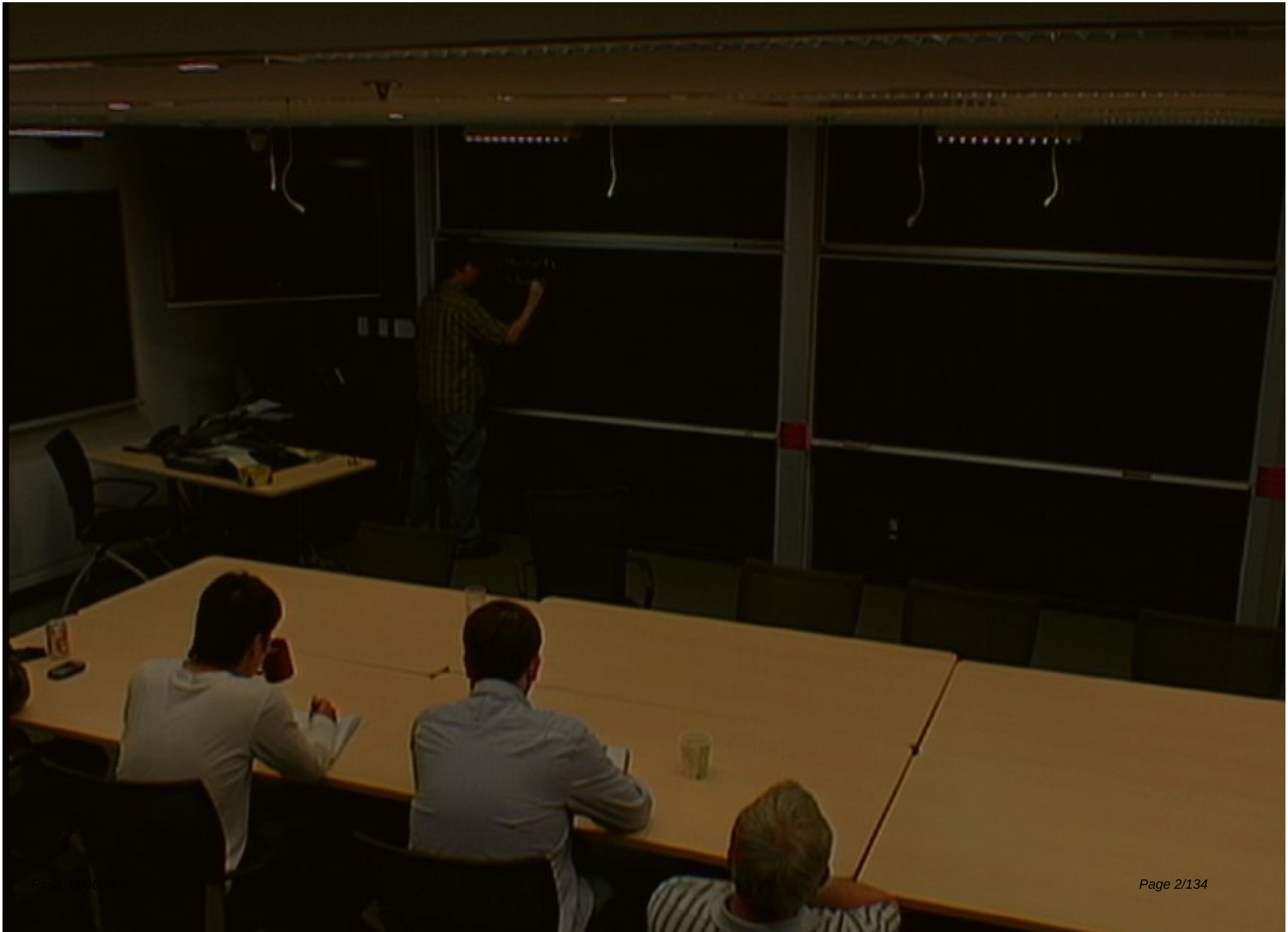
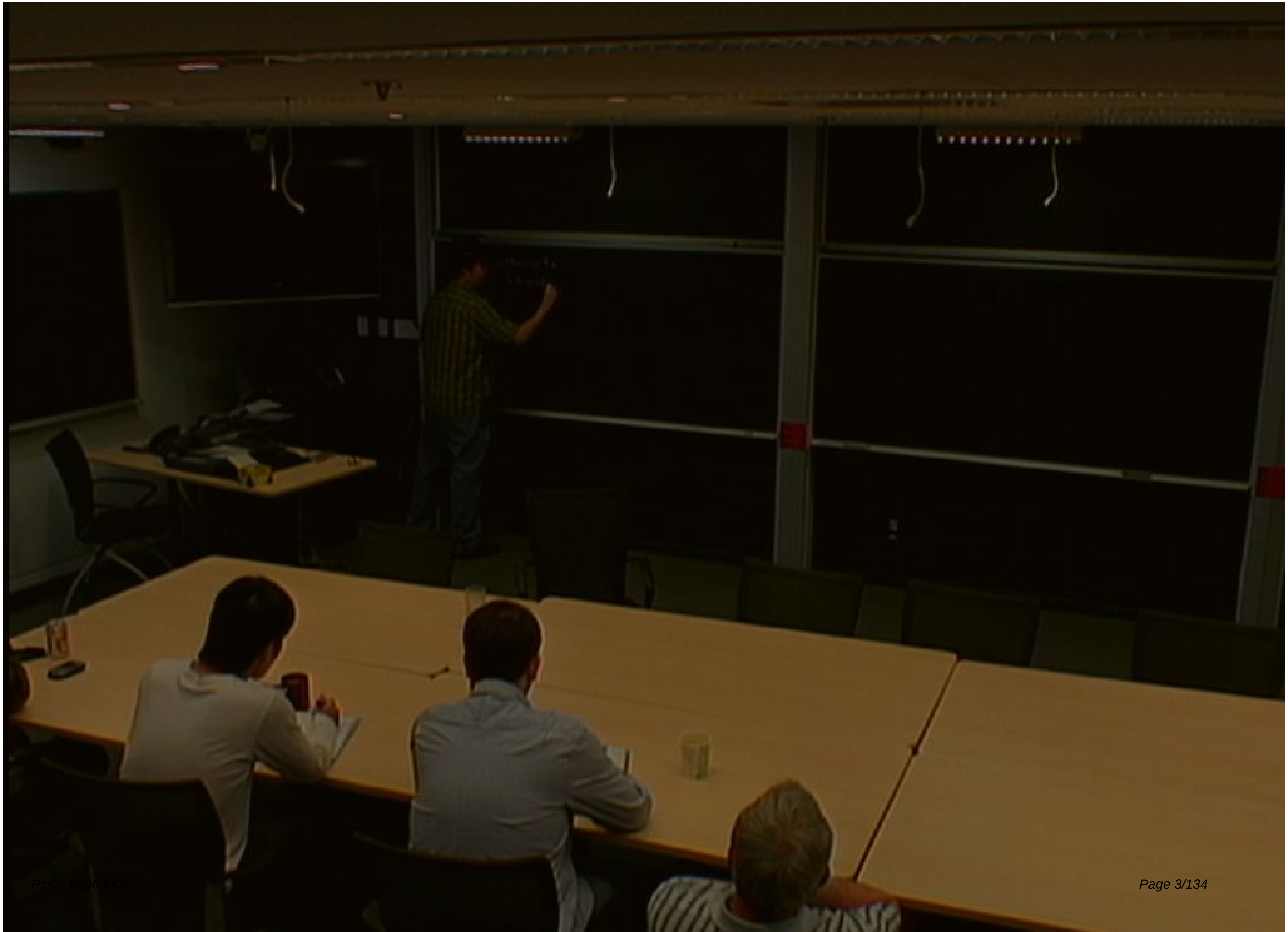
Title: Looking behind the horizon with D-brane probes

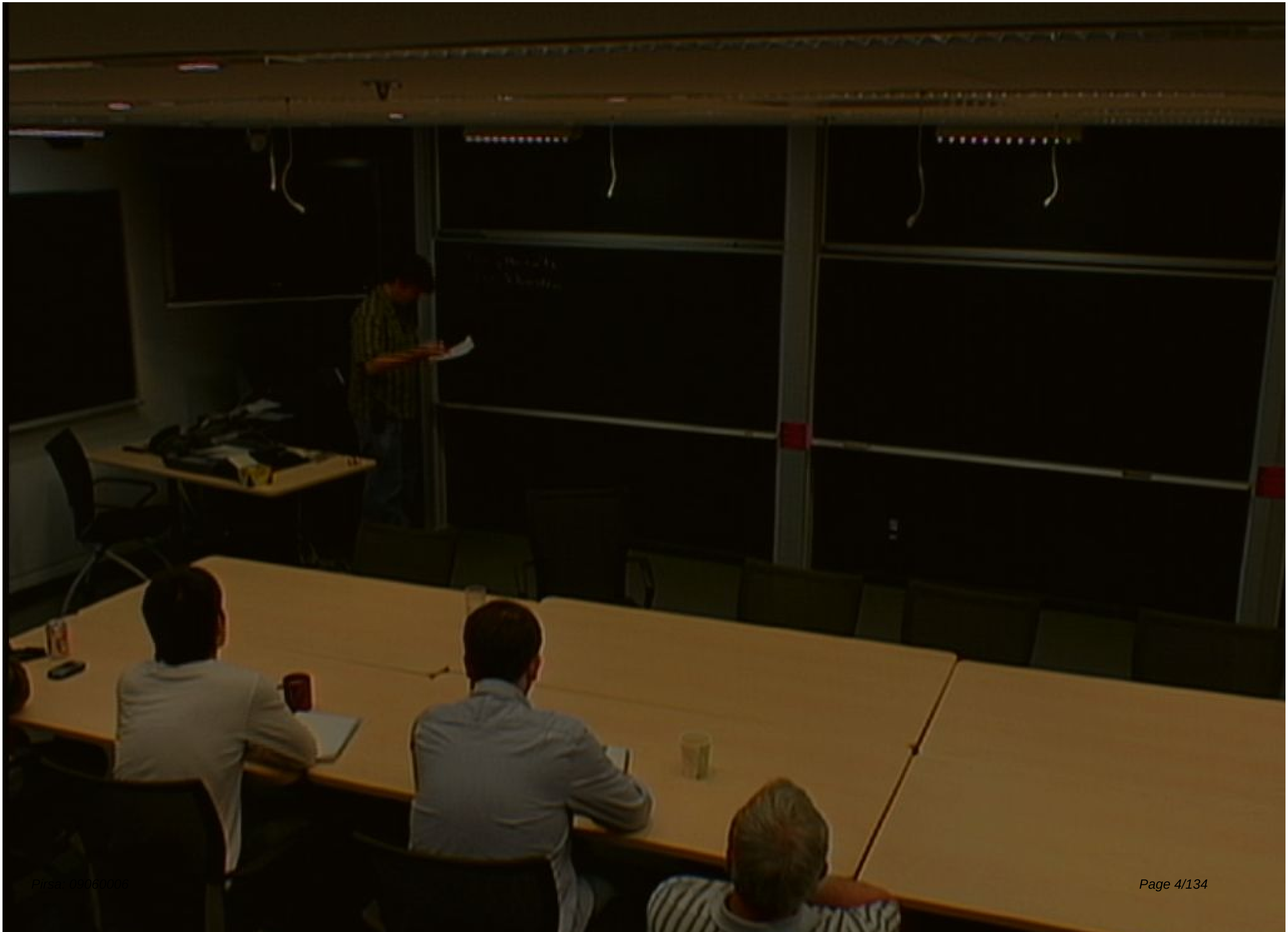
Date: Jun 02, 2009 11:00 AM

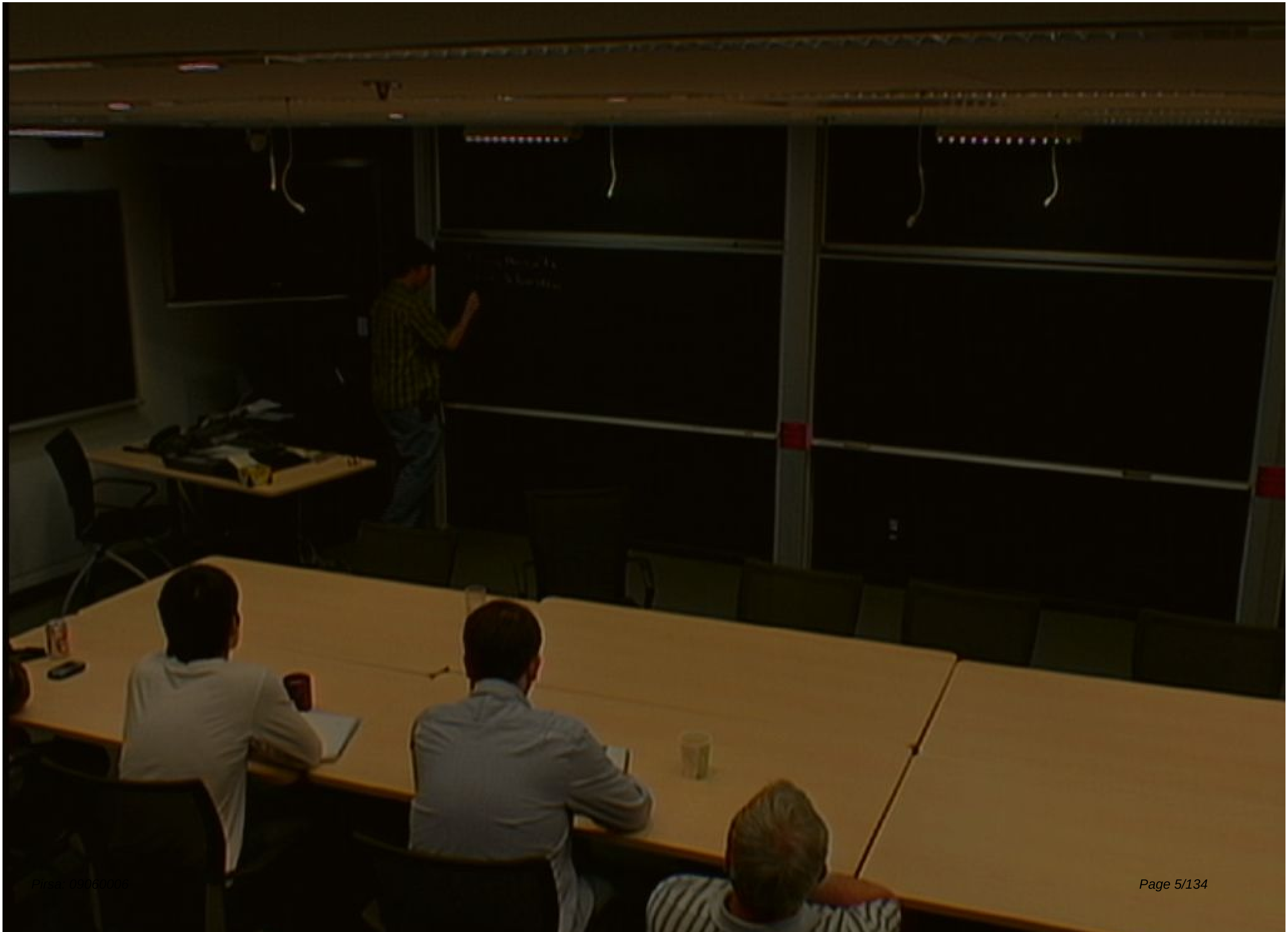
URL: http://pirsa.org/09060006

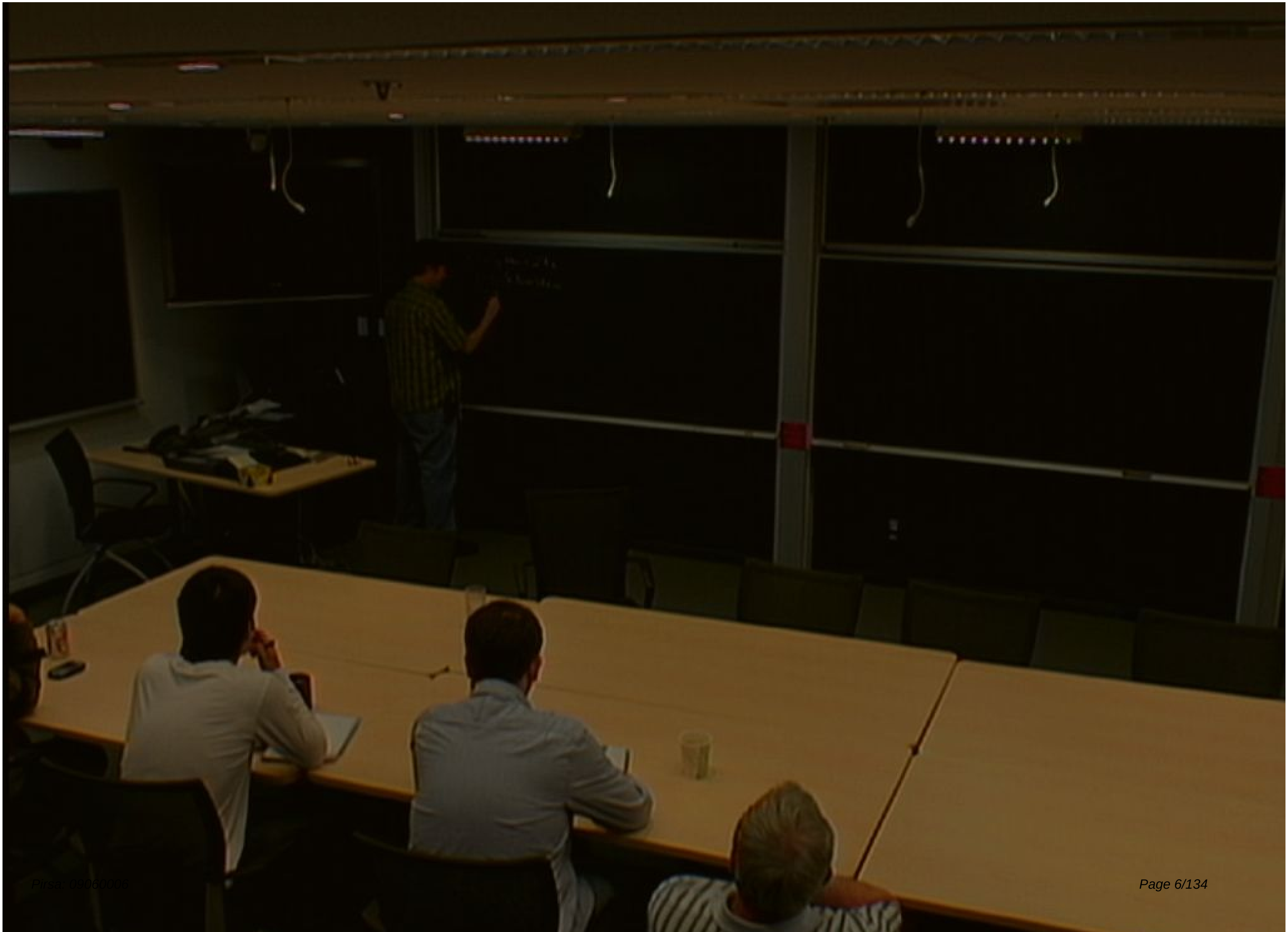
Abstract: We study a simple model of a black hole in AdS and obtain a holographic description of the region inside the horizon,as seen by an infalling observer. For D-brane probes, we construct a map from physics seen by an infalling observer to physics seen by an asymptotic observer that can be generalized to other AdS black holes.

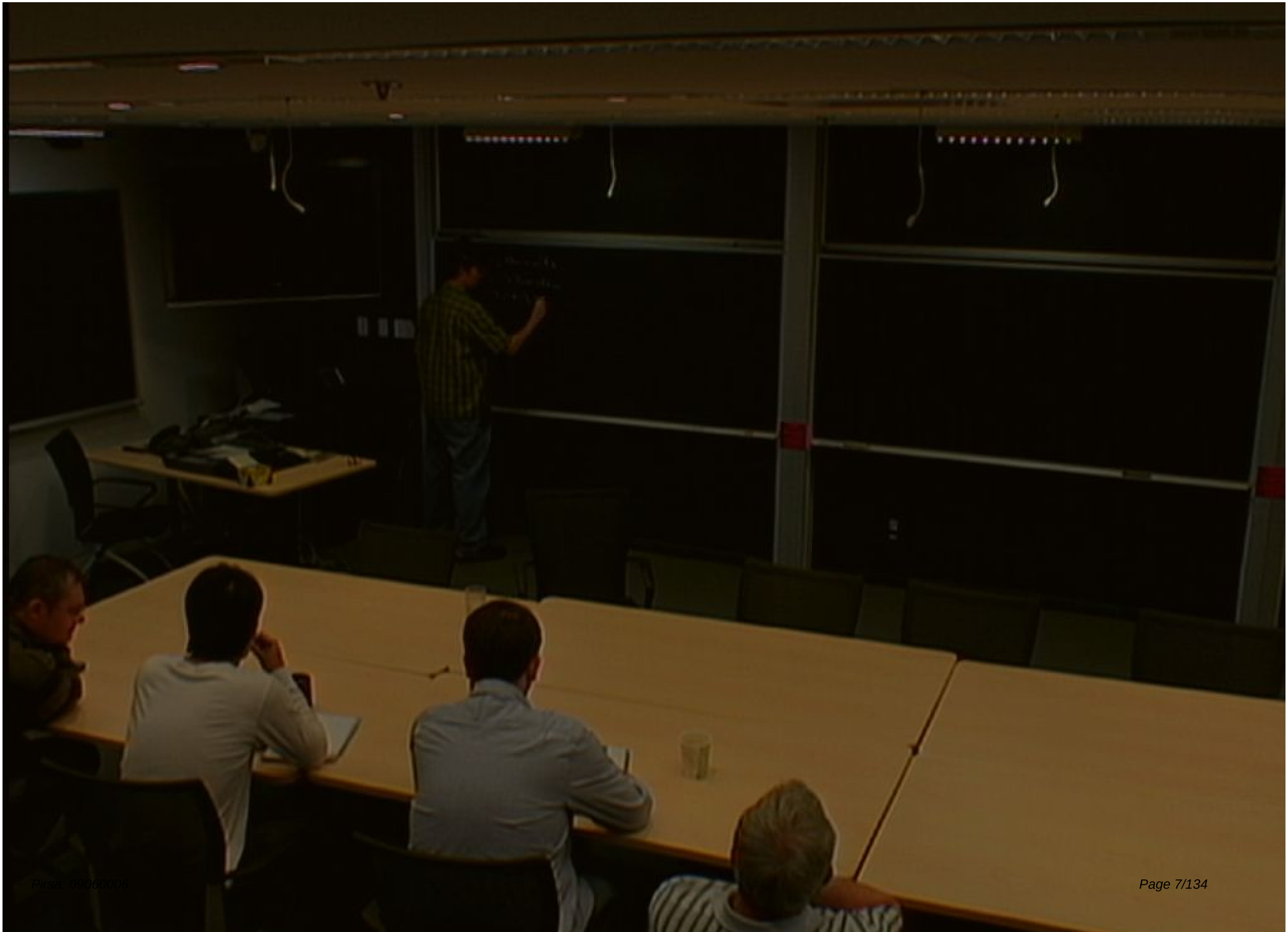


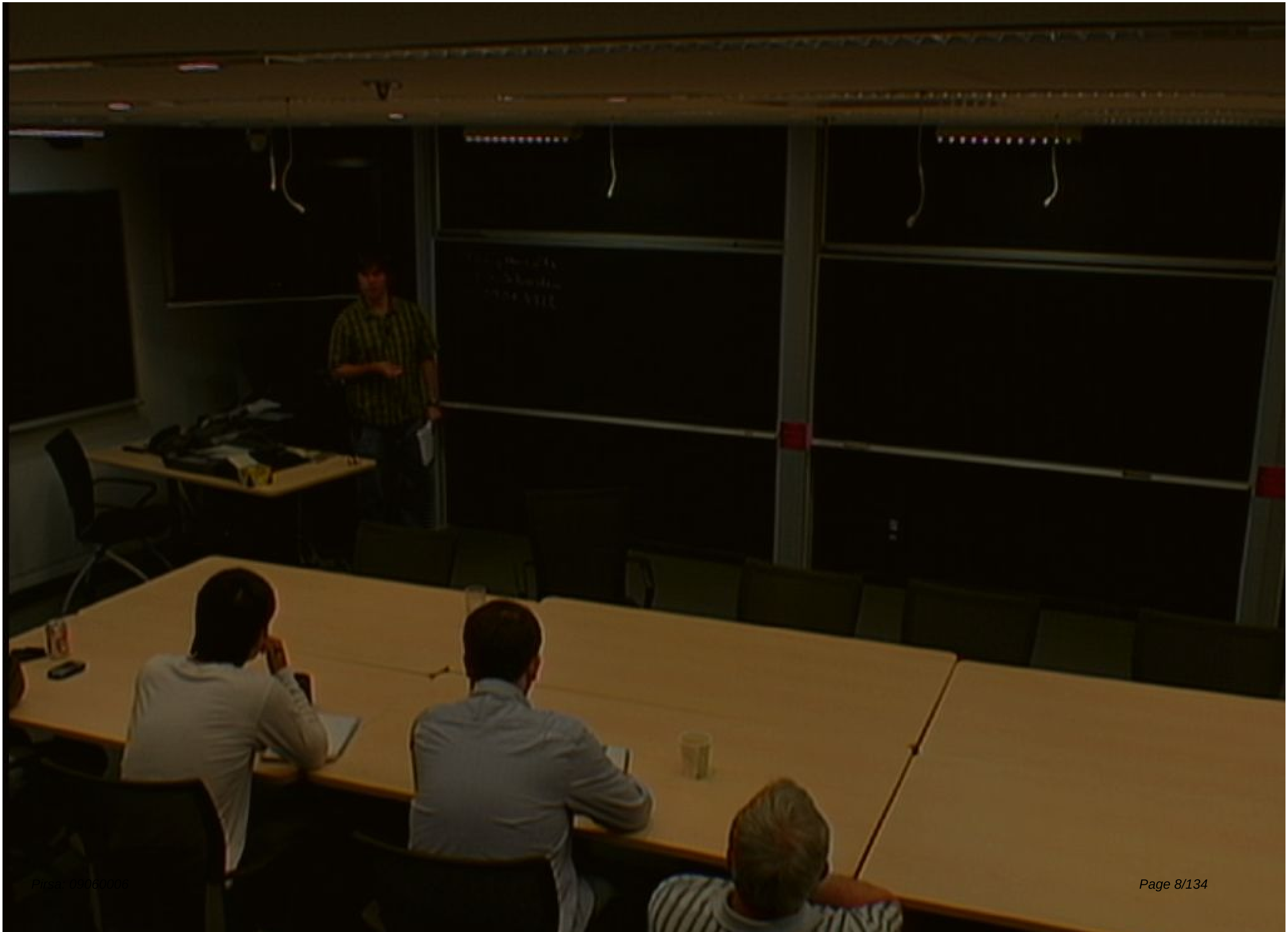












w/ Gary Horowitz
Eva Silverstein
0904.3922

I. Introduction

w/ Gary Horowitz
Eva Silverstein
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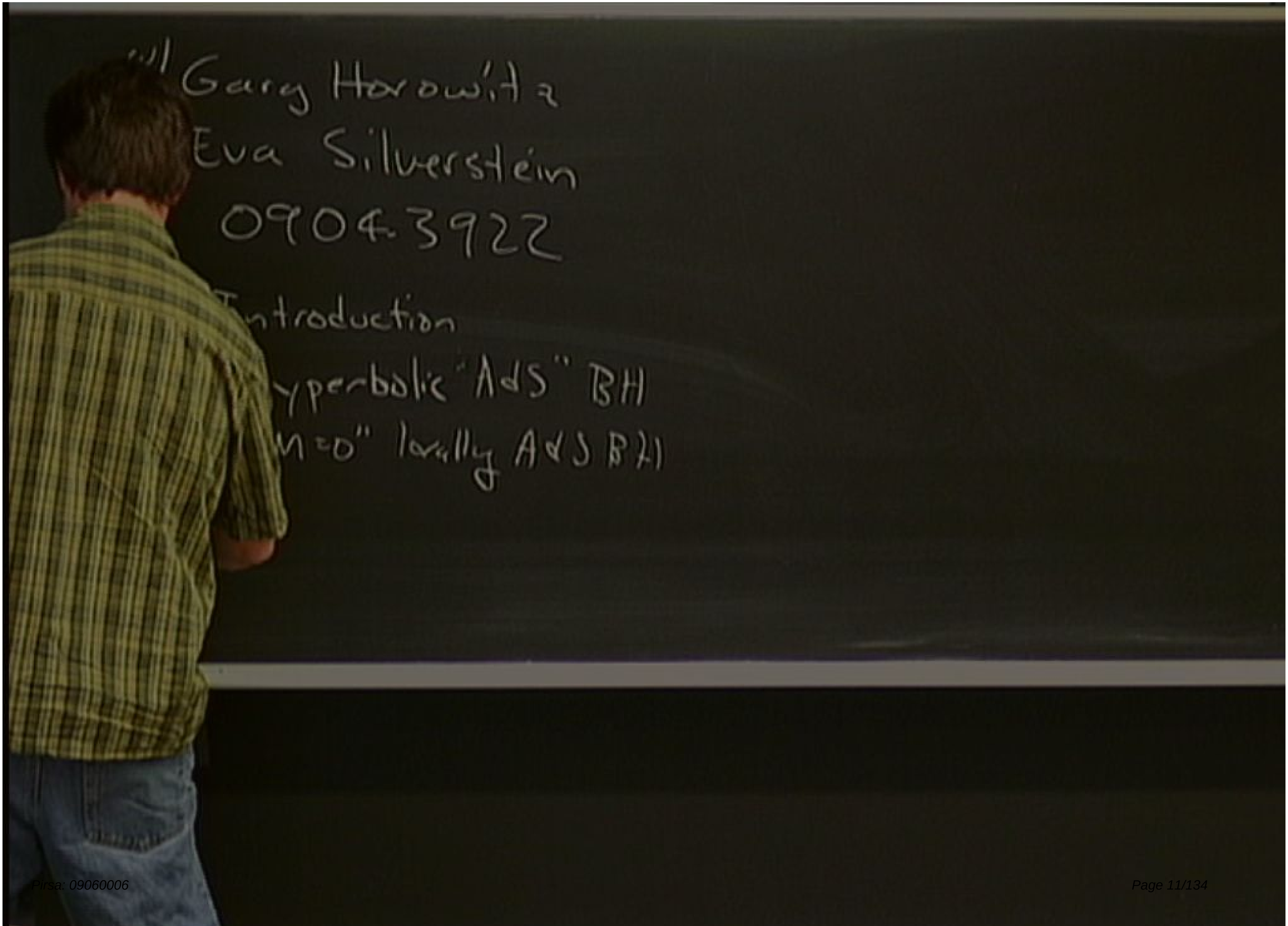
I. Introduction

II. "Hyperbolic" AdS" BH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Introduction
yperbolic "AdS" BH
M=0" locally AdS BH

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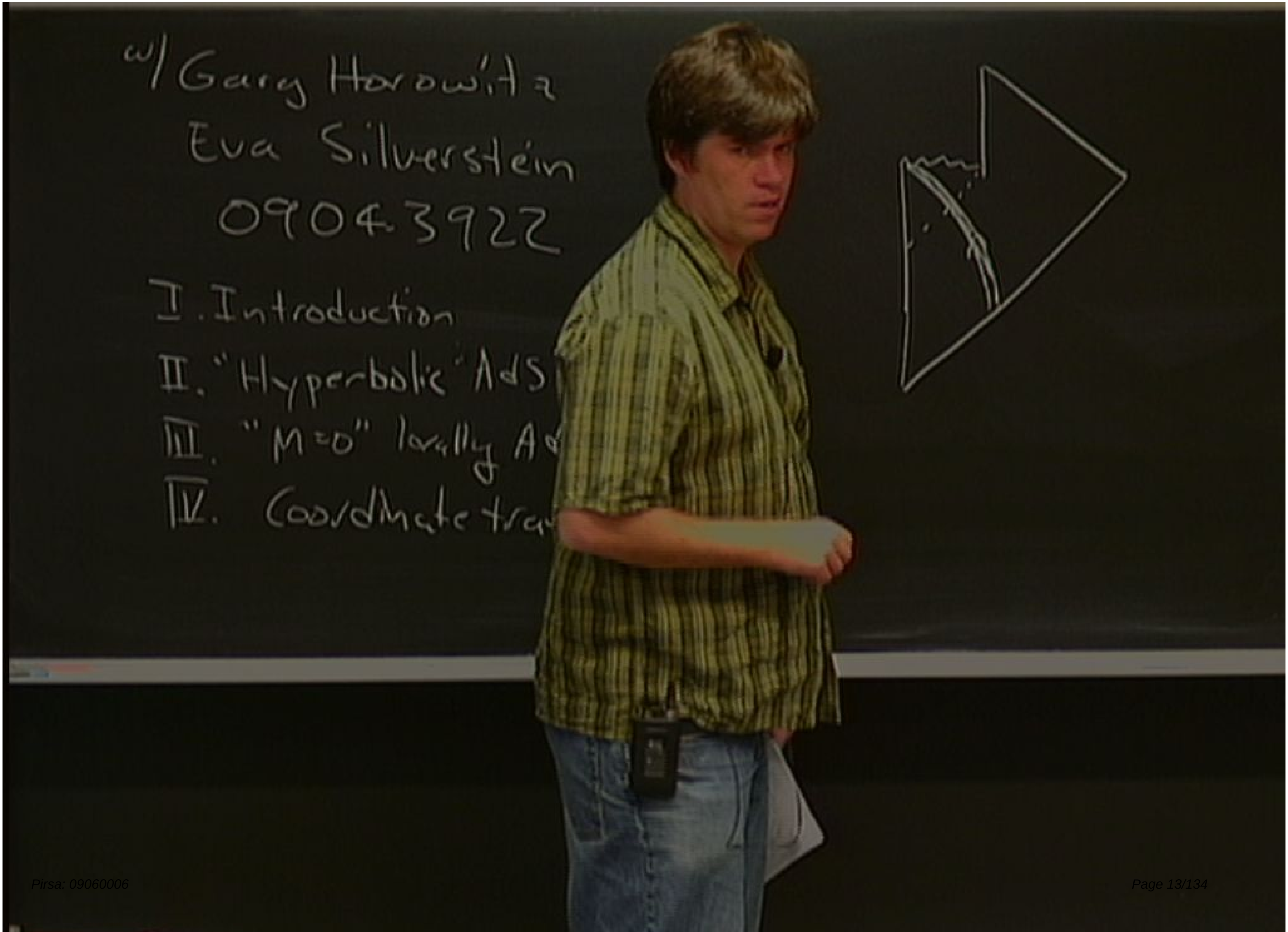
I. Introduction

II. "Hyperbolic AdS" BH

III. "M=0" locally AdS BH

IV. Coordinate transf. in QFT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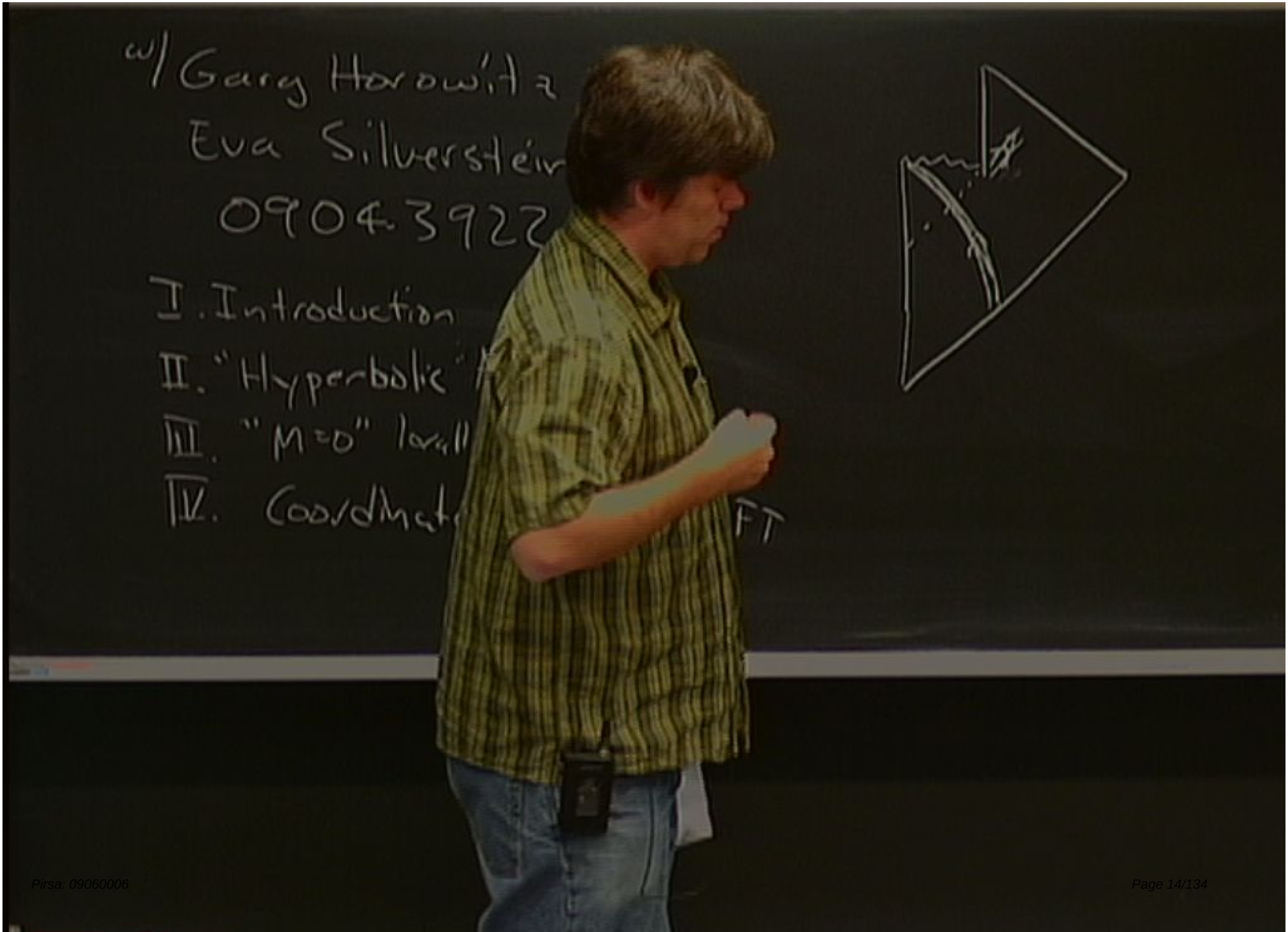
II. "Hyperbolic" AdS
III. "M=0" locally
IV. Coordinate



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III. "M=0"
IV.

