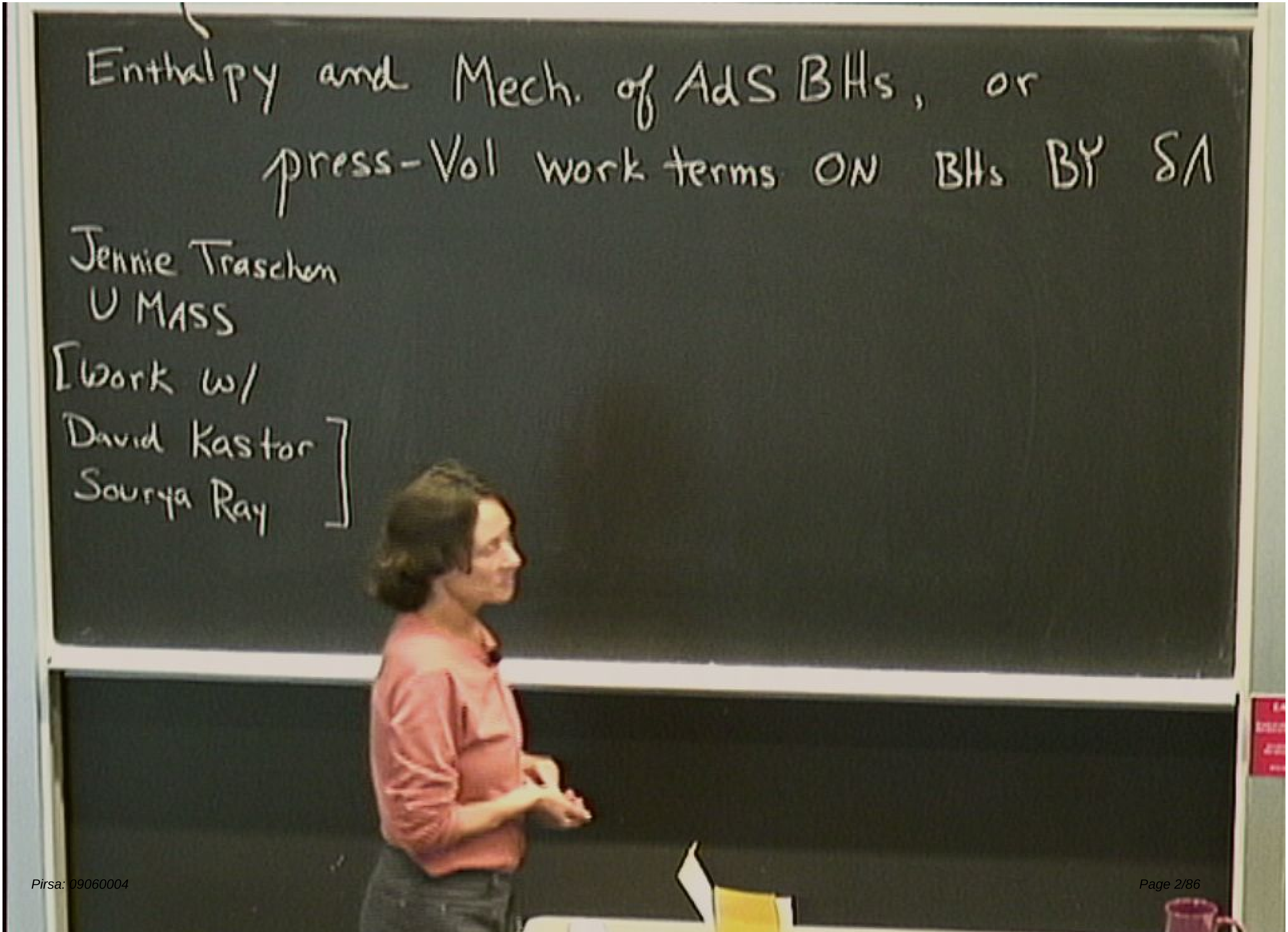
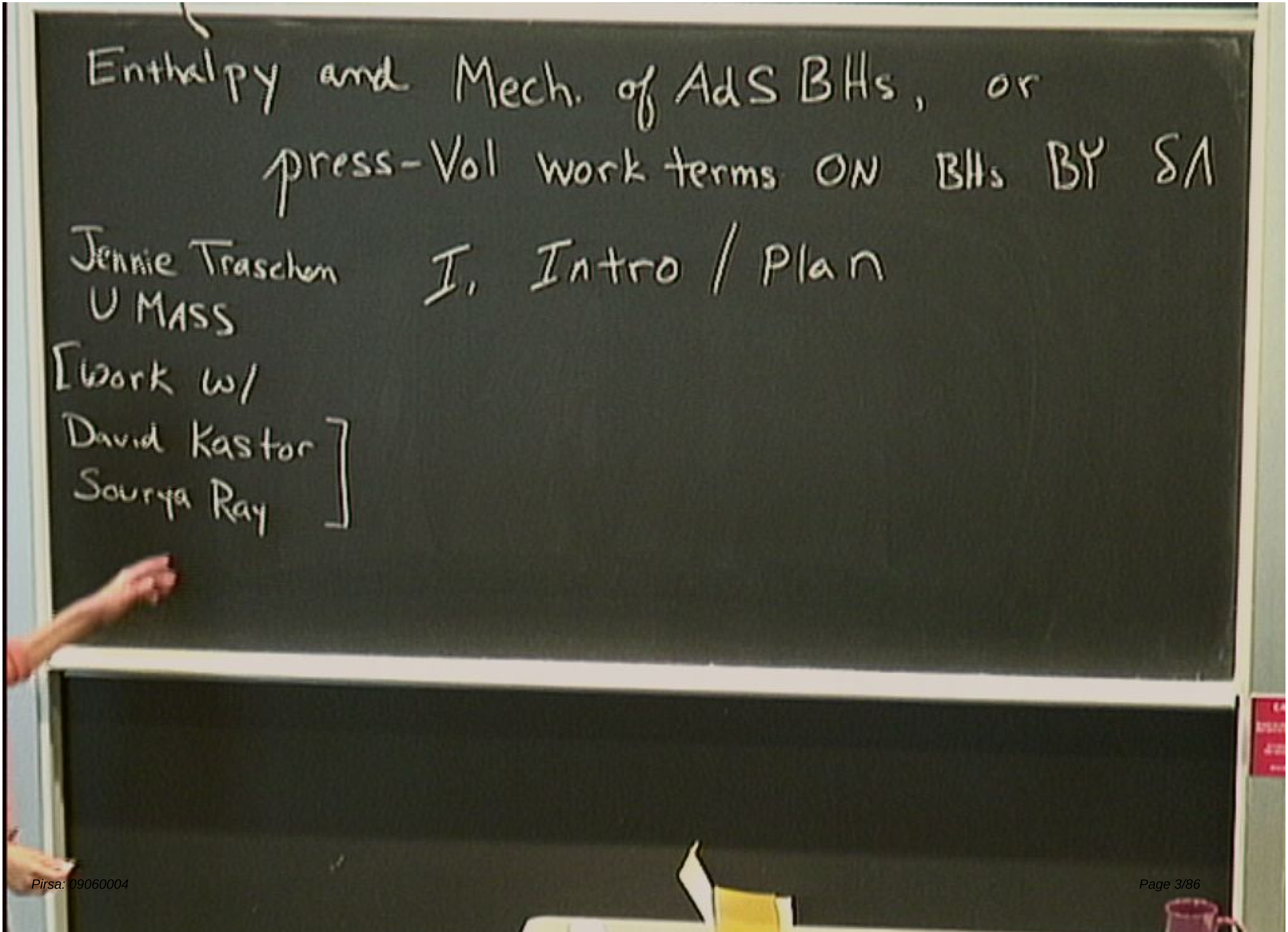
Title: Enthalpy and the Mechanics of AdS Black Holes

Date: Jun 10, 2009 04:00 PM

URL: http://pirsa.org/09060004

Abstract: I will discuss the contribution to black hole thermodynamics from a variation in the cosmological constant. The description of black hole with a cosmological constant is facilitated by introducing a two-form potential for the static Killing field. The resulting Smarr formula then includes a term proportional to the cosmological constant times an effective volume, which arises as the difference between the Killing potential on the horizon and the boundary at infinity. This volume is shown to be equal to the difference between the (infinite) volume of AdS and the (infinite) volume outside the black hole horizon of AdS containing a black hole--and so can be interpreted as the volume occupied by the black hole. I will outline the derivation for the first law for AdS black holes including a variation in the cosmological constant. This yields a new work term, the change in the cosmological constant times the effective volume. Hence this is analogous to a "volume times change in pressure" work term in classical thermodynamics. This suggests that the usual change in mass term is better interpreted as a change in the enthalpy, the mass plus the pressure times the volume. In the AdS/CFT correspondence a change in the cosmological constant corresponds to a change in the t'Hooft coupling, for example, a change in the number of degrees of freedom. The effective volume multiplier then looks like a chemical potential. Members of the audience will be asked to contribute their own interpretations at this point.


Enthalpy and Mech. of AdS BHs, or
press-Vol work terms ON BHs BY δΛ
Jennie Traschen
U MASS
[Work w/
David Kastor
Sourya Ray]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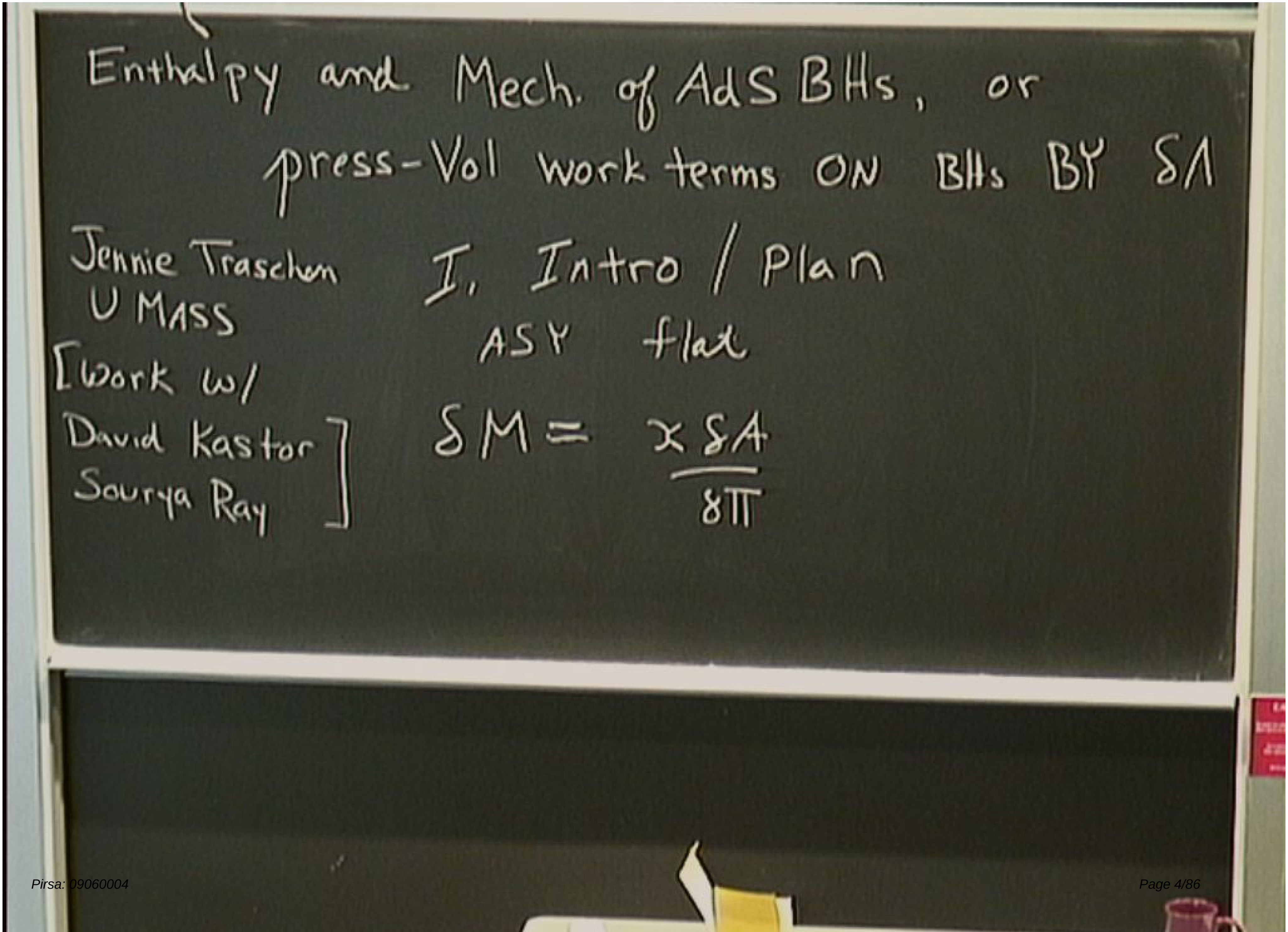
I. Intro / Plan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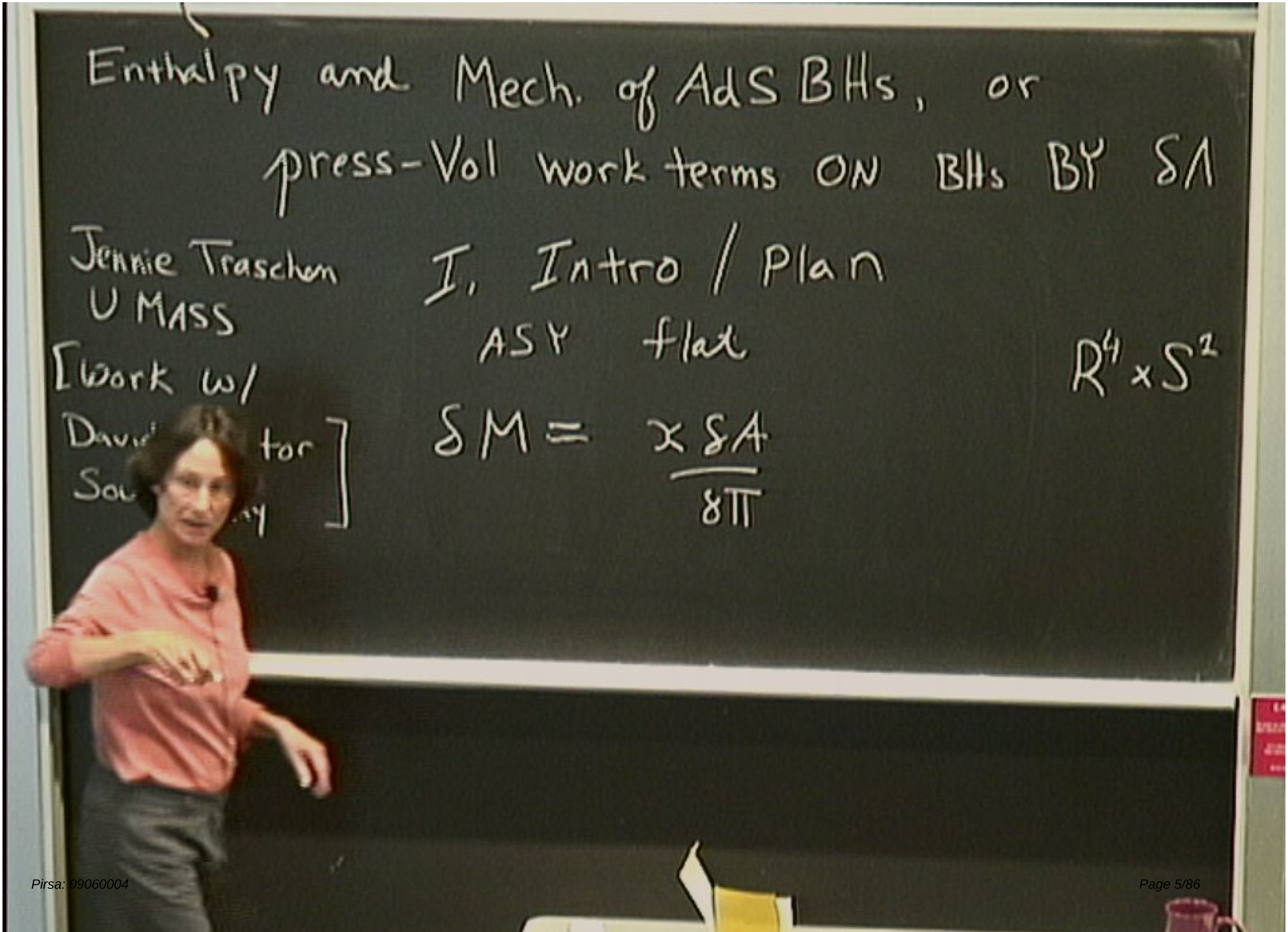
I. Intro / Plan
ASY flat
δM = κδA / 8π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[Work w/
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I. Intro / Plan
ASY flat
R⁴ × S²
δM = κδA / 8π

Enthalpy and Mech. of AdS BHs, or press-Vol work terms ON BHs BY $\delta\Lambda$

Jennie Traschen
U MASS
[Work w/
David Kastor
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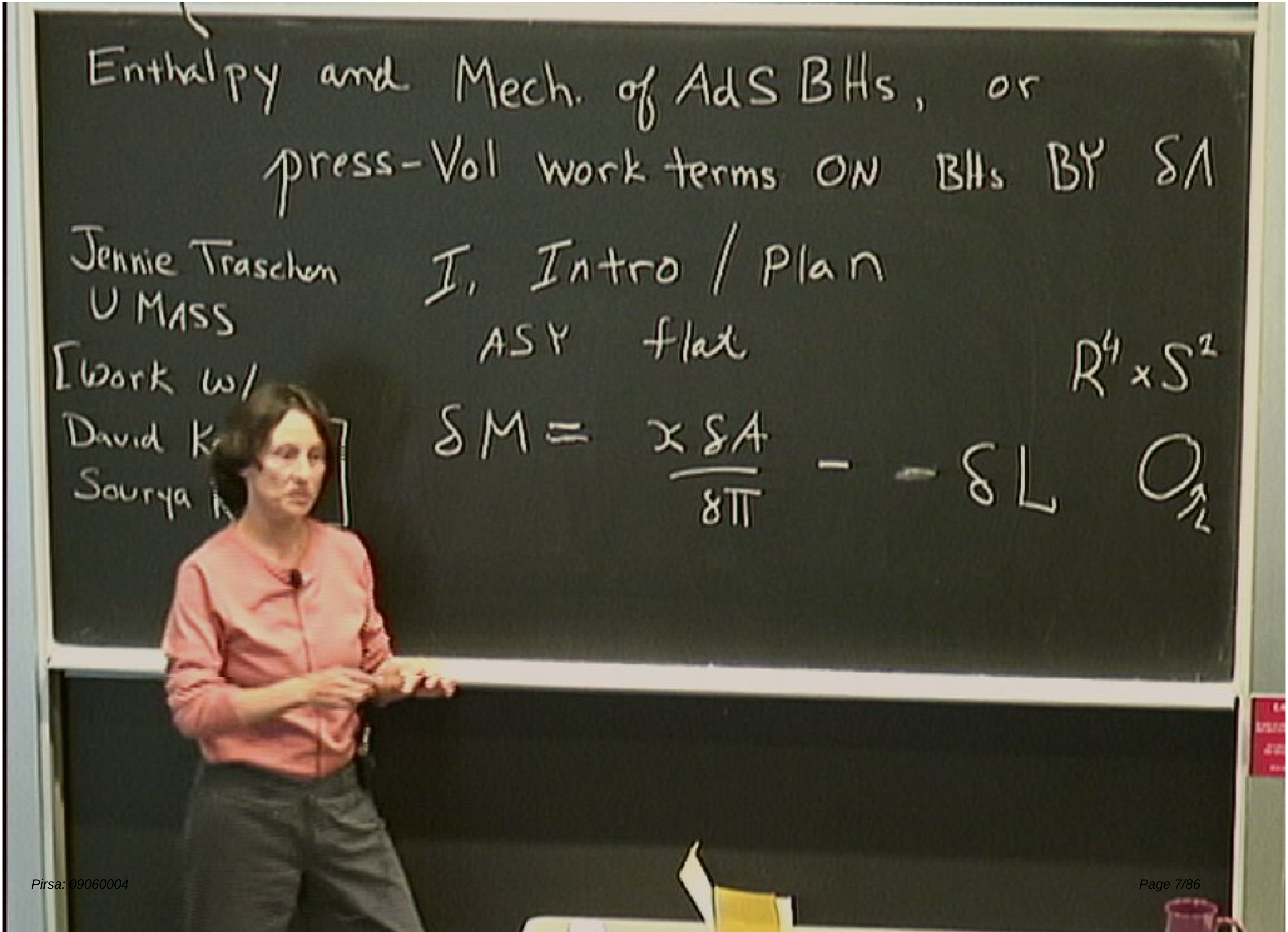
I. Intro / Plan

ASY flat

$R^4 \times S^1$

$$\delta M = \frac{\kappa\,\delta A}{8\pi} - \delta L \ \text{[illegible]}$$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Enthalpy and Mech. of AdS BHs, or

press-Vol work terms ON BHs BY $\delta\Lambda$

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[Work w/
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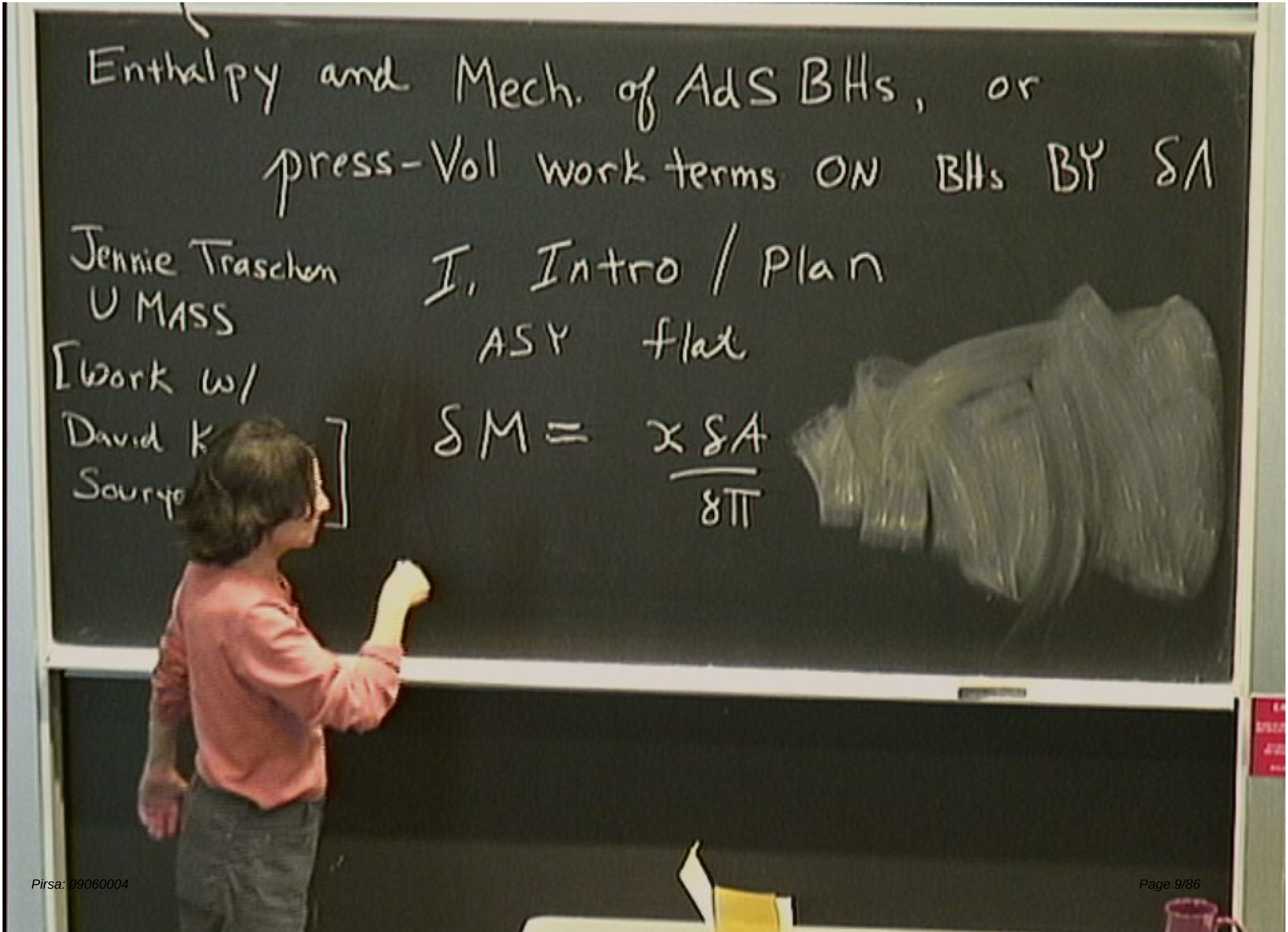
I. Intro / Plan

