Title: A natural origin of primordial density perturbations

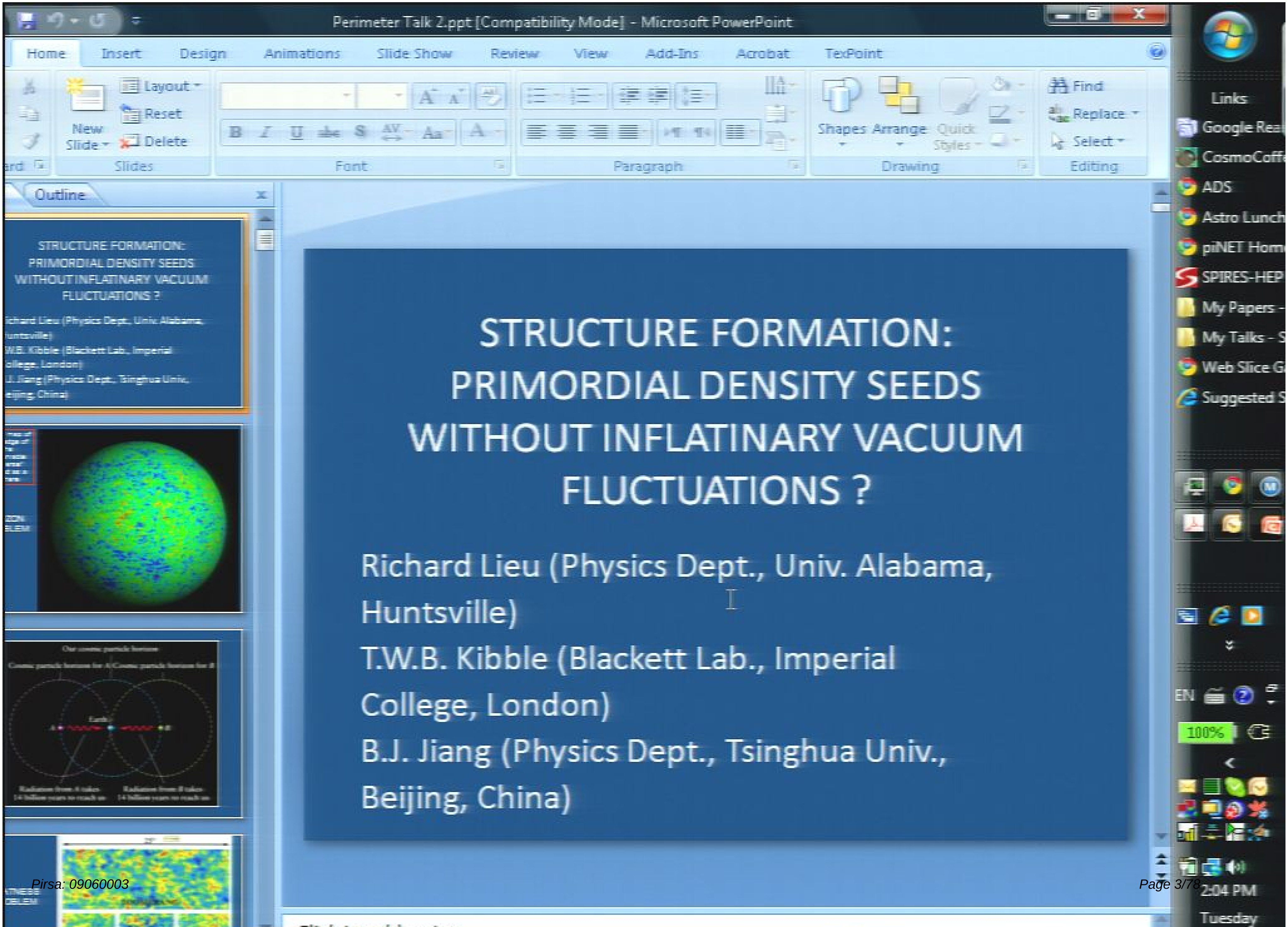
Date: Jun 09, 2009 02:00 PM

URL: http://pirsa.org/09060003

Abstract: We suggest here a mechanism for the seeding of the primordial density fluctuations. We point out that a process like reheating at the end of inflation will inevitably generate perturbations, even on superhorizon scales, by the local diffusion of energy. Provided that the final temperature is of order the GUT scale, the density contrast δ_R for spheres of radius R will be of order 10^{-5} at horizon entry, consistent with the values measured by \texttt{WMAP}. If this were a purely classical process, δ_R^2 would fall as $1/R^4$ beyond the horizon, and the resulting primordial density power spectrum would be $P(k) \propto k^n$ with $n=4$. However, as shown by Gabrielli et al, a quantum diffusion process can generate a power spectrum with any index in the range $0<N\LEQ $n="1$" for R^4$ $1 and $n<1$ R^{3+n}$ 1 $\propto be then will ($\delta_R^2$ observed the to close values including 4$,>1$). Thus, the two characteristic parameters that determine the appearance of present day structures could be natural consequences of this mechanism. These are in any case the minimum density variations that must have formed if the universe was rapidly heated to GUT temperatures by the decay of a `false vacuum'. There is then no \emph{a priori} necessity to postulate additional (and fine tuned) quantum fluctuations in the `false vacuum', nor a pre-inflationary period. Given also the very stringent pre-conditions required to trigger a satisfactory period of inflation, altogether it seems at least as natural to assume that the universe began in a flat and homogeneously expanding phase.

STRUCTURE FORMATION: PRIMORDIAL DENSITY SEEDS WITHOUT INFLATINARY VACUUM FLUCTUATIONS ?

Richard Lieu (Physics Dept., Univ. Alabama, Huntsville)
T.W.B. Kibble (Blackett Lab., Imperial College, London)
B.J. Jiang (Physics Dept., Tsinghua Univ., Beijing, China)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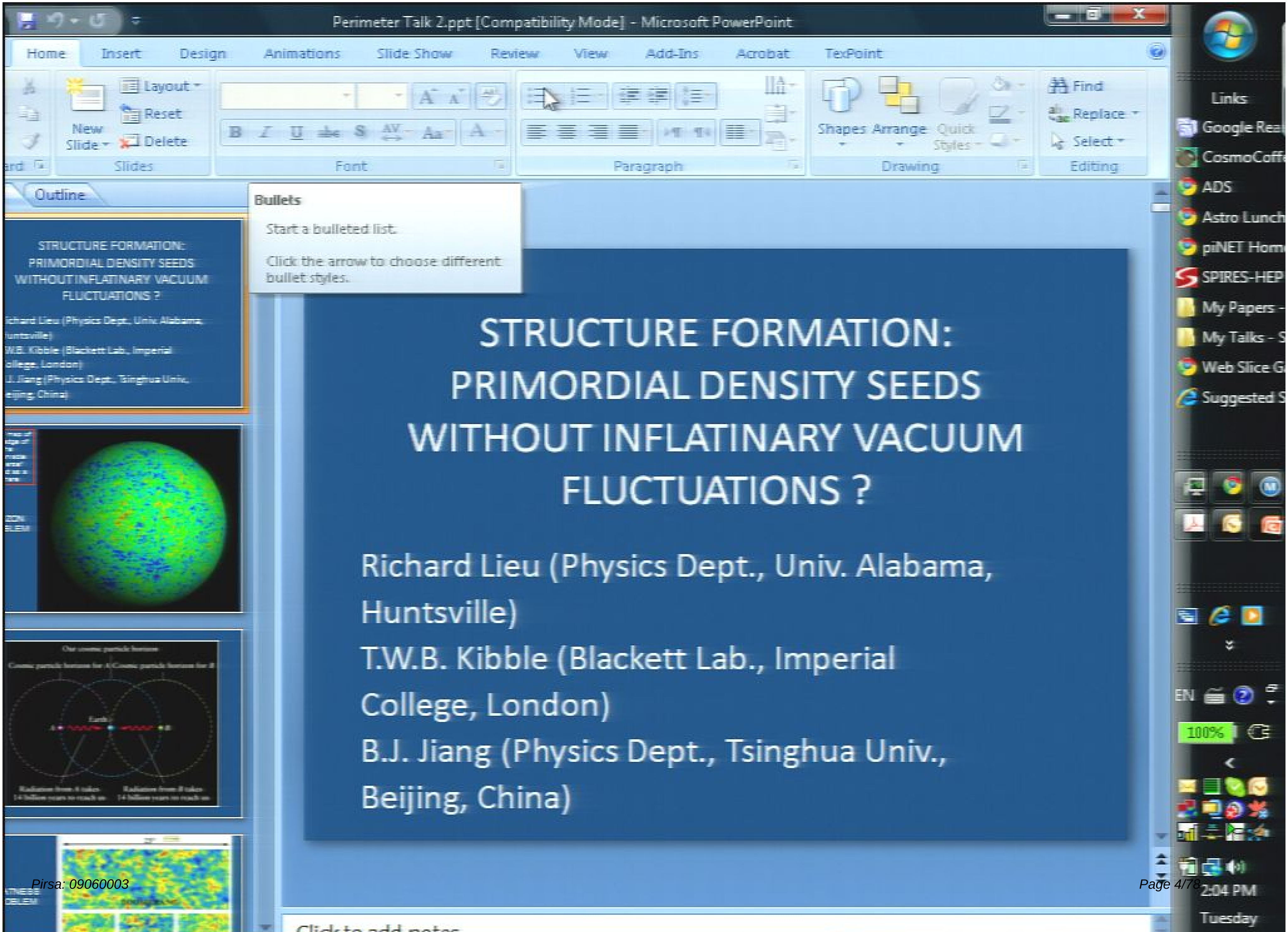
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Bullets
Start a bulleted list.
Click the arrow to choose different bullet styles.

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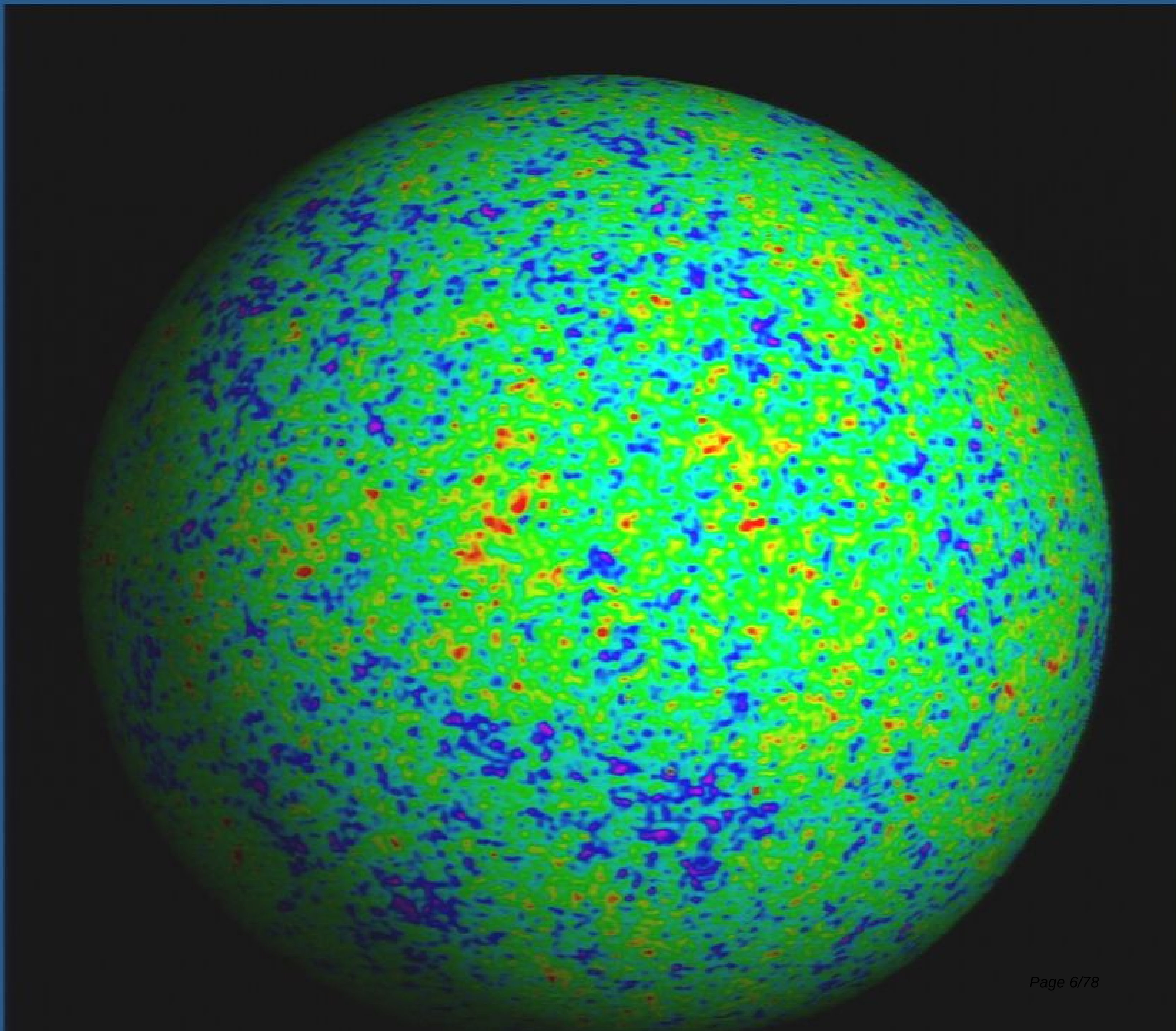
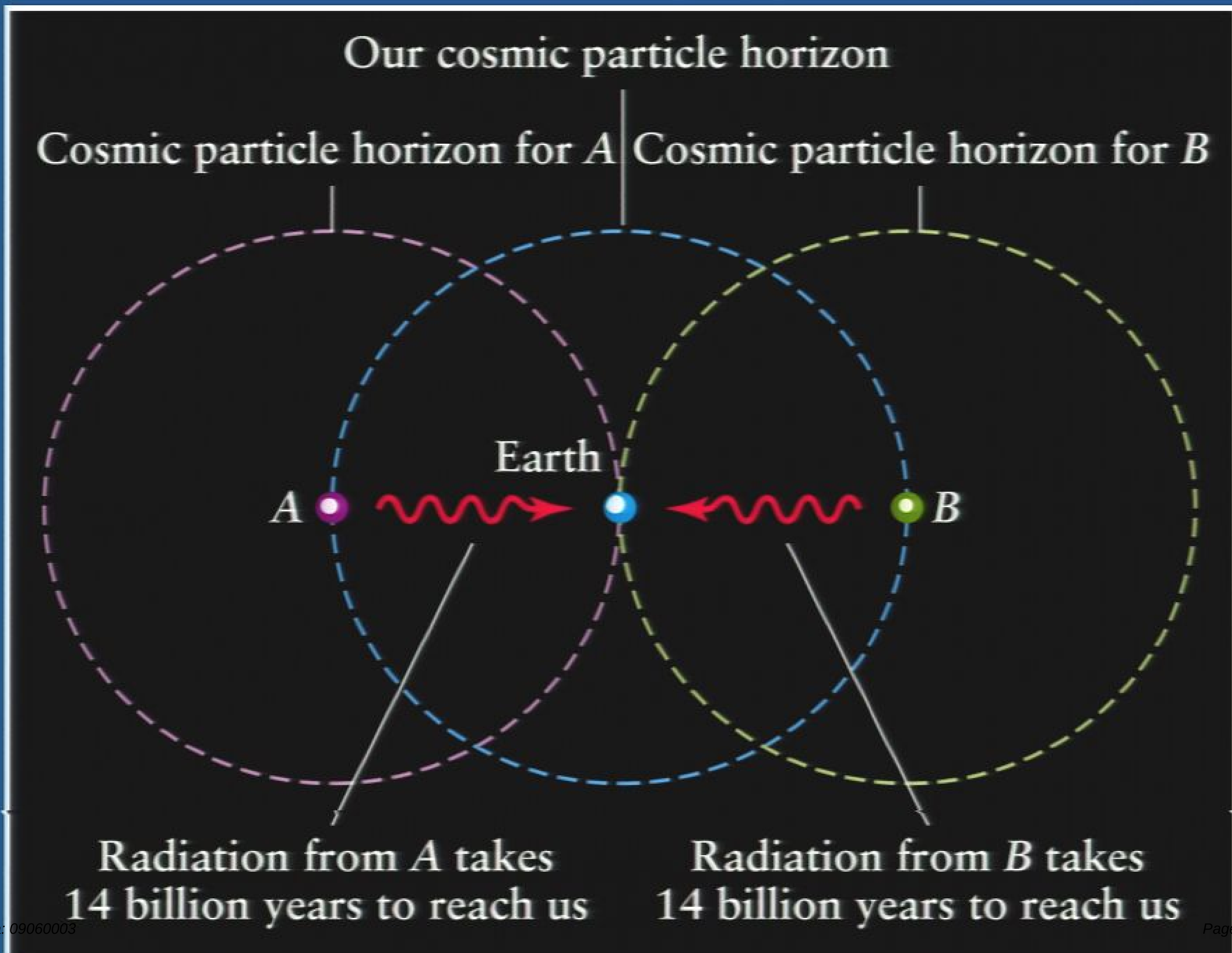
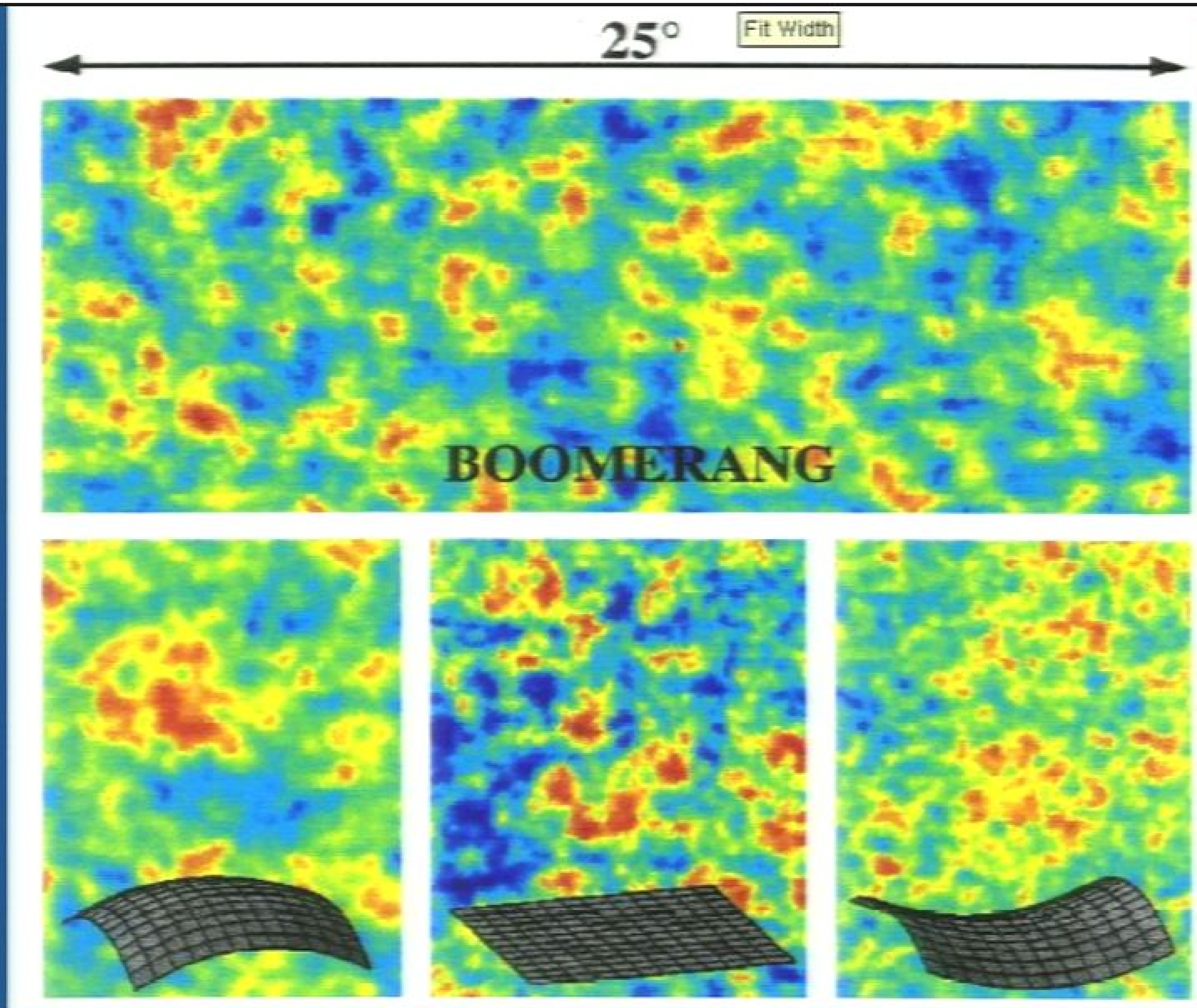
ORIZON
ROBLEM