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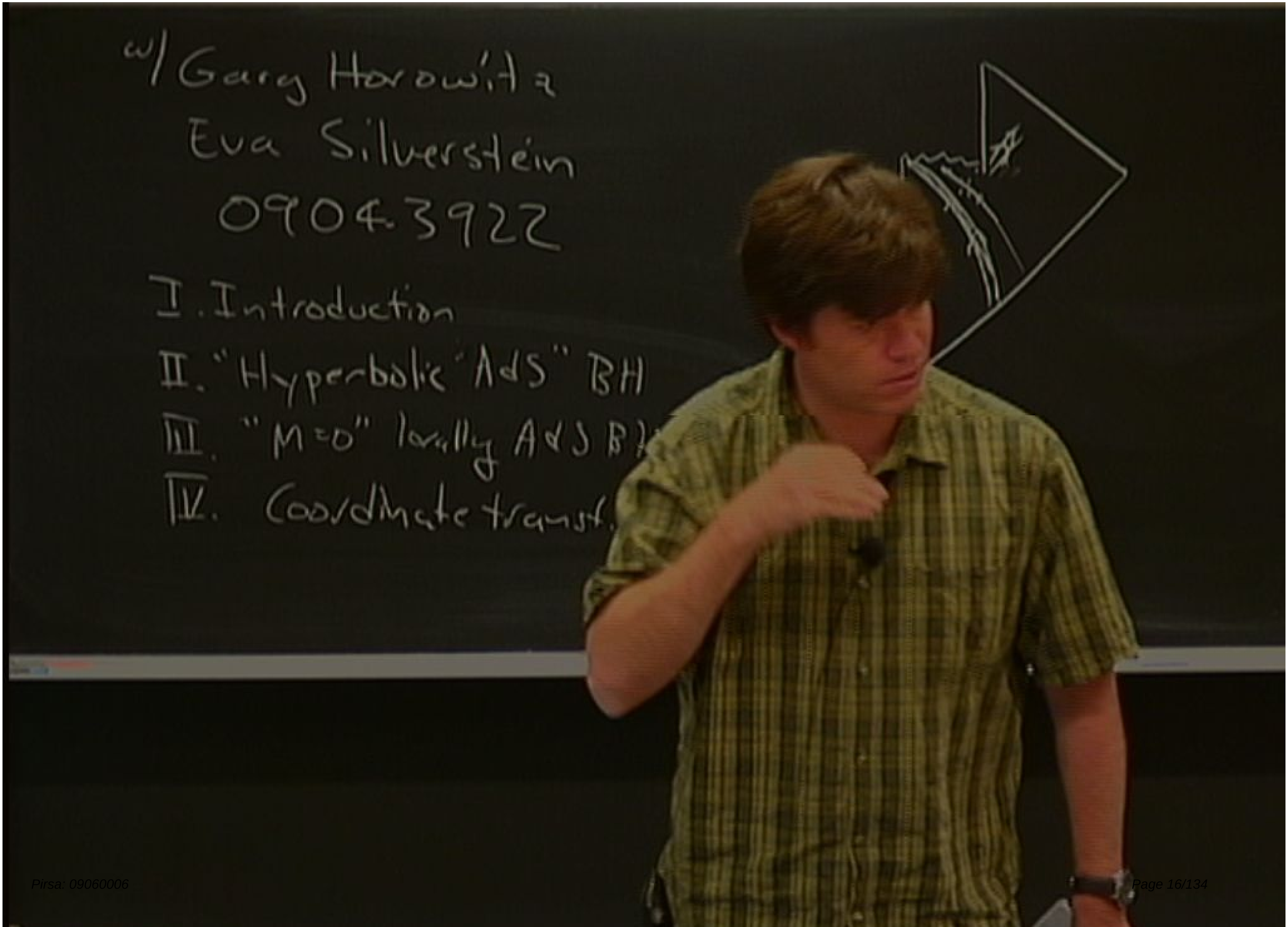

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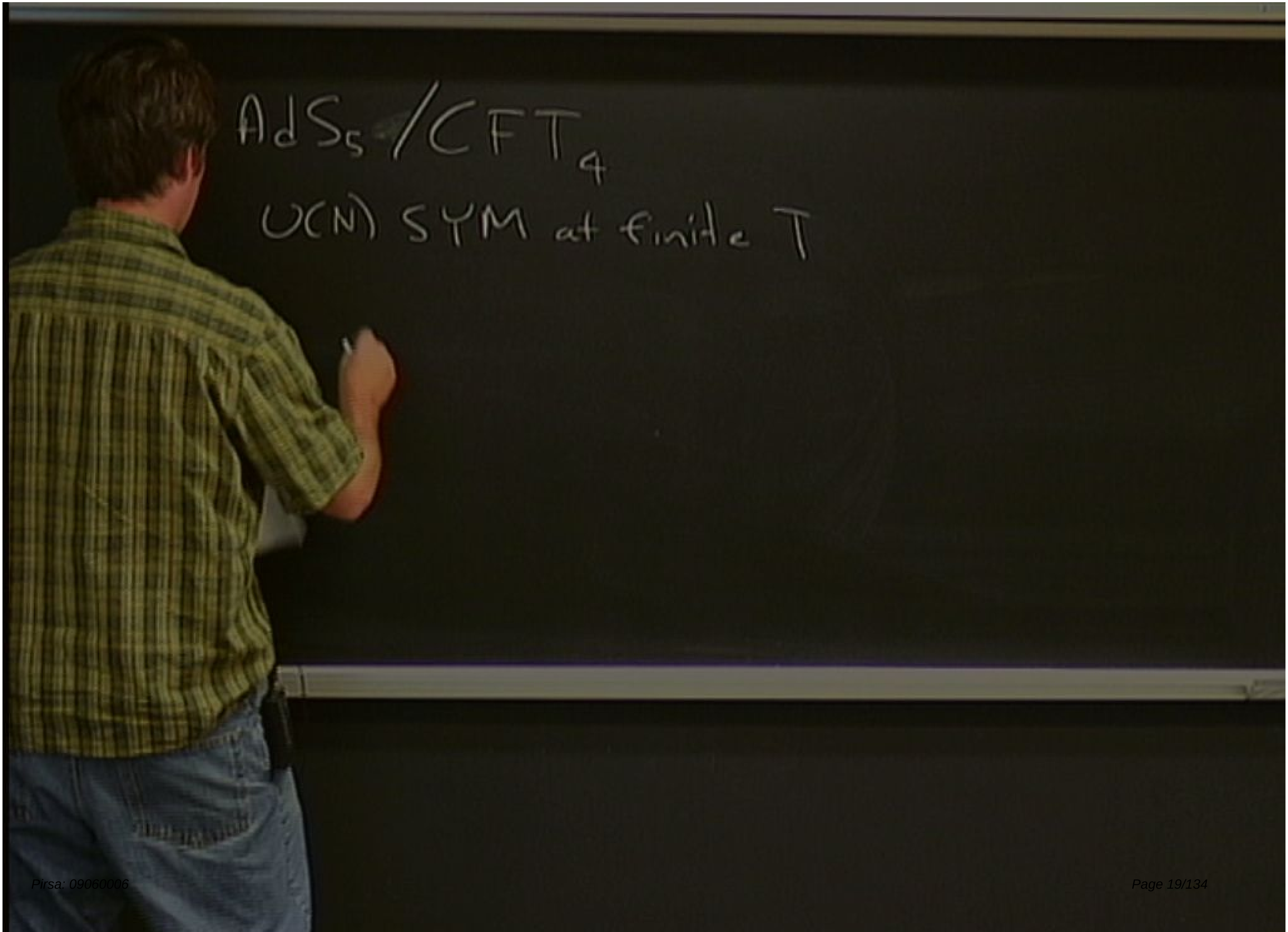
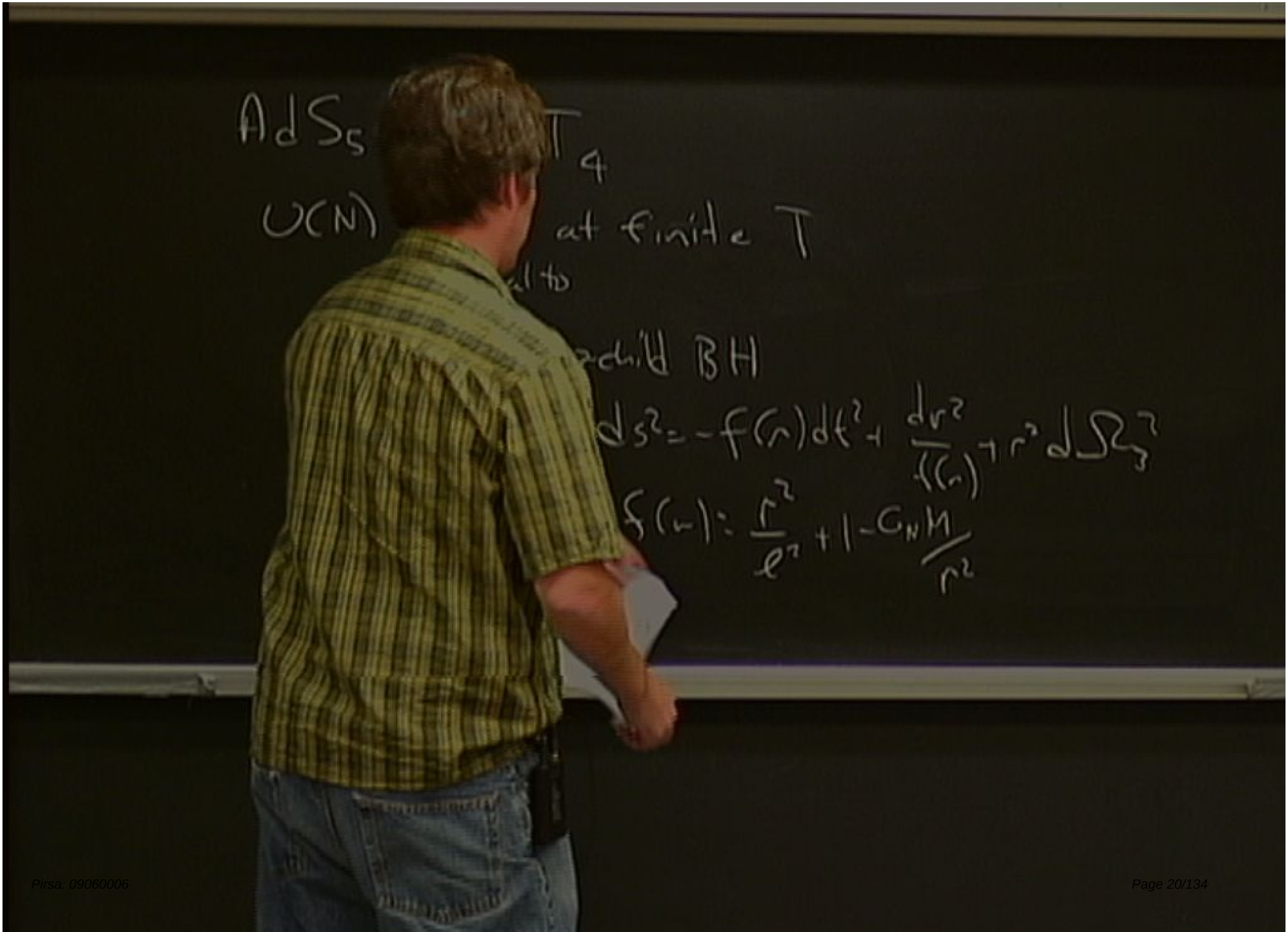
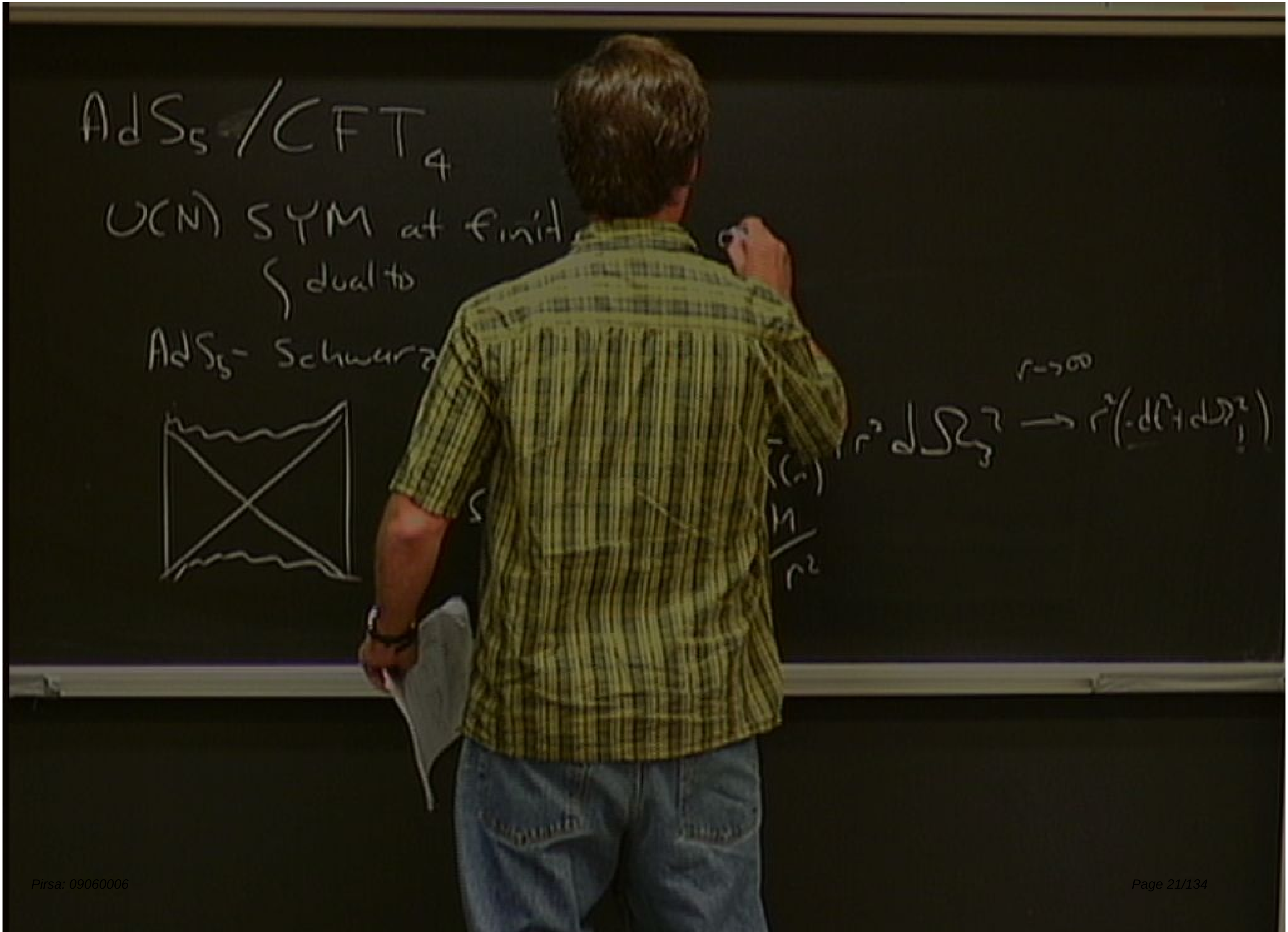
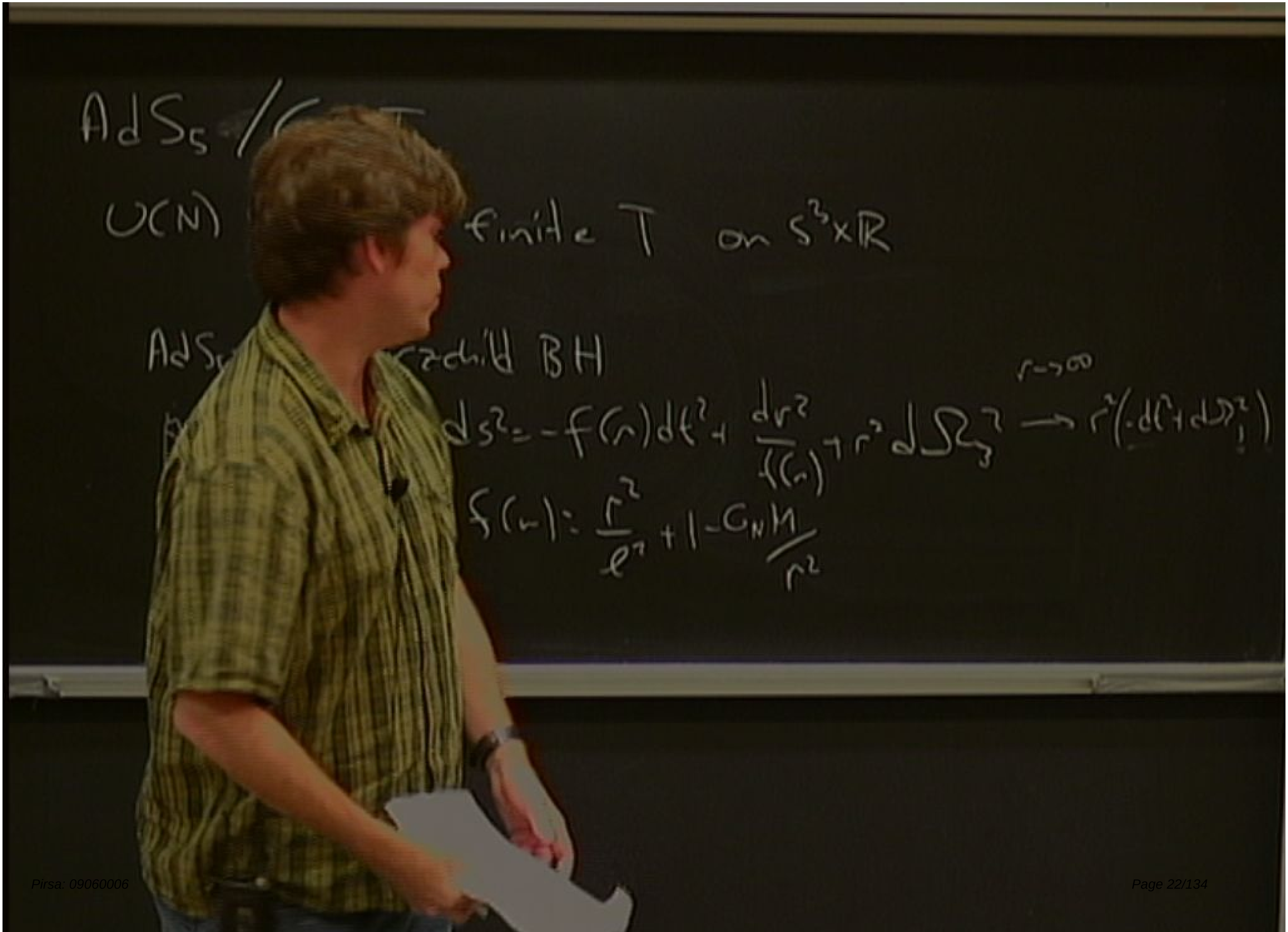
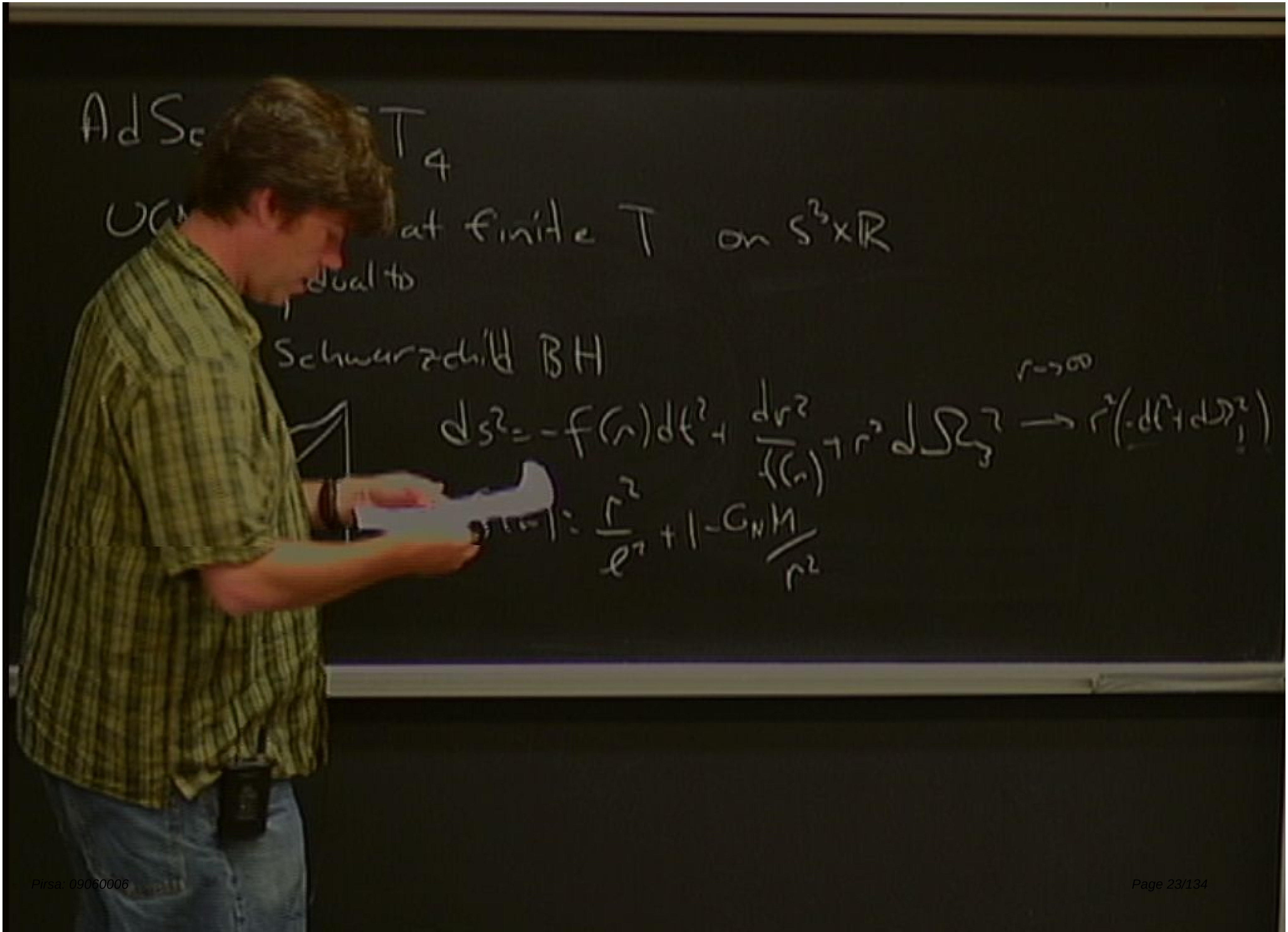

AdS5/CFT4
U(N) SYM at finite T


AdS_5
T_4
$U(N)$
at finite T
al to
child BH
$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2$
$f(r) = \frac{r^2}{\ell^2} + 1 - \frac{G_N M}{r^2}$

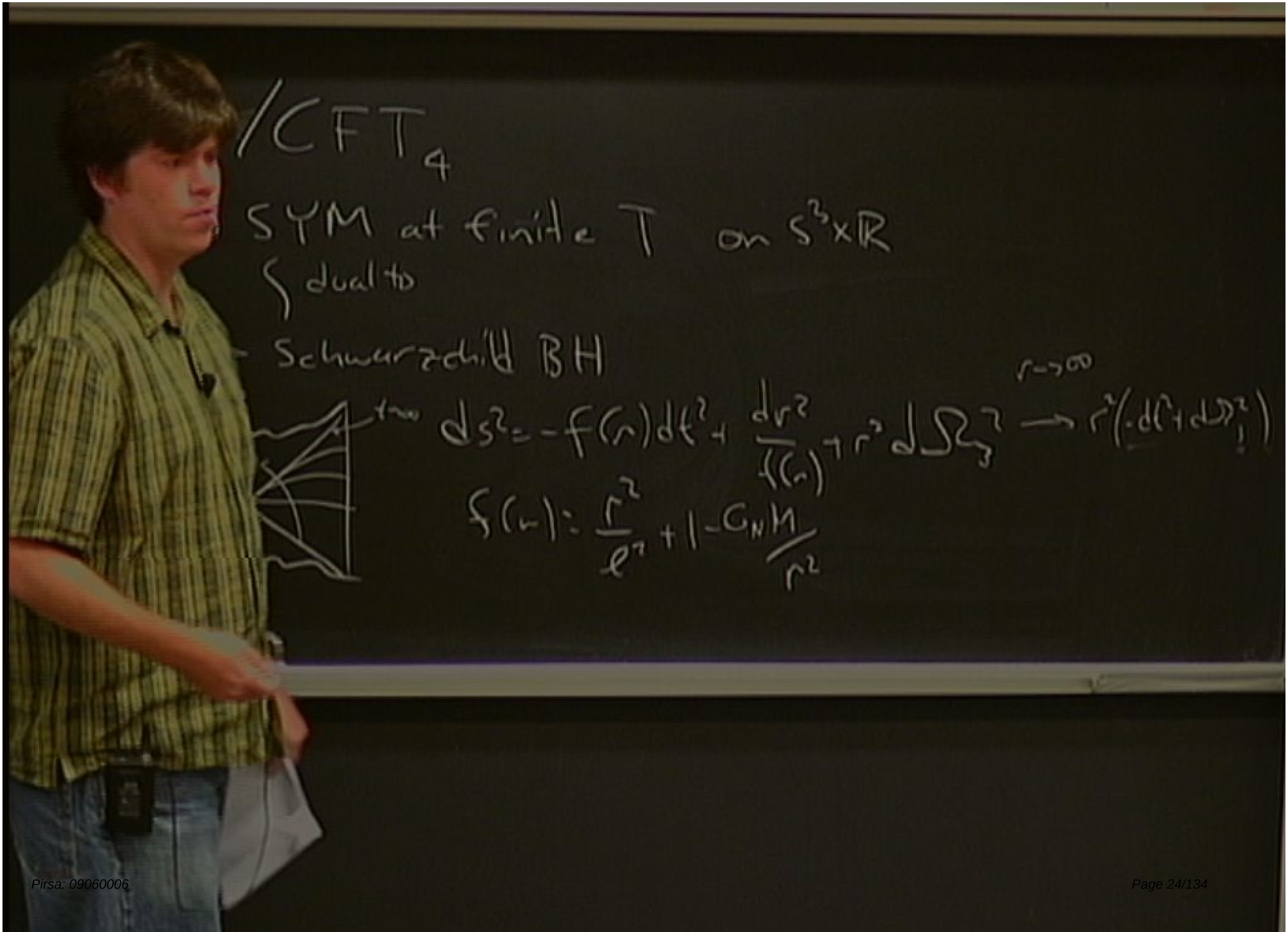

AdS5/CFT4
U(N) SYM at finit
dual to
AdS5- Schwarz
r→∞



AdS5 /
U(N)
finite T on S3xR
AdS5
BH
r→∞
ds2 = -f(r)dt2 + dr2/f(r) + r2dΩ32 → r2(-dt2+dΩ32)
f(r) = r2/l2 + 1 - GNM/r2



at finite T on S³xR
dual to
Schwarzschild BH
r→∞
ds² = -f(r)dt² + dr²/f(r) + r²dΩ₃² → r²(-dt²+dΩ₃²)
= r²/ℓ² + 1 - G_N M/r²


/CFT4
SYM at finite T on S3xR
dual to
Schwarzchild BH
t=∞
r→∞
ds2 = -f(r)dt2 + dr2/f(r) + r2dΩ32 → r2(-dt2+dΩ32)
f(r) = r2/ℓ2 + 1 - GNM/r2

AdS_5/CFT_4

$U(N)$ SYM at finite T on $S^3 \times \mathbb{R}$

dual to

AdS_5-Schwarzschild BH

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \xrightarrow{r \to \infty} r^2(-dt^2 + d\Omega_3^2)$$

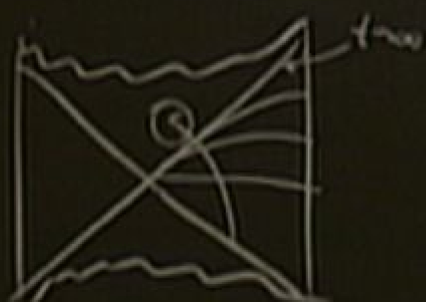
$$f(r) = \frac{r^2}{\ell^2} + 1 - \frac{G_N M}{r^2}$$

AdS_5/CFT_4

$U(N)$ SYM at finite T on $S^3 \times \mathbb{R}$

dual to

AdS_5- Schwarzchild BH



$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \xrightarrow{r \to \infty} r^2(-dt^2 + d\Omega_3^2)$$

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dual to

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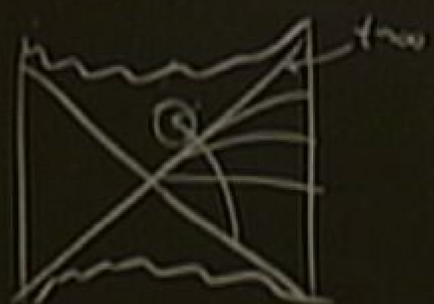


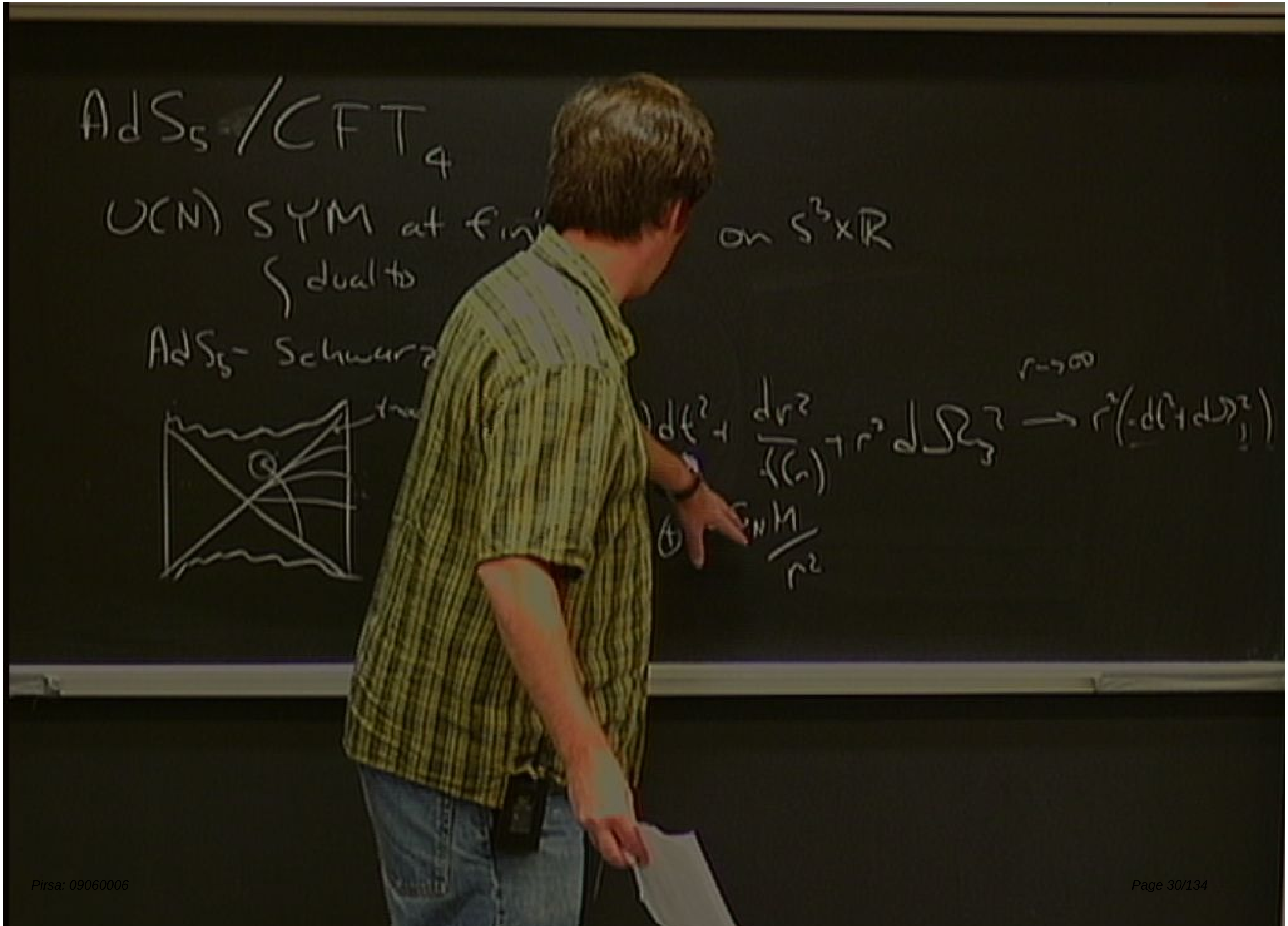
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \xrightarrow{r\to\infty} r^2(-dt^2 + d\Omega_3^2)$$

$$f(r) = \frac{r^2}{\ell^2} + 1 - \frac{G_N M}{r^2}$$

III. "Hyperbolic AdS" (or
"topological" BH

II. "Hyperbolic AdS" (or "topological" BH

A. Spacetime solutions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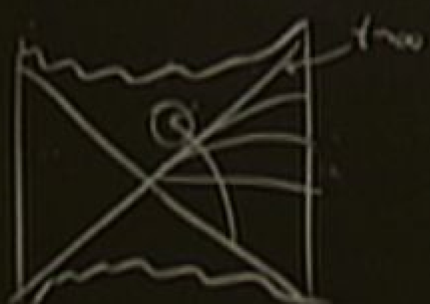
AdS_5/CFT_4