ASY flat $\quad R^4 \times S^2$

$$\delta M = \frac{\kappa \, \delta A}{8\pi} + \mathcal{T} \, \delta L$$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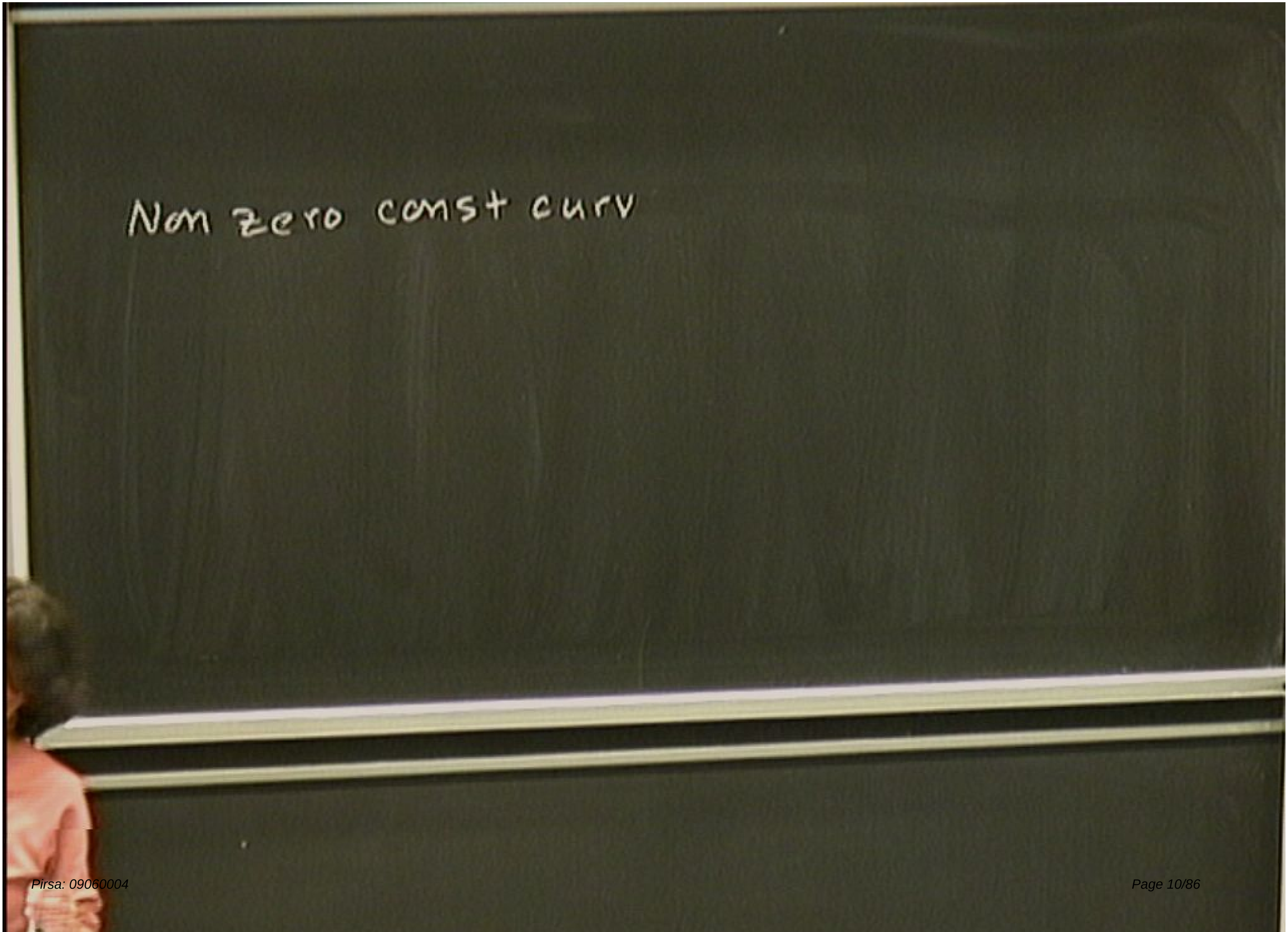
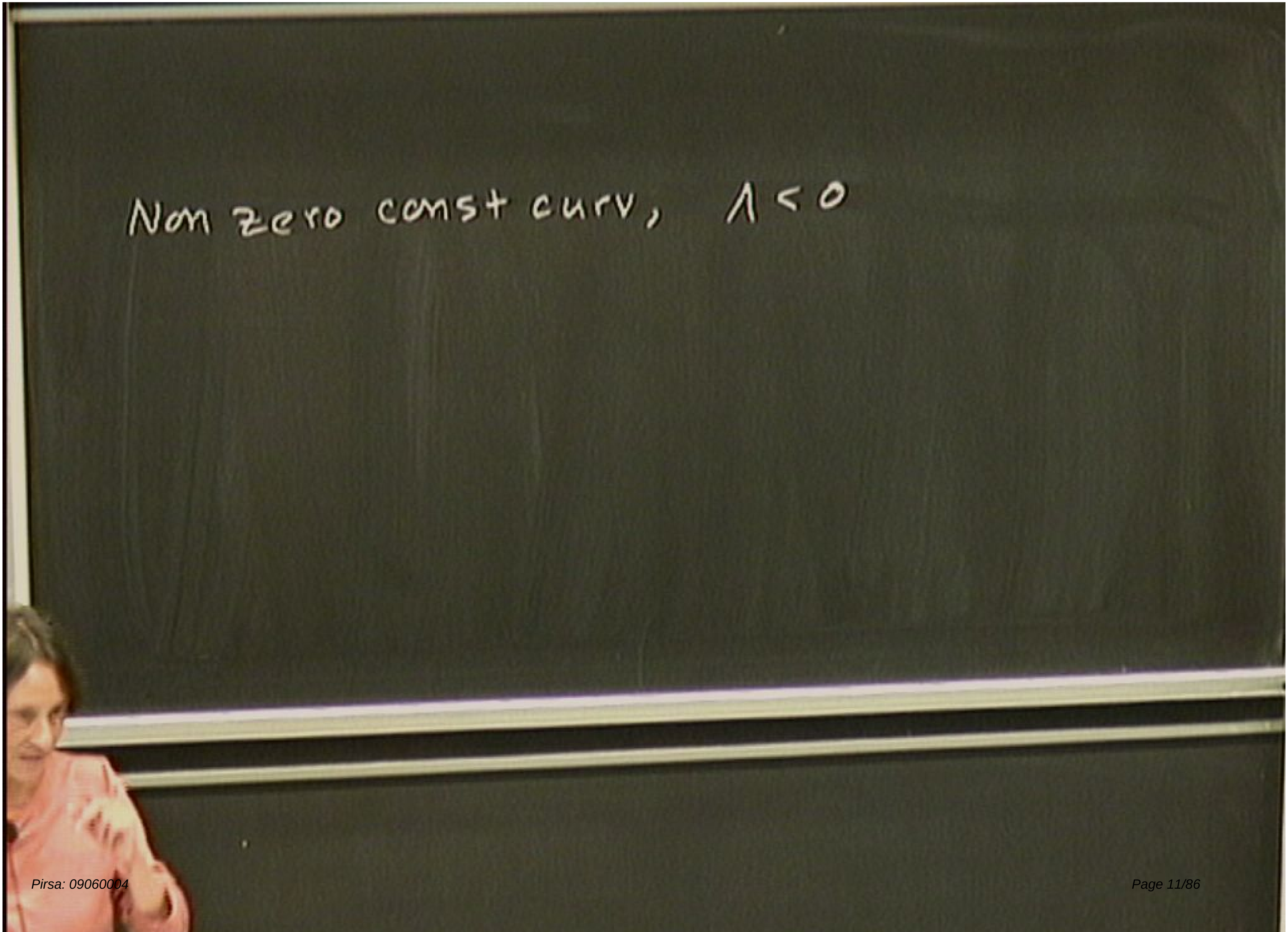
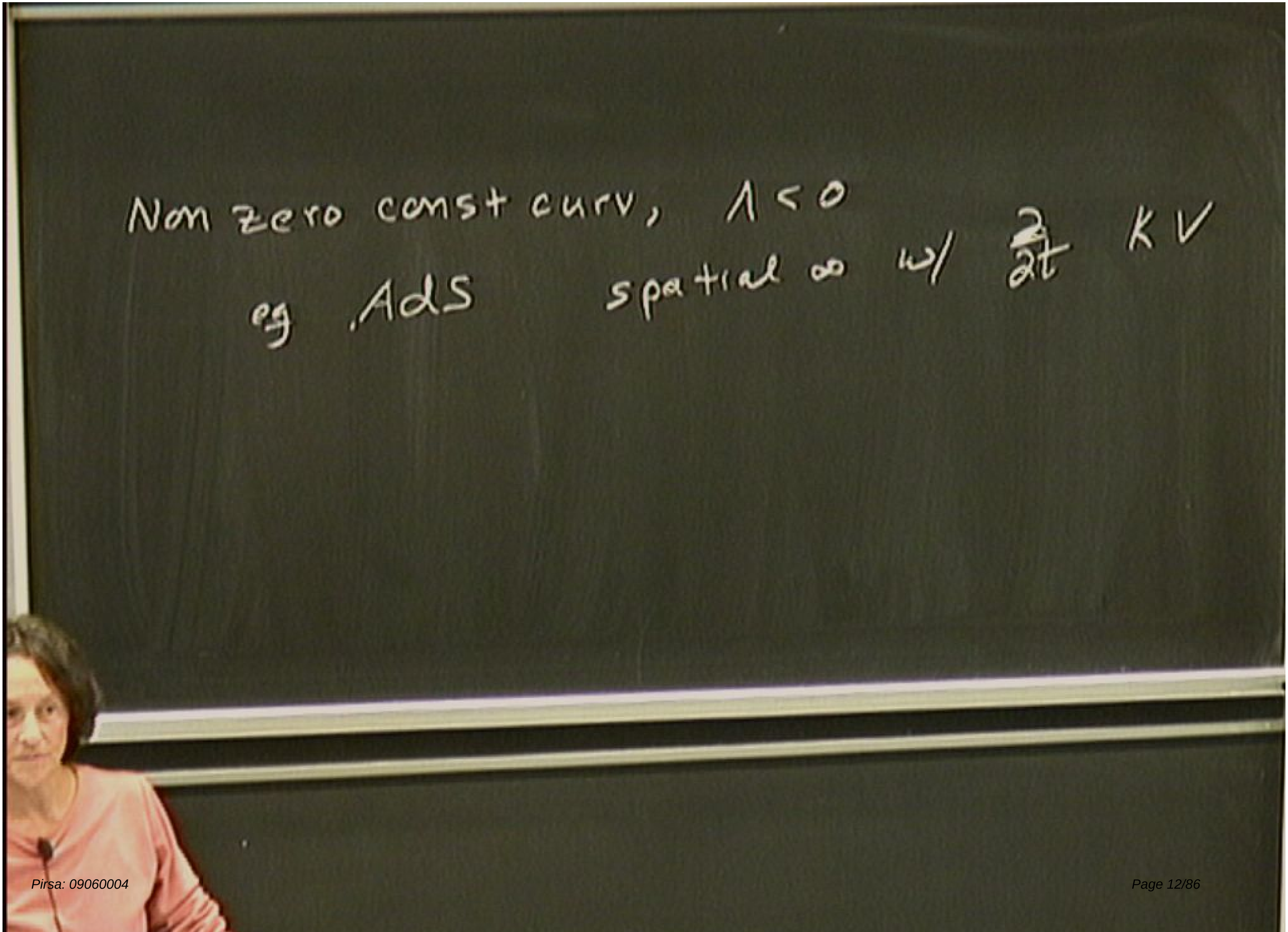

Non zero const curv


Non zero const curv, Λ < 0


Non zero const curv, Λ < 0
eg AdS
spatial ∞ w/ ∂/∂t KV

Non zero const curv, $\Lambda < 0$

eg AdS

spatial ∞ w/ $\frac{\partial}{\partial t}$ KV

$\rightarrow$

Non zero const curv, $\Lambda < 0$

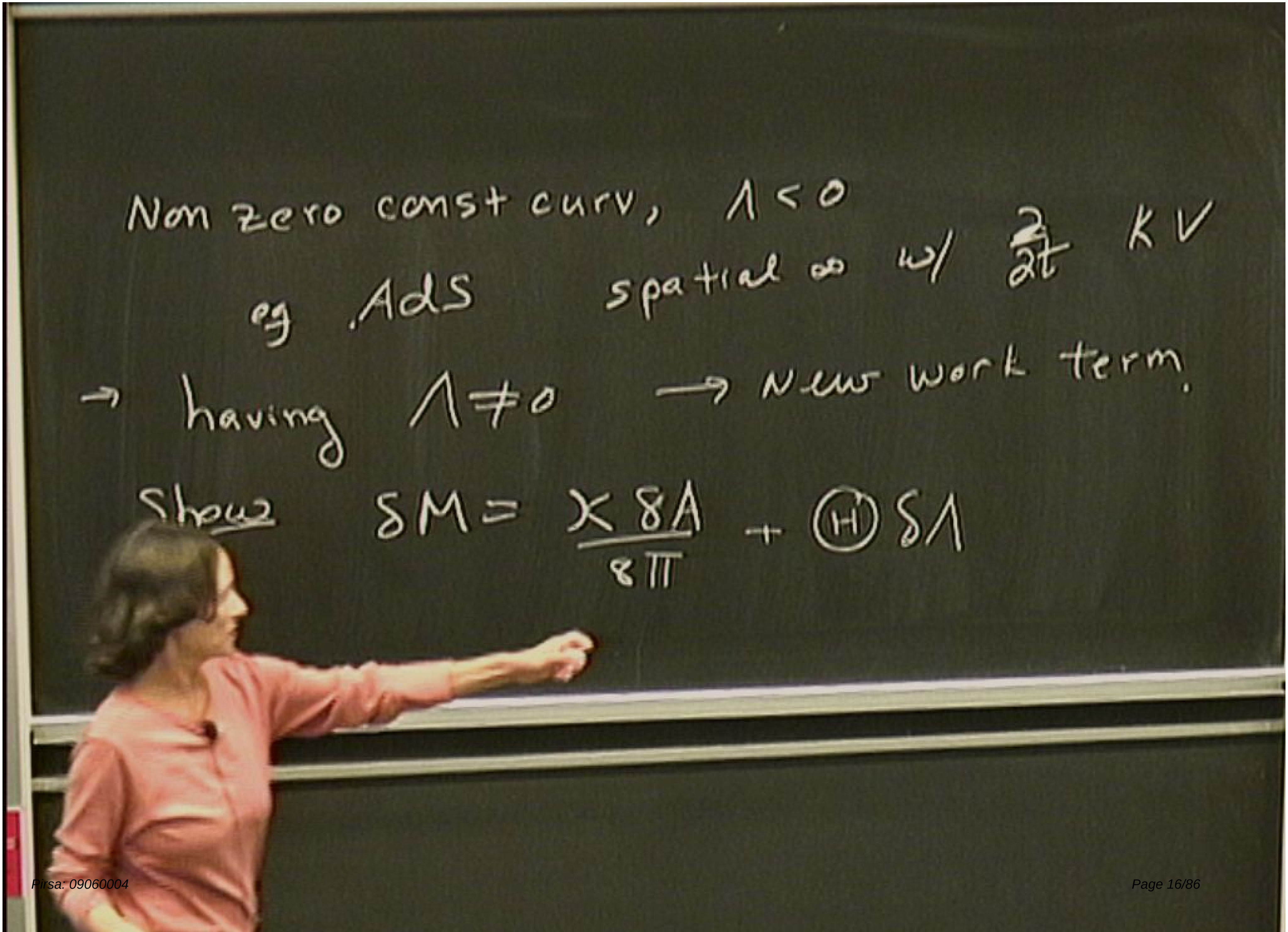
eg AdS spatial ∞ w/ $\frac{\partial}{\partial t}$ KV

→ having $\Lambda \neq 0$

Non zero const curv, $\Lambda < 0$

eg AdS   spatial ∞ w/ $\frac{\partial}{\partial t}$ KV

→ having $\Lambda \neq 0$ → New work term.


Non zero const curv, Λ < 0
eg AdS   spatial ∞ w/ ∂/∂t KV
→ having Λ ≠ 0 → New work term.
Show δM = κ δA / 8π + Ⓗ δΛ

eg AdS spatial ∞ w/ $\frac{\partial}{\partial t}$

→ having $\Lambda \neq 0$ → New work term.

Show $\delta M = \frac{\kappa \, \delta A}{8\pi} + \Theta \, \delta\Lambda$

$-\Theta = \int_\infty (p^K_0 +) - \int (K \text{ pot})$

eg AdS spatial ∞ w/ $\frac{\partial}{\partial t}$ KV

→ having $\Lambda \neq 0$ → New work term.

Show $\delta M = \frac{\kappa \, \delta A}{8\pi} + \Theta \, \delta\Lambda$

$- \Theta = \int_{\infty} (\text{pot}^{k}) - \int_{\partial\Sigma} (\not{k}\ \text{pot})$

$= V$

eg AdS spatial ∞ w/ $\frac{\partial}{\partial t}$ KV

→ having $\Lambda \neq 0$ → New work term.

Show $\delta M = \frac{\kappa \, \delta A}{8\pi} + \textcircled{H} \, \delta\Lambda$

$-\textcircled{H} = \int_{\infty} (p_0^K +) - \int_{\mathcal{H}} (K \text{ pot})$

$= V_\Lambda - V_{\Lambda, BH}$

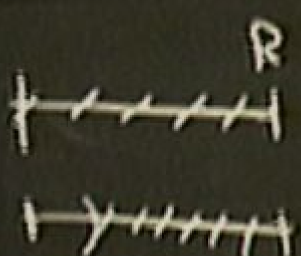
$\frac{\partial}{\partial t}$ KV

eg AdS spatial ∞ w/ $\frac{\partial}{\partial t}$

→ having $\Lambda \neq 0$ → new work term.

Show $\delta M = \frac{\kappa\, \delta A}{8\pi} + \textcircled{H}\, \delta\Lambda$

$-\textcircled{H} = \int_{\infty} (p^{K}_{ot}) - \int_{\partial H} (K\ pot)$



$= V_{\Lambda} - V_{\Lambda, BH}$

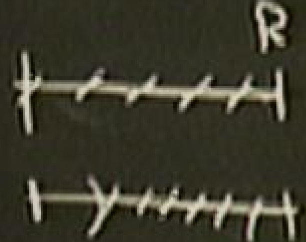
$\frac{\partial}{\partial t}$ kV

eg AdS spatial ∞ w/ $\frac{\partial}{\partial t}$

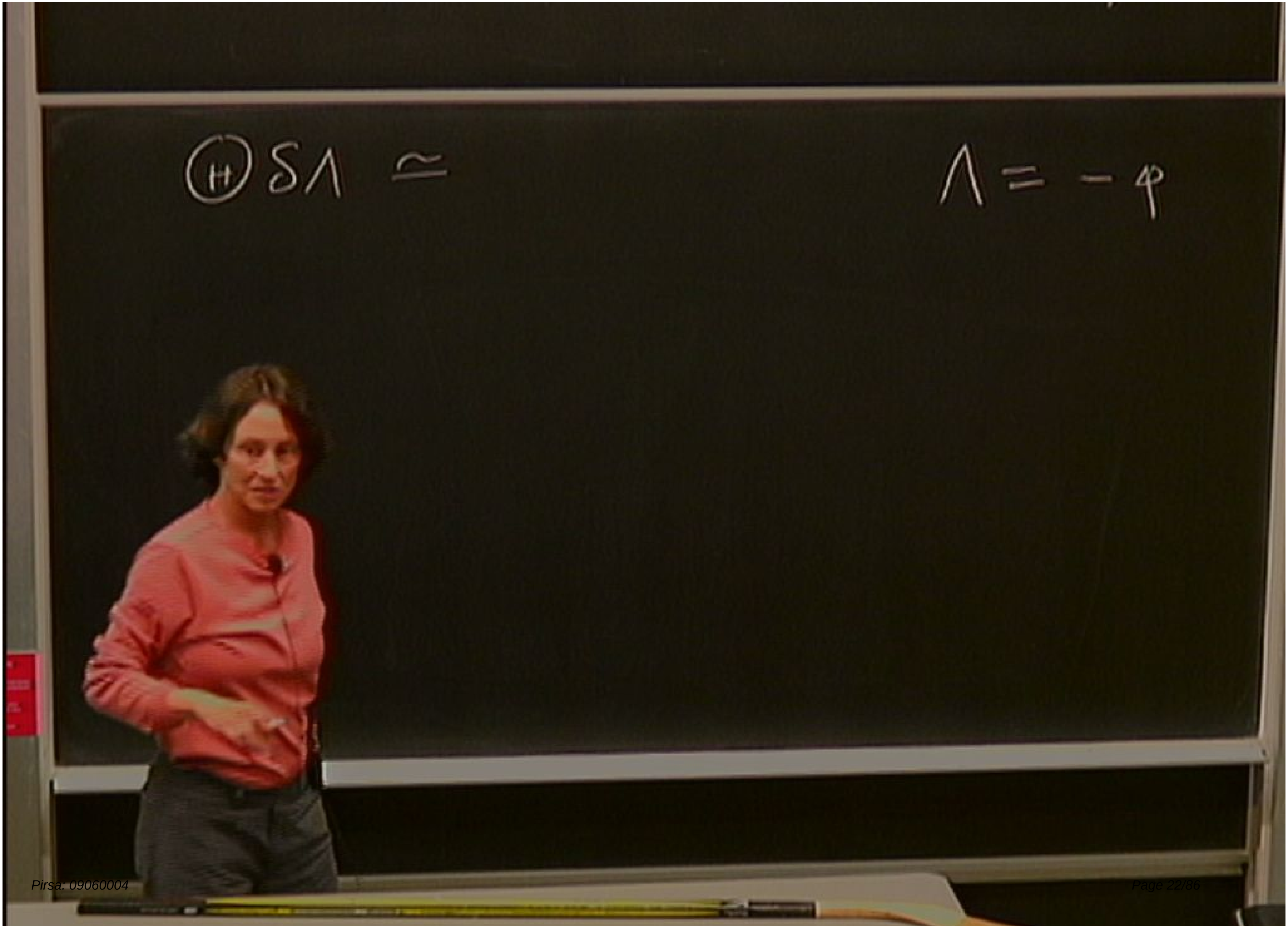
→ having $\Lambda \neq 0$ → New work term.