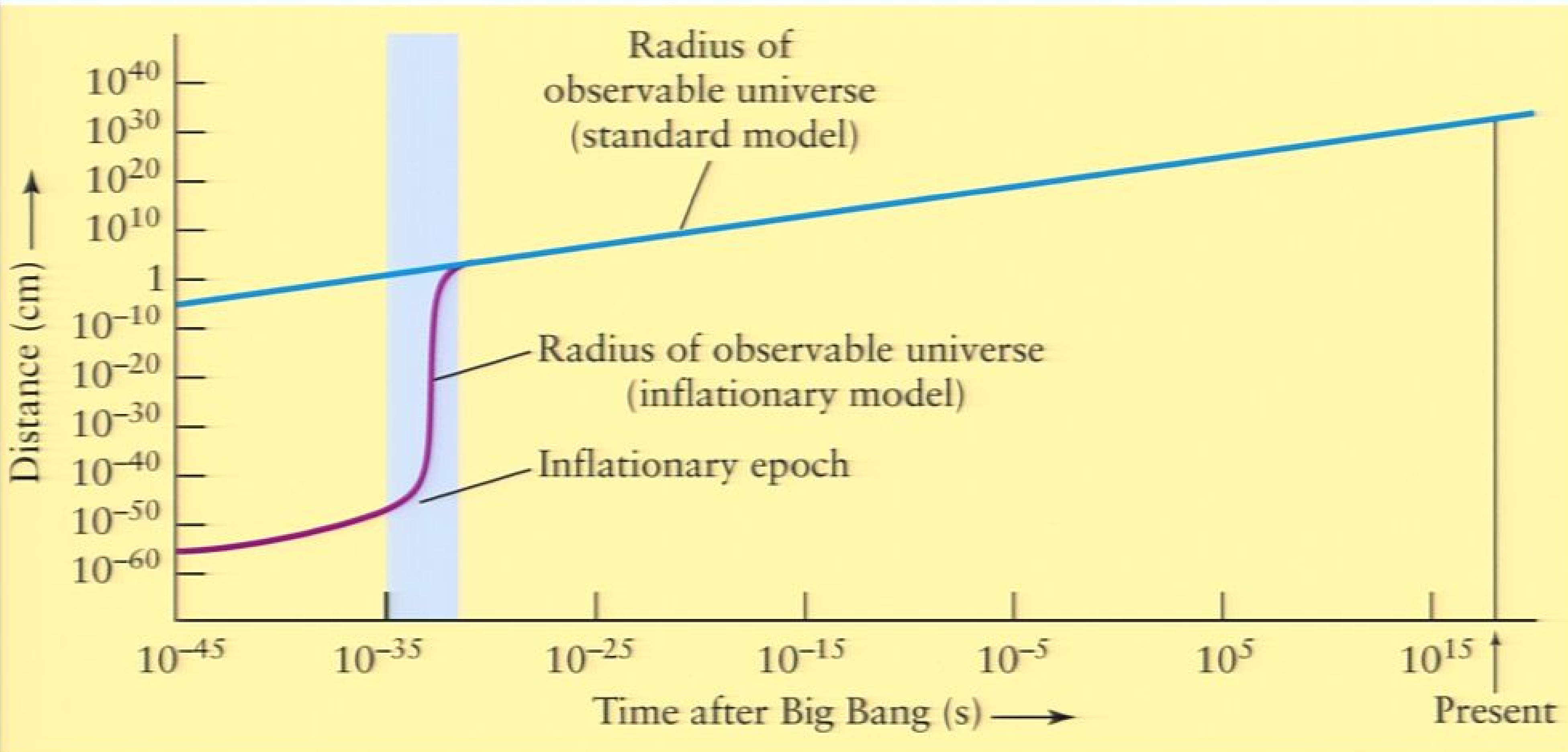

Our cosmic particle horizon
Cosmic particle horizon for A
Cosmic particle horizon for B
Earth
A
B
Radiation from A takes 14 billion years to reach us
Radiation from B takes 14 billion years to reach us

FLATNESS PROBLEM

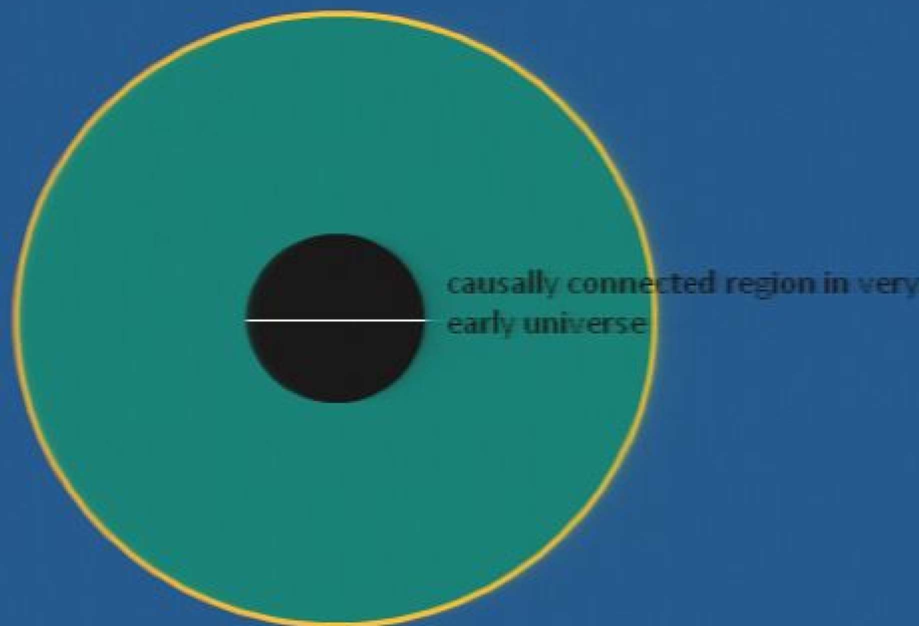


+ve curvature Flat -ve curvature

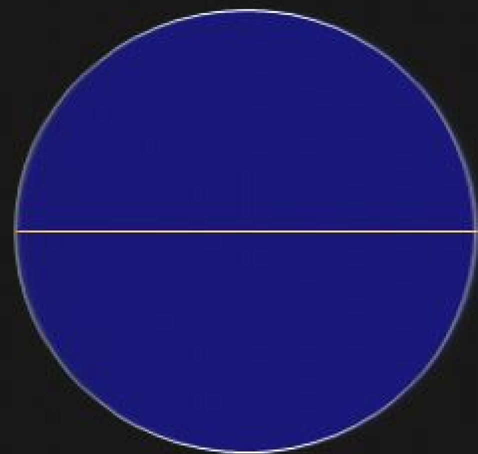
INFLATION TO THE RESCUE



Inflation

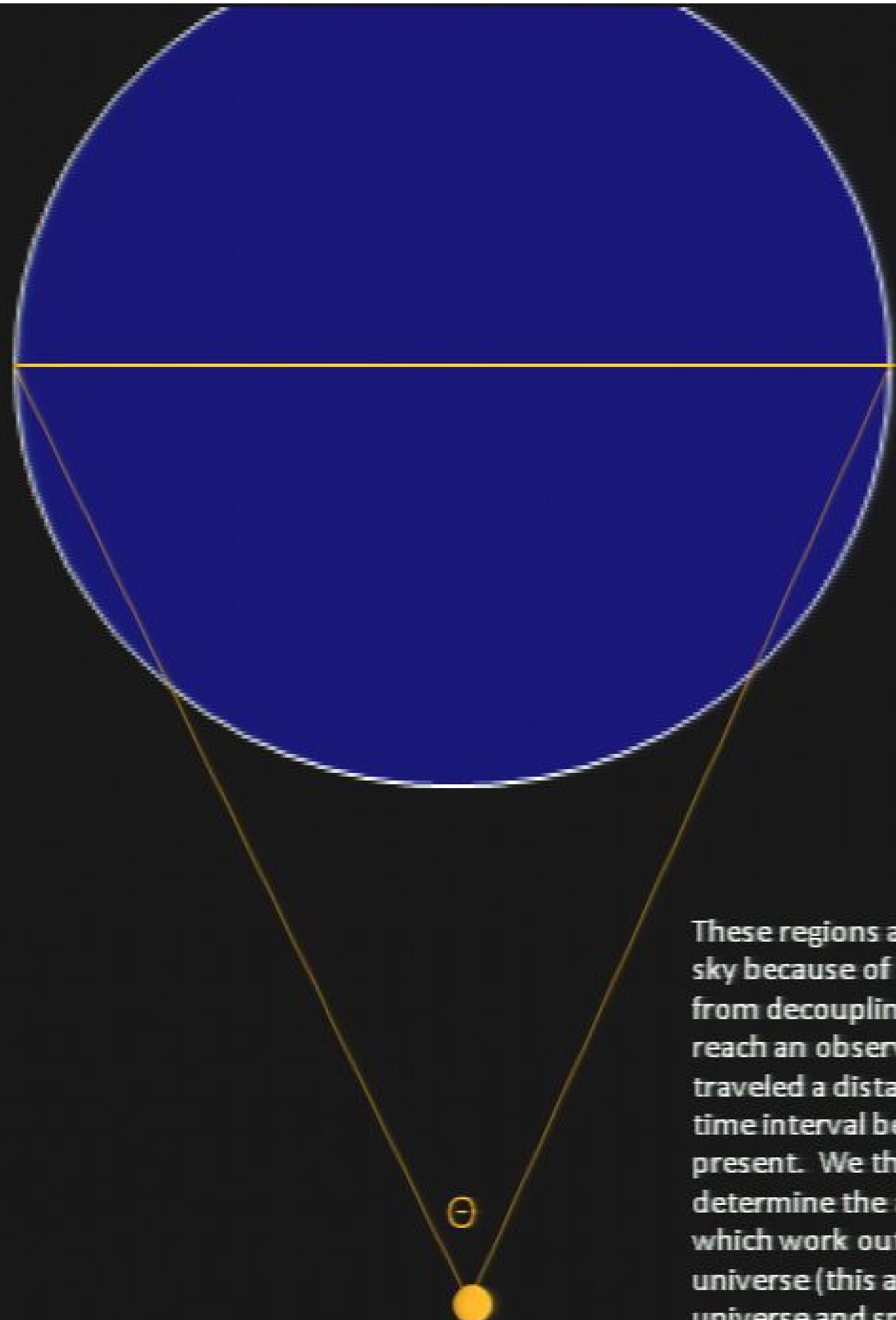


: Pre-Inflation



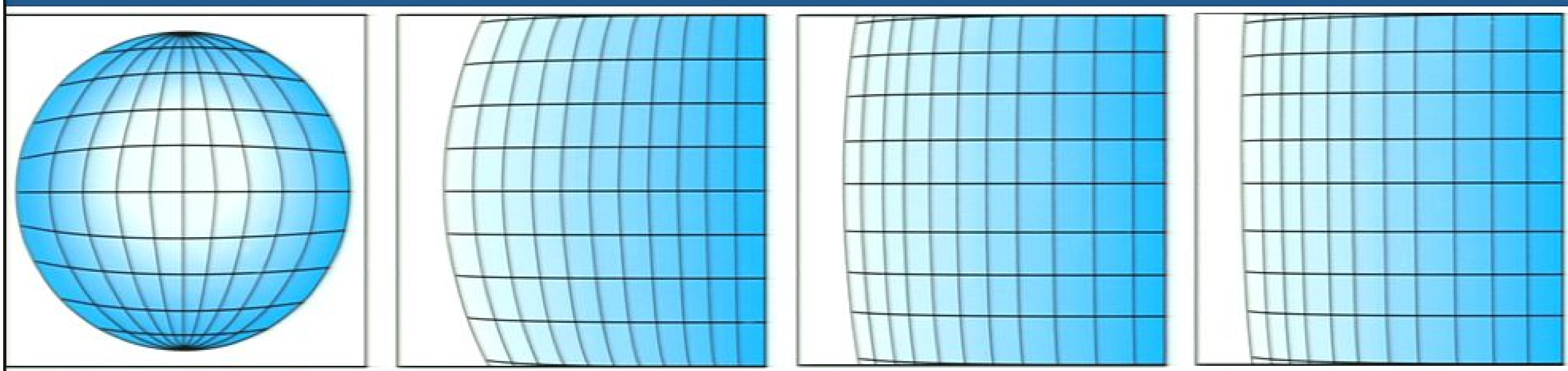
The size of the universe increased so fast and became so large, that our entire observable universe is made up of a region that was causally connected in the pre-inflation universe

: Decoupling



These regions appear 1100 times larger on our sky because of the expansion of the universe from decoupling to the present. For photons to reach an observer on earth they will have traveled a distance of the speed of light times the time interval between decoupling and the present. We then can use simple geometry to determine the angle of these regions on our sky, which work out to about one degree for a flat universe (this angle would be larger for a close universe and smaller for and open universe).