$U(N)$ SYM at finite T on $S^3 \times \mathbb{R}$

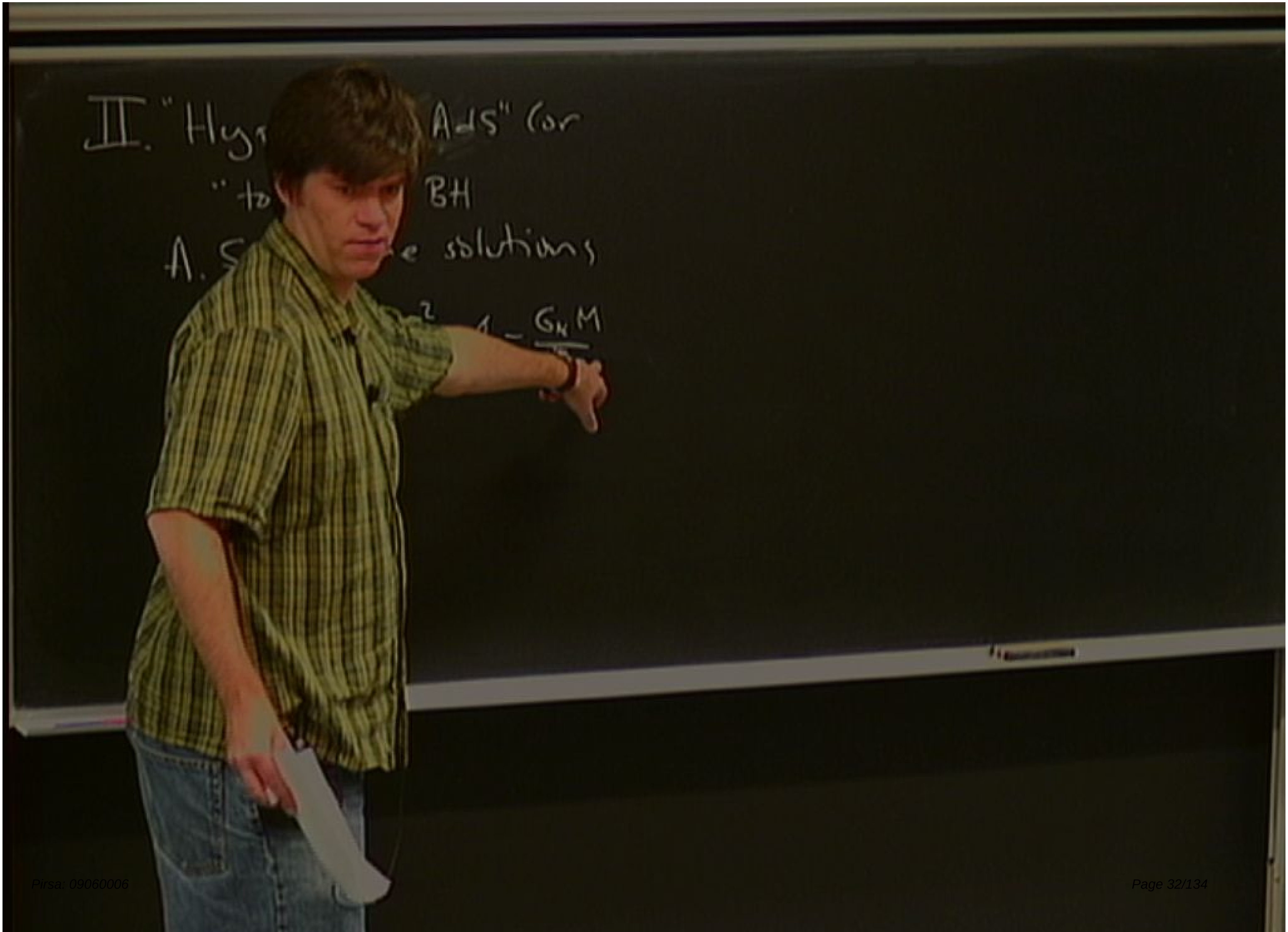
dual to

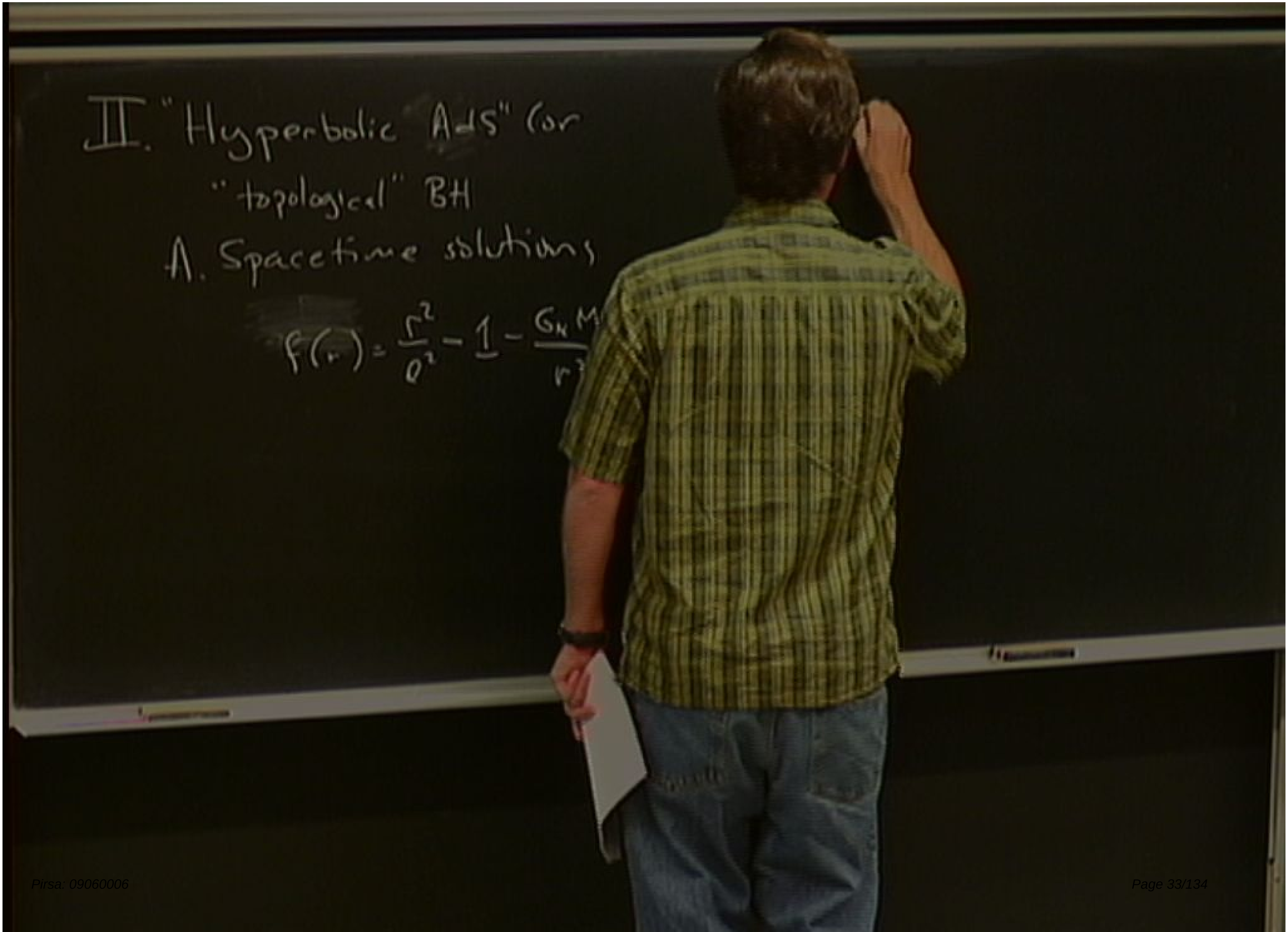
AdS_5- Schwarzchild BH

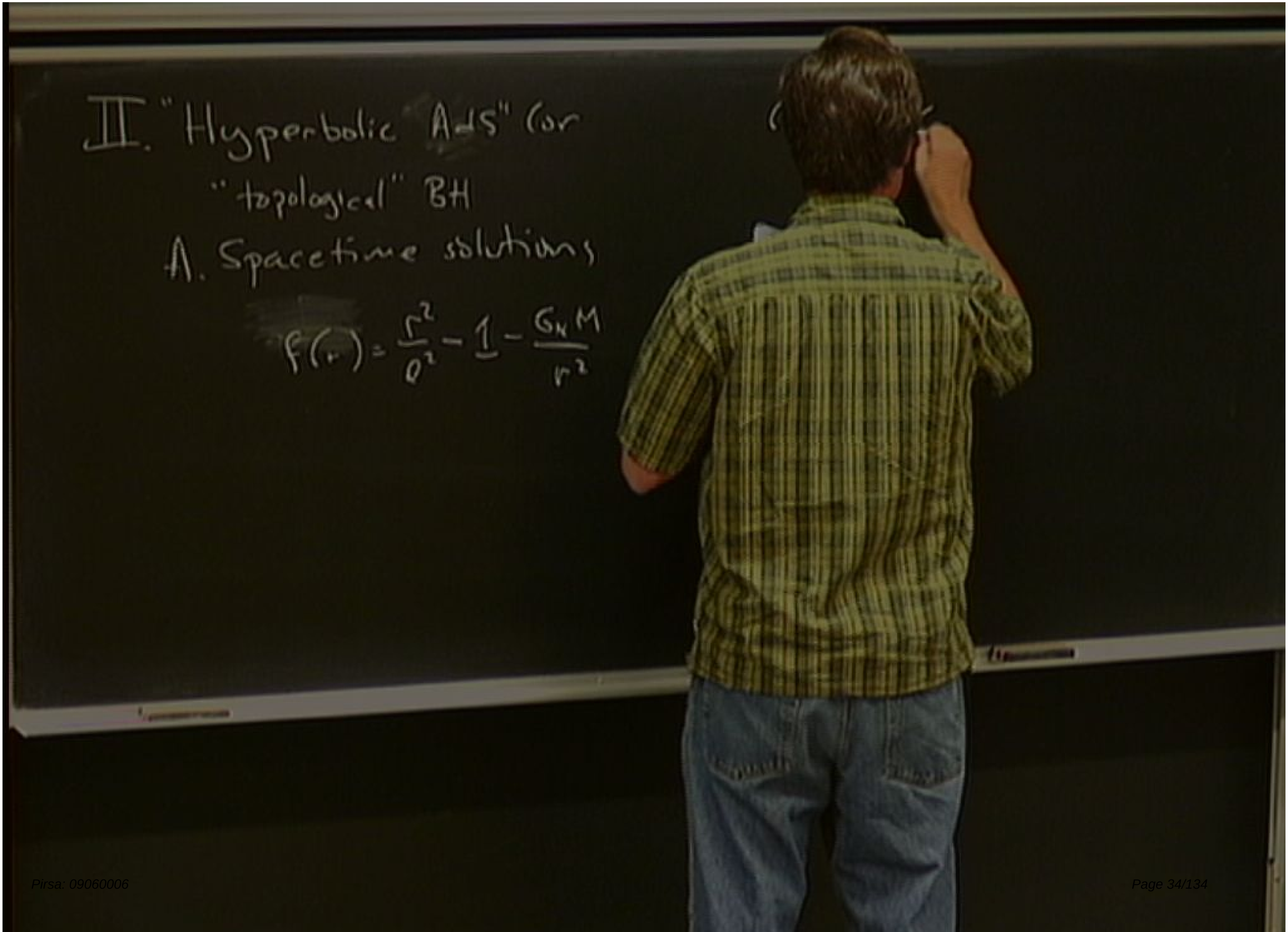


$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \xrightarrow{r\to\infty} r^2(-dt^2 + d\Omega_3^2)$$

$$f(r) = \frac{r^2}{\ell^2} + 1 - \frac{G_N M}{r^2}$$


AdS" for
BH
solutions


II. "Hyperbolic AdS" (or
"topological" BH
A. Spacetime solutions
$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^3}$


II. "Hyperbolic AdS" (or
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A. Spacetime solutions
$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^2}$

II. "Hyperbolic AdS" for "topological" BH

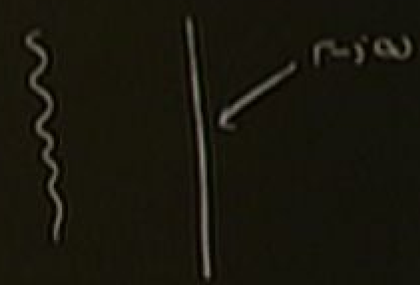
A. Spacetime solutions

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^2}$$

$$ds^2 = -f\, dt^2 + \frac{dr^2}{f} + \frac{r^2}{\ell^2}\, d\sigma^2$$

H_3/Γ

(i) $G_N M < \mu_{ext} = -\frac{\ell^2}{4}$



II "Hyperbolic AdS" (or "topological" BH) spacetime solutions

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^2}$$

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + \frac{r^2}{\ell^2}\,d\sigma^2$$

$\mathbb{H}_3/\Gamma$

(i) $G_N M < \mu_{ext} = -\frac{\ell^2}{4}$

$r = \infty$

II. "Hyperbolic AdS" (or "topological" BH

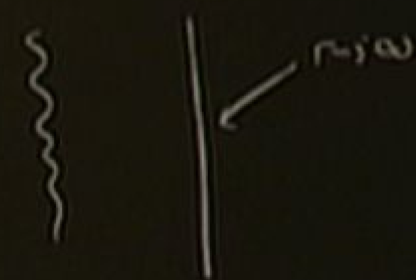
A. Spacetime solutions

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^2}$$

$$H_3 / \Gamma$$

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + \frac{r^2}{\ell^2}\,d\sigma^2$$

(i) $G_N M < \mu_{ext} = -\frac{\ell^2}{4}$



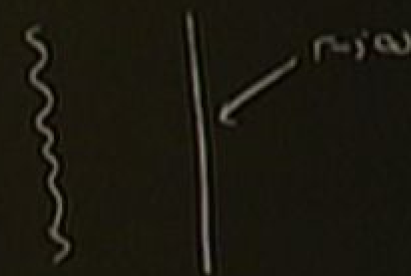
II. "Hyperbolic AdS" for "topological" BH

A. Spacetime solutions

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^2}$$

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + \frac{r^2}{\ell^2}\,d\sigma^2$$

(i) $G_N M < \mu_{ext} = -\frac{\ell^2}{4}$



II. "Hyperbolic AdS" (or "topological" BH

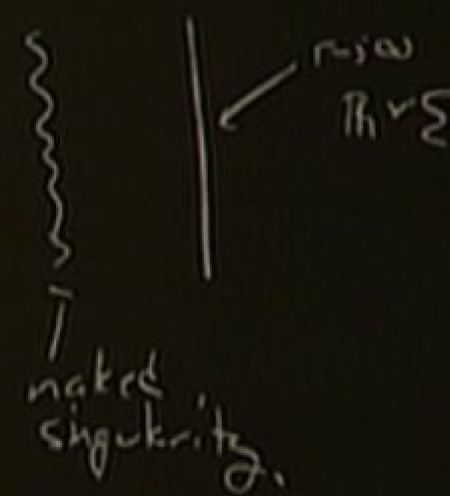
A. Spacetime solutions

$$f(r) = \frac{r^2}{\ell^2} - 1 - \frac{G_N M}{r^2}$$

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + \frac{r^2}{\ell^2}\,d\sigma^2$$

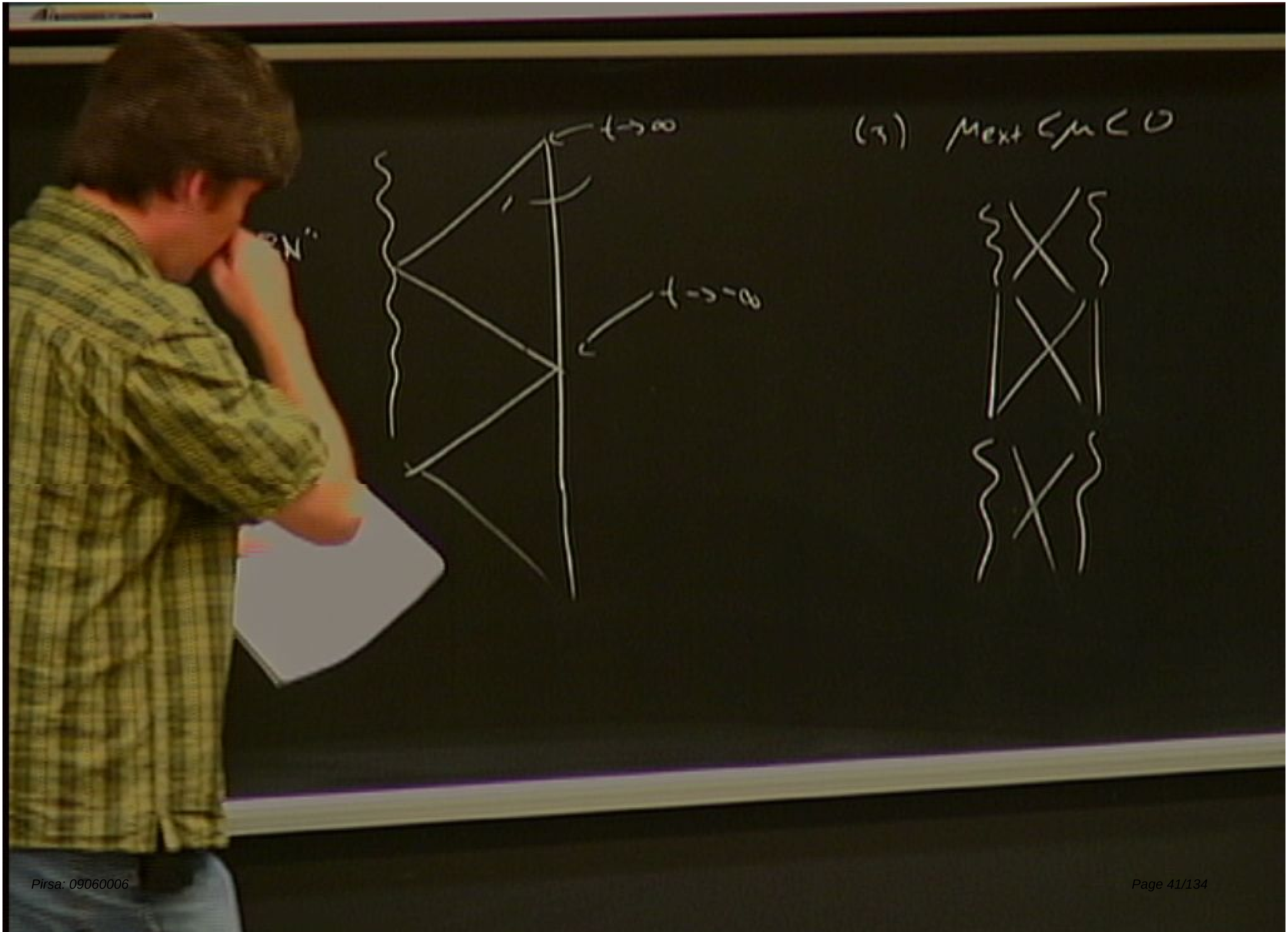
$$\mathbb{H}^3/\Gamma = \Sigma$$

(1) $G_N M < \mu_{ext} = -\frac{\ell^2}{4}$



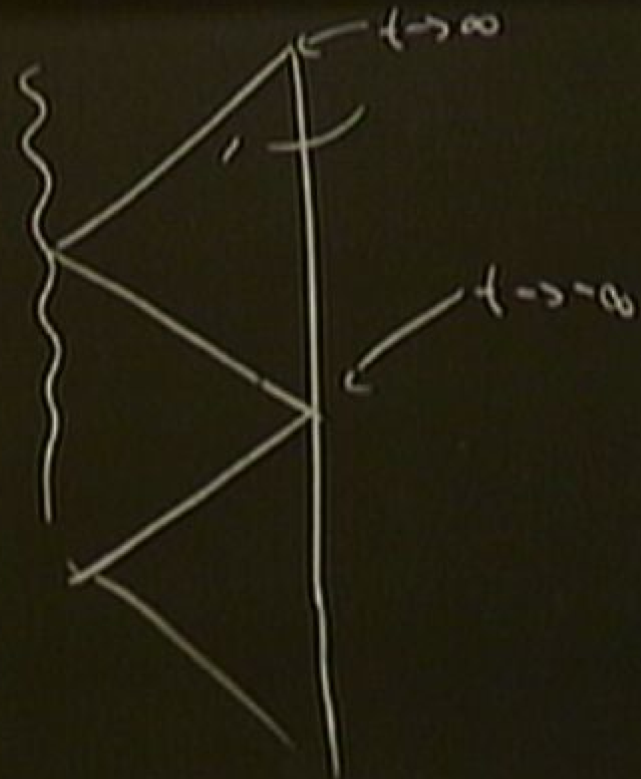
(2) $\mu = \mu_{ext}$

"extremal RN"

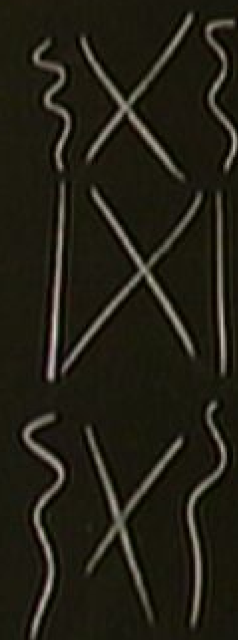

$t \to \infty$
$t \to -\infty$
(3) $M_{ext} < \mu < 0$

(2) $\mu = \mu_{ext}$

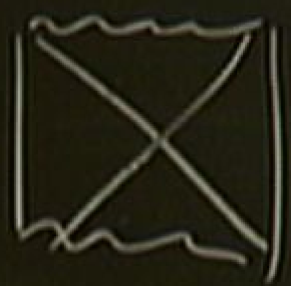
"extremal RN"



(3) $\mu_{ext} < \mu < 0$



(1) $\mu > 0$



(1) $\mu \geq 0$

all of these $\xrightarrow{r \to \infty} r^2(-dt^2 + dx^2)$

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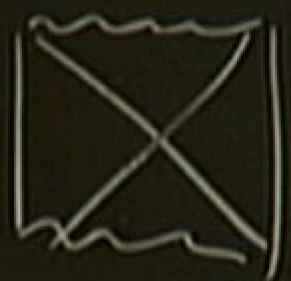
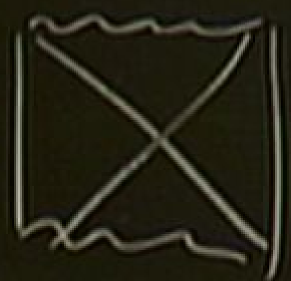


all of these $\xrightarrow{r\to\infty}$ $r^2(-dt^2+ds^2)$

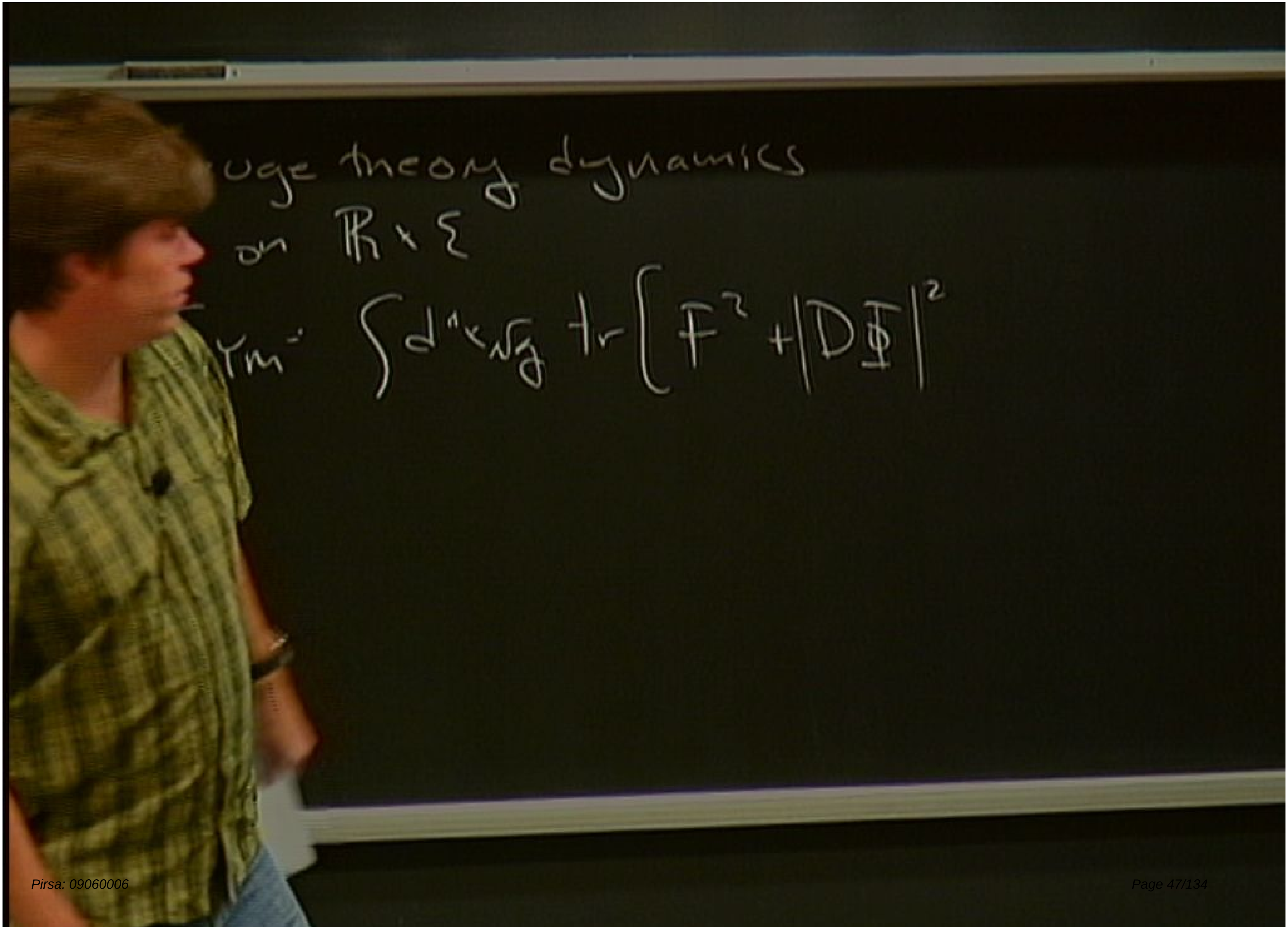
$\mathbb{R}\times$

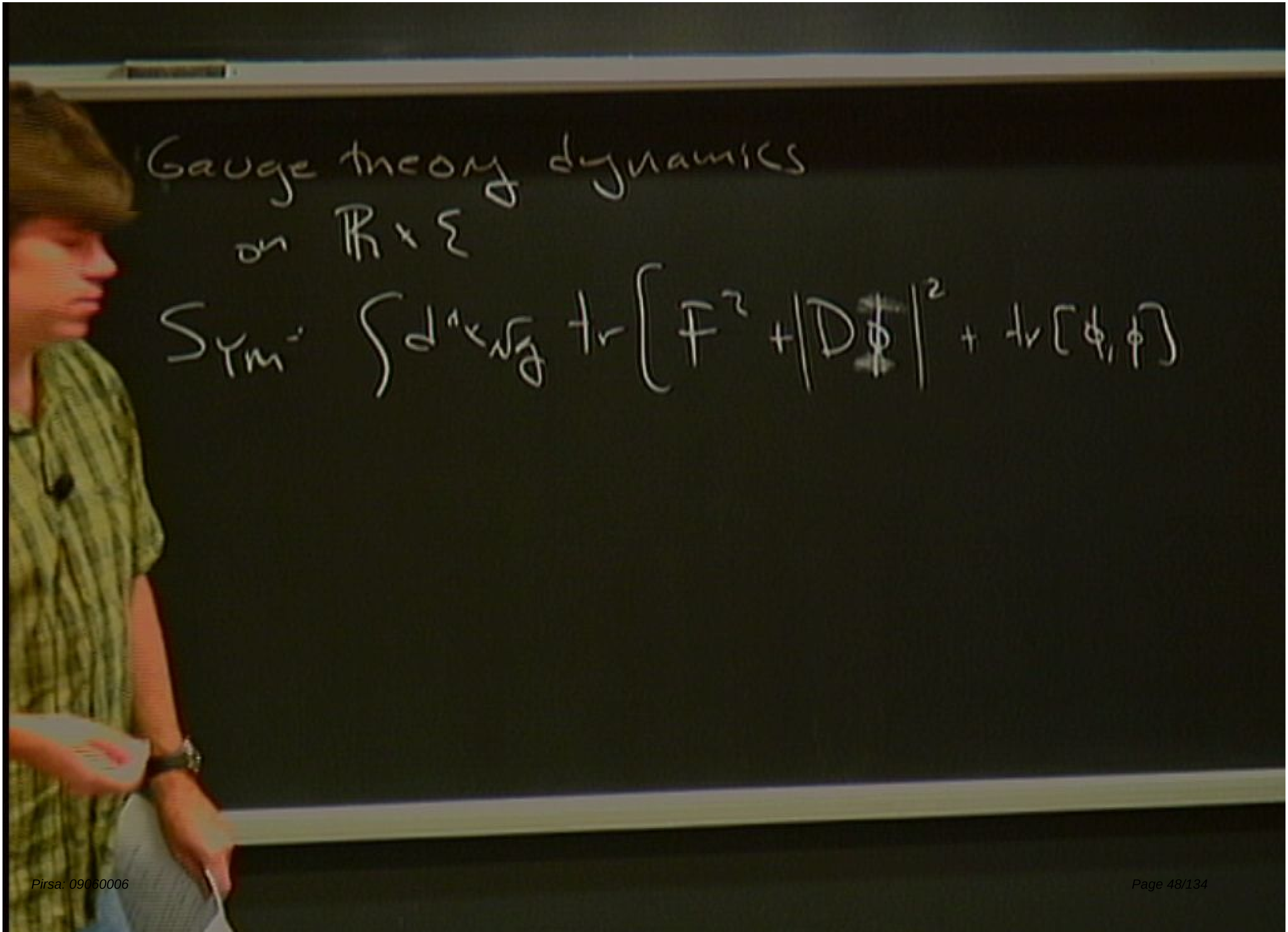
(1) $\mu > 0$



all of these $\xrightarrow{r\to\infty}$ $r^2(-dt^2 + d\Sigma^2)$

$\mathbb{R} \times \Sigma$


uge theory dynamics
on ℝ × Σ
Ym = ∫ d^n x √g tr [F^2 + |DΦ|^2


Gauge theory dynamics
on $\mathbb{R} \times \Sigma$
$S_{YM} = \int d^4x \sqrt{g}\, \mathrm{tr}\left[F^2 + |D\Phi|^2 + \mathrm{tr}[\phi, \phi]\right]$

B. Gauge theory dynamics

on $\mathbb{R} \times \Sigma$

$$S_{YM} = \int d^4x \sqrt{g}\, \mathrm{tr} \left(F^2 + |D\phi|^2 + ([\phi,\phi])^2 - R\, \mathrm{tr}\, \phi^2 \right)$$

B Gauge theory dynamics

on $\mathbb{R} \times \Sigma$

$$S_{YM} = \int d^4x \sqrt{g}\, \mathrm{tr}\left(F^2 + |D\phi|^2 + ([\phi,\phi])^2 - R\, \mathrm{tr}\, \phi^2 \right)$$

Σ

All modes except for

B. Gauge theory dynamics
on $\mathbb{R} \times \Sigma$

$$S_{YM} = \int d^n x \sqrt{g}\, \mathrm{tr}\left(F^2 + |D\phi|^2 + ([\phi,\phi])^2 - R\, \mathrm{tr}\,\phi^2 \right)$$

Σ compact: All modes <u>except</u> for 0-mode
(maybe few KK modes)
are unstable)

B. Gauge theory dynamics

on $\mathbb{R} \times \Sigma$

$$S_{YM} = \int d^n x \sqrt{g}\, \mathrm{tr} \left(F^2 + |D\phi|^2 + ([\phi,\phi])^2 - R\, \mathrm{tr}\, \phi^2 \right)$$

Σ (compact): All modes except for 0-mode

(maybe few KK modes are unstable)

B Gauge theory dynamics
on $\mathbb{R} \times \Sigma$

$$S_{YM} = \int d^n x \sqrt{g}\, \mathrm{tr}\left(F^2 + |D\phi|^2 + ([\phi,\phi])^2 - R\,\mathrm{tr}\,\phi^2 \right)$$