<u>Show</u> $\delta M = \frac{\kappa\, \delta A}{8\pi} + \Theta\, \delta\Lambda$

$-\Theta = \int_{\infty} (\text{pot}^{K}) - \int_{H} (K\ \text{pot})$



$= V_{\Lambda} - V_{\Lambda, BH} = V_{eff, BH}$


(H) δΛ ≃
Λ = − φ

(H) $\delta\Lambda \simeq V_{BH}\,\delta p$ $\Lambda = -p$

(H) $\delta\Lambda \simeq V_{BH}\, \delta p$ $\Lambda = -p$

Assume Asympt AdS, $\frac{\partial}{\partial t}$ KV of $g^{(0)}_{ab}$

(H) $\delta\Lambda \simeq V_{BH}\, \delta p$ $\Lambda = -p$

<u>Assume</u> Asympt AdS, $\frac{\partial}{\partial t}$ KV of $g^{(0)}_{ab}$

<u>Outline</u>

Ⓗ $\delta\Lambda \simeq V_{BH}\,\delta P \qquad \Lambda = -P$

<u>Assume</u> Asympt AdS, $\frac{\partial}{\partial t}$ KV of $g^{(0)}_{ab}$

BH

<u>Outline</u>

(H) $\delta\Lambda \simeq V_{BH}\,\delta P$ $\Lambda = -P$

Assume Asympt AdS, $\frac{\partial}{\partial t}$ KV of $g^{(0)}_{ab}$

Outline BH, Einst $+\Lambda$

Ⓗ $\delta\Lambda \simeq V_{BH}\,\delta P$ $\qquad \Lambda = -P$

Assume [Static] Asympt AdS, $\frac{\partial}{\partial t}$ KV of $g^{(0)}_{ab}$

BH, Einst $+\Lambda$

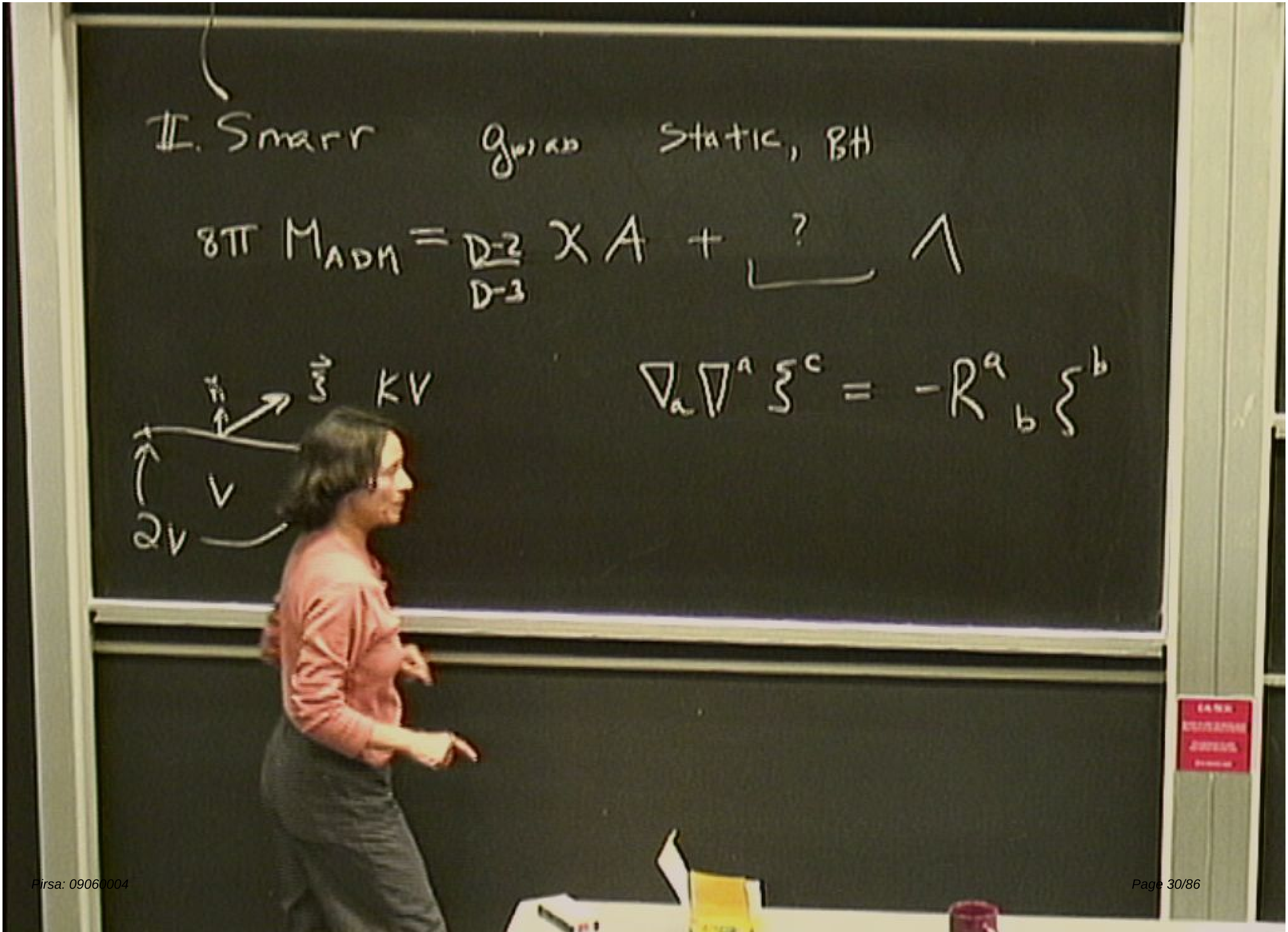
Outline

1. Smarr

BH, Einst $+\Lambda$

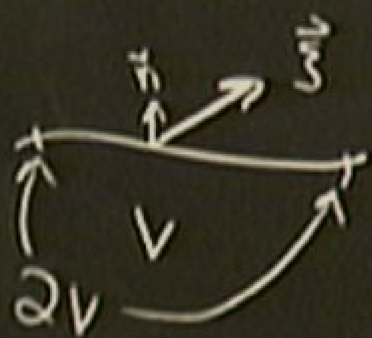
<u>Outline</u>

1. Smarr $\leftarrow$ Killing Pot ω_{ab}
2. Sols ω
3. 1^{st} Law


II. Smarr
static, BH
8π M_ADM = (D-2)/(D-3) κ A + ? Λ
ξ KV
V
∂V
∇_a ∇^a ξ^c = -R^a_b ξ^b

II. Smarr g_{ab} static, BH

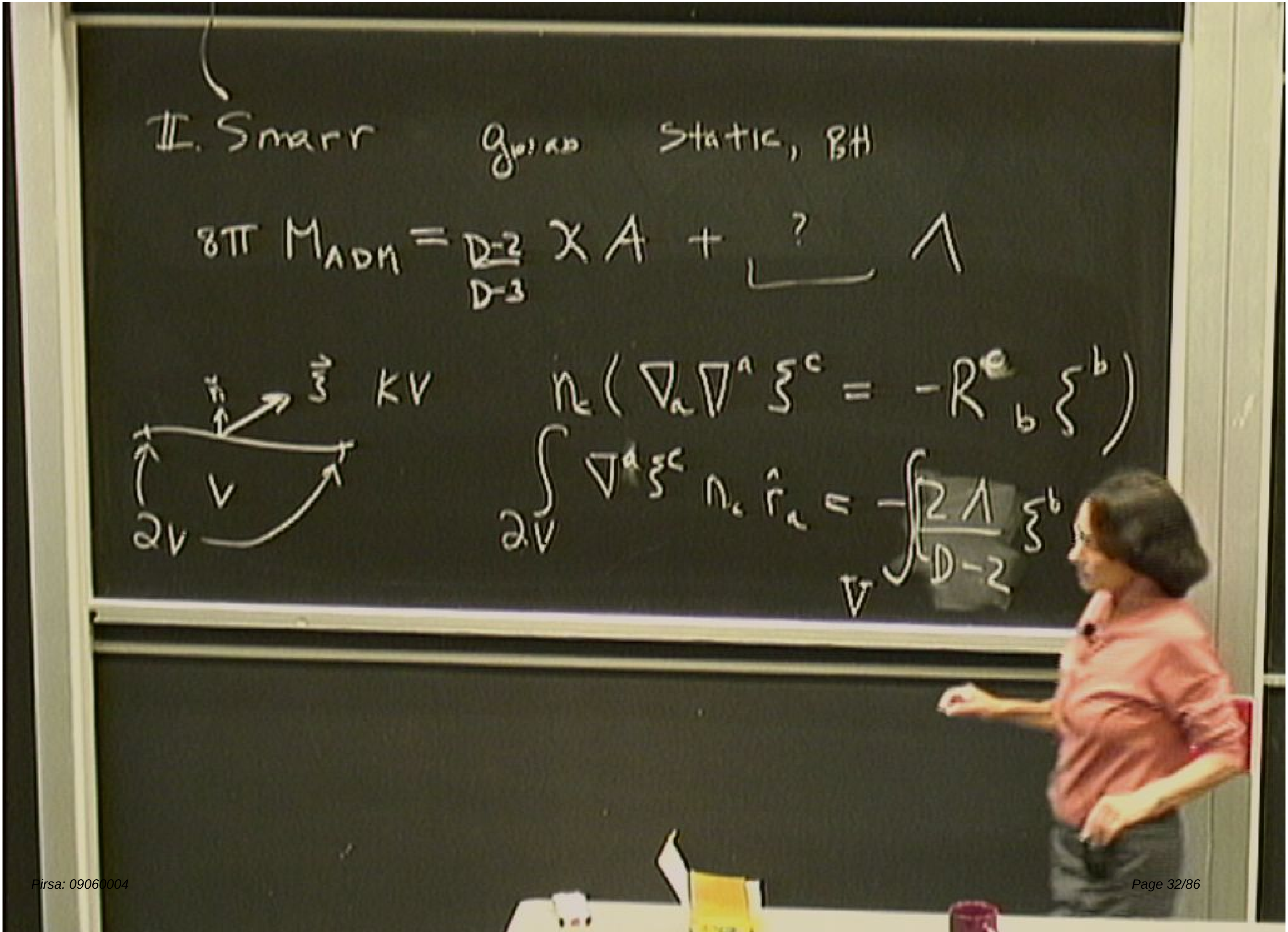
$$8\pi M_{ADM} = \frac{D-2}{D-3}\,\kappa A + \underline{\quad ? \quad}\,\Lambda$$

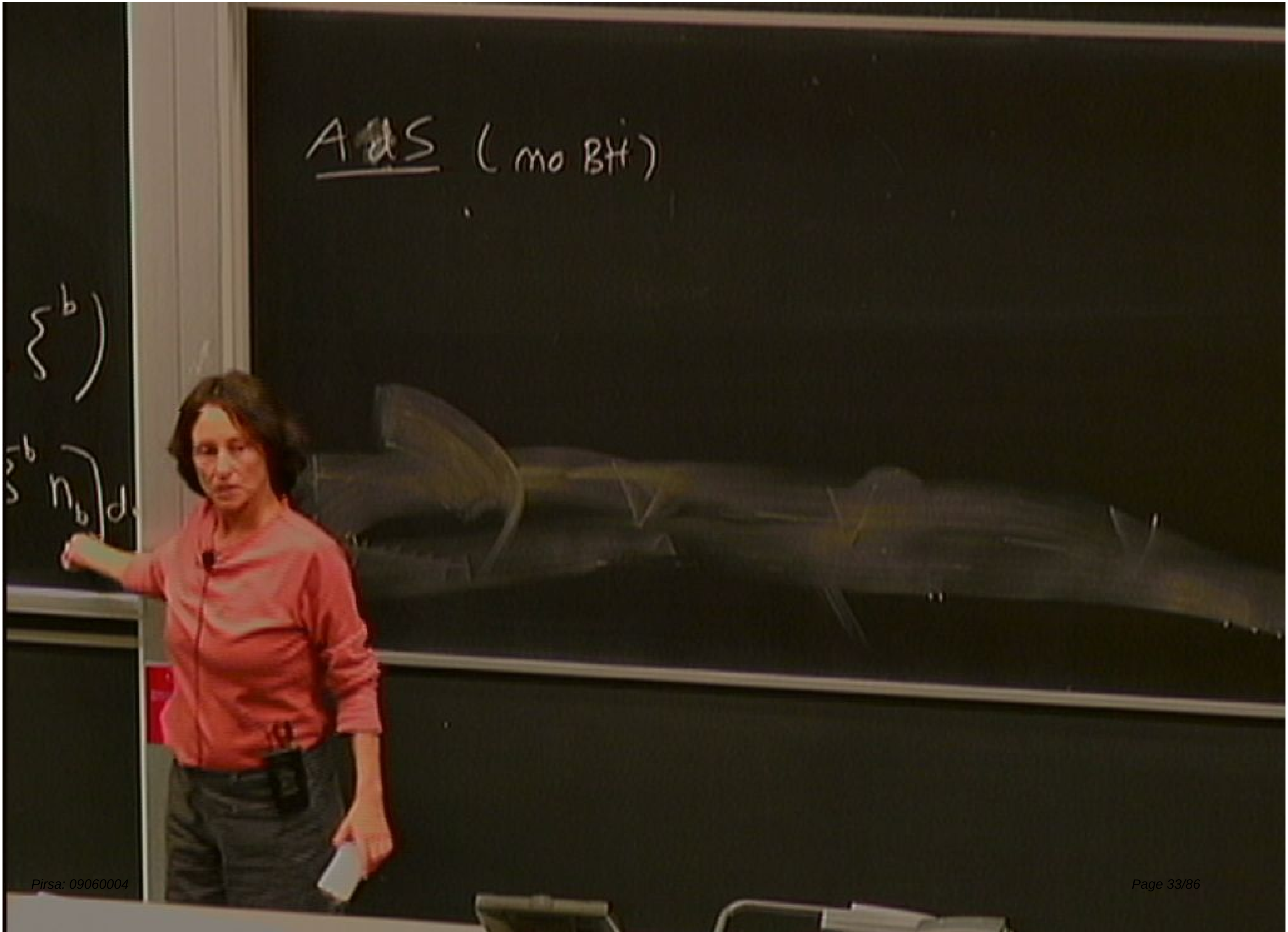


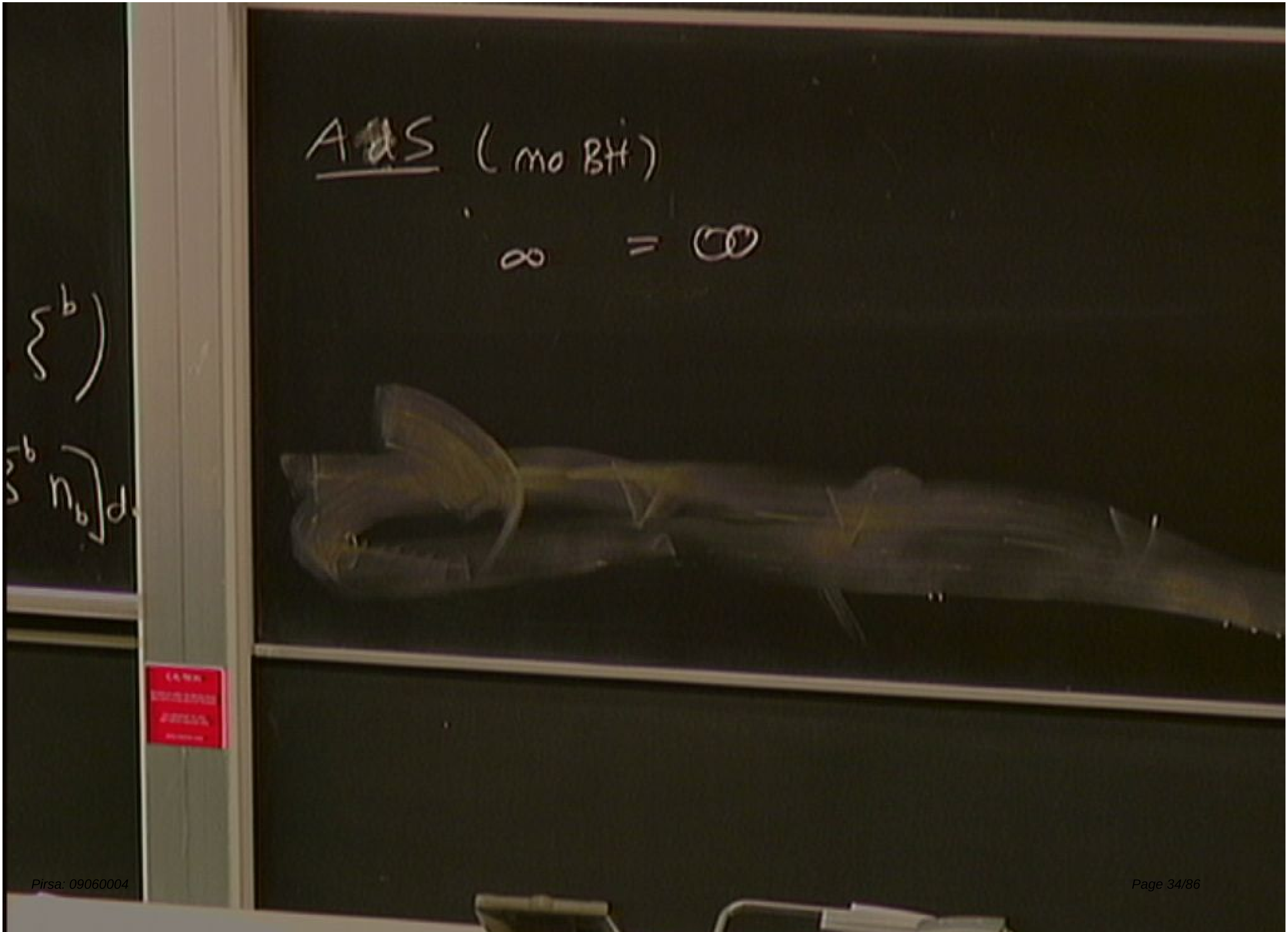
$\vec{\xi}$ KV

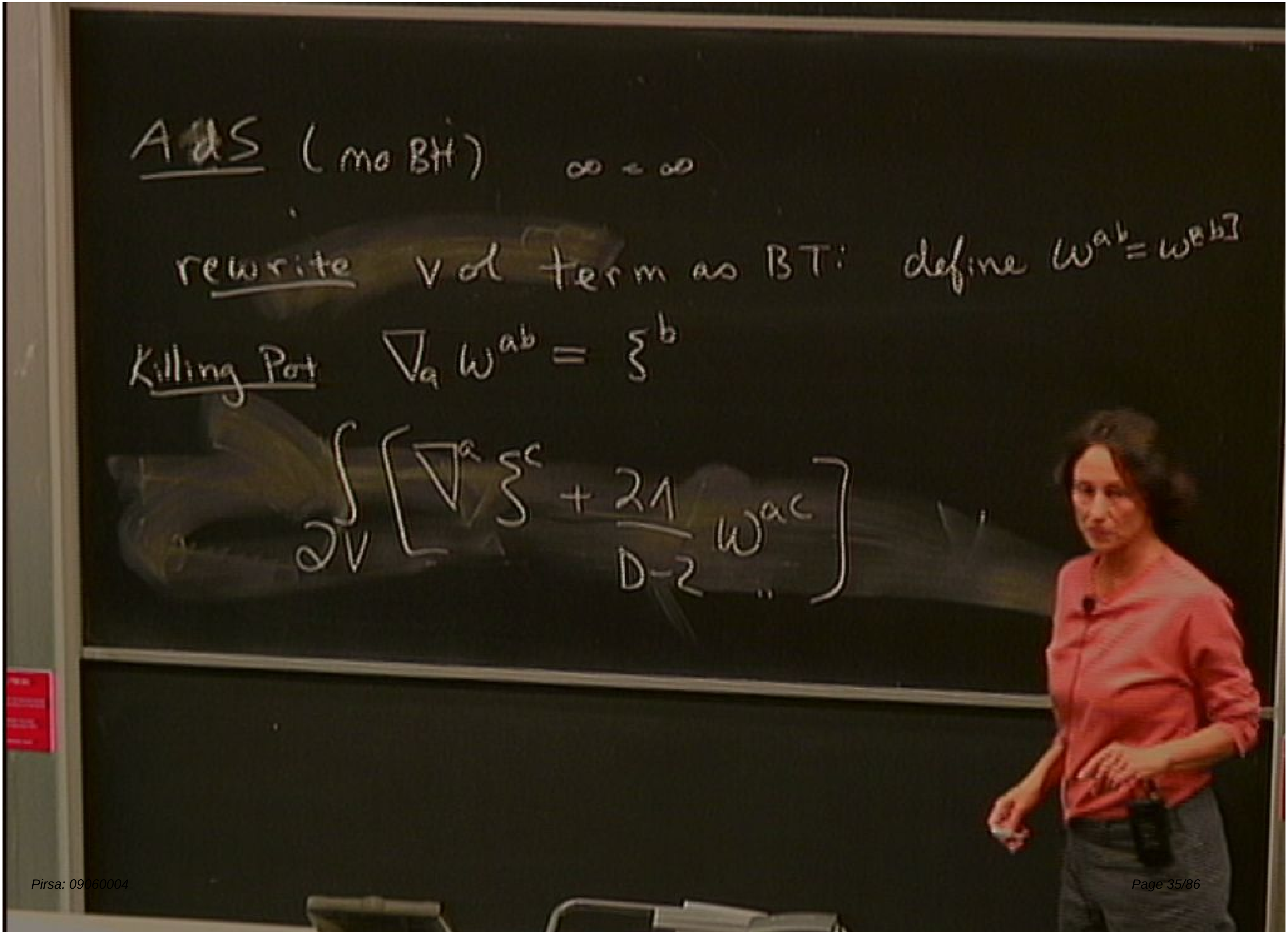
$$n_c\left(\nabla_a \nabla^a \xi^c = -R^c{}_b\, \xi^b\right)$$

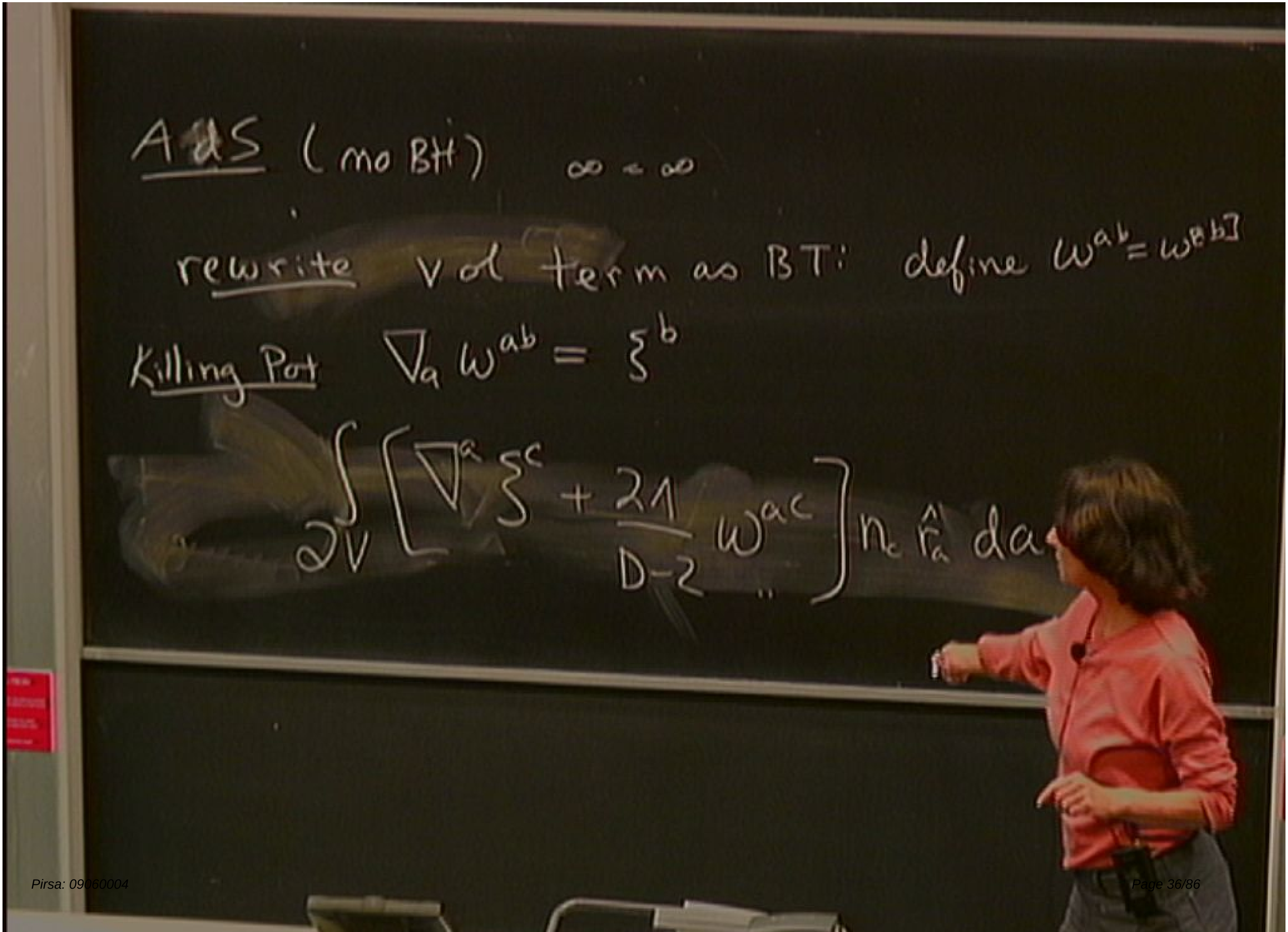
$$\int_{\partial V} \nabla^a \xi^c\, n_c\, \hat{r}_a =$$