: Present



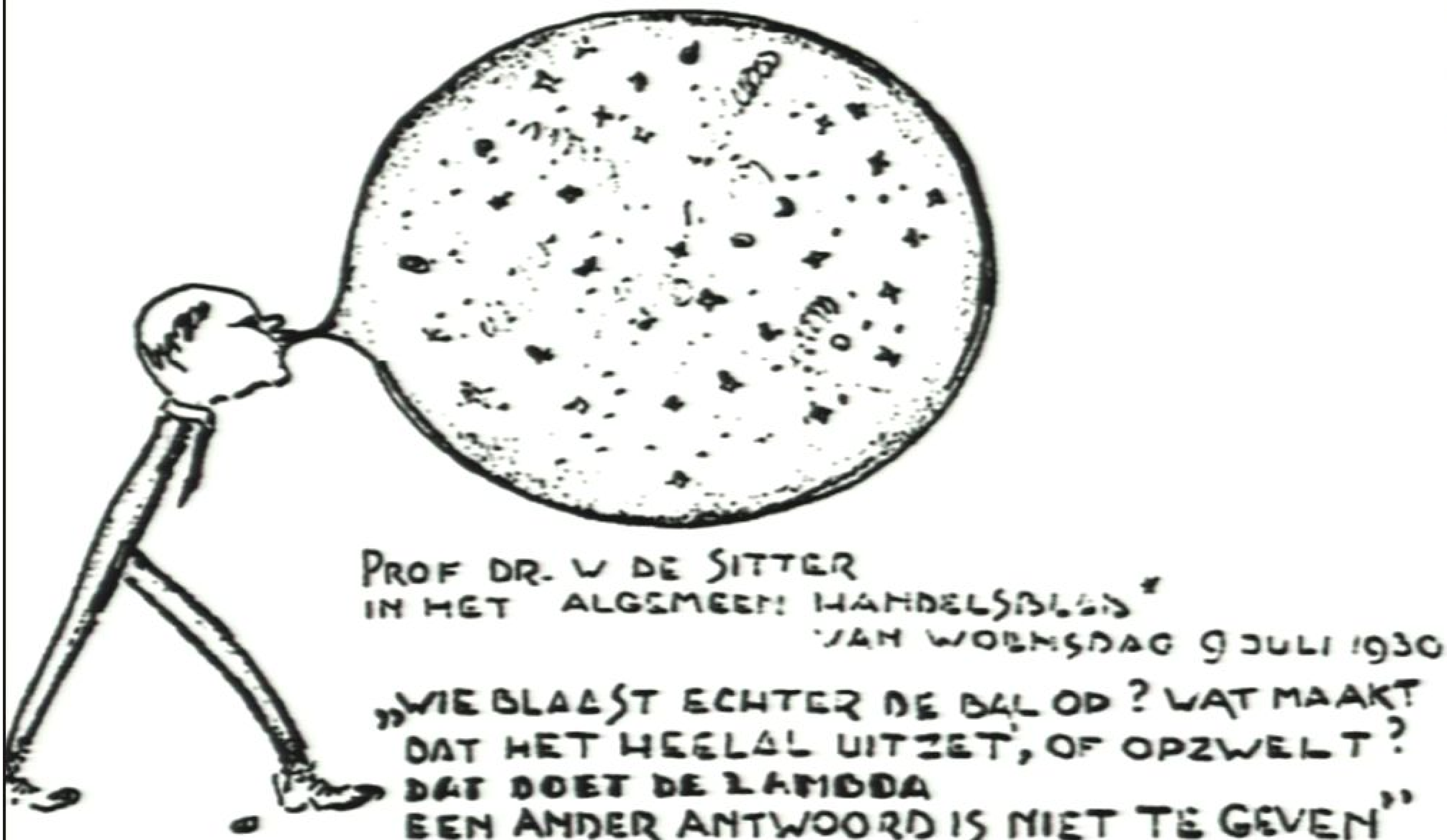
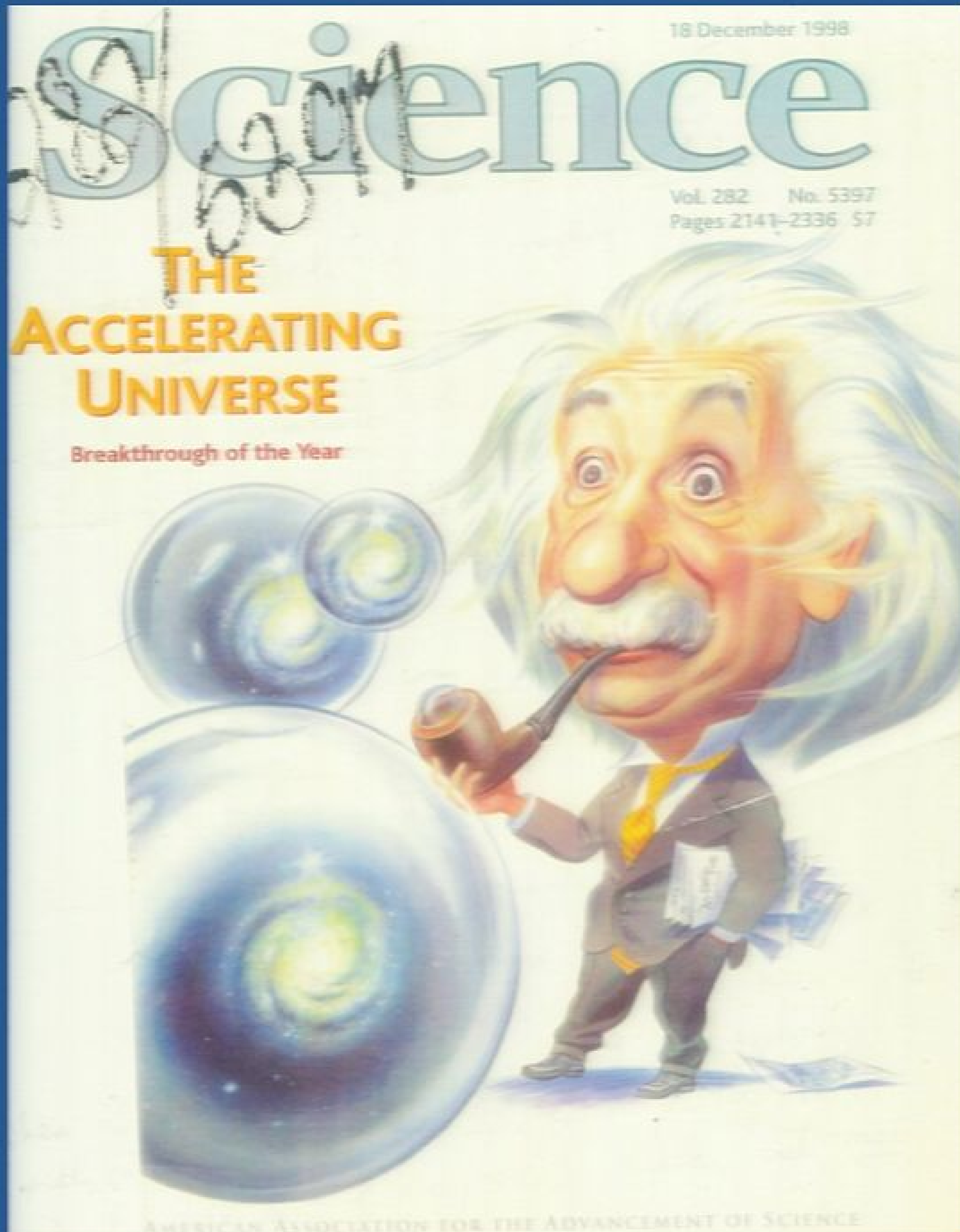


Figure 5.2. This sketch appeared following an interview of de Sitter published in a Dutch newspaper. The quote is translated by van der Laan as: "What, however, blows up the ball? What makes the universe expand or swell up? That is done by the Lambda. Another answer cannot be given."


$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$
A. EINSTEIN


18 December 1998
Science
Vol. 282 No. 5397
Pages 2141–2336 $7
THE ACCELERATING UNIVERSE
Breakthrough of the Year
American Association for the Advancement of Science

• If the universe is dominated by vacuum energy it will expand exponentially and unceasingly, i.e. the Hubble constant $\dot{a}/a = \sqrt{\Omega_r(1+z)^4 + \Omega_\Lambda}$ $\to \sqrt{\Omega_\Lambda} =$ constant $\Longrightarrow a(t) \sim \exp(Ht)$.

• Some mechanism is needed to end this era and create a thermal state, i.e. the energy of the 'false vacuum' must dissipate.

• Yet the energy level of the 'true vacuum' coincides with today's zero point energy to one part per 10^{120} !!

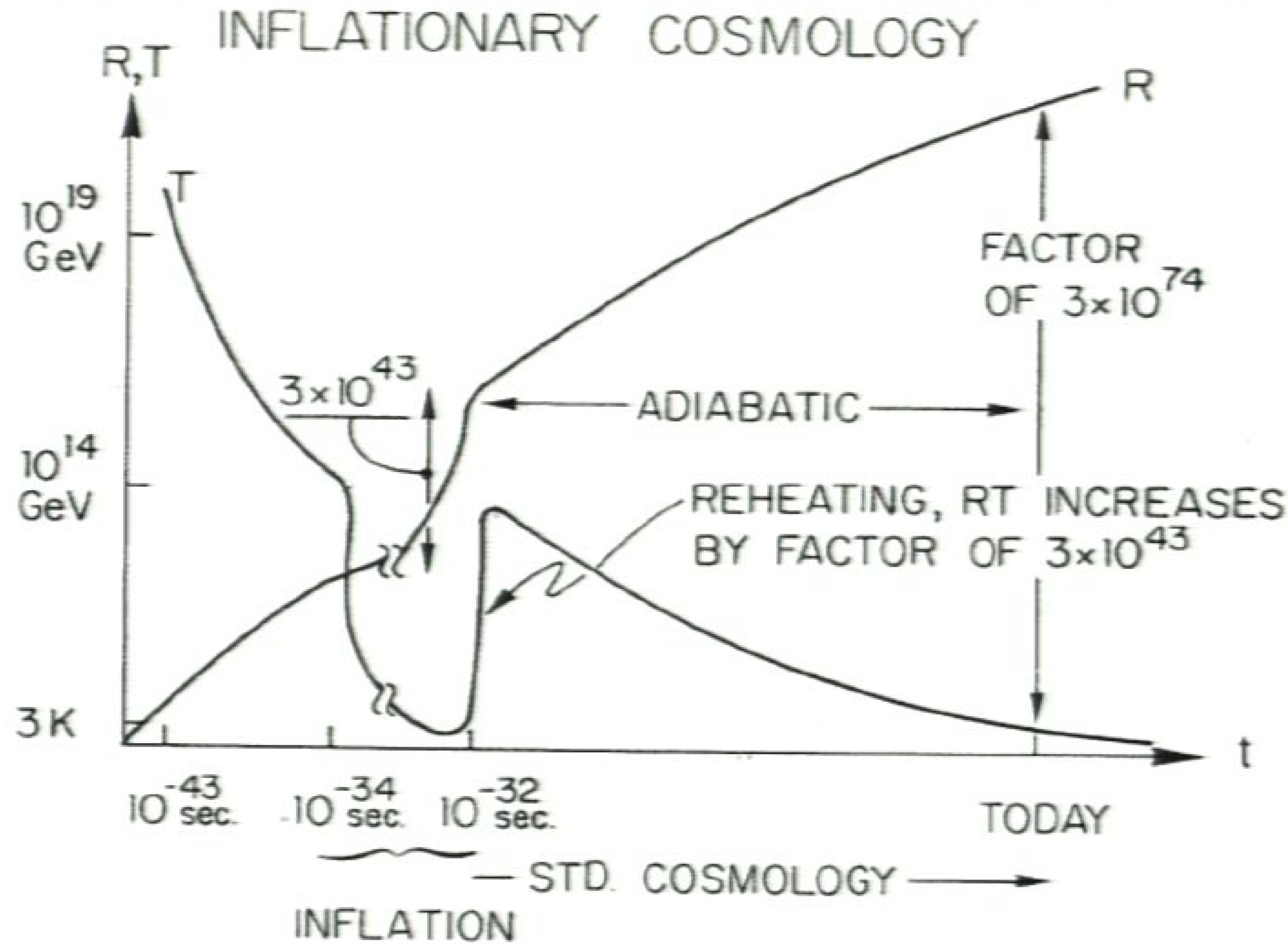
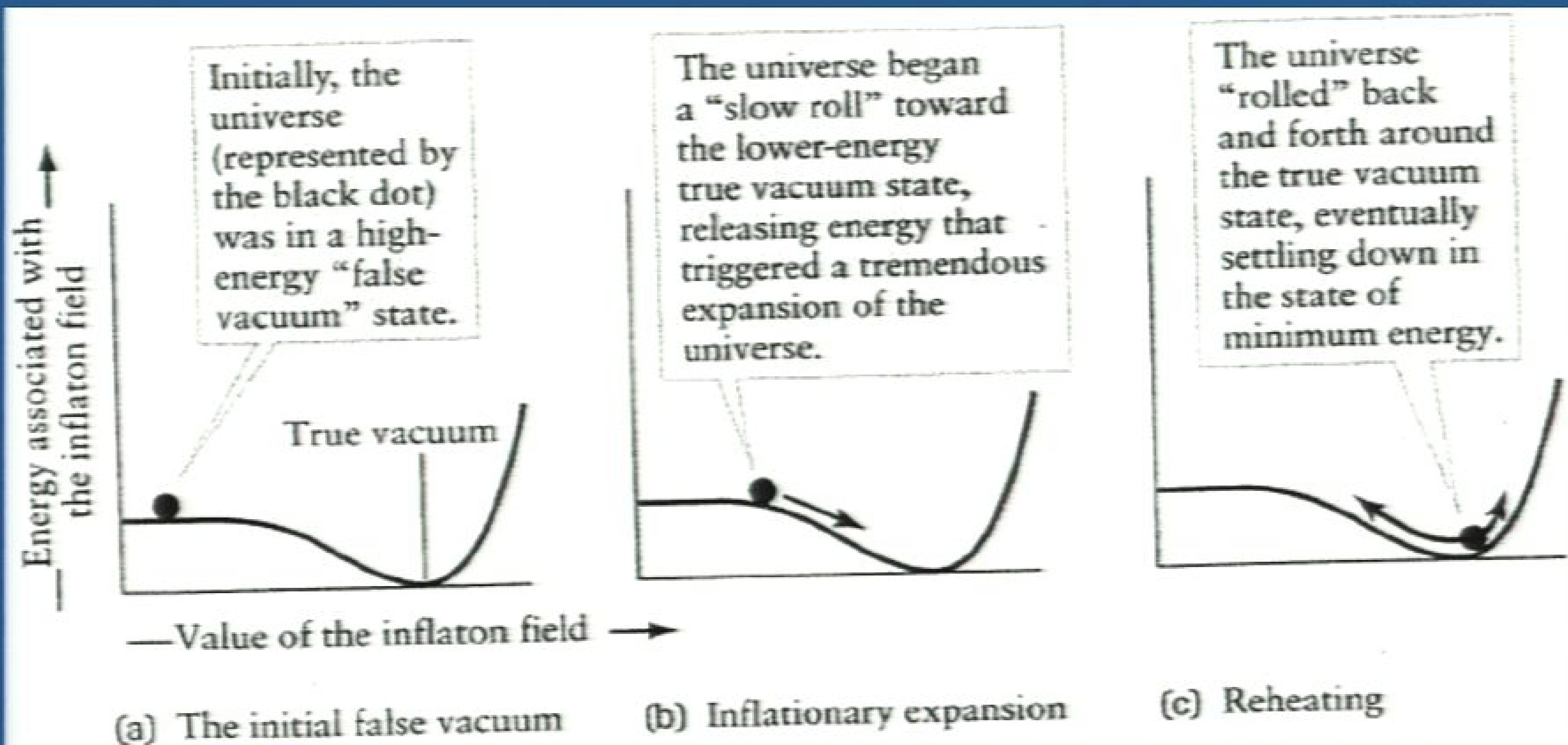
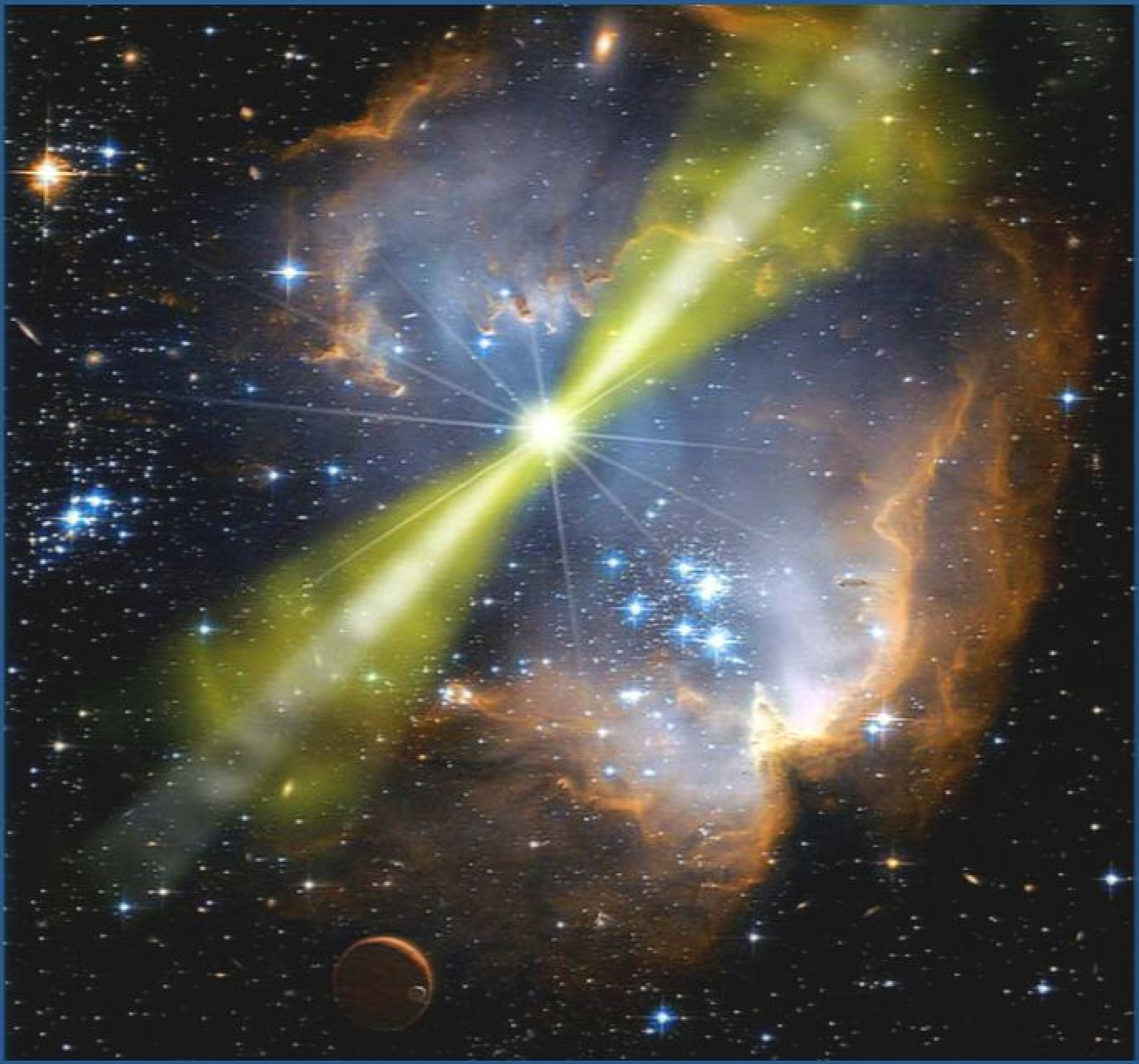


Fig. 8.2: Comparison of the evolution of R and T in the standard and inflationary cosmologies. Note the enormous jump in entropy ($S \propto R^3T^3$) at the end of inflation.