Σ
(compact: All modes except for 0-mode
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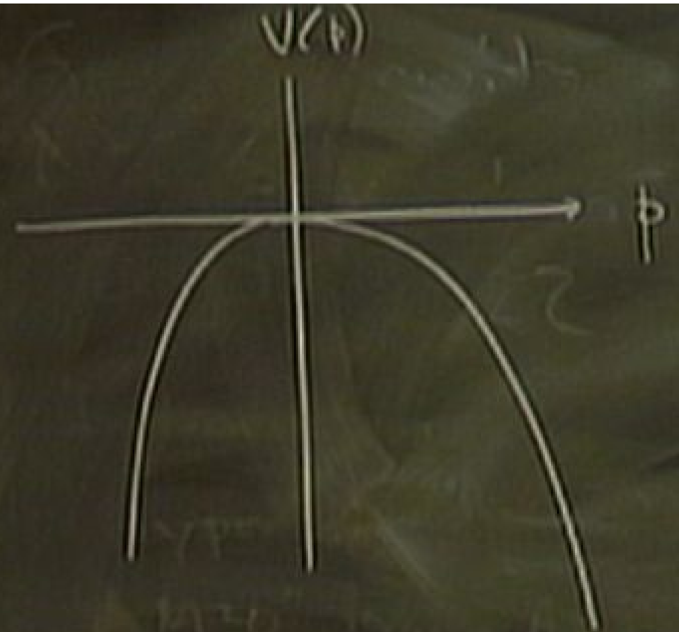
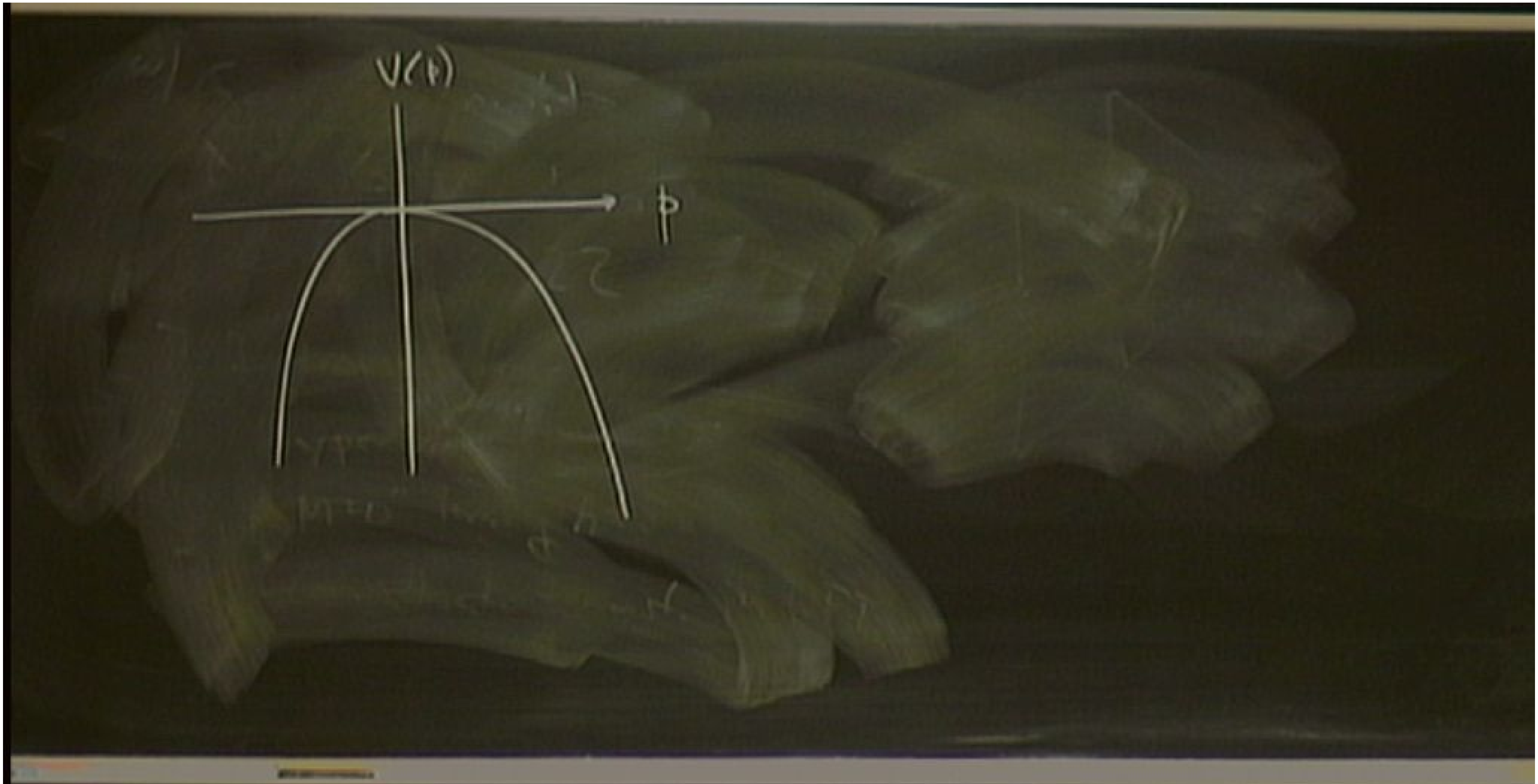
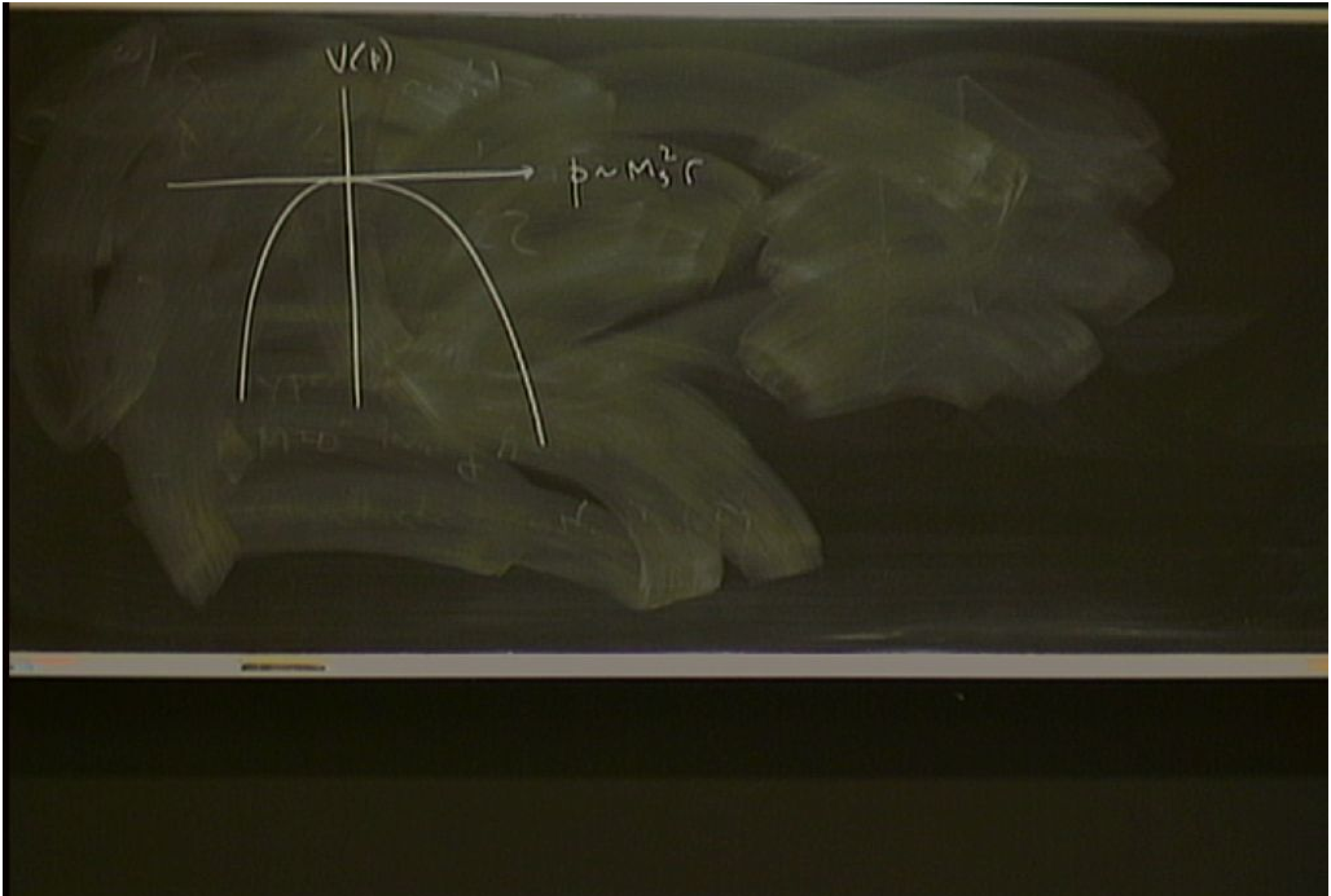
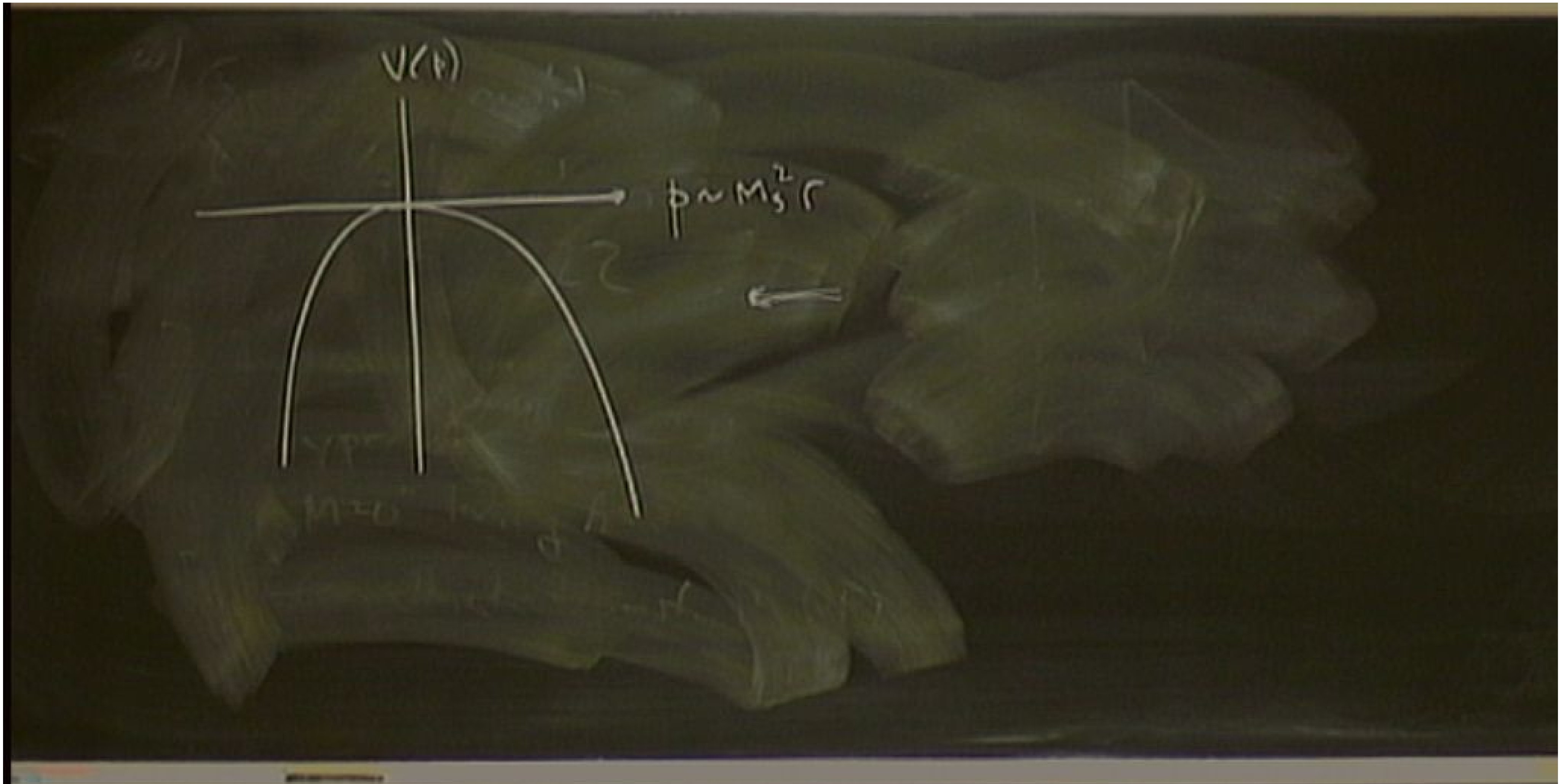
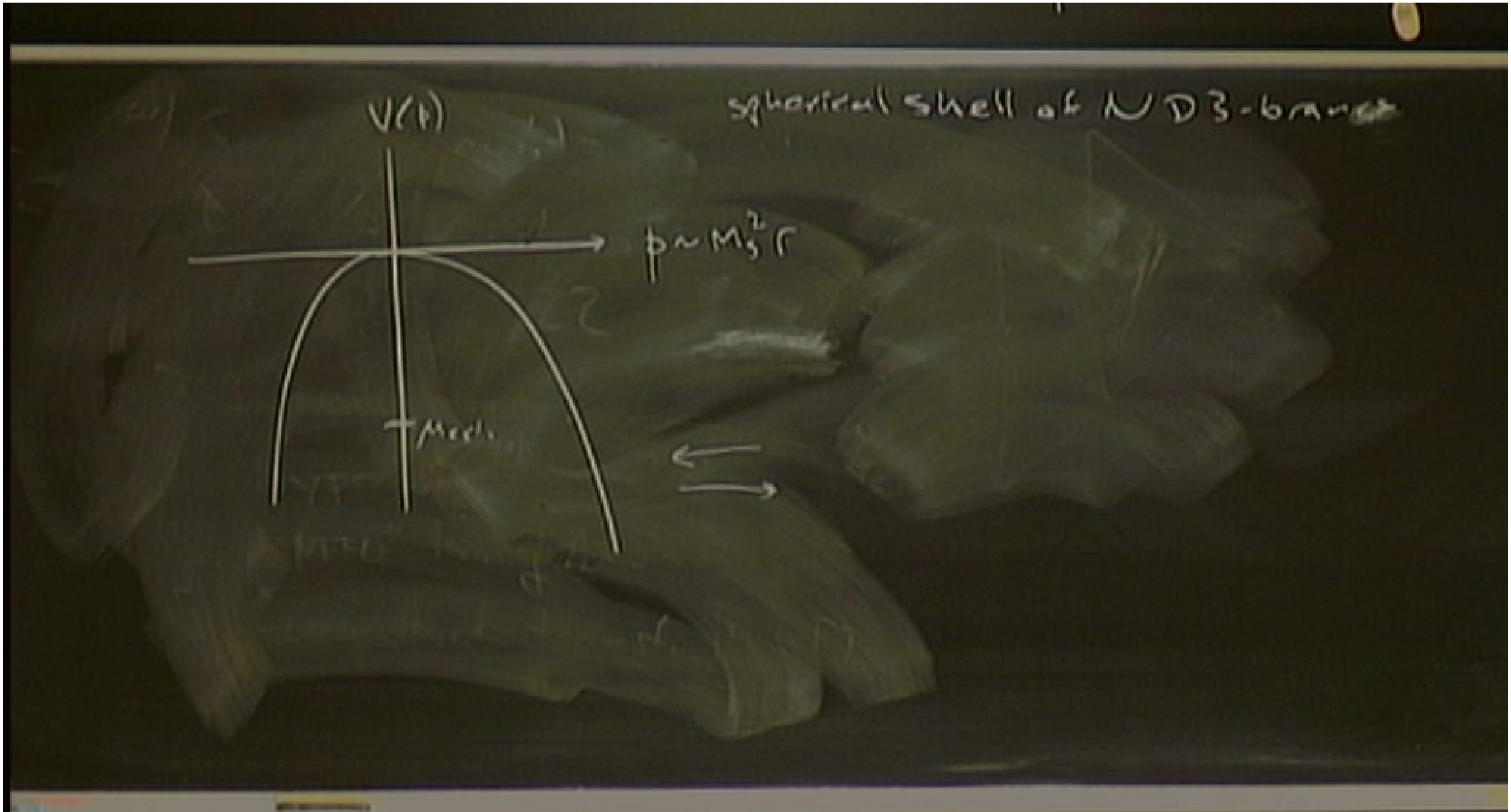

V(ϕ)
ϕ


V(φ)
φ


$V(p)$
$p \sim M_5^2 r$


V(φ)
φ ~ M_s^2 r

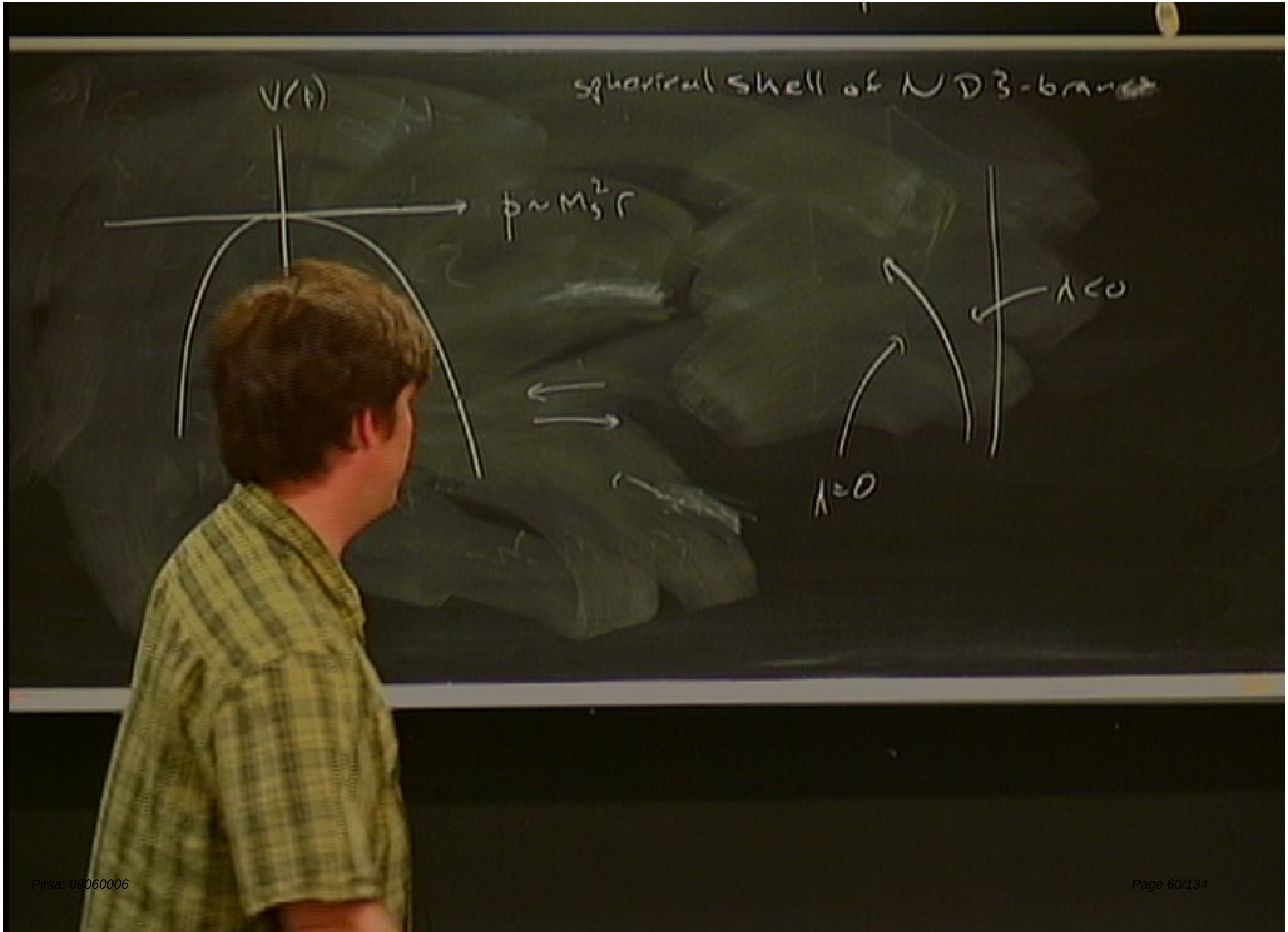

V(φ)
φ ~ M_s^2 r
spherical shell of N D3-branes

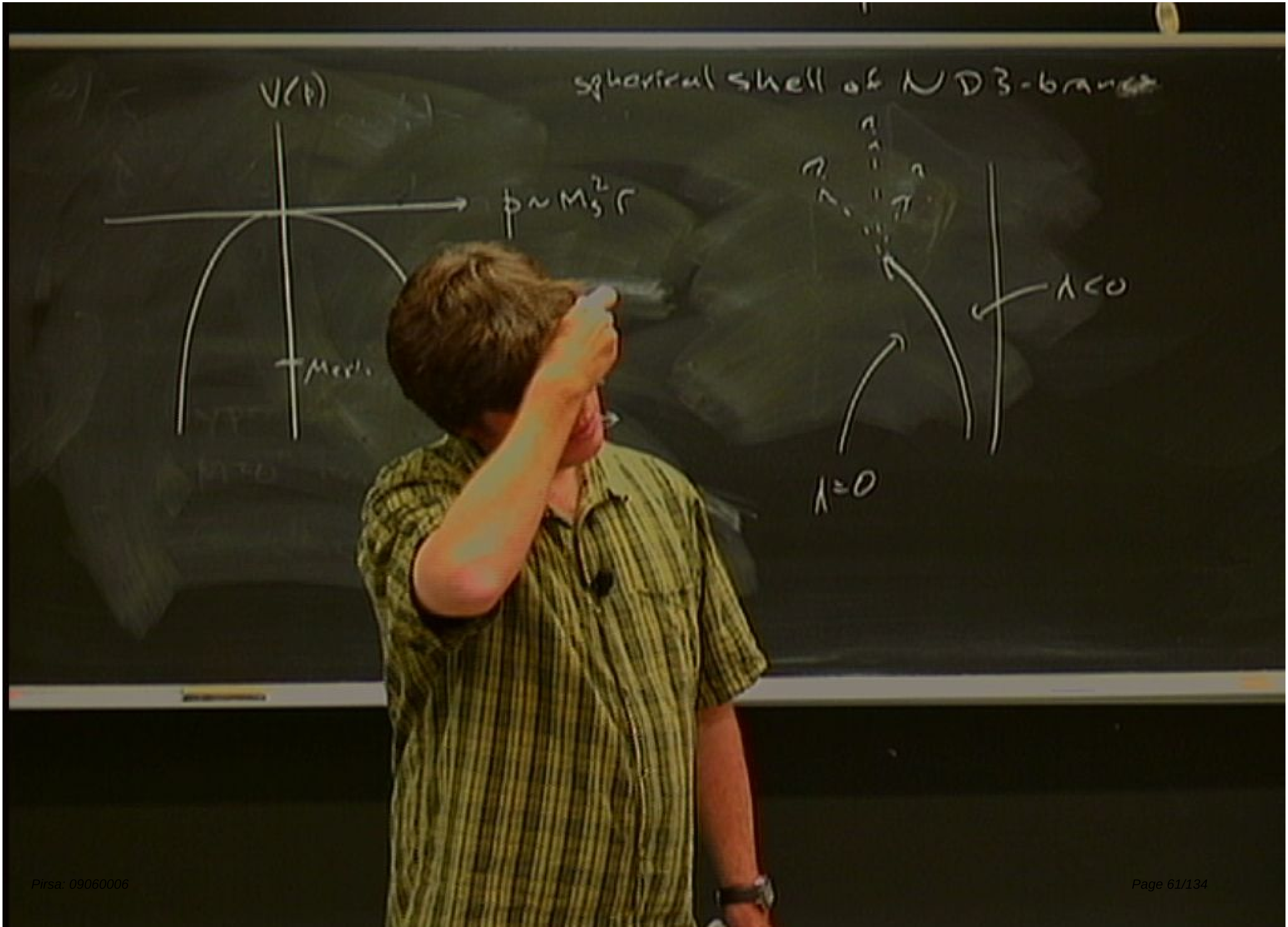
B. Gauge theory dynamics

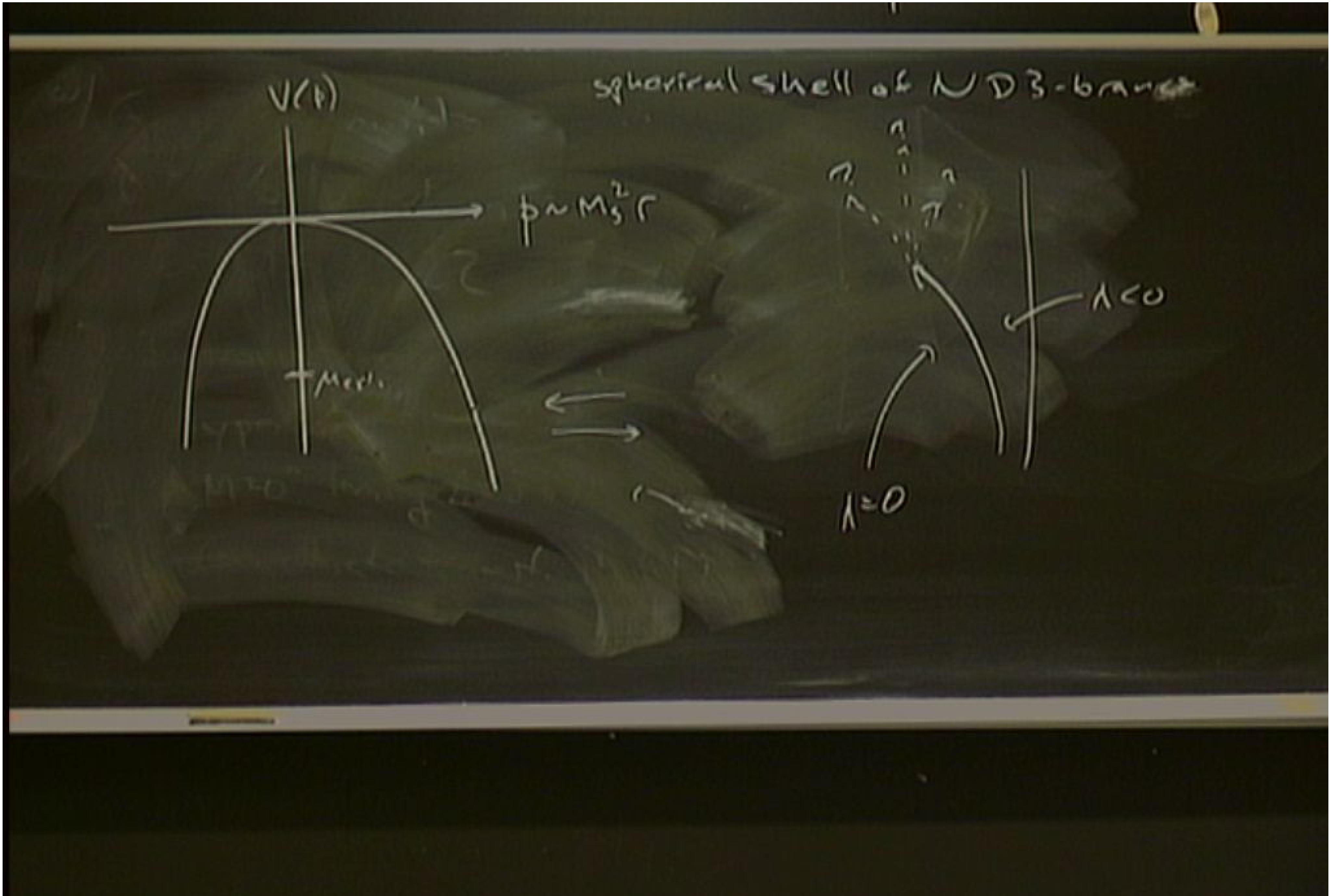
on $\mathbb{R}_t \times \Sigma$

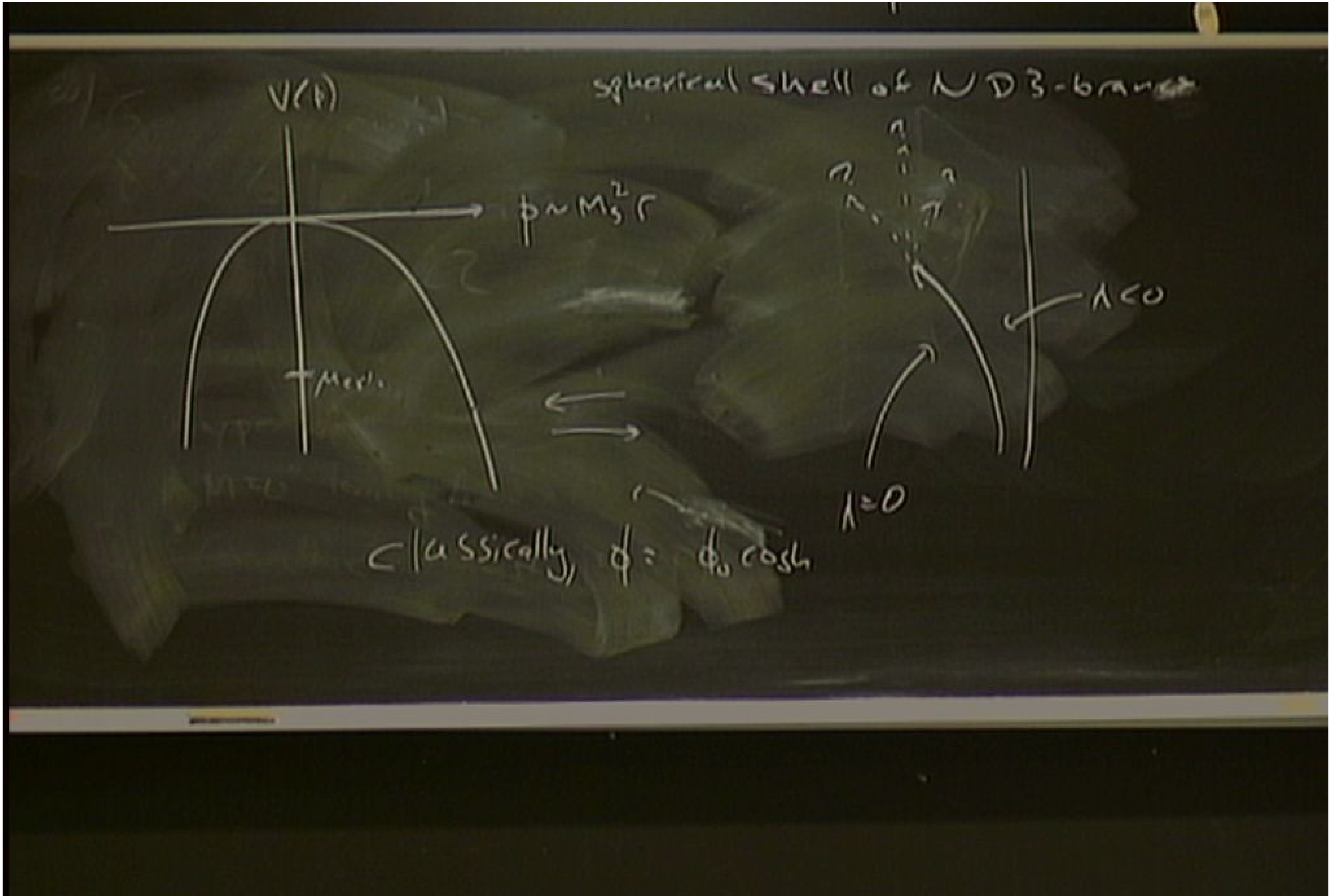
$$S_{YM} = \int d^4x \sqrt{g}\, \mathrm{tr}\left[F^2 + |D\Phi|^2 + \frac{1}{4}([\phi,\phi])^2 - R^{(4)} \mathrm{tr}\,\phi^2 \right)$$