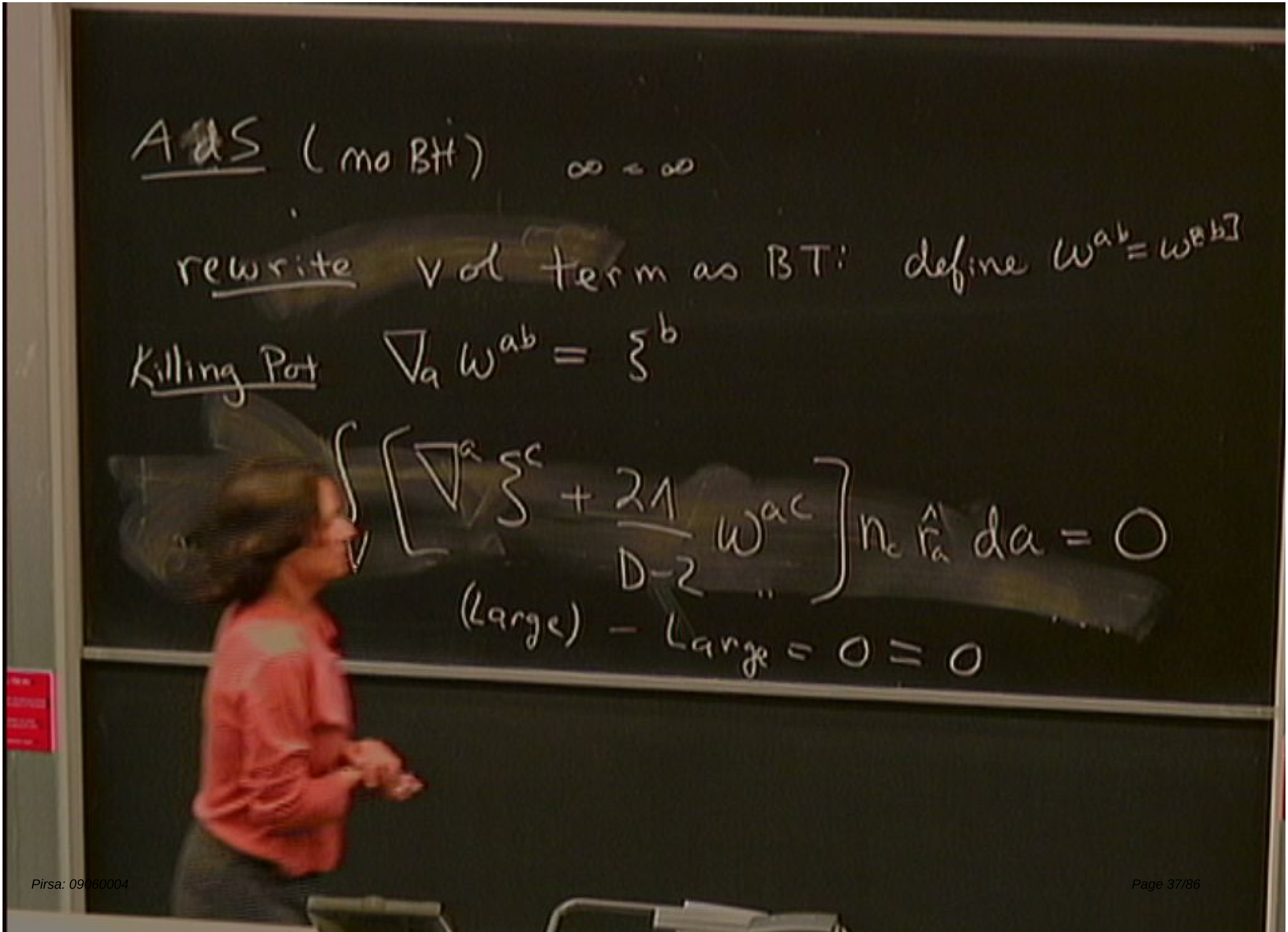

II. Smarr
Static, BH
8π M_ADM = (D-2)/(D-3) κ A + ? Λ
ξ KV
V
∂V
n_c (∇_a ∇^a ξ^c = -R^c_b ξ^b)
∫_∂V ∇^a ξ^c n_c r̂_a = -∫_V 2Λ/(D-2) ξ^b


AdS (no BH)


AdS (no BH)
∞ = ∞


AdS (no BH) ∞ − ∞
rewrite vol term as BT: define ω^ab = ω^[ab]
Killing Pot ∇_a ω^ab = ξ^b
∫_∂V [∇^a ξ^c + 2Λ/(D−2) ω^ac]


AdS (no BH)
∞ - ∞
rewrite vol term as BT: define ω^ab = ω^[ab]
Killing Pot ∇_a ω^ab = ξ^b
∫_∂V [∇^a ξ^c + 2Λ/(D-2) ω^ac] n_c r̂_a da


AdS (no BH)
∞ - ∞
rewrite vol term as BT: define ω^ab = ω^[ab]
Killing Pot ∇_a ω^ab = ξ^b
∫ [∇^a ξ^c + 2Λ/(D-2) ω^ac] n_c r̂_a da = 0
(Large) - Large = 0 = 0

AdS (no BH) $\infty - \infty$

rewrite vol term as BT: define $\omega^{ab} = \omega^{[ab]}$

Killing Pot $\nabla_a \omega^{ab} = \xi^b$

$$\int_{\partial V} \left[\nabla^a \xi^c + \frac{2\Lambda}{D-2} \omega^{ac} \right] n_c \hat{r}_a \, da = 0$$

(Large) − Large = 0 = 0

AdS (no BH) $\infty - \infty$

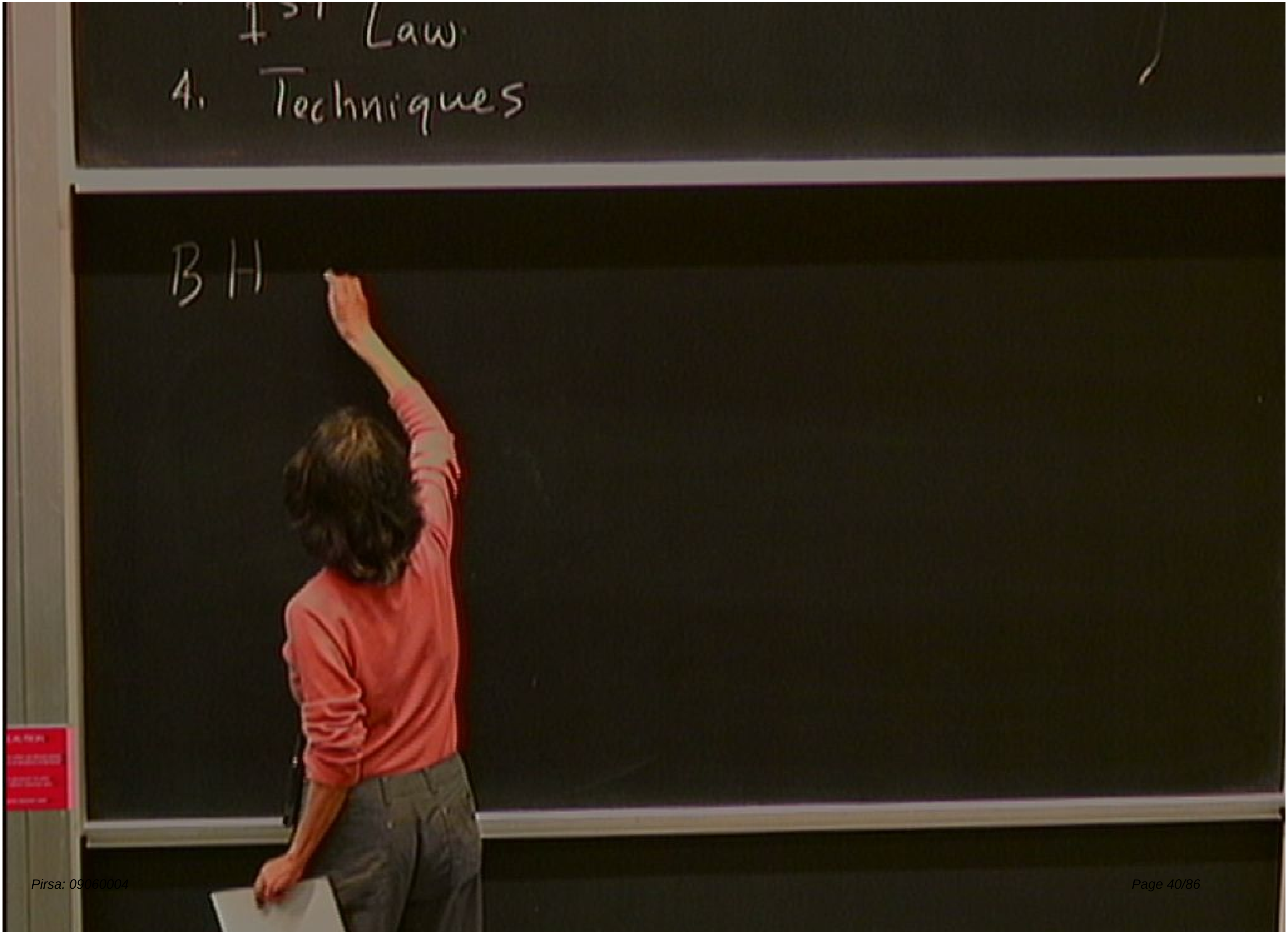
rewrite vol term as BT: define $\omega^{ab} = \omega^{[ab]}$

ω^{rt}

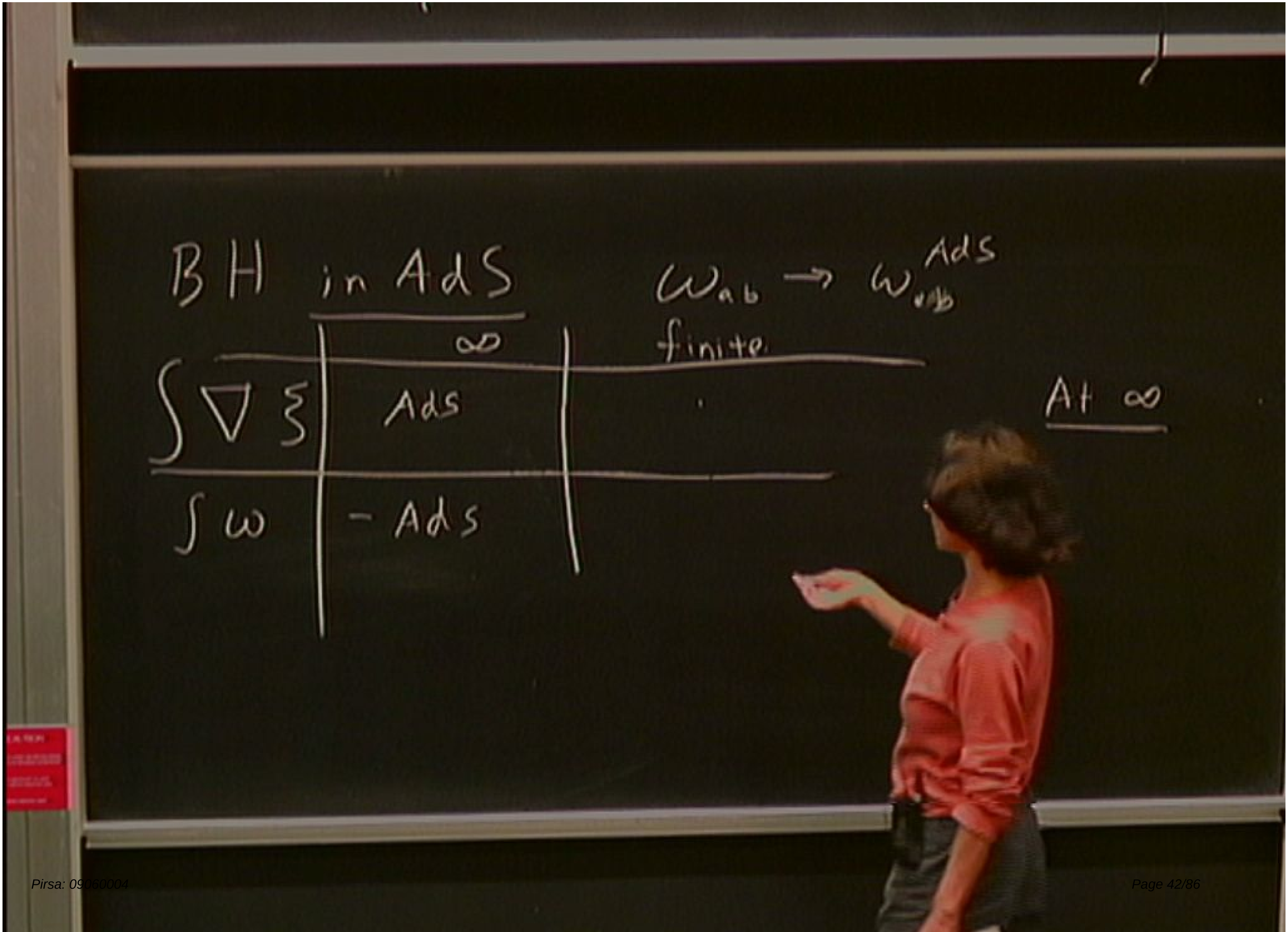
Killing Pot $\nabla_a \omega^{ab} = \xi^b$

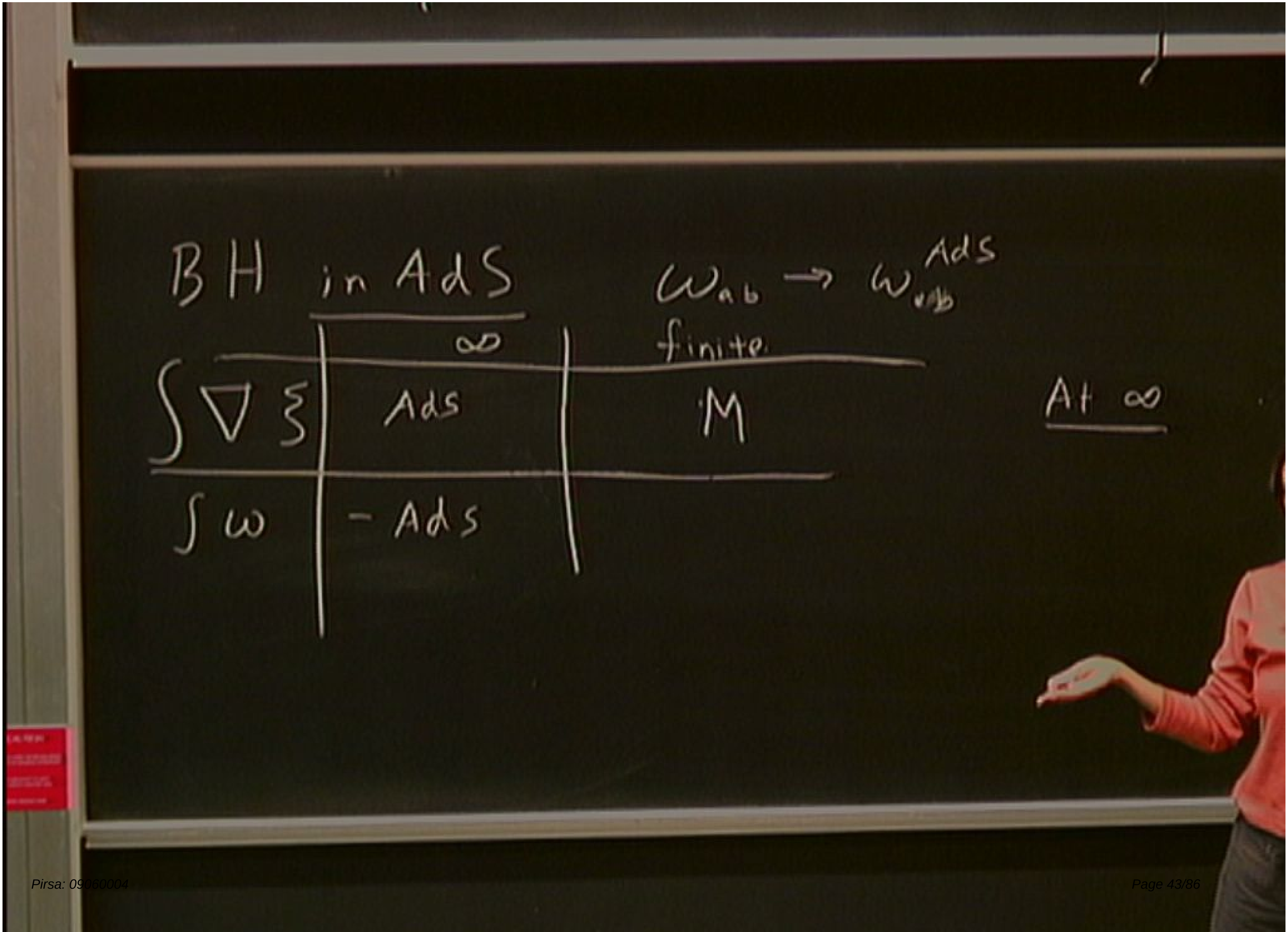
$$\int_{\partial V} \left[\nabla^a \xi^c + \frac{2\Lambda}{D-2} \omega^{ac} \right] n_c \hat{r}_a \, da = 0$$

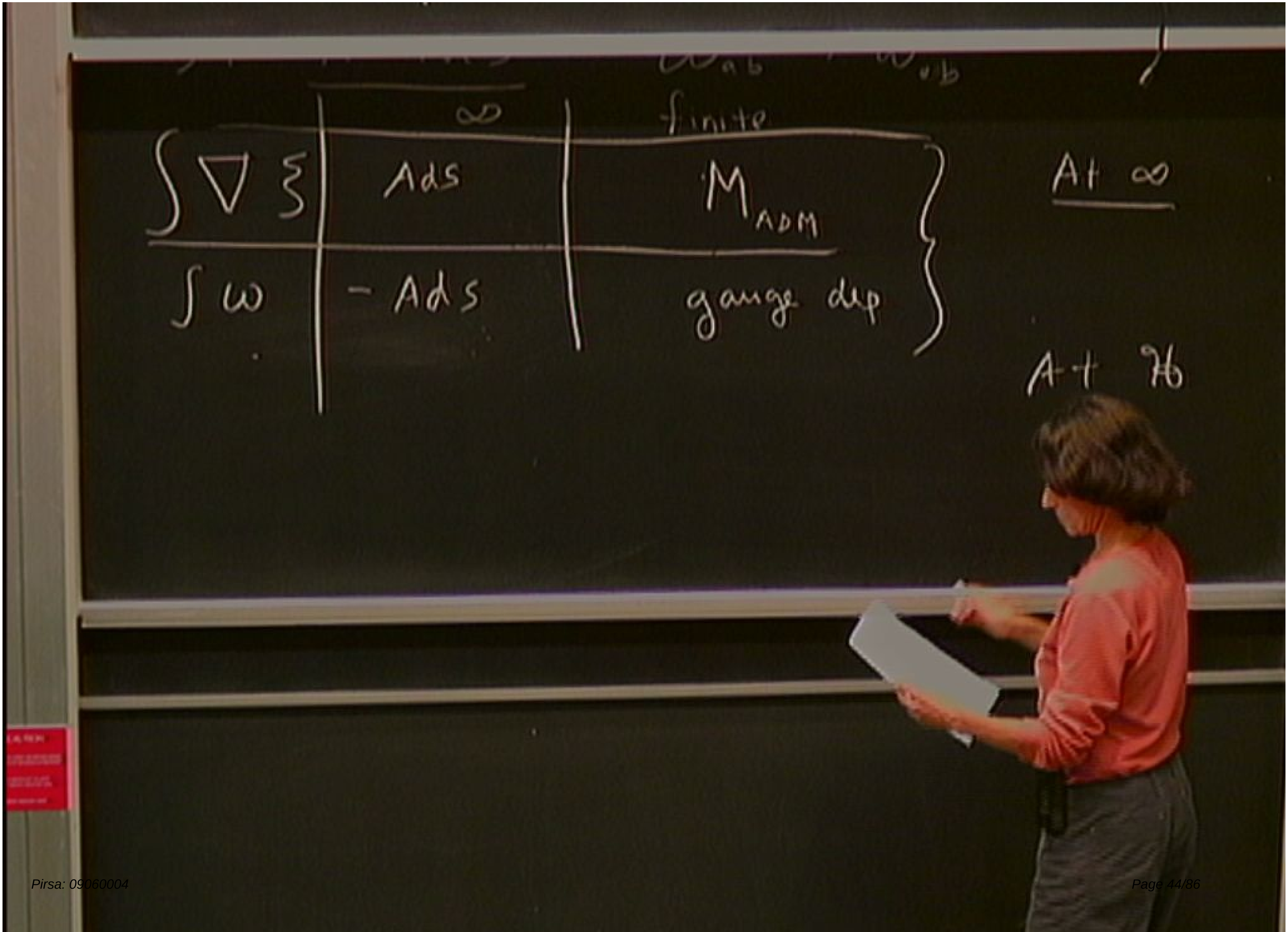
(Large) − Large = 0 = 0


1st Law
4. Techniques
BH

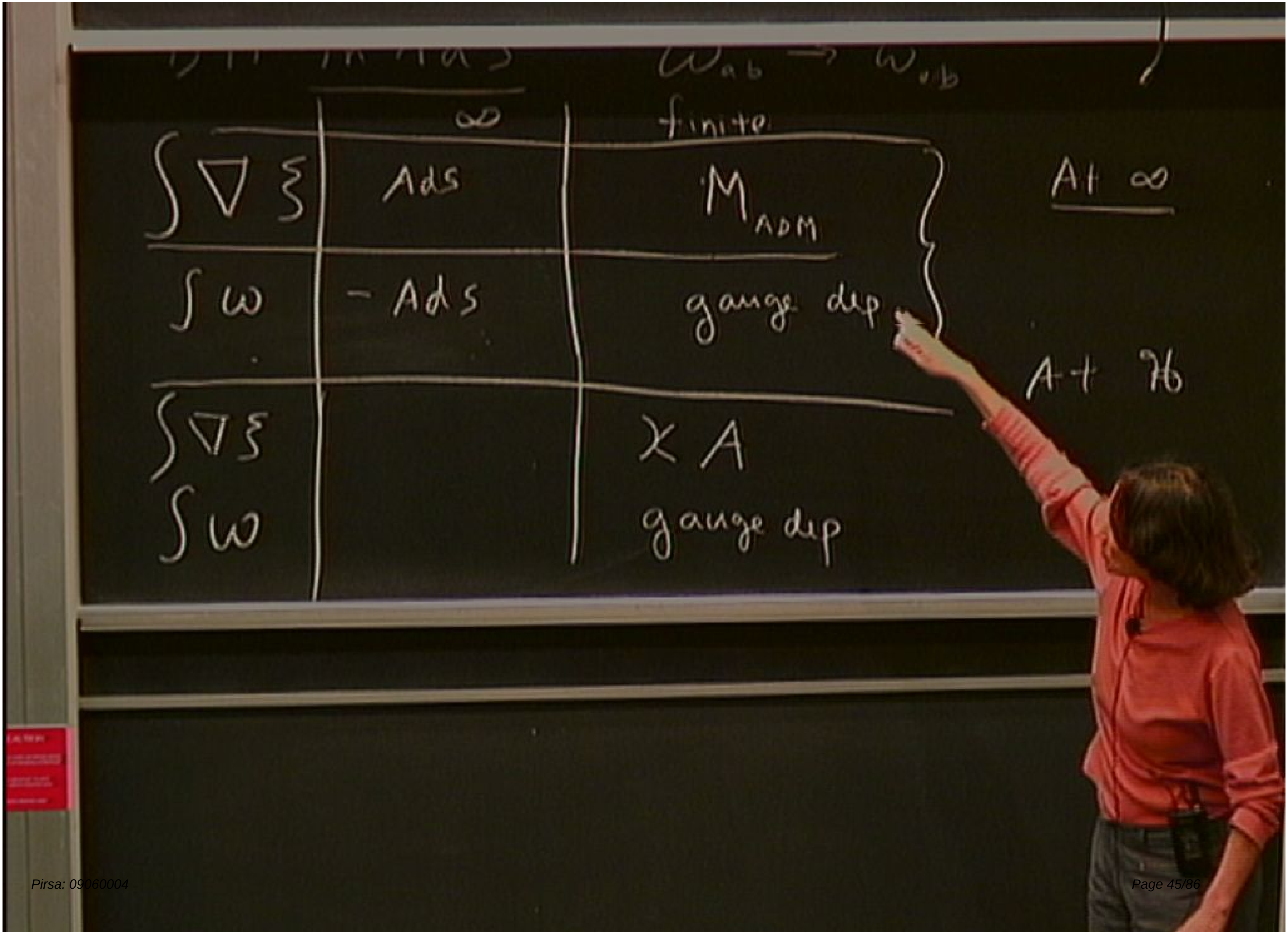
BH in AdS $\qquad \omega_{ab} \rightarrow \omega_{ab}^{AdS}$


