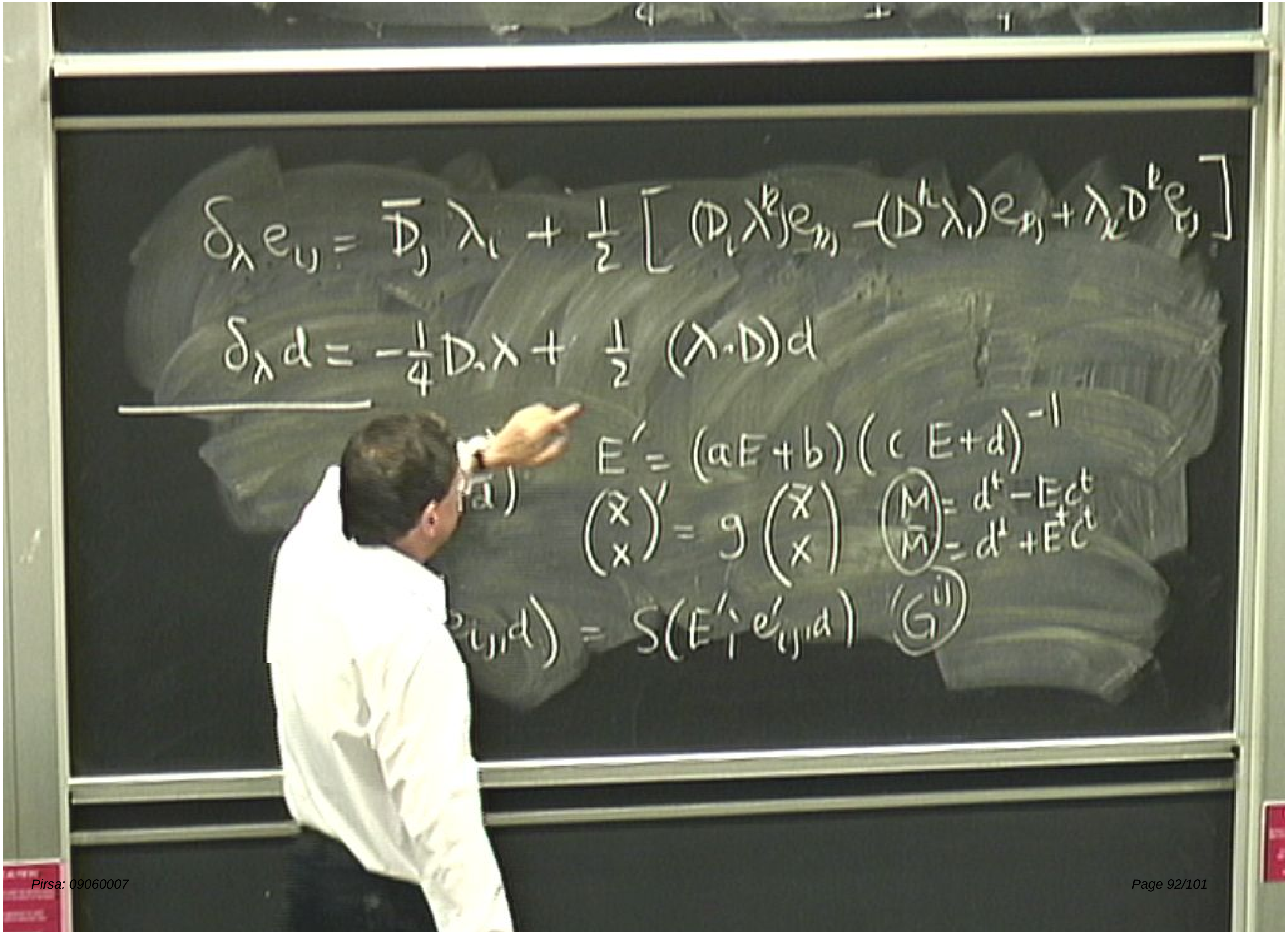
$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{(kj)} - (D^k \lambda_i) e_{(kj)} + \lambda_k D^k e_{(ij)} \right]$$