Initially, the universe (represented by the black dot) was in a high-energy "false vacuum" state.
True vacuum
The universe began a "slow roll" toward the lower-energy true vacuum state, releasing energy that triggered a tremendous expansion of the universe.
The universe "rolled" back and forth around the true vacuum state, eventually settling down in the state of minimum energy.
Energy associated with the inflaton field
Value of the inflaton field
(a) The initial false vacuum
(b) Inflationary expansion
(c) Reheating



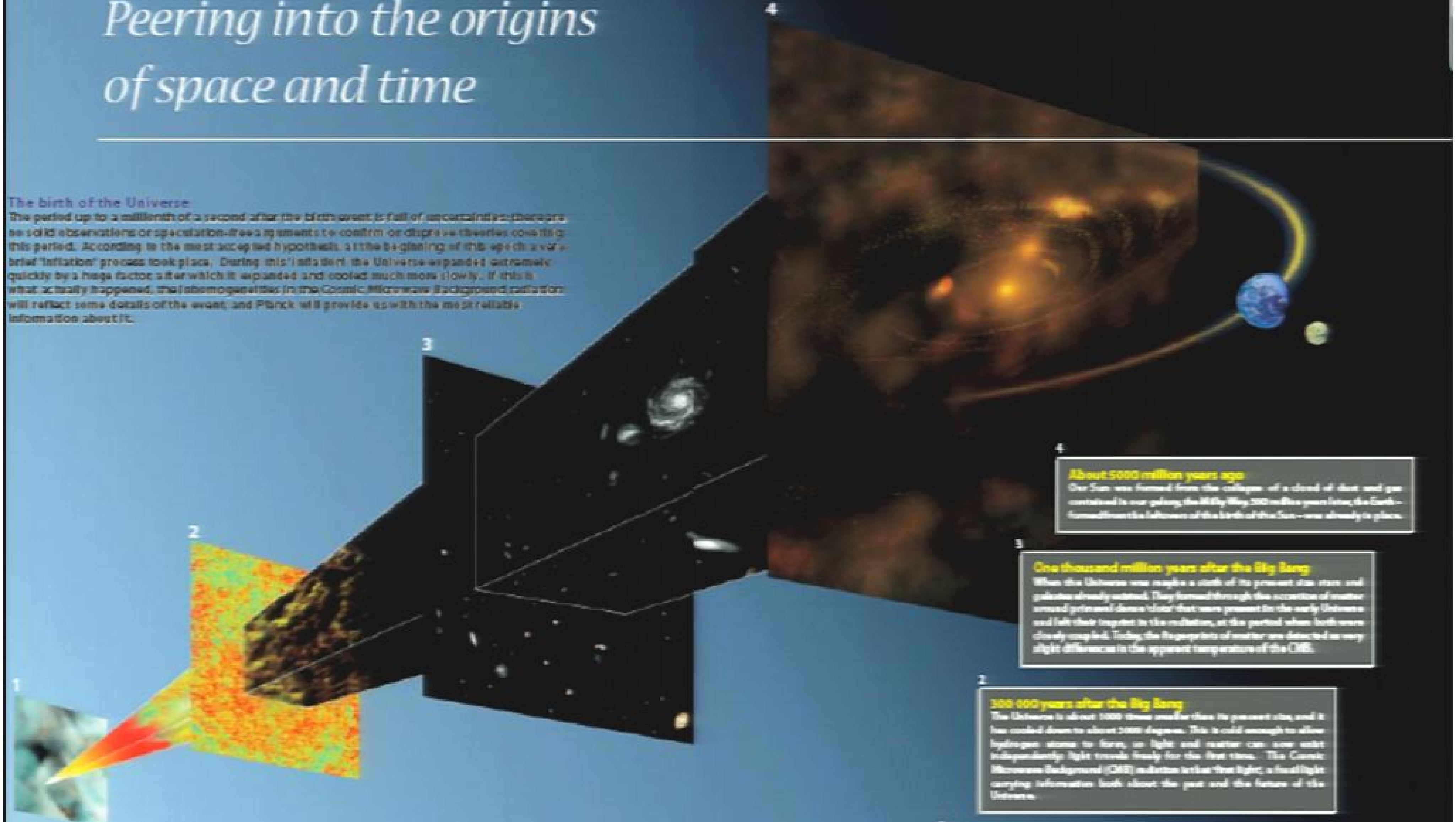
- Yet all these are still not enough!

WHAT ABOUT STRUCTURES ?

Peering into the origins of space and time

The birth of the Universe

The period up to a millionth of a second after the birth event is full of uncertainties: there are no solid observations or speculation-free arguments to confirm or disprove theories covering this period. According to the most accepted hypothesis, at the beginning of this epoch a very brief 'inflation' process took place. During this 'inflation' the Universe expanded extremely quickly by a huge factor, after which it expanded and cooled much more slowly. If this is what actually happened, the inhomogeneities in the Cosmic Microwave Background radiation will reflect some details of the event, and Planck will provide us with the most reliable information about it.



4

About 5000 million years ago
Our Sun was formed from the collapse of a cloud of dust and gas contained in our galaxy, the Milky Way. 200 million years later, the Earth – formed from the leftovers of the birth of the Sun – was already in place.

3

One thousand million years after the Big Bang
When the Universe was maybe a sixth of its present size stars and galaxies already existed. They formed through the accretion of matter around primeval dense 'clots' that were present in the early Universe and left their imprint in the radiation, at the period when both were closely coupled. Today the fingerprints of matter are detected as very slight differences in the apparent temperature of the CMB.

2

300 000 years after the Big Bang
The Universe is about 1000 times smaller than its present size, and it has cooled down to about 3000 degrees. This is cold enough to allow hydrogen atoms to form, so light and matter can now exist independently: light travels freely for the first time. The Cosmic Microwave Background (CMB) radiation is that 'first light', a fossil light carrying information both about the past and the future of the Universe.

1

From one second until three minutes after the Big Bang
One second after its birth, the temperature of the Universe has dropped to 10 000 million degrees. The first atomic nuclei are formed. Meanwhile the Universe keeps expanding and cooling. But it is too hot yet for neutral atoms to form: electrons roam about freely and interact strongly with radiation. As a result, matter and radiation are closely coupled together.

MAP map of
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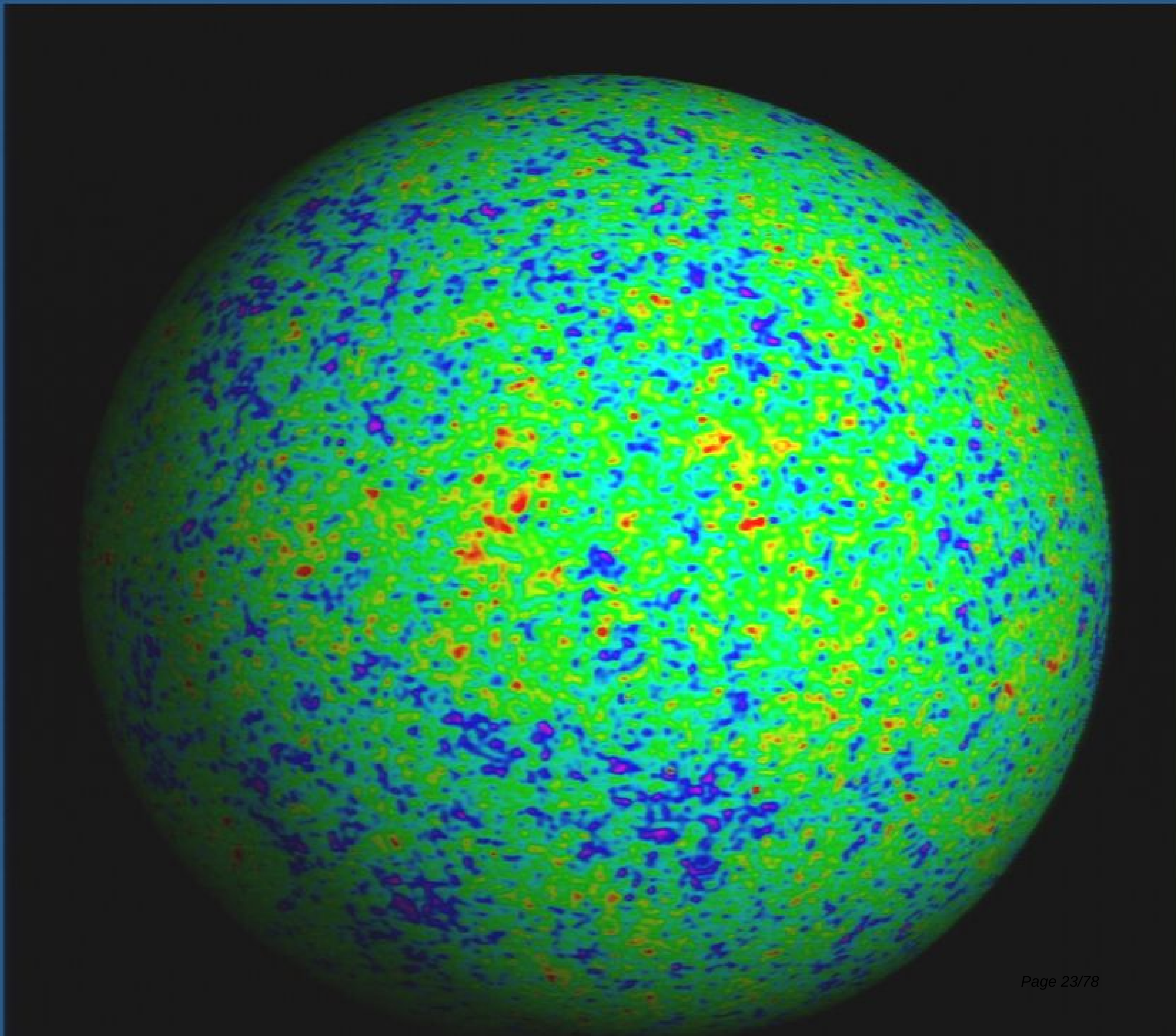

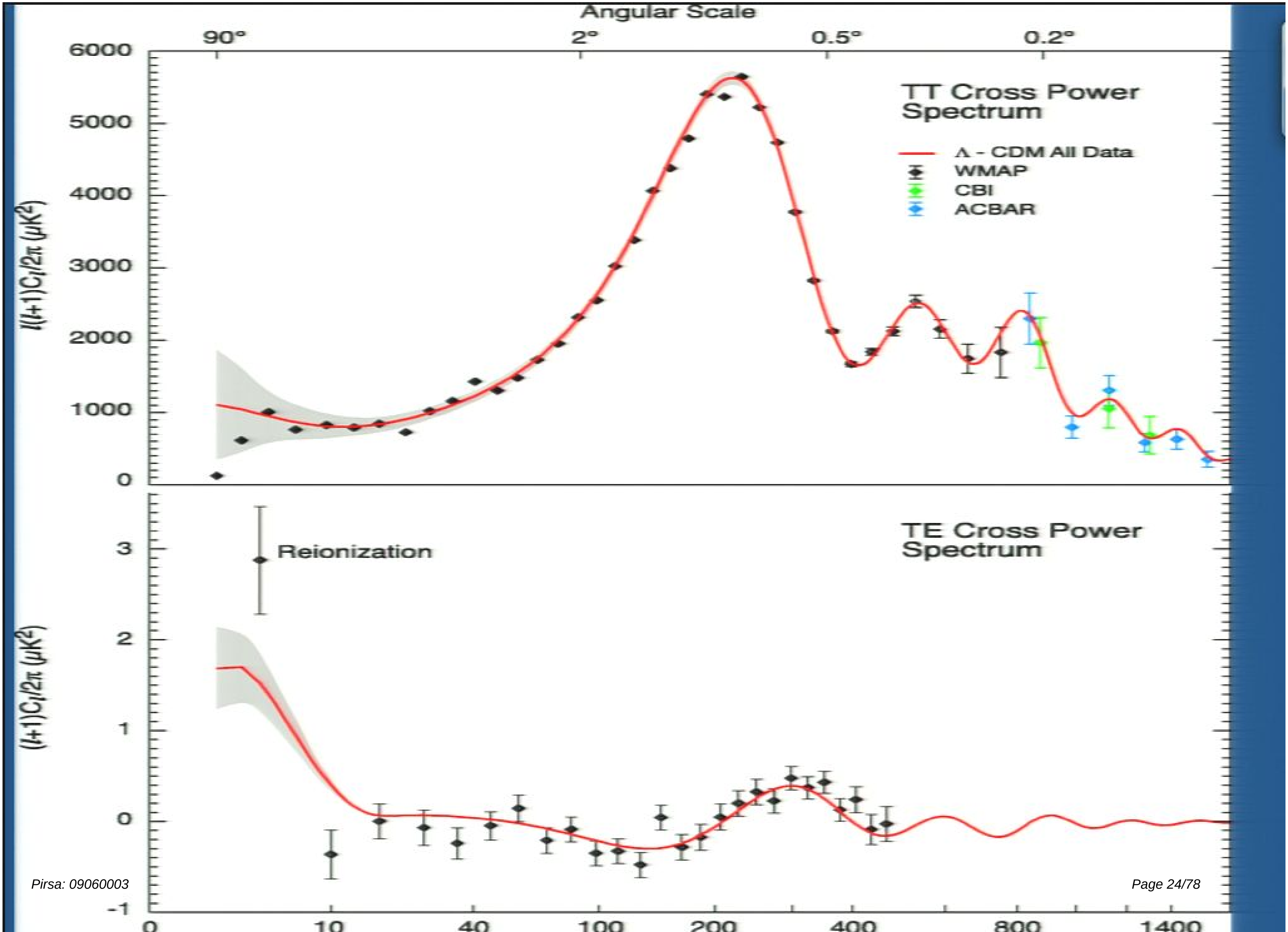

Angular Scale
90°
2°
0.5°
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TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
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$(l+1)C_l/2\pi\ (\mu K^2)$
Reionization
TE Cross Power Spectrum
0
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40
100
200
400
800
1400

Harrison Zel'dovich power spectrum of fluctuations: working in the comoving system of osmic expansion,

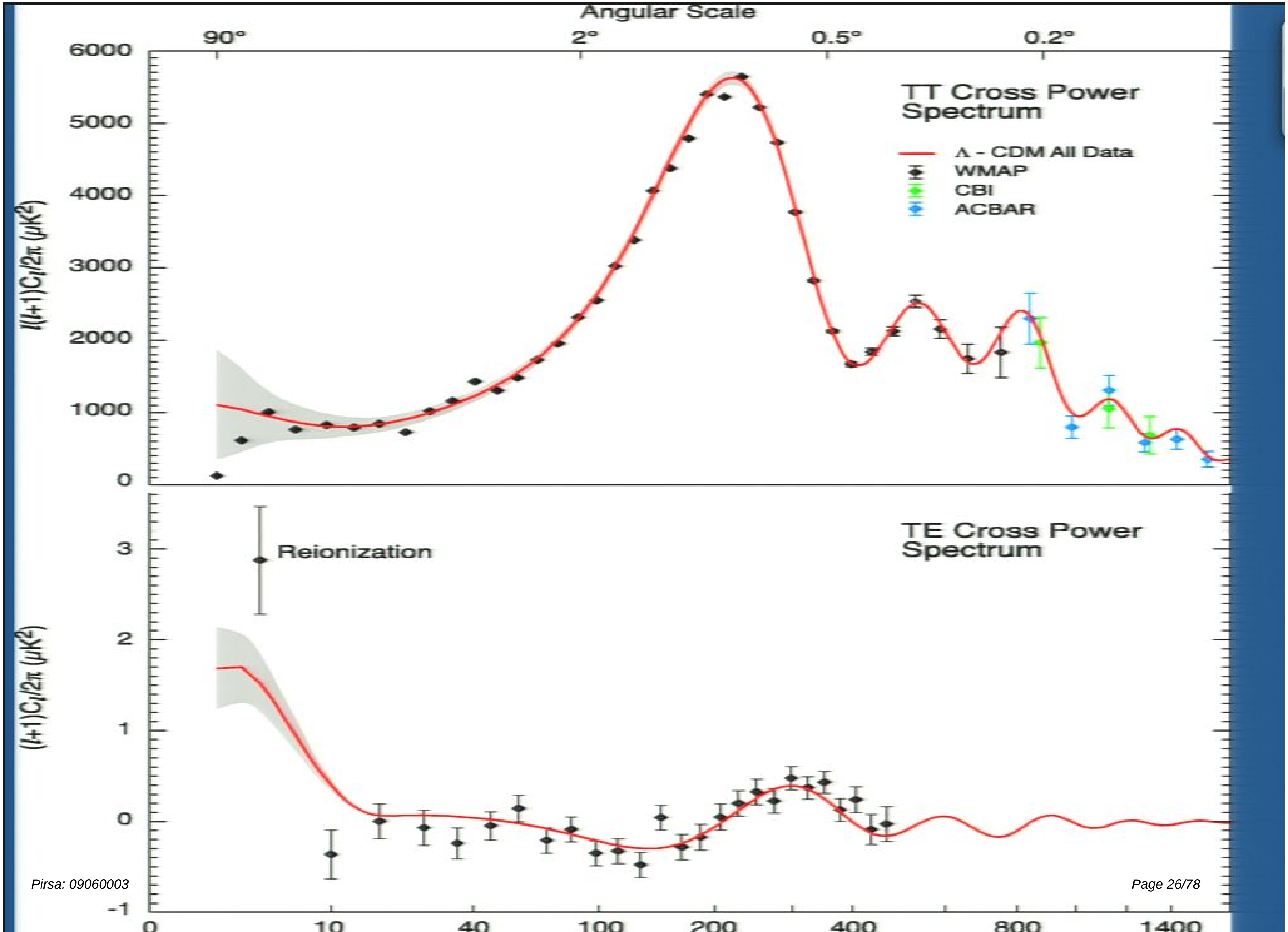
$$P(k) = |\delta_k|^2 \propto k^n, \text{ with } n \approx 1.$$