BH in AdS
ωab → ωab AdS
finite
∞
∫ ∇ ξ
AdS
∫ ω
- AdS
At ∞


BH in AdS
ωab → ωab AdS
∞
finite
∫∇ξ
AdS
M
At ∞
∫ω
- AdS



∞
finite
∫ ∇ ξ
AdS
M_ADM
∫ ω
− AdS
gauge dep
At ∞
At ℋ


∞
finite
∫∇ξ
AdS
M_{ADM}
At ∞
∫ω
− AdS
gauge dep
At ℋ
∫∇ξ
× A
∫ω
gauge dep

BH in AdS $\omega_{ab} \to \omega_{ab}$

	∞	finite
$\int \nabla \xi$	AdS	M_{ADM}
$\int \omega$	$-$ AdS	gauge dep
$\int \nabla \xi$		$\times A$
$\int \omega$		gauge dep

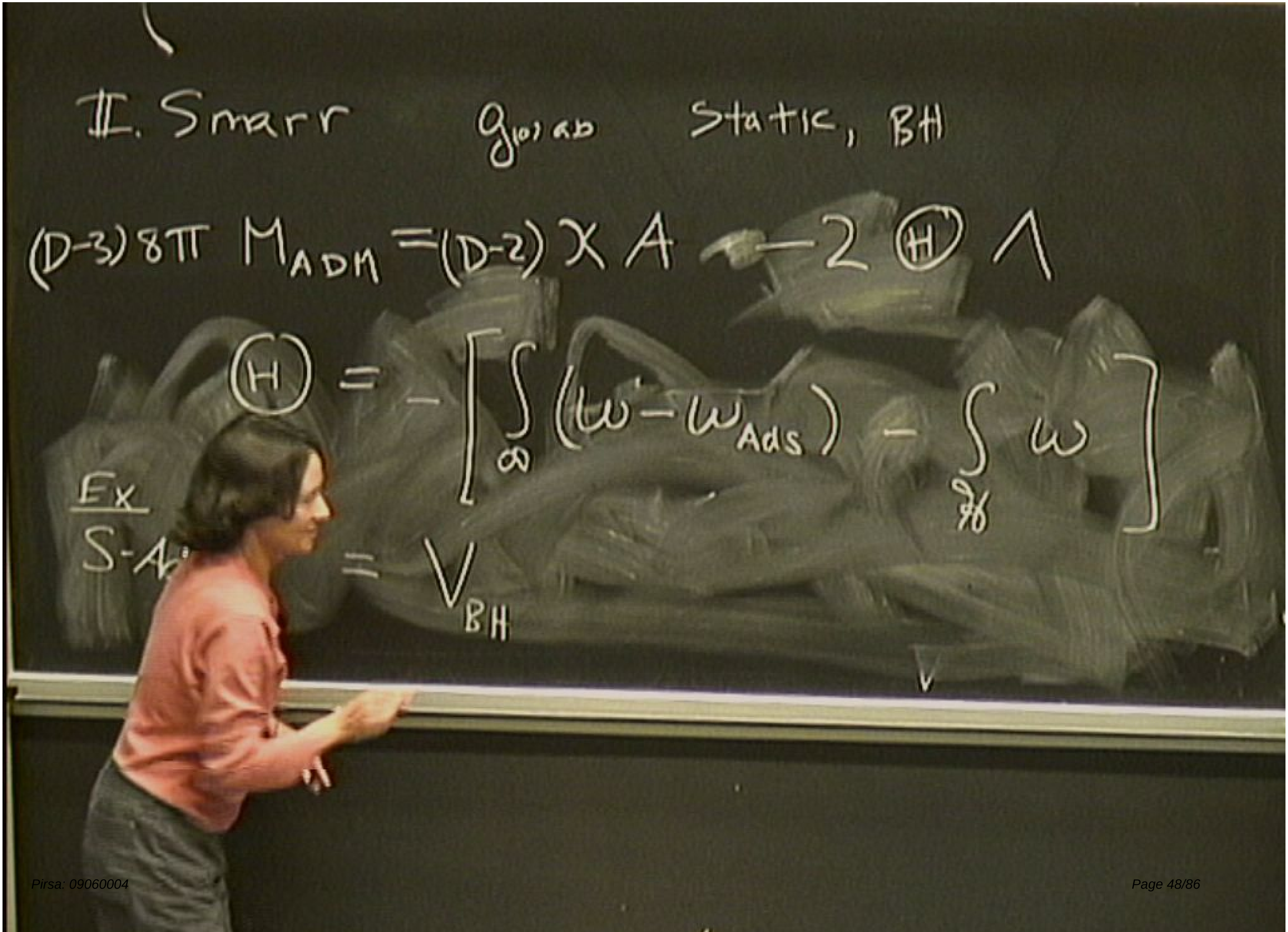
At ∞

At $\mathcal{H}$

II. Smarr $\quad g_{(0)ab}$ Static, BH

$$(D-3)\, 8\pi\, M_{ADM} = (D-2)\, \kappa A - 2\, \textcircled{H}\, \Lambda$$

$$\textcircled{H} = \int_{\infty} \left(\omega - \omega_{AdS} \right)$$


II. Smarr g(0) ab Static, BH
(D-3) 8π M_ADM = (D-2) κ A − 2 Θ Λ
Θ = − [∫∞ (ω − ω_AdS) − ∫∂H ω]
Ex
S-A
= V_BH

II. Smarr $\quad g_{(0)ab}$ static, BH

$$(D-3)\,8\pi M_{ADM} = (D-2)\,\kappa A - 2\,\textcircled{H}\,\Lambda$$

$$\textcircled{H} = -\left[\int_{\infty} (\omega - \omega_{AdS}) - \int_{\mathcal{H}} \omega\right]$$

Ex: S-AdS $= V_{BH}$ using $\int_V \sqrt{-g}$

II. Smarr $\quad g_{(0)ab}$ Static, BH

$$(D-3)\, 8\pi\, M_{ADM} = (D-2)\, \chi A - 2\, \textcircled{H}\, \Lambda$$

$$\textcircled{H} = -\left[\int_{\infty} (\omega - \omega_{AdS}) - \int_{\mathcal{H}} \omega\right]$$

Ex
S-AdS $= V_{BH}$ using $\int \sqrt{-g}$

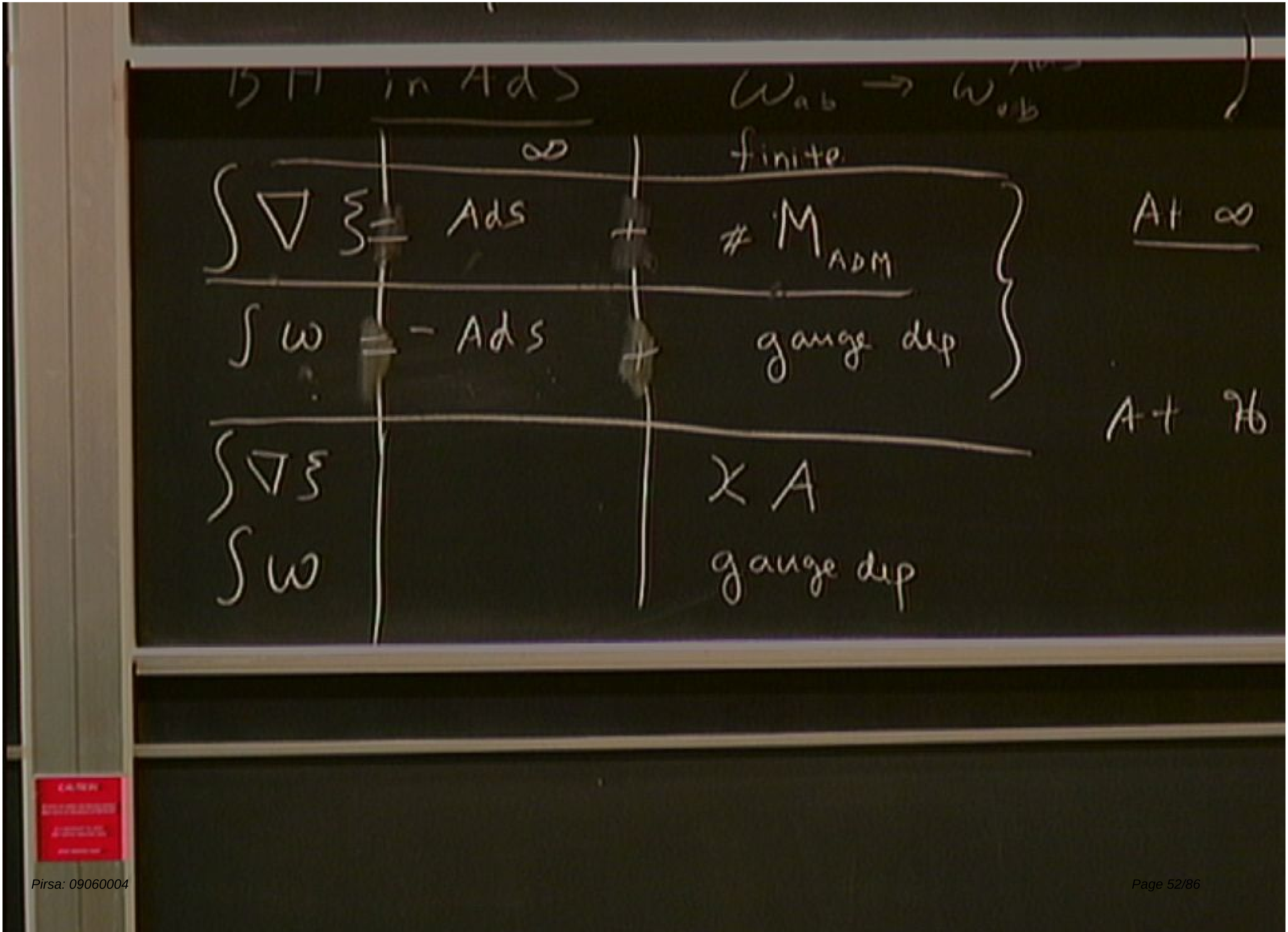
II. Smarr $\quad g_{(0)ab}$ static, BH

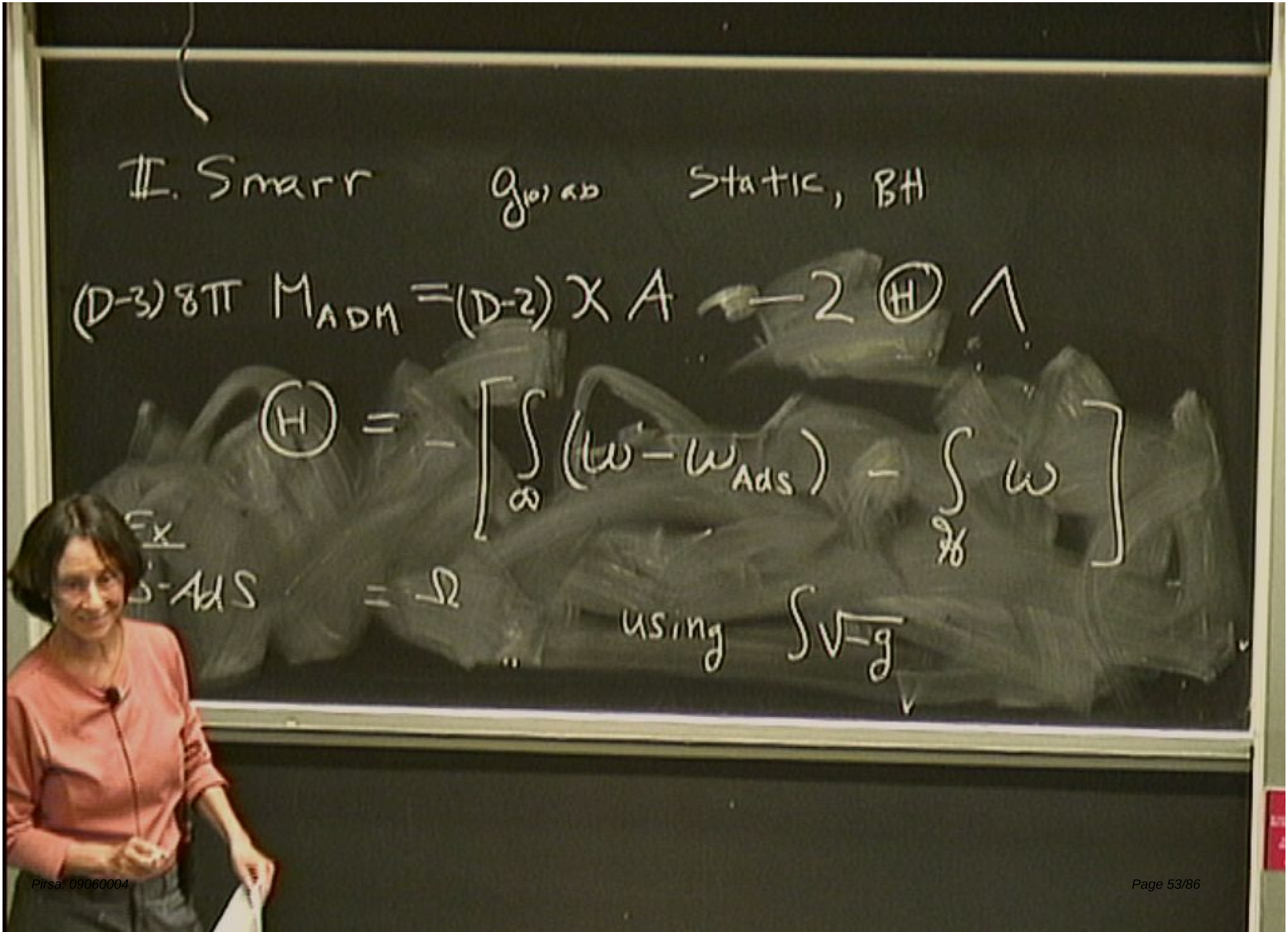
$$(D-3)\,8\pi M_{ADM} = (D-2)\,\kappa A - 2\,\textcircled{H}\,\Lambda$$

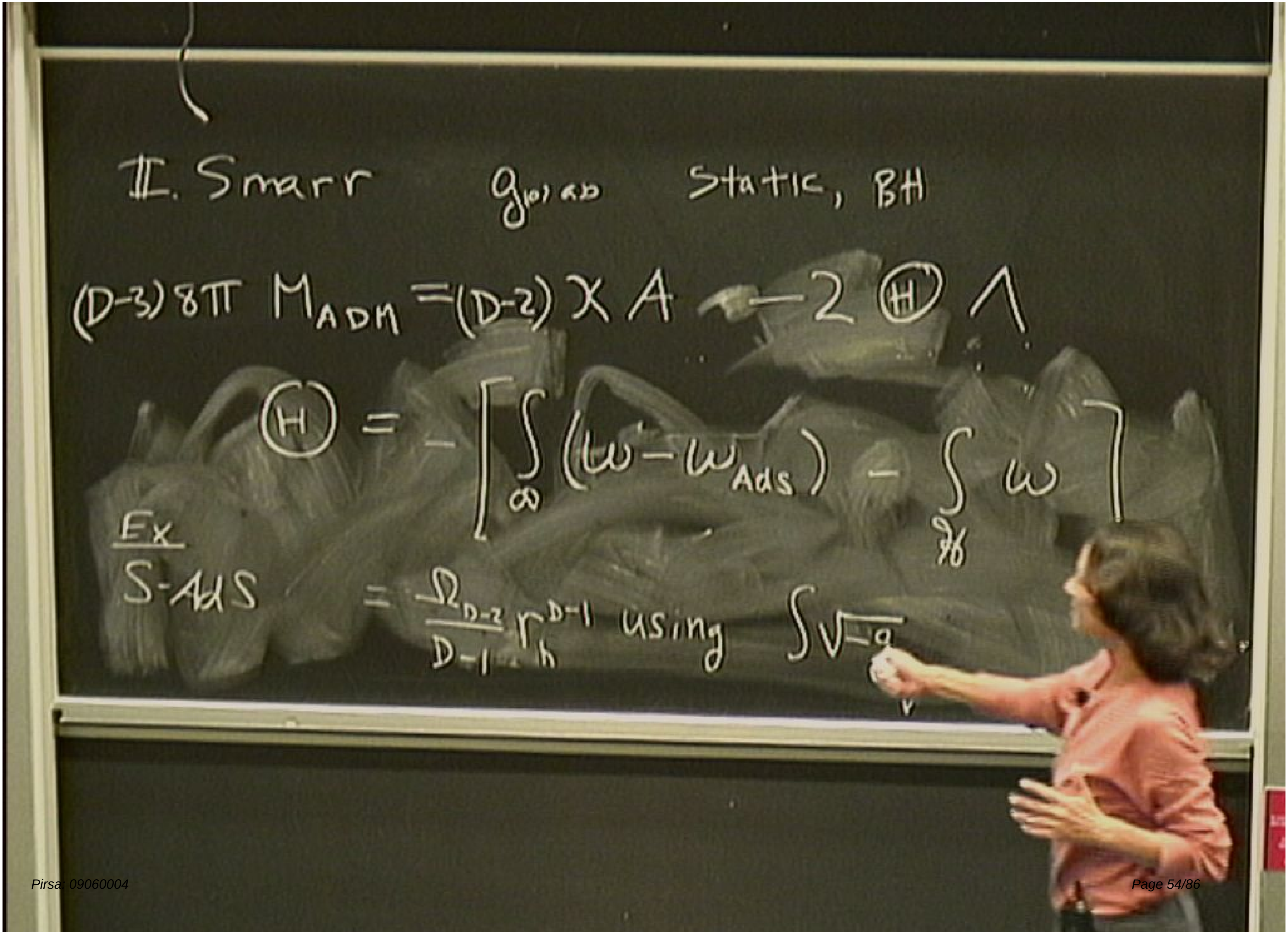
$$\textcircled{H} = -\left[\int_{\infty} (\omega - \omega_{AdS}) - \int_{\mathcal{H}} \omega\right]$$

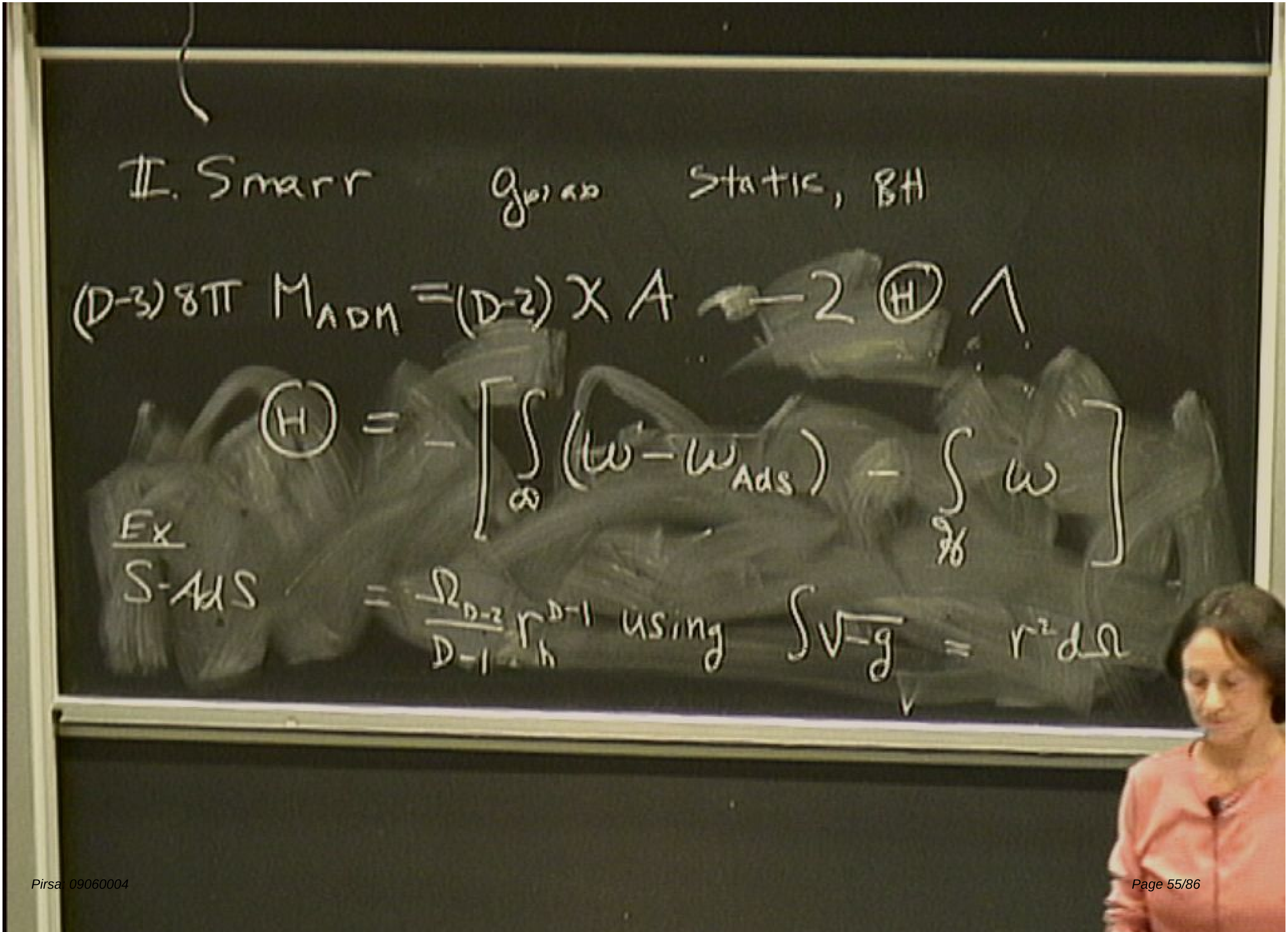
Ex

S-AdS $\quad = V_{BH}$ using $\int \sqrt{-g}$


BH in AdS
$\omega_{ab} \to \omega_{ab}^{AdS}$
∞
finite
$\int \nabla \xi$ − AdS ≠ # M_{ADM}
$\int \omega$ − AdS ≠ gauge dep
At ∞
At $\mathcal{H}$
$\int \nabla \xi$ χA
$\int \omega$ gauge dep


II. Smarr
Static, BH
(D-3) 8π M_ADM = (D-2) κ A − 2 Θ Λ
Θ = − [∫∞ (ω − ω_AdS) − ∫ ω]
Ex
S-AdS
= −Ω
using ∫√−g


II. Smarr g(0) ab Static, BH
(D-3) 8π M_ADM = (D-2) κ A - 2 Θ Λ
Θ = - [∫∞ (ω - ω_AdS) - ∫H ω]
Ex
S-AdS = - Ω_(D-2)/(D-1) r_h^(D-1) using ∫√-g