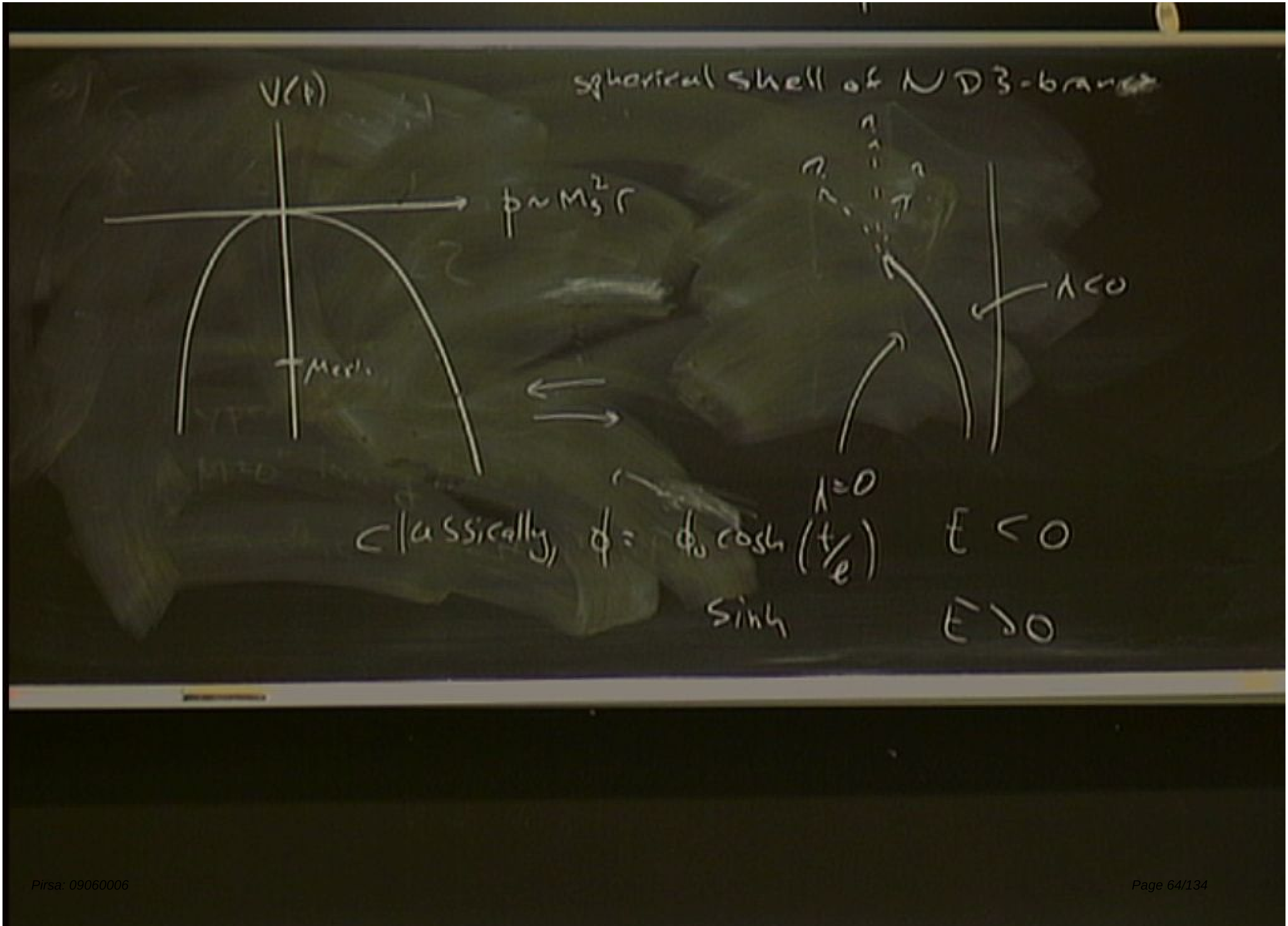
Σ compact: All modes <u>except</u> for 0-mode (maybe few KK modes) are stable)


spherical shell of N D3-branes
V(ϕ)
ϕ ~ M_s^2 r
λ < 0
λ = 0


V(ϕ)
ϕ~M_s^2 r
spherical shell of N D3-branes
λ<0
λ=0

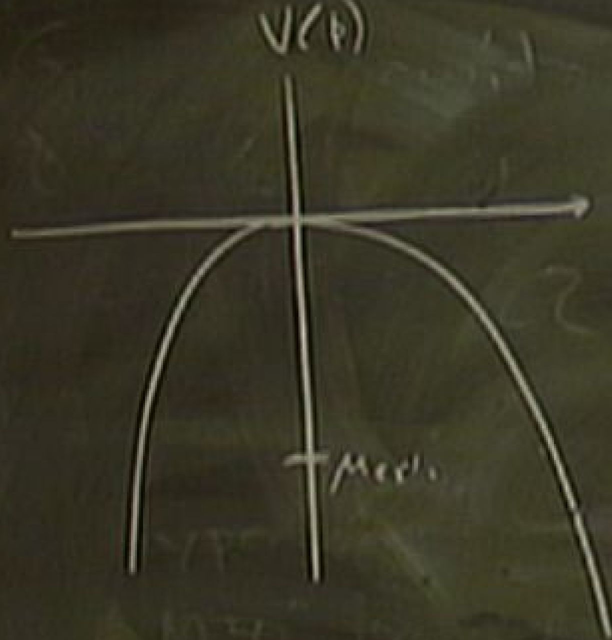

spherical shell of N D3-branes
V(p)
p ~ M_s^2 r
λ<0
λ=0

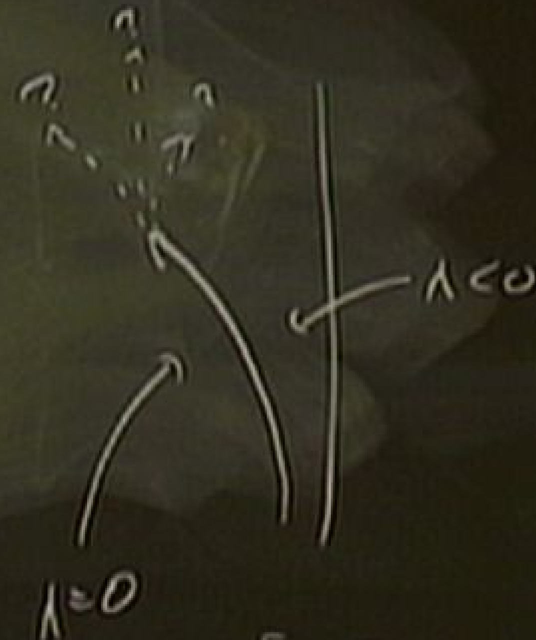

spherical shell of N D3-branes
V(ϕ)
ϕ ~ M_s^2 r
λ<0
λ=0
classically, ϕ = ϕ_0 cosh



spherical shell of N D3-branes
V(φ)
φ ~ M_s^2 r
Merl.
λ<0
λ=0
classically, φ = φ_0 cosh(t/ℓ)   E<0
sinh   E>0

spherical shell of N D3-branes





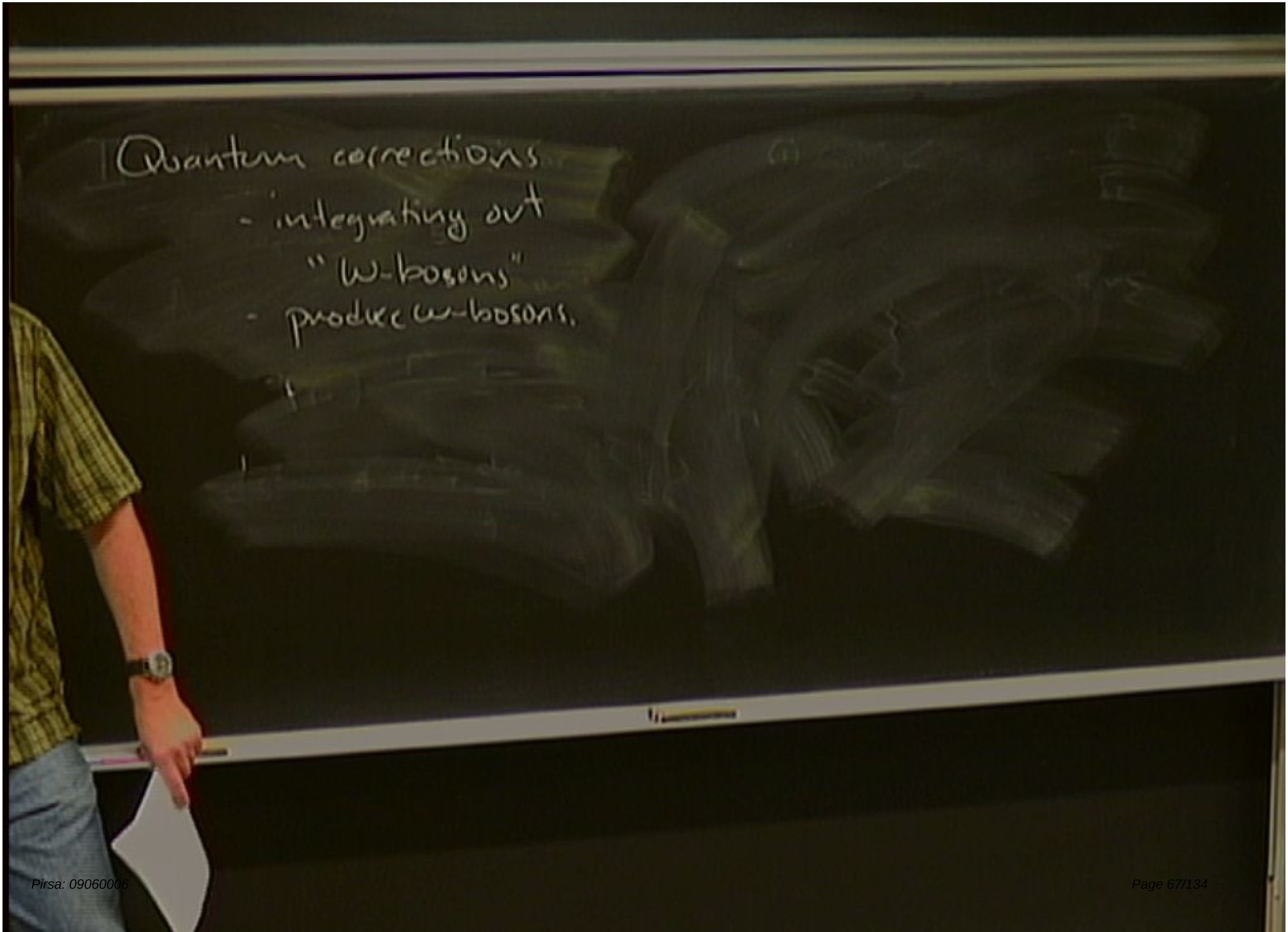
classically, $\phi = \phi_0 \cosh(t/\ell)$ $\quad E < 0$

$\sinh$ $\quad E > 0$

Quantum corrections

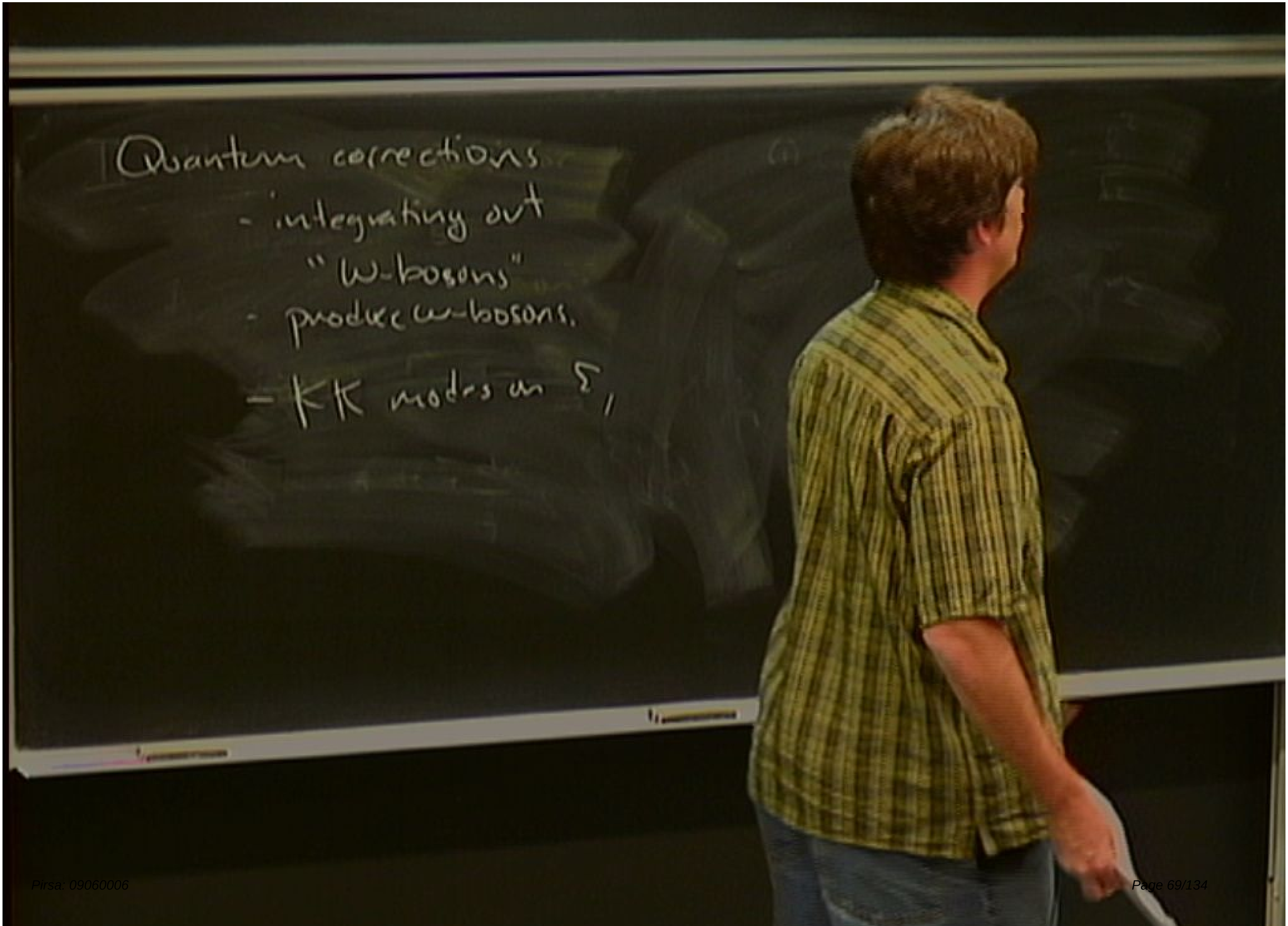
- "integrating out

"W-bosons"


Quantum corrections
- integrating out
"W-bosons"
- produce W-bosons.

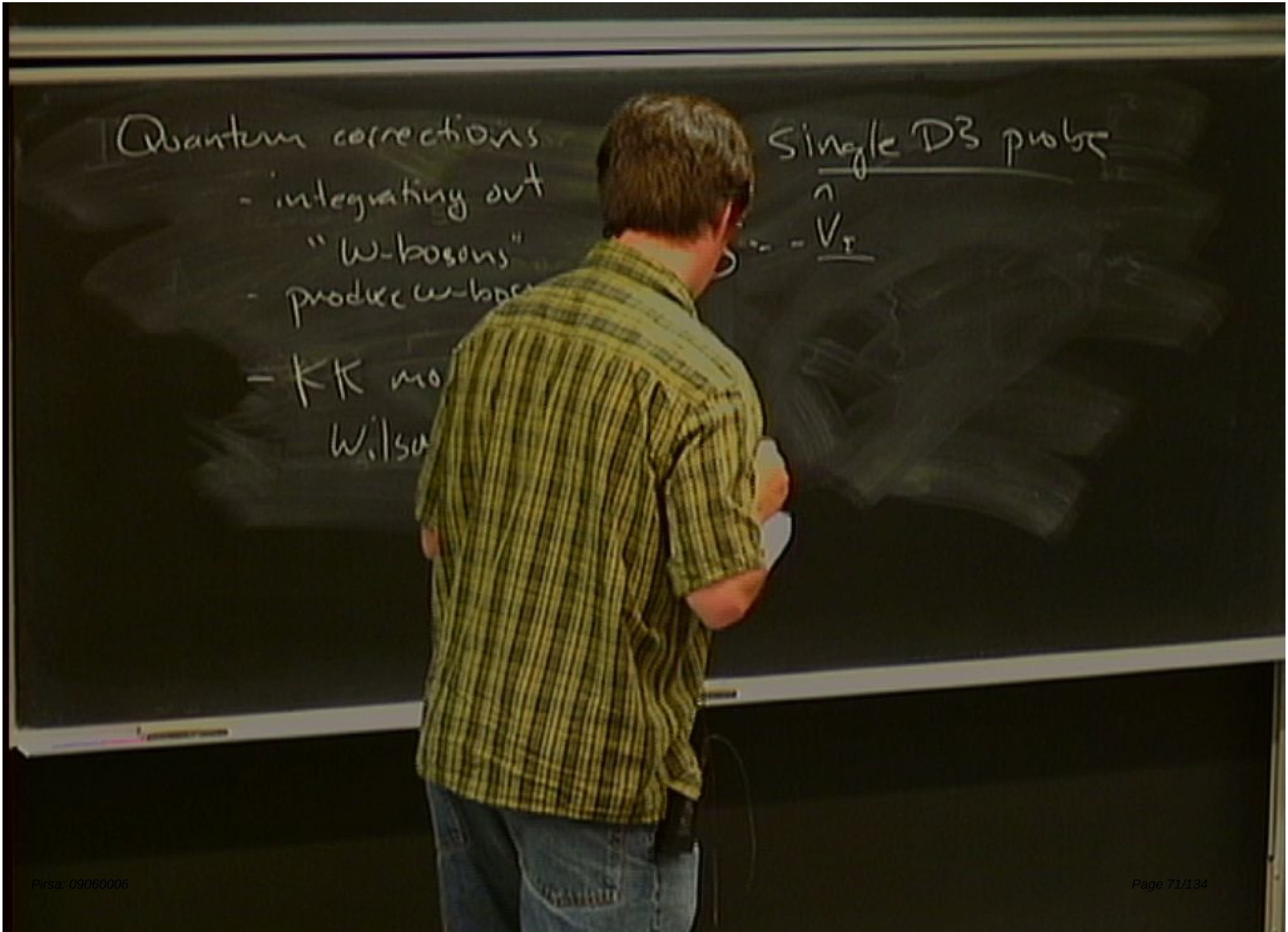
Quantum corrections

- integrating out "W-bosons"
- produce W-bosons.


Quantum corrections
- integrating out
"W-bosons"
- produce W-bosons.
- KK modes on Σ,

Quantum corrections

- integrating out "W-bosons"
- produce W-bosons.
- KK modes on Σ,
  Wilson lines on Σ


Quantum corrections
- integrating out
"W-bosons"
single D3 probe

Quantum corrections

- integrating out "W-bosons"
- produce W-bosons.
- KK modes on Σ, Wilson lines on Σ

Single D3 probe

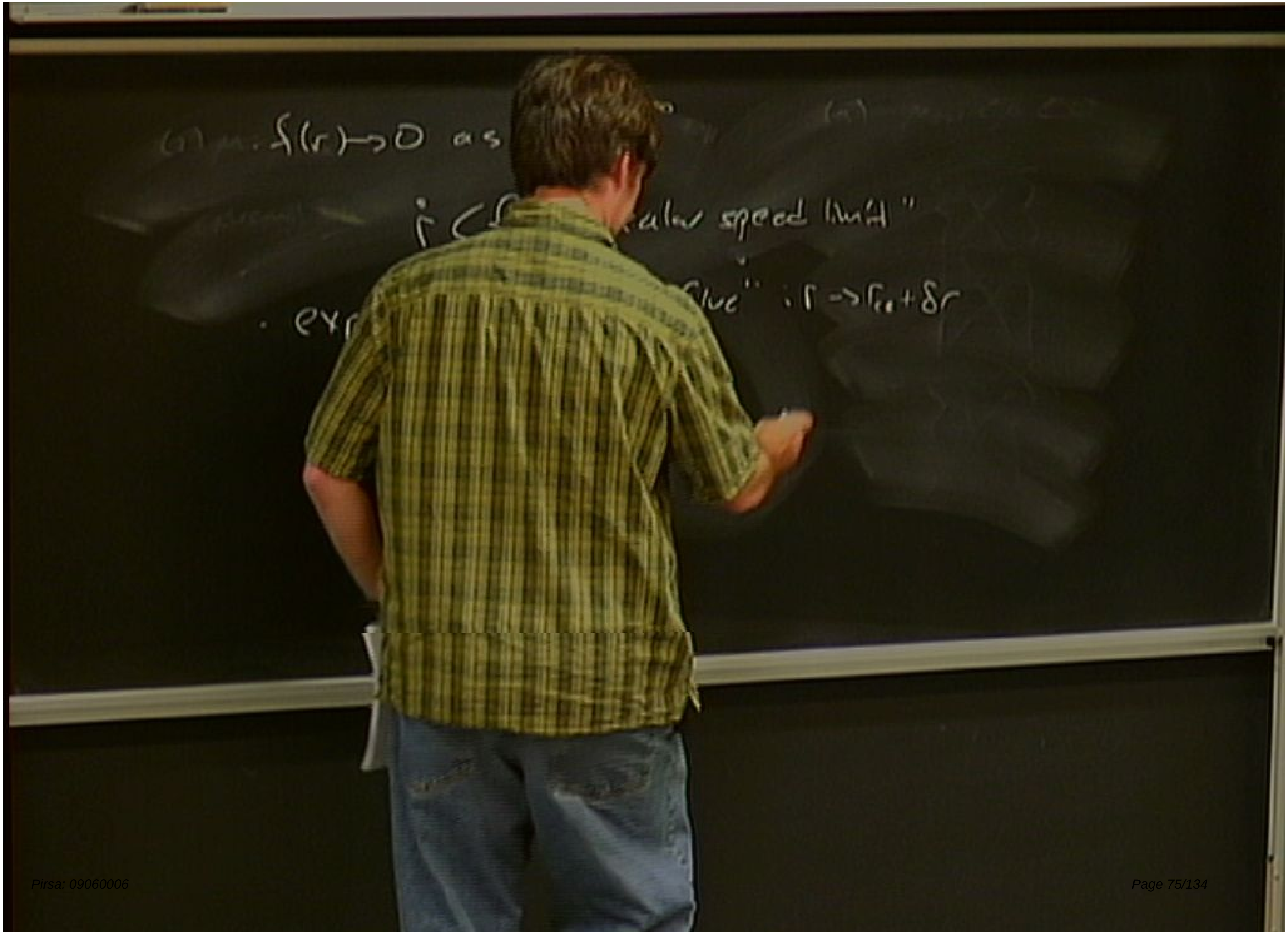
$$S = -\frac{\hat{V}_3}{g_s(\alpha')^2}\int dt\left[-r^4\sqrt{f(r)-\frac{\dot{r}^2}{f(r)}}+\frac{r^4-r_h^4}{\ell^4}\right]$$

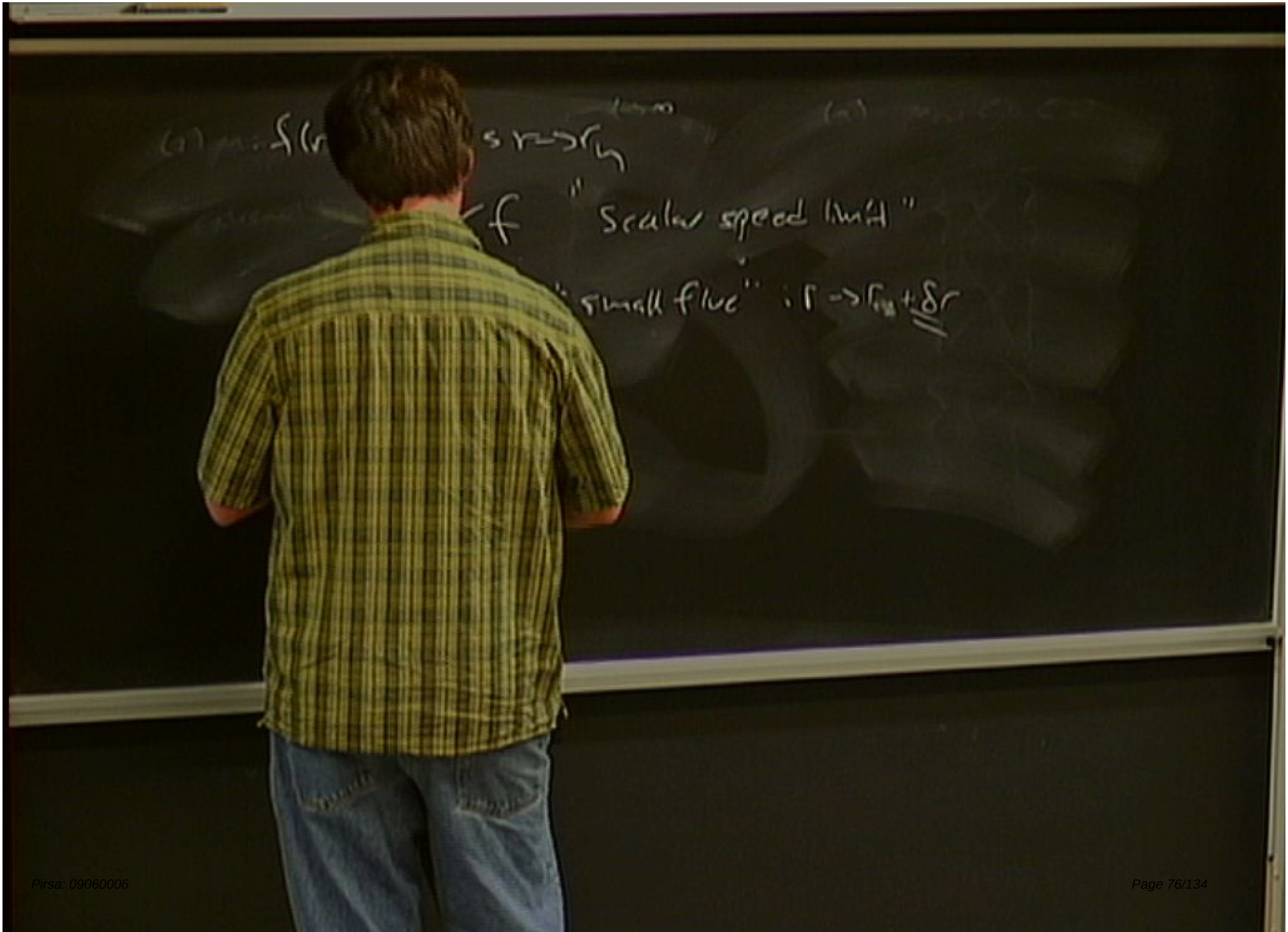
$f(r) \to 0$ as $r \to r_h$

$\dot{r} < f$ "Scalar speed limit"

$f(r) \to 0$ as $r \to r_h$

$\dot{r} < f$ "Scalar speed limit"


f(r) → 0 as
speed limit"
r → r₀ + δr

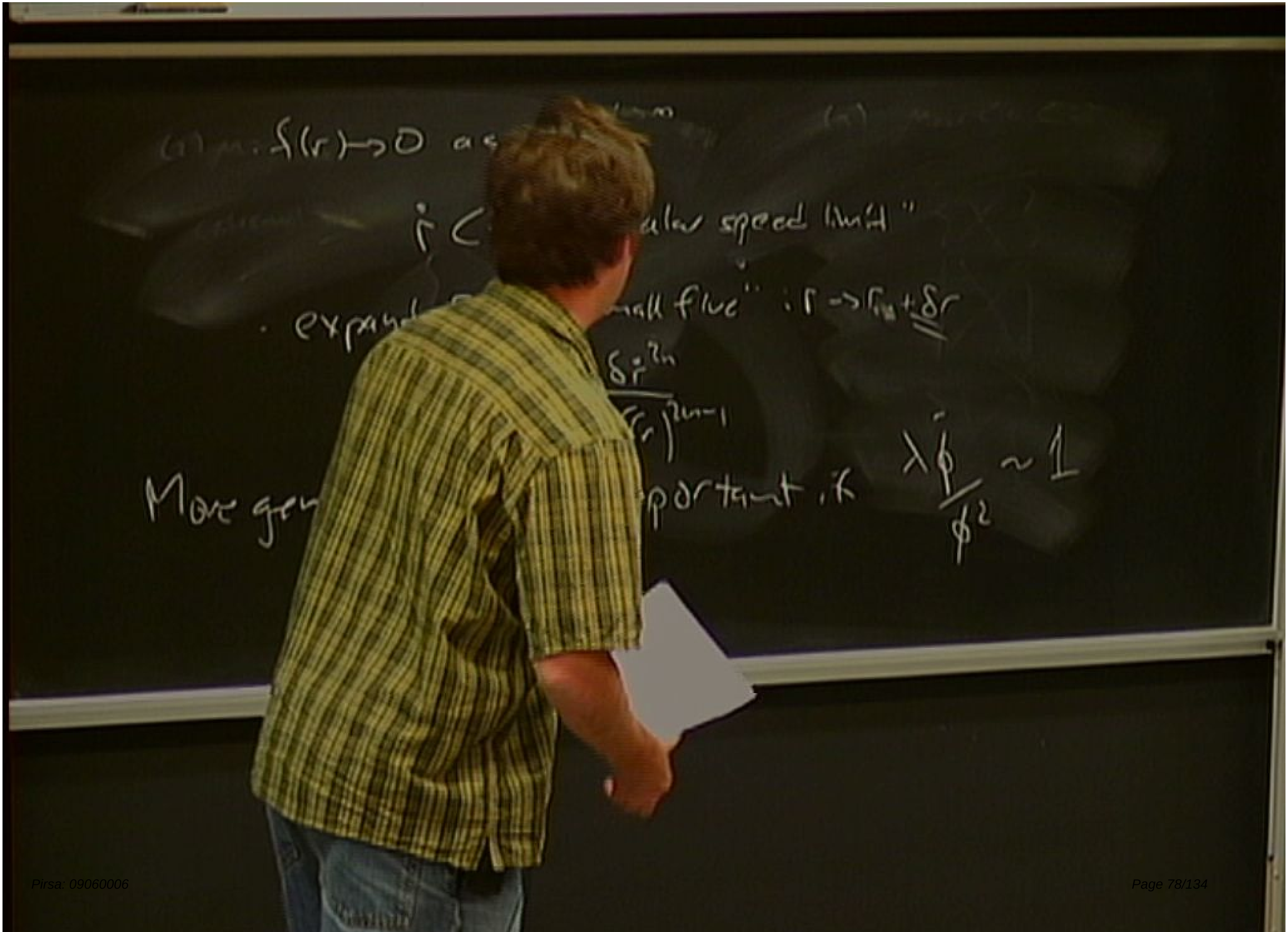

"Scalar speed limit"

(a) ... $f(r) \to 0$ as $r \to r_h$

$\dot{r} < f$ "scalar speed limit"

· expand S in "small flve" : $r \to r_h + \underline{\delta r}$

$$L \sim \sum_n \frac{\delta \dot{r}^{2n}}{f(r)^{2n-1}}$$


f(r) → 0 as
ular speed limit"
expand
mall five"
More gen
portant if
~ 1

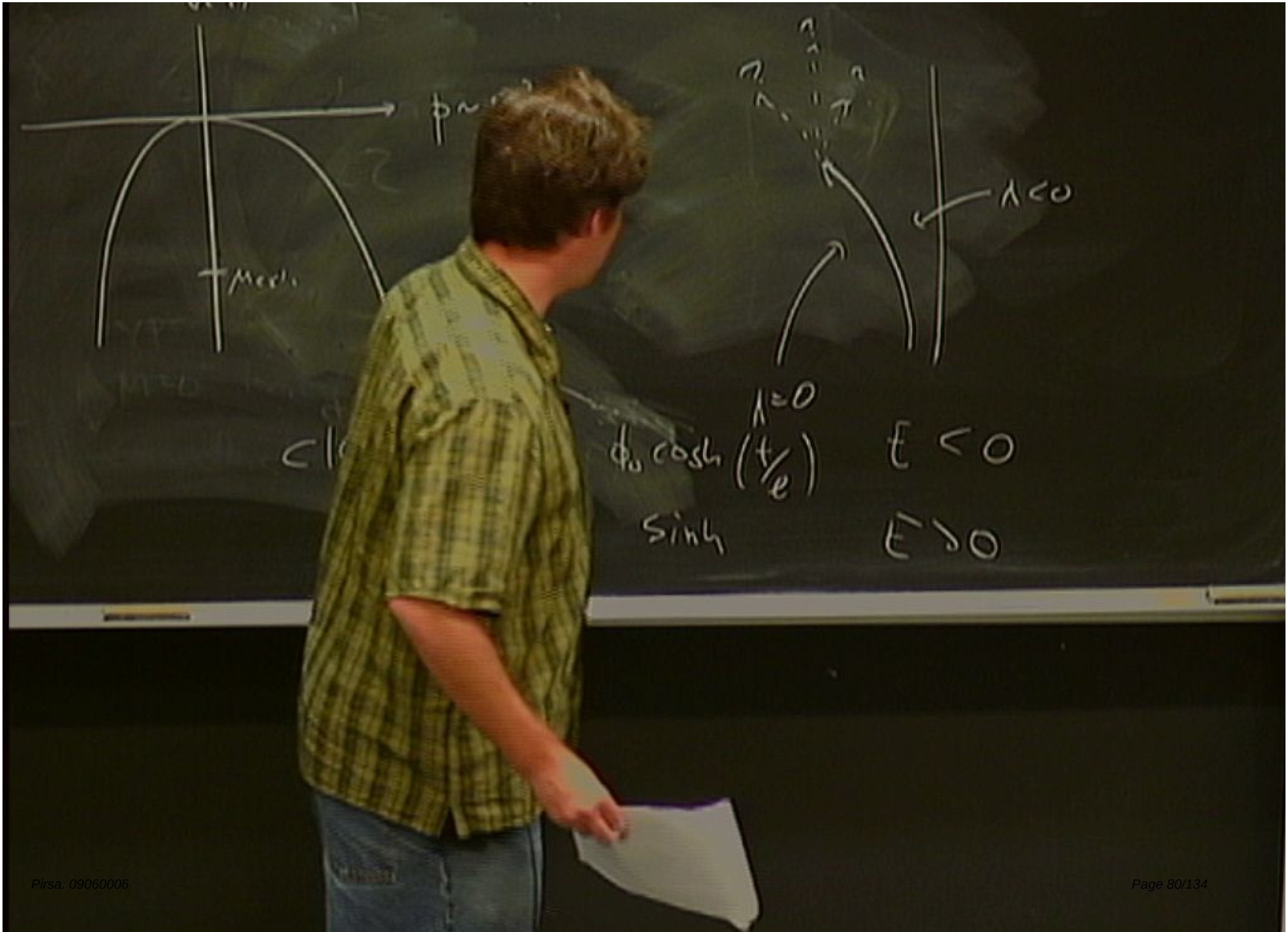
(1) $f(r) \to 0$ as $r \to r_h$

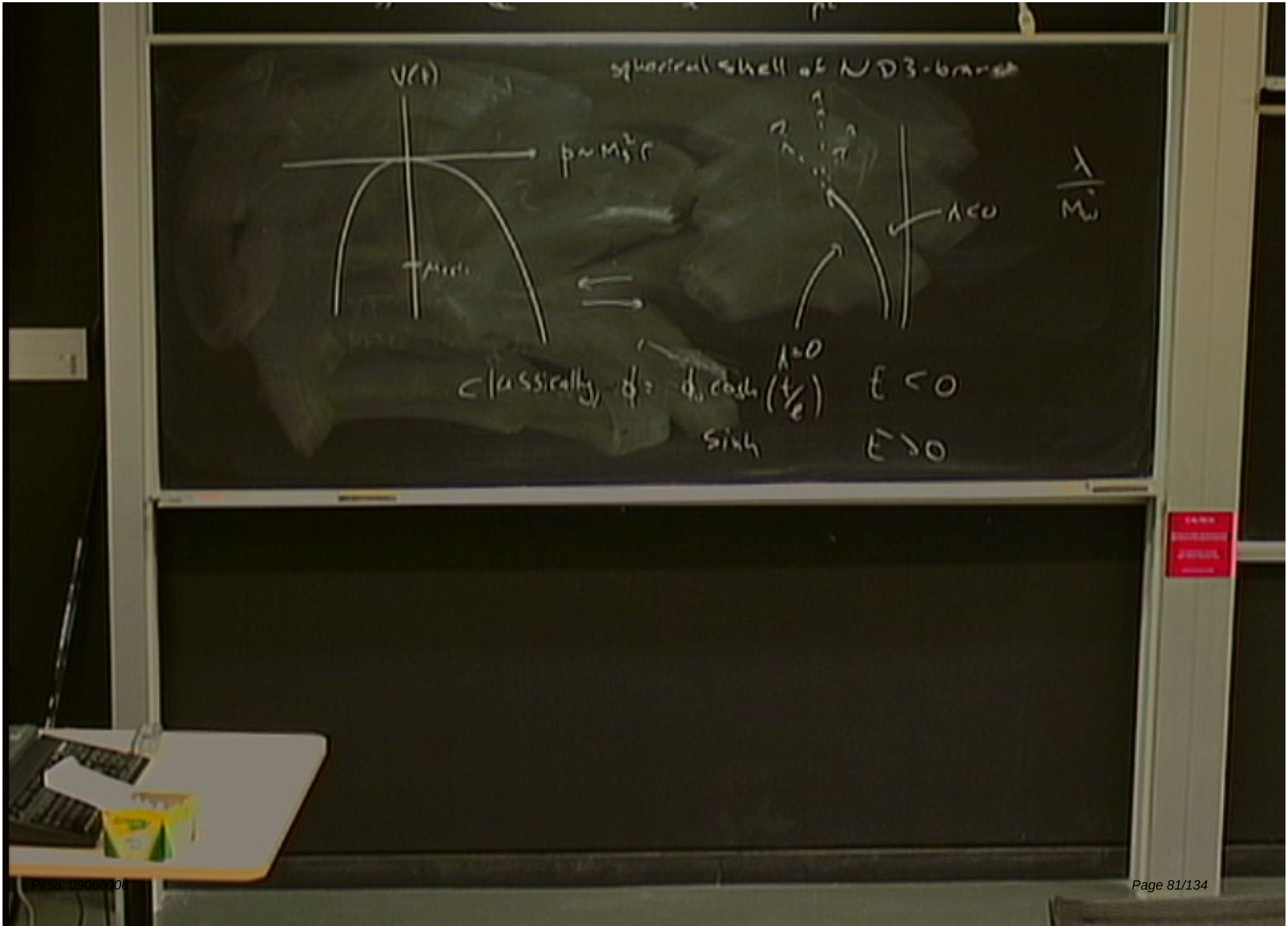
$$\dot{r} < f \quad \text{"scalar speed limit"}$$