Provided $P(k)$ 'levels off' at $k \approx 1/r_H$ where r_H is the comoving radius of the horizon, he density contrast δ_R within a sphere of radius R is given by

$$\delta_R^2 = \frac{\langle \delta u^2 \rangle}{\bar{u}^2} = \frac{1}{V^2} \int d^3k |W(k)|^2 P(k) \propto \begin{cases} \frac{1}{R^{3+n}}, & n < 1, \\ \frac{1}{R^4}, & n > 1, \end{cases} \tag{1}$$

vhere $\bar{u}$ is the mean density of the universe, $\bar{u} + \delta u$ the mean density within the sphere, $V = 4\pi R^3/3$, and

$$W(k) \text{ is the 3D F.T. of } W(r) = \begin{cases} 1 & \text{for } r \leq R, \\ 0 & \text{for } r > R. \end{cases} \tag{2}$$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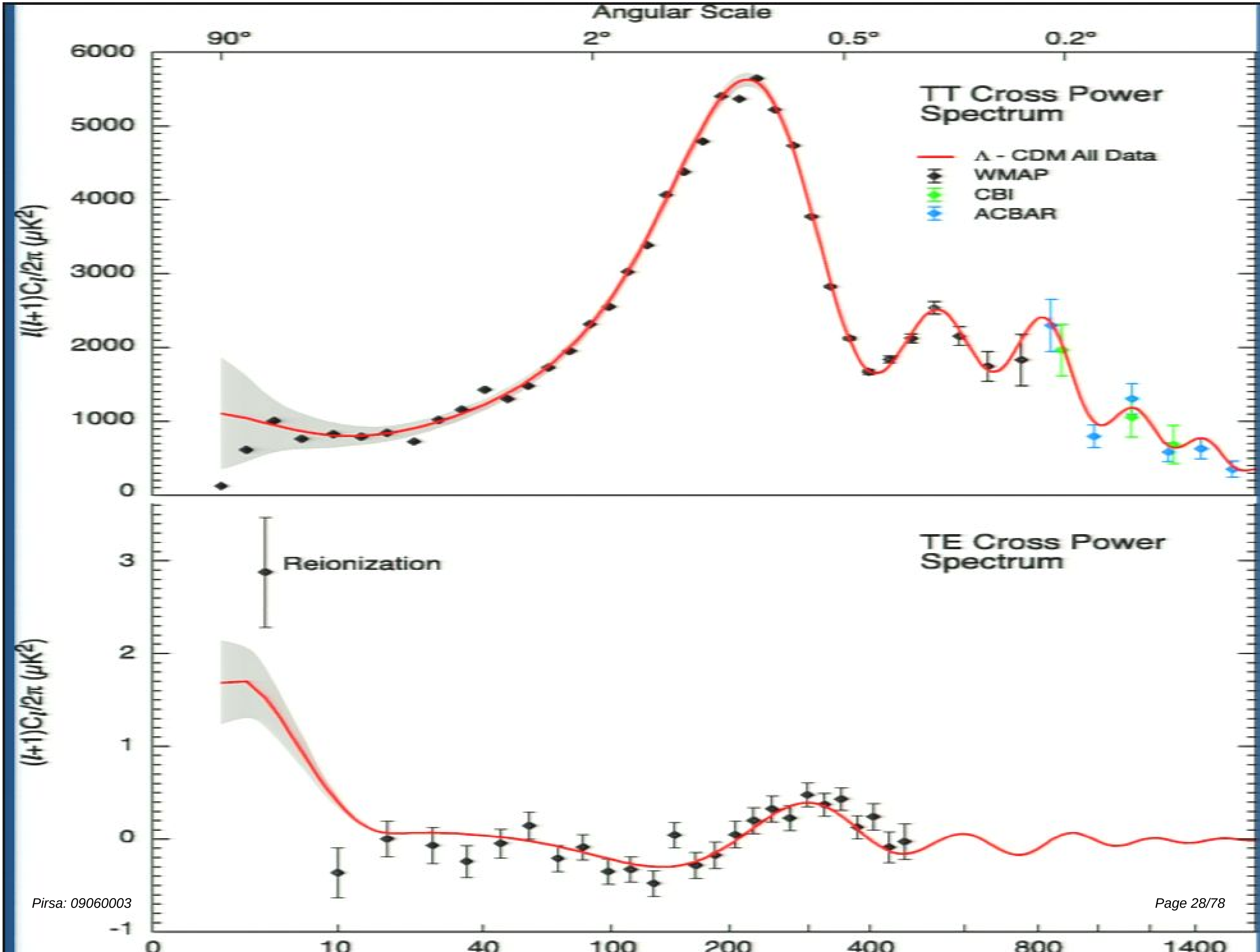
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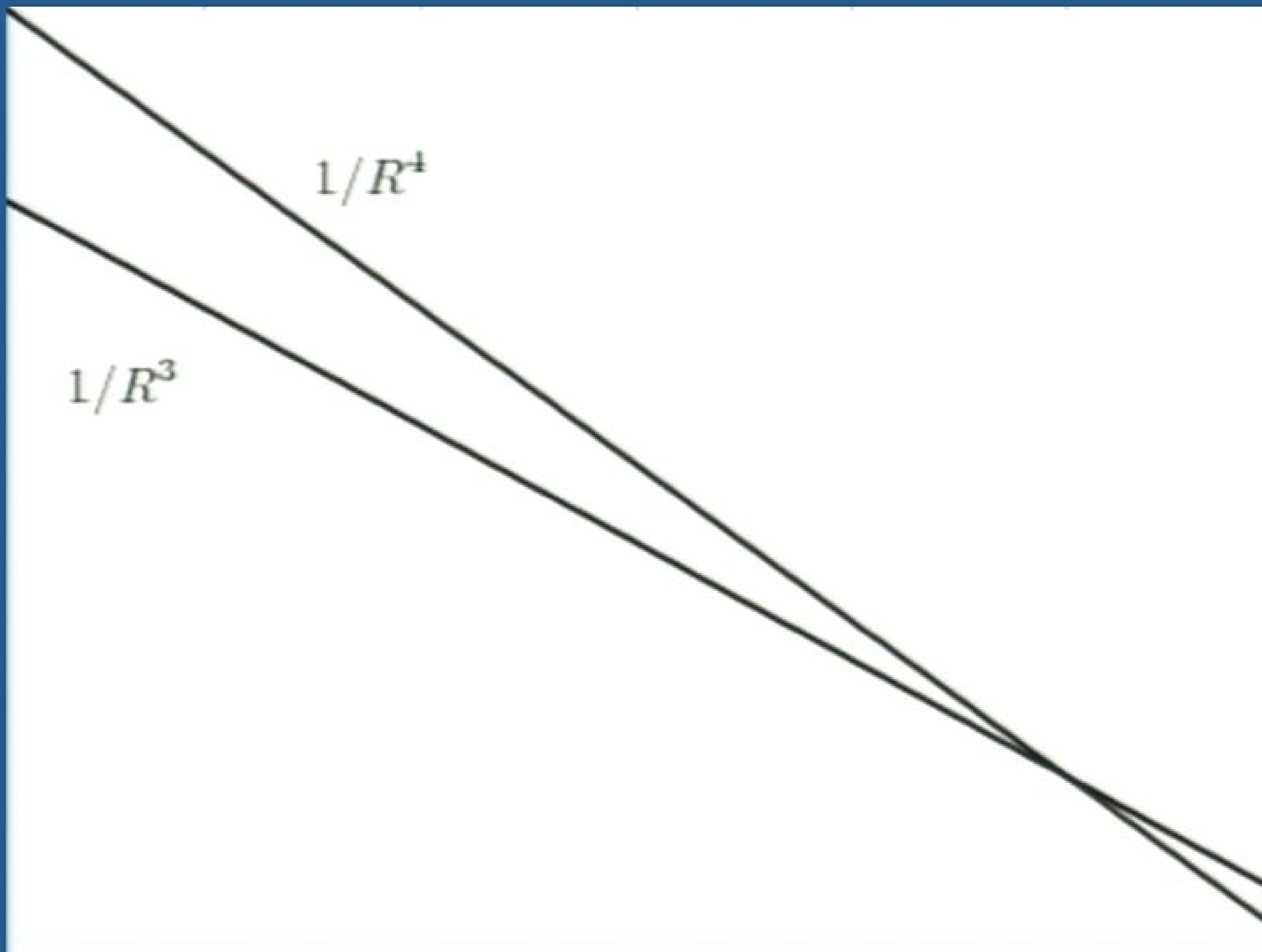
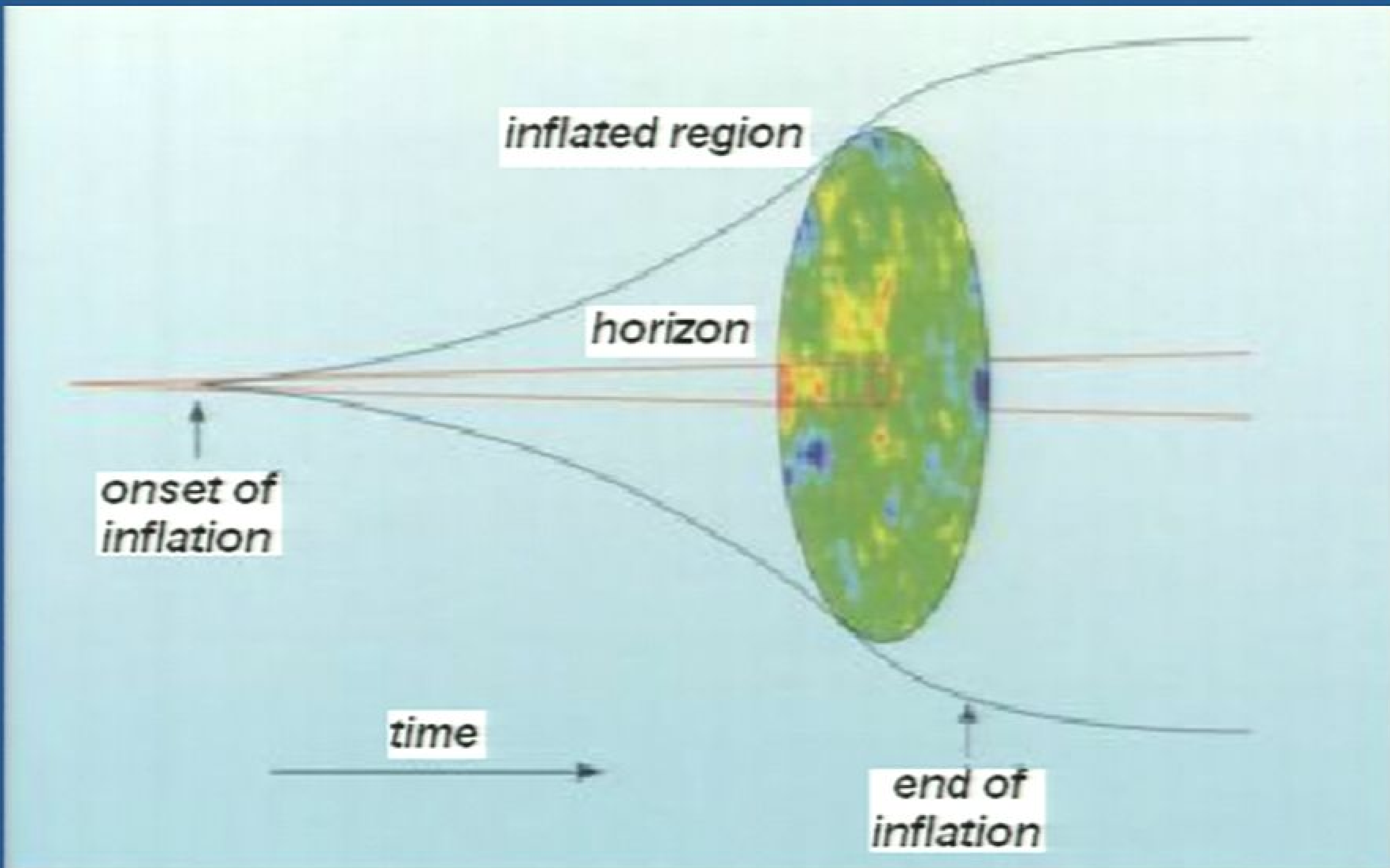
- `Seeds' of density contrast must have been planted well before the epoch of equality.


$1/R^4$
$1/R^3$


inflated region
horizon
onset of
inflation
time
end of
inflation

COSMOLOGY

henomenon	Explanation	Based on laboratory establised physics?	Verifiable in future experiments?
edhsift	Expansion of Space	No	Unverifiable (Chodorowski 2007)
MB	Big Bang	No	Far Future
otation Curves	Dark Matter	No	Near Future (As always)
istant Supernovae	Dark Energy	No	Far Future
latness and Isotropy	Inflation	No	Remote Future
tructure Formation	Inflationary vacuum fluctuations	No	Remote Future

ASTROPHYSICS

Phenomenon	Explanation	Seminal Paper	Based on Laboratory Established Physics?
Planetary orbits	Universal gravitation	Newton	Yes
Tides	Universal gravitation	Newton	Yes
X-ray Bursts	Thermonuclear Flashes	Wosley, Taam	Yes
Her X-1	Accretion	Hayakawa, Matsuoka, Prendergast, Burlidge	Yes/Maybe
Superluminal Motion	Special Relativity	Martin, Rees, Albert Einstein	Yes
White Dwarf Star	Quantum Physics Meets Gravity	Chandrasekhar	Known Physics Individually Verified

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- CAN WE STRIKE OUT SOME ASSUMPTIONS?
- Could the universe have started with heating by a uniform vacuum – the classical cosmological constant?
- Initially then, space was homogeneous. It could also be flat.

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`Seeds' of density contrast must have been planted well before the epoch of equality.

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NOT SO FAST: what is the intrinsic density contrast on the horizon at the end of inflation?

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NOT SO FAST: what is the intrinsic density contrast on the horizon at the end of inflation?

The answer turns out to be …

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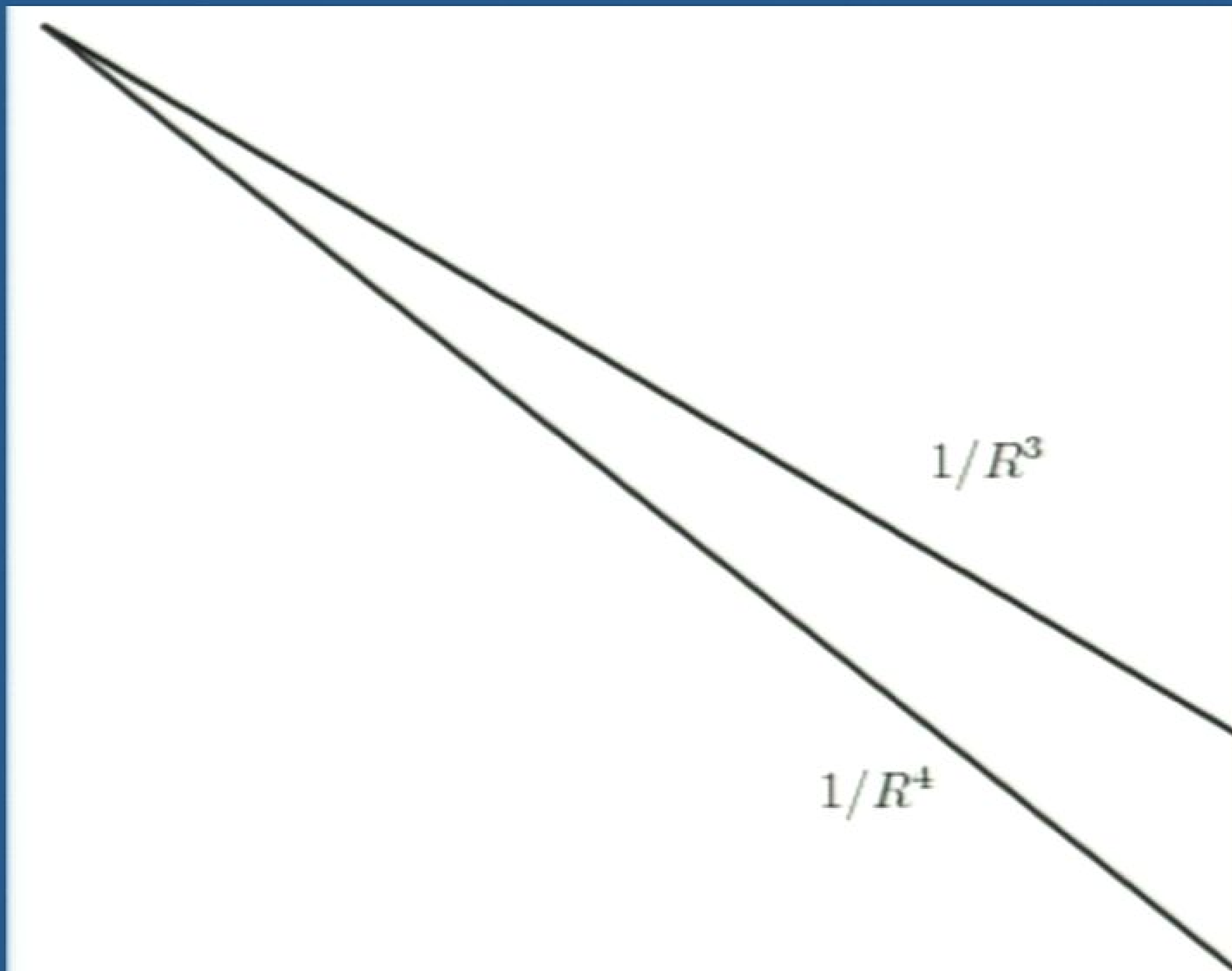
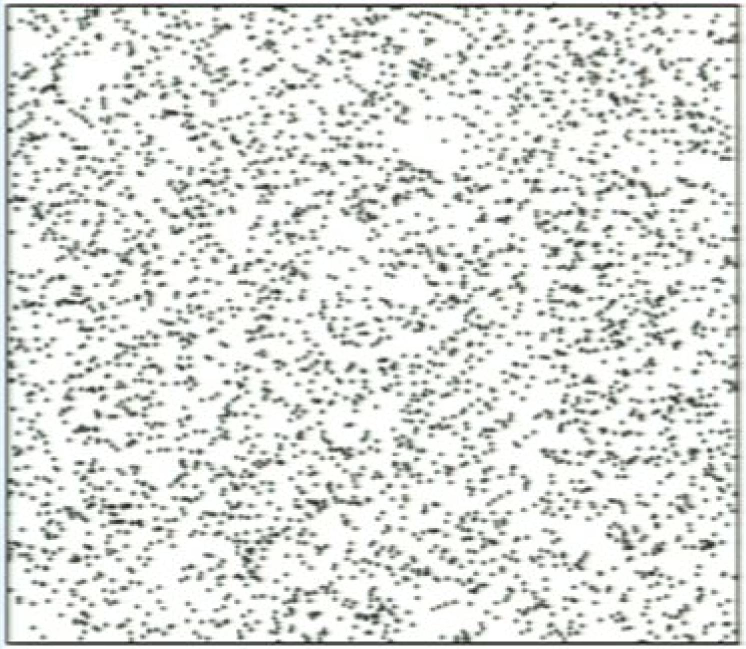
NOT SO FAST: what is the intrinsic density contrast on the horizon at the end of inflation?