II. Smarr $g_{D, AdS}$ Static, BH
$(D-3)\, 8\pi\, M_{ADM} = (D-2)\, \kappa A - 2\, \Theta\, \Lambda$
$\Theta = -\left[\int_{\infty} (\omega - \omega_{AdS}) - \int_{H} \omega \right]$
Ex
S-AdS $= \frac{\Omega_{D-2}}{D-1} r_h^{D-1}$ using $\int \sqrt{-g} = r^2 d\Omega$

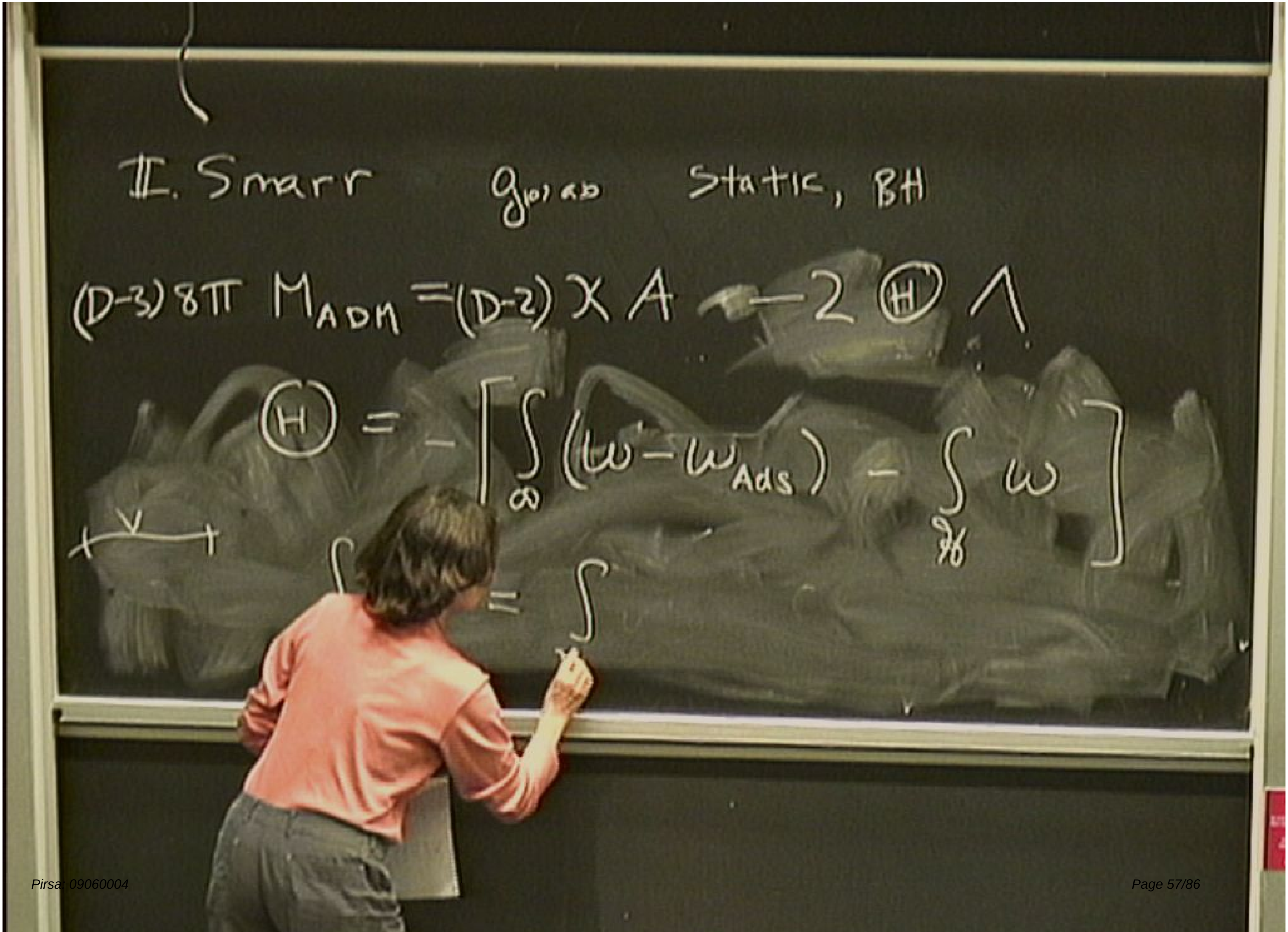
II. Smarr $g_{(0)ab}$ Static, BH

$$(D-3)\,8\pi\, M_{ADM} = (D-2)\,\chi A - 2\,\textcircled{H}\,\Lambda$$

$$\textcircled{H} = -\left[\int_{\infty}(\omega - \omega_{AdS}) - \int_{\mathcal{H}} \omega\right]$$

Ex

S-AdS $= -\frac{\Omega_{D-2}}{D-1} r_h^{D-1}$ using $\int \sqrt{-g} = r^2 d\Omega$

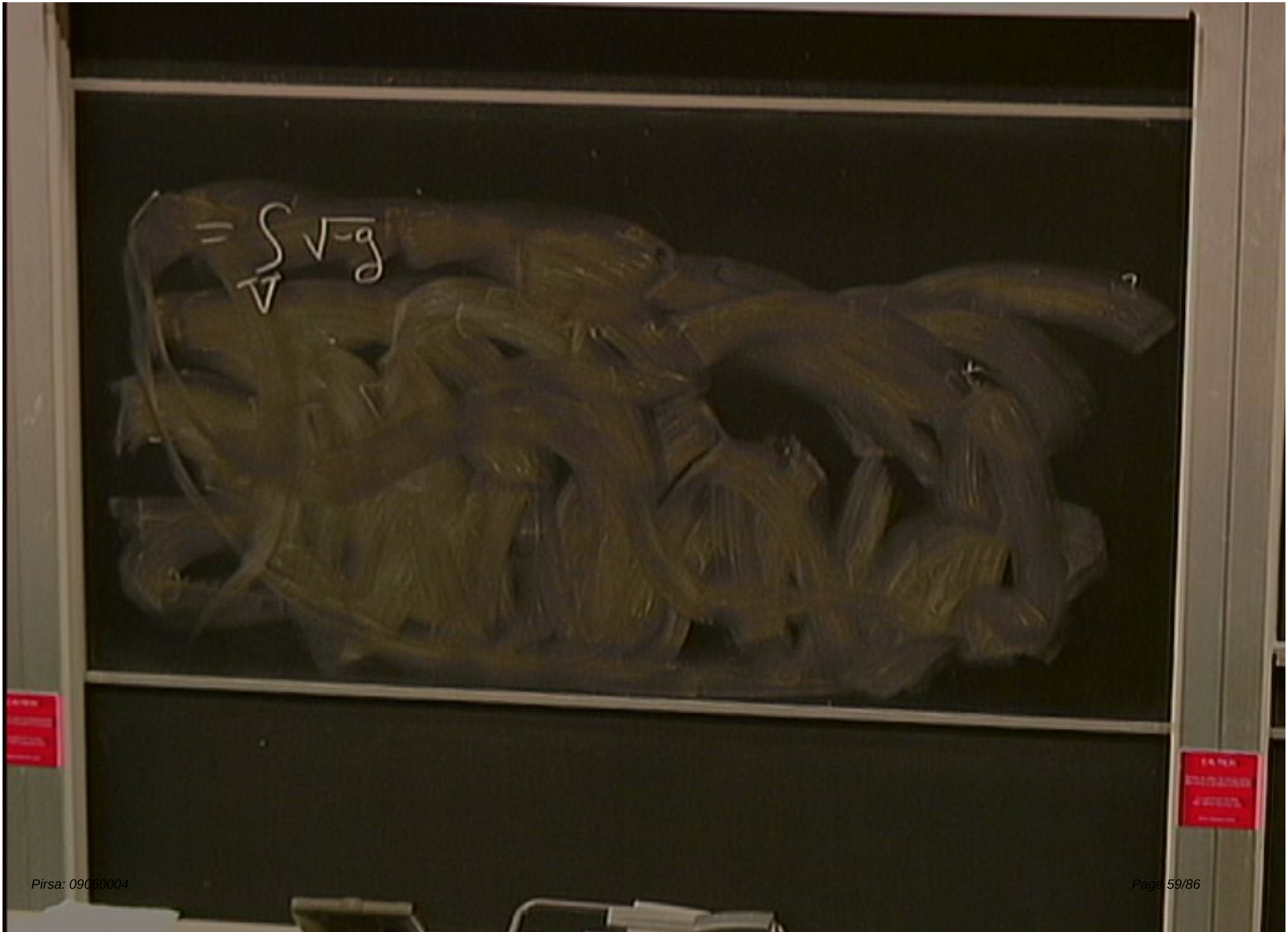

II. Smarr   g(0) ab   Static, BH
(D-3) 8π M_ADM = (D-2) κ A − 2 Ⓗ Λ
Ⓗ = − [∫_∞ (ω − ω_Ads) − ∫_H ω]

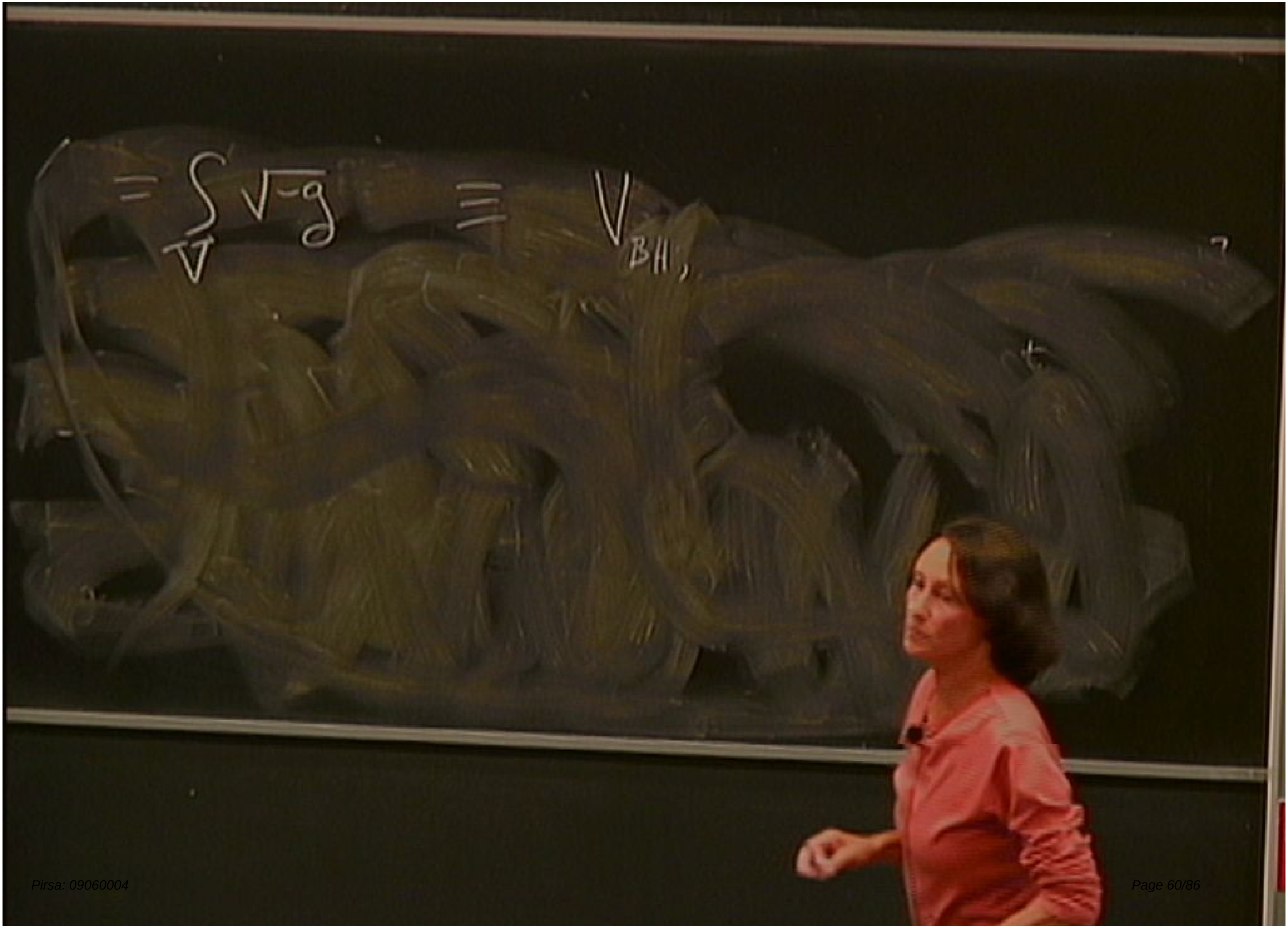
II. Smarr $\quad g_{(0)ab}$ static, BH

$$(D-3)\,8\pi\, M_{ADM} = (D-2)\,\kappa A - 2\,\Theta\,\Lambda$$

$$\Theta = -\left[\int_{\infty} (\omega - \omega_{AdS}) - \int_{\mathcal{H}} \omega\right]$$

$$\int_{\partial V} \omega = \int_{V} D_a(\omega^{ab} n_b)\, dv = \int_{V} \xi^b n_b\, dv$$


$= \int_V \sqrt{-g}$


$= \int_V \sqrt{-g} \equiv V_{BH},$

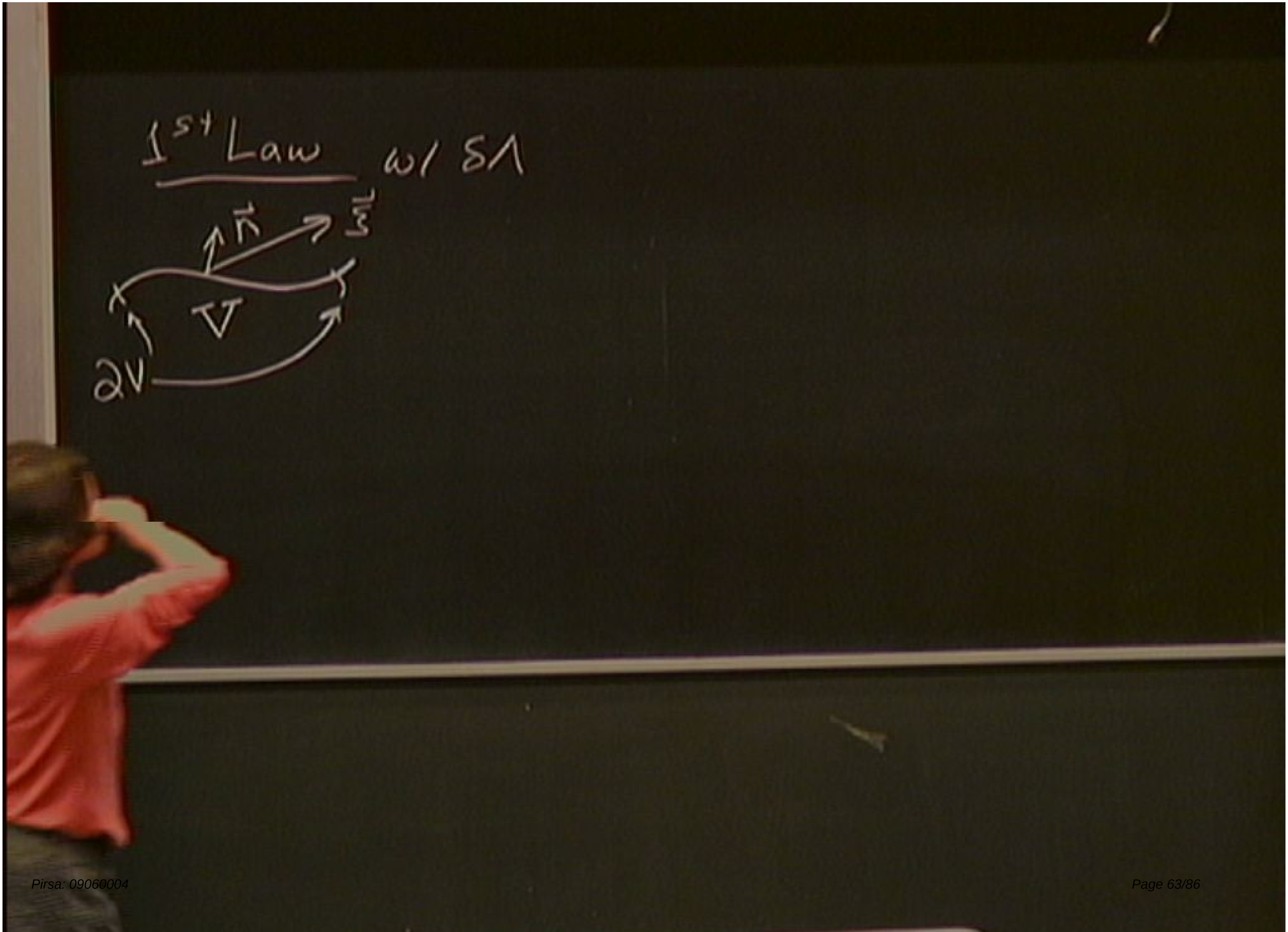
$$= \int_V \sqrt{-g} \equiv V_{BH,\Lambda}$$

$$\textcircled{H} = \left(V_{Ads} - V_{Ads,\, BH} \right)$$

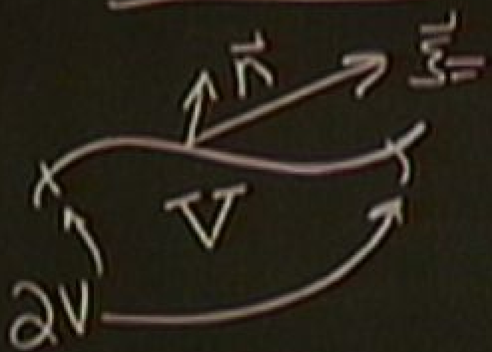
$$= \int_V \sqrt{-g} \equiv V_{BH,\Lambda}$$