- expand S in "small flue": $\dot{r} \to r_{h} + \delta \dot{r}$

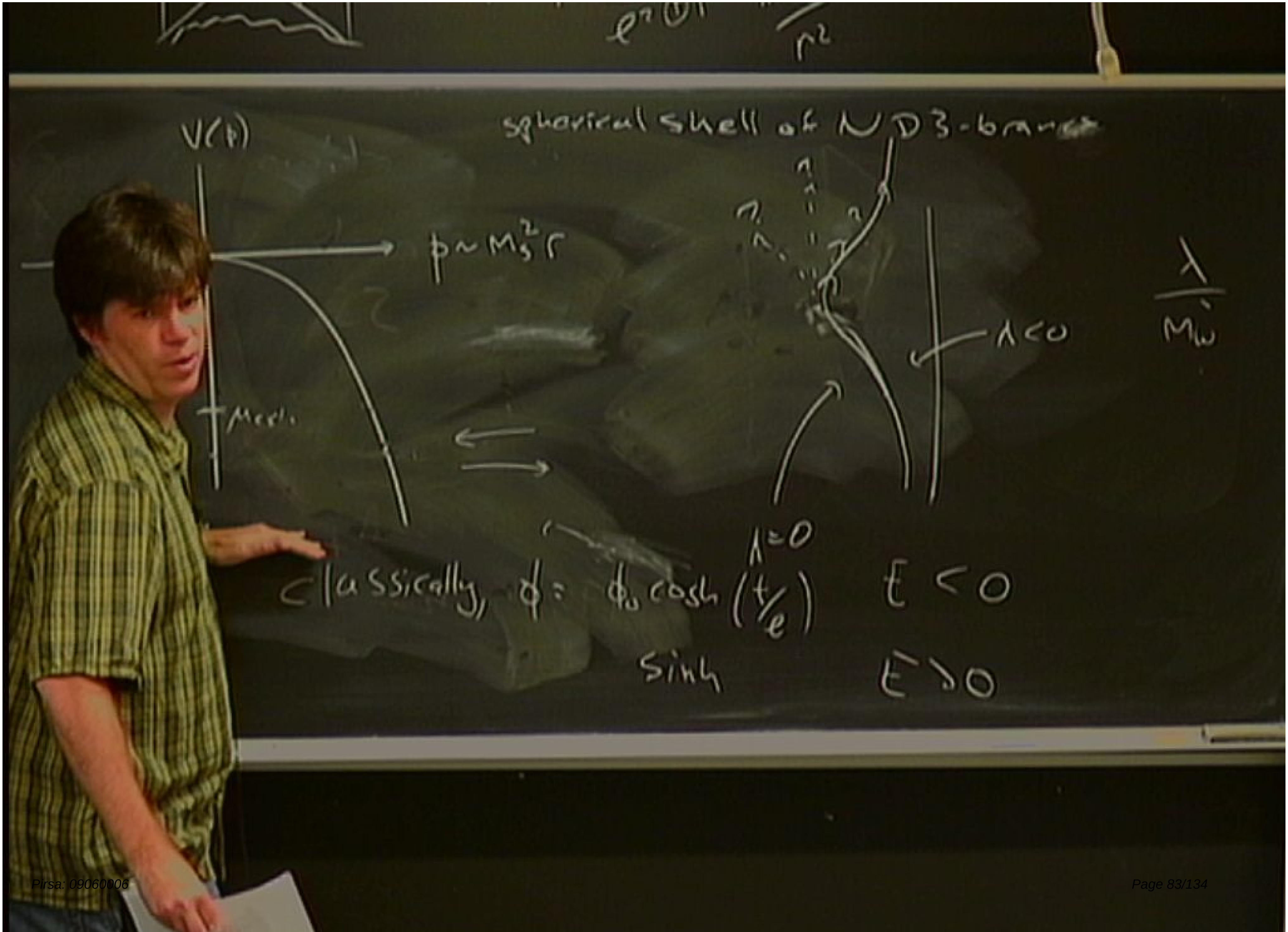
$$L \sim \sum_n \frac{\delta \dot{r}^{2n}}{f(r)^{2n-1}}$$

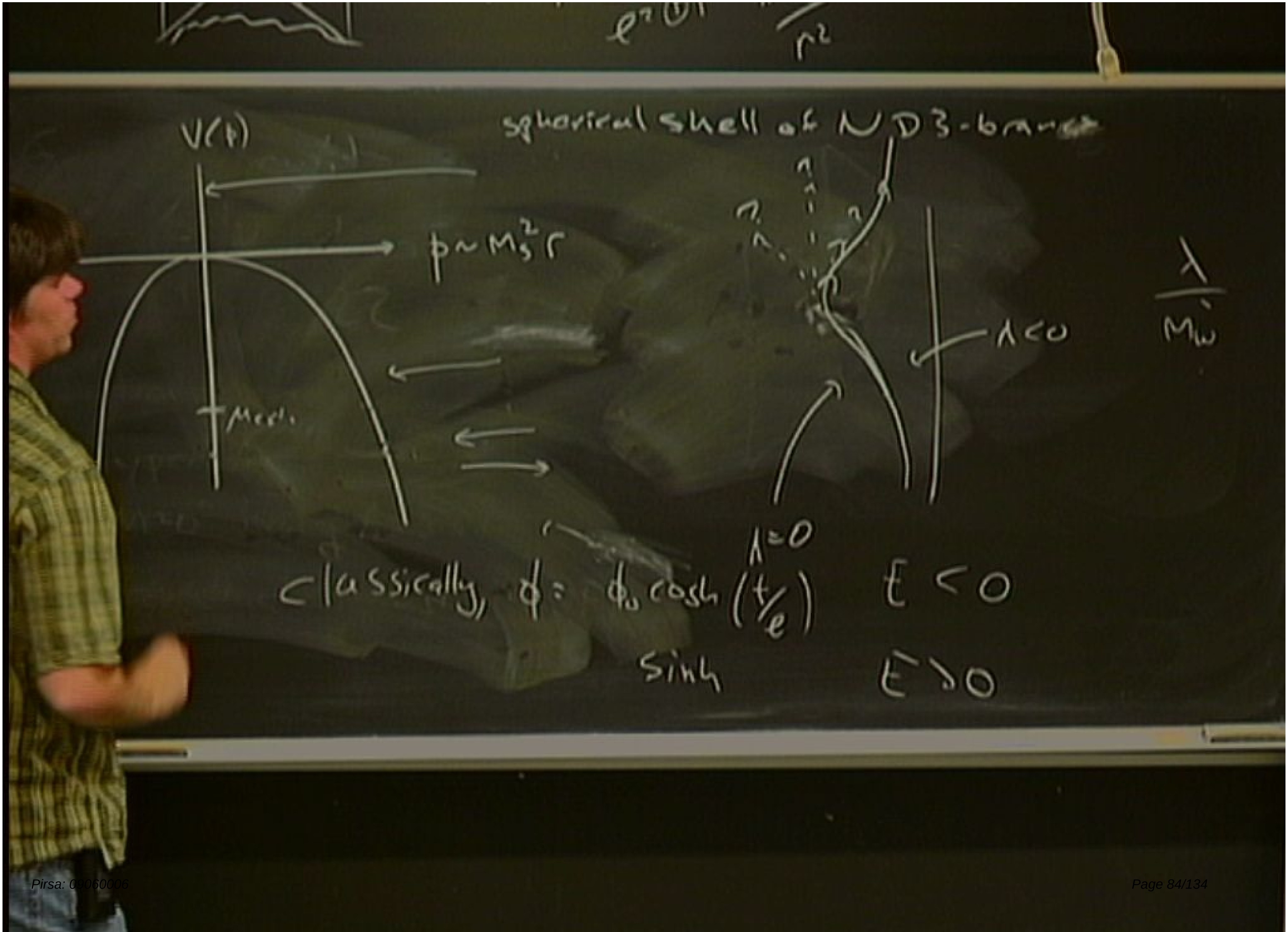
More generally, loops important if $\lambda \frac{\dot{\phi}}{\phi^2} \sim 1$

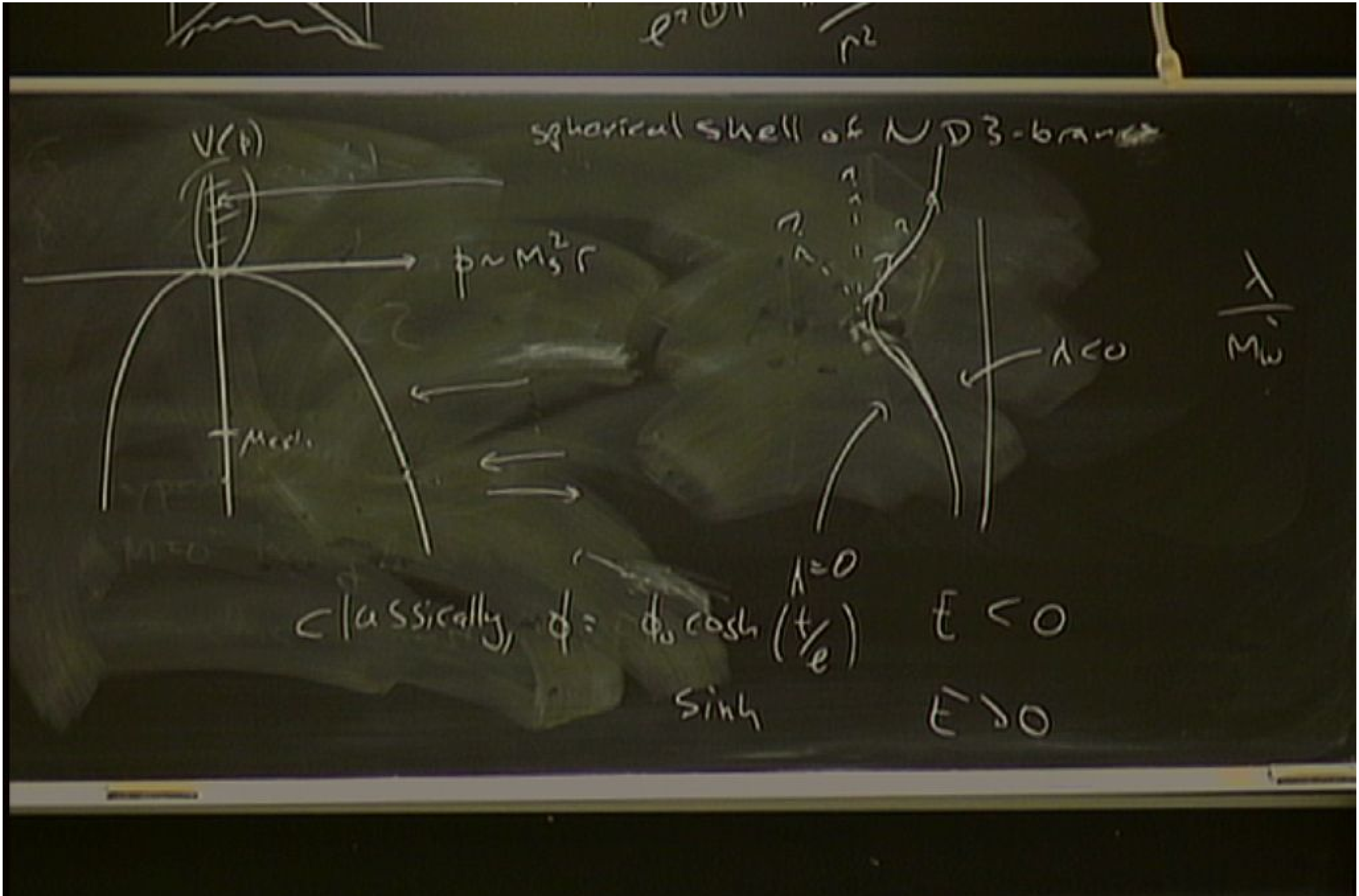

$\Lambda < 0$
$\Lambda = 0$
$\phi_0 \cosh(t/\ell)$
sinh
$E < 0$
$E > 0$


spherical shell of N D3-branes
V(φ)
φ ~ M_s^2 r
λ<0
λ>0
λ / M_w
classically, φ = φ_0 cosh(t/ℓ)   E < 0
sinh   E > 0

$\mu < \mu_{ext}$: loops never important.

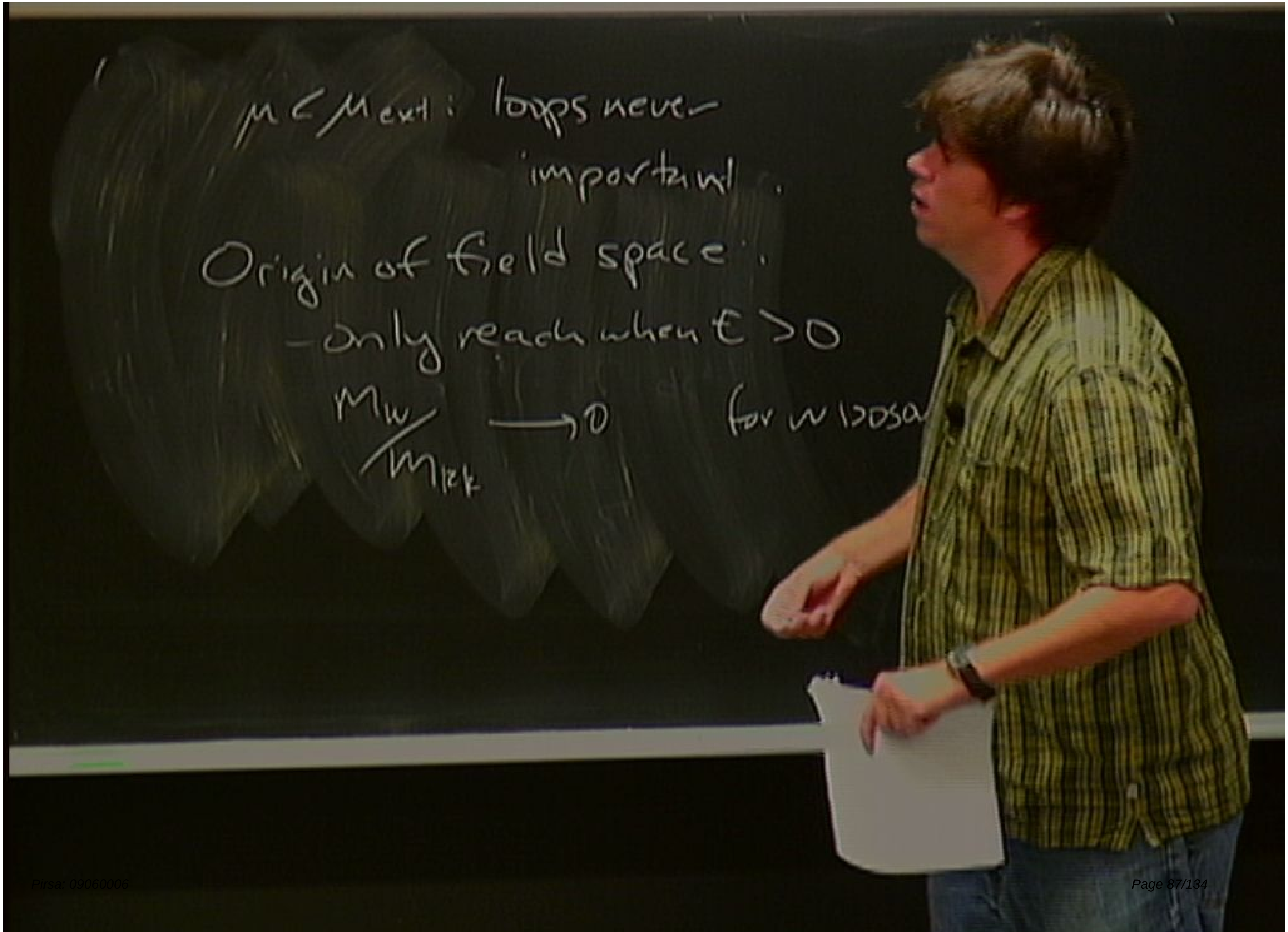

V(ϕ)
ϕ ~ M_s^2 r
spherical shell of N D3-branes
λ<0
λ/M_w
λ=0
classically, ϕ = ϕ_0 cosh(t/ℓ)
E < 0
sinh
E > 0


spherical shell of N D3-branes
V(φ)
φ ~ M_s^2 r
λ<0
λ/M_W
λ=0
classically, φ = φ_0 cosh (t/ℓ)   E < 0
sinh   E > 0


spherical shell of N D3-branes
V(ϕ)
ϕ ~ M_s^2 r
Mech
λ<0
λ/M_ω
λ=0
classically, ϕ = ϕ_0 cosh (t/ℓ)   E < 0
sinh   E > 0

$\mu < \mu_{ext}$: loops never important.

Origin of field space.


μ < μext : loops never important.
Origin of field space:
- only reach when ε > 0
mW/mKK → 0
for w

$\mu < M_{ext}$: loops never important.

Origin of field space :

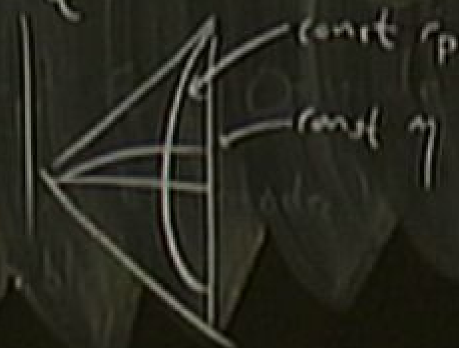
- only reach when $\epsilon > 0$

$M_W / M_{KK} \longrightarrow 0$ for W bosons

III. M = 0 black hole

A. An orbifold of AdS5

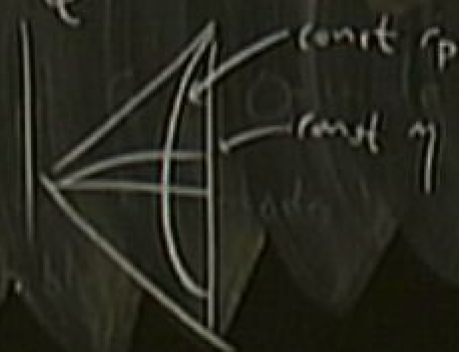
(1) $$ds^2 = \frac{r_p^2}{\ell^2}\left(-d\eta^2 + d\vec{x}_3^2\right) + \frac{\ell^2}{r_p^2}\,dr_p^2 + \ldots$$



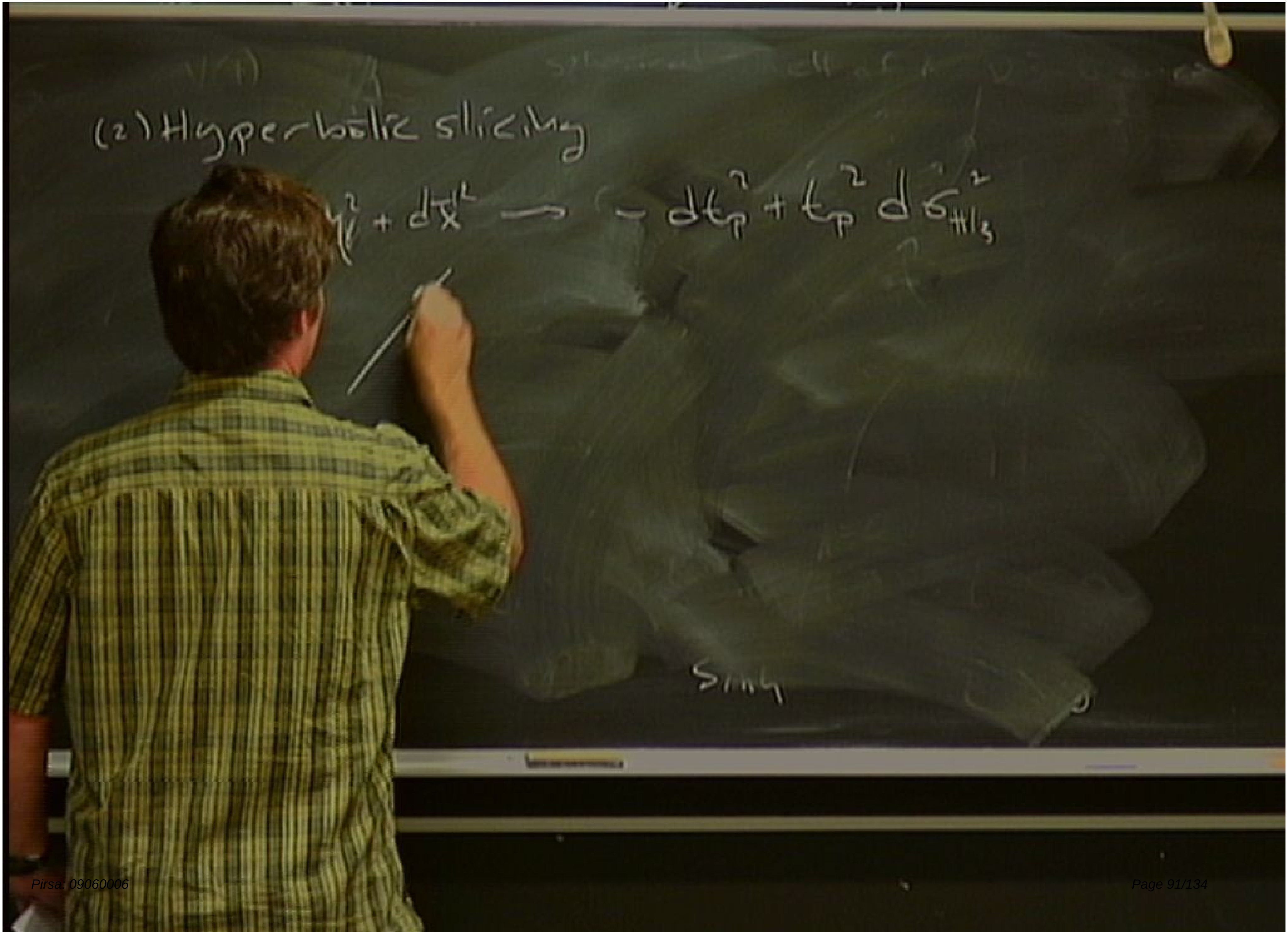
III. $M=0$ black hole

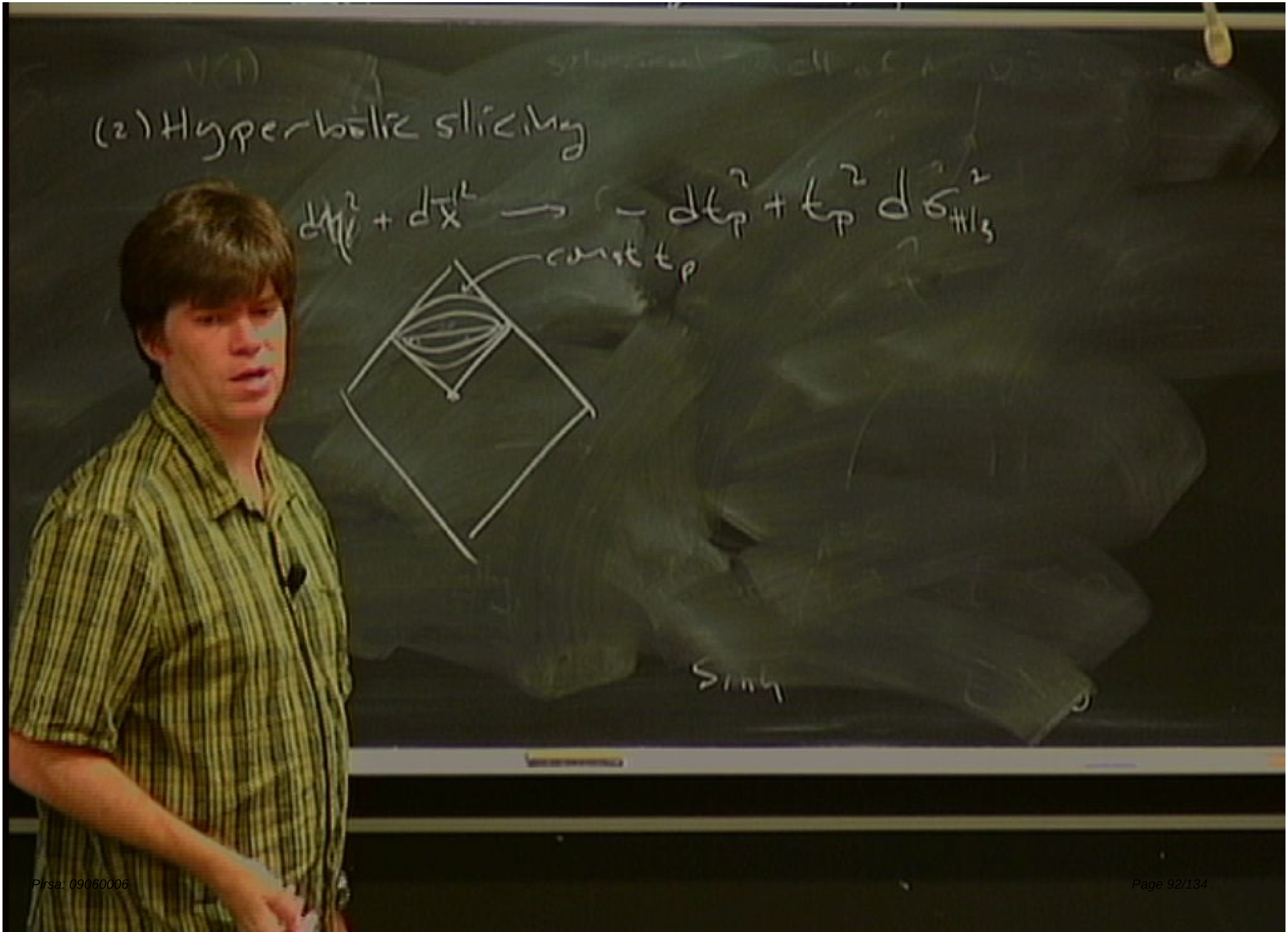
A. An orbifold of AdS5

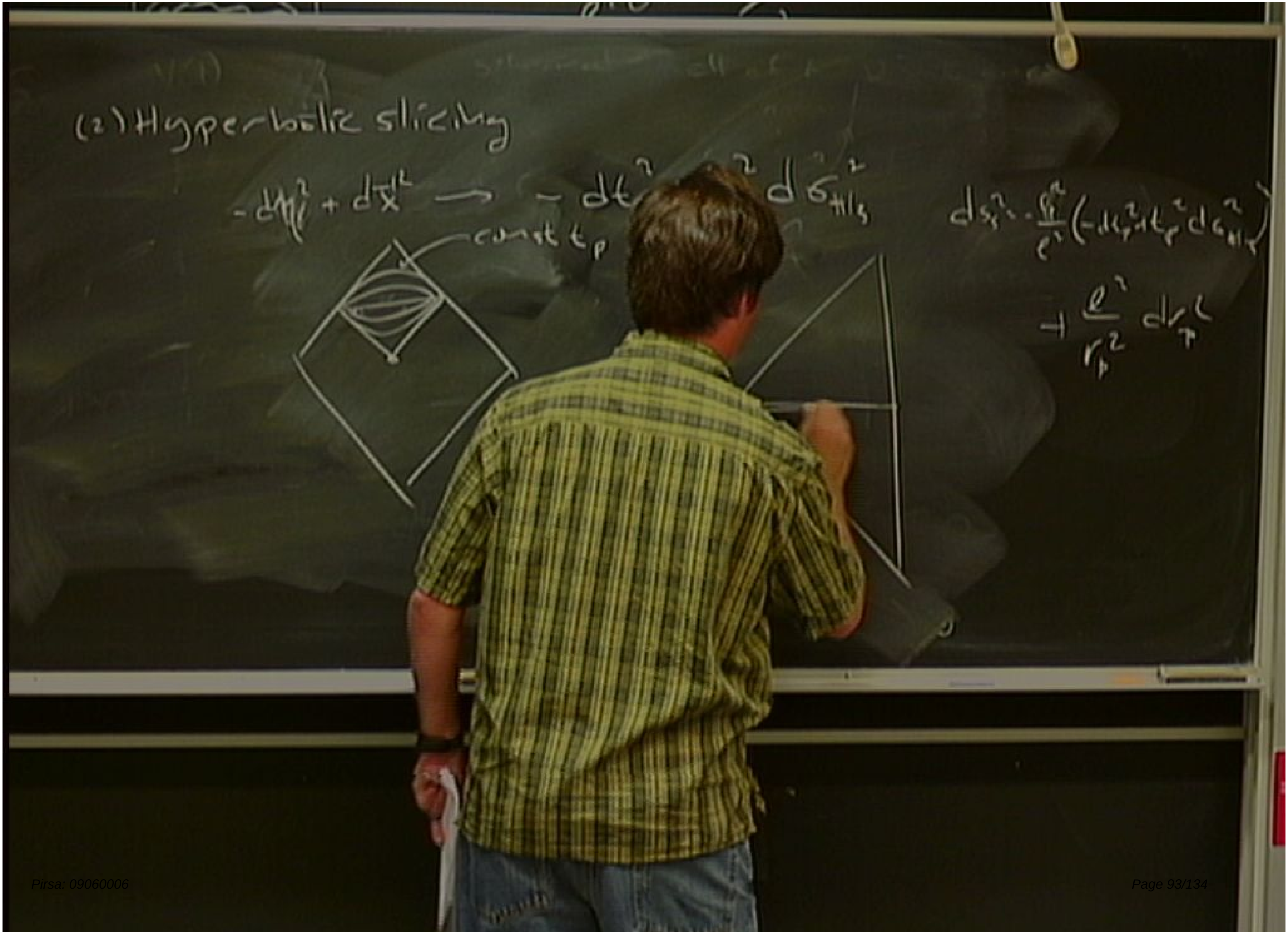
$$(1) \qquad ds^2 = \frac{r_p^2}{\ell^2}\Big(\overbrace{-d\eta^2 + d\vec{X}_3^2}^{\mathbb{R}^4}\Big) + \frac{\ell^2}{r_p^2}\, dr_p^2 + \cdots$$

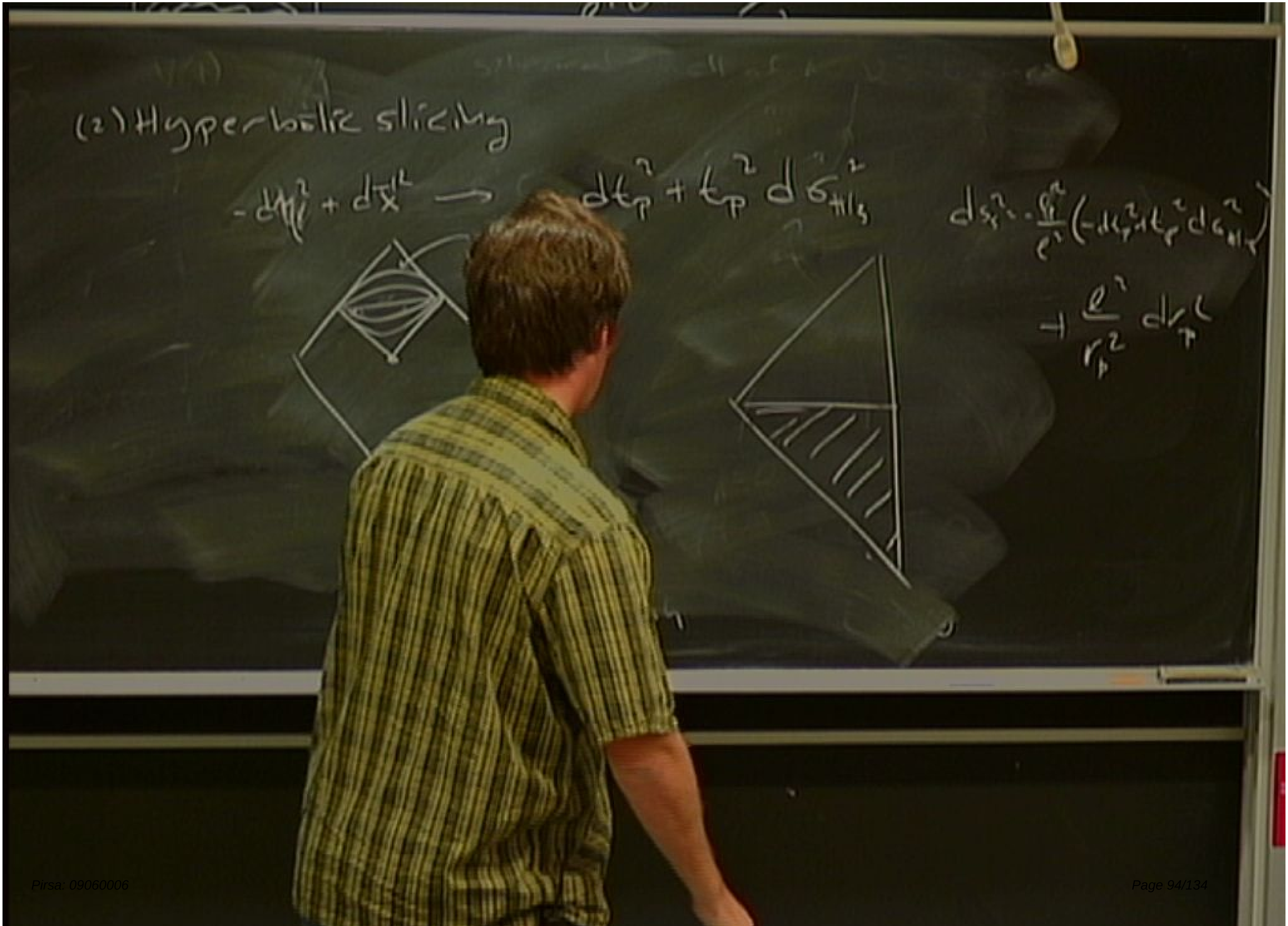


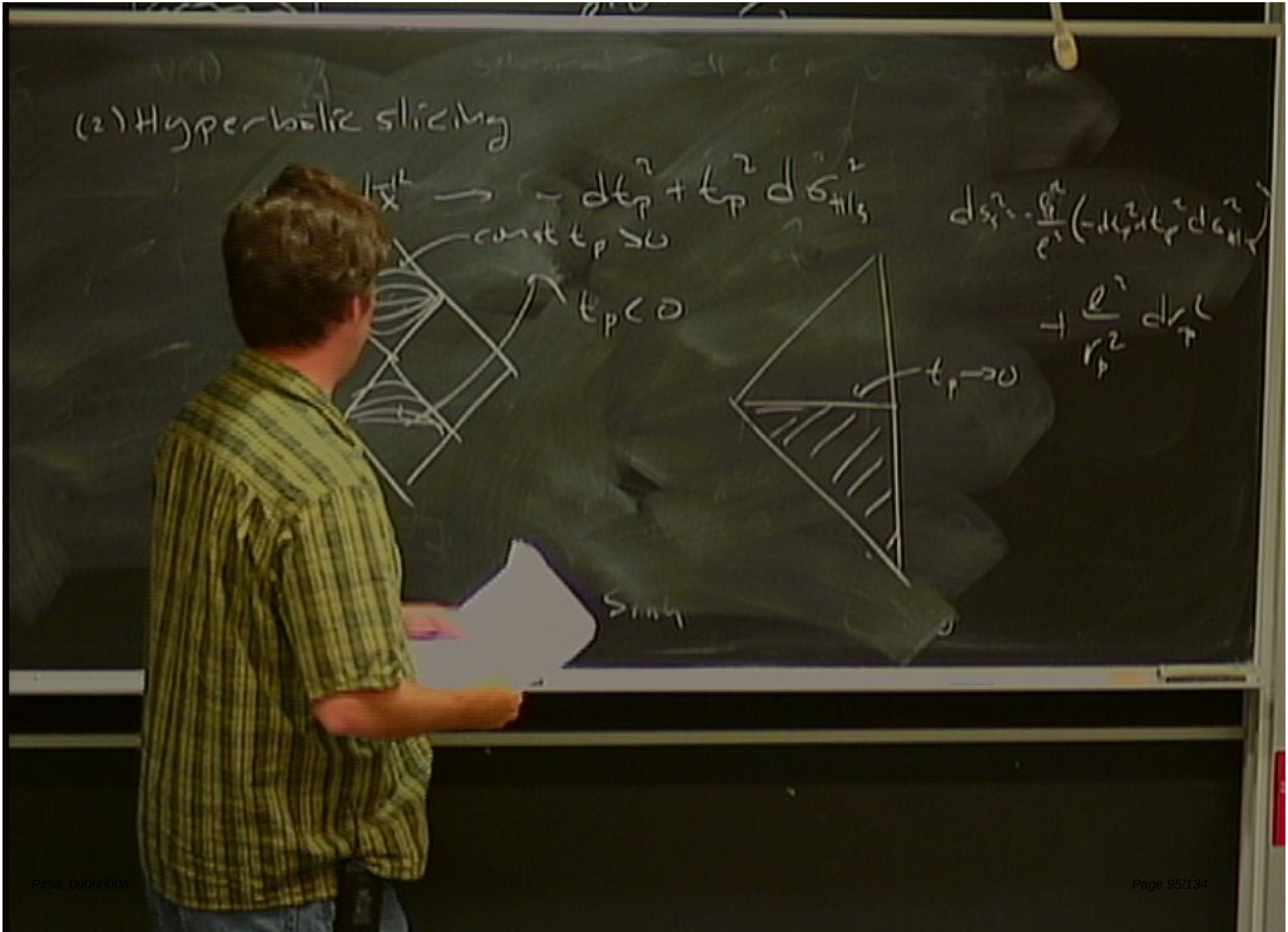
dual to $U(N)$ on $\mathbb{R}^3 \times \mathbb{R}$