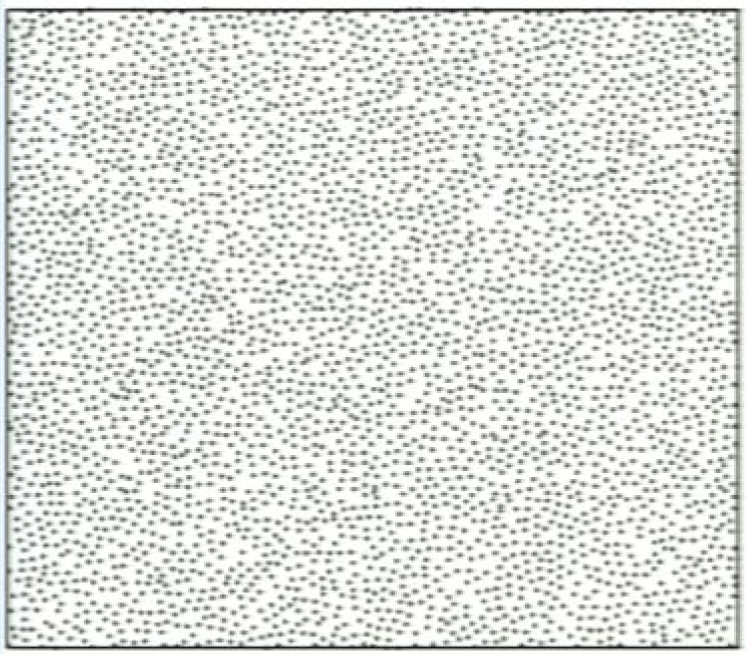
The answer turns out to be … one part per 10^5!

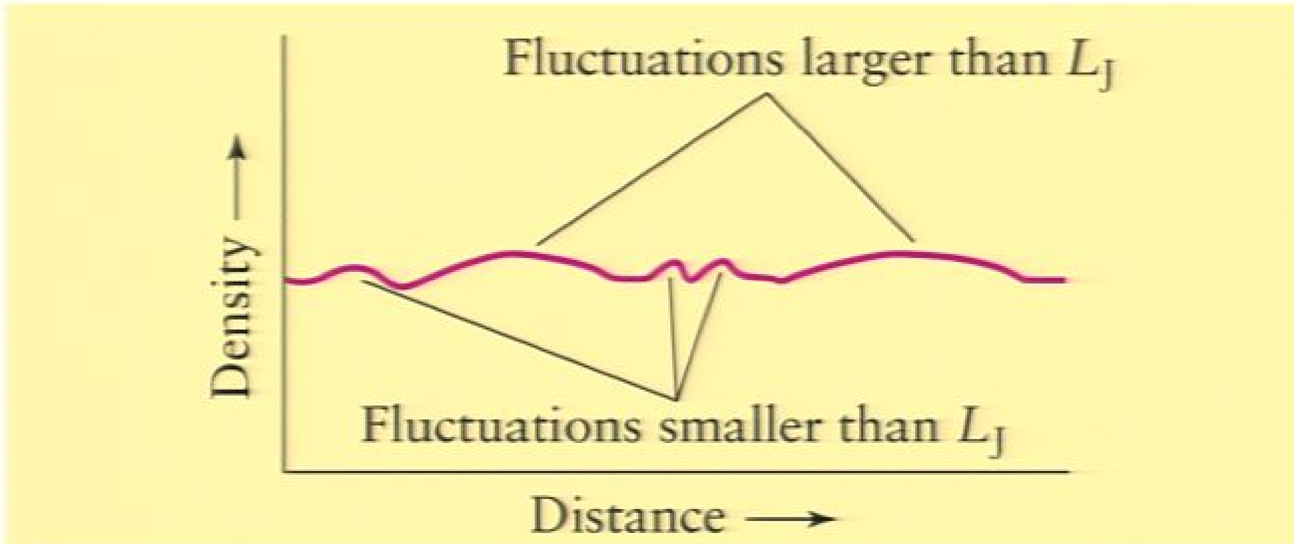

$1/R^3$
$1/R^4$

Poisson

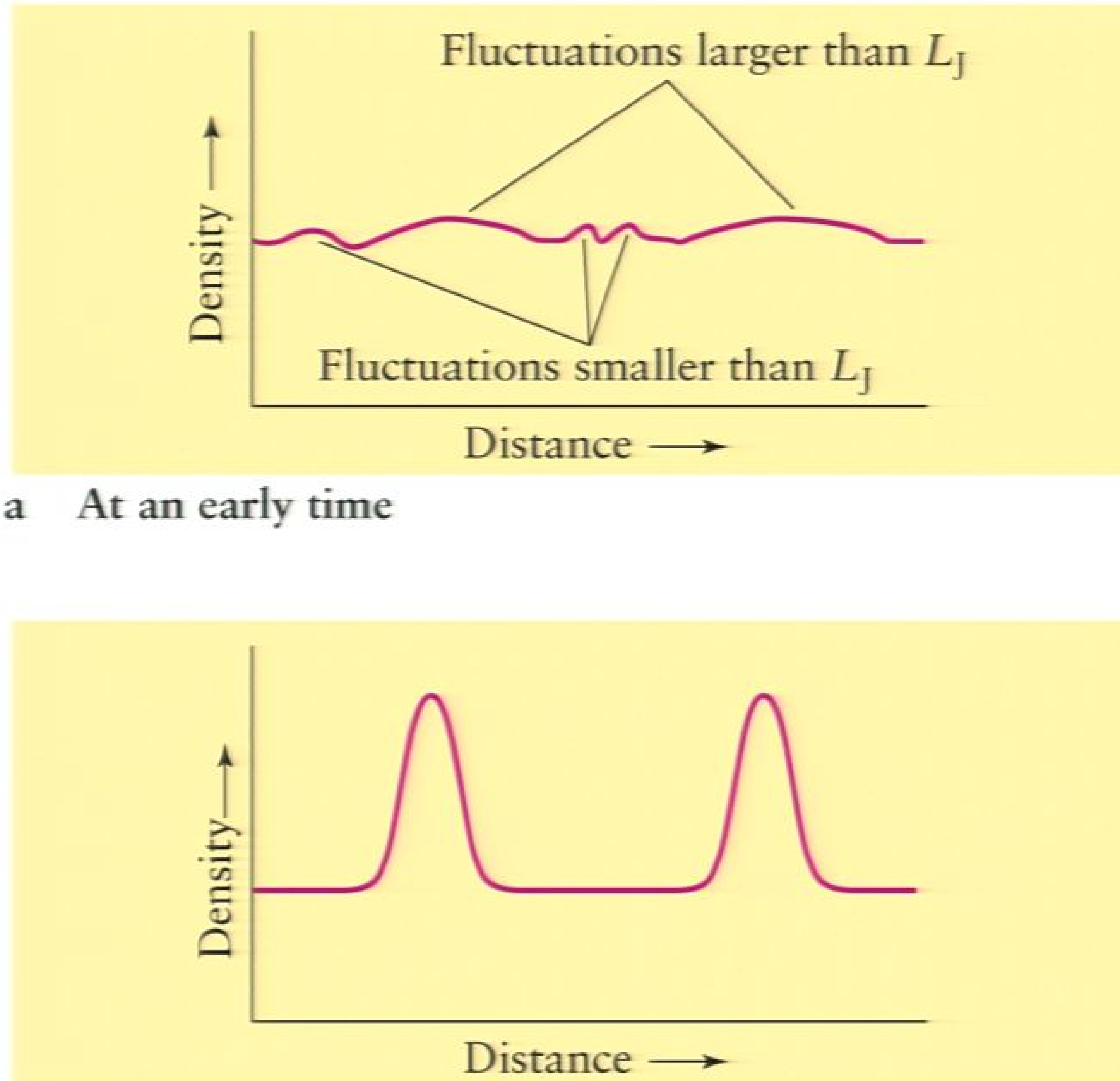


Harrison-
Zel'dovich





a At an early time



a At an early time

b At a later time

rowth rate applies only to superhorizon
odes in the synchronous gauge.

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j],$$

i.e. $d\tau = dt/a(t)$ with $c = 1$ for now.

But in other gauges the CMB parameters also
assume values different from unperturbed FRW
ones. Thus we use the synch gauge henceforth.

$$T_{\mu\nu} = 2\int (-g)^{-\frac{1}{2}} \frac{dP_1 dP_2 dP_3}{h^3} \frac{P_\mu P_\nu}{P^0} n(\mathbf{P}, T_0)$$

$$T^0{}_0 = -2\int \frac{(p^0)^3 dp^0 d\Omega}{h^3} \frac{1}{\exp\left(\frac{p^0}{k_{\rm B}T}\right) - 1}$$

uring the epochs after reheating, $\delta_k \sim r_H^2$ for
$< 1/r_H$. This means, as r_H grows with time
the radiation and matter eras a mode 're-
nters' the horizon with the same amplitude
that of any other superhorizon mode (just
at the time of 're-entry' is different, modes
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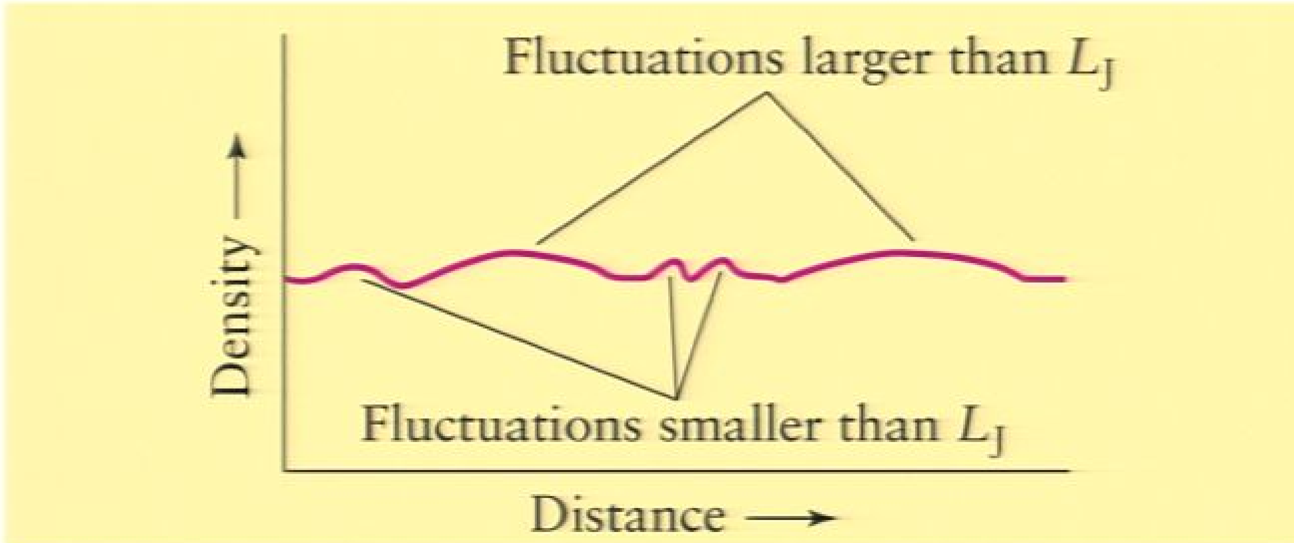
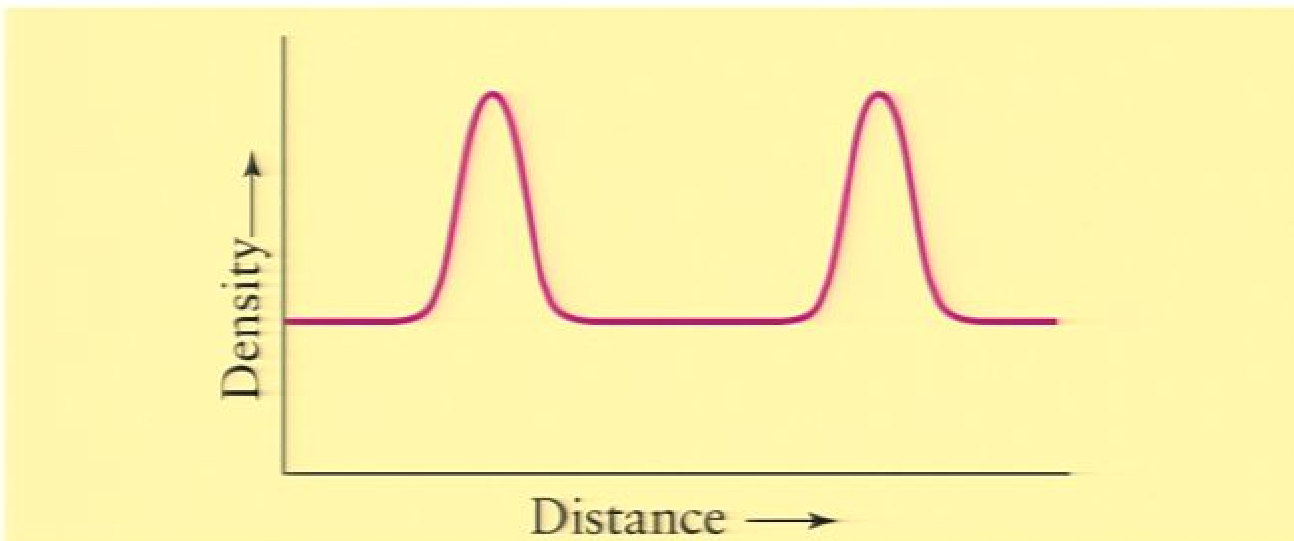


a At an early time



b At a later time

rowth rate applies only to superhorizon
nodes in the synchronous gauge.

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j],$$

i.e. $d\tau = dt/a(t)$ with $c = 1$ for now.