$$\text{(H)} = -\left(V_{AdS} - V_{AdS,BH}\right)^{t}$$

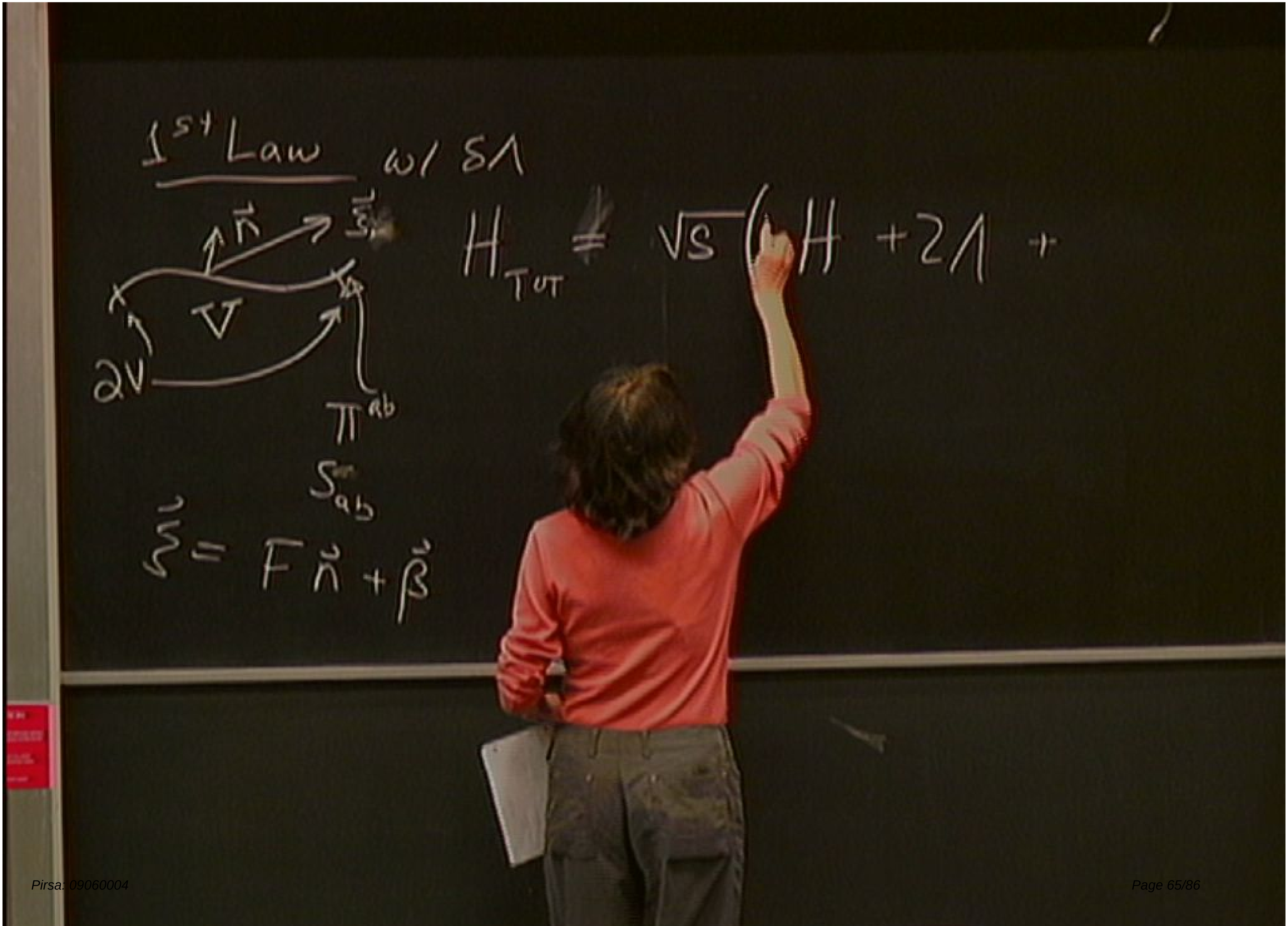
$$= - V_{eff,BH}$$

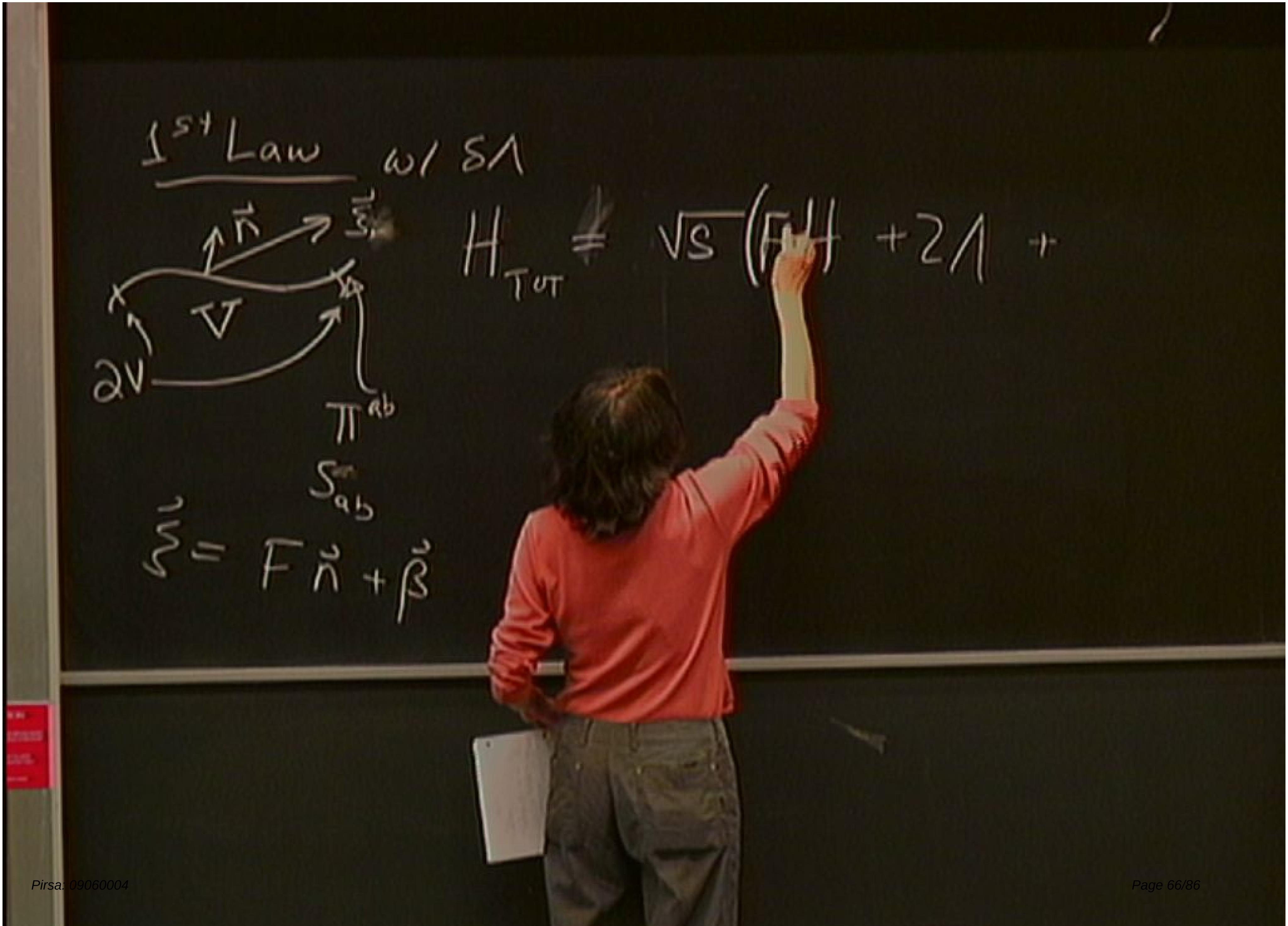

1st Law w/ δΛ
n
V
∂V

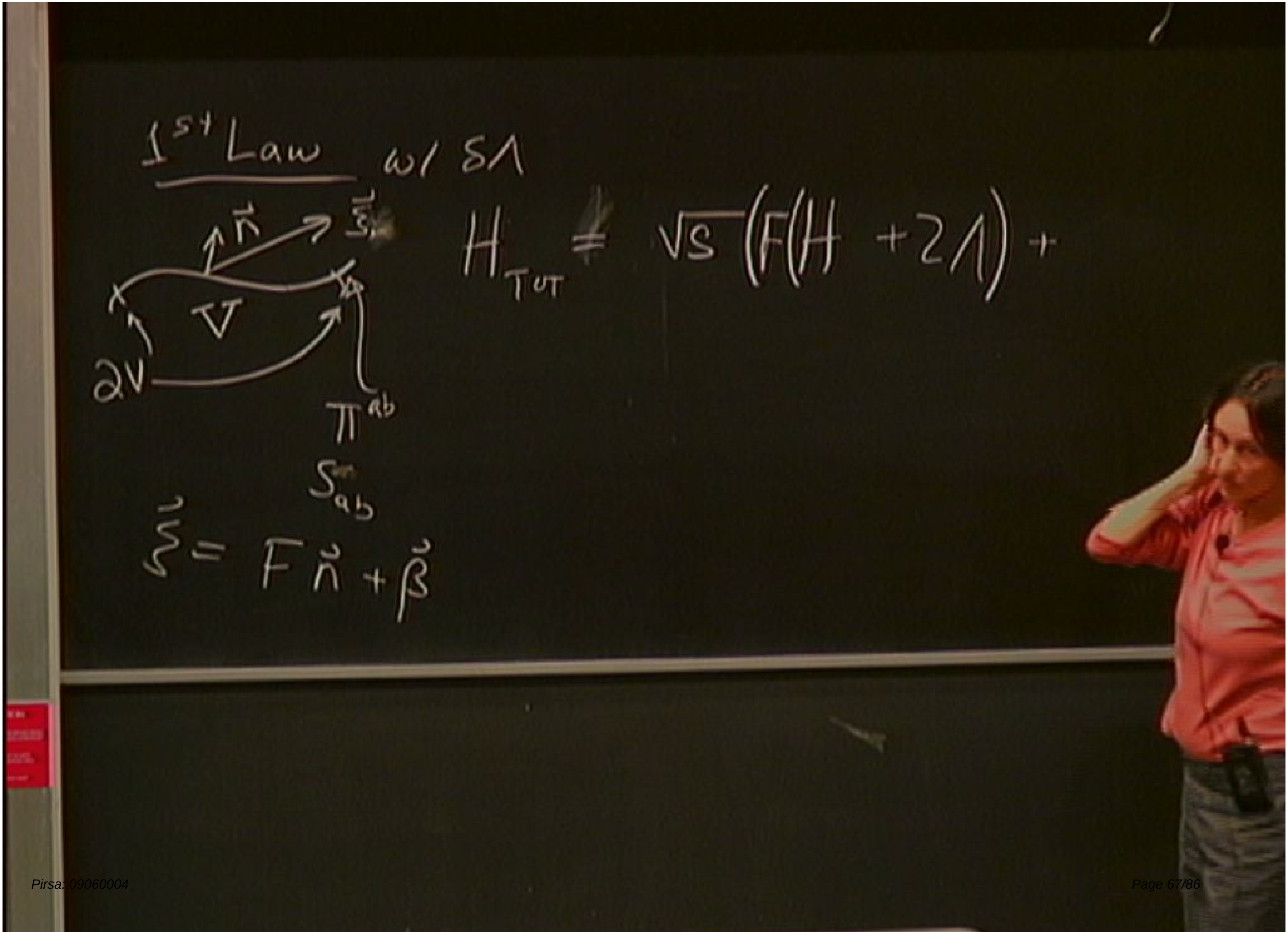
1^{st} Law w/ $\delta\Lambda$



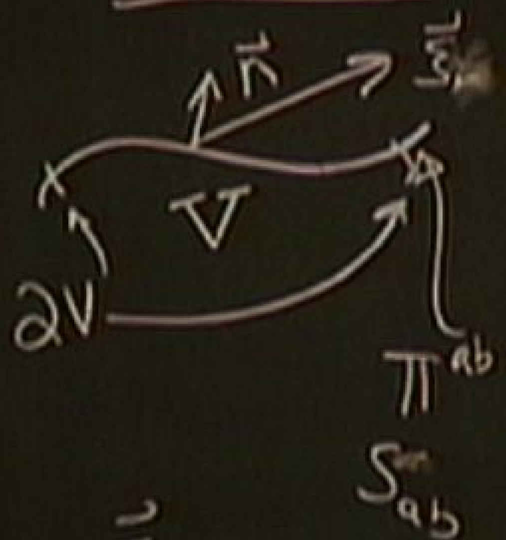
$$H_{TOT} = \sqrt{s}\,(H + 2\Lambda +$$


1st Law w/ δΛ
H_TOT = √s (H + 2Λ +
n
ξ
V
∂V
π^ab
S_ab
ξ = F n + β


1st Law w/ δΛ
n
ξ
V
∂V
π^ab
S_ab
ξ = F n + β
H_TOT = √s (FH + 2Λ +

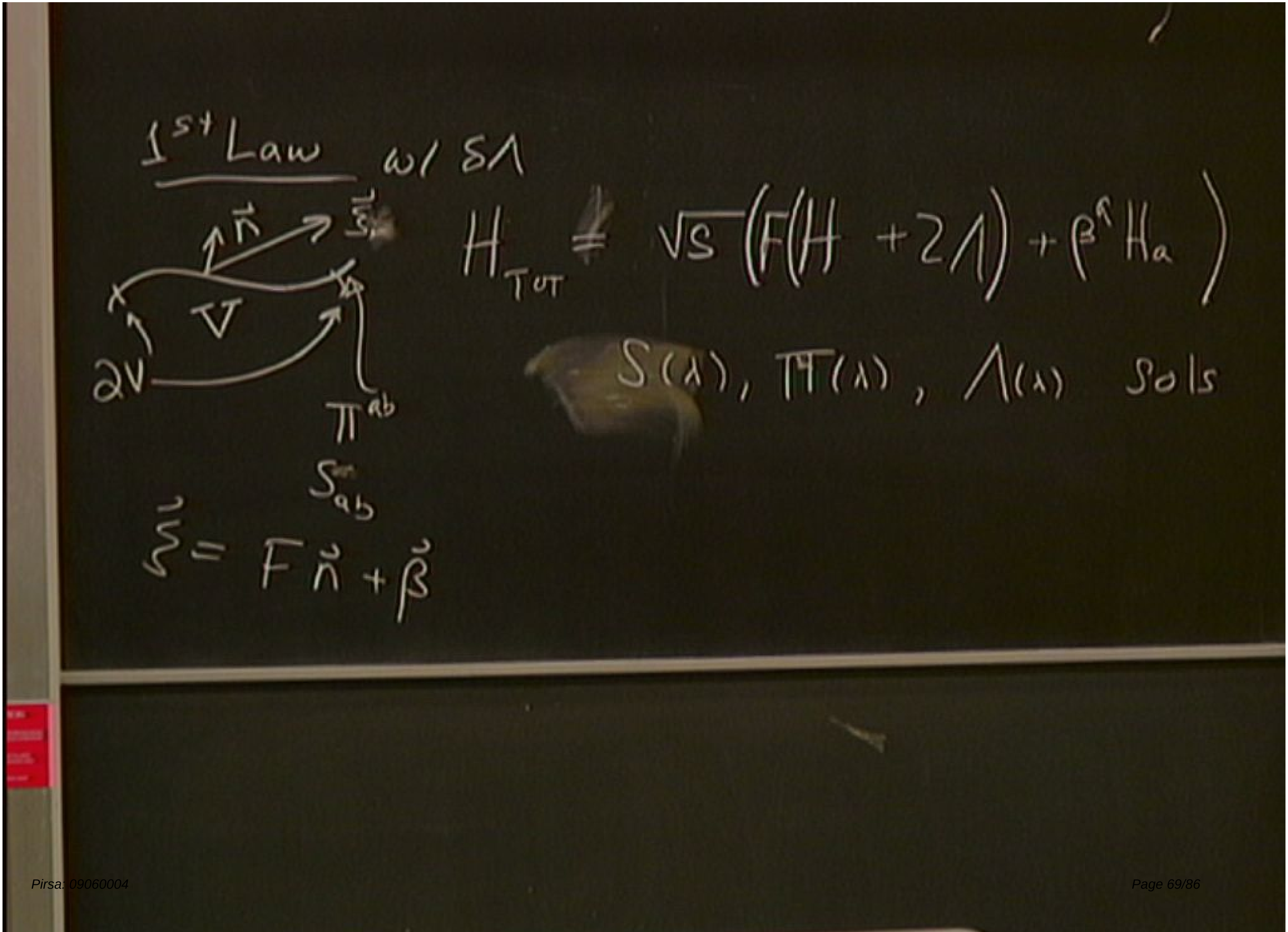

1st Law w/ δΛ
H_TOT = √s (F(H + 2Λ) +
n
ξ
V
∂V
π^ab
S_ab
ξ = F n + β

1^{st} Law w/ $\delta\Lambda$

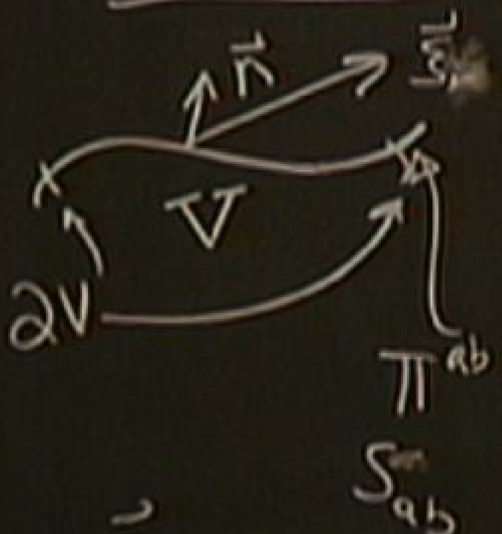


$$H_{TOT} = \sqrt{s}\left(F\left(H + 2\Lambda\right) + \beta^a H_a\right)$$

$$\vec{\xi} = F\vec{n} + \vec{\beta}$$


1st Law w/ δΛ
H_TOT = √s (F(H + 2Λ) + β^a H_a)
S(λ), π(λ), Λ(λ) sols
∂V
V
π^ab
S_ab
ξ = F n + β

1^{st} Law w/ $\delta\Lambda$



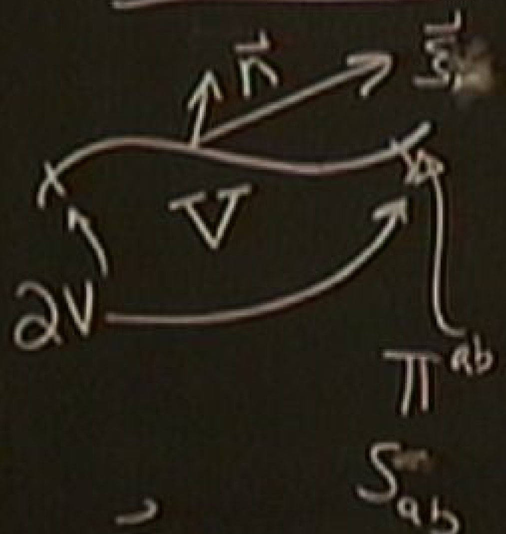
$$H_{TOT} = \sqrt{s}\,\big(F(H + 2\Lambda) + \beta^a H_a\big)$$

$S(\lambda)$, $\Pi(\lambda)$, $\Lambda(\lambda)$ sols

$$H_{TOT}(s, \pi, \Lambda) = 0$$

$$\vec{\xi} = F\vec{n} + \vec{\beta}$$

1^{st} Law w/ $\delta\Lambda$

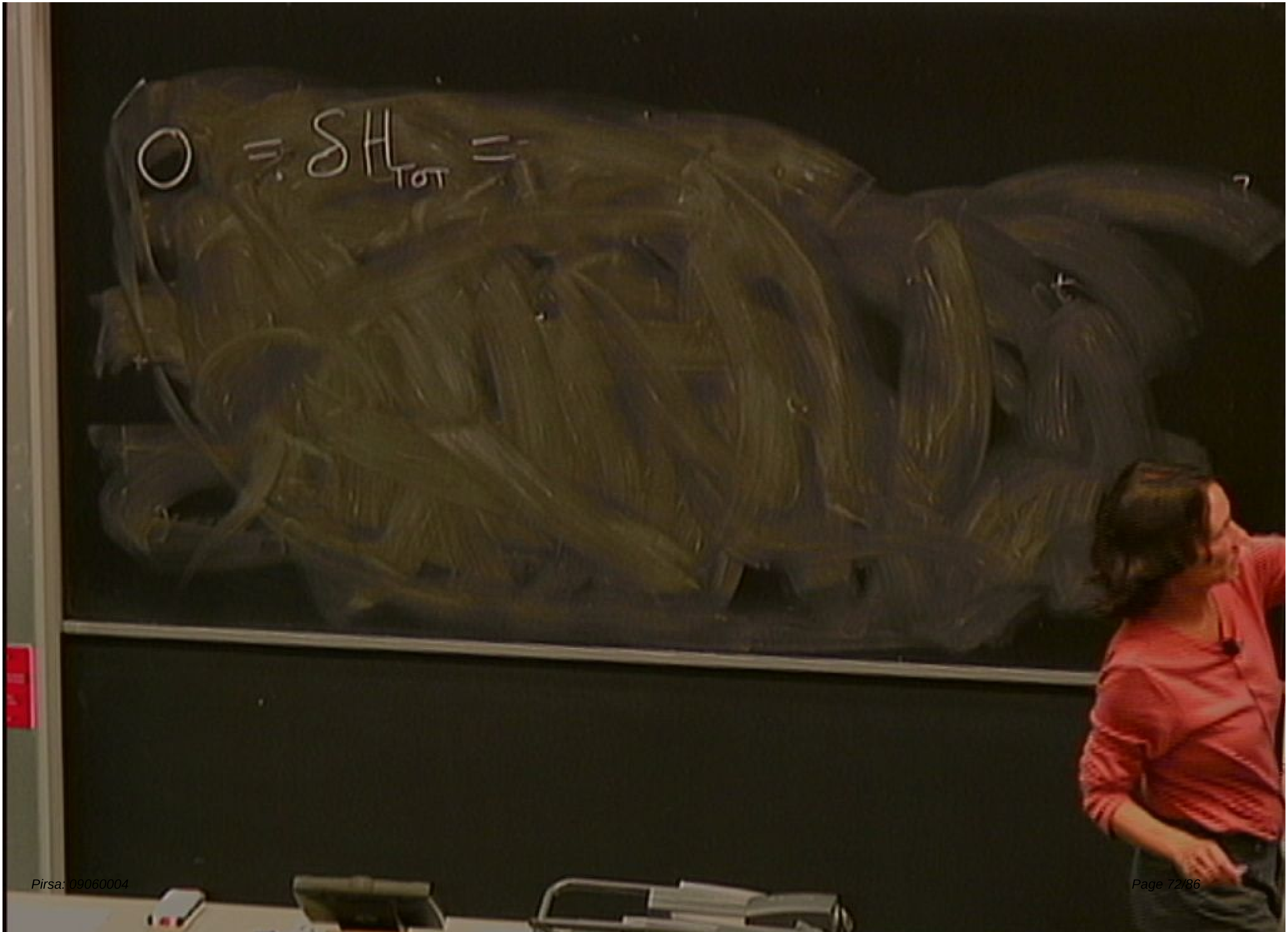


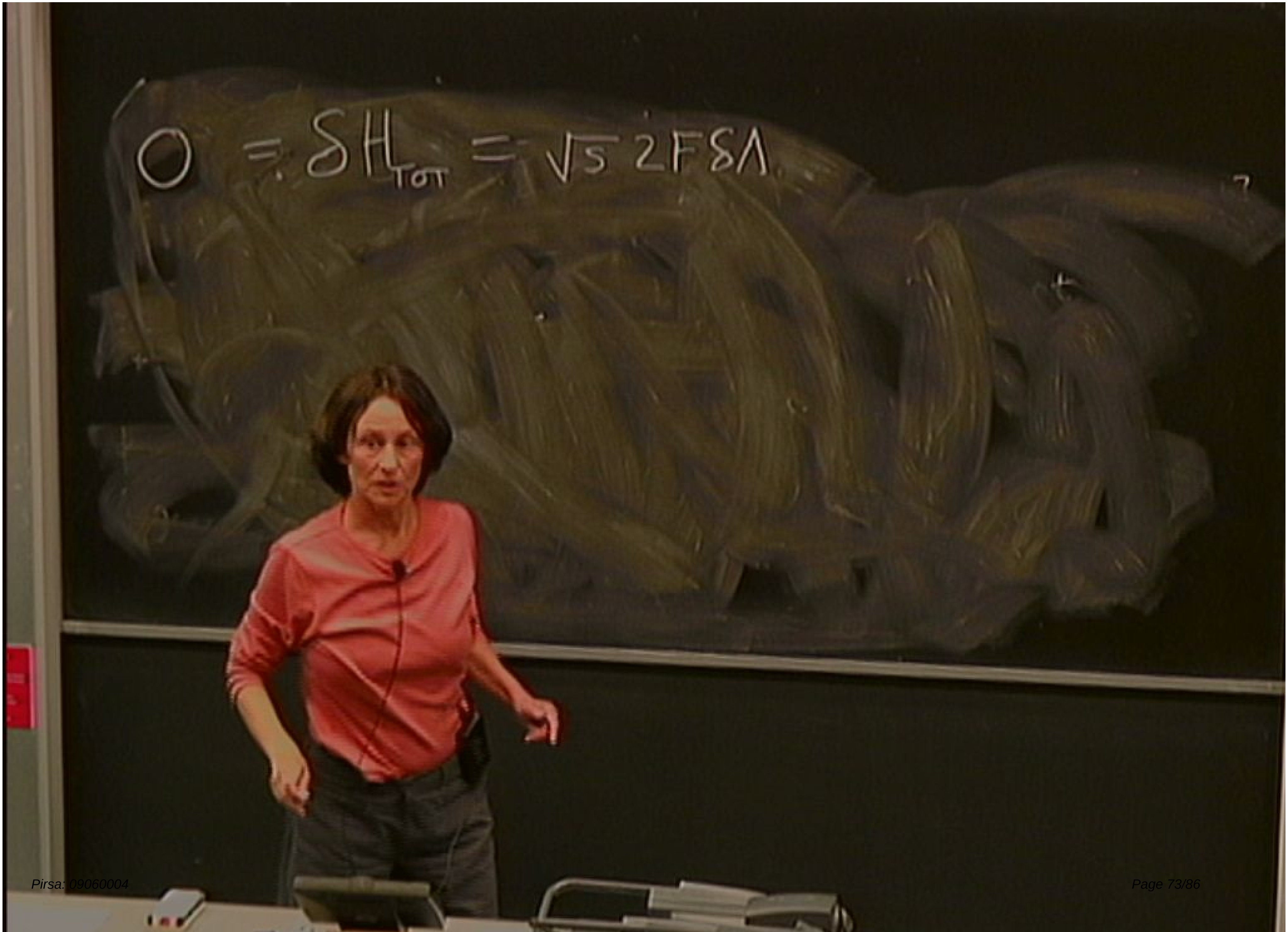
$$\vec{\xi} = F\vec{n} + \vec{\beta}$$

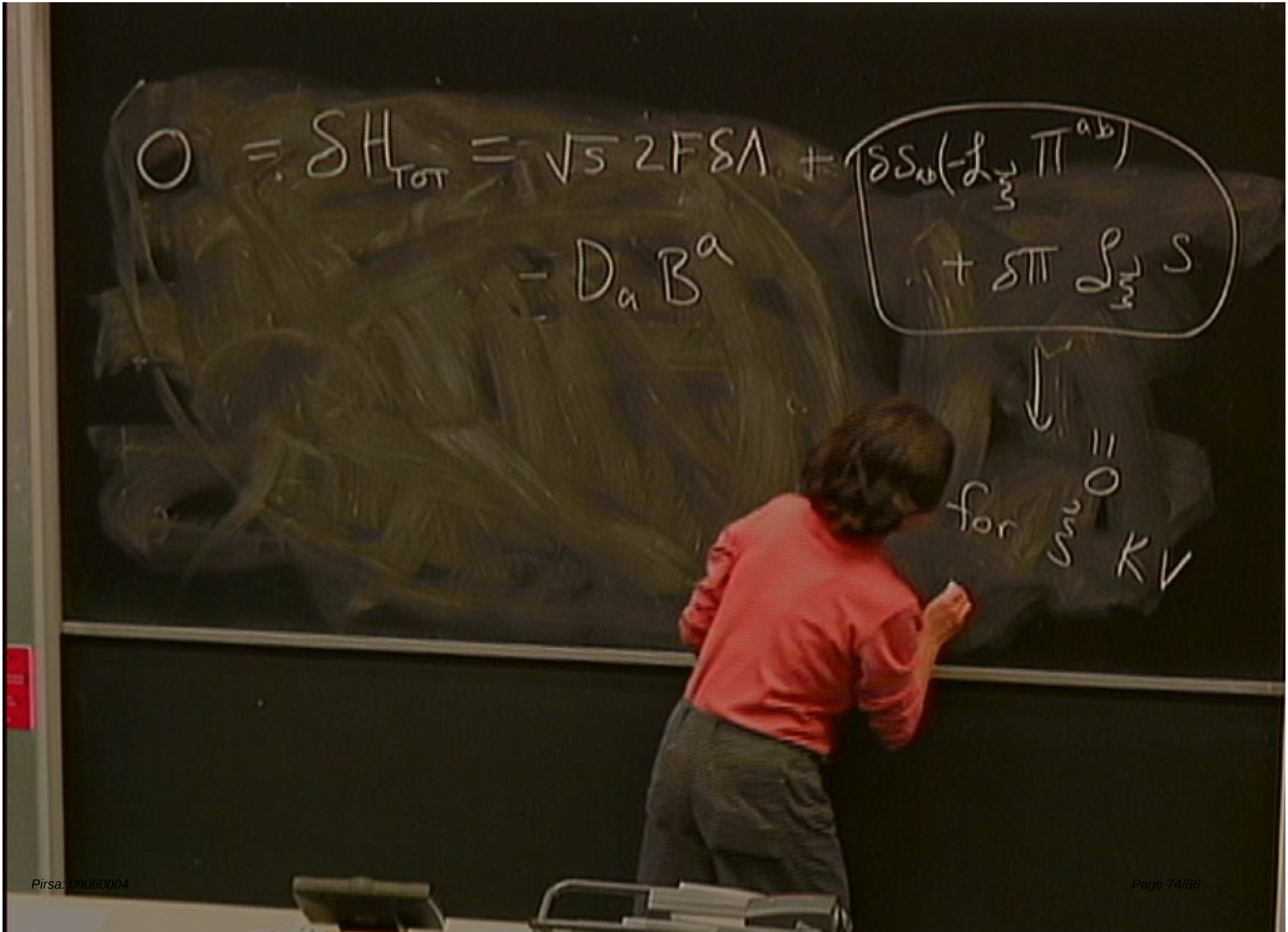
$$H_{TOT} = \sqrt{s}\left(F(H + 2\Lambda) + \beta^a H_a\right)$$