$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} (\lambda \cdot D) d$$

$$g = \left(\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right) \qquad E' = (aE + b)(cE + d)^{-1}$$

$$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix} \qquad M = d^t - E c^t \qquad \bar{M} = d^t + E^t c^t$$

$$S(E, e_{(ij)}, d) = S(E', e'_{(ij)}, d) \qquad (G^{ij})$$


$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{(kj)} - (D^k \lambda_i) e_{(kj)} + \lambda_k D^k e_{(ij)} \right]$
$\delta_\lambda d = -\frac{1}{4} D\cdot\lambda + \frac{1}{2}(\lambda\cdot D) d$
$E' = (aE+b)(cE+d)^{-1}$
$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix}$
$M = d^t - E c^t$
$\bar{M} = d^t + E^t c^t$
$= S(E'; e'_{(ij)}, d)$
G^{ij}

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2} [\![(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{(ij)}]\!]$$

$$\delta_\lambda d = -\frac{1}{4} D\cdot\lambda + \frac{1}{2} [\![(\lambda\cdot D) d]\!]$$

$$g = \left(\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right) \qquad E' = (aE+b)(cE+d)^{-1}$$

$$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix} \qquad M = d^t - E c^t$$

$$\bar{M} = d^t + E^t c^t$$

$$S(E; e_{(ij)}, d) = S(E'; e'_{(ij)}, d) \qquad G^{ij}$$

$$
\begin{aligned}
S^{(3)} = \int dx\, d\tilde{x} \Big[\tfrac{1}{4} e_{ij} \Big(& (D^i e_{kl})(\bar{D}^j e^{kl}) \\
& - (D^i e_{kl})(\bar{D}^l e^{kj}) \\
& - (\bar{D}^k e^{il})(\bar{D}^j e_{kl}) \Big) \\
& + \tfrac{1}{2} d \big[(D^i e_{ij})^2 + \cdots \\
& + 4 e_{ij}\, d\, D^i \bar{D}^j d + 4 d^2 \Box d \Big]
\end{aligned}
$$

$$\delta_\lambda e_{ij} = \bar{D}_j \lambda_i + \frac{1}{2}\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]\!]$$

$$\delta_\lambda d = -\frac{1}{4} D\cdot\lambda + \frac{1}{2}[\![(\lambda\cdot D) d]\!]$$

$$g = \left(\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right) \qquad E' = (aE + b)(cE + d)^{-1}$$

$$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix} \qquad \begin{aligned} M &= d^t - E c^t \\ \bar{M} &= d^t + E^t c^t \end{aligned}$$

$$S(E; e_{ij}, d) = S(E'; e'_{ij}, d) \qquad G^{ij}$$

$$\delta_\lambda e_{ij} = \bar{D}_j \lambda_i + \frac{1}{2} \left[\!\left[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e_{ij} \right]\!\right]$$

$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} \left[\!\left[(\lambda \cdot D) d \right]\!\right]$$

$$g = \left(\begin{array}{c|c} a & b \\ \hline c & d \end{array} \right) \qquad E' = (aE + b)(cE + d)^{-1}$$

$$\begin{pmatrix} \tilde{x} \\ x \end{pmatrix}' = g \begin{pmatrix} \tilde{x} \\ x \end{pmatrix} \qquad \begin{array}{l} M = d^t - E c^t \\ \bar{M} = d^t + E^t c^t \end{array}$$

$$S(E, e_{ij}, d) = S(E', e'_{ij}, d) \qquad (G^{ii})$$

$$S^{(3)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e_{ij} \Big((D^i e_{kl})(\bar{D}^j e^{kl}) - (D^i e_{kl})(\bar{D}^l e^{kj}) - (\bar{D}^k e^{il})(\bar{D}^j e_{kl}) \Big)$$

$$+ \frac{1}{2} d \Big[(D^i e_{ij})^2 + \cdots$$

$$+ 4 e_{ij}\, d\, D^i \bar{D}^j d + 4 d^2 \Box d \Big]$$

$$e_{ij}(x) \qquad e_{ij}(x, \tilde{x})$$

$$\delta_\lambda e_{(ij)} = \bar{D}_j \lambda_i + \frac{1}{2}\Big[(D_i \lambda^k) e_{kj} - (D^k \lambda_i) e_{kj} + \lambda_k D^k e \ldots$$