But in other gauges the CMB parameters also
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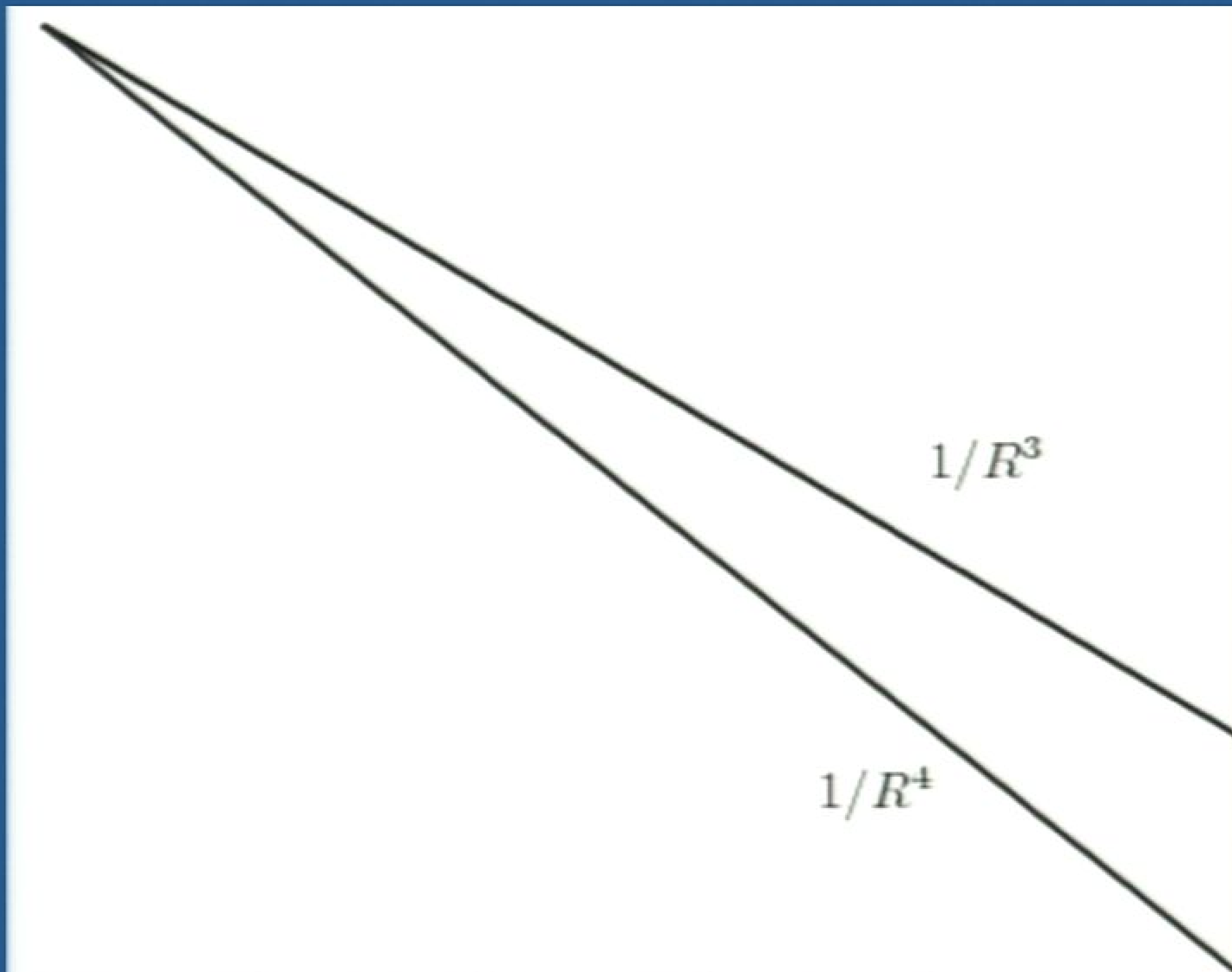
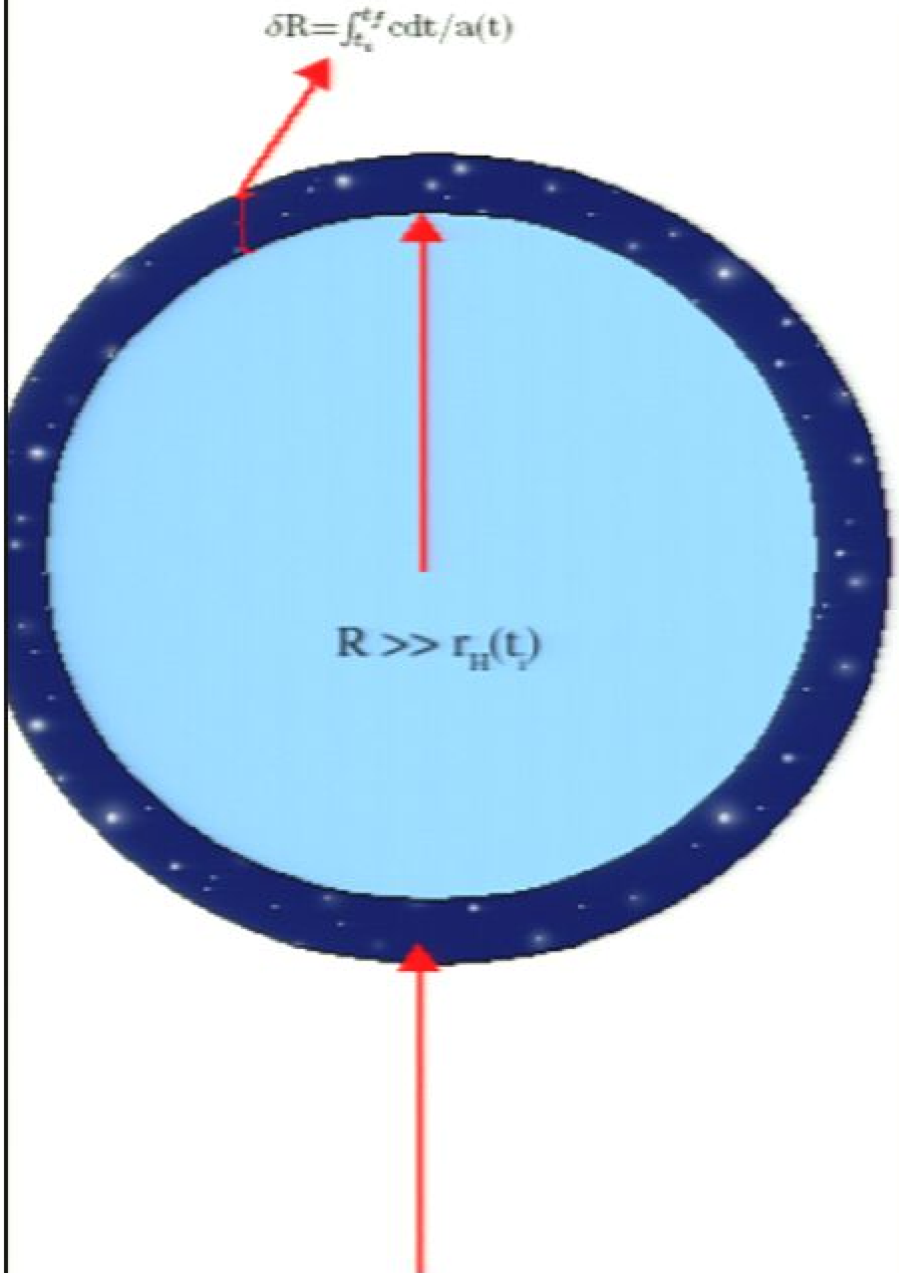
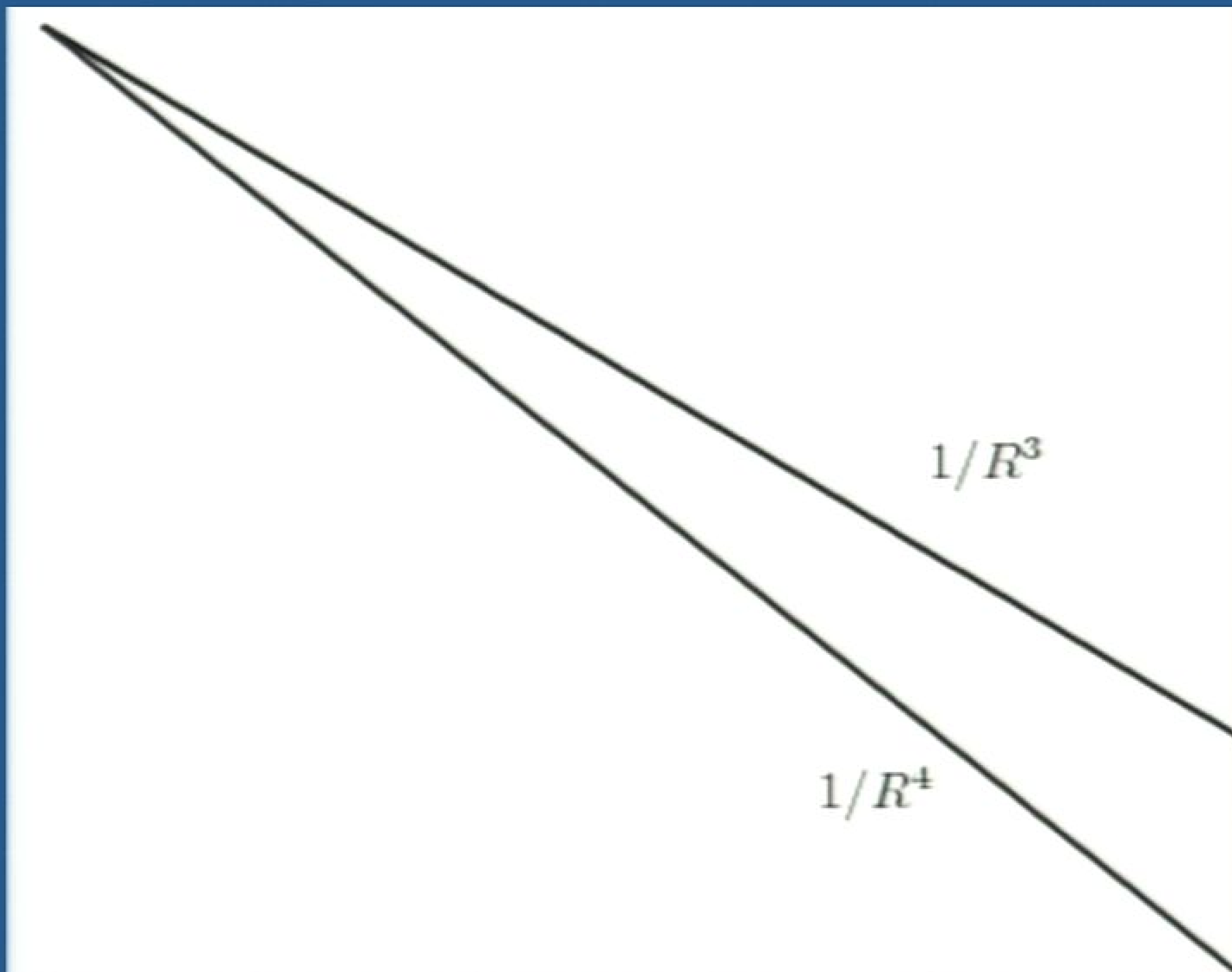

$1/R^3$
$1/R^4$



Number of particles in shell = $N' \sim 4\pi R^2 \delta R$

Variance in N = N' (Poisson)

Total number in the sphere $N \sim 4/3\pi R^3$


$1/R^3$
$1/R^4$

uring the epochs after reheating, $\delta_k \sim r_H^2$ for
$< 1/r_H$. This means, as r_H grows with time
the radiation and matter eras a mode 're-
nters' the horizon with the same amplitude
; that of any other superhorizon mode (just
at the time of 're-entry' is different, modes
ith smaller k re-enter later).

he consequence is: the WMAP observed hori-
on entry amplitude of $\delta_k \approx 10^{-5}$ *might* have
een fixed by the thermalization of horizon-
zed spheres at the time of reheating.

ut do we have $P(k) \sim k$ for modes outside

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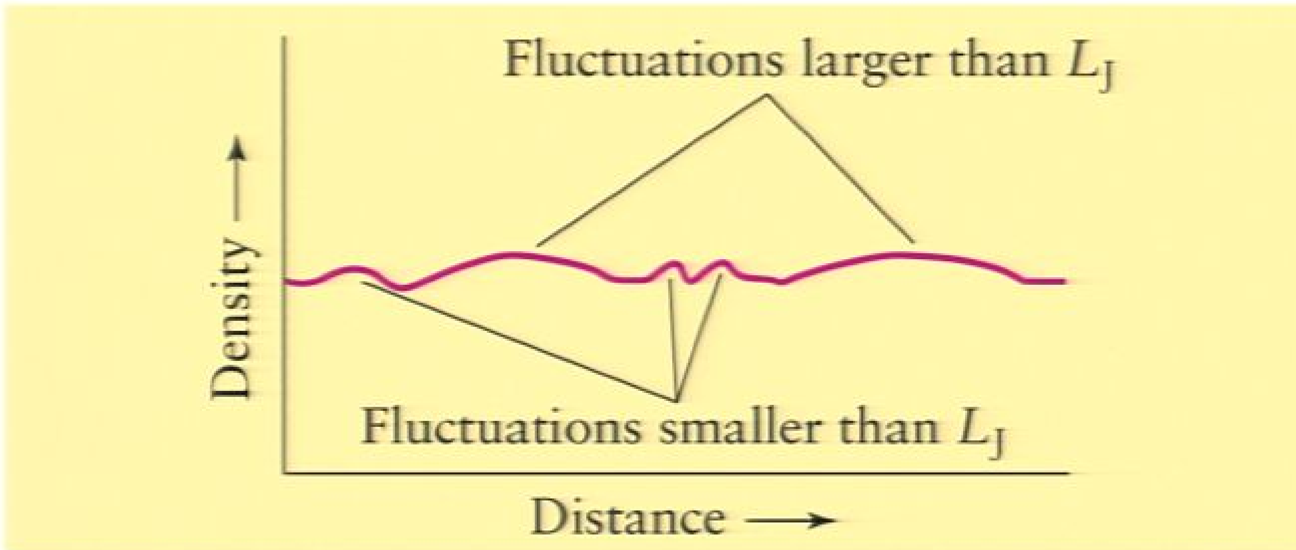
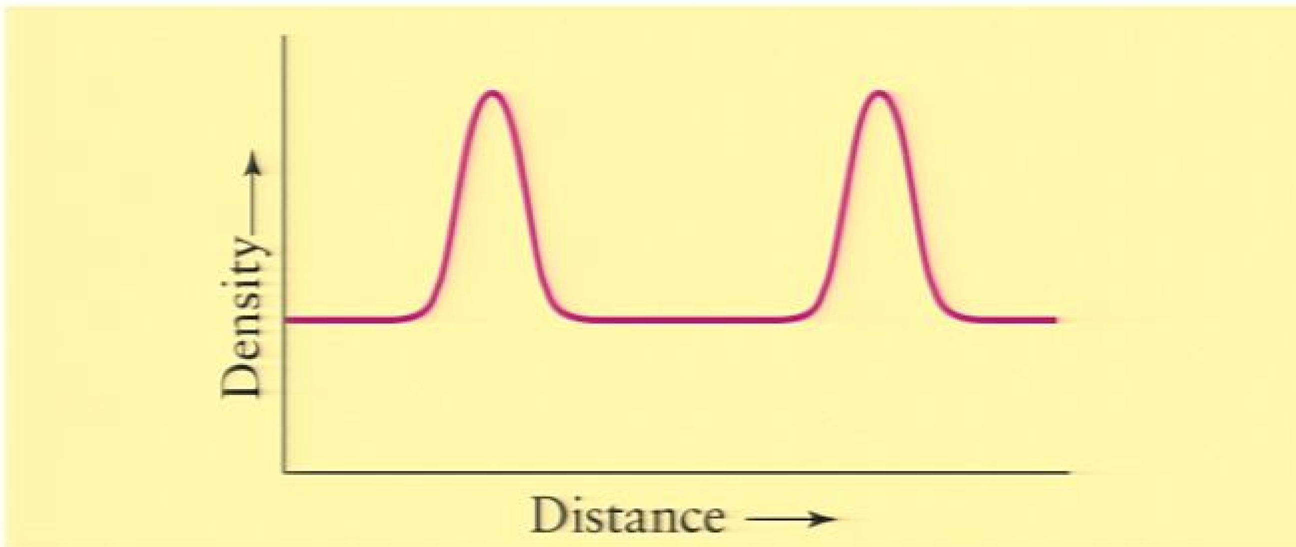


a At an early time



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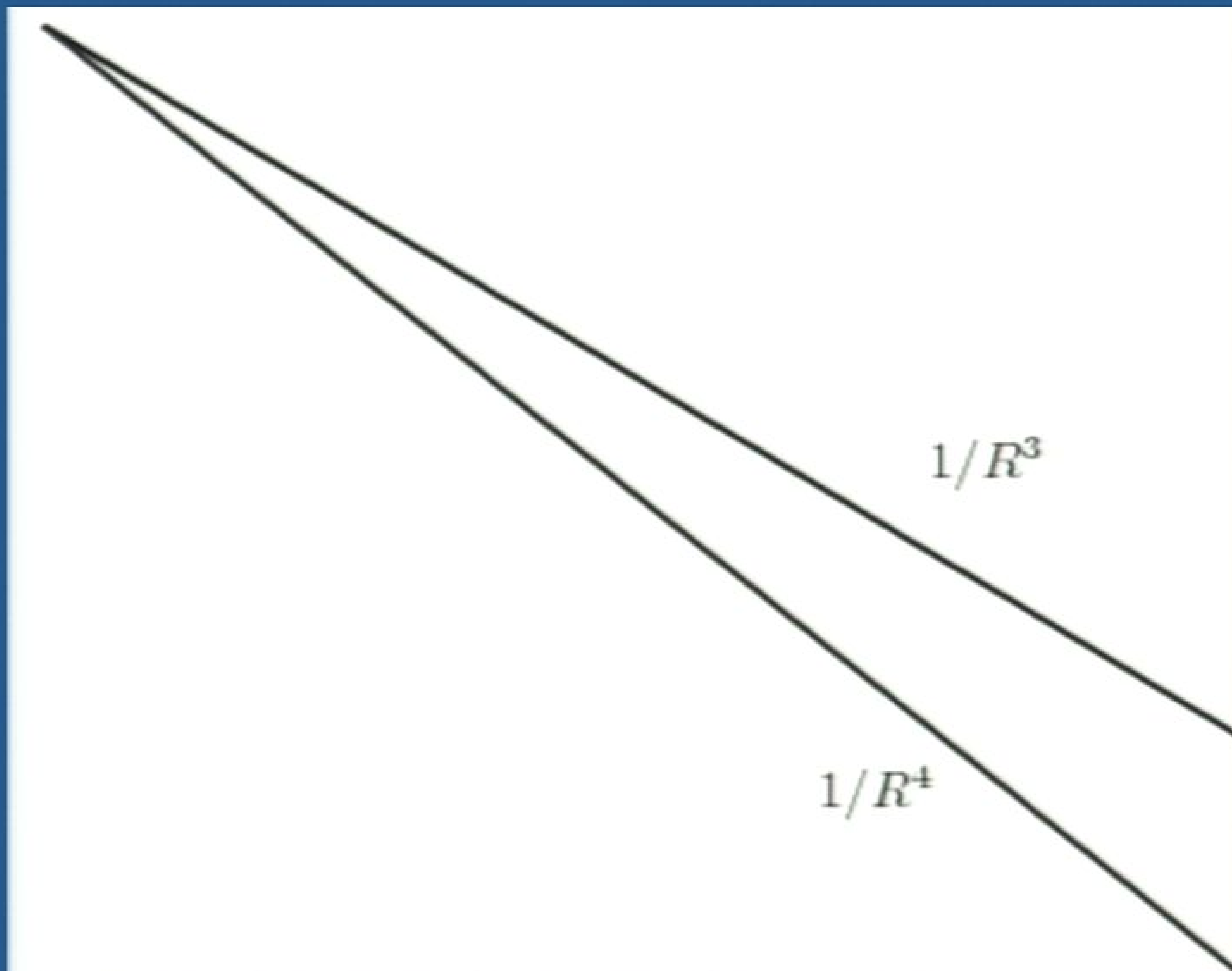
$1/R^3$
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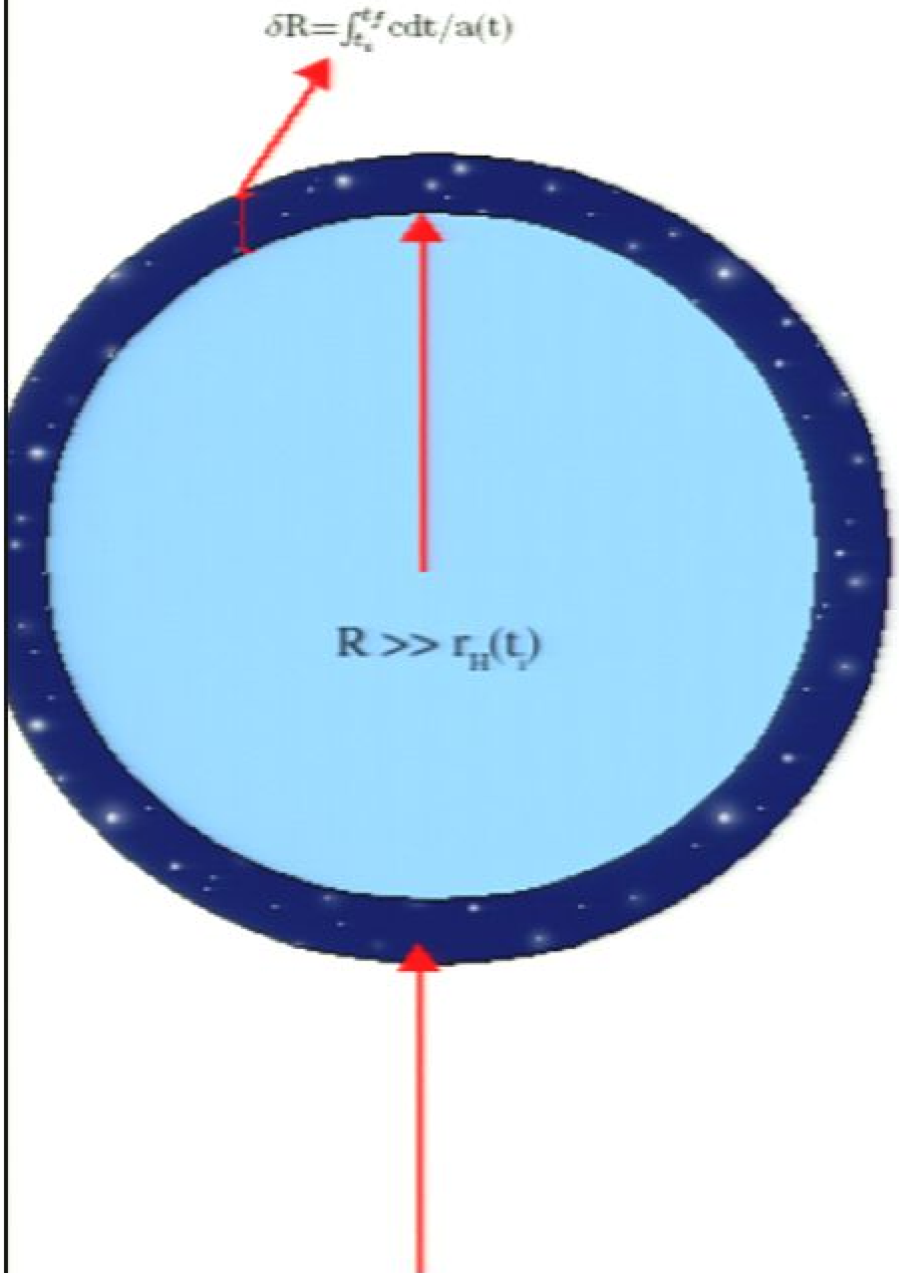