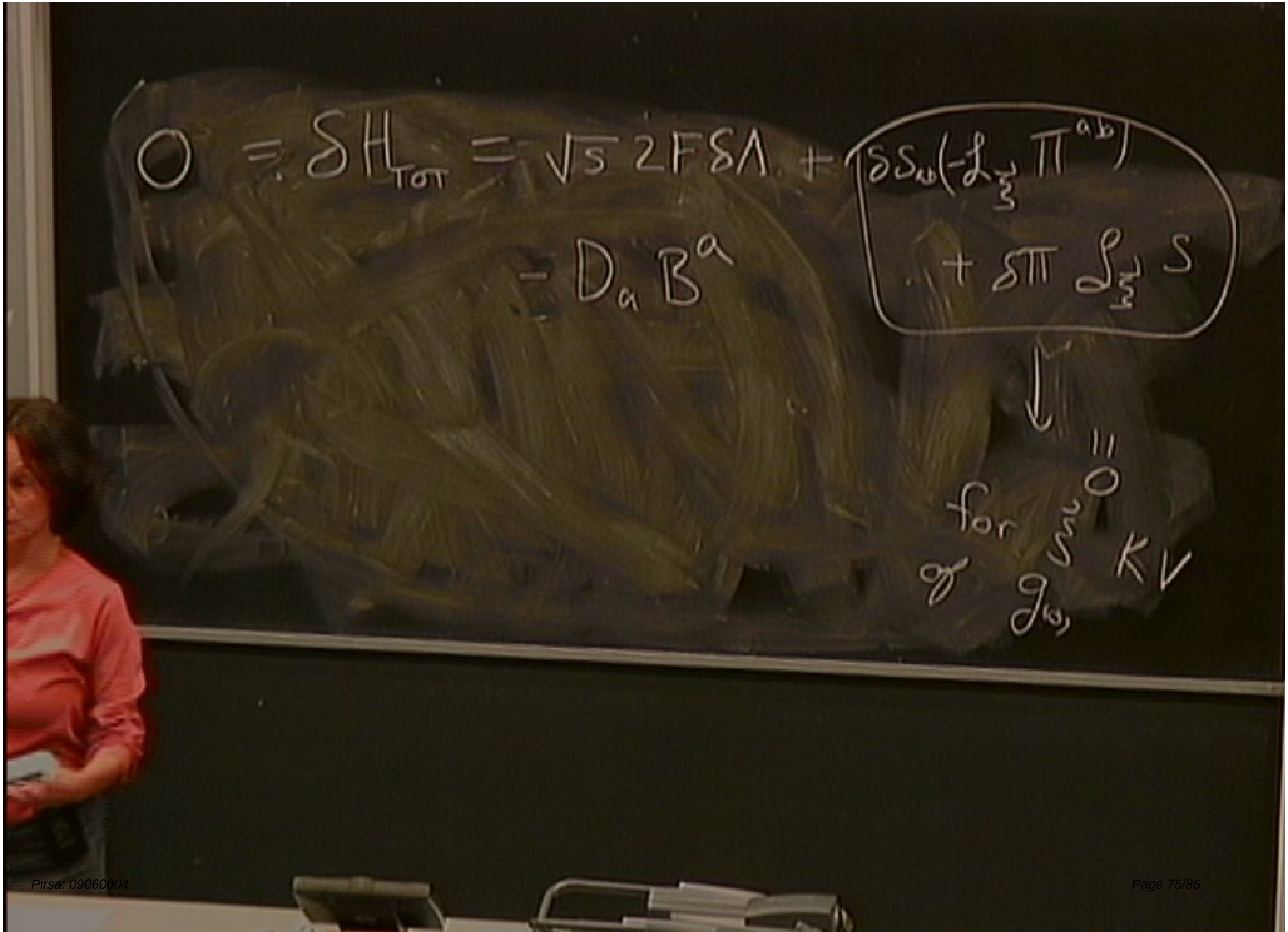
$s(\lambda),\ \pi(\lambda),\ \Lambda(\lambda)$ sols

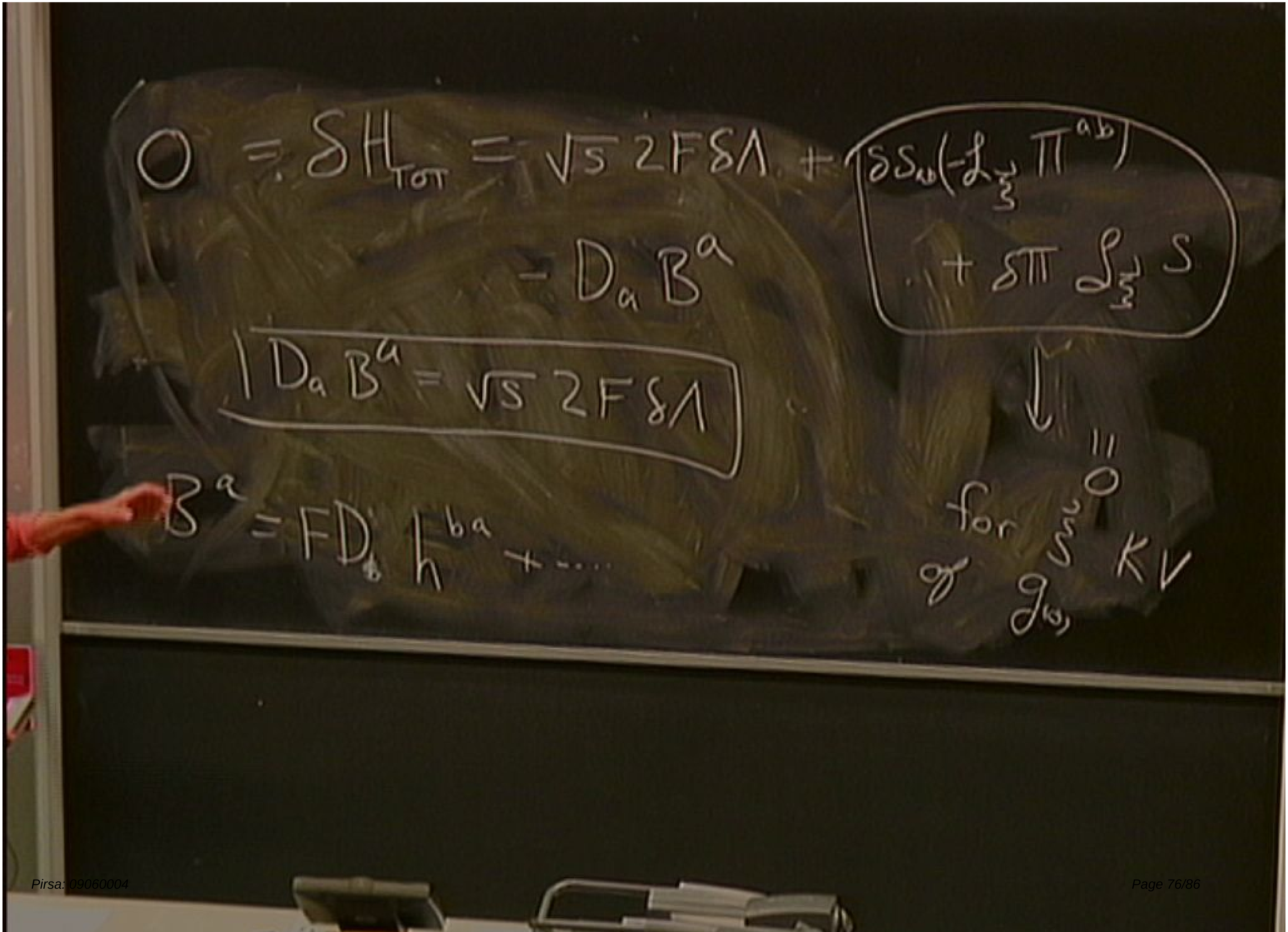
$$H_{TOT}(s, \pi, \Lambda) = 0$$


0 = δH_TOT =


0 = δH_TOT = √5 2FδΛ


0 = δH_TOT = √s 2FδΛ + δS_ab(−£_ξ π^ab)
+ δπ £_ξ S
= D_a B^a
= 0
for ξ KV


0 = δH_TOT = √s 2FδΛ + δS_ab(-L_ξ π^ab)
+ δπ L_ξ S
= D_a B^a
= 0
for ξ KV of g_(0)


0 = δH_TOT = √s 2FδΛ + δs_ab(−L_ξ π^ab)
− D_a B^a + δπ L_ξ s
D_a B^a = √s 2FδΛ
B^a = F D_b h^ba + ...
= 0
for ξ KV
of g_(0)

$$0 \overset{?}{=} \delta H_{TOT} = \sqrt{s}\, 2F\delta\Lambda + \delta s_{ab}\left(-\mathcal{L}_{\xi}\pi^{ab}\right) + \delta\pi\, \mathcal{L}_{\xi} s - D_a B^a$$

$$\boxed{D_a B^a = \sqrt{s}\, 2F\delta\Lambda}$$

$$B^a = F D_b h^{ba} + \ldots$$

$$\delta s_{ab}\left(-\mathcal{L}_{\xi}\pi^{ab}\right) + \delta\pi\, \mathcal{L}_{\xi} s = 0 \quad \text{for } \xi \text{ KV of } g_{(0)}$$

$$0 = \delta H_{TOT} = \sqrt{s}\, 2F\delta\Lambda + \delta s_{ab}(-\mathcal{L}_{\xi}\pi^{ab}) + \delta\pi\,\mathcal{L}_{\xi} s - D_a B^a$$

$$D_a B^a = \sqrt{s}\, 2F\delta\Lambda = -2\delta\Lambda\, \xi^a n_a$$

$$0 = \delta H_{TOT} = \sqrt{s}\, 2F\delta\Lambda + \left[\delta s_{ab}\left(-\mathcal{L}_{\xi}\, \pi^{ab}\right) + \delta\pi\, \mathcal{L}_{\xi}\, s\right]$$

$$- D_a B^a$$

$$D_a B^a = \sqrt{s}\, 2F\,\delta\Lambda = -2\,\delta\Lambda\, \xi^a n_a$$

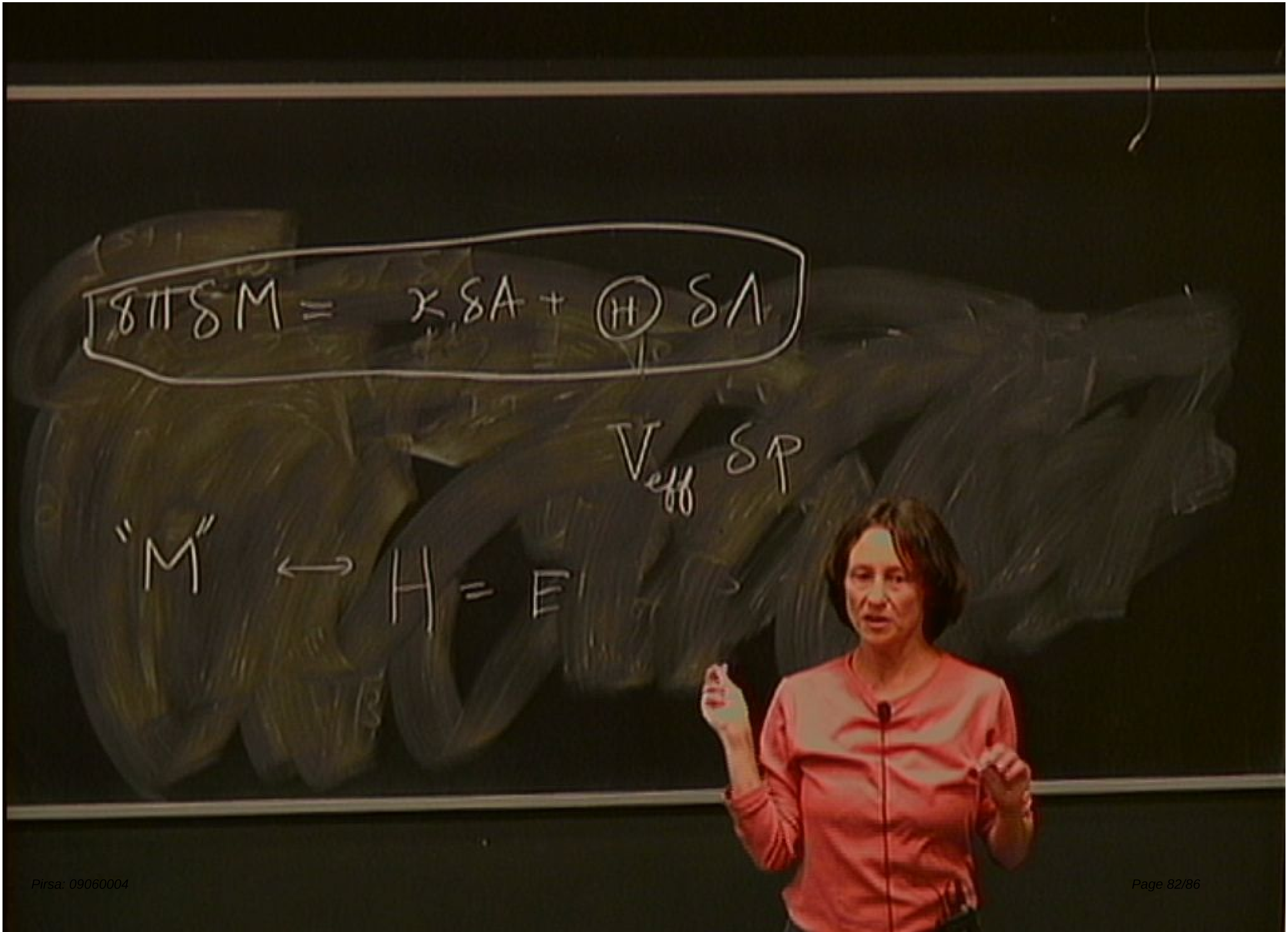
$$\int_{\partial V} \left[B^a + 2\,\delta\Lambda\, \omega \right.$$

$$0 = \delta H_{TOT} = \sqrt{s}\, 2F\delta\Lambda + \left(\delta s_{ab}(-\mathcal{L}_\xi \pi^{ab}) + \delta\pi\, \mathcal{L}_\xi s \right) - D_a B^a$$

$$D_a B^a = \sqrt{s}\, 2F\delta\Lambda = -2\delta\Lambda\, \xi^a n_a$$

$$\int_{\partial V} [B^a + 2\,\delta\Lambda\, \omega^{ac} n_c]\, da_a$$

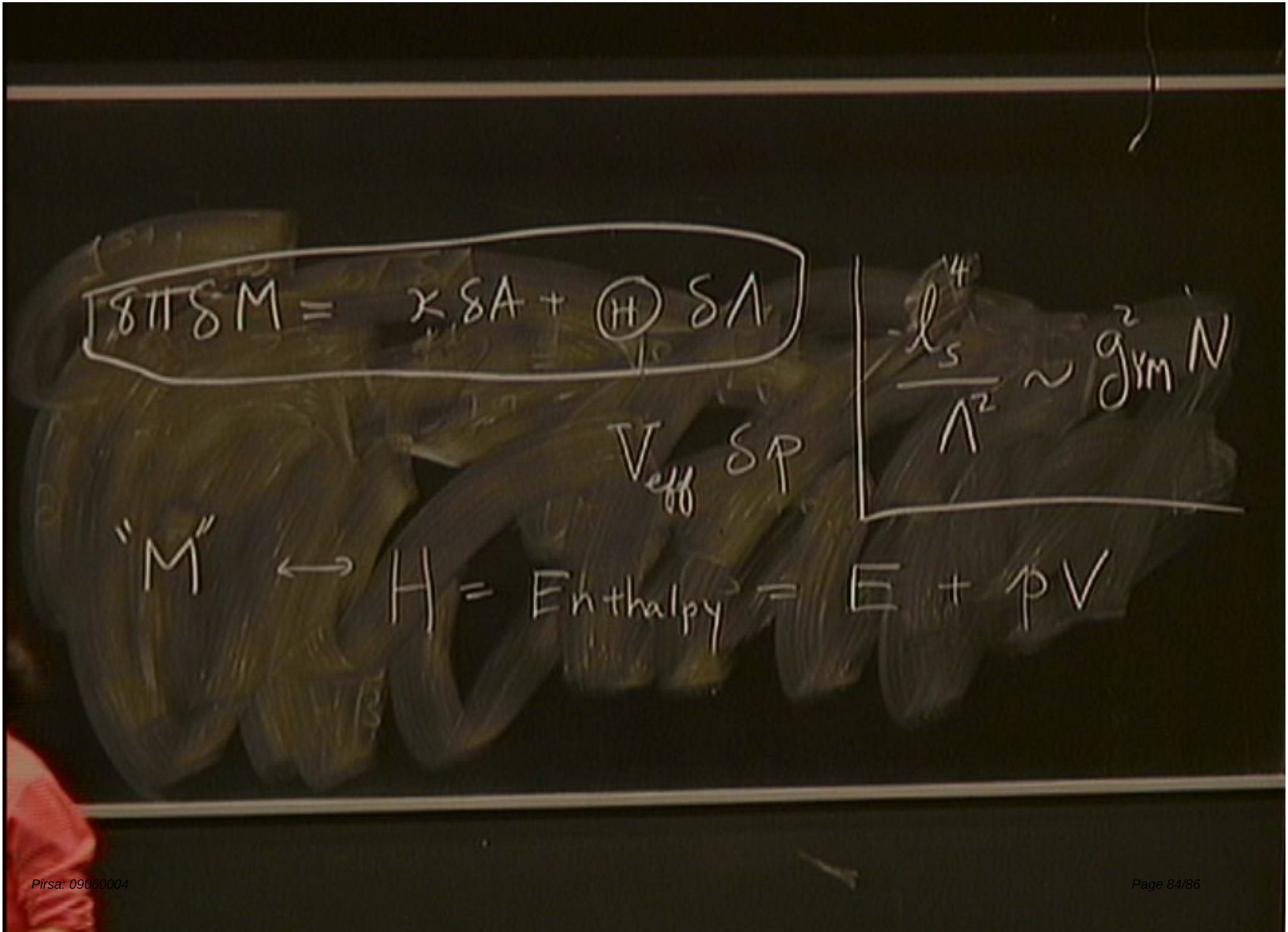
$$8\pi \delta M = \kappa \delta A + \Theta \delta \Lambda$$

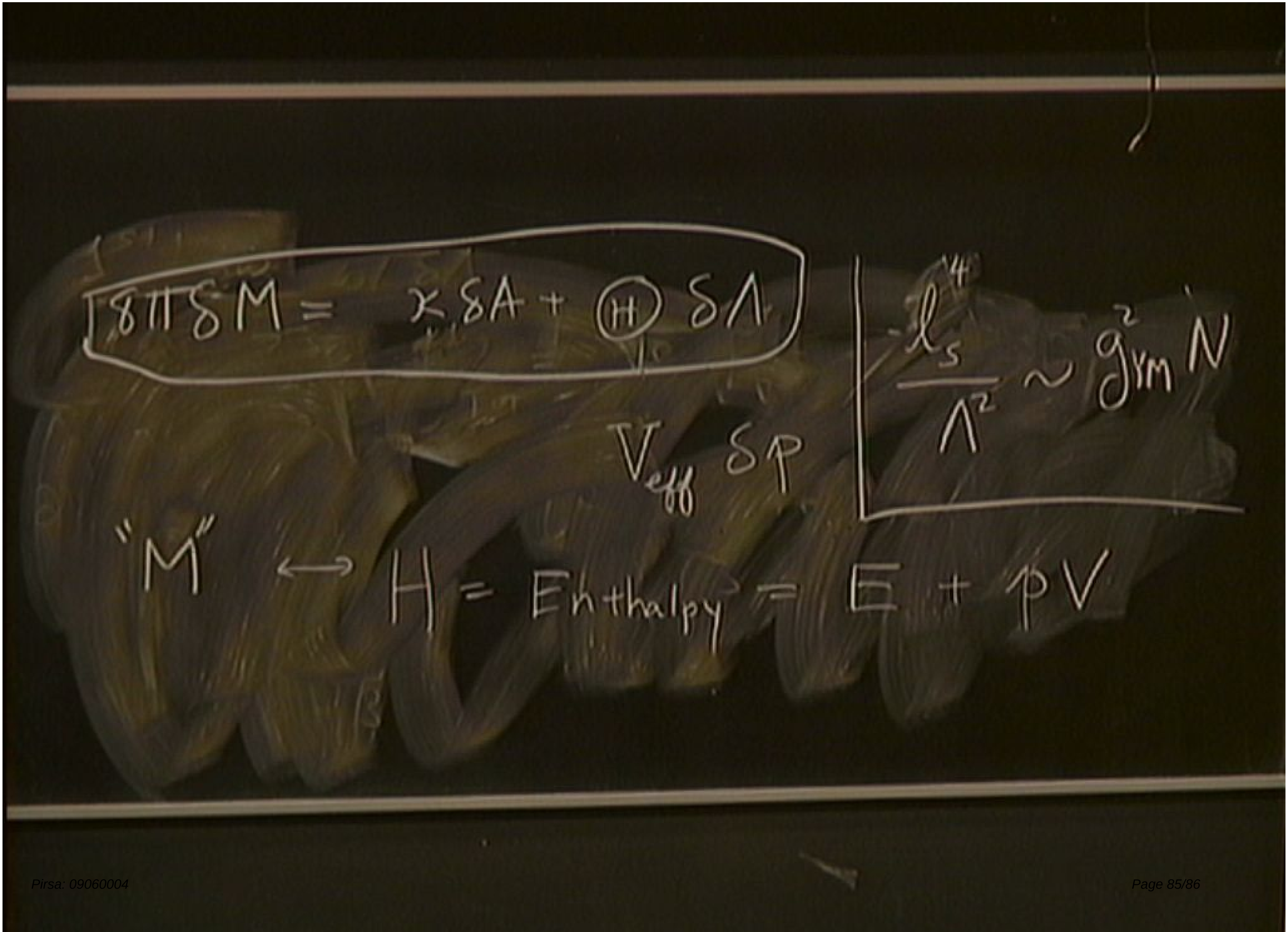

8π δM = κ δA + Θ δΛ
V_eff δp
"M" ⟷ H = E

$$8\pi\,\delta M = \kappa\,\delta A + \textcircled{H}\,\delta\Lambda$$

$$V_{eff}\,\delta p$$

"M" $\longleftrightarrow$ H = Enthalpy = $E + pV$


$8\pi\,\delta M = \kappa\,\delta A + H\,\delta\Lambda$
$V_{eff}\,\delta p$
$\frac{l_s^4}{\Lambda^2} \sim g_{YM}^2 N$
"M" $\longleftrightarrow$ H = Enthalpy = E + pV


$8\pi\,\delta M = \kappa\,\delta A + H\,\delta\Lambda$
$V_{eff}\,\delta p$
$\frac{l_s^4}{\Lambda^2} \sim g_{YM}^2 N$
"M" $\longleftrightarrow$ H = Enthalpy = E + pV

$$8\pi \, \delta M = \kappa \, \delta A + H \, \delta \Lambda$$

$$V_{eff} \, \delta p$$

$$\frac{l_s^4}{\Lambda^2} \sim g_{YM}^2 N$$

$$\text{“}M\text{”} \longleftrightarrow H = \text{Enthalpy} = E + pV$$