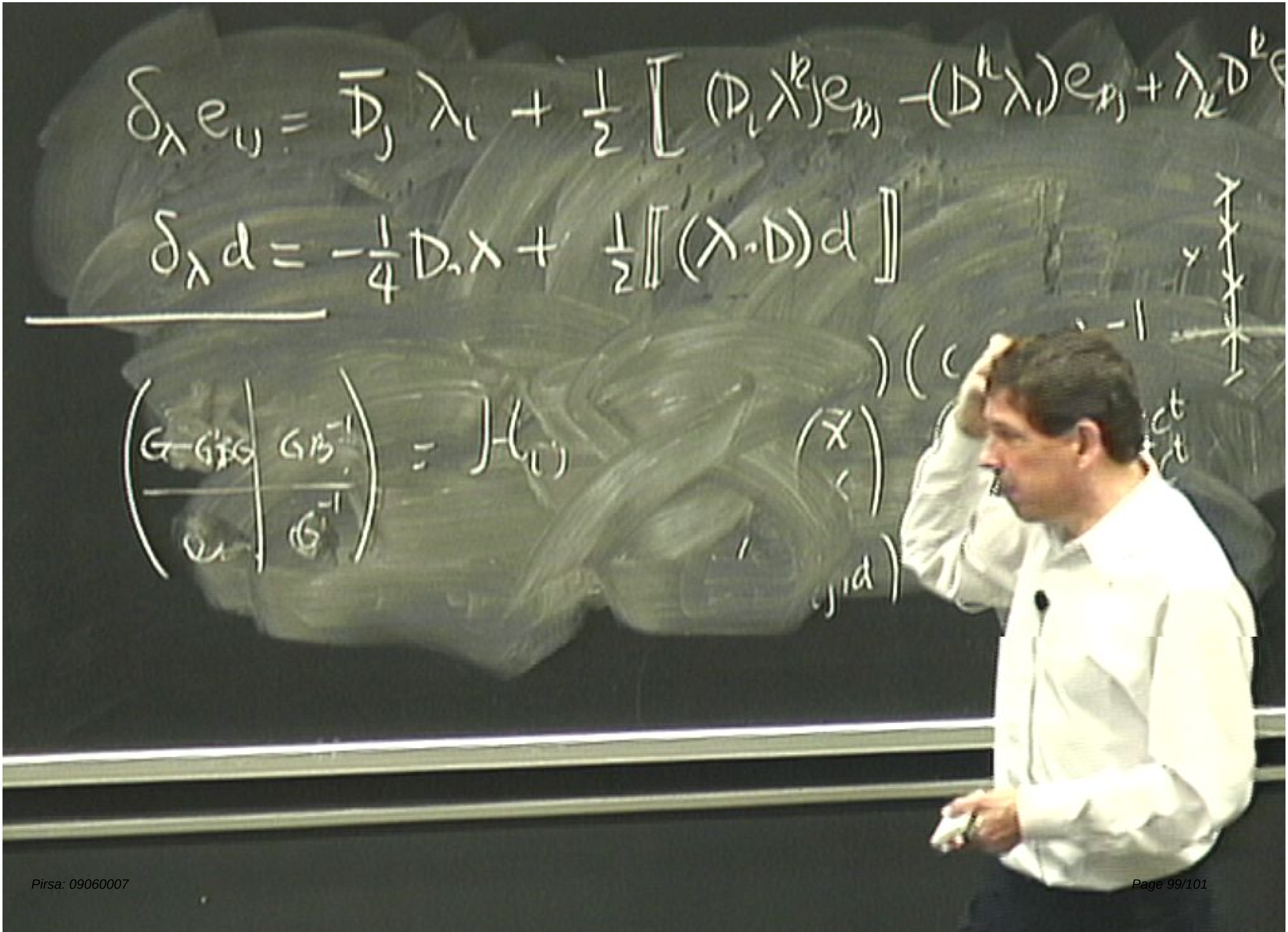
$$\delta_\lambda d = -\frac{1}{4} D \cdot \lambda + \frac{1}{2} [\![(\lambda \cdot D) d]\!]$$