Number of particles in shell = $N' \sim 4\pi R^2 \delta R$

Variance in $N = N'$ (Poisson)

Total number in the sphere $N \sim 4/3\pi R^3$

We assume that $\delta R \sim$ one horizon size at reheating.

After reheating and during radiation era, $a(t) \sim t^{\frac{1}{2}} \Longrightarrow \delta R \sim t^{\frac{1}{2}} \Longrightarrow \delta_R \sim t^{\frac{1}{4}}$.

This is much slower than the linear growth rate of $\delta_R \sim r_H^2 \sim t$.

Thus the role of the diffusion is significant only *during* reheating - it plants the

Thermal (black body) noise

$$\delta_R^{\rm th} = \left(\frac{88 k_{\rm B} T_0}{43 u_0 V}\right)^{\frac{1}{2}} = \left(\frac{165 h^3 c^3}{43 \pi^5 V k_{\rm B}^3 T_0^3}\right)^{\frac{1}{2}} = 2.10 \times 10^{-2} \left(\frac{R}{1\ {\rm cm}}\right)^{-\frac{3}{2}}$$

$$\left(\frac{\delta E}{\bar{E}}\right)^2 = \left(\frac{\delta u}{u_0}\right)^2 \approx \frac{1}{N} = \frac{1}{8\pi V}\left(\frac{hc}{k_{\rm B} T_0}\right)^3 \left(\int_0^\infty \frac{x^2 dx}{e^x - 1}\right)^{-1}$$

The skin layer perturbing the superhorizon sphere has thickness

$$\delta R \approx \int_{t_i}^{t_f} \frac{c\,dt}{a(t)} \approx -\frac{c\delta z}{H(t_f)} \approx \frac{c(z_i - z_f)}{H_0 z_f^2}\sqrt{\frac{2}{g\Omega_\gamma}}$$

And it leads to the density contrast

$$\delta_R = \frac{\delta u}{u_0} = \frac{\delta E}{\bar{E}} \approx \delta_R^{\rm th}\sqrt{\frac{N'}{N}} \approx \delta_R^{\rm th}\sqrt{\frac{3\delta R}{R}} \propto \frac{1}{R^2}, \text{ for } R \geq r_H(t_i) \approx r_H(t_f)$$

Encouraging, but does not prove uniquely that P(k) ~ k, only P(k) ~ k^n with n>=1.

$$r_H \approx \frac{c}{a_{\rm reheat} H_{\rm reheat}} = \frac{T_0}{T_{\rm reheat}} \frac{c}{H_0} \sqrt{\frac{2}{g\Omega_\gamma}} \approx 200 \left(\frac{k_{\rm B} T_{\rm reheat}}{10^{15}\ {\rm GeV}} \right)^{-1} \ {\rm cm}$$

$$\delta_R \approx 1.29 \times 10^{-5} \left(\frac{k_{\rm B} T_{\rm reheat}}{10^{15}\ {\rm GeV}} \right)^{\frac{3}{2}} \left(\frac{g}{100} \right)^{\frac{3}{4}} \left(\frac{z_i - z_f}{z_f} \right)^{\frac{1}{2}} \left(\frac{R}{r_H} \right)^{-2} \ {\rm for}\ R \geq r_H$$

In fact, Zel'dovich 1965 and Peebles 1974: start with particle positions in a near homogeneous configruation

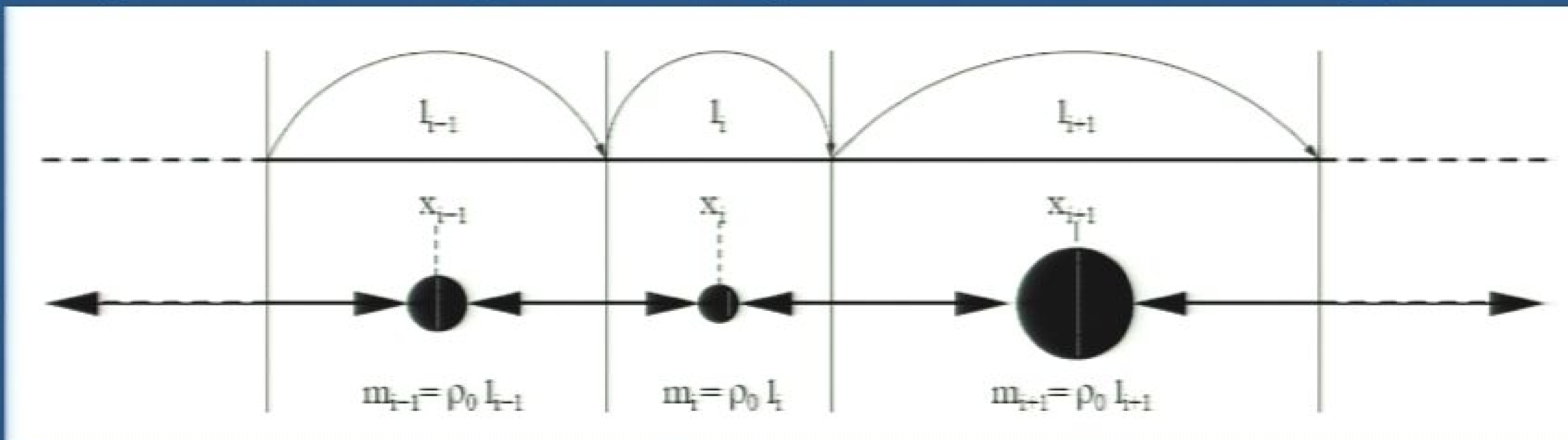
$$\delta_{\mathbf{k}} \propto \sum_j m_j e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

Then displace the particles slightly

$$\begin{aligned}\Delta\delta_{\mathbf{k}} &\propto \sum m_j [e^{i\mathbf{k}\cdot(\mathbf{x}_j+\Delta\mathbf{x}_j)} - e^{i\mathbf{k}\cdot\mathbf{x}_j}] \\ &= \sum m_j [i\mathbf{k}\cdot\Delta\mathbf{x}_j - (\mathbf{k}\cdot\Delta\mathbf{x}_j)^2/2 + \cdots] e^{i\mathbf{k}\cdot\mathbf{x}_j}\end{aligned}$$

Momentum conservation (center of mass) annihilates the first term. Next order is P(k) ~ k^4.

Gabrielli et al 2004: the Zel'dovich bound assumes the localization of classical particles (billiard balls), hence sharp cutoff of probability for displacements > r_H.



In quantum theory, particles are not localised.
And reheating is NOT a classical process!
If the superhorizon cutoff is power-law, one can reinstate P(k) ~ k^n with n >=0.

Even black body noise in laboratory temperatures cannot be explained purely by the random motion of billiard balls, or classical Poisson particles.

Start with photons as bosons

$$P_n = \left(1 - e^{-\beta\hbar\omega_j}\right) e^{-n\beta\hbar\omega_j}$$

$$\bar{n}_j = \frac{1}{e^{\beta\hbar\omega_j} - 1}, \text{ and } (\delta n_j)^2 = \frac{e^{\beta\hbar\omega_j}}{e^{\beta\hbar\omega_j} - 1}$$

$$(\delta n)^2 = \bar{n}(\bar{n} + 1), \text{ or}$$

$$(\delta n)^2 = \frac{e^x}{(e^x - 1)^2} = \frac{1}{e^x - 1} + \frac{1}{(e^x - 1)^2}$$

Phase space continuum

$$\sum_j \to 2 \times \frac{V}{(2\pi)^3}\int d^3\mathbf{k}, \text{ or } \frac{dn_{\text{mode}}}{d\omega} = 2 \times \frac{\omega^2 V}{2\pi^2 c^3}.$$

yielding

$$\bar{\epsilon} = \frac{8\pi V (k_B T)^4}{(hc)^3}\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{8\pi^5 V (k_B T)^4}{15(hc)^3},$$

and

$$(\delta\epsilon)^2 = \frac{8\pi V (k_B T)^5}{(hc)^3}\int_0^\infty \frac{x^4 e^x dx}{(e^x - 1)^2} = \frac{32\pi^5 k_B^5 T^5 V}{15 h^3 c^3}.$$

Or

$$\left(\frac{\delta\epsilon}{\epsilon}\right)^2 = \left(\frac{\delta u}{u}\right)^2 = \frac{15 h^3 c^3}{2\pi^5 V k^3 T^3} = \frac{16 k_B}{3S}.$$

Consider 1D problem of temporal noise of a collimated beam of black body radiation

$$I(\tau) = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} c|\mathbf{E}(t)|^2 dt,$$

$$\mathbf{E}(t) = \sum_j \mathbf{E}_j(t)$$

where

$$\mathbf{E}_j(\mathbf{r},t) = |E_j|\mathbf{e}_j \exp i(\mathbf{k}_j \cdot \mathbf{r} - \omega_j t + \phi_j)$$

$$I_{\mathrm{DC}} = \sum_j B_j;$$

$$I_{\mathrm{AC}}(\tau) = \sum_{\mathbf{k},m} \sum_{(\mathbf{k}',n)\neq(\mathbf{k},m)} \mathbf{e}_{\mathbf{k},m} \cdot \mathbf{e}_{\mathbf{k}',n} \sqrt{B_k B_{k'}} \frac{\sin\left[\frac{(\omega_k - \omega_{k'})\tau}{2}\right]}{(\omega_k - \omega_{k'})\tau} e^{i\phi_{\mathbf{k},\mathbf{k}',m,n}}$$

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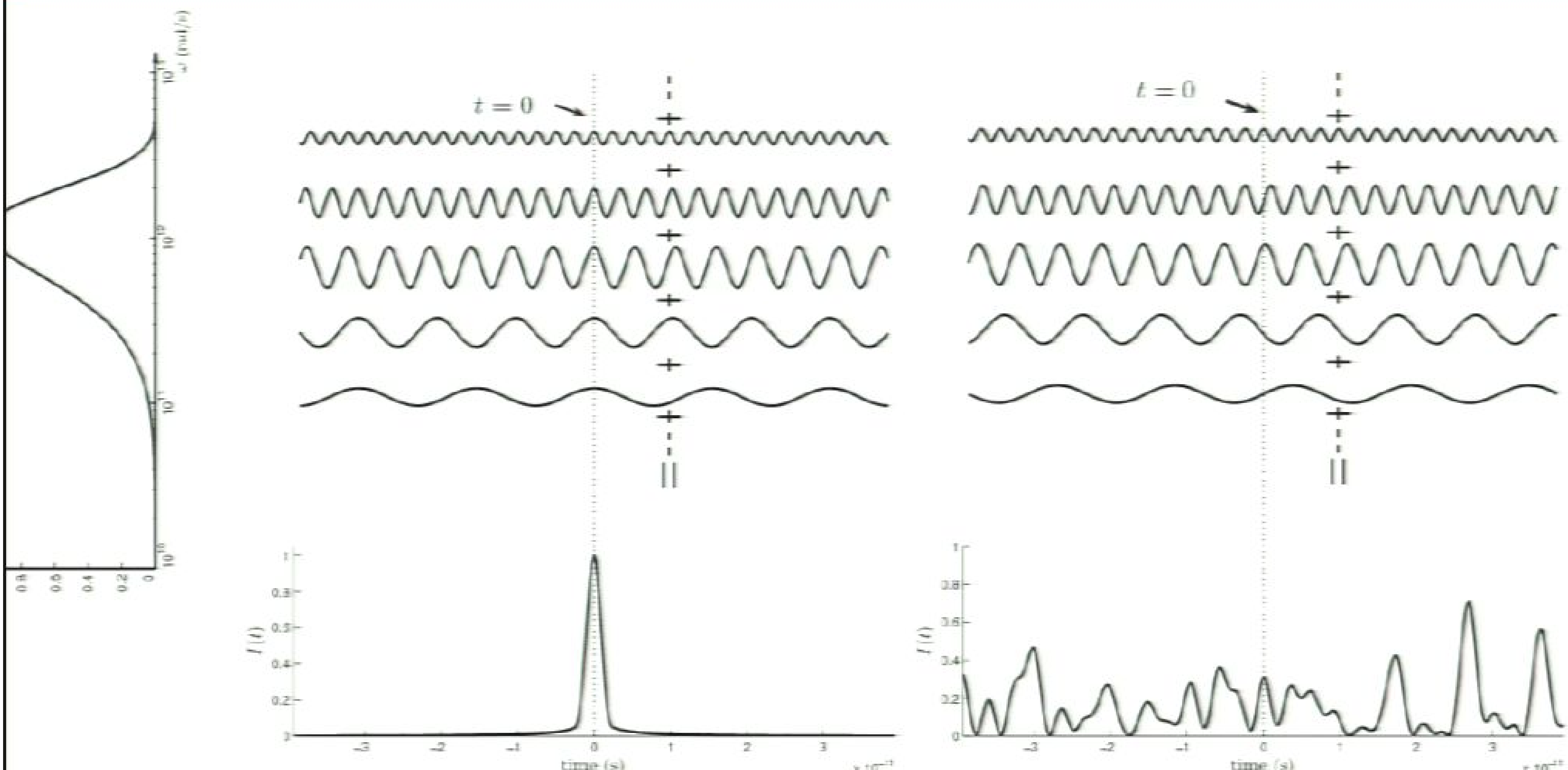
Figure 1



Black body intensity variation on different timescales.

$$\left(\frac{I_\tau}{I}\right)^2 = \frac{225h}{2\pi^8 \tau kT}\int_0^\infty \frac{x^6 dx}{(e^x-1)^2}$$

$$= 7.3 \times 10^2 \times \frac{h}{\pi^8 \tau kT}$$

$$= 0.0123 \left(\frac{\tau}{10^{-10}\ \text{s}}\right)^{-1} \quad \text{for } T = 3\ \text{K}$$

he variance of the phase noise is, then,

$$\delta u)^2 = \frac{8\pi k^5 T^5}{h^3 c^3 V} \int_0^\infty \frac{x^4 dx}{(e^x - 1)^2}$$

ecall that

$$\epsilon)^2 = \frac{8\pi V (k_B T)^5}{(hc)^3} \int_0^\infty \frac{x^4 e^x dx}{(e^x - 1)^2} = \frac{32\pi^5 k_B^5 T^5 V}{15 h^3 c^3}$$

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Even laboratory black body noise has a non-localized component, due to the superposition of normal modes of random phases. This component cannot be explained in terms of random placement of billiards.

Photon noise = Poisson noise + wave noise

CONCLUSION

It may not be necessary to invoke quantum fluctuations in the inflationary vacuum to explain structures

Thermal physics at the end of reheating can account for the observed amplitude of density perturbations on the horizon, and potentially also $P(k) \sim k$.

In this case, perhaps even inflation is unnecessary: why not axiomatize a universe that began in a flat and homogeneous phase at the time of GUT symmetry breaking?