(2) Hyperbolic slicing


(2) Hyperbolic slicing
$dt_M^2 + d\vec{x}^2 \to -dt_p^2 + t_p^2 d\sigma^2_{H_3}$
const t_p
Sinh


(2) Hyperbolic slicing
const t_p


(2) Hyperbolic slicing

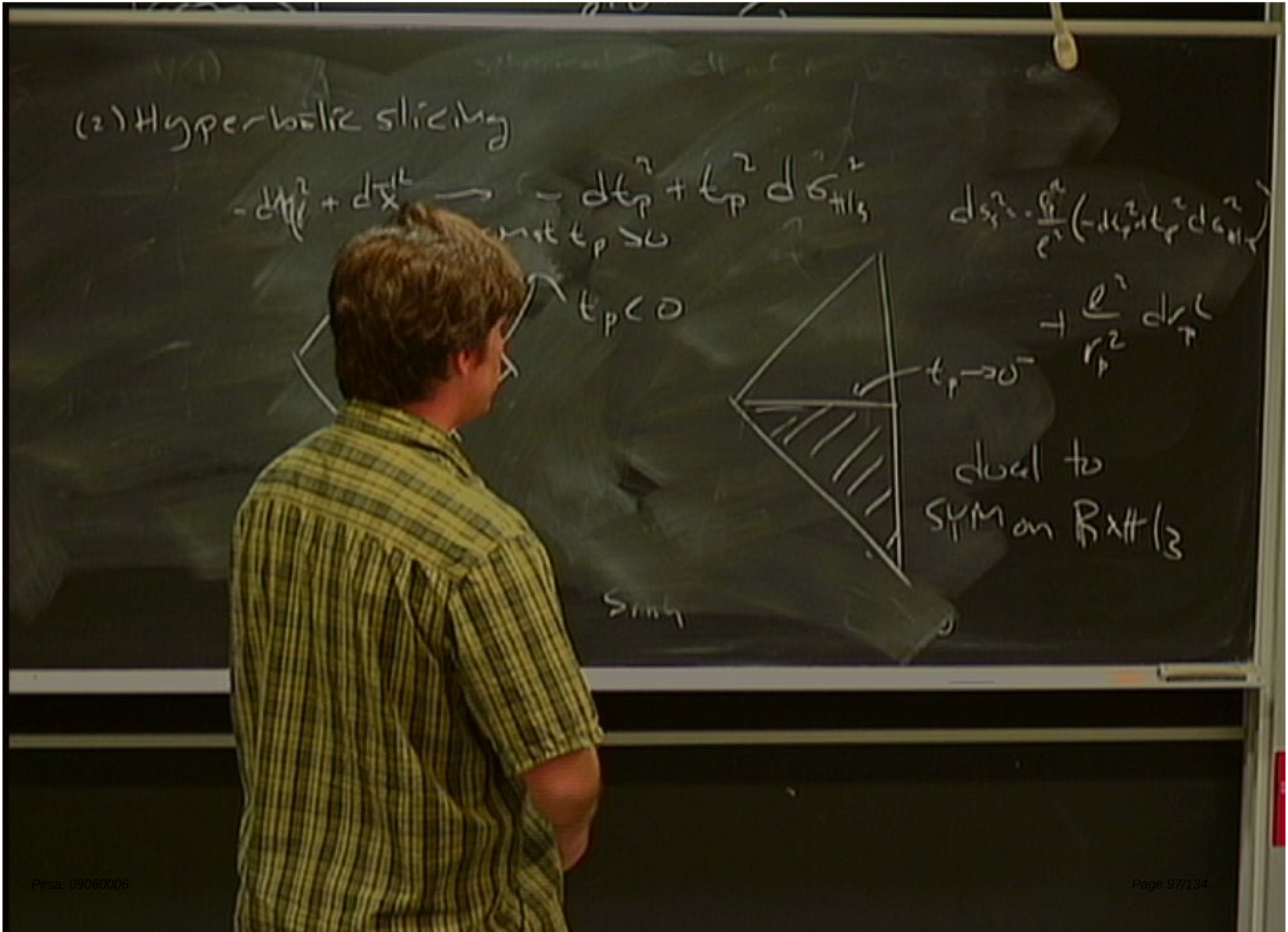


(2) Hyperbolic slicing
$\to -dt_p^2 + t_p^2 d\sigma_{H_3}^2$
const $t_p > 0$
$t_p < 0$
$+ \frac{\ell^2}{r_p^2} dr_p^2$
$t_p \to 0$
Sing

(2) Hyperbolic slicing

$$-dt_M^2 + d\vec{x}^2 \longrightarrow -dt_p^2 + t_p^2\, d\sigma_{H_3}^2$$

$$ds_5^2 = \frac{\ell^2}{r_p^2}\left(-dt_p^2 + t_p^2\, d\sigma_{H_3}^2\right) + \frac{\ell^2}{r_p^2}\, dr_p^2$$


(2) Hyperbolic slicing
dual to
SYM on R×H3

(2) Hyperbolic slicing

$$-dt_\ell^2 + d\vec{x}^2 \longrightarrow -dt_p^2 + t_p^2\, d\sigma_{H_3}^2$$

const $t_p > 0$

$t_p < 0$

$t_p \to 0^-$

Sing

$$ds_5^2 = \frac{\ell^2}{r^2}\left(-dt_p^2 + t_p^2\, d\sigma_{H_3}^2\right) + \frac{\ell^2}{r_p^2}\, dr_p^2$$

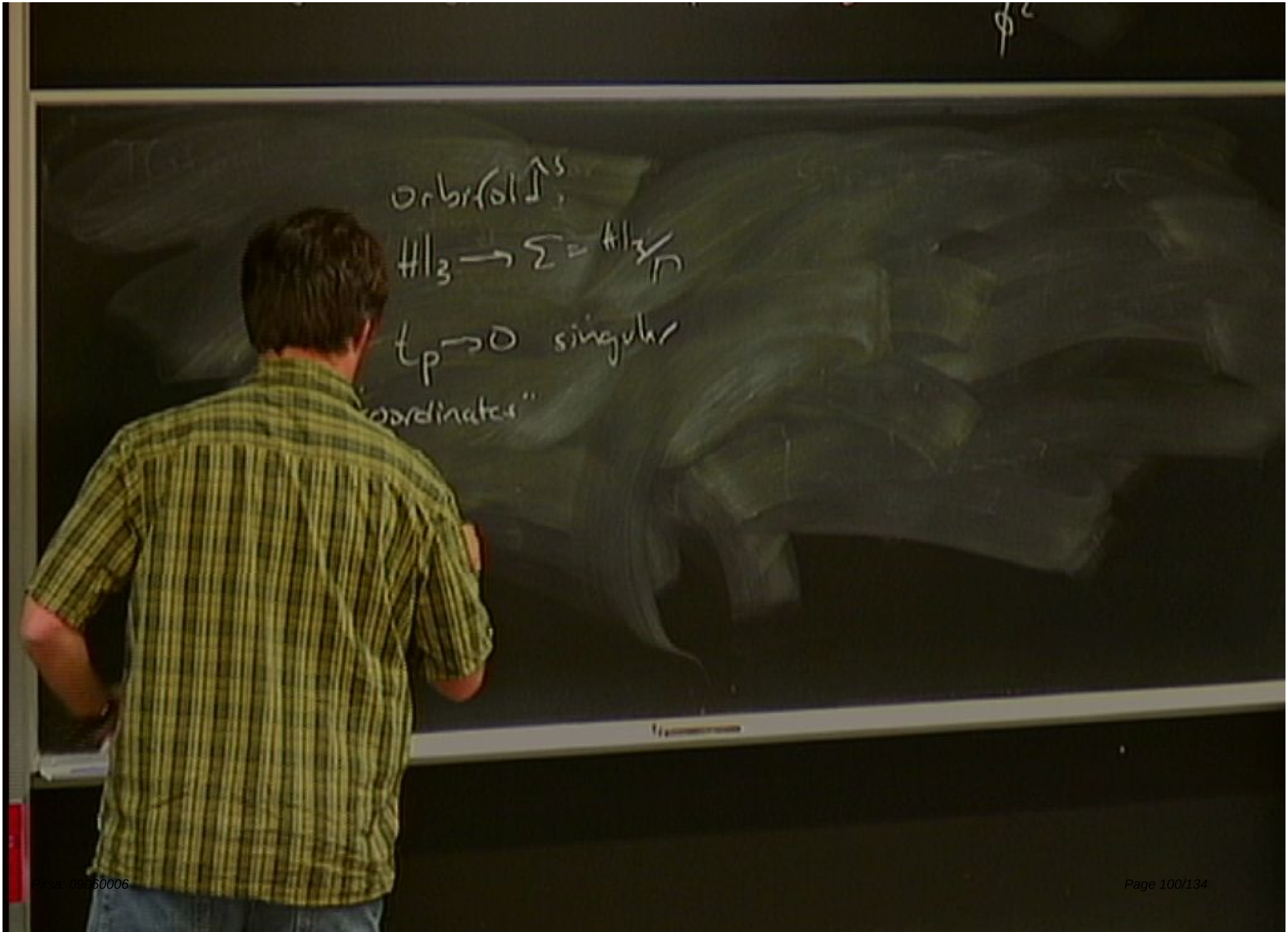
dual to SYM on Milne

$$-dt_p^2 + t_p^2\, d\sigma^2$$

Orbifold Σ's:

$\mathbb{H}_3 \to \Sigma = \mathbb{H}_3/\Gamma$

$t_p \to 0$ singular


Orbifold Σ's:
H3 → Σ = H3/Γ
tp → 0 singular
oordinates"

ϕ^2

Orbifold Γ's:

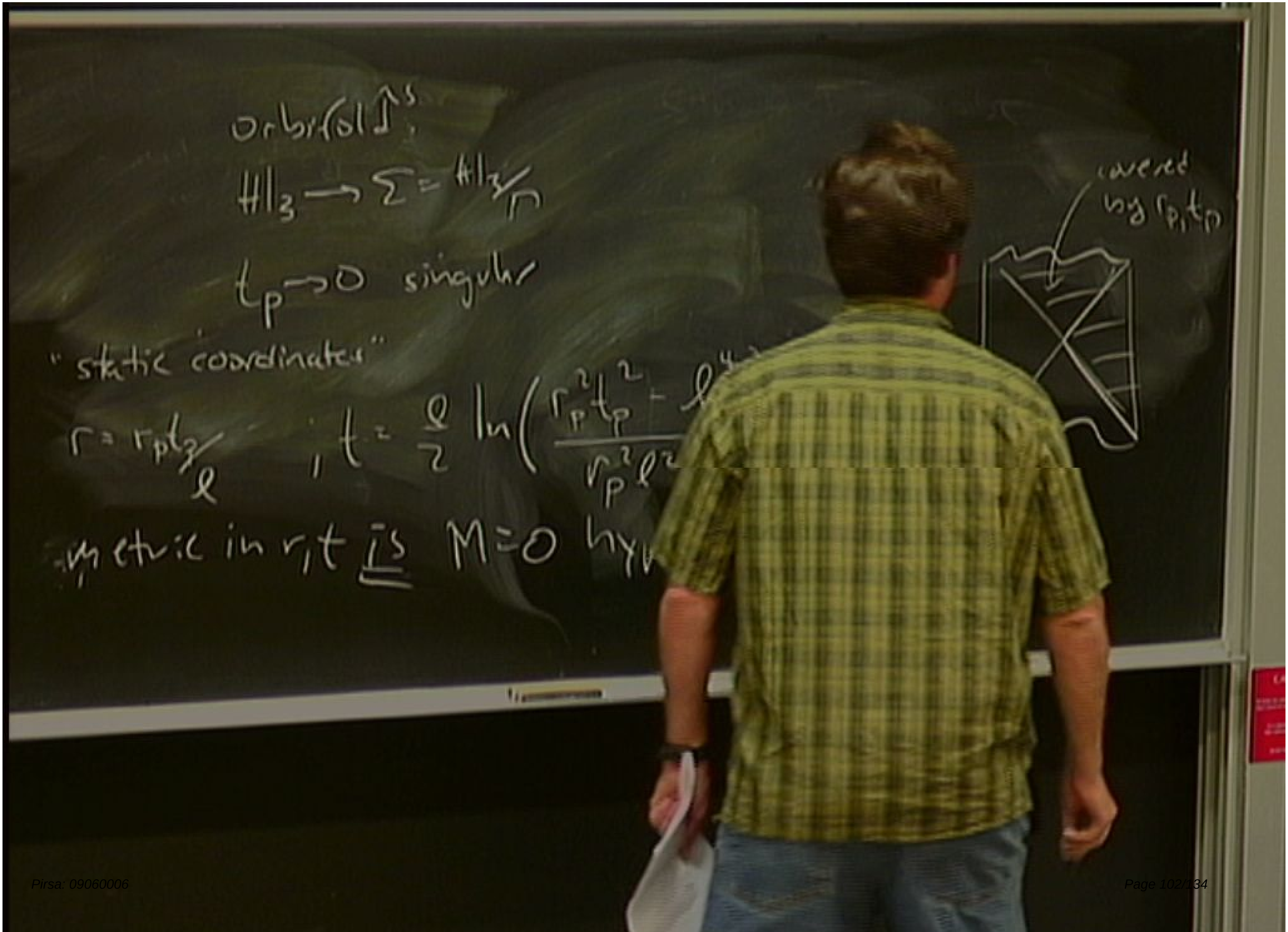
$\mathbb{H}_3 \to \Sigma = \mathbb{H}_3/\Gamma$

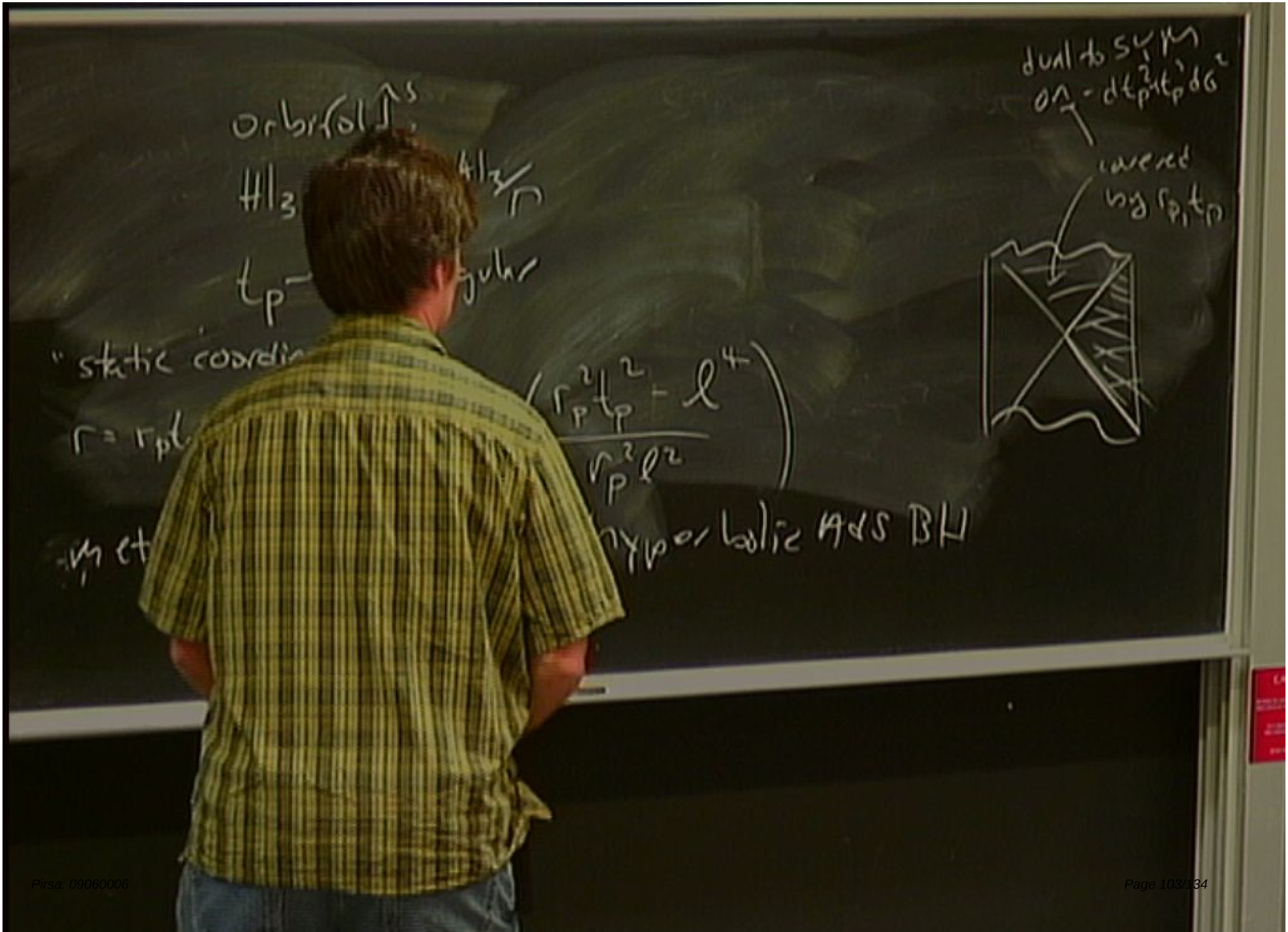
$t_p \to 0$ singular

"static coordinates"

$r = r_p t_p/\ell$, $t = \frac{\ell}{2} \ln\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$

$\to$ metric in r, t <u>is</u> $M=0$ hyperbolic AdS BH


Orbifold Γ's:
$\mathbb{H}_3 \to \Sigma = \mathbb{H}_3/\Gamma$
$t_p \to 0$ singular
"static coordinates"
$r = r_p t_p/\ell$, $t = \frac{\ell}{2} \ln($
metric in r, t is M=0 hyp
covered by r_p, t_p


dual to SYM
on - dtp^2 + tp^2 dσ^2
covered
by rp, tp
Orbifold
H3
static coordin
hyperbolic AdS BH

orbifold Σ's

$\mathbb{H}_3 \to \Sigma = \mathbb{H}_3/\Gamma$

$t_p \to 0$ singular

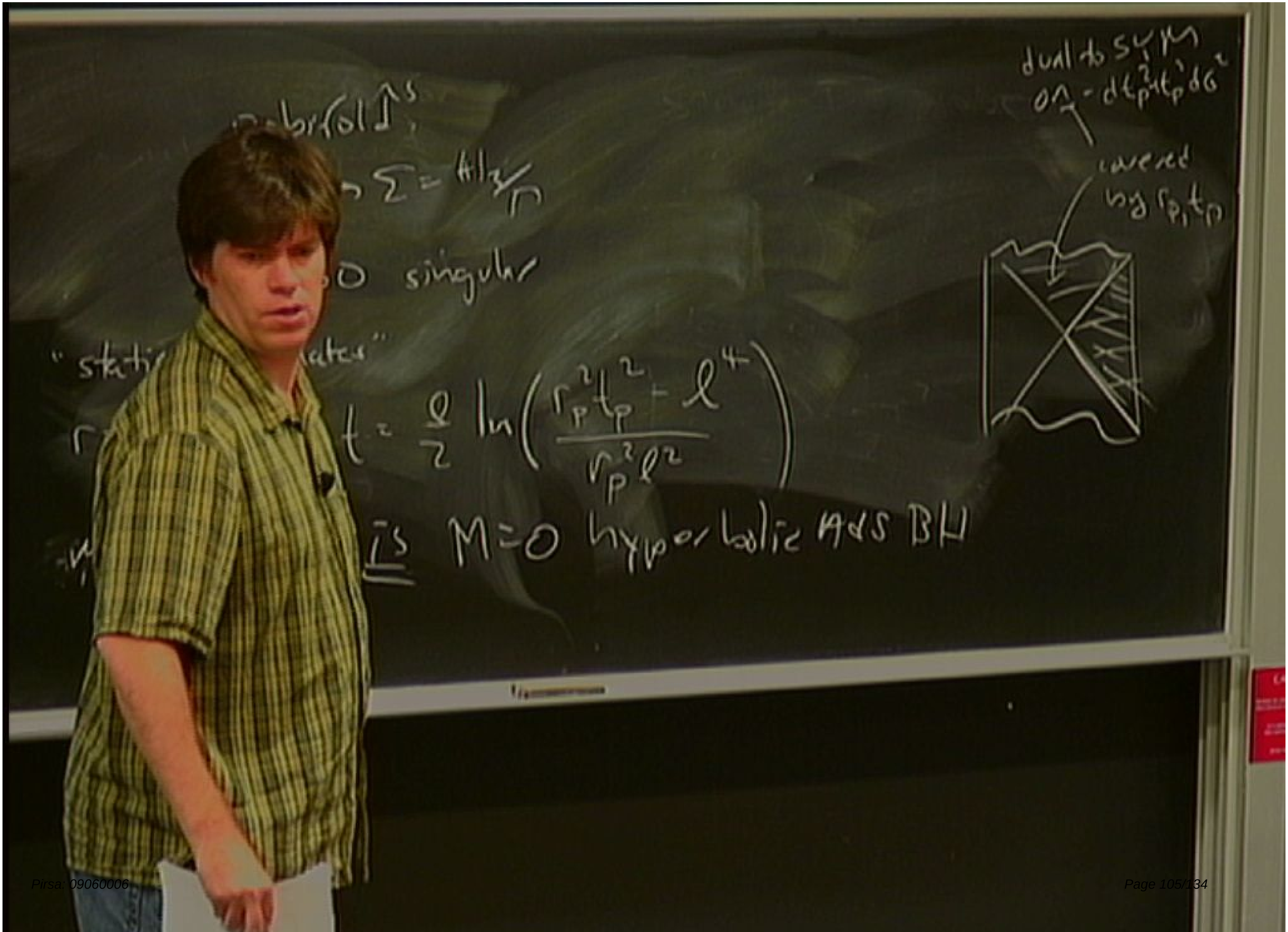
"static coordinates"

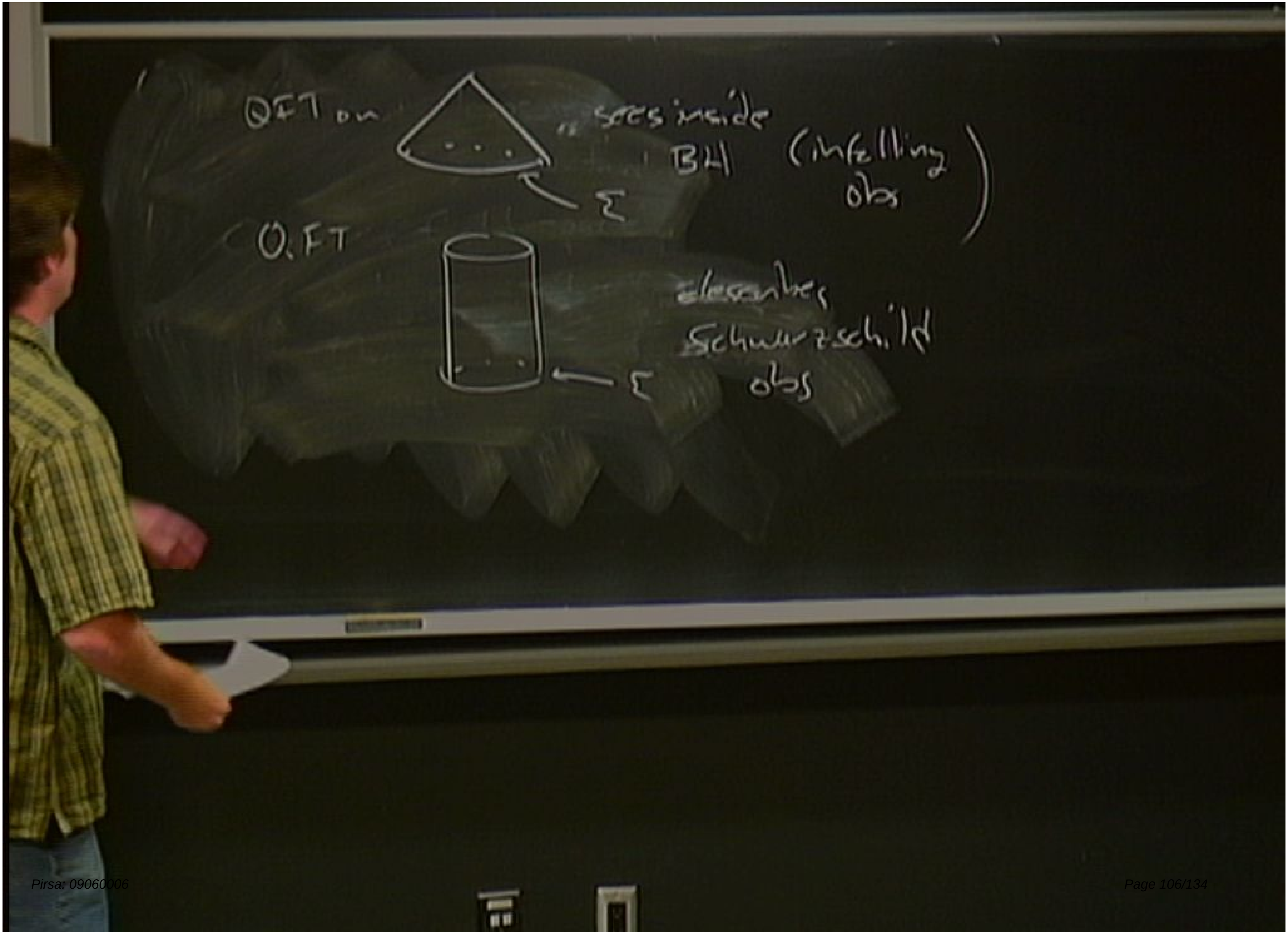
$$r = r_p t_p/\ell \ , \quad t = \frac{\ell}{2}\ln\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$$

metric in r, t is $M=0$ hyperbolic AdS BH

dual to SYM on $-dt_p^2 + t_p^2 d\sigma^2$

covered by r_p, t_p

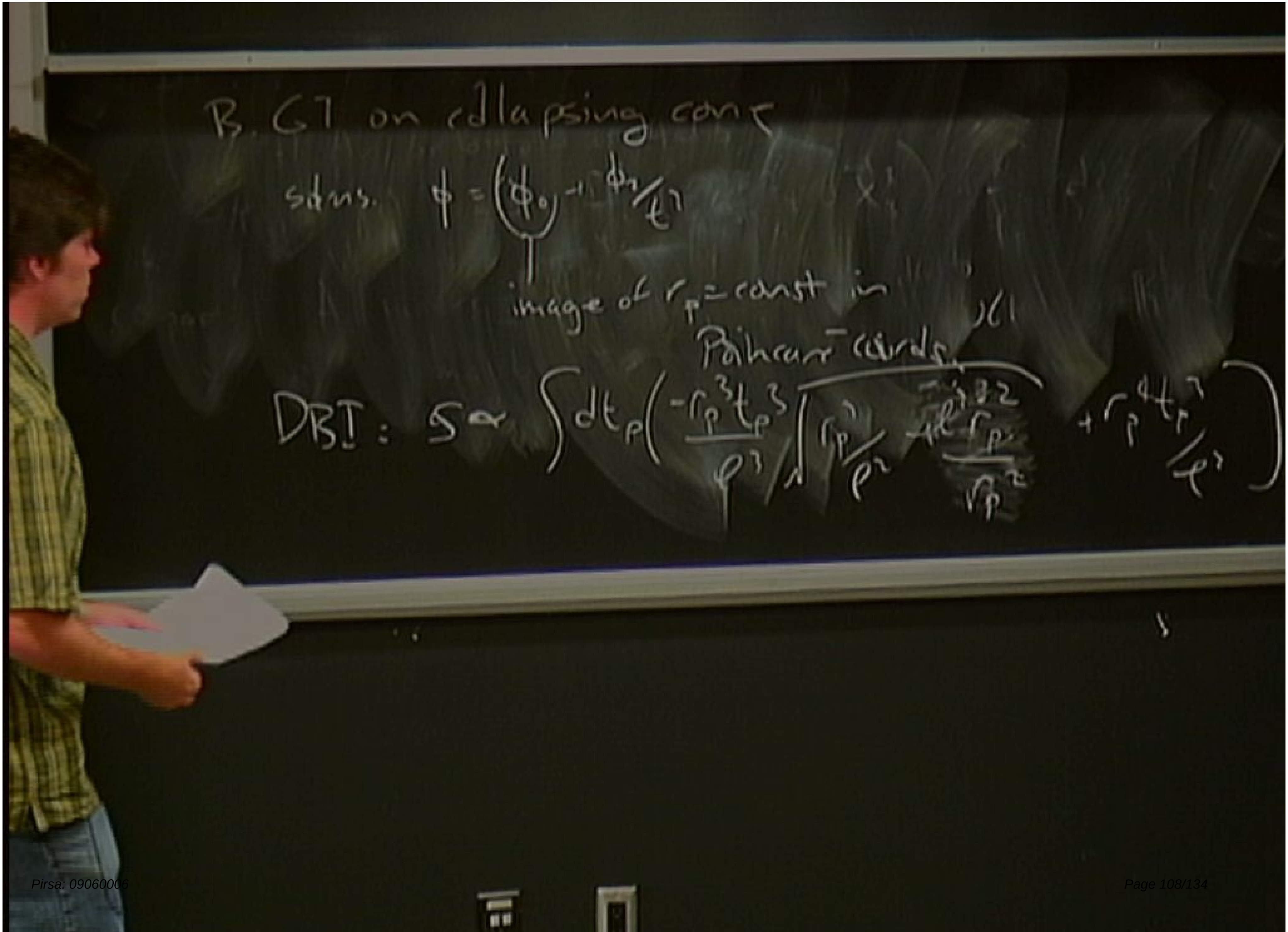

dual to SYM
covered
singular
M=0 hyperbolic AdS BH


QFT on
sees inside
BH (infalling
obs)
Σ
QFT
elsewhere
Schwarzschild
obs
Σ

B. GT on collapsing cone

soln. $\phi = \phi_0 + \phi_1 / t^?$

image of r_p = const in Poincaré coords.


B. GT on collapsing cone
soln. φ = (φ₀ + φ₁/t²)
image of r_p = const in
Poincaré coords.
DBI: S ∝ ∫dt_p(

B. GT on collapsing cone

soln. $\phi = (\phi_0 + \phi_2/t^2$

horizon:
$r_p t_p = \ell^2$

image of $r_p = $ const in Poincaré coords.

DBI: $S \propto \int dt_p \left(-\frac{r_p^3 t_p^3}{\ell^3} \sqrt{\frac{r_p^2}{\ell^2} - \frac{\ell^2 \dot{r}_p^2}{r_p^2}} + r_p^4 t_p^2 / \ell^2 \right)$

B. GT on collapsing cone

soln. $\phi = \left(\phi_0 + \phi_2 / t^2 \right)$

horizon: $r_p t_p = \ell^2$

image of $r_p = \text{const}$ in Poincaré coords

DBI: $S \propto \int dt_p \left(-\frac{r_p^3 t_p^3}{\ell^3} \sqrt{\frac{r_p^2}{\ell^2} - \frac{\ell^2 \dot{r}_p^2}{r_p^2}} + r_p^4 t_p^2 / \ell^2 \right)$

B. GT on collapsing cone

soln. $\phi = \left(\phi_0 - i\, \phi_2 / t^2\right)$

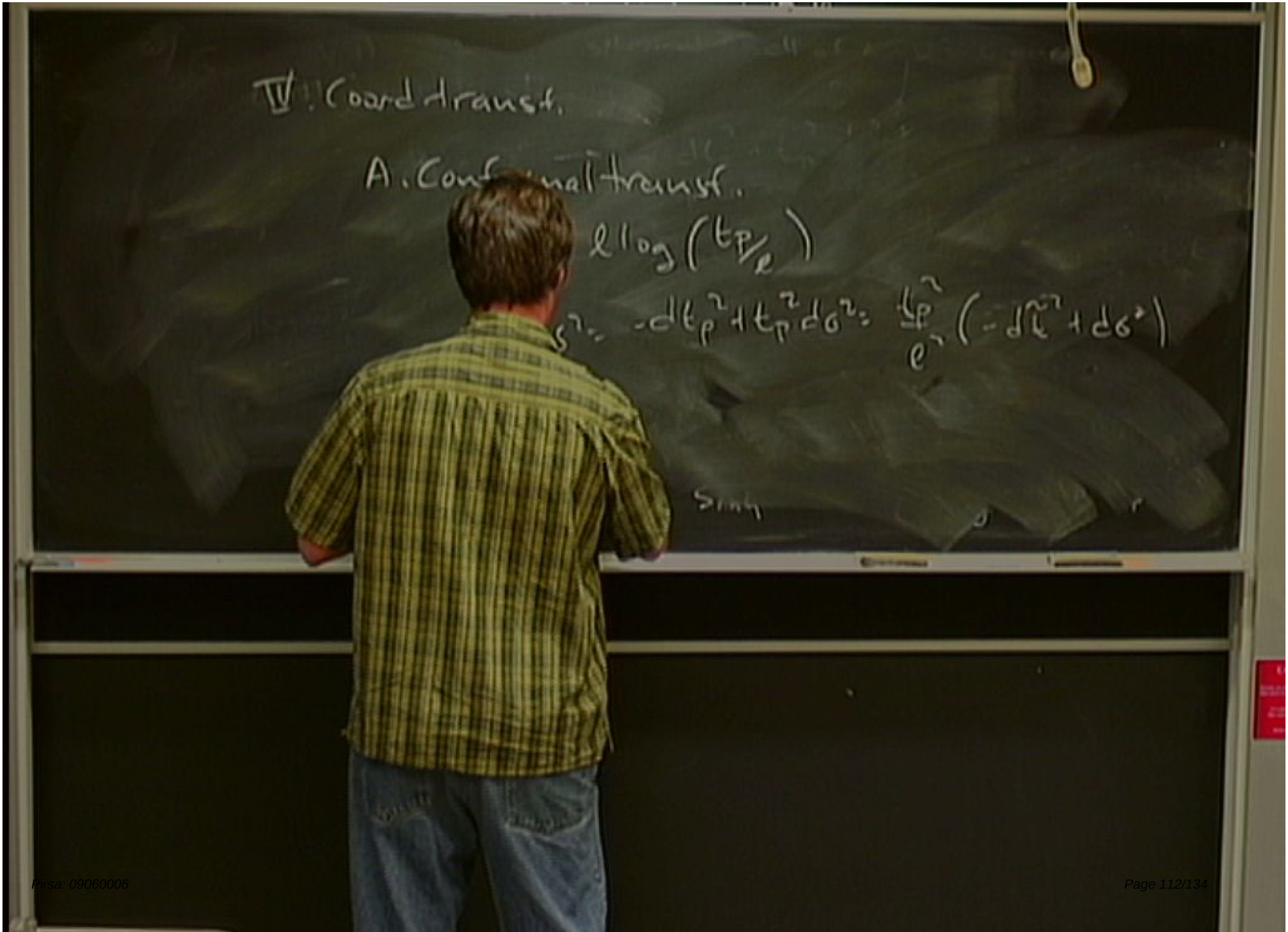
horizon:
$r_p t_p = \ell^2$

image of $r_p = $ const in Poincaré coords.