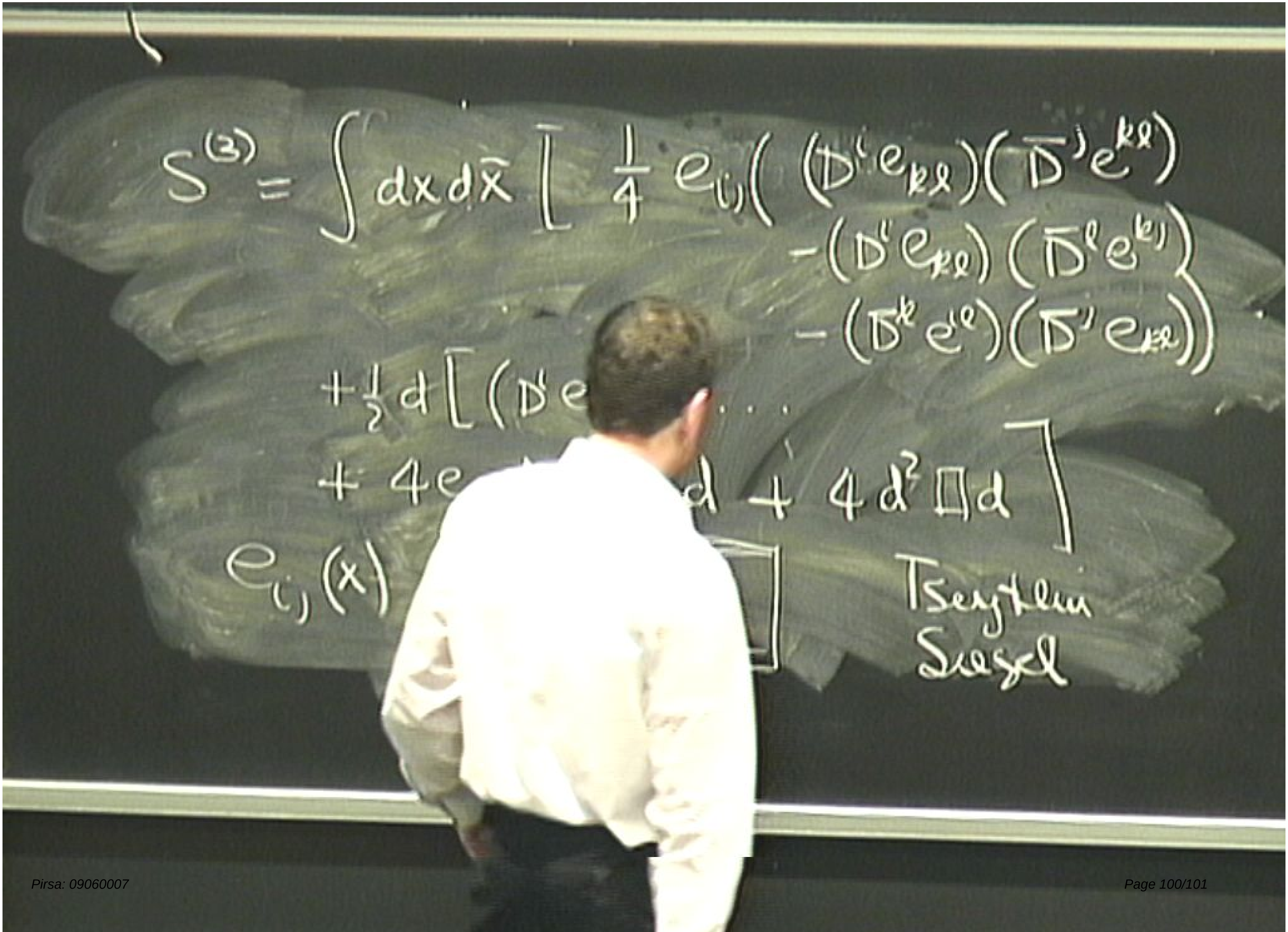
$$)\,(c E + d)^{-1}$$

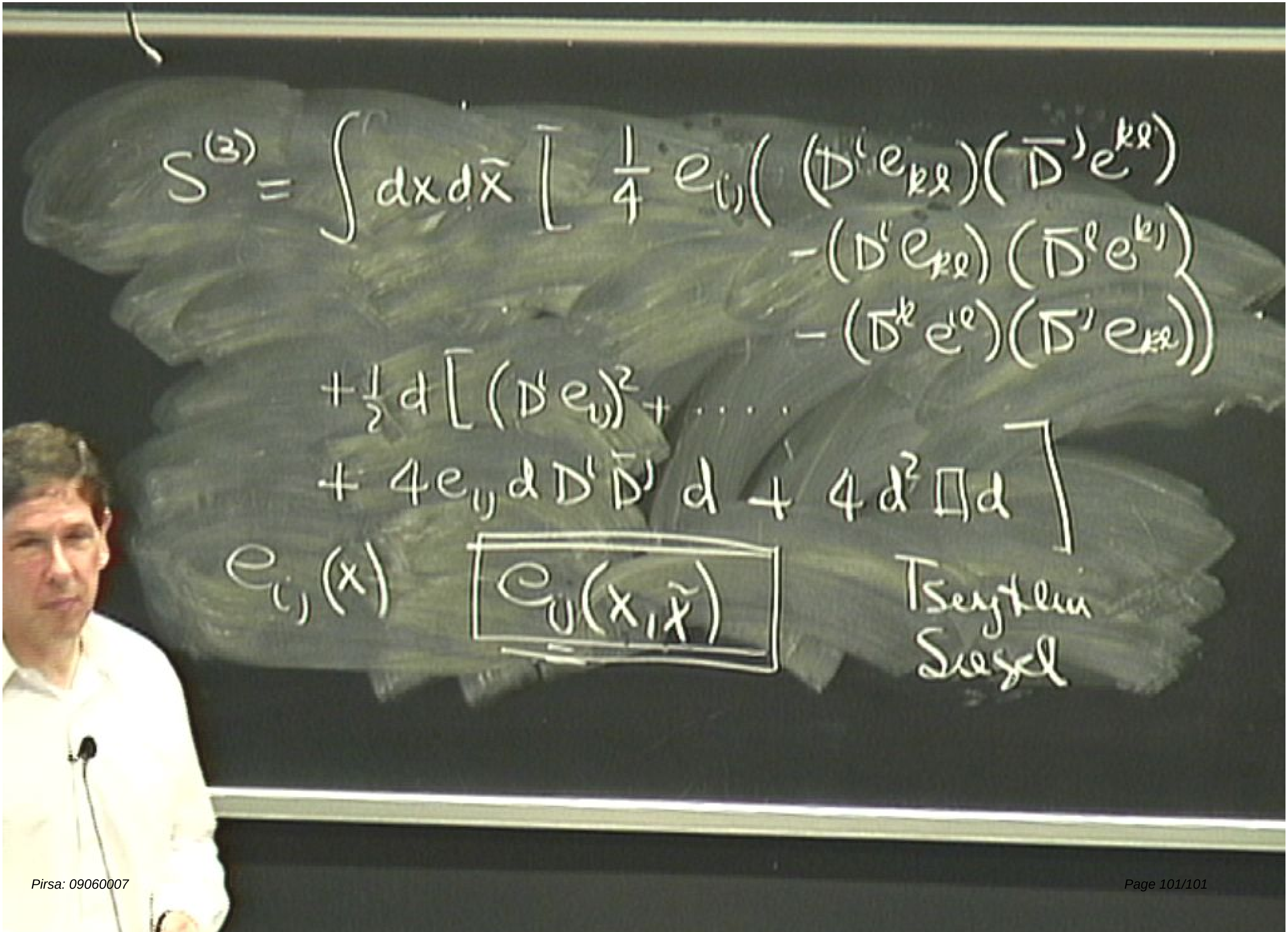
$$M = d^t - E c^t$$

$$\bar{M} = d^t + E c^t$$

$$G^{ij}$$


δλ e(ij) = D̄j λi + ½ [(Di λk) ek)j − (Dk λi) ek)j + λk Dk e
δλ d = −¼ D·λ + ½ [[(λ·D) d]]


$S^{(3)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e_{ij} \big((D^i e_{kl})(\bar{D}^j e^{kl})$
$-(D^i e_{kl})(\bar{D}^l e^{kj})$
$-(\bar{D}^k e^{il})(\bar{D}^j e_{kl})\big)$
$+\frac{1}{2} d \big[(D^i e \ldots$
$+ 4e \ldots d + 4 d^2 \Box d \Big]$
$e_{ij}(x)$
Tseytlin
Siegel


$S^{(3)} = \int dx\, d\tilde{x} \Big[\frac{1}{4} e_{ij} \Big((D^i e_{kl})(\bar{D}^j e^{kl})$
$- (D^i e_{kl})(\bar{D}^l e^{kj})$
$- (\bar{D}^k e^{il})(\bar{D}^j e_{kl}) \Big)$
$+ \frac{1}{2} d \big[(D^i e_{ij})^2 + \dots$
$+ 4 e_{ij}\, d\, D^i \bar{D}^j d + 4 d^2 \Box d \Big]$
$e_{ij}(x)$
$\boxed{e_{ij}(x, \tilde{x})}$
Tseytlin
Siegel