DBI: $S \propto \int dt_p \left(-\frac{r_p^3 t_p^3}{\ell^3} \sqrt{\frac{r_p^2}{\ell^2} \ldots \frac{r_p}{r_p^2}} + r_p^4 t_p^2 / \ell^2 \right)$

$\phi = \phi_0$: $m_W = $ const

$m_W / m_{HR} \to 0$ at $t_p \to 0^-$


IV. Coord transf.
A. Conf nal transf.
$\ell \log(t_p/\ell)$
$-dt_p^2 + t_p^2 d\sigma^2 = \frac{t_p^2}{\ell^2}(-d\tau^2 + d\sigma^2)$

IV: Coord transf.

A. Conformal transf.

$$\tilde{t} = -\ell \log\left(t_p/\ell\right)$$

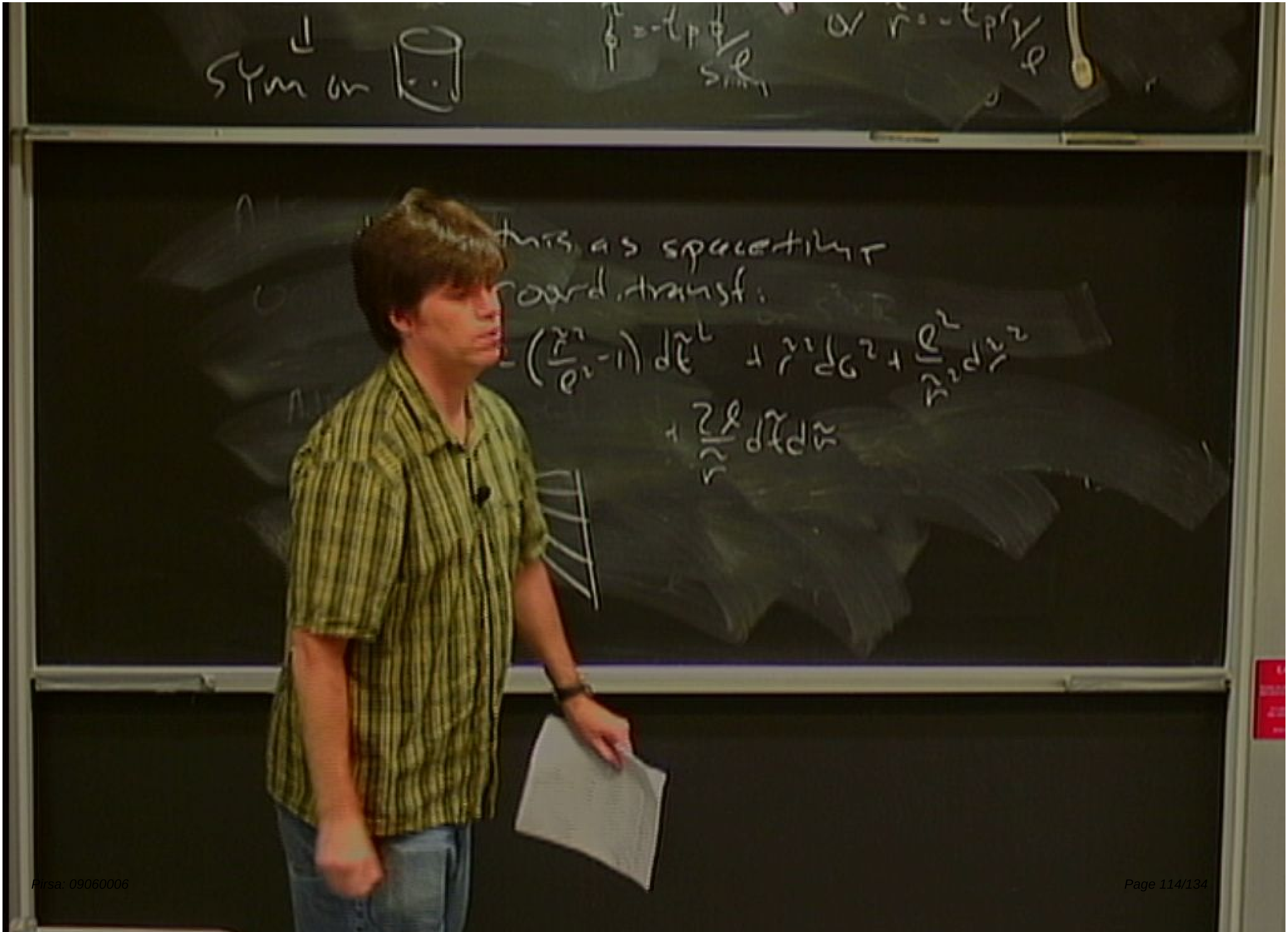
takes sym. $\quad ds^2 = -dt_p^2 + t_p^2 d\sigma^2 = \frac{t_p^2}{\ell^2}\left(-d\tilde{t}^2 + d\sigma^2\right)$

$ds^2 \to d\tilde{s}^2$

$\tilde{\phi} = -t_p \phi/\ell \qquad$ or $\tilde{r} = -t_p r/\ell$

sym

Sym on 


Sym on
as spacetime
coord. transf.

$\Downarrow$

Sym on 

$\hat\phi = -t_p \phi/\ell$

S_{ℓ}

or $\hat r = -t_p r \gamma/\ell$

treat this as spacetime coord. transf.:

$$ds^2 = -\left(\frac{\tilde r^2}{\ell^2}-1\right)d\tilde t^2 + \tilde r^2 d\sigma^2 + \frac{\ell^2}{\tilde r^2}\, d\tilde r^2 + \frac{2\ell}{\tilde r}\, d\tilde t\, d\tilde r$$

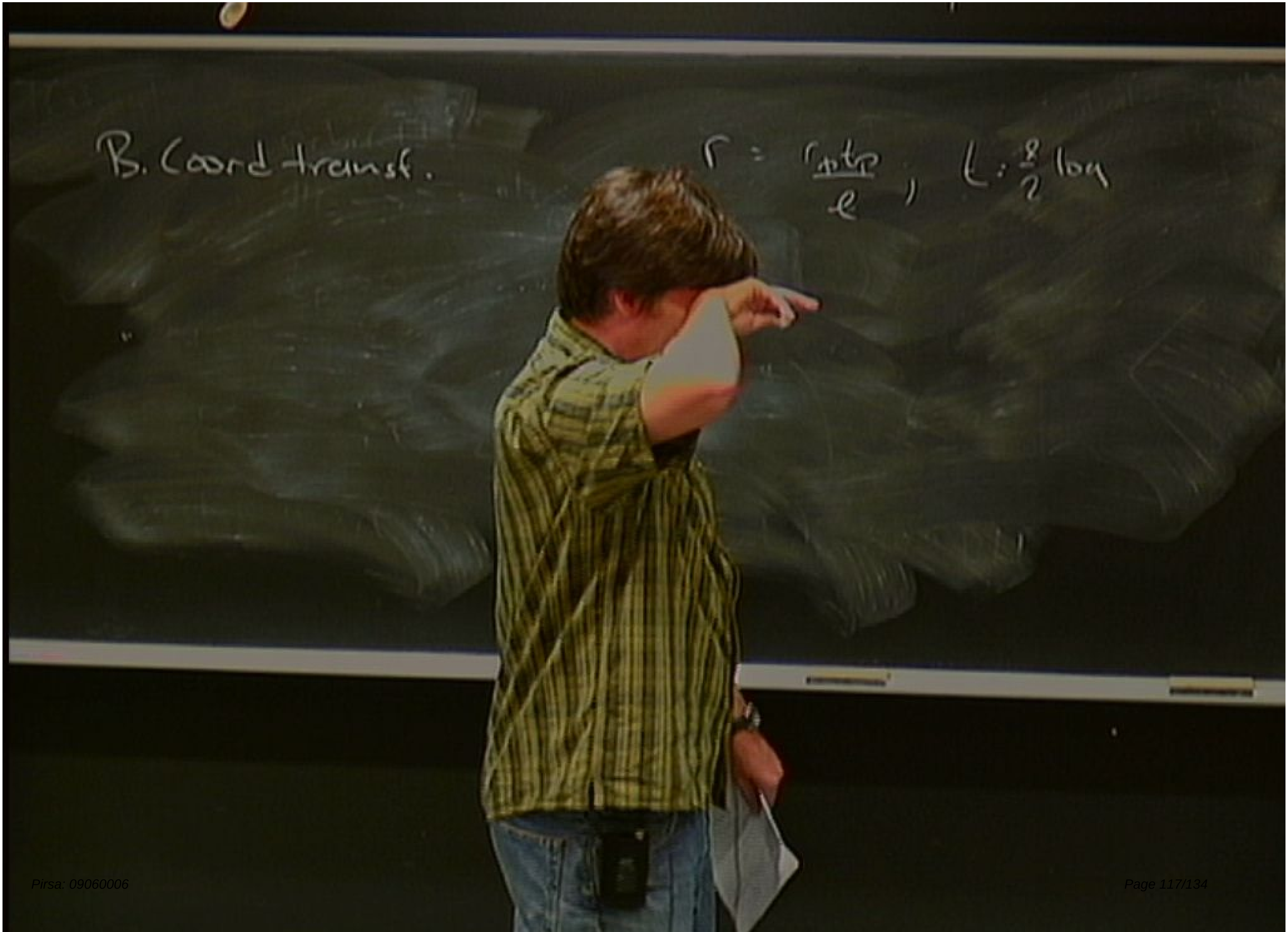
SYM on

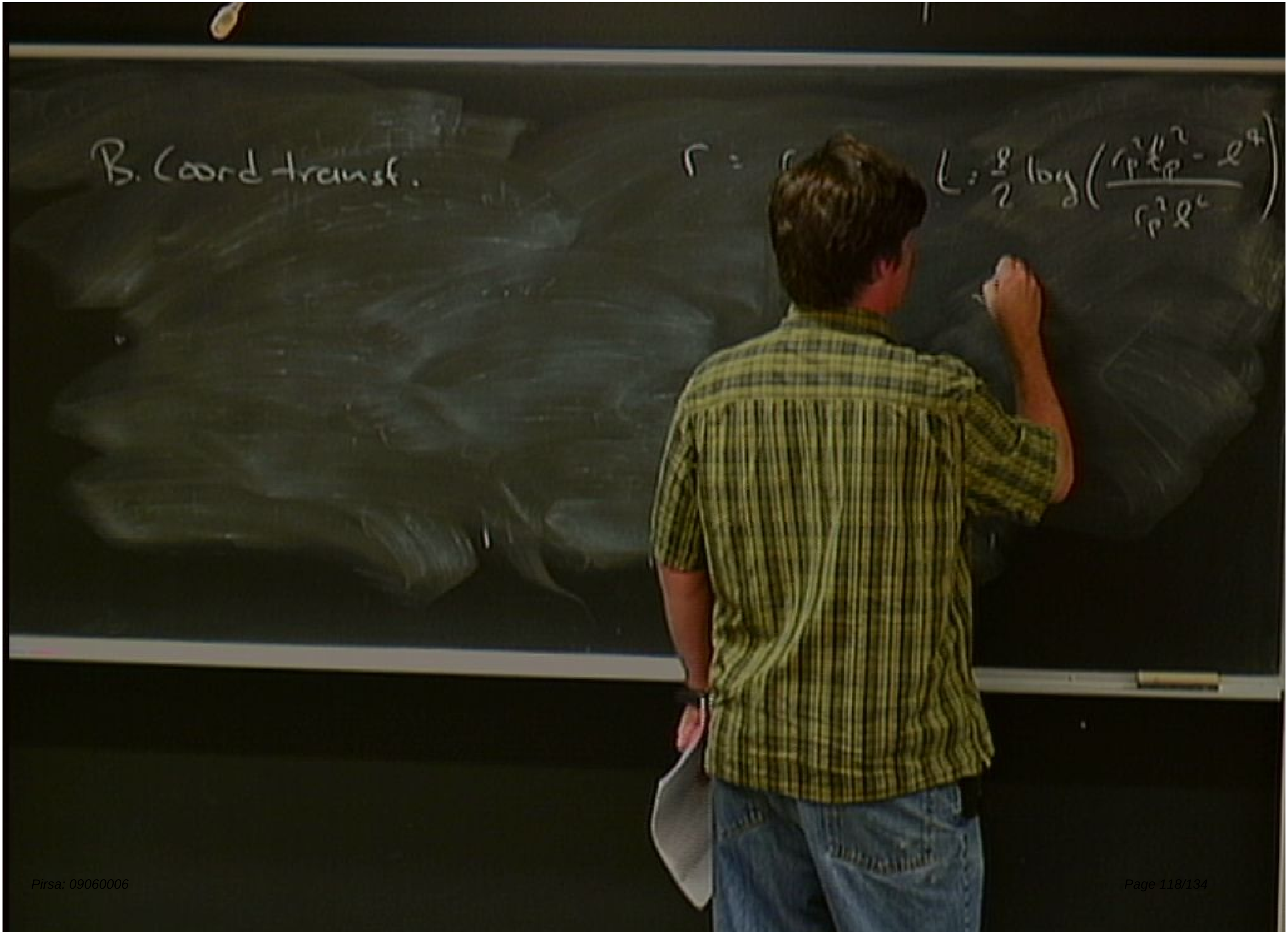
$\phi = -t_p \phi / \ell$ or $r = -t_p r / \ell$

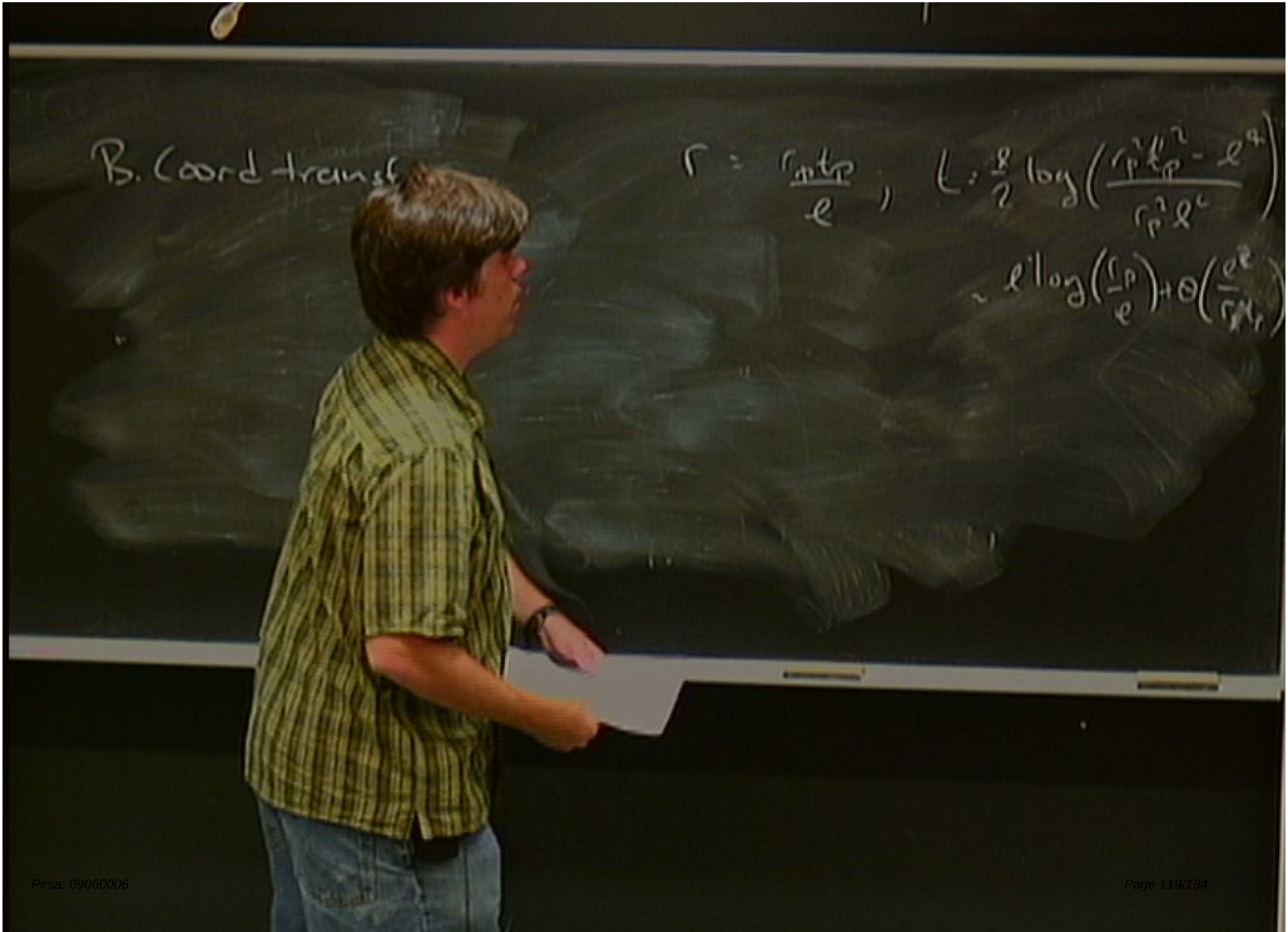
string

treat this as spacetime coord. transf.:

$$ds^2 = -\left(\frac{\tilde r^2}{\ell^2} - 1\right) d\tilde t^2 + \tilde r^2 d\sigma^2 + \frac{\ell^2}{\tilde r^2} d\tilde r^2 + \frac{2\ell}{\tilde r}\, d\tilde t\, d\tilde r$$


B. Coord transf.

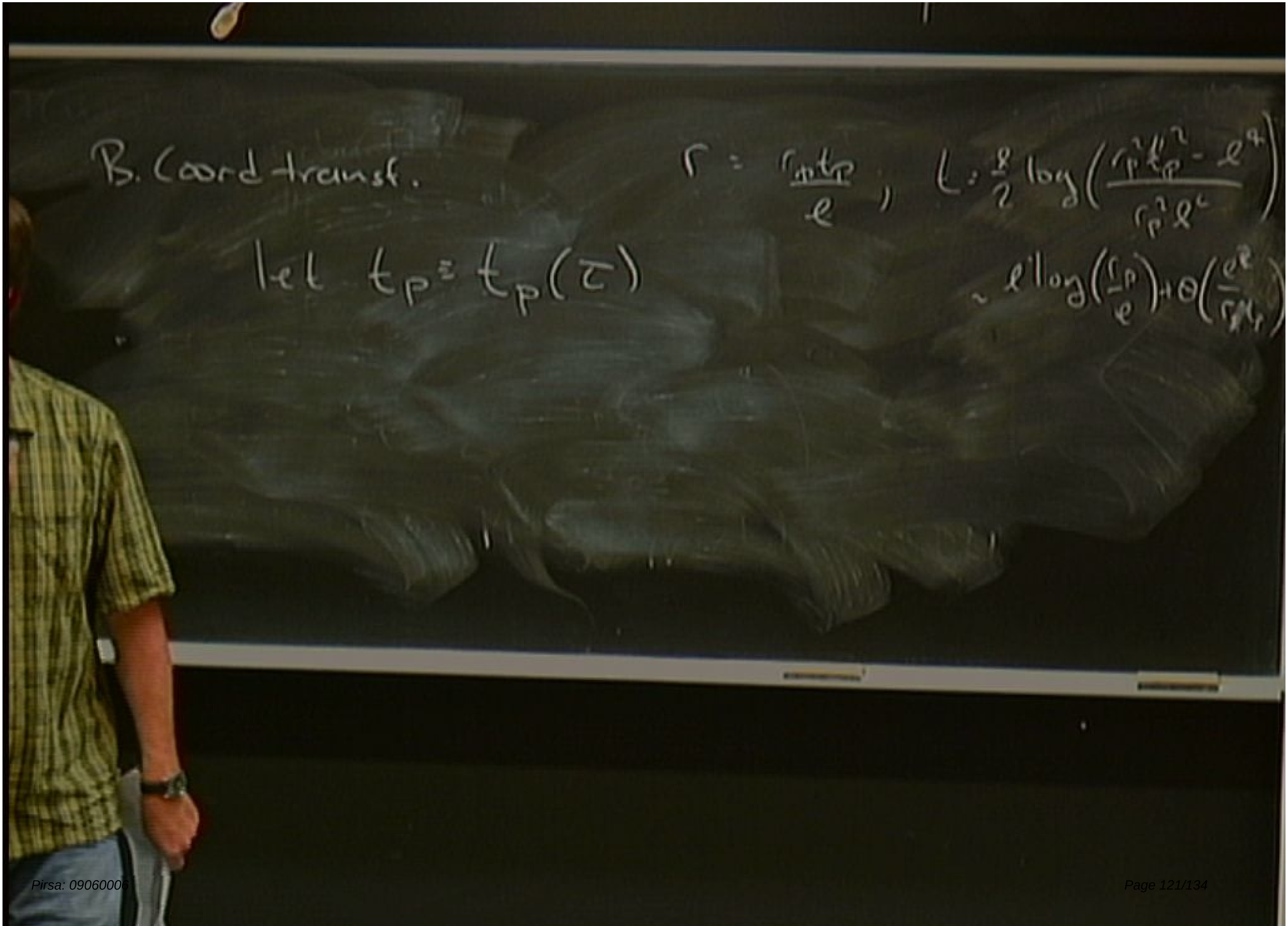

B. Coord transf.


B. Coord transf
$r = \frac{r_p t_p}{\ell}$, $L = \frac{\ell}{2} \log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$
$\sim \ell \log\left(\frac{r_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$

B. Coord transf.

$$r = \frac{r_p t_p}{\ell}, \quad t = \frac{\ell}{2}\log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$$

$$\sim \ell \log\left(\frac{r_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$$


B. Coord transf.
$r = \frac{r_p t_p}{\ell}$, $L = \frac{\ell}{2}\log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$
let $t_p = t_p(\tau)$
$\sim \ell \log\left(\frac{r_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$

B. Coord transf.

$$r = \frac{r_p t_p}{\ell}, \quad L = \frac{\ell}{2}\log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$$

$$\approx \ell \log\left(\frac{r_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$$

let $t_p = t_p(\tau)$

adds a gauge invariance:

fix $t_p = \tau$

B. Coord transf.

$$r = \frac{r_p t_p}{\ell}, \quad t = \frac{\ell}{2} \log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$$

$$\approx \ell \log\left(\frac{r_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$$

let $t_p = t_p(\tau)$

adds a gauge invariance.

fix $t_p = \tau$

B. Coord transf.

$$r = \frac{r_p t_p}{\ell}, \quad t = \frac{\ell}{2}\log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$$

$$\sim \ell \log\left(\frac{t_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$$

let $t_p = t_p(\tau)$

adds a gauge invariance:

fix $t_p = \tau$

B. Coord transf.

$$r = \frac{r_p t_p}{\ell}, \quad L = \frac{\ell}{2}\log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right)$$

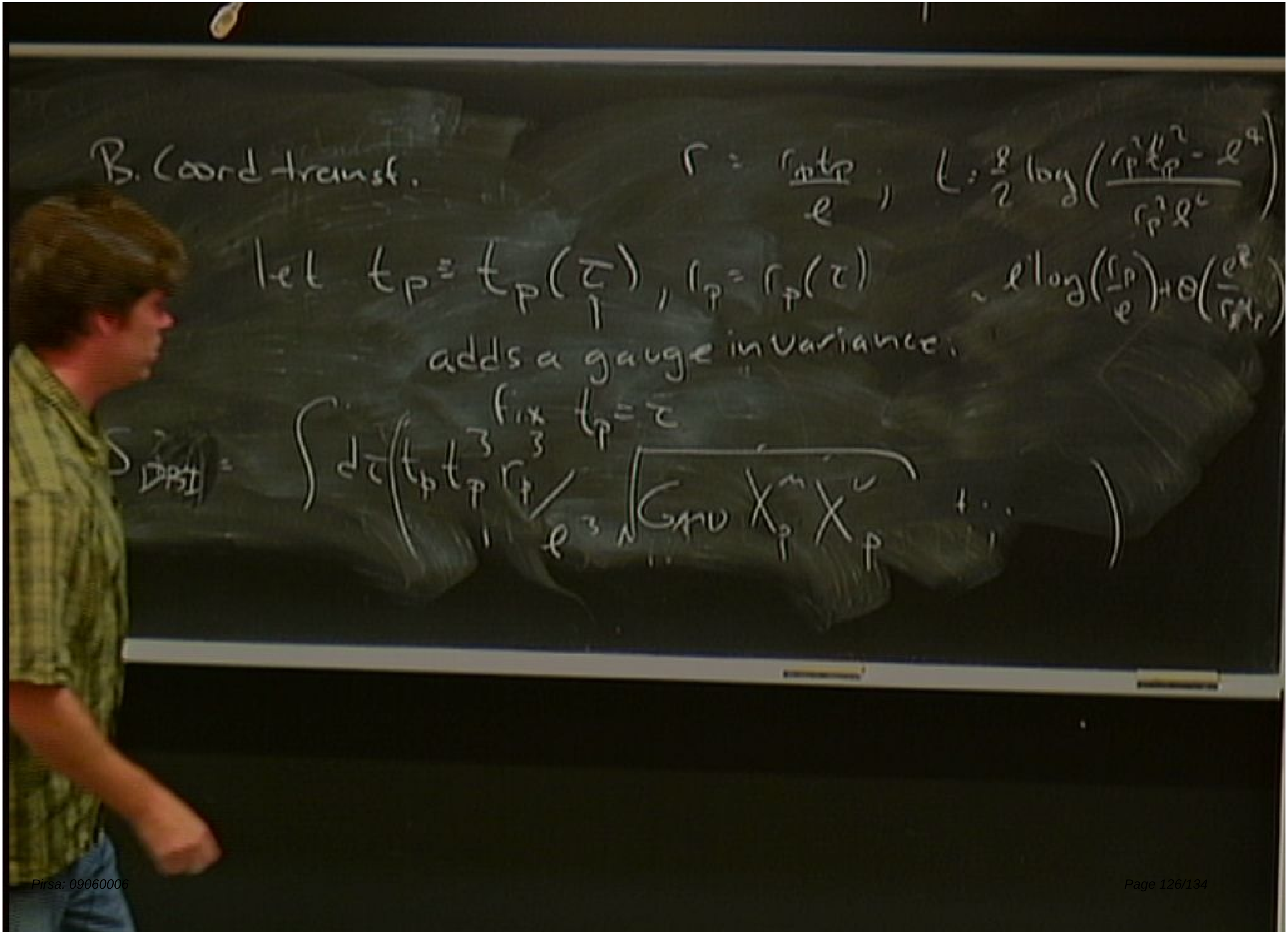
let $t_p = t_p(\tau)$

$$\sim \ell \log\left(\frac{r_p}{\ell}\right) + O\left(\frac{\ell^2}{r_p t_p}\right)$$

adds a gauge invariance.

fix $t_p = \tau$

$$S_{DBI} = \int d\tau \left(t_p t_p^3 r_p^3 / \ell^3 \sqrt{G_{\mu\nu} \dot{X}_p^\mu \dot{X}_p^\nu} + \dots \right)$$


B. Coord transf.
r = r_p t_p / ℓ , t = ℓ/2 log((r_p^2 t_p^2 - ℓ^4) / (r_p^2 ℓ^2))
≈ ℓ log(r_p/ℓ) + O(ℓ^2/(r_p t_p))
let t_p = t_p(τ), r_p = r_p(τ)
adds a gauge invariance.
fix t_p = τ
S_DBI = ∫dτ (t_p t_p^3 r_p^3 / ℓ^3 √(G_μν X_p^μ X_p^ν) + ...)

B. Coord transf.

$$r = \frac{r_p t_p}{\ell}, \quad t = \frac{\ell}{2}\log\left(\frac{r_p^2 t_p^2 - \ell^4}{r_p^2 \ell^2}\right) \sim \ell \log\left(\frac{t_p}{\ell}\right) + \theta\left(\frac{\ell^2}{r_p t_p}\right)$$

let $t_p = t_p(\tau)$, $r_p = r_p(\tau)$

adds a gauge invariance:

fix $t_p = \tau$

$$S_{DBI} = \int d\tau \left(t_p t_p^3 r_p^3 / \ell^3 \sqrt{G_{\mu\nu} \dot{X}_p^\mu \dot{X}_p^\nu} + \dots \right)$$

$$S_{DBI} = \int d\tau\, d^n\sigma \sqrt{G_{\mu\nu}\dot X^\mu \dot X^\nu}$$

$$S_{DBI} = \int d^4\tau\, r^3 \sqrt{G_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}$$

$$t = \tau$$

$$\int dt\, r^3 \sqrt{f - \frac{\dot{r}^2}{f}}$$

$$S_{DBI} = \int d^4\sigma\, r^3 \sqrt{G_{\mu\nu}\dot{X}^\mu \dot{X}^\nu}$$

$$\Big\downarrow\ t = \sigma$$

$$\int dt\, r^3 \sqrt{f - \frac{\dot{r}^2}{f}}$$

treat this as spacetime

coord. transf:

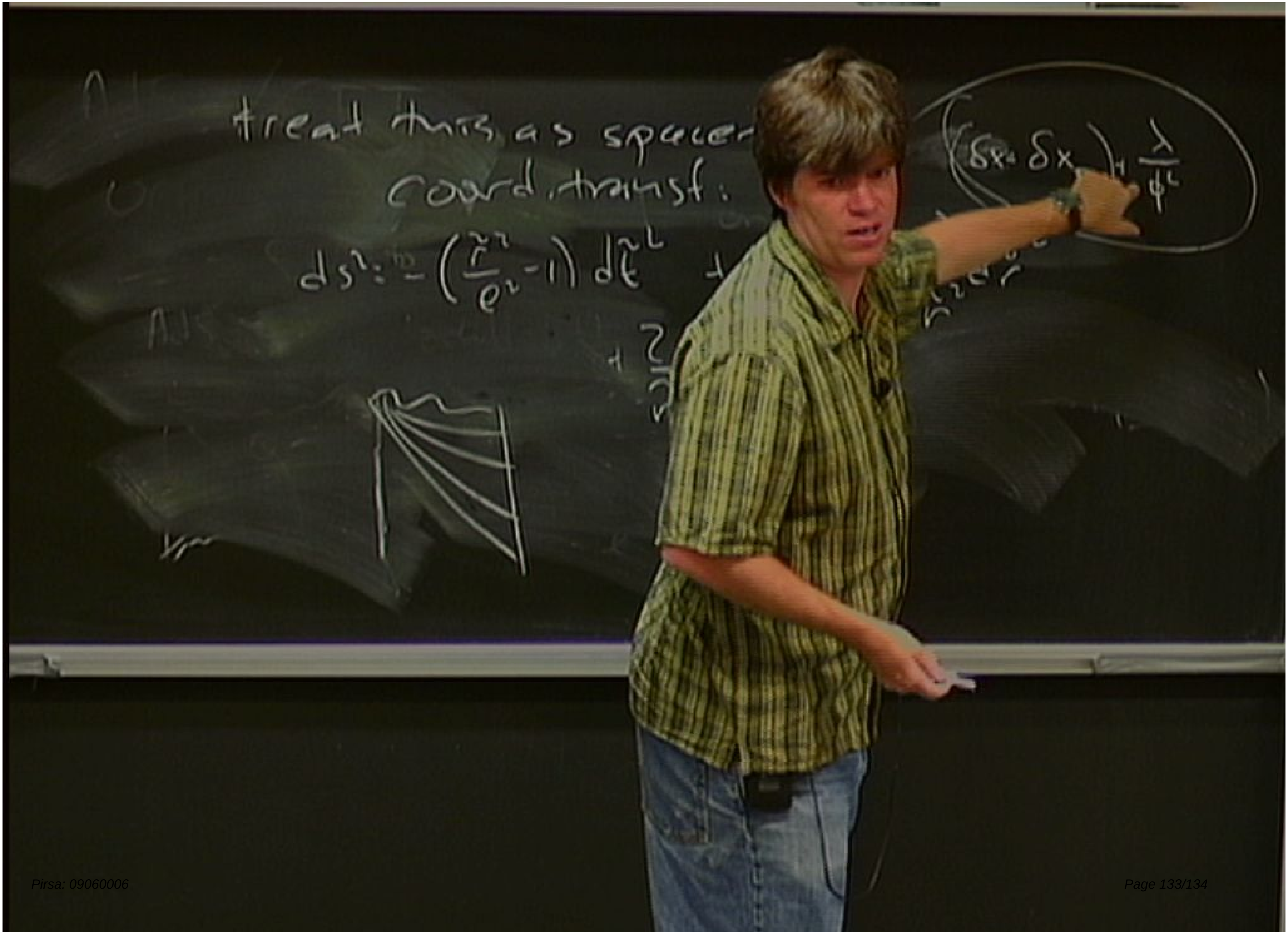
$$ds^2 = -\left(\frac{\tilde{r}^2}{\ell^2} - 1\right) d\tilde{t}^2 + \tilde{r}^2 d\sigma^2 + \frac{\ell^2}{\tilde{r}^2} d\tilde{r}^2 + \frac{2\ell}{\tilde{r}} d\tilde{t}\, d\tilde{r}$$

treat this as spacetime

coord. transf.:

$$\delta x = \delta x_{sen} + \frac{\lambda}{\phi^2}$$

$$ds^2 = -\left(\frac{\tilde r^2}{\ell^2}-1\right)d\tilde t^2 + \tilde r^2 d\sigma^2 + \frac{\ell^2}{\tilde r^2}d\tilde r^2 + \frac{2\ell}{\tilde r}\,d\tilde t\,d\tilde r$$


treat this as
coord. transf :

treat this as spacetime coord. transf.:

$$\left(\delta x \cdot \delta x_{str}\right)^{4} \frac{\lambda}{\phi^{4}}$$

$$ds^2 = -\left(\frac{\tilde r^2}{\ell^2}-1\right)d\tilde t^2 + \tilde r^2 d\sigma^2 + \frac{\ell^2}{\tilde r^2}d\tilde r^2 + \frac{2\ell}{\tilde r}\,d\tilde t\,d\tilde